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The application of Taguchi's method of parameter design to the design of mechanical systems

Robert A. Zambanini
Lehigh University

AUTHOR:

Zambanini, Robert a. , Jr.

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THE APPLICATION OF TAGUCHI'S METHOD OF PARAMETER DESIGN TO THE DESIGN OF MECHANICAL SYSTEMS

by

Robert A. Zambanini, Jr.

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Professor in Charge
Professor John B. Ochs

Chairman of the Department
Professor Robert P. Wei

This document is dedicated with sincere appreciation to my quality "mentor", Joseph L. Pascuzzi, Sr. (also known as "The Wise One"), who was not afraid to go against the grain in order to stress the true meaning and application of the term "quality improvement".

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LIST OF SYMBOLS, ACRONYMS, AND TERMS

CAD	Computer-Aided Design
CAM	Computer-Aided Manufacturing
N/C	Numerically-controlled
SED	Statistical Experiment Design
SPC	Statistical Process Control
TQC	Total Quality Control
TQM	Total Quality Management
ANOVA	Analysis of Variance
DOP	Design Optimization Procedure
ESOI	Experimental Set(s) of Iterations
DOE	Design of Experiments
FDE	Factorial Design of Experiments
QFD	Quality Function Deployment
RA	Reliability Analysis
FMA	Failure Mode Analysis
RSM	Response Surface Methodology
S/N	Signal-to-noise ratio
MSD	Mean-squared deviation
TM	Taguchi Methods
T	Transmissibility (measure of displacement transmitted from a support to a particular object)
lbm	pound-mass (English unit of mass)
lbf	pound-force (English unit of force)
ton	a unit of mass equal to 2000 lbm
kip	kilopound (a unit of force equal to 1000 lbf)
ft	foot (English unit of length)
sec	second (English unit of time)
TL	Transmission loss (measure of reduction of energy in an acoustic muffler)
in	inch (a unit of length equal to 1/12 of a foot)
rpm	revolutions per minute (unit of rotational speed)
hp	horsepower (English unit of power)
dB	decibel (conversion of unit to a logarithmic scale)
cps	cycles per second (unit of frequency)
Hz	Hertz (unit of frequency)
psi,ksi	pounds per square inch, kilopounds per square inch (English units of stress)
$	dollar (American unit of currency)
rad	radian (unit of angular measure)
μsec	unit of time equal to one-millionth of a second
nsec	unit of time equal to one-billionth of a second

ABSTRACT

Taguchi's method of parameter design is successfully applied to five mechanical systems - a dynamic absorber, an acoustic muffler, a gear/pinion system, a spring, and an electro-hydraulic servosystem. In each system, the design parameters to be optimized are identified, along with the desired response. Equations relating the system response as a function of the design parameters are programmed and become the basis for the experiments. The Taguchi method of obtaining the optimum values of the design parameters is performed by computer in an iterative process. The iterative procedures converged to values of the design parameters that satisfied the response criteria (including constrained response and multi-variable response systems) and were shown to be robust. For one system, a confirmation experiment was performed and the results verified.

CHAPTER ONE - INTRODUCTION

Section 1.1 - Problem Statement

A product that is designed, marketed, and manufactured travels through various stages in its development. This progression, often called the product life cycle or product development cycle, is shown in Figure 1-1 [1]. In this figure, the typical steps in the evolution of a new product are shown. The product life cycle

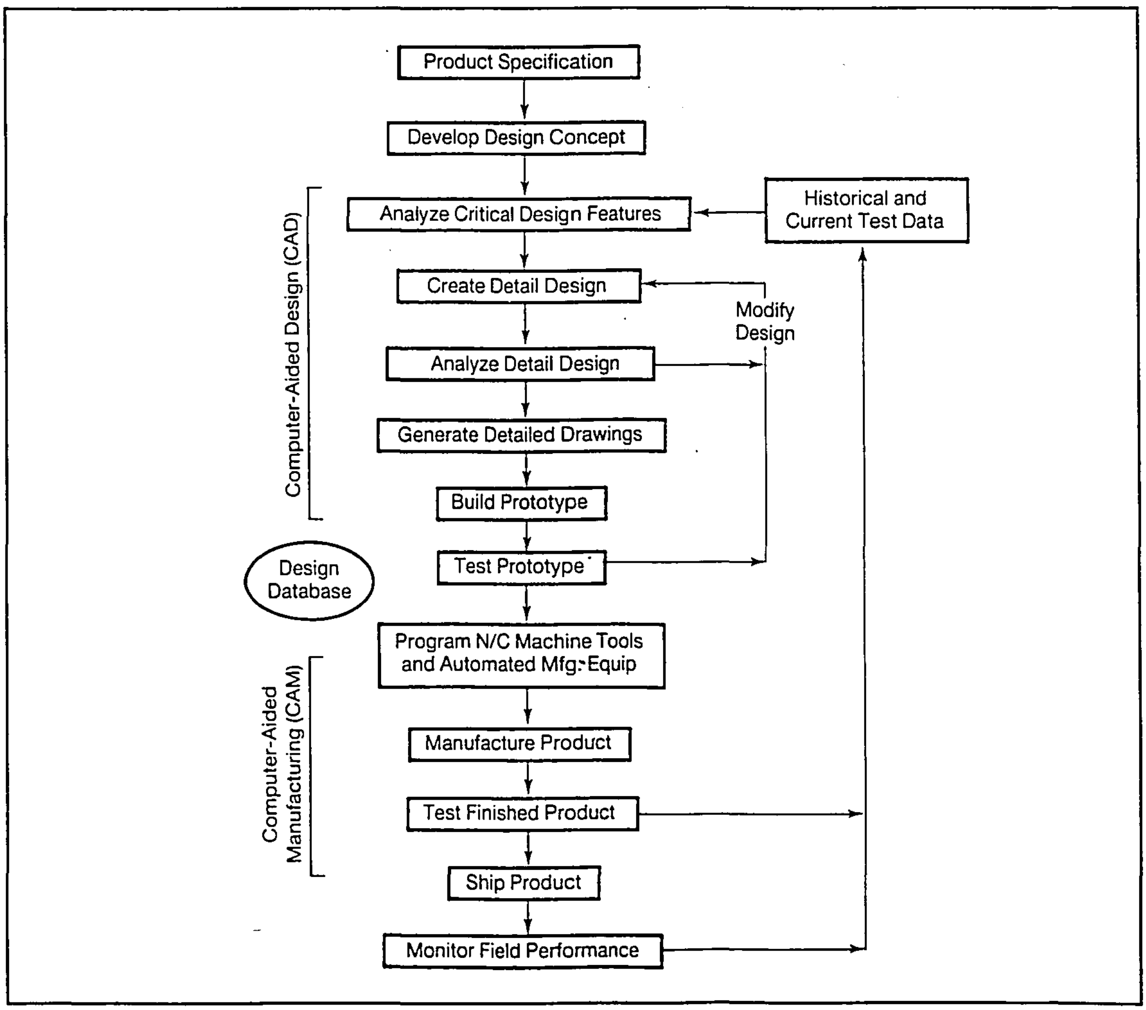

Figure 1-1 - The product life cycle.

begins with product specification, which is usually the result of much brainstorming, and the development of the design concept. At this early point in the cycle, Computer-Aided Design (CAD) becomes an important tool. CAD provides a quick and accurate way of analyzing critical design features, creating detail designs, analyzing the detail design, and generating detailed drawings for the construction and testing of a prototype. In the later part of the product life cycle, Computer-Aided Manufacturing becomes an important tool. CAM provides an accurate way to program numerically-controlled (N/C) machines to perform various operations as well as a means to design the manufacturing process and the testing of finished product. In summary, most of the design effort is generally concentrated in the early phases of the product life cycle, while most of the manufacturing effort is generally concentrated in the later phases of the product life cycle.

It is often thought that the responsibility for quality and general product performance is the responsibility of the later phases of the product life cycle. The designers in the earlier phases often will send the design to the manufacturing organization for fabrication, production, and final shipment to the customer. At that point, however, the designers often unconsciously divorce themselves from the rest of the product life cycle and quite frequently moves on to another project.

With this removal of the designers from the remainder of the product life cycle, the manufacturing organization is expected to assume the responsibility of quality improvement. Statistical Experimental Design (SED), developed in the 1920's by Sir Ronald Fisher, and Statistical Process Control (SPC), developed in the 1920's by Walter Shewhart, became the backbone of the traditional quality improvement approaches. There have been other approaches to quality improvement, such as component swapping and zero defects, but the basic philosophy for quality improvement has remained the same: using SED and SPC, the manufacturing organization must analyze its various processes in order to improve quality.

There was one "other approach" to quality, however, that represented a radical shift of thought from traditional quality improvement ideas. Genichi Taguchi postulated that quality improvement should begin during the design phases of the product life cycle. Products could be designed with the values of the various parameters set such that they were *robust* - that is, the performance of such products or processes were insensitive to the various sources of variation in the performance of the product or process. He claimed that the design phase was the lone part of the product life cycle where this improvement could be effected. He referred to this process as *parameter design*.

Since the introduction of Taguchi's ideas, parameter design applied to the design of mechanical systems has been limited in current literature. It has been applied to other disciplines, such as business and finance. Christine Rivers, in fact, applied Taguchi's methods to a behavioral science problem within the business world in a statistical study of the "five known factors of job satisfaction: work, supervision, salary, promotional opportunities, and co-workers" to develop a measure of job satisfaction for management [2].

Parameter design has been applied to electrical systems, including a treatment by Filippone [3], who uses a computer model of an electrical system to compare parameter design to Suh's Design Axioms. These axioms of

1. Maintain the independence of functional requirements and
2. Minimize the information content

were proposed by Professor Nam Suh and his colleagues at the Massachusetts Institute of Technology [4] and provide a "framework for evaluating and extending design concepts" to manufacturing [3].

The contribution of the research contained in this thesis is to investigate the application of Taguchi's method of parameter design to the design of several typical mechanical systems. The goal is to be able to predict the optimum values for the system parameters - that is, the values that will satisfy the prescribed engineering requirements so that the system is robust (insensitive to the causes of variation of the desired response). Analytical models, design guides, and computer simulations are used to simulate the necessary "experiments".

Section 1.2 - Current Statistical Techniques as a Measure of Quality

As indicated above, the statistical techniques currently used in the majority of American industries emphasize improvements to the manufacturing process. It is assumed that the values of the design parameters that are prescribed are the "ideal" values that will automatically give the best performance of the product; any poor performance of the product must be caused by variation in the manufacturing process.

In addition, some current quality improvement techniques - as well as much current American thinking on quality in industry - are motivated by that philosophy that states that any product that is manufactured such that all of its features lie within the permitted tolerances is of high quality, regardless if the product's performance is exactly equal to the nominal performance value or if it barely lies within the tolerance limit. Shown graphically in Figure 1-2, this philosophy, which Ross refers to as the "Goalpost Philosophy" [5], was supported by Crosby [6], but is criticized by Phadke (who refers to this concept as "The Fraction Defective Fallacy") [7]. The approach assumes that all products that are within the tolerance limits

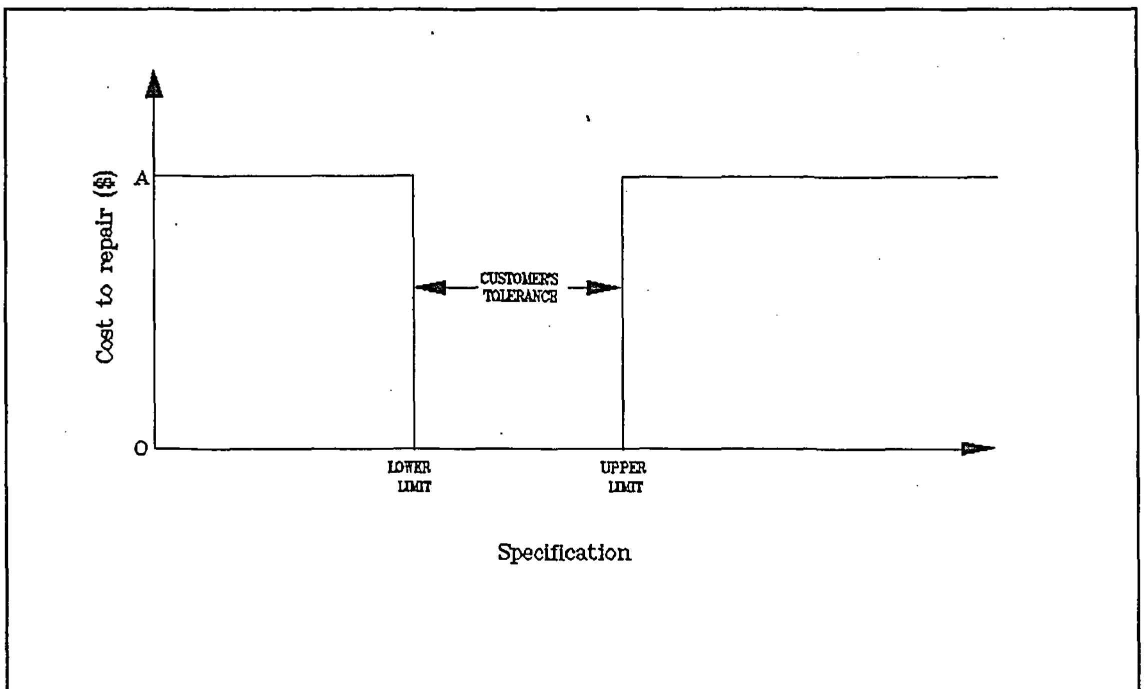

Figure 1-2 - The goalpost philosophy.

impart no deterioration in quality to the customer (the cost of repair of these products equals $0, as shown in Figure 1-2). Furthermore, this philosophy also assumes that all products that are outside of the tolerance limits impart an equal deterioration in quality to the customer (the cost of repair of these products is some cost $A, where "A" is a predetermined, average maintenance expense) as indicated in Figure 1-2.

A good example of the fallacies of the Goalpost Philosophy can be seen in the popular example of the case study conducted by SONY and published by a Japanese newspaper *The Asahi* [8], which was can be found in several textbooks (see, for example, Roy [9]). In this case study, the quality of the color pictures of television sets manufactured in SONY plants in the United States was compared to the quality of the color pictures of television sets manufactured abroad. Even though both locations manufactured the television sets according to the same specifications and tolerances, customers continually preferred the quality of the color pictures of television sets manufactured abroad. The reason for this difference in quality perception was the fact that the American plants concentrated on manufacturing television sets whose picture quality fell within the tolerance limits, whereas the plants abroad concentrated on manufacturing television sets whose picture quality fell as close as possible to the nominal value. By concentrating only on meeting the tolerance limits, the American plants produced television sets that, on the average, had picture qualities that were further from the preferred picture quality than did the plants abroad [7].

Lochner and Matar [10] outlined another modern quality system. They divided this system into two parts: quality of design (to ensure "that new...products...are designed to meet customer needs...and are economically achievable") and quality of conformance ("manufacturing products or providing services which meet previously determined...specifications") [10]. In addition, they outlined several characteristics of a modern quality control system:

1. It is customer driven;
2. It is led by top management;
3. Everyone in the organization understands their specific responsibilities for quality;
4. It is defect-prevention oriented versus defect-detection oriented ("Quality cannot be inspected out of a product, it must be built in.");
5. Quality is a way of life in the organization [7].

This system stresses that quality improvement must be a total effort, a position held by other, more management-based philosophies as Feigenbaum's Total Quality Control (TQC) and the Total Quality Management (TQM) philosophy proposed by Tenner and DeToro, but one that is also held by many of the leading quality improvement leaders.

As mentioned in Section 1.1, the quality improvement procedure widely used in American industry today is Statistical Process Control (SPC). SPC uses control charts to monitor the performance of a manufacturing process or product for purposes of maintaining control of a process within limits. Deviations from the nominal value can be readily observed, and when the process deviates such that its value is either greater than the upper control limit (usually the nominal value of the process plus some tolerance that represents the limit of process capability) or its value is lower than the lower control limit (usually the nominal value of the process minus the same tolerance used in the upper control limit), the process is flagged as being "out of control", and can be shut down until corrective action is taken. This corrective action may be a simple adjustment to the process or product. However, the corrective action may not be readily apparent. In such cases, the search for the cause of the problem (and hence the appropriate corrective action) will involve Statistical Experimental Design (SED). SED was developed in the 1920s by Sir Ronald Fisher [11], who used it in England to improve yields in agricultural applications. Fisher also developed a technique to analyze the data from these experiments called Analysis of Variance (ANOVA) [4]. Excellent descriptions of ANOVA can be found in Ross [5] and Roy [9]. Factorial designs have been used in many applications [9].

Together, SQC and SED form TQC - Total Quality Control (different from Feigenbaum's similarly-titled philosophy) - which is the most frequently used quality control technique used in industry. SPC, according to Taylor [12], "has proven to be an extremely effective practice of manufacturing variation." However, he later adds that SPC is effective up to a point, that much of the variation already exists in the product before the manufacturing process is reached [12]. "To finish the job one must go up front and address variation during product design" [12].

Taguchi's Methods provide a means of "going up front" when addressing variation, since they enable quality to be designed into the process when the product can be made insensitive to external, uncontrollable sources of variation (called *noise* *factors*). In this regard, Taguchi's Methods represent a drastic change in traditional thinking about quality improvement. However, others have offered new philosophies that have been accepted and absorbed into current quality improvement thinking. Dorian Shainin postulated the components swapping technique [13]. In this technique, components are interchanged in an attempt to isolate the component that causes variation. George Box, meanwhile, has collaborated with several authors to link the science of statistical experiment design to response surface methodology [7] and has done considerable research in the SED field. Walter Shewhart and Joseph Juran have both made significant contributions to the field of SPC, with Juran the author of the widely used <u>Quality Control Handbook</u> [14].

But perhaps the most significant contributor to current quality improvement thinking is W. Edwards Deming. Deming proposed fourteen philosophical points of quality improvement. Through such a philosophical approach to quality improvement, Deming has been given (along with Juran, to a lesser extent) the majority of credit for helping Japan develop into a leading manufacturing power. Deming's ideas were reflected in a comment made by AT&T's Chief Executive Officer, James E. Olson in the October, 1985 edition of *Quality* and reprinted in Lochner and Matar [10]. The late Olson said:

> We believe that quality is not something you add to a product, rather something you put into a product or service right from design on through customer feedback. When people say that quality is going to cost money - I don't believe that. When you do it right the first time, you'll end up with a satisfied customer. [10]

Deming's impact on Japanese industry was further evidenced when JUSE (Union of Japanese Scientists and Engineers) created an annual quality award and named it the Deming Award [15]. This award is a highly prestigious award that many companies strive to earn. In 1987, the United States created an American version of this award, the Malcolm Baldridge National Quality Award.

In summary, the approaches to quality listed in this section are more philosophical than statistical in nature. Nevertheless, these philosophies represent important benchmarks in the science of quality improvement in industry.

Section 1.3 - Specific Objectives

The specific objective of this research is to analyze the application of Taguchi's parameter design to five different mechanical systems. In the each application, several issues were studied, such as the number of iterations required for the method to converge to the optimum solutions and several techniques that were involved in using the iteration scheme, the uniqueness of the optimum solutions, the effect of different numbers of control factors, and, of course, whether or not the "optimum solutions" actually satisfied the design criteria. In one example, an actual physical experiment was performed to "confirm" the results of that application. In all of the applications, the iteration process was repeated at least once with different target output responses. Henceforth, each of the five applications will be referred to as a "design optimization procedure" (DOP), while each set of iterations that was performed within each design optimization procedure will be referred to as an "experimental set of iterations" (ESOI).

Section 1.4 - General Approach

Figure 1-3 shows a block diagram of a product or process. The individual components of the diagram are described in detail in Section 2.2.3. This diagram is called a "P-diagram" by Phadke [7].

In each of the five examples described in this study, the design optimization procedures, a nine step approach has been taken. The outline of this approach is as follows:

1. Once the individual system is chosen, the system characteristics that are to remain constant throughout the optimization process are identified;
2. The *output response* and its desired value are determined;
3. The *control factors* (i.e., the system parameters whose optimum values are to be obtained through the optimization process), including the number of such factors and the number of levels of each factor, are determined;
4. The proper starting range for each of the values is determined based on the system characteristics noted in Step One above as well as other engineering constraints;
5. The proper theoretical equation(s) relating the control factors to the output response are identified;
6. These data are transferred into computer code;
7. The computer program is executed to identify the optimum values of the control factors;
8. The optimum values are examined for practicality and other features, using ANOVA if necessary;
9. Repeat steps 2-7 as needed to converge to an optimum value.

In Step One, certain system characteristics that are to remain constant are identified. For example, in the second design optimization procedure (the acoustic

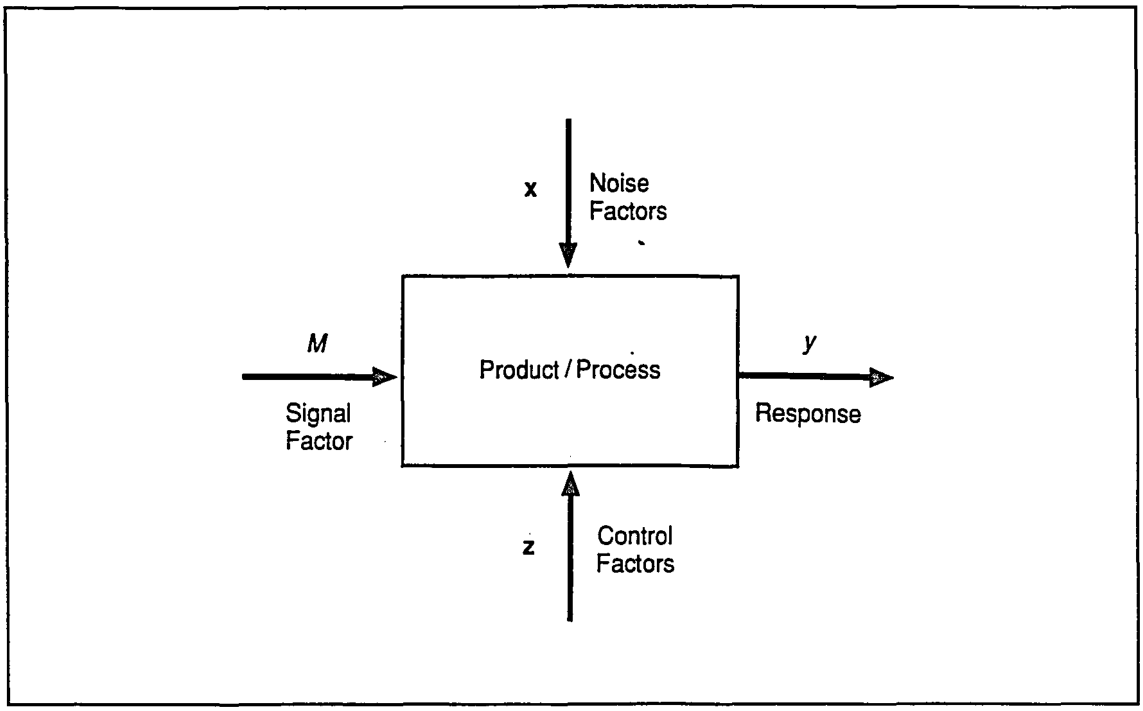

Figure 1-3 - Block diagram of a process or product.

muffler), it is desired to reduce the noise produced by the engine when the turbocharger is used. The turbocharger is not utilized until the engine speed reaches 3000 rpm and the corresponding engine output is 146 hp. These two conditions (3000 rpm, 146 hp) are thus identified as system characteristics that are to remain constant throughout the optimization process.

In Step Two, the desired output response is identified. The output response is simply the "goal" of the experiment. Usually, it is desired to have the output response have a certain value. In the case of the acoustic muffler, the goal was to have a 5 dB reduction in the noise produced by the engine at the "constant characteristics" identified in Step One. Thus, the output response is not the acoustic energy produced by the engine, but the change in the value of this energy.

In Step Three, the number of control factors are identified. Control factors are parameters that the system designer prescribes for the system [7]. In the acoustic muffler, for example, two control factors were identified: the length of the enlarged section of the muffler, and the ratio of the cross-sectional areas of the enlarged portion of the pipe to the regular section of the pipe. In Step Four, the initial starting ranges are prescribed for the design optimization procedure. In the case of the acoustic muffler, the starting ranges were based on physical constraints, rather than from conditions imposed by the theoretical equations. That is, the amount of space underneath a car in which an acoustic muffler can be installed is limited. If the value of the area ratios is too high, for example, the acoustic muffler will be dragging on the ground, or will need to "penetrate" the ground in order to be undeformed. Obviously, this condition is not desirable. Similarly, other design considerations may limit the length of the enlarged section of the acoustic muffler. These considerations must be taken into account when setting the initial range of the control factors. In the acoustic muffler, the initial range of the control factor "l_e" (the length of the enlarged section of pipe) was constrained to be 6" to 48", where as the initial range of the control factor "N" (the area ratios) was constrained to be 6 to 100 (see Chapter 4 for the criteria used to establish these initial ranges).

Step Five merely involves selecting the proper theoretical equation(s) that relate the output response as a function of the control factors. If the equations do not relate the output response as a function of the control factors, then the equation must be solved for the control factors. If this procedure is not possible or practical, then the output response is included as an extra control factor with a fixed response and the new target response is zero (since the equation has now been transformed from the desired expression $y=f(x_1,x_2,..,x_n)$, where y = output response and $x_1,x_2,...,x_n$ = control factors, to the expression $F(x_1,x_2,...,x_n,y)=0$).

Steps Six and Seven involve computer programming. In Step Six, the above data is transferred to a computer program, whereas in Step Seven, the program is executed to determine the optimum values of the control factors. In this thesis, a standard set of computer code was in each design optimization procedure. This code automatically calculated the necessary statistics as required by the Taguchi's methods (see Chapter 2 for more details). The lone changes to the code that were required in Step 6 were the modifications of the necessary theoretical equations done in Step 5. It was also necessary to use different programs based on the number of control factors and the number of levels (possible values) of each factor. Additional information about the computer programs can be found in Appendix A.

Step Seven of the design optimization procedure involves selecting an initial range of control factors, finding an initial set of optimum values of these factors, and then continually reiterating until each successive range for each control factor is very small (that is, the difference is negligible based on the number of practical, significant figures that could be used in the values). This step is covered in more detail in the chapters for each design optimization procedure.

In Step Eight, the values are examined for practicality and other features, such as possible interactions of the control factors. This latter case can be studied using

ANOVA. In the case of practicality, one must be certain that the optimum values obtained through the design optimization procedure, in essence, make sense. For example, if, in the acoustic muffler example, the values for the optimum area ratios converged through the iterations to a value of one, then the procedure would indicate that the optimum design of the muffler would have a tailpipe of constant cross-sectional area with no area difference. In this case, no acoustic muffler would be present and no reduction in energy could result. For this result, the procedure went awry and must be rechecked for possible errors.

The last step of the process involves additional iterations. For example, several of the design optimization procedures were run using two control factors, then were rerun using three control factors and possibly even four control factors. The purpose was to examine the effect of the extra control factor(s) on the optimum values of the other control factors.

Section 1.5 - Organization of this Thesis

Chapter Two contains a detailed background of Taguchi's methods, including his philosophies of quality and how those philosophies effect his use of statistical tools. In addition, a brief history of Taguchi's methods and their reception by leading statistical thinkers is reviewed. The chapter is concluded by a study of the current literature for contemporary studies of parameter design and its applications.

The next five chapters (Chapters 3 through 7, inclusive) detail the results of the research. Each of these five chapters is devoted to a single design optimization procedure, as outlined in the Table of Contents. In each chapter, a background of each mechanical system is included, as well as the initial conditions and control factors, as outlined in Steps One through Six above. The results are then listed and a brief discussion follows. Additional experimental sets of iterations as described in Step Nine above are then included and described, with corresponding results discussed. It should be noted that the mechanical systems studied in the five design optimization procedures - the dynamic absorber, the acoustic muffler, the gear/pinion system, the spring, and the electro-hydraulic servosystem - had no particular significance except for the fact they were representative of a class of mechanical systems for which there existed closed-form solutions for the system responses of these systems as a function of the system parameters.

Chapter Eight is an in-depth discussion and analysis of the results. Chapter Nine is a summary of the research as well as recommendations for further research.

CHAPTER TWO - LITERATURE REVIEW OF AND GENERAL BACKGROUND ON TAGUCHI'S METHODS

Section 2.1 - The Uses of Statistics

Section 2.1.1 - Background on Statistics

Statistics is that branch of mathematics that analyzes numerical data. Statistics helps transform this data into an understandable form. In such a form, the statistics can be studied to determine the causes of the discovered statistical patterns more effectively. A cause of the pattern can then be proposed, studied, and hopefully confirmed, and an action to correct the pattern (if desired) can then be similarly proposed, studied, and implemented. Each of these subsequent steps - identifying the cause of the statistical patterns, studying the patterns, etc. - are in turn studied with the use of statistics. Many text books are available that present the introductory principles and mathematics of statistics (see Dowdy and Wearden [16] and Milton and Arnold [17], the latter of which covers topics such as ANOVA, FDE, parameter design, and Shewhart's control charts) and as such will not be covered in this paper.

According to the World Book Encyclopedia, statistics as a science had their origin in eighteenth-century and nineteenth-century Germany. Governments utilized statistical techniques to conduct censuses, which provided a basis from which taxes could be levied [18]. The first census in the United States, 1790, enabled statistical techniques to be brought to the United States.

Since then, the uses of statistics have spread to practically all the sciences (physical, engineering, business, social, and biological). However, the first widespread exposure of the general public to statistics probably occurred through the first sport to be declared "America's National Pastime", namely, baseball. The first professional baseball league, the National Association of Professional Baseball Players, was formed in 1871 and the first professional league to be defined as "major", the National League, formed in 1876. Public interest in the sport, which had descended from various ball-and-stick games (such as the British games cricket and "rounders"), was such that statistics were used to measure player performances. On October 25, 1845, the New York *Herald* published an account of a game between the New York Club and the Brooklyn Club that included one of the first boxscores (a listing of the statistics for each player in the game) [19]. Henry Chadwick was the first news reporter to cover baseball games on a regular basis, and developed the boxscore in its current form. He also published the first baseball guide (which published baseball statistics for the entire year) in 1860 and edited every baseball guide until his death in 1908 [20]. Due in large part to Chadwick's efforts, in 1879 the National League officially released such statistics as percentage of base hits per time at bat (now known as batting average: a statistic that measures the effectiveness of a batter) and many of the common baseball statistics that are used today [21]. Baseball statistics have since developed beyond an art form into a science, with such annual publications as <u>The Bill James Baseball Book</u> (formerly <u>The Baseball Abstract</u>) and <u>The Elias Baseball Analyst</u> publishing new and innovative baseball statistics for each player, not to mention in-depth, statistical analyses of certain controversial issues.

Section 2.1.2 - Initial Applications of Statistics to Industry

Despite this popularity in the sports world, statistics did not readily spread to American industry which, by the end of the nineteenth century "had become far and away the colossus among world manufacturers" [22]. Lochner and Matar comment:

> Prior to the industrial revolution, quality was built into products as they were made. It was just too expensive to make a product that was unacceptable. People involved in manufacturing knew their products inside and out. Each item was produced, start to finish, either by one person or by a small team of craftspeople who knew what customers expected of the product...With the industrial revolution there were suddenly thousands of unskilled workers involved in high-speed manufacturing operations. For the most part, they did not understand the manufacturing processes or the technology behind them. [10]

In response to this situation, Frederick W. Taylor (1865-1915) used a scientific approach that he called Scientific Management. Among his other axioms, Taylor believed in subdividing a particular process into pieces such that each worker had his own particular task on which to concentrate. Each worker would also be provided with the resources that he or she needed to complete the task, including detailed instructions. This approach improved productivity, but it also reduced quality: since each worker was, in essence, "mentally confined" to his or her own task, the worker was isolated from the responsibility to improve quality [23]. "...Taylor delivered a devastating blow to craftsmanship" stated Juran, who also added that this isolation resulted in separate, "customer-representing" inspection organizations that further divorced the individual laborer from his responsibility for quality [24]. Indeed, G.S. Radford echoed some of Taylor's ideas in his book The Control of Quality in Manufacturing, in which he "identified quality as a distinct management responsibility and even advocated the need for different departments in a company..." [25].

The first significant effort to improve the quality of American Industrial products coincided with the first application of statistics to industrial processes. On May 16, 1924, Walter A. Shewhart formally presented a memo to his supervisor at Bell Labs [26]. In the memo, he proposed a method of using statistics to improve quality. The memo also included a sketch of the first control chart. The idea of control charts was an innovative and revolutionary idea. By tracking the performance of a process or product, and setting limits for acceptable and non-acceptable values of this performance (control limits), control charts enabled the workers to determine when their work was of bad quality. At this condition, the process or product was given the name *out-of-control.* Shewhart had, however, essentially anticipated the goalpost philosophy, since all work that was within the control limits was considered to be "good".

About the same time, Sir Ronald Fisher discovered the basic principles of SED in an effort to improve agricultural yields [11]. Statistical experiment design is alternately called Design of Experiments (DOE), factorial design, or Factorial Design of Experiments (FDE) [9], since it uses all possible combinations of parameters and their associated levels (a fact that can lead to an impractical number of experiments as the number of parameters and/or levels increase). Fisher also founded a technique called ANOVA (Analysis of Variance) to analyze the data obtained from the planned experiments.

The two concepts - SPC and SED - worked hand-in-hand to provide an excellent way to improve quality in the American industries of the pre-World War II decades. SPC would be used to identify abnormal variations in a process or product, and SED would be used to identify the causes of these variations. Corrective action could then be taken to remove the variation from the process or product. As a result, Shewhart's ideas were implemented at Bell Labs and its associated manufacturing wing, Western Electric. In fact, Western published an in-house book in 1956 entitled Statistical Quality Control. This comprehensive text (which is recommended as an excellent reference for details on the basics of both SQC and SED) was one of the first books to combine SPC and SED in one book, coining the title of the book as the new term for this combination. The text further defined this new term by simply defining each of the three words in the term as follows:

> 1. Statistical = having to do with numbers;
> 2. Quality = qualities and characteristics of the process or product being studied;
> 3. Control = to keep something within boundaries, or to make something behave the way we want it to behave;
>
> Statistical Process Control: with the help of numbers, we study the characteristics of our process [or product] in order to make it behave the way we want it to behave. [27]

From these beginnings, SQC grew. In 1936, probability sampling, a scientific method of collecting data within SQC, was applied to a poll conducted by the Gallup organization that attempted to predict the winner of the 1936 Presidential election [25]. That the Gallup poll correctly forecasted Franklin D. Roosevelt to win the election (he won rather easily) while the then premier pollster, *Literary Digest*, selected Alfred E. Landon helped establish the Gallup poll as the premier pollster (it continues to be so to this day). However, it also validated the technique of scientific sampling.

Contributions were also made to the field of SED. These contributions were outlined by Namini (1989) as follows: D.J.Finney (1945) introduced fractional replication; R.C. Bose (1947) developed the "mathematical theory of symmetrical factorial experiments"; C.R. Rao (1946) presented the use of arrays in factorial experiments; R.L. Plackett and J.P. Burmam (1946) introduced saturated orthogonal fractional factorial plans [28].

In addition, other statisticians such as Dodge and Romig added embellishments to SQC, such as sampling plans that "put receiving inspection and final product inspection sampling on a firm statistical foundation" [10]. However, they also note that the focus was still "on quality through inspection, or at best, quality through monitoring production processes." [7] Nevertheless, SQC was a major factor in the American industrial effort to produce the materials and supplies needed for the Allies to win World War II. In addition, the emphasis was still on quality improvement.

After World War II, this emphasis took a change for the worse. With most of the world's industries in disarray, American industry - now in the process of converting to the production of peacetime, everyday products - assumed a monopolistic position in the world. A major industrial philosophy was that "if you could make it you could sell it." [10] Since there were no other major manufacturers to compete with American manufacturers, any defective products were simply repaired or replaced.

