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Transient hygrothermal stress in composites: Coupling of moisture and heat with temperature varying diffusivity.

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TRANSIENT HYGROTHERMAL STRESS IN COMPOSITES: COUPLING OF MOISTURE AND HEAT WITH TEMPERATURE VARYING DIFFUSIVITY

by

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NOMENCLATURE

C - concentration of water vapor in the air spaces

C_i - initial value of C

C_f - final value of C

$C(t)$ - moisture content at time t

$\overline{C}(t)$ - average moisture content

D_m - moisture diffusivity

D_h - thermal diffusivity

D_o - moisture diffusion constant

D - moisture diffusion coefficient

$\mathcal{D}$ - thermal diffusion coefficient

E_o - the energy required for one unit of mass to move into the solid

E - Young's modulus

$\underset{\sim}{f}$ - the amount of moisture leaving a unit of volume of void space

h - thickness of the plate

k - thermal conductivity

M - the amount of moisture absorbed by unit mass of solid

$\overline{M}$ - average amount of moisture absorbed by the solid

m - the mass of moisture contained in the volume of the composite per unit mass of solid

$\overline{m}$ - average amount of moisture abosrbed by the composite per unit mass of solid

$\overline{m}(\infty)$ - equilibrium moisture content

N - moles of the vapor

P_{w1} - vapor pressure of air at temperature T_1

P_{w2} - vapor pressure of air at temperature T_2

$\underset{\sim}{q}$ - heat flux out per unit area

R - gas constant

RH - relative humidity

T - absolute temperature

t - time in hours

T_i - initial temperature

T(t) - temperature at time t

T_f - final temperature

$\bar{T}(t)$ - average temperature through the plate

u - $D/\mathcal{D}$

u_o - $D_o/\mathcal{D}$

V - volume occupied by the void per unit volume

w(t) - weight of specimen at time t

w_i - initial dry weight of the specimen

z - axis in thickness direction

α - coefficient of thermal expansion

β - coefficient of moisture expansion

γ_s - specific humidity

ε_{ij} - strains

θ - dimensionless time

λ - constant

ν - constant

ν_p - Poisson's ratio

ξ - dimensionless space

ρ - density of composite

ρ_s - density of solid

ρ_a - density of air

σ - constant

σ_{ij} - stresses

ω - constant

ABSTRACT

The influence of coupled diffusion of heat and moisture on the transient moisture and stress distribution in a composite is investigated analytically. The moisture diffusion coefficient is assumed to be temperature dependent while the thermal diffusion coefficient is kept constant.

A study of the coupled diffusion equations was made by application of the finite-difference scheme allowing time-dependent changes in the humidity and temperature of the environment. The appropriate transient moisture and/or temperature boundary conditions are specified on the surfaces of an infinite plate such that the problem is one-dimensional with changes occurring only in the thickness direction.

Degradation of mechanical properties and dimensional changes due to moisture are analogous to those caused by thermal effects. For the most part, hygro-elastic stress analysis is similar to thermal stress analysis. In an epoxy system, the moisture diffusion process is several orders of magnitude slower than the thermal conduction process. With reference to the exposure time, thermal shock is considered to be rare in practice whereas moisture shock is rather common.

Numerical calculations were carried out for the T300/5208 graphite fiber-reinforced epoxy matrix composite in which the nonuniformity of moisture and temperature is evaluated for sudden changes in the surface moisture and/or temperature. The coupling effect between temperature and moisture is found to be most significant when the plate

undergoes a sudden change in surface temperature while the surface moisture concentration is held constant. For a sudden change in the surface temperature, the results indicate that the stresses due to coupling can deviate from the uncoupled results anywhere from 20 to 80 percent depending on the surface temperature gradient. This suggests the need to perform additional experiments for evaluating the coupled diffusion phenomenon and its influence on the mechanical behavior of epoxy-resin-composites.

I. INTRODUCTION

Owing to their favorable performance characteristics, composite materials have been gaining wide use in industry because of their low weight and high strength characteristics. Absorption of moisture by composites causes dimensional changes through non-uniform expansion and/or contraction of material elements which, in turn, leads to internal stresses and strains. Therefore, in order to utilize the full potential of composite materials, their response to moist environments should be understood.

Classical solutions assume that the moisture and temperature effects are uncoupled. Refer to the studies of Shen and Springer, Pipes et al and Tenny [1-3]. Other solutions consider the time varying diffusivity such as those by Weitsman and Harris et al [4,5].

However, little attention has been given to the coupling of moisture and heat which, in turn, can affect the transient stresses and strains in the composite materials. The significant variables in such a study must, at least, involve time, relative humidity of the environment, temperature, relevant physical constants, etc. The governing equations for coupled moisture and temperature effects were studied by Henry [6] and Hartranft and Sih [7]. Five different physical models that led to the same type of governing equations were discussed by Hartranft and Sih [8]. The coefficients in these models are associated with the basic thermodynamic properties of the solid and can be related to one another.

The purpose of this investigation is to develop an analytical model and technique for calculating the nonuniform moisture, temperature and stresses in a composite system.

The properties of the T300/5208 graphite/epoxy system were used in the numerical calculation as the diffusion data and the variation of the moisture diffusion coefficient with temperature for this material is readily available [9]. A finite difference computer program was developed for solving the coupled diffusion equations with transient boundary conditions on moisture and/or temperature.

Examples and numerical calculations are provided for moisture and/or temperature diffusing into a plate from its surfaces. The plate is initially at a uniform temperature with a given moisture content distributed uniformly throughout the plate. Suddenly, the temperature and/or moisture at the plate surfaces are changed and maintained constant thereafter. The corresponding stresses are also calculated as a function of time while the numerical results for other quantities of interest are displayed graphically.

II. MOISTURE CONTENT

II.1 Coupled Diffusion Equations of Moisture and Heat

In this study, an element of the composite material will be modeled by a medium that is occupied partly by the solid and partly by air spaces or voids. Assume that the solid portion can always be considered as in equilibrium with its immediate surroundings such that a linear dependence on both temperature and moisture can be taken:

$$M = \sigma C - \omega T + \text{const} \tag{1}$$

where C is the concentration of water vapor in the air spaces expressed in g/cm^{+3}, M is the amount of moisture absorbed by unit mass of solid, σ and ω are constant (may be functions of C and T), and T is the temperature.

Henry [6], altered the classic uncoupled equations for diffusion of heat and moisture to incorporate effects due to porosity and thermodynamics. The heat conduction equation is given by

$$\nabla \cdot \underset{\sim}{q} = - \rho C_p \frac{\partial T}{\partial t} + \rho \eta \frac{\partial M}{\partial t} \tag{2}$$

where

$$\underset{\sim}{q} = - k \nabla T \tag{3}$$

Here, $\underset{\sim}{q}$ is heat flux out per unit area, k is thermal conductivity, and ρ is the density of the composite. The unit of space consists of void space, occupying volume V, and the solid occupies the volume 1-V. The density for the solid is ρ_s:

$$\rho_s = \rho/(1-V) \tag{4}$$

and the thermal diffusivity is

$$D_h = k/(\rho C_p) \tag{5}$$

while η is the heat lost by the solid when it absorbs a unit mass of moisture.

The amount of moisture leaving a unit of volume of void space is

$$\nabla\cdot\underset{\sim}{f} = -\frac{\partial C}{\partial t} - \frac{\rho}{V}\frac{\partial M}{\partial t} \tag{6}$$

where

$$\underset{\sim}{f} = -\ gD_m\nabla C \tag{7}$$

and g is a correction factor accounting for the intricacy of the paths through the void space. D_m is the moisture diffusivity. When both D_h and D_m are constants, M may be eliminated from equations (2) and (6). With the aid of equations (3) and (7), the following system of coupled equations are obtained [7]:

$$\begin{aligned} D\nabla^2 C - \frac{\partial}{\partial t}(C-\lambda T) &= 0 \\ \mathcal{D}\nabla^2 T - \frac{\partial}{\partial t}(T-\nu C) &= 0 \end{aligned} \tag{8}$$

where

$$\begin{aligned} D &= gD_m/[1 + \rho\sigma/V] \\ \mathcal{D} &= D_h/[1 + \eta\omega/C_p] \\ \nu &= \sigma/[\omega + C_p/\eta] \\ \lambda &= \omega/[\sigma + V/\rho] \end{aligned} \tag{9}$$

and ∇^2 is the Laplacian operator in the space variables. The diffusion coefficients D and $\mathcal{D}$ have units of area per unit time. The parameters λ and ν have units of mass per unit volume per unit temperature and the reciprocal, respectively. These equations are relatively easy to solve when the coefficients are constant and boundary values of temperature and moisture content are held constant [7].

It has been noted experimentally that the moisture diffusion coefficient, D, depends on temperature by a relation of the form [10]

$$D = D_o \exp(-E_o/RT) \tag{10}$$

in which E_o is the energy required for one unit of mass to move into the solid, R is the gas constant and T is the absolute temperature. The form of D in equation (10) can be incorporated into the coupled theory giving

$$\nabla \cdot (D\nabla C) - \frac{\partial}{\partial t}(C-\lambda T) = 0$$

$$\mathcal{D}\nabla^2 T - \frac{\partial}{\partial t}(T-\nu C) = 0 \tag{11}$$

where $\mathcal{D}$ is constant throughout this study. When D is a function of temperature, equations (11) are nonlinear and a numerical scheme for solving equations (11) is required.

For the problem at hand, only the moisture and temperature changes in the plate thickness or z direction is considered, Figure 1, and hence $\nabla^2 = \partial^2/\partial z^2$. It is expedient to introduce the dimensionless space and time variables

$$\xi = \frac{2z}{h}, \quad \theta = \frac{4D_o t}{h^2} \tag{12}$$

in which h stands for the plate thickness. Equations (11) can thus be expressed in terms of ξ and θ as (Appendix 1)

$$\left[\frac{\partial^2 C}{\partial \xi^2} + \frac{E_o}{RT^2}\left(\frac{\partial C}{\partial \xi}\right)\left(\frac{\partial T}{\partial \xi}\right)\right] \exp\left(-\frac{E_o}{RT}\right) - \left(\frac{\partial C}{\partial \theta} - \lambda \frac{\partial T}{\partial \theta}\right) = 0$$

$$\mathcal{D}\frac{\partial^2 T}{\partial \xi^2} - D_o\left(\frac{\partial T}{\partial \theta} - \nu \frac{\partial C}{\partial \theta}\right) = 0 \tag{13}$$

In what follows, two types of transient boundary conditions will be treated, namely sudden change in moisture and temperature.

II.2 Sudden Change in Moisture

II.2.1 Diffusion Equations

Consider the problem of diffusion into an infinite plate as shown in Figure 1. The temperature and moisture concentration are initially uniform at the values T_i and C_i, respectively. At time t=0, the moisture at both surfaces $z = \pm h/2$ are suddenly changed to C_f, and maintained constant thereafter. The surface temperature of the plate is always kept at T_i. These conditions may be stated as

$$T(z,o) = T_i, \; C(z,o) = C_i \tag{14}$$

and

$$T(\pm h/2,t) = T_i, \; C(\pm h/2,t) = C_f \text{ for } t>0 \tag{15}$$

In terms of the nondimensional variables ξ and θ in equations (12), the solution for moisture and temperature may be expressed in the forms

$$\begin{aligned} C(t) &= C_i + (C_f-C_i)f(\xi,\theta) \\ T(t) &= T_i + \nu(C_f-C_i)g(\xi,\theta) \end{aligned} \tag{16}$$

in which $f(\xi,\theta)$ and $g(\xi,\theta)$ are functions to be determined from the conditions in equations (14) and (15). Substituting equations (16) into equations (13) yields (Appendix 2)

$$\frac{\partial f}{\partial \theta} = \frac{1}{1-\lambda\nu}\left\{\frac{\lambda\nu}{u_o}\frac{\partial^2 g}{\partial \xi^2} + F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right]\right\}$$

$$\frac{\partial g}{\partial \theta} = \frac{1}{1-\lambda\nu}\left\{\frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2} + F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right]\right\} \qquad (17)$$

where

$$u_o = \frac{D_o}{\mathcal{D}}$$

$$F = \exp\left(-\frac{A}{1+Bg}\right)$$

$$A = \frac{E_o}{RT_i}, \quad B = \frac{\nu(C_f - C_i)}{T_i}$$

II.2.2 Finite Difference Method

Since equations (17) cannot be solved analytically, it is necessary to resort to approximate numerical methods. The method of finite difference is adopted to replace the governing partial differential equations. This then reduces the problem to a set of simultaneous algebraic equations which can be easily solved. Referring to the space and time interval in Figure 2, the first of equations (17) may be written in difference form as

$$\frac{f_{m,n+1}-f_{m,n}}{\Delta\theta} = \frac{1}{1-\lambda\nu}\{\frac{\lambda\nu}{u_o}(\frac{g_{m+1,n}-2g_{m,n}+g_{m-1,n}}{(\Delta\xi)^2})$$

$$+ F(m,n)[\frac{f_{m+1,n}-2f_{m,n}+f_{m-1,n}}{(\Delta\xi)^2} + \frac{AB}{(1+Bg_{m,n})^2} \times$$

$$\times (\frac{f_{m+1,n}-f_{m,n}}{\Delta\xi})(\frac{g_{m+1,n}-g_{m,n}}{\Delta\xi})]\} \qquad (18)$$

while the second of equations (17) becomes

$$\frac{g_{m,n+1}-g_{m,n}}{\Delta\theta} = \frac{1}{1-\lambda\nu}\{\frac{1}{u_o}(\frac{g_{m+1,n}-2g_{m,n}+g_{m-1,n}}{(\Delta\xi)^2})$$

$$+ F(m,n)[\frac{f_{m+1,n}-2f_{m,n}+f_{m-1}}{(\Delta\xi)^2} + \frac{AB}{(1+Bg_{m,n})^2} \times$$

$$\times (\frac{f_{m+1,n}-f_{m,n}}{\Delta\xi})(\frac{g_{m+1,n}-g_{m,n}}{\Delta\xi})]\} \qquad (19)$$

In order to achieve acceptable accuracy in the finite difference calculations, the grid size in space, $\Delta\xi$, and time, $\Delta\theta$, must be sufficiently small and satisfy the stability requirement that

$$\Delta t \leq \frac{(\Delta z)^2}{4D_o} \exp(E_o/RT) \qquad (20)$$

The boundary conditions in equations (14) and (15) may then be written in terms of $f(\xi,\theta)$ and $g(\xi,\theta)$. They become

$$f(\xi,o) = 0, \; g(\xi,o) = 0 \qquad (21)$$

and

$$f(\pm 1,\theta) = 1, \; g(\pm 1,\theta) = 0 \tag{22}$$

for $\theta > 0$.

A computer program was developed to solve equations (18) through (22) for the functions $f(\xi,\theta)$ and $g(\xi,\theta)$ from which the moisture and temperature throughout the solid can be determined.

II.2.3 Average Moisture Quantities

Referring to equation (1), the mass of moisture contained in the volume of the composite per unit mass of solid m, is given by

$$m = \frac{v}{\rho} C + M = \omega\left(\frac{C}{\lambda} - T\right) + \text{constant} \tag{23}$$

The average values of these moisture content quantities are defined as

$$\overline{C} = \frac{1}{V} \int_V C dV, \; \overline{M} = \frac{1}{V} \int_V M dV, \; \overline{m} = \frac{1}{V} \int_V m dV \tag{24}$$

The total moisture in the voids, solid and composite are, respectively, $vV\overline{C}$, $\rho V\overline{M}$, and $\rho V\overline{m}$. Now, let the average values of T, C, and m be defined by the integrals

$$\begin{aligned} \overline{T}(t) &= \frac{1}{h} \int_{-h/2}^{h/2} T(z,t) dz \\ \overline{C}(t) &= \frac{1}{h} \int_{-h/2}^{h/2} C(z,t) dz \\ \overline{m}(t) &= \frac{1}{h} \int_{-h/2}^{h/2} m(z,t) dz \end{aligned} \tag{25}$$

In terms of $f(\xi,\theta)$ and $g(\xi,\theta)$, equations (25) become

$$\bar{T}(t) - T_i = \frac{\nu}{2}(C_f - C_i)\int_{-1}^{1} g(\xi,\theta)d\xi$$

$$\bar{C}(t) - C_i = \frac{1}{2}(C_f - C_i)\int_{-1}^{1} f(\xi,\theta)d\xi \qquad (26)$$

$$\bar{m}(t) - m_i = \frac{\omega}{2\lambda}(C_f - C_i)\int_{-1}^{1} [f(\xi,\theta) - \lambda\nu g(\xi,\theta)]d\xi$$

In view of equations (14) and (15), the third of equations (26) may be put into the dimensionless form $[\bar{m}(t) - m_i]/(m_f - m_i)$ which, when approximated by Simpson's rule for a fixed time θ_o, gives

$$\frac{\bar{m}(t)-m_i}{m_f-m_i} = \frac{1}{2}\left(\frac{\Delta\xi}{3}\right)\{[f(1,\theta_o) + 4f(2,\theta_o) + 2f(3,\theta_o)$$

$$+ \ldots + 4f(n-1,\theta_o) + f(n,\theta_o)] - \lambda\nu[g(1,\theta_o)$$

$$+ 4g(2,\theta_o) + \ldots + 4g(n-1,\theta_o) + g(n,\theta_o)]\} \qquad (27)$$

II.2.4 Numerical Examples

Numerical calculations are made for a T300/5208 epoxy resin plate with thickness h = 0.2 cm. The constants $D_o = 1.53 \times 10^3$ cm^2/hr and $E_o = 1.25 \times 10^4$ cal/g·mole are obtained from [9]. For the coupled diffusion problem, the particular values of $u = 0.1$, $\lambda = 0.5$ and $\nu = 0.5$ are chosen by comparing the analytical prediction of the percent moisture content as a function of $\sqrt{t}$ with the experimental data in [9]. Note that $u = D/\mathcal{D}$ should be distinguished from u_o in equation (17). The constants A and B in equation (17) are determined from an initial temperature of T_i = 21°C = 294°K, C_i = 0 and a gas constant of R = 1.986. Hence

$$A = \frac{E_o}{RT_i} = \frac{1.25x10^4}{1.986(294)} = 21.41$$

$$B = \frac{\nu C_f}{T_i} = \frac{0.5\ C_f}{294} = 1.7 \times 10^{-3}\ C_f$$

in which C_f, the equilibrium moisture concentration, can be obtained from

$$C_f = \frac{\rho_a \gamma_s}{1+\gamma_s} \tag{28}$$

for different relative humidity of the environment. In equation (28), γ_s is the specific humidity measured in grams of water per lb_m of dry air and ρ_a is the density of the ambient air in units of g/cm³. Another important quantity in the diffusion analysis is the relationship between equilibrium moisture content $\overline{m}(\infty)$ of the composite and RH of the environment. For the post-cured T300/5208 epoxy resin, the relation [9]

$$\overline{m}(\infty) = 0.0155\ (RH) \tag{29}$$

may be used in which RH is expressed in percent. There remains the appropriate selection of the time and space interval before carrying out the finite difference calculations. For example, if the plate in the z-direction is divided into seven segments and hence $\Delta z = h/7$ while Δt must satisfy the stability condition in equation (20). Results are expressed in terms of percent moisture content $\bar{m}(t)$ as manifested by the weight gain of the composite:

$$\bar{m}(t) = \frac{w(t)-w_i}{w_i} \times 100 \tag{30}$$

where $w(t)$ is the weight of the specimen at time t and w_i is the initial dry weight of the specimen.

