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# A Study of Membership Functions on Mamdani-Type Fuzzy Inference System for Industrial Decision-Making

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# A STUDY OF MEMBERSHIP FUNCTIONS ON MAMDANI-TYPE FUZZY INFERENCE SYSTEM FOR INDUSTRIAL DECISION-MAKING

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Abstract

The complexity of product design in industry has been continuously increasing. More factors are required to be taken into account simultaneously before a decision about the new product could be determined. For this reason, decision-making process costs much more time and it may even be impossible to determine the optimal decision by normal calculations. Therefore, Fuzzy Inference System based on Fuzzy Logic is introduced as a quick decision-making tool to arrive at a good decision within much shorter time.

This thesis focuses on studying the features of membership functions in Mamdani-type fuzzy inference process. It is aimed at making the black box of fuzzy inference system to be transparent by adjusting the membership functions to control the relations between input and output variables. Systematic trial and error is implemented based on the Fuzzy Logic Toolbox from MATLAB, and conclusions developed from experiments help eliminate the uncertainties of membership functions, so that the inference process turns to be more precise and reliable. Firstly, Single-Input Single-Output (SISO) Fuzzy Inference System is discussed through the adjustment of membership functions, and the influence on input-output relations are concluded. Next, Two-Input Single-Output (TISO) Fuzzy Inference System is simulated to verify the conclusions from SISO Fuzzy Inference System, and general features of membership functions on affecting input-output relation are developed. Then, an approach using weights on input variables, for practical decision-making process, is derived. Finally, a design problem of timing system of automobile engine is chosen as case study to examine the validity of conclusions on practical decision-making problem.

# 1. Introduction of Fuzzy Logic and Fuzzy Inference Process

In this chapter the definition and foundational concepts of fuzzy logic, including the meaning of fuzzy set, membership function, fuzzy logical operation and If-Then rule, are explained. This chapter also illustrates the fuzzy inference process and the features of Mamdani-type fuzzy inference system. Meanwhile specific procedures of Mamdani fuzzy inference are discussed in brief, and the critical concern during this process on which this thesis focuses is pointed out, and it is followed by the motivation of this study. Then the last part explains the literature survey and the thesis organization.

## 1.1 What is Fuzzy Logic?

In a wide sense, Fuzzy Logic is a form of soft computing method which accommodates the imprecision of the real world. As the antonym of the traditional, hard computing, soft computing exploits the tolerance for imprecision, uncertainty, and partial truth to achieve tractability, robustness, and low solution cost. In a more specific sense, Fuzzy Logic is an extension of multivalued logic whose objective is approximate reasoning rather than exact solution. Unlike traditional Crisp Logic, such as Binary Logic where variables may only take on truth values true and false represented by 1 and 0 respectively, the variables in Fuzzy Logic may have a truth value that ranges in degree between 0 and 1. Instead of describing absolute yes or no, the truth value, or membership in Fuzzy Logic explains a matter of degree. 0 shows completely false, while 1 expresses completely true, and any value within the range indicates the degree of true. Furthermore, the concept of membership in Fuzzy Logic is close to human words and intuition, so the number and variety of applications of Fuzzy Logic have increased significantly in recent years.

## 1.2 Basic Conceptions of Fuzzy Logic

This thesis focuses on fuzzy inference which is a primary application of fuzzy logic. The main approach of fuzzy inference is taking input variables through a mechanism which is comprised by parallel If-Then rules and fuzzy logical operations, and then reach the output space. The If-Then rules are expressed directly by human words, and each of the word is regarded as a fuzzy set. All of these fuzzy sets are required to be defined by membership functions before they are used to build If-Then rules.

### 1.2.1 Fuzzy Set

Fuzzy set is an extension of the classical set. In classical crisp set theory, the membership of elements complies with a binary logic --- either the element belongs to the crisp set or the element does not belong to the set. While in fuzzy set theory, it can contain elements with degree of membership between completely belonging to the set and completely not belonging to the set. This is because a fuzzy set does not have a crisp, clearly defined boundary, and its fuzzy boundary is described by membership functions which make the degree of membership of elements range from 0 to 1.

A brief example for fuzzy set is showed in Figure 1.1. In the following fuzzy set which describes a criterion of fuel-efficient automobiles, a model whose fuel consumption is equal or greater than 28 mile per gallon (mpg) is defined as an element in this set with full degree of membership. In this case, an automobile with 33 mpg has a full degree of membership, in other words, completed belongs to the set. Another model with 18 mpg is far away from the criterion, so it seems have zero degree of membership, in other words, completed does not belong to the set. And a car with 25 mpg is fairly close to the criterion, so it is reasonable to say it has partial degree of membership, and the value of membership (e.g. 0.6) is decided by the feature of membership functions.

### 1.2.2 Membership Functions

A membership function (MF) is a curve that defines the feature of fuzzy set by assigning to each element the corresponding membership value, or degree of membership. It maps each point in the input space to a membership value in a closed unit interval [0, 1]. Figure 1.2 shows a general membership function curve. The horizontal axis represents an input variable $x$, and the vertical axis defines the corresponding membership value $\mu(x)$ of the input variable $x$. The Support of membership function curve explains the range where the input variable will have nonzero membership value. In this figure, $\mu(x) \neq 0$ when $x$ is any point located between point $a$ and point $d$. While the Core of membership function curve interprets the range where the input variable $x$ will have full degree of membership ( $\mu(x) = 1$ ), in other words the arbitrary point within the interval [$b$, $c$] completely belongs to a fuzzy set which is defined by this membership function.

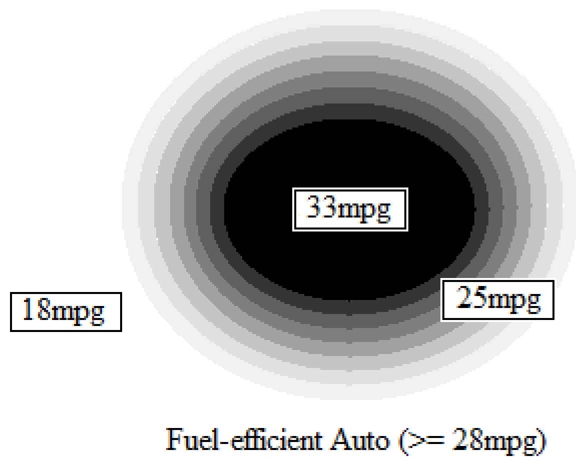


Figure 1.1 Fuzzy set ‘Fuel-efficient Auto’

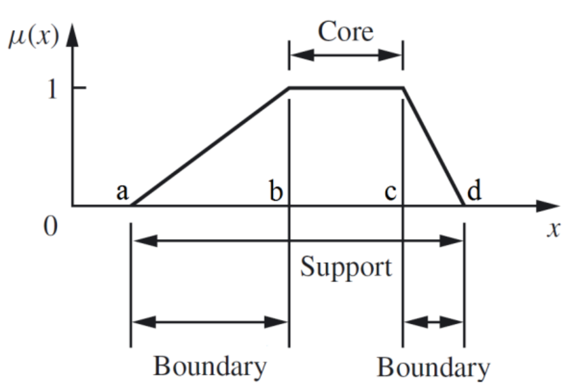


Figure 1.2 A sample of membership function

Generally, there are five common shapes of membership function: Triangle MF, Trapezoidal MF, Gaussian MF, Generalized Bell MF, and Sigmoidal MF. Regardless of the shape, a single MF may only define one fuzzy set. Usually, more than one MF are used to describe a single input variable. Taking the fuel consumption of automobile for instance, a three-level fuzzy system with fuzzy sets 'Low', 'Medium' and 'High' is applicable to represent the whole situation.

### 1.2.3 Logical Operation

Because the standard binary logic is a special case of fuzzy logic where the membership values are always 1 (completely true) or 0 (completely false), fuzzy logic must hold the consistent logical operations as the standard logical operations. The most foundational logical operations are AND, OR and NOT. Unlike standard logical operation, the operands *A* and *B* are membership values within the interval [0, 1]. In fuzzy logical operations, logical AND is expressed by function *min*, so the statement *A* AND *B* is equal to *min* (*A*, *B*). Logical OR is defined by function *max*, thus *A* OR *B* becomes equivalent to *max* (*A*, *B*). And logical NOT makes operation NOT *A* become the operation $1 - A$.

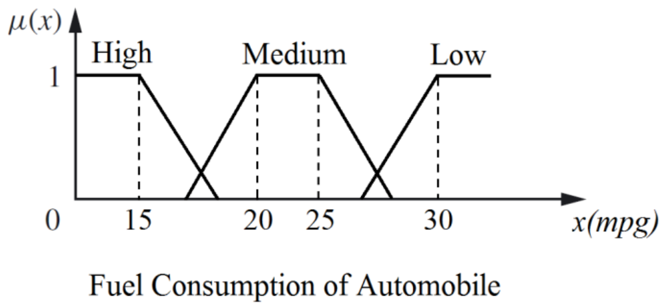


Figure 1.3 Variable "fuel consumption of automobile" represented by membership functions

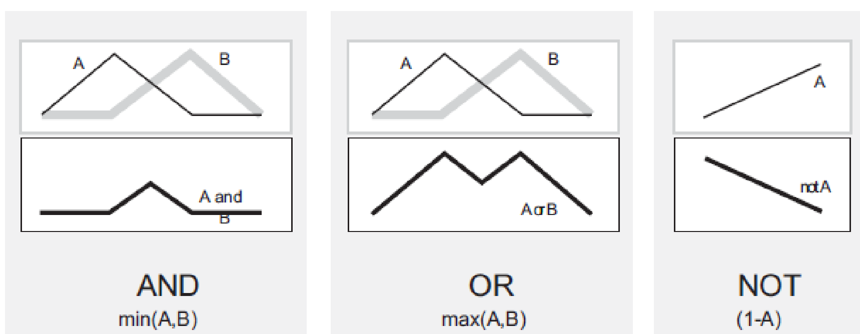


Figure 1.4 Fuzzy logical operations

### 1.2.4 If-Then Rules

In fuzzy inference process, parallel If-Then rules form the deducing mechanism which indicates how to project input variables onto output space. A single fuzzy If-Then rule follows the form

If $x$ is $A$, Then $y$ is $B$

The first If-part is called the antecedent, where $x$ is input variable. The rest Then-part is called the consequent, and $y$ is output variable. The reason why If-Then conditional statements are universally applicable is because both $A$ and $B$ are linguistic values, or adjectives in most cases, and this form of conditional statement works the concordant way with human judgment. For example, an appropriate If-Then rule might be "If *material hardness* is *hard*, Then *cutting speed* is s*low*". $A$ can be regarded as fuzzy set and defined by specific membership function, and $B$ can be either a fuzzy set or a polynomial with respect to input $x$ depending on specific fuzzy inference method. In the antecedent, the If-part is aimed at working out the membership value of input variable $x$ corresponding to fuzzy set $A$. While in the consequent, the Then-part assigns a crisp value back to the output variable $y$.

## 1.3 Fuzzy Inference and Mamdani-Type Fuzzy Inference

Fuzzy inference is the process of mapping the given input variables to an output space via fuzzy logic based deducing mechanism which is comprised by If-Then rules, membership functions and fuzzy logical operations. Because the form of If-Then rule fits in human reasoning, and fuzzy logic approximates to people's linguistic habits, this inference process projecting crisp quantities onto human language and promptly yielding a precise value as result is widely adopted.

Generally, three types of fuzzy inference methods are proposed in literature: *Mamdani* fuzzy inference, *Sugeno* fuzzy inference, and *Tsukamoto* fuzzy inference. All of these three methods can be divided into two processes. The first process is fuzzifying the crisp values of input variables into membership values according to appropriate fuzzy sets, and these three methods are exactly the same in this process. While the differences occur in the second process when the results of all rules are integrated into a single precise value for output. In Mamdani inference, the consequent of If-Then rule is defined by fuzzy set. The output fuzzy set of each rule will be reshaped by a matching number, and defuzzification is required after aggregating all of these reshaped fuzzy sets. But in Sugeno inference, the consequent of If-Then rule is explained by a polynomial with respect to input variables, thus the output of each rule is a single number. Then a weighting mechanism is implemented to work out the final crisp output. Although Sugeno inference avoids the complex defuzzification, the work of determining the parameters of polynomials is inefficient and less straightforward than defining the output fuzzy sets for Mamdani inference. Thus Mamdani inference is more popular and this thesis only focuses on Mamdani inference method. Tsukamoto inference seems like a combination of Mamdani and Sugeno method, but it is even less transparent than these two models, and it will not be discussed in this thesis neither.

## 1.4 Mamdani-Type Fuzzy Inference Process

Mamdani-type fuzzy inference process consists of five steps:

Step 1: Fuzzify input variables

Step 2: Apply fuzzy operator

Step 3: Apply implication method

Step 4: Apply aggregation method

Step 5: Defuzzification

An example of "Climate Comfortability" is introduced to illustrate the complete process of fuzzy inference. As the logical flows showed in Figure 1.5, two input variables, temperature and humidity, are taken through the fuzzy reasoning process with three If-Then rules, then the results from the rules are combined and transformed to a crisp evaluating number about climate comfortability.

### 1.4.1 Fuzzify Input Variables

The first step is to transform the crisp numerical values of input variables into the equivalent membership values of the appropriate fuzzy sets via membership functions. No matter what the input variables describe, through the fuzzification process the output is usually degree of membership in the related fuzzy linguistic sets within the interval between 0 and 1.

In the example mentioned above, three If-Then rules totally present four different fuzzy

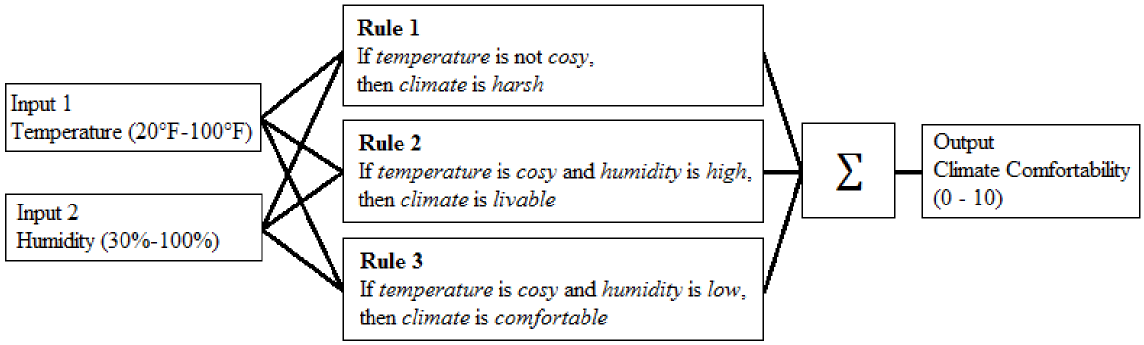


Figure 1.5 Logic flow of "Climate Comfortability" example

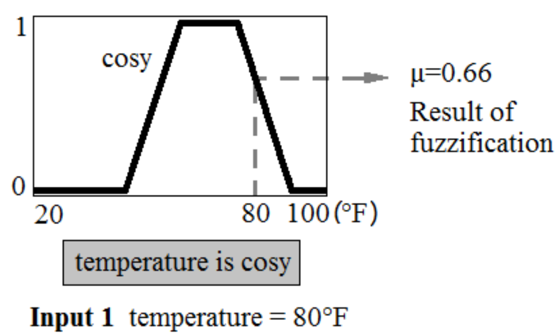


Figure 1.6 Fuzzifying input variable "temperature"

linguistic sets: ‘temperature is cosy’, ‘temperature is not cosy’, ‘humidity is high’, and ‘humidity is low’. Two input variables, temperature and humidity, must be fuzzified according to the membership functions of these linguistic sets. For example, Figure 1.6 shows that the point representing 80°F is projected onto the membership function curve which describes the linguistic set ‘temperature is cosy’, and reach the membership value $\mu = 0.66$ for the fuzzy set ‘temperature is cosy’.

### 1.4.2 Apply Fuzzy Operator

When the fuzzy inference system contains more than one input variable, the antecedent of If-Then rule might always be defined by more than one fuzzy linguistic set, because in most cases each input variable has one corresponding fuzzy set based on which to figure out degree of membership. In the above example, the antecedent of Rule 2 consists of two fuzzy linguistic sets --- ‘temperature is cosy’ and ‘humidity is high’. Here the fuzzy operator is required to combine the two membership values from set ‘temperature is cosy’ and set ‘humidity is high’ respectively, and then obtain one numerical value that represents the result of the antecedent for this rule. The most common fuzzy operators are AND operation and OR operation. To formulate these logical operation, function *min* and function *max* are applied. Although other functions, such as *product* and *probabilistic OR*, are also applicable in expressing these fuzzy operators, function *min* and function *max* are always simple, effective and widely used, so the formulating method based on *min* and *max* will run through the whole thesis. The following Figure 1.7 shows the AND operation via function *max*. Two different fuzzy sets of the antecedent in Rule 2 yielded the fuzzy membership values 0. 66 and 0.33 respectively, and the maximum of the two values, 0.33, is picked out as the result of antecedent of Rule 2.

### 1.4.3 Apply Implication Method

The consequent part of If-Then rule is another fuzzy linguistic set defined by an appropriate membership function. Unlike the result from antecedent part of If-Then rule that a single numerical value is generated, the inference method in Then-part is to reshape the fuzzy set of consequent part according to the result associated with the antecedent, or say the single number. This process is called implication method. The AND operation is implemented which truncates the fuzzy set of consequent part. The extent of deformation of the output fuzzy set in each rule should depend on the specific single number coming from the matching antecedent of the rule.

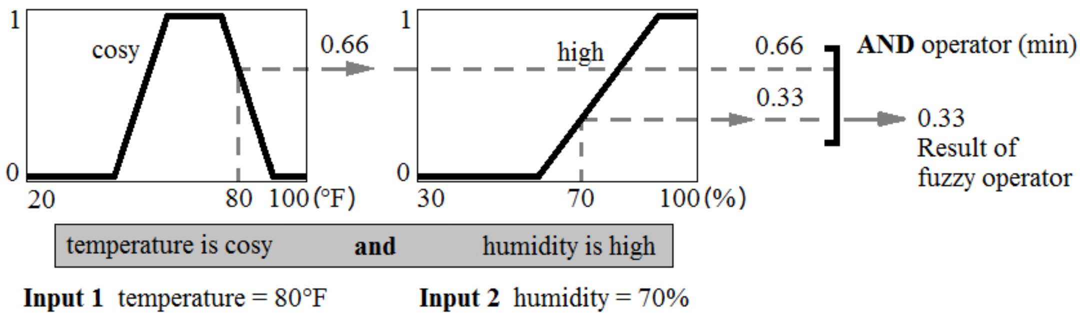


Figure 1.7 Applying fuzzy operator

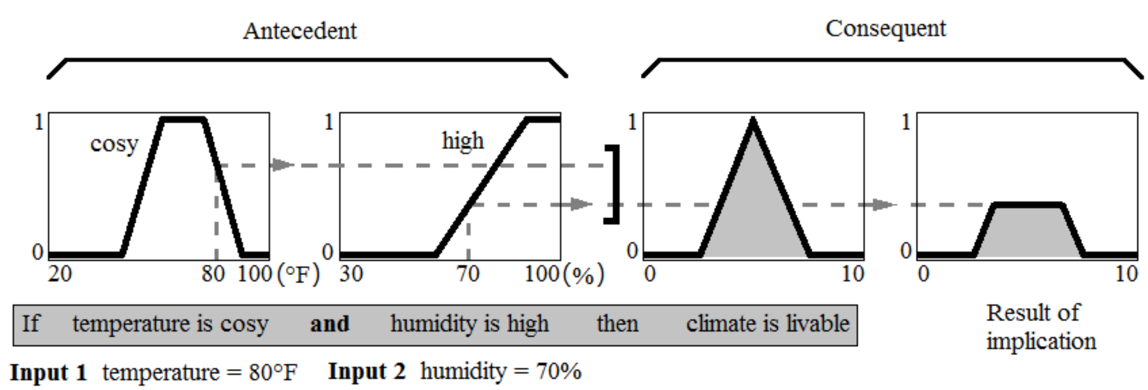


Figure 1.8 Applying implication method

### 1.4.4 Apply Aggregation Method

After each of the If-Then rules generating a modified fuzzy set as output, the aggregation method is implemented to combine these fuzzy sets that represent the outputs of rules into a single fuzzy set in order to make a decision. The final combined fuzzy set is the output of the aggregation process, and every output variable of the fuzzy inference system will have a single matching combined fuzzy set for reference. Function *max*, *sum*, and *probabilistic OR* are all applicable for aggregation operation, but function *max* is chosen for all discussion in this thesis because it is more straightforward and well accepted.

In the climate comfortability example, three truncated fuzzy sets coming from three rules respectively are operated through aggregation method by function *max*, and a combined new fuzzy set representing the outcome for output variable 'climate evaluation' is ready for the last defuzzification process.

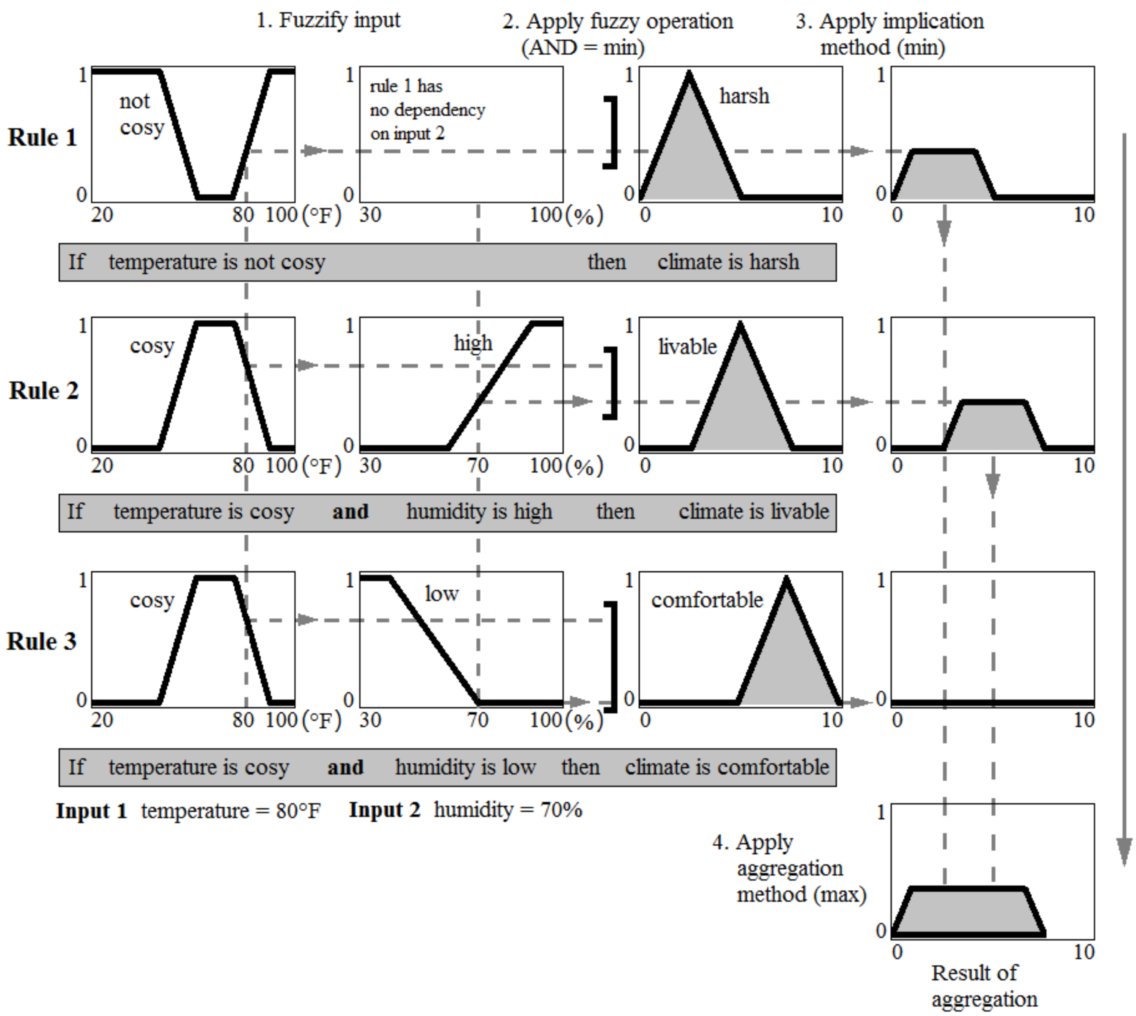


Figure 1.9 Applying aggregation method

### 1.4.5 Defuzzification

The last step of fuzzy inference process is defuzzification, through which the combined fuzzy set from aggregation process will output a single scalar quantity. As the name implies, defuzzification is the opposite operation of fuzzification. Since in the first procedure the crisp values of input variables are fuzzified into degree of membership with respect to fuzzy sets, the last procedure extracts a precise quantity out of the range of fuzzy set to the output variable. Among the many defuzzification methods that have been proposed in the literature, the *Centroid Method* (also called *center of area* or *center of gravity*) which is the most prevalent and physically appealing of all the defuzzification methods (Sugeno, 1985; Lee, 1990), is the only adopted method in this thesis. It is given by the algebraic expression

$$z_{COA} = \frac{\int_z \ \mu_A(z) \cdot z dz}{\int_z \ \mu_A(z)}$$

where $z$ is the output variable, and $\mu_A(z)$ is the membership function of the aggregated fuzzy set $A$ with respect to $z$. The following figure shows the result of the climate comfortability example calculated via *Centroid Method*. This indicates that when temperature is 80°F and humidity reaches 70%, the fuzzy inference system rates the climate comfortability 3.75 points, which means the climate is not appropriate for long-time living.

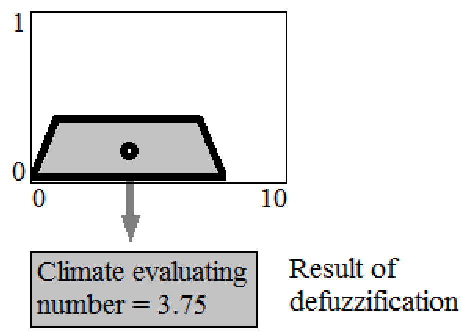


Figure 1.10 Applying Centroid Method for defuzzification

## 1.5 Background and Motivation

The applications of fuzzy inference method can be found in numerous aspects of industrial production or manufacture. Fuzzy inference system can be an efficient tool to help make decision on manufacturing reengineering, optimize the process parameters for drilling process, realize a better batch process scheduling, or design an injection-molded part with optimal process. All of these applications come with a common point that they are all required to take several factors into account at the same time before reaching a final result. In this kind of problems, the relations between input and output and relations between each input factor are fairly complicated, and it is difficult to formulate interactive relations or indirect relations. Therefore the advantages of fuzzy inference system stand out, because the If-Then rule based inference mechanism can be defined directly by practical experience. Although a most optimal result from accurate mathematical expressions is hard to get, fuzzy inference greatly simplifies and accelerates the computing process, and produces a result which is good enough for reference.

Before we admit fuzzy inference system as an ideal decision-making tool, a potential problem must be pointed out --- the fuzzy inference system is not absolutely reliable because of a couple of uncertainties during setting up the system. One of the most critical uncertainties is how to build up membership functions for the fuzzy sets of If-Then rules. Even though the rich experience may guarantee rigorous rules and form precise inference mechanism, inappropriate membership functions will mislead the reasoning process, and cause a input-output relation with unexpected error. This may reduce the system's sensitivity. When the difference between input values keeps decreasing, the inference system turns to be invalid to analyze the difference very soon. As in the climate example, several uncertainties needs to be settled down when describing input variables. What kind of membership function is appropriate? How many MFs are necessary to define input 'temperature'? Should the MFs for a same input overlap with each other? And all of these questions are required to be asked about output variable as well. In many cases, the MFs for independent input variable or output variable can be reasonable, but the input-output relations are always concealed in black box. To make the fuzzy inference system, especially the Mamdani inference method, more transparent and controllable, the question that how membership functions influence input-output relation of fuzzy inference system is worthy to be studied.

## 1.6 Literature

Numerous literatures demonstrate specific applications of fuzzy inference system in industry, but very few literatures introduce detailed thinking process about setting up appropriate membership functions, and it is even much more difficult to search a single literature discussing the influence of membership functions during fuzzy inference. Hashmi (2000) restudied the data selection problem for drilling process via fuzzy inference. In his Single-Input Single-Output fuzzy inference system, both input and output variable was represented by six identical triangle membership functions, and each of them overlapped the adjacent ones by 50%. Lucian (2006) applied fuzzy inference system to study manufacturing reengineering problem. Five input variables are defined by two or three MFs respectively. Gaussian MFs, trapezoidal MFs and general bell MFs with different support and core are used. This inference system showed a high reliance on professional experience. Srinoi (2008) developed a fuzzy system with four input variables and one output variable for sequencing determination in flexible manufacturing system. Every input variable was defined by three triangle MFs which overlap with contiguous ones in different extents, while the output variable was subdivided by nine triangle MFs. Totally 81 (3*3*3*3 = 81) kinds of combinations of input variables are expected, and the more finely the output range was divided, the higher the system's resolution could be. And Gheorghe (2013) also used triangle MFs to perform fuzzy inference for typical parts manufacturing. For all input and output variables, three full-triangle MFs described the middle range of the universe of discourse and two half-triangle MFs represented the two ends of the domain of discourse respectively. And again, these neighboring MFs overlapped with each other by 50%.

Although these authors did not expatiate on establishing MFs, their consideration about the shape and number of MF for a single input or output variable and the overlap ratio must be positive to the performance of input-output relation. Admittedly, practical experience and related knowledge guide the specifics of fuzzy sets and corresponding membership functions, also a well performed fuzzy inference system must be adjusted repeatedly, but a systematic study on the relation between the property of Mamdani fuzzy inference system and membership function features is still worthy to be performed.

## 1.7 Objective of Thesis

The main objective of this thesis is to study the influence of membership function on Mamdani-type fuzzy inference system and research the significant factors of membership function which can determine input-output relation, meanwhile develop appropriate membership functions for ideal linear inference system and classical non-linear inference systems. The approach here is to implement trial and error by using Fuzzy Logic Toolbox of MATLAB. Experimental Mamdani fuzzy inference systems will keep every condition identical except changing the characteristics of membership function, including the shape, quantity, overlap ratio between neighboring MFs, etc. Single-Input Single-Output Mamdani fuzzy inference system, as the simplest model, will be studied at first to extract the traits of membership functions on adjusting input-output curves. Two-Input Single-Output inference system will be discussed as the second stage to summarize the consistent effects of membership function on both SISO inference system and TISO inference system. A method of introducing weight into Multi-Input Single-Output system will be demonstrated and a practical application will be expected to verify the applicability of conclusions.

## 1.8 Thesis Outline

The first chapter of this thesis is an introduction to fuzzy logic and fuzzy inference process. Overview of fuzzy logic, types of fuzzy inference system, and full procedures of Mamdani inference are presented. The chapter ends with background, literature and objective. Chapter 2 discusses Single-Input Single-Output (SISO) Mamdani Fuzzy Inference System through the adjustment of membership functions, and comes with conclusions on the relation between membership function and system performance. In chapter 3, same trials are implemented on Two-Input Single-Output (TISO) Fuzzy Inference System, and consistent features of membership function are screened out. Chapter 4 develops an approach to apply weight mechanism on Multi-Input Single-Output (MISO) Fuzzy Inference System, and finally chapter 5 displays a decision-making problem related to design of timing system of automobile engine to test the applicability of the concluded features of membership function on Mamdani fuzzy inference system based decision-making problem.

# 2. Adjustment of Membership Functions in Single-Input Single-Output (SISO) Mamdani Fuzzy Inference System

This thesis starts with the simplest model, Single-Input Single-Output Mamdani Fuzzy Inference System, to study the influence of membership function on fuzzy inference performance. Trial and error is performed via Fuzzy Logic Toolbox from MATLAB, and a number of SISO Mamdani fuzzy inference systems are set up following the same assumptions and identical constrains except for the details of membership function. A list of questions related to uncertain factors of membership function include

(a). How many MFs are needed to describe a single input variable?

(b). How many MFs are needed to describe a single output variable?

(c). How the shape of MFs for input variable affects inference system?

(d). How the shape of MFs for output variable affects inference system?

(e). How the overlap percentage between adjacent input MFs impacts the result?

(f). How the overlap percentage between adjacent output MFs impacts the result?

These uncertain factors work on membership functions simultaneously, thus comprehensive consideration is necessary in order to regulate input-output relation.

## 2.1 Assumptions and Expectation

Besides all of these questions mentioned above, many other factors which can still make important contributions to the fuzzy inference process will not be discussed in this thesis but need to be settled, for avoiding their disturbance. Thus preconditions and assumptions are indispensable before discussing the deep laws of membership function.

(1). For the purpose of studying the characteristics of MF in a general situation, the universe of discourse for both input and output variables are normalized into interval [0,10]. The input and output values are transformed to numbers within rate range from 0 to 10, where a bigger number usually represents a preferable choice. This precondition is aimed at providing a general circumstance for discussion and eliminate the difference among the layouts of MF curves which is caused by different domains of discourse.

(2). The criterion for the performance of fuzzy inference model is based on the monotonicity of input-output relation. The output value yielded from defuzzification should hold positive correlation with the input value. In SISO fuzzy inference model discussed in this chapter, a rising curve representing input-output relation is expected as normal system performance. The peak of input-output curve should located at the point where input value is the maxima.

