# An application of classifier systems to the reduction of finite state machines 

Bijan Marjan<br>Lehigh University

Follow this and additional works at: http:// preserve.lehigh.edu/etd

[^0]
## AUTHOR: Marjan, Bijan

## TITLE:

An Application of Classifier Systems to the Reduction of Finite State Machines

## DATE: May 31, 1992

# AN APPLICATION OF CLASSIFIER SYSTEMS TO THE REDUCTION OF FINITE STATE MACHINES 

by

## Bijan Marjan

A Thesis<br>Presented to the Graduate Committee of Lehigh University<br>in Candidacy for the Degree of<br>Master of Science<br>in

- Computer Science

Lehigh University
1992

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Computer Science.

Date: $5 / 12 / 92$

Advisor in charge,
S. David Wu

Eleftrical Engineering and Cormputer Science Department Chairperson,

Kenneth K. Tzeng

## ACKNOWLEDGMENTS

I would like to thank all those who inspired and aided me in completing this paper. First of all;I am grateful to Dr. S. David Wu, my advisor, for having spent much time with me, giving me numerous suggestions and hints on how to proceed. I would also like to thank Rick L. Riolo for having generously provided his software for my usage and allowed me to consult him when I had difficulties. My thanks are also extended to Minho Jo and Mona A. El-Gayar for all the ideas and support they gave me on possible applications. I also owe many thanks to the Lehigh University Interlibrary Loan Service for all the patience they had with me in obtaining materials that I needed. Special thanks to my parents and brother Alessandro for all the support, both financial and moral, they have given me, allowing me to pursue my education. Finally, I would like to thank all those whose names I have not mentioned, but who have greatly helped in writing my thesis.

## TABLE OF CONTENTS

ABSTRACT ..... 1
CHAPTER 1 INTRODUCTION ..... 2
1.0 Adaptive Systems ..... 3
1.1 Classifier Systems Applications ..... 7
1.3 Classifier Systems and other Artificial Intelligence Systems ..... 9
1.4 Classifier Systems and Finite State Machines ..... 11
1.5 Classifier Systems, Finite State Machines, and Communication Protocols ..... 11
CHAPTER 2 CLASSIFIER SYSTEMS ..... 14
2.0 Classifier Systems and Induction ..... 14
2.1 Elements of Classifier Systems ..... 16
2.1.1 Types of Classifier Systems ..... 17
2.1.2 The Rule-Base ..... 20
2.3 Schematas and Mental Models ..... 21
2.3.1 Schemata and Building Blocks ..... 22
2.3.2 Mental Models and Q-Morphisms ..... 24
2.4 The Classifier System Major Cycle ..... 28
2.5 Modifying the Classifier List ..... 32
2.5.1 Credit Assignment: the Bucket Brigade Algorithm ..... 32
2.5.2 Taxes ..... 35
2.6 Genetic Algorithms: The Discovery Method ..... 36
2.6.1 Genetic Algorithms and their Operation ..... 37
2.6.2 Other Operators that Complement the Genetic Algorithm ..... 41
2.6.3 Genetic Algorithms and Appropriate Usage ..... 43
2.7 Parallelism and Emergence ..... 44
2.7.1 Emergence ..... 44
2.7.2 Parallelism ..... 46
2.8 Trade-Off Between Knowledge and Search ..... 47
CHAPTER 3 FINITE STATE MACHINE REDUCTION AND COMMUNICATIONS PROTOCOL APPLICATION ..... 50
3.0 The Finite State Machine World ..... 51
3.0.1 The Markov Process ..... 51
3.0.2 The Leaming Process ..... 54
3.1 The Finite Space Reduction Task ..... 56
;3.1.1 The Reduction Task Approach ..... 56
3:2 The Protocol Environment ..... 57
3.2.1 The Protocol Specifications ..... 58
3.2.2 Reduction of the Protocol ..... 59
CHAPTER 4 EMPIRICAL RESULTS FOR THE FINITE STATE MACHINE REDUCTION TASK ..... 62
4.0 Testing Procedures for the Finite State Machine Task ..... 62
4.0.1 Preliminary Tests ..... 63
4.0.2 Parameter Settings ..... 65
4.0.3 Results with Modified Parameter Settings ..... 67
4.0.4 Selecting the best Parameter Settings ..... 71
4.0.5 Using Different Heuristics ..... 75
4.1 Using the Classifier System Appropriately ..... 77
4.2 Protocol Analysis ..... 79
4.2.1 Protocol Reduction ..... 79
4.2.2 Protocol State Convergence ..... 83
4.2.3 Perturbation and its Effects ..... 84
4.2.4 Rewards ..... 86
CHAPTER 5 CONCLUSION ..... 90
5.0 Results ..... 90
5.1 Communication Protocols Used as the Environment ..... 92
5.2 Classifier Systems Revisited ..... 93
REFERENCES ..... 96
APPENDIX A PROTOCOL ENVIRONMENT SPECIFICATION ..... 104
APPENDIX B THE CLASSIFIER LIST ..... 116
VITA ..... 125

## LIST OF TABLES

Table 4.1 Parameter Settings for the Classifier System ..... 63
Table 4.2 Test Set for FSM ..... 66
Table 4.3 Payoff Values for FSM States at Major Cycle 12800 Using Different Values for mutprop, bid_k; and crowdfac ..... 68
Table 4.4 Deviations from the Mean of Payoff Values for the FSM at Major Cycle 12800 ..... 70
Table 4.5 Best Parameter Settings ..... 71
Table 4.6 Parameter Settings for FSM 1 Problem 1, Including Deviations from the Mean ..... 73
Table 4.7 Results of Discriminatory Analysis for Different Parameter Settings and State Sets from FSM 1 Problem 1 ..... 74
Table 4.8 Results of Discriminatory Analysis for all Tests ..... 75
Table 4.9 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 327] for Payoffs and [0, 24] for Transitions ..... 76
Table 4.10 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 250] for Payoffs and [0, 18] for Transitions ..... 76
Table 4.11 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 150] for Payoffs and [0, 12] for Transitions ..... 77
Table 5.1 Gap Sizes as a Percentage of the Range of Deviation of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 327] for Payoffs and [0,24] for Transitions ..... 91
Table 5.2 Gap Sizes'as a Percentage of the Range of Deviation of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0,250] for Payoffs and [0, 18] for Transitions ..... 91
Table 5.3 Gap Sizes as a Percentage of the Range of Deviation
of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 150] for Payoffs and [0, 12] for Transitions ..... 92
Table A. 1 Data-Link Layer State Descriptions ..... 105

## LIST OF FIGURES

Figure 1.1: An Adaptive System ..... 3
Figure 1.2: Amount of epitasis for different search and optimization techniques ..... 6
Figure 1.3 A Finite State Machine Representation of a
Figure 2.1: A Classifier System ..... 18
Figure 2.2: A Classifier System with Long-Term Memory ..... 19
Figure 2.3: A transition function which detects the input from the environment and outputs some behavior that affects the environment. ..... 25
Figure 2.4: A transition function which detects the input from the environment and and produces the approximate internal category ..... 26
Figure 2.5: A Q-morphism showing the two levels of abstraction that are used to model the environment, using function $P$ ..... 27
Figure 2.6: The Hertzian Model ..... 28
Figure 2.7 The Genetic Algorithm Behavior Mechanism ..... 38
Figure 2.8 Genetic Algorithms in Classifier Systems ..... 39
Figure 2.9: Trade-off between Knowledge (Rules) and Search (Tasks) ..... 49
Figure 3.1 A Simple Finite State Machine Defining a Markov Process ..... 53
Figure 3.2 The Open Systems Interconnection ..... 58
Figure 3.3 Reduction of a Communications Protocol ..... 60
Figure 4.1 Final Payoff Values for a Finite State Machine Using Five Different Initial Payoff Values ..... 64
Figure 4.2 Final Payoff Values for a Finite State Machine Using Five Different Initial Payoff Values ..... 65
Figure 4.3 Payoff Values at Major Cycle 12,800 Using Eight Different Parameter Setting Combinations ..... 67.
Figure 4.4 Deviations from the Mean of Payoff Values for FSM State 12 ..... 69
Figure 4.5 Deviations from the Mean of Payoff Values for FSM State 15 ..... 69
Figure 4.6 Payoff Values at Major Cycle 12,800 for FSM 1 with Payoff Set 1 ..... 72
Figure 4.7 Payoff Values at Major Cycle 12,800 for FSM 1 with Payoff Set 1 ..... 72
Figure 4.8 Convergence of State Payoff Values Using a Threshold of 1000 ..... 78
Figure 4.9 Payoff Values at Major Cycle 12,800 using the Communications Protocol ..... 80
Figure 4.10 Number of Classifier Schemata in the Classifier List Supporting Each State at Cycle Step 1000 ..... 81
Figure 4.11 Number of Classifier Schemata in the Classifier List Supporting Each State at Cycle Step 12,000 Superimposed with the Payoff for those States at the Same Time ..... 82
Figure 4.12 Flow of Strength in Classifier List ..... 82
Figure 4.13 Convergence of Payoff Values for Protocol Using Three Different Heuristics ..... 83
Figure 4.14 Increase in Payoff Values for Protocol ..... 84
Figure 4.15 Comparison of Fitness of two Classifiers Supporting States 2 and 6.1 with no Perturbation ..... 85
Figure 4.16 Comparison of Fitness of Classifiers
Supporting States 2 and 6.1 in Presence of Perturbation ..... 86
Figure 4.17 Flow of Strength in Classifier List with no Rewards Received ..... 87
Figure 4.18 Difference in Fitness Between a Strategy which Shared Rewards Among all Active Classifiers and One that Does Not ..... 88

## ABSTRACT

Classifier systems are massively parallel, rule-based adaptive systems that use genetic algorithms for the generation of new rules and the production of dynamic behavior. Classifier systems, which are now emerging as new computational models of learning, use intermittent feedback from the environment as a guide to future behavior generation. One way in which this environment can be modeled is via Finite State Machines (FSMs). FSMs can properly represent an environment since they are characterized by a number of states and transitions among those states which designate the condition of the environment at any given time: FSMs are an ideal tool for representing a multitude of synthetic environments such as communication protocols. One problem an FSM often encounters is its size, since the computation involved in evaluating an FSM is directly affected by its size. In this thesis, we proposed a method which uses classifier systems to reduce and refine a given FSM. The classifier learns the relevance of each state and gives rewards to the states based on the frequency of visitation. At the end of the computation, the classifier system identifies a specific subset of states that demonstrates the highest level of relevancy. In this research, a number of FSMs tested using various parameter settings in the classifier system. A specific FSM of a communications protocol was used to demonstrate the efficacy of this approach.

## CHAPTER 1

## INTRODUCTION

In the recent history of artificial intelligence (AI), new methods have emerged that allow machines to operate on the basis of cognitive models. Much effort has been placed in creating intelligent systems that can leam based on the environment with which they interact. Such systems have to be able to analyze large knowledge bases and evaluate the significance of the residing information therein. Connectionist systems have been very popular in this area for a number of years. However, they are now flanked with a new computational model that is also an isomorphic paradigm to connectionist systems. Classifier systems, which like connectionist systems are used to classify real-world data in some fashion, are rising as a new powerful tool in the cognitive arena. Classifier systems are rulesbased, massively parallel, message-passing systems that learn via intermittent environmental feedback. They constitute a bridge that links not only researchers in artificial intelligence, but also those in parallel distributed processing, cognitive science, machine leaming, and artificial life (AL). This last category is really a combination of all the previous ones since it introduces the concept of adaptive nonlinear networks (ANNs). John Holland introduced this category which incorporates classifier systems, connectionist systems (cognitive psychology), immune system, economic systems, ecologies, genetic systems, and so on. In addition, a number of other AL research programs have recently emerged that attempt to produce the natural living system in artificial forms. These have extensive simulation capabilities and include computer viruses, evolving computer processes, biomorphs and ontogenetically realistic processes, robotics, autocatalytic networks, cellular automata, artificial nucleotides, cultural evolution, and evolutionary reinforcement learning (ERL) [1]. A good description of these is given by Taylor [72].

### 1.0 Adaptive Systems

In order for computer systems to be able to produce intelligent behavior, they must be capable of interacting with the environment and learning from it. This is a continuous knowledge acquisition task that we as human beings always indulge in. Adaptive systems gain knowledge via environmental feedback. Figure 1.1 shows how an adaptive system might look like [33]. Note that the system gathers input from the environment, processes


The scene shown is classified as $C^{+}$because $\sum_{i=1}^{16} w_{1} \delta_{2}(t)=\delta_{1}(t)+2 \delta_{2}(t)+2 \delta_{5}(t)$ $+\delta_{4}(t)+2 \delta_{5}(t)+4 \delta_{6}(t)+\cdots>4$. where $\delta$ is a weight given to each cell in the sensor array.

Figure 1.1: An Adaptive System
From: Holland, John H. Adaption in Natural and Artificial Systems. Ann Arbor, MI: The University of Michigan Press (1975)
the input (in this case represented by a summation function), and then generates output. This adaptive system resembles a classifier system. The sensor array would consist of a number
of classifiers that have weights assigned to them called fitnesses or strength. These values designate the importance of a classifier in the environment being probed. This mechanism provides a leaming mechanism since the classifiers are constantly modified given the input that is received from the environment. Any change in the environment should be reflected by a change in the behavior of the classifier system.

Classifier systems are ANN systems. To fit this category, certain criteria have to be met, as Holland requires [37]. First of all, an ANN system exhibits hierarchical organization. This implies that building blocks exist that combine intelligent units at lower levels to produce intelligent units at higher levels. Such behavior can be found in all living systems (as seen with meiosis in biological systems). In classifier systems, classifiers are built via a process that begins with the establishment of certain building blocks called schemata and the successive refinement of them in time. A second characteristic of ANN systems is that a certain level of competition exists that drives the formation of schemata and the eventual formation of system behavior. The fitness value is the commanding element in such competition. Third, the environment with which the system interacts provides incentives for the classifier system to produce desirable outputs. This is done via the assignment of payoff values or system rewards. A fourth criteria is that of the exploitation of environmental regularities. In other words, the environment produces behavior that is often used by a system as a basis for its own behavior. A classifier system that represents a robot probing some environment would, hence, use light as its tool in its examination. This restraint causes the system to function at non-optimal levels since the robot could well be using other, more efficient means of driving its behavior. It is therefore difficult to exploit all possible environmental regularities for the generating system behavior. Fifth, there is a tradeoff between exploitation and exploration. To explore an entire search space, such as the robot trying to find which direction to move in, is very expensive since it requires much time. Exploitation, however, may not consider all desirable movements but focus on a single one and would, therefore, produce
less than optimal behavior. Hence, a trade-off exists between long searches but higher levels of confidence and brief searches and lower levels of confidence.

A sixth factor is denoted by a tradeoff between tracking and averaging. The environment may generate behavior that changes too fast in accordance to the systems behavior, therefore the environmental data has to be averaged in time to gather qualitative information. But if the behavior is synchronous with the system behavior, then the data can be tracked. A seventh momentous characteristic of ANN systems resides in the nonlinear feature. In a natural system, such as a genetic system, which resembles classifier systems, genes depend on other genes during the building block process [15]. In genetic terms, a - A phenotype, which would be the resulting organism from the interaction of genes, emerges. But genes tend to inhibit other genes from acting in given ways or activate them at other times. This creates the condition called epitasis, where one gene affects the behavior of another one. The genes, interact together to produce some higher level organization (a new organism). But due to epitasis, it is not possible to determine from the genes alone the constituency of the higher level organism (via the schemata building block). This factor introduces nonlinearity in the system. In classifier system, this effect is produces via the search technique çalled genetic algorithms. The amount of nonlinearity present in the functioning of genetic algorithms is important to control since too much epitasis can lead to very complex schemata that are hard to understand while too little epitasis does not provide an adequate building block mechanism. Davidor [15] compares genetic algorithms in comparison to other search techniques and identifies how much epitasis each method should have, as displayed in Figure 1.2. Note that chaotic behavior of the environment is also an endogenous factor, beyond linear estimation. The functioning of the classifier system depends on the environment and how the environment interacts with the system. If the environment displays nonlinear behavior, where it acts in bursts rather than in a deterministic fashion, then the classifier system's behavior will also be additionally unpredictable.


Figure 1.2: Amount of epitasis for different search and optimization techniques From: Davidor, Yuval. Genetic Algorithms and Robotics: A Heuristic Strategy for Optimization. Teaneck, NJ: World Scientific (1991)

An eight facet to ANNs is coupling. This is somewhat related to hierarchical organization since the combination of classifier constituents gives rise to different classifiers. Two classifiers can be coupled when the action of one satisfies the condition of another one, resulting in the organization or coupling of classifiers. Ninth, there can be certain building blocks of a general nature (generalists), those upon which many classifiers can be built, and there can be specialists, restricting themselves to a small pool of classifiers. Tenth, the cooperation of different building blocks in producing system behavior within a certain environment can lead to more flexible and complete system behavior. The eleventh point concems the construction of internal models. This puts ANNs in the perspective of cognitive models, where systems learn from the environment. Internal models provide a better understanding of how the system adapts and develops. They define what the system is getting from the environment, what is it doing to the input, and what output does it produce in any given . time step. Such topic has also been examined in the neural network area [20]. Finally, ANN systems exhibit a growth in entropy [15]. As building blocks are created within a system, that system has to use more resources in order to continue its building process. In this fashion, the amount of (finite) resources increase as the system complexity increases. Entropy puts a ceiling on how much a system can evolve due to limited resources. Note that this is recur-
ring problem in the design of information systems, where sometimes the amount of information to be processed is onerous. In cases where the amount of data becomes unmanageable, the haystack syndrome results, where it becomes difficult to determine what data is really useful.

Once the above requirements are met, the adaptive system can be created. Genetic algorithms provide a useful means to manipulating and adapting classifier systems.. The mechanisms employed by genetic algorithms focus on chromosomes, which are the structures that encode the traits of living beings. These beings take shape given the decoded chromosomes. Genetic algorithms have four key features. First of all, the chromosomes are the means by which we evolve and therefore any adaption that takes place directly affects these structures. Second, evolution via natural selection is based on the strength of the chromosomes (their ability to survive in time). This happens because certain chromosome "blue-prints" (or schemata) are better fit for the environment being tested. Third, during reproduction, new offsprings evolve that-are hopefully more fit than their parents (in the given environment). Fourth, since biological evolution has no memory, the certain chromosome blue-prints will be forgotten and surpassed in time [16].

### 1.1 Classifier Systems Applications

Classifier systems represent a computational and cognitive model that can be easily applied to a great number of domains. This is done by changing the environmental interface of the classifier system to support new applications. The application used in this thesis used a package that provides a facilitated construction of a specific domain.[56].

Classifier systems have been employed in a number of domains. Theese include strategy acquisition [30], learning models of consumer behavior [27], discovering scheduling heuristics [32], leaming sequential decision rules [29], detecting events in syn-
thetic images [47], recognizing letters [22], negotiating [43], natural language processing [3] and various others, some of which are explained very well by Goldberg [23].

A number of different systems have evolved for developing classifier system applications. They are based on two programmatic approaches to classifier systems [17]. One of them encapsulates all the rules in the classifier system within one data structure which is the "Pitt" approach, after the University of Pittsburgh, indoctrinated by DeJong, while the other one takes a less holistic approach where each rule is a separate data structure. This latter approach is characteristic of classifier systems of the "Michigan" type, after the University of Michigan, designed by Holland.

Among the different systems used in implementing classifier systems we find the LS-1 (Learning System-1) family of programs [64] [65] the ARGOT system [63], the GOFER family of systems [8], the classifier system with long-term memory (CSM) [76], the KL-ONE family of classifier systems [21] which use LISP as the programming language, the $C S-1$ (Cognitive System One) program [34] which was the first classifier system, the ANIMAT system [73], the BOOLE system 4 [74], and CFS-C [56], which is used in this thesis and is written in C. CFS-C also uses the "Michigan" approach to classifier systems.

Although various programs have been developed for classifier systems, there still rests much work to be done in the area. Currently provided are the basic classifier system computational mechanisms, which are not yet capable of handling commercial-scale problems. A number of refinement processes for classifier architectures and genetic algorithms are still necessary. In addition, user interfaces are primitive and have to be enhanced.

Programming in classifier systems has been done in a number of languages. The ideal language is one that does not necessarily provide symbolic processing (as in the case of Prolog or OPS5), but one that allows good numerical computations. CFS-C has been designed in C , which provides sufficient computational power.

Classifier systems are well suited for implementations on parallel machines.

One such implementation was created with the CFS-C package on à Connection Machine [60]. A number of other systems have been designed for parallel implementations [14] [50] [61] [39] [71] [24]. Most of these incorporate the parallelism inherent in genetic algorithms.

### 1.3 Classifier Systems and other Artificial Intelligence Systems

Classifier systems would seem to have a number of disadvantages at first glance. First of all, the rule-base seems to consist of a great number of rules encoded in binary format ( 0 's and 1 's), giving classifier systems very little descriptive power. In addition, classifier systems support parallelism where many rules can be active simultaneously. There is a problem in the management of rule activations in such a case since somehow the rules that should be activated must be found and selected in some qualitative fashion [10]. However, the classifier system language allows one to build representations using the binary encodings. In fact, schemata (or building blocks or blue-prints) permit the creation of hierarchical associations among rules. A good example of this is given by Holland and Booker ${ }^{1}$ where networks are build via associations among different rules. Another description of high-level hierarchical associations is given by Antonisse [2]. Rule clustering is also becoming of great relevance in classifier systems. Such clustering is useful especially when the system attempts to solve problems via associations or analogy [76]. In such case, rules that are similar in some ways are recalled in attempting to solve the problem at hand. Another concept is that of rule corporations [74]. This idea involves having rule clusters cooperate together in solving problems (just like corporate collusion).

Rule clustering and combination can also be represented via the disjunction of complementary rules. Booker [11] also argues that using complementary feature manifolds, where a chromosome can be used to represent one value, such as the color blue or red

1. This is done via the tags assigned to rules (which will be discussed later). Holland and Booker give a good description in [10] on page 249
or green, a number of disjunctive generalizations can be obtained. Thus, for the three colors above from which the other colors of the spectrum can be obtained, a generalization can be made using disjunctive operators that could create a new color. Thus, blue and red may produce purple as a result (this would also have its own chromosome 'encoding different from all others).

In addition to being able to produce qualitative representations, classifier system provide parallelism which is crucial in building combinations of rules that would otherwise be impossible to configure in a serial system. Specifically, the firing of rules simultaneously greatly enriches the pool of facts (or messages in classifier systems) that exist in working memory and that can be accessed for the matching of classifier conditions.

Classifier systems use sub-symbolic processing, similar to connectionist systems. In such models of computation, each "unit of intelligence" is in itself not very meaningful [21]. For example, a single synapse in a neural network or a classifier in a classifier system by themselves do not have much informative power. But the combinations and interactions of these units at low-level produces very complex high-level representations, as supported by the schemata mechanism in classifier systems. On the other hand, knowledge can be represented in rule-base format which is more easily interpretable and accessible to investigation that the hidden layers of the connectionist systems. Classifier systems, hence, provide a middle ground to connectionist systems and traditional symbolic processing systems such as expert systems.

Furthermore, the schemata mechanism of genetic algorithms has been used to provide a better understanding of sub-symbolic processing in connectionist systems. Via the schemata mechanisms, hierarchies are built to represent different symbols. An excellent example of the is given by Dolan [19]. Indeed, understanding neural network hidden layers and what those layers represent (what is the network learning at any given time) can be quite complicated [20].

Antonisse and Keller [2] provide yet another example of how classifier systems can help build high-level representations via the schemata processing. They build a production system for building trees representing hierarchies.

### 1.4 Classifier Systems and Finite State Machines

A Finite State Machine (FSM) can be used to model a number of domains with classifier systems. The reasgn for this is that the environment with which the classifier system intermittently transacts can be represented as a state space system. Each state within the FSM can represent a state the environment can be in at a given time. Once the classifier system receives input from the environment, processes it, and outputs its behavior, the environment's state changes. FSM, hence, make the problem of representation easier. The classifier system is a computer program with no inherent cognitive traits. In order for it to be able to process the information it receives, it must know how the environment "looks like."

The use of the FSM in classifier systems is not a unique concoction. In fact, FSMs are the main design element of cellular automata. They have also been used in a number of new artificial life systems, underlining the fact that FSMs are an appropriate cognitive model, capable of representing environmental states in a simple while complete fashion. They are also used as a main counter-example of artificial neural networks. ${ }^{2}$

### 1.5 Classifier Systems, Finite State Machines, and Communication Protocols

It has been shown that classifier systems are an important tool in developing problem solving systems that can successfully handle tasks within a particular domain. One
2. For a good sampling of Finite State Machines and their use in intelligent systems, the Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity volume can be a good guide.
such area is that of protocol design. A protocol defines the functions of a communication system. Specifically, a protocol delineates all the rules and conventions that are used in communication among various systems that are connected to a network.