Figure 3 gives a plot of $\bar{m}(t)$ versus $\sqrt{t}$ for different relative humidities of RH = 13, 33, 52, 75 and 100 percent at T_i = 21°C. The moisture diffusion coefficient D is assumed to be temperature dependent. The dotted curves represent solutions for the uncoupled theory in which $\lambda = \omega = B = 0$, $\lambda\nu = 0$ and u can be arbitrary. They differ very little from the curves for the coupled theory. Similar results can also be obtained for T_i = 43°C, 63°C and 82°C. Figures 4 and 5 show the variations of moisture content $\bar{m}(t)$ with the normalized thickness coordinate 2z/h for RH = 13, 33. Initially, i.e., for small time t, only the region close to the plate surface experiences moisture while the center region of the plate is not affected. As time increases, moisture is penetrated into all the material elements with the minimum influence at z=0. The difference of $\bar{m}(t)$ between z=0 and $z = \pm h/2$ increases with increasing RH.

The effect of initial temperature on the penetration of moisture is shown in Figure 6 for a sudden change of RH from 0 percent to 100 percent. Coupling is neglected and D is taken to be a constant. The time at which the plate reaches moisture equilibrium is seen to decrease as T_i is increased for a fixed value of h = 0.2 cm. Table 1 gives the comparison of the moisture variation at 2z/h = - 1.0, - 0.66, - 0.33 and 0.0, for symmetric and nonsymmetric* sudden moisture change at 21°C from $(RH)_i$ = 0 percent to 75 percent. The average moisture gain for nonsymmetric case is half of that for symmetric case. The moisture gain at $z = -\frac{h}{2}$ for both cases are equal while at 2z/h = 0, the deviation is about 50 percent.

II.3 Sudden Change in Temperature

II.3.1 Diffusion Equations

Suppose that the surface temperature on the plate in Figure 1 is changed from an initial value of T_i to a final value T_f and the moisture concentrations at z = ±h/2 are kept constant at all time. Then, in addition to equations (14), the following conditions must also prevail:

$$T(\pm h/2,t) = T_f, \quad C(\pm h/2,t) = C_i \tag{31}$$

The form of the solution expressed in terms of the variables ξ and θ defined in equations (12) is

*Nonsymmetric boundary conditions are given by $T(\pm \frac{h}{2},t) = T_i$, $C(-\frac{h}{2},t) = C_f$, $C(\frac{h}{2},t) = C_i$ for t>0.

$$C(t) = C_i + \lambda(T_f - T_i) f(\xi,\theta)$$
$$T(t) = T_i + (T_f - T_i) g(\xi,\theta) \tag{32}$$

Substituting equations (32) into equations (13) yield (Appendix 3)

$$\frac{\partial f}{\partial \theta} = \frac{F}{1-\lambda\nu}\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+g)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right] + \frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2}$$
$$\frac{\partial g}{\partial \theta} = \frac{1+\lambda\nu}{u_o}\frac{\partial^2 g}{\partial \xi^2} + \frac{\lambda\nu}{1-\lambda\nu} F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right] \tag{33}$$

where

$$u_o = \frac{D_o}{\mathcal{D}},\ A = \frac{E_o}{RT_i},\ B = \frac{T_f - T_i}{T_i},\ F = \exp\left(-\frac{A}{1+Bg}\right)$$

Using equations (32), the conditions in equations (31) may be written as

$$f(\pm 1,\theta) = 0,\ g(\pm 1,\theta) = 1 \text{ for } \theta \geq 0$$

As in the previous example, equations (33) will be solved numerically by the finite difference method.

II.3.2 Finite Difference Equations

Equations (33) will now be cast into the finite difference form. With the nondimensional time and space interval as chosen in Figure 2, the following expressions are obtained

$$\frac{f_{m,n+1}-f_{m,n}}{\Delta\theta} = \frac{F(m,n)}{1-\lambda\nu}\left[\frac{f_{m+1}-2f_{m,n}+f_{m-1,n}}{(\Delta\xi)^2} + \frac{AB}{(1+Bg_{m,n})^2} \times \right.$$

$$\left. \times \left(\frac{f_{m+1,n}-f_{m,n}}{\Delta\xi}\right)\left(\frac{g_{m+1,n}-g_{m,n}}{\Delta\xi}\right)\right] + \frac{1}{u_o} \times$$

$$\times \left[\frac{g_{m+1,n}-2g_{m,n}+g_{m-1,n}}{(\Delta\xi)^2}\right] \tag{34}$$

and

$$\frac{g_{m,n+1}-g_{m,n}}{\Delta\theta} = \frac{1+\lambda\nu}{u_o}\left[\frac{g_{m+1,n}-2g_{m,n}+g_{m-1,n}}{(\Delta\xi)^2}\right] + \frac{\lambda\nu}{1-\lambda\nu} F(m,n) \times$$

$$\times \left[\frac{f_{m+1,n}-2f_{m,n}+f_{m-1,n}}{(\Delta\xi)^2} + \frac{AB}{(1+Bg_{m,n})^2} \times \right.$$

$$\left. \times \left(\frac{f_{m+1,n}-f_{m,n}}{\Delta\xi}\right)\left(\frac{g_{m+1,n}-g_{m,n}}{\Delta\xi}\right)\right] \tag{35}$$

The stability requirement for selecting the relative size of Δt and Δz in the numerical calculation is the same as that state in equation (20).

II.3.3 Moisture Content

Following the definitions of the various moisture parameters as discussed earlier, the average moisture content in the composite per unit mass of solid is

$$\overline{m}(t) - m_i = \frac{\omega}{2}(T_f - T_i) \int_{-1}^{1} [f(\xi,\theta) - g(\xi,\theta)]d\xi \tag{36}$$

From equation (23), it can be shown that (Appendix 4)

$$m_f - m_i = -\omega(T_f - T_i) \quad (37)$$

which when substituted into equation (36) yields

$$\frac{\overline{m}(t)-m_i}{m_f-m_i} = \frac{1}{2}\left(\frac{\Delta\xi}{3}\right)\{[g(1,\theta_o) + 4g(2,\theta_o) + \ldots + g(n,\theta)]$$

$$- [f(1,\theta_o) + 4f(2,\theta_o) + \ldots + f(n,\theta)]\} \quad (38)$$

Here, Simpson's rule has been applied for evaluating the integral in equation (36) at $\theta = \theta_o$.

II.3.4 Physical Meaning of Sudden Temperature Change

From equation (1)

$$M = \sigma C - \omega T + \text{const}$$

and equation (23)

$$m = \frac{V}{\rho} C + M = \omega\left(\frac{C}{\lambda} - T\right) + \text{const}$$

where

$$\omega = \frac{\partial M}{\partial T}, \quad \sigma = \frac{\partial M}{\partial C}$$

Equations (11) read as

$$\nabla \cdot (D\nabla C) - \frac{\partial}{\partial t}(C-\lambda T) = 0$$

$$\mathcal{D}\nabla^2 T - \frac{\partial}{\partial t}(T-\nu C) = 0$$

For uncoupling case, we required $\lambda=0$, $\nu=0$ since $\lambda=0$, for m to be finite, we have to require

$$\omega = \frac{\partial M}{\partial T} = 0 \tag{39}$$

Note that the classical theory cannot address the influence of temperature change on the moisture content in the medium. Thus, for sudden surface temperature change, it is necessary to use coupled theory.

For the coupling cases, assume λ, ν are constants. After a sudden temperature change, if the vapor pressure is kept constant, then the volume of the ambient air will change which affects C_f. Since C_i is to be kept constant on the surface, the volume must also be a constant. Making use of the ideal gas equation PV = NRT, vapor pressure of moisture will increase, and consequently, the relative humidity of the ambient air is also affected as follows:

$$Pw_1 V = NRT_1$$
$$Pw_2 V = NRT_2 \tag{40}$$

By definition of relative humidity

$$\text{RH percent} = \frac{Pw}{Pg} \times 100 \text{ percent} \tag{41}$$

in which Pw is the vapor pressure and Pg is the saturate vapor pressure at certain temperature. Thus

$$\text{RH percent} = \frac{Pw_2}{Pg_2} \times 100 \text{ percent} = \frac{Pw_1}{Pg_2} \times \frac{T_2}{T_1} \times 100 \text{ percent} \tag{42}$$

From equation (42), it is seen that when the moisture content of the environment is kept constant, the relative humidity of the ambient air will decrease with increasing temperature and vice versa. Hence, an increase in the ambient temperature will cause desorption and a decrease of the temperature will lead to absorption of moisture.

II.3.5 Numerical Examples

Referring to the boundary conditions prescribed by equations (14) and (31), the moisture concentrations at $z = \pm h/2$ are to be kept at C_i while the surface temperature will be increased from T_i to T_f. Assuming that the mass of moisture contained in the voids per unit volume of void space on the boundary is constant, the relative humidity of the ambient air will decrease as the temperature is raised. The opposite occurs when T_f is decreased. As it is to be expected, an increase in the ambient temperature will cause moisture desorption, while a decrease in the ambient temperature leads to moisture absorption. The results are summarized in graphical form for T300/5208 epoxy resin with the coupling constants $u = 0.1$, $\lambda = 0.5$ and $\nu = 0.5$ as determined earlier.

Figure 7 gives a plot of $\overline{m}(t)$ as a function of $2z/h$ for $T_i = 21°C$, $(RH)_i = 52$ percent and $T_f = 10°C$. In this case, ΔT is negative and the moisture level in the plate will increase with time until the equilibrium condition is approached. Figures 8 and 9 display the results for ΔT positive where $T_f = 43°C$ and $63°C$ are all greater than $T_i = 21°C$. The opposite trend is observed, i.e., the moisture level in the plate will now decrease with time until an equilibrium state is reached. The influence of ΔT on $\overline{m}(t)$ can be best illustrated by plotting $\overline{m}(t)$ versus $\sqrt{t}$ as shown in Figure 10. The curves for the six different values of T_f offer a quantitative assessment of moisture absorption and desorption as ΔT changes sign.

Tables 2 and 3 give a comparison of the moisture variation at $2z/h = -1.0, -0.66, -0.33$ and 0.0 for the symmetric and nonsymmetric* sudden temperature change from 21°C with $(RH)_i = 52$ percent to 10°C and 43°C. For a sudden temperature drop, the difference between average moisture uptake is about 15 percent, while for a sudden temperature rise, the difference may become as great as 70 percent.

II.3.6 Influence of Coupling

An attempt is made in Figure 11 to illustrate the difference between the coupled solution obtained from equations (11) and the

*Nonsymmetric boundary conditions are given by $T(-\frac{h}{2},t) = T_f$, $T(\frac{h}{2},t) = T_i$, $C(\pm h/2,t) = C_i$ for $t>0$.

uncoupled equation

$$\frac{\partial}{\partial z}\left(D \frac{\partial C}{\partial z}\right) - \frac{\partial C}{\partial t} = 0 \quad (43)$$

Note that for $(RH)_i$ = 52 percent, the dotted curves based on equation (43) can differ significantly from a solid curve of the coupled theory.

The foregoing results reveal that the coupling of moisture and heat is inherent in the study of sudden temperature change in composites at a given moisture level. The extent to which coupling influences the mechanical behavior of the composites can be evaluated by calculating for the stress and/or strains.

III. TRANSIENT STRESSES

The mechanical behavior of composites may be altered when exposed to high temperature and/or moisture environments. Their behavior should be understood before the full potential of composites can be realized. For the resin-base composite treated earlier, moisture is assumed to diffuse into the solid in much the same way as heat. It tends to degrade the mechanical properties and introduce dimensional changes of the composite similar to those caused by thermal changes. Thus, for the most part, hygroelastic stress analysis is similar to thermal stress analysis.

III.1 Basic Equations

III.1.1 Basic Equation for Symmetric Boundary Conditions

Let the plate in Figure 1 extend to infinity in both x and y directions and be free from mechanical loads. The stresses induced by mechanical loads can simply be added onto those due to moisture and heat. The material of the plate is assumed to be isotropic and homogeneous. Only hygrothermal stresses will be treated, for symmetric boundary conditions

$$\sigma_{ij} = E(\varepsilon_{ij} - \alpha\Delta T - \beta\Delta m) \tag{44}$$

where E is the Young's modulus of elasticity, α the coefficient of thermal expansion and β the coefficient of moisture expansion. The quantity, m, is defined by equation (23). Since the stress state is a function of the thickness variable z only, shear stresses vanish everywhere and σ_{ij} and ε_{ij} consist of normal components only. Hence, the strain and stress relations may be written as

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu_p(\sigma_y+\sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu_p(\sigma_x+\sigma_z)] \tag{45}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu_p(\sigma_x+\sigma_y)]$$

where ν_p is the Poisson's ratio and $\sigma_z = 0$ will be assumed. For the isotropic and homogeneous material, the stresses induced by the strains

$$\varepsilon_x^o = \varepsilon_y^o = -\alpha\Delta T - \beta\Delta m \tag{46}$$

are given by

$$\sigma_x^o = \sigma_y^o = -\frac{\alpha E\Delta T}{1-\nu_p} - \frac{\beta E\Delta m}{1-\nu_p} \tag{47}$$

which prevails everywhere in the plate.

In order to free the plate edges from external stresses, it is necessary to apply stresses, equal in magnitude and opposite in direction, to those of equation (47). The following average tensile stresses

$$\bar{\sigma}'_x = \bar{\sigma}'_y = \frac{1}{(1-\nu_p)h} [\alpha E \int_{-h/2}^{h/2} \Delta T dz + \beta E \int_{-h/2}^{h/2} \Delta m dz] \tag{48}$$

are thus introduced. The results for a free edge plate are

$$\sigma_x = \sigma_y = - \frac{\alpha E \Delta T}{1-\nu_p} - \frac{\beta E \Delta m}{1-\nu_p} + \frac{\alpha E}{(1-\nu_p)h} \int_{-h/2}^{h/2} \Delta T dz$$

$$+ \frac{\beta E}{(1-\nu_p)h} \int_{-h/2}^{h/2} \Delta m dz \tag{49}$$

or

$$\sigma_x = \sigma_y = \frac{\alpha E}{1-\nu_p} (\bar{T}-T) + \frac{\beta E}{1-\nu_p} (\bar{m}-m) \tag{50}$$

in which $\bar{T}$ and $\bar{m}$ are the temperature and moisture average through the plate thickness.

Using equation (50), if the distribution of temperature and moisture through the thickness of the plate are known, we can calculate the hygrothermal stress within the plate.