Section 2.1.3 - Five Major Statistical Gurus

"Just choke off the competition. Never mind about the customer, he doesn't enter into this at all." [25] So spoke Dr. W. Edwards Deming about the prevailing attitude of the management of American Industry. Deming was one of the four major American statistical experts (Juran, Feigenbaum, and Crosby were the others) who would appear on the scene during the post-World War II period, a crucial one for the development of the quality of manufactured products in America - and in the world. Excellent background information on these figures can be found in Tenner & DeToro (pp. 14-26) [23], and Dobyns & Crawford-Mason (Chapter 3, "Teachers and Sensei", pp. 52-87, which includes background information on the two Japanese statistical giants, Kaoru Ishikawa and Genichi Taguchi) [26]. For the sake of continuity in the narrative, a synopsis of five of these six giants - the four Americans and Ishikawa - conclude this section of the thesis.

Deming, who was born in 1900 in Sioux City, Iowa [25], lived in poverty as a child in the deserts of Wyoming [26], but had the privilege of working at Western Electric about the same time that Shewhart was introducing SPC to the company (and subsequently the world). He eventually met Shewhart and became familiar with

SPC (early 1930's), met Fisher and studied statistical theory under him (mid-1930's) [25], and was recruited to help with the war effort. After the War, he tried to return quality to the forefront of the minds of American management. Unfortunately, Deming observed that "Quality in those postwar years took a back seat to production...getting those numbers out." [29] General Douglas MacArthur, the Supreme Commander of the Allied Powers, invited Deming to Japan in 1947 to assist in preparing the 1951 census, a census that had, as one its functions, to determine the destruction caused by World War II. In 1950, the Union of Japanese Scientists and Engineers (JUSE) invited him back to their country. Deming brought with him Shewhart's ideas of SPC and Fisher's ideas of SED, ideas Deming felt "that American management, though familiar with...in principle, had largely ignored...in practice." [30] Japan had been devastated in the war, and was in extreme economic peril, since most of the country's industries were in ruin and several prominent Japanese colonies (such as Manchuria and Taiwan) were lost. Deming, along with MacArthur, possessed "a deep respect for the abilities of the Japanese, a dogged determination to rebuild their country, and a keen curiosity about Japanese culture" (the latter which he satisfied by his extensive touring of the Japanese countryside [25]. He presented his theories on quality and SQC. The result was that Japan embraced, developed, and applied SQC to their industries to such an extent that it "...institutionalized the use of statistical analysis to control variation and to bring about improvement throughout every sector of Japanese industry, and in every management discipline." [25] At the end of 1980, Japan had become the world's second leading industrial power as well as a country with the third highest per capita GNP in the world - all in a country that lay in ruins 35 years earlier, that has "no natural resources and insufficient land to grow enough crops to feed its people" [23]. Deming was - and is - held in such high regard that he received two overwhelming honors: in 1951, a newly-created, annual quality award was named The Deming Prize (the reception of which has been "an obsession, at one time or another, of the biggest names in Japanese industry"); and in 1960, Deming received the Second Class Sacred Treasure from Emperor Hirohito [25].

Deming, who has been known to be less diplomatic and more direct in his sayings, has been described as a harsh teacher but a very humanistic person [26]. He has also been described as a philosopher [26] who is driven by "less-than-scientific convictions, in particular a religious, puritanical belief system." [25] He summarized his philosophies of quality in his famous "14 Points and 7 Deadly Sins". These 21 philosophies have appeared in slightly different forms, but can be summarized succinctly as follows:

Deming's 14 Points

1. Create constancy of purpose;
2. Adopt the new philosophy;
3. Do not use mass inspection to achieve quality;
4. End the practice of awarding business on price tag alone;
5. Constantly improve the system of production and service;
6. Institute on-the-job training;
7. Institute and teach leadership;
8. Drive out fear, create trust, create an environment that fosters innovation;
9. Break down barriers to team unity;

10. Eliminate slogans;
11. Eliminate work quotas and management by objectives;
12. Remove barriers that rob people of pride of workmanship;
13. Encourage education and self-improvement for everyone;
14. Everyone in the organization - from hourly workers to the CEO - should take action to accomplish the transformation.

Deming's 7 Deadly Sins:

1. Lack of constancy of purpose;
2. Emphasis on short-term profits;
3. Evaluation of performance, merit rating, or annual review/appraisal;
4. Mobility of management;
5. Management by use of visible figures;
6. Excessive medical costs;
7. Excessive costs of liability.

After earning his fame in Japan, Deming finally was received in the United States in the early 1980s.

The second of the leading American statistical thinkers, Dr. Joseph M. Juran had a career that was similar to Deming's in many ways. Born on December 24, 1904, in what is now northern Romania, Juran likewise came from a poor family. Like Deming, he lived in the frontier (Minnesota), and like Deming, he worked within the massive Bell System (Bell Telephone's Hawthorne Works). He also was not initially received in the United States (his first book, which dealt with quality management, was rejected by several publishers until finally published in 1951). And, Juran went to Japan in 1954, where he earned his fame.

The similarities between Deming and Juran are less pronounced after those facts. Juran, for example, places more of his emphasis on quality improvement with management than does Deming. He also feels that while quality is not easy, it is not as hard to attain as Deming seems to suggest, saying that "managing for quality is analogous to managing for finance and does not require a revolution [26]", although he does feel that a revolutionary rate of quality improvement is required [26]. In fact, in his quality improvement philosophy summary, called The Juran Trilogy ("The Juran Trilogy" is a registered trademark of Juran Institute, Inc.), Juran borrowed managerial processes that had been traditionally applied to finance management. Juran used three steps (not fourteen) in his Trilogy, as listed below:

1. Quality planning: a process that identifies the customers, their requirements, the product and service features the customers expect, and the processes that will deliver those products and services with the correct attributes and then facilitates the transfer of this knowledge to the producing arm of the organization.
2. Quality control: a process in which the product is actually examined and evaluated against the original requirements expressed by the customer. Problems detected are then corrected.
3. Quality improvement: a process in which the sustaining mechanisms are put in place so that quality can be achieved on a continuous basis. This includes allocating resources, assigning people to pursue quality projects, training those involved in pursuing projects, and in general establishing a permanent structure to pursue quality and maintain the gains secured.

Juran's emphasis on management led to the term Total Quality Control (TQC) being coined in Japan. Ironically, Juran, who placed less emphasis on the statistical side of quality than perhaps any of the other quality giants, is often credited with introducing SPC to Japan even though he did not receive the Second Class Order

Medal until 1981. Nevertheless, Juran's stature is such that Christopher Hart, a quality consultant and a former professor at the Harvard Business School, said that Juran "...like W.Edwards Deming, is a national treasure. Never has there been a person in a discipline who has such a broad conceptualization of what the field is about...it's a flexible approach, it's not dogmatic." [26]

Kaoru Ishikawa, the Japanese representative in the group of "Statistical Gurus" (Taguchi is not included in this group simply because he and his ideas are discussed in the rest of this chapter), was born of an aristocratic family. His father headed the industrial and engineering groups after World War II, and was responsible for Deming's early journeys and lectures in Japan. Ishikawa perhaps more than Juran promoted the term Total Quality Control, since he believed strongly in the ninth and fourteenth Deming points, namely, removing barriers to team unity and having everybody in the organization taking part in the quality movement. Ishikawa stressed the customer in his philosophies; and, in the spirit of total organization participation, he said that all employees should identify a customer who needs to be satisfied with their work.

Ishikawa authored a book, What is Total Quality Control?, 1985. In it, he gives credit for the term TQC to Dr. Armand V. Feigenbaum. Born in 1920, Feigenbaum, the third major American statistical guru, interacted with Shewhart occasionally, and worked at General Electric for 26 years. He started his career at General Electric in 1942, and by 1944 he had become the top quality expert in the corporation. He is generally recognized as the first person to consider formally the cost of quality, saying that he needed to stress "to a disbelieving market that quality and costs are partners, not adversaries...because the industrial mythology of the time was that better quality has to cost more..." [26]. He claimed that the cost of failures and the rework associated with these failures represents 10%-40% of the annual sales of companies [23].

Feigenbaum said that he tried various quality improvement techniques, all of which improved not only quality, but his entire organization, as well. However, he encountered the same resistance that years later would be known as "culture change" - that is, various departments within the organization would resist the change. "So I began to realize, basically, that quality was not a group of individual techniques or tools...it was, instead, a total field...and I called it Total Quality Control. I did that in the latter part of the 1940s, and nobody would listen." [26] In his book, he outlines six parts to his version of TQC:

1. Business Quality Management;
2. The Total Quality System;
3. Management Strategies of Quality;
4. Engineering Technology of Quality;
5. Statistical Technology of Quality;
6. Applying TQC in the Company. [31]

The primary difference between the TQC described by Ishikawa and the TQC described by Feigenbaum is that Feigenbaum tends to drift toward the Taylor approach of restricting the planning and implementation of quality control to the quality organization, with the remaining workers in their organization merely "just doing their jobs." Ishikawa follows Deming's fourteenth point in that everybody in the organization is involved, at least with the implementation of quality (a total effort of quality control).

The fourth of the American statistical gurus, Philip Crosby, was the youngest, having been born in 1926. He worked for several companies, including Crosley Corp. (Indiana), Martin Marietta, and ITT, before retiring in 1979 and forming the Quality College in Winter Park, Florida. As such, he became the last of the quality gurus to begin independent consulting.

Crosby stated his theories in quality in his first book, Quality Is Free. In it, he, like Deming, summarized his theories in fourteen steps, which in turn were built around four fundamental beliefs, or absolutes (as he called them). They are:

Four Absolutes

1. Conformance to requirements;
2. Prevention (do it right the first time);
3. Zero defects;
4. Price of nonconformance.

Fourteen Steps

1. Management commitment;
2. Quality improvement team;
3. Measurement;
4. Cost of quality;
5. Quality awareness;
6. Corrective action;
7. Zero defects planning;
8. Employee education;
9. Zero defects day;
10. Goal setting;
11. Error cause removal;
12. Recognition;
13. Quality councils;
14. Do it all over again.

If there is any one of the four American gurus who is not like the others in his ideas, it is Crosby. For example, Crosby's idea of zero defects is in sharp disagreement with the other gurus, who hold that some variation is inevitable. Furthermore, zero defects - which was supposed to be a way to make management believe that it did not have to accept that defects always had to exist - ended up being twisted into an excuse for management to place the blame for defects on the worker. In essence, it was deformed into a motivational tool for management. For example, in the beginning of his book, Zero Defects, Halpin lists three causes of defects: lack of knowledge, lack of proper facilities, and lack of attention [32]. He continues by saying that the first two causes are easily correctable. The third cause, however, is the subject of the entire book. He adds, "...it is the subject of improper employee attitudes...Management can show the individual how to do it and provide the best possible tools to accomplish the task properly, and if the employee doesn't care whether he or not he makes a mistake, he will probably err." [32] Crosby supports Deming's tenth point, not using slogans or other exhortations to motivate workers (even though his concept of Zero Defects is itself a sloganish phrase), and claims that the idea of zero defects was twisted by the Defense Department [26]. Finally, the biggest controversy with Crosby's philosophies lies with his very first absolute: conformance to requirements. Crosby, as mentioned in Chapter 1, claims that any product or process is of high quality if it meets the specifications, even if it is barely within the tolerances (see Figure 1-2). This "goalpost" philosophy shifts the focus away from producing a product that is as good as it can be to one that is merely good enough.

There is some merit to these ideas: "do it right the first time" does help the individual worker recognize his or her role in quality improvement, not to mention the fact that the emphasis is on prevention, not inspection. Similarly, conformance to requirements implies focusing on the customer and the expectations that the customer has for the product.

Crosby also authored the 1984 book Quality Without Tears, in which he reiterates the four "absolutes" (one short chapter for each) as well as the fourteen points [33]. He also makes an interesting analogy, comparing an organization with a quality problem to a sick patient. He lists symptoms of the "patient" in one chapter, then spends a chapter describing the "serum" with which the organization can be cured.

All of these gurus have differences, but there are common threads to their theories. All four stress continuous improvement by removing unwanted variations [23]. They also moved industrial thinking from inspecting in quality to preventing defects in the first place. Occasionally, they commented on the work of the others. Deming, for example, respected Juran and thought that his 1954 visit to Japan was very beneficial to the Japanese [26], but was openly critical of Crosby, philosophically disagreeing with him vehemently on such issues as inspecting out defects to ensure zero defects and the goalpost philosophy.

Section 2.1.4 - Other Contributors to the Use of Statistics in Industry

In addition to the major quality gurus listed above, several other persons have made contributions to the statistical field that are worth mentioning. Quality function deployment (QFD) was developed in the early 1970s at the Kobe Shipyard in Japan [10]. This highly structured technique, described in detail by Bossert [34], includes a planning matrix, in which customer requirements are listed in the rows and final product characteristics are listed in the columns, and a deployment matrix, which lists customer requirements and product control characteristics. The main intent of these matrices is to ensure that the customer has a loud voice in the product development process. Similarly, Winner, Pennell, Bertrand, and Slusarczuk (1988) [35] outlined a process called concurrent engineering, in which development, manufacturing, and support organizations are integrated into one unit for a particular product. Reliability analysis (RA) arose to test the reliability of a product over time, and failure mode analysis (FMA) arose as a systematic approach to determine the various modes of failure for a particular product (a sort of particular set of SED). Dorian Shainin was a leading proponent of component swapping (see Taylor [12], pp. 267-276 for additional details), in which components in units of product with consistent differences are switched and the effects of this switch are studied in order to isolate the component responsible for the cause of the variation [10]. Dr. George E.P. Box and others developed the area of Response Surface Methodology (RSM) in the 1950s (see references [36] and [37] for more details on this technique). Box has contributed greatly to the field of SED. Finally, on the management philosophy side of quality control, Tenner and DeToro have taken TQC one step further. They have proposed TQM (Total Quality Management). TQM uses three fundamental principles (customer focus, process improvement, and total involvement) and six supporting elements (leadership, education and training, supportive structure, communications, reward and recognition, and measurement) to "utilize the talents of all employees, to the benefit of the organization in particular and society in general, and...[provide]...a positive financial return to the shareholders." [23]

Section 2.2 - Taguchi's Methods

Section 2.2.1 - Background

The ideas of Genichi Taguchi, the sixth of the major statistical gurus, are revolutionary because they emphasize the placement of quality improvement techniques upstream to the early (design) phases of the product life cycle as opposed to the later (manufacturing) phases of the product life cycle (see Figure 1.1). Furthermore, he does not propose the elimination of sources of variation. Rather he emphasizes that products should be designed that are insensitive to these sources of variation - a term he calls *robust*. If products can be designed to be robust, then lower-grade components can be used (since the variation of the final product due to the low-grade component would have been minimized), and quality can be achieved with a cost savings, not an increase in cost.

But Taguchi's methods do not only impact on the SPC side of TQC - they impact on the SED side, as well. By using orthogonal (or "balanced") arrays, the number of experiments needed to provide the experimenter with essentially the same amount of significant data as in traditional, factorial designs is reduced, thus reducing experimentation costs.

Since TQC is the sum of SQC and SED, the contribution of Taguchi's Methods to TQC is the sum of their contributions to both SQC and SED:

> *Taguchi's methods provide a means to determine the optimum values of the characteristics of a product or process such that the product is robust (insensitive to sources of variation) while requiring less experiments than is required by traditional methods.*

Obviously, since Taguchi's methods reduce materials cost and experimentation costs while providing higher quality, Taguchi's methods also provide an enormous cost savings to the entire organization.

Section 2.2.2 - A Brief History of Taguchi's Methods

Taguchi developed his methods in the 1950s and 1960s parallel to the developments of Deming and Juran. Working in the same ravaged country, with a shortage of high-grade, raw materials, Taguchi was given the task of developing a methodology to develop high-quality products. Taguchi, who at the time was a product development manager at the Electric Communications Laboratories at Nippon Telephone and Telegraph Company, succeeded by developing the principles of robust design and was awarded the individual Deming Award in 1962.

A practical application of Taguchi's Methods occurred at the Ina Tile Company in the late 1950s [7]. The company faced a quality crisis due to wide variances in dimensions on the tiles produced. Screening the defective tiles was rejected as too expensive of a solution. A fact-finding team studied the problem and discovered that the major cause of the dimensional variation was a similar variation in temperature within the kilns that baked the tiles. To eliminate the cause of the variation, the company would have had to redesign and rebuild the kiln itself, a half-million dollar project. However, the team decided to minimize the sensitivity of the tile dimensions to the temperature variation in the kiln. They discovered, through "...a small set of well-planned experiments according to Robust Design methodology..." [7] that by changing the lime content of the tiles, the dimensional variation was reduced significantly with a minimal cost (since lime also happened to be the least expensive ingredient in the tiles). The Ina Tile Company obtained a significant improvement in quality at a significantly lower cost by using robust design techniques.

Section 2.2.3 - The Parts of Taguchi's Methods

Taguchi's methods begin with the definition of the word quality. Taguchi employs a revolutionary definition: "Quality is the loss imparted to society from the time a product is shipped." [38] Phadke further defines the ideal quality that a customer can expect is that "every product delivers the target performance each time that the product is used, under all intended operating conditions, with no side effects." [7] Note that this definition goes beyond the traditional ideas of Crosby, who measured quality in terms of the cost of correcting imperfections in the factory, and Feigenbaum, who included the customer in his definition ("quality is whatever the customer says it is" [26]). Taguchi includes *all* the costs associated with a bad product. For example, if an important part malfunctioned on a weekend, the total loss to society would include the value of lost production and the cost of the repairman's wages (at an overtime rate). However, it would also include the loss of the repairman's rest and relaxation, perhaps a chance to teach his or her son how to play baseball, and extra wear on the repairman's car.

It was for all of these extra losses to society that Deming claimed that the total cost of quality could not be calculated [26]. Taguchi, however, claimed that the cost could and should be calculated. He proposed the quadratic loss function as an approximate measure of the quality loss. This function L(y) is given by

$$L(y) = k * (y-m)^2 , \tag{2-1}$$

where m is the target value of the product or process, y is the actual value of the product or process, and k is a constant, the quality loss coefficient. Note that if the product or process is operating at the nominal value, the quality loss is $0.

The quadratic loss function also provides a way to quantifiably reject the goalpost philosophy of "just meeting the tolerance limits" which caused the difference in preferences among the SONY televisions that was reported in Chapter 1. In Figure 2-1, the quadratic loss function is plotted, along with the approximate distributions of the quality of color pictures in SONY television sets made in the United States and those made abroad [7]. Note that more sets made in the U.S. were closer to the tolerance limits, where the quality loss was very high, compared to sets made abroad, most of which were close to the nominal value, where the quality loss was low.

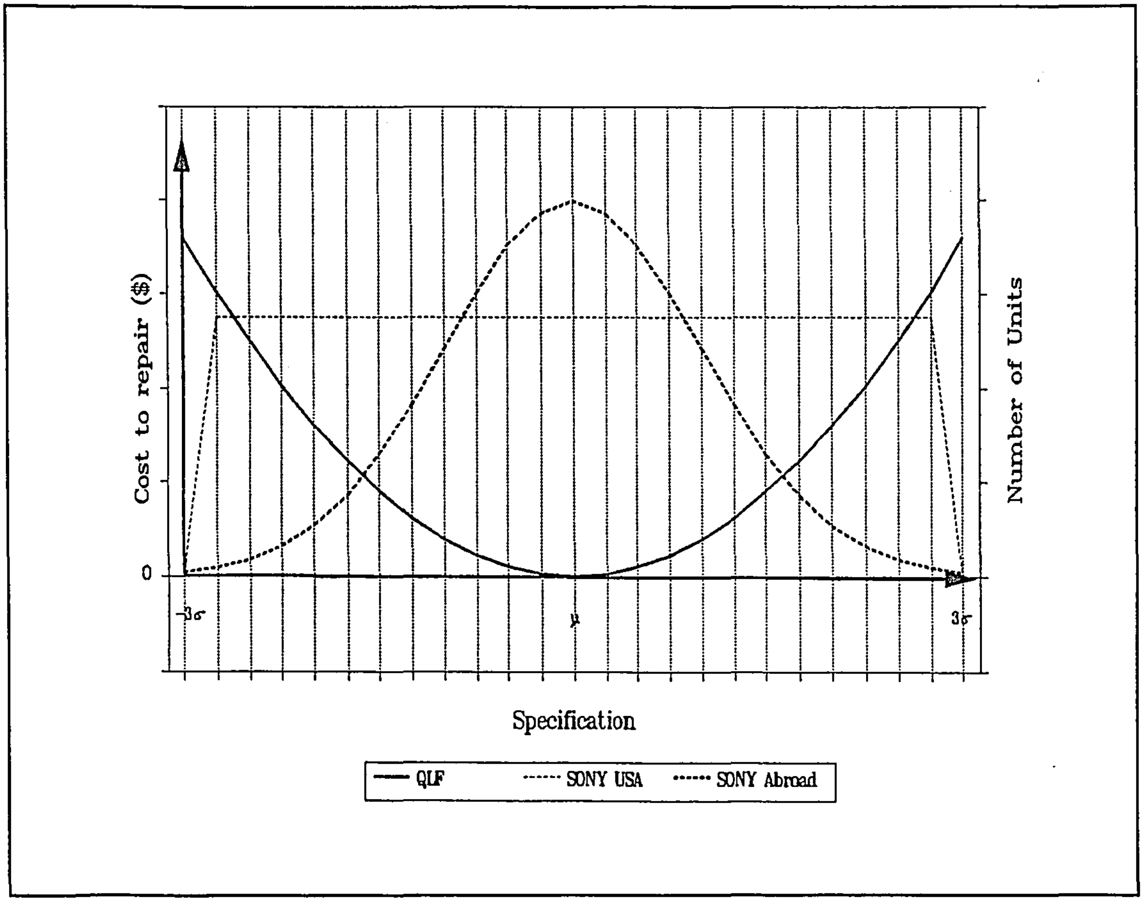

Figure 2-1 - Quadratic loss function, distribution of SONY television sets in U.S.A. and abroad.

In Taguchi's methods, sources of variation are called noise factors. There are three types of noise factors: external (such as the environment); unit-to-unit (the variation that all of the major statistical gurus claim is inherent to all manufacturing processes); and deterioration (decreased performance over time) [4].

Noise factors form one type of input to a product or process. The other input factors are signal factors and control factors, while the lone output signal is usually the response. This relationship was shown in Figure 1-3. The types of factors are best defined by using the example of an automobile. One of the car's responses is its acceleration. In this case, the signal factor (factor that is set by the user of the product to achieve a desired response) would be the gas pedal. The control factors (factors that are specified freely by the designer and unalterable by the user of the product) would include engine horsepower, number of cylinders, transmission type, and many others. Noise factors would include such items as wind resistance (external), wet versus dry pavement (external), wear of the engine parts (deterioration), and the fact that the gas pedal might be slightly thicker than its nominal value (unit-to-unit).

Taguchi's methods consist of three different methods: system/concept design, parameter design, and tolerance design. These three methods are summarized in Figure 2-2 (which is reproduced from Chen [39]), and are described below in more detail.

1. System/concept design refers to the design of the overall system. In the example of the automobile, one potential system would be a turbocharged, 146-hp standard transmission engine, while another potential system would be a V8, 200-hp automatic transmission engine.
2. Parameter design refers to the design of the actual components of the system. In this step, the overall system is now fixed, and the designer must decide what type and level of control factors will be used in order to minimize quality loss. In the automobile example, if the first system above were chosen, one of the parameters would be to choose among a two-inch diameter cylinder or a three-inch diameter cylinder.
3. Tolerance design refers to the determination of the acceptable variation in the individual components. Usually, this step is performed only if the first two steps are unsuccessful. Since lower tolerances mean higher manufacturing costs, this step involves a tradeoff. In the example of the automobile, if all of the possible cylinder

diameters did not enable the engine to achieve a required acceleration, then the tolerances of its diameter would have to be made smaller.

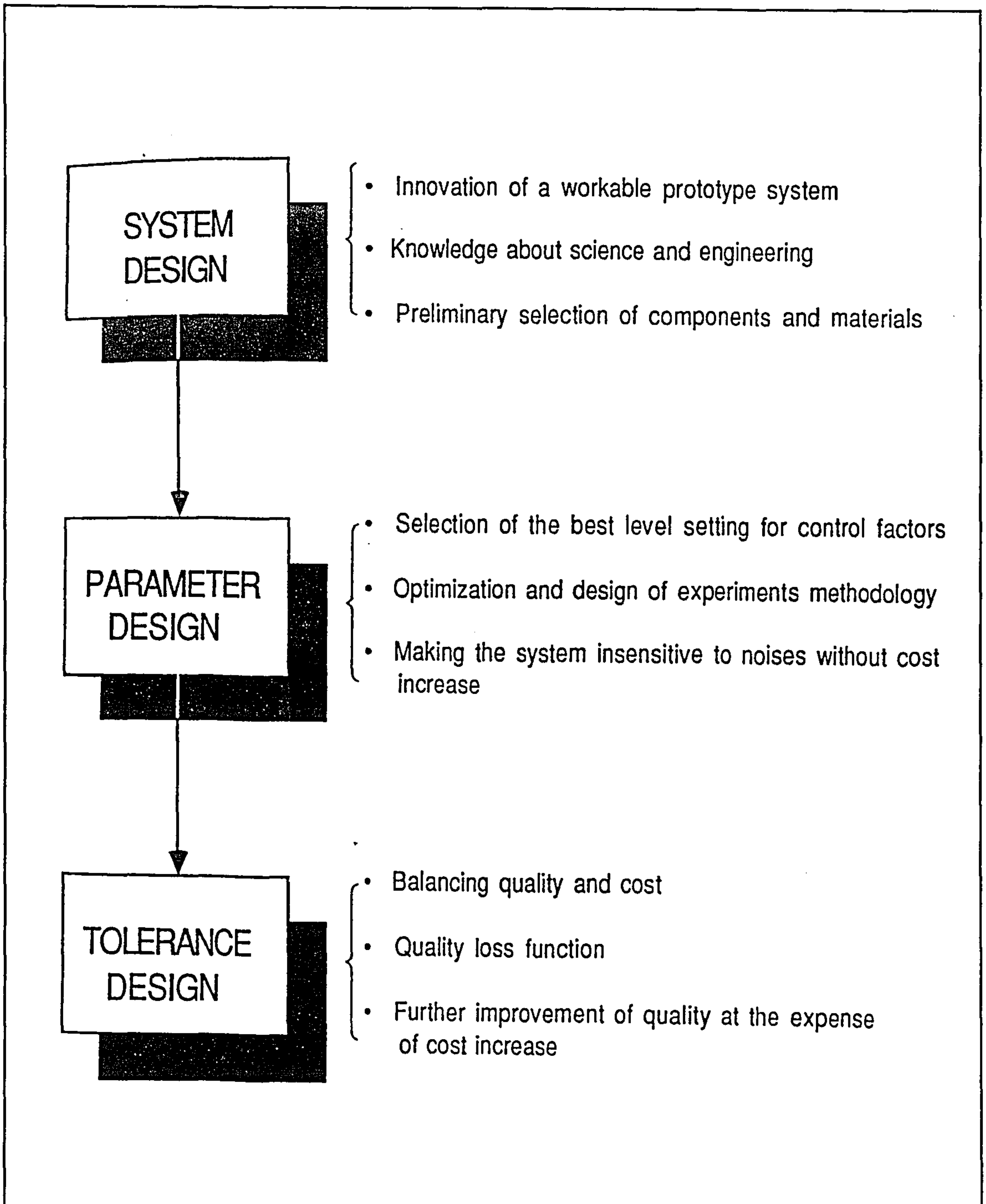

Figure 2-2 - The three methods of Taguchi's robust design.

Note that the ideal Taguchi method is parameter design, since robust designs can be achieved easily and at low costs in this step. System/concept design requires significant breakthroughs to make significant improvements, and these breakthroughs, which cannot be scheduled or predicted, can lead to long development cycles. Tolerance design, as mentioned, involves added cost to the manufacturing cycle.

In Table 2-1 (which is reproduced from Phadke [4]), the effect of the three Taguchi methods on the each of the four product realization steps is summarized. Note that it is only during product design that all of Taguchi's methods can reduce the effects of all of the types of noise factors. Hence, Taguchi stresses the application of his methods to the design phase. It is this application, though it is the application of only the parameter design method, that will be studied in this thesis.

The actual steps in using Taguchi's method of parameter design (which will be called robust design for the remainder of this thesis) are divided into three groups: planning the experiment, performing the experiment, and analyzing the data from the experiment. Phadke summarizes these steps as follows:

PLANNING THE EXPERIMENT:

1. identify the target response, side effects, and failure modes;
2. identify noise factors and testing conditions;
3. identify the quality characteristic to be observed and the objective function to be observed;
4. identify the control factors and their levels;
5. design the matrix experiment and define the data analysis procedure;

PERFORMING THE EXPERIMENT:

6. conduct the matrix experiment;

Table 2-1 - The effect of the three methods of Taguchi's robust design on three phases of the product life cycle.

Product Realization Step	Quality Control Activity	Ability to Reduce Effect of Noise Factors			Comments
		External	Unit to Unit	Deterioration	
Product design	a) Concept design	Yes	Yes	Yes	Involves innovation to reduce sensitivity to all noise factors.
	b) Parameter design	Yes	Yes	Yes	Most important step for reducing sensitivity to all noise factors. Uses Robust Design method.
	c) Tolerance design	Yes	Yes	Yes	Method for selecting most economical grades of materials, components and manufacturing equipment, and operating environment for the product.
Manufacturing process design	a) Concept design	No	Yes	No	Involves innovation to reduce unit-to-unit variation.
	b) Parameter design	No	Yes	No	Important for reducing sensitivity of unit-to-unit variation to manufacturing variations.
	c) Tolerance design	No	Yes	No	Method for determining tolerances on manufacturing process parameters.
Manufacturing	a) Detection and correction	No	Yes	No	Method of detecting problems when they occur and correcting them.
	b) Feedforward control	No	Yes	No	Method of compensating for known problems.
	c) Screening	No	Yes	No	Last alternative, useful when process capability is poor.
Customer usage	Warranty and Repair	No	No	No	

Source: Adapted from G. Taguchi, "Off-line and On-line Quality Control System," International Conference on Quality Control, Tokyo, Japan, 1978.

ANALYZING THE RESULTS:

7. analyze the data to determine the optimum levels, and predict the product/process response under these optimum levels;
8. conduct a confirmation experiment to verify the optimum levels. [4]

This procedure is the one that was outlined in Section 1.4 and was used in the entire research, hence, an example at this point is not necessary. However, two explanations are in order. The quality characteristic is also called the signal-to-noise (S/N) ratio. It is defined for a nominal-the-best procedure as

$$S/N = -10*\log_{10}(\frac{\mu^2}{\sigma^2}) \tag{2-2}$$

and as such, is a measure of the signal of the process (its response) to the noise factors in the process. According to Phadke, maximizing this ratio is equivalent to minimizing the quality loss after adjustment, as well as maximizing the robustness of the product. Note that Equation 2-2 can be rewritten as

$$S/N = -10*\log_{10}(MSD)^2, \tag{2-3}$$

where MSD is the actual response minus target response. Minimizing the MSD maximizes the S/N ratio. Equation 2-3 is used extensively in this research.

The other term to be explained is that of the orthogonal array. An orthogonal array, the L9 for example, is shown in Table 2-2, where there are four control factors each with three levels that will be combined to form nine trials. The orthogonal array is a device that lists the particular levels of each control factor for each trial of the iteration. As Sandgren points out, "the term experiment [trial] need not refer

Table 2-2 - The L9 orthogonal array.

TRIAL	FACTORS 1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	1	2	3
9	3	3	2	1

to a test of existing parts as it may also be the result of a computational operation." [40] Each column represents a control factor. A control factor might be temperature, time, size, etc; each control factor is assigned levels, such as low, medium, and high. Each row of Table 2-2 represents the individual trial and the levels of each control factor to be used in that trial. The three possible levels for control factor one could represent, for example, a low (level one), medium (level two), and high (level three) temperature setting for a thermoforming machine. Similarly, control factor two might represent forming time, with levels of 1 second (low), 20 seconds (medium), and 40 seconds (high). Again for example, the fifth trial would be conducted with control factors one and two at level two, control factor three at level three, and control factor four at level one. Note that between any two columns, every possible pair of control factor levels is found and no pairs are repeated. Thus, the array is orthogonal or balanced. Note also that only nine trials are required to conduct this experiment, whereas $3^4 = 81$ trials would have been needed to conduct the corresponding, factorial experiment (where each factor is varied, one at a time, while all of the other factors remain constant). Finally, it is assumed in this array that the control factors are independent variables - that is, the control factors do not interact. For systems where interactions do exist, a larger orthogonal array must be used in order to estimate the effect of such interactions.

With the concept of the orthogonal array presented, the entire framework of the experimental procedure can be defined. Each row of Table 2-2 represents a trial. The group of nine trials in Table 2-2 represents an iteration. A group of iterations that converged to optimum values of the control factors constitutes an experimental set of iterations (ESOI). Finally, experimental sets of iterations, each based on the same mechanical system, constitute a design optimization procedure (DOP). This terminology will be used throughout the remainder of the text.

Figure 2-3 is a visible representation of the concept of robust design taken from Dehnad [32]. In this diagram, output voltage of a particular circuit is shown as a function of the gain of a particular transistor in the circuit. Note that if the value of the transistor gain is varied to a small degree about a target value x_0, the value of the output voltage changes dramatically. However, if the transistor gain were to vary, even by a larger amount, about a target value x_1, the corresponding change in output voltage is much smaller than before. Thus, the transistor gain x_1 provides a more robust process/product than does the transistor value x_0. The optimum value of the transistor gain is the value that produces the most robust process/product.

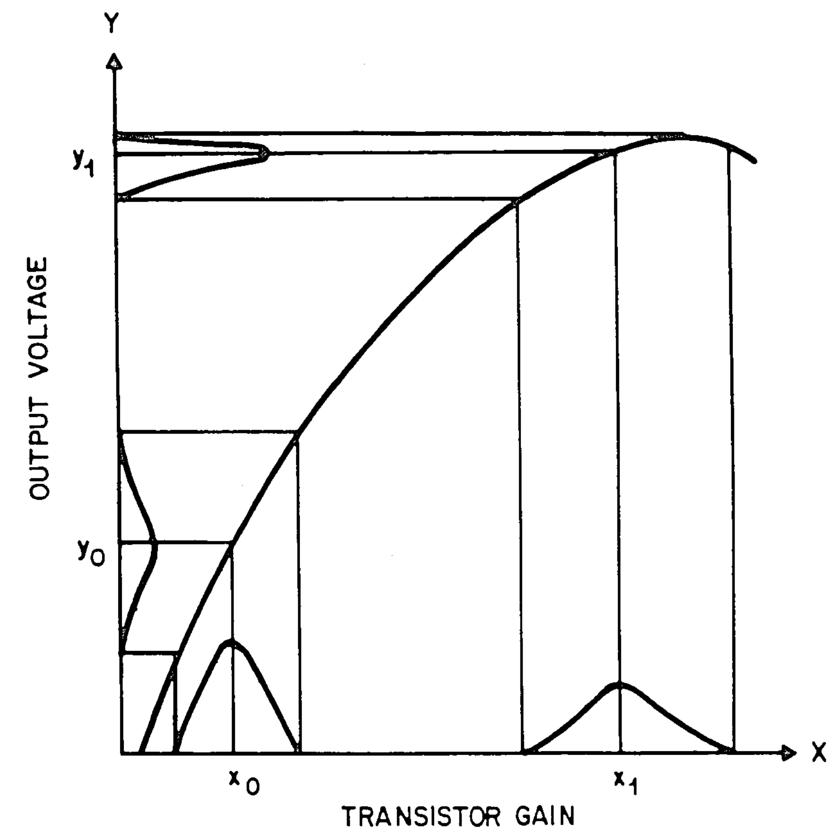

Figure 2-3 - Visual representation of robust design.