Communication protocols are specified as finite state machines, or protocol machines [25], and incorporate the various states that the communications system may be in at any given time. The FSM concept is especially appropriate in protocol specifications since a communications network consists of a number of input and output sequences that change the state of the network. Hence, at one point of communication between two nodes (such as two computer terminals, using Data Connecting Equipment (DCE)) in a network, one node may be sending a message or it may be waiting for an acknowledgement for a message that was sent to another node. An example of such a state diagram is seen in Figure 1.3, as defined by Bochmann [7]:

Very often the number of states specified in a protocol machine can be superfluous or-redundant. In-such a-case, inefficiencies are created in communication since the protocol system has to transit to additional states when this not really necessary. To avoid such a plethora of states, a method has to be devised to analyze whether all the states are necessary and whether any of the states can be eliminated, while not comprising the functioning or efficiency of the protocol. The classifier system in this paper analyzes a protocol that defines the Integrated Services Digital Network link-access protocol on D-channel (LAPD), which is defined in the data-link layer of the Open System Interconnection (OSI) Reference Model of the International Organization for Standardization (ISO). In doing so, it learns which states appear to be bottlenecks within the protocol specification. In addition, to support empirical results, some randomly generated finite states machines are produced and tested to check the efficacy of the approach taken.

This thesis intends to support the concept of system reliability optimization. To produce systems that are reliable and perform well, redundancies have to be eliminated


Figure 1.3 A Finite State Machine Representation of a Communications Protocol
From: Bochmann, Gregor V. "A General Transition Model for Protocols and Communication Service" in IEEE Transactions on Communications, vol. COM-28, no. 4, (April 1980)
or highly sophisticated and reliable components have to be manufactured to support the system. It seems far less expensive to take the former approach, minimizing costs in doing so [18]. The approach taken here is of optimizing protocol reliability.

## CHAPTER 2

## CLASSIFIER SYSTEMS

Classifier systems are dynamic, rule-based systems that incorporate mechanisms that permit learning via environmental feedback. Such a system is based on inferential learning aided by continuous interaction with the environment. By inference here we mean the capability of deriving knowledge from very broad concepts and applying them to specific situations. Learning in classifier systems is provided by credit assignment and discovery algorithms, which modify the repository-of information at any given time and produce an updated knowledge base at every successive time step that is more adept at the problem being solved. The learning process follows a model of evolution whereby each knowledge "unit" or rule competes for survival within the knowledge base using a "fitmess" value as its leading survival indicator. In doing so, the system develops an internal model reflecting what the environment looks like.

Classifier systems, in addition to being a good computational model of cognition, are also a good example of parallel architectures. This allows the knowledge search to be performed in a more efficient and accelerated fashion. Parallelism also supports the framework of emergent computation, where "units" of intelligence compete with each other to produce knowledge about the environment.

### 2.0 Classifier Systems and Induction

Classifier systems are examples of inductive systems, where learning occurs as a result of exposure to a given environment. An inductive system is defined in these terms, as explained by Holland et al. [36]:

1) contains a knowledge base that encodes environmental data; each knowledge rule (classifiers in classifier systems) can be represented in terms of conditions and actions resulting from those actions. Therefore, the system is capable of perceiving a certain event and reacts to that event in an appropriate fashion (system behayior). Note that chaining of actions is allowed here where the action of one condition-action rule leads to the fulfillment of the condition of another rule;
2) such rules encode a relation between the current state of the system (diachronic relation) and a modification of categories and their associations (synchronic relation) where diachronic relations are used in the search for new rules and synchronic relations permit their modification as a result of the learning process. Categories here represent certain common features encoded by the rules in the knowledge base;
3) a building block mechanism whereby categories can be refined and specialized. This occurs only after sufficient interactions with the environment;
4) the construction of default hierarchies, which represent the first state of any building block and that are refined given relations among various categories within the knowledge base and the environmental transactions;
5) an emergent model based on a number of diachronic and synchronic rules that permit system behavior (i.e., the reinforcement of information in the knowledge base);
6) the existence of parallelism, by which a number of rules are active at the same time in the system (their conditions are satisfied) and where these rules complement each other by providing further refinement of the categories they support;
7) a mechanism for modifying rule strength (credit assignment algorithm) and a mechanism for generating new rules given those strengths (discovery algorithm);
8) new rules are generated given their past failures in supporting a given category;
9) the inductive system must learn about any changes that occur in the environment and adjust to them appropriately, by altering the knowledge base.

As it will be shown, classifier systems comply to the specifications delineated by inductive systems, permitting dynamic behavior as a result of its interactions with the environment and successive modification of its knowledge base.

### 2.1 Elements of Classifier Systems

A classifier system is a rule-based system that interfaces with the outside world. In doing so, it learns from the environment and generates appropriate behavior. In expert system language, a classifier system is made of condition/action rules, where conditions are matched by messages detected from the environment and actions are fired as a consequence. The classifier system consists of the following element:

1) an input interface that extracts information from the environment and encodes it into binary form, the classifier-system operating language. These-binary strings are-called chromosomes for their use in the genetic algorithm (the discovery algorithm used by classifier systems). These chromosomes are called detector messages since they detect the current state of the environment.
2) a rule-base or classifier-list consisting of the condition/action rules. The number of classifiers in the rule-base is called the population, once more in genetic algorithm terminology.
3) a message list, much like the agenda of an expert system, that contains the detector messages that have been taken from the input interface. The message list is updated constantly to represent the state of the environment. Note that the classifier system is capable of performing real-time operations thanks to its environmental interface.
4) an output interface that generates system behavior. The output is also a chromosome, called effector, which is then decoded to affect the environment in some way. For example, if in a packet-switched networking system a certain data packet is not received at a given
time, the classifier system must learn to request re-transmission of the data packet using its output interface.

### 2.1.1 Types of Classifier Systems

The rule-base of the classifier system consists of short-term memory of facts that lasts as long as the classifier system is operating to solve-a-certain task. Once the task is accomplished, or leaming has occurred, this volatile memory is shut down. In order to prevent such loss, a classifier system with long-term memory has been developed [77] that allows old rules to be recalled. In order to do so, the classifier system matches its detector messages with rules residing in this long-term memory and builds a population of classifiers from those rules. This type of association can be referred to as learning by analogy, since the classifier system retrieves facts from previous experiences and uses them for current problems. Note also that this noticeably improves system performance since the system does not need to reconstruct those same rules again. Once the system has accomplished its tasks, it saves those rules in long-term memory in order to fetch them at some future time.

Figure 2.1 shows a classifier system with short-term memory. Note that whatever resides in the classifier list is then lost when the task is accomplished. The input interface gathers data from the environment in the form of a message which it places on the message list. The message list is also updated with any other messages that are fired from the classifier population. When the classifiers generate effector messages, these messages are delivered via the output mechanism of the system, generating system behavior.

Figure 2.2 shows a classifier system with long-term memory which is initialized with a classifier population stored from previous runs of the system. Associations are made between what is in the message list and what resorts in the long-term memory via the matcher. Specifically, as soon as the classifier system receives some initial environmental


Figure 2.1: A Classifier System
data from the input mechanism, it compares the data to the rules in the long-term memory, which are grouped into clusters related to the different problem-solving tasks dealt with in the past, and initializes a population consisting of all the rules in that cluster. If the number of rules in that cluster is not as big as the limit of the classifier list, additional rules are generated randomly. The remaining operations are similar to the short-term memory classifier


Figure 2.2: A Classifier System with Long-Term Memory
system.
The classifier system, like any other adaptive system, is supposed to be a dynamic system which changes over time given its interaction with the "outside world" and improves its performance as it learns more about it [17]. The adaptive mechanism is an encapsulated "black box" to which access is limited and all interfacing is done via a special
broadcast or interface language. In addition, in order to improve performance, it has to be able to extract new environmental data into a format that it can understand (which is called schema). The formation of such abstract forms is an ongoing process since the environment is rarely in ${ }^{\circ}$ a static state, changing in time as a result of its interaction with the classifier system or other source.

### 2.1.2 The Rule-Base

All rules within a classifier system are composed of bits ( 1,0 ) and a don't care symbol (\#). The condition and action strings of a rule are composed of a sequence of such symbols. The syntax used in can be written in Bakus-Naur Form (BNF) [21]:

```
<classifier system> ::= <classifier>
<classifier> ::=<condition><condition> \(=>\) <action>
\(<\) condition> \(::=<\) alphabet \(_{n} \mid \sim\) alphabet \(>_{n}\)
<action> ::=<alphabet> \({ }_{\mathrm{n}}\)
<alphabet> ::=110|\#.
```

Note here that we have used two condition strings which is the default number used by the software library [56] used in this thesis and have set $n$ to 16 . The tilda in the condition statement indicates that if no other strings matches that condition, then the rule is fired, or accepted, given that the first condition is also satisfied.

Therefore, a condition or action may look like 00101010101010110 or ~1010100001010101010 or \#100\#\#11010\#11111. The latter case is that of a generalist message, which will match many different messages (approximately $4^{3}$ messages) since there are 4 don't cares. The former messages, on the other hand, are specialists and will match only
one other message, genotypically identical to them.
There are four different tags associated with the messages. A message ${ }^{3}$ that starts with " 00 " in its left-most bits designates a detector message. Therefore, such a message will only match messages from the input interface. Messages that start with " 11 " or " 01 " are used only within the classifier list. Finally, those messages that start with a " 10 " in their left-most bits represent effector messages. For example, when a classifier fires an action with " 11 " or " 01 " in it, that action will be matched only by the condition of another classifier. If that action has " 00 " in its left-most bits, then it will match messages on the message-list. And if the action has " 10 " in left-most bits, then it will match effectors. In fact, there are different effectors that can be defined, each defining the system behavior that is desired. This will be discussed later on.

Tags are also used to produce networks of activations of classifiers. For example, a tag like 1001, embedded in the action of a classifier, can designate an action to be takenby the effectormechanism:But that action,-when matched by aconditioncangenerate another action with a different tag which would only occur if the previous rule had fired an action. In this fashion, a hierarchy of classifiers is formed giving the classifier system a knowledge structure that not only provides maneuverability of the knowledge and how it should be used, but also flexibility in how to use that knowledge. A more accurate account of networks is given by Booker et al. [10].

### 2.3 Schematas and Mental Models

A schema is the classifier mechanism for encoding categories. Schemata are the building blocks of the system. From the schema, the classifier system builds an internal
3. Note that terms such as message, chromosome, bit string are used interchangeably since they are standard classifier system terminology.
representation of the system. In other words, the schemata define the most relevant characteristics of the environment that are being considered. The schemata constitute a part of the conditions or actions of a classifier. For example, if one of the classifiers in the rule-base looks like the following: 00110000\#1\#10101 ~1010101\#01\#0001111 $\rightarrow$ \#\#100\#\#\#0101\#\#\#\#\#\#, then the schemata (in boldface) designate those attributes that are significant in identifying a particular concept. Note also that the pound signs ("\#") here serves as pass-through symbols since the message that the action of this rule will match will satisfy the message regardless of the values in those positions.

### 2.3.1 Schemata and Building Blocks

Each of the bits in a chromosome can represent a certain concept. For example, in a system that diagnoses communication network problems and a string is posted in the classifier system-such as 0000000000000001 , the right-most bit (turned on, or set to 1 ) may indicate that the last data packet in the a series has not been received. But one may need to capture more data once a problem has been detected. In such a case, a number of bits can be used to represent such a concept. For example, in the bit string 000000000001010 , the sequence 1010 can represent the network state network load high and rising, where bit 2 could represent rising load and bit 4 could represent load high. Note that since the alphabet is also composed of a third symbol, the 1010 sequence of bits is actually only one in a set of four distinct possibilities (\{1010, 1110, 1011, 1111\}). This schema, 1\#1\#, permits the classifier system to represent many different concepts having a similar building block. Therefore, the network example could be expanded to represent network load high, rising, and breakdown point reached, using the schema 1110 , for example. A schema can be of any length and appear anywhere in the chromosome. In the above example, the schema can be represented as $* * * * * * * * * * * * 1$ \#1\# since the other bits are not part of it. Therefore, in more
generic terms, a schema can be in the set $\left\{1,0, \#, *^{n}\right.$, where $n$ is the length of the string (set to 16 in this application). Radcliffe [52] uses this more generic definition of schemata theory to develop his formae.

This phenomena, where an $n$-bit string is an instance of $2^{n}$ schemata, has been called implicit parallelism. In a sense, the schemata and all the strings that can be derived from them, are active at the same time. This gives greater computational power to the classifier system and a greater number of concept formation strategies.

A schema, then, serves as a building block for the classifier system. Holland, however, suggests that there are various obstacles to adaption related to schemata that have to be overcome if learning is to be efficient [33]. First of all, the cardinality of the various schemata is very high. There can be a great number of such structures and the search for the most fit can be non-exhaustive. Second, due to the fact that biological evolution has no memory, the apportionment of credit may occur on the basis of incomplete knowledge, based solely on the information currently in the classifier list. Third, the strength (fitness) of the classifiers have high dimensionalities, removing much influence from the other variables that are at the basis of the optimization process provided by the genetic algorithm. The strength measure follows a nonlinear (chaotic) behavior due to epitasis, causing the optimization process to be locked in suboptimal areas of the total search space. Fifth, search and exploitation are not mutually exclusive. the search for new structures interferes with the generation of above-average schemata. Finally, there is more information than payoff values that the environment provides that has to be taken into consideration. This is especially true with such parameters as system rewards. Holland also explains that such problems can be overcome by progressively exploiting the best schemata, ensuring that the schemata being generated are the best ones, and testing a large number of new schemata in order to ameliorate the chances of exploiting the entire search space.

### 2.3.2 Mental Models and Q-Morphisms

Mental models are used to emulate the structures by which we represent the world in our brains. These structures, categories, or schemata are certainly not unique to classifier systems and genetic algorithms. Indeed, Roger Schank and Marvin Minsky developed most of the concepts related to such models. Schank developed the concept of scripts and later on Memory Organization Packages (MOPs) while Minsky improvised frames [62] [45].

Schemata and similar structures permit adaptive systems to gather environmental data from the outside world, decode them, and represent them in internal format. This is what the detectors do in a classifier system on a continual basis since the environment keeps on changing with time. Once a classifier system has detected the information from the environment, performed its classification, and output its behavior, the environment has clearly changed its state. A mathematical model of such state transition is given by Holland et al. [36]. This is composed of a transition function $T[\mathrm{~S}(t), \mathrm{O}(t)$ ], where $T$ is the transition function, $\mathrm{S}(t)$ is the current state of the environment at time $t$, and $\mathrm{O}(t)$ is the output of the classifier system at that time step.

Once the time step is concluded, the transition will be $T[\mathrm{~S}(t+1), \mathrm{O}(t+1)]$. This transformation process is represented in Figure 2.3. Note that information about the outside world is mapped into the classifier system at any given time step.

Since the environment is highly complex and difficult to convert into an internal state using an isomorphic function, only approximations can be made. This is done via a homomorphism. In doing so, the most relevant characteristics of the environment are extracted and mapped into the input interface of the classifier system. Recall that in the classifier system being used here, these characteristics are mapped into 16-bit messages. A better


Figure 2.3: A transition function which detects the input from the environment and outputs some behavior that affects the environment.

From: Holland, John H., Holyoak, Keith J., Nisbett, Richard E., and Thagard, Paul R. Induction: Processes of Inference, Learning, and Discovery. Cambridge, MA: Massachusetts Institute of Technology Press (1986)
representation of this is given in Figure 2.4 Note that for the different characteristics, certain schemata are formed.

The schemata are the initial building blocks of the classifier systems. They constitute default hierarchies from which additional refinements can be made to fully represent the state of the environment. In order to do so, the current mathematical model has to be expanded to take into consideration the default hierarchies and the models that can be derived from them. Figure 2.5 displays a quasi-morphism, or $Q$-morphism, as defined by Holland et al. [36] which not only brings the previous two figures into one, but also models the internal state into two levels. Level 1 represents the default hierarchy while the Level 2 represents a specific instance of that hierarchy. Therefore, a condition within one rule in the classifier system could undergo the following changes (the schemata is in boldface):

## Time $t$

condition (Level 1): 01010\#1\#00\#01010
condition (Level 2): 0101011100101010

Time $t+1$
01010\#0\#00\#01010
0101000100101010


Figure 2.4: A transition function which detects the input from the environment and and produces the approximate internal category
Adapted from: Holland, John H., Holyoak, Keith J., Nisbett, Richard E., and Thagard,
Paul R. Induction: Processes of Inference, Learning, and Discovery. Cambridge, MA: Massachusetts Institute of Technology Press (1986)

Note that pound signs have been replaced by bits, indicating that a specific concept has been formed.

Accordingly, mental models undergo a continuous process of alteration. It is via this mathematical process that learning takes place and refinement occurs. Schank [61] argues that learning occurs thanks to the iteration of interfaces with the environment. And this is what occurs in a classifier system since the more instances of a schema that are encountered, the stronger and less abstract that schemata becomes, representing the current state of the environment at any given time.


Figure 2.5: A Q-morphism showing the two levels of abstraction that are used to model the environment, using function $P$
From: Holland, John H., Holyoak, Keith J., Nisbett, Richard E., and Thagard, Paul R. Induction: Processes of Inference, Learning, and Discovery. Cambridge, MA: Massachusetts Institute of Technology Press (1986)

An interesting analogy can be derived from the works of Cariani and the Hertzian model of emergence he presents [13]. This model embraces the concept that emergence in a system takes place by encapsulating and encoding information from the environment, processing it via some rules (or functions), and then generating a prediction of what the environment would look like in a successive time step. This is also clearly a mental model of how a system behaves, with the difference that a predictive element is used rather than a deterministic, after the fact formula. The Hertzian model given by Cariani is displayed in

Figure 2.6 and shows how the data from the world is gathered and understood (semantics) and how it is transformed from one time step to the other via internal functions (syntax).


Figure 2.6: The Hertzian Model
From: Cariani, Peter "Emergence and Artificial Life" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992), pp. 775-779

### 2.4 The Classifier System Major Cycle

The classifier system performs its task by adapting to the environment. In order to do so, it keeps on getting information from the environment and processing it. The classifier system, therefore, uses the following algorithm [56]:

1. Add messages that represent the current state of the environment to the message list;
2. Compare all the messages that are on the message list with the conditions of the classifiers in the classifier list;
3. Calculate bids for all the classifiers that have their conditions satisfied, run a competition among the bidding classifiers and select those classifiers that win;
4. Generate new message from the actions of those classifiers that won the bidding competition (add the messages to the message list);
5. Process the new messages to the output interface (effectors) and get the reward from the environment.;
6. Apply the credit assignment algorithm to reallocate the strength of the classifiers;
7. Use the discovery algorithm to modify the classifier rules given the new strengths; this will produce new rules representing the current state of the environment;
8. Replace of the message list with the new messages from the new classifiers;
9. Go to Step 1.

Messiages are added to the message list by a classifier and have a value associated with them called intensity. The intensity of the message is the amount a classifier bids in order to get its message (the action it fired) posted. The bidding system will be discussed later, but it is important to keep in mind that the message intensities affect how much classifiers bid for their messages. After every major cycle, the messages are added to a list
of messages along with their intensity values.
All classifiers undergo a Darwinian survival process, via the discovery algorithm, where they must survive given their use and strength. Therefore, the more a classifier is used'(the greater its use), the more its strength will increase. Classifiers also have abidratio associated with them. The higher the bidratio, the more the classifier will bid during the bidding competition. The bidratio can be seen as the classifier's willingness to post messages. The classifier specification can now be expanded to:
<condition> <condition> $\rightarrow$ <action>, bidratio, strength
The default value for the bidratio of a classifier is called the specificity, in case a bidratio is not assigned. The bidratio is computed in this fashion ${ }^{4}$ :

$$
\begin{equation*}
\text { specificity }=\frac{(S(\text { condition } 1)+S(\text { condition } 2))}{(\text { chromosome size } * 2)} \tag{2.1}
\end{equation*}
$$

where

$$
S(\text { condition } \mathrm{i})=\left\{\begin{array}{c}
\text { number of } 1 \text { or } 0 \text { symbols in match conditions } \\
\text { number of \# symbols in non-match conditions (those with } \\
\text { tildas }(\sim))
\end{array}\right.
$$

Another value that is used in the bidding contest is the support value for a classifier. It is the summation of all the intensities of the messages on the message list that match the classifier's conditions. This can be viewed as:

$$
\begin{equation*}
V_{i}(t)=\sum_{m^{\prime}} I_{m^{\prime}}(t) \tag{2.2}
\end{equation*}
$$

4. The parameters and formulae which appear are taken from Riolo in [56].
where $m^{\prime}$ represents all the messages that match classifier $i$ at time step $t$ and $I_{m^{\prime}}$ is the intensity associated with those messages. Support is used to compute bids for classifiers. The greater the support for a certain classifier, the greater the probability that that classifier will still be active in the next time step.

The bidding process of the classifier system permits those classifiers that have their conditions satisfied to bid in order to become active. The bid for a classifier $i$ at time step $t$ is $B_{i}(t)$ and is computed in the following fashion:

$$
\begin{equation*}
B_{i}(t)=k * S_{i}(t) * \text { bidratio }_{i} \text { power } * \text { avgsupport }_{i}(t) \tag{2.3}
\end{equation*}
$$

where $k$ is the parameter that determines how much of a classifier's strength will be used in bidding, $S_{i}(t)$ is classifier $i$ 's strength at time step $t$, bidratio is classifier $i$ 's specificity, power increases the importance of the specificity in the bidding process, and avgsupport $_{i}(t)$ is the support for classifier $i$ divided by the summation of the supports of the other bidding classifiers. Note that the bidding process heavily relies on the relevance of the classifier to the current situation the classifier system concerned with and also on how effective the classifier was in the past.

Once the bids for all the classifiers that want to be active are computed, a competition is run to determine which classifiers will become active, in case there is a threshold to the number of classifiers that can be active at any given time step. The probability that a classifier can then become active is:

$$
\begin{equation*}
P(i)=\frac{\beta_{i}(t)}{\sum_{j} \beta_{j}(t)} \tag{2.4}
\end{equation*}
$$

where $P(i)$ is the probability that classifier $i$ will win the competition, $j$ ranges over all the
bidding classifiers at time step $t, \beta_{i}(t)$ is the effective bid of a classifier and is defined as:
where bidpower is used to change the probability distribution used to selecting classifiers and effpower is used to allow specialist classifiers to win as opposed to generalists. The higher the bidpower, the higher the probability that those classifiers that have highest bid rates will win the competition. The effective bid is used to introduce some additional randomness into the selection process so that all classifiers get a chance to post messages, providing for a redistribution of strength among the bidding classifiers (otherwise the selection would be deterministic and biased). It is important to note that all classifiers should be given a chance to post messages since even though they may not have high strengths, they may contribute to providing a useful solution to the problem on hand.

### 2.5 Modifying the Classifier List

In order for adaption to occur, the classifier system must have a mechanism that will change its knowledge base. This knowledge encoded in the chromosomes has to be modified. Two mechanisms are used for this. One is the credit assignment algorithm and the other is the discovery algorithm. As mentioned earlier, each classifier has a strength parameter associated with it. The greater the strength, the greater the "survival probabilities" of that classifier. Classifiers with low strength values are considered as not being useful to the system's learning process and are thus discarded. In addition to credit assignment and discovery algorithms, taxes are applied to classifiers to reallocate strength.

### 2.5.1 Credit Assignment: the Bucket Brigade Algorithm

Classifier system have to reward those classifiers that have shown some utility to the task being learned. However, when doing this, two very important factors have to be taken into consideration [10]. First of, the classifiers in the classifier list in a time step (or major cycle) $t$ are "stage-setters" for other classifiers in time step $t+1, t+2, \ldots$. In other words, current classifiers determine what the future classifier list will look like since they encode the attributes of any future chromosomes, as defined by the schema model. As the classifier list is refined, certain schemata become stronger than others and their presence within the knowledge base increases rapidly until there is complete convergence towards those schemata. The chromosomes that contain those schemata become stronger since they generate profits by paying out part of their strength in order to post messages but getting more heavily recompensed when they have posted their messages.

Another issue is that the environment with which the classifier system interfaces with is of a dynamic sort, making it difficult to maintain a knowledge base that can handle abrupt changes. This element of uncertainty is present within all of the rules and must be handled successfully.