III.1.2 Basic Equation for Nonsymmetric Boundary Conditions

In the nonsymmetric cases, the stresses induced by the strains in equations (46) can still be expressed by equation (47). In order to free the plate edges from external stresses, a set of compressive stresses must be superimposed, equation (47), together with the uniform tensile stresses in equation (48), the bending stresses

$$\bar{\sigma}''_x = \bar{\sigma}''_y = 2\sigma z/h \tag{51}$$

can be determined from the condition that the moment of the forces distributed over a cross section must be zero. Hence

$$\int_{-h/2}^{h/2} (2\sigma z/h)dz - \int_{-h/2}^{h/2} \frac{\alpha E\Delta Tz}{1-\nu_p} dz - \int_{-h/2}^{h/2} \frac{\beta E\Delta mz}{1-\nu_p} dz = 0 \tag{52}$$

from which

$$\frac{2\sigma}{h} = \frac{12}{h^3(1-\nu_p)} \left(\int_{-h/2}^{h/2} \alpha E\Delta Tzdz + \int_{-h/2}^{h/2} \beta E\Delta mzdz \right) \tag{53}$$

It follows that

$$\bar{\sigma}''_x = \bar{\sigma}''_y = \frac{12z}{h^3(1-\nu_p)} \left(\int_{-h/2}^{h/2} \alpha E\Delta Tzdz + \int_{-h/2}^{h/2} \beta E\Delta mzdz \right) \tag{54}$$

and the stresses that correspond to a plate with free edges are

$$\begin{aligned} \sigma_x = \sigma_y = & - \frac{\alpha E\Delta T}{1-\nu_p} - \frac{\beta E\Delta m}{1-\nu_p} + \frac{\alpha E}{(1-\nu_p)h} \int_{-h/2}^{h/2} \Delta Tdz \\ & + \frac{\beta E}{(1-\nu_p)h} \int_{-h/2}^{h/2} \Delta mdz + \frac{12\alpha Ez}{(1-\nu_p)h^3} \int_{-h/2}^{h/2} \Delta Tzdz \\ & + \frac{12\beta Ez}{(1-\nu_p)h^3} \int_{-h/2}^{h/2} \Delta mzdz \end{aligned} \tag{55}$$

III.2 Hygrothermal Stresses for Sudden Moisture Change

Based on the diffusion results obtained earlier, equation (50) is applied to find the stresses for a sudden moisture change that occurs symmetrically. The following material properties for the T300/5208 resin are used:

$$\alpha = 45 \times 10^{-6} \text{ cm/cm/°C } (25 \times 10^{-6} \text{ cm/cm/°F})$$
$$\beta = 2.68 \times 10^{-3} \text{ cm/cm/m(t) percent } H_2O$$
$$\nu_p = 0.34 \qquad (56)$$
$$E = 3.45 \text{ GP}_a \text{ } (0.5 \times 10^6 \text{ psi) at 21°C}$$

The hygrothermal stress field induced by sudden moisture change is

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}(T-T) + \frac{E\beta}{1-\nu_p}[\overline{m}(t) \text{ percent} - m(t) \text{ percent}] \qquad (57)$$

Using $f(\xi,\theta)$, $g(\xi,\theta)$, $\overline{m}(t)$ as defined earlier, equation (57) can be written as (Appendix 5)

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}\{[T_i + \nu(C_f-C_i) - \frac{1}{2}\int_{-1}^{1} g(\xi,\theta)d\xi]$$
$$- [T_i + \nu(C_f-C_i)g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p}\{\overline{m}(\infty) \times \frac{\overline{m}(t)-m_i}{m_f-m_i}$$
$$- \overline{m}(\infty)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\} \qquad (58)$$

Carrying out the finite difference calculation as in equations (18), (19) and (27), the stresses σ_x, σ_y can be obtained by numerical means.

For the case of nonsymmetric moisture change, equation (55) can be written as (Appendix 6)

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p} \{[T_i + \nu(C_f-C_i) \frac{1}{2} \int_{-1}^{1} g(\xi,\theta)d\xi]$$

$$- [T_i + \nu(C_f-C_i)g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p} \{m(\infty) \times \frac{\overline{m}(t)-m_i}{m_f-m_i}$$

$$- \overline{m}(\infty)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\} + \frac{3\xi\alpha E\nu(C_f-C_i)}{2(1-\nu_p)}$$

$$\times \int_{-1}^{1} \xi g(\xi,\theta)d\xi + \frac{3\xi\beta E\overline{m}(\infty)}{2(1-\nu_p)} \int_{-1}^{1} [f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\xi d\xi \qquad (59)$$

III.2.1 Numerical Example

The hygrothermal stress distribution through the plate thickness for a sudden moisture change (symmetric case) is shown in Figure 12 as RH is changed from 0 percent to 75 percent with $T_i = T_f = 21°C$. Initially, both moisture and temperature are at the equilibrium state and hence give rise to no stress. As the relative humidity on the plate surfaces is altered, moisture absorption begins. This causes contraction or expansion of the material elements and leads to hygrothermal stresses that vary as a function of z and t. It can be easily seen from the graph that the stresses near the surface are compressive and their magnitude decrease as the plate thickness is increased. These stresses become tensile in the region close to the center of the plate. The stress at the center increases in magnitude reaching the maximum value at t ≈ 1,571 hour and then begins to decrease settling at the zero equilibrium state. This trend is similar to the results of the uncoupled theory [12] and is to be expected since the influence of coupling due to diffusion was weak for the case of sudden moisture change. The variations of the stresses σ_x (or σ_y) for z=0 and ±h/2 with time are summarized in Figures 13 and 14, respectively. Figure 13 shows clearly that the stresses at the midplane are tensile. They rise quickly to a peak and then decay. Their amplitude increases with the relative humidity of the environment. The time variation of the compressive stresses at the plate surfaces is similar except that the peaks are much more pronounced. Table 4 gives the percent deviation of stresses in T300/5208 for symmetric and nonsymmetric conditions for sudden moisture change with T_i = 21°C and $(RH)_i$ = 0 percent. The stresses for nonsymmetric cases at the midplane are exactly 50 percent smaller than those for the symmetric cases, while the deviations at the surface is about 37.20 percent for t ≃ 1961 hour.

III.3 Hygrothermal Stresses for Sudden Temperature Change

Let the moisture concentration on the plate be a constant and change the surface temperature which is initially kept at the ambient condition. With $\overline{T}(t)$, $\overline{m}(t)$, as defined in equations (25), using equations (32), it is found that

$$\overline{T}(t) - T_i = (T_f - T_i) - \frac{1}{2} \int_{-1}^{1} g(\xi, \theta) d\xi \qquad (60)$$

Equations (36) and (37) yield

$$\overline{m}(t) \text{ percent} = \left(\frac{m_f - m_i}{w_i} \times \frac{\overline{m}(t) - m_i}{m_f - m_i} + \frac{m_i}{w_i}\right) \times 100 \text{ percent} \tag{61}$$

$$m(t) \text{ percent} = \frac{m_i}{w_i} \times 100 - \left(\frac{m_i}{w_i} - \frac{m_f}{w_i}\right) \times 100 \times [g(\xi,\theta) - f(\xi,\theta)] \tag{62}$$

Substituting equations (60), (61) and (62) into equation (50), the stress field for the symmetric case of a sudden temperature change is found:

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}\{(T_f - T_i)[\frac{1}{2}\int_{-1}^{1} g(\xi,\theta)d\xi - g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p}\{(\frac{m_i - m_f}{w_i} \times 100)[g(\xi,\theta) - f(\xi,\theta) - \frac{\overline{m}(t) - m_i}{m_f - m_i}]\} \tag{63}$$

Using equation (38), σ_x, σ_y can be obtained numerically. The stress field for the nonsymmetric case of a sudden temperature change can be derived from equation (55). The result is (Appendix 7)

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}\{(T_f - T_i)[\frac{1}{2}\int_{-1}^{1} g(\xi,\theta)d\xi - g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p}\{(\frac{m_i - m_f}{w_i} \times 100)[g(\xi,\theta) - f(\xi,\theta) - \frac{\overline{m}(t) - m_i}{m_f - m_i}]\} + \frac{3\alpha E\xi(T_f - T_i)}{2(1-\nu_p)}\int_{-1}^{1} g(\xi,\theta)\xi d\xi + \frac{3\beta E\xi}{2(1-\nu_p)} \times \frac{m_f - m_i}{w_i} \times 100 \int_{-1}^{1} [g(\xi,\theta) - f(\xi,\theta)]\xi d\xi \tag{64}$$

III.3.1 Numerical Examples

Figures 15 and 16 show the results of σ_x (or σ_y) plotted against 2z/h for a symmetric temperature change from T_i = 21°C and $(RH)_i$ = 52 percent to T_f = 10°C, 12.78°C. This corresponds to a temperature drop. The tensile stresses in the interior increase in magnitude while the compressive stresses near the plate surface decrease in magnitude. The peak tensile stress occurs at t ≈ 1,961 hour. The opposite trend is observed when the surface temperature is raised. Figures 17 and 18 give the results for T_f = 43°C and 63°C. The stresses at the center region now become compressive and those near the surface are tensile. Maximum value of the compressive stress at z=0 occurs at t ≈ 791 and 402 hour.

The time-dependent character of the stresses is exhibited in Figures 19 and 20 for z=0 and z = ±h/2. For z=0, σ_x (or σ_y) increases in amplitude to a peak and then decreases for negative ΔT while σ_x (or σ_y) attains an oscillatory character when ΔT is positive. On the surface where z = ±h/2, all the stresses, whether tensile or compressive, reach a peak and then reduce to the equilibrium condition of zero stress. Table 5 gives the percent deviation of stress in T300/5208 for symmetric and nonsymmetric conditions for sudden temperature change with T_i = 21°C and $(RH)_i$ = 52 percent. The stresses at the midplane of nonsymmetric case are about 50 percent smaller than those of the symmetric case for temperature drop, while for temperature rise, the deviations vary from about 20 percent to more than 100 percent. The stress deviations at the surface of nonsymmetric case vary from

0 percent to about 35 percent for temperature drop; for temperature rise, the deviations can be larger than 100 percent and the signs even change from negative to positive.

III.3.2 Comparison With Uncoupled Theory

The stress results for the uncoupled theory are also observed such that a comparison with the coupled theory can be made. Figures 21 to 24 for the uncoupled case correspond, respectively, to the results in Figures 15 to 18 for the coupled case. Although the general trend of the curves may be similar, there are noticeable differences in the stress amplitudes. In order to be more specific, Table 6 gives a comparison of the stresses at $z=0$ and $z = \pm h/2$ for 6 different values of the final temperature. The percent of deviation between the results of the coupled and uncoupled theory is calculated for elapsed time $t = 11.7$ hr., 402 hr. and 1961 hr. The largest deviation occurs at $t \approx 1961$ hr. Note that for positive ΔT, i.e., temperature increase, the coupling of moisture and heat can alter the stress anywhere from 40 to 80 percent depending on ΔT. In such cases, the stresses predicted from the uncoupled theory may not adequately model the physical problem.

IV. CONCLUSIONS

For the T300/5208 epoxy resin composite material treated in this study, it is seen that the interaction of moisture and heat can significantly alter the stress distribution in the composite. Although thermal and moisture diffusion do not peak simultaneously because of the wide margin of difference between the coefficients α and β, the way in which moisture and heat interact in a solid is complicated and cannot be disposed on intuitive grounds. In particular, when a composite is subjected to a sudden temperature change on its surface, the transient stresses predicted from the coupled and uncoupled theory can differ appreciably.

In this preliminary analysis, material isotropy and homogeneity have been assumed. These simplifications should be further investigated by incorporating the real structure of the composite. What lies ahead is the formulation of a finite difference or finite element method that treats three independent variables: two in space (x,y) and one in time (t). This will permit an evaluation on the effect of material anisotropy, the presence of cavities and non-uniform temperature and/or moisture boundary conditions. These additional influences will also interact with moisture and heat and should be assessed quantitatively such that their individual contribution on the overall mechanical behavior of the composite can be understood.

TABLE 1 - PERCENT DEVIATION OF MOISTURE VARIATION IN T300/5208 FOR SYMMETRIC AND NONSYMMETRIC SUDDEN MOISTURE CHANGE AT 21°C FROM $(RH)_i$ = 0 PERCENT TO 75 PERCENT

$\sqrt{t}$ $(hr)^{1/2}$		$\bar{m}(t)$ average	$m_1(t)$ $2z/h = -1.0$	$m_3(t)$ $2z/h = -0.66$	$m_5(t)$ $2z/h = -0.33$	$m_7(t)$ $2z/h = 0.0$
3.42	sym.	0.069	1.162	0.000	0.000	0.000
	nonsym.	0.034	1.162	0.000	0.000	0.000
	dev. %	50	0	0	0	0
20.04	sym.	0.232	1.162	0.211	0.015	0.002
	nonsym.	0.116	1.162	0.211	0.015	0.001
	dev. %	50	0	0	0	0
28.13	sym.	0.323	1.162	0.387	0.070	0.015
	nonsym.	0.161	1.162	0.387	0.069	0.008
	dev. %	50	0	0	1.43	50
34.37	sym.	0.393	1.162	0.497	0.140	0.049
	nonsym.	0.197	1.162	0.497	0.136	0.025
	dev. %	50	0	0	2.86	50
39.64	sym.	0.453	1.162	0.573	0.209	0.100
	nonsym.	0.226	1.162	0.571	0.200	0.050
	dev. %	50	0	0.17	4.3	50
44.28	sym.	0.505	1.162	0.629	0.275	0.158
	nonsym.	0.253	1.162	0.625	0.256	0.079
	dev. %	50	0	0.64	6.91	50

TABLE 2 - PERCENT DEVIATION OF MOISTURE VARIATION IN T300/5208 FOR SYMMETRIC AND NONSYMMETRIC SUDDEN TEMPERATURE CHANGE FROM 21°C, $(RH)_i$ = 52 PERCENT TO 10°C

$\sqrt{t}$ $(hr)^{1/2}$		$\bar{m}(t)$ average	$m_1(t)$ 2z/h = -1.0	$m_3(t)$ 2z/h = -0.66	$m_5(t)$ 2z/h = -0.33	$m_7(t)$ 2z/h = 0.0
3.42	sym.	0.852	1.518	0.807	0.806	0.806
	nonsym.	0.829	1.518	0.807	0.806	0.806
	dev. %	2.70	0	0	0	0
20.04	sym.	0.972	1.518	0.954	0.869	0.858
	nonsym.	0.891	1.518	0.950	0.857	0.832
	dev. %	8.33	0	0.40	1.38	3.03
28.13	sym.	1.026	1.518	1.032	0.908	0.893
	nonsym.	0.919	1.518	1.024	0.887	0.851
	dev. %	10.43	0	0.78	2.30	4.70
34.37	sym.	1.059	1.518	1.084	0.934	0.910
	nonsym.	0.938	1.518	1.076	0.911	0.862
	dev. %	11.43	0	0.74	2.46	5.27
39.64	sym.	1.083	1.518	1.123	0.955	0.921
	nonsym.	0.953	1.518	1.115	0.933	0.872
	dev. %	12.00	0	0.71	2.30	5.38
44.28	sym.	1.103	1.518	1.152	0.973	0.932
	nonsym.	0.965	1.518	1.145	0.954	0.882
	dev. %	12.51	0	0.61	1.95	5.36

TABLE 3 - PERCENT DEVIATION OF MOISTURE VARIATION IN T300/5208 FOR SYMMETRIC AND NONSYMMETRIC SUDDEN TEMPERATURE CHANGE FROM 21°C, $(RH)_i$ = 52 PERCENT TO 43°C

$\sqrt{t}$ $(hr)^{1/2}$		$\bar{m}(t)$ average	$m_1(t)$ 2z/h = -1.0	$m_3(t)$ 2z/h = -0.66	$m_5(t)$ 2z/h = -0.33	$m_7(t)$ 2z/h = 0.0
	sym.	0.769	0.237	0.805	0.806	0.806
3.42	nonsym.	0.788	0.237	0.805	0.806	0.806
	dev. %	2.47	0	0	0	0
	sym.	0.612	0.237	0.567	0.749	0.779
20.04	nonsym.	0.713	0.237	0.573	0.760	0.793
	dev. %	16.5	0	1.05	1.47	1.80
	sym.	0.527	0.237	0.465	0.640	0.670
28.13	nonsym.	0.677	0.237	0.477	0.675	0.771
	dev. %	28.46	0	2.58	5.47	15.07
	sym.	0.459	0.237	0.409	0.544	0.593
34.37	nonsym.	0.652	0.237	0.433	0.614	0.735
	dev. %	42.04	0	5.87	12.87	23.95
	sym.	0.404	0.237	0.367	0.467	0.503
39.64	nonsym.	0.632	0.237	0.408	0.572	0.700
	dev. %	56.44	0	11.17	22.48	39.17
	sym.	0.362	0.237	0.334	0.408	0.435
44.28	nonsym.	0.616	0.237	0.391	0.543	0.672
	dev. %	70.17	0	17.07	33.10	54.48

TABLE 4 - PERCENT DEVIATION OF STRESSES IN T300/5208 FOR SYMMETRIC AND NONSYMMETRIC CONDITIONS FOR SUDDEN MOISTURE CHANGE WITH T_i = 21°C AND $(RH)_i$ = 0 PERCENT