Obviously, this value appears to occur just to the right of x_1. This novel approach takes advantage of the non-linear relationship between the output response (output voltage) and the control factor (transistor gain) and suggests that the levels of the control factor under consideration (i.e., x_o and x_1) should be widely spread.

Section 2.2.4 - Taguchi Methods versus Other Quality Methods

Taguchi's methods are similar to other quality methods in several ways. For example, Taguchi tries to compute the cost of quality, as does Feigenbaum and Crosby. In addition, Taguchi emphasizes, as do the other quality gurus, the need to reduce the final, overall variation of the desired product/process response. Finally, Taguchi's Methods make use of SPC, SED, and ANOVA, not to mention QFD (which can be used in system/concept design).

Taguchi's methods differ from traditional methods in the way that they are applied. Taguchi insists (for reasons demonstrated above) that his methods be applied in the design phase, whereas most SQC techniques are applied in the manufacturing phase. In addition, Taguchi measures the total loss of quality as the loss incurred by all of society, not only the manufacturing plant or the customer. By attempting to quantify this loss, Taguchi has mathematically shown the failings of the goalpost philosophy. In fact, he himself commented that "it was in opposition to Zero Defects that Taguchi Methods emerged." [41] Perhaps the biggest difference between Taguchi's methods (particularly parameter design) and traditional methods is that Taguchi does not propose to eliminate the sources of the overall variation of the product or process, but rather proposes to reduce the sensitivity of the product or process to these sources. The sources of variation are still present, but the actual product or process variation can be eliminated (or reduced).

Section 2.2.5 - Current Standing of Taguchi's Methods

Perhaps the most accurate way of summarizing the current status of Taguchi's methods is that some of Taguchi's ideas are very controversial. Taguchi's methods have been implemented and proved successful, but western statisticians disagree with some of his tenets.

Section 2.2.5.1 - Acceptance/Rejection of Taguchi's Methods

Taguchi's methods have been used in many companies with success. Dehnad comments "Many Japanese companies have experienced success with Taguchi's strategy for off-line quality." [38] Phadke lists seven particular applications and one more general application of Taguchi's methods to processes at AT&T [7]. Phadke also mentions that the American Supplier Institute and Ford Motor Company have enabled several automobile manufacturers to achieve quality and cost improvement using the method [7]. Ashley comments that among those world-class manufacturers that have been aided by the use of Taguchi's Techniques include Hitachi Ltd., NEC Corp., and Toshiba Corp [42]. Finally, Yokoyama and Taguchi showed that the methods could be successfully applied to profit-planning in business, cash-flow optimization in banking, and government policymaking [43]. Although criticized, Taguchi's methods have not been flat out rejected, although his concept of an optimum value implies that, theoretically at least, continual improvement has a limit, an idea that is contrary to Deming's fifth point.

Section 2.2.5.2 - Compliments/Criticism of Taguchi's Methods

Taguchi has received compliments from several different sources, particularly from the authors of statistical and quality methods texts. Ross, Roy, Phadke, and Dehnad all speak favorably of the method in their books on Taguchi's methods. Lochner and Matar, for example, used for their book, Designing for Quality, the subtitle "An Introduction to the Best of Taguchi and Western Methods of Statistical Experimental Design" [10]. Dobyns and Crawford-Mason include Taguchi in their chapter entitled "Teachers and Sensei", which includes the other quality gurus (Deming, Juran, Feigenbaum, Crosby, and Ishikawa) as well as Shewhart [26]. Taylor lists Taguchi Methods ("TM") alongside of SPC as the "two important approaches to optimization and variation reduction that have received special notoriety in recent years." [12] Gabor comments that Deming "...is a great admirer of Taguchi..." [25]. And Taguchi received perhaps the ultimate compliment in 1962, when he won the individual Deming Award.

However, Taguchi has received much criticism of his methods. Deming disagrees with Taguchi's attempt to quantify quality loss [26]. Crosby says that Taguchi "is impossible to understand unless you have a Ph.D. in mathematics or economics" [26], an interesting comment since Taguchi does not have even a bachelor's degree in those fields. Crosby also says that "he doesn't see how Taguchi's theories would help an American manager run a quality company." [26]

Other stronger criticisms exist. The most notable criticisms appeared in the October, 1985 issue of *Journal of Quality Technology*, where Box, Easterling, Freund, Lucas, and Pignatiello and Ramberg responded in a panel-type discussion to Kackar's article, "Off-Line Quality Control, Parameter Design, and the Taguchi Method." [44] This issue is an ideal reference for contemporary criticism of Taguchi's methods. As a brief summary, the following criticisms are made:

1. The sequential nature of investigation is not exploited (Box);
2. Simpler and better-researched constrained optimization procedures are passed over (Box);
3. Data transformation techniques are not used (Box);

4. Interactions receive little attention (Easterling);
5. Disagreement on using an overall performance statistic (Lucas).

Perhaps the best concise summary of the criticisms of Taguchi's methods was given by Richie (1989), who said "The consensus is that Taguchi's philosophy is an excellent vehicle for incorporating quality considerations into the design of a product, but the analytical methods leave something to be desired." [45]

Despite these criticisms, it seems that Taguchi's methods are usable means to achieve quality and cost improvement. As Hsiang noted in a review of Dehnad's book, "Although acceptance has been slowed by statistical controversy, Taguchi methods are beginning to have a greater effect on U.S. products than any one concept or method yet devised." [46]

Section 2.3 - Literature Search - The Use of Parameter Design

Section 2.3.1 - Current Applications

Filippone (1989) presented an application of Taguchi's parameter design method to an RC Filter Network [3]. Filippone targets two objectives for the system, but his overall purpose is "to illustrate the application of Taguchi methods with an axiomatic approach rather than detailed design of a complex circuit." [3] He reports that "the two design axioms agree in principle with the methods of engineering described by Taguchi". [3] He adds that "the language of axiomatics is different from that of Taguchi methods, but the principles are the same: maintain independence of functional requirements and minimize the information content of the design", [3] and also lists a table comparing the language of axiomatics and Taguchi's methods.

Namini (1989) presents new results "in the comparison of influence of sets of observations under robust designs..." [28]. This mathematically-intense paper is directed more towards the theoretical mathematics behind the experimental arrays. Another mathematically-intense paper is presented by Jo (1991), who studies "the estimation of robustness for dynamic systems with structured uncertainties." [47]

Chang (1991) applies Taguchi's methods to optimize producibility for manufacturing systems. He "introduces the new concept of producibility loss and proposes the signal-to-noise ratio to control producibility." [48] And Rivers (1990), as noted in Section 1.1, applies Taguchi's methods "to investigate job satisfaction and its effect on productivity", thus showing how Taguchi's methods can be used "in solving complex behavioral problems." [2]

Section 2.3.2 - Applications to Mechanical Systems

Chen (1990) considers the optimization of dynamic systems (systems where the ideal value can vary) by examining various quality loss functions and tolerance design. He studies the validity and limitations of the S/N ratio and proposes an optimization model that balances quality and cost in tolerance design. He presents a flow chart for the optimization procedure using parameter design. Finally, he also examines multivariate loss functions for systems with multiple quality characteristics. [39]

However, this procedure is a mathematically complicated procedure with no immediate, practical application to mechanical systems.

Kim (1988) presents a standard parameter design approach, with the emphasis on new techniques "to investigate two-factor interactions for 2^t and 3^t parameter designs" (designs with control factors with only two or three levels), and the major objective "to be able to identify influential two-factor interactions and take those into account in properly assessing the optimum setting of the control factors." [49] Also, "an extension of the parameter design to several quality characteristics is also developed by devising suitable statistics to be analyzed, depending on whether a proper loss function can be specified or not." [49] This goal is achieved using a vector/matrix approach, in which the output responses, for example, are formulated in a vector **Y**. This paper, like those of Namini and Chen, is theoretical in nature and has few examples of the application of the research.

Chao (1990), however, presents one of the more significant applications of parameter design to a mechanical system, namely, to articulating linkage mechanisms in robotic systems. He presents "a strategy for the application of...set theory to multi-objective optimal design requirements...to choose the compromise solution." [50] In addition, he accomplishes "the formulation of the...multi-objective optimization design problem...by integrating design variables and by defining the...domains corresponding to the objective functions and the constraints." [50] Finally, "the...multi-objective optimization problem is solved by using conventional single-objective function programming techniques. [50]

Ritchie (1988) presents a methodology for design optimization. He uses a multiple-objective approach, rather than "collapsing the data into a single summary statistic" [45] using linear weighting of multivariate criteria or a true multivariate model to form a desired performance. Finally, he fits a "response surface" to the data, to give "a better understanding of what is happening to the process, in terms of how the responses of interest are affected by changes in the design parameters." [45] With this surface, he adds, "one can locate an exact optimum rather than just improved points." [45] Benjamin (1991) conducts similar studies, constructing a "Multiple Criteria Optimization Approach to formulate the problem and an RSM procedure to solve it." [51] This paper also features the added use of computer simulation to perform the Taguchi trials.

In his October, 1985 article in *Journal of Quality Control* mentioned above, Kackar presents a well-written article on Taguchi's parameter design method. He discusses the method in enough detail to warrant complimentary comments from a panel of statistical authorities in subsequent articles. Taguchi's method for conducting the parameter design experiments is discussed. No recent physical applications are explained in detail. Also in this issue is an article by Hunter that describes parameter design ("Product Design") and covers such topics as sensitivity analysis, experimental design, orthogonal arrays, 2^k and 2^{k-1} factorial designs, second-order and three-level designs, experimental design geometry, and the concerns listed earlier in the journal and listed above, such as interactions, data transformations, and signal-to-noise ratio as an effective measure of quality. [52]

The most recent, large-scale discussion of the method in the literature, and thus the most up-to-date with respect to current trends in parameter design, occurred in the May, 1992 issue of *Technometrics*. Although no physical applications were discussed, the most recent opinions of statisticians such as Abraham, MacKay, Box, Kacker, Lorenza, Lucas, Myers, Vining, Nelder, Phadke, Sacks, Welch, Shoemaker, Tsui, Shin Taguchi, and Wu are presented, edited with introductory comments by Nair. [53]

Finally, the best source of information on current applications of Taguchi's methods in the literature is the *Symposium on Taguchi Methods*. The symposium is hosted by The American Supplier Institute Center for Taguchi Methods once every month. The proceedings from the symposium are published in the form of a manual which is over 600 pages in length. Each symposium contains presentations of over 40 applications of Taguchi's methods to various industrial applications, such as "Capstan Motor Assembly Optimization Via Taguchi Techniques", "Improving a Gold Plating Process Using Taguchi Methods", and "Wave Soldering Optimization by Use of Taguchi Methods" [54]. It should be noted, however, that the symposiums concentrate principally on the Manufacturing and Manufacturing design phases of the product life cycle.

Section 2.3.3 - Other Topics in the Literature Worthy of Mention

Two other sources in the literature were found to be of worthy note. Taylor's previously referenced Optimization & Variation Reduction in Quality (1991) provides an excellent, concise method of comparing Taguchi's methods of improving quality through robust design to other optimization techniques as SPC, SED, and components swapping [12]. And, Arora (1989) provides an entire textbook on mathematical constrained and unconstrained optimum design theory entitled Introduction to Optimum Design [55]. These techniques suggest a way to circumvent Ritchie's use of a multivariate, linearly weighted objective function, and are the techniques used in this research to develop appropriate objective functions.

Although the application of the method has been studied in the literature, the application of Taguchi's parameter design to specific mechanical systems with closed-form solutions appears sporadically in the literature. In addition, iteration schemes for convergence to optimum solutions are not covered. Thus, the research in this paper, while following along the lines of the research of Ritchie, Chao, Kim and Benjamin, constitutes an original contribution to the field.

CHAPTER 3 - DESIGN OPTIMIZATION PROCEDURE EXAMPLE ONE: THE DYNAMIC ABSORBER

Section 3.1 - Background

The first system to be used as an example of design optimization is the dynamic absorber. In this design optimization procedure, the desired response is a level of vibrational amplitude; the control factors are the mass ratio n, the damping ratio δ_{2R}, and the tuning ratio μ (which are described in detail later in this chapter); and the levels of the control factors are low, medium, and high. The experimental sets of iterations were performed using closed-formed equations developed by Snowdon [56] and described later in this chapter. It was assumed that the operating speed of the piece of machinery was essentially the frequency of the input force. The assumptions that the spring constants remain constant as well as the fact that the springs underwent linear deformations were also made.

The practical application of this design optimization procedure was to find the optimum values for a dynamic absorber that would provide a desired reduction in the transmission of vibrational energy to a machine of a given mass when a force with a certain frequency is imparted to the system.

Section 3.1.1 - Description of the System

Many mechanical systems can be modeled as a single degree of freedom system as shown in Figure 3-1 [56]. The system consists of a mass M_1 that is attached to a base by means of springs with an equivalent spring constant k_1. Either

M_1 or the base (or perhaps both) is driven by a forcing function that is either a continuous force or an impulsive force of some sort. Due to external excitation the system may experience large amplitudes or vibrations due to its natural tendency to oscillate at its natural frequency. The natural frequency of the initial system is given by the expression $(k_1/M_1)^{\frac{1}{2}}$. At this condition, known as *resonance*, considerable damage could be done to sensitive instruments that are included in the term M_1. Additionally, continuous systems oscillate at multiple natural frequencies but can be modeled simply as a single degree of freedom system. From a vibration isolation point of view there is a need to reduce vibration amplitude near resonance.

With those facts in mind, a dynamic absorber is added as shown in Figure 3-2 [56]. This absorber consists of mass M_2 that is attached to M_1 by means of a spring of stiffness k_2 and a dashpot of viscosity C. As Snowdon point out, the values k_2 and

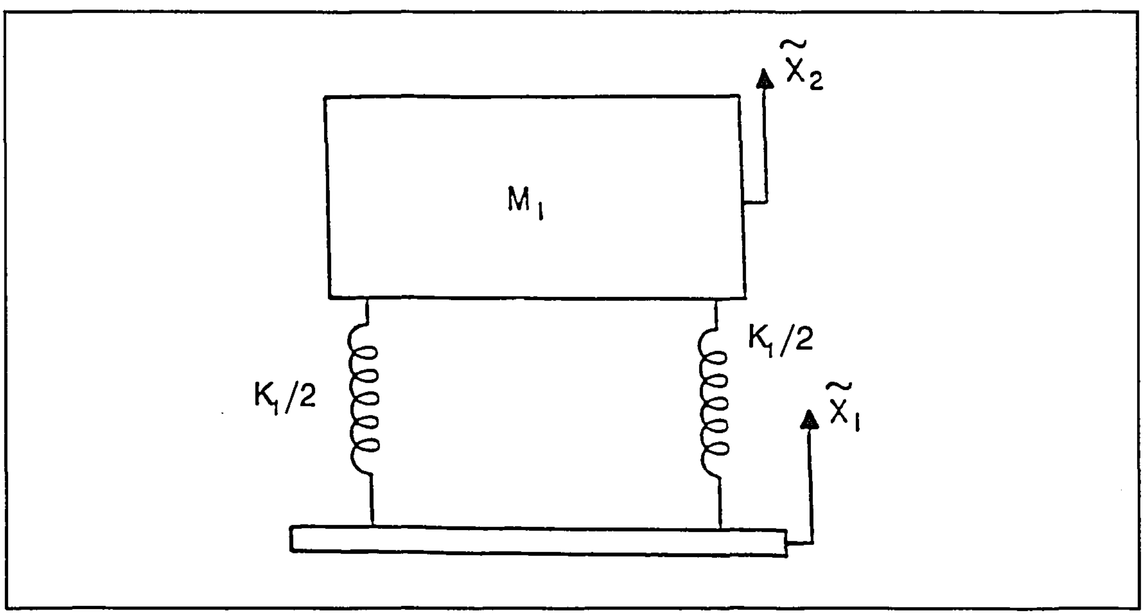

Figure 3-1 - A single degree of freedom system.

C are set such that the dynamic absorber experiences resonance at a frequency close to the resonant frequency of the original system, with the result that "The motion of M_1 is then reduced because, in effect, the absorber mass M_2 is greatly magnified in the neighborhood of this frequency and it adds considerably to the inertia of M_1." [56]

The plots shown in Figure 3-3 and Figure 3-4 may help to further clarify the concept of the dynamic absorber. Each of these plots feature the quantity of transmissibility on the ordinate axis. *Transmissibility* is defined as the ratio of the displacement of M_1 to the displacement of the base. In Figure 3-3, a plot of transmissibility (also called "T") versus the frequency ratio Ω for various situations is shown. The frequency ratio Ω is defined as the ratio of the frequency of the driving force to the natural frequency of the system [56]. The original system (i.e., with M_1

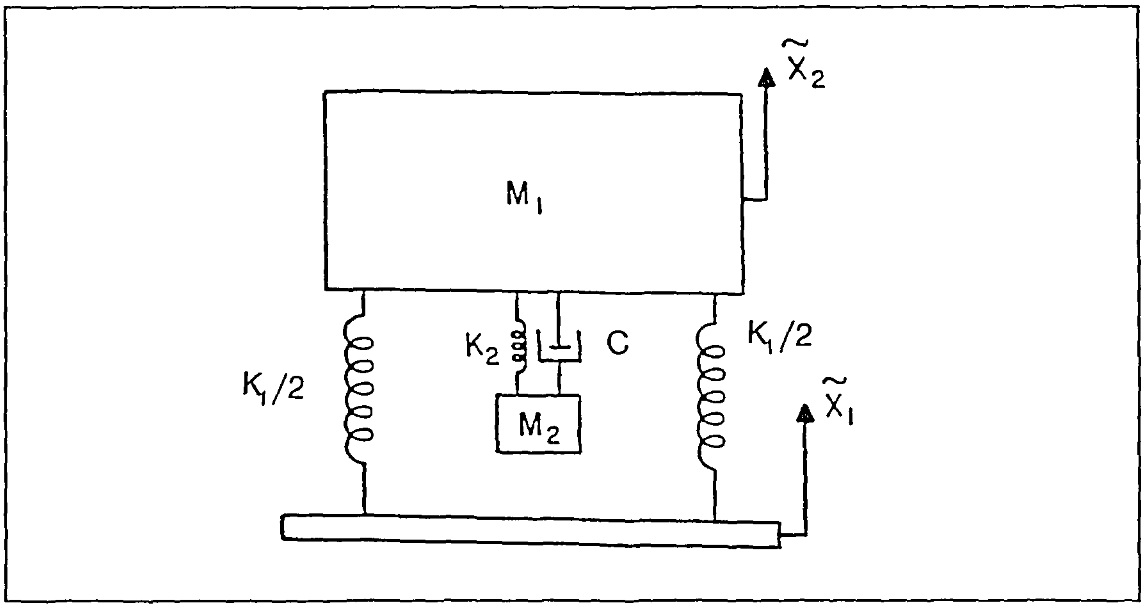

Figure 3-2 - Vibrating item with dynamic absorber attached.

and k_1 only) is plotted with the label "infinite damping" for reasons that will be explained below. Note that the curve goes to infinity at resonance, where $\Omega = 1$. The plot of T vs. Ω for the dynamic absorber that exerts no damping (i.e., there is merely an extra mass M_2, and no damper) is shown with the label "no damping". As expected, there are now resonant peaks on each side of $\Omega = 1$ corresponding to the effects of the two masses, but at $\Omega = 1$, T is greatly reduced - in fact, it is infinitely small. In order to reduce the infinitely large peaks at the new resonance frequencies, the damping effect of the absorber is increased (that is, C is increased from 0). The tradeoff is that the "trough" at $\Omega = 1$ is not as deep (in other words, the transmissibility increases at $\Omega = 1$). The other extreme, T versus Ω for a system with

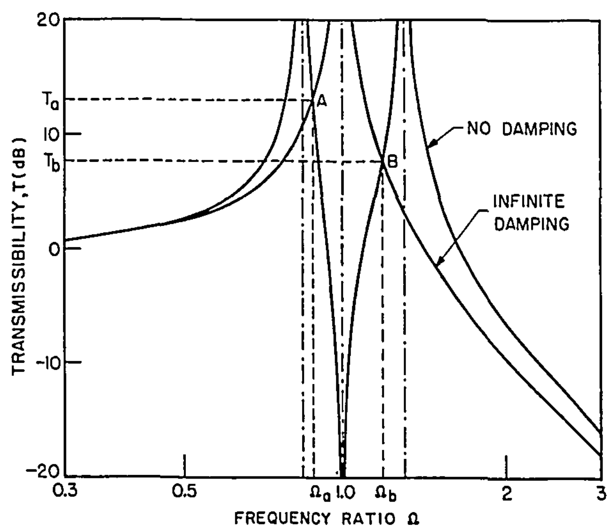

Figure 3-3 - T versus Ω for several damping arrangements.

infinitely large damping, is shown with the label "infinite damping". It can be shown mathematically that in this case the system behaves as if M_2 is not present, hence the curve for infinite damping is the same as the curve of the original system, with a result being that resonance again occurs at $\Omega = 1$. Finally, Figure 3-4 summarizes the effect of damping by showing the effects of different degrees of damping in a plot of T versus Ω for a mass ratio of 5/6. The mass ratio, also called μ, is simply $M_1/(M_1+M_2)$. Note that as the degree of damping decreases, T decreases at $\Omega = 1$, but resonant peaks on each side of $\Omega = 1$ form and become larger. This tradeoff condition is one that must be taken into consideration when designing a dynamic absorber - how severe can the side resonances be, how "shallow" can the trough at $\Omega = 1$ be, etc.

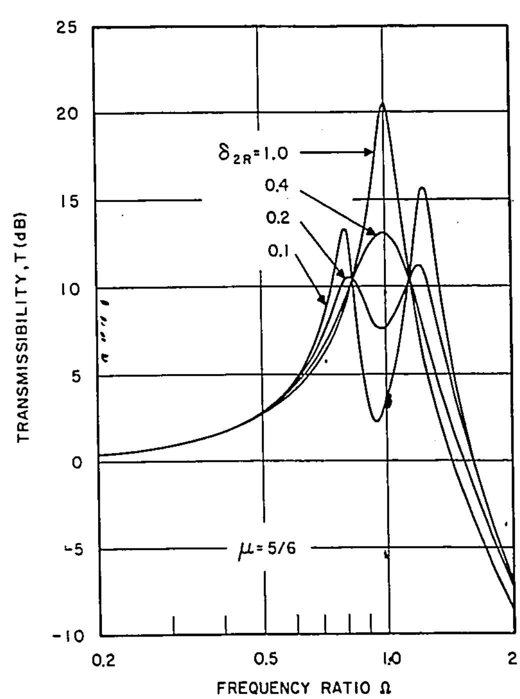

Figure 3-4 - T versus Ω for $\mu = 5/6$.

Although compound dynamic absorbers can be formulated, Snowdon basically divides dynamic absorbers into three classes based on the type of damping used. Viscous-type dynamic absorbers are those with some type of dashpot that uses friction or some other type of energy dissipation to achieve a damping effect. Solid Type I (hysteretic damping) dynamic absorbers are those that are made of rubbers or other rubber-type solids where the damping effect of the material varies rather slowly with frequency, and as such, can be safely approximated as constant in the range of frequencies commonly used in most applications. In addition, the degree of shear deformation of the material can also be approximated as constant over these frequency ranges. Solid Type II dynamic absorbers are those made of materials where the damping effect is larger than in Solid Type I dynamic absorbers, but is still essentially constant over the range of frequencies used in most applications. However, the degree of shear deformation increases rapidly with frequency.

In this design optimization procedure, only viscous-type dynamic absorbers were considered.

Section 3.1.2 - Practical Applications

There are many practical applications of dynamic absorbers. The most widely used application is in machine design and the layout of machines in an industrial workspace. The mass M_1 often represents a piece of machinery with sensitive instruments. It may also be a piece of high-precision machinery where any vibrational displacements may cause flawed production or even physical injury to the operator of the machine. The base often represents the floor of the workspace, and the driving force in the floor often represents vibrational forces caused by the operation of another piece of machinery. The dynamic absorber to be added to M_1 may very well be a rubber-type support attached to the base of M_1. By adding the dynamic absorber to M_1, the vibration transmitted to M_1 is reduced.

Quite often, the dynamic absorber is used to reduce the effects of resonance. Usually, machines are designed so as not to operate with a frequency equal to its own natural frequency. However, when the machine is powered on, and the operating speed is increased from 0 to the preferred operating speed, it may briefly pass through the point of resonance, with the result being that large vibrations or load noise may be generated. Dynamic absorbers can be inserted to reduce these effects.

Another common application of dynamic absorbers is the shock absorber found on automobiles. In this case, M_1 is each passenger in the car (along with the body of the car), while the base is the roadway and the driving vibrational force is caused by bumps and other discontinuities in the surface of the roadway. The shock absorber helps to dampen the transmission of these vibrations, resulting in a "smoother ride".

Finally, the concept of the dynamic absorber has been used to explain phenomena that have been encountered in the design of earphones and the shock testing of large-scale beam-mass structures [56].

Section 3.1.3 - Theoretical Equations

The derivation of the expression for transmissibility is performed in detail in Snowdon [56] and will be omitted here, except for certain details. In deriving this expression, certain parameters need to be defined. These parameters are [56]:

$$\omega = \textit{driving frequency}; \tag{3-1}$$

$$\omega_o = \sqrt{\frac{k_1}{M_1}} = \textit{natural frequency of undamped system}; \tag{3-2}$$

$$\omega_a = \sqrt{\frac{k_2}{M_2}} = \textit{natural frequency of absorber}; \tag{3-3}$$

$$n = \frac{\omega_a}{\omega_o} = \textit{tuning ratio}; \tag{3-4}$$

$$\Omega = \frac{\omega}{\omega_o}; \tag{3-5}$$

$$\mu = \frac{M_1}{M_1+M_2} = \textit{mass ratio}; \tag{3-6}$$

$$\delta_{2R} = \frac{\omega_a * \eta}{2 * k_2} = \textit{damping ratio}; \tag{3-7}$$

$$N = \sqrt{\frac{1-\mu}{1+\mu}}; \tag{3-8}$$

With these parameters defined, Snowdon obtains an expression for T as [56]

$$T^2 = \frac{(-\Omega^2+n^2)^2 + (2*n*\Omega*\delta_{2R})^2}{[\mu*\Omega^4-\Omega^2*(1+n^2)+n^2]^2 + (2*n*\Omega*\delta_{2R})^2*(1-\Omega^2)^2} . \qquad \textbf{(3-9)}$$

Snowdon also derives expressions for *optimum tuning* and *optimum damping*. Optimum tuning is the condition whereby the values of T at the so-called "fixed points" Ω_a and Ω_b (see Figure 3-3) are equal [56]. The expression for optimum tuning [56] is

$$n_o = \sqrt{\mu} . \qquad \textbf{(3-10)}$$

Optimum damping is the condition that the maximum values of T occur at the fixed points [56]. The expression for optimum damping is [56]

$$\delta_{2R} = \sqrt{\frac{3}{8}*(1-\mu)} . \qquad \textbf{(3-11)}$$

These conditions are displayed in Figure 3-5, which is Figure 3-4 repeated with the addition of a curve representing the optimum damping value of 0.25.

Section 3.2 - Goals and Targets

As outlined in Section 1.4, a nine-step process was followed in each design optimization procedure. The application of this process to this design optimization procedure will now be described in more detail, although it will not be detailed in subsequent chapters.

1. STEP ONE - the dynamic absorber was chosen as the first system to be optimized. A constant value of M_1 (1 ton or 2000 lbm) was assumed. It was

also assumed that the driving force had a frequency of 30 Hz or 1800 rpm. These values were chosen arbitrarily, but are realistic.

2. STEP TWO - transmissibility was chosen as the output factor and a desired value of -6 dB was established. This value was also chosen arbitrarily, but is realistic.
3. STEP THREE - M_2, k_2, and C were chosen as control factors, since these values constitute the characteristics of the dynamic absorber. Three levels of each control factor were used, necessitating the use of an L9 orthogonal array.
4. STEP FOUR - the initial ranges for each of the control factors were established as 88 lbm $\leq M_2 \leq$ 4409 lbm, 3.4 lbf/ft $\leq k_2 \leq$ 1030 kip/ft, and 5.1 lbf·sec/ft $\leq$ C $\leq$ 103 lbf·sec/ft. The range on

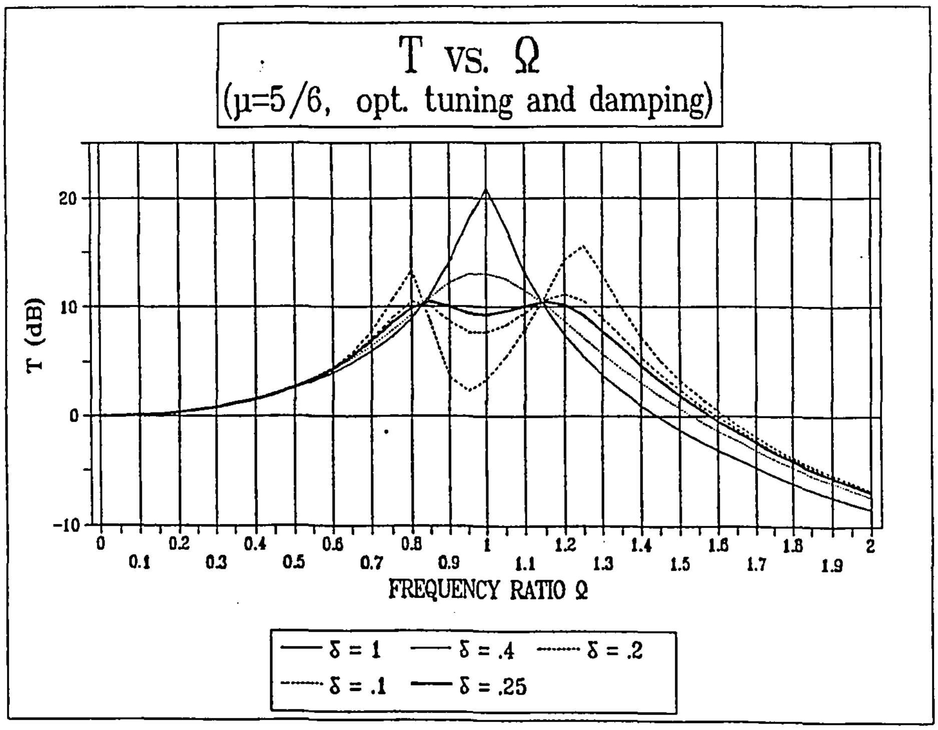

Figure 3-5 - T versus Ω for $\mu = 5/6$, optimum tuning and optimum damping.

M_2 was established by the constraint that $0 < M_2 \leq M_1$. The other ranges were chosen to be as large as possible, as recommended by Taguchi [7].

5. STEP FIVE - Equations 3-1 through 3-9 above were used as the theoretical equations. Equations 3-1 through 3-8 were used to help "set up" Equation 3-9.
6. STEP SIX - These control factors, constant parameters, initial ranges, theoretical equations, and Taguchi equations were programmed. Details on the computer program can be found in Appendix A. It should be noted that the program was set up so that the various constant system parameters were placed in the beginning so that they could be quickly changed in order to observe the resulting effect on the optimum control factor values.
7. STEP SEVEN - The first experimental set of iterations were run. Several false starts were encountered that resulted in the repetition of Steps One through Six. This procedure is described in more detail in Section 3.3.
8. STEP EIGHT - Several sets of results did not make physical sense. ANOVA was then performed on the results to examine possible interactions. This procedure is described in more detail in Section 3.3 and is discussed in Section 3.4.
9. STEP NINE - As a result of Steps Seven and Eight, the entire process was repeated. A total of six experimental sets of iterations were run. This procedure is described in more detail Section 3.3.

Section 3.3 - Details and Results of the Experiment

Since this design optimization procedure was the first one attempted, several "false starts" were encountered, although all of these false starts resulted in important corrections being made to the procedure. One of the biggest mistakes that was made in the execution of this design optimization procedure was the initial assumption that the experimental set of iterations would converge to values that would satisfy **both** the conditions of optimum tuning **and** optimum damping. For example, the first iteration (that is, the first run of the computer program to compute the S/N ratios for each trial, then determine the optimum values as outlined in Section 2.2.3) gave a solution trio such that Equations 3-10 and 3-11 (for optimum conditions) were not satisfied. A separate program was written to simply find any (M_2,k_2,C) trios that would satisfy the optimality conditions, but even though such trios were found, none would satisfy the criterion on the output response that T = -6 dB at resonance.

Then, it was determined that at optimum conditions, M_2 and k_2 are dependent. The value of M_2 fixes μ (Equation 3-6), which in turn fixes n_o (Equation 3-10) at optimum tuning; with w_o fixed, w_a can be determined (Equation 3-4) and from that value, k_2 is fixed (Equation 3-3). Since the term "optimum" had become, at this early stage of the research, ambiguous (c.f. the terms "optimum tuning" and "optimum values of the control factors"), it was decided to drop the unwritten requirement that the control factor values meet the optimality conditions (as defined by Snowdon).

In determining the dependence of M_2 and k_2, it was noticed that none of the equations used C. When Equation 3-9 was reexamined, it was observed that T is only a function of n, δ_{2R}, and μ, since the value of Ω is fixed, Ω = 1, at resonance. In addition, the observation was made when iterating using the extra program that n and δ_{2R} had the most impact on altering the value of T. Thus, it was decided to change the three control factors to the trio (n, δ_{2R}, μ). Also it was decided the use the MSD concept from Roy [9], and compute the S/N ratio by the expression

$$S/N = -10 * \log_{10}(T_{actual} - T_{target})^2, \tag{3-12}$$

where T_{actual} is the value of T using the values of the control factors for that particular Taguchi trial and T_{target} = -6 dB.

The usage of the "wrong" trio of control factors may have contributed to the success of the design optimization procedure by pointing out the "correct" trio of control factors, but it also contributed to the success of the design optimization procedure in that it provided an opportunity to try different array sizes as well as an ANOVA. In addition to the original, 3-factor 3-level approach ("3x3"), a 3-factor, 2-level approach ("3x2"), and a 3-factor, 4-level approach ("3x4") were used. The ANOVA was performed on the 3x2 case and is included in Appendix B as Table B-2. Note that an L8 array was used for this iteration, since it was the smallest size array that could be used with the highest possible *resolution* - that is, the effects of the individual control factors and all of the possible interactions could be estimated from this array. The first set of iterations with the newly selected control factors was conducted with μ fixed at 5/6. As such, it was a 2x2 experiment with the initial range of control factors set at $.5 \leq n \leq .99$ and $.01 \leq \delta_{2R} \leq .433$. These ranges were determined based on constraint that $0 \leq M_1 \leq M_2$. An ANOVA was performed on the results, suggesting that δ_{2R} contributed most significantly to T, with n and the $n x \delta_{2R}$ contributing almost evenly (see Appendix B, Table B-4). An L4 array (highest resolution) was used.

Next, μ was brought back into the analysis and another set of iterations were performed with a 3x2 orthogonal array, with the initial range on n and δ_{2R} the same as before, while the initial range on μ was $0.5 \leq \mu \leq 0.9$ (based on the M_2 constraint). An ANOVA was once again performed on the results, suggesting that the individual effect of n, δ_{2R}, and μ as well as the n$\times\delta_{2R}$ interaction were significant (see Appendix B, Table B-5). Since n and δ_{2R} were both functions of μ (though not the *same* function), it was decided that these results made sense.

With these changes in control factors made, another experimental set of iterations were performed. The three control factors remained the same for each problem, and their initial ranges were left intact. However, each control factor was studied with three levels, with the extra level being the midpoint between the values comprising the range. This conversion to a 3x3 problem necessitated the usage of an L9 array. The use of this array meant that interactive effects could not be accurately estimated, but it was felt that that was not important, since the interactive effects had already been estimated.