(3). This thesis only studies a few properties of MFs and the rest of them are fixed as premises. To meet the expectation of monotonic input-output relation, all the MFs which describe the same input or output variable are constrained by identical geometrical characteristics, and translated to fill in the range [0, 10]. All of the MFs are normal and convex.

(4). The If-Then rules can always make significant contribution to the relation of input and output variables. To exclude the interference from rules, the rules in the fuzzy inference models must be complete and symmetric.

The prime expectation is setting up a SISO fuzzy inference system with ideal linear input-output relation via adjusting the geometrical characteristics of MFs discussed in this thesis. Then take this as starting point to study the controllability of fuzzy inference system, modifying the ideal linear fuzzy inference model into classical non-linear models, and conclude the direct factors of MF related to performance of fuzzy inference system.

## 2.2 System Modeling

In this chapter, the basic SISO Mamdani fuzzy inference model consists of input variable *A* and output variable *B*. Several parallel If-Then rules make up the inference mechanism, and these rules are expected to be complete and symmetric. The MFs which are supposed to define rules are regulated by the need of experimental trials. In the fuzzy inference process, the implication process and aggregation process are implemented by function *min* and function *max* respectively, and *Centroid Method* is adopted for defuzzification process.

The following experimental trials start with the simplest SISO Mamdani fuzzy inference system in which the input variable *A* has two fuzzy sets 'Low' and 'High'. Each fuzzy set

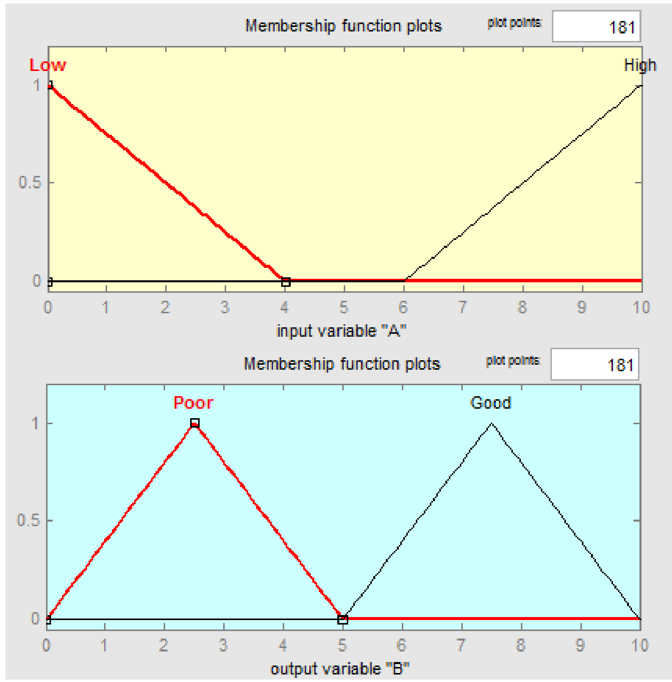


Figure 2.1 Sample membership functions for SISO fuzzy inference system

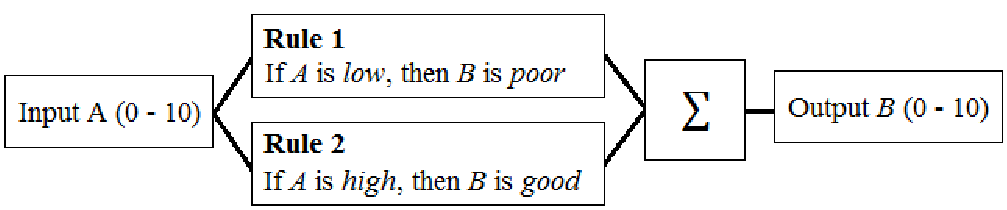


Figure 2.2 Logic flow of SISO fuzzy inference system

is defined by a specific MF, and as the Figure 2.1 shows that it is fairly reasonable to locate the vertexes of both MF curves at the appropriate end of the domain of discourse. Totally the antecedent of If-Then rule has two kinds of possibilities --- 'If *A* is Low' and 'If *A* is High', therefore an output variable with two fuzzy set 'Poor' and 'Good' greatly fits the requirement of editing complete and symmetric rules. Figure 2.2 shows the logic flow of the sample model of SISO Mamdani fuzzy inference system.

## 2.3 Overlap Ratio of Adjacent Membership Functions

In many applications described in literatures, overlap between neighboring membership functions in a same input or output variable is more or less considered in fuzzy inference systems. We have reason to speculate that overlap ratio (OR) of adjacent MFs has direct or indirect impact on input-output relation of fuzzy system, thus a series of trials based on SISO Mamdani fuzzy inference model are created for studying the potential influence from overlap ratio.

As mentioned above, experimental trial starts with the simplest SISO inference model where input variable *A* is represented by two fuzzy sets 'Low' and 'High', and output variable *B* is also defined by two fuzzy sets 'Poor' and 'Good'. To exclude the disturbance of rules, the complete and symmetric If-Then rules are set as

Rule 1: If *A* is Low, then *B* is Poor

Rule 2: If *A* is High, then *B* is Good

### 2.3.1 Membership Functions for Input Variable

**Trial 2-1:** Input A: 2 half-triangle MFs (0% OR)

Output B: 2 full-triangle MFs (0% OR)

**Trial 2-2:** Input *A*: 2 half-triangle MFs (50% OR)

Output *B*: 2 full-triangle MFs (0% OR)

**Trial 2-3:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR)

In Trials 2-1, 2-2 and 2-3, the MFs for output variable *B* remain unchanged, while the overlap ratio between adjacent MFs for input variable *A* is modulated. It is reasonable to use two half-triangle MFs representing the linguistic fuzzy sets 'Low' and 'High' which match to the two ends of domain of discourse, '0' and '10', respectively.

In Trial 2-1, the support of each MF for input *A* spans half of the universe of discourse, and encounter with each other at middle point with 0% of overlap. In this case, the input-output relation performs a stair-shaped line. When the value range of input *A* is limited in interval [0 5), the defuzzified value for output *B* remains at 2.5. And when input *A* takes arbitrary value within interval (5 10], defuzzification process only produces another single number 7.5. Because the two half-triangle MFs are completed non-overlapping, it equally invalidates the aggregation process, and makes the two If-Then rules become completely independent. As Figure 2.6 shows below, when input *A* takes value 2, only the first MF of output *B* is truncated and projected down to be defuzzified. While when input *A* is 4, the

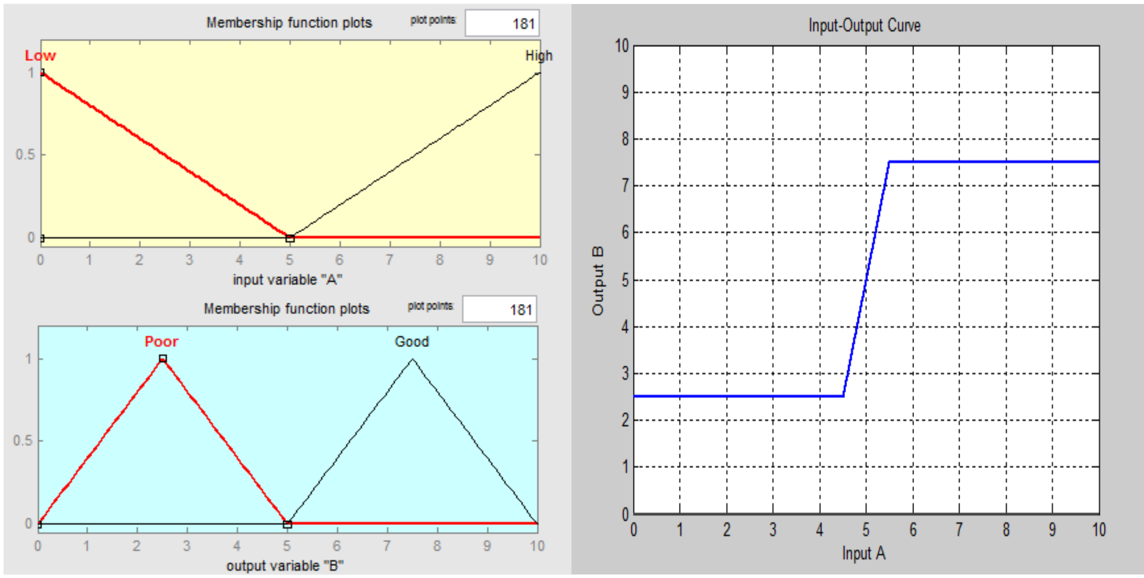


Figure 2.3 Membership function arrangement and input-output relation for Trial 2-1

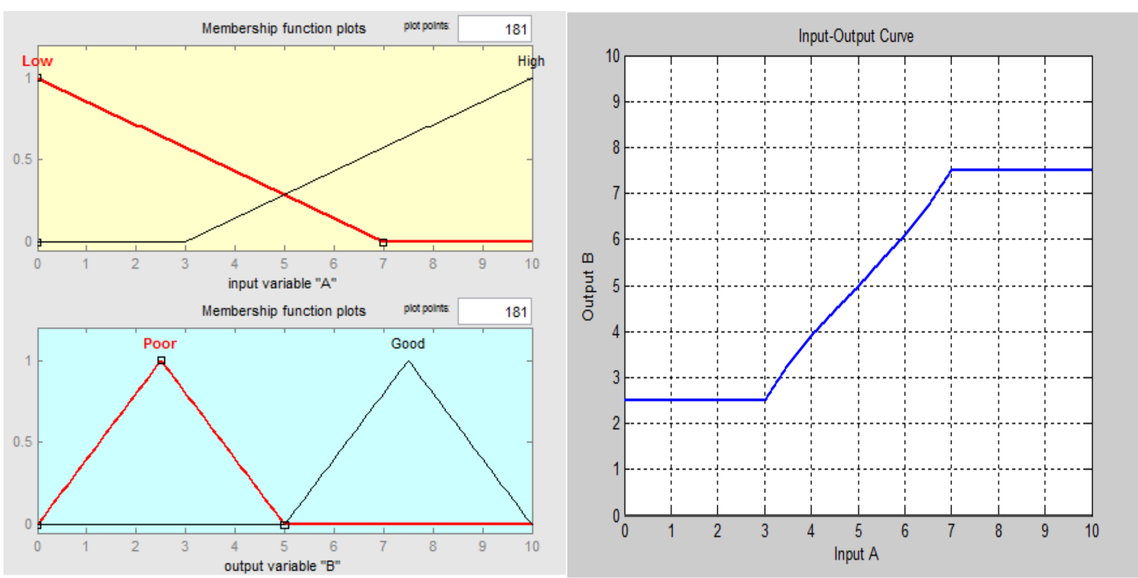


Figure 2.4 Membership function arrangement and input-output relation for Trial 2-2

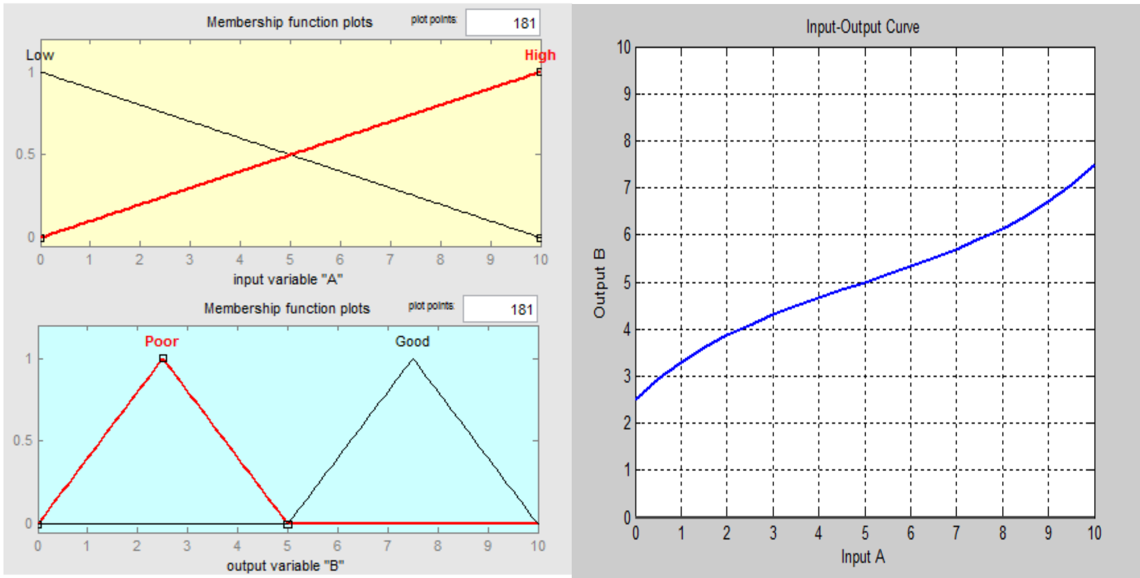


Figure 2.5 Membership function arrangement and input-output relation for Trial 2-3

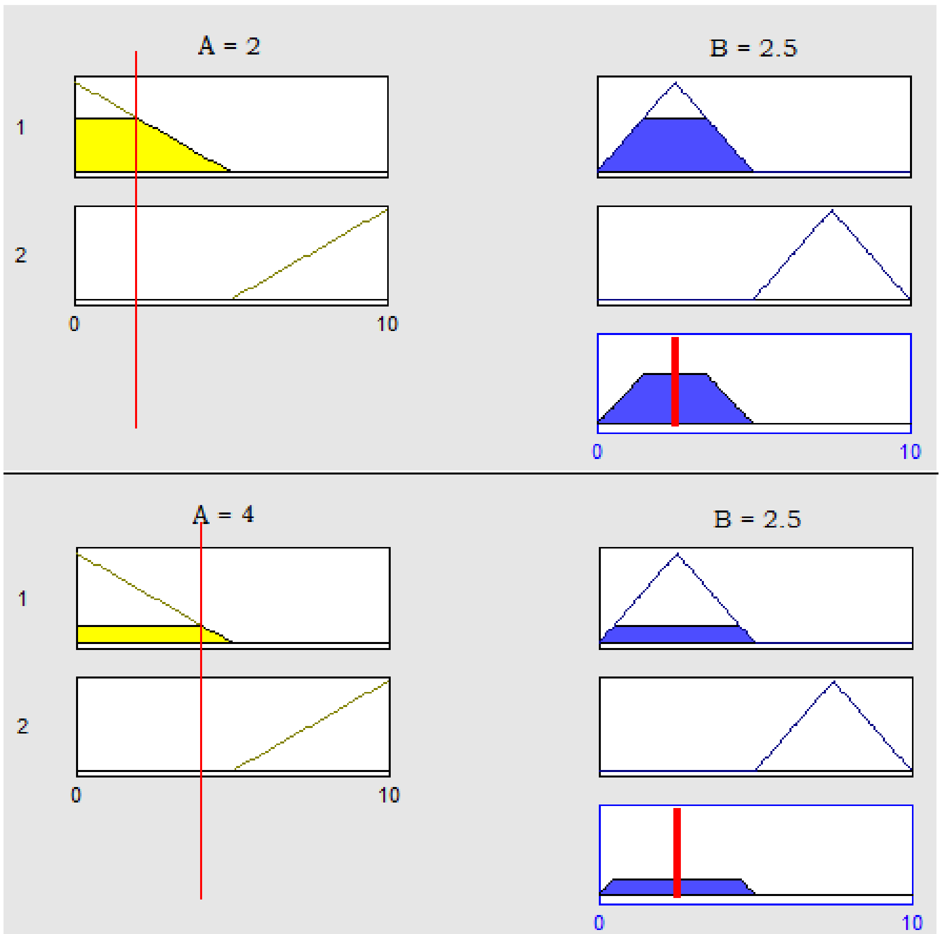


Figure 2.6 Implication and aggregation processes for Trial 2-1

same MF for output *B* is reshaped depending on a new membership value. Although these two truncated MFs have different geometric shapes, the *Centroid method* locates the centers of two areas on a same spot, 2.5. Therefore the output *B* only provides a single feedback value for the intervals in which the MFs for input variable *A* don't overlap with each other.

In Trial 2-2, the support of both MFs for input *A* are extended, and overlapped with each other by 50 percent. By overlapping neighboring MFs, the input-output relation performs a smooth curve in middle range of domain of discourse, but stair-shaped lines still turn up within the intervals where the MF does not overlap with adjacent one. When the support of both MFs for input *A* span the whole range of scale in Trial 2-3, the support of both MFs are completed overlapped. As we can expect, in this case a continuous curve with good monotonicity presents an ideal input-output relation. Because two rules become fully correlated in Trial 2-3, both MFs for output *B* will be reshaped referring to a same number from input *A* at the same time. Then as Figure 2.7 expresses, the aggregation process will yield unique geometric shape with particular center of area for every numerical value of input *A*.

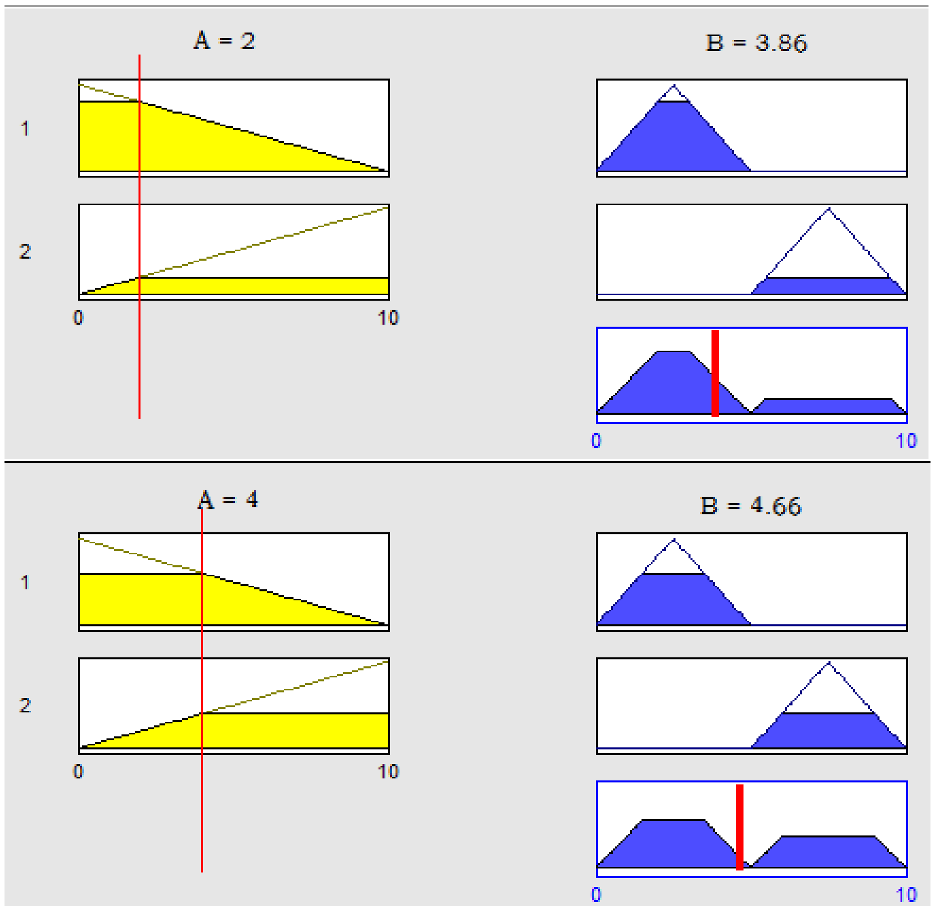


Figure 2.7 Implication and aggregation processes for Trial 2-3

If a new MF, 'Medium', is introduced to input variable *A*, and a corresponding MF, 'Fair' is added into output *B*, the similar system performance related to overlap ration between adjacent MFs should be conceivable. Following the assumptions for system modeling mentioned above, two half-triangle MFs representing fuzzy linguistic sets 'Low' and 'High' and one full-triangle MF for set 'Medium' are appropriate for defining input variable *A*. Three full-triangle MFs are used to describe output *B*. And a symmetric inference mechanism should consists of three rules

Rule 1: If *A* is Low, then *B* is Poor

Rule 2: If *A* is Medium, then *B* is Fair

Rule 3: If *A* is High, then *B* is Good

**Trial 2-4:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 0% OR)

Output *B*: 3 full-triangle MFs (0% OR)

**Trial 2-5:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 full-triangle MFs (0% OR)

As we expected, a stair-shaped line representing input-output relation appears in the Trial 2-4 where the overlap ratio is 0% among MFs for input *A*. While Trial 2-5 comes with a smooth curve, because the support of each MF is fully overlapped. The full-triangle MF, 'Medium', adjoins two half-triangle MFs, so it is reasonable to make the MF 'Medium' symmetrically overlap with both half-triangle MFs.

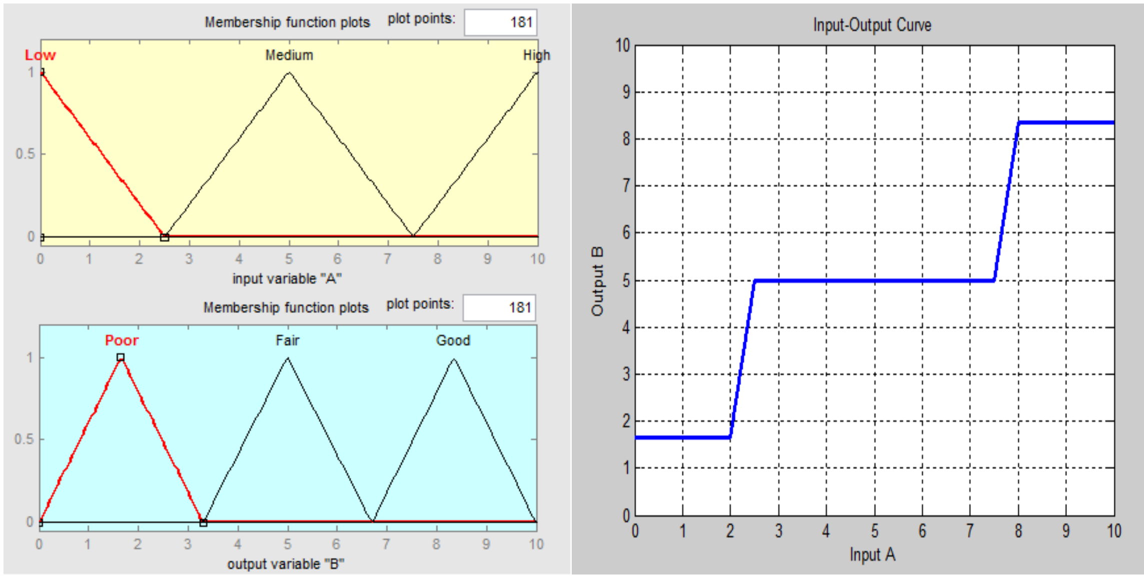


Figure 2.8 Membership function arrangement and input-output relation for Trial 2-4

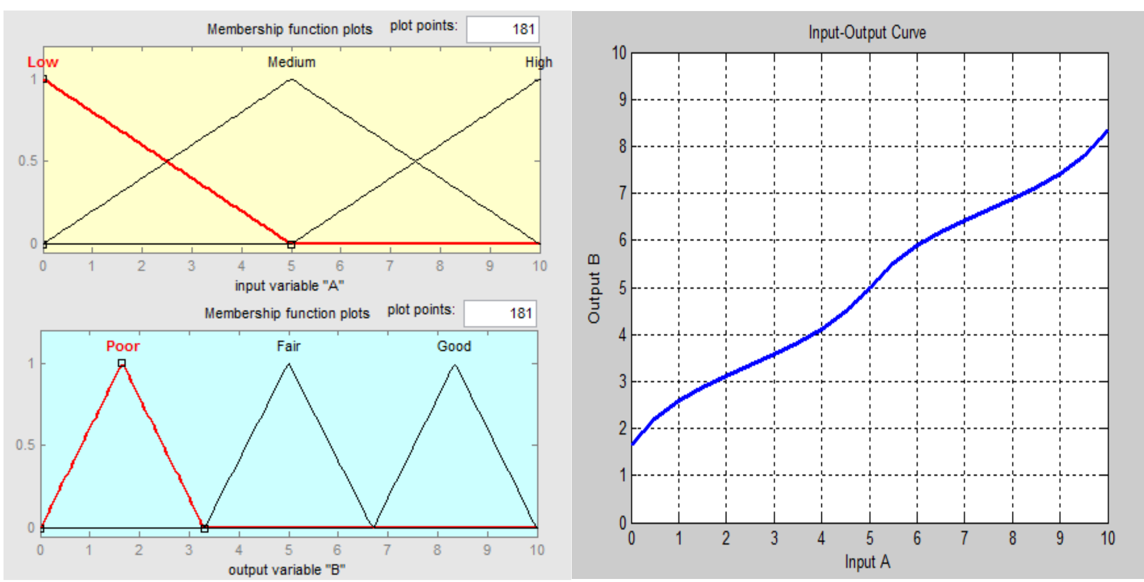


Figure 2.9 Membership function arrangement and input-output relation for Trial 2-5

### 2.3.2 Membership Functions for Output Variable

**Trial 2-6:** Input A: 2 half-triangle MFs (100% OR)

Output B: 2 full-triangle MFs (50% OR)

**Trial 2-7:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR)

**Trial 2-8:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR, separated)

**Trial 2-9:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR, separated)

From Trial 2-6 to 2-9, the MFs for input variable *A* remain unchanged, but the overlap ratio between adjacent MFs for output variable *B* is adjusted by shortening the base lines of triangle MFs. Compare with other three trials, in Trial 2-6 where each MF for output *B* is overlapped by 50%, the slope of input-output curve is the most flat one. Because when overlap happens between two contiguous MFs, the centers of two areas always move toward each other. Because of the property of Centroid method, the location of center of area with respect to abscissa exactly determines the output value, as a result the range of output value is shrunk and the slope of input-output curve decreases. In extreme case when the areas of two MFs thoroughly overlap, the center of area will never be shifted and the input-output relation turns to be a horizontal line.

On the contrary, the input-output curve with the steepest slope occurs in Trial 2-9 in which

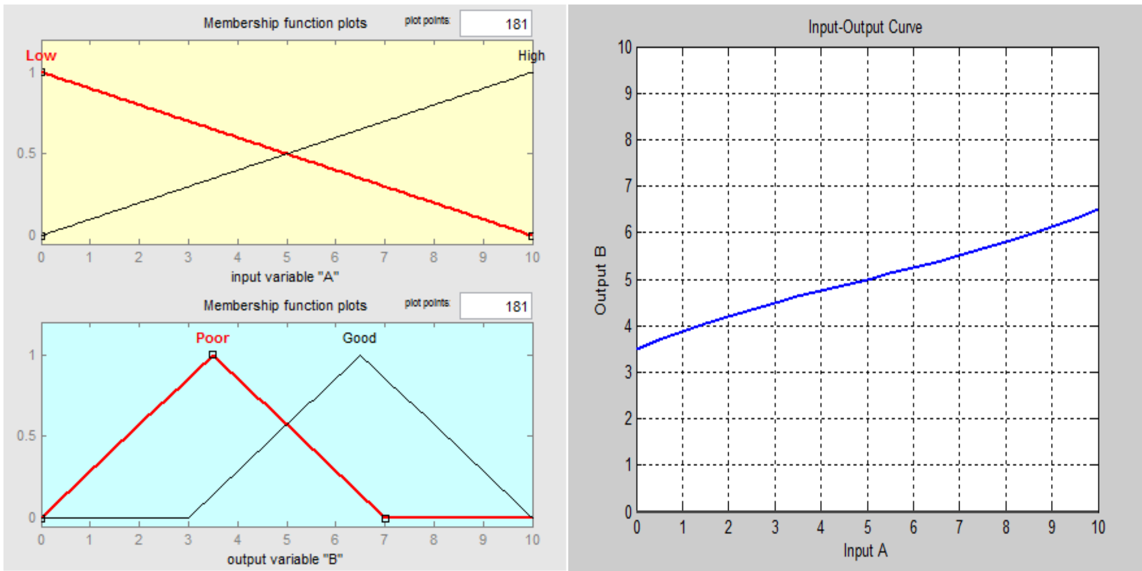


Figure 2.10 Membership function arrangement and input-output relation for Trial 2-6

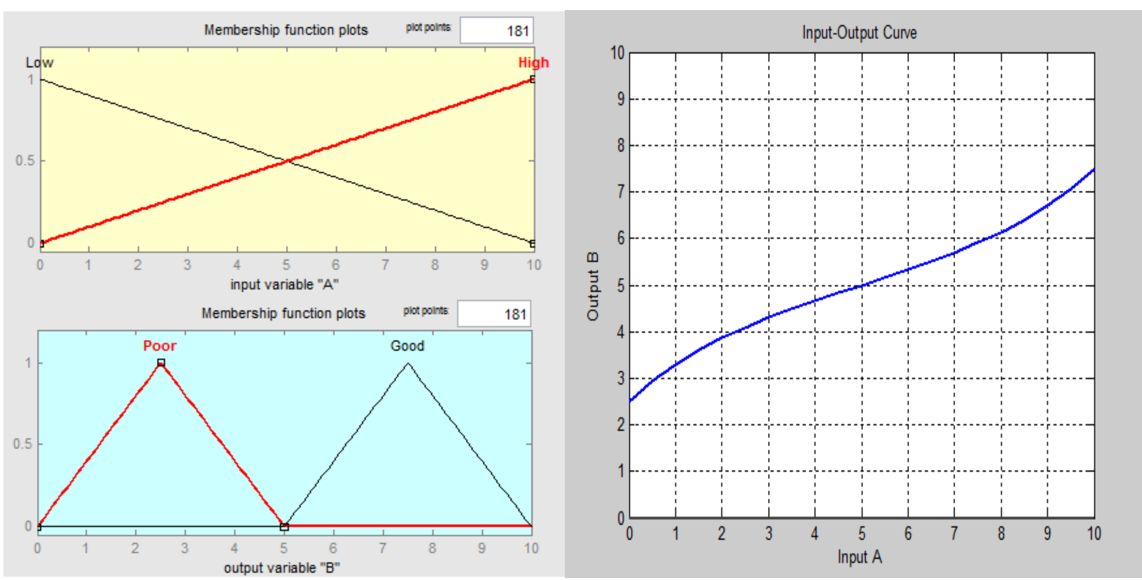


Figure 2.11 Membership function arrangement and input-output relation for Trial 2-7

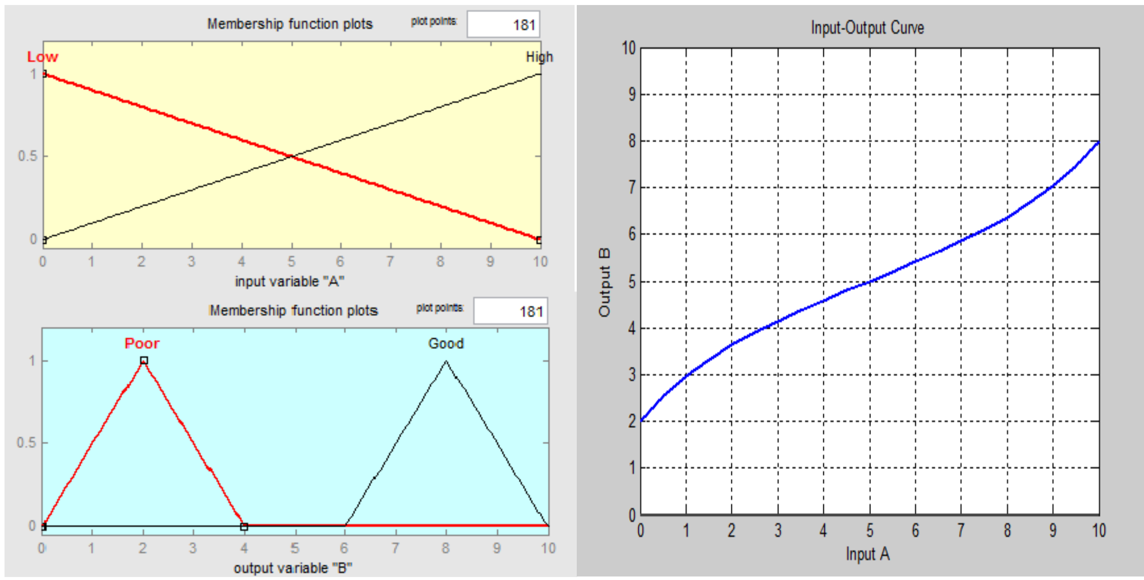


Figure 2.12 Membership function arrangement and input-output relation for Trial 2-8

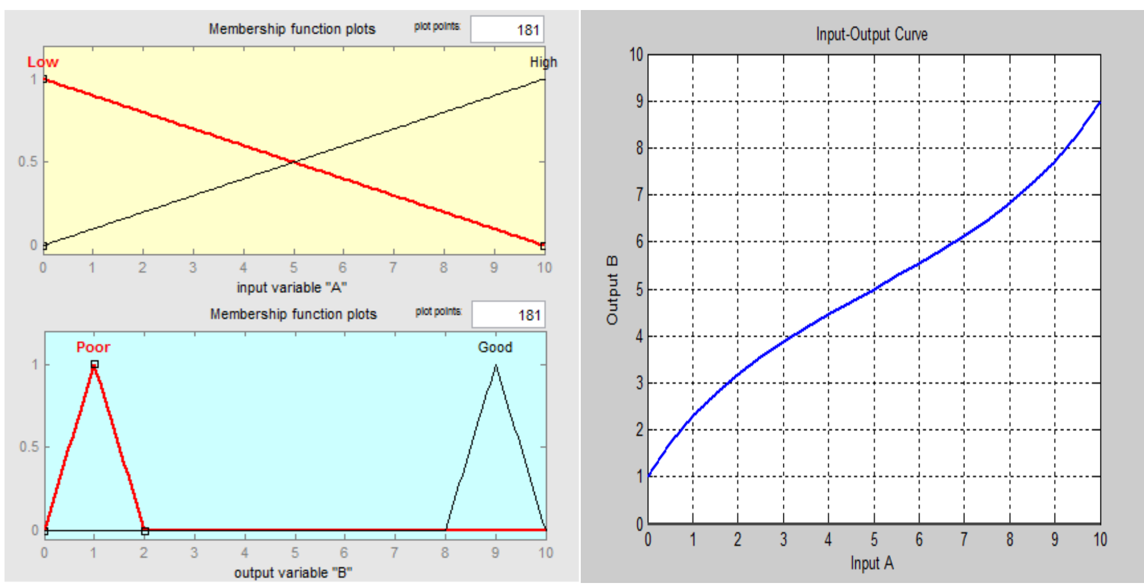


Figure 2.13 Membership function arrangement and input-output relation for Trial 2-9

the two MFs for output *B* are far separated to the ends of the universe of discourse. Because the centers of two areas are fairly close to the boundaries of output scale, the input-output curve shows a desirable monotonicity. Even if the length of base line of triangle MF keeps decreasing, the center of area can never be shifted to the endpoint, in other words the output value can never reach 0 or 10, but a similar system performance like that in Trial 2-9 where the output values span a range from 1 to 9 is good enough to express positive correlation.