Final Relative Humidity	Time (hrs)	Stress at Surface (psi) Symmetric	Stress at Surface (psi) Nonsymmetric	Dev. %	Stress at Midplane (psi) Symmetric	Stress at Midplane (psi) Nonsymmetric	Dev. %
13%	11.70	- 384.92	- 361.09	- 6.20	24.19	12.09	-50.0
	402	- 327.63	- 266.20	-18.75	80.72	40.36	-50.0
	1961	- 231.25	- 145.23	- 37.20	122.13	61.07	-50.0
33%	11.70	- 977.09	- 916.62	- 6.20	61.41	30.70	-50.0
	402	- 831.68	- 675.73	- 18.75	204.91	102.46	-50.0
	1961	- 587.03	- 368.66	- 37.20	310.03	155.02	-50.0
52%	11.70	-1539.66	-1444.36	- 6.20	96.76	48.38	-50.0
	402	-1310.52	-1064.78	- 18.75	322.89	161.45	-50.0
	1961	- 925.01	- 580.92	- 37.20	488.53	244.27	-50.0
75%	11.70	-2220.67	-2083.22	- 6.20	139.56	69.78	-50.0
	402	-1890.17	-1535.74	- 18.75	465.71	232.85	-50.0
	1961	-1334.16	- 837.87	- 37.20	704.61	352.31	-50.0
98%	11.70	-2901.67	-2722.08	- 6.20	182.36	91.18	-50.0
	402	-2469.83	-2006.70	- 18.75	608.52	304.26	-50.0
	1961	-1743.30	-1094.82	- 37.20	920.68	460.35	-50.0
100%	11.70	-2960.89	-2777.63	- 6.20	186.08	93.04	-50.0
	402	-2520.23	-2047.65	- 18.75	620.94	310.47	-50.0
	1961	-1778.87	-1117.16	- 37.20	939.48	469.74	-50.0

TABLE 5 - PERCENT DEVIATION OF STRESSES IN T300/5208 FOR SYMMETRIC AND NONSYMMETRIC CONDITIONS FOR SUDDEN TEMPERATURE CHANGE WITH T_i = 21°C AND $(RH)_i$ = 52 PERCENT

		Stress at Surface			Stress at Midplane		
Final Temperature °C	Time (hrs)	Symmetric psi	Nonsymmetric psi	Dev. %	Symmetric	Nonsymmetric	Dev. %
0	11.70	-1622.81	-1534.50	- 5.40	85.46	42.85	- 49.86
	402	-1685.47	-1495.76	- 11.26	196.41	106.41	- 45.82
	1961	-1566.77	-1170.50	- 25.30	436.03	220.50	- 49.43
10	11.70	-1015.07	- 957.92	- 5.63	56.13	28.10	- 49.94
	402	- 989.52	- 847.23	- 14.38	166.18	86.67	- 47.85
	1961	- 837.88	- 570.30	- 31.94	345.44	166.84	- 51.70
12.78	11.70	- 608.12	- 575.31	- 5.40	31.77	15.90	- 49.95
	402	- 607.79	- 519.45	- 14.53	107.10	55.35	- 48.32
	1961	- 509.24	- 337.42	- 33.74	229.71	110.43	- 51.93
43	11.70	402.05	395.85	- 1.5	- 0.964	- 0.404	- 58.10
	402	513.22	416.85	- 18.78	-203.48	- 96.15	- 52.75
	1961	226.98	149.56	- 34.10	-133.22	-106.70	- 19.90
63	11.70	51.81	87.42	+ 68.73	46.97	23.71	- 49.52
	402	358.09	292.84	- 18.22	-238.40	-123.84	- 48.05
	1961	47.36	57.91	+ 22.28	- 28.15	- 56.43	+100.46
83	11.70	- 465.34	- 376.78	- 19.03	106.22	53.53	- 49.60
	402	64.35	85.49	+ 32.85	-138.60	-106.20	- 23.38
	1961	- 5.28	0.129	+102.44	2.82	- 8.78	+411.34

TABLE 6 - PERCENT DEVIATION OF STRESSES IN T300/5208 FOR COUPLED AND UNCOUPLED THEORY

WITH T_i = 21°C AND $(RH)_i$ = 52 PERCENT

Final Temperature °C	Time (hrs)	Stress at Surface (psi) Coupled	Uncoupled	Dev. %	Stress at Midplane (psi) Coupled	Uncoupled	Dev. %
0	11.70	-1622.82	-1637.52	0.91	85.46	70.75	17.2
	401.5	-1685.47	-1931.90	14.63	196.41	137.76	29.9
	1961	-1566.77	-1966.08	25.49	436.03	536.38	23.0
10	11.70	-1015.07	-1024.26	0.91	56.13	46.94	16.37
	401.5	- 989.52	-1134.84	14.69	166.18	139.11	16.29
	1961	- 837.88	-1060.13	26.53	345.44	427.36	23.71
12.78	11.70	- 608.12	- 613.64	0.91	31.77	26.25	17.37
	401.5	- 607.79	- 696.35	14.57	107.10	89.68	16.26
	1961	- 509.24	- 646.47	26.95	229.71	285.67	24.36
43	11.70	402.05	405.88	0.95	- 0.96	2.87	
	401.5	513.22	586.51	14.28	-203.48	-192.23	5.53
	1961	226.98	317.31	39.8	-133.22	-184.71	38.65
63	11.70	51.81	52.78	1.87	46.97	47.94	2.06
	401.5	358.10	420.71	17.48	-238.39	-252.01	5.71
	1961	47.35	67.25	42.03	- 28.15	- 39.39	39.92
83	11.70	- 465.37	- 468.67	0.71	106.22	102.92	3.11
	401.5	64.35	113.84	76.9	-138.60	-190.21	37.24
	1961	- 5.28	- 0.84	84.1	2.82	0.299	89.40

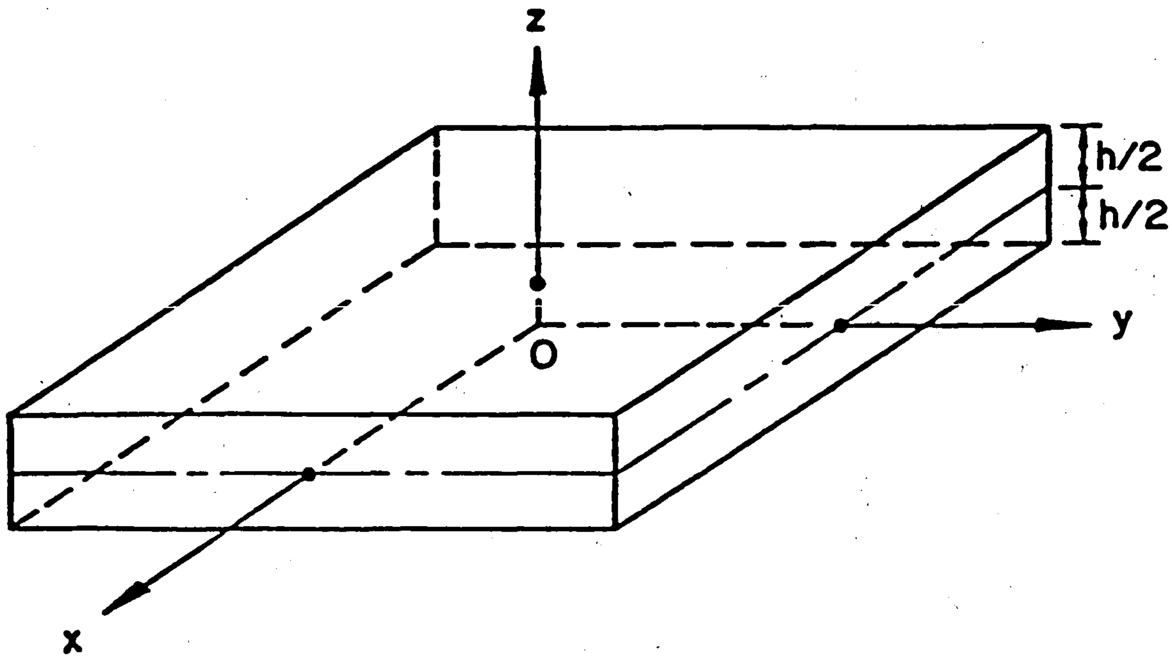

Figure 1 - Diffusion of moisture and/or temperature in an infinite plate with finite thickness

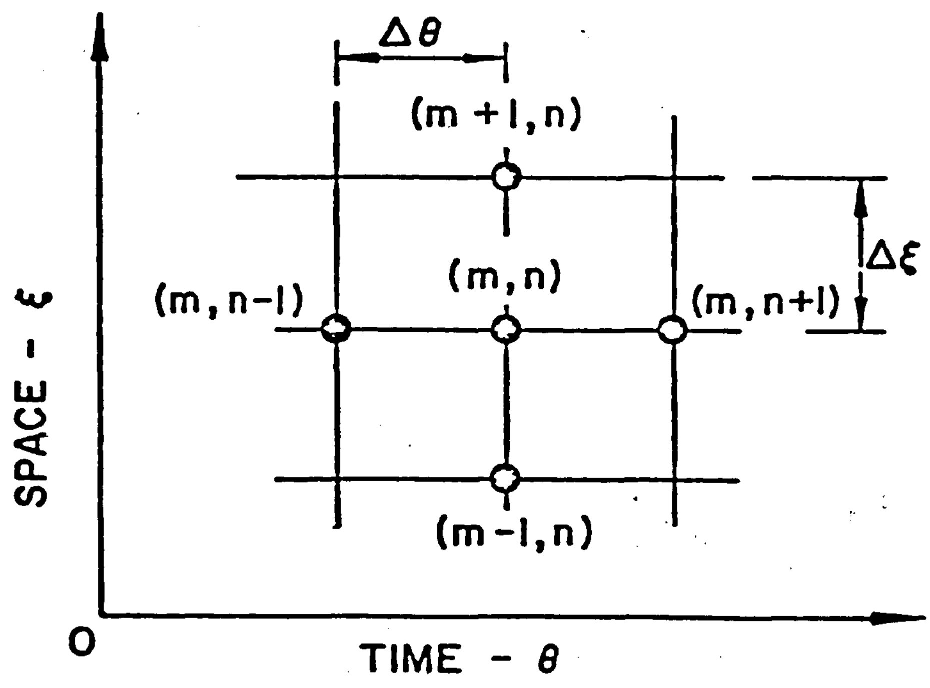

Figure 2 - Finite difference mesh in dimensionless space and time variables

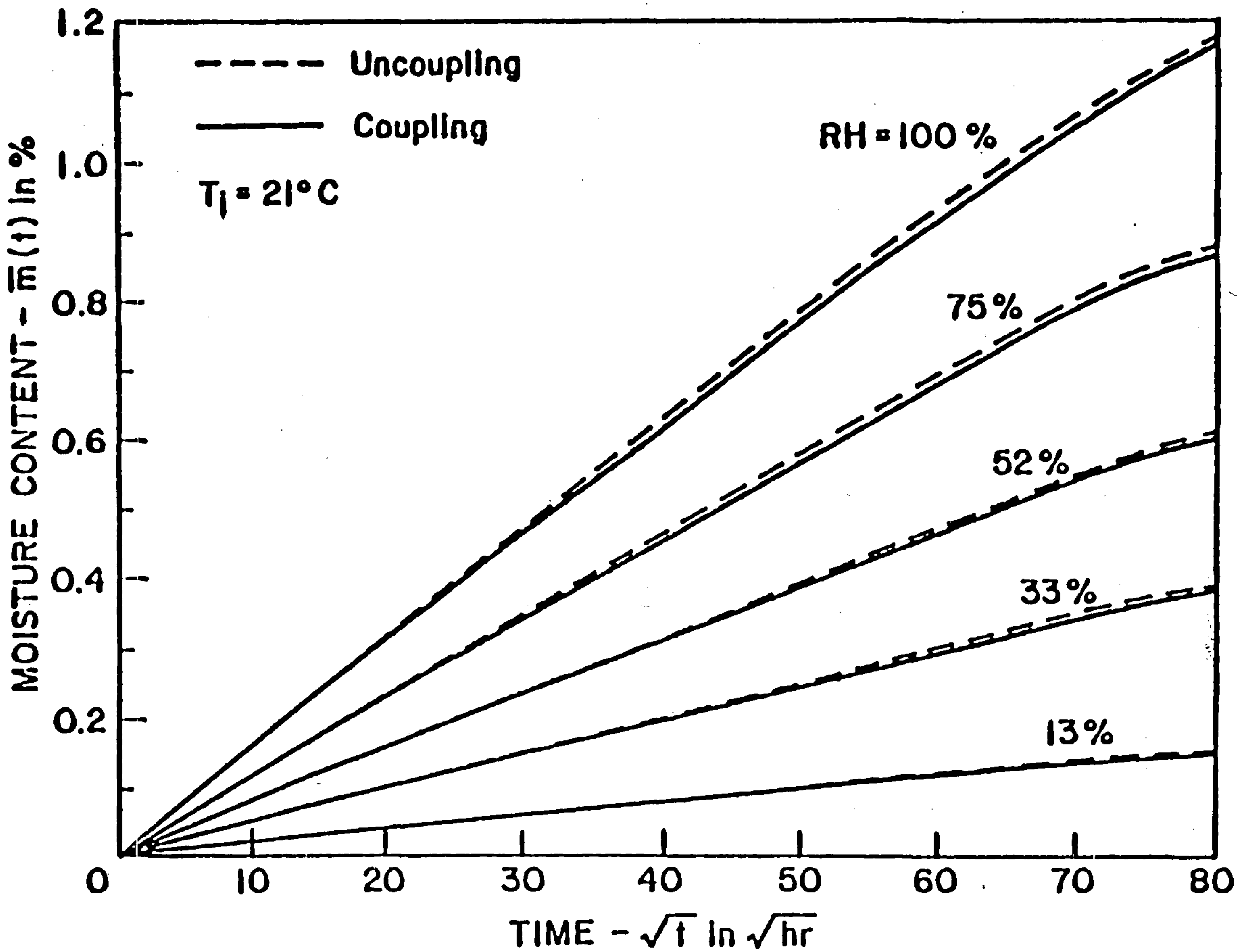

Figure 3 - Variations of moisture content with time caused by sudden change in moisture for T300/5208