The iteration procedure consisted of running the 9 trials of the L9 array with the levels of the control factors set according to the standard setup of an L9 array (see Table 2-2). Table 3-1 shows the values obtained in the first iteration. The average S/N for each level of each control factor was then computed based on the S/N ratio of the trial that contained that level. For example, the second (middle) level of δ_{2R} was used in trials 2, 5, and 8. Thus, the average S/N for the second

Table 3-1 - Results of the first iteration of the first experimental set of iterations of the first design optimization procedure (target: T = -6 dB at resonance).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.5000	.0100	.5000	3.523	-19.58
2	.5000	.2500	.7000	8.416	-23.18
3	.5000	.4330	.9000	18.75	-27.87
4	.7500	.0100	.7000	3.282	-19.35
5	.7500	.2500	.9000	15.22	-26.53
6	.7500	.4330	.5000	3.897	-19.91
7	.9900	.0100	.9000	-11.03	-14.04
8	.9900	.2500	.5000	-.0803	-15.45
9	.9900	.4330	.7000	9.123	-23.59
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	-23.54	-17.66	-18.31	
	2	-21.93	-21.72	-22.04	
	3	-17.69	-23.79	-22.81	
SECONDARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	k_1 (lbf/ft)	w_a (rad/sec)
1	4409.	1.217×10^6	258.3	9.737×10^6	94.25
2	1890.	5.217×10^5	2768.	6.955×10^6	94.25
3	489.9	1.353×10^5	1243.	5.411×10^6	94.25
4	1890.	1.174×10^6	166.1	6.955×10^6	141.4
5	489.9	3.043×10^5	1076.	5.411×10^6	141.4
6	4409.	2.739×10^6	16770.	9.737×10^6	141.4
7	489.9	5.302×10^5	56.83	5.411×10^6	186.6
8	4409.	4.773×10^6	12790.	9.737×10^6	186.6
9	1890.	2.045×10^6	9491.	6.955×10^6	186.6

control factor (δ_{2R}) at the second level (the middle level) was S/N_{22} = -1/3 * (23.18+26.53+15.45) = -21.72.

The next step after finding the levels of the control factors with the highest average S/N ratios was to perform the iteration again, with the new range for each particular control factor consisting of the value that produced the highest S/N ratio, the adjacent level with the next highest S/N ratio, and the midpoint of this range. The intent was to repeat this iterative process until the ranges of the control factors converged to a single value. As a result of the experience gained in performing the experimental sets of iterations, three rules were developed to guide the subdivision process:

1. the level with the highest average S/N ratio would be the midpoint of the new range, with the difference between each level in the new iteration being half the difference between each level in the previous iteration;
2. if the level with the highest average S/N ratio was an endpoint of the original range (that is, a starting value), then it remained at that end of the range, with the next range simply halved;
3. if the level with highest average S/N ratio was an endpoint of the original range, and the level with the lowest average S/N ratio was the second level, then the range would remain unaltered.

As an example illustrating both rules, if the original levels of a particular control factor were 10, 20, and 30, and the first iteration gave average S/N ratios of 13, 16, and 14, respectively, then the levels for the next iteration would be 15, 20, and 25. Since level 20 gave the best average S/N ratio, it would be the center value for the new levels with -5 on each side. However, if the first iteration would have given average S/N ratios of 16, 14, and 13, then the levels for the next iteration would be 10, 15, and 20.

With a method for determining the next levels of the control factors, the iterations were performed and continued until a substantial convergence occurred. This criterion for "substantial convergence" was that each control factor had to have at least one level with an average S/N of 60 or higher. This criterion was modified where necessary (as will be noted in later chapters). Nevertheless, iteration procedures continued in all optimization procedures conducted in this research until either the levels of all of the control factors or all of the S/N averages were the same to the appropriate number of significant figures.

The first official experimental set of iterations, then, for the dynamic absorber, converged after 16 iterations - that is, 16 sets of nine trials. The results of the iterations are shown in Table 3.2, which includes the primary values of the experiment (the values of the control factors, the value of the output response, and the MSD for each trial), the values of the average S/N ratios for each level of each factor, and the secondary values (the values of the various system components based on the equations in Section 3.1.3). Note that the iterations were actually continued until every control factor had at least one level with an average S/N ratio of 66.68 or more. And, the levels of the control factors were identical to four decimal places. The component values given were of similar accuracy, the difference in the largest and smallest non-rounded-off values of k_2, for example, being .0112%.

Table 3-2 - Results of the first experimental set of iterations of the first design optimization procedure (target: T = -6 dB at resonance).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.9252	.1089	.5057	-6.000	116.9
2	.9252	.1089	.5057	-6.000	66.10
3	.9252	.1089	.5057	-6.001	60.10
4	.9252	.1089	.5057	-6.000	69.08
5	.9252	.1089	.5057	-6.000	76.95
6	.9252	.1089	.5057	-6.000	70.04
7	.9252	.1089	.5057	-6.001	63.00
8	.9252	.1089	.5057	-6.001	65.44
9	.9252	.1089	.5057	-6.000	87.54
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	81.03	82.99	84.12	
	2	72.02	69.50	74.12	
	3	71.99	72.56	66.68	
SECONDARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	k_1 (lbf/ft)	w_a (rad/sec)
1	4310.	4.074×10^6	5089.	9.628×10^6	174.4
2	4310.	4.074×10^6	5089.	9.628×10^6	174.4
3	4310.	4.074×10^6	5089.	9.628×10^6	174.4
4	4310.	4.074×10^6	5089.	9.628×10^6	174.4
5	4310.	4.074×10^6	5089.	9.628×10^6	174.4
6	4310.	4.074×10^6	5089.	9.628×10^6	174.4
7	4310.	4.074×10^6	5089.	9.628×10^6	174.4
8	4310.	4.074×10^6	5089.	9.628×10^6	174.4
9	4310.	4.074×10^6	5089.	9.628×10^6	174.4

The iterations were then performed again with the new target output response of T = 0 dB. This experimental set of iterations converged after 18 iterations. The values from the final iteration are shown in Table 3-3. Notice that two of the nine levels have average S/N ratios greater than 82. Such an S/N ratio corresponds to a absolute difference of less than $8x10^{-5}$ between the actual value and the target value.

Having thus converged to optimum (in the robust sense) values for arbitrarily selected target values of transmission, it was decided to run another set of iterations in an attempt to converge on optimum (as defined by Snowdon) values (Figure 3-5). Since a plot of T vs Ω for μ = 5/6 was used as an importance reference, μ was set at 5/6. At optimal tuning and damping conditions, using equations 3-10 and 3-11 above, n = .9129 and δ_{2R} = .2500. Substituting these values into Equation 3-9 yields a target value of T = 9.294. Accordingly, the equation for S/N (Equation 3-12) was modified by setting T_{target} = 9.294 dB. The iterations should converge to (n, δ_{2R}, μ) = (.9129, .2500, .8333). The initial ranges were kept constant.

Unfortunately, the iterations did not converge to these values. After 16 iterations, it had appeared that the actual output response had converged, namely, to a range of [9.294, 9.295] dB. However, the control factors converged to (.9400, .3713, .7573), which were off from the optimum values as predicted by Equations 3-9 and 3-10 by as much as 48.52 %. The results of this third experimental set of iterations is shown in Table 3-4. Since n and δ_{2R}, as noted above, are both

Table 3-3 - Results of the second experimental set of iterations of the first design optimization procedure (target: T = 0 dB at resonance).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.7488	.07259	.5475	-0.0000	95.09
2	.7488	.07259	.5475	0.0001	84.58
3	.7488	.07259	.5475	0.0001	79.96
4	.7488	.07259	.5475	-0.0001	85.13
5	.7488	.07259	.5475	0.0000	97.09
6	.7488	.07259	.5475	-0.0001	84.66
7	.7488	.07259	.5475	-0.0001	77.97
8	.7488	.07259	.5475	-0.0002	75.32
9	.7488	.07259	.5475	-0.0001	77.72
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	86.54	86.06	85.02	
	2	88.96	85.66	82.48	
	3	77.01	80.78	85.01	
SECONDARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	k_1 (lbf/ft)	w_a (rad/sec)
1	3644.	2.256×10^6	2321.	8.894×10^6	141.2
2	3644.	2.256×10^6	2321.	8.894×10^6	141.2
3	3644.	2.256×10^6	2322.	8.894×10^6	141.2
4	3644.	2.256×10^6	2321.	8.894×10^6	141.2
5	3644.	2.256×10^6	2321.	8.894×10^6	141.2
6	3644.	2.256×10^6	2322.	8.894×10^6	141.2
7	3644.	2.256×10^6	2321.	8.894×10^6	141.2
8	3644.	2.256×10^6	2322.	8.894×10^6	141.2
9	3644.	2.256×10^6	2322.	8.894×10^6	141.2

Table 3-4 - Results of the third experimental set of iterations of the first design optimization procedure (target: T = 9.294 dB at resonance).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.9400	.3713	.7573	9.294	68.75
2	.9400	.3713	.7573	9.294	93.97
3	.9400	.3713	.7573	9.295	69.78
4	.9400	.3713	.7573	9.294	104.9
5	.9400	.3713	.7573	9.295	69.10
6	.9400	.3713	.7573	9.294	87.35
7	.9400	.3713	.7573	9.295	68.28
8	.9400	.3713	.7573	9.294	82.35
9	.9400	.3713	.7573	9.295	67.52
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	77.50	80.63	79.48	
	2	87.10	81.80	88.78	
	3	72.72	74.88	69.05	
SECONDARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	k_1 (lbf/ft)	w_a (rad/sec)
1	1413.	1.379×10^6	5781.	6.430×10^6	177.2
2	1413.	1.379×10^6	5780.	6.430×10^6	177.2
3	1413.	1.379×10^6	5780.	6.430×10^6	177.2
4	1413.	1.379×10^6	5781.	6.430×10^6	177.2
5	1413.	1.379×10^6	5780.	6.430×10^6	177.2
6	1413.	1.379×10^6	5781.	6.430×10^6	177.2
7	1413.	1.379×10^6	5781.	6.430×10^6	177.2
8	1413.	1.379×10^6	5781.	6.430×10^6	177.2
9	1413.	1.379×10^6	5781.	6.430×10^6	177.2

functions of μ (though not the <u>same</u> function of μ), it was decided to fix μ and run the iterations as a 2x3 case, keeping the L9 array, which by now had turned into the standard array to be used. Once again, it was predicted that the iterations should converge to $(n,\delta_{2R}) = (.9129, .2500)$.

Unfortunately, the iterations did not converge to these values. By the fourth iteration, the levels of the control factors had converged to $.75 \leq n \leq .81$ and $.19 \leq \delta_{2R} \leq .25$ and the output response had converged to $9.919 \leq T \leq 10.77$. Although somewhat close to the predicted range, these ranges were still significantly different.

In a last attempt at reaching some type of approximate, Snowdon-optimum condition, one additional experimental set of iterations was undertaken. In the fifth experimental set of iterations, the target was not a specific value of T at resonance, but rather that this value (whatever it would be) would be a local minimum.

The results of this marginally successful experimental set of iterations are summarized in Table 3-5. The iterations were set up by assuming that the value of T at equal distances from resonance are equal when T at resonance is a local minimum. An output response was then defined to be

$$T_{dif} = T_{\Omega=1.05} - T_{\Omega=.95}, \qquad \textbf{(3-13)}$$

and, as a result,

$$S/N = -10*\log_{10}(T_{dif}). \qquad \textbf{(3-14)}$$

This experimental set of iterations converged in 15 iterations to the values $(n,\delta_{2R},\mu) = (.7386, .4574, .5333)$. Note that for this value of μ, $n_o = .7303$, and $(\delta_{2R})_o = .4183$.

Table 3-5 - Results of the fifth experimental set of iterations of the first design optimization procedure (target: T_{dif} as a minimum).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.7386	.4574	.5333	-.0001	83.15
2	.7386	.4574	.5333	-.0001	83.57
3	.7386	.4574	.5333	-.0001	84.76
4	.7386	.4574	.5333	-.0001	78.68
5	.7386	.4574	.5333	-.0001	78.86
6	.7386	.4574	.5333	-.0002	72.33
7	.7386	.4574	.5333	-.0002	75.65
8	.7386	.4574	.5333	-.0003	70.61
9	.7386	.4574	.5333	-.0003	70.68
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	83.49	79.16	75.36	
	2	76.63	77.68	77.65	
	3	72.32	75.59	79.43	
SECONDARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	k_1 (lbf/ft)	w_a (rad/sec)
1	3858.	2.108×10^6	14540.	8.278×10^6	132.6
2	3858.	2.108×10^6	14540.	8.278×10^6	132.6
3	3858.	2.108×10^6	14540.	8.278×10^6	132.6
4	3858.	2.108×10^6	14540.	8.278×10^6	132.6
5	3858.	2.108×10^6	14540.	8.278×10^6	132.6
6	3858.	2.108×10^6	14550.	8.278×10^6	132.6
7	3858.	2.108×10^6	14540.	8.278×10^6	132.6
8	3858.	2.108×10^6	14550.	8.278×10^6	132.6
9	3858.	2.108×10^6	14550.	8.278×10^6	132.6

Section 3.4 - Discussion of Results

One of the first objectives of the analysis of the results was to determine if the values obtained by this method were accurate. To accomplish this check, several plots were studied. Figure 3-6 shows a plot a T vs. Ω for the results of the first experimental set of iterations (T=-6 dB), where "ESOI" in the title box represents "experimental set of iterations". Five different plots are shown: one plot for the value of δ_{2R} obtained in the first experimental set of iterations, and the remaining

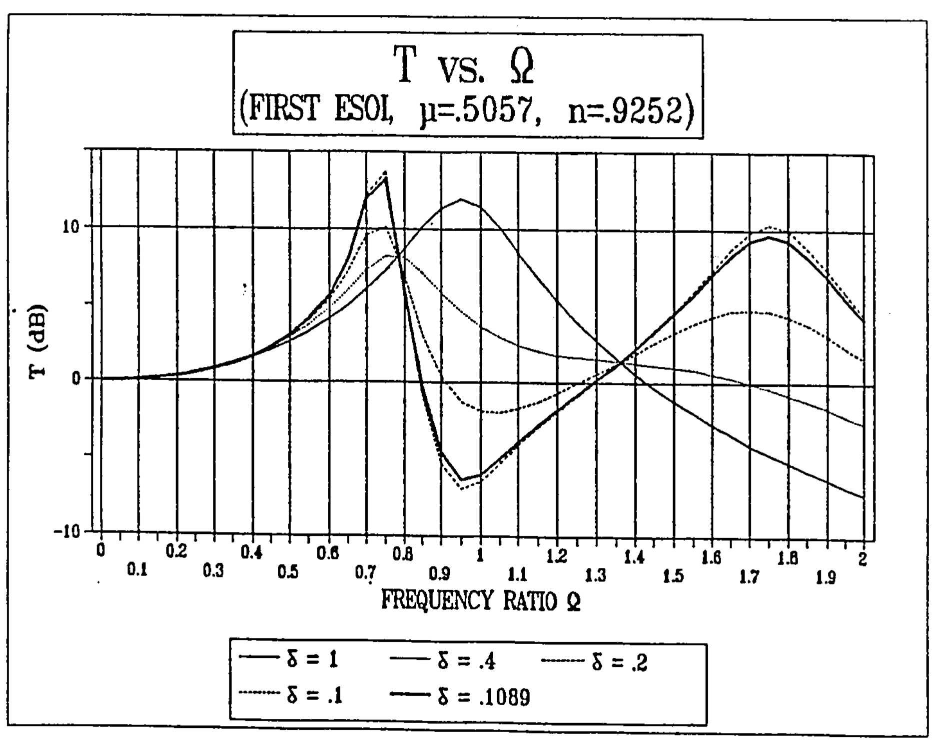

Figure 3-6 - T versus Ω for the first experimental set of iterations of the first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

four plots for four different values of δ_{2R}. Note that the two fixed points exist, the curve for δ_{2R}=.1089 passes through T=-6 at Ω=1, and the troughs at resonance deepen at a damping ratio of 2.0. These observations make sense, and confirm the accuracy of the values obtained in the first experimental set of iterations. Figure 3-7 shows similar plots for the results of the second experimental set of iterations (T=0 dB). The same observations can be made about this plot: the fixed points are present, the troughs (though not existing at resonance) deepen at a damping ratio of

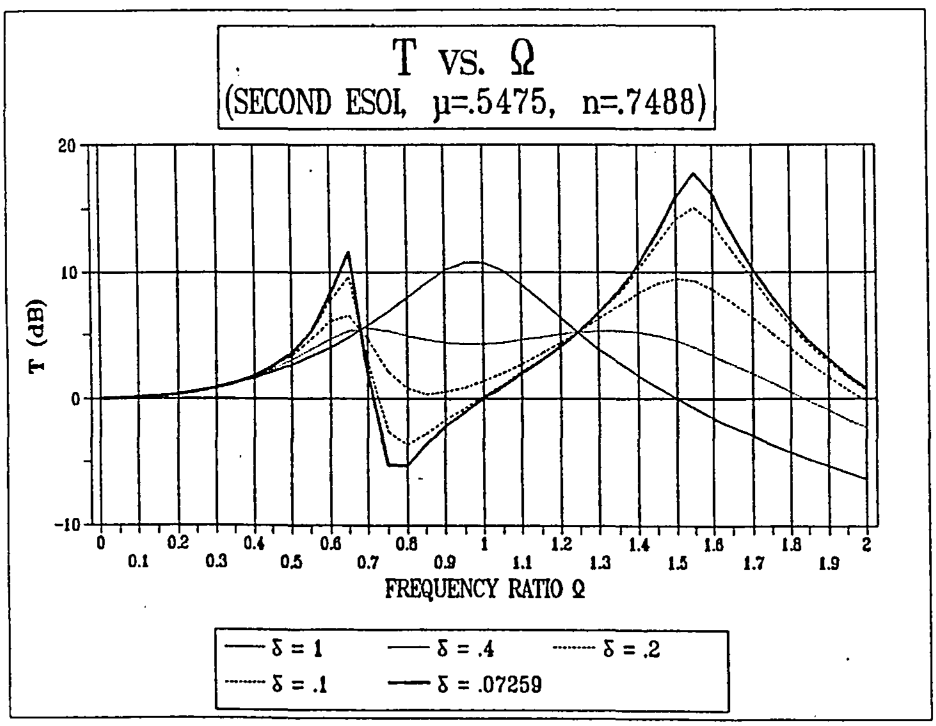

Figure 3-7 - T versus Ω for the second experimental set of iterations of the first design optimization procedure (target: T = 0 dB at resonance, Ω = 1).

.2, and the plot for the appropriate damping curve passes through the target output point. Similar observations can be made regarding Figure 3-8, which shows the plots for the four reference values of δ_{2R} as well as the value of δ_{2R} obtained in the third experimental set of iterations.

For the fourth experimental set of iterations (the one in which $\mu = 5/6$ and $T_{target} = 9.2942$), a comparison was made with Figure 3-5. No comparison was available for the results of the third experimental set of iterations, since there were

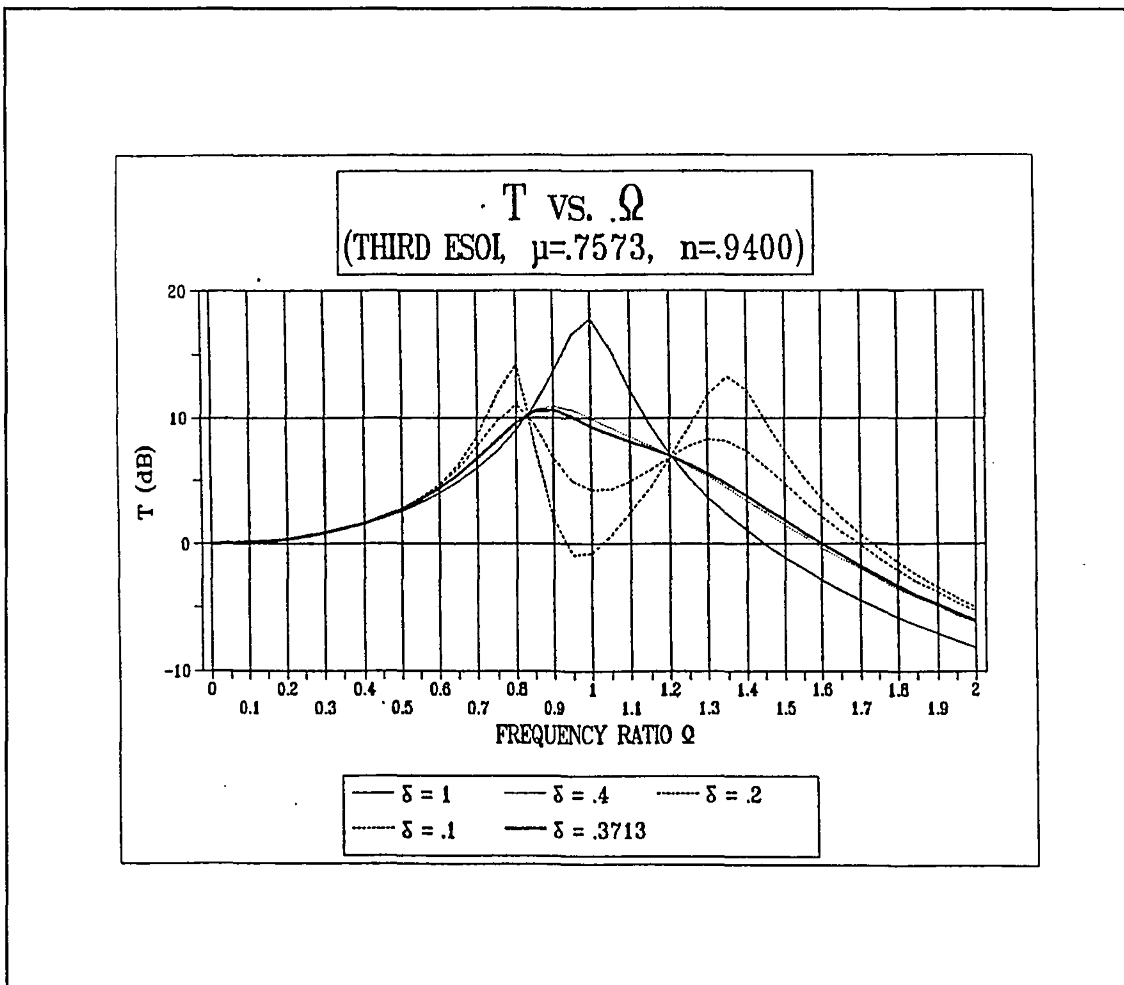

Figure 3-8 - T versus Ω for the third experimental set of iterations of the first design optimization procedure (target: T = 9.294 dB at resonance, Ω = 1).

no available plots of T versus Ω for the values of μ and δ_{2R} obtained in this experimental set of iterations. This figure is reproduced as Figure 3-9 with the addition of the curve corresponding to the value δ_{2R} = .1900 obtained in the fourth experimental set of iterations. Note that the δ_{2R}=.1900 curve "fits in" properly in the graph, but the conditions of optimum tuning and damping are not met.

Finally, Figure 3-10 is a plot of T versus Ω for the values of δ_{2R} obtained in the fifth experimental set of iterations, as well as the traditional reference values of

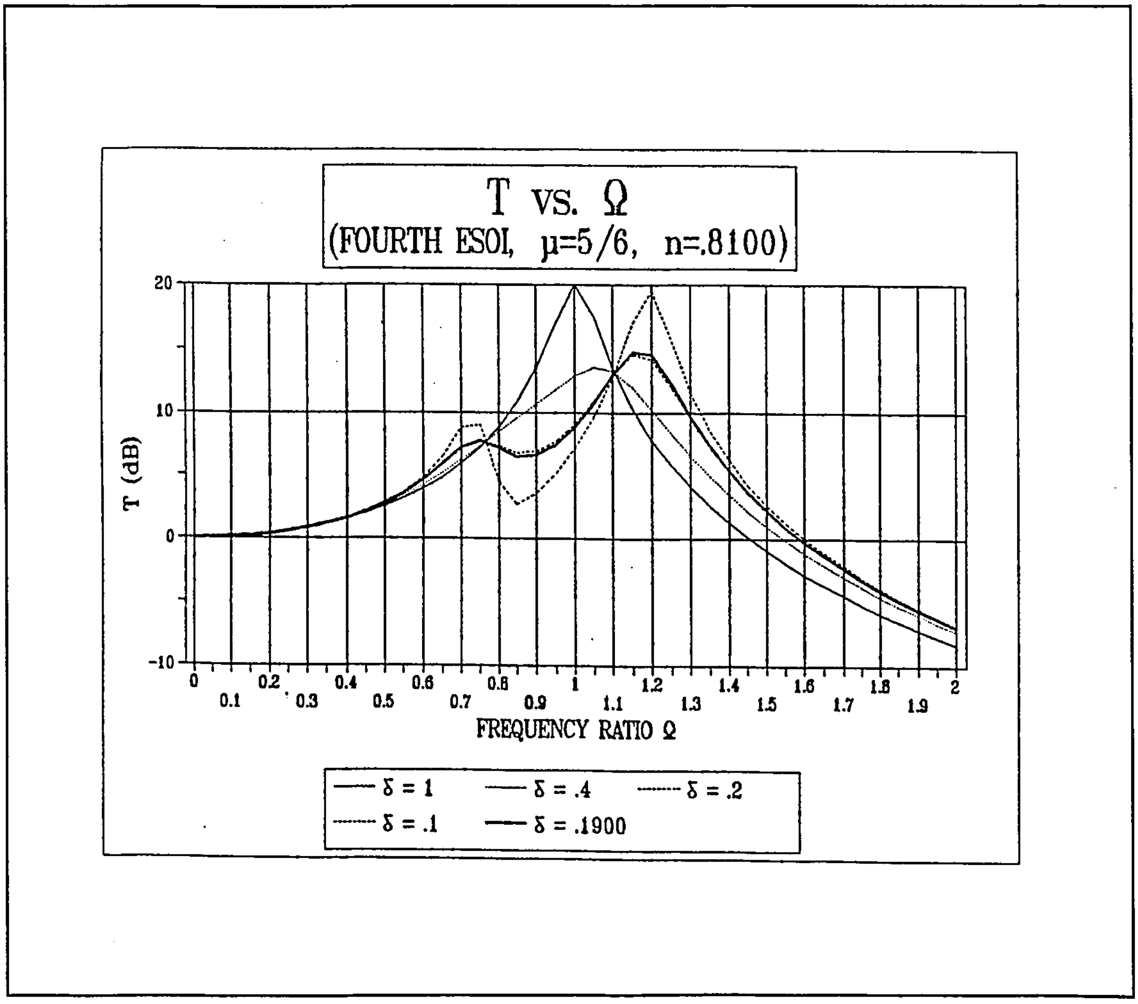

Figure 3-9 - T versus Ω for the fourth experimental set of iterations of the first design optimization procedure (target: T = 9.294 dB at resonance, Ω = 1).

δ_{2R} = 0.1, 0.2, 0.4, and 1.0. Note that in this plots, the target criterion appears to have been met: there is a local minimum at resonance on the δ_{2R}=.4547 curve in Figure 3-10. In addition, the added curve "fits" in its proper places with respect to the others, while the fixed points are once again present. Finally, the final values of the trio (n,δ_{2R},μ) are all within 10 % of the values for optimum tuning and damping ratios as predicted by Equations 3-10. An interesting experiment for further study would be to try to impose either of these two conditions while at the same time

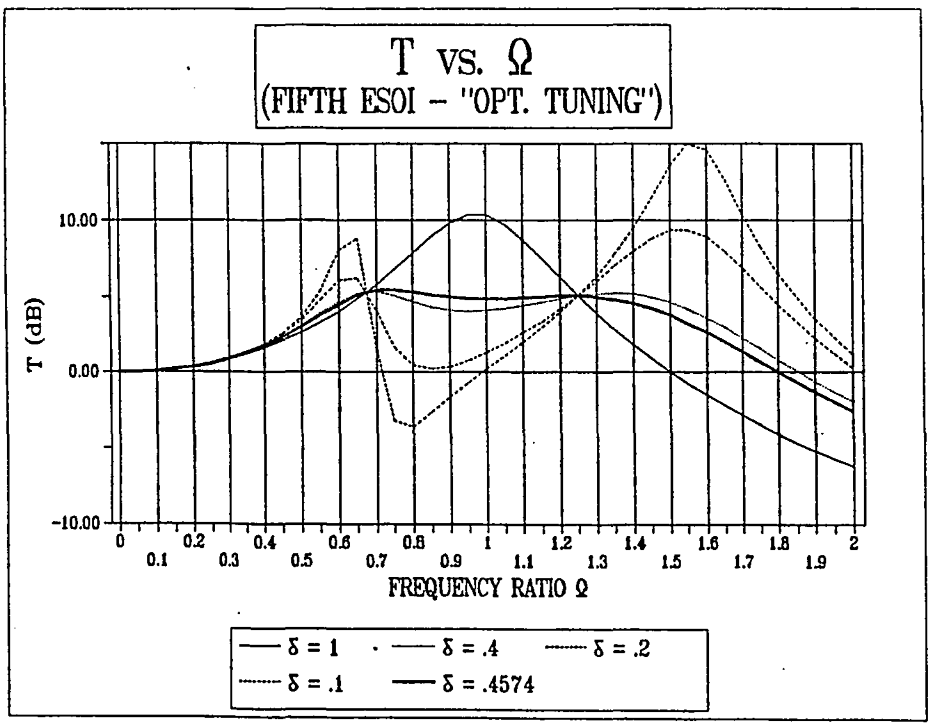

Figure 3-10 - T versus Ω for the fifth experimental set of iterations of the first design optimization procedure (target: T_{dif} a minimum at resonance).

establishing a required value for T_{target} at resonance (since the values of T_{res} from these graphs of 4.835 is unacceptable from an engineering application point of view).

Checking that the values are robust is a more difficult matter. According to Phadke [7], a product with a output response described by w = f(x,y,z) is mathematically robust when the *sensitivity coefficients* are smallest. The sensitivity coefficients are the squares of the partial derivatives of f(x,y,z) with respect to each of the control factors x, y, and z. However, for mathematically complex output response functions such as Equation 3-9, finding these partial derivatives is tedious. As a result, it was beyond the scope of this research to mathematically prove the robustness of these values for this particular design optimization procedure.

Finally, the nagging question of why the iterations would not converge to the values of control factors necessary for optimum tuning and damping, even when the values of T and μ for these conditions were given, remains unanswered. It was possible to "flatten" the curve and isolate a local minimum at resonance using iterations, but in both cases, the criteria for optimum tuning and damping (Equations 3-10 and 3-11) were not satisfied.

The conclusion of Design Optimization Procedure Example One, the dynamic absorber, is that the Taguchi's method of parameter design was applicable to simple, practical cases of finding the optimum values of the control factors to achieve a specified reduction in transmissibility at resonance. However, the method proved unsuccessful when applied to the conditions of optimum tuning and damping, perhaps due to the interrelationship of the parameters at these conditions.

CHAPTER 4 - DESIGN OPTIMIZATION PROCEDURE EXAMPLE TWO: THE ACOUSTIC MUFFLER

Section 4.1 - Background

The second system to be used as an example of design optimization is the acoustic muffler. In this design optimization procedure, the desired response is a reduction in transmitted noise energy; the control factors are the area ratio m and the muffler length l_e (which are described later in this chapter); and the levels of the control factors are low, medium, and high. The experimental sets of iterations were performed using closed-form equations developed by Davis [57] and described later in this chapter. Assumptions that were made in this design optimization procedure are also described later in this chapter.

An acoustic muffler is, in essence, an acoustical filter, whose primary purpose is to reduce the transmission of acoustical energy (commonly called "noise") that is produced by a mechanical system, such as a car engine or air conditioner. Examples of the usage of acoustic mufflers are considered in Section 4.1.2.

The practical application of this design optimization procedure was to find the optimum geometry for the muffler that would enable the noise generated by the engine of the author's car to be reduced by the 5 dB, 10 dB, or 15 dB when the turbocharger was used.

Section 4.1.1 - Description of the System

Acoustic mufflers are classified by the mechanism that is used to filter the unwanted noise and as such are classified into two types. *Dissipative*, or *absorption*, mufflers are those "whose acoustical performance is determined mainly by the presence of sound-absorbing...material." [58] These mufflers suffer a decrease in filtering ability when the temperature or velocity of the fluid in the passageway (such as a duct or pipe) is very high [57]. *Reactive*, or *reflection*, mufflers are those "whose performance is determined mainly by its geometrical shape." [58] Included in this "geometrical shape" is a discontinuity in the characteristic cross sectional area of the passageway. This discontinuity - whether it consists of a wider section of passageway or a narrower section of passageway - restricts the amount of acoustic energy that can continue along the passageway, with the remainder of the energy forming a standing wave that is reflected back towards the source of the acoustic energy. This reflection of the standing wave back towards the source can reduce the performance of the source, the one drawback of a reflection muffler. In this design optimization procedure, reflection-type mufflers were considered.

Reflection mufflers are themselves classified into several different categories. A *low-pass acoustic filter* is shown in Figure 4-1. It consists of a passageway of cross-sectional area A_1 into which a larger or smaller section of cross-sectional area A_2 and length l_e is inserted [59]. This type of muffler permits the continuation of low-frequency noise, but impedes the progress of high-frequency noise. A *high-pass acoustic filter* is shown in Figure 4-2. It consists of a passageway of cross-sectional

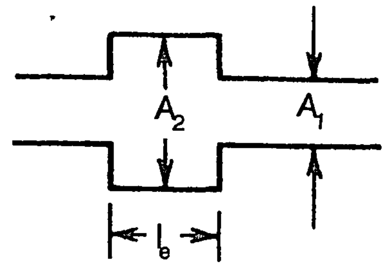

Figure 4-1 - A low-pass acoustic filter.

area A_1 onto which a side branch of pipe of cross-sectional area A_2 and length l_e is attached [59]. This type of muffler permits the continuation of high-frequency noise, but impedes the progress of low-frequency noise. Finally, a third type of reflection muffler that is not shown here is a *band-pass acoustic filter.* It permits the passage of noise whose frequency lies within a certain range who endpoint frequencies are called *stop bands* [57].

The concept of frequency-passage that is employed in all of these reflection mufflers is exactly analogous to electrical circuits, and thus the equations of power output can be obtained from analysis of the equivalent electrical circuits. Crocker and Price [60] provide an excellent reference for corresponding circuit equations. In addition, it is possible to construct more complex versions of the above mufflers. Low-pass filters (also called series filters by Harris [57]) can be constructed with multiple sections of differing cross sectional areas, with the most complex, for

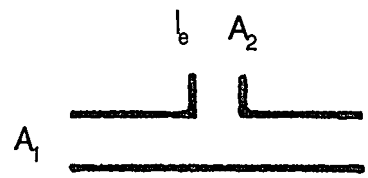

Figure 4-2 - A high-pass acoustic filter.

example, having sections of reduced cross-section within a section of increased cross-sectional area. Similarly, high-pass filters (also called parallel filters by Harris [57]) can be constructed with multiple side-branches and open-ended side-branches. Davis, Stokes, Moore, and Stevens [61] provide an outstanding summary of many different versions of reflection mufflers along with performance curves. In this design optimization procedure, a simple, low-pass muffler was considered.

Section 4.1.2 - Practical Applications of Acoustic Filters

Acoustic filters have many applications. The most common application of low-pass, reflection muffler is in automobiles, where the muffler is used to reduce the noise generated by exhaust gases from the engine. Similarly, low-pass reflection mufflers are used in air-conditioning systems to prevent noise generated at blowers or at the conditioning unit itself to enter the air-conditioned rooms, and in gun silencers, to muffle the sound of the explosion.

In these cases, mufflers are used to prevent the transmission of noise for aesthetical purposes. However, mufflers are also used "to reduce pressure pulsations that endanger engineering structures" [57]. In this application, the dynamic absorber from Chapter 3 could be considered a vibrational muffler. Harris adds that in such cases, the actual parameters of the muffler be such "that the static pressure drop not be excessive." [57]

Finally, high-pass mufflers have their most common application in musical instruments such as flutes, clarinets, oboes, bassoons, and saxophones. In this case, the fluid is air provided by the musician. The body of the instrument provides the passageway, and the holes and/or keys of the instrument provide the side-branches attached to the passageway.