The bucket brigade algorithm is designed to cope with these problems. The algorithm can be understood in economic terms. The economy is composed of a great number of classifiers where each classifier performs transactions with its peers. Specifically, there are certain suppliers for a classifier, those that post messages that satisfy the classifiers' conditions, and there are consumers for a classifier, those who have their conditions matched when the classifier fires its action and posts a message on the message list. Once a classifier wins a bidding competition, it has to pay some of its strength to those classifiers that supplied the matching messages. Once the classifier becomes active and posts messages, it is remunerated by those classifiers that will have their conditions satisfied by it.

In addition to the payments (bids) that the classifier gets when it posts messages, it also gets an environmental reward. In other words, the environment can be specified
so that after producing a beneficial effect, the classifier system as a whole gets rewarded for that. For example, if a classifier system correctly detects a possible network failure before the disaster takes place, the system may get rewarded by an amount that compensates it for having prevented a significant loss. Hence, the strength of a given classifier is modified in the following manner:

$$
\begin{equation*}
S_{i}(t+l)=S_{i}(t)+R(t)+P_{i}(t)-B(i) \tag{2.6}
\end{equation*}
$$

where $S_{i}(t+1)$ is the strength of the classifier at time step $t+1, S(t)$ is the strength at the current time step, $R(t)$ is the reward received from the environment, $P_{i}(t)$ is the summation of the bids made to classifier $i$ for having matched their conditions, and $B_{i}(t)$ is the amount the classifier has to pay to its suppliers. The above equation can also be seen in the following fashion:

$$
\begin{align*}
& \begin{array}{l}
S_{i}(t+1)=S_{i}(t)+R_{i}(t)-B(i) \\
\\
\text { for the classifier posting messages and } \\
S_{j}(t+1)=S_{j}(t)+a B(j) \\
\text { for the suppliers to the winning classifier. }
\end{array} \text {. }
\end{align*}
$$

Notice that a chain of producers and consumers is formed here [10]. Specifically, the classifier list becomes more robust and complete as consumers and their respective consumers build the classifiers and become stronger. The last consumers in this chain are those that transact directly with the environment and receive payoff from it. If this chain of consumers is not one that can successfully work with the environment, then that chain is broken up and eventually replaced by stronger, more profitable rules. Thus, the bucket brigade attempts to maintain a repository of highly profitable and remunerative rules in the knowledge base at
all times.

### 2.5.2 Taxes

There are a number of taxes that classifiers have to pay at a given time step (after each major cycle) that cause classifiers to remain productive and generate actions ap- : propriate to the learning process. Taxes are applied at a rate $R$ and and the strength of a classifier is affected in the following manner after each time step:

$$
\begin{equation*}
\operatorname{strength}_{i}(t+1)=\operatorname{strength}_{i}(t)-\left[\operatorname{strength}_{i}(t) * R\right] \tag{2.9}
\end{equation*}
$$

where $i$ is the classifier being taxed.
The first tax that is applied is the head tax which is used to stimulate classifiers that do not typically bid to do so. These classifiers are not contributing in any way to the task being solved. Therefore, their strength is reduced in order to warn them of their inefficiency. If such warning will be of no use, these classifiers will be replaced by other classifiers introduced by the discovery algorithms.

A bid tax is applied to all bidding classifiers at a given time step. The purpose of this tax is to promote specialized classifiers over generalized classifiers that may bid all the time since their conditions get satisfied easily at every major cycle. In this way, specialized classifiers that may actually contribute useful information about the current state of the environment will have chance to enter the bidding competition. This also controls the premature convergence problem towards the generalized rules by permitting other, more pertinent rules to participate.

The third tax that is applied, the producer tax, is paid by those classifiers that post messages during a time step. This tax tries to accomplish the same goal as the bid tax,
attempting to discredit overly generalized classifiers that tend to remain in the classifier list for a long time without getting environmental rewards. This sort of permanent memory is eliminated by applying to classifier $i$ the $\operatorname{tax} R$ :

$$
\begin{equation*}
R_{i}(t)=\text { maxproducertax } *\left[\frac{m_{i}(t)}{M}\right]^{\text {producertaxpow }} \tag{2.10}
\end{equation*}
$$

where maxproducertax is the maximum producer tax that can be applied, $m_{i}$ is the number of messages classifier $i$ produced at time $t, M$ is the maximum size of the message list, and producertaxpow is used to magnify the effect of the tax.

### 2.6 Genetic Algorithms: The Discovery Method

In order for learning to take place within the classifier system, there must be some mechanism to provide dynamic behavior. Genetic algorithms accomplish this by taking the classifier list at every time step and selecting those classifiers that are the strongest within the classifier list. Those classifiers are then modified, creating new ones which would replace the weakest ones.

Genetic algorithms are an appropriate discovery approach for classifier systems since they are robust and can handle a number of different problems [23]. In other words, genetic algorithms provide a trade-off between exploitation and exploration of various domains. While trying to be explore new concepts, genetic algorithms also attempt to exploit those that are available. In this way, more domains can be searched without worrying about problems that commonly face optimization methods. One such problem is that of premature convergence. Optimizing too quickly often causes the search to end surreptitiously without finding the desired optima. Holland [35] gives a good description of genetic algo-
rithms and their function in the optimization of the schema search space.

### 2.6.1 Genetic Algorithms and their Operation

The classifier system consists of a population of classifiers, each composed of a condition and action strings, called chromosomes. The chromosomes are defined by the genotype. The genotype defines the structure of a chromosome (i.e., how the chromosome looks like, or its gene structure). Each classifier can be evaluated (or decoded) to a given value. This is called the phenotype of the classifier and is determined by the genotype. The phenotype is also used to determine the behavior of the classifier. Each position in the chromosome is called a gene and the value of the gene is the called the allele and the position of the gene is called the locus. Figure 2.7 shows such a relationship, that Stork et al. describe [69], among the genetic algorithm constituents and their effects on the functioning of the algorithm. Note how this figure shows the effects of adaption in the system. The system gets input from the environment, matches it with its internal "memory" which is encoded in the form of genotypes. These in tum produce system behavior which then affects the structure of the "memory".

In such a system, the classifiers within the population have to compete in order to survive (to preserve the best classifiers in the system). The genetic algorithm process consists mainly of three steps:

1. Select a pair of classifiers from the population according to their strength (select those classifiers that have greatest strengths). The genotype of the parents (those classifiers being selected) defines the offsprings' traits
2. Apply the genetic operators on the selected pairs.


Figure 2.7 The Genetic Algorithm Behavior Mechanism
From: Stork, David G., Jackson, Bernie and Scott Walker "Non-Optimality" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992)
3. Replace the weakest classifiers by the resulting classifiers from

Step 2.

Figure 2.8 gives a graphical illustration of how this is done. Assume that a set $G(t)$ of $M$ classifiers $\left\{C_{1}, C_{2}, C_{3}, \ldots, C_{M}\right\}$ is in the classifier system at time $t$. Each classifier has strength


Figure 2.8 Genetic Algorithms in Classifier Systems
$S\left(C_{i}, t\right)$. The genetic algorithm then proceeds to-modify the classifier list in the following manner [10]:

1. Compute the average strength $S(t)$ from all the classifiers in $G(t)$ and assign the value $S\left(C_{i}, t / S(t)\right.$ to the classifiers.
2. Assign a probability $p_{i}$ proportional to the above value to each classifier in $G(t)$. Use $p_{i}$ to select $n$ pairs of classifiers from $G(t)$.
3. Apply the crossover and mutation operators on the pairs of classifiers selected. Crossover is applied by randomly selecting a locus $j$ and then switching the remaining bits in the string between the two pairs. The CFS-C systems allows switching to occur in the entire classifier (so that the two conditions strings and action string are crossed over as one string) or just in the condition strings. Then apply the mutation operator which selects an allele and replaces it given a Poisson distribution.
4. Replace the classifiers in the classifier list that have the least strength with the new ones generated.
5. Go to Step 1.

By modifying the classifier list, genetic algorithms handle schemata and therefore affect the building block mechanism of classifier systems. A schemata $H$ (sometimes called hyperplane in literature) may have $m$ samples in the population $G(t)$ at any given time, denoted as $m=m(H, t)$. After each major cycle, probabilities ( $p_{i} ; s$ ) of being selected for being crossed over and mutated are computed and applied to the schemata survival. Therefore, at the next time step $t+1$ the number of schemata in the classifier list is going to be:

$$
\begin{equation*}
m(H, t+1)=m(H, t) * S(H, t) / S(t) * n \tag{2.11}
\end{equation*}
$$

where $n$ is the number of classifiers that are selected for replacement.

### 2.6.2 Other Operators that Complement the Genetic Algorithm

There are a number of other discovery algorithms that aid the genetic algorithm and the generation of classifiers. The reasoning here is that the classifier system must be able to be updated with environmental feedback at all times, and the genetic algorithm sometimes cannot handle novel situations.

The Cover Detector Messages algorithm is used when there are no detector messages that can be matched in a major cycle [56]. In such a case, which occurs especially when a population is initialized, the classifier list is modified to accommodate the detector messages. The task is managed by checking if at a major cycle rules bid for a message in the message list $M$ but no message is matched by the conditions of the classifiers and then making a copy of all those classifiers and assigning a probability relative to the bid. If no classifiers bid for any messages in $M$, then select a parent classifier that matches more closely a message in $M$. To do this, a counter is kept of the number of loci that are similar between the message and the condition of the parent classifier. The higher the count for a classifier, the greater the chances that that classifier will be selected. Then, modify the conditions of the copied classifiers to match the detector message. This will, of course, occur in the successive time step.

Another algorithm that is used by Riolo is the Cover Effectors Operator. This operator is triggered when the system is incapable of producing behavior (matching any of the effectors) or when the actions of the classifiers are too similar. In this latter case, those actions would keep on matching the conditions of certain classifiers, making them the stron-
gest and leading to the premature convergence of the knowledge base. When, during a time step, the classifier system fails to produce system behavior (i.e., emit an effector), the Cover Effectors Operator will select the classifiers that bid to have their actions posted. It copies them and randomly modifies the action chromosome by randomly changing a don't care symbol to a zero or one so that they will more closely match an effector.

There also two operators for chaining classifiers together. This concept was purported by Booker et al. [10] and is implemented in CFS-C. The chaining of classifiers is a useful building block strategy. It is not always possible to have one classifier represent an entire concept in a problem-solving situation. Rather, as supported by the concept of emergence in classifier systems, a number of classifiers permit the construction of "trains of thought." Triggered Chaining Operators cause one classifier to be coupled with another one. This means that the action of one classifier will satisfy the condition of another one. To accomplish this, a classifier $C_{2}$ must make a profit (which is the reward the classifier received minus how much it paid in bids) at time step $t$ and another classifier $C_{l}$ must have been active at time step $t-1$. The chaining operator would thus modify the action string of $C_{1}$ to make it match one of the conditions of $C_{2}$.

Another operator that is used is the Low-Bidders Operator. The genetic algorithm modifies those rules that have the highest strengths. But there can be cases where classifiers exist that do not have much strength and are never modified, keeping on producing system behavior even when not desired. In such case, the classifier system will check to see which rules bid below a certain threshold (determined using the strength of the classifier). The amount a classifier bids is a reflection of its strength. Thus, in selecting the low-bidders, the Low-Bidders Operator selects the poorly performing classifiers and modifies their actions randomly. The reasoning here is that the classifiers' conditions may be adequate to the problem being solved, but the action that the classifier is generating is not appropriate and
therefore does not produce acceptable behavior.

### 2.6.3 Genetic Algorithms and Appropriate Usage

Genetic algorithms are appropriate for usage in classifier systems for their robustness in optimizing a number of domains, as I explained before. However, genetic algorithms are also better suited for learning in nonlinear environments since they do not require a monitor during the learning process. Specifically, the environment often displays chaotic behavior and the learning systems must be able to deal with such realistic problems. Some systems, such as neural networks, require a monitor to evaluate the output of the network. A feedback mechanism is needed to analyze the output and send it back for re-processing in case the output was not acceptable (via such devices as back-propagation). Genetic algorithms, on the other hand, have no such need and perform well in those cases. This clearly adds to their efficacy and robustness.

Genetic algorithms have, in fact, been used in developing hybrid systems to aid connectionist systems in this aspect. Various models have been developed and seem to be very successful [6] [66]. The advantages tracked in such cases also include the fact that genetic algorithms are good for global sampling rather than local sampling. This is beneficial iflarge search spaces have to be searched, which is very possible especially in cases with neural networks with large hidden units, leading to a high dimensionality. Genetic algorithms have also been employed in optimizing the networks [51].

Finally, genetic algorithms are appropriate for usage in classifier systems and other artificial life_(evolutionary) systems since they provide the-needed flexibility and dy= namics to develop complex systems. Other optimization techniques tend to be less efficient from the evolutionary standpoint, due to the data structures that genetic algorithms use and the operations that can be performed on them. The data structures allow the encapsulation
of many different types of problems and their manipulation. This flexibility was desired by John von Neumann, claiming that for a complex system to evolve, the mechanisms on which it is built should not be delimiters 48]. Bec [5] and others followed the von Neumann ideal.

### 2.7 Parallelism and Emergence

Classifier systems fit the category of biological, physical, psychological computational systems found in connectionist systems, cellular automata, ecologies, etc. These systems purport the view that some mechanism exists that produces growth of knowledge in time. This view is further enhanced when the concept of sub-symbolic processing is understood. Classifier systems, like connectionist systems, are sub-symbolic systems since computation is performed on very low-level elements rather than high-level symbols as in expert systems. Specifically, classifier systems build their representations on sequences of bits rather than words.

To build these representations, the appropriate architecture is indispensable. The bits of information are to fit like pieces in a puzzle, and time is an important factor. Parallel architectures become important in classifier systems since they can process the classifiers in a distributed fashion. This leads to the emergence of knowledge within a given domain.

### 2.7.1 Emergence

The concept of emergence is not new. Roger Schank has addressed the issue in the concept of storytelling. According to him, the mind stored information in distributed fashion. Each bit of information is in itself not significant, but when recalled for telling stories, all of the information is put together in a perspicacious fashion, producing knowledge.

Marvin Minsky also confirmed such a resolution when he wrote:

How can intelligence emerge from non-intelligence?... One can build a mind from many little parts each mindless by itself. I'll call "Society of Mind" this scheme in which a mind is made of smaller processes. We'll call them agents. Each agent by itself can do some simple thing which needs no mind or thought at all. Yet where we join those agents in societiesin certain very special ways-that leads to true intelligence. ${ }^{5}$

In classifier systems, each classifier is one intelligent unit. The classifier encodes information about the environment at a given time. However, the information that it provides is clearly not complete since one classifier cannot encode the state of the entire environment. This is due to the dynamics of the environment and the many possible state the environment could be in. The classifier list could, therefore, be thought of a short-term memory containing different representations (schemata) of how the environment might look like. In order to select the real or actual state of the environment and represent it in behavioral terms, the classifier system has to extract the appropriate representations and, in a sense, converge towards one solution. This is an emergent process since the system accumulates information and places it together in a comprehensible fashion.

Self-organization of the classifier system occurs via emergence, where information is gathered and converged to represent the environment as closely as possible. The classifier list is continuously updated until it is fully organized.

Emergence can be thought of a selective process. It is an operation which requires gathering those pieces of knowledge which are most appropriate for representing the environment and generating the appropriate behavior (matching conditions and firing actions). The more information is available for scrutiny, the more focused and precise will the

[^1]representation be and more adequate the resulting behavior.

### 2.7.2 Parallelism

Parallelism in classifier systems occurs in a number of ways [21]. First of all, the bits of messages being matched are done so simultaneously. Second, the messages match classifier conditions at the same time (one or more messages on the classifier list matching one or more classifier conditions). Third, more than one classifier can be active at any given time step once the competition is run for the strongest classifiers. Fourth, parallelism occurs at the genetic algorithms level. Here, implicit parallelism identifies the situation where each $n$-gene chromosome can actually represent $2^{n}$ schematas. Specifically, consider a schema $H$ from a population of classifiers $G(t)$ at time step $t$. If $m(H, t)$ denotes the number of schema $H$ in $G(t)$ at time $t$, then the selection of the schema can be written as [27]:

$$
\begin{equation*}
M(H, t+1)=\frac{S(H, t)}{S(G, t)} M(H, t) \tag{2.12}
\end{equation*}
$$

where $S(H, t)$ is the average strength of the chromosomes that are in both the schema $H$ and the classifier population $G(t)$ and $S(G, t)$ are those bit strings that are present only in the population $G(t)$. This equation shows that the as chromosomes are found in' $H, H$ grows exponentially. Thus, for $n$-gene chromosomes in a population of $N$ chromosomes, from $2^{n}$ to $N 2^{\mathrm{n}}$ schemata can be found.

The ideal architecture for a parallel implementation of a classifier system would be Multiple Instruction Stream Multiple Data Stream (MIMD) rather than Single Instruction Stream Multiple Data Stream (SIMD). Forrest [21] explains that SIMD architectures are "coarse-grained", where each processor (CPU) in the system is powerful enough
and can compute large amounts of data ${ }^{6}$ and the processing among units is done synchronously. In a MIMD (or "fine-grained") architecture, each processor is not very powerful and the inter-process communication is not rigidly defined. The MIMD architecture would favor the concept of emergence. Each processor is small enough to process some information and interact with other processes in reaching a cooperative solution. This removes the requirement that all classifiers have to be simultaneously recalled (due to the synchronous characteristic of SIMD systems). Only certain classifiers will be activated, given the nonlinear behavior of the environment and the interactions among the activated classifiers cannot be in any way anticipated. As a result, "fine-grained" implementation also tends to remove any bias that there may be in favor of specific classifiers in the system in reaching system behavior.

### 2.8 Trade-Off Between Knowledge and Search

Raymond Kurzweil [41] justly argues that the human brain is a very large repository of information, where such information is retrieved and accessed via the parallel architecture provided by the many neurons ( 100 billion) and the many more interconnections that exist among them $(1,000)$. However, the analog computation provided by the brain falls short of the digital power of modern computers. This limitation forces the brain to retrieve information that is pertinent to the current situation being considered on an analogical (or associative) level in order to be able to face problems on a real-time basis. The brain, is in addition, capable of putting this knowledge together in a sensible fashion [62]. Computers, on the other hand, have a much computational power, capable of performing searches within a given state space much faster than the human brain. However, they lack the necessary
6. The nCUBE might represent such a machine with each processor having the power of an Intel 386, while the MMMD architecture might be more like the Connection Machine.
amount of information in order to produce applicable or significant results.
Classifier systems seem to provide a way in between these two extremes. First of all, classifier systems are capable of encoding vast amounts of knowledge within the rules. This is allowéd by having a sequences of bits within each condition or action represent a number of concepts (schemata). Schemata augment the number of possible representations and provide for an even greater repository of information. Finally, chaining among various rules further enriches the knowledge base of the classifier system, providing for interrelationships among different concepts.

But classifier systems are also adept at performing fast searches. Through their parallel architecture, having many rules active simultaneously, classifier systems perform searches of the state space that consent real-time performance even given the large repository of knowledge. The rules appropriate to the problem being solved are extracted and applied. The genetic algorithm also aids in this search effort, modifying rules so that many possible alterations are analyzed in a rapid and efficient fashion. An example of a parallel implementation of classifier systems is *CFS, the implementation of the CFS-C program on Thinking Machine's Connection Machine [59]. In this case, 65,000 classifiers were generated and each one was processed on a single processor. Each processor in this system does not have extensive processing powers, but the combination of processors make *CFS gain noticeable performance, illustrating the concept of emergent computing.

Figure 2.9, modified from Kurzweil [41], shows where classifier systems might stand in relation to some of the more traditional AI approaches. They would move to a higher equiperformance isobar thanks to the greater computational power provided by the parallel mechanisms. Classifier systems would move up along the equicost isobars due to their additional capabilities in terms of the amount of the knowledge that can be manipulated. The outcome is greater search power accompanied by additional informative power, a goal artificial intelligence systems of the future have to achieve more and more.


Figure 2.9: Trade-off between Knowledge (Rules) and Search (Tasks)
From: Kurzweil, Raymond. The Age of Intelligent Machines. Cambridge, MA: Massachusetts Institute of Technology Press (1990)

## CHAPTER 3

## FINITE STATE MACHINE REDUCTION AND COMMUNICATIONS PROTOCOL APPLICATION

Finite State Machines (FSMs) provide a good cognitive model of learning. The FSM mechanism describes a model where certain states designate the condition of the environment at any given time. It, thus, serves as an excellent example of how to simulate an environment for leaming purposes. The FSM receives inputs which make it change the state it is in and produce an output (behavior). The desirable outcome in an FSM world is to have the FSM always move to those states which provide the highest rewards or that are simply more attractive. A reward can be viewed as a mechanism by which the environment encourages further desirable behavior. For example, if a person invests money in a company's stock option in the stock market (the environment), any improvement in the company's performance would lead to a rise in the stock's value. The stock market has generated a reward.

Accordingly, a classifier system interacting with an FSM would use the reward as an indication of how it should act in the future. In using such a scheme, where the classifier system can interact with an FSM, many domains can be simulated. Some interesting applications include Riolo's system where human category learning is studied [59] and Zhou's system which teaches a robot how to move in an FSM world and avoid obstacles [76].

Finite State Machines, however, sometimes tend to have a great number of states that may not be consequential or of any use. In addition, the environment contains a number of states that can be represented by other ones. It is important to attempt to design an environment (when possible) which performs better while not compromising its function. Various procedures exist for the minimization of FSMs such as the $k$-equivalence analysis
[40]. Methods such as this one, however, use procedures that analyze the machines a priori and are not capable of capturing the rather dynamic behavior of the environment. In order to accomplish this, the classifier system presented here attempts to analyze which states are more useful in the FSM and which ones are not. A simple heuristic is used to function as the judging factor in the FSM reduction task.

One such design task is found in communication protocol design. Communication protocols are designed as FSMs and often have a great number of states that represent the protocol status. The more states there are, the more expensive the design in terms of performance and cost. It is thus important to attempt to reduce as much as possible the design complexity. This is a concern in many such information systems, where the system at times reaches burdening levels of complexity.

### 3.0 The Finite State Machine World

The FSM used here is the FSW1 model designed by Riolo [55]. The FSM consists of a finite Markov process consisting of a number of states and payoff values associated with those states. The Markov process's current state is used as a detector message that is placed on the message list. Once the classifier system processes the input, it generates output which causes it to modify the transition it will take next. In order to do this, the classifier system will internally "bid" for certain transitions as opposed to others, affecting the movement in the world. Each state has a certain payoff associated with it and that payoff is given to the classifier system as system reward.

### 3.0.1 The Markov Process

The Markov process is described in the following fashion:

1. A set of $n$ states $S_{i}$ where $i=0,1, \ldots, n$
2. A set of payoff values $v\left(S_{i}\right) \in \Re$ assigned to the states $S_{i}$
3. An alphabet $\Sigma$ defined by the value $r \in \Re$
4. A set of probability matrices $P(r)$ where each entry $p_{i j}(r)$ in $P(r)$ gives the probability of making a transition to state $S_{j}$, given that the classifier is in state $S_{i}$ and the value $r$ has been emitted by the classifier system
5. A set of transitions $T_{i j}$ for each state $S_{i}$ defining the transition from state $S_{i}$ to state $S_{j}$

Each state $S_{i}$ in the Markov process is defined as a bit string in the classifier system. Specifically, each state has a unique value associated with it that identifies it from the other states and this value is in bit-string (chromosomal) encoded format. This attribute string is used as the detection mechanism. At every time step, the current state's attribute string is placed on the message list indicating what state the system is in (of course, other messages that classifiers fire for satisfying conditions of other classifiers are also placed on the message list). Each state has also associated with it a next best state the system may move to. This is useful when designing a protocol since sequencing of states becomes important.

The system transits to a given state whenever the $r$ value of the edge leading to that state is emitted. To see how this works, the state diagram, derived from the one in Chapter 1, can be used as an example. Figure 3.1 shows a FSM consisting of five states. Note that each state is defined by a bit string (attribute list) used for placing it on the message list.

Whenever the system activates a effector messages (those starting with " 10 " and serving as the system's output mechanism), the system will transit to a given state. Specifically, the effector message will be decoded into an integer value representing the $r$ value.