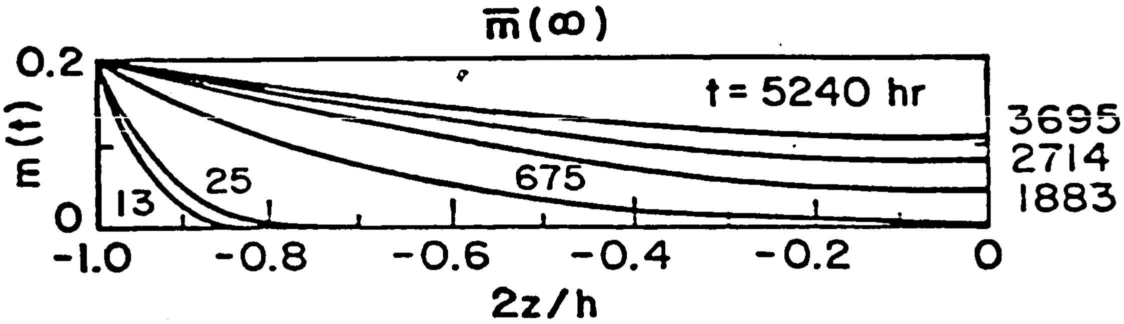

Figure 4 - Sudden moisture change from RH = 0% to 13% at 21°C for T300/5208

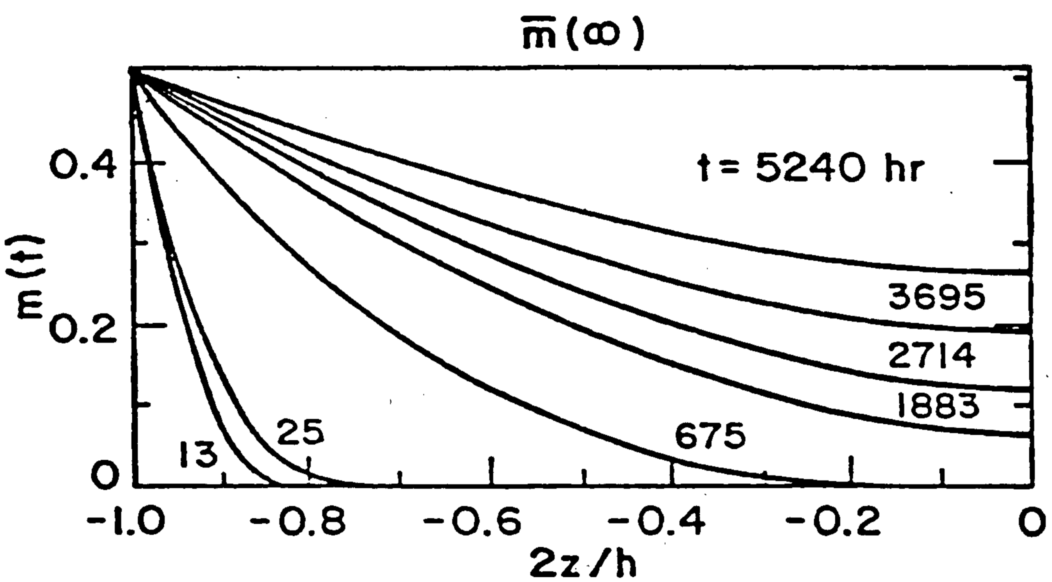

Figure 5 - Sudden moisture change from RH = 0% to 33% at 21°C for T300/5208

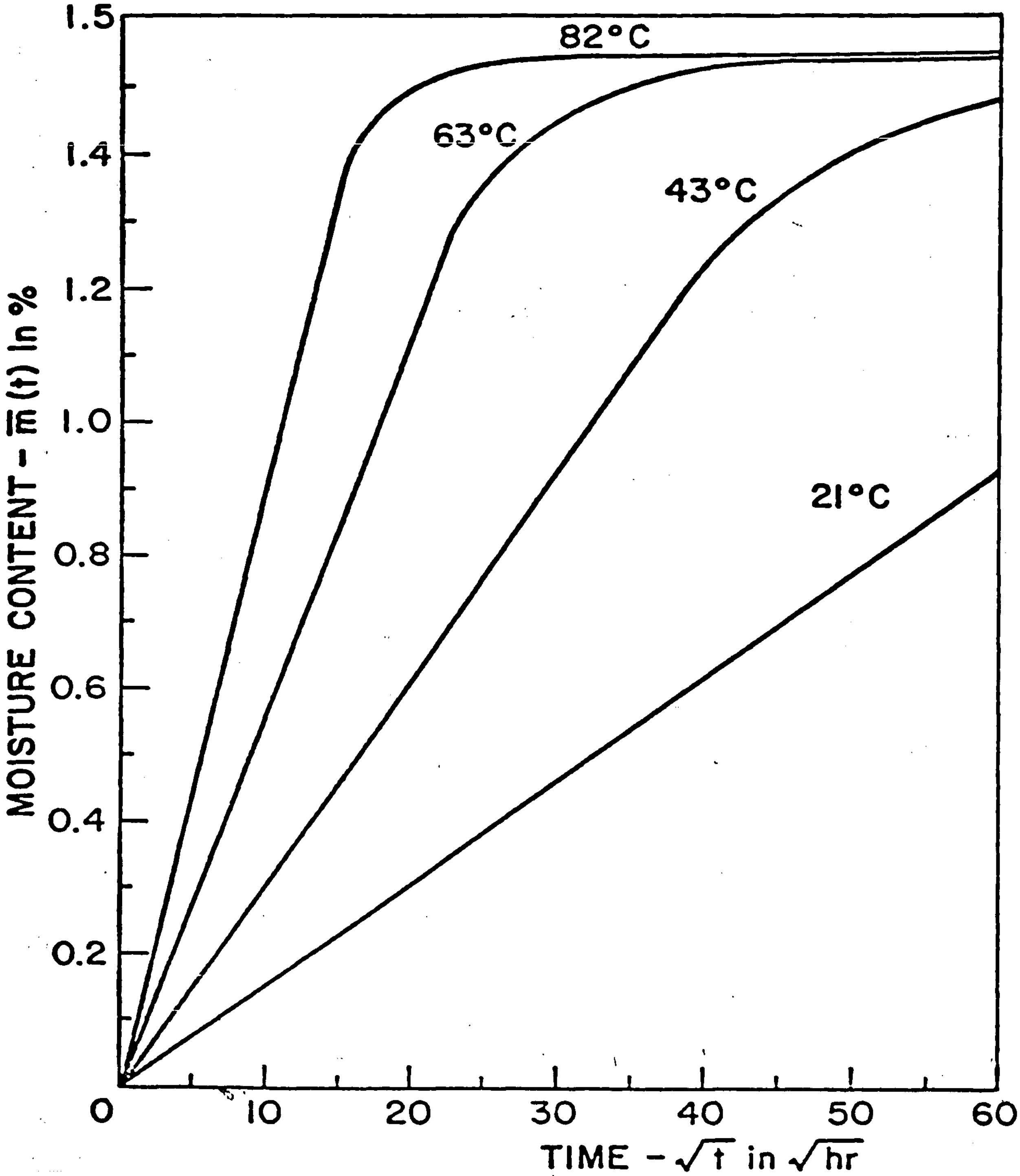

Figure 6 - Moisture absorption speed for T300/5208 in RH = 100% air with different temperature based on uncoupled theory

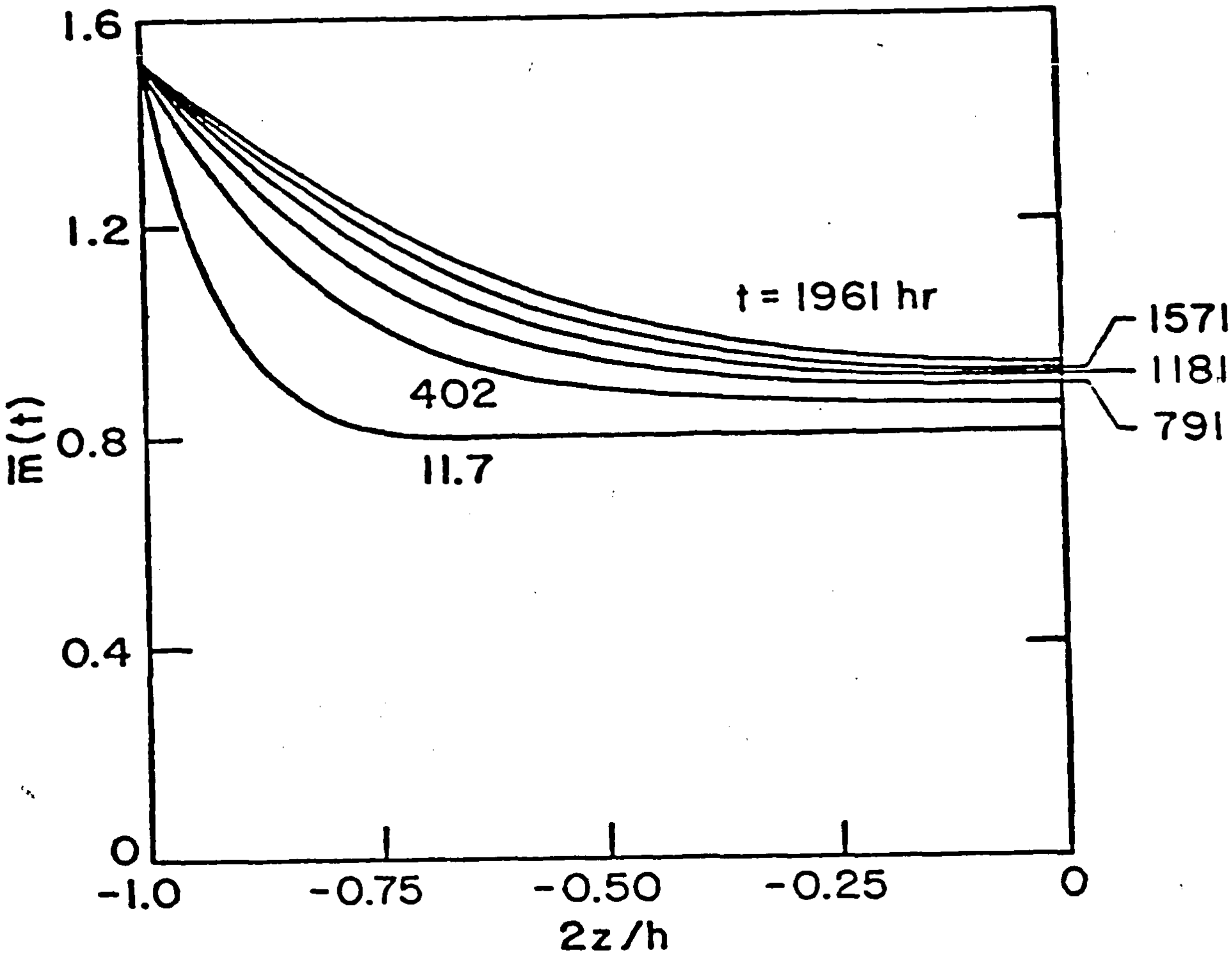

Figure 7 - Sudden temperature change from 21°C (RH = 52%) to 10°C for T300/5208

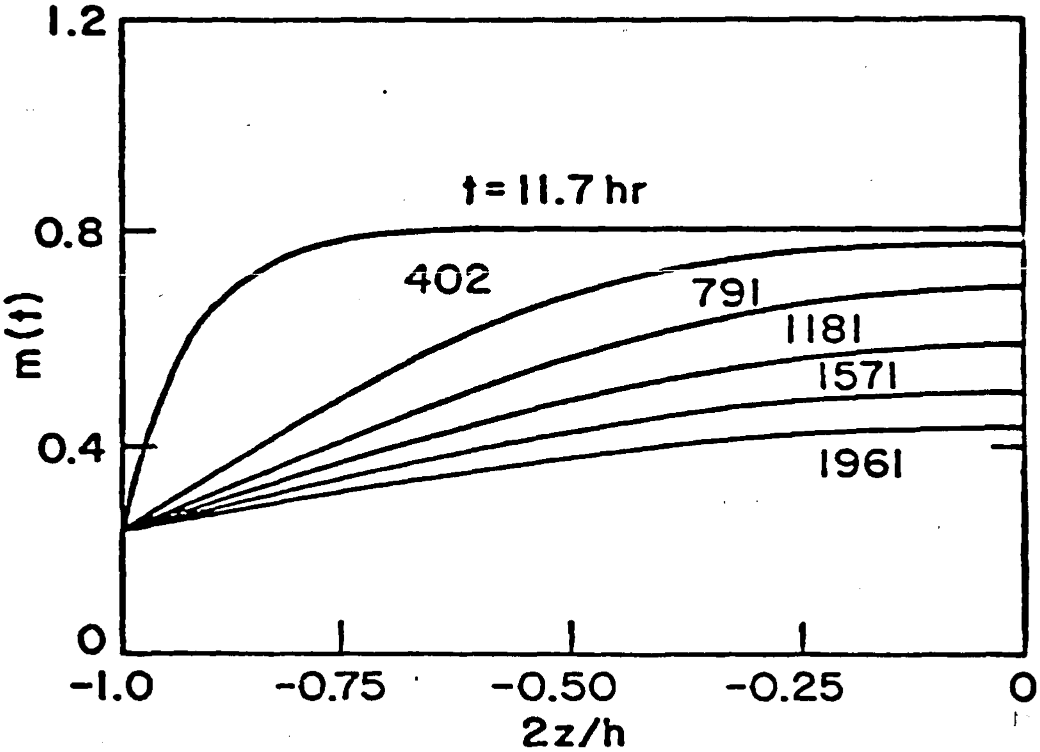

Figure 8 - Sudden temperature change from 21°C (RH = 52%) to 43°C for T300/5208

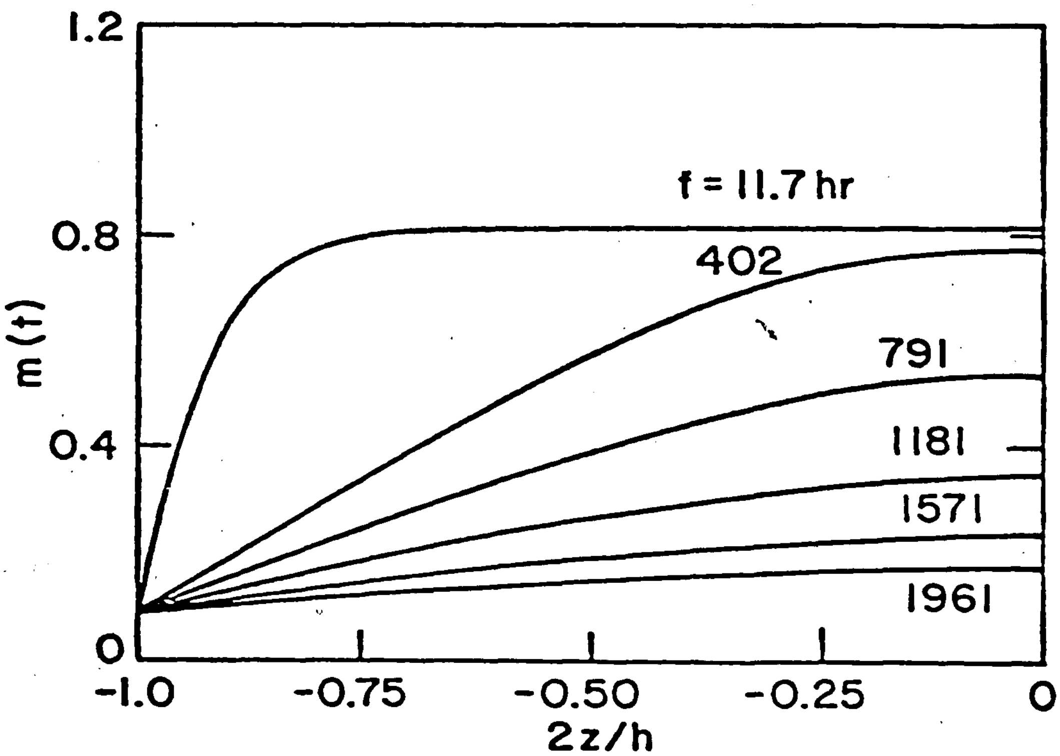

Figure 9 - Sudden temperature change from 21°C (RH = 52%) to 63°C for T300/5208

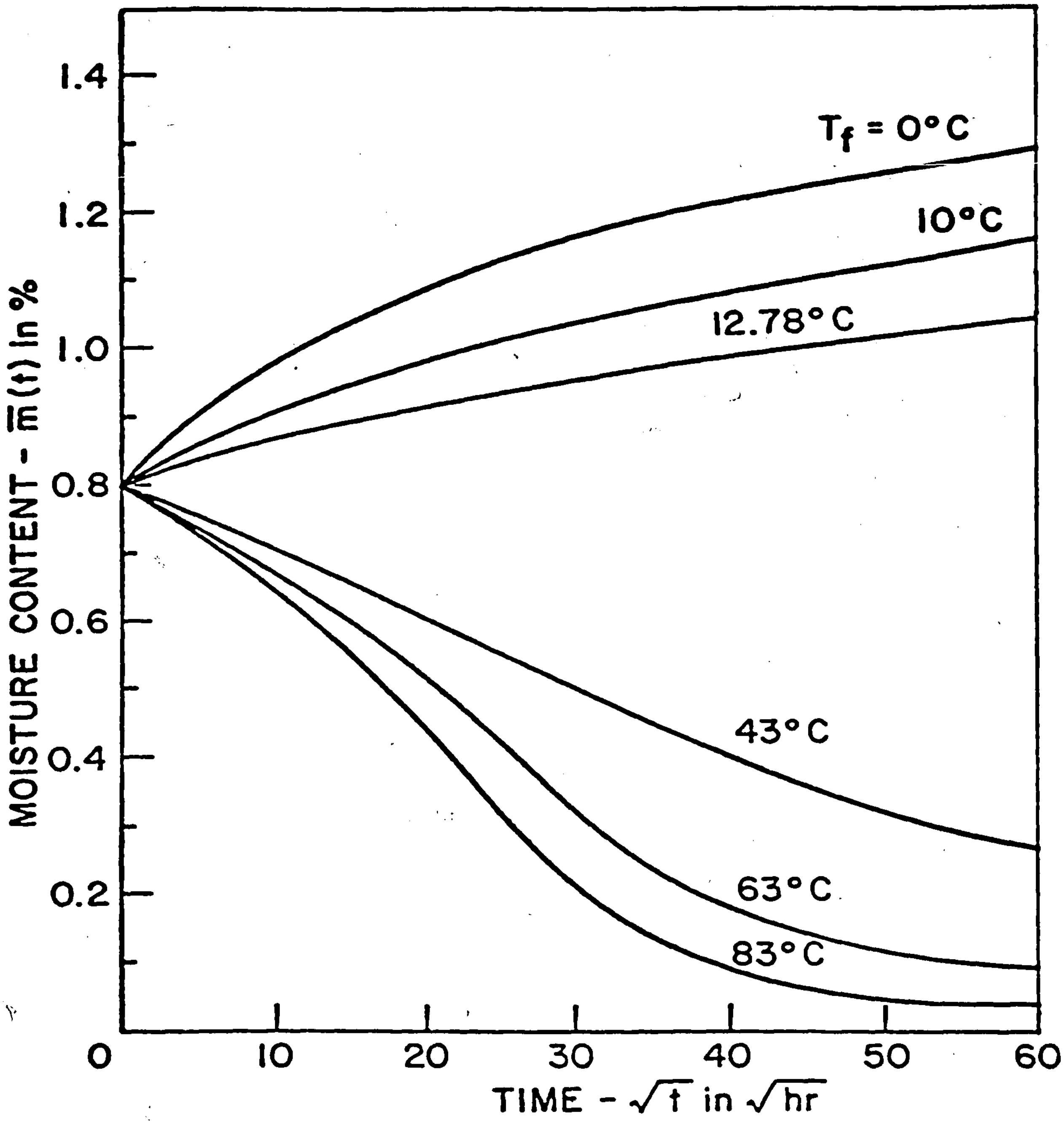

Figure 10 - Moisture absorption and desorption speed for different temperature gradients at ambient condition of 21°C and RH = 52% (T300/5208)