Section 4.1.3 - Theoretical Equations

The transmission loss of a pipe/duct system is defined by Cook and Chrzanowski as "the number of decibels by which sound energy which is randomly incident on a partition is reduced in transmission through it." [57] Davis [57] obtains an equation for transmission loss for the acoustic muffler shown in Figure 4-1 as

$$TL = 10*\log_{10}[1+\frac{1}{4}*(m-\frac{1}{m})^2*\sin^2(k*l_e)] \tag{4-1}$$

(for details on the derivation of this equation, see Reference 57). In this equation, m is simply the ratio of the cross sectional areas (that is, $m=A_2/A_1$) and l_e is the length of the enlarged section of pipe or duct. Also,

$$k = \frac{2*\pi*f}{c},\qquad (4\text{-}2)$$

where f is the frequency of the fluid in cycles per second and c is the velocity of sound at the temperature considered. The performance curves referenced in Davis, Stokes, Moore, and Stevens and mentioned above are given as plots of TL vs. f, but plots of TL vs. $k*l_e$ are also given.

Section 4.2 - Experimental Setup

Section 4.2.1 - Goals and Targets

The goal of the Acoustic Muffler Design Optimization Procedure was to use Taguchi's method of parameter design to select the optimum values of the control factors m and l_e of a low-pass, reflection muffler such that:

1. The transmission loss was a specified target value when the turbocharger on the author's personal car was used;
2. The process was robust.

These goals established the target output response as "transmission loss = x dB", where "x" was a specified reduction in engine noise, and the control factors as m and l_e (Steps 2 and 3 of Section 1.4). The starting ranges for each variable were chosen to be $4.0 \leq m \leq 100.0$ and $6.0'' \leq l_e \leq 48.0''$. The range on l_e was chosen based on the fact that data was available for muffler lengths of 6 inches up to an including 48 inches at 6 inch intervals, thus providing a means to check the reasonableness of the solutions obtained. The range on m was chosen arbitrarily (Step 4). Equation 4-1 was the principal equation needed, since it equated the transmission loss as a direct function of the area ratio and the muffler length (Step 5). This equation was programmed, and the S/N ratio (which was to be maximized) was calculated using the MSD concept by the following equation:

$$SN = -10*\log_{10}(TL_{actual}-TL_{target})^2 , \qquad \text{(4-3)}$$

where TL_{actual} was the calculated value of the transmission loss based on the levels of the control factors for that particular trial (Step 6). Before performing Step 7, the actual execution of the design optimization procedure, it was decided to run the design optimization procedure three times, using different target values of transmission loss each time (Step 9). These values were chosen as 5 dB, 10 dB, and 15 dB. Step 7 is discussed in Section 4.3, and Step 8 is discussed briefly in Section 4.4.

Section 4.2.2 - Other Assumptions, Modifications

In addition to the equations used above, several other relationships and assumptions were used. First, it was assumed that carbon monoxide exhaust gas was used as the fluid in the tailpipe. Second, it was necessary to determine the speed of sound. The equation used for this purpose was the equation

$$c = \sqrt{\gamma*R*T}, \qquad \text{(4-4)}$$

where

$$R_u = 1545. \frac{ft \cdot lbf}{mole \cdot {}^\circ R} \qquad \text{(4-5)}$$

and

$$\gamma = \frac{c_p}{c_v} = 1.333 \text{ for } CO \tag{4-6}$$

and

$$M = molecular\ mass\ of\ CO = 28.01 \frac{lbm}{lbmole}. \tag{4-7}$$

It was assumed that the exhaust gases pass through the muffler at 1400.°F or 1860. R. This assumption was made based on information from several sources. Substituting this value, Equations 4-5, 4-6, and 4-7, the identity

$$lbm \equiv \frac{lbf \cdot \sec^2}{32.174 ft} \tag{4-8}$$

and the conversion factor of inches to feet into Equation 4-4 yielded a value of 25,170 in/sec as the speed of sound. In addition, it was assumed that the value of the frequency of the fluid was equal to the engine speed, or 3000 rpm (value obtained from car manual). Finally, it was assumed that the original section of tailpipe was round in shape with a diameter of two inches, while the enlarged section (the muffler) was square in shape with a side width to be determined from the optimum value of the area ratio.

Section 4.3 - Details and Results of the Experiment

The first experimental set of iterations, with TL_{target} = 5 dB, was run. After 16 iterations, the procedure had converged to the ordered pair (m,l_e) = (12.0, 19.99"). The data for this first experimental set of iterations is shown in Table 4-1,

Table 4-1 - Results of the first experimental set of iterations of the second design optimization procedure (target: TL = 5 dB).

PRIMARY VALUES					
TRIAL	m	l_e (in)	TL (dB)	MSD	
1	12.0	19.99	5.00	73.8	
2	12.0	19.99	5.00	65.3	
3	12.0	19.99	5.00	57.8	
4	12.0	19.99	5.00	65.5	
5	12.0	19.99	5.00	57.9	
6	12.0	19.99	5.00	53.9	
7	12.0	19.99	5.00	57.9	
8	12.0	19.99	5.00	53.9	
9	12.0	19.99	5.00	51.2	
AVERAGE S/N RATIOS					
	SETTING	m	l_e		
	1	65.7	65.8		
	2	59.1	59.0		
	3	54.3	54.3		
SECONDARY VALUES					
TRIAL	A_1 (in^2)	A_2 (in^2)	k (1/in)	D_2 (in)	S_2 (in)
1	3.14	37.7	.01248	6.93	6.14
2	3.14	37.7	.01248	6.93	6.14
3	3.14	37.7	.01248	6.93	6.14
4	3.14	37.7	.01248	6.93	6.14
5	3.14	37.7	.01248	6.93	6.14
6	3.14	37.7	.01248	6.93	6.14
7	3.14	37.7	.01248	6.93	6.14
8	3.14	37.7	.01248	6.93	6.14
9	3.14	37.7	.01248	6.93	6.14

which includes the primary values of the experiment (the values of the control factors, the value of the output response, and the MSD for each trial), the values of the average S/N ratios for each level of each factor, and the secondary values (the values of the various system components based on the equations in Section 4.1.3). Included in the secondary values are the length of the side of the enlarged section of the muffler if it were a square cross section and the length of the diameter of the enlarged section if it were a round cross section.

The second experimental set of iterations, with TL_{target} = 10 dB, was then run. The same initial ranges of the control factor values were used. After 16 iterations, the procedure had converged to the ordered pair (m,l_e) = (34.8, 13.90"). The corresponding data is shown in Table 4-2.

The third experimental set of iterations, with TL_{target} = 15 dB, was run. The same initial ranges of the control factor values were used. After 14 iterations, the procedure had converged to the ordered pair (m,l_e) = (50.8, 17.62"). The corresponding data for this set of iterations is shown in Table 4-3.

Table 4-2 - Results of the second experimental set of iterations of the second design optimization procedure (target: TL = 10 dB).

PRIMARY VALUES					
TRIAL	m	l_e (in)	TL (dB)	MSD	
1	34.8	13.90	10.0	63.4	
2	34.8	13.90	10.0	69.9	
3	34.8	13.90	10.0	89.0	
4	34.8	13.90	10.0	69.2	
5	34.8	13.90	10.0	102.	
6	34.8	13.90	10.0	68.7	
7	34.8	13.90	10.0	94.8	
8	34.8	13.90	10.0	69.4	
9	34.8	13.90	10.0	63.2	
AVERAGE S/N RATIOS					
	SETTING	m	l_e		
	1	74.1	75.8		
	2	80.1	80.5		
	3	75.8	73.7		
SECONDARY VALUES					
TRIAL	A_1 (in^2)	A_2 (in^2)	k (1/in)	D_2 (in)	S_2 (in)
1	3.14	109.3	.01248	11.80	10.46
2	3.14	109.3	.01248	11.80	10.46
3	3.14	109.3	.01248	11.80	10.46
4	3.14	109.3	.01248	11.80	10.46
5	3.14	109.3	.01248	11.80	10.46
6	3.14	109.3	.01248	11.80	10.46
7	3.14	109.3	.01248	11.80	10.46
8	3.14	109.3	.01248	11.80	10.46
9	3.14	109.3	.01248	11.80	10.46

Table 4-3 - Results of the third experimental set of iterations of the second design optimization procedure (target: TL = 15 dB).

PRIMARY VALUES					
TRIAL	m	l_e (in)	TL (dB)	MSD	
1	50.8	17.62	15.0	53.3	
2	50.8	17.62	15.0	60.3	
3	50.8	17.62	15.0	72.5	
4	50.8	17.62	15.0	58.4	
5	50.8	17.62	15.0	105.	
6	50.8	17.62	15.0	58.3	
7	50.8	17.62	15.0	72.9	
8	50.8	17.62	15.0	60.2	
9	50.8	17.62	15.0	53.2	
AVERAGE S/N RATIOS					
	SETTING	m	l_e		
	1	62.0	61.5		
	2	73.9	75.1		
	3	62.1	61.3		
SECONDARY VALUES					
TRIAL	A_1 (in^2)	A_2 (in^2)	k (1/in)	D_2 (in)	S_2 (in)
1	3.14	159.4	.01248	14.25	12.63
2	3.14	159.4	.01248	14.25	12.63
3	3.14	159.4	.01248	14.25	12.63
4	3.14	159.4	.01248	14.25	12.63
5	3.14	159.4	.01248	14.25	12.63
6	3.14	159.4	.01248	14.25	12.63
7	3.14	159.4	.01248	14.25	12.63
8	3.14	159.4	.01248	14.25	12.63
9	3.14	159.4	.01248	14.25	12.63

Section 4.4 - Discussion of Results

Each of the three sets of iterations converged to ordered pairs (m,l_e), as desired. The computed values of TL were equal to the desired values of TL (using the proper number of significant figures) in each case. As an added check of the accuracy of the values, a comparison was made to Figure 21.20 in Harris [57]. In that figure, transmission loss is plotted versus $k*l_e$ (the argument of the sine function in Equation 4-1), with separate curves drawn for m = 4, 9, 16, 25, 49, and 100. This figure is reproduced as Figure 4-3 with the modifications that the curves for m = 16, 25, and 49 are replaced by curves for the m values obtained in the iteration procedures (that is, m = 12.0, 34.8, and 50.8). Note that all three replacement curves "fit in" with the others - that is, each is in its proper place relative to the others. Also, by examining where each of these curves cross the corresponding target transmission loss, it can be seen that the value of $k*l_e$ corresponds well to the corresponding calculated values of 0.2494, 0.1735, and 0.2199, thus verifying the accuracy of the solutions obtained.

As an added iteration procedure, it was decided to run the design optimization procedure using frequency (f) as a third control factor. The target value of the transmission loss was set at 5 dB. The results of this set of iterations were to be compared to those obtained when only two control factors were used. The initial ranges on m and l_e were kept constant. The initial range on f was 10 cps $\leq$ f $\leq$ 750 cps, with these end values based on the fact that available data [61] was given only in this range.

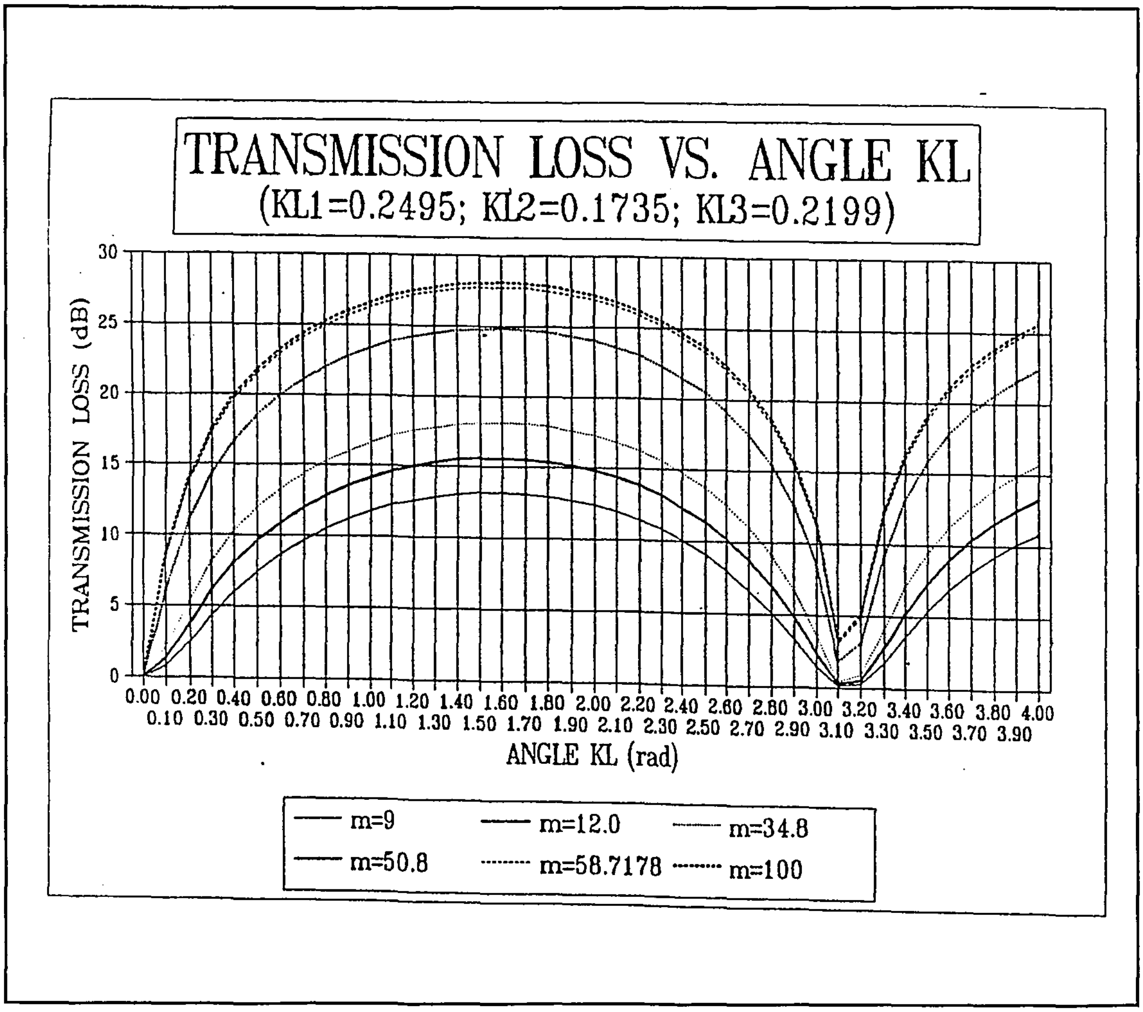

Figure 4-3 - TL versus k^*l_e for several values of m, including those from the first three experimental sets of iterations of the second design optimization procedure.

The results of the fourth experimental set of iterations are shown in Table 4-4. After 16 iterations, the procedure converged to the ordered triplet (m,l_e,f) = (4.75, 45.49", 338.8 cps). The m,l_e pair of 4.75, 45.49" was different than any of the pairs obtained in the first three experimental sets of iterations. Nevertheless, when the curve of TL vs k^*l_e for m = 4.75 was added to the curves for m = 4, 9, 16, 25, and 49, in Figure 4-4, the values were confirmed. The m = 4.75 curve was located just

Table 4-4 - Results of the fourth experimental set of iterations of the second design optimization procedure (target: TL = 5 dB).

PRIMARY VALUES					
TRIAL	m	l_e (in)	F (cps)	TL (dB)	MSD
1	4.75	45.49	338.8	4.99	44.0
2	4.75	45.49	338.8	5.00	51.8
3	4.75	45.49	338.8	5.00	58.8
4	4.75	45.49	338.8	5.00	60.5
5	4.75	45.49	338.8	5.00	51.2
6	4.75	45.49	338.8	5.00	49.0
7	4.75	45.49	338.8	5.00	47.1
8	4.75	45.49	338.8	5.00	54.5
9	4.75	45.49	338.8	5.00	54.8
AVERAGE S/N RATIOS					
	SETTING	m	l_e	F	
	1	51.6	50.5	49.1	
	2	53.6	52.4	55.7	
	3	52.1	54.2	52.4	
SECONDARY VALUES					
TRIAL	A_1 (in^2)	A_2 (in^2)	k (1/in)	D_2 (in)	S_2 (in)
1	3.14	14.9	.08455	4.36	3.86
2	3.14	14.9	.08455	4.36	3.86
3	3.14	14.9	.08455	4.36	3.86
4	3.14	14.9	.08455	4.36	3.86
5	3.14	14.9	.08455	4.36	3.86
6	3.14	14.9	.08455	4.36	3.86
7	3.14	14.9	.08455	4.36	3.86
8	3.14	14.9	.08455	4.36	3.86
9	3.14	14.9	.08455	4.36	3.86

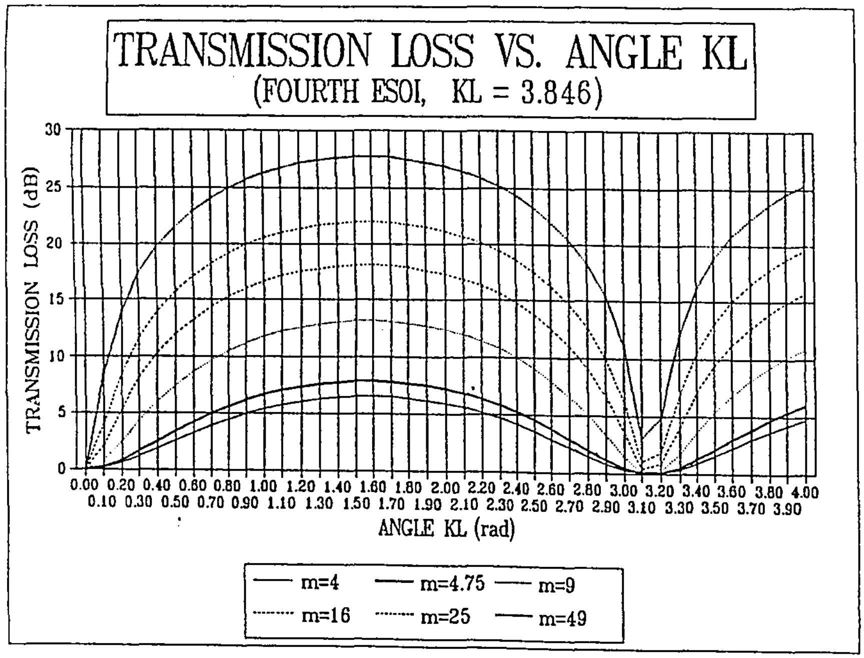

Figure 4-4 - TL versus k^*l_e for several values of m, including the value obtained in the fourth experimental set of iterations of the second design optimization procedure.

above the m = 4 curve, thus validating the results of the fourth experimental set of iterations. In addition, the value of k^*l_e at the point where the m = 4.75 curve crossed the horizontal line TL = 5 dB was approximately 3.85, which is exactly the calculated value of k^*l_e = 3.846 rounded to two decimal places.

However, a question remains. In the experimental set of iterations with two control factors and TL_{target} = 5 dB, the ordered triplet would have been (12.0, 19.99", 314.2 cps), if we consider f to be a "constant" control factor. But when f was allowed to vary, the ordered triplet above (m,l_e,f) = (4.75, 45.49", 338.8 cps) was obtained. It is not known why this difference in values existed.

Finally, an attempt was made to analyze the values obtained in the two-control factor iterations for robustness. As in the case of the first design optimization procedure, however, the relationship between the output response and the control factors was too complicated to permit a calculation of the sensitivity coefficients. Thus, this procedure was not completed.

The conclusion of Design Optimization Procedure Example Two, the acoustic muffler, is that the Taguchi's method of parameter design was applicable to the two-control factor case, converging to values for the control factors that satisfied the output response through the theoretical equations and were robust, but the method converged to different values when a third control factor was added to the design optimization procedure.

CHAPTER 5 - DESIGN OPTIMIZATION PROCEDURE EXAMPLE THREE: THE GEAR/PINION SYSTEM

Section 5.1 - Background

The third system to be used as an example of design optimization is the gear/pinion system. In this design optimization procedure, the desired response is that the gear geometry satisfy a geometric constraint; the control factors are the diametral pitch P and the permissible stress σ_p (which are described later in this chapter); and the levels of the control factors are low, medium, and high. The experimental sets of iterations were performed using closed-form equations developed by Shigley [62] and described later in this chapter. Assumptions that were made in this design optimization procedure are also described later in this chapter.

Gears are mechanical devices that transmit rotary motion from one shaft to another. Gears also are used to change speed or motion of a rotary shaft. This transmission or change is accomplished by placing a gear at each of the shafts and then having the two gears "mesh", with one of the gears turning the other gear by the power that is present in the shaft onto which it is mounted. Examples of the usage of gears are considered in Section 5.1.2.

The practical application of this design optimization procedure was to find the optimum gear geometry for the gear that would be able to transmit rotary energy to the wheels (and hence the tires) of the author's car when the turbocharger was used.

Section 5.1.1 - Description of the System

There are several types of gears. *Spur gears* are the most common type of gears. These gears, which are often cylindrical-shaped with straight teeth that are parallel to the rotation axis, provide for rotary transmission between parallel shafts. *Helical gears* are similar to spur gears except that the teeth are not parallel to the axis of rotation, but rather are in the shape of an involute helicoid. *Bevel gears* are used to transmit rotary motion between intersecting shafts. Subsets of bevel gears include straight bevel gears, spiral bevel gears, hypoid gears, and spiroid gears. *Worm gears* are used to transmit rotary motion between shafts that do not intersect.

When two gears of different sizes are used, the smaller gear is often called the *pinion*. The larger gear is simply called the gear. In this design optimization procedure, a spur gear and pinion were considered.

Section 5.1.2 - Practical Applications of Gears

There are many practical applications of gears and gear/pinion systems. The most common application of gear/pinion systems is their use in automobile drive trains, where a series of gear/pinion systems is used in specified reduction ratios to transmit engine power to rotational energy that drives the axles onto which are mounted the car's wheels and tires. Gears are also used in machine design, as well as in common household appliances, such as can openers and manual egg beaters.

Shigley points out that gear design is at the same time a science and an art:

> When you realize that the gears in...your automobile differential can be made to run 100,000 miles or more before they need to be replaced, and when you count the actual number of revolutions, you begin to appreciate

> the fact that the design and manufacture of these gears is really a remarkable accomplishment. People do not generally realize how highly developed the design, engineering, and manufacturing of gear elements has become because gears are such ordinary machine elements. [62]

He adds that much can be learned about engineering through the study of gears [62].

Section 5.1.3 - Theoretical Equations

Before reviewing any theoretical equations for gears, a review of gear terminology is necessary. Figure 5-1 shows two gear teeth and corresponding terminology. The *pitch circle* is the point at which mating teeth are tangent to one another. The diameter of this circle is usually called "d". The *circular pitch*, called

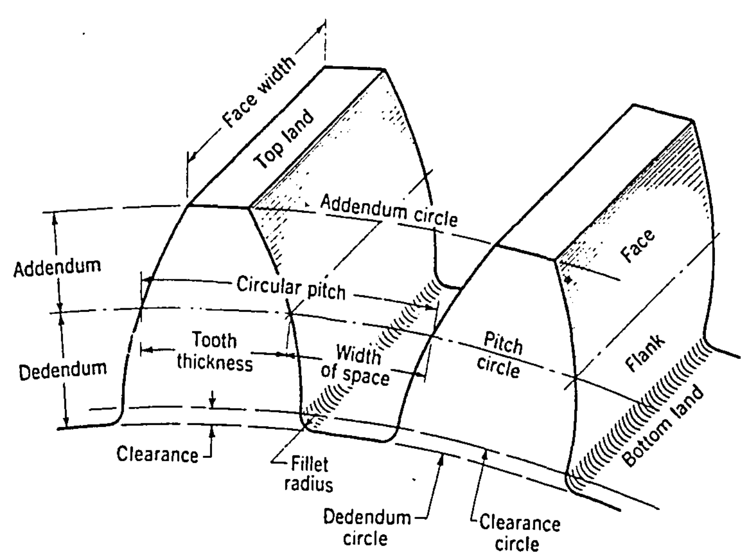

Figure 5-1 - Gear terminology.

"p", is the distance, along this circle, between corresponding points of adjacent teeth. From the Figure 5-1, it can be deduced that circular pitch equals tooth thickness plus width of space. The ratio of "d" to the number of teeth (often called "N") is called the *module*, while the ratio of "N" to "p" is called the *diametral pitch* "P" (note the difference in letter casing).

There are several other terms that are useful. As shown in Figure 5-1, the distance from the top of the tooth (the *top land*) to the pitch circle is the *addendum*, while the distance from the pitch circle to the base of the tooth (the *bottom land*) is called the *dedendum*. The sum of the addendum and dedendum is called the *whole depth* (h_t). The *clearance circle* is the circle formed by the addendum of the mating clear. The distance between this circle and the dedendum of the gear is called the *clearance* ("c"). Finally, the *backlash* is the distance on the pitch circle by which the width of a tooth space is greater than the thickness of the tooth on the mating gear.

With these terminologies defined, the following relationships can be derived:

$$P = \frac{N}{d}; \qquad (5\text{-}1)$$

$$m = \frac{d}{N}; \qquad (5\text{-}2)$$

$$p = \pi * m; \qquad (5\text{-}3)$$

$$\pi = p * P. \qquad (5\text{-}4)$$

In addition, the following relationships are given by Shigley [62] and are necessary for this design optimization procedure:

$$V = \frac{\pi * d * n}{12} = pitch\text{-}line\ velocity; \tag{5-5}$$

$$W_t = 33{,}000 \frac{H}{V} = work\ transmitted; \tag{5-6}$$

$$K_v = \frac{1200}{1200+V} = velocity\ factor; \tag{5-7}$$

$$F = \frac{W_t * P}{K_v * Y * \sigma_p} = face\ width. \tag{5-8}$$

In Equation 5-6, H is the power imparted to the shaft. In Equation 5-8, σ_p is the permissible bending stress for the gear material, while Y is the Agma Lewis Form Factor, whose value varies as several tooth parameters vary. Values of Y are tabulated in various handbooks.

Finally, Shigley [62] recommends a design approach whereby face widths for prospective gear teeth lie in the range $3p \leq F \leq 5p$. Teeth with $F > 5p$ are likely "to have a nonuniform distribution of the load across the face of the tooth because of the torsional deflection of the gear and shaft" [62]. Shigley reasons that teeth with $F < 3p$ will require a larger gear that is not cost-effective to manufacture, and thus sets 3p as a lower limit. And, Shigley adds that "When estimating gear sizes, it is a good idea to use a factor of safety of 3 or more, depending upon the material and application." [62]

Section 5.2 - Experimental Setup

Section 5.2.1 - Goals and Targets

The goal of the gear/pinion system design optimization procedure was to use Taguchi's method of parameter design to select the optimum values of the control factors P and σ_p of a spur gear such that:

1. The engine horsepower and speed was a specified target value when the turbocharger on the author's personal car was used;
2. Shigley's design criteria of $3p \leq F \leq 5p$ was met;
3. The process was robust.

These goals established the target output response as a range instead of a specified value ($3p \leq F \leq 5p$), while the control factors were specified as P and σ_p (Steps 2 and 3 of Section 1.4). The starting ranges for each variable were chosen to be $3.0 \leq m \leq 5.0$ and 6.0 ksi $\leq \sigma_p \leq$ 58.5 ksi. The range on σ_p was chosen based on the fact that data available for various gear steel showed a range of approximately 24 ksi to 234 ksi, which translates into the range above when a factor of safety of 4 is used. The range on P was chose arbitrarily (Step 4).

The actual process of obtaining the final face width as described by Shigley [62] was itself an iterative process. The appropriate value of N was determined, then used in Equation 5-1 to determine d, which was used in Equation 5-5 to determine V. This value was used to determine K_v from Equation 5-7 and was used, along with the appropriate value of H (the engine horsepower), in Equation 5-6 to determine W_t. Finally, the value of Y was determined from tables and was used, along with the values of P and σ_p for that particular trial and the values of K_v and W_t found above to determine the value of the face width (Step 5). These equations have been programmed. In order to accommodate the fact that the desired output response was a range instead of a single value, the S/N ratio was set using a "plateau-type" method whereby any value of F in the desired design range $3p \leq F \leq 5p$ was given an arbitrarily high S/N ratio (99.9882), while the S/N ratio for all other values of F was determined by the following equation:

$$S/N = -10*\log_{10}(F_{actual}-F_{target})^2 , \tag{5-9}$$

where F_{actual} was the calculated value of the face width based on the levels of the control factors for that particular trial (Step 6) and F_{target} was the closest value of the preferred design range (that is, $F_{target} = 3p$ for $F_{actual} < 3p$, and $F_{target} = 5p$ for $F_{actual} > 5p$). It may seem that the concept of arbitrarily giving all (P,σ_p) values that produce an F-value in the desired range the same, high S/N value represents a form of the goalpost philosophy discussed in Chapters One and Two, since it is being assumed that all values of F in the desired range are equally good. In this case, however, all values of F in the desired range *are* equally good, since they will satisfy the power transmission and engine speed requirements to similar degrees. In addition, a range of P and σ_p values was produced (as will be discussed in Section 5.3). Since all values in these ranges are equally good, the midpoint values should be chosen, thus permitting wider tolerances (consistent with robust thinking) to be used.

Section 5.2.2 - Other Assumption, Modifications

In addition to the equations used above, several other relationships and assumptions were used. First, it was assumed that the entire gear train would be reduced to a pair of 4:1 reduction gears. Next, it was assumed that the gears would be 20° full-depth gears with clearances of .250/P. These facts enabled the Y value to be determined as 0.29327 and the number of teeth necessary to avoid undercutting as 18 on the pinion. Both assumptions were made in order to approximate the iteration procedure given in Shigley [62].

It was also assumed that the engine speed (3000 rpm obtained from car manual) and engine power (146 hp, also obtained from car manual) were constant throughout the system with minimal friction losses. Finally, it was assumed that a factor of safety of 4 would be used, consistent with the procedure given in Shigley [62].

Section 5.3 - Details and Results of the Experiment

After only five iterations, the procedure had converged such that all levels of both control factors had average S/N values equal to 100. At that point, the range on the levels of P were 4.38" ≤ P ≤ 4.63", while the range on the levels of σ_p was 27.6 ksi ≤ σ_p ≤ 30.9 ksi. The data for this first experimental set of iterations is shown in Table 5-1, which includes the primary values of the experiment (the values of the control factors, the value of the output response F, the value of d, and the MSD for each trial), the values of the average S/N ratios for each level of each factor, and the

Table 5-1 - Results of the first experimental set of iterations of the third design optimization procedure (target: $3p \leq F \leq 5p$).

PRIMARY VALUES					
TRIAL	P (teeth/in)	σ_p (ksi)	F (in)	d (in)	MSD
1	4.38	27.6	2.97	4.11	100.
2	4.38	29.3	2.81	4.11	100.
3	4.38	30.9	2.66	4.11	100.
4	4.50	27.6	3.08	4.00	100.
5	4.50	29.3	2.91	4.00	100.
6	4.50	30.9	2.76	4.00	100.
7	4.63	27.6	3.19	3.89	100.
8	4.63	29.3	3.01	3.89	100.
9	4.63	30.9	2.86	3.89	100.
AVERAGE S/N RATIOS					
	SETTING	P	σ_p		
	1	100.	100.		
	2	100.	100.		
	3	100.	100.		
SECONDARY VALUES					
TRIAL	K_v	V (ft/min)	W_t (lbf)	p (in)	F range (in)
1	.271	3230.	1490.	.718	2.15-3.59
2	.271	3230.	1490.	.718	2.15-3.59
3	.271	3230.	1490.	.718	2.15-3.59
4	.276	3140.	1530.	.698	2.09-3.49
5	.276	3140.	1530.	.698	2.09-3.49
6	.276	3140.	1530.	.698	2.09-3.49
7	.282	3060.	1580.	.679	2.04-3.40
8	.282	3060.	1580.	.679	2.04-3.40
9	.282	3060.	1580.	.679	2.04-3.40

secondary values (the values of the various system components based on the equations in Section 5.1.3). Included in the secondary values are the circular pitch p, the range of face allowable face widths based on Shigley's criteria, and several parameters calculated in the process of computing the face width (such as the velocity factor K_v). Note that an nearly non-fractional value for F appears in one of the trials, namely, a face width of three inches (Trial 8). This value would require a circular pitch of 4.63 teeth per inch and a steel with a yield strength of 117.2 ksi, such as G10500 3130 steel drawn at 1000 °F. However, it is more robust to choose the midpoint values of the final ranges on P and σ_p, namely P = 4.5 teeth/in and σ_p = 117.2 ksi (this value of σ_p happens to be the one used in Trial 8, while the pair (4.5 teeth/in., 117.2 ksi) occurs in Trial 5). These two values produce a face width of 2.91 inches, not far from 2.79 inches, which is the exact midpoint of the range [3p,5p]. Thus, a steel with a yield strength close to 117.2 ksi, such as G10500 3130 drawn at 1000 °F (σ_p = 120 ksi) should be used, with a face width of approximately 2.91 inches.

Section 5.4 - Discussion of Results

The iteration procedure converged to a range of ordered pairs (P,σ_p), as desired. The values of F obtained by using any of the ordered pairs in this range were within the design criteria established by Shigley. However, it is not known if all possible ordered pairs are included in this range. It is possible that the lower or upper limit on σ_p, for example, should be lower or higher, respectively, to include all pairs that would satisfy the design criteria.

As an added experiment, it was decided to repeat the iteration procedure using three control factor, with F as the third control factor. An initial range of 1" $\leq$ F $\leq$ 9" was used, with these values based on practicality. This experimental set of iterations converged in only five iterations. Again, the average S/N ratios for all levels of all three control factors was 100 at convergence. The ranges on the control factors was 3.31 teeth/in $\leq$ P $\leq$ 3.44 teeth/in, 40.6 ksi $\leq \sigma_p \leq$ 43.9 ksi, and 3.25" $\leq$ F $\leq$ 3.75". Using the same reasoning as in the first experimental set of iterations, the most robust values to use would be the midpoint of these ranges. Thus, the optimum choice of material would be a steel with a yield strength of 169.2 ksi (such as G46200 4640 drawn at 800 °F, σ_y = 170 ksi), with a face width of 3.75 inches and a diametral pitch of 3.38 teeth per inch. These values are tabulated in Table 5-2. As was the case in the acoustic muffler, the addition of the third control factor caused the original two control factors to converge to new optimum values, although the value of F changed, as well.

Table 5-2 - Results of the second experimental set of iterations of the third design optimization procedure (target: $3p \le F \le 5p$).