Figure 3.1 A Simple Finite State Machine Defining a Markov Process
Adapted from: Bochmann, Gregor V. "A General Transition Model for Protocols and Communication Service" in IEEE Transactions on Communications, vol. COM-28, no. 4, (April 1980)

Each two bits of the message are translated into one bit using the following scheme:

$$
\begin{array}{r}
00 \rightarrow 0 \\
01 \rightarrow 1 \\
10 \rightarrow \# \\
10 \rightarrow \#
\end{array}
$$

Hence, when a detector is fired from the classifier list such as 1000000000010001 , that translates to (using all the bits except the left-most two which serve as the tag for effector messages) 0000101 , which decodes to the integer value 4 (using binary to decimal conversion).

Note that if a don't care symbol were used, such as in the message 100000000000000110 , that would produce more that one $r$ value. This last effector could set $r$ to a value in the set
$\{2,3\}$ since the effector decodes to $0000001 \#$, allowing both " 10 " and " 11 ".
The value $r=4$ will be set only if a bidding competition is won. Since there may be a number of classifiers that will fire an effector message in any time step, the system must determine which $r$ value is the most desired at that time. In order to accomplish this, the system uses a probabilistic approach whereby the amount each classifier bid to have its message posted (or its intensity) is used as a determinant in whether that $r$ value will be used. As an example, if the system is in state 4 , and two effectors are on the message list, one which would like to set $r$ to 3 with an intensity of 100 , while the other wants to set it to 4 with an intensity of 200 , then it will be set to 4 .

### 3.0.2 The Learning Process

The classifier system must learn what states are the most rewarding ones. In fact, when the system goes to a given state, it will receive reward in the form of payoff values. Each state has a reward associated with it. The system then distributes the reward to all the classifiers that are active (i.e., fire actions) during that time step. In doing this, the system is trying to stimulate classifiers to generate $r$ values that move the system to the higher paying states within the FSM. Hence, once the detection mechanism inputs the attribute list of a state which is then matched by some classifier, the system must learn to generate an action string that supports setting the $r$ value to the next reachable high-paying state. Each classifier could also set the action string to retum to the same state in the condition string in case that state also provides substantial rewards.

The learning process is also reflect in the emergence of coupled chains [57]. Specifically, a number of classifiers are chained together when the action of one satisfies the condition of another. In the FSM domain, learning is reinforced when a number of classifiers satisfy the condition of another classifier that would fire an effector that would produce de-
sirable $r$ values. Hence, if a classifier $C$ would set the $r$ value to 4 by firing its effector, then classifier B might want to satisfy the condition of that classifier in order to make $\mathbf{C}$ fire its action. At the same time, another classifier A might want to satisfy $B$ so that $B$ will fire its action. These chains will last as long as their importance to the task is safeguarded. Once the states that classifier C supports lose relative payoff in relation to other states, the chain may break down.

As the system is exposed to more environmental feedback, it discovers which particular states provide the highest rewards of all. In such a case, it may well be that if there is one specific state that has rewards greater than anyone else's, then the system will always try to move to that state. This all depends on what the environmental (Markov process) input is. Since the classifier system's behavior affects the environment directly, the environment will tend to react in harmony to the classifier system, unless some chaotic element is introduced.

Such a behavior is clearly synchronous to a cognitive approach since the system attempts to model the environment according to its own needs. The more the system learns from the Markov process, the greater its ability to manipulate that process in its own favor.

An important factor in any learning process is that it should not be precipitous. The classifier system should not evaluate a subset of all states as the best states only after a few time steps. It should have enough time to evaluate all states. In addition, the environment can change any time due to anomalous behavior, heavily influencing the relevance of certain states to the Markov process. Hence, if 2 out of 5 states were important up to time step $t$, those same states may become completely useless in the problem-solving task after time step $t$ due to a some event that modified the environment's behavior.

Classifier systems are equipped with a number of mechanisms that prevent rapid convergence to a solution. The most important of these is incorporated in the Bucket

Brigade Algorithm (BBA). The BBA increments the strengths of classifiers gradually in order to support incremental learning. Hence, the system will not only learn gradually, but it will also have time to modify its behavior if later on the environment modifies its behavior. A good description of the BBA and its traits as a graceful credit assignment method are given by Sutton [70].

### 3.1 The Finite Space Reduction Task

Any Finite State Machine may be composed of a number of states that perform a certain function. However, it often happens that some states might perform overlapping function as other states and their functionality can, therefore, be encapsulated by other states. Such a discovery would lead to a more simplified design of an FSM system, reducing not only the number of states, but also the transitions to those states.

In order to accomplish this task, a method has to be developed to learn which states are really important to the FSM and how can the FSM be reduced. The classifier system here is put to test this problem and designate those states in the FSM that may not be needed.

### 3.1.1 The Reduction Task Approach

To approach taken here is to make the states prove their usefulness. In order to do so, the state's visit count, or the number of times a state is transited to, is used to determine it worthiness. The classifier system increases the payoff attributed to each state, depending on how many times the state has been visited. Therefore, the payoff also determines state needs. The following equation shows how this occurs:

$$
\begin{equation*}
P_{i}=P_{i}+\left(\text { Const } * \frac{\left(V_{i}-V_{\min }\right)^{k}}{\sum_{j}\left(V_{i}-V_{\min }\right)^{k}}\right) \tag{3.1}
\end{equation*}
$$

$P_{i}$ is the payoff value for state $i$, Const is some constant value, $V_{i}$ is the visit count for state $i, V_{\min }$ is the lowest visit count among all states, and $k$ is some value used to calibrate increases in the payoff. The payoff value of the states are increased to give a qualitative estimate of the worthiness of that state. At the end of a task, the payoff values of the states are analyzed and those $n$ states that have the greatest payoff values are classified as being the ones useful to the FSM. Note, however, that such a conclusion can only be preliminary, since there are a number of other facts that determine the legitimacy of a certain state in addition to the payoff value. One such momentous element is state transition. Our model attempts to reach a resolved based on the visits made to a state, which is not necessarily a conclusive factor. It suggests, however, that the FSM might have to be redesigned to incorporate the functions of less frequently visited states into those that are most often visited. There are a number of applications that use the FSM paradigm which may find such suggestion useful since there is a cost associated with having additional states.

### 3.2 The Protocol Environment

A communications protocol defines the rules of a network that connects various communicating systems together. There are various layers of communications that exist within a certain protocol specification. One such specification is defined by the Open Systems Interconnection (OSI)Reference model of the International Organization for Standardization (ISO) [38]. The functionality of the protocol is divided into 7 layers, each of which deals with specific aspects of a communication between two connected stations. Figure 3.2 shows how the OSI looks like and what the various layers perform.

Each layer performs a certain task. For example, the presentation layer defines the data formats and codes in an exchange, while the data-link layer assures reliable


Figure 3.2 The Open Systems Interconnection
transfer of data via error detection and error recovery. A good description of the OSI model is given by Green [26].

Each protocol can be specified as a Finite State Machine, defining exactly in what stage of communication the system is in. Figure 3.1 illustrates such a protocol.

### 3.2.1 The Protocol Specifications

The communications protocol models the environment that the classifier system uses. The protocol is defined as a rule set that contains a series of input/output sequences. There are a number of possible inputs and outputs that can be generated, depending on the protocol, causing the environment to change its state as a consequence. The classifier system relies on such changes to display its behavior.

As an example, this thesis uses an FSM protocol that was developed by Hashem et al [31] which is used for conformance testing of the data-link layer of the ISDN D channel. The protocol defined by Hashem at al. consists of a total of 12 states, 3 of which
exist only for testing the reliability of the protocol. There are a number of possible inputs to each state and a number of possible outputs. There is also a list of transitions. The transitions are specified as:
current_state: next_state WHEN (input, output).

The inputs and outputs represent actual data that the data-link layers manipulates. Therefore, the data must meet certain syntactic and semantic requirements. In a communication session, data is sent from one station to another in a sequential fashion, such as requesting to send data, sending it, and acknowledging it. The data also has to conform to certain specifications such as size and content.

Transitions can be either valid, inopportune, or illegal. Valid transitions exist when a certain input is expected, while inopportune inputs are received that are correct syntactically, but do not arrive at the desired time. Illegal inputs are received when the input does not contain the desired data. Appendix A defines some of the data and the state definitions.

### 3.2.2 Reduction of the Protocol

There are great number of protocols defining communication between two entities. The more complex the protocol for each entity, the more computational overhead there is and the higher delay in communication. It, thus, becomes necessary to reduce the protocol to some design that could provide a more efficient form to represent the protocol.

Such a design would focus on combining certain states that may be transited to given the same or similar conditions or combining a number of states infrequently visited. The classifier system must determine which states are not transited to frequently, and there-
fore do not provide high returns to the protocols performance. The task then involves deter-


Figure 3.3 Reduction of a Communications Protocol In $(a)$ the original protocol is shown while in $(b)$ the reduced protocol is derived.
Adapted from: Bochmann, Gregor V. "A General Transition Model for Protocols and Communication Service" in IEEE Transactions on Communications, vol. COM-28, no. 4, (April 1980)
mining whether those states that have been tagged as not generating high payoffs can be combined, as a result of similar transitions. It will also be able to study whether certain transitions
can be modified so that the protocol still performs its functions as set forth initially, but does so with a fewer number of states. In other words, rather than having more states, it may be better to have one state performing the functions of a number of other states with additional transitions to that state. A very simple example, just for illustrative purposes, is the one shown in Figure 3.3. Here, the protocol that was shown in Chapter 1 is displayed in part (a). This protocol has two states that define the acknowledgment of some message. However, the two states could be combined into one that captures the functionality of the two separate states. The transitions then are left to decide when to leave those states. Some logic could then be built within the state that would allow the input/output sequences that define the protocol to function as desired. In any case, unique input/output sequences exist for all the states of the protocol, and only the logic within the states defines which output to take given the input received.

## CHAPTER 4

## EMPIRICAL RESULTS FOR THE FINITE STATE MACHINE REDUCTION TASK

The concepts for which this work is done have now been laid out. It is appropriate to observe what results can be derived from the prescribed methodology. The classifier system is put to test to see if the Finite State Machines to which it presents can be reduced.

The FSM, which models a Markov process, serves as the environment with which the classifier system interacts. The Markov process uses probabilistic transitions which have been set to 1 in all cases in order to model a communications protocol, which is of deterministic nature.

This chapter presents a number of different results that were derived both from the Finite State Machine minimization perspective and also from classifier systems and their performance in this domain. Due to the fact that no other machine learning systems have been employed in this domain, the results that are derived cannot be compared to other leaming systems and therefore, one cannot readily assess how classifier systems perform here. There are, however, a number of different test sets that have been generated to give a better grasp of the value and quality of the results gathered. Specifically, such test sets provide benchmarks to test the efficacy and robustness of the classifier system in this domain.

### 4.0 Testing Procedures for the Finite State Machine Task

A gradual refinement strategy was used in this paper to identify a method by which FSMs could be generated and appropriate tests could be performed on them. First of
all, in order to generalize our contribution, the FSM is not restricted to a specific domain. In order to do that, FSMs have been generated randomly using uniform distributions. This includes random initial payoffs, random state attribute values, and random number of transitions.

### 4.0.1 Preliminary Tests

The test sets that were developed were based on randomly generated FSMs using uniform distributions. The number of states generated was set to 18 for computational purposes. The software was run on an IBM RS/6000 model 950 and could handle 18 states comfortably. The attribute strings for the 18 states were also randomly generated using binary values. The payoffs were generated using a uniform distribution [0, 327]. The number of transitions were generated using a uniform distribution from 0 to 24 . Finally, the next best states were produced using a uniform distribution from 0 to 18 .

Initially, various machines were generated to test the robustness of the classifier system in its interaction with the FSM environment. The classifier system's major cycle

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Population | 200 | Sharerew | 1 |
| Bid_k | 0.10 | Crowdfax | 1 |
| Bidpow | 1 | Mutprop | 0.05 |
| Brpow | 1 | $k$ | 1.0 |
| Effpow | 6 | Const | 2.0 |
| Headtax | 0.0005 | Maxproducertax | 0.00 |
| Bidtax | 0.000 | Producertaxpow | 1.5 |
| Fbidtax | 0.02 | Crossover prob. | 0.5 |

Table 4.1 Parameter Settings for the Classifier System
was run for 12,800 generation in order to make it interact sufficiently with the FSM domain.
Some of the parameter settings that were used are shown in Table 4.1.
All the parameters are described in Chapter 2, except for $k$ and Const which are described in equation 3.1. The crossover probability is for crossing over entire classifiers rather than each chromosome. Figure 4.1 shows the results for the 18 states that were tested.


Figure 4.1 Final Payoff Values for a Finite State Machine Using Five Different Initial Payoff Values

The results reflect the payoff values at cycle 12,800 for a randomly generated FSM with five different randomly generated initial payoff values. One will notice that the classifier system demonstrates quite robust behavior to the initial payoff values.

The graph in Figure 4.1 helps in easily distinguishing useful states. For example, states 3,6 , and 15 appear to produce very low payoffs, raising questions as to their usefulness to the FSM. The FSM could be reduced by removing those states and attempting a new design that would incorporate the functions of those states in other states.

There are occasions, however, that its become difficult to select the states that may be removed. One such instance is illustrated in Figure 4.2. The results in this case show that there exists only one state (i.e., 18) that can be possibly discarded, but the automata is


Figure 4.2 Final Payoff Values for a Finite State Machine Using Five Different Initial Payoff Values
quite robust otherwise. Therefore, such machine may already be reduced to start with.

### 4.0.2 Parameter Settings

In order to develop a method that is capable of confidently discriminating useful states from "useless" ones, the greatest difference in payoff values must be sought. In other words, the greater the difference between the payoff values of the high-payoff states and the low-payoff states, the greater the reliance in eliminating those states.

A number of tests were developed to provide this additional tool. The different test sets utilized a number of different parameter settings of the classifier system. These are shown in Table 4.2. The parameters were selected on the basis of their ability to successfully discriminate among states. They include:

1. mutprop - The mutation probability which controls how often a chromosome's allele will be mutated and therefore additionally diversified.
2. bid_ $k$ - The bid factor represents the percentage of the strength a classifier will lose in case it bids for another classifier but that classifier fails to post a message.
3. crowdfac - The crowding factor is the variable that controls how how many classifiers will be selected for each classifier to be replaced. When a weak classifier is to be removed from the classifier list, more than one can be chosen as a substitute.

| 134kSum |  | arame |  |
| :---: | :---: | :---: | :---: |
|  | mutprop | bid_k | crowdfac |
| 1 | 0.05 | 0.10 | 1 |
| 2 | 0.05 | 0.10 | 6 |
| 3 | 0.05 | 0.20 | 1 |
| 4 | 0.05 | 0.20 | 6 |
| 5 | 0.10 | 0.10 | 1 |
| 6 | 0.10 | 0.10 | 6 |
| 7 | 0.10 | 0.20 | 1 |
| 8 | 0.10 | 0.20 | 6 |
| 9 | Average of above results |  |  |

Table 4.2 Test Set for FSM

All three parameters have the capability of strongly affecting the premature convergence problem. In other words, there must be someway of avoiding the classifier list of quickly converging towards a solution. This criteria is indispensable if the best possible FSM must be designed. The environment exhibits nonlinear behavior and in order to capture any alterations in its conduct, the classifier system must be exposed to sufficient interaction. It may
happen, for example, that a states $x$ might be initially very relevant to the FSM, but then, due to some endogenous force, state $x$ 's strength drops due to changed circumstances. The classifier system must be able to adjust to such abrupt phenomena.

Eight different parameter settings have been generated as seen in Table 4.1. From these, the best settings have to selected.

### 4.0.3 Results with Modified Parameter Settings



Figure 4.3 Payoff Values at Major Cycle 12,800 Using Eight Different Parameter Setting Combinations

The parameter settings strongly affect the behavior of the classifier system and its ability to determine what states are relevant to the FSM. Figure 4.3 gives the results

| 4 \#" |  |  | Paramelers Setimg |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1678 | 1794 | 1749 | 1548 | 1881 | 1634 | 1653 | 1714 | 1706 |
| 2 | 3071 | 3118 | 3159 | 3023 | 2929 | 3303 | 3246 | 3083 | 3216 |
| 3 | 454 | 249 | 521 | 332 | 439 | 340 | 511 | 297 | 392 |
| 4 | 954 | 964 | 1165 | 995 | 1045 | 961 | 889 | 986 | 994 |
| 5 | 1332 | 1384 | 1563 | 1347 | 1616 | 1491 | 1644 | 1632 | 1501 |
| 6 | 324 | 384 | 320 | 426 | 351 | 404 | 321 | 336 | 358 |
| 7 | 1200 | 1123 | 1203 | 1172 | 996 | 1421 | 1118 | 1205 | 1179 |
| 8 | 1217 | 1252 | 1165 | 1149 | 1254 | 1232 | 1148 | 1304 | 1215 |
| 9 | 772 | 783 | 948 | 797 | 831 | 610 | 912 | 629 | 785 |
| 10 | 1374 | 1521 | 1353 | 1266 | 1198 | 1587 | 1414 | 1457 | 1396 |
| 11 | 1639 | 1966 | 1814 | 1919 | 1709 | 1427 | 1788 | 1479 | 1717 |
| 12 | 3195 | 3551 | 3394 | 3255 | 3731 | 3548 | 3337 | 3487 | 3437 |
| 13 | 3403 | 3331 | 2874 | 3316 | 3087 | 2933 | 3182 | 3164 | 3161 |
| 14 | 1828 | 1595 | 1391 | 1526 | 1461 | 1485 | 1540 | 1466 | 1536 |
| 15 | 292 | 313 | 416 | 238 | 264 | 262 | 320 | 328 | 304 |
| 16 | 1957 | 1729 | 1619 | 1623 | 2039 | 1629 | 1574 | 1430 | 1700 |
| 17 | 2603 | 2584 | 2753 | 2523 | 2492 | 2630 | 2532 | 2511 | 2578 |
| 19 | 618 | 725 | 658 | 623 | 633 | 562 | 874 | 778 | 683 |

Table 4.3 Payoff Values for FSM States at Major Cycle 12800 Using Different Values for mutprop, bid_k, and crowdfac


Figure 4.4 Deviations from the Mean of Payoff Values for FSM State 12


Figure 4.5 Deviations from the Mean of Payoff Values for FSM State 15
for a randomly generated FSM and its payoff values at major cycle 12,800 using the parameter settings in Table 4.2.Note that there are some observable differences among the different settings. To better view the above result, the mean value for the 8 combinations was computed and served as a basis for the variance analysis. Table 4.3 displays the numerical results for Figure 4.3.

Using the mean value (parameter setting 9 of Table 4.2), the deviations from that mean were computed. Column 9 of Table 4.3 gives the mean values. The deviation from
the mean values for state 12 , with the highest mean payoff value, and state 15 , with the lowest mean payoff value, are shown in Figures 4.4 and 4.5, respectively. These figures show which parameter setting provided the greatest discriminatory effects. A complete analysis for all

| 4s! 5if: | state\# |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | -28 | -145 | 62 | -40 | -169 | -34 | 21 | 2 | -13 |
| 2 | 88 | -98 | -143 | -30 | -117 | 26 | -56 | 37 | -2 |
| 3 | 43 | -57 | 129 | 171 | 62 | -38 | 24 | -50 | 163 |
| 4 | -158 | -193 | -60 | 1 | -154 | 68 | -7 | -66 | 12 |
| 5 | 175 | -287 | 47 | 51 | 115 | -7 | -183 | 39 | 46 |
| 6 | -72 | 87 | -52 | -33 | -10 | 46 | 242 | 17 | -175 |
| 7 | -53 | 30 | 119 | -105 | 143 | -37 | -61 | -67 | 127 |
| 8 | 8 | -133 | -95 | -8 | 131 | -22 | 26 | 89 | -156 |
|  | 10 | 11 | d 2 | 13 | 14 | 15 | 16 | 17 | 18 |
| 1 | -22 | -78 | -242 | 242 | 292 | -12 | 257 | 25 | -65 |
| 2 | 125 | 249 | 114 | 170 | 59 | 9 | 286 | 6 | 42 |
| 3 | -43 | 97 | -43 | -287 | -145 | 112 | -81 | 175 | -25 |
| 4 | -130 | 202 | -182 | 155 | -10 | -66 | -77 | -55 | -60 |
| 5 | -198 | -8 | 294 | -74 | -75 | -40 | 339 | -86 | -50 |
| 6 | 191 | -290 | 111 | -228 | -51 | -42 | -71 | 52 | -121 |
| 7 | 18 | 71 | $-100$ | 21 | 4 | 16 | -126 | -46 | 241 |
| 8 | 61 | -238 | 50 | 3 | -70 | 24 | -270 | -67 | 95 |

Table 4.4 Deviations from the Mean of Payoff Values for the FSM at Major Cycle 12800

18 states is shown in Table 4．4．Using this latter table and computing the sum of deviation from the mean，three of the 8 parameter setting combinations were chosen to guide future experimentation．These are shown in Table 4．5．

| 至乡0等 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Above Mean | 901 | 1211 | 976 | 438 | 1106 | 746 | 790 | 487 |
| Below Mean | 1130 | 446 | 769 | 1218 | 1018 | 1145 | 595 | 1059 |
| Variance | 2\％＊乡 | 1657 | 1745 | 1656 | 424 | ¢¢\％ | 1385 | 1546 |

Table 4．5 Best Parameter Settings

## 4．0．4 Selecting the best Parameter Settings

Now that the number of parameter setting combinations has been narrowed down，the best one among them has to be selected．Specifically，the three parameter setting combinations were put to the test again using three randomly generated FSMs and three dif－ ferent randomly generated payoff values for each FSM．The best combination of settings is found by the testing．

First of all，a heuristic was used here to discriminate among states．Out of the 18 states used，those ten with the highest mean payoff values were regarded as useful and the remaining eight as not contributing to the FSM design．This formed two sets of states． Figures 4.6 and 4.7 show the results for the first FSM tested using two out of three of the initial payoff values used．It is easy to see that no matter what the initial payoff values，almost the same results are achieved．This result was observed in all three automata．From these re－ sults，a variance analysis was carried out to identify the best parameter setting combination．

Table 4.6 shows the deviation results for the first FSM used and its first set of payoff values． The deviations are from the mean values in the fifth column．Using this table，all the positive


Figure 4.6 Payoff Values at Major Cycle 12,800 for FSM 1 with Payoff Set 1
deviations and negative deviations were summed up for both sets of states (i.e., the best 10 and worst 8 ) and for each of the different parameter settings. The mean values were then computed for the summations. Finally, deviations from the mean were calculated.

The results for the same FSM of Table 4.6 are displayed in Table 4.7. For each


Figure 4.7 Payoff Values at Major Cycle 12,800 for FSM 1 with Payoff Set 1
parameter setting combination, 1,5 , and 6 , the mean values where computed for the 8 worst
states, the 10 best, and the two sets combined. The deviations were then used to carry out the discriminatory analysis.

| State \# |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 5 | 6 | Mean | 1 | 5 | 6 |
| 1 | 1678 | 1881 | 1634 | 1731 | -53 | 150 | -97 |
| 2 | 3071 | 2929 | 3303 | 3101 | -30 | -172 | 202 |
| 3 | 454 | 439 | 340 | 411 | 43 | 28 | -71 |
| 4 | 954 | 1045 | 961 | 986 | -32 | 59 | -25 |
| 5 | 1332 | 1616 | 1491 | 1479 | -147 | 137 | 12 |
| 6 | 324 | 351 | 404 | 359 | -35 | -8 | 45 |
| 7 | 1200 | 996 | 1421 | 1205 | -5 | -209 | 216 |
| 8 | 1217 | 1254 | 1232 | 1234 | -17 | 20 | -2 |
| 9 | 772 | 831 | 610 | 737 | 35 | 94 | -127 |
| 10 | 1374 | 1198 | 1587 | 1386 | -12 | -188 | 201 |
| -11 | 1639 | 1709 | 1437 | 1595 | 44 | 114 | -158 |
| 12 | 3195 | 3731 | 3548 | 3491 | -296 | 240 | 57 |
| 13 | 3403 | 3087 | 2933 | 3141 | 262 | -54 | -208 |
| 14 | 1828 | 1461 | 1485 | 1591 | 237 | -130 | -106 |
| 15 | 292 | 264 | 262 | 272 | 20 | -8 | -10 |
| 16 | 1957 | 2039 | 1629 | 1875 | 82 | 164 | -246 |
| 17 | 2603 | 2492 | 2630 | 2575 | 28 | -83 | 55 |
| 18 | 618 | 633 | 562 | 604 | 14 | 29 | -4 |

Table 4.6 Parameter Settings for FSM 1 Problem 1, Including Deviations from the Mean

Thus, for parameter setting 1 and state set 8 , the mean payoff was 728 and the deviation 369. The deviation ranged from $359(728-369)$ to $1097(728+369)$. For the same parameter setting but state set 10 , the mean was 2208 with a standard deviation of 787 .

| Paraniter Setings | Statesel | Mean Fayगt | Standard Devition | Ramge of Deyation |  | ©menh |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 : | 8 | 728 | 369 | 359 | 1097 | $324$ |
|  | 10 | 2208 | 787 | 1421 | 2995 |  |
|  | 18 | 1550 | 977 | 573 | 2527 |  |
| 5 | 8 | 719 | 349 | 370 | 1068 | 342 |
|  | 10 | 2219 | 809 | 1410 | 3028 |  |
|  | 18 | 1553 | 992 | 561 | 2545 |  |
| 6 | 8 | 724 | 431 | 293 | 1155 | 182 |
|  | 10 | 2167 | 840 | 1337 | 3007 |  |
|  | 18 | 1526 | 997 | 529 | 2523 |  |

Table 4.7 Results of Discriminatory Analysis for Different Parameter Settings and State Sets from FSM 1 Problem 1

The range in this case was from 1421 to 2995 . The upper range of state set 8 was subtracted from the upper range of state set $10(1421-1097)$ giving the gap of deviation between the two states which was 324 in this case. The greater the gap, the more the classifier system discriminates between the higher-payoff value states and the lower-payoff value states. This is, of course, a desired factor since the more discrimination there is, the better the confidence in eliminating states. Hence, in Table 4.7, parameter setting 5 gives the best results since it differentiates the most between the two sets.