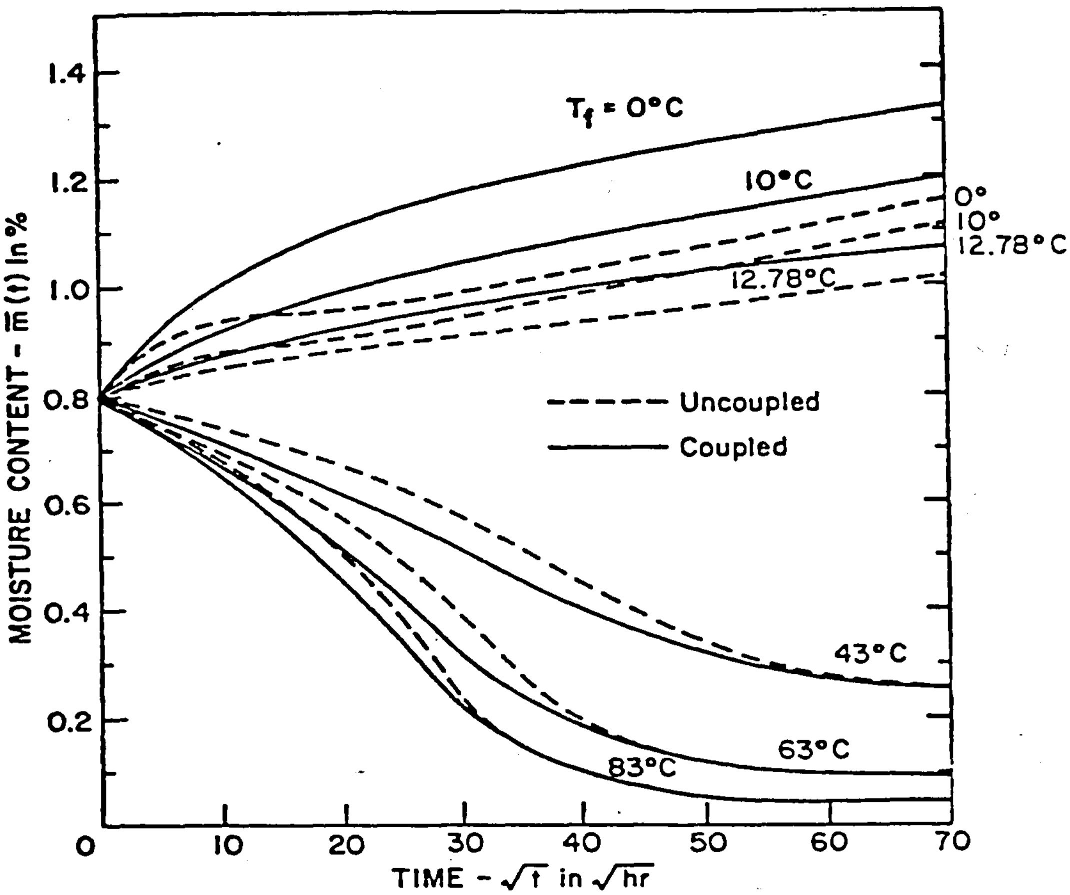

Figure 11 - Comparison of coupled and uncoupled results for moisture absorption and desorption speed with ambient condition of 21°C and RH = 52% (T300/5208)