PRIMARY VALUES					
TRIAL	P (teeth/in)	σ_p (ksi)	F (in)	d (in)	MSD
1	3.31	40.6	3.25	5.43	100.
2	3.31	42.3	3.50	5.43	100.
3	3.31	43.9	3.75	5.43	100.
4	3.38	40.6	3.50	5.33	100.
5	3.38	42.3	3.75	5.33	100.
6	3.38	43.9	3.25	5.33	100.
7	3.44	40.6	3.75	5.24	100.
8	3.44	42.3	3.25	5.24	100.
9	3.44	43.9	3.50	5.24	100.
AVERAGE S/N RATIOS					
	SETTING	P	σ_p	F	
	1	100.	100.	100.	
	2	100.	100.	100.	
	3	100.	100.	100.	
SECONDARY VALUES					
TRIAL	K_v	V (ft/min)	W_t (lbf)	p (in)	F range (in)
1	.219	4270.	113.	.948	2.85-4.74
2	.219	4270.	113.	.948	2.85-4.74
3	.219	4270.	113.	.948	2.85-4.74
4	.223	4190.	115.	.931	2.79-4.65
5	.223	4190.	115.	.931	2.79-4.65
6	.223	4190.	115.	.931	2.79-4.65
7	.226	4110.	117.	.914	2.74-4.57
8	.226	4110.	117.	.914	2.74-4.57
9	.226	4110.	117.	.914	2.74-4.57

Finally, the values obtained in the two-control factor iterations were analyzed for robustness. For the third consecutive design optimization procedure, the sensitivity coefficients were too complex to calculate and examine. However, an alternate approach similar to the one used in Section 2.2.3 and shown in Figure 2-3 was used to examine the values obtained in the first experimental set of iterations for this design optimization procedure. In Figure 5-2, the output response F is plotted as a function of P, with σ_p held constant at the value obtained in the iteration

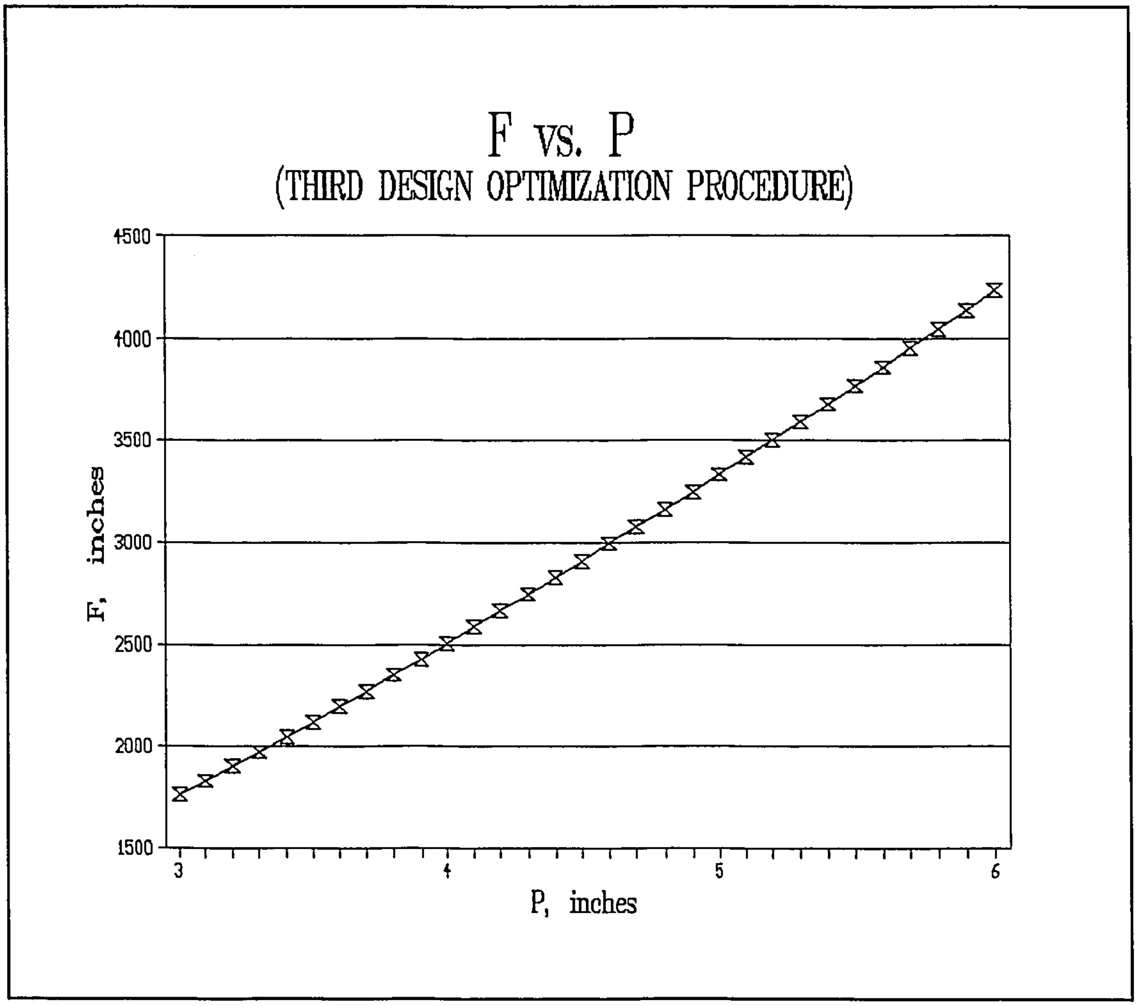

Figure 5-2 - F versus P holding σ_p constant, third design optimization procedure.

procedure (29.3 ksi). Note that even though the actual expression for F as a function of P and σ_p is complicated, the relationship between F and P appears to be a simple, linear one. In Figure 5-3, the output response F is plotted as a function of σ_p, with P held constant at the value obtained in the iteration procedure (4.50). Note that this relationship is *not* linear. Thus, it can be observed that at the value of σ_p = 29.3 ksi obtained in the iteration process, the system is robust: large changes in σ_p will produce small changes in F. Note that values of σ_p > 29.3 ksi appear to be even

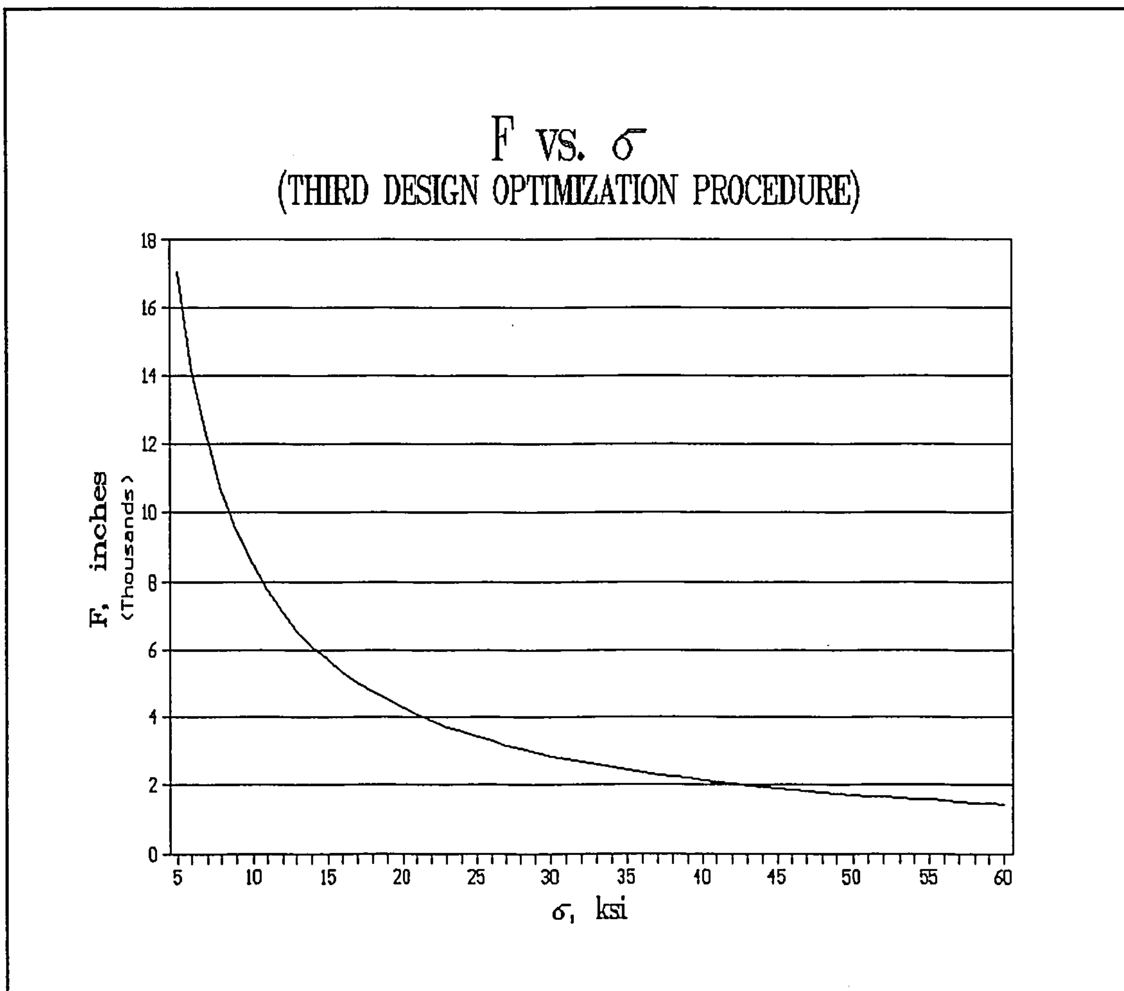

Figure 5-3 - F versus σ_p holding P constant, third design optimization procedure.

more robust that the obtained value, but only by a small amount, since the plot of F versus σ_p is approximately linear as σ_p increases. Since the relationship between F and P is linear, P can be used as an adjustment factor. *An adjustment factor* is one that has little influence on the sensitivity of the process to noise factors, but which does have an influence on the mean value of the process. Thus, once the optimum value of σ_p is obtained, the proper value of P can be chosen to bring the output response back to the desired value.

The conclusion of Design Optimization Procedure Example Three, the gear/pinion system, is that the Taguchi's method of parameter design was applicable to the two-control factor case, converging to values for the control factors that satisfied the output response through the theoretical equations and were graphically shown to be robust. However, the method once again converged to different values when an extra control factor was added to the analysis.

CHAPTER 6 - DESIGN OPTIMIZATION PROCEDURE EXAMPLE FOUR: THE SPRING

Section 6.1 - Background

The fourth system to be used as an example of design optimization is the spring. In this design optimization procedure, the desired response is a minimum material cost while satisfying geometric constraints; the control factors are the wire diameter d and the spring index C (which are described later in this chapter) in three experimental sets of iterations and the trio of d, C, and the number of turns of the spring (N) in another experimental set of iterations; and the levels of the control factors are low, medium, and high. The experimental sets of iterations were performed using closed-form equations developed by Shigley [62] and described later in this chapter. Assumptions that were made in this design optimization procedure are also described later in this chapter.

Springs are mechanical elements that are used in various applications. Springs exert forced, provide flexibility, and store energy. Usually, springs take the form of a helical or coiled wire and obey Hooke's Law,

$$F = k*x, \qquad \textbf{(6-1)}$$

where F is the force applied to the spring, x is the displacement that the spring undergoes from its non-stretched position, and k is a constant based on the material of the spring. Examples of the usage of springs are considered in Section 6.1.2.

The practical applications of this design optimization procedure were to find the optimum size of spring given a certain type of steel spring and a 1-ton load (such as an engine block) such that the spring will not fail while minimizing material cost, and find the optimum size of spring given a certain type of steel spring and a 1-ton load (such as an engine block) to be suspended with enough of a deflection from the ceiling of a garage or industrial plant such that the engine is not dragging on the floor but is reachable (sub-optimization procedure two) while minimizing the cost of material.

Section 6.1.1 - Description of the System

According to Shigley [62], wires are classified into three groups. *Wire springs* are made of wire that is either round or square in cross-section. Included in this group are the familiar helical springs. *Flat springs* include cantilever, elliptical, wound-motor, clock-type, and flat spring washer (Belleville) springs. *Special-shaped springs* include all other springs, such as constant-force springs. Springs are also classified by the type of forces that they resist: tension, compression, or torsional. In this design optimization procedure, a tension wire helical spring was considered.

Section 6.1.2 - Practical Applications of Springs

As noted above, springs have many applications. One common application of the torsional-type helical spring is in the commercial mousetrap. This spring resists torsional forces (to violently throw the mechanism down towards the piece of wood) and also stores potential energy (which causes this wild thrust of the mechanism). Well-calibrated springs are used in grocery stores to weigh certain produce items (since the weight of the object will cause a deflection predictable from Hooke's Law). Large helical springs are used in automobiles to provide rigid support to the body frame of the car. The most recent application of springs is in the exercise equipment business, where torsional springs are used to provide resistive forces that the muscles of the body must overcome, such as in finger exercisers. The strength of the spring used in the device determines how severe of an exercise will be undertaken.

But perhaps the largest application of springs is in industrial applications. Springs are primarily used to exert force to hold objects in place. A lever may be used, for example, to restrict the flow of objects. In its natural, non-stretched position, the lever allows the objects to pass; if the flow needs to be stopped, the machine exerts a force on the lever and moves it to a position where it blocks the flow, thus stretching the spring. When the flow is ready to continue, the machine force is removed, and the spring pulls the lever back to its original position and holds it there, allowing the flow to continue. This type of mechanism is used in machines that test electronic devices. The flow of devices, in these machines, may be halted due to a device that was not able to leave the machine due to a jamming problem. Conversely, springs are used in some applications to keep a certain passage closed. Finally, springs may be used to suspend objects for observation (since the objects can be maneuvered much better while suspended on the spring) and, in some cases, for calibration of other instruments.

Section 6.1.3 - Theoretical Equations

The cross section of a helical spring is shown in Figure 6-1. The diameter "D" is the mean spring diameter while the diameter "d" is merely the thickness of the wire used in the spring. If a force "F" is applied to the ends of the spring, it can be shown through elementary mechanics of materials that the maximum shear stress in the wire is give by

$$\tau_{max} = \frac{8*K_s*F*D}{\pi*d^3}. \qquad \textbf{(6-2)}$$

In this equation, K_s is the *shear-stress multiplication factor* and is found by the relationship

$$K_s = 1 + \frac{0.5}{C}, \qquad \textbf{(6-3)}$$

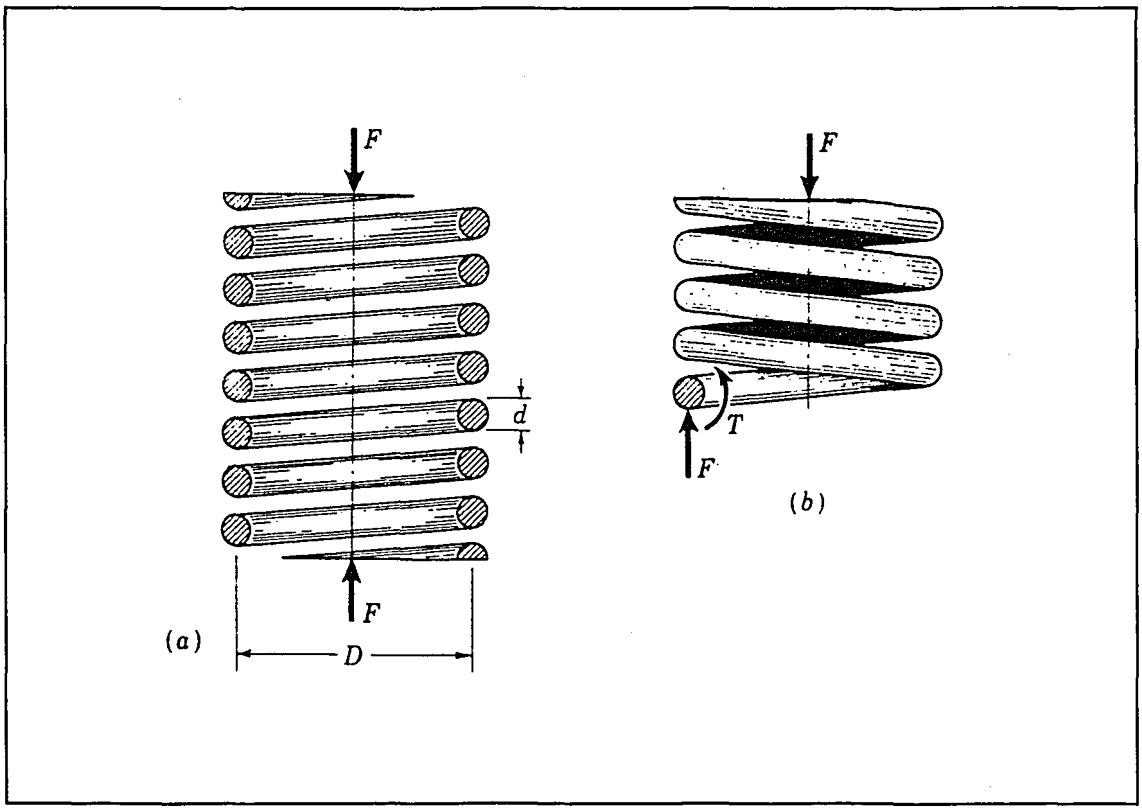

Figure 6-1 - The cross section of a helical spring.

where "C" in turn is the spring index, defined as the ratio of "D" to "d". Similarly, by using strain-energy methods, it can be shown that the deflection of the spring from its non-stretched state is given by

$$y = \frac{8*F*D^3*N}{d^4*G}, \tag{6-4}$$

where "N" is the number of active coils of the spring and G is the torsional modulus of elasticity for the material used. It should be noted that the value of the spring constant could also be found from the expression

$$k = \frac{d^4*G}{8*D^3*N}. \tag{6-5}$$

Section 6.2 - Experimental Setup

Section 6.2.1 - Goals and Targets

Due to the relatively simple nature of this system, the spring design optimization procedure was divided into two "sub-optimization procedures", both of which contained an experimental set of iterations that had its own target output responses. These two sub-procedures were similar in that all had the same general goal: to use Taguchi's method of parameter design to select the optimum values of the control factors of a helical, wire spring such that a certain target output value was reached subject to certain constraints, and the process was robust. Specifically, the sub-procedures varied as follows:

> Sub-Procedure 1: select optimum values of the control factors C and d such that a one ton load can be safely

hung from a spring made of steel with a yield strength of 104 ksi while minimizing the material cost of the spring, knowing that the cost of the steel used is to be a certain dollar amount per cubic inch.

Sub-Procedure 2: assuming that the one-ton load is to be suspended from a 20' ceiling in order to be examined, find the optimum values of the control factors C, d, and N such that the load is suspended off the ground but no higher than 4' above the ground while at the same time minimizing the cost of the material, given the fact that the cost of the steel to be used is a certain dollar amount per cubic inch.

In both sub-procedures, constrained optimization type problems and problems with multiple target output responses were examined.

Section 6.2.2 - Other Assumptions, Modifications

The starting ranges for the control factors and the identification of the control factors themselves underwent several modifications before reaching final status. Originally, D and d were used as the control factors in sub-procedure one, and a convergence was reached. However, when the load was increased to a value of 10 tons as a side-experiment, the iterations converged to values such that $D < d$, which is physically impossible.

The next attempt was to use C and σ_p as control factors. However, by inspection of Equation 6-2, one of either D or d must be a control factor. Thus, it was decided to use C and d (since it was felt that d is a more practical parameter than D) as the control factors. The initial ranges of the variables were set at $2.0 \leq C \leq 102.0$, $1.0 \leq d \leq 13.0$, and $6.0 \text{ ksi} \leq \sigma_p \leq 58.5$ ksi. The range on d was set based on engineering judgement, and the range on C was chosen such based on engineering judgement and the physical constraint that $D \geq 2d$.

In sub-procedure two, the height of the ceiling was initially set at a standard, 8 feet. However, even though the iterations converged, the criteria that the load be suspended no higher than 4' was violated (in fact, the load was not suspended, but was in effect resting on the ground). Several changes were made to the initial ranges before realizing that a spring with the number of turns that was initially specified in the initial range of N and a wire diameter d could not have physically fit in the room even when not stretched. Thus, the ceiling was raised to a height of 20 feet (to approximate the height of a garage or industrial work area) and the initial ranges were chose to make sense. These ranges were chosen as $2.0 \leq C \leq 12.0$, $1.0" \leq d \leq 11.0"$, and $1 \leq N \leq 11$ turns. The total non-stretched length of the spring is given by the equation

$$L = \pi * D * N . \tag{6-6}$$

Thus, the longest non-stretched spring that is possible from these initial ranges is 380 feet, which is obviously too long to fit in this room. However, the implementation in the computer code of the constraint that the total length of the spring with the load applied must be between 16 feet (inclusive) and 20 feet will prevent the iteration procedure from converging to this combination of control factor values as well as any combination of values that would combine to form an oversize spring. Even though this trio of control factor values and other trios may not physical sense, the initial range was held constant so as to be as large as is possible.

Four final assumptions were made. A factor of safety of 4.0 was used in all computations of permissible stress. In addition, a total cost of $10/cubic inch of material was assumed. This value was obtained from a commercial catalogue for steels. Also, in both sub-optimization procedures, it was assumed that the only available material was a steel with a yield strength of 104 ksi (26 ksi after adjusting for the factor of safety). Finally, a value of $G = 11.5 \times 10^6$ psi was assumed.

Section 6.3 - Details and Results of the Experiment

Once the initial problems outlined in Section 6.2.2 above were corrected, the iterations were quite successful. In sub-optimization procedure one, the iterations converged to S/N ratios that were actually very negative (most were in the -50s in the final iteration). The final values of the control factors were (C,d) = (2.00, 1.00"). These values were reached after 15 iterations. The data for this first experimental set of iterations is shown in Table 6-1, which includes the primary values of the experiment (the values of the control factors, the value of the output response total cost, and the MSD for each trial), the values of the average S/N ratios for each level of each factor, and the secondary values (the values of the various system components based on the equations in Section 6.1.3). Included in the secondary values are the outer diameter D, the curvature factor K_s, and the actual maximum shear stress τ. Note that the maximum shear stress in the spring was approximately 12.7 ksi, thus permitting the material on hand to be safely used with the desired factor of safety. Note also that since the final values of both C and d were the same as their original, lower limits, it is deducible that even lower values of C and d could give desired results. In this particular case, the minimum value of d was ascertained to have to be at least 1 inch. Nevertheless, the sub-optimization procedure could

Table 6-1 - Results of the first experimental set of iterations of the fourth design optimization procedure (target: minimize total cost).

PRIMARY VALUES					
TRIAL	C	d (in)	Total Cost ($)	MSD	
1	2.00	1.00	988.	-59.9	
2	2.00	1.00	989.	-59.9	
3	2.00	1.00	990.	-59.9	
4	2.00	1.00	990.	-59.9	
5	2.00	1.00	991.	-59.9	
6	2.00	1.00	992.	-59.9	
7	2.00	1.00	991.	-59.9	
8	2.00	1.00	992.	-59.9	
9	2.00	1.00	993.	-59.9	
AVERAGE S/N RATIOS					
	SETTING	C	d		
	1	-59.9	-59.9		
	2	-59.9	-59.9		
	3	-59.9	-59.9		
SECONDARY VALUES					
TRIAL		D (in)	K_s	τ_{actual} (psi)	
1		2.00	1.25	12700.	
2		2.00	1.25	12700.	
3		2.00	1.25	12700.	
4		2.00	1.25	12700.	
5		2.00	1.25	12700.	
6		2.00	1.25	12700.	
7		2.00	1.25	12700.	
8		2.00	1.25	12700.	
9		2.00	1.25	12700.	

have been run with a lower initial minimum for d, with the predicted result being that the design optimization procedure would have converged to an optimum value of d less than 1.0".

Before performing the second sub-optimization procedure, two additional sets of experimental set of iterations were performed as variations to the first set. Since the actual maximum shear stress in the spring was approximately 12.7 ksi in the first experimental set of iterations, it was decided to repeat the first sub-optimization procedure using an allowable maximum shear stress of 12 ksi (after adjusting for the factor of safety). The results of this iteration are shown in Table 6-2. As expected, the size of the spring increased so that the actual maximum shear stress in the material would be less. In turn, the cost also increased. The actual maximum shear stress in the wire was just under 800 psi. Then, it was decided to run the sub-optimization procedure using 800 psi as an allowable maximum shear stress (after adjusting for the factor of safety). As expected, these results were identical to the ones obtained in the second experimental set of iterations, since the second set had already satisfied the constraint on the actual maximum shear stress.

In sub-optimization procedure two, the S/N ratios once again converged to negative values. The convergence took 16 iterations, with the final triplet being (C,d,N) = (5.80, 5.39", 1.95). The results of this third experimental set of iterations are summarized in Table 6-3. Note that the total length of the spring with the load attached is 16', satisfying the deflection constraint, while the actual shear stress in the wire (1.1 ksi) is well below the allowable limit of 26 ksi.

Table 6-2 - Results of the second (and third) experimental set of iterations of the fourth design optimization procedure (target: minimize cost).

PRIMARY VALUES					
TRIAL	C	d (in)	Total Cost ($)	MSD	
1	2.00	4.00	63200.	-96.0	
2	2.00	4.00	63200.	-96.0	
3	2.00	4.00	63200.	-96.0	
4	2.00	4.00	63200.	-96.0	
5	2.00	4.00	63200.	-96.0	
6	2.00	4.00	63200.	-96.0	
7	2.00	4.00	63300.	-96.0	
8	2.00	4.00	63300.	-96.0	
9	2.00	4.00	63300.	-96.0	
AVERAGE S/N RATIOS					
	SETTING	C	d		
	1	-96.0	-96.0		
	2	-96.0	-96.0		
	3	-96.0	-96.0		
SECONDARY VALUES					
TRIAL		D (in)	K_s	τ_{actual} (psi)	
1		8.00	1.25	796.	
2		8.00	1.25	796.	
3		8.00	1.25	796.	
4		8.00	1.25	796.	
5		8.00	1.25	796.	
6		8.00	1.25	796.	
7		8.00	1.25	796.	
8		8.00	1.25	796.	
9		8.00	1.25	796.	

Table 6-3 - Results of the fourth experimental set of iterations of the fourth design optimization procedure (target: minimize cost).

PRIMARY VALUES					
TRIAL	C	d (in)	N	Total Cost ($)	MSD
1	5.80	5.39	1.95	43600.	-92.8
2	5.80	5.39	1.95	43600.	-92.8
3	5.80	5.39	1.95	43600.	-92.8
4	5.80	5.39	1.95	43600.	-92.8
5	5.80	5.39	1.95	43600.	-92.8
6	5.80	5.39	1.95	43600.	-92.8
7	5.80	5.39	1.95	43600.	-92.8
8	5.80	5.39	1.95	43600.	-92.8
9	5.80	5.39	1.95	43600.	-92.8
AVERAGE S/N RATIOS					
	SETTING	C	d	N	
	1	-92.8	-92.8	-92.8	
	2	-92.8	-92.8	-92.8	
	3	-92.8	-92.8	-92.8	
SECONDARY VALUES					
TRIAL	D (in)	K_s	τ_{actual} (psi)	Total Length (feet)	
1	31.2	1.09	1100.	16.0	
2	31.2	1.09	1100.	16.0	
3	31.2	1.09	1100.	16.0	
4	31.2	1.09	1100.	16.0	
5	31.2	1.09	1100.	16.0	
6	31.2	1.09	1100.	16.0	
7	31.2	1.09	1100.	16.0	
8	31.2	1.09	1100.	16.0	
9	31.2	1.09	1100.	16.0	

Section 6.4 - Discussion of and Results

The spring design optimization procedure enabled several facets of Taguchi's method of parameter design as well as optimum design in general to be examined. One such item was constrained optimization problems. In both sub-optimization procedures, for example, one of the target output responses was an actual maximum shear stress of 26 ksi - but no more than 26 ksi. Thus, the output response was constrained. In addition, the target output response was cost, which was to be minimized, but with the proviso that the material be able to support the weight within the initial ranges of C and d. Thus, constrained optimization techniques were mixed with the concept of multiple response systems, which was the second major facet of Taguchi's method of parameter design to be examined. In this example, there were three target output responses: deflection, maximum shear stress, and cost. All of these target output responses were constrained: the deflection had to be such that the total length of the spring lay in the interval [16',20'); the maximum shear stress had to be 26 ksi, but not greater; and the cost was to be a minimum.

In all of these cases, the constraints were taken into account by manipulating computer code. This approach may not have been as effective nor appropriate as the techniques given by Arora [55], but they seemed to have been successful. The "manipulation of computer code" was, in essence the assignment to any trial that produced output responses outside the desired range to be given an artificially low S/N ratio, somewhat the opposite of the approach taken in design optimization procedure 3 ("The Gear/Pinion System"), where values that were inside the desired range were given artificially high S/N ratios. This difference arose due to the fact that in the latter case, any value in the range was desirable, where as in the case of the spring, merely being inside the desired range was not enough; there were other conditions to satisfy.

The lone concern from this entire design optimization procedure concerned the S/N values, since they were very, very low. The explanation for these low S/N values can be explained simply by observing that although the total material cost was a minimum for the range of control factors considered, it was still significantly greater than the target value of $0. One way to avoid the negative signs may be to simply take the absolute value of the S/N ratio and consider the S/N measure itself as one to be minimized, rather than one to be maximized. Another way to avoid the negative signs is to set the target cost at a non-zero value which could represent the amount of funding budgeted for the purchase of the spring material.

Nevertheless, the conclusion of Design Optimization Procedure Example Four, the spring, is that Taguchi's method of parameter design was applicable to the entire system. In addition, it was applicable to multiple-response and constrained optimization conditions by the use of simple but appropriate modification of computer code.

CHAPTER 7 - DESIGN OPTIMIZATION PROCEDURE EXAMPLE FIVE: THE ELECTRO-HYDRAULIC SERVOSYSTEM

Section 7.1 - Background

The fifth system to be used as an example of design optimization is the electro-hydraulic servosystem. In this design optimization procedure, the desired response is a minimum response time while satisfying a geometrical constraint; the control factors are the attenuator settings K1 and K2; and the levels of the control factors are low, medium, and high. The experimental sets of iterations were performed using closed-form equations developed by from control systems theory and described later in this chapter. Assumptions that were made in this design optimization procedure are also described later in this chapter.

An electro-hydraulic servosystem is simply a servomechanism whose basic medium of power transmission is liquid pressure. A *servomechanism* is a power controlling device that uses the measurement of the difference between the actual output signal of a device and the desired output signal of the device to control the input signal, with the end result being the reduction of this difference in the output signal. The electro-hydraulic servosystem uses electrical signals to control the hydraulics, which in turn use liquid pressure to transmit the power to perform the actual mechanical act (in this case, rotary and linear motion). In summary, the electro-hydraulic servosystem is a control-system device that controls its own performance by measuring the error in its own performance, then sending electrical signals to a hydraulic mechanism to correct this performance. Examples of the usage of control systems in general are considered in Section 7.1.2.

The practical application of this design optimization procedure was to find the optimum attenuator values for a particular servomechanism that would minimize the rise time while keeping the overshoot less than 15%.

Section 7.1.1 - Description of the System

There are many different types of control systems, but the two principal types are closed-loop and open-loop systems. In *open-loop control systems*, there is no feedback of the output signal back to the control unit in order to modify the input signal. An example of such a system is a dishwasher, where the various cycles occur without regard to the output (the cleanliness of the dishes). In a *closed-loop control system*, such as the electro-hydraulic servosystem that was used in this design optimization procedure, the output signal is sent back to the control unit, where it is compared to the desired value. The difference between the actual and the desired value (often called the error signal) is used to adjust the input signal.

Section 7.1.2 - Practical Applications

The engineering concept of control plays a vital role in engineering systems. The first control system is generally considered to be the flyball centrifugal governor introduced by Watt in the seventeenth century. In this century, several key contributors were Minorsky, Nyquist, and Hazen. Minorsky applied the concept to the steering of ships and determined stability criterion; Nyquist proposed a method for evaluating the stability of closed-loop systems; and Hazen introduced the term servomechanism while developing relay servomechanism that could follow a changing input [63].

Today, control theory is used in spacecraft systems, robotics, fluid systems, plant design, missile-guidance technology, and autopiloting systems in airplanes. In particular, the unit used in this design optimization procedure, the Electro-Hydraulic Servomechanism Type EHS160, provides close-loop control of the linear and rotary motion of an attached shaft. It uses hydraulic power control, "the most common form of precise power manipulation used in modern technology." [64]

Section 7.1.3 - Theoretical Equations

The theoretical equations used in this design optimization procedure were developed based on control theory, and as such, require a development beyond the scope of this paper. The reader is advised to consult Okata [63] for an in-depth treatment of control theory. A somewhat abridged condensed review of the process used to obtain the proper theoretical equation in given in Appendix C. It is summarized in bullet form as follows:

1. The system was represented in block diagram form;
2. The block diagram was reduced to its simplest form, thus giving an expression for the transfer function g(s);
3. Preparatory experiments were conducted to find the values of the various parameters in the expression for g(s);
4. The expression for the output response per unit of input response was determined in the form $C(s)=R(s)*g(s)=1/s*g(s)$;
5. The inverse Laplace transform of C(s) was taken to obtain the expression for the output response as a function of time (c(t)).

This tedious, somewhat inaccurate procedure (see Appendix C for details on inaccuracies) eventually yielded the following expression for c(t):

$$c(t) = \frac{L}{K} - \frac{L*J}{K}*e^{-\alpha*t}*\sin(\omega*t) - \frac{L*\beta}{K*\omega}*e^{-\alpha*t}*\sin(\omega*t-\phi)\,, \qquad (7\text{-}1)$$

where all of the parameters except t (time) are parameters for the various system components that have been collected to facilitate the taking of the inverse Laplace transform (see Appendix C for complete details on these parameters).

Embedded in the many parameters of Equation 7-1 are the parameters K1 and K2. These values represent the values of the two system attenuators, which will be described in the proceeding section. These values are important, since they were identified as the control factors.

Finally, three other terms need to be defined. The *steady-state response* of the system is the value of the response to which the system will converge after long time. In mathematical terms, the steady state response "$c(\infty)$" is the limit of c(t) as $t \rightarrow \infty$. The *rise time* (t_r), an quantitative measure of the system's quickness in responding to the input signal, is defined as the time needed for the output response to change from its initial value to the time that it first passes through the steady-state value. Finally, the *overshoot* is a measure of the stability of the system at the given attenuator settings. Overshoot is found by the expression

$$\delta = \frac{c(\delta)-c(\infty)}{c(\infty)} * 100, \qquad (7\text{-}2)$$

where δ is given as a percent of the steady-state response. The value of $c(\delta)$ is found by setting the derivative of Equation 7-1 equal to zero and solving for "t", which is t_δ, then substituting t_δ in Equation 7-1.

Section 7.2 - Experimental Setup

Section 7.2.1 - Apparatus

The actual apparatus (the EHS160) is shown in Figure 7-1. Additional information on the EHS160 can be found in the EHS Technical Information Manual

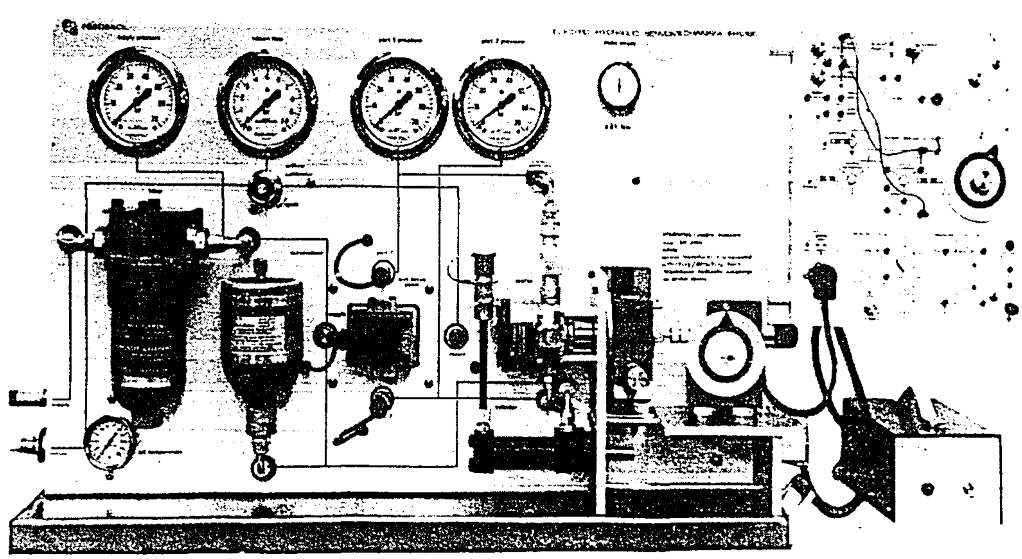

Figure 7-1 - The Electro-Hydraulic Servomechanism EHS160.