Table 4.8 contains the discrimination results for all three FSMs and the different payoff values. Note that parameter setting 1 provides the greatest amount of discrimination between the two state sets since the gap in the payoff values of the two sets is the greatest
using that setting. In addition, parameter setting 1 also proves to be the most robust since the variation in gap sizes is more constant than in the other two cases. For example, parameter settings 5 and 6 have values as low as 9 and 19 for problem 3-2, while setting 1 remains much higher.

| Warameter Setimp. |  |  |  |  |  |  |  |  |  | Suni |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1-1 | 1-2 | 1-3 | 2-1 | 2-2 | 2-3 | 3-1 | 3-2 | 3-3 |  |
| 1 | 324 | 291 | 345 | 230 | 205 | 201 | 116 | 124 | 50 | 1886 |
| 5 | 342 | 246 | 340 | 172 | 122 | 249 | 105 | 9 | 37 | 1622 |
| 6 | 182 | 306 | 350 | 153 | 288 | 282 | 47 | 19 | 83 | 1710 |

Table 4.8 Results of Discriminatory Analysis for all Tests

Therefore, the best parameter setting for the tests thus conducted consists of using a bid_ $k$ of 0.10 , a mutprop of .05 , and a crowdfac of 1 . Using these values, an FSM can be designed containing only essential states. The states that the classifier system will identify as "below mean", should be eliminated, if possible.

### 4.0.5 Using Different Heuristics

The machines that have been used up to now have used a number of heuristics. In order to increase the confidence in the results provided by the classifier system, a number of different machines were generated using a number of heuristics.

First of all, up to now the states of the FSM were divided into two sets consisting of those with the lower payoff values (8 of them) and those with higher payoff values ( 10 of them). In order to modify the above configuration of sets, the combination was modified to include 10 states in the low payoff set and 8 states in the high payoff set. Such a diversification would allow checking the robustness of the classifier system in discriminating states,
even when they are subdivided into variable sets.
In addition, different uniform probabilities were used for generating the FSMs. Table 4.9 displays the two different set sizes using the uniform probability used up to now, namely, 0 to 327 for the payoff values and 0 to 24 for the number of transitions. Table 4.10, on the other hand, uses a range of 0 to 250 for payoff values and 0 to 18 for the number

| FSM | Gap Sizes |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1-1-1 | 1-1-2 | 1-1-3 | 1-2-1 | 1-2-2 | 1-2-3 | 1-3-1 | 1-3-2 | 1-3-3 |
| 8/10 | 386 | 486 | 157 | 154 | 79 | 55 | 415 | 442 | 341 |
| 10/8 | 486 | 439 | 259 | 212 | 48 | 126 | 389 | 344 | 311 |

Table 4.9 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms $[0,327]$ for Payoffs and $[0,24]$ for Transitions
of transitions. Finally, Table 4.11 uses a uniform probability of 0 to 150 for the payoff values and 0 to 12 for the number of transitions. Each of the tables are divided into three sub-groups of machines representing different, randomly generated FSMs. Hence, in Table 4.9, FSMs $1-1-1,1-1-2$, and $1-1-3$ all use the same randomly generated machine but have different payoff values, while FSMs 1-2-1,1-2-2, and 1-2-3 use a different randomly generated machine, and so on.

| ESM | GapSines |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2-1-1 | 2-1-2 | 2-1-3 | $2-2-1$ | 2-2-2 | 2-2-3 | 2-3-1 | 2-3-2 | 2-3-3 |
| 8/10 | 114 | 157 | 202 | 574 | 595 | 365 | 347 | 320 | 351 |
| 10/8 | 109 | 164 | 221 | 460 | 458 | 255 | 367 | 348 | 424 |

Table 4.10 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0,250] for Payoffs and [0, 18] for Transitions

The results show that whether the $8 / 10$ combination is used (which is the one used in all the tests) or the $10 / 8$ combination, the gap sizes remain positive. However, the

10/8 combination (which reduced the FSM even further) provides greater discriminatory power. Thus, it could be expected that the more the FSM is reduced, the more the discrimination increases.


Table 4.11 Gap Sizes for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms $[0,150]$ for Payoffs and $[0,12]$ for Transitions

Notice that the results also show that the $8 / 10$ combination, which would reduce the FSM from 18 states to 10 , generates sufficient discrimination, not requiring additional reduction, which would most probably compromise the functioning of the machine (i.e., with the $10 / 8$ combination).

### 4.1 Using the Classifier System Appropriately

There are a number of factors to consider when carrying out the tests. The classifier system must be set up appropriately so that it will carefully consider all possible environmental data.

One way to assure that this happens is to avoid the premature convergence problem. If the system very quickly discriminates among states, then it may not have gathered all the needed data from the environment. To achieve some control over this problem, the classifier system provides a few tools. One is that of crowding factors.

When the weakest classifier in the classifier list is to be replaced, a stronger classifier will be used as a replacement. But this may be problematic in some problems since that would cause the strongest classifiers to quickly dominate all the coupled sequences and
default hierarchies, since those classifiers would keep on being chosen as replacements. To avoid this problem, a number of rules can be selected for replacement randomly, so that the bias is reduced and other classifiers that may appear to be trivial to the task being solved will have a chance to remain in the classifier list for future use. These latter classifiers could indeed become very useful at some future time. The crowding factor specifies how many of these classifiers will be entered in the replacement pool.

Figure 4.8 shows how the different parameter settings fared with the conver-


Figure 4.8 Convergence of State Payoff Values Using a Threshold of 1000
gence problem. The graph shows the number of states whose payoff value exceeded a threshold of 1000 at each cycle step. Thus, the more the graph is skewed to the left, the worse the premature convergence problem. From the figure, it is possible to see that the winning pa-
rameter setting combination (in black) converges nicely, representing almost an average of the other ones. Another tool is that of sharing rewards. Each time the classifier system produces a desired output, such as going to a state that has a higher payoff value, then the environment generates a reward so that the system will continue to produce such behavior. The classifier system can be controlled so that the reward will be shared only by those classifiers that posted the message and activated an effector or it will be shared among all the classifiers that were active when the system produced its behavior. Sharing among all active classifiers may help some of the weaker classifiers to remain in case they may be needed for future use.

### 4.2 Protocol Analysis

Now that the parameter setting combination has been selected, the testing can be done on the communications protocol. Appendix A gives a brief description of the protocol and its design. There are 12 states in the model. A number of the states represent timers that are activated when data is not received. The states that represent these timers are clearly not as important as those that actually represent the current state of the communication, but must be included in the analysis since they still represent the status of the protocol (e.g., data not received).

### 4.2.1 Protocol Reduction

The classifier system, using the parameter settings 1 , was run using the protocol as its environment. Two different initial payoff strategies were used. One with fixed payoff values (set to 100.00) and the other using a random uniform strategy from 0 to 327. The results are shown in Figure 4.9. Note that the difference between the two is almost trivial, signalling that the classifier system can recognize useful states even in the presence of some


Figure 4.9 Payoff Values at Major Cycle 12,800 using the Communications Protocol

## Random initial payoff values <br> Non-random initial payoff values

"noise". This robust behavior can be attributed to the fact that certain states are rarely used, no matter what their initial payoff values are.

The LAPD protocol defined here has a number of states that could be classified as being used infrequently. States 5 through 9 in this case might be considered as not contributing to the protocol design as a whole. These states are transited to when a data packet is not received in sequence (states 5 and 5.1), when a busy signal is received from the receiving end (states 6 and 6.1 ), or when a timer expires.(states 7,8 , and 9 ). The timer could probably be combined into a single state and a logic could be set up to still provide the appropriate transitions. Minimizing the protocol would save on design costs in this case.

The result here achieved can also be used as a test for the classifier system's
learning ability. It would be logical to assume that states such as the ones that have been pinpointed by the classifier system would be rarely visited. Hence, the classifier system has learned to identify them correctly. The learning that has taken place can be observed by the number of schemata in the classifier list that support each state. Figure 4.10 shows the num-


Figure 4.10 Number of Classifier Schemata in the Classifier List Supporting Each State at Cycle Step 1000
ber of schemata supporting the protocol states while Figure 4.11 shows the same figure superimposed payoff values in the non-random case. Figure 4.11 shows that once learning is complete, the classifier list converges towards the states with the highest payoff values. Again, we must specify that the result that is produced by the classifier system is based on the transitions and visitations to those states. The "support" for the winning states can then be used for actually deciding whether the reduction of those states would be acceptable and plausible.

Related to the support for certain states by the classifiers is the increase in the strengths (or fitnesses) of the classifiers. A growth in strength is a sign that learning is taking


Figure 4.11 Number of Classifier Schemata in the Classifier List Supporting Each State at Cycle Step 12,000 Superimposed with the Payoff for those States at the Same Time


Figure 4.12 Flow of Strength in Classifier List
place and that the classifier system is "confident" in the solution it is reaching. The growth
in classifier strength is mapped in Figure 4.12. Note that the graph increases constantly with no disruption, marking the confidence the classifier system has in the problem solving task.

### 4.2.2 Protocol State Convergence

As it was with the generic automata that were tested earlier in this chapter, in minimizing the communications protocol attention must be paid to rapid convergence. Figure 4.13 shows the convergence of the classifier list towards the final set of states using


Figure 4.13 Convergence of Payoff Values for Protocol Using Three Different Heuristics
a threshold value of 500 .
The figure shows two different curves each representing the heuristic that was used in increasing the payoff value. The first one, which uses equation 3.1 , is used in all the testing done so far. It is the one that provides a very gradual convergence and does
not discriminate as much as the other one. In fact, the other equation, $P_{i}=P_{i}+1$, causes the system to very quickly converge towards a solution which would eliminate most of the states. Such a solution would not be very realistic. It is desirable to reduce the number of states in the design, but up to a certain limit. Otherwise, the functionality would certainly be compromised. Figure 4.14 gives a second way of looking at the convergence problem. The figure


Figure 4.14 Increase in Payoff Values for Protocol
shows how quickly the payoff values for the states increase. Note that three states very quickly receive high payoffs for they are the most commonly visited ones. Others increase gradually while a few remaining ones have almost no growth in payoff.

### 4.2.3 Perturbation and its Effects

- It was mentioned that the environment at times displays nonlinear behavior, which affects the performance of the classifier system. The performance is especially affected when the system has been interacting with the environment for a long time and learning has taken place to a great extent. If the environment is perturbed, the classifier system must adapt to the change and learn again, even though the classifier list may have already converged to a certain extent.

To see how such an event can affect the system's behavior, the payoff values of the states were randomly changed and re-mapped using the same probabilistic approach used to assign them initially. The simulated perturbation was injected half way through the run. Figure 4.15 shows the growth in fitness of classifiers that support two states. State 2 is


Figure 4.15 Comparison of Fitness of two Classifiers Supporting States 2 and 6.1 with no Perturbation
the state that receives the highest payoff during a normal run while state 6.1 is one of the low-payoff states. State 2 receives payoff quickly and its fitness increases rapidly while state
6.1 increases more slowly. In addition, the area that is dashed represents a discontinuous growth in state 6.1's curve since no classifiers existed in the classifier list in support of that state.

Figure 4.16 shows the same two states but with the perturbation introduced


Figure 4.16 Comparison of Fitness of Classifiers Supporting States 2 and 6.1 in Presence of Perturbation
after cycle 6000 . Note that the fitnesses fall to almost their initial values and then start increasing again. But, in addition, state 6.1 's curve rises almost as fast as state 2 . That signifies that the initial payoff may have moved in favor of state 6.1. But the classifier soon learns of the greater importance of state 2 and that curve starts growing faster.

### 4.2.4 Rewards

Rewards are essential for the classifier system to carry out its task properly. If no reward is received, the classifier system cannot determine whether it is discriminating properly among states. This is a problem that exists in many learning systems. Connectionist systems, for example, include a "tutor" that check the output that the network produces and see if the output is correct.

Figure 4.17 shows how the average fitness of the classifiers in the classifier


Figure 4.17 Flow of Strength in Classifier List with no Rewards Received
list in the absence of rewards. Clearly, when no rewards are received, the system loses a sense of the worthiness of states and cannot successfully determine which states should be allowed into the protocol design. If on the other hand a reward mechanism exists and guides the system, then the design benefits from the advice that the system produces.

Sharing rewards can also affects system performance to. When rewards are shared among all active classifiers in the classifier list, the fitness of the list is enhanced on
the average, allowing classifiers that are weaker to remain in the list for a longer period of time. A more aggressive strategy would allocate the reward only to classifiers that post their actions and produce effectors. Such a "greedy" approach would quickly eliminate weak classifiers and lead to a much more rapid convergence of the classifier list. This may be desired in environments that are deterministic and are not affected by abrupt changes. Such environments are not too commonplace, however.

The communications protocol, in fact, can often be affected by sudden changes and the classifier system must adapt to them. Figure 4.18 shows the difference in the average fitness of the classifiers between an approach that recompenses all active classifi-


Figure 4.18 Difference in Fitness Between a Strategy which Shared Rewards Among all Active Classifiers and One that Does Not
-ers and one that does not. For the most part, sharing rewards allows the average fitness to be higher than in the case were no rewards are shared. The tests that have been performed
up to now have utilized the greedy approach since the intent has been to discriminate the most among protocol states. In addition, in this simulated environment, the amount of perturbation is trivial, even though it could be induced, as was shown before.

## CHAPTER 5

## CONCLUSION

The results thus achieved demonstrate that the classifier system learning methodology can successfully identify states within a Finite State Machines that can be eliminated, reducing the complexity of the machine. As specified earlier, the classifier system functions more as an advisor than an implementor. In other words, the classifier system can only advise an FSM designer (for a specified domain such as communication protocols) what are the states in the system that could be removed without compromising the functioning of the machine. If the machine is then affected in its functionality, then the reduction cannot take place.

### 5.0 Results

The reduction procedure consisted of identifying what parameter settings would provide the greatest discriminatory results between the two sets of states (low payoff and high payoff) in the FSM.

Successful reduction requires that the classifier system be able to bias more towards those states that generate the higher payoffs. The greater the discrimination, the more confident the results. In Chapter 4 a discriminatory analysis was carried out which served to test this confidence. The mean payoff were computed for 9 different FSM, each with three different payoff sets. From these, the standard deviation were computed and the gap separating the lower range of the high payoff states and the higher range of the low payoff states was computed. The gap served to identify the amount of discrimination displayed by the classifier system.

Another way of analyzing this information is to consider what percentage of the range of deviation for all 18 states the gap covers. In other words, how much does the classifier system separate the two sets of states from one another? The greater the gap, the more the two sets of states are apart and the more confident the solution.

This type of analysis is carried out in Tables 5.1,5.2, and 5.3 which are related to Tables $4.9,4.10$, and 4.11 , respectively. The tables show the percentage that the discriminatory gap covers range of deviation for the payoff values for the 18 states of the FSM. Hence, if the mean for one of the machines for all 18 states were 1000 which a standard deviation of 250 , then the total range of deviation would be $(1000-250)+(1000+250)=2000$.

| FSM | Percent of Beviation |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1-1-1 | 1-1-2 | 1-1-3 | 1-2-1 | 1-2-2 | 1-2-3 | 1-3-1 | 1-3-2 | 1-3-3 |
| 8/10 | 12 | 15 | 5 | 5 | 3 | 2 | 13 | 14 | 11 |
| 10/8 | 15 | 14 | 8 | 7 | 2 | 4 | 12 | 11 | 10 |

Table 5.1 Gap Sizes as a Percentage of the Range of Deviation of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 327] for Payoffs and [0,24] for Transitions

If the gap were 200 , then the percentage that the gap covers is $10 \%$. This number gives a qualitative estimate of how much discriminatory power the classifier system produces.

| FSM | Percempof bemation |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2-1-1 | 2-1-2 | 2-1-3 | 2-2-1 | 2-2-2 | 2-2-3 | 2-3-1 | 2-3-2 | 2-3-3 |
| 8/10 | 3 | 5 | 6 | 19 | 19 | 12 | 11 | 10 | 11 |
| 10/8 | 4 | 5 | 7 | 15 | 15 | 8 | 11 | 11 | 13 |

Table 5.2 Gap Sizes as a Percentage of the Range of Deviation of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 250] for Payoffs and [0, 18] for Transitions

Hence, the greater the percentage, the greater the discrimination. The $10 / 8$ combination provides more discriminatory power than the $8 / 10$ combination, on the average.

Once these tests were completed and the best parameter setting was selected for usage on the FSM, the communications protocol was tested. This machine consisted of 12 states. The minimization procedure that was carried out showed that there are a number of states in the protocol that could be tentatively eliminated, given that the functionality of the protocol is not compromised. Any reduction in the size of that protocol could contribute to a reduction in the cost of design and a noticeable increase in performance.

| ISM | Mercento. Merator |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3-1-1 | $3-1-2$ | 3-1-3 | 3-2-1 | 3-2-2 | 3-2-3 | 3-3-1 | 3-3-2 | 3-3-3 |
| 8/10 | 3 | 3 | 3 | 4 | 4 | 4 | 12 | 9 | 8 |
| 10/8 | 6 | 10 | 6 | 5 | 7 | 5 | 24 | 20 | 19 |

Table 5.3 Gap Sizes as a Percentage of the Range of Deviation of Payoff Values for Three Different FSMs Using Two Different Payoff Sets with Random Uniforms [0, 150] for Payoffs and [ 0,12 ] for Transitions

Hence, it was suggested that some of the states representing timers in the protocol be reduced to one state generalizing the functions of all of them, but producing the same outputs.

### 5.1 Communication Protocols Used as the Environment

Clearly, the method developed here can be expanded to usage in other FSMs and protocols. Another type of environment that could be constructed would consist of a number of different communicating protocols (i.e., each representing a different node in a communication network). Hence, each protocol would represent a separate state in the environment. The system could then find the optimal number of nodes in such a communication system. The classifier system could determine what the load is on such a system and try to
find a better configuration. The system could then learn to identify such problems in a number networks and attempt to provide similar solutions, if acceptable.

A more challenging task, derived from the above, would be to find out how the network could be expanded in case the network is already at high capacity. The classifier system would have to analyze the load and determine an appropriate new configuration.

### 5.2 Classifier Systems Revisited

The above results that have been achieved are encouraging and demonstrate the capabilities that classifier systems have in interacting with different environments and solving different types of problems. However, there are a number of ways to make them work more effectively.

First of all, a number of different strategies could be used for crossover and mutation, that have recently shown greater success than the traditional operators used in CFS-C. A number of different crossover operators have been proposed, including single arithmetic crossover, whole arithmetic crossover [44], partially mapped crossover, order crossover, cycle crossover, and position-based crossover [67]. Some mutation techniques include boundary mutation, non-uniform mutation [44], swap, reverse, and remove-and-reinsert [41]. These techniques could prove to be more appropriate for the learning process, generating more robust behavior.

A second way of improving classifier system performance might be of that of incorporating the concepts of synthetic systems [52]. Classifier systems demonstrate simulation systems, where the genetic classifiers and the environment with which they interact are simulated. Specifically, classifier systems are designed a priori with an understanding of what the intemal configuration of the system is and what the environment looks like. But this may not be sufficient. Rather, there may be a need to introduce some elasticity, allowing
classifier systems to be independent of the design specifications. One example might be that of the population size in the classifier system, which is fixed. But fixing the number of classifier allowed might greatly influence the performance of the system, because at a given time the classifier system may need some of the classifiers that are being replaced. This is especially true when an elitist strategy is used, attempting to maintain the strongest classifiers at every time step. But as it was mentioned, very often weal classifiers may be needed in the future, and a simple heuristic that randomly keeps a few for future use may not be sufficient. Rather, the classifier list should be able to grow and shrink, depending on the status of the environment with which it interacts and the relevance of the classifiers.

The environment can also be synthesized. The environment does not change only its state or condition, but also its appearance. In such an environment, the classifier system must be able to update its representation of the environment at all times. This is one way of learning too, characterized by learning to recognize diverse phenomena.

Third, the incorporation of long-term memory would be helpful in the classifier systems. Zhou [77] already implemented such a system using an associative memory. Another mechanism that can be used is that developed by Beale et al. [4]. This implementation uses a distributed associative memory for storing information that is classified during a run of their connectionist system. The memory is composed of à matrix that consist of a number of vectors (representing the size of the memory) to which the input vector (or in the classifier system, the detector) would be matched. If a perfect match occurs between the input vector and a vector that was stored, that vector is recalled. Otherwise, the closest match is used by counting the number of bits that match between the two vectors. The vector with the highest match is then recalled. Beale et al. describe the memory in the following fashion:

1. A memory matrix $M_{i j}$
2. An input vector (detector) $A_{i}$
3. A class vector (the vector in memory) $C_{j}$

When the system is learning, the memory stores all the vectors in memory:

$$
M_{i j}=\left\{\begin{array}{ll}
1 & A_{i}, C_{j}=1 \\
0 & \text { otherwise }
\end{array} \quad \forall i, j\right.
$$

and during recall, the vectors are fetched back:

$$
\text { Recalled vector } R_{j}=\sum_{p=0}^{i} M_{p} j^{*} A_{p} \quad \forall j
$$

This can easily be applied to classifier systems since the population could be initialized using the above method. Rather than randomly generating the population, a detector could capture the current status of the environment (used as the input vector to the memory) and recall stored classifiers. The above equations would have to be modified to include the don't care symbol \#. If enough classifiers cannot be recalled using this method, the remaining members of the population could be generated randomly. Even in such case, the learning process could be greatly improved.

These and other extensions to classifier systems may prove to greatly increase their learning capability and enhance their robustness in a number of different domain. The ultimate goal would be to have a classifier system that can adapt to a number of environments. The technology to date does not provide such flexibility, even though its generality could be a powerful tool.

## REFERENCES

[1] Ackley, David, and Micheal Littman "Interactions Between Learning and Evolution" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992), pp. 487-510
[2] Antonisse, Hendrik James and K. S. Keller "Genetic Operators for High-Level Knowledge Representations" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[3] Antonisse, Hendrik James "A Grammar-Based Genetic Algorithm" in Gregory J.E. Rawlins (Editor) Foundations of Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991)
[4] Beale, Russell, Finlay, Janet, Austin, James, and Micheal Harrison "User Modelling by Classification: a Neural-based Approach" in Taylor, J. G. and C. L. T. Mannion (Editors) New Developments in Neural Computing IOP (1989), pp. 103-110
[5] Bec, Louis "Eléments d'Epistemologie Fabulatoire" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addi-son-Wesley (1992), pp. 799-812
[6] Belew, Richard K., McInerney, John, and Nicol N: Schraudolph. "Evolving Networks: Using the Genetic Algorithm with Connectionist Learning". in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II; A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992), pp.511-548
[7] Bochmann, Gregor V. "A General Transition Model for Protocols and Communication Service" in IEEE Transactions on Communications, vol. COM-28, no. 4, (April 1980)
[8] Booker, Lashon B. "Classifier Systems that Learn Internal World Models" in Machine Learning, 3, (1988), pp.161-192
[9] Booker, Lashon B. "Improving the Performance of Genetic Algorithms in Classifier Sys-
tems" in Proceedings of the Third International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufman Publishers (1989)
[10] Booker, Lashon B., Goldberg, David E., and John H. Holland "Classifier Systems and Genetic Algorithms" in Artificial Intelligence, 40 (1989) pp. 235-282
[11] Booker, Lashon B. "Representing Attribute-Based Concepts in a Classifier System" in Gregory J.E. Rawlins (Editor) Foundations of Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991)
[12] Carbonell, J.G. "Learning by Analogy: Formulating and Generalizing Plans from Past Experience" In Michalski, R.S., Carbonell, J.G., and Mitchell, T.M. (Editors). Machine Learning: An Artificial Intelligence Approach, Vol. 1, San Mateo, CA: Morgan Kaufman Publishing (1983)
[13] Cariani, Peter "Emergence and Artificial Life" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II; A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addi-son-Wesley (1992), pp. 775-779
[14] Cohoon, J. P., Hegde, S.U., Martin, W. N., and D. Richards "Punctuated Equilibria: A Parallel Genetic Algorithm" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[15] Davidor, Yuval. Genetic Algorithms and Robotics: A Heuristic Strategy for Optimization. Teaneck, NJ: World Scientific (1991)
[16] Davis, Lawrence. Handbook of Genetic Algorithms. New York: Van Nostrand Reinhold (1991)
[17] DeJong, Kenneth A. "Genetic-Algorithm-Based Learning" in Michalski, R.S., and Y. Kodratoff (Editors). Machine Learning: An Artificial Intelligence Approach, Vol. 3, San Mateo, CA: Morgan Kaufman Publishing (1990)
[18] Dhillon, Balbir S. Reliability Engineering in Systems Design and Operation. New York, NY: Van Nostrand Reinhold (1983)
[19] Dolan, Charles P. and Micheal G. Dyer "Towards the Evolution of Symbols" in John J.

Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[20] Elman, Jeffrey L. "Distributed Representations, Simple Recurrent Networks, and Grammatical Structures" in Machine Learning, 7, (1991), pp. 195-225
[21] Forrest, Stephanie. Parallelism and Programming in Classifier Systems. San Mateo, CA: Morgan Kaufmann Publishers (1991)
[22] Frey, Peter W. and David J. Slate "Letter Recognition Using Holland-Style Adaptive Classifiers" in Machine Learning, 6, (1991), pp. 161-182
[23] Goldberg, D. E. Genetic Algorithms in Search. Optimization, and Machine Learning, Reading, MA: Addison-Wesley (1989)
[24] Goldberg, David E., Earickson, Jeff A., and Robert E. Smith "SGA-Cube: A Simple Genetic Algorithm fornCUBE 2Hypercube Parallel Computers" The Clearinghouse for Genetic Algorithms TCGA Report No. 91005 , Department of Engineering Mechanics, University of Alabama, Tuscaloosa, Alabama (1991)
[25] Goldberg, S.H. and J. A. Mouton, Jr. "A Base for Portable Communications Software" in IBM Systems Journal, vol. 30 no. 3 (1991)
[26] Green, Paul E. (Editor) Computer Network Architectures and Protocols. New York, NY: Plenum Press (1982)
[27] Greene, David Perry and Stephen F. Smith "A Genetic System for Learning Models of Consumer Choice" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[28] Grefenstette, John J. "Genetic Algorithms and Their Applications" in Kent, A. and J.G. Williams (Editors). The Encyclopedia of Computer Science and Technology volume 21, Marcel Dekker (1990)
[29] Grefenstette, John J., Ramsey, Connie L., and Schultz, Alan C. "Learning Sequential Decision Rules Using Simulation Models and Competition" in Machine Learning, 5 (1990), pp. 355-381
[30] Grefenstette, John J. "Strategy Acquisition with Genetic Algorithms" in Handbook of Genetic Algorithms. New York: Van Nostrand Reinhold (1991)
[31] Hashem, Mostafa S. and M. Ümit Uyar "Protocol Modeling for Conformance Testing: Case Study for the ISDN LAPD Protocol" in AT\&T Technical Journal, vol 69, no 1. (January/February 1990)
[32] Hillard, M. R., Liepins, G. E., Palmer, Mark, Morrow, Micheal, and Jon Richardson "A Classifier-based System for Discovering Scheduling Heuristics" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[33] Holland, John H. Adaption in Natural and Artificial Systems. Ann Arbor, MI: The University of Michigan Press (1975)
[34] Holland, John J. "A Cognitive System with Powers of Generalization and Adaption". Unpublished manuscript, Department of Computer and Communication Sciences, University of Michigan, Ann Arbor, MI (1977)
[35] Holland, John H. "Escaping Brittleness: The Possibilities of General Purpose LearningAlgorithms Applied to Parallel Rule-Based Systems" in Michalski, R.S., Carbonell, J.G., and Mitchell, T.M. (Editors). Machine Learning: An Artificial Intelligence Approach, Vol. 2, San Mateo, CA: Morgan Kaufman Publishing (1986)
[36] Holland, John H., Holyoak, Keith J., Nisbett, Richard E., and Thagard, Paul R. Induction: Processes of Inference, Learning, and Discovery. Cambridge, MA: Massachusetts Institute of Technology Press (1986)
[37] Holland, John H. "Genetic Algorithms and Classifier Systems: Foundations and Future Directions" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[38] ISO/TC97/SC21, "Information processing systems - Open systems interconnection, Basic Reference Model" in ISO 7498. Geneva, Switzerland: International Organization for Standardization (1984)
[39] Jog, Prasanna, and Dirk Van Gucht "Parallelisation of Probabilistic Sequential Search

Algorithms" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications; Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[40] Kohavi, Zvi Switching and Finite Automata Theory. New York, NY: McGraw-Hill Book Compay (1978)
[41] Kurzweil, Raymond. The Age of Intelligent Machines. Cambridge, MA: Massachusetts Institute of Technology Press (1990)
[42] Manderick, Bernard, de Weger, Mark, and Pief Spiessens "The Genetic Algorithm and the Structure of the Fitness Landscape" in Belew, Richard K. and Lashon B. Booker (Editors). Proceedings of the Fourth International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991), pp. 324-333
[43] Matwin, Stan, Szapiro, Tom, and Haigh, Karen "Genetic Algorithms Approach to a Negotiation Support System" IEEE Transactions on Systems, Man, and Cybernetics, vol. 21, No. 1 (January/February 1991), pp. 102-114
[44] Michalewicz, Zbigniew, and Cezary Z. Janikow "Handling Constraints in Genetic Algorithms" in Belew, Richard K. and Lashon B. Booker (Editors). Proceedings of the Fourth International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991), pp. 324-333
[45] Minsky, Marvin. "A Framework for Representing Knowledge" in P.H. Winston (Editor). The Psychology of Computer Vision. New York: McGraw-Hill (1975)
[46] Minsky, Marvin. Society of Mind. New York: Simon and Schuster Publishers (1986)
[47] Montana, David J. "Empirical Learning Using Rule Threshold Optimization for Detection of Events in Synthetic Images" in Machine Learning, 5, (1990), pp. 427-450
[48] von Neumann, John Theory of Self-Reproducing Automata. Edited by Arthur W. Burks. Urbana, IL: University of Illinois Press (1966)
[49] Nussbaumer, Henri. Computer Communication Systems. Volume 1, Data Circuits, Error Detection, Data Links. New York: John Wiley \& Sons (1990)
[50] Pettey, Chrisila B., Leuze, Micheal R., and John J. Grefenstette "A Parallel Genetic Al-
gorithm" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[51] Radcliffe, Nicholas J. "Genetic Set Recombination and its Application to Neural Network Topology Optimization" Techinal Report EPCC TR9121 of the Edinburgh Parallel Computing Center: University of Edinburgh (1991)
[52] Radcliffe, Nicholas J. "Equivalence Class Analysis of Genetic Algorithms" Techinal Report EPCC TR9003 of the Edinburgh Parallel Computing Center: University of Edinburgh (1990)
[53] Ray, Thomas S. "An Approach to the Synthesis of Life" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II; A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992), pp. 371-408
[54] Riolo, Rick L. "LETSEQ1: An Implementation of the CFS-C Classifier System in a Task Domain that Involves Learning to Predict Letter Sequences" Logic of Computers Group: University of Michigan (1988)
[55] Riolo, Rick L. "CFS-C/FSW1: An Implementation of the CFS-C Classifier Systemin a task Domain that Involves Learning to Traverse a Finite State World" Logic of Computers Group: University of Michigan (1988)
[56] Riolo, Rick L. "CFS-C: A Package of Domain Independent Subroutines for Implementing Classifier Systems in Arbitrary, User-Defined Environments" Logic of Computers Group:University of Michigan (1988)
[57] Riolo, Rick L. "The Emergence of Coupled Sequences of Classifiers" in Proceedings of the Third International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufman Publishers (1989)
[58] Riolo, Rick L. "The Emergence of Default Hierarchies in Learning Classifier Systems" in Proceedings of the Third International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufman Publishers (1989)
[59] Riolo, Rick L. "Modeling Simple Human Category Learning with a Classifier System" in Belew, Richard K. and Lashon B. Booker (Editors). Proceedings of the Fourth Interna-
tional Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991), pp. 324-333
[60] Robertson, George G. "Parallel Implementation of Genetic Algorithms in a Classifier System" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications; Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[61] Sannier, Adrain V. II, and Erik D. Goodman "Genetic Learning Procedures in Distributed Environments" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[62] Schank. Roger C. Real and Artificial Memories. New York, NY: Scribner's \& Sons (1990)
[63] Shaefer, Craig G. "The ARGOT Strategy: Adaptive Representation Genetic Optimizer Technique" in John J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[64] Smith, S.F. "A Learning System Based on Genetic Adaptive Algorithms" Ph.D. Dissertation, Department of Computer Science, University of Pittsburgh, Pittsburg, PA. (1980)
[65] Smith, S.F. "Flexible Leaming of Problem Solving Heuristics Through Adaptive Search" in Proceedings of the Eighth International Joint Conference on Artificial Intelligence, Karlsruhe, Germany (1983)
[66] Spears, William M. and Kenneth DeJong "Using Neural Networks and Genetic Algorithms as Heuristic for NP-complete Problems" in International Joint Conference on Neural Networks, volume I, Hillsdale, NJ: Lawrence Erlbaum Associates (1990), pp. 118-125
[67] Starkweather, T, McDaniel, S. Mathias, K. Whitley D., and C. Whitley "A Comparison of Genetic Sequencing Operators" in Belew, Richard K. and Lashon B. Booker (Editors). Proceedings of the Fourth International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufmann Publishers (1991), pp. 324-333
[68] Stein, Richard M. "Real Artificial Life" Byte (January 1991), pp. 289-297
[69] Stork, David G., Jackson, Bernie and Scott Walker "Non-Optimality" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Addison-Wesley (1992), pp. 409-430
[70] Sutton, Richard S. "Learning to Predict by the Methods of Temporal Differences" in Machine Learning, 3 (1988), pp. 9-44
[71] Tanese, Reiko "Parallel Genetic Algorithm for a Hypercube" in ohn J. Grefenstette (Editor). Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms. Hillsdale, NJ: Lawrence Erlbaum Associates (1987)
[72] Taylor, Charles E. "'Fleshing Out' Artificial Life II" in Langton, Christopher G., Taylor, Charles, Farmer, J. Doyne, and Steen Rasmussen. Artificial Life II: A Proceedings Volume in the Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, CA: Ad-dison-Wesley (1992), pp. 25-38
[73] Wilson, Stewart W. "Knowlegde Growth in an Artificial Animat" in Proceedings of the Fourth Yale Workshop on Applications of Adaptive Systems (1985)
[74] Wilson, Stewart W. "Classifier System Learning of a Boolean Function" Research Memo RIS-27r, Cambridge, MA: Rowland Institute for Science (1986)
[75] Wilson, Stewart W and David E. Goldberg "A Critical Review of Classifier Systems" in Proceedings of the Third International Conference on Genetic Algorithms. San Mateo, CA: Morgan Kaufman Publishers (1989)
[76] Zhou, Harry H. "CSM: A Computational Model of Cumulative Learning" Machine Learning, 5 (1990), pp. 383-406
[77] Zhou, Harry H. "CSM: A Genetic Classifier System with Memory for Learning by Analogy" Ph.D. Dissertation, Computer Science Department: Vanderbilt University (1987)

## APPENDIX A

## PROTOCOL ENVIRONMENT SPECIFICATION

The specifications for the FSM used in this thesis are given by Hashem et al [31]. Here a few relevant features are described.

The protocol defines a networking system where information is carried via packets, or frames. However, different kinds of frames exist. One frame is sent to the receiver (i.e., user A sends to user B) to establish connection, called sabme. Another frame, the I-frame, contains the actual packet of information being sent. Each I-frame is usually part of sequence of such frames part of an entire message being sent across the network. Each I-frame contains a number, $N(S)$, identifying the frame's position in the sequence of frames. Finally, a disc frame exists to disconnect the communication, or logical link, which defines the medium where the transfer of the data actually occurs. There exist also a number of timers that assure system functioning. The T200 timer assures that once an I-frame is sent to the receiver, the receiver acknowledges its receipt after some time. The T203 timer is used to prevent the system from remaining idle indefinitely. Bad frames can also be transmitted for a number of reasons [31]. Table A. 1 defines the states as they are defined by Hashem et al.

A number of frames are defined for this protocol. There also a number of input and output service primitives that designate the state of the communication at any stage. A number of other frames are added by Hashem et al. for the testing of the protocol. The specifics of the above will not be discussed here. The intent, rather, is to give a general idea of what the protocol looks like and how it is defined in the classifier system.

The text that follows shows how the environment is defined in CFS-C and for this particle application. Please note that in the transitions section, the text in italics is not part of the file designating the environment. It is included here to show the transitions defined
in the protocol. All the state specifications and the transitions are derived from Hashem et al. [31].
(

Table A. 1 Data-Link Layer State Descriptions

Each state is represented as a chromosome consisting of 16 bits. The left-most two bits, which are set to ' 00 ' define the type of the chromosome, which in this case are detectors since the system updates its classifier list and by checking what the current state of the FSM is.

Each state definition is given as num1, num2, chromosome, where numl is the identification of the state, num 2 is the visit count, which is set to 0 for all states except for the first one, which is the initial state. The chromosome follows.

The payoffs follow as a pair state, payoffl, where the payoff in this case is equal across all states since all the states in the protocol are important to start with.

The transitions of the FSM are defined as $s l, r, s 2, p$, where $s l$ is the current state the system is in, $r$ is the $r$ value that leads to that state (edge value), $s 2$ is the state the edge leads to, and $p$ is the probability that the system will move to that state.

Finally, the effector values are given. The first effector defines the effector that usually determines the $r$ value. There are cases, however, that the system will not generate an effector. In such cases, a default effector is used that will keep the classifier system interacting with the environment.
; The number of states followed by the initial state followed by a threshold value 12, 0, 500
; These are the states and their equivalents in the LAPD model:
; Classifier System State LAPD State Description

| 0 | s1 | Initial state |
| :---: | :---: | :---: |
| 1 | s2 | Await EST |
| 2 | s3 | Await REL |
| 3 | s4 | MF EST NORM |
| 4 | s4.1 | MF EST NORM WITH WINDOW CLOSED |
| 5 | s5 | MF EST REJ |
| 6 | s5.1 | . MF EST REJ WITH WINDOW CLOSED |
| 7 | s6 | MF EST BUSY |
| 8 | s6.1 | MF EST BUSY WITH WINDOW CLOSED |


| 9 | s7 TM REC NORM |  |  |
| :---: | :---: | :---: | :---: |
| 10 | s8 | TM REC REJ |  |
| 11 | s9 | TM REC BUSY |  |
| 0, 1, 0000000000100001 |  |  |  |
| 1,0,0000 000001000001 |  |  |  |
| 2, 0,0000000001010101 |  |  |  |
| 3, 0, 0000010101010101 |  |  |  |
| 4, 0, 0000010100010100 |  |  |  |
| 5, 0, 0001000101000101 |  |  |  |
| 6, 0,0001 010100010101 |  |  |  |
| 7, 0, 0000010000010101 |  |  |  |
| 8, 0, 0001010100010101 |  |  |  |
| 9, 0, 0000000000100110 |  |  |  |
| 10, 0, 0000000001000001 |  |  |  |
| 11, 0, 0000000001000011 |  |  |  |
| ENDSTATES |  |  |  |
| 0,100/ 1,100/2,100/3,100/4,100/5,100/6,100/7,100/8,100/9,100/10,100/11,100 |  |  |  |
| ENDPAYOFFS |  |  |  |
| ; State s1 |  |  |  |
| 0,1, 0,1 | WHEN (disc | $d m f()$ | inopportune |
| 0,2, 0,1 | WHEN (disc | $d m f 1)$ | inopportune |
| 0,3, 3,1 | WHEN (sabme | , ua_fl_13_e8) | valid |
| 0,4, 3,1 | WHEN (sabm | , ua_f0_13_e8) | valid |
| 0,5, 1,1 | WHEN (13_ | me_pl) | valid |
| 0,6, 1,1 | WHEN (dm | bme pl) | valid |
| 0,7, 0,1 | WHEN (time | 200, null) | timer |
| 0,8, 0,1 | WHEN (time | 203, null) | timer |
| ; State s2 |  |  |  |
| 1,9, 0,1 | WHEN (dm | _e7) | valid |
| 1,10, 1,1 | WHEN (sabm | 0, ua f0) | inopportune |
| 1,11, 1,1 | WHEN (sabm | 1, uaf1) | inopportune |
| 1,1, 1,1 | WHEN (disc | $d m$ f0) | inopportune |
| 1,2, 1,1 | WHEN (disc | $d m$ fl) | inopportune |
| 1,12, 3,1 | WHEN (ua | -e9) | valid |
| 1,13, 1,1 | WHEN (time | 200, sabme_pl) | timer |
| 1,14, 0,1 | WHEN (time | 200_N200, 13_e7) | timer |
| ; State 3 - - |  |  |  |
| 2,15, 0,1 | WHEN (ua | e10) | valid |
| 2,16, 0,1 | WHEN (dm | _el0) | valid |
| 2,17, 2,1 | WHEN (sabme | , dm fl) | inopportune |
| 2,18, 2,1 | WHEN (sab | $0, d m f 0)$ | inopportune |
| 2,19, 2,1 | WHEN (disc | ua fl) | inopportune |
| 2,20, 2,1 | WHEN (disc | uaf0) | inopportune |
| 2,21, 2,1 | WHEN (time | 200, 13_e7) | timer |


| 2,22, 0,1 | WHEN (timeout_t200_N200, disc_p1) | timer |
| :---: | :---: | :---: |
| ; State 4 - - |  |  |
| 3,4, 3,1 | WHEN (sabme p0, ua_f0_13_e8) | valid |
| 3,3, 3,1 | WHEN (sabme pl, ua_fl_13_e8) | valid |
| 3,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | valid |
| 3,24, 0,1 | WHEN (disc_pl, ua_fl_13_e7) | valid |
| 3,5, 1,1 | WHEN (13_el, sábme $\bar{p} 1)$ | valid |
| 3,25, 2,1 | WHEN (13_e2, disc_p1) | valid |
| 3,26, 5,1 | WHEN (13_e3, rnr_p0) | valid |
| 3,27, 3,1 | WHEN (13_e5, ip0) | valid |
| 3,28, 4,1 | WHEN (13_e6_max, $i$ p 0 ) | valid |
| 3,29, 3,1 | WHEN (13_e6,i_p0) | valid |
| 3,6, 1,1 | WHEN (dm_f0, sabme p 1 ) | inopportune |
| 3,30, 2,1 | WHEN (dm_f1, sabme_p1) | inopportune |
| 3,31, 1,1 | WHEN (frmr f0, sabme_p1) | inopportune |
| 3,32, 1,1 | WHEN (frmr fl, sabme_pl) | inopportune |
| 3,33, 3,1 | WHEN (i_pl,rr_fl_13_ell) | valid |
| 3,34, 3,1 | WHEN (i_p0, rrfo_13_ell) | valid |
| 3,35, 3,1 | WHEN (i_p0_cond, i_p0_13_ell) | valid |
| 3,36, 4,1 | WHEN (i_pO_unexp_ns, $\mathrm{rej}_{j} \bar{f} 0$ ) | inopportune |
| 3,37, 4,1 | WHEN (i_pl_unexp_ns, rej_fl) | inopportune |
| 3,38, 3,1 | WHEN (rrp $\overline{0}$, null) ${ }^{-}$ | valid |
| 3,39, 3,1 | WHEN (rr_f0, null) | valid |
| 3,40, 3,1 | WHEN (rrfl, null) | inopportune |
| 3,41, 3,1 | WHEN (rrpl, rr fl) | valid |
| 3,42, 3,1 | WHEN (rnr_p0, null) | valid |
| 3,43, 3,1 | WHEN (rnrff0, null) | valid |
| 3,44, 3,1 | WHEN (rnr fl, null) | inopportune |
| 3,45, 3,1 | WHEN (rnr_pl, rr_fl) | valid |
| 3,46, 3,1 | WHEN (rej p0, null) | valid |
| 3,47, 3,1 | WHEN (rej fo, null) | valid |
| 3,48, 3,1 | WHEN (rej fl, null) | inopportune |
| 3,49, 3,1 | WHEN (rej_pl, rr_fl) | valid |
| 3,50, 1,1 | WHEN (bad frame, sabme pl) | illegal |
| 3,51, 9,1 | WHEN (timeout_t200, rr_pl) | timer |
| 3,52, 3,1 | WHEN (timeout_t203, rr_pl) | timer |
| ; State s4.1 |  |  |
| 4,4, 3,1 | WHEN (sabme_p0, ua f0_13_e8) | valid |
| 4,3, 3,1 | WHEN (sabme pl, ua_fl-13-e8) | valid |
| 4,23, 0,1 | WHEN (disc p0, ua_f0_13_e7) | inopportune |
| 4,24, 0,1 | WHEN (disc_pl, ua_fl_13_e7) | inopportune |
| 4,5, 1,1 | WHEN (13_el, sabme_pl) | inopportune |
| 4,25, 2,1 | WHEN (13_e2, disc_pl) | inopportune |
| 4,26, 8,1 | WHEN (13_e3, rnr p0) | inopportune |


| 4,53, 4,1 | WHEN (13_e6, null) | inopportune |
| :---: | :---: | :---: |
| 4,6, 1,1 | WHEN (dm_f0, sabme_pl) | inopportune |
| 4,30, 1,1 | WHEN (dm fl, sabme pl) | inopportune |
| 4,31, 1,1 | WHEN (frmr_f0, sabme_pl) | inopportune |
| 4,32, 1,1 | WHEN (frmr fl, sabme_pl) | inopportune |
| 4,33, 4,1 | WHEN (i_pl,rr_fl_13_ell) | inopportune |
| 4,34, 4,1 | WHEN (i p0, rr_f0_13_el1) | inopportune |
| 4,54, 3,1 | WHEN (i_pl_max, rr_fl_13_ell) | valid |
| 4,55, 3,1 | WHEN (ip0_max, rr_f0_13_ell) | valid |
| 4,56, 3,1 | WHEN (rr_p0_max, null) | valid |
| 4,57, 3,1 | WHEN (rr_f0 max, null) | valid |
| 4,58, 3,1 | WHEN (rr_pI_max, null) | valid |
| 4,59, 3,1 | WHEN (rnr_p0_max, null) | valid |
| 4,60, 3,1 | WHEN (rnr f0 max, null) | valid |
| 4,61, 3,1 | WHEN (rnr_pl_max, null) | valid |
| 4,62, 3,1 | WHEN (rej_p0_max, null) | valid |
| 4,63, 3,1 | WHEN (rej,fo_max, null) | valid |
| 4,64, 3,1 | WHEN (rej_pl_max, null) | valid |
| 4,36, 6,1 | WHEN (i p0_unexp_ns, rej_f0) | inopportune |
| 4,37, 6,1 | WHEN (i_pl_unexp_ns, rej_fl) | inopportune |
| 4,38, 4,1 | WHEN (rr_p0, null) | inopportune |
| 4,39, 4,1 | WHEN (rr f0, null) | inopportune |
| 4,40, 4,1 | WHEN (rrff, null) | inopportune |
| 4,41, 4,1 | WHEN (rrpl, rrff) | inopportune |
| 4,42, 4,1 | WHEN (rnr_p0, null) | inopportune |
| 4,43, 4,1 | WHEN (rnr_f0, null) | inopportune |
| 4,44, 4,1 | WHEN (rnr fl, null) | inopportune |
| 4,45, 4,1 | WHEN (rnr_pl, rr fl) | inopportune |
| 4,46, 4,1 | WHEN (rej p0, null) | inopportune |
| 4,47, 4,1 | WHEN (rej fo, null) | inopportune |
| 4,48, 4,1 | WHEN (rej fl, null) | inopportune |
| 4,49, 4,1 | WHEN (rej_pl, rr_fl) | inopportune |
| 4,50, 1,1 | WHEN (bad frame, sabme pl) | illegal |
| 4,51, 6,1 | WHEN (timeout_t200, rr_pl) | timer |
| 4,52, 4,1 | WHEN (timeout_t203, rr_pl) | timer |
| ; State s5 |  |  |
| 5,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | valid |
| 5,24, 0,1 | WHEN (disc_p1, ua_fl_13_e7) | valid |
| 5,4, 3,1 | WHEN (sabme p0, ua fo_13_e8) | valid |
| 5,3, 3,1 | WHEN (sabme pl, ua_fl_13_e8). | valid |
| 5,6, 1,1 | WHEN (dm_f0, sabme pl) | inopportune |
| 5,50, 1,1 | WHEN (dm_fl, sabme pl) | inopportune |
| 5,31, 1,1 | WHEN (frmr f0, sabme_pl) | inopportune |
| 5,32, 1,1 | WHEN (frmr_fl, sabme_pl) | inopportune |