## 2.4 Influence from Different Type of Membership Functions

Different kinds of MFs can undoubtedly impact the performance of fuzzy inference system. This section discusses about possible combinations of different input MFs and output MFs based on SISO Mamdani fuzzy inference model with two fuzzy sets for both input variable *A* and output variable *B*. As mentioned in former section, the two fuzzy sets for input *A*, 'Low' and 'High', will be represented by two half-shape MFs whose support cover the full range of discourse, in order to avert the stair-shaped input-output curve. For the output MFs, two full-shape MFs separated to the ends of output scale are desirable. Because in complex MISO (Multi-Input Single-Output) inference system, more MFs are required to finely subdivide the range of output scale. For the purpose of maintaining consistent monotonicity, all output MFs should be constrained by same geometric characteristics. Thus for output variable, the full-shape MFs are appropriate to meet the condition.

The following trials focus on three general types of MFs, Triangle MF, Gaussian MF, and Trapezoidal MF for both input *A* and output *B*. Meanwhile, a special type of trapezoidal

MF --- Rectangular MF is tested for representing output *B*.

**Trial 2-10 (same as Trial 2-9):** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR)

**Trial 2-11:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-Gaussian MFs (0% OR)

**Trial 2-12:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 full-trapezoidal MFs (0% OR)

**Trial 2-13:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

**Trial 2-14:** Input *A*: 2 half-trapezoidal MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR)

**Trial 2-15:** Input *A*: 2 half-trapezoidal MFs (100% OR)

Output *B*: 2 full-Gaussian MFs (0% OR)

**Trial 2-16:** Input *A*: 2 half-trapezoidal MFs (100% OR)

Output *B*: 2 full-trapezoidal MFs (0% OR)

**Trial 2-17:** Input *A*: 2 half-trapezoidal MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

**Trial 2-18:** Input *A*: 2 half-Gaussian MFs (100% OR)

Output *B*: 2 full-triangle MFs (0% OR)

**Trial 2-19:** Input *A*: 2 half-Gaussian MFs (100% OR)

Output *B*: 2 full-Gaussian MFs (0% OR)

**Trial 2-20:** Input *A*: 2 half-Gaussian MFs (100% OR)

Output *B*: 2 full-trapezoidal MFs (0% OR)

**Trial 2-21:** Input *A*: 2 half-Gaussian MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

From Trial 2-10 to 2-21, a total twelve possible combinations with 3 kinds of MFs for input *A* and 4 types of MFs for output *B* are performed. Following the conclusions about overlap ratio from the previous section, all of these twelve models generate input-output relations with distinct monotonicity, and all ranges of output values are restricted to the identical interval [1, 9].

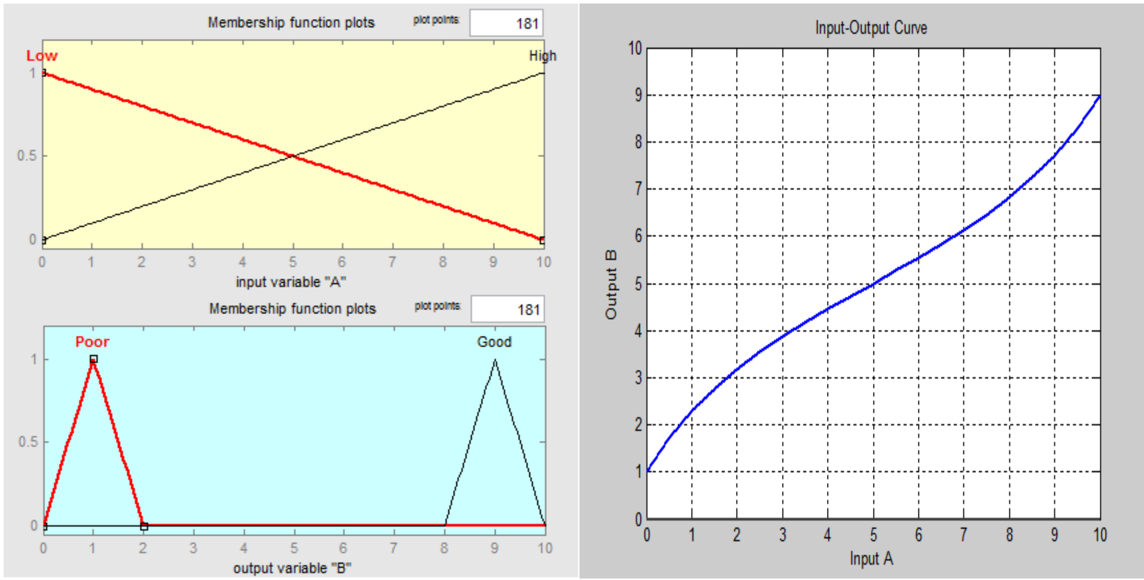


Figure 2.14 Membership function arrangement and input-output relation for Trial 2-10

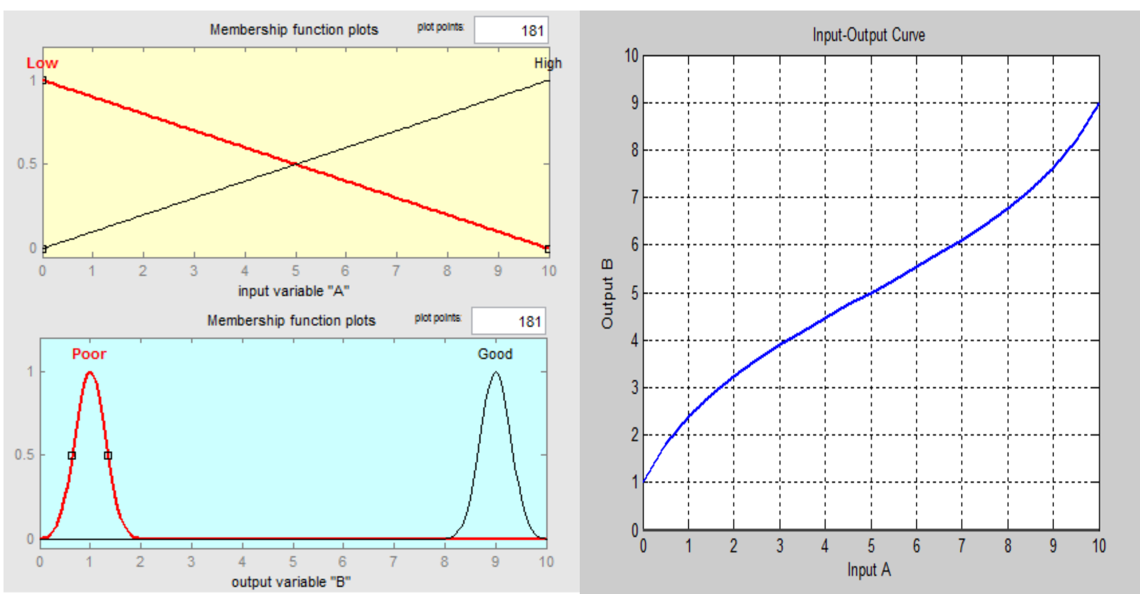


Figure 2.15 Membership function arrangement and input-output relation for Trial 2-11

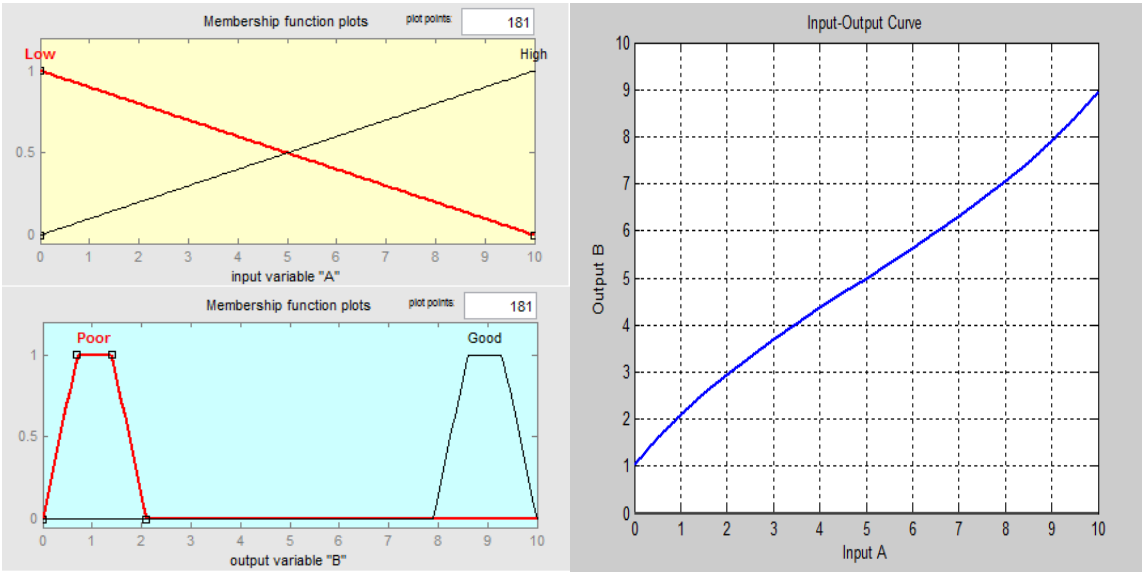


Figure 2.16 Membership function arrangement and input-output relation for Trial 2-12

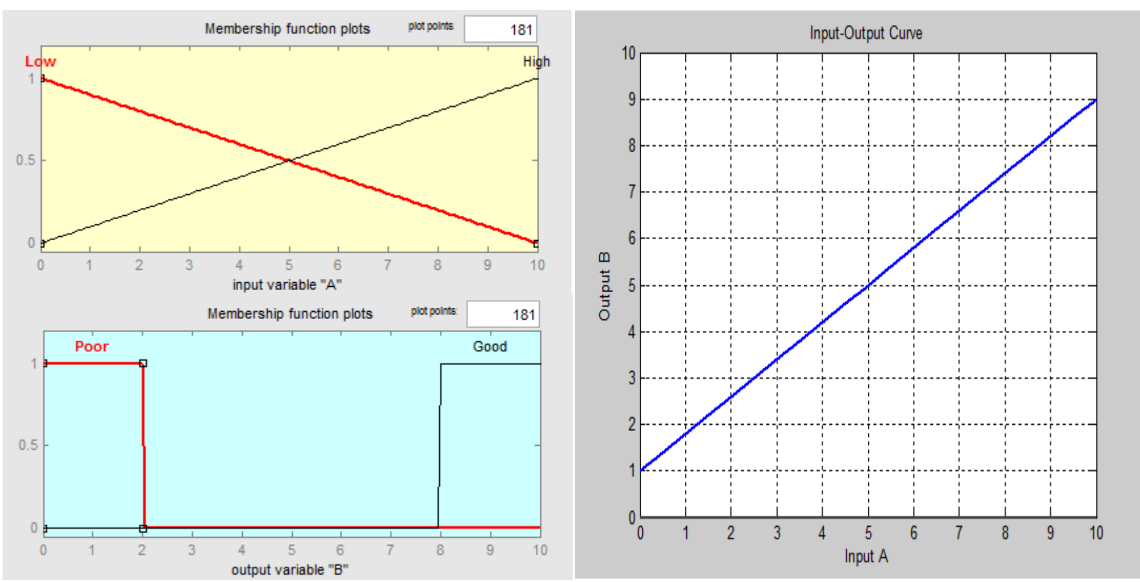


Figure 2.17 Membership function arrangement and input-output relation for Trial 2-13

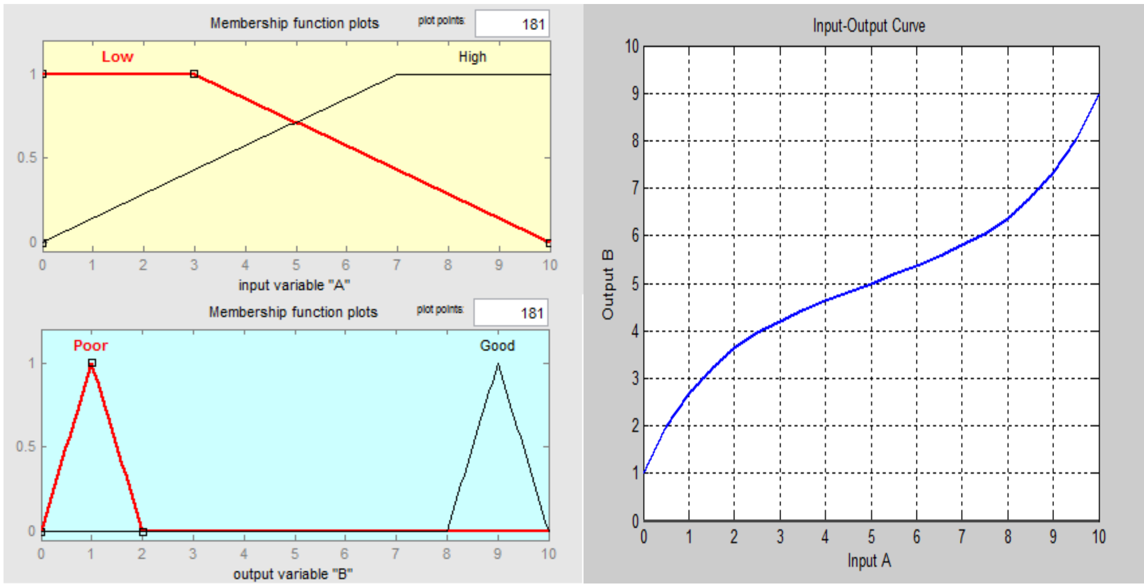


Figure 2.18 Membership function arrangement and input-output relation for Trial 2-14

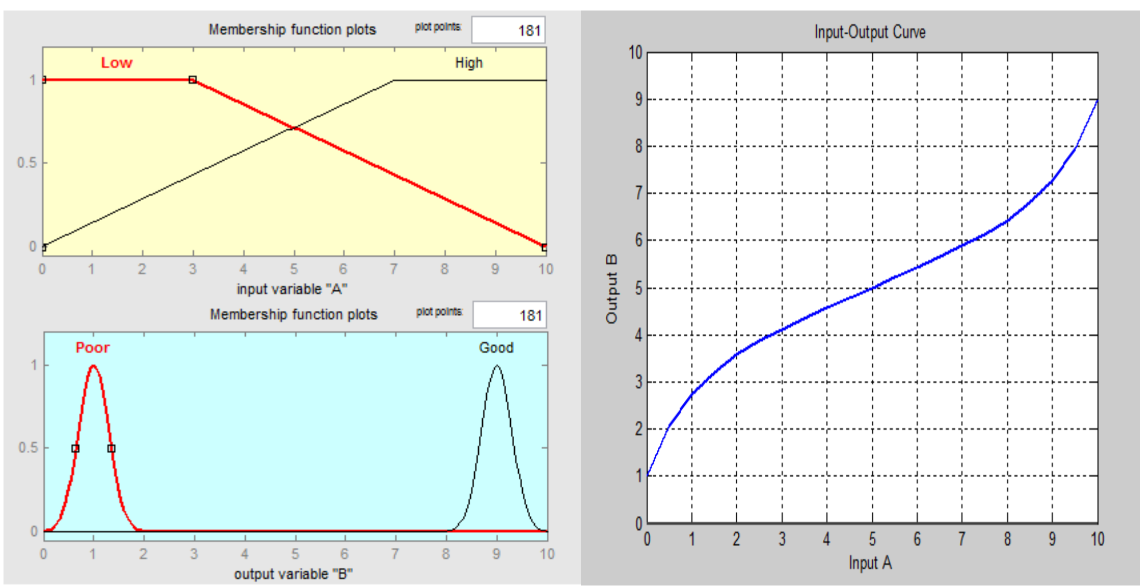


Figure 2.19 Membership function arrangement and input-output relation for Trial 2-15

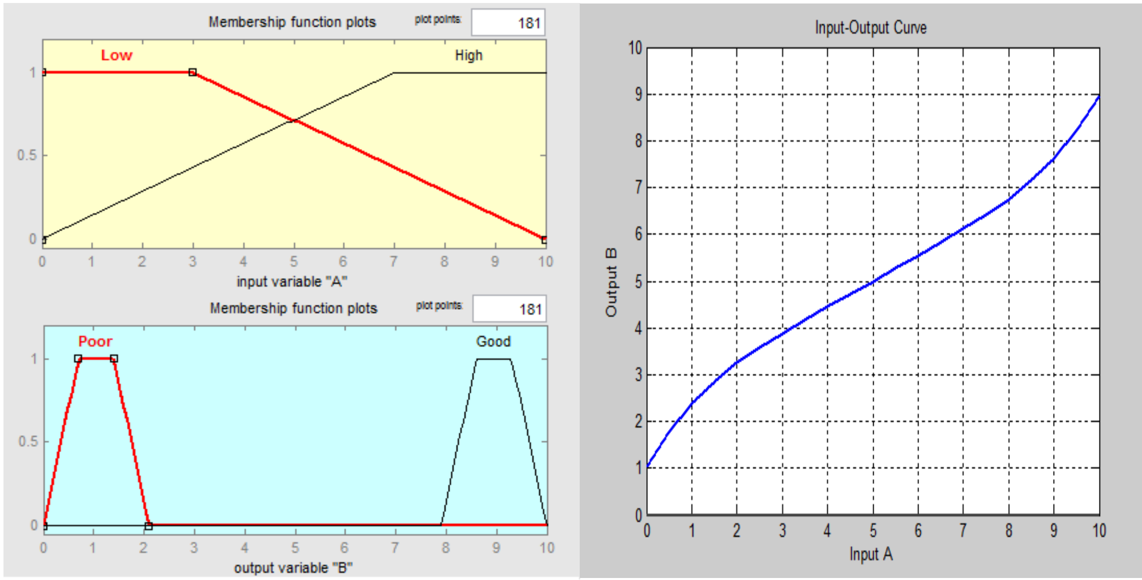


Figure 2.20 Membership function arrangement and input-output relation for Trial 2-16

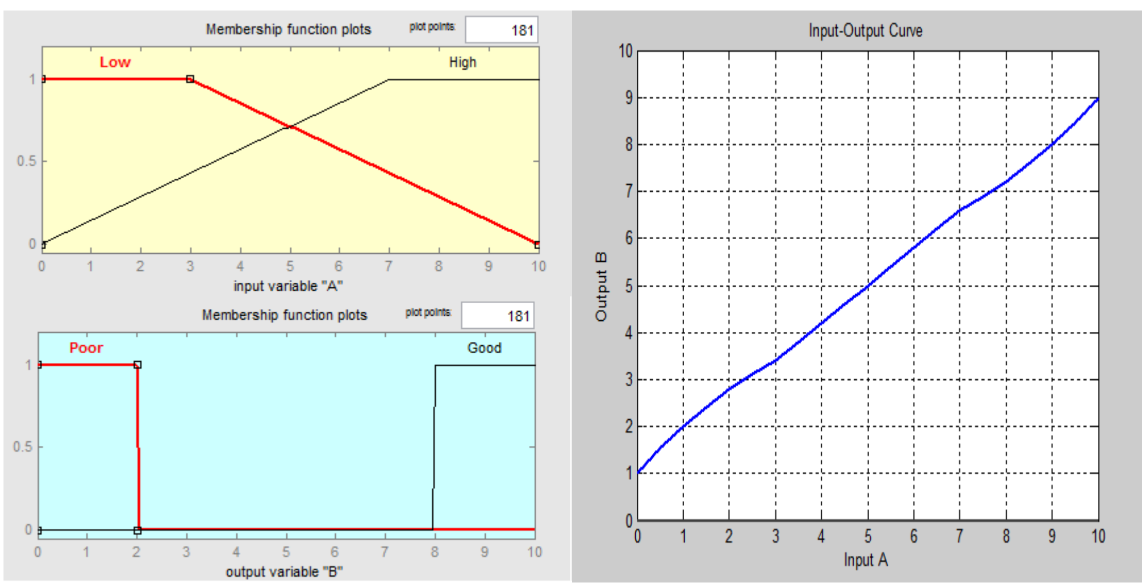


Figure 2.21 Membership function arrangement and input-output relation for Trial 2-17

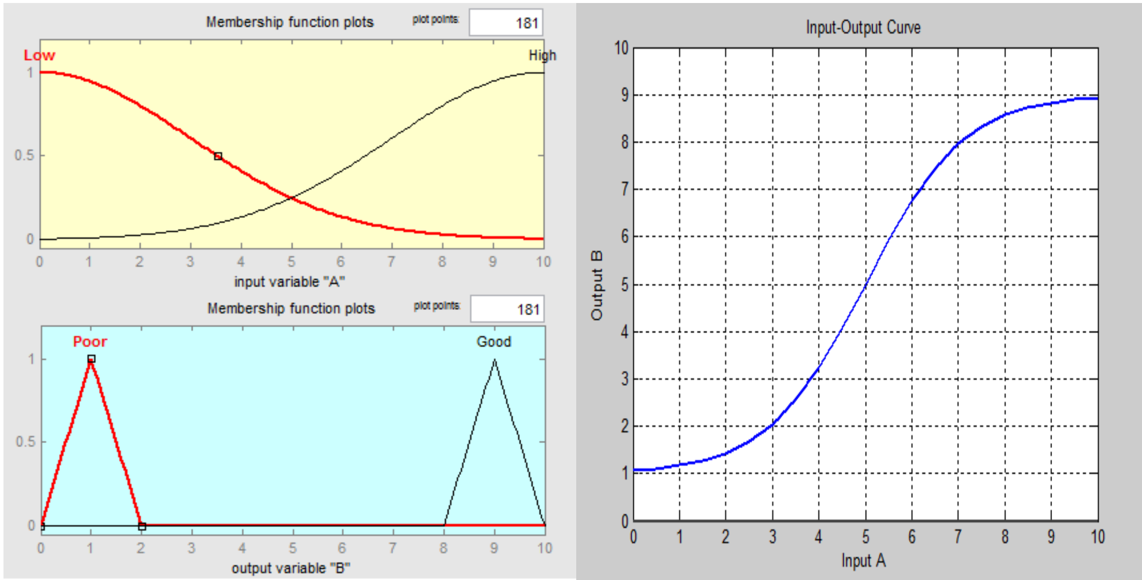


Figure 2.22 Membership function arrangement and input-output relation for Trial 2-18

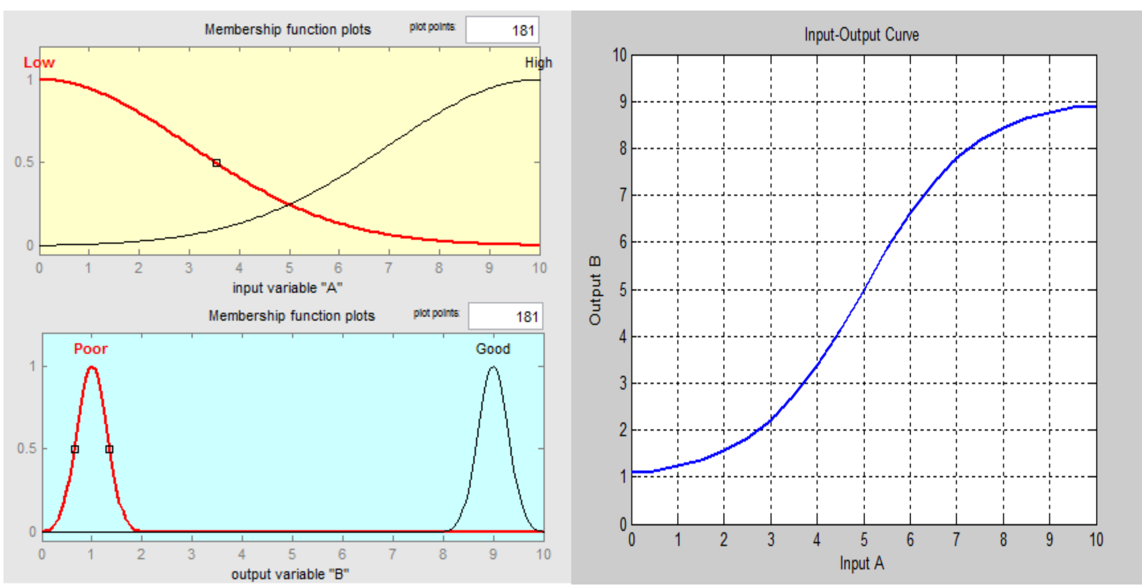


Figure 2.23 Membership function arrangement and input-output relation for Trial 2-19

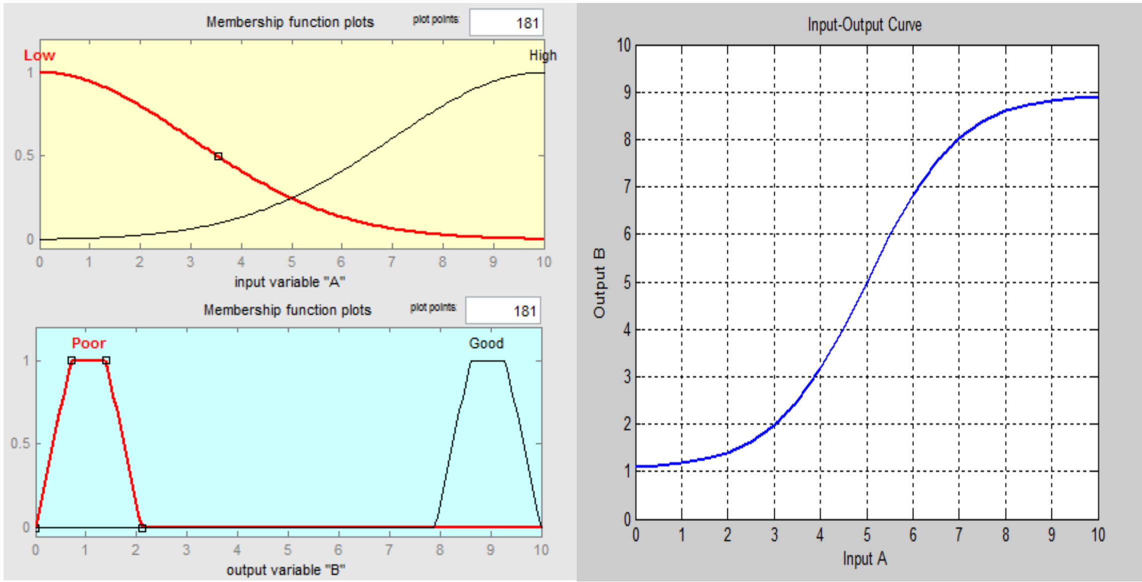


Figure 2.24 Membership function arrangement and input-output relation for Trial 2-20

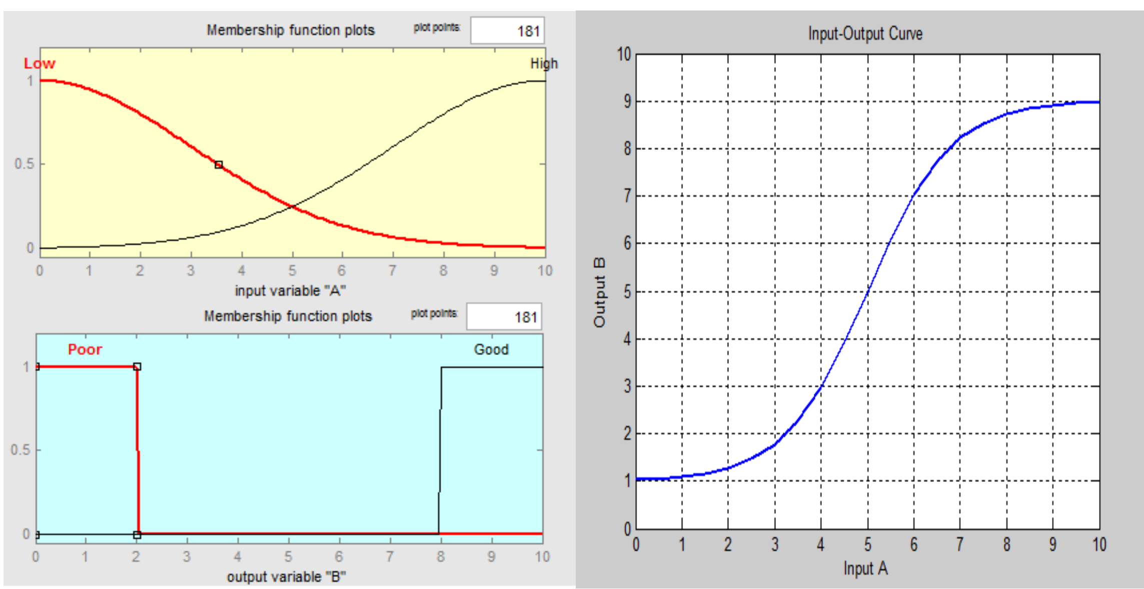


Figure 2.25 Membership function arrangement and input-output relation for Trial 2-21

By comparing the different situations in the following input-output relation chart, the models comprised by same MFs for input and different MFs for output display a common rising trend in the input-output curves. On the other hand, the models constituted by same type of output MFs and different types of input MFs make different tendencies in their input-output curves. Because the area of output MFs are confined in order to extend the span of output values, the role that MFs play in output variable is weakened.

From these twelve combinations, it is observed that the trapezoidal MFs for input variable increasingly steepen the slope of input-output curve on both ends, while oppositely the Gaussian MFs gradually flatten the slope of the curve on the two ends. And the most surprising performance is produced from the model that half-triangle MFs for input and rectangular MFs for output contribute the perfect linear input-output relation. Because triangle MF performs linear gradients, and rectangular MFs directly transfer this linear relation gradients onto output space, it is imaginable for SISO fuzzy inference system with this combination to express an ideal linear input-output relation. In the latter section which studies the controllability of fuzzy inference system will also be based on this linear model from Trial 2-13.