[63] and in Appendix C. Other apparatuses were used for the preliminary experiments used to determine constant system parameters. These apparatuses included a Fluke 21 Multimeter, a Health 2718 Tri-Power Supply, a Strobe Tachometer, a Tektronix 2200 portable plotter, and connecting wires. The role of each apparatus is described in detail in Appendix C. It should be noted that the actual output signal of the system is the angular position, measured in radians, of a rotary shaft.

Section 7.2.2 - Goals and Targets

The goal of the electro-hydraulic servosystem design optimization procedure was to use Taguchi's method of parameter design to select the optimum values of the control factors "K1" and "K2" of a the servosystem such that:

1. The rise time was a minimum;
2. The overshoot was less than 15%;
3. The process was robust.

These goals established dual target output responses, one of which was a range: rise time was to be as small as possible, while overshoot was to be less than 15%. In essence the problem was a constrained optimization problem. Since the range on the attenuator dials was 0 to 1, the initial range on both K1 and K2 was 0 to 1, inclusive.

As outlined in Section 7.1.3 above, several preliminary experiments were performed in order to determine the system parameters that were to remain constant. This process, which took longer to complete than the rest of the entire design optimization procedure by far, is detailed in Appendix C.

Once the transfer function g(s) was obtained and the output response c(t) was determined, expressions for $c(\delta)$, $c(\infty)$, and t_r were obtained by using the computer program MAPLE [65]. These expressions were implemented directly into the computer code, along with the additional expression for δ (Equation 7-2).

An expression for the S/N ratio was then constructed based on the MSD concept as

$$S/N = -10 * \log_{10} (t_r)^2 . \tag{7-3}$$

The condition that the overshoot be less than 15% was implemented by the use of the computer code modification described in Section 6.3. In this design optimization procedure, the modification was that the S/N trial for any $\delta \geq 15$ was set to an artificially low S/N value of -99.9982.

Section 7.2.3 - Other Assumption, Modifications

The principle assumption used in this design optimization procedure was the assumption that the system behaved as a first-order system. This assumption greatly simplified the mathematics involved in the procedure, but introduced some inaccuracies. A secondary assumption was that the values for the various systems parameters obtained in the preliminary experiments remained constant throughout the design optimization procedure and did not vary significantly, a generous assumption since numerous sources of "drift" of these values was present in the apparatus (such as slightly fluctuating hydraulic fluid pressure). Finally, a significant assumption that was used was the assumption that these parameter values were accurate in the first place (see Appendix C for additional details on inaccuracies in these determinations).

Section 7.3 - Details and Results of the Experiment

When the experiment was initially performed, it was mistakenly thought that the initial range on K1 and K2 could have a maximum value as high as 100. As a result, the initial ranges were [1,100]. When all of the average S/N ratios were at least 115 after only one iteration, it was concluded that the optimum values of K1 and K2 lay above the maximum value of 100. Thus, the experiment was repeated with an initial range of [1,49]. Again, all of the average S/N ratios were at least 115 after only one iteration. The experiment was repeated a third time, with an initial range of [1,101]. This time, all of the average S/N ratios were at least 123. In addition, values of t_r were steadily increasing with increasing K1 and K2 to the point that the pair (K1,K2) = (101, 101) produced a rise time of 79 nsec.

At that point, the correct possible range of attenuator settings was determined to be [0,1]. Nevertheless, the first three unofficial experimental sets of iterations showed rather conclusively that the optimum settings for K1 and K2 such that the output factor criteria were met were simply the largest ones possible. This observation was confirmed by qualified personnel in the controls field. After nine iterations using the significant figures rule (see Section 3.3), the only official experimental set of iterations for this design optimization procedure converged to the expected pair (K1,K2) = (1,1), the maximum values of both attenuator settings. The results of this experimental iteration are given in Table 7-1. Note that t_r for the optimum pair is 6.8 μsec, while δ was .18 %.

Since The Electro-Hydraulic Servosystem was the last design optimization procedure to be studied as part of this particular research, it was decided to run a physical confirmation experiment. The system was set up with seven different (K1,K2) pairs. These pairs were chosen as (1.0,1.0), (1.0,0.75), (0.75,1.0), (0.75,0.75), (1.0,0.55), (0.55,1.0), and (0.55,0.55). Plots of the output response versus time were made for several of these pairs of attenuator values and are given in Appendix C.

Section 7.4 - Discussion of Results

Since personnel in the controls laboratories commented that, from experience with the equipment, it could have been predicted that the highest attenuator values would satisfy the output response criteria, the entire fifth design optimization procedure was in essence a verification of the applicability of Taguchi's method of parameter design. The fact that the physical verification experiment confirmed the results (ref. conversation with Dr.Stanley Johnson) that the optimum values (K1,K2) = (1.0, 1.0) did give the smallest value of t_r provided an even more solid means of verification. In this physical application, the use of Taguchi's method of parameter design may not have been necessary - but it was still applicable. And the fact that many sources of inaccuracies were present did not prevent the method from converging to the expected and reasonable values.

Table 7-1 - Results of the first experimental set of iterations of the fifth design optimization procedure (target: minimum rise time).

PRIMARY VALUES					
TRIAL	KA1	KA2	t_r (nsec)	MSD	
1	.996	.996	6.8	103.	
2	.996	.998	6.8	103.	
3	.996	1.00	6.8	103.	
4	.998	.996	6.8	103.	
5	.998	.998	6.8	103.	
6	.998	1.00	6.8	103.	
7	1.00	.996	6.8	103.	
8	1.00	.998	6.8	103.	
9	1.00	1.00	6.8	103.	
AVERAGE S/N RATIOS					
	SETTING	KA1	KA2		
	1	103.	103.		
	2	103.	103.		
	3	103.	103.		
SECONDARY VALUES					
TRIAL		δ	c (∞) (rad)	c (max) (rad)	
1		.0018	.24	.24	
2		.0018	.24	.24	
3		.0018	.24	.24	
4		.0018	.24	.24	
5		.0018	.24	.24	
6		.0018	.24	.24	
7		.0018	.24	.24	
8		.0018	.24	.24	
9		.0018	.24	.24	

The conclusion of Design Optimization Procedure Example Five, the electro-hydraulic servosystem, is that Taguchi's method of parameter design was applicable to the entire system, although its use was probably not necessary. It was applicable to the multiple-response and constrained optimization conditions by the use of simple but appropriate modification of computer code (as in design optimization procedure four). It produced values for the control factors that satisfied the output response(s) and constraints on the (those) response(s) through the theoretical equations and were robust.

CHAPTER 8 - DISCUSSION OF OVERALL RESULTS

Section 8.1 - General Topics

The actual application of Taguchi's parameter design to the five mechanical systems was a rather straightforward task. The nine-step process outlined in Sections 1.4 and 3.2.1 provided an excellent road map for the procedure. The desired output target was established, control factors and their levels were determined, and the theoretical equations relating these control factors to the output response were obtained and coded. Although it was straightforward in this research, problems can be encountered with this procedure if no theoretical equations exist that relate the control factors to the output response for a given mechanical system. In such a case, additional experiments may be needed to determine empirically an approximate functional relationship between the control factors and the output response.

Problems may also be encountered in the selection of the system parameters to be appropriate control factors. In several of the design optimization procedures, the initial selection of control factors was incorrect. In these cases, false starts featuring unrealistic results were encountered which necessitated changing the control factors. No better example existed than in the first design optimization procedure, the dynamic absorber, where the control factors had to be changed from three system parameters to three ratios of system parameters. A reason for such a change seems to be that the original control factors were not independent variables in the equation relating the system response as a function of the system parameters. In the first design procedure, for example, the independent variables in Equation 3-9 are the ratios n, δ_{2R}, and μ (since Ω is fixed). The original choice of control factors -M_2, k_2, and C - are not present in Equation 3-9. Thus, the results of the research seem to suggest that, in order to be control factors, a system parameter must be an independent variable in the equation relating system response to the system parameters.

The process of successive iterations was used to converge to the optimum levels of the control factors. It is highly recommended that these iterations be performed by the process of computer simulation, i.e., having the computer perform the iterations. These iterations can be performed manually, but this procedure is more tedious and prone to errors. However, some observations were made in the initial stages of the research, when manual iterations were still being performed. When a particular control factor, for example, has one level with an S/N ratio much higher than the other levels, then it was possible to "collapse" the range of levels around that level. That is, instead of merely reducing the range in half, the range could have been reduced by a factor of four. A much higher S/N ratio for a particular level of a control factor implies that the actual robust value of that particular control factor lies in the near vicinity of the value of that level. In addition, the continual appearance, after each iteration, of the highest S/N ratio at a control factor level that was one of the original range endpoints implies that the actual optimum level of the control factor lies outside the original range. In this case, it may be necessary to rerun the iterations with an expanded initial range on that particular control factor. This expanded range, however, may not be physically possible or feasible. In that case, the original endpoint value will have to suffice as the optimum level.

An interesting situation occurred when the levels L_n of a control factor "a" such that $L_1 < L_2 < L_3$ had corresponding average S/N ratios such that $S/N_{a1} > S/N_{a3} > S/N_{a2}$ - that is, the smallest average S/N corresponded to the control factor level in the middle of the range. This situation was particularly confusing when S/N_{a1} and S/N_{a3} were equal or close to being equal. If by that point in the iterations L_1, L_2, and L_3 were approximately equal, then there was no concern. However, if these levels were significantly different, then the phenomena seemed puzzling. It implies that the S/N function for a range of control factor levels is not necessarily monotonically increasing or decreasing, but rather is subject to several local extrema, each of which may be local optimum points as well. This topic is an excellent one for further research.

It was also interesting to note the effect of choosing the wrong subsequent range in which to iterate. Suppose, for example, the control factor levels above had average S/N ratios such that $S/N_{a1} > S/N_{a2} > S/N_{a3}$, but the next range of L_n was incorrectly chosen as $[L_2, L_2+\frac{1}{2}(L_3-L_2), L_3]$ instead of $[L_1, L_1+\frac{1}{2}(L_2-L_1), L_2]$. The result was, as might be predicted, that the highest S/N ratios in subsequent iterations would be S/N_{a1}. Based on the observation above, one should then realize that the optimum level lies below L_2, and the range would be opened up such that the values between

L_1 and L_2 when the incorrect range determination was made would then be evaluated.

It was also interesting to note that some particular trials had the exact desired value of the output response before the actual convergence was achieved. The question of whether the particular trio that produced this value is any less robust than the value of the trio at convergence is then raised. Consider, however, that it was observed that even back at this iteration, where the procedure was not yet considered to have converged, the ranges on the control factors and other parameters were somewhat small. Thus, it is quite probable that the iterations had already converged to the degree of engineering accuracy needed.

This issue raises another question, namely, the determination of when convergence is achieved. Usually, the average S/N ratios continued to increase without bound, and the iterations were stopped after these average S/N ratios reached a certain level (at least one average S/N ratio for each control factor had to be at least 60). However, in one experimental set of iterations, the average S/N ratios converged to a limiting value of approximately 50. On another occasion, the average S/N ratios had converged to artificially high values but the ranges of the control factor levels had not converged (design optimization procedure three). This situation occurred, however, because there was a range of optimum control factor levels that satisfied the design criteria. And, of course, there was the situation where the average S/N ratios converged to negative values (design optimization procedure four).

The question of knowing when the iterations have converged, then, is actually a question of how much accuracy is needed in determining the values of system parameters. Beer and Johnston comment that "an accuracy of greater than 0.2% is seldom necessary or meaningful in the solution of practical engineering problems." [66] Thus, convergence of the iterations to the point that system parameters differ only in the fourth decimal spot is unnecessary. Consider the dynamic absorber. If M_1 is known with an accuracy of .2% or less, then the values of such system parameters as k_2 and M_2 cannot have an accuracy greater than .2%. In Section 3.3, it was reported that, at convergence, the accuracy of the value of k_2 was .0012%. Obviously, this value is unnecessarily accurate, and the iterations could have been stopped earlier. Thus, it developed as a general rule in the iteration procedures performed in this research that the iterations should not be stopped based on average S/N ratio values, but rather when the range on the individual control factor levels reaches the appropriate number of significant figures for that value based either on the tolerance that can be held on the parameter or on the rules of significant figures in the operations of addition and multiplication. If the length of the muffler in design optimization procedure two, for example, can only be determined with a tolerance of ± .04", then the iterations could be stopped when the range on the length is [35.20",35.20",35.20"], but not at [35.20",35.20",35.21"]. If the various control factors have different numbers of significant figures, then the iterations should not be stopped until all of the levels under each control factor are the same to the number of significant figures for that control factor. For example, consider a system with two control factors where the tolerance on control factor 1 is 1.0 and the tolerance on control factor 2 is .5. After a certain iteration, the range on control factor 1 might be [25.5,26.0,26.5], which would be [26.,26.,26.] using the correct number of significant figures; meanwhile, the range on control factor 2 might be, [16.0,16.5,17.0]. The three levels of control factor 1 are already the same to the correct number of significant figures, but the same cannot be said regarding control factor 2. Thus, the iteration procedure must continue, although it is a foregone conclusion that the optimum value of control factor 1 is already know as 26.

Care should also be taken when performing calculations to determine auxiliary parameters based on these control factors. The correct number of significant figures should be observed (see Taylor for a summary of significant figures [67]). For example, in design optimization procedure one, the control factor μ is not a physical system parameter, but a ratio of such parameters, namely, the two masses. Thus, the number of significant figures used in representing the masses should be used to calculate the correct number of significant figures to be used in μ consistent with the rules pertaining to the use of significant figures. If the resulting range of an auxiliary factor is too wide (although these ranges in this research were rarely non-zero using the correct number of significant figures), then perhaps Taguchi's method of tolerance design should be employed. As noted in Section 2.3.3, however, this method results in increased costs.

As a final note on the topic of when to conclude that the iterations have converged, it should be remembered that the concept of robust design and optimum values allows for wider tolerances on the control factors and other system parameters, since the robustness of the design means that the output response is not sensitive to such unit-to-unit noise factors as wide tolerances. Thus, the iterations do not have to be run to an overly high degree of accuracy.

Finally, one last general topic that was not fully researched was the mathematical robustness of the optimum values obtained. Mathematical robustness involves minimizing the sensitivity coefficients, which in turn involves calculating partial derivatives. This tedious procedure was not performed for the design optimization procedures due to the complexity of the theoretical equations. The more practical approach to examine the robustness of control factors is to examine a plot of the output response versus that particular control factor. If the optimum value lies at a position where the output response does not vary greatly when the control factor level does, then the value is robust. However, this test can only be used if the output response varies non-linearly with that particular control factor and linearly with the other control factors. It is not known if this procedure works if the output response is a linear function of all of the control factors, or if the output response is a non-linear function of more than one control factor. Fortunately, this procedure could be performed in one of the design optimization procedures considered in this research, and in this case, the robustness of the values obtained was verified. Nevertheless, it must be remembered that this verification was a graphical one, not a mathematical one.

Section 8.2 - Specific Topics

Several interesting observations were made in terms of specific results. From a practical point of view, it was interesting to note that the optimum muffler for the author's car such that a 5 dB reduction in noise when the turbocharger is used had an enlarged section that was approximately 20 inches long and 7 inches in diameter (design optimization procedure two, iteration set one). The pinion gear in this case (design optimization procedure three, iteration set two) had a face width of 3.5" and was to be made of a steel with a yield stress of 117 ksi.

Several of the systems were constructed with constraints (design optimization procedure four), while others had multiple objectives, one of which may have been, in turn, a constraint. Unlike the approach used by Richie [45] involving weighting functions, these conditions were implemented through the computer program, where trials that did not meet a certain constraint condition were arbitrarily set to an artificially low S/N value. If the target output response was itself a range, then any trial whose output response fell in that desired range was given an artificially high S/N ratio. These techniques provide shortcuts to the multiple objective and constrained optimization situations.

The failure of design optimization procedure one to converge to optimum tuning and optimum damping control factor values is perplexing. This observation is especially true when considering that in one experimental set of iterations, the value of μ was fixed, and the theoretical value of the output response was determined and used as the target output response, yet the iterations not only converged to the improper values of the other control factors, but also did not converge to the proper target output response. Furthermore, the values of the optimum control factors did not satisfy the equations of optimum tuning and damping (Equations 3-10 and 3-11, respectively). It is given as a reason for this lack of proper convergence that the control factors contained a significant interaction, since they are both functions of μ and hence can be determined in terms of the other.

It was also of interest to examine the effect of adding a control factor on the optimum values of the previously existing control factors. For example, consider design optimization procedure two (the acoustic muffler). The parameters m and l_e were chosen as control factors and their optimum values were (m,l_e) = (15.94,8.29"). However, when the frequency f was introduced as a third control factors, the optimum values for m and l_e became (m,l_e) = (8.09,47.06"). This difference was the motivation for the change in the procedure of selecting the new interval in which to iterate (see Section 3.3). However, the differences were still noted: with only two control factors, (m,l_e)=(12.0,19.99"), while with three control factors, $(m_o,l_{e,o})$=(34.8,13.89").

Nevertheless, for each design optimization procedure, the iterations did produce optimum values for the control factors that satisfied the target output response and other engineering constraints (although mathematical optimality was not shown). And in each case, the Taguchi method of parameter design was applied to a mechanical system, thus highlighting the practical application of the method. In this sense, the research was successful.

CHAPTER 9 - SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Section 9.1 - Review of this Thesis

With the Quality Revolution in the United States in the last decade, the goal has been to improve the quality of products manufactured in the United States. Traditional methods of quality control have focused on inspecting quality into a process or product. Better methods of quality control exist that focus on improving the manufacturing process particularly by removing the sources of variation of a product or process. But an even better method of quality improvement is to focus on the design stage of the product life cycle and design the product to be robust, or less sensitive to the sources of variation without actually eliminating the sources of the variation. Genichi Taguchi has developed a method for accomplishing robust designs called parameter design, in which the nominal values of certain design parameters of the product or system are set so as to make the process robust, thus enabling wider tolerances and lower grade materials to be used. The overall result is quality improvement without a subsequent increase in cost.

Taguchi's method of parameter design has been applied to five mechanical systems - the dynamic absorber, the acoustic muffler, the gear and pinion system, the spring, and the electro-hydraulic servosystem. In each application, called an design optimization procedure, a target (desired) value of the output of the system was determined, and the number and levels of control factors were obtained. Using the appropriate orthogonal array to set up the experimental trials for each iteration, Taguchi's method of parameter design was used to obtain the values of the identified control factors such that this output target was met. For one of the systems, the robustness of the values obtained was shown graphically. An ANOVA was performed in certain cases to determine the effect of interactions.

Optimum values were determined through an iterative process in which the preceding range on the control factor was reduced in half around the control factor level that produced the highest S/N value. This iteration process continued until suitable convergence was achieved. Suitable convergence was established as convergence of the levels of the control factors such that all levels of each control factor were the same using the correct number of significant figures (since additional accuracy is not necessary). The iteration procedure was performed by means of a computer simulation that included the theoretical equations to calculate the output response as a function of the control factors as well as the equations postulated by Taguchi in the use of his parameter design method. Several observations were made regarding iteration procedures and rules were established for proper subdividing of the control factor ranges for subsequent iterations.

Systems were studied that required certain constraints (a constrained optimization problem), while other systems featured multiple output targets where one of the output target was a range of values or a constraint. Both cases were dealt with by using appropriate modification of the signal-to-noise ratio in the computer code. The optimum values obtained through the analysis successfully satisfied the target output responses and the constraints.

Tolerance design (another method of quality improvement proposed by Taguchi) and a mathematical analysis of the robustness of the values obtained was beyond the scope of this research and was not studied, although the robustness for a set of optimum control factors was shown graphically for one design optimization procedure.

Section 9.2 - Conclusion of the Research

It is the conclusion of this research that Taguchi's method of parameter design can be effectively and rather easily applied to mechanical systems with known equations that predict the desired output response as a function of the control factors. Taguchi's method of parameter design will converge in an iterative procedure to the values of control factors that will satisfy the target output, including multiple output responses and constrained responses, such that the system will be robust - insensitive to sources of variation. This procedure can be very easily achieved through the use of unsophisticated computer code. The control factors must be the independent variables in the equation that predicts the desired output response as a function of the system parameters to avoid meaningless results.

Section 9.3 - Recommendations for Further Research

Several topics were not addressed as part of this research and as such they represent excellent topics for further research. The mathematical robustness of mechanical systems was not addressed. This topic is important so as to provide a means of proving that the optimum values obtained are indeed robust. This process involves the study of sensitivity coefficients, each of which are functions of the partial derivative of the output response with respect to each control factor.

The application of Taguchi's method of parameter design to mechanical systems with no theoretical equations to predict the system output response was not studied. This topic constitutes an important area of research, since most developing technology involves systems for which theoretical equations are still being developed. In this case, empirical data would have to be used to approximate a functional relationship between the output response and the control factors.

In this research, a method to deal with multiple output response requirements where one of the requirements is a constrained response was studied. However, a systematic approach to multiple output criteria where multiple output responses were required at different conditions was not studied. The application of such topics to mechanical systems thus remains an excellent area for additional research. A multivariate loss function was proposed by Chen [39] and by Ritchie [45]; however, as noted in Section 2.3.2, examples of applications of the method to mechanical systems were not given. Ritchie also presents the concept of control surfaces for multivalued functions. These surfaces provide a three-dimensional view of the behavior of the output response as a function of the control factors. The application of this topic to mechanical systems also was not studied in this research and as such remains an excellent area for additional research. The papers of Chen and Ritchie would serve as an excellent starting place for research on these topics.

Finally, Taguchi's method of parameter design for static mechanical systems (systems with constant signal factors) was studied. However, the study of the method to dynamic mechanical systems was not studied. Dynamic systems are those where the signal factors do not remain constant. Since these mechanical systems have important applications, it is important to study the application of Taguchi's method of parameter design to these mechanical systems in order to provide a means of applying this important method of long-term, quality improvement that was shown to be applicable to static mechanical systems in this research.

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APPENDICES

APPENDIX A - COMMENTS ON THE COMPUTER CODE USED IN THIS RESEARCH

This appendix contains notes, comments, and details on the computer programs that were used in this research. These programs were important tools in the research, since the execution of the computer code provided the backbone of the research effort.

Although a separate computer program was used for each design optimization procedure, the structure of the programs was the same throughout the research. A typical program was divided into five parts:

1. **INTRODUCTION SECTION** - program name, included explanatory comments (such as purpose of the program), and declaration of variable types.
2. **SETUP SECTION** - sets the initial values of several system constants and variables; also sets the initial ranges the control factors.
3. **ITERATION SECTION** - computes the value of the output response and other system values based on the values of the setup section; also determines the MSD and the average S/N ratios for the control factors and stores them in subscripted variables.
4. **NEXT-ITERATION SECTION** - uses the stored S/N values and stored control factor levels to determine the subsequent range of the control factors; the iteration rules proposed in Section 3.3 are implemented in this section.

A detailed description of each section now follows.

The introduction section is somewhat self-explanatory. The first line of the program lists the program name. The next group of lines consist of comments describing the purpose of the program, such as the mechanical system in that particular design optimization procedure, the number of control factors, and the desired output response. The section concludes with the declaration of variable type (real, integer, etc.) for all of the variables used in the program. Variable type for all variables are declared in these lines (even though HP-FORTRAN [68] recognizes implicit variable declaration) for organizational purposes.

The setup section of the program is an important section. As mentioned, the initial values of certain system constants are set here. One such constant is the target output response. For example, in the first official experimental set of iterations of the first design optimization procedure, the target output response was a transmission loss of -6 dB. In the computer code, the line

```
TARGET = -6.0
```

established the target output response as -6 dB. In subsequent sets of iterations where the target output response was different, it was necessary to change this line only before re-running the program.

Calculations were also performed in this section. For example, in all of the programs in the second design optimization procedure, the speed of sound was needed. According to Equation 4-4, this value was a function of several other values (such as the universal gas constant R_u and the molecular mass of carbon monoxide). Thus, these other values were set, and then the speed of sound was calculated. The convenience was that when a more accurate value of one of the other values was obtained, only one line would have to be changed; all of the other values would be updated immediately. It should be noted that calculations were only performed in this section to determine values that were to remain constant throughout the design optimization procedure.

Many system values were set in this section. These include, but were not limited to, mass M_1 and spring k_1 (first DOP); gamma, temperature of exhaust gases, molecular mass of carbon monoxide, and the universal gas constant (second DOP); and the value of pi (several DOP).

The iteration section was the principal section of the computer program. In this section, secondary values that changed as the control factor levels changed were calculated. For example, in the first design optimization program, the value of M_2 changed as the mass ratio (a control factor) changed; thus, M_2 was calculated in this section. The most important values calculated in this section were the output response, the MSD, and the average S/N ratios. In most cases, the calculation of these values was a case of merely copying the appropriate equation from this document into the HP FORTRAN language [68]. However, in several of the constrained optimization problems, an if-loop was needed to assign an arbitrarily low value to the MSD value for a value of the output response that was outside the preferred range or which did not satisfy the constraint. For example, in the fourth design optimization procedure, one of the output responses had the constraint that the actual maximum shear stress in the spring had to be less than 26 ksi. If this condition was met, then the MSD was calculated based on the deviation of the actual material cost from the target cost (the second output response). These conditions were coded as follows:

```
IF (SA.GT.26000.) THEN
     MSD=LOWLIM
ELSE
     MSD=-10*LOG(TCOST-TARGET)**2.0/LOG(10.0)
ENDIF
```

Note that "LOWLIM" was an artificially low S/N value (-500.0) that was set in the setup section. This example of computer code is the so-called "manipulation of computer code" to which was alluded in earlier sections of this document for achieving constrained optimization results (c.f. Section 6.4).

This entire section was constructed in the frame work of an infinite or continuous loop that began with the incrementing of a counting variable that was appropriately called "ITERATION". Within this loop was a loop that went from 1 to LIM, where LIM was the number of trials in the orthogonal array to be used (and was set in the setup section). The infinite loop was broken by issuing a control-C command when convergence had been achieved. It is possible to program the convergence criteria, but it was felt that this procedure would be unnecessarily time-consuming for this research.

The next-iteration section was a rather long and involved section that simply determined the subsequent values of the control factor levels. This section was a rather long, laborious, and mechanical section that was more of a lesson on computer logic than a section of Taguchi-related information; readers interested in details on this section should contact the author for copies of the software, since it would be unnecessarily time consuming to discuss the details here.

The actual procedure of running these programs involved running the program and observing the results (slowing down the scrolling speed of the screen output if possible), then manually stopping the program when convergence was achieved. At that point, the value of the final iteration was noted. The program was then copied, it name altered through the addition of a "p", and code was added so that only the final iteration would print. For example, the first program for the second design optimization procedure was program 2a.f. This program was copied to program 2ap.f. Modifications were made to program 2ap.f so that only the last iteration was printed. These modifications consisted simply of setting an integer variable FINIT (for FINal Iteration) to the number of the last iteration, then surrounding the various WRITE statements with an IF-loop such that the WRITE statements would be executed only if ITERATION=FINIT. The output from 2ap.f, then, was obtained an reported as the official results.

Finally, it should be noted that these programs were entirely the creation of the author and were not copied or modified in any way from any other source. The files are located on the Lehigh University Department of Mechanical Engineering and Mechanics CAD/CAM Lab HP System (Dr. John B. Ochs, director, and Mr. Fred J. Wehden, manager), in the directory /user1/users/grad/graz/thesis. The catalogue of programs used in this research and their purposes in the research is given in Table A-1.

Table A-1 - Catalogue of computer programs used in this research.

PROGRAM	PURPOSE OF PROGRAM
1g.f	First DOP, first official ESOI.
1gp.f	Prints results, first DOP, first official ESOI.
1h.f	First DOP, second official ESOI.
1hp.f	Prints results, first DOP, second official ESOI.
1i.f	First DOP, third official ESOI.
1ip.f	Prints results, first DOP, third official ESOI.
1j.f	First DOP, fourth official ESOI.
1jp.f	Prints results, first DOP, fourth official ESOI.
1k.f	First DOP, fifth official ESOI.
1kp.f	Prints results, first DOP, fifth official ESOI.
2a.f	Second DOP, first official ESOI.
2ap.f	Prints results, second DOP, first official ESOI.
2b.f	Second DOP, second official ESOI.
2bp.f	Prints results, second DOP, second official ESOI.
2c.f	Second DOP, third official ESOI.
2cp.f	Prints results, second DOP, third official ESOI.
2d.f	Second DOP, fourth official ESOI.
2dp.f	Prints results, second DOP, fourth official ESOI.
3a.f	Third DOP, first official ESOI.
3ap.f	Prints results, third DOP, first official ESOI.
3b.f	Third DOP, second official ESOI.
3bp.d	Prints results, third DOP, second official ESOI.
3c.f	Third DOP, third official ESOI.
3cp.f	Prints results, third DOP, third official ESOI.
3d.f	Third DOP, fourth official ESOI.
3dp.f	Prints results, third DOP, fourth official ESOI.
4a.f	Fourth DOP, first official ESOI.
4ap.f	Prints results, first DOP, first official ESOI.

APPENDIX B - SOME DETAILED RESULTS FROM THE FIRST DESIGN OPTIMIZATION PROCEDURE

This appendix contains additional detailed results from the first design optimization procedure. As noted in Section 3.3, several false starts were encountered in the execution of the first design optimization procedure. Since these results were "false starts", their inclusion in the narrative in Chapter 3 was not justified. However, since valuable knowledge and experience with several aspects of Taguchi's methods were obtained, their mentioning in this document was justified.

In its earliest stages, several aspects of the research were different. First, the process of iterating until all of the control factor levels were the same to the correct number of significant figures was not done. One iteration of the orthogonal array was made, and the results were taken as final. The iteration process did not begin until several initial runs were made. These initial runs are described in this appendix. Second, in this particular design optimization procedure, the control factors were initially (M_2, k_2, C) instead of the trio (n, δ_{2R}, μ). And third, several different orthogonal array sizes were used in an effort to understand interrelationships among the control factors.

The first iteration was performed with a 3x3 array (3 control factors, each with 3 levels). However, in order to analyze all of the effects of the control factors and all of their interactions at the highest resolution (see Section 3.3 for a description of the term resolution), the iteration was rerun using an expanded 3x3 array. This array

- the L27 - was the smallest that could be used for the desired analysis. The initial ranges were 88.18 lbm $\leq M_2 \leq$ 4409. lbm, 729.6 lbf/ft $\leq k_2 \leq$ 1028. kip/ft, and 5.139 lbf·sec/ft $\leq C \leq$ 102.8 lbf·sec/ft. The results of this iteration are shown in Table B-1.

As mentioned, it was desired to perform an ANOVA on these results so as to study the effect of interactions; however, it was not known how to handle the fact that the various interactions had two degrees of freedom. Thus, the iteration was rerun using a simple, 3x2 array, the L8. In this case, the main effects of the control factors and the effects of the interactions could be easily studied at highest resolution using ANOVA. The results of this iteration are shown in Table B-2.

The results of the ANOVA performed on this iteration are also included in Table B-2. The ANOVA suggests that the first control factor, M_2, has the greatest effect on the output response T (88.76%), while the second control factor (k_2) and the interaction between M_2 and k_2 (denoted M_2xk_2) contribute somewhat. All of the other effects (C, M_2xC, k_2xC, M_2xk_2xC) are negligible. For details on the performance of an ANOVA, see Roy [9].

Finally, the iteration was repeated using a 3x4 array, the L16 orthogonal array. The results of this iteration are shown in Table B-3. No ANOVA was performed on these results, since the array was not of highest resolution. It should be noted that in all of the iterations presented in these first three tables, the target output response was a transmissibility of -6 dB, as it was is the first official experimental set of iterations.

Table B-1 - Results of the expanded 3x3 (L27 array) iteration, first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

PRIMARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	T (dB)	MSD
1	88.18	3.426	5.139	34.15	-32.07
2	88.18	3.426	68.52	34.23	-32.09
3	88.18	3.426	102.8	34.32	-32.11
4	88.18	3.426×10^5	5.139	42.18	-33.66
5	88.18	3.426×10^5	68.52	42.19	-33.66
6	88.18	3.426×10^5	102.8	42.20	-33.66
7	88.18	1.028×10^6	5.139	53.76	-35.53
8	88.18	1.028×10^6	68.52	53.76	-35.53
9	88.18	1.028×10^6	102.8	53.76	-35.53
10	2205.	3.426	5.139	9.542	-23.83
11	2205.	3.426	68.52	9.542	-23.83
12	2205.	3.426	102.8	9.543	-23.83
13	2205.	3.426×10^5	5.139	8.224	-23.06
14	2205.	3.426×10^5	68.52	8.225	-23.06
15	2205.	3.426×10^5	102.8	8.225	-23.06
16	2205.	1.028×10^6	5.139	4.776	-20.65
17	2205.	1.028×10^6	68.52	4.776	-20.65
18	2205.	1.028×10^6	102.8	4.777	-20.65
19	4409.	3.426	5.139	6.021	-21.60
20	4409.	3.426	68.52	6.021	-21.60
21	4409.	3.426	102.8	6.021	-21.60
22	4409.	3.426×10^5	5.139	4.387	-21.13
23	4409.	3.426×10^5	68.52	5.387	-21.13
24	4409.	3.426×10^5	102.8	5.387	-21.13
25	4409.	1.028×10^6	5.139	3.960	-19.97
26	4409.	1.028×10^6	68.52	3.960	-19.97
27	4409.	1.028×10^6	102.8	3.960	-19.97
AVERAGE S/N RATIOS					
	SETTING	M_2	k_2	C	
	1	-33.76	-25.84	-25.72	
	2	-22.51	-25.95	-25.72	
	3	-20.90	-25.38	-25.73	

Table B-2 - Results of the 3x2 iteration and ANOVA, first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

PRIMARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	T (dB)	MSD
1	88.18	3.426	5.139	34.15	-32.07
2	88.18	3.426	102.8	34.23	-32.11
3	88.18	$1.028x10^6$	5.139	53.76	-35.53
4	88.18	$1.028x10^6$	102.8	53.76	-35.53
5	4409.	3.426	5.139	6.021	-21.60
6	4409.	3.426	102.8	6.021	-21.60
7	4409.	$1.028x10^6$	5.139	3.960	-19.97
8	4409.	$1.028x10^6$	102.8	3.960	-19.97
AVERAGE S/N RATIOS					
	SETTING	M_2	k_2	C	
	1	-33.81	-26.84	-27.29	
	2	-20.78	-27.75	-27.30	
Analysis of Variance (ANOVA)					
Source	Sum of Squares	Degrees of Freedom	Variance	Effect (%)	
M_2	3043.14	1	3043.14	88.76	
k_2	152.50	1	152.50	4.45	
M_2xk_2	232.94	1	232.94	6.79	
error*	0.02	4	0.005	0.0001	
TOTAL	3428.6	7	3428.585	100.0	
* - C, M_2xC, k_2xC, M_2xk_2xC, and error pooled.					

Table B-3 - Results of the 3x4 iteration, first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

PRIMARY VALUES					
TRIAL	M_2 (lbm)	k_2 (lbf/ft)	C (lbf·sec/ft)	T (dB)	MSD
1	88.18	3.426	5.139	34.15	-32.07
2	88.18	3.426×10^5	34.26	42.18	-33.66
3	88.18	6.852×10^5	68.52	49.77	-34.93
4	88.18	1.028×10^6	102.8	53.76	-35.53
5	2205.	3.426	34.26	9.542	-23.83
6	2205.	3.426×10^5	5.139	8.224	-23.06
7	2205.	6.852×10^5	102.8	6.671	-22.06
8	2205.	1.028×10^6	68.52	4.776	-20.65
9	3307.	3.426	68.52	7.360	-22.52
10	3307.	3.426×10^5	102.8	6.504	-21.94
11	3307.	6.852×10^5	5.139	5.554	-21.25
12	3307.	1.028×10^6	34.26	4.487	-20.41
13	4409.	3.426	102.8	6.021	-21.60
14	4409.	3.426×10^5	68.52	5.387	-21.13
15	4409.	6.852×10^5	34.26	4.703	-20.59
16	4409.	1.028×10^6	5.139	3.960	-19.97
AVERAGE S/N RATIOS					
	SETTING	M_2	k_2	C	
	1	-34.05	-25.00	-24.09	
	2	-22.40	-24.98	-24.62	
	3	-21.53	-24.71	-24.81	
	4	-20.82	-24.14	-25.28	

As noted in Section 3.3, the initial presumption was that the execution of the trials would give values of the control factors such that Snowdon's conditions for optimum tuning and optimum damping (Equations 3-10 and 3-11) would be satisfied. These initial iterations did not satisfy the conditions, thus providing extra motivation to perform an ANOVA to study possible interactions.