| 5,5, 1,1 | WHEN (13_el, sabme_pl) | valid |
| :---: | :---: | :---: |
| 5,25, 2,1 | WHEN (13_e2, disc_pl) | valid |
| 5,26, 7,1 | WHEN (13_e3, rnr_p0) | valid |
| 5,27, 5,1 | WHEN (13_e5, ip0) | valid |
| 5,28, 5,1 | WHEN (13_e6, ip 0 ) | valid |
| 5,29, 6,1 | WHEN (13_e6_max, i p 0 ) | valid |
| 5,33, 3,1 | WHEN (i_p1_rr_f1_13_ell) | valid |
| 5,34, 3,1 | WHEN (i_p0, rr_f0_13_el1) | valid |
| 5,35, 3,1 | WHEN (i_p0_cond, i_p0_13_el1) | valid |
| 5,65, 5,1 | WHEN (i p0_unexp_ns, null) | inopportune |
| 5,66, 5,1 | WHEN (i_pl_unexp_ns, rr_fl) | inopportune |
| 5,38, 5,1 | WHEN (rr_po, null) | inopportune |
| 5,39, 5,1 | WHEN (rrpl, rr_fl) | inopportune |
| 5,40, 5,1 | WHEN (rr f0, null) | inopportune |
| 5,41, 5,1 | WHEN (rr_fl, null) | inopportune |
| 5,42, 5,1 | WHEN (rnr_p0, null) | inopportune |
| 5,43, 5,1 | WHEN (rnr_f0, null) | inopportune |
| 5,44, 5,1 | WHEN (rnr fl, null) | inopportune |
| 5,45, 5,1 | WHEN (rnr_pl, rr fl) | inopportune |
| 5,46, 5,1 | WHEN (rej f0, null) | inopportune |
| 5,47, 5,1 | WHEN (rej_fl, null) | inopportune |
| 5,48, 5,1 | WHEN (rejp0, null) | inopportune |
| 5,49, 5,1 | WHEN (rej_pl, rr_fl) | inopportune |
| 5,50, 1,1 | WHEN (bad_frame, sabme pl) | illegal |
| 5,51, 10,1 | WHEN (timeout_t200, rr pl) | timer |
| 5,52, 5,1 | WHEN (timeout_t203, rr_pl) | timer |
| ; State s5.1 |  |  |
| 6,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | inopportune |
| 6,24, 0,1 | WHEN (disc p1, ua_fl_13_e7) | inopportune |
| 6,4, 3,1 | WHEN (sabme p0, ua_fo_l3_e8) | valid |
| 6,3, 3,1 | WHEN (sabme_p1, ua_fl_13_e8) | valid |
| 6,54, 5,1 | WHEN (i_pl_max, rr_fl_13_ell) | valid |
| 6,55, 5,1 | WHEN (i_p0_max, rr f0_13_ell) | valid |
| 6,56, 5,1 | WHEN (rr_p0_max, null) | valid |
| 6,57, 5,1 | WHEN (rr_f0_max, null) | valid |
| 6,58, 5,1 | WHEN (rr_pl_max, null) | valid |
| 6,59, 5,1 | WHEN (rnr_p0_max, null) | valid |
| 6,60, 5,1 | WHEN (rnr_f0_max, null) | valid |
| 6,61, 5,1 | WHEN (rnr_pI_max, null) | valid |
| 6,62, 5,1 | WHEN (rej_p0_max, null) | valid |
| 6,63, 5,1 | WHEN (rej_f0_max, null) | valid |
| 6,64, 5,1 | WHEN (rej pl_max, null) | valid |
| 6,6, 1,1 | WHEN (dm_f0, sabme pl) | inopportune |
| 6,30, 1,1 | WHEN (dm fl, sabme pl) | inopportune |


| 6,31, 1,1 | WHEN (13_el, sabme_pl) | inopportune |
| :---: | :---: | :---: |
| 6,25, 2,1 | WHEN (13-e2, disc pl $)$ | inopportune |
| 6,26,8,1 | WHEN (13_e3, rnr_p0) | inopportune |
| 6,53, 6,1 | WHEN (13_e6, disc pl) | inopportune |
| 6,33, 4,1 | WHEN (i_pl, rrfl_l3_ell) | inopportune |
| 6,34, 4,1 | WHEN ( ip 0, rrfo_13_ell) | inopportune |
| 6,65, 6,1 | WHEN (i_p0_unexp_ns, null) | inopportune |
| 6,66, 6,1 | WHEN (i_pl_unexp_ns, rr_fl) | inopportune |
| 6,38, 6,1 | WHEN (rr $p \overline{0}$, null) | inopportune |
| 6,39, 6,1 | WHEN (rrpl, rrfl) | inopportune |
| 6,40, 6,1 | WHEN (rr_f0, null) | inopportune |
| 6,41, 6,1 | WHEN (rrfl 1 , null) | inopportune |
| 6,42, 6,1 | WHEN (rnr_p0, null) | inopportune |
| 6,43, 6,1 | WHEN (rnr f0, null). | inopportune |
| 6,44, 6,1 | WHEN (rnr_fl, null) | inopportune |
| 6,45, 6,1 | WHEN (rnr_pl, rrff) | inopportune |
| 6,46, 6,1 | WHEN (rej f0, null) | inopportune |
| 6,47, 6,1 | WHEN (rej fl, null) | inopportune |
| 6,48, 6,1 | WHEN (rej p0, null) | inopportune |
| 6,49, 6,1 | WHEN (rej_pl, rr_fl) | inopportune |
| 6,50, 1,1 | WHEN (bad frame, sabme p 1 ) | illegal |
| 6,51, 10,1 | WHEN (timeout t200, rr_pl) | timer |
| 6,52, 6,1 | WHEN (timeout_ t203, rr_pl) | timer |
| ; State s6 |  |  |
| 7,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | valid |
| 7,24, 0,1 | WHEN (disc pl, uafl_13-e7) | valid |
| 7,5,1,1 | WHEN (13_el, sabme_p 1) | valid |
| 7,25, 2,1 | WHEN (13_e2, disc_p1) | valid |
| 7,67, 3, 1 | WHEN (13_et_condl, rr_p 0 ) | valid |
| 7,68, 5,1 | WHEN (13_e4_cond2, rej p 0 ) | valid |
| 7,69,5,1 | WHEN (13_e4_cond3, rr_p0) | valid |
| 7,70, 7,1 | WHEN (13_e5, i_p0) | valid |
| 7,71,7,1 | WHEN (13_e6, i_p0) | valid |
| 7,72, 8,1 | WHEN (13_e6_max, i_p0) | valid |
| 7,6, 1,1 | WHEN (dm_f0, sabme $p$ 1) | inopportune |
| 7,30, 1,1 | WHEN (dm_fl, sabme_pl) | inopportune |
| 7,31, 1,1 | WHEN (frmr fl, sabme_pl) | inopportune |
| 7,32, 1,1 | WHEN (frmr fo, sabme pl) | inopportune |
| 7,4,3,1 | WHEN ( sabme_p0, ua_fo_13_e8) | valid |
| 7,3,3,1 | WHEN (säbme pl, ua fl_13_ell) | valid |
| 7,33, 7,1 | WHEN (i_pl, rnr_fl_13-ell) | inopportune |
| 7,34, 7,1 | WHEN (i_p0,rnr_f0_13_ell) | inopportune |
| 7,73,7,1 | WHEN (i_p0_unexp_ns, rnr_f0) | inopportune |
| 7,74, 7,1 | WHEN (ipl_unexp_ns, rnr_fl) | inopportune |




| 9,50, 1,1 | WHEN (bad frame, sabme p1) | illegal |
| :---: | :---: | :---: |
| 9,77, 1,1 | WHEN (timeout_t200_N200, sabme pl) | timer |
| 9,78, 9,1 | WHEN (timeout_t200, rr_pl) | timer |
| ; State 88 |  |  |
| 10,6, 1,1 | WHEN (dm f0, sabme $\mathrm{p}^{\text {1 }}$ ) | inopportune |
| 10,30, 1,1 | WHEN (dm_fl, sabme pl) | inopportune |
| 10,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | inopportune |
| 10,24, 0,1 | WHEN (disc_p1, ua_fl_13_e7) | inopportune |
| 10,4, 3,1 | WHEN (sabme_p0, uaffo_l3_e8) | inopportune |
| 10,3, 3,1 | WHEN (sabme pl, ua_fl_13_e8) | inopportune |
| 10,31, 1,1 | WHEN (frmr_fl, sabme_pl) | inopportune |
| 10,32, 1,1 | WHEN (frmr p0, sabme pl) | inopportune |
| 10,5, 1,1 | WHEN (13_el, sabme pl) | valid |
| 10,25, 2, 1 | WHEN (13_e2, disc_pl) | valid |
| 10,26, 11,1 | WHEN (13_e3, rnr p0) | valid |
| 10,33, 9,1 | WHEN (i_pl, rnr_fl_13_ell) | valid |
| 10,34, 9,1 | WHEN (ip0, rnr-f0_13_ell) | valid |
| 10,65, 10,1 | WHEN (i_p0_unexp_ns, null) | inopportune |
| 10,66, 10,1 | WHEN (i_pl_unexp_ns, rr_fl) | inopportune |
| 10,38, 10,1 | WHEN (rrppo,null) | inopportune |
| 10,39, 10,1 | WHEN (rrpl, rrafl) | inopportune |
| 10,40, 5,1 | WHEN (rr_f0, null) | valid |
| 10,41, 10,1 | - WHEN (rr fl, null) | inopportune |
| 10,42, 10,1 | WHEN (rnr_p0, null) | inopportune |
| 10,43, 10,1 | WHEN (rnr_f0, null) | inopportune |
| 10,44, 5,1 | WHEN (rnr fl, null) | valid |
| 10,45, 10,1 | WHEN (rnr pl, rnr fl) | inopportune |
| 10,46, 10,1 | WHEN (rej f0, null) | inopportune |
| 10,47, 10,1 | WHEN (rej fl, null) | inopportune |
| 10,48, 10,1 | WHEN (rej_p0, null) | valid |
| 10,49, 10,1 | WHEN (rej_pl, rr fl) | inopportune |
| 10,50, 1,1 | WHEN (bad frame, sabme pl) | illegal |
| 10,77, 1,1 | WHEN (timeout_t200_N200, sabme pl) | timer |
| 10,78, 10,1 | WHEN (timeout_t200, rr_pl) | timer |
| ; State s9 |  |  |
| 11,23, 0,1 | WHEN (disc_p0, ua_f0_13_e7) | valid |
| 11,24, 0,1 | WHEN (disc_pl, uaffl_13_e7) | valid |
| 11,6, 1,1 | WHEN (dm f0, sabme pl) | inopportune |
| 11,30, 1,1 | WHEN (dm-f1, sabme_pl) | inopportune |
| 11,31, 1,1 | WHEN (frmr fl, sabme pl) | inopportune |
| 11,32, 1,1 | WHEN (frmr p0, sabme pl) | inopportune |
| 11,4,3,1 | WHEN (sabme_p0, ua_f0_13_e8) | valid |
| 11,3, 3,1 | WHEN (sabme_p1, ua_fl_13_e8) | valid |
| 11,5, 1,1 | WHEN (13_el, sabme_pl) - | valid |


| 11,25, 2,1 | WHEN (13_e2, disc_p1) | valid |
| :---: | :---: | :---: |
| 11,67, 9,1 | WHEN (13_e4_condl, rr_p0) | valid |
| 11,68, 10, 1 | WHEN (13_e4_cond2, rej_p 0 ) | valid |
| 11,69, 10,1 | WHEN (13_e4_cond3, rr_p0) | valid |
| 11,33, 11,1 | WHEN (i_pl, rnr_f1_13_ell) | valid |
| 11,34, 11,1 | WHEN (i $\left.p 0, r n r-f 0 \_13-11\right)$ | valid |
| 11,73, 11,1 |  | inopportune |
| 11,74, 11,1 | WHEN (i_pl_unexp_ns, rnr_fl) | inopportune |
| 11,38, 11,1 | WHEN (rr_ $\overline{0}$, null $)$ | inopportune |
| 11,39, 7,1 | WHEN (rrpl, rrfl) | valid |
| 11,40, 11,1 | WHEN (rr f0, null) | inopportune |
| 11,41, 11,1 | WHEN (rrfl, null) | inopportune |
| 11,42, 11,1 | WHEN (rnr p 0 , null) | inopportune |
| 11,43, 7,1 | WHEN (rnrffo, null) | valid |
| 11,44, 11,1 | WHEN (rnr fl, null) | inopportune |
| 11,45, 11,1 | WHEN (rnr pli, rnr fl) | inopportune |
| 11,46, 10,1 | WHEN (rej f0, null) | inopportune |
| 11,47, 7, 1 | WHEN (rej fl, null) | valid |
| 11,48, 10,1 | WHEN (rej. p 0 , null) | inopportune |
| 11,49, 11, 1 | WHEN (rej_pl, rr_fl) | inopportune |
| 11,50, 1,1 | WHEN (bad frame, sabme ¢ 1 ) | illegal |
| 11,79, 11,1 | WHEN (timeout_t200, rnr p 1) | timer |
| 11,80, 1,1 | WHEN (timeout_t200_N200, sabme p1) | timer |
| ENDTRANS |  |  |
| 10\#\# \#\#\#\# \#\# \#\# \#\# \#\#, Effector1, 0,0 |  |  |
| ~10\#\# \#\#\#\# \#\# \#\# \#\# \#\#, Default, 0, 0 |  |  |
| END-EFFS |  |  |

## APPENDIX B

## THE CLASSIFIER LIST

The following text shows the classifier list at major cycle 1000. The first number is an identification number assigned to the classifier. It is followed by the first condition string, the state in the Finite State Machine that chromosome supports, the second condition string, and the state that chromosome supports. The action sting follows the arrow and is followed by the strength (fitness) of the classifier, its bidratio, and the support.

Current Classifiers (cycle-step 1000):
$819>0000 \# 00100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ $987>0000 \# 0$, 100 \#001 [1,10]; 011000 1\#11 1101[]$\rightarrow 10110000101101$ 405> $000101010101 \# 1$ [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ $373>0000 \# 00100 \# 001[1,10] ; 10110000101101[] \rightarrow 01100010111101$ $578>0001010101$ 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ 877>0001010101 01\#1 [3]; 0\# 0\#01 0101 O\#01 [3] $\rightarrow 111001111110111$ $648>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 10111010101111$ $282>0000 \# 00100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ $333>0000 \# 00100 \# 001[1,10] ; 01100010111101[] \rightarrow 10110000101101$ $531>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ $278>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ $992>0000 \# 00100$ \#001 [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ 626>00 01010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ 835> $0100010 \# \# 00101$ [5]; 010001 010\# 0101 [5] $\rightarrow 11001110101111$ $1131>000101010101 \# 1$ [3]; 11100111110111[]$\rightarrow 11000000110110$ $431>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 10111010101111$ $520>0000 \# 00100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $891>0000 \# 00100 \# 001$ [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $747>000101010101 \# 1$ [3]; 0\# O\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ $217>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 10111010101111$ $785>0000 \# 00100 \# 001$ [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $1018>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100010111101$ $1012>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 11011010101111$ $1064>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 11111010111111$ $553>0000 \# 00100 \# 001$ [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $1023>0000 \# 00100 \# 001[1,10] ; 01100010111101$ [] $\rightarrow 10110000101101$ 311>00 00\#0 0100 \#001 [1,10]; 10110000101101 [] $\rightarrow 01100010111101$ $799>000101010101 \# 1$ [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ 953>000000010101\#1 [2]; 00000001010101 [2] $\rightarrow 0011101110$ 11\#1 $787>0000 \# 00100 \# 001[1,10] ; 10110000101101[] \rightarrow 10111110011101$
$1043>0000 \# 0.0100 \# 001$ [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $1135>00000000100001$ [0]; $0000000010000 \#$ [0] $\rightarrow 11111011111111$ $532>0000 \# 00100$ \#001 [1,10]; 01100010111101[]$\rightarrow 10110000101101$ 372> 00 00\#\# 0100 \#001 [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ 997>0000\#0 0100 \#001 [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $352>0000 \# 00100$ \#001 [1,10]; 10101001010011[]$\rightarrow 11110001011101$ $986>0000 \# 00100$ \#001 [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ 622> $000000010101 \# 1$ [2]; $00000001010101[2] \rightarrow 1010101111$ 11\#1 $671>0000 \# 00100$ \#001 [1,10]; 01100010111101[]$\rightarrow 10110000101101$ 909> 010001 0\#\#0 0101 [5]; 010001 010\# 0101 [5] $\rightarrow 11101001101111$ $985>000101000101 \# 0$ [4]; $00 \# 1010 \# 010100[4] \rightarrow 00010111011001$ $749>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 1010101111$ 11\#1 $988>0000 \# 00100$ \#001 [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $436>00000000100001$ [0]; $0000000010000 \#$ [0] $\rightarrow 10000111111111$ 1080> 0001010001 01\#0 [4]; $00 \# 1010 \# 010100$ [4] $\rightarrow 00110101110111$ 1099> $0000 \# 00100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $1081>000101000101 \# 0$ [4]; $00 \# 1010 \# 010100$ [4] $\rightarrow 01000100000001$ $1020>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 11001101111111$ $378>0000 \# 001000001$ [1,10]; 10101001010011[]$\rightarrow 11110001011101$ $376>00000000100001$ [0]; $00000 \# 0010000 \#$ [0] $\rightarrow 10101001010011$ $1151>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 10100110101011$ $812>0000 \# 00100 \# 001$ [1,10]; 10110000101101[]$\rightarrow 10111110011101$ $769>00$ 00\#0 0100 \#001 [1,10]; 10110000101101[]$\rightarrow 01100010111101$ 644> 00 00\#0 $0010 \# 110$ [9]; $00000000100 \# \# 0$ [9] $\rightarrow 10001100010111$ 435> 00 00\#0 0010\#110 [9]; $00000000100 \# \# 0$ [9] $\rightarrow 10001100010111$ $822>0000 \# 00100 \# 001$ [1,10]; 10110000101101 [] $\rightarrow 10111110011101$ $1106>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01010010110011$ 366> 00 00\#0 0100 \#001 [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $811>0000 \# 00100$ \#001 [1,10]; 01100010111101 [] $\rightarrow 10110000101101$ $1048>00$ 00\#0 0100 \#001 [1,10]; $00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100000111101$ $1161>000101010101 \# 1$ [3];; 0011110000 111\# [] $\rightarrow 10111111110110$ $1160>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 00111100001111$ $1154>0000 \# 00010 \# 110$ [9]; $00000000100 \# \# 0$ [9] $\rightarrow 10001100010111$ $1152>0000 \# 00100$ \#001 [1,10]; 0\# 0\#01 0101 0\#01 [] $\rightarrow 10110000101101$ $1156>00000000100001$ [0]; 10001100100111[]$\rightarrow 10111010101111$ 1153> $000101010101 \# 1$ [3]; 01100010111101[]$\rightarrow 00110110110111$ 1157>0000000101 01\#1 [2]; 00000001010101 [2] $\rightarrow 00000101000111$ $1147>000101010101 \# 1$ [3]; 11100111110111[]$\rightarrow 11000000110110$ $959>010101000101 \# 1[6,8] ; 01010100010101[6,8] \rightarrow 1010100100$ 11\#1 1143>000101010101\#1 [3]; 11001110101111[]$\rightarrow 11100100010100$ $386>000101010101 \# 1$ [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 10111111110110$ $1136>0000 \# 00100 \# 001[1,10] ; 01010010110011[\rightarrow 01100010111101$ $575>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 1010101111$ 11\#1 $1159>0000 \# 00100 \# 001[1,10] ; 01100010111101 \square \rightarrow 10110000101101$ 1155> $0000 \# 00100$ \#001 [1,10]; 10110000101101$] \rightarrow 01100010111101$ $1158>0000 \# 00100 \# 001$ [1,10]; 01100010111101 ] $\rightarrow 00110000101101$ 1055> 00 00\#0 0100 \#001 [1,10]; 011000 1\#11 1101 [] $\rightarrow 11001000110111$ $1126>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 00000101000111$ $1128>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 1000100101$ 11\#1 $1127>0000000101$ 01\#1 [2]; 00000001010101 [2] $\rightarrow 11000001110001$ $1063>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01111110100001$