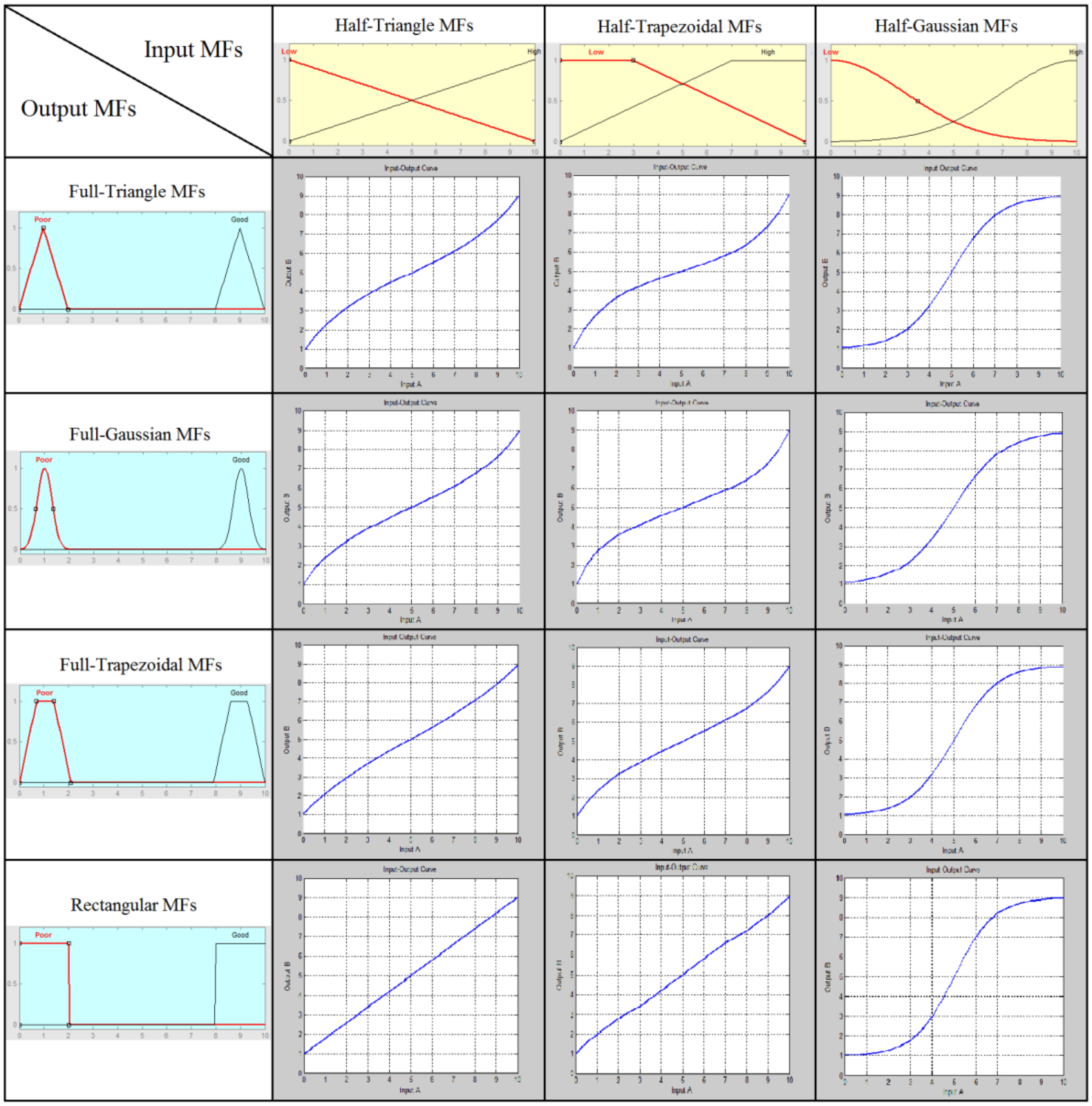


Figure 2.26 Chart of input-output relations from different membership functions for SISO models

## 2.5 Number of Membership Functions in Variables

In diverse applications of fuzzy inference system, the quantity of membership functions describing a single input or output variable is different, and no general standard exists. For instance, to define the range of input variable 'temperature', a mechanism of two fuzzy sets, 'Low' and 'High', is applicable, and a series of fuzzy sets with three levels, 'Low', 'Medium' and 'High', is also reasonable. This section studies the internal connection between the quantity of MFs for single variable and inference system performance.

Based on the SISO Mamdani fuzzy inference model with two MFs for each variable, one more MF is introduced into input variable *A*. Input *A* with three fuzzy sets, 'Low', 'Medium' and 'High', will be defined by two half-shape MF and one full-shape MF. To ensure a complete and symmetric rule mechanism, three matching MFs for output *B*, 'Poor', 'Fair' and 'Good', are required. All MFs for output will be expressed by identical full-shape MFs. The appropriate rules for new model are

Rule 1: If *A* is Low, then *B* is Poor

Rule 2: If *A* is Medium, then *B* is Fair

Rule 3: If *A* is High, then *B* is Good

Then a model with five MFs for each variable is considered. Input *A* comes with levels 'Very Low', 'Low', 'Medium', 'High' and 'Very High' represented by two half-shape MFs and three full-shape MFs. Correspondingly, five MFs, 'Very Poor', 'Poor', 'Fair', 'Good' and 'Very Good' are expected for output *B*. Rules for this model are compiled as

Rule 1: If *A* is Very Low, then *B* is Very Poor

Rule 2: If *A* is Low, then *B* is Poor

Rule 3: If *A* is Medium, then *B* is Fair

Rule 4: If *A* is High, then *B* is Good

Rule 5: If *A* is Very High, then *B* is Very Good

**Trial 2-22:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 full-triangle MFs (0% OR)

**Trial 2-23:** Input *A*: 5 triangle MFs (2 half MFs + 3 full MF, 100% OR)

Output *B*: 5 full-triangle MFs (0% OR)

In Trial 2-22, the same model in Trial 2-10 is rebuilt by constructing input *A* with three fuzzy sets. And in Trial 2-23, two more MFs are brought in to further subdivide the range of input *A*. The following figure shows a clear relation among the models whose input and output variable are expressed by two, three and five MFs, respectively. The input-output curve for Trial 2-22 repeats the input-output curve for Trial 2-10 once more, while the input-output curve for Trial 2-23 repeats the curve of the two-MF defined model three times. When the times of repeat keeps growing, because the range of input-output curve remains unchanged, and the original curve coming from Trial 2-10 is shrunk in proportion, the variance of the new curve decreases and the linearity is improved.

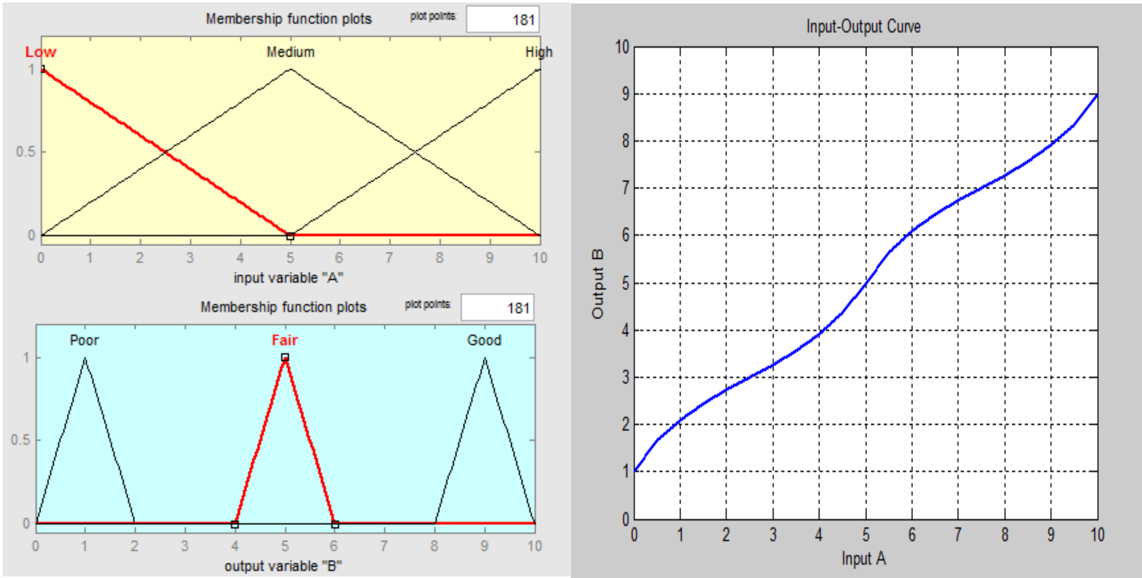


Figure 2.27 Membership function arrangement and input-output relation for Trial 2-22

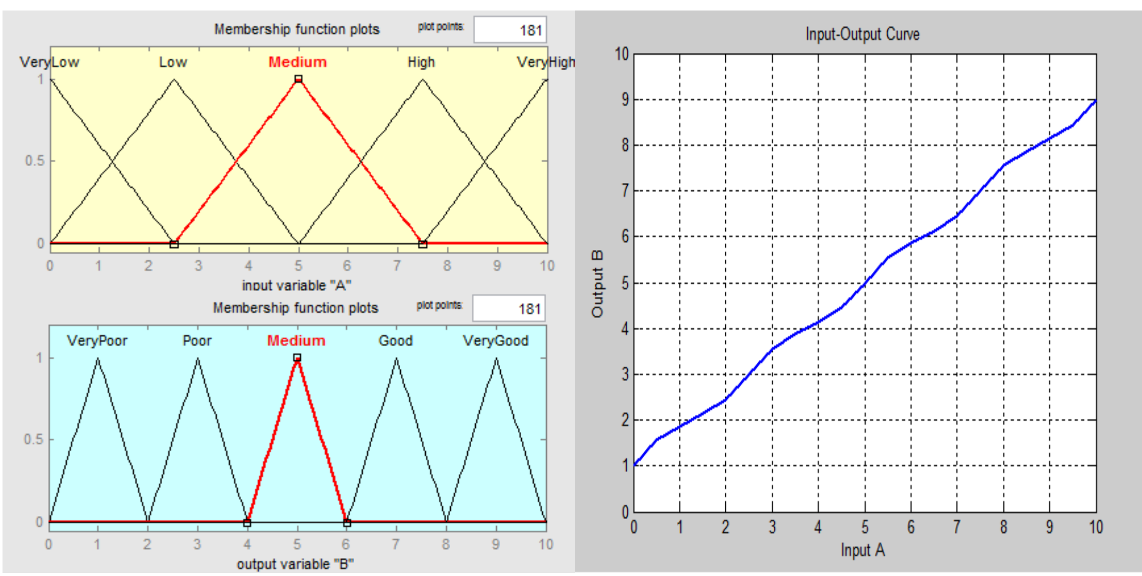


Figure 2.28 Membership function arrangement and input-output relation for Trial 2-23

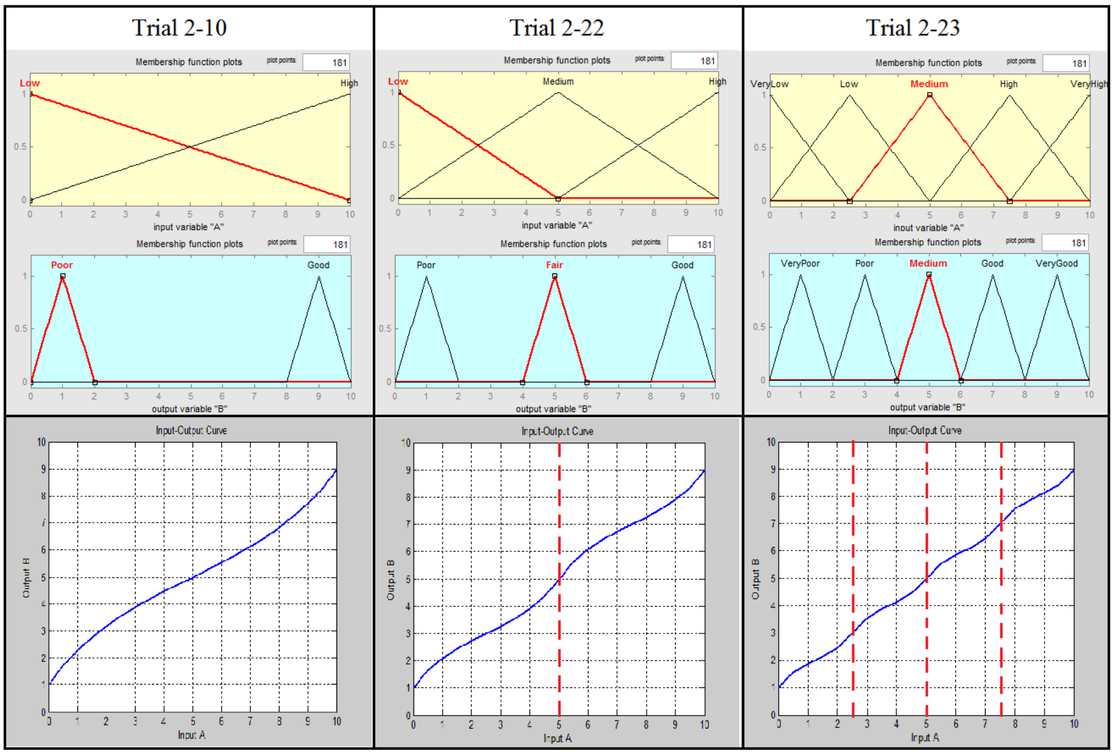


Figure 2.29 Comparison among Trials 2-10, 2-22 and 2-23

**Trial 2-24:** Input *A*: 3 Gaussian MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 full-triangle MFs (0% OR)

**Trial 2-25:** Input *A*: 5 Gaussian MFs (2 half MFs + 3 full MF, 100% OR)

Output *B*: 5 full-triangle MFs (0% OR)

Similar to Trials 2-22 and 2-23, in Trials 2-24 and 2-25 the MFs for input *A* are replaced by Gaussian MFs. A same tendency is displayed in the figure below. Compared with the input-output curve from Trial 2-18, the curve for Trial 2-24 repeats the original one for one more time, and another curve for Trial 2-25 repeats three times.

Thus, we have reasons to imagine that when the quantity of MFs for a single variable increases, the new input-output curve will repeat the original curve which comes from an inference model with each variable defined by two MFs appropriate times, and the same situation will happen in any SISO fuzzy inference system no matter what types of MFs are adopted for expressing input or output variable.

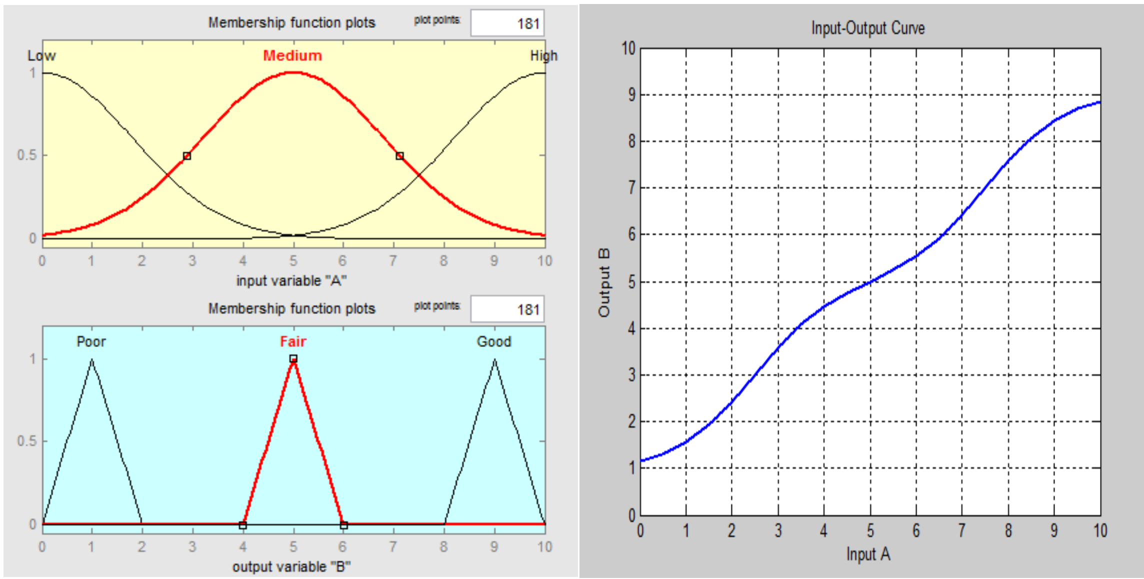


Figure 2.30 Membership function arrangement and input-output relation for Trial 2-24

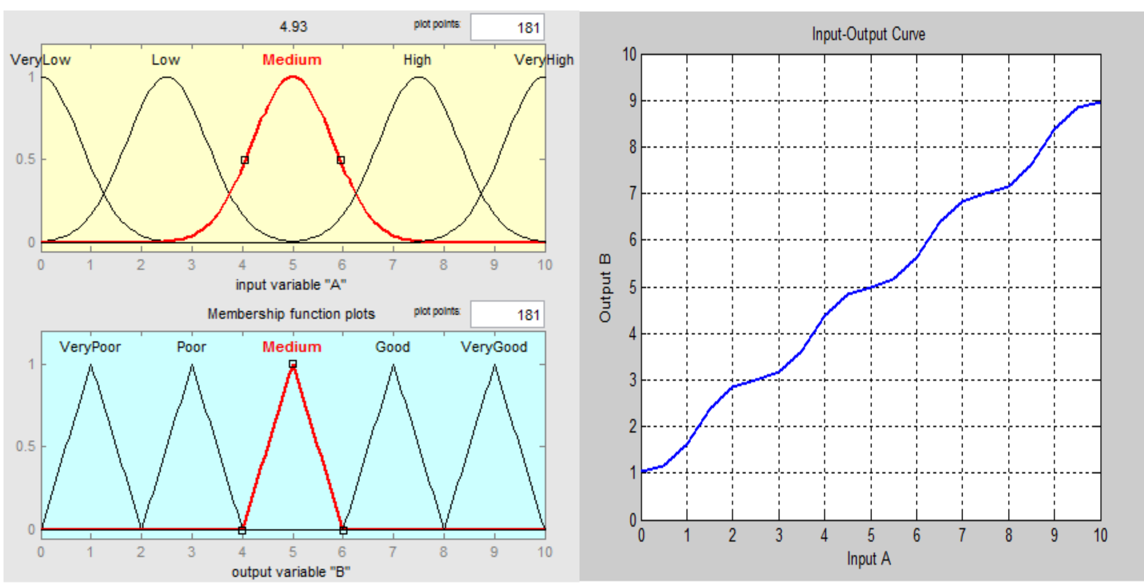


Figure 2.31 Membership function arrangement and input-output relation for Trial 2-25

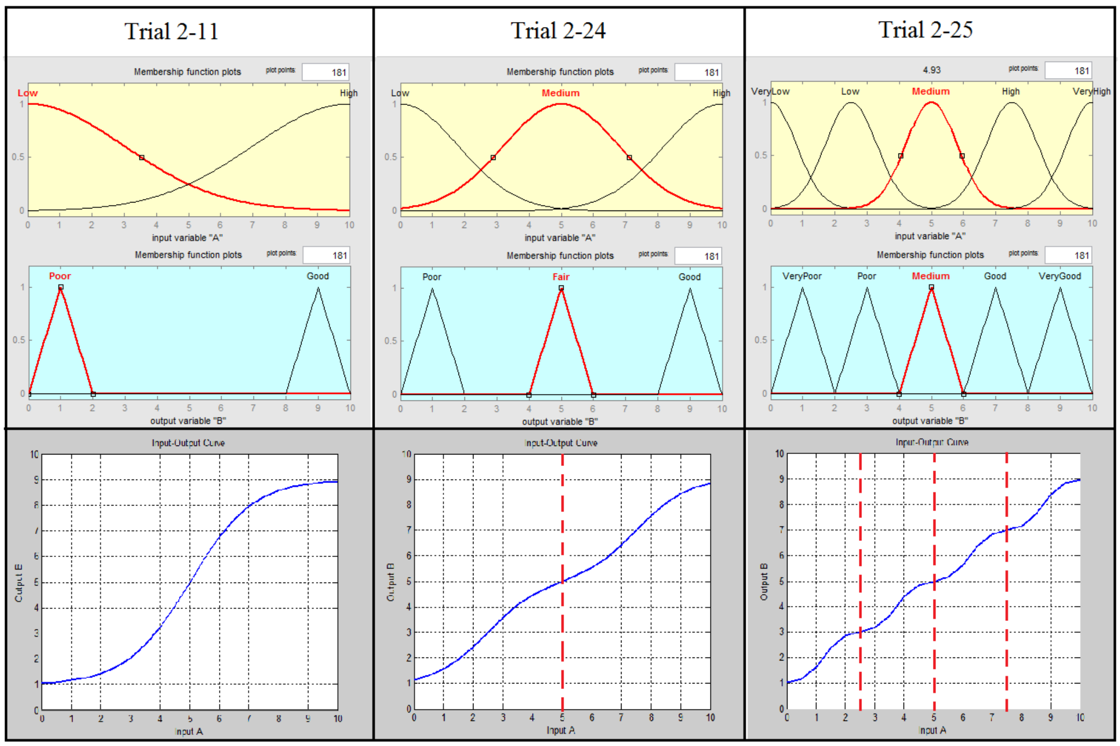


Figure 2.32 Comparison among Trials 2-11, 2-24 and 2-25

**Trial 2-26:** Input *A*: 5 triangle MFs (2 half MFs + 3 full MF, 100% OR)

Output *B*: 5 rectangular MFs (0% OR)

In Trial 2-26, it is proved one more time that when more MFs are used to describe input variable and its matching output variable in SISO Mamdani fuzzy inference system, the new input-output curve turns to duplicate a certain curve from a same model with less MFs for input and output. Since in Trial 2-13, triangle input MFs and rectangular output MFs generate the input-output relation with perfect linearity, the input-output relation from Trial 2-26 still maintains the same linear performance.

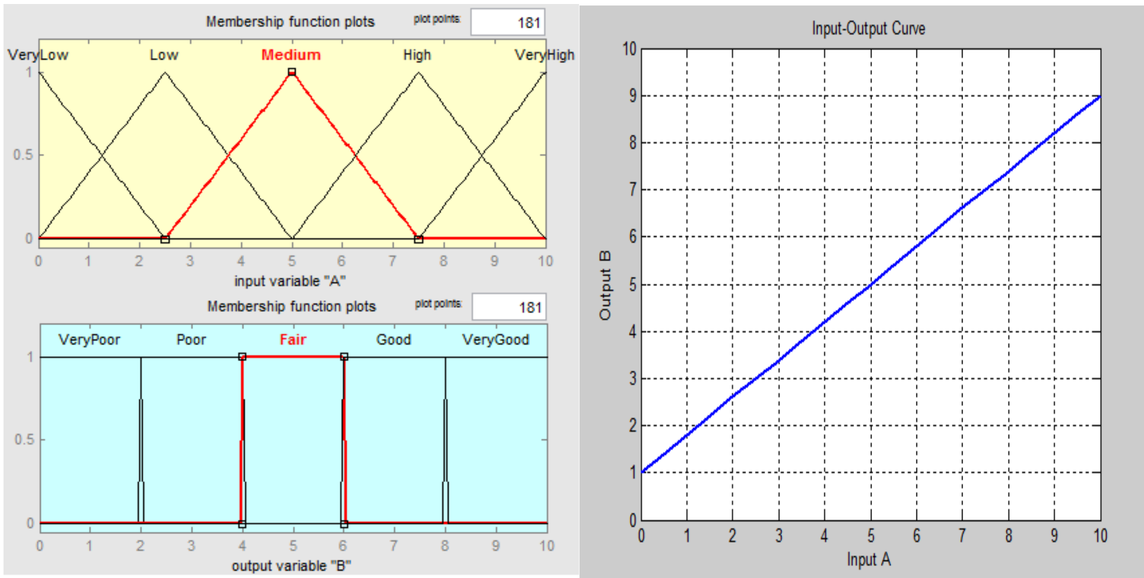


Figure 2.33 Membership function arrangement and input-output relation for Trial 2-26

## 2.6 Control of Input-Output Relation

In fuzzy inference systems for practical purpose, not only the linear input-output relation is desired, but also some non-linear input-output relations are indispensable. This section focuses on setting up classical non-linear fuzzy inference systems via adjusting linear model. As it is discussed in former section, triangle input MFs and rectangular output MFs compose the SISO inference system with perfect linear performance. Because the layouts of MFs for input *A* in preceding trials are equally distributed and generally applicable, and it is prone to manipulate rectangular MFs than adjust triangle ones, all non-linear inference models in this section consist of unique rectangular MFs for output *B* and common triangle MFs for input *A*.

### 2.6.1 Variable with Two Membership Functions

Input *A*: Low / High   Output *B*: Poor / Good

Rule 1: If *A* is Low, then *B* is Poor

Rule 2: If *A* is High, then *B* is Good

**Trial 2-27 (same as Trial 2-13):** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

**Trial 2-28:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

**Trial 2-29:** Input *A*: 2 half-triangle MFs (100% OR)

Output *B*: 2 rectangular MFs (0% OR)

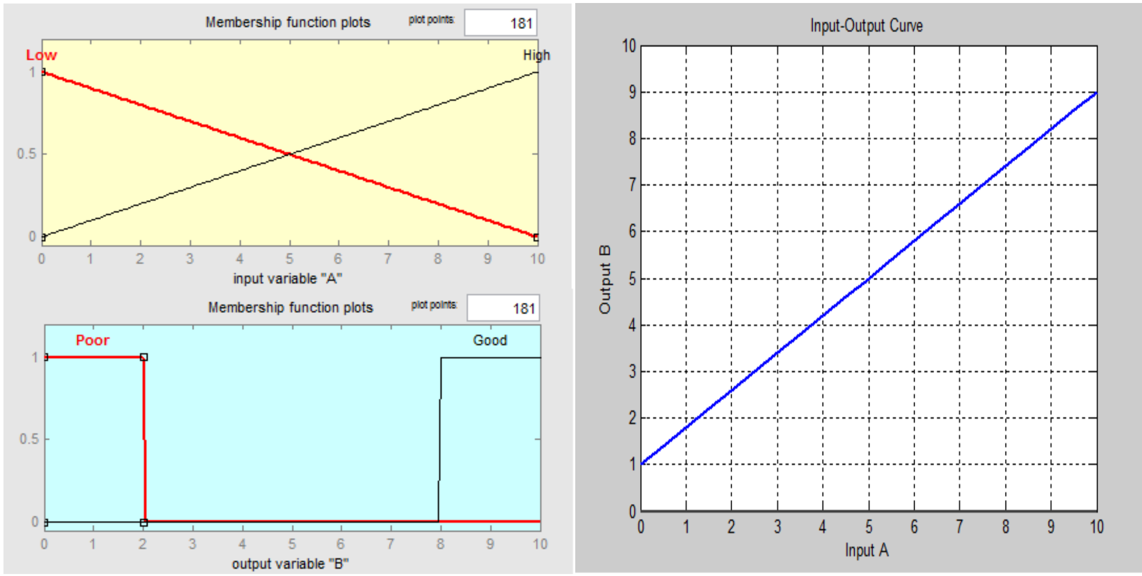


Figure 2.34 Membership function arrangement and input-output relation for Trial 2-27

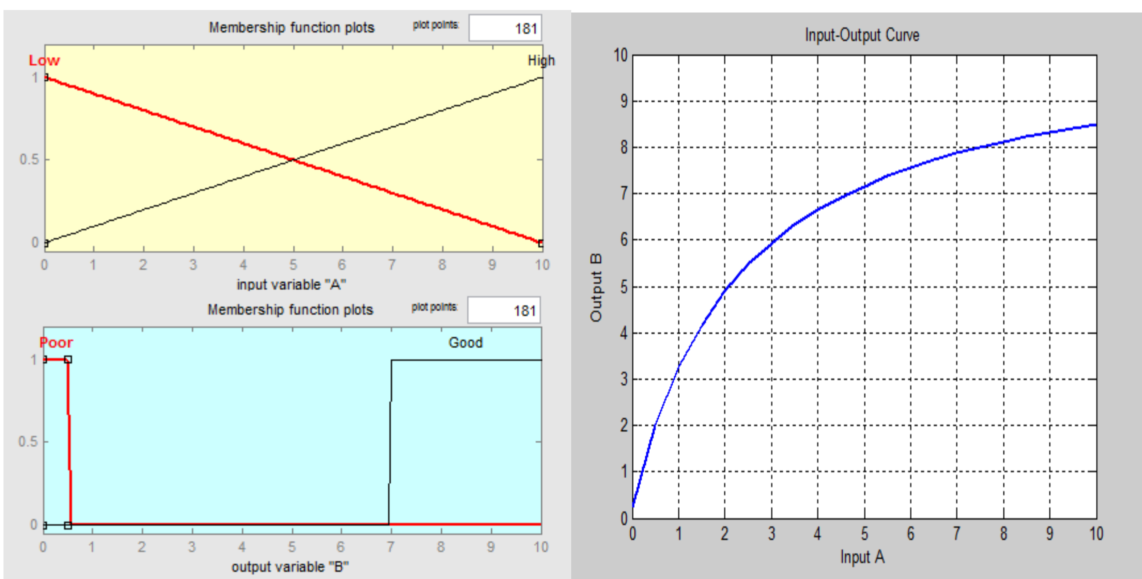


Figure 2.35 Membership function arrangement and input-output relation for Trial 2-28

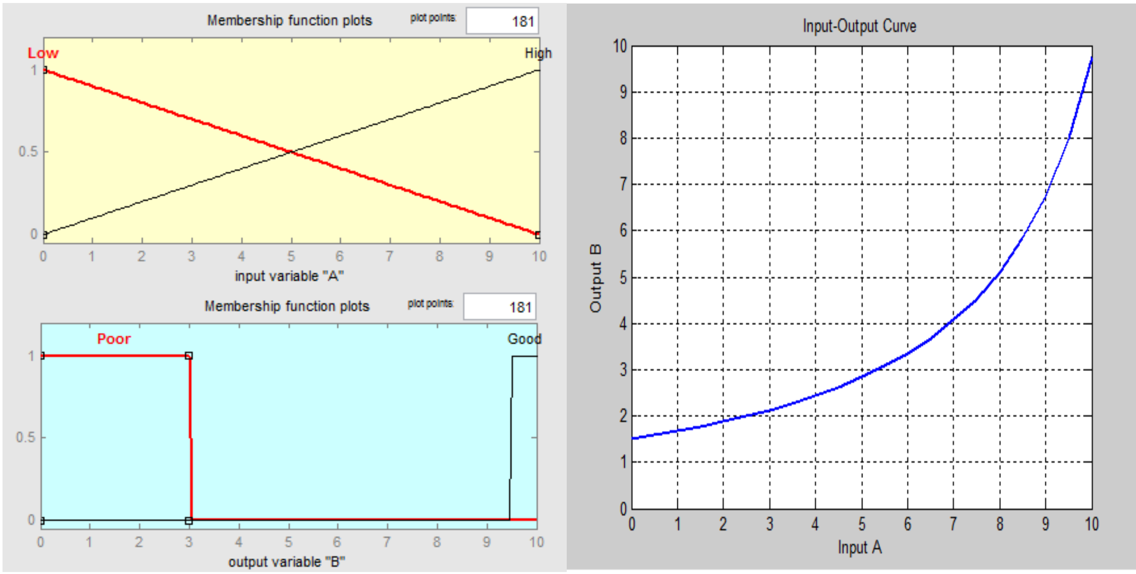


Figure 2.36 Membership function arrangement and input-output relation for Trial 2-29

Based on the Trial 2-27 whose system performance shows an ideal linearity, the Models 2-28 and 2-29 are created by modifying the support of rectangular MFs for output. In Trial 2-28, the area of MF ‘Poor’ is less than that of MF ‘Good’. Because the MF with larger area plays dominant role in inference process, the output value increases sharply at beginning and the grow rate gradually slows down along with input value rises. In this case, the inference performance of Trial 2-28 seems close to a logarithmic curve. The opposite situation happens in Trial 2-29, where MF ‘Poor’ has larger area, and as a result the input-output curve of Trial 2-29 looks similar to exponential curve. Because only two rules form the inference mechanism and each variable simply comes with two MFs, it has many limitations to constitute more complex non-linear inference systems.

### 2.6.2 Variable with Three Membership Functions

Input *A*: Low / Medium / High   Output *B*: Poor / Fair / Good

Rule 1: If *A* is Low, then *B* is Poor

Rule 2: If *A* is Medium, then *B* is Fair

Rule 3: If *A* is High, then *B* is Good

**Trial 2-30:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-31:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-32:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-33:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-34:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-35:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-36:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

**Trial 2-37:** Input *A*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *B*: 3 rectangular MFs (0% OR)

By modulating the rectangular output MFs of Model 2-30, a collection of various non-linear SISO fuzzy inference models are produced from Trial 2-31 to 2-37. Compared with inference model whose variable is defined by only two MFs, a model with three MFs for each variable undoubtedly has more degree of freedom to control system performance.