The failure to satisfy Equations 3-10 and 3-11 also provided the motivation to study the relationship among the system parameters, and, as outlined in Section 3.3, it was eventually decided to change the control factors to (n,δ_{2R},μ). Having done so, an initial, 2x2 iteration was performed using an L4 orthogonal array, with μ being held constant. The results of this iteration, along with an accompanying ANOVA, are shown in Table B-4.

Then it was decided to let μ vary as well. Thus, the iteration was rerun as a 3x2 iteration using an L8 array, with the highest resolution available. The results of this iteration, along with an accompanying ANOVA, are shown in Table B-5.

The results of these ANOVA show that some interactions, even after the new control factors were established, were significant. When μ was treated as a constant, the $n\mathrm{x}\delta_{2R}$ interaction had almost as much of an effect on the output response as the factor n did. In the 3x2 case, the main effects of the three control factors as well as the $n\mathrm{x}\delta_{2R}$ interaction had an effect on the output response within 5% of each other. The fact that this interaction had so significant of an effect on the output response was initially thought to expected, at least in terms of the conditions of optimum tuning and damping. As was noted in Section 3.3, μ is a function of δ_{2R} and of

Table B-4 - Results of the 2x2 iteration and ANOVA, first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.5000	.0100	.8333	4.500	-25.60
2	.5000	.4330	.8333	5.196	-26.16
3	.9900	.0100	.8333	0.1684	-19.53
4	.9900	.4330	.8333	5.145	-26.12
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	μ	
	1	-25.88	-22.57	-25.86	
	2	-22.82	-26.14	-22.84	
Analysis of Variance (ANOVA)					
Source	Sum of Squares	Degrees of Freedom	Variance	Effect (%)	
N	9.6040	1	9.6040	27.55	
δ_{2R}	16.0898	1	16.0898	46.16	
$N x \delta_{2R}$	9.1648	1	9.1648	26.29	
error*	0.0000	4	0.0000	0.00	
TOTAL	34.8586	7	34.8586	100.0	
* - unpooled.					

Table B-5 - Results of the 3x2 iteration and ANOVA, first design optimization procedure (target: T = -6 dB at resonance, Ω = 1).

PRIMARY VALUES					
TRIAL	N	δ_{2R}	μ	T (dB)	MSD
1	.5000	.0100	.5000	3.523	-19.58
2	.5000	.0100	.9000	17.50	-27.42
3	.5000	.4330	.5000	4.771	-20.64
4	.5000	.4330	.9000	18.75	-27.87
5	.9900	.0100	.5000	-25.01	-25.58
6	.9900	.0100	.9000	-11.03	-14.04
7	.9900	.4330	.5000	4.686	-20.58
8	.9900	.4330	.9000	18.67	-27.84
AVERAGE S/N RATIOS					
	SETTING	N	δ_{2R}	C	
	1	-23.88	-21.65	-21.60	
	2	-22.01	-24.23	-24.29	
Analysis of Variance (ANOVA)					
Source	Sum of Squares	Degrees of Freedom	Variance	Effect (%)	
N	409.5980	1	409.5980	24.32	
δ_{2R}	478.8894	1	478.8894	28.44	
μ	390.8472	1	390.8472	23.23	
N$\times\delta_{2R}$	381.9020	1	381.9020	22.68	
N$\times\delta_{2R}\times\mu$	22.8390	1	22.8390	1.36	
error*	0.0125	3	0.0042	0.00	
TOTAL	1684.0881	8	1684.079	100.0	
* - N$\times\mu$, $\delta_{2R}\times$C, and error pooled.					

n, though not the same function, at optimum tuning and optimum damping, according to Equations 3-10 and 3-11. Thus, an equation can be written relating δ_{2R} and n, which are then functions of one another and are not independent of one another. However, that fact does not mean that μ is independent of δ_{2R} and n. Rather, μ is a dependent variable at optimum tuning and damping. In fact, only one of the three variables - either n or δ_{2R} - are actually independent. Thus, there should be at least two significant two-factor interactions, not just one.

The only explanation that can be given for these ANOVA results is that the ANOVA was performed on the results in general, and not just at optimum tuning and damping. When not at those conditions, the three control factors are not related by any theoretical equations. However, ANOVA shows that the nxδ_{2R} interaction is significant, even when not at optimum tuning and damping (this interaction was also shown to be significant in the 2x2 case where μ was held fixed). Thus, the results of Chapter 3 may not be entirely accurate, since this interaction was not adequately dealt with and studied.

APPENDIX C - DERIVATION OF TRANSFER FUNCTION AND OTHER DETAILS FOR THE FIFTH DESIGN OPTIMIZATION PROCEDURE

This appendix contains the mathematical details of the derivation of the transfer function from the fifth design optimization procedure. Included here is a description of the experimental procedure in which important parameters of the electro-hydraulic servosystem were obtained en route to calculating this transfer function.

The electro-hydraulic servomechanism can be represented in block-diagram form as shown in Figure C-1. Based on the rules of linear first-order systems, the system can be consolidated to a simple system in which output response equals input response times some transfer function (see Ogata [63] for additional details on block-diagram representations of control systems). Based on these consolidation rules, an expression for the transfer function g(s) was obtained as:

$$g(s) = \frac{K_1 * K_I}{s * K_v * [(T * s + 1) + K_{A2} * K_1] + K_{A1} * K_P * K_1 * K_I}. \qquad \text{(C-1)}$$

In order to simplify this expression, the following definitions of terms were made:

$$A = K_I * K_1 ; \tag{C-2}$$

$$B = K_p * K_{A1} ; \tag{C-3}$$

$$C = K_v * K_1 ; \tag{C-4}$$

$$D = A * B ; \tag{C-5}$$

$$E = K_v * T ; \tag{C-6}$$

$$F = K_v + C * K_{A2} . \tag{C-7}$$

When Equations C-2 through C-7 were substituted into Equation C-1, the transfer

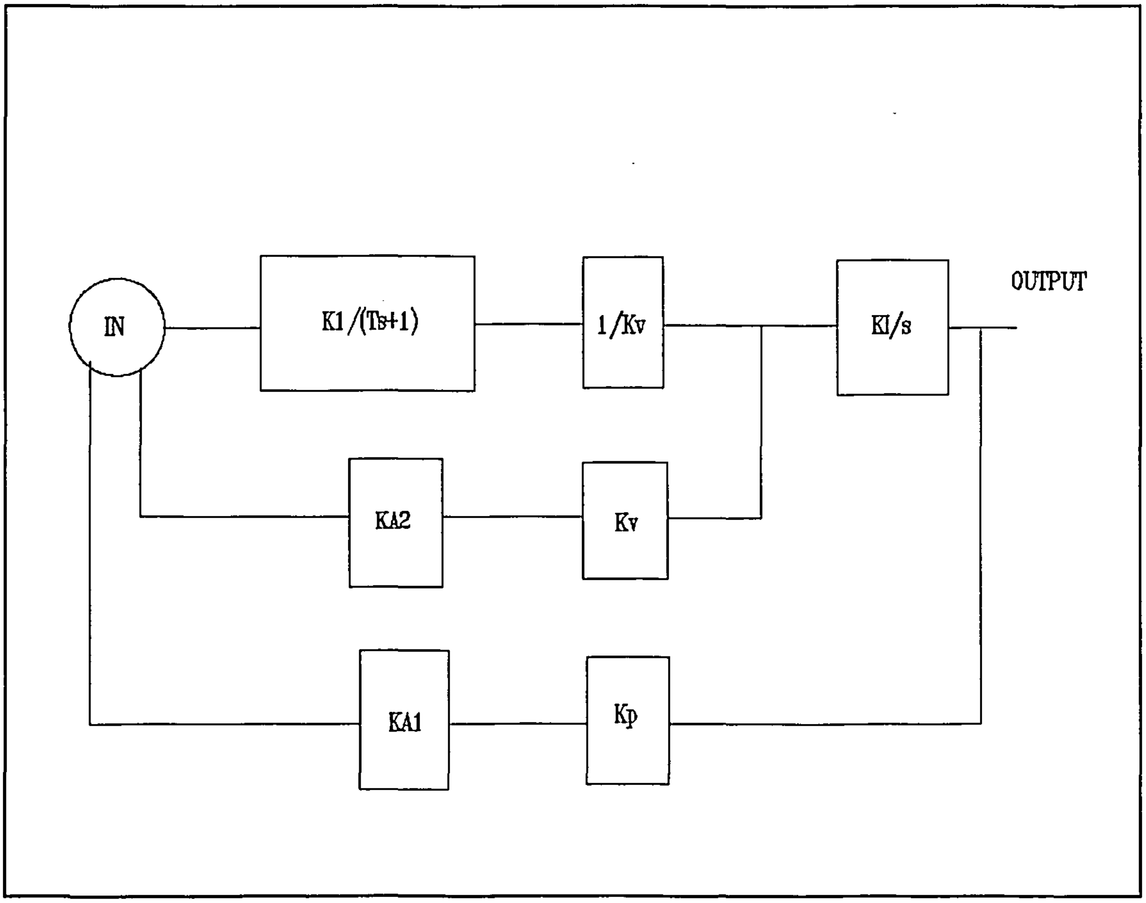

Figure C-1 - Block diagram of the electro-hydraulic servosystem.

function became

$$g(s) = \frac{A}{E*s^2 + F*s + D} \,. \qquad \textbf{(C-8)}$$

The expression for output as a function of input could then be written as

$$C(s) = R(s) * g(s) \,, \qquad \textbf{(C-9)}$$

where g(s) is given by Equation C-8 and R(s) = 1/s. The output response as a function of the time "t" was then determined by the equation

$$c(t) = \mathcal{L}^{(-1)}\,[C(s)] \,. \qquad \textbf{(C-10)}$$

The inverse Laplace transform was obtained using the partial fractions technique (see Kreysig [69] or O'Neil [70] for mathematical details). In order to use this technique, Equation C-8 was modified by dividing through by E. In addition, the following additional definitions were made:

$$J = \frac{F}{E} \,; \qquad \textbf{(C-11)}$$

$$K = \frac{D}{E} \,; \qquad \textbf{(C-12)}$$

$$L = \frac{A}{E} \,. \qquad \textbf{(C-13)}$$

With these definitions, Equation C-9 becomes

$$C(s) = \frac{L}{s*(s^2 + J*s + K)}. \qquad \text{(C-14)}$$

The inverse Laplace transform was then determined to be

$$c(t) = \frac{L}{K} - \frac{L*J}{K}*e^{-\alpha*t}*\sin(\omega*t) - \frac{L*\beta}{K*\omega}*e^{-\alpha*t}*\sin(\omega*t-\phi), \qquad \text{(C-15)}$$

which is the same equation as Equation 7-1, where

$$\alpha = \frac{J}{2}, \qquad \text{(C-16)}$$

$$\beta = \sqrt{K}, \qquad \text{(C-17)}$$

$$\omega = \beta^2 - \alpha^2, \qquad \text{(C-18)}$$

and

$$\phi = \arctan(\frac{\omega}{\alpha}). \qquad \text{(C-19)}$$

The rest of the procedure consisted of determining the various system parameters from equation C-1. The parameter K_1, the plant gain, was determined by applying a simple unit step impulse to the system without using any of the control-theory feedback mechanisms. The plant gain was then simply the ratio of the output to the input, 5.3/1.4 = 3.8. The parameter K_I was the conversion factor from radians to degrees, or 360/(2*π).

Finding the remaining parameters involved several fairly tedious experiments. The simple unit step impulse that enabled the plant gain to be determined was plotted; it is shown in Figure C-2. From this plot, the inverse of the slope at the point where the output response begins to increase is the value of the parameter T, which was determined to be approximately .10 sec/V.

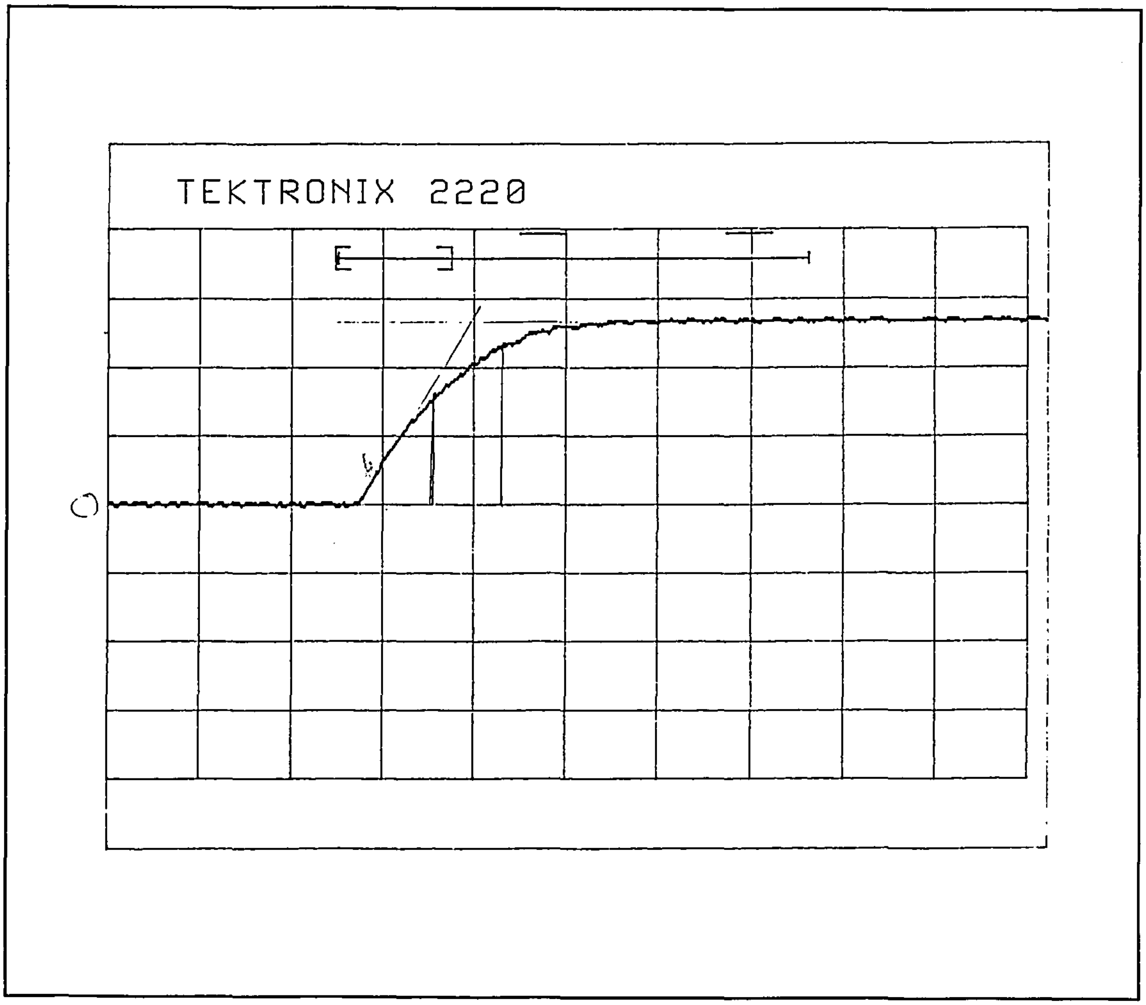

Figure C-2 - System response to unit step impulse.

To determine K_p, measurements of the input voltage versus the output position angle were taken (as the attenuator setting were varied) and plotted; this plot is shown in Figure C-3. The value of K_p was then determined as the average of the absolute values of the slopes of the curve as it crossed the origin. This value was determined to be 0.072 V/degree, or 4.125 V/rad.

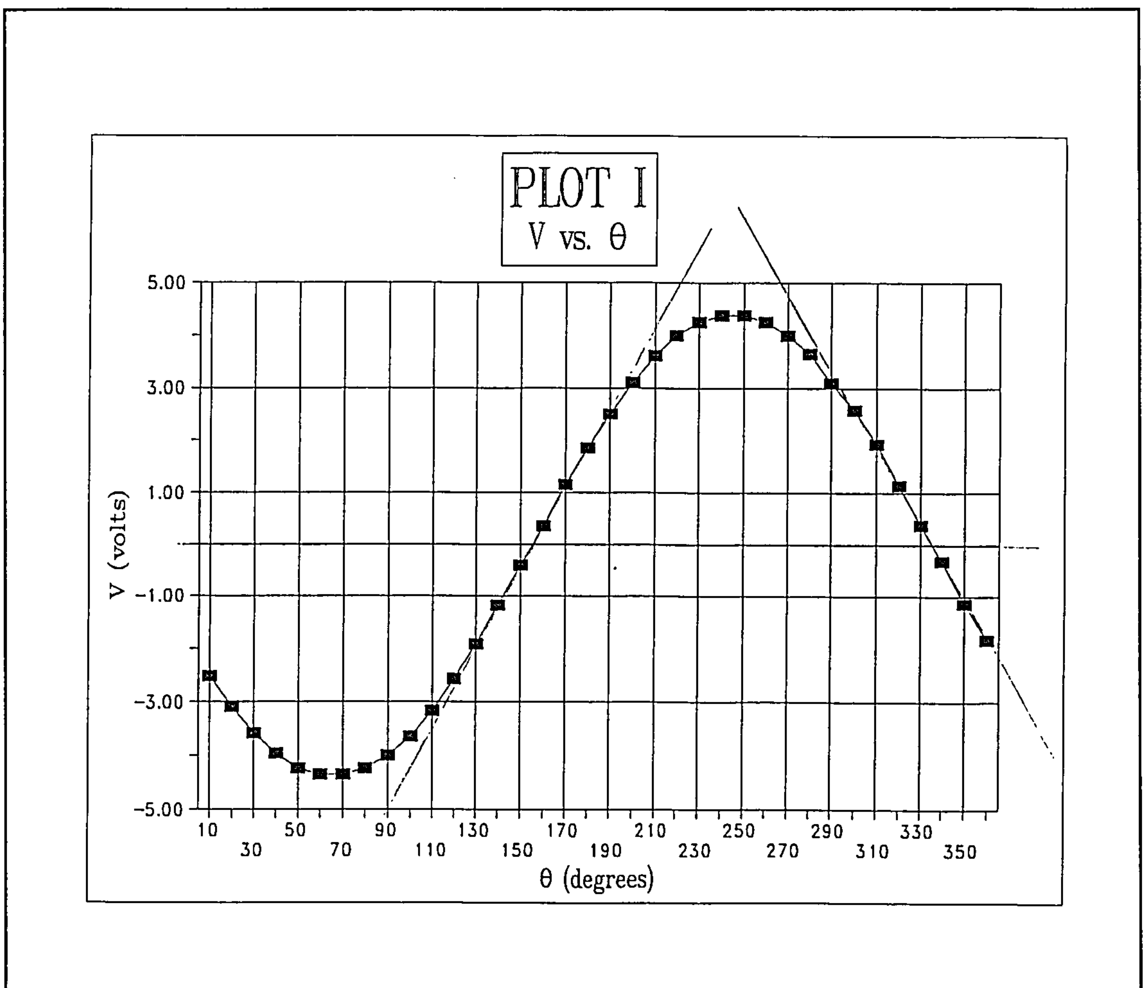

Figure C-3 - Voltage versus output position.

Finally, the value of K_v was determined by taking measurements of the speed of the rotating shaft as the input voltage was varied. These measurements are shown graphically in Figure C-4. The approximate slope of the curve at the point where the curve crosses the origin is the value of K_v; this value was determined to be 0.00856 (V·sec)/rad.

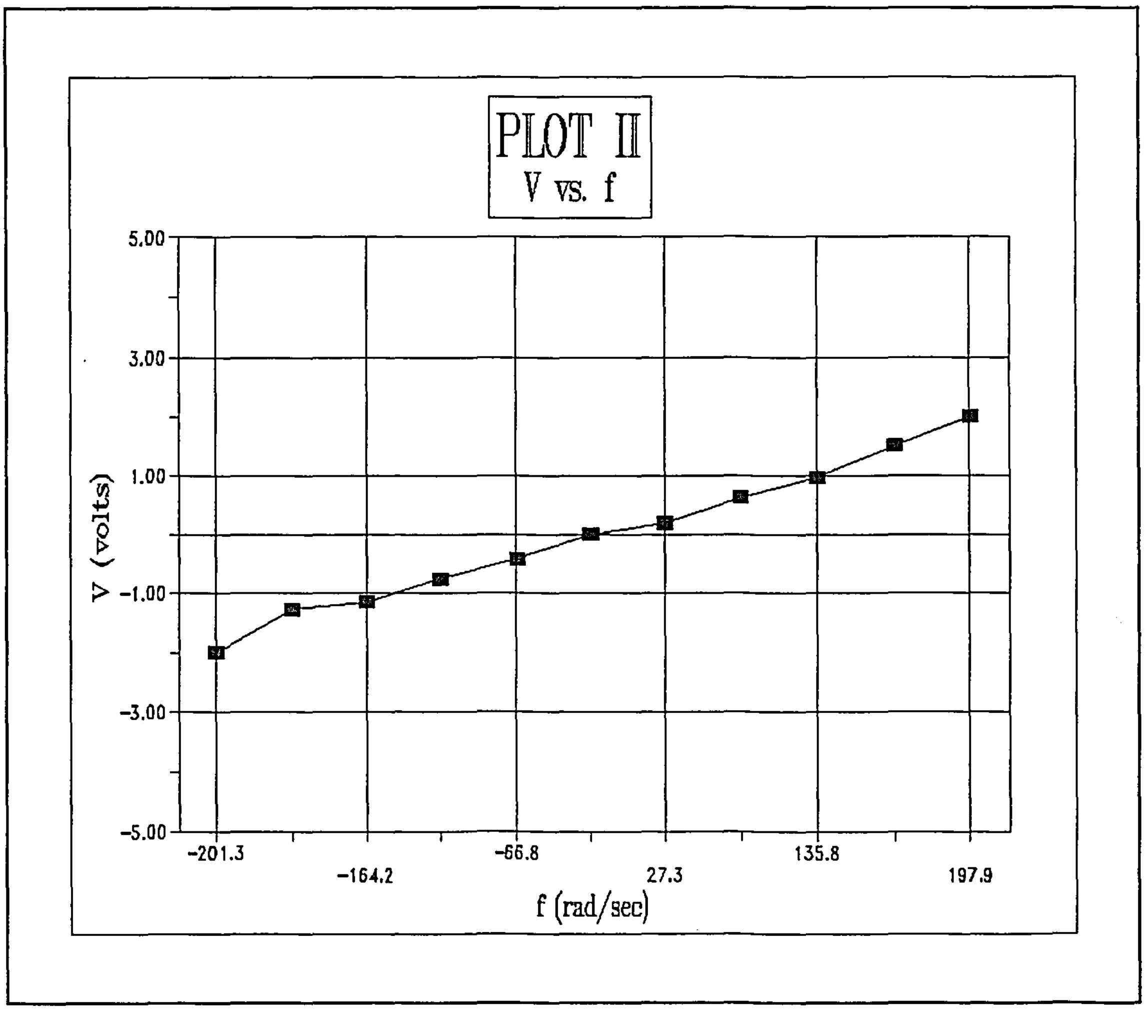

Figure C-4 - Voltage versus output frequency.

Several additional apparatuses were used to take these measurements. A Fluke 21 Multimeter was used to take the voltage measurements, with an appropriate calibration to determine the scale of the readout. A Health 2718 Tri-Power Supply was used to supply the actual input voltage. The Strobe Tachometer was used in the measurement of output rotation frequency as a function of input voltage; this procedure is described in more detail below. Finally, a Tektronix 2200 portable plotter was used for several of the plots, and connecting wires were used to connect the auxiliary apparatuses with the electro-hydraulic servosystem (for additional details, see the manual for The Electro-Hydraulic Servomechanism Type EHS 160 Technical Information Manual [64]).

Obviously, these values were subject to error. Merely determining the slopes of the various graphs introduced error, since the plots did not have overly accurate scales (some, in fact, did not even have scales). In addition, there were the traditional rounding errors in the calculations. But the largest source of error was in the actual taking of the measurements. For example, in determining K_v, measurements of output rotation frequency as the input voltage was varied were taken. The output rotation frequency, however, was determined by a method in which a strobe light was directed at the rotating shaft, and the frequency of the light adjusted until a one-inch wide strip of paper "appeared" to be stationary. This procedure was not exact. Similar errors existed in reading the values of the attenuators, as well.

Nevertheless, despite these large sources of errors, Taguchi's method of parameter design gave the values of the attenuator settings that were expected to be given. As a validation of the results, a confirmation experiment was performed. As described in Section 7.3, seven sets of attenuator settings were sampled and plotted. Due to the inaccuracies of the width of the curve, and the fact that the time scales are not small enough to show the small differences in rise times, it may not be readily apparent in all of the plots that the setting (1.0,1.0) gives the smallest rise time. Nevertheless, this result was given by the experimental procedure and confirmed by personnel in the Interdisciplinary Controls Laboratory. Three of the plots are shown in Figures C-5, C-6, and C-7. These plots show a plot of output position versus time for the attenuator settings (1,1), (.75,.75), and (.55,.55), respectively.

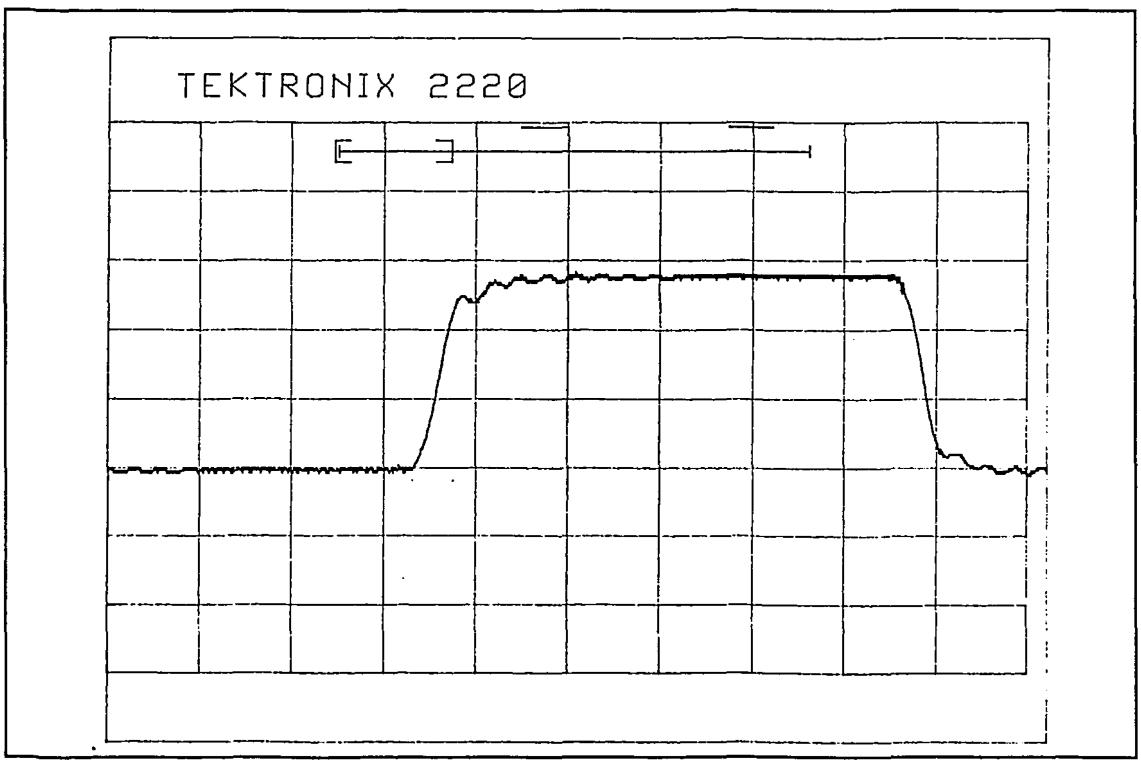

Figure C-5 - Output position versus time, (K1,K2) = (1.0,1.0).

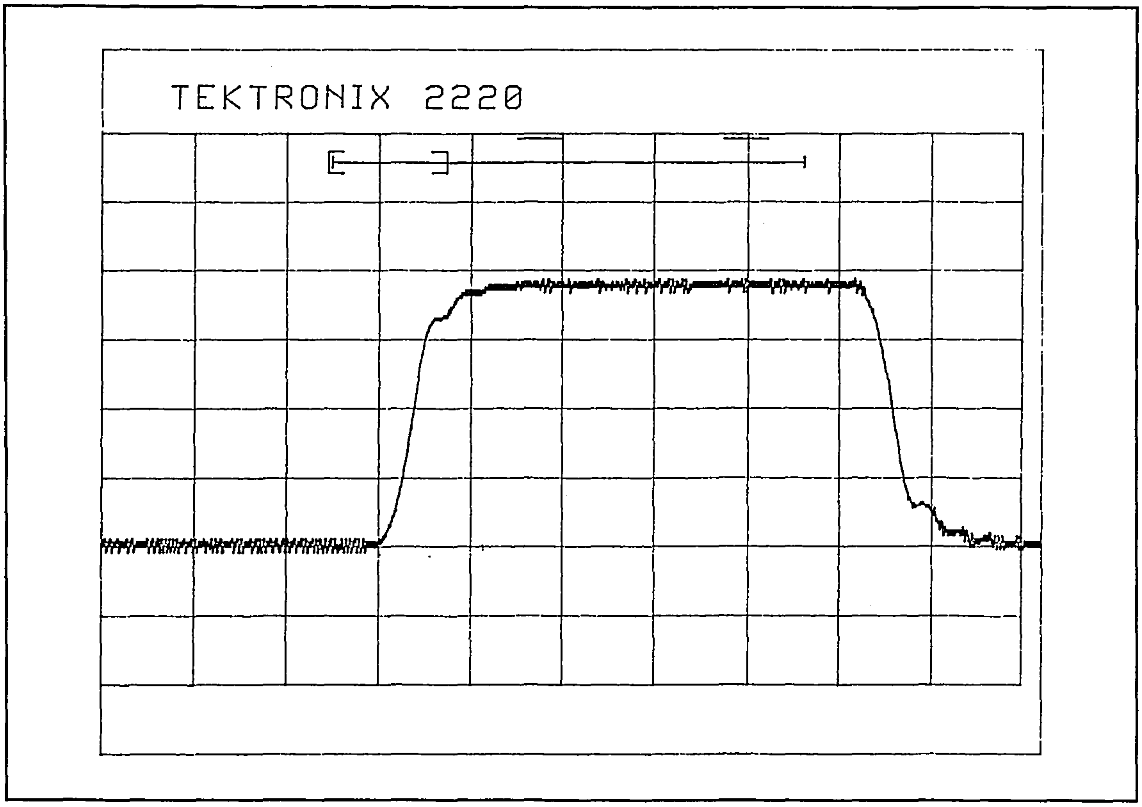

Figure C-6 - Output position versus time, (K1,K2) = (.75,.75).

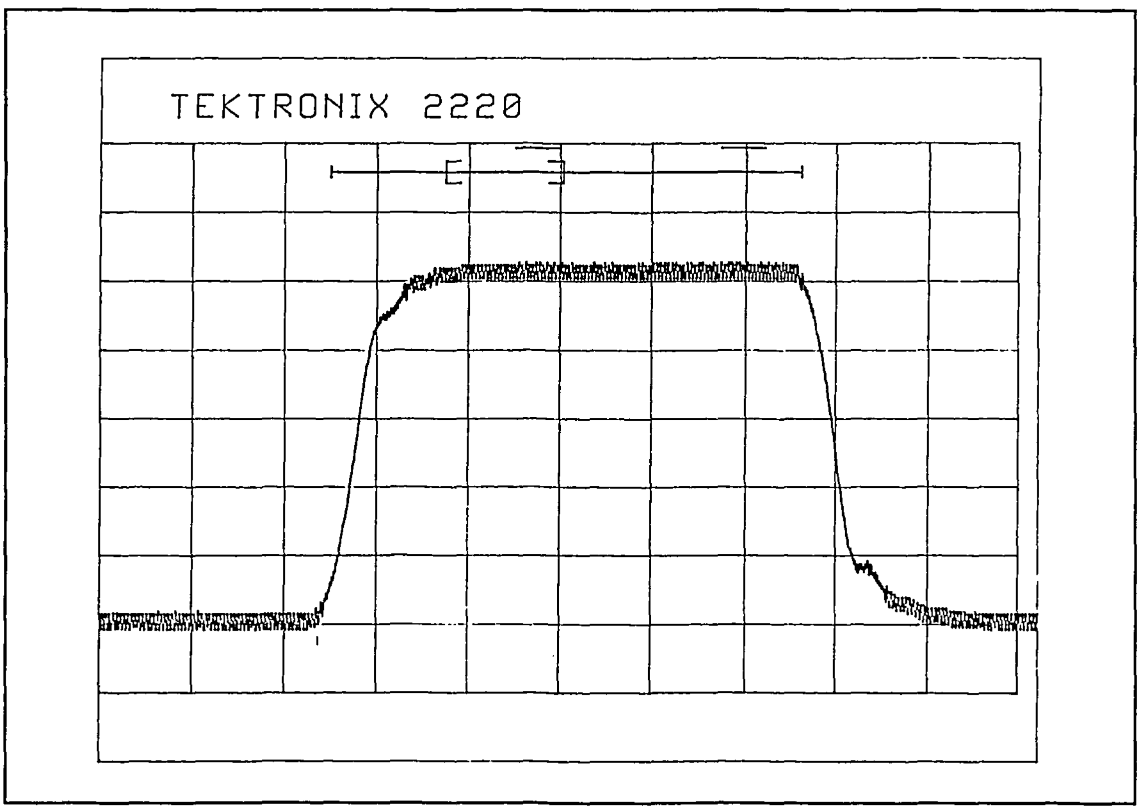

Figure C-7 - Output position versus time, (K1,K2) = (.55,.55).

VITA

Robert Arthur Zambanini, Jr., was born on Friday, September 3, 1965, in Philadelphia, Pennsylvania, the son of Robert Arthur Zambanini, Sr., and Sylvia Joan Harp. After attending Immaculate Conception Roman Catholic Elementary School in Levittown, Pennsylvania and Most Blessed Sacrament Regional Roman Catholic Elementary School, in Bally, Pennsylvania, he graduated as Valedictorian of the Class of 1983 at Boyertown Area High School. After spending a semester at The United States Military Academy at West Point, New York, Zambanini attended Lehigh University and graduated himself in 1987 with a Bachelor of Science in Mechanical Engineering.

Zambanini then spent four years as a mechanical engineer at AT&T. The first nine months of this tenure was spent in the research and design wing of AT&T, namely, Bell Labs, where he performed design support activities such as the design of mechanical devices and finite element analyses, including the publication of a technical memorandum on the temperature distributions in electronic receivers with laser chips. The final twenty-seven months were spent in the manufacturing arm of AT&T, AT&T Microelectronics, where he was a process engineer in charge of the mechanical responsibilities of integrated circuit chip testing and transfer.

Zambanini was enrolled in August, 1988, as a part-time graduate student in the Engineering Mathematics curriculum at Lehigh University. On November 18, 1989, he married Luann Kathryn Riegner. In August, 1991, he became a full-time graduate student. He and his wife reside in Bally, Pennsylvania.

END

OF

TITLE