1137> 00 00\#0 0100 \#001 [1,10]; 011000 1\#11 1101[]$\rightarrow 10110000101101$ 1112>000101 0100 \#1\#1 [; 0\# 0\#01 $01010 \# 01$ [3] $\rightarrow 11100111110111$ $1113>00$ 10\#0 $01010001[; 00000001000 \# \# 1[1,10-11] \rightarrow 0110 \# 000000000$ $1111>0000 \# 00100 \# 001$ [1,10]; $0100000010000 \#[] \rightarrow 10111010101111$ $1110>00000000100001$ [0]; 00100010111101[]$\rightarrow 10110000111101$ 1116>0001010101 01\#1 [3]; 0\# 0\#01 0100 0\#\#1[5] $\rightarrow 01100010111111$ 1146>000101010101\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ $1122>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 00100110010000$ $359>0000 \# 00100 \# 001[1,10] ; 01100010111101[] \rightarrow 10110000101101$ $1124>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 11100000110001$ $794>0000 \# 00100$ \#001 [1,10]; 10101001010011[]$\rightarrow 11110001011101$ $1101>000101010101 \# 1$ [3]; 11000001011101[]$\rightarrow 11100111110111$ $1093>00000000100001$ [0]; 10111101001101[]$\rightarrow 10111010101111$ $1115>0000 \# 00100 \# 001[1,10] ; 10110000101101[\rightarrow 01100010111101$ $1109>0000 \# 00100 \# 001[1,10] ; 10110000101101 \square \rightarrow 10111110011101$ $1123>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 00101100001110$ $943>0000 \# 00010 \# 110$ [9]; $00000000100 \# \# 0[9] \rightarrow 11000010110111$ 1089>010001 0\#\#0 0101 [5]; $0011010111 \# 110[] \rightarrow 11001110101111$ $1141>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 0001000011$ \#001 $1120>000101010101 \# 1[3] ; 11100111110111[] \rightarrow 11000000110110$ $1138>0000 \# 00100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 00100101010010$ 1142> $0100010 \# \# 0101$ [5]; 010001 010\# 0101 [5] $\rightarrow 11001110101111$ 1133> $0100010 \# \# 0101$ [5]; $010001010 \# 0101$ [5] $\rightarrow 11011011101000$ 885>00 00\#0 0100 \#001 [1,10]; 10110000101101[]$\rightarrow 01100010111111$ $964>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 001011011011 \# 1$ $1100>0000 \# 00100 \# 001[1,10] ; 01100010111101[\square \rightarrow 11000001011101$ $1150>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 00111000100001$ 1075>0001010101 01\#1 [3]; O\# 100001010101[]$\rightarrow 1011101111$ 11\#1 1074>00 $0101010101 \# 1$ [3]; $110 \# 0101010 \# 01 \square \rightarrow 11100111110111$ 1094>000101000101\#0 [4]; $00 \# 1010 \# 010100[4] \rightarrow 10010000111010$ $1062>0010 \# 00100 \# 001$ [; $00000001000 \# \# 1[1,10-11] \rightarrow 0110 \# 000000000$
1088>000101010101\#1 [3]; 11100111110111[]$\rightarrow 00110101110110$ 1144>0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 01000100000111$ $1046>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 01110011101111$ $1061>000101010101 \# 1$ [3]; 0001110011 \#110 [] $\rightarrow 11100111110111$ $1077>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01100010101101$ $1149>00000000100001[0] ; 0000000010000 \#[0] \rightarrow 01010110101111$ $1148>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 10010101111010$ 1049>000101010101\#1 [3]; 01100000111101[]$\rightarrow 11100111110111$ $1130>000101010101 \# 1$ [3]; 0\# O\#01 0101 0\#01 [3] $\rightarrow 10010110010111$ 1134>000101 0101 01\#1 [3]; 0\# 0\#01 0101 O\#01 [3] $\rightarrow 11100111110111$ 1132> 0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 11100111110111$ $1035>00000000100001$ [0]; 0001011001 111\# [] $\rightarrow 00000001010011$ 1086> $000101.010101 \# 1$ [3]; 0\# 0\#01 0101 O\#01 [3] $\rightarrow 11100111110111$ $1033>0001010101$ 01\#1 [3];-01 00010100 111\# [ $\rightarrow 11100111110111$ $1031>00000000100001[0] ; 11001000011101[] \rightarrow 10000111111111$ $1076>0000 \# 00100 \# 001[1,10] ; 01100010111101[] \rightarrow 10110000111101$ $1041>00000000100001$ [0]; 1101101010 111\# [] $\rightarrow 10111010101111$ $678>0000 \# 00100 \# 001[1,10] ; 10101001010011[] \rightarrow 11110001011101$ 1119>0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 10110111110001$ 1087>000101010101\#1 [3]; 11100111110111[]$\rightarrow 10111111110110$
$1103>0000 \# 00100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 00001101100100$
1060> $000101010101 \# 1$ [3]; 11 100111110111[]$\rightarrow 00011100110110$ 1054> $0000 \# 00100 \# 001[1,10] ; 01100010111101[\rightarrow 00101011001010$ 1011>000101010101\#1 [3]; 11100001010101[]$\rightarrow 1011101111$ 11\#1 1010> $000000010101 \# 1$ [2]; 00000111110111[]$\rightarrow 11101011100101$ $1040>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 11011010101111$ 1098> 00 00\#0 $0100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$ 1118> 0001010101 01\#1 [3]; 0\# 0\#01 $01010 \# 01$ [3] $\rightarrow 10010110010111$ 991> 00 00\#0 0100 \#001 [1,10]; 01010101101101[]$\rightarrow 01100010111101$ 1117>00 01010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00110110110111$ $993>0000 \# 00010 \# 110$ [9]; $00000000100 \# \#[9] \rightarrow 11000010110111$ 989> 00 00\#0 0100 \#001 [1,10]; 10110000101101 [] $\rightarrow 10111110011101$ $977>0100010 \# \# 0101$ [5]; $10010011111111 \square \rightarrow 01100010101100$ $569>0000 \# 00100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $981>0000010 \# 00101$ []; $01000 \# 0010000 \#[] \rightarrow 01000000001100$ $978>0000 \# 0001101 \# 1[] ; 11100111110111[] \rightarrow 10101011110110$
$796>0000 \# 00100 \# 001$ [1,10]; 10110000101101[]$\rightarrow 01100010111101$ $917>0000 \# 00100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $911>0100010 \# \# 0101$ [5]; $010001010 \# 0101$ [5] $\rightarrow 00111101011111$ 1036> 0000 000 0100 \#001 [1,10]; 01100010111101 [] $\rightarrow 00011111101101$ $505>0000 \# 00100 \# 001[1,10] ; 00000001000 \# \# 1[1,10-11] \rightarrow 01100010111101$
$938>00000000100001$ [0]; $10001100100111 \square \rightarrow 10111010101111$ $901>000101010101 \# 1$ [3]; $11001110101111[\rightarrow 11100100010100$ $928>000000010101 \# 1$ [2]; 0000101111 \#110 $\rightarrow 00011100100010$ 1104> 0001010101 01\#1 [3]; 0\# 0\#01 \#101 0\#01 [3] $\rightarrow 11100111110111$ $983>0100010 \# \# 00101$ [5]; $010001010 \# 01 \# 1$ [5] $\rightarrow 00110001111111$ $509>0000 \# 00100 \# 001[1,10] ; 01100010111101[] \rightarrow 10110000101101$ 974> $0000 \# 00100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 10110000101101$ $1070>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 01101000111111$ 904> 00000000100001 [0]; 1110101101 11\#1 [] $\rightarrow 10000111111111$ 1027> $0000 \# 00100$ \#001 [1,10]; $0110001 \# 111101[] \rightarrow 10110000101101$ 1009> $0000 \# 00100 \# 001[1,10] ; 10110000101101[] \rightarrow 10000010100100$ 886>0100010\#0 0101 [5]; $01010001101111 \square \rightarrow 00110001111101$ 888> $000101010101 \# 1$ [3]; 11100011110111[] 10111111110110 -961>0100010\#\#0 0101 [5]; $010001010 \# 0101[5] \rightarrow 11011011101000$ $976>0100010 \# \# 0101$ [5]; $010001010 \# 0101$ [5] $\rightarrow 10010011111111$ $918>0000 \# 00100 \# 001$ [1,10]; $101100001011 \# 1[] \rightarrow 00100010111101$ $1068>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 11110011110010$ $676>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 1010100100$ 11\#1 1006> $000101010101 \# 1$ [3]; 11100111110111[]$\rightarrow 10010000100110$ $875>0000 \# 00100 \# 001[1,10] ; 10101001010011[] \rightarrow 10010101011101$ $874>0000000101$ 11\#1 []; $010001000011 \# 1[] \rightarrow 101010111111$ \#1 862> $000101010101 \# 1$ [3]; $11100111110111 \square \rightarrow 11101011100101$ $277>0000 \# 00100 \# 001[1,10] ; 11110001011101[] \rightarrow 01100010111101$ $915>01.00010 \# \# 0.0101$ [5]; $010001010 \# 0101$ [5] $\rightarrow 10010010111001$ $1030>0000 \# 00100 \# 001[1,10] ; 00000001000 \# 1[1,10-11] \rightarrow 11001000011101$ 844> $000101010101 \# 1$ [3]; $11100111110111 \mathrm{D} \rightarrow 10111111110110$ 927>000101010101\#1 [3]; $11100111110111[\rightarrow 00001011110110$ $990>0000 \# 00100 \# 001$ [1,10]; 10110000101101[]$\rightarrow 01010101101101$ $274>00000000100001$ [0]; 00 000\# 0010 000\# [0] $\rightarrow 1010 \pm 001010011$ $828>00000000100001$ [0]; 0101010010 111\# [] $\rightarrow 10110101110011$

1082> 0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 1001010001$ 10\#0 916> $0100010 \# \# 0101$ [5]; $010001010 \# 0101$ [5] $\rightarrow 01010111011010$ 1004> 00000000100001 [0]; $0000000010000 \#[0] \rightarrow 01000111001111$ $1121>000101010101 \# 1[3] ; 1100000011$ \#110 [ $\rightarrow 11100111110111$ 1059> $000101010101 \# 1$ [3]; 0\# 0\#01 $01010 \# 01$ [3] $\rightarrow 01000010001001$ $903>000000010101 \# 1$ [2]; 00000001010101 [2] $\rightarrow 1110101101$ 11\#1 $522>00000000100001$ [0]; $00000 \# 0010000 \#[0] \rightarrow 10101001010011$ $1032>00000000100001$ [0]; 0000000010 000\# [0] $\rightarrow 01000101001111$ 1067> 0001010101 01\#1 [3]; 0\# 0\#01 0101 O\#01 [3] $\rightarrow 01110100101011$ 1038> $0000 \# 00100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 00110101000110$ $994>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 10001011000100$ 1025> 00000000100001 [0]; $0000000010000 \#[0] \rightarrow 00000111101101$ $718>0000 \# 00100 \# 011$ [11]; 00000001000011 [11] $\rightarrow 10110000101101$ 292> $0000 \# 00100 \# 001[1,10] ; 01100010111101[] \rightarrow 10110000101101$ $770>00000000100001$ [0]; $00000 \# 0010000 \#$ [0] $\rightarrow 10101001010011$ $857>0000 \# 00100 \# 001$ [1,10]; 0000000 0r00 (\#\#1 [1,10-11] $\rightarrow 01100010111101$ 829> 00000000100001 [0]; 00 000\# 0010 000\# [0] $\rightarrow 01011010011011$

Number of Classifiers: 200. Ave. strength 2129.47 (total 425894.94 ).

The following portion shows the classifier list at major cycle 12,000 . Note that the list has converged towards a much smaller number of states.

Current Classifiers (cycle-step 12000):
$11058>00000000100001$ [0]; 1110101011 111\# $] \rightarrow 11101010111111$ $9279>0000 \# 00100 \# 001$ [1,10]; $1110101011111 \# \square \rightarrow 11101010111111$ 10906> $000101010101 \# 1$ [3]; $10000011111011 \square \rightarrow 00010011110111$ $10719>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 11101010111111$ $11238>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 11101010111111$ 11435> $0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 10994> 0000000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11574>000101010101\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ $11363>0001010101$ 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ 11436>0000000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11324>000101010101\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ 10736> $0000000100 \# 001$ [1,10]; $00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100010111101$ $11478>0000 \# 00100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11303>0000000100 \# 001$ [1,10]; $00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100010111101$ 11125> $0000 \# 00101$ \#101 [2]; $0000000101010 \#[2] \rightarrow 11101010111111$ $11349>0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11332> 00 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $9861>0000000100$ \#001 [1,10]; 01100010111101[]$\rightarrow 11101010111111$ 11576>0000000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 9176> $0000 \# 00101$ \#101 [2]; $0000000101010 \#$ [2] $\rightarrow 11101010111111$ 10348>000101010101\#1 [3]; $10000011111011 \mathrm{D} \rightarrow 00010011110111$
$11388>000101010101 \# 1$ [3]; $10000011111011[\rightarrow 00010011110111$
$10251>0001010101$ 01\#1 [3]; 10000011111011$] \rightarrow 00010011110111$ $11337>00000000100001$ [0]; 1110101011 111\# $] \rightarrow 11101010111111$ $11212>00000000100001$ [0]; 1110101011 111\# $\square \rightarrow 11101010111111$ 11246>0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ $10576>0000000100 \# 001$ [1,10]; $00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100010111101$ 11358> $010001010001 \# 1$ [5]; 01000101000101 [5] $\rightarrow 10000011111011$ $11567>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ 11538>000000 $00100001[0] ; 1110101011$ 111\# [] $\rightarrow 11101010111111$ $11376>00000000100001$ [0]; 1110101011 111\# $\square \rightarrow 11101010111111$ $11442>000101010101 \# 1[3] ; 10000011111011] \rightarrow 00010011110111$ 11495> $0000 \# 00100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11142>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 11101010111111$ $11560>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 11101010111111$ 10835> $0000 \# 00101$ \#101 [2]; 0000000101 010\# [2] $\rightarrow 11101010111111$ 11440>000101010101\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ $11476>0001010101$ 01\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ $10213>0000000100$ \#001 [1,10]; 01100010111101[]$\rightarrow 11101010111111$ 10474>000101010101\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ 11329>000101010101\#1 [3]; $10000011111011 \square \rightarrow 00010011110111$ $11526>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11432>000101010101 \# 1$ [3]; $10000011111011 \square \rightarrow 00010011110111$ $11610>00000000100001$ [0]; $0000000010000 \#[0] \rightarrow 11101010111111$ $11005>000101010101 \# 1$ [3]; $10000011111011 \square \rightarrow 00010011110111$ $11578>0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11566>000101010101 \# 1$ [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ $11326>000101010101 \# 1$ [3]; $10000011111011[\rightarrow 00010011110111$ $11345>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 11101010111111$ $11260>00000000100001$ [0]; $1110101011111 \#[] \rightarrow 11101010111111$ $11499>0000 \# 00101$ \#101 [2]; 0000000101 010\# [2] $\rightarrow 11101010111111$ $11579>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11564>0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $9699>0000 \# 00100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11028>000101010101\#1 [3]; 10000011111011[] 00010011110111 $11447>000101010101 \# 1$ [3]; $10000011111011 \square \rightarrow 00010011110111$ $11503>00000000100001[0] ; 1110101011111 \# \square \rightarrow 11101010111111$ $10852>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11130>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 11101010111111$ $9596>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $9538>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 11101010111111$ $11545>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 111010$ 10111 1111 $11571>000101010101 \# 1$ [3]; $10000011111011[\rightarrow 00010011110111$ $11620>000101010101 \# 1$ [3]; 0\# O\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ 10985> 00 00\#0 0101 \#101 [2]; 0000000101 010\# [2] $\rightarrow 11101010111111$ $11249>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 11101010111111$ $11581>00000000100001$ [0]; 11 1010-1011 111\# $] \rightarrow 11101010111111$ $11535>00000000100001$ [0]; $1110101011111 \# \square \rightarrow 11101010111111$ $11516>000101010101 \# 1$ [3]; O\# O\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ $11612>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11540>0000 \# 00100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 10673> $000101010101 \# 1$ [3]; 10000011111011$] \rightarrow 00010011110111$ $11187>000101010101 \# 1$ [3]; 0\# O\#01 0101 0\#01 [3] $\rightarrow 00010011110111$
$11623>00000000100001$ [0]; 11001111101011[]$\rightarrow 11101010111111$ 11622> $000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 11001111101011$ 11625> 00000000100001 [0]; 1110101011 111\# $\square \rightarrow 11101010111111$ $11624>0000 \# 00100 \# 001$ [1,10]; 1110101011 111\# [ $\rightarrow 11101010111111$ 11544> 00000000100001 [0]; 1110101011 111\# $\square \rightarrow 11101010111111$ $11615>00010100010100$ [4]; $00001111001011[\rightarrow 01100001010101$ $11609>0000000100$ \#001 [1,10]; 1010010101 \#011 [] $\rightarrow 01010111101000$ 10266> 00000000100110 [9]; $0000000010011 \#$ [9] $\rightarrow 11101010111110$ 11554> 0000000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11605> $000101010101 \# 1$ [3]; 0110101101 111\# D $\rightarrow 00010011110111$ 11616> 0001010101 01\#1 [3]; 00010011110111[]$\rightarrow 00101101100110$ 11392> 00 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11599> 0000000100 \#001 [1,10]; 01100010111101[]$\rightarrow 11101010111111$ 11597> $0000000100 \# 001[1,10] ; 01110000101011[] \rightarrow 01100010111101$ 11595> $0000000100 \# 001$ [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 1010100111$ \#011 11548> $0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11621>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 11110001010001$ 11617> $000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10011110101011$ 11570> $010001010001 \# 1$ [5]; 01000101000101 [5] $\rightarrow 10000011111011$ $11569>000101010101 \# 1$ [3]; $10000011111011 \mathrm{D} \rightarrow 00010011110111$ 11594> $0000000100 \# 001$ [1,10]; $00000001000 \# \#$ [1,10-11] $\rightarrow 01010111101000$ 11583> $000101010101 \# 1$ [3]; 01000101000101[]$\rightarrow 10000011111111$ 11582> 0100010100 01\#1 [5]; 0001010110 111\# $\square \rightarrow 00010011110011$ 11472> 00000000100001 [0]; 1110101011 111\# $\square \rightarrow 11101010111111$ 11614> $000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 00001111001011$ $11568>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11618>0000000100 \# 001$ [1,10]; $00000001000 \# \#$ [1,10-11] $\rightarrow 0001111101$ \#000 $11561>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 11101010111111$ 10956> $000101010101 \# 1$ [3]; $10000011111011 \square \rightarrow 00010011110111$ 11078> 0001010101 01\#1 [3]; 0\# 0\#01 0101 0\#1 [3] $\rightarrow 00010011110111$ 10831> $0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11533> $0000000100 \# 001$ [1,10]; 00111111001011[]$\rightarrow 00010011110111$ $11529>00000001000001[1,10] ; 11101010111101[] \rightarrow 11101010111111$ $11528>0000000010 \# 001$ [0]; 0110001011 111\# [] $\rightarrow 11101010111111$ $11530>0000000100 \# 001$ [1,10]; $01100001010 \# 01[] \rightarrow 11101010111111$ $11448>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11525>0000000100 \# 001[1,10] ; 00101010111011[] \rightarrow 01100010111101$ $11438>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ 11613> 00 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11606>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10100010110111$ $11523>0000000100 \# 001[1,10] ; 11101000001011[] \rightarrow 01100010111101$ 11519> $000101010101 \# 1$ [3]; 11101011111011[]$\rightarrow 00010011110111$ 11521> 0000000010 \#001 [0]; 0110001011 111\# $] \rightarrow 11101010111111$ $11512>00000000100001$ [0]; $11000110111101 \square \rightarrow 11101010111111$ $11602>00000000100001$ [0]; $1110101011111 \#[\rightarrow 11101010111111$ 11506> $000101010101 \# 1$ [3]; 0001010110 111\# $] \rightarrow 00010011110111$ $11443>000101010101 \# 1[3] ; 10000011111011 \square \rightarrow 00010011110111$ $11598>0000000100$ \#001 [1,10]; $00000001000 \# \# 1$ [1,10-11] $\rightarrow 01100010111101$ 11607> $000101010101 \# 1$ [3]; 00010011110111 [] $\rightarrow 01100001011010$ 11604> 00000000100001 [0]; 1110101011 111\# $] \rightarrow 01101011011111$ $11493>000101010101 \# 1[3] ; 01100011111011 \square \rightarrow 00010011110111$
$11562>00000000100001[0] ; 1110101011$ 111\# [ $\rightarrow 11101010111111$ $11539>00000000100001[0] ; 1110101011111 \# \square \rightarrow 11101010111111$ $10947>000101010101 \# 1[3] ; 10000011111011] \rightarrow 00010011110111$ 11464>0000\#0 $0100 \# 001$ [1,10]; 10100101011011[]$\rightarrow 11010110101001$ 11549> 0000000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11234>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11454>0000000100 \# 001[1,10] ; 10000010111101 \square \rightarrow 11101010111111$ 11593>00 01010101 01\#1 [3]; 00010011110111[]$\rightarrow 01000110000011$ $11288>000101010101 \# 1$ [3]; $00010011110111[\rightarrow 10000011111011$ $11343>0000000100 \# 001[1,10] ; 00111111001011[] \rightarrow 01100010111101$ $11603>00000000100001$ [0]; $1110101011111 \#[] \rightarrow 11101010111111$ $11600>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 01100101101010$ $11515>00010100010100$ [4]; 0\# 01010001 010\# [4,6,8] $\rightarrow 01100001010101$ $11527>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 11101010111111$ $11194>0001010101$ 01\#1 [3]; 0\# 0\#01 0101 0\#01 [3] $\rightarrow 00010011110111$ $11592>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 101111100101 \# 0$ $11477>0000000100$ \#001 [1,10]; 01100010111101[]$\rightarrow 11101010111111$ 11589>00 00000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11588>0000 \# 00100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11601>00000000100001[0] ; 1110101011111 \#[] \rightarrow 00101000000011$ 11423>0000\#0 0101 \#101 [2]; 1110100101 010\# [] $\rightarrow 11101010111111$ $11409>0000000100$ \#1\#1 []; 00010011110111$] \rightarrow 01100010111101$ $11608>0000000100$ \#001 [1,10]; $00000001000 \# \# 1[1,10-11] \rightarrow 1010010101$ \#011 11585>00 00000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010001111$ $11041>00$ 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11552>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 10111001010111$ 10886> 00 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ 11415>010001010001\#1 [5]; 01000101000101 [5] $\rightarrow 11101010101001$ 11584> 0000000100 \#001 [1,10]; $1110101011111 \#[] \rightarrow 11100100111111$ $11395>00000000100001[0] ; 1110101011111 \# \square \rightarrow 11101010111111$ $11586>00000000100001[0] ; 1110101011$ 111\# [] $\rightarrow 11101010111111$ $11587>00000000100001[0] ; 1110101011111 \# \square \rightarrow 11101010111111$ $11517>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11522>000101010101 \# 1[3] ; 10000011111011 \square \rightarrow 00010011110111$ 11416>010001010001\#1 [5]; 01000101000101 [5] $\rightarrow 01000110011011$ $11441>0001010101$ 01\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ $11417>0000000100$ \#001 [1,10]; 01100010111101[]$\rightarrow 01111011101101$ 11406> 0000000100 \#001 [1,10]; 01100010111101 [] $\rightarrow 11101010111111$ $11580>00000000100001[0] ; 1110101011111 \#[] \rightarrow 11101010111111$ $11439>0001010101$ 01\#1 [3]; 10000011111011$] \rightarrow 00010011110111$ $11590>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11235>000101010101 \# 1$ [3]; 10000011111011$] \rightarrow 00010011110111$ $11460>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11299>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11396>00-00 \# 00100 \# 001[1,10] ; 1110101011111 \#[] \rightarrow 11101010111111$ $11543>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 10000011111011$ $11563>00000000100001[0] ; 1110101011111 \# \square \rightarrow 11101010111111$ $11354>000101010101 \# 1[3] ; 10000011111011 \square \rightarrow 00010011110111$ $11456>000101010101 \# 1[3] ; 10000011111011 \square \rightarrow 00010011110111$ 11397>00 00\#0 0100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11471>00000000100001[0] ; 1110101011111 \#[] \rightarrow 11101010111111$
$11553>0000000100$ \#001 [1,10]; 11 101\# 1011 111\# [] $\rightarrow 10010000010000$
$11450>0001010101$ 01\#1 [3]; $10000011111011 \square \rightarrow 00010011110111$ $11558>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 10001011110101$ $11551>0000 \# 00100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 10001111100001$ $11550>0000 \# 00100 \# 001[1,10] ; 1110101011111 \#[] \rightarrow 01001100010100$ $11158>0000 \# 00101$ \#101 [2]; $0000000101010 \#[2] \rightarrow 00011010011111$ $11497>0001010101$ 01\#1 [3]; 00010011110111[]$\rightarrow 10000011111011$ $11559>00000000100001$ [0]; 1110101011 111\# [] $\rightarrow 10001100000000$ $11458>000101010101 \# 1$ [3]; 000100111$] 0111[] \rightarrow 11110001010001$ 11403>00 00000100 \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11101010111111$ $11556>00000000100001$ [0]; 7110101011 111\# [] $\rightarrow 11101010111111$ $11141>0000000100 \# 001$ [1,10]; 01100010111101[]$\rightarrow 1100000110 \# 101$ $11485>0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 11000000000101$ $11536>00000000100001[0] ; 1110101011111 \#[] \rightarrow 11101010111111$ $11489>0000000100$ \#001 [1,10]; 1110101011 111\# [] $\rightarrow 11100100001111$ $11486>0000000100 \# 001$ [1,10]; 1110101011 111\# [] $\rightarrow 10100110100101$ $11484>0000 \# 00100 \# 001[1,10] ; 1110101011111 \#[] \rightarrow 00000111010011$ $11268>00000000100001[0] ; 1110101011$ 111\# [] $\rightarrow 11101010111111$ $10836>00000000100001[0] ; 1110101011111 \#[] 11101010111111$ $11502>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 11100111100101$ $11228>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 11101000001011$ 11429>000101010101\#1 [3]; 00010011110111[]$\rightarrow 11010101001100$ $11371>000101010101 \# 1$ [3]; 00010011110111[]$\rightarrow 11110001010001$ 11106>00 $00000100 \# 001$ [1,10]; $1110101011111 \#[] \rightarrow 11101010111111$ $11333>00000000100001$ [0]; 1110101011 111\# [ $\rightarrow 11101010111111$

Number of Classifiers: 200. Ave. strength 44151.51 (total 8830302.00).

## VITA

Mr. Bijan Marjan was born in Teheran, Iran on June 5, 1967 to Amir Houshang Marjan and Rosaria Salomone Marjan. He earned a B.A. in Computer Science and a B.A. in Economics from Lehigh University in 1990 . He has worked on a number of projects while at Lehigh. These include computer-aided design for Ford Corporation while with the Intelligent Systems Lab and computer-aided software engineering for McNEIL Consumer Products Company, a Johnson and Johnson Company while with the Computer Integrated Manufacturing Lab. Mr. Marjan intends to join Technology Systems Corporation in Bethlehem, Pennsylvania and return to his home, Rome, Italy, after the completion of some training in this country. Mr. Marjan's interests reside in artificial intelligence, machine learning, and intelligent information systems. Mr. Marjan received a number of awards while at Lehigh including the Graduate Student Leadership Award and is a member of IEEE, IEEE Computer Society, ACM, and ACM-SIGART, for which he served as Vice-Chairman of the Lehigh Chapter.



[^0]:    Recommended Citation
    Marjan, Bijan, "An application of classifier systems to the reduction of finite state machines" (1992). Theses and Dissertations. Paper 31.

[^1]:    5. Minsky, Marvin. Society of Mind. p. 17