Trial 2-31, as Trial 2-28 does, performs a model with logarithmic-shaped input-output curve, while Trial 2-32 repeats an exponential curve which is similar to the result of Trial 2-29. More than that, Trial 2-33 proves that compressing the support of MF ‘Fair’ and extending the support of MF ‘Poor’ and ‘Good’ can make an input-output relation

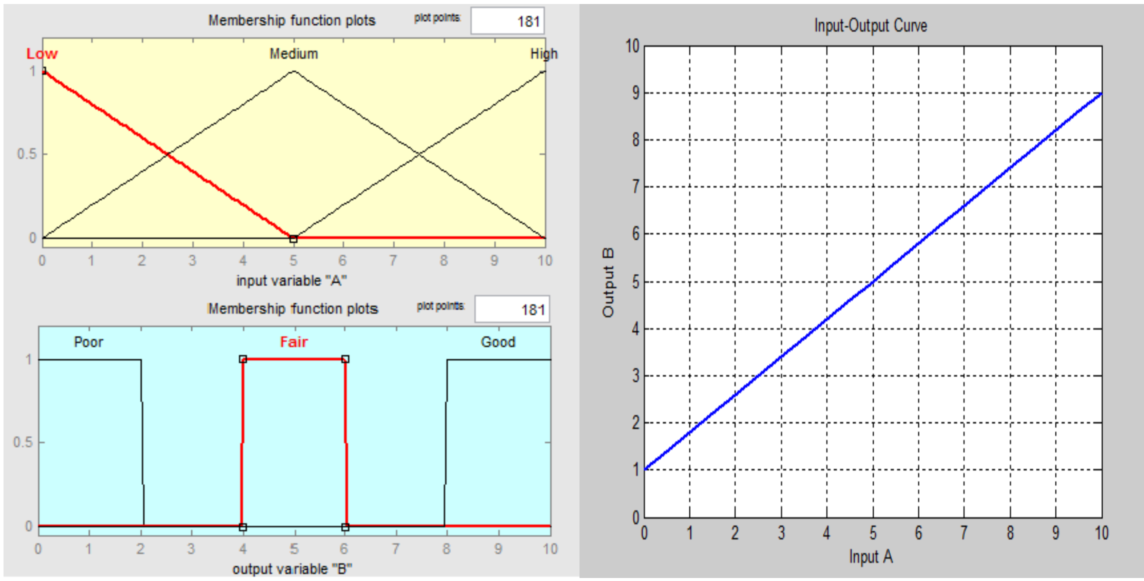


Figure 2.37 Membership function arrangement and input-output relation for Trial 2-30

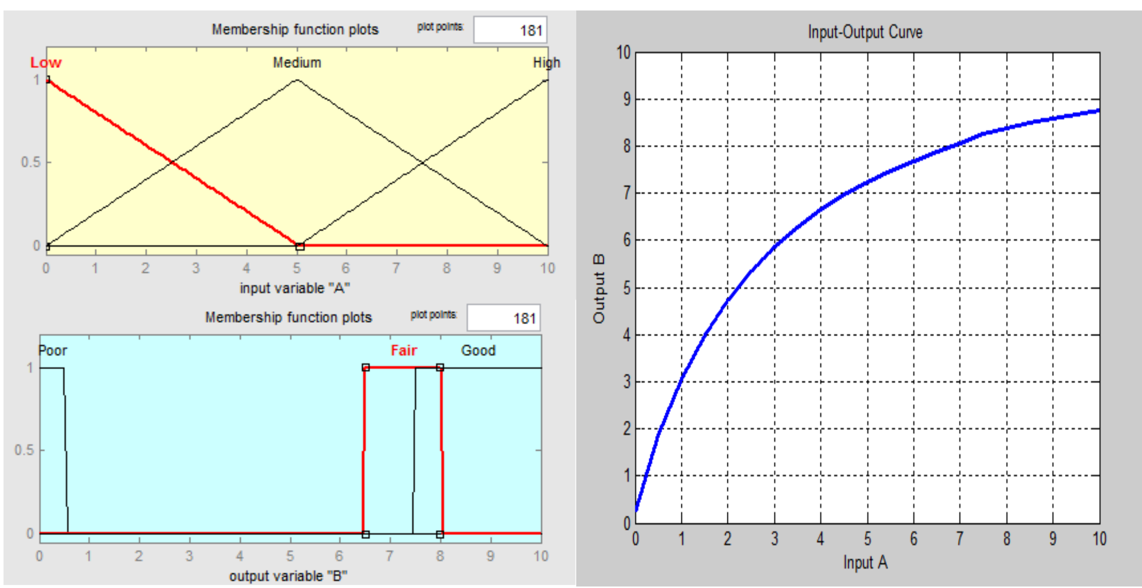


Figure 2.38 Membership function arrangement and input-output relation for Trial 2-31

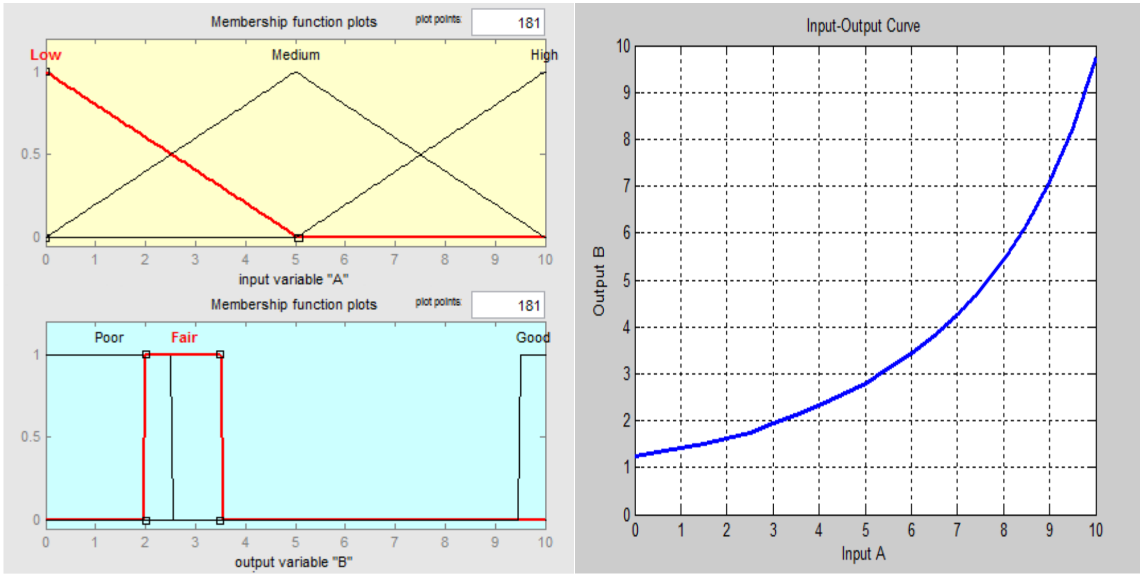


Figure 2.39 Membership function arrangement and input-output relation for Trial 2-32

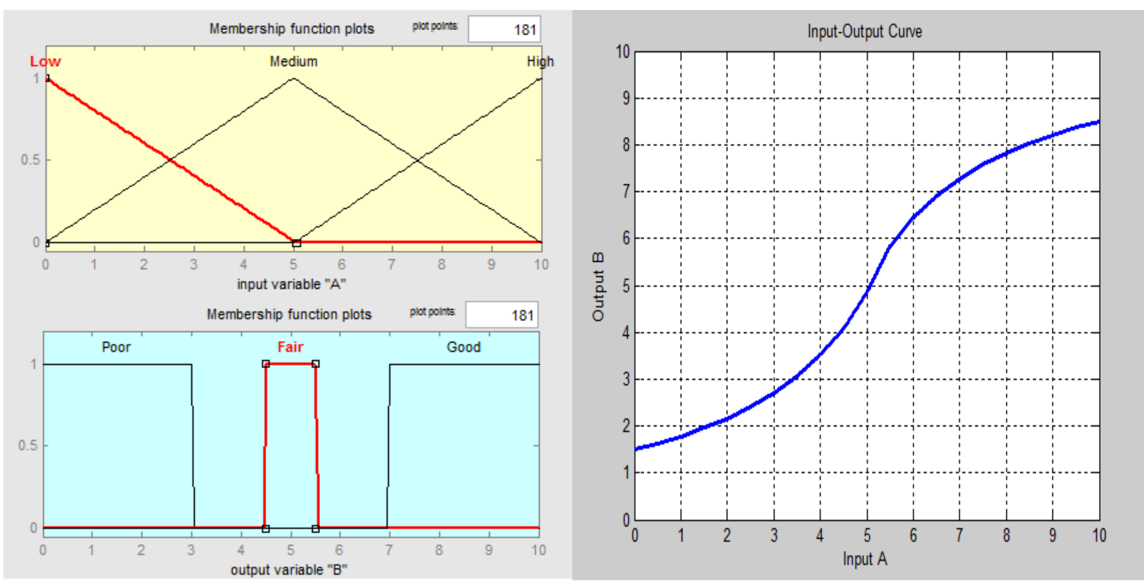


Figure 2.40 Membership function arrangement and input-output relation for Trial 2-33

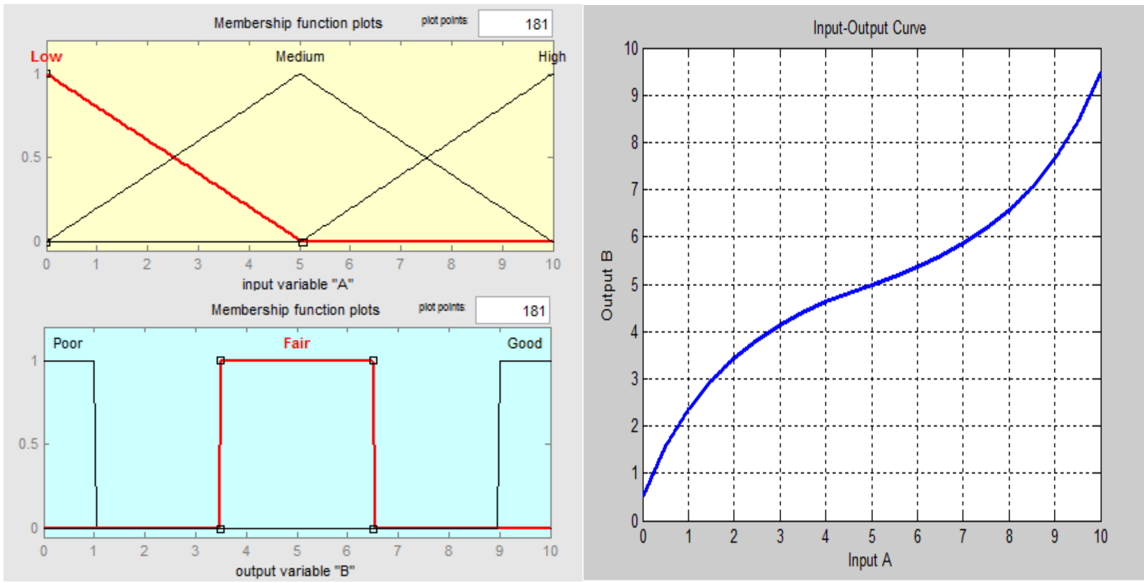


Figure 2.41 Membership function arrangement and input-output relation for Trial 2-34

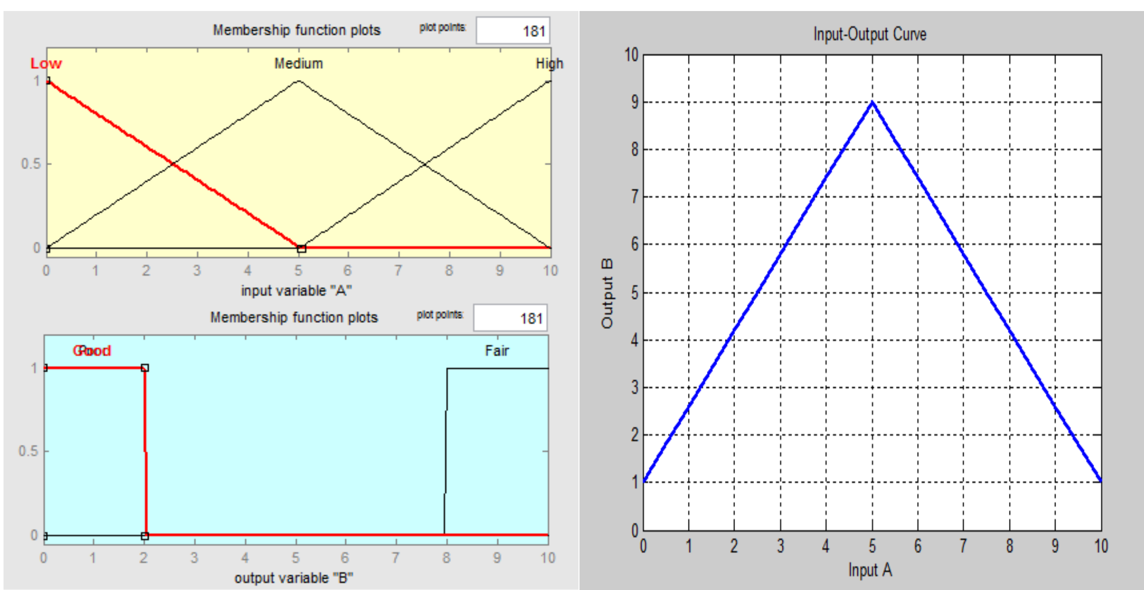


Figure 2.42 Membership function arrangement and input-output relation for Trial 2-35

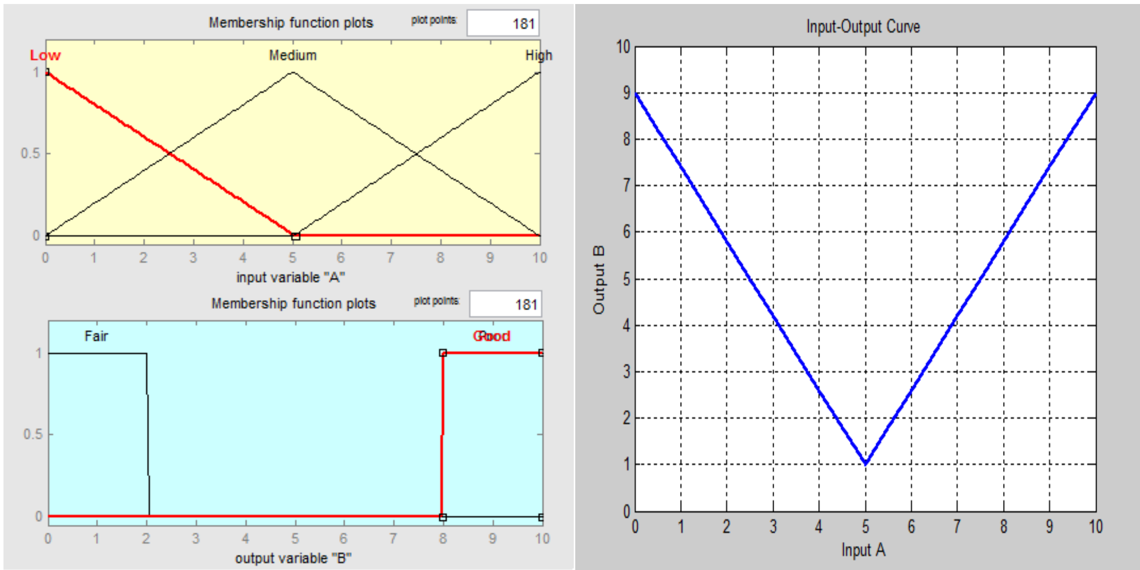


Figure 2.43 Membership function arrangement and input-output relation for Trial 2-36

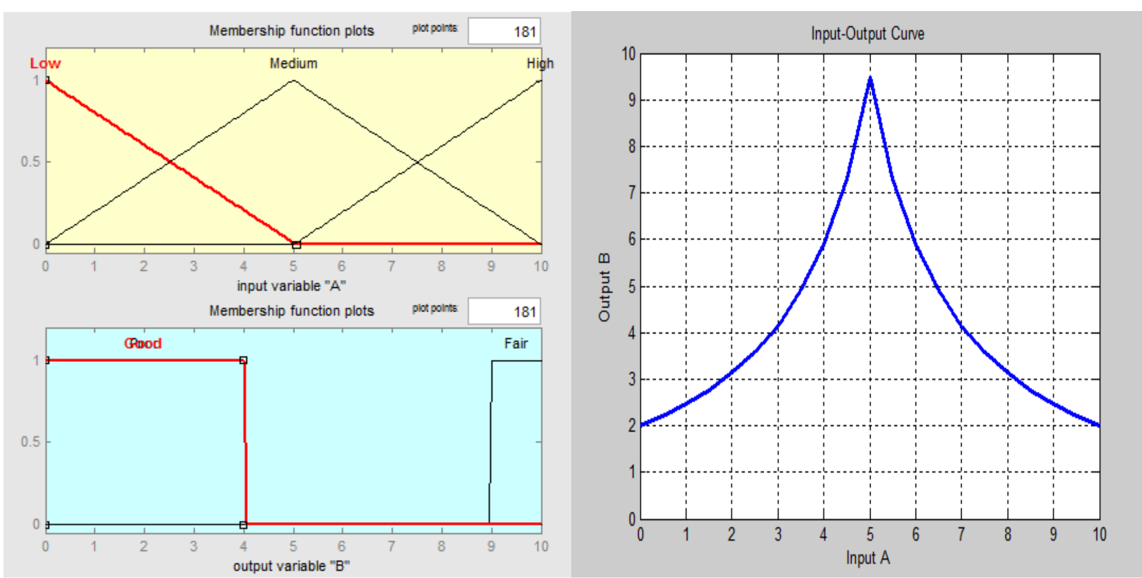


Figure 2.44 Membership function arrangement and input-output relation for Trial 2-37

approximate a sigmoid function. Similarly, if lengthening the support of MF 'Fair' and shortening the support of MF 'Poor' and 'Good', as it does in Trial 2-34, the former sigmoid curve will be turned over along the diagonal. In Trial 2-35, when the MF 'Good' fully overlaps with MF 'Poor' on the left side of output scale and the MF 'Fair' is transferred to the right side, a triangular piecewise function is achievable. And in Trial 2-36, if MFs 'Good' and 'Poor' switch their position with MF 'Fair', the input variable will be mapped onto output space through an inverse triangular piecewise function. Furthermore, based on Trial 2-35, a nonlinear piecewise function can be created in Trial 2-37 by changing the width of MFs.

From above trials, the fuzzy inference system whose input and output variables are represented by 3 MFs is flexible enough to modulate the input-output relation to a number of various non-linear functions, but the degree of freedom still cannot satisfy the expectation of meliorating details of system performance. If more MFs are introduced, it might be feasible to finely modulate input-output curve within smaller region, but at the same time, all output MFs must be well coordinated with each other, and more time consuming is expected.

### 2.6.3 Variable with Five Membership Functions

Input $A$: Very Low / Low / Medium / High / Very High

Output $B$: Very Poor / Poor / Fair/ Good / Very Good

Rule 1: If *A* is Very Low, then *B* is Very Poor

Rule 2: If *A* is Low, then *B* is Poor

Rule 3: If *A* is Medium, then *B* is Fair

Rule 4: If *A* is High, then *B* is Good

Rule 5: If *A* is Very High, then *B* is Very Good

**Trial 2-38:** Input *A*: 5 triangle MFs (2 half MFs + 3 full MFs, 100% OR)

Output *B*: 5 rectangular MFs (0% OR)

In Trial 2-38, a total five triangle MFs are introduced to divide the scale of input variable *A* and correspondingly five rectangular MFs are used to represent the output *B*. With more degree of freedom, the input-output relation comes with improved controllability. The following figure shows the comparison between Trial 2-38 and Trial 2-31 whose variables are described by three MFs. Based on the exact geometric characteristics of rectangular MFs in Trial 2-31, two more independent rectangular MFs are added into Trial 2-38 for output space. Because of that, the input-output curve in the right range of the chart is modulated and the output variable tends to the maximum with higher growth rate.

**Trial 2-39:** Input *A*: 5 triangle MFs (2 half MFs + 3 full MFs, 100% OR)

Output *B*: 5 rectangular MFs (0% OR)

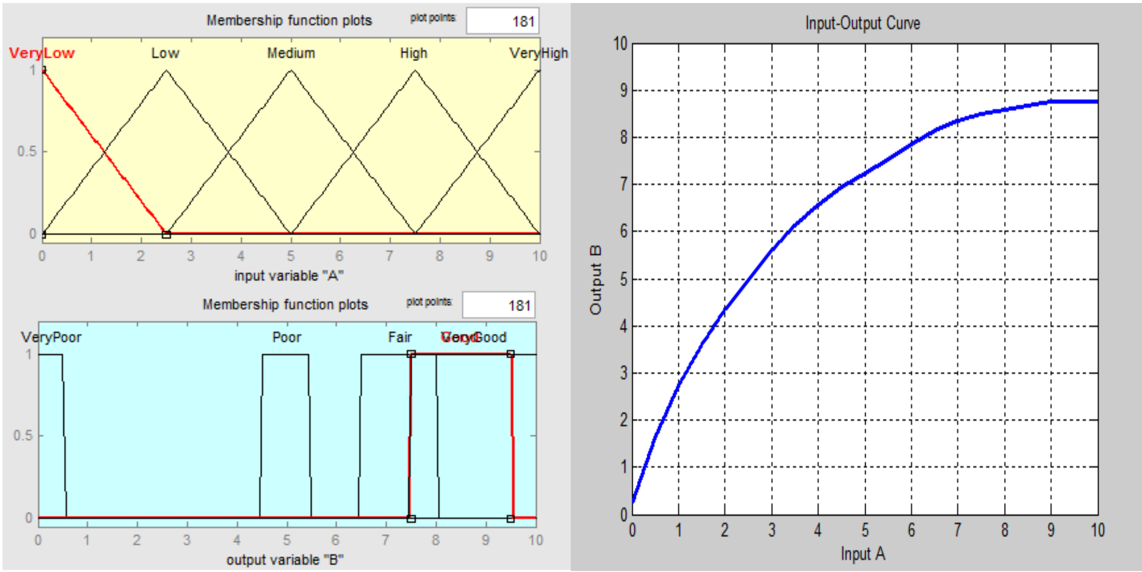


Figure 2.45 Membership function arrangement and input-output relation for Trial 2-38

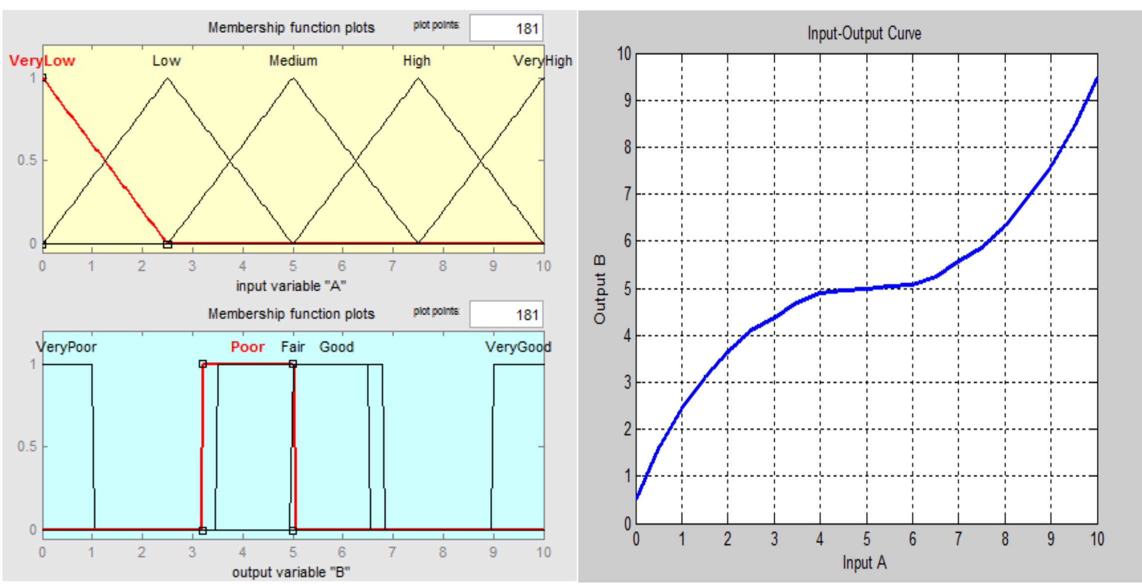


Figure 2.46 Membership function arrangement and input-output relation for Trial 2-39

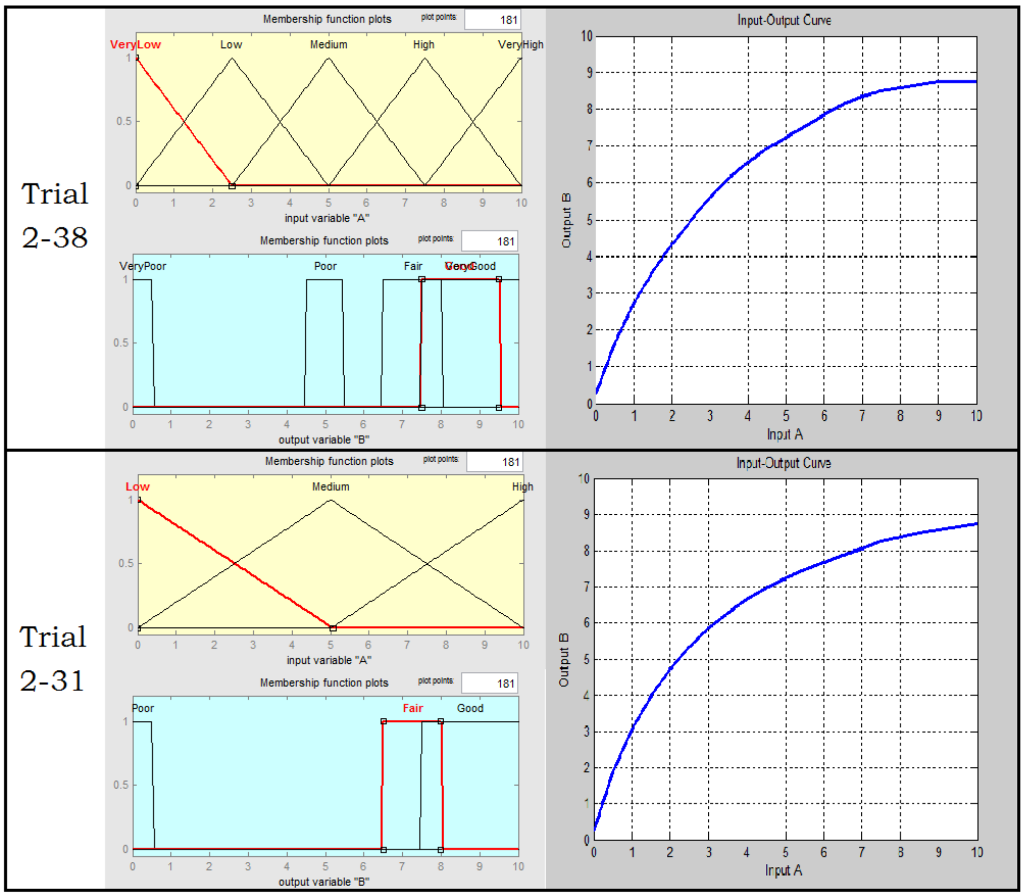


Figure 2.47 Comparison between Trial 2-38 and Trial 2-31

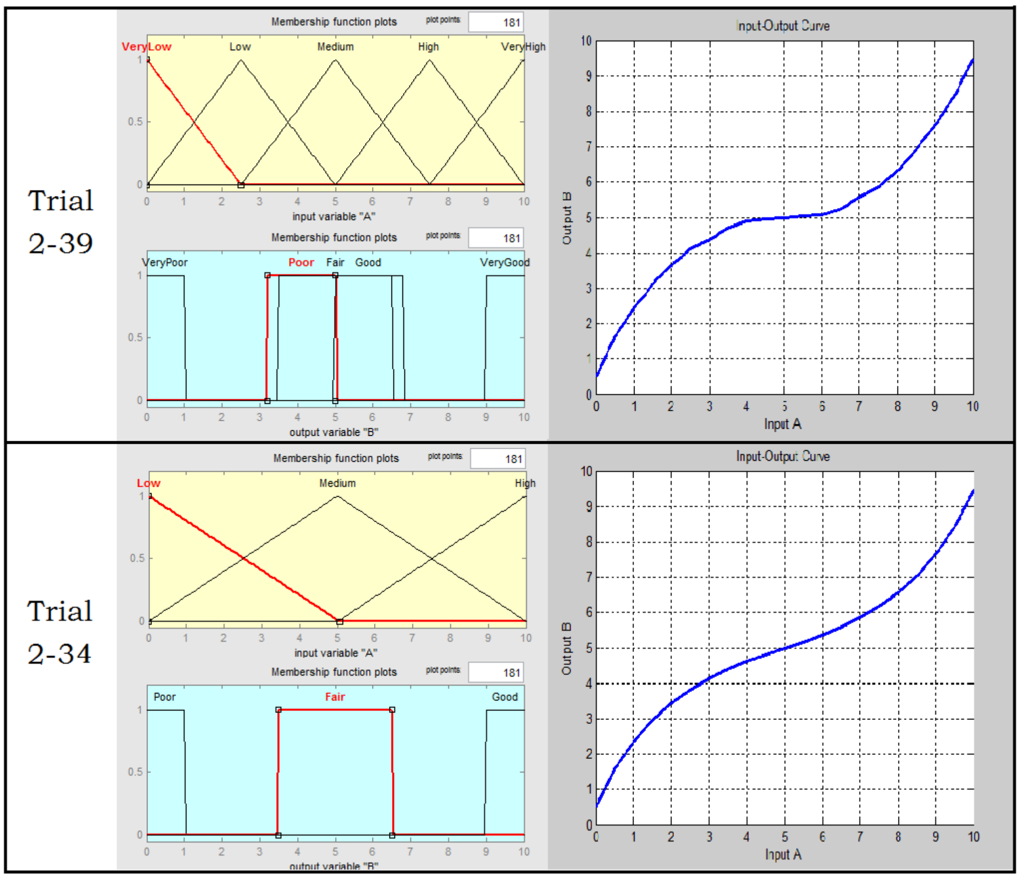


Figure 2.48 Comparison between Trial 2-39 and Trial 2-34

Same as Trials 2-38 and 2-31, the variables in Trial 2-39 are defined by two more MFs than the variables in Trial 2-34 are. As a result, it turns to be realizable to finely adjust the input-output curve within smaller region, and makes it possible to reduce the rate of increase of output variable around the middle range of input-output curve.

## 2.7 Summary

Chapter 2 focuses on studying the impact of membership functions on SISO Mamdani fuzzy inference model. Based on the assumptions proposed in the beginning, totally 39 trials are performed. From Trial 2-1 to 2-9, the influence of overlap ratio between adjacent MFs is discussed. To ensure an inference model with continuous input-output relation, it is necessary to make a half-shape MF for input variable to be fully overlap by the contiguous MF, or a full-shape MF for input is required to be symmetrically overlapped by its two neighboring MFs. In order to make output values to be distributed in a wide range, the overlap between adjacent MFs for output is unwished. As a matter of fact, the farther the MFs for output are separated to each other, the better the monotonicity of input-output curve will be. From Trial 2-10 to 2-21, twelve possible combinations with 3 kinds of MFs for input $A$ and 4 types of MFs for output $B$ are provided to study the potential effect from the shape of MF. It turns out that the inference model with triangle MFs for input and rectangular MFs for output produces input-output relation with ideal linearity. Then Trial 2-22 to Trial 2-26 are introduced to compare the different system performance among inference models whose variables are defined by different number of MFs. When the quantity of MF for each variable increases, the input-output curve becomes to repeat a certain curve or straight line periodically, and to some extent the linearity of input-output curve is improved. Finally based upon the optimal linear inference system in trial 2-13, the rectangular MFs for output variable are adjusted to attempt non-linear system performance in Trials 2-28 and 2-29. Meanwhile, the same experiments are duplicated on models with more MFs used for single variable. The quantity of MF determines the controllability of inference system. Generally, a SISO inference system with more MFs for each variable is achievable to finely modulate input-output curve, but this is at the cost of a large amount of time.

# 3. Adjustment of Membership Functions in Two-Input Single-Output (TISO) Mamdani Fuzzy Inference System

In the previous chapter, the potential impacts of MFs on SISO fuzzy inference system are discussed. With the simplest fuzzy inference model, SISO system can be used as starting point to study membership functions, even though Multi-Input Single-Output models are more commonly applied than SISO systems. In this chapter, experimental trials are implemented on Two-Input Single-Output (TISO) Mamdani fuzzy inference system. All TISO models observe the same assumptions and identical constrains. Similar characteristics of membership function are discussed in this chapter, including overlap ratios between adjacent MFs, the shape of MFs and the quantity of MFs for describing a single input variable or output variable.

## 3.1 Assumptions and Expectation

As the previous chapter, many characteristics of membership function which will not be considered in this thesis are required to be fixed, in order to exclude their unexpected interference. TISO fuzzy inference systems comply with the same assumptions proposed in Chapter 2.

(1). The universe of discourse for both input and output variables are normalized into interval [0, 10].

(2). The monotonicity of input-output relation is the basic criterion for inference system performance.

(3). All of the MFs are normal and convex. All MFs which define the same input or output variable are constructed with identical geometrical characteristics, and are translated to fill in the domain of discourse [0, 10].

(4). In TISO fuzzy inference models, input variable *A* and input variable *B* are identically defined, for simplifying discussion process.

(5). The If-Then rules in fuzzy inference models must be complete and symmetric.

In Chapter 3, the main expectation is testing and verifying the conclusions from SISO fuzzy inference model in Chapter 2 on TISO inference model, and summarizing the effects of membership function on inference system performance which are universally effective in both SISO and TISO fuzzy inference models. The same approach, regulating the geometrical features and quantity of MF, will be applied for building a TISO fuzzy inference system with optimal linear input-output relation, and study the controllability of TISO system via modulating the MFs for output variable *C* only.

## 3.2 System Modeling

In this chapter, the TISO Mamdani fuzzy inference model consists of two input variables, *A* and *B*, and one output variable *C*. Likewise, If-Then rules are required to be complete and symmetric, so the antecedent part of each rule must have dependency on both input *A* and input *B*. The number of complete rules is equal to the product of the number of fuzzy sets (or MFs) for input *A* and the number of fuzzy sets (or MFs) for input *B*. To meet the demand of monotonicity of input-output surface, antecedent parts having the same score will match a common MF for output *C*. As always, the implication process and aggregation process are implemented by function *min* and function *max* respectively, and the *Centroid Method* is chose for defuzzification process.

The following experimental trials start with the simplest TISO Mamdani fuzzy inference system where both input variable *A* and *B* have two fuzzy sets 'Low' and 'High', and it is reasonable to represent fuzzy sets 'Low' and 'High' by two half-shape MFs respectively. And for the purpose of ensuring a consistent input-output relation, only full-shape MFs are considered for output variable *C*. Totally, four possible antecedents for If-Then rules exist: "If *A* is Low and *B* is Low", "If *A* is High and *B* is Low", "If *A* is Low and *B* is High", and "If *A* is High and *B* is High". When score 1 is assigned to fuzzy set 'Low' and score 2 is allocated to fuzzy set 'High', the score for each antecedent part is the sum of the scores of two fuzzy sets .Thus the score for "If *A* is Low and *B* is Low" is 2, score for "If *A* is High and *B* is High" is 4, and scores for "If *A* is High and *B* is Low" and "If *A* is Low and *B* is High" are equal to 3. Accordingly, three MFs, 'Poor', 'Fair' and 'Good' are expected to match these possible antecedents with three different scores. Figure 3.2 shows the logic flow of a sample TISO Mamdani fuzzy inference system.