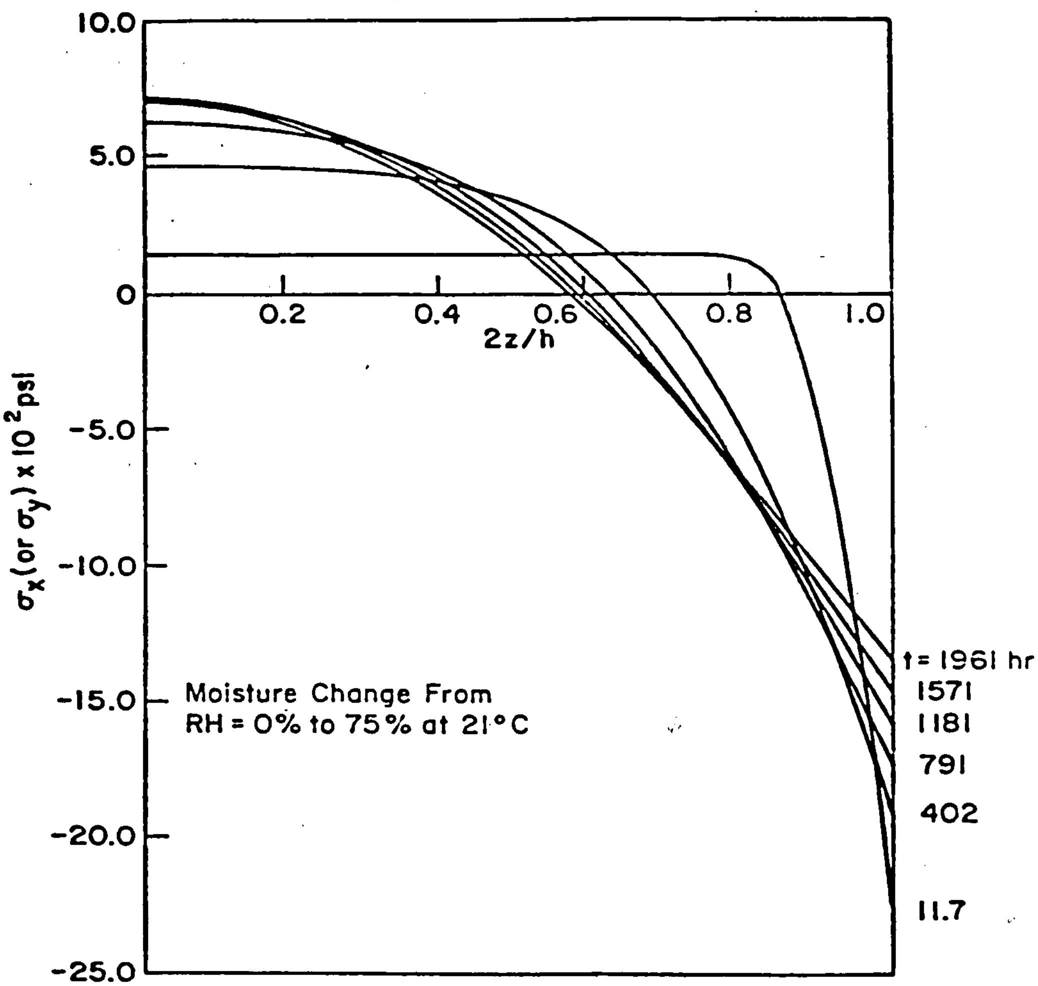

Figure 12 - Stress variations for $(RH)_f = 75\%$

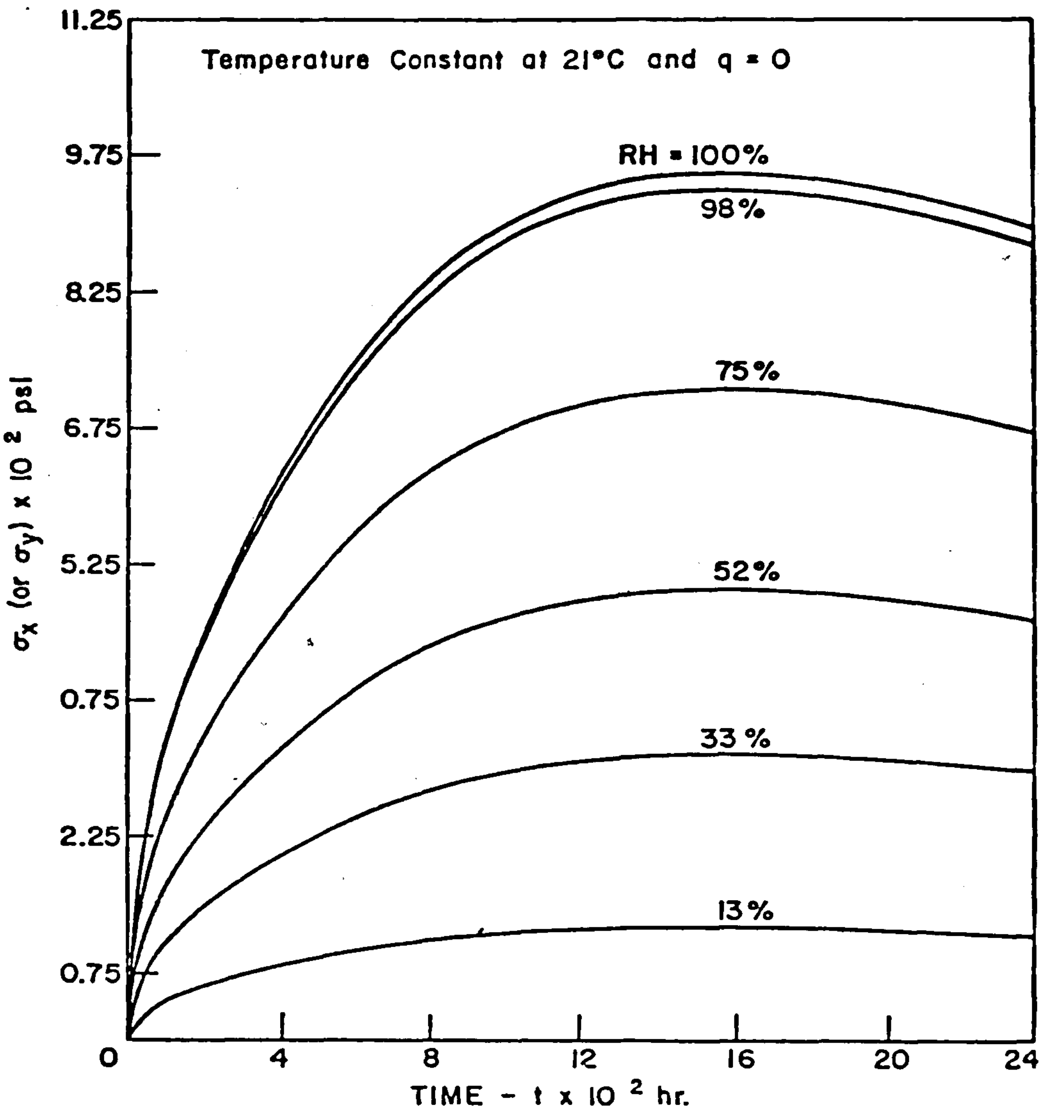

Figure 13 - Stress at midplane as a function of time

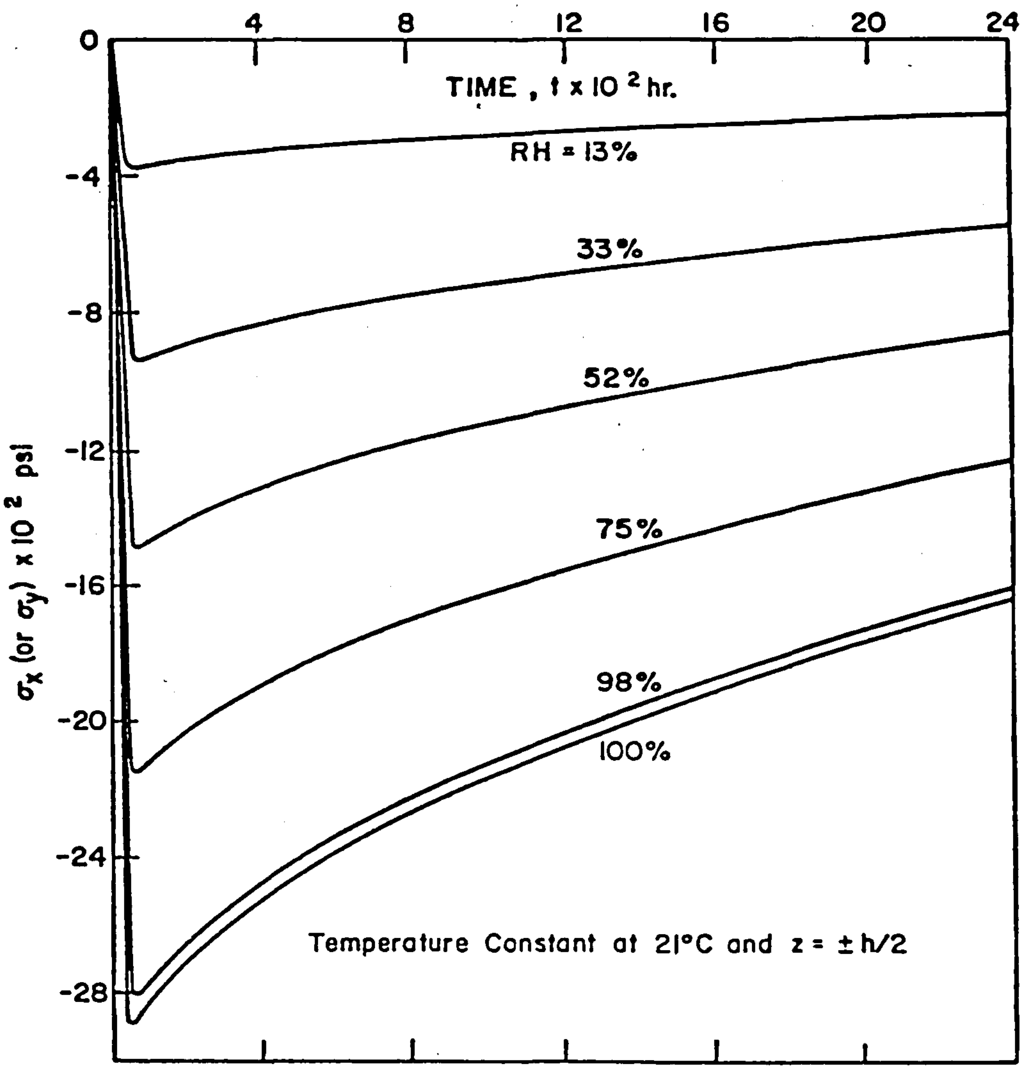

Figure 14 - Stress on plate surface as a function of time

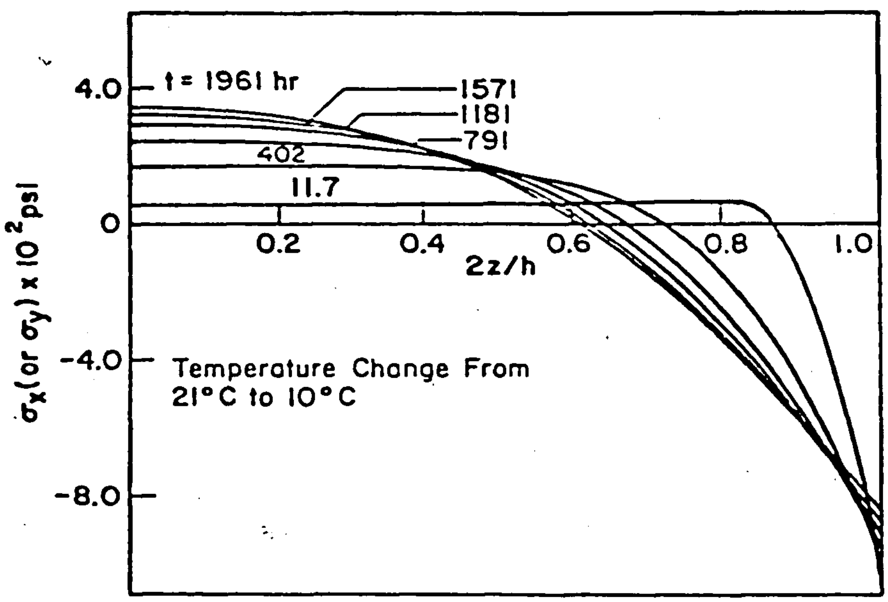

Figure 15 - Stress variations for temperature drop with T_f = 10°C

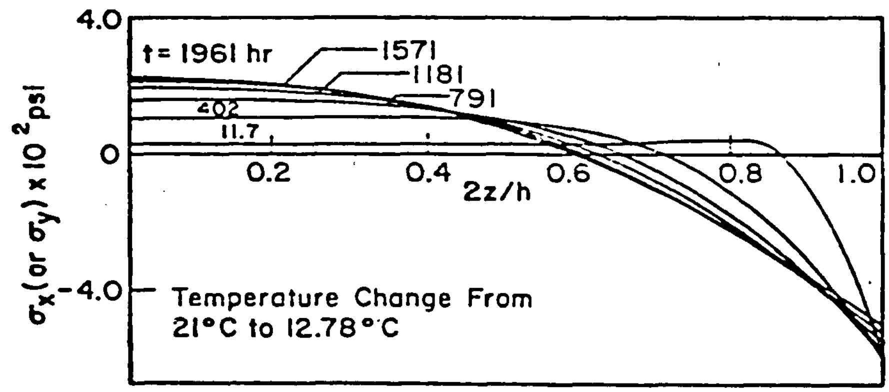

Figure 16 - Stress variations for temperature drop with T_f = 12.78°C

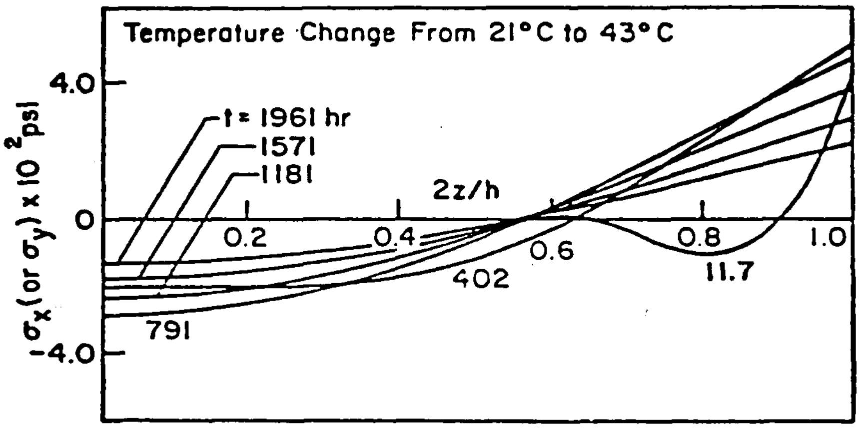

Figure 17 - Stress variations for temperature increase with T_f = 43°C

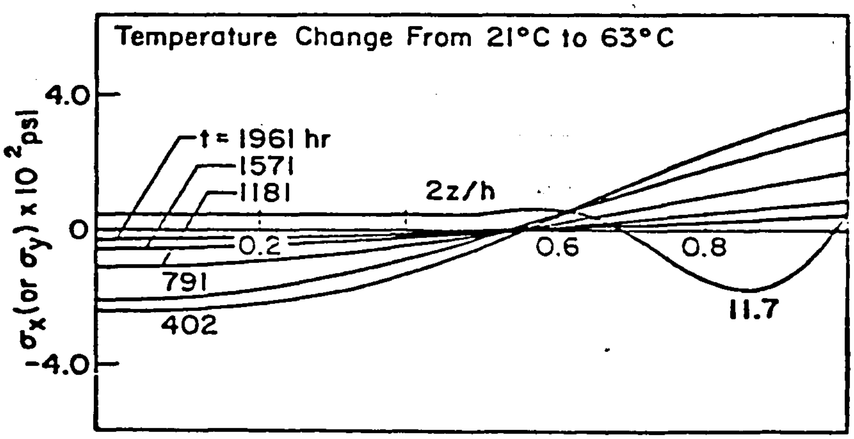

Figure 18 - Stress variations for temperature increase with T_f = 63°C

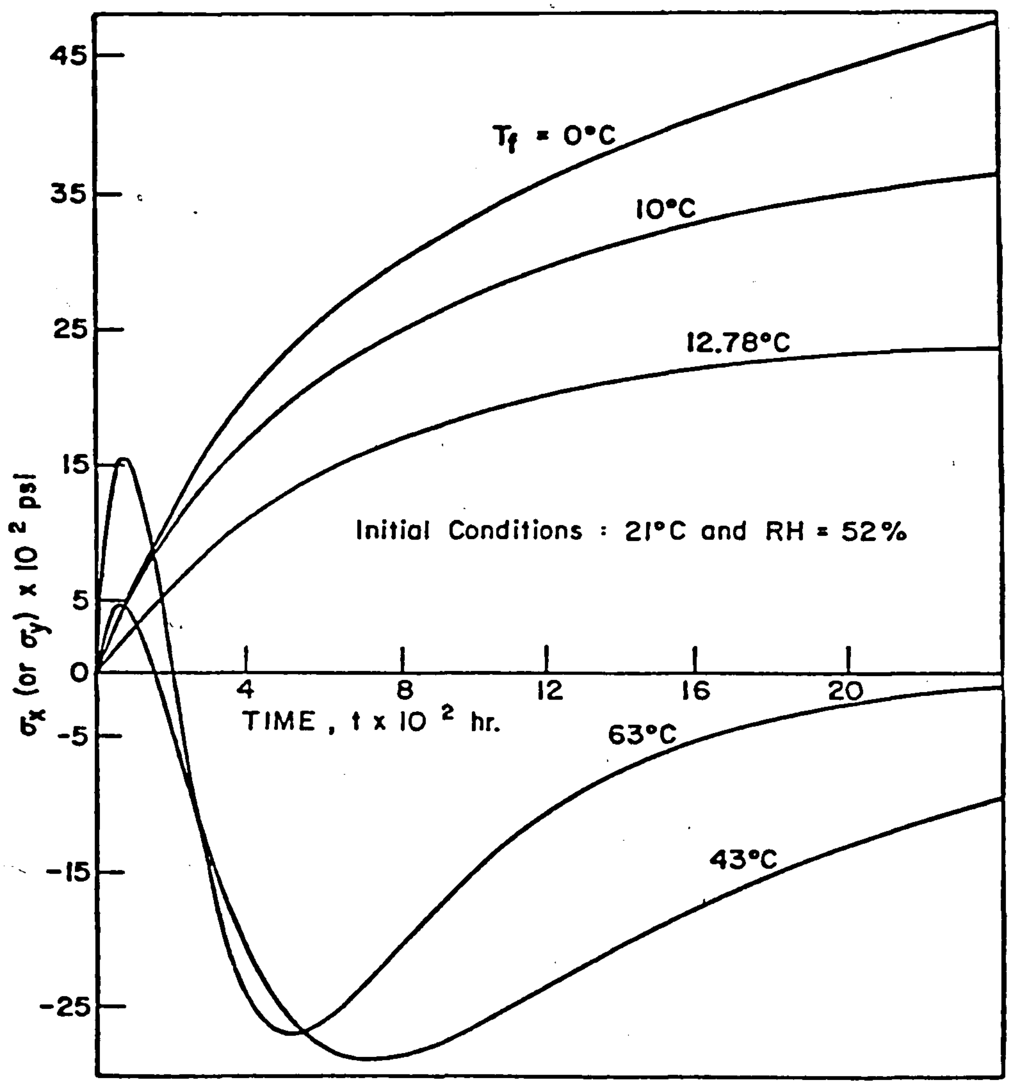

Figure 19 - Time dependent stress at the midplane due to different temperature gradients

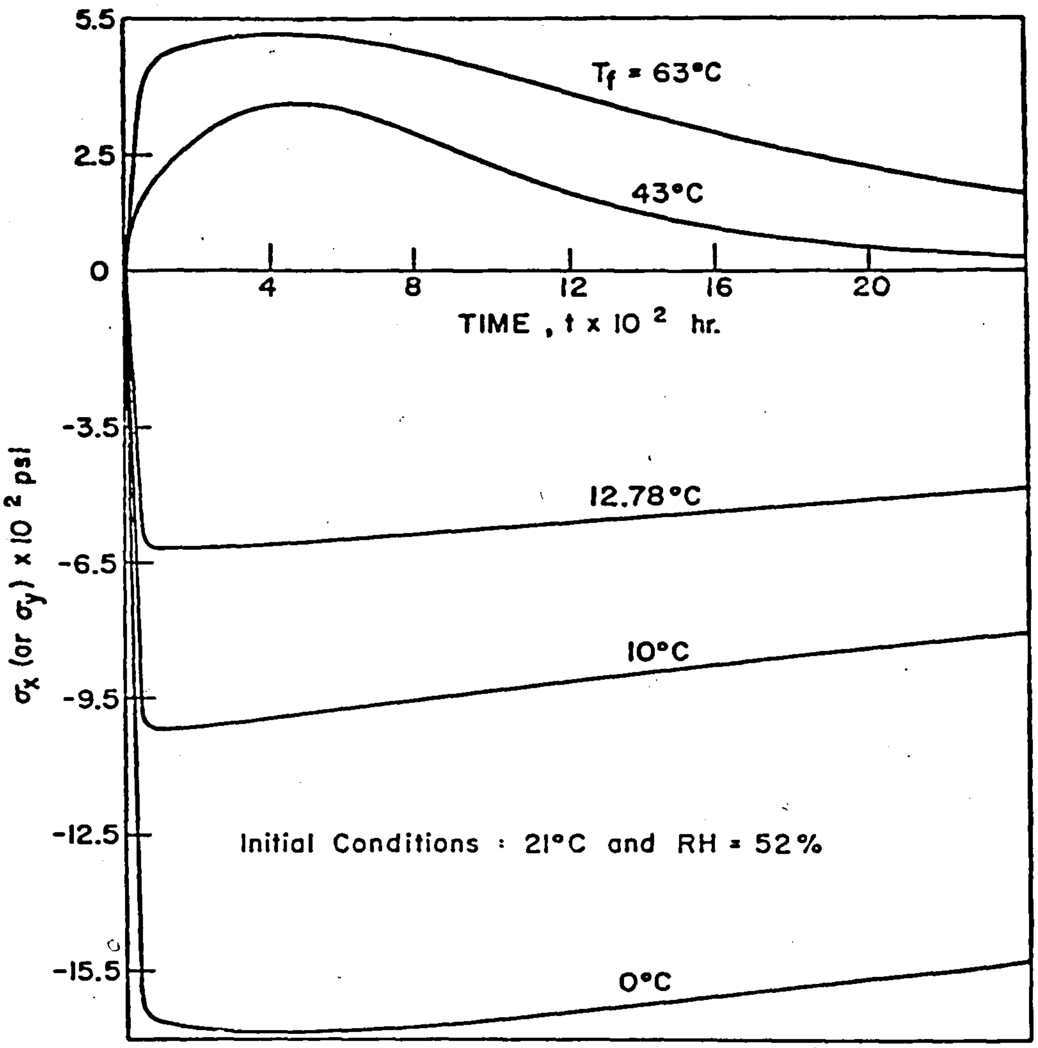

Figure 20– Time dependent stresses on plate surface due to different temperature gradients

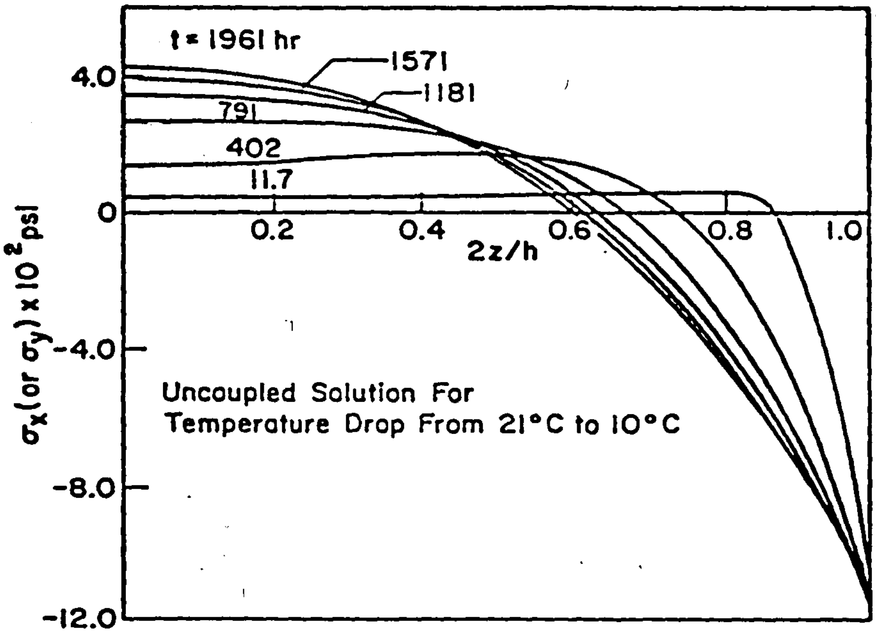

Figure 21- Uncoupled stress solution for ΔT = -11°C

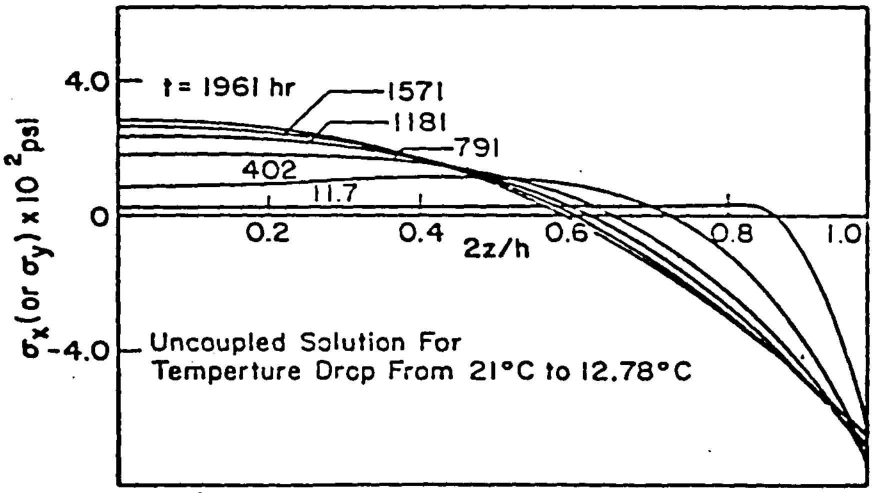

Figure 22- Uncoupled stress solution for ΔT = -8.22°C

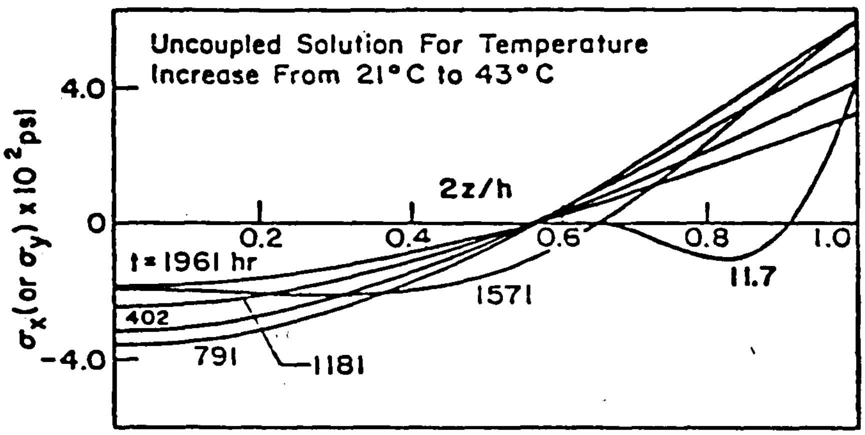

Figure 23- Uncoupled stress solution for ΔT = 22°C

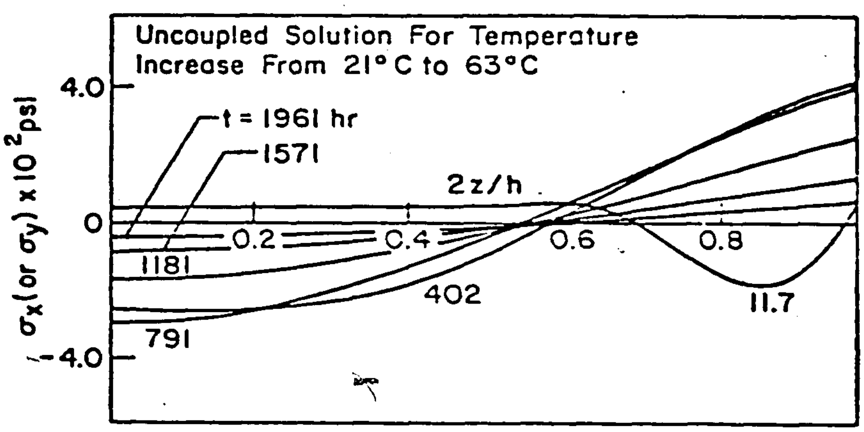

Figure 24- Uncoupled stress solution for ΔT = 42°C

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APPENDIX 1

DIMENSIONLESS EQUATIONS OF DIFFUSION

$$\nabla \cdot (D\nabla C) - \frac{\partial}{\partial t}(C-\lambda T) = 0$$

$$\nabla \cdot (\mathcal{D}\nabla T) - \frac{\partial}{\partial t}(T-\nu C) = 0 \qquad (11)$$

$$D = D_o \exp(-E_o/RT)$$

Let

$$\xi = \frac{2z}{h}, \quad \theta = \frac{4D_o t}{h^2}$$

$$\frac{\partial C}{\partial z} = \frac{\partial C}{\partial \xi}\frac{\partial \xi}{\partial z} = \frac{2}{h}\frac{\partial C}{\partial \xi}$$

$$\frac{\partial^2 C}{\partial z^2} = \left(\frac{2}{h}\right)^2 \frac{\partial^2 C}{\partial \xi^2}$$

$$\frac{\partial T}{\partial z} = \frac{2}{h}\frac{\partial T}{\partial \xi}$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{4}{h^2}\frac{\partial^2 T}{\partial \xi^2}$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial \theta}\frac{\partial \theta}{\partial t} = \frac{\partial C}{\partial \theta}\frac{4D_o}{h^2}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \theta} - \frac{4D_o}{h^2}$$

Substituting into equation (11)

$$\frac{\partial}{\partial z} [D_o \exp(-E_o/RT) \frac{\partial C}{\partial z}] - \frac{\partial}{\partial t} (C-\lambda T) = 0$$

$$D_o \exp(-E_o/RT) \frac{\partial^2 C}{\partial z^2} + D_o \frac{\partial C}{\partial z} \exp(-E_o/RT) \cdot \frac{E_o}{RT} \frac{\partial T}{\partial x}$$

$$- \frac{\partial}{\partial t} (C-\lambda T) = 0$$

$$D_o \exp(-E_o/RT)[\frac{\partial^2 C}{\partial \xi^2} + \frac{E_o}{RT^2} (\frac{\partial C}{\partial \xi})(\frac{\partial T}{\partial \xi})] - (\frac{\partial C}{\partial \theta} - \lambda \frac{\partial T}{\partial \theta}) = 0$$

Similarly, we can get

$$\mathcal{D} \frac{\partial^2 T}{\partial \xi^2} - D_o(\frac{\partial T}{\partial \theta} - \nu \frac{\partial C}{\partial \theta}) = 0$$

APPENDIX 2

DERIVATION OF EQUATIONS (17)

From equations (13)

$$\left[\frac{\partial^2 C}{\partial \xi^2} + \frac{E_o}{RT^2}\left(\frac{\partial C}{\partial \xi}\right)\left(\frac{\partial T}{\partial \xi}\right)\right] \exp\left(-\frac{E_o}{RT}\right) - \left(\frac{\partial C}{\partial \theta} - \lambda \frac{\partial T}{\partial \theta}\right) = 0$$

$$\mathcal{D} \frac{\partial^2 T}{\partial \xi^2} - D_o\left(\frac{\partial T}{\partial \theta} - \nu \frac{\partial C}{\partial \theta}\right) = 0 \qquad (13)$$

the solution for moisture and temperature are

$$C(t) = C_i + (C_f - C_i) f(\xi, \theta)$$

$$T(t) = T_i + \nu(C_f - C_i) g(\xi, \theta) \qquad (16)$$

$$\frac{\partial C}{\partial \xi} = (C_f - C_i) \frac{\partial f}{\partial \xi}, \quad \frac{\partial T}{\partial \xi} = \nu(C_f - C_i) \frac{\partial g}{\partial \xi}$$

$$\frac{\partial C}{\partial \theta} = (C_f - C_i) \frac{\partial f}{\partial \theta}, \quad \frac{\partial T}{\partial \theta} = \nu(C_f - C_i) \frac{\partial g}{\partial \theta}$$