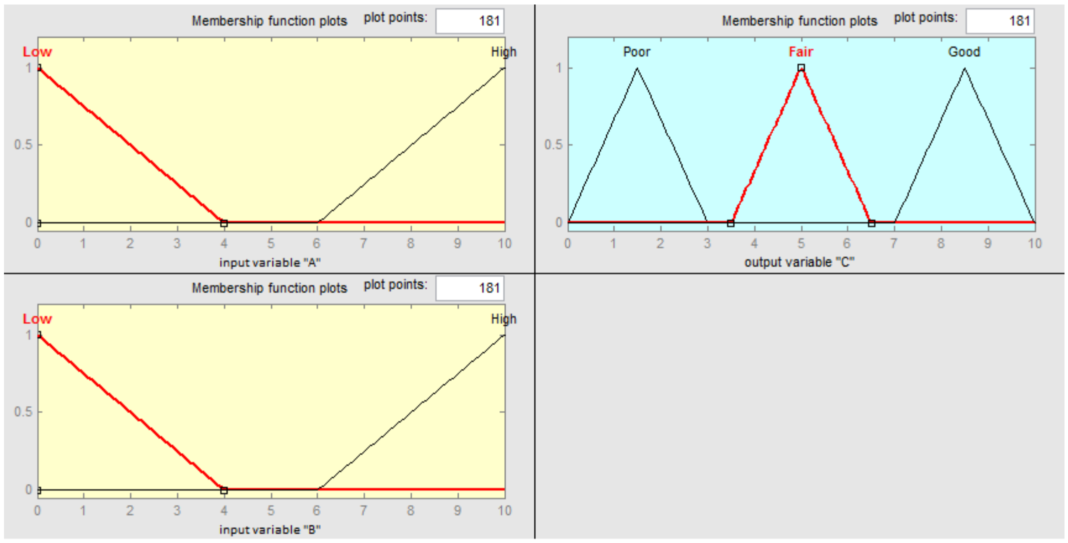


Figure 3.1 Membership function arrangement for sample TISO fuzzy inference system

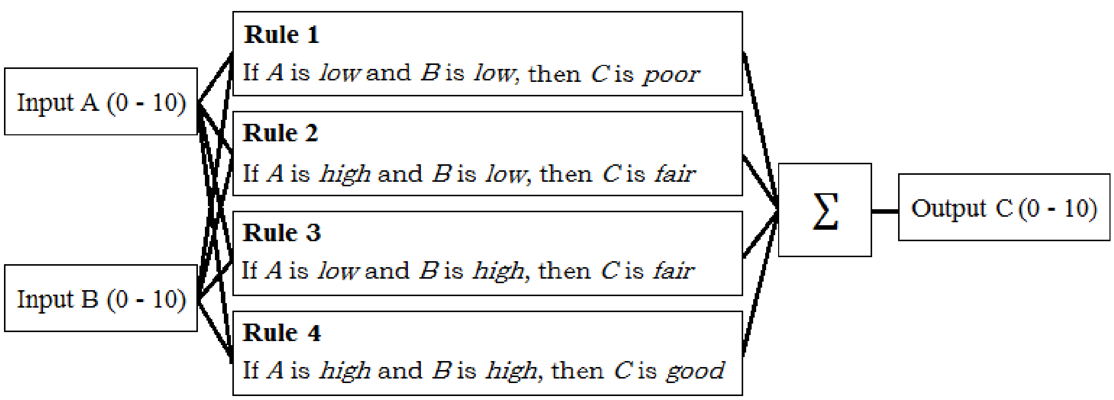


Figure 3.2 Logic flow of TISO fuzzy inference system

## 3.3 Overlap Ratio of Adjacent Membership Functions

In SISO Mamdani fuzzy inference model, it is proved that overlap the ratio between adjacent MFs plays important role on the monotonicity of input-output relation. Therefore, we have a reason to speculate that this feature of MF will affect the system performance of TISO inference model in the same way.

The following experimental trials for this section consist of two input variables *A* and *B*, and one output variable *C*. Two fuzzy sets, 'Low' and 'High', are used to define both input *A* and input *B*, while three fuzzy sets, 'Poor', 'Fair' and 'Good' are introduced for output *C*. A complete and symmetric inference mechanism with four If-Then rules are expected.

Rule 1: If *A* is Low and *B* is Low, then *C* is Poor

Rule 2: If *A* is High and *B* is Low, then *C* is Fair

Rule 3: If *A* is Low and *B* is High, then *C* is Fair

Rule 4: If *A* is High and *B* is High, then *C* is Good

### 3.3.1 Membership Functions for Input Variable

**Trial 3-1:** Inputs *A&B*: 2 half-triangle MFs (0% OR)

Output *C*: 3 full-triangle MFs (0% OR)

**Trial 3-2:** Inputs *A&B*: 2 half-triangle MFs (50% OR)

Output *C*: 3 full-triangle MFs (0% OR)

**Trial 3-3:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-triangle MFs (0% OR)

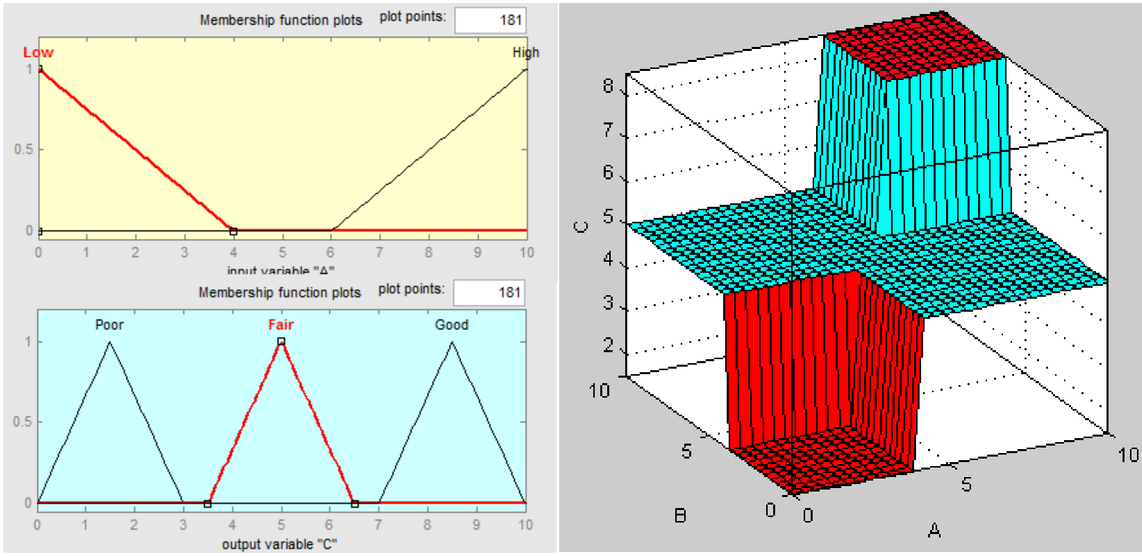


Figure 3.3 Membership function arrangement and input-output surface for Trial 3-1

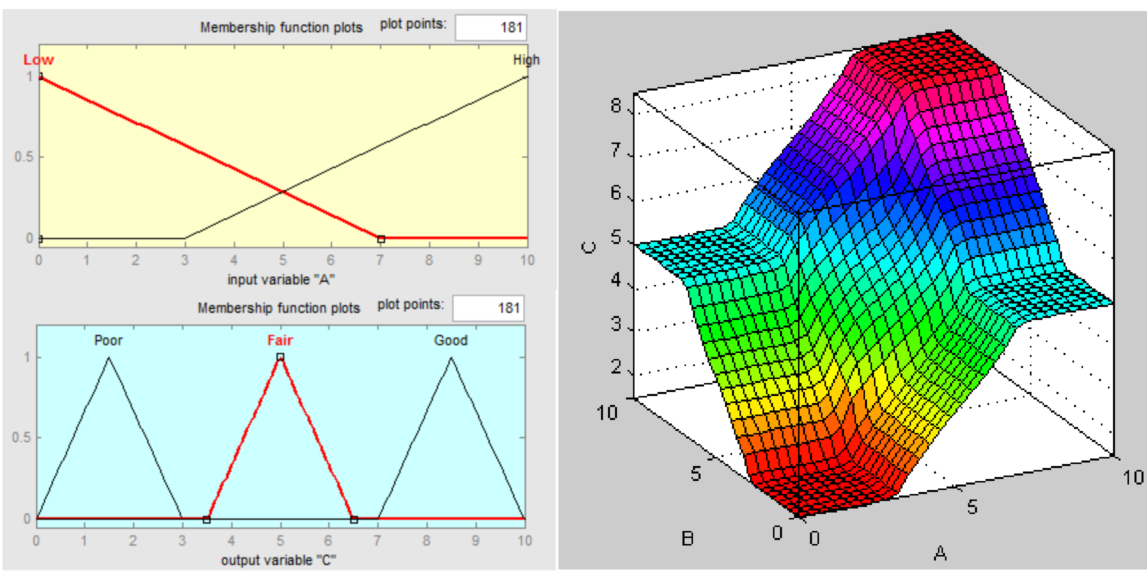


Figure 3.4 Membership function arrangement and input-output surface for Trial 3-2

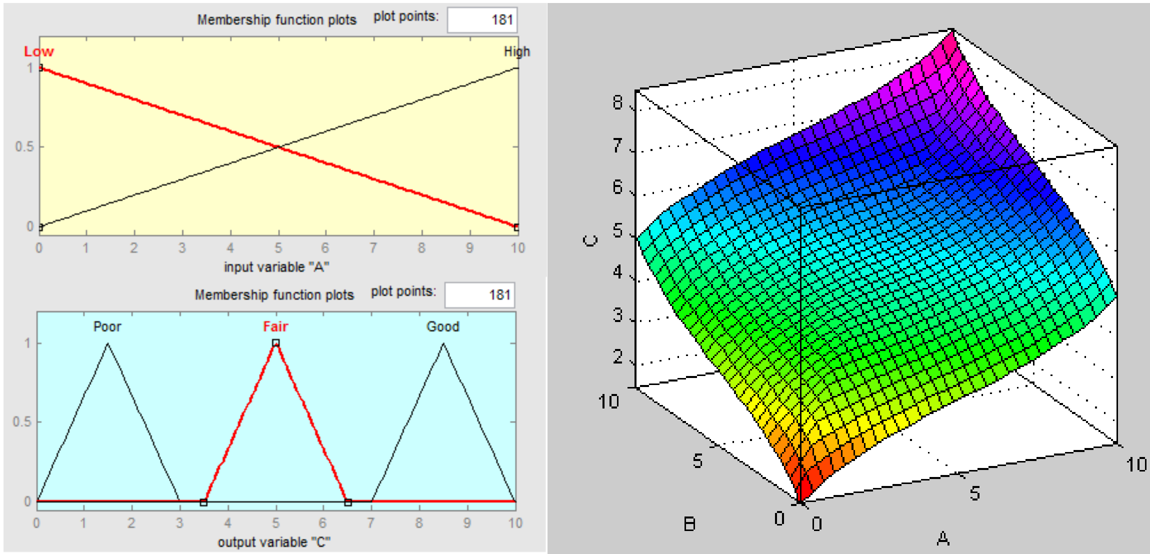


Figure 3.5 Membership function arrangement and input-output surface for Trial 3-3

From Trial 3-1 to 3-3, the support of MFs for input variables *A* and *B* are simultaneously adjusted, while the MFs for output *C* remain unchanged. Because input variables *A* and *B* are identically defined, only one figure of input MFs is showed above for presenting both inputs *A* and *B*. Compared with SISO fuzzy inference system, one more input variable brings in a new dimension for input space, thus a 3-dimensional surface where x-axis and y-axis measure inputs A and B and then the z-direction represents the value of output C is a desired expression for the input-output relation.

In Trial 3-1 when the MFs for inputs *A* and *B* are not overlapped, a stair-shaped surface expresses the input-output relation. Similarly for the situation in Trial 2-1, four If-Then rules become completely independent in this case. When the values of inputs A and B make anyone of the rules valid, the other three rules will be rejected in the meantime and the *Centroid method* can only retrieve the center of area of the output MF matching the valid rule. As Figure 3.6 shows when two groups of values for inputs A and B, (1, 1) and (3, 3), are introduced into Model 3-1, only Rule 1 is activated. Although the two numbers from Rule 1 truncate the corresponding output MF into different shapes, the two centers of area with respect to the abscissa are represented by the same number. Thus the output values for *C* are the same, damaging the monotonicity of input-output relation.

In Trial 3-2, the input MF overlaps with adjacent MF by 50%, but the flat input-output relation with no sensitivity still happens in the regions where the input MFs do not work collectively. While in Trial 3-3, input MFs are fully overlapped with each other, so that the

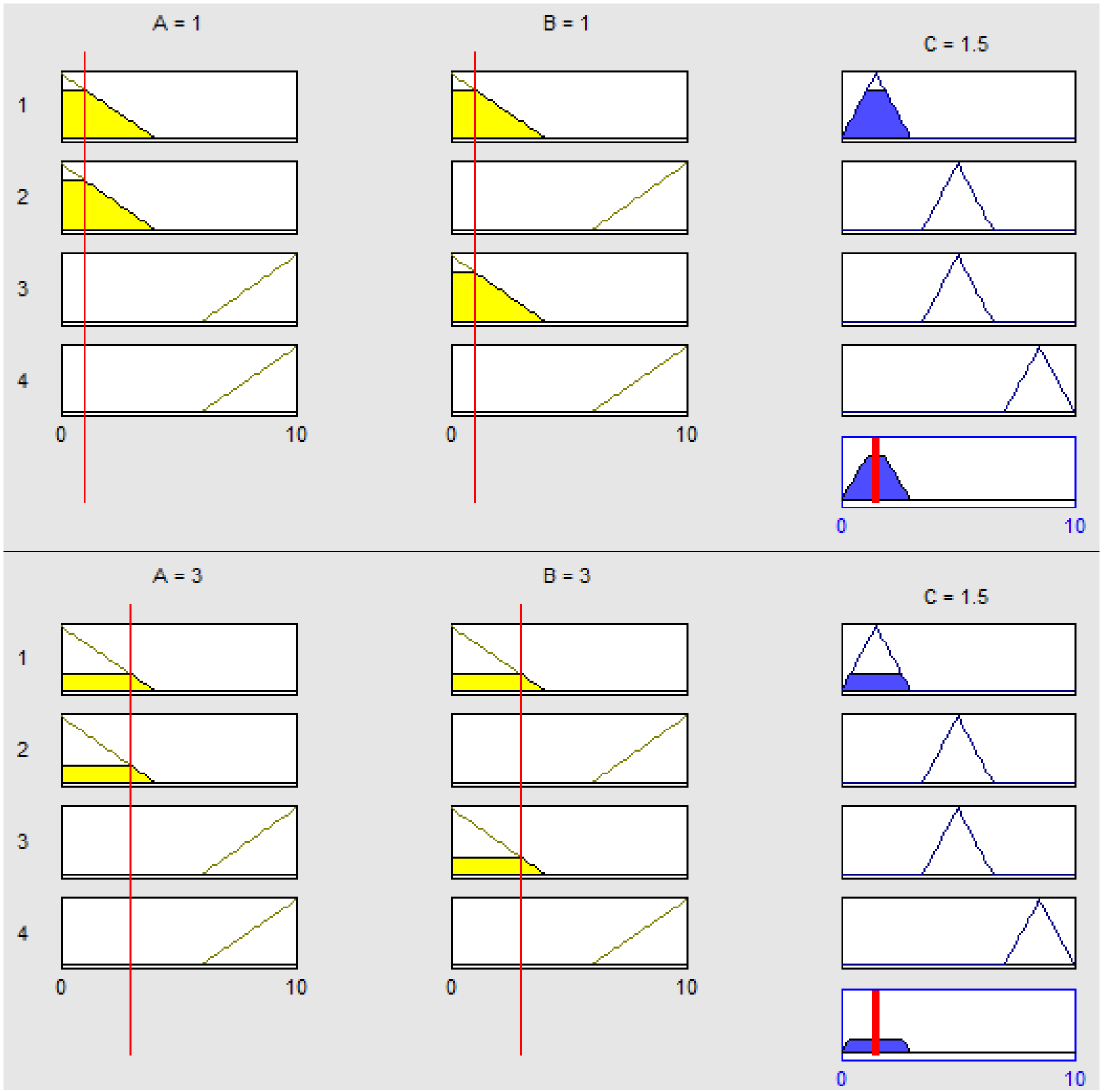


Figure 3.6 Implication and aggregation processes for Trial 3-1

four rules become thoroughly correlated. As a result, the input-output relation shows a smooth and continuous surface with desired monotonicity.

### 3.3.2 Membership Functions for Output Variable

**Trial 3-4:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-triangle MFs (50% OR)

**Trial 3-5:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-triangle MFs (0% OR)

In Trial 3-4 and 3-5, the MFs for input variable *A* and *B* remain unchanged, and the overlap ratio between adjacent MFs for output variable *C* is adjusted. In Trial 3-4, the output MF 'Fair' is symmetrically overlapped with MFs 'Poor' and 'Good' by 50% respectively, and in Trial 3-5 all MFs are shrunk and evenly distributed to the whole output scale with 0% overlap. Comparing the two surfaces of input-output relation, the one coming with separated output MFs performs a wider range for output *C*, and the overlap situation among output MFs tends to reduce the distance between the centers of area of output MFs, and then curtails the span of output value. Because of the feature of the Centroid method, the center of area of every MF can never reach the endpoints of output scale, so that the extreme values of input-output surface can never be 0 or 10.

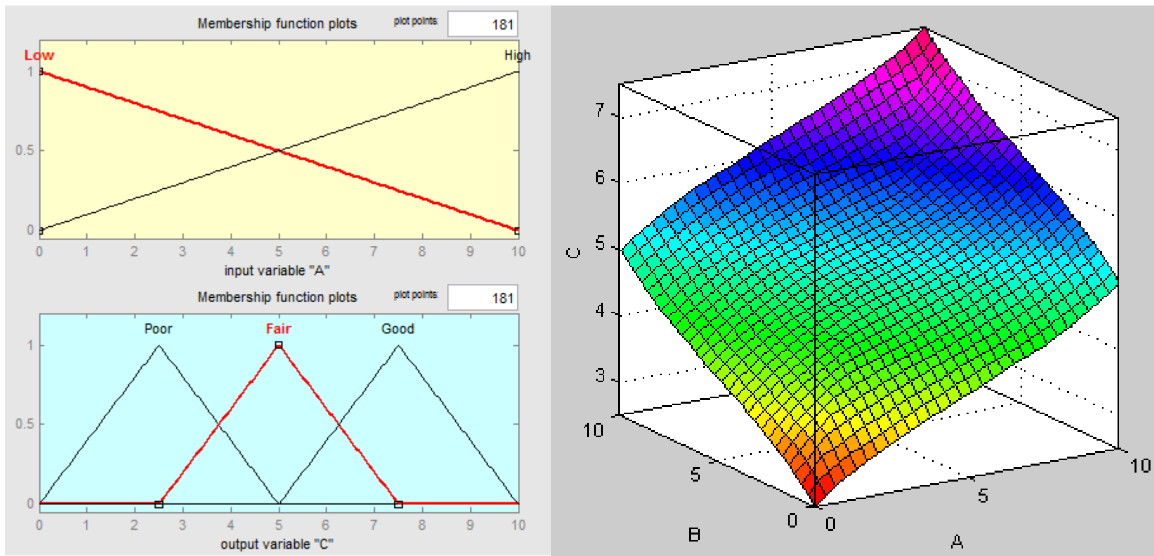


Figure 3.7 Membership function arrangement and input-output surface for Trial 3-4

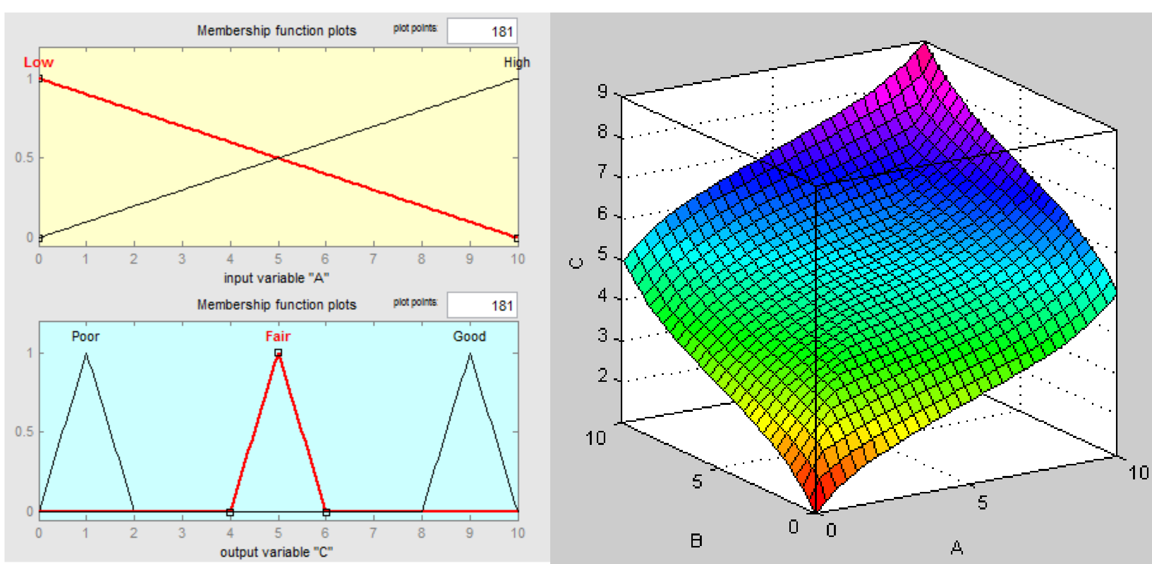


Figure 3.8 Membership function arrangement and input-output surface for Trial 3-5

## 3.4 Influence from Different Type of Membership Functions

In this section, a total 12 trials are implemented to test the influence from the shape of MF on TISO system performance. As within the above trials in Chapter 3, input variables *A* and *B* are identically defined, and only one graph will be displayed in each trial to present the MF arrangement for both inputs A and B. The basic model for this section is comprised by input variables which are defined by two half-shape MFs for fuzzy sets 'Low' and 'High', and one output variable which is represented by three full-shape MFs for sets 'Poor', 'Fair' and 'Good'. If-Then rules are same as those in last section.

Rule 1: If *A* is Low and *B* is Low, then *C* is Poor

Rule 2: If *A* is High and *B* is Low, then *C* is Fair

Rule 3: If *A* is Low and *B* is High, then *C* is Fair

Rule 4: If *A* is High and *B* is High, then *C* is Good

Three general types of MFs, Triangle MF, Gaussian MF, and Trapezoidal MF for inputs A and B, and four kinds of MFs, Triangle MF, Gaussian MF, Trapezoidal MF and Rectangular MF for output *C* are considered below.

**Trial 3-6:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-triangle MFs (0% OR)

**Trial 3-7:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-Gaussian MFs (0% OR)

**Trial 3-8:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 full-trapezoidal MFs (0% OR)

**Trial 3-9:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

**Trial 3-10:** Inputs *A&B*: 2 half-trapezoidal MFs (100% OR)

Output *C*: 3 full-triangle MFs (0% OR)

**Trial 3-11:** Inputs *A&B*: 2 half-trapezoidal MFs (100% OR)

Output *C*: 3 full-Gaussian MFs (0% OR)

**Trial 3-12:** Inputs *A&B*: 2 half-trapezoidal MFs (100% OR)

Output *C*: 3 full-trapezoidal MFs (0% OR)

**Trial 3-13:** Inputs *A&B*: 2 half-trapezoidal MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

**Trial 3-14:** Inputs *A&B*: 2 half-Gaussian MFs (100% OR)

Output *C*: 3 full-triangle MFs (0% OR)

**Trial 3-15:** Inputs *A&B*: 2 half-Gaussian MFs (100% OR)

Output *C*: 3 full-Gaussian MFs (0% OR)

**Trial 3-16:** Inputs *A&B*: 2 half-Gaussian MFs (100% OR)

Output *C*: 3 full-trapezoidal MFs (0% OR)