Let

$$F = \exp\left[-\frac{E_o/R}{T_i + \nu(C_f - C_i) g}\right]$$

then the first equation of equation (13) becomes

$$F\{(C_f-C_i)\frac{\partial^2 f}{\partial\xi^2} + \frac{\frac{R}{E_o}\nu(C_f-C_i)^2\frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\xi}}{[\frac{R}{E_o}T_i + \frac{R}{E_o}\nu(C_f-C_i)g]^2}\}$$

$$- [(C_f-C_i)\frac{\partial f}{\partial\theta} - \lambda\nu(C_f-C_i)\frac{\partial g}{\partial\theta}] = 0$$

$$F\{\frac{\partial^2 f}{\partial\xi^2} + \frac{\frac{R}{E_o}\nu(C_f-C_i)\frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\xi}}{[\frac{R}{E_o}T_i + \frac{R}{E_o}\nu(C_f-C_i)g]^2}\} - (\frac{\partial f}{\partial\theta} - \lambda\nu\frac{\partial g}{\partial\theta}) = 0$$

Let

$$A = \frac{E_o}{RT_i}, \quad B = \frac{\nu(C_f-C_i)}{T_i}$$

then

$$F = \exp(-\frac{A}{1+Bg})$$

so we may have

$$F[\frac{\partial^2 f}{\partial\xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\xi}] - (\frac{\partial f}{\partial\theta} - \lambda\nu\frac{\partial g}{\partial\theta}) = 0 \qquad \textbf{(A)}$$

and similarly, the second equation of equation (13) becomes

$$\frac{\partial^2 g}{\partial\xi^2} - \frac{D_o}{D}(\frac{\partial g}{\partial\theta} - \frac{\partial f}{\partial\theta}) = 0$$

Let

$$\frac{D_o}{\mathcal{D}} = u_o$$

then

$$\frac{\partial^2 g}{\partial \xi^2} - u_o\left(\frac{\partial g}{\partial \theta} - \frac{\partial f}{\partial \theta}\right) = 0$$

or

$$\frac{\partial g}{\partial \theta} = \left(\frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2}\right) + \frac{\partial f}{\partial \theta} \qquad \text{(B)}$$

Substituting (A) into (B), we can have

$$\frac{\partial f}{\partial \theta} = \frac{1}{1-\lambda\nu}\left\{\frac{\lambda\nu}{u_o}\frac{\partial^2 g}{\partial \xi^2} + F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right]\right\}$$

$$\text{(17)}$$

$$\frac{\partial g}{\partial \theta} = \frac{1}{1-\lambda\nu}\left\{\frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2} + F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right]\right\}$$

APPENDIX 3

DERIVATION OF EQUATIONS (33)

$$\left[\frac{\partial^2 C}{\partial \xi^2} + \frac{E_o}{RT^2}\left(\frac{\partial C}{\partial \xi}\right)\left(\frac{\partial T}{\partial \xi}\right)\right] \exp\left(-\frac{E_o}{RT}\right) - \left(\frac{\partial C}{\partial \theta} - \lambda \frac{\partial T}{\partial \theta}\right) = 0$$

$$\mathcal{D} \frac{\partial^2 T}{\partial \xi^2} - D_o\left(\frac{\partial T}{\partial \theta} - \nu \frac{\partial C}{\partial \theta}\right) = 0 \qquad (13)$$

The assumed solution for moisture and temperature are

$$C(t) = C_i + \lambda(T_f - T_i) f(\xi,\theta)$$

$$T(t) = T_i + (T_f - T_i) g(\xi,\theta) \qquad (32)$$

and

$$\frac{\partial C}{\partial \xi} = \lambda(T_f - T_i)\frac{\partial f}{\partial \xi}, \quad \frac{\partial^2 C}{\partial \xi^2} = \lambda(T_f - T_i)\frac{\partial^2 f}{\partial \xi^2}$$

$$\frac{\partial T}{\partial \xi} = (T_f - T_i)\frac{\partial g}{\partial \xi}, \quad \frac{\partial^2 T}{\partial \xi^2} = (T_f - T_i)\frac{\partial^2 g}{\partial \xi^2}$$

$$\frac{\partial C}{\partial \theta} = \lambda(T_f - T_i)\frac{\partial f}{\partial \theta}, \quad \frac{\partial T}{\partial \theta} = (T_f - T_i)\frac{\partial g}{\partial \theta}$$

Substituting the above equations into the first equation of equations (13)

$$\left[\lambda(T_f - T_i)\frac{\partial^2 f}{\partial \xi^2} + \frac{E_o}{RT^2}\lambda(T_f - T_i)(T_f - T_i)\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right] \exp\left(-\frac{E_o}{RT}\right)$$

$$- [\lambda(T_f - T_i) \frac{\partial f}{\partial \theta} - \lambda(T_f - T_i) \frac{\partial g}{\partial \theta}] = 0$$

Thus

$$[\frac{\partial^2 f}{\partial \xi^2} + \frac{E_o}{RT^2} (T_f - T_i) \frac{\partial f}{\partial \xi} \frac{\partial g}{\partial \xi}] \exp(-\frac{E_o}{RT}) - [\frac{\partial f}{\partial \theta} - \frac{\partial g}{\partial \theta}] = 0 \qquad \text{(A)}$$

and the second equation of equations (13) becomes

$$\mathcal{D}(T_f - T_i) \frac{\partial^2 g}{\partial \xi^2} - D_o[(T_f - T_i) \frac{\partial g}{\partial \theta} - \lambda\nu(T_f - T_i) \frac{\partial f}{\partial \theta}] = 0$$

$$\mathcal{D} \frac{\partial^2 g}{\partial \xi^2} - D_o[\frac{\partial g}{\partial \theta} - \lambda\nu \frac{\partial f}{\partial \theta}] = 0 \qquad \text{(B)}$$

Let

$$\frac{D_o}{\mathcal{D}} = u_o$$

Equation (B) becomes

$$\frac{\partial^2 g}{\partial \xi^2} - u_o[\frac{\partial g}{\partial \theta} - \lambda\nu \frac{\partial f}{\partial \theta}] = 0$$

or

$$\frac{\partial g}{\partial \theta} = \frac{1}{u_o} \frac{\partial^2 g}{\partial \xi^2} + \lambda\nu \frac{\partial f}{\partial \theta} \qquad \text{(C)}$$

Substituting (C) into (A), equation (A) becomes

$$\frac{\partial f}{\partial \theta} = \frac{1}{1-\lambda\nu}\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{E_o}{RT^2}(T_f - T_i)\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right]\exp\left(-\frac{E_o}{RT}\right) + \frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2} \tag{D}$$

Let

$$F = \exp\left[-\frac{E_o/R}{T_i + (T_f - T_i)g}\right]$$

Then equation (D) becomes

$$\frac{\partial f}{\partial \theta} = \frac{F}{1-\lambda\nu}\left\{\frac{\partial^2 f}{\partial \xi^2} + \frac{\frac{R}{E_o}(T_f - T_i)}{\left[\frac{R}{E_o}T_i + \frac{R}{E_o}(T_f - T_i)g\right]^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right\} + \frac{1}{u}\frac{\partial^2 g}{\partial \xi^2} \tag{E}$$

Let

$$A = \frac{E_o}{RT_i}, \quad B = \frac{T_f - T_i}{T_i}$$

then

$$F = \exp\left(-\frac{A}{1+Bg}\right)$$

and equation (E) becomes

$$\frac{\partial f}{\partial \theta} = \frac{F}{1-\lambda\nu}\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+g)^2}\frac{\partial f}{\partial \xi}\frac{\partial g}{\partial \xi}\right] + \frac{1}{u_o}\frac{\partial^2 g}{\partial \xi^2}$$

Equation (C) becomes

$$\frac{\partial g}{\partial \theta} = \frac{1+\lambda\nu}{u_o} \frac{\partial^2 g}{\partial \xi^2} + \frac{\lambda\nu}{1-\lambda\nu} F\left[\frac{\partial^2 f}{\partial \xi^2} + \frac{AB}{(1+Bg)^2} \frac{\partial f}{\partial \xi} \frac{\partial g}{\partial \xi}\right]$$

APPENDIX 4

DERIVATION OF EQUATIONS (36) AND (37)

From equation (23)

$$m = \frac{V}{\rho} C + M = \omega(\frac{C}{\lambda} - T) + \text{constant}$$

$$\overline{m}(t) = \frac{1}{h} \int_{-h/2}^{h/2} [\frac{\omega}{\lambda} (C-\lambda T) + \text{const}]dz = \frac{1}{2} \int_{-1}^{1} \frac{\omega}{\lambda} \{[C_i + \lambda(T_f-T_i) \times$$

$$\times f(\xi,\theta)] - \lambda[T_i + (T_f-T_i)g(\xi,\theta)]\}d\xi = \frac{1}{2} \int_{-1}^{1} \frac{\omega}{\lambda} \{(C_i-\lambda T_i)$$

$$+ \lambda(T_f-T_i)[f(\xi,\theta) - g(\xi,\theta)]\}d\xi$$

Thus

$$\overline{m}(t) - m_i = \frac{\omega}{2} (T_f-T_i) \int_{-1}^{1} [f(\xi,\theta) - g(\xi,\theta)]d\xi \qquad (36)$$

and

$$m_f - m_i = [\frac{\omega}{\lambda} (C_f-\lambda T_f) + \text{const}] - [\frac{\omega}{\lambda} (C_i-\lambda T_i) + \text{const}]$$

$$= \frac{\omega}{\lambda} [-\lambda T_f - \lambda T_i] \text{ for } C_i = C_f = - \omega(T_f-T_i) \qquad (37)$$

APPENDIX 5

DERIVATION OF EQUATION (58)

Using ξ, θ as defined in equation (12), the boundary conditions in equations (14), (15), $\overline{T}(t)$, m(t) in equation (26), dimensionless moisture uptake, $[\overline{m}(t) - m_i]/(m_f - m_i)$ in equation (27), and

$$m(t) = m_i + \frac{\omega}{\lambda}(C_f - C_i)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)] \tag{A}$$

For $m_i = 0$, $C_i = 0$

$$m(t) = \frac{\omega}{\lambda}(C_f - C_i)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)] = \overline{m}(\infty)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)] \tag{B}$$

and

$$\overline{m}(t) = \overline{m}(\infty) \times \frac{\overline{m}(t) - m_i}{m_f - m_i} \tag{C}$$

and equation (50) can be written as

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}\{[T_i + \nu(C_f - C_i) - \frac{1}{2}\int_{-1}^{1} g(\xi,\theta)d\xi] - [T_i + \nu(C_f - C_i) \times g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p}\{\overline{m}(\infty) \times \frac{\overline{m}(t) - m_i}{m_f - m_i} - \overline{m}(\infty)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\} \tag{58}$$

APPENDIX 6

DERIVATION OF EQUATION (59)

Using equations (57) and (58), equation (55) becomes

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}(\bar{T}-T) + \frac{E\beta}{1-\nu_p}[\bar{m}(t) \text{ percent} - m(t) \text{ percent}]$$

$$+ \frac{12z\alpha E}{h^3(1-\nu_p)} \int_{-h/2}^{h/2} \Delta Tzdz + \frac{12z\beta E}{h^3(1-\nu_p)} \int_{-h/2}^{h/2} \Delta mzdz \qquad \text{(A)}$$

where

$$\frac{E\alpha}{1-\nu_p}(\bar{T}-T) + \frac{E\beta}{1-\nu_p}[\bar{m}(t) \text{ percent} - m(t) \text{ percent}]$$

was given by equation (58) and using ξ, θ as defined before

$$\frac{12z\alpha E}{h^3(1-\nu_p)} \int_{-h/2}^{h/2} \Delta Tzdz = \frac{12\xi\alpha E}{2^3(1-\nu_p)} \int_{-1}^{1} \nu(C_f-C_i)g(\xi,\theta)\xi d\xi$$

$$= \frac{3\xi\alpha E\nu(C_f-C_i)}{2(1-\nu_p)} \int_{-1}^{1} \xi g(\xi,\theta)d\xi \qquad \text{(B)}$$

$$\frac{12z\beta E}{h^3(1-\nu_p)} \int_{-h/2}^{h/2} \Delta mzdz = \frac{12\xi\beta E}{2^3(1-\nu_p)} \int_{-1}^{1} \frac{\omega}{\lambda}(C_f-C_i)[f(\xi,\theta)$$

$$- \lambda\nu g(\xi,\theta)]\xi d\xi = \frac{3\xi\beta E\cdot\omega(C_f-C_i)}{2(1-\nu_p)\lambda}$$

$$\times \int_{-1}^{1} [f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\xi d\xi = \frac{3\xi\beta E\cdot\bar{m}(\infty)}{2(1-\nu_p)}$$

$$\times \int_{-1}^{1} [f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\xi d\xi \qquad \text{(C)}$$

Thus, (A) becomes

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p} \{[T_i + \nu(C_f - C_i) - \frac{1}{2} \int_{-1}^{1} g(\xi,\theta)d\xi]$$

$$- [T_i + \nu(C_f - C_i)g(\xi,\theta)]\} + \frac{E\beta}{1-\nu_p} \{\overline{m}(\infty) \times \frac{\overline{m}(t) - m_i}{m_f - m_i}$$

$$- \overline{m}(\infty)[f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\} + \frac{3\xi\alpha E\nu(C_f - C_i)}{2(1-\nu_p)} \times$$

$$\times \int_{-1}^{1} \xi g(\xi,\theta)d\xi + \frac{3\xi\beta E\overline{m}(\infty)}{2(1-\nu_p)} \int_{-1}^{1} [f(\xi,\theta) - \lambda\nu g(\xi,\theta)]\xi d\xi \qquad (59)$$

APPENDIX 7

DERIVATION OF EQUATION (64)

As in Appendix 6, equation (55) can be written as

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}(\bar{T}-T) + \frac{E\beta}{1-\nu_p}[\bar{m}(t) \text{ percent} - m(t) \text{ percent}]$$

$$+ \frac{12z\alpha E}{h^3(1-\nu_p)}\int_{-h/2}^{h/2} \Delta Tzdz + \frac{12z\beta E}{h^3(1-\nu_p)}\int_{-h/2}^{h/2} \Delta mzdz \qquad \text{(A)}$$

when expressed as a function of ξ, θ

$$\frac{12z\alpha E}{h^3(1-\nu_p)}\int_{-h/2}^{h/2} \Delta Tzdz = \frac{3\alpha E\xi}{2(1-\nu_p)}\int_{-1}^{1} (T_f - T_i)g(\xi,\theta)\xi d\xi$$

$$= \frac{3\alpha E\xi(T_f - T_i)}{2(1-\nu_p)}\int_{-1}^{1} g(\xi,\theta)\xi d\xi \qquad \text{(B)}$$

$$\frac{12z\beta E}{h^3(1-\nu_p)}\int_{-h/2}^{h/2} \Delta mzdz = \frac{3\alpha E\xi}{2(1-\nu_p)}\int_{-1}^{1} \left(\frac{m_f - m_i}{w_i} \times 100\right)[g(\xi,\theta)$$

$$- f(\xi,\theta)]\xi d\xi = \frac{3\alpha E\xi}{2(1-\nu_p)} \times \frac{m_f - m_i}{w_i} \times 100$$

$$\times \int_{-1}^{1} [g(\xi,\theta) - f(\xi,\theta)]\xi d\xi \qquad \text{(C)}$$

Thus, (A) becomes

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p}(\bar{T}-T) + \frac{E\beta}{1-\nu_p}[\bar{m}(t) \text{ percent} - m(t) \text{ percent}]$$

$$+ \frac{3\alpha E\xi(T_f-T_i)}{2(1-\nu_p)} \int_{-1}^{1} g(\xi,\theta)\xi d\xi$$

$$+ \frac{3\beta E\xi}{2(1-\nu_p)} \times \frac{m_f-m_i}{w_i} \times 100 \int_{-1}^{1} [g(\xi,\theta) - f(\xi,\theta)]\xi d\xi \qquad \text{(D)}$$

Using equation (63), (D) becomes

$$\sigma_x = \sigma_y = \frac{E\alpha}{1-\nu_p} \{(T_f-T_i)[\frac{1}{2} \int_{-1}^{1} g(\xi,\theta)d\xi - g(\xi,\theta)]\}$$

$$+ \frac{E\beta}{1-\nu_p} \{(\frac{m_i-m_f}{w_i} \times 100)[g(\xi,\theta) - f(\xi,\theta) - \frac{\overline{m}(t)-m_i}{m_f-m_i}]\}$$

$$+ \frac{3\alpha E\xi(T_f-T_i)}{2(1-\nu_p)} \int_{-1}^{1} g(\xi,\theta)\xi d\xi + \frac{3\beta E\xi}{2(1-\nu_p)} \times \frac{m_f-m_i}{w_i} \times 100$$

$$\times \int_{-1}^{1} [g(\xi,\theta) - f(\xi,\theta)]\xi d\xi \qquad \text{(64)}$$

VITA

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