**Trial 3-17:** Inputs *A&B*: 2 half-Gaussian MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

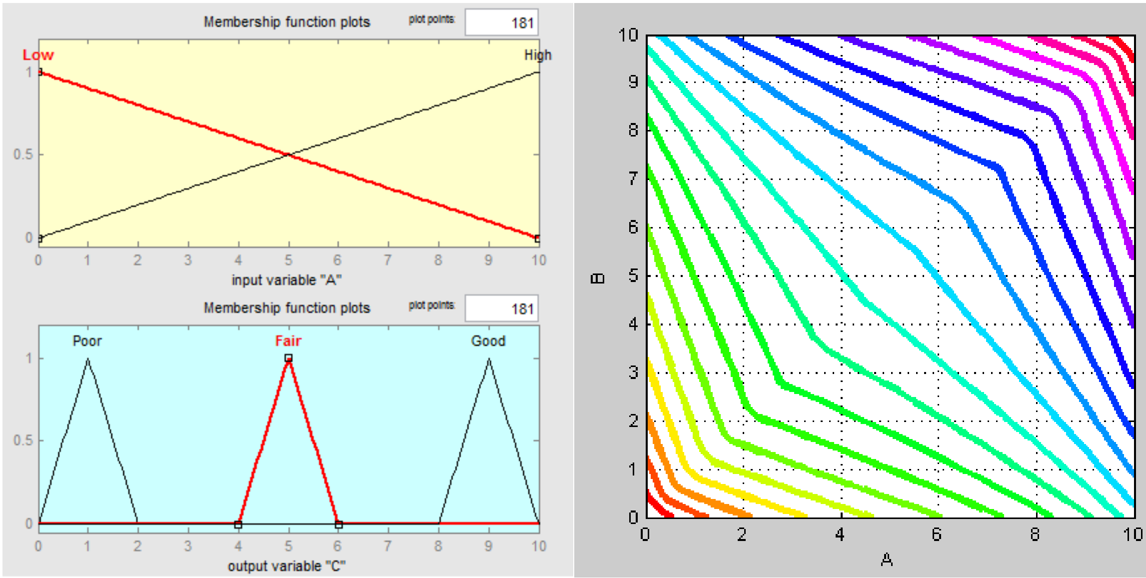


Figure 3.9 Membership function arrangement and contour graph for Trial 3-6

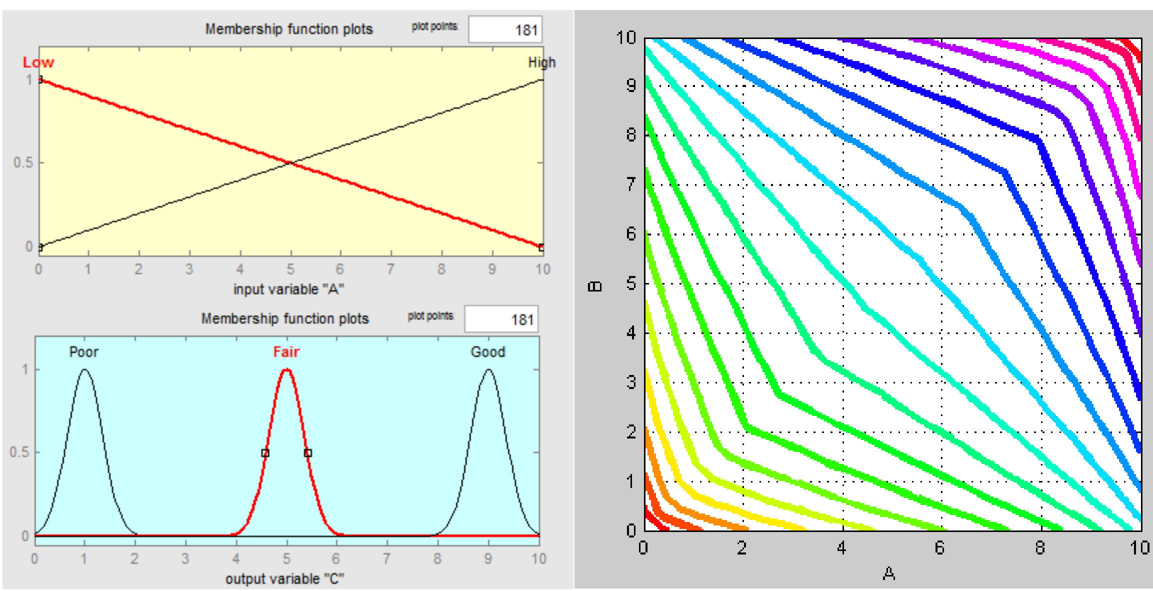


Figure 3.10 Membership function arrangement and contour graph for Trial 3-7

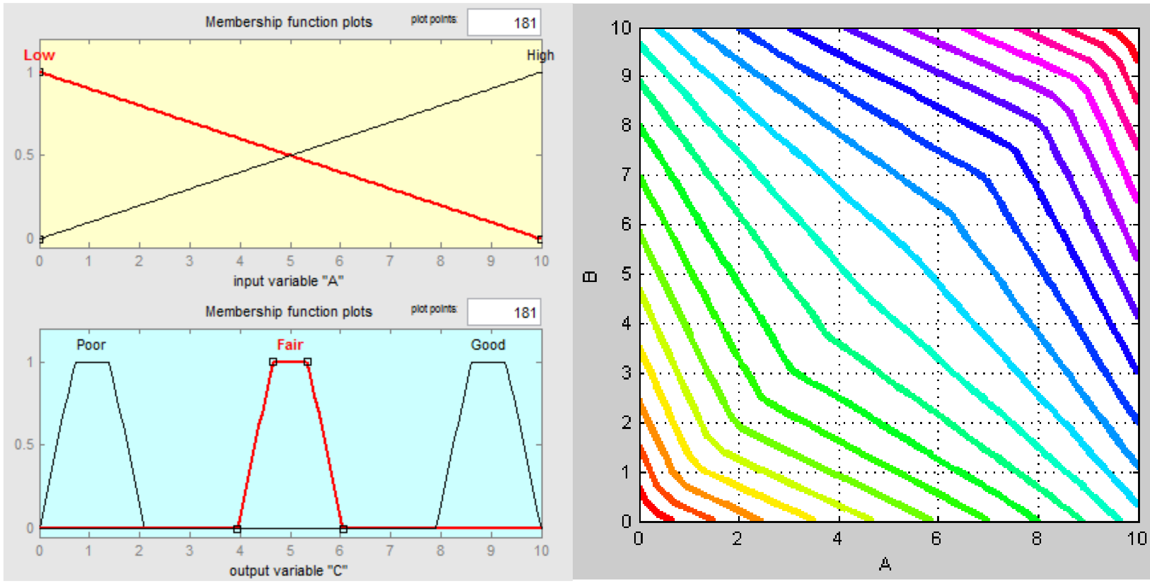


Figure 3.11 Membership function arrangement and contour graph for Trial 3-8

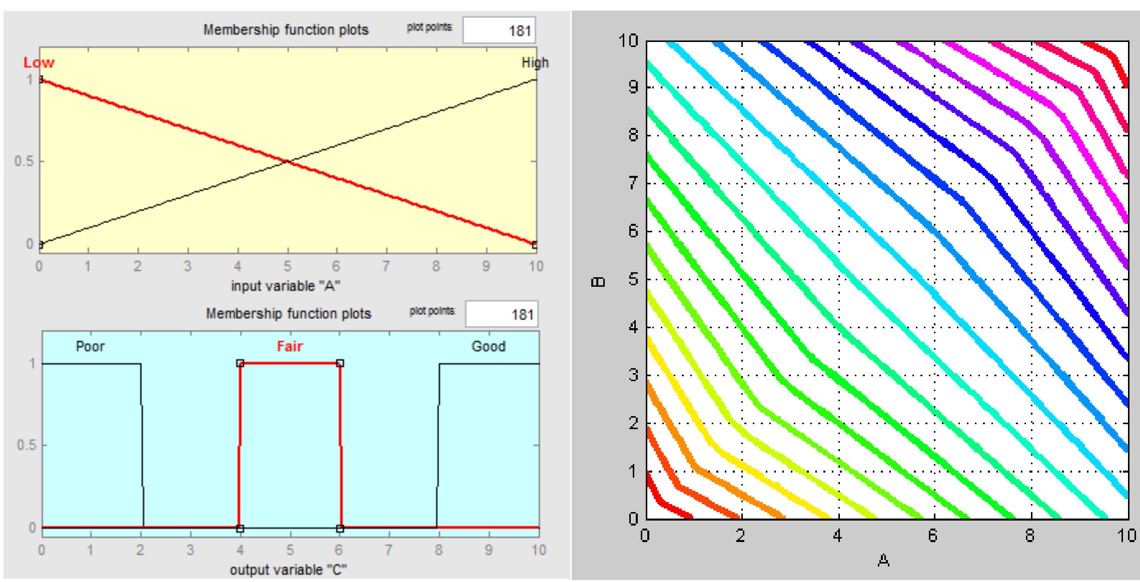


Figure 3.12 Membership function arrangement and contour graph for Trial 3-9

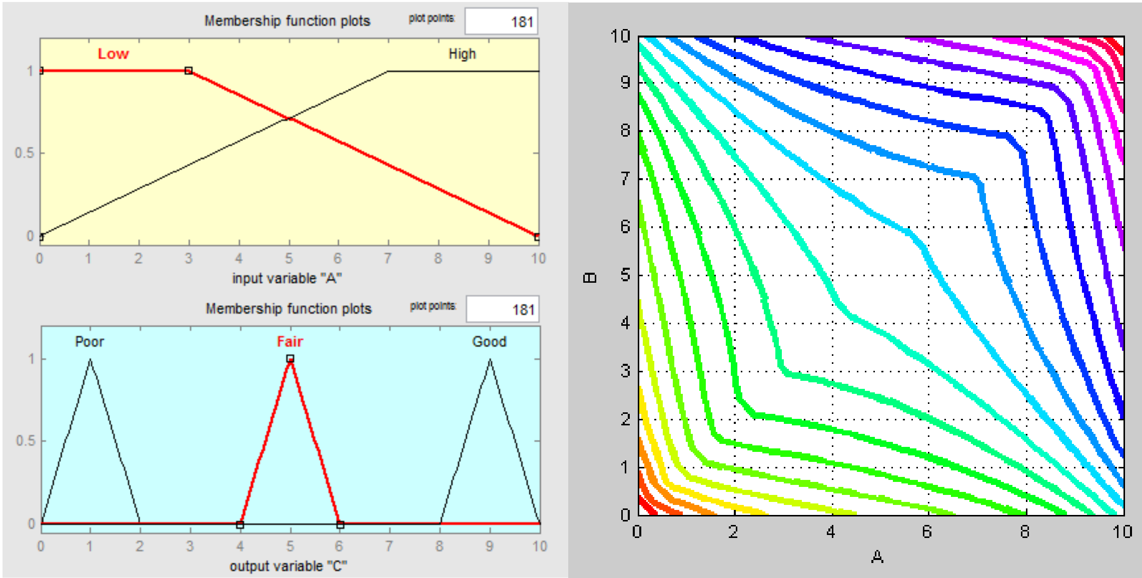


Figure 3.13 Membership function arrangement and contour graph for Trial 3-10

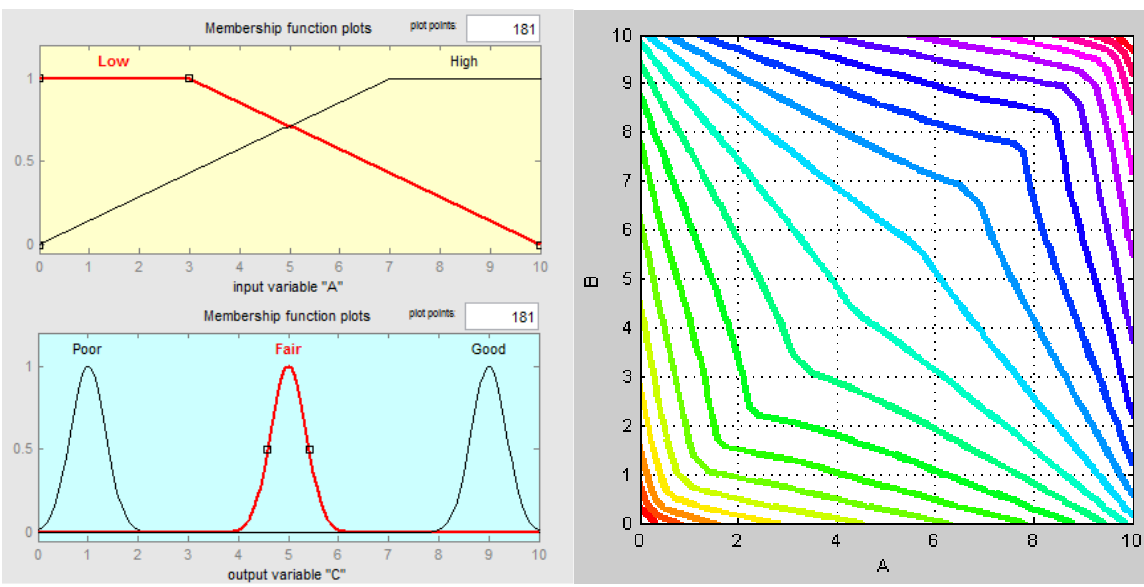


Figure 3.14 Membership function arrangement and contour graph for Trial 3-11

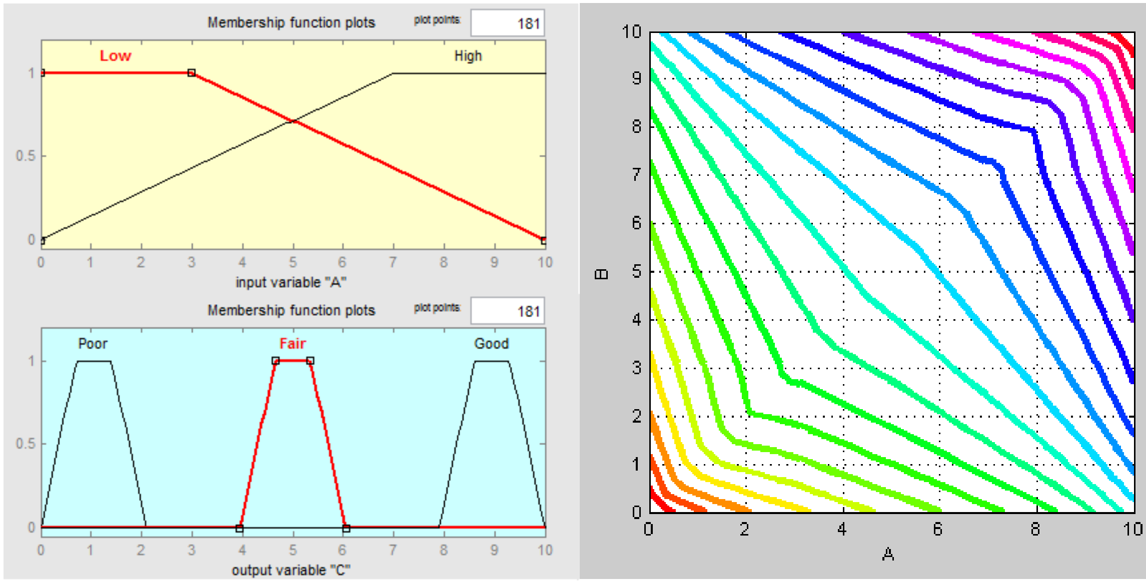


Figure 3.15 Membership function arrangement and contour graph for Trial 3-12

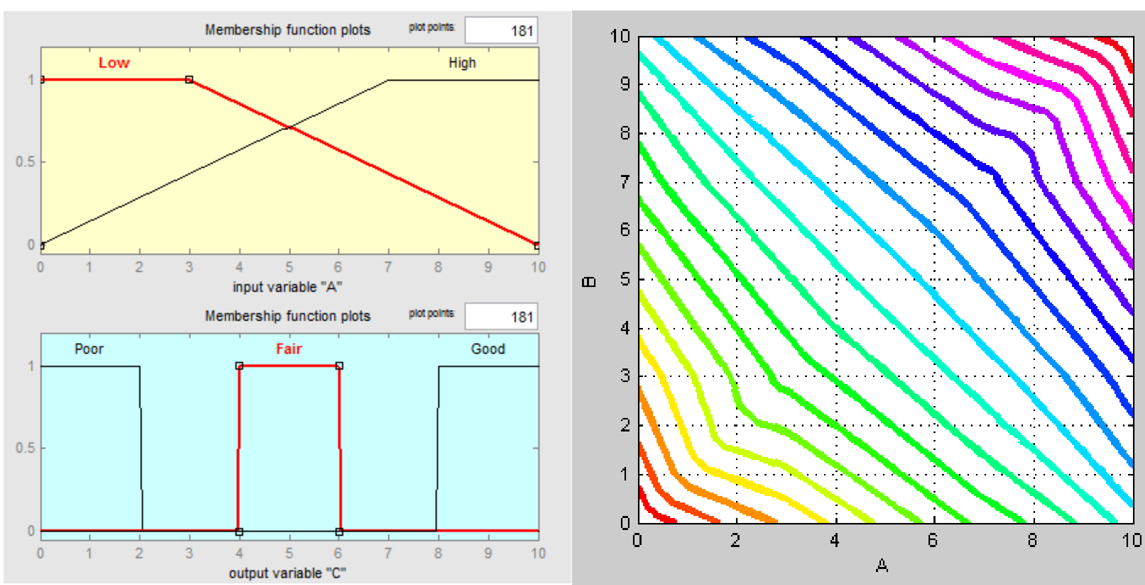


Figure 3.16 Membership function arrangement and contour graph for Trial 3-13

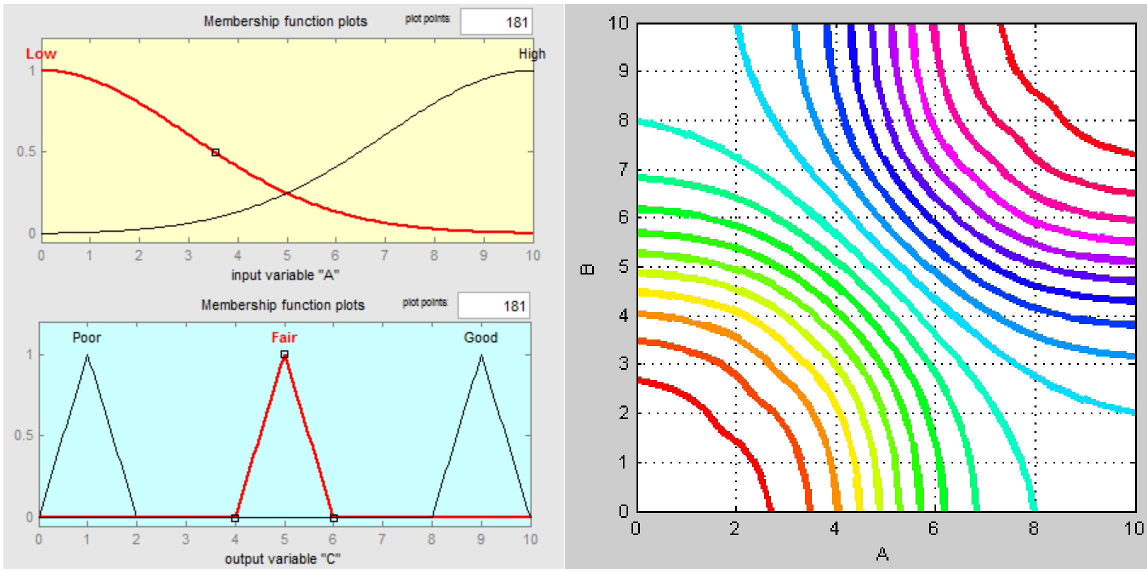


Figure 3.17 Membership function arrangement and contour graph for Trial 3-14

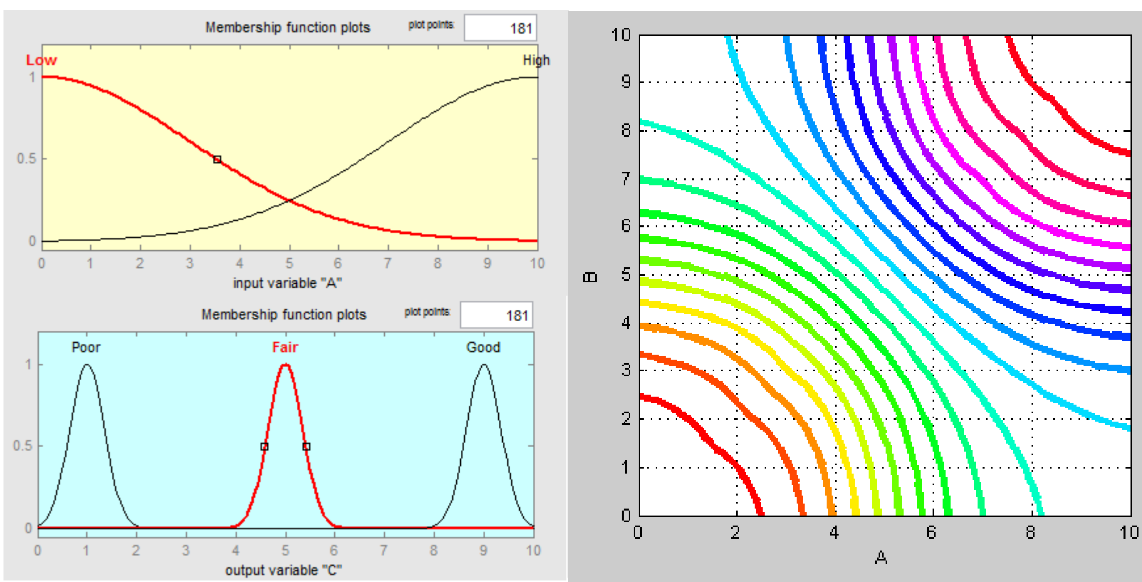


Figure 3.18 Membership function arrangement and contour graph for Trial 3-15

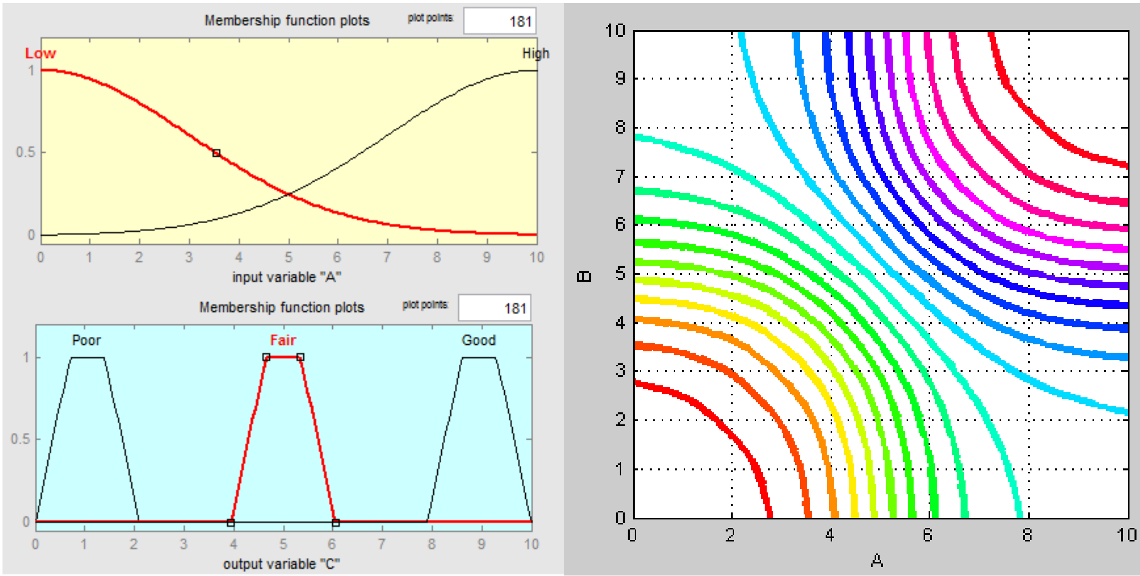


Figure 3.19 Membership function arrangement and contour graph for Trial 3-16

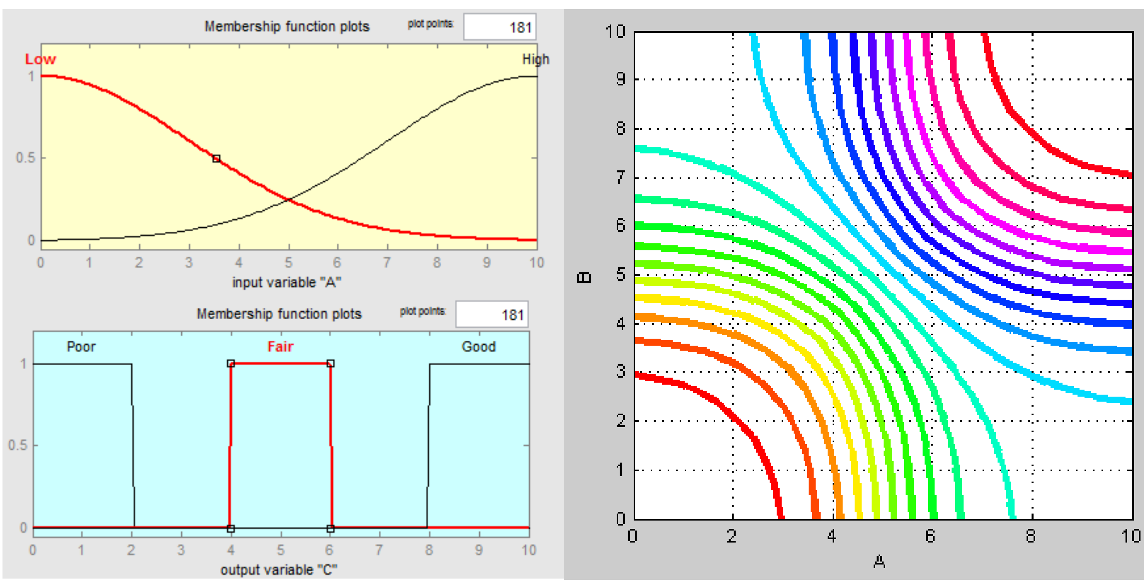


Figure 3.20 Membership function arrangement and contour graph for Trial 3-17

Although a 3-dimensional surface is the most straightforward way to display the input-output relation for TISO fuzzy inference system, sometimes it is less recognizable to tell the difference among similar TISO inference models. In this section, the contour of input-output relation is adopted to present the system performance of each experimental trail. It is easy to observe that from Trial 3-6 to 3-17, the models with same MFs for input variable share a similar pattern in the contours, while the models with same MFs for output variable do not have a common tendency in their contours.

In a contour graph, each curve (or straight line) represents a single value for output variable *C*, and the difference value between any two adjacent curves (or straight lines) is constant. If the gap between two neighboring lines is large, the rate of change between two values represented by these lines is low. Oppositely, if two adjacent lines are fairly close to each other, the rate of change between two values represented by the lines is high. Taking diagonal from (0, 0) to (10, 10) for reference, in the models whose input variables are defined by Gaussian MFs, the gaps between lines around the two ends of the diagonals are wider than those around the middle part of diagonals. If one curve can be used to express the input-output relation along the diagonal, the maximum slope will locate at the middle point, and the slope will gradually decrease with input-output curve tending to the endpoints. This phenomenon in TISO trials from 3-14 to 3-17 is surprisingly coincident with that in SISO trials from 2-18 to 2-21. On the other hand, from Trial 3-10 to 3-13, trapezoidal MFs are used for input variables. Along the diagonal direction on the graphs of contour, the gaps between contiguous lines around middle part are wider than those close to the ends of diagonals. If we use another curve to describe the input-output relation along the diagonal, this time the minimum slope will happen at the middle point, and the slope will gradually increase with the input-output curve extending to the endpoints. This characteristic of TISO trials from 3-10 to 3-13 manifests the accordance with that in SISO trials from 2-14 to 2-17. In Figure 3.21 and Figure 3.22, the similar input-output relations between Trial 3-14 and 2-18, and Trial 3-10 and 2-14 are displayed respectively.

Moreover, this internal relation between SISO and TISO fuzzy inference systems is strongly validated via Trial 3-9. Same as in Trial 2-13, the model in Trial 3-9 consists of triangle MFs for input variables and rectangular MFs for output. As always, model 3-9 displays input-output relation with ideal linearity as Model 2-13 does. Because the lines in the contour of Trial 3-9 are almost straight and parallel, and the distances between any adjacent lines are approximately equal, the input-output surface becomes similar to flat with optimal monotonicity. Therefore, in the latter section where the controllability of TISO fuzzy inference system will be discussed, Model 3-9 will be regarded as basic linear model.

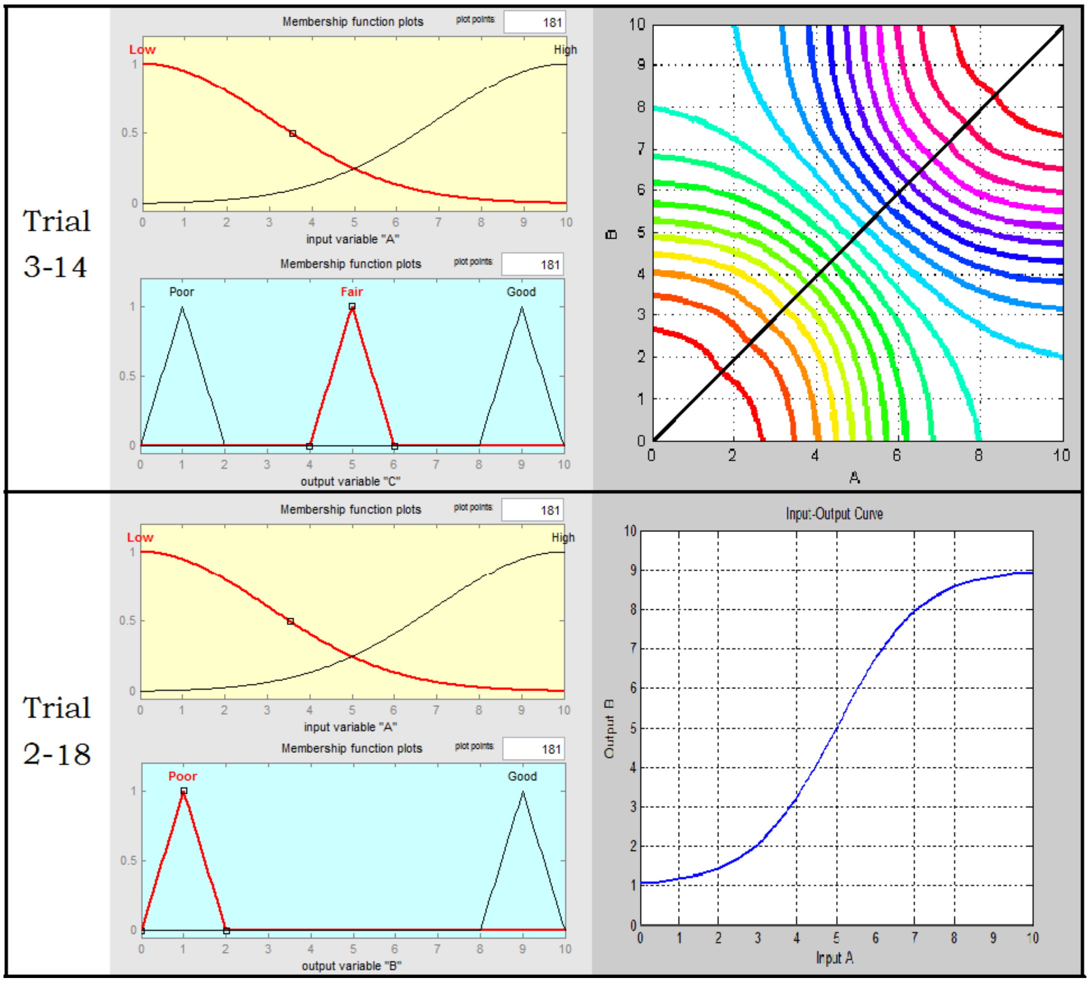


Figure 3.21 Reference figure between Trial 3-14 and Trial 2-18

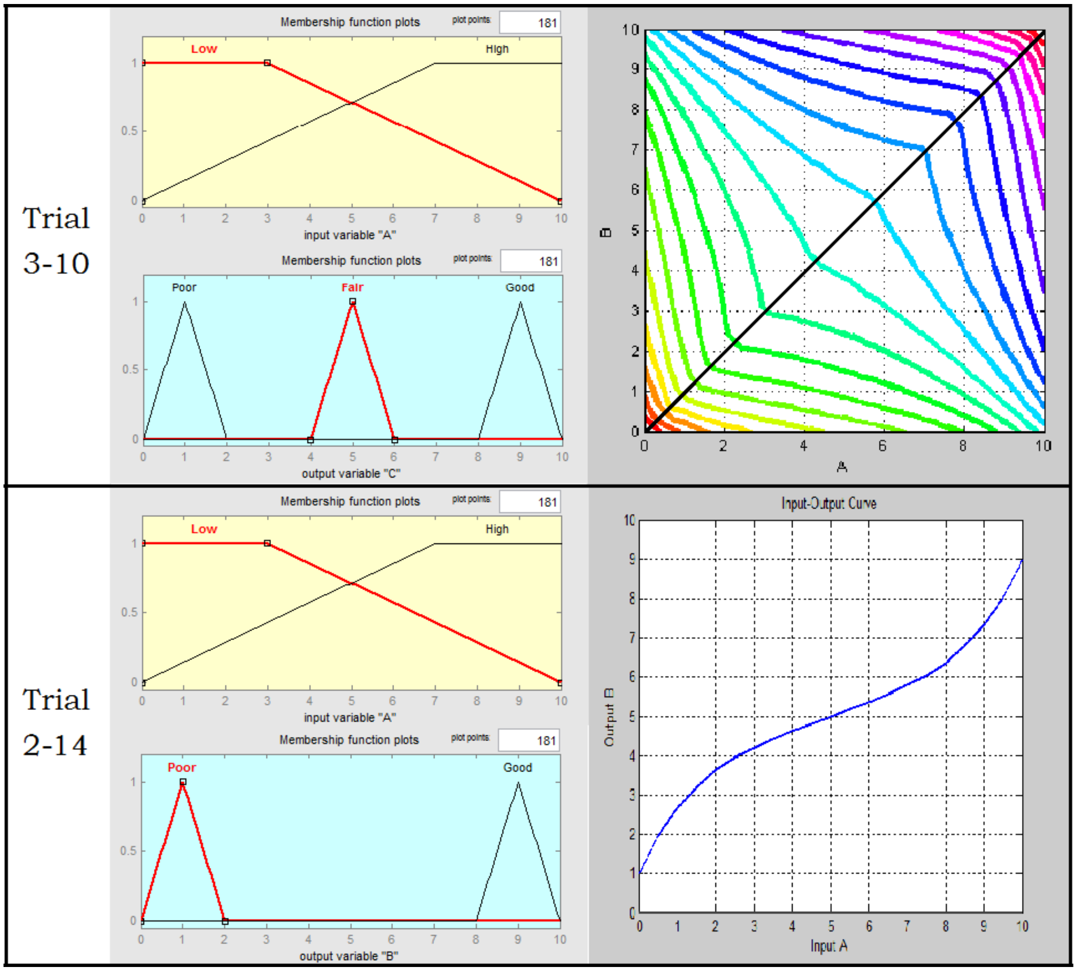


Figure 3.22 Reference figure between Trial 3-10 and Trial 2-14

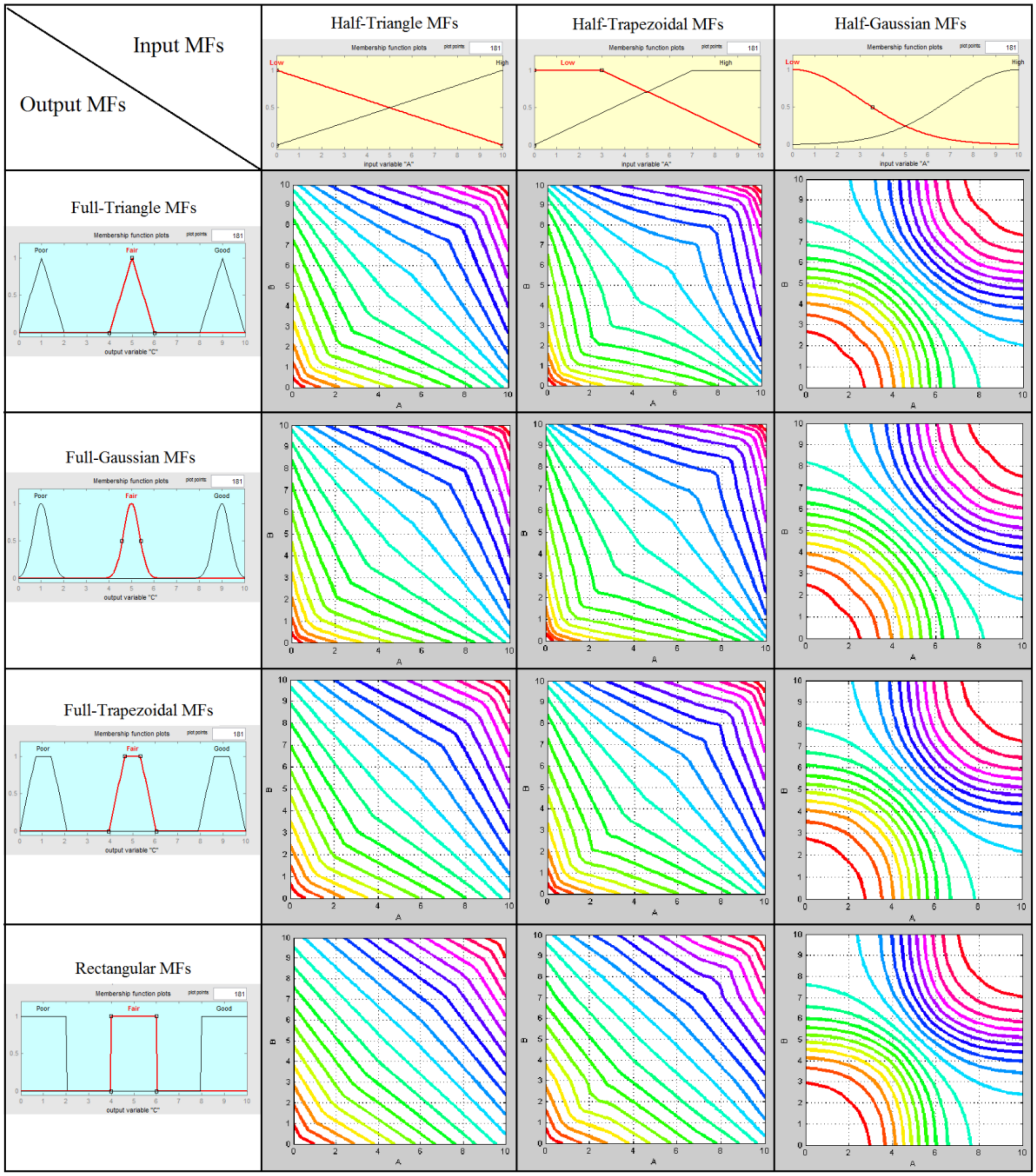


Figure 3.23 Chart of input-output relations from different membership functions for TISO models

## 3.5 Number of Membership Function in Variables

In the corresponding section of Chapter 2, the effect from the number of MFs for a single variable is studied through SISO fuzzy inference models. For an identical situation, when the quantity of MFs rises, the new derived input-output curve tends to repeat an original pattern of previous input-output curve. In this case, the original input-output curve is proportionally shrunk and meanwhile the amplitude diminishes, then the new input-output curve performs an improved linearity. In this section, TISO fuzzy inference models substitute former SISO inference models to retest the characteristic from the quantity of MFs.

Firstly, three fuzzy sets, 'Low', 'Medium' and 'High', are conceived to divide both input variable *A* and *B*. Same forms of MFs are applied within fuzzy sets as former trials do, two half-shape MFs stand for sets 'Low' and 'High', while one full-shape MF is utilized for set 'Medium'. In this situation, a completed inference mechanism requires totally 9 parallel If-Then rules. Accordingly, the quantity of MFs for output variable *C* is determined by the amount of different scores of antecedent-parts from rules. For easy configuring, score 1 is assigned for fuzzy set 'Low', while scores 2 and 3 are considered for rating sets 'Medium' and 'High' respectively. Since the antecedent "If *A* is Low and *B* is Low" contributes the lowest score 2, and antecedent "If *A* is High and *B* is High" produces maximum score 6, five different numbers are collected within interval [2, 6], and then five full-shape MFs for output variable *C* are expected.

Inputs *A* & *B*: Low / Medium / High

Output *C*: Very Poor / Poor/ Fair / Good / Very Good

Rule 1: If *A* is Low and *B* is Low, then *C* is Very Poor

Rule 2: If *A* is Low and *B* is Medium, then *C* is Poor

Rule 3: If *A* is Medium and *B* is Low, then *C* is Poor

Rule 4: If *A* is Low and *B* is High, then *C* is Fair

Rule 5: If *A* is High and *B* is Low, then *C* is Fair

Rule 6: If *A* is Medium and *B* is Medium, then *C* is Fair

Rule 7: If *A* is Medium and *B* is High, then *C* is Good

Rule 8: If *A* is High and *B* is Medium, then *C* is Good

Rule 9: If *A* is High and *B* is High, then *C* is Very Good

Then, a more complex model with five fuzzy sets for each input variable is tested. With five sets 'Very Low', 'Low', 'Medium', 'High' and 'Very High', two half-shape MFs and three full-shape MFs are required to constitute both inputs A and B. In this situation, a more complicated inference mechanism with 25 If-Then rules is anticipated. The lowest score, 2, comes from antecedent "If *A* is Very Low and *B* is Very Low", and the highest score, 10, is from antecedent "If *A* is Very High and *B* is Very High". Totally 9 numbers are collected within in interval [2, 10], thus 9 MFs for output *C* is demanded. For easy naming these 9 MFs, the default name 'mf+number' from MATLAB Fuzzy Logic Toolbox is remained. Completed rules are listed as follow

Inputs *A & B*: Very Low / Low / Medium / High / Very High

Output *C*: mf1 / mf2 / mf3 / mf4 / mf5 / mf6 / mf7 / mf8 / mf9

Rule 1: If *A* is Very Low and *B* is Very Low, then *C* is mf1

Rule 2: If *A* is Very Low and *B* is Low, then *C* is mf2

Rule 3: If *A* is Low and *B* is Very Low, then *C* is mf2

Rule 4: If *A* is Very Low and *B* is Medium, then *C* is mf3

Rule 5: If *A* is Medium and *B* is Very Low, then *C* is mf3

Rule 6: If *A* is Low and *B* is Low, then *C* is mf3

Rule 7: If *A* is Very Low and *B* is High, then *C* is mf4

Rule 8: If *A* is High and *B* is Very Low, then *C* is mf4

Rule 9: If *A* is Low and *B* is Medium, then *C* is mf4

Rule 10: If *A* is Medium and *B* is Low, then *C* is mf4

Rule 11: If *A* is Very Low and *B* is Very High, then *C* is mf5

Rule 12: If *A* is Very High and *B* is High Low, then *C* is mf5

Rule 13: If *A* is Low and *B* is High, then *C* is mf5

Rule 14: If *A* is High and *B* is Low, then *C* is mf5

Rule 15: If *A* is Medium and *B* is Medium, then *C* is mf5

Rule 16: If *A* is Low and *B* is Very High, then *C* is mf6

Rule 17: If *A* is Very High and *B* is Low, then *C* is mf6

Rule 18: If *A* is Medium and *B* is High, then *C* is mf6

Rule 19: If *A* is High and *B* is Medium, then *C* is mf6

Rule 20: If *A* is Medium and *B* is Very High, then *C* is mf7

Rule 21: If *A* is Very High and *B* is Medium, then *C* is mf7

Rule 22: If *A* is High and *B* is High, then *C* is mf7

Rule 23: If *A* is High and *B* is Very High, then *C* is mf8

Rule 24: If *A* is Very High and *B* is High, then *C* is mf8

Rule 25: If *A* is Very High and *B* is Very High, then *C* is mf9

**Trial 3-18:** Inputs *A&B*: 3 rectangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *C*: 5 rectangular MFs (0% OR)

**Trial 3-19:** Inputs *A&B*: 5 triangle MFs (2 half MFs + 3 full MFs, 100% OR)

Output *C*: 9 rectangular MFs (0% OR)

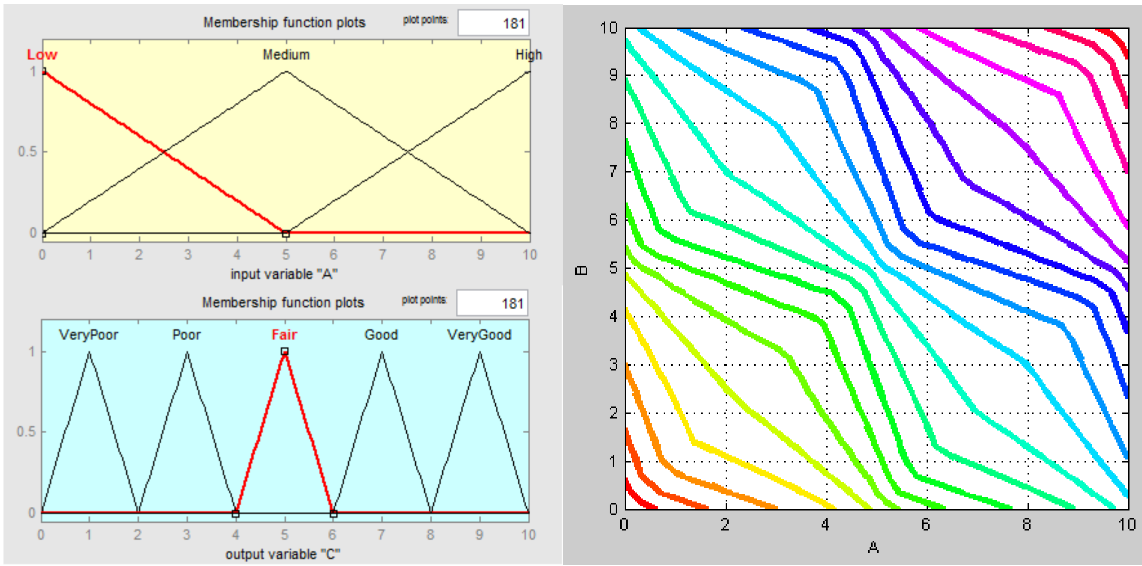


Figure 3.24 Membership function arrangement and contour graph for Trial 3-18

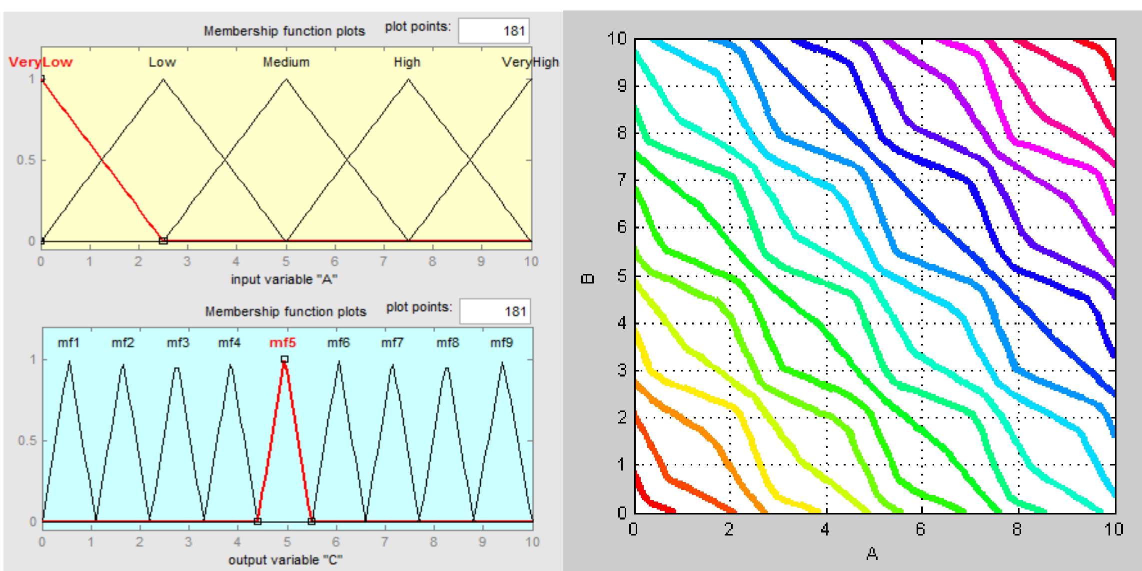


Figure 3.25 Membership function arrangement and contour graph for Trial 3-19

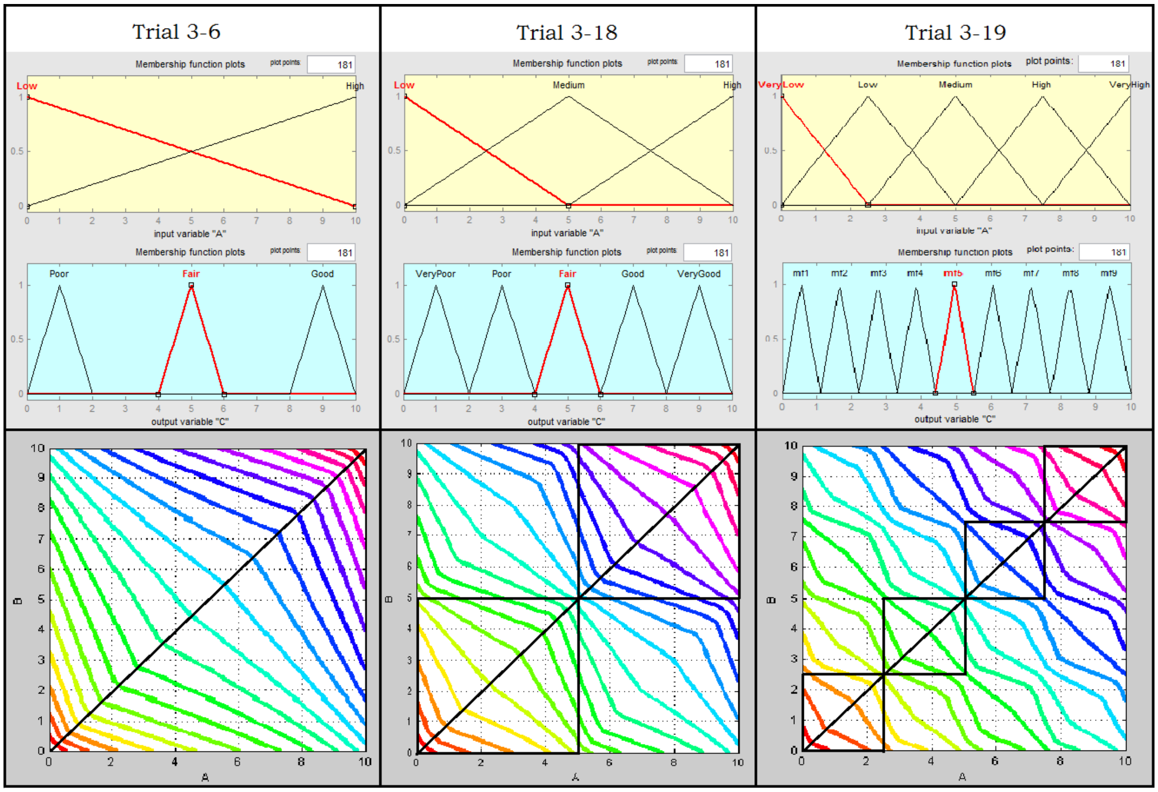


Figure 3.26 Reference figure among Trial 3-6, 3-18 and 3-19

The same model in Trial 3-6 is reconstructed in Trial 3-18 and 3-19. In Trial 3-18, the original model in Trial 3-6 is modified by introducing one more fuzzy set ‘Medium’ into both inputs A and B, and matching MFs for output *C* are adjusted. In Figure 3.26, it is clearly to demonstrate that the contour pattern from Model 3-6 is repeated one more time along the diagonal of contour graph of Trial 3-18. While in Trial 3-19, the original model with 2MFs-defined input is extended to new one with 5MFs-defined input. Same as the trend in SISO models, the contour pattern from Model 3-6 is repeated three more times along the diagonal of contour graph of Trial 3-19. Because the scales of contour graphs remain unchanged within Trial 3-6, 3-18 and 3-18, and the original contour pattern is proportionally contracted when the quantity of MFs for single input variable increases, then the input-output surface grows to be more smooth and displays improved linearity.

**Trial 3-20:** Inputs *A&B*: 3 Gaussian MFs (2 half MFs + 1 full MF, 100% OR)

Output *C*: 5 rectangular MFs (0% OR)

**Trial 3-21:** Inputs *A&B*: 5 Gaussian MFs (2 half MFs + 3 full MFs, 100% OR)

Output *C*: 9 rectangular MFs (0% OR)

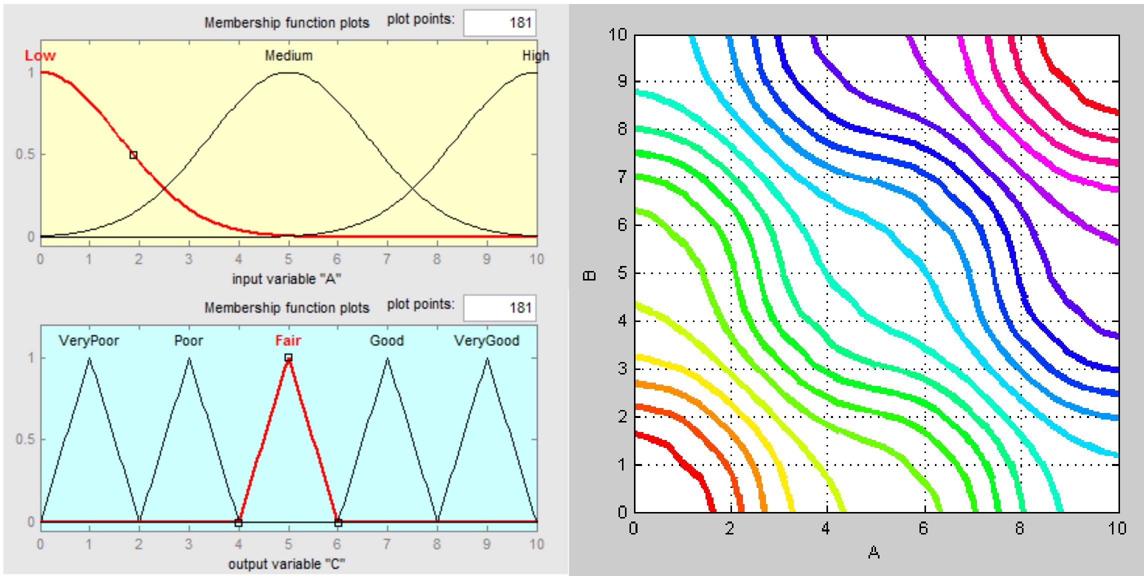


Figure 3.27 Membership function arrangement and contour graph for Trial 3-20

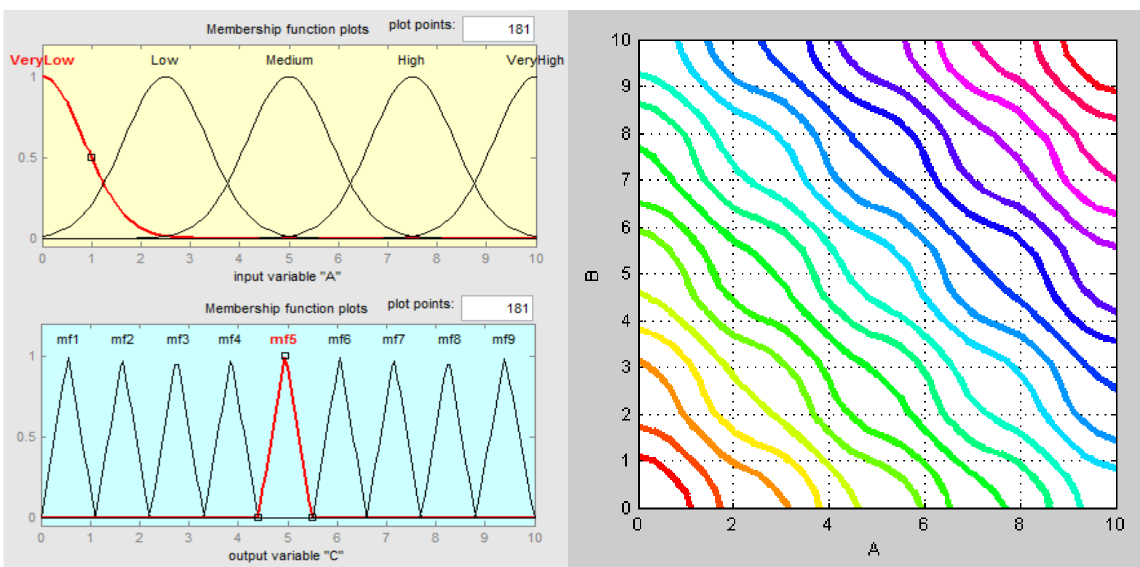


Figure 3.28 Membership function arrangement and contour graph for Trial 3-21

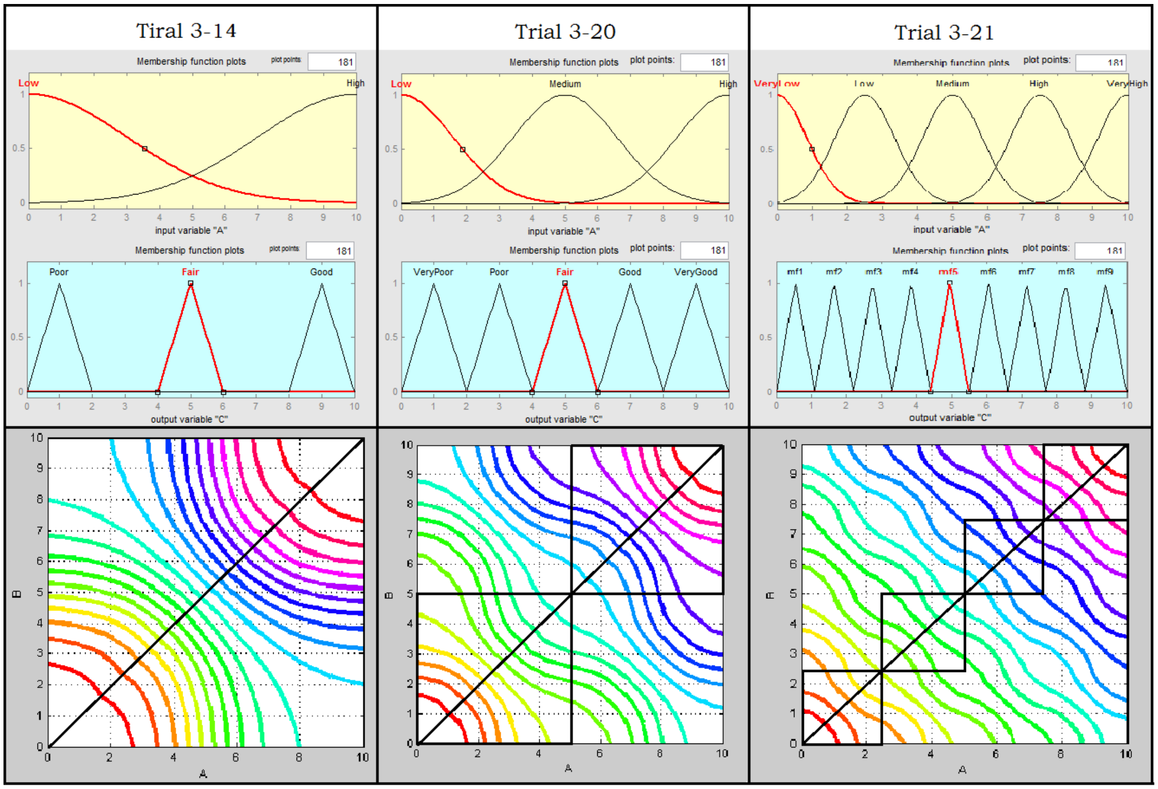


Figure 3.29 Reference figure among Trial 3-14, 3-20 and 3-21

In Trial 3-20 and Trial 3-21, the models of Trial 3-18 and 3-19 are revised by substituting the Gaussian MFs for triangle MFs of inputs A and B. Along with an increased number of MFs for input, as Figure 3.29 shows, a similar repetitive tendency appears among Trials 3-14, 3-20 and 3-21. As it will always be, when the quantity of MFs for input variable rises, the contour lines tend to be straight and parallel to each other with consistent gaps. Consequently the input-output surface becomes more even, and the output variable C possesses an approximate linear relation with either input variables *A* or *B*.

It is reasonable to conjecture that for the most ideal TISO model in Trial 3-9 which consists of triangle input MFs and rectangular output MFs, the increase of MFs for inputs A and B will also improve the system performance of linearity. As Trial 3-22 proves, the model with five triangle input MFs and nine rectangular output MFs expresses a more desirable input-output surface. Summarily, when more MFs are utilized to define a same scale, the support of each MF must be shrunk. Along with this adjustment, the differences among various types of MFs are diminished as well. Since the If-Then rules are configured symmetrically, all SISO and TISO models will gradually transform to linear fuzzy inference systems. Nevertheless, because TISO model with triangle input MFs and rectangular output MFs can produce satisfactory linear performance with few MFs for input variable, it is still an ideal reference for forming non-linear TISO fuzzy inference models in the next section.

**Trial 3-22:** Inputs *A&B*: 5 triangle MFs (2 half MFs + 3 full MFs, 100% OR)

Output *C*: 9 rectangular MFs (0% OR)

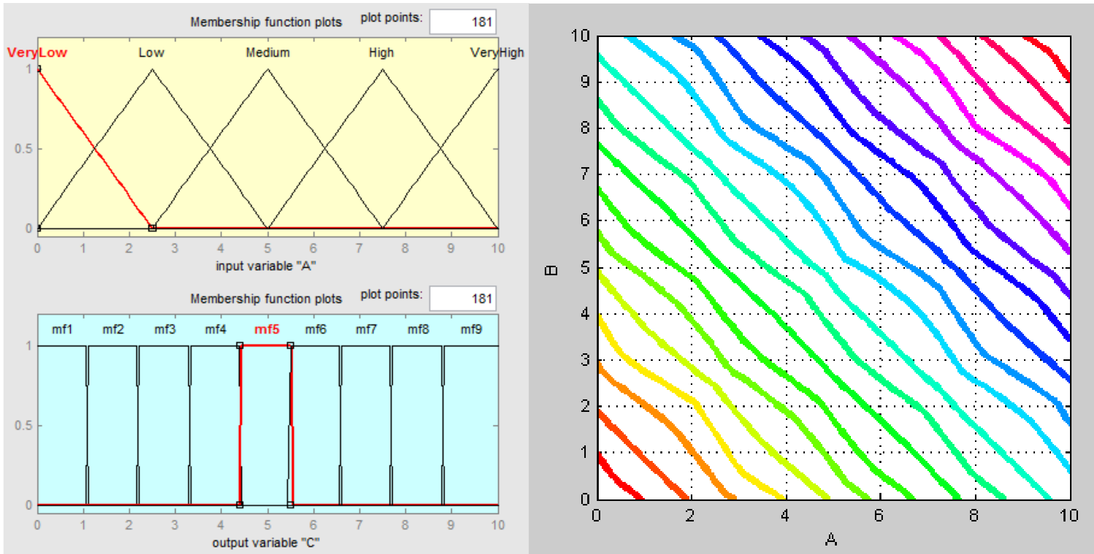


Figure 3.30 Membership function arrangement and contour graph for Trial 3-22

### 3.6 Control of Input-Output Relation

This section focuses on building non-linear TISO fuzzy inference systems by adjusting approximate-linear TISO model with triangle input MFs and rectangular output MFs. Because modulating output MFs is more transparent than adjusting input MFs, meanwhile rectangular MF has less geometric characteristics than triangle MF does, it is more effective and easier to configuring non-linear TISO systems via converting output rectangular MFs.

#### 3.6.1 Action Spot of If-Then Rules

Unlike SISO fuzzy inference model where each If-Then rule corresponds to a unique output MF, many rules in TISO systems and MISO systems must be matched with a common output MF in most cases. In this situation, all the rules will be affected when their common output MF is modulated. For easy discussing, the concept of action spot of rule is introduced. Taking TISO model with two MFs for input variable as example, four rules

Rule 1: If *A* is Low and *B* is Low, then *C* is Poor

Rule 2: If *A* is High and *B* is Low, then *C* is Fair

Rule 3: If *A* is Low and *B* is High, then *C* is Fair

Rule 4: If *A* is High and *B* is High, then *C* is Good

are demanded, and Rule 2 and Rule 3 go together with same output MF ‘Fair’. The full effect of each rule can only concentrate on small region of this 2-dimensional input space. Because of the geometric feature of half-shape MFs, the membership value of fuzzy set ‘Low’ or ‘High’ can only reach 1 when it takes input value 0 or 10, thus showed as Figure 3.31, the action spot of antecedent “*A* is Low and *B* is Low” is indicated by the dot on the

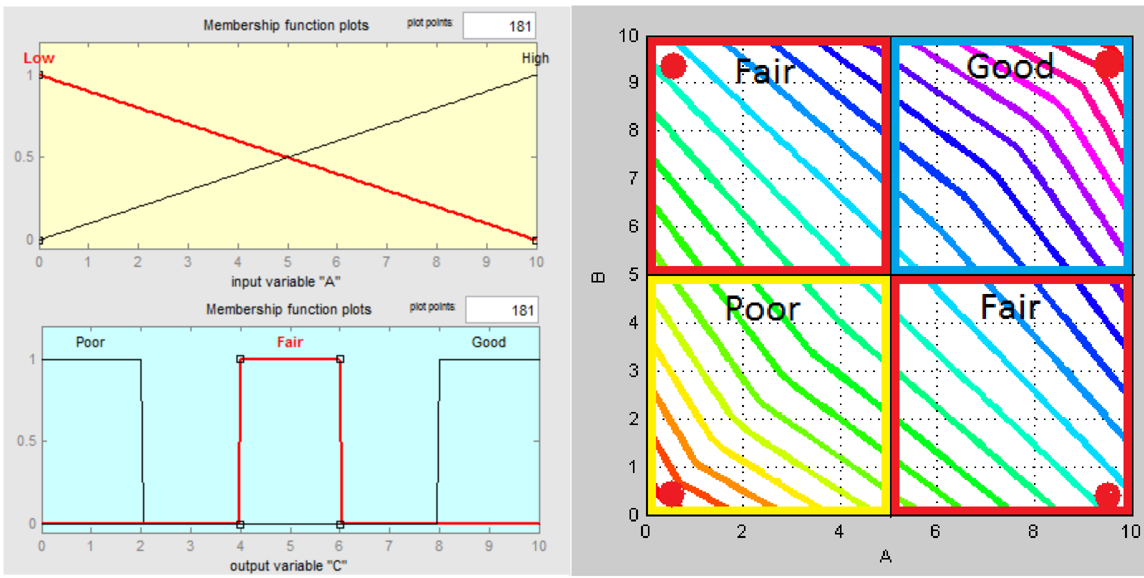


Figure 3.31 Membership function arrangement and action spots distribution for sample TISO fuzzy inference system with 4 rules

lower-left corner of contour graph where both input values of *A* and *B* are close to 0. Similarly, the dot on the upper-right corner of contour graph shows the action spot of antecedent "*A* is High and *B* is High" where both input values of *A* and *B* are close to 10. The geometric characteristics of output MF can be expressed by the density and linearity of contour lines around the corresponding action spot, and the transition between action spots will also manifest the features of output MFs.

### 3.6.2 Input Variables with Two Membership Functions

Inputs *A & B*: Low / High   Output *C*: Poor / Fair / Good

Rule 1: If *A* is Low and *B* is Low, then *C* is Poor

Rule 2: If *A* is High and *B* is Low, then *C* is Fair

Rule 3: If *A* is Low and *B* is High, then *C* is Fair

Rule 4: If *A* is High and *B* is High, then *C* is Good

**Trial 3-23 (same as Trial 3-9):** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

**Trial 3-24**: Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

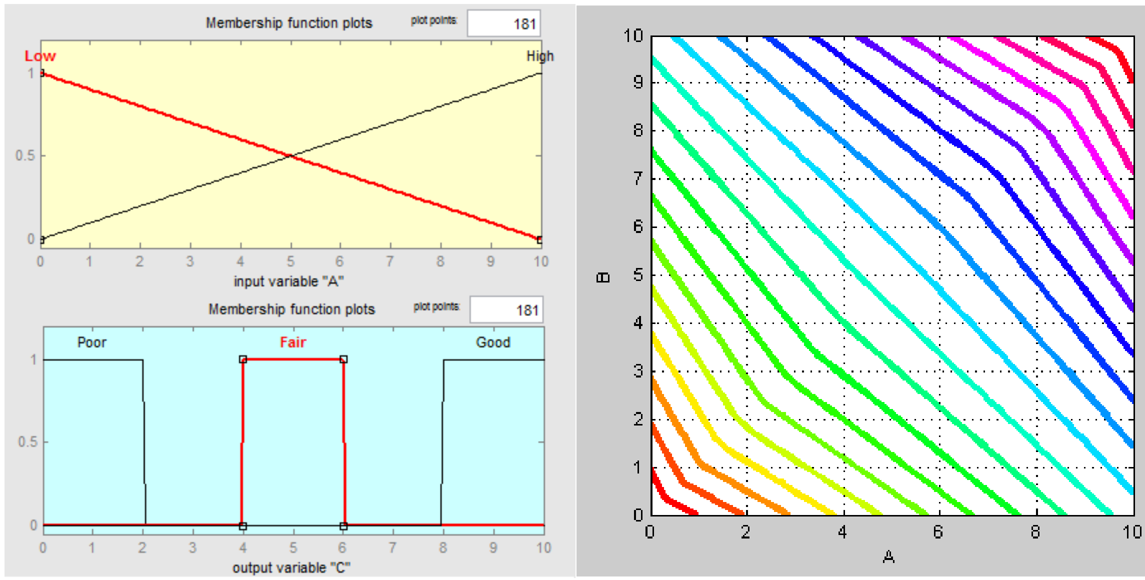


Figure 3.32 Membership function arrangement and contour graph for Trial 3-23

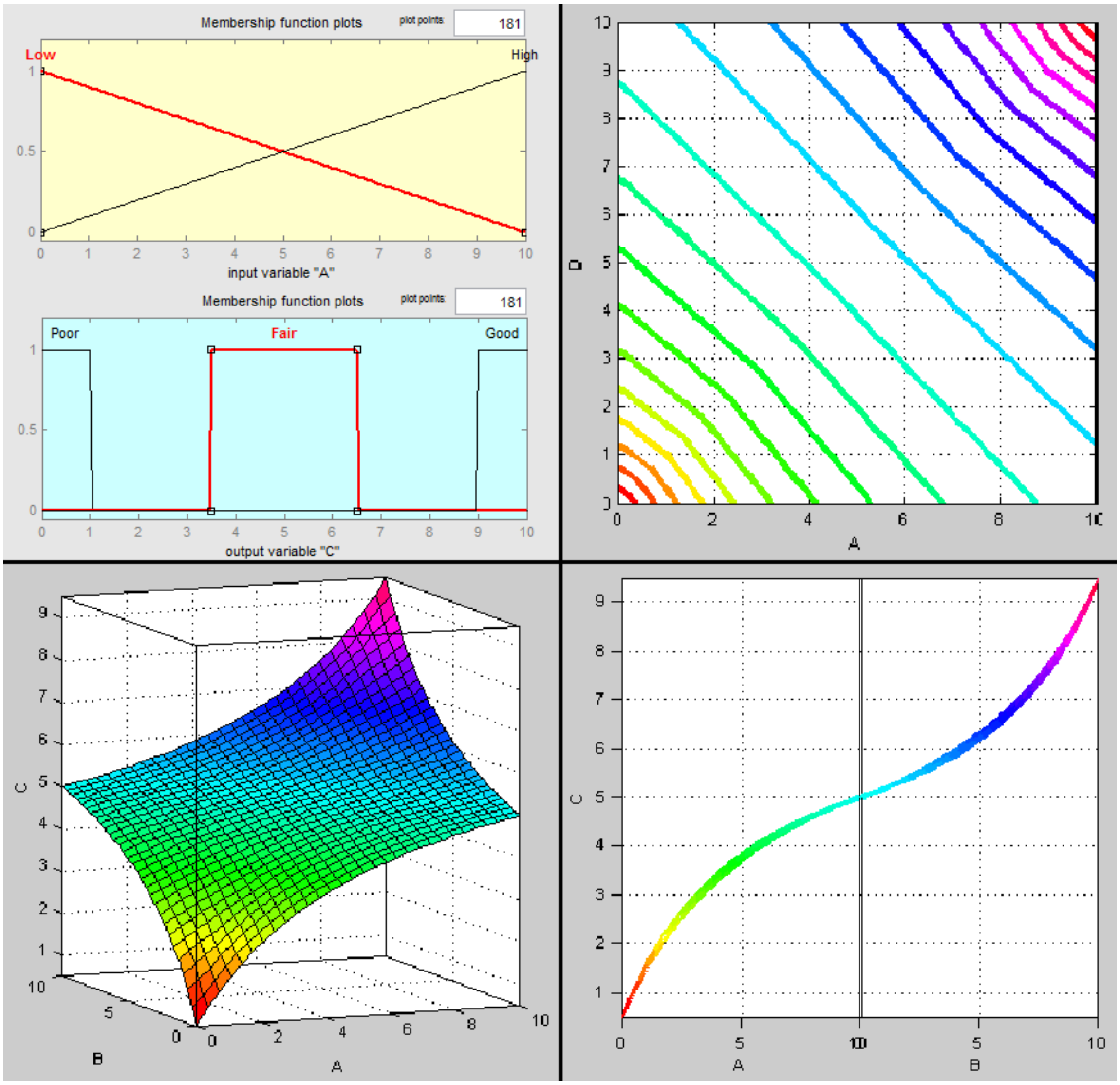


Figure 3.33 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-24

Compared with Trial 3-23, in Trial 3-24, the support of MFs 'Poor' and 'Good' are contracted and the support of MF 'Fair' is extended. By doing that, the density of contour lines turns to be thicker at endpoints (0, 0) and (10, 10) than that around middle region of input space. As showed in Figure 3.34, because these two output MFs with small area match action spot (0, 0) and (10, 10), the MF with large area will not make contribution when both input values are close to a same boundary of input scale. However, if each of the input value moves one small step toward middle point, the area of MF 'Fair' will be get involved and markedly alter the defuzzified value. Thus as the contour lines express, the rate of change of output value is high at the action point (0, 0) and (10, 10). Contrarily, in Figure 3.35, when both input values are located around mid-value of input scale, or they are far separated to different ends, the MF 'Fair' with large area will play the determinant role. Even though a small change of each input value will impact the implication process for all output MFs simultaneously, the geometric center of aggregated area will not be changed observably. As a result the density of contour lines within those regions, including at action spots (0, 10) and (10, 0), seems thinner, so that the rate of change of output variable $C$ is low. The side view of surface on the lower-right corner of Figure 3.33 is imaged through diagonal from (10, 0) to (0, 10). It shows a distinct growth tendency of output variable $C$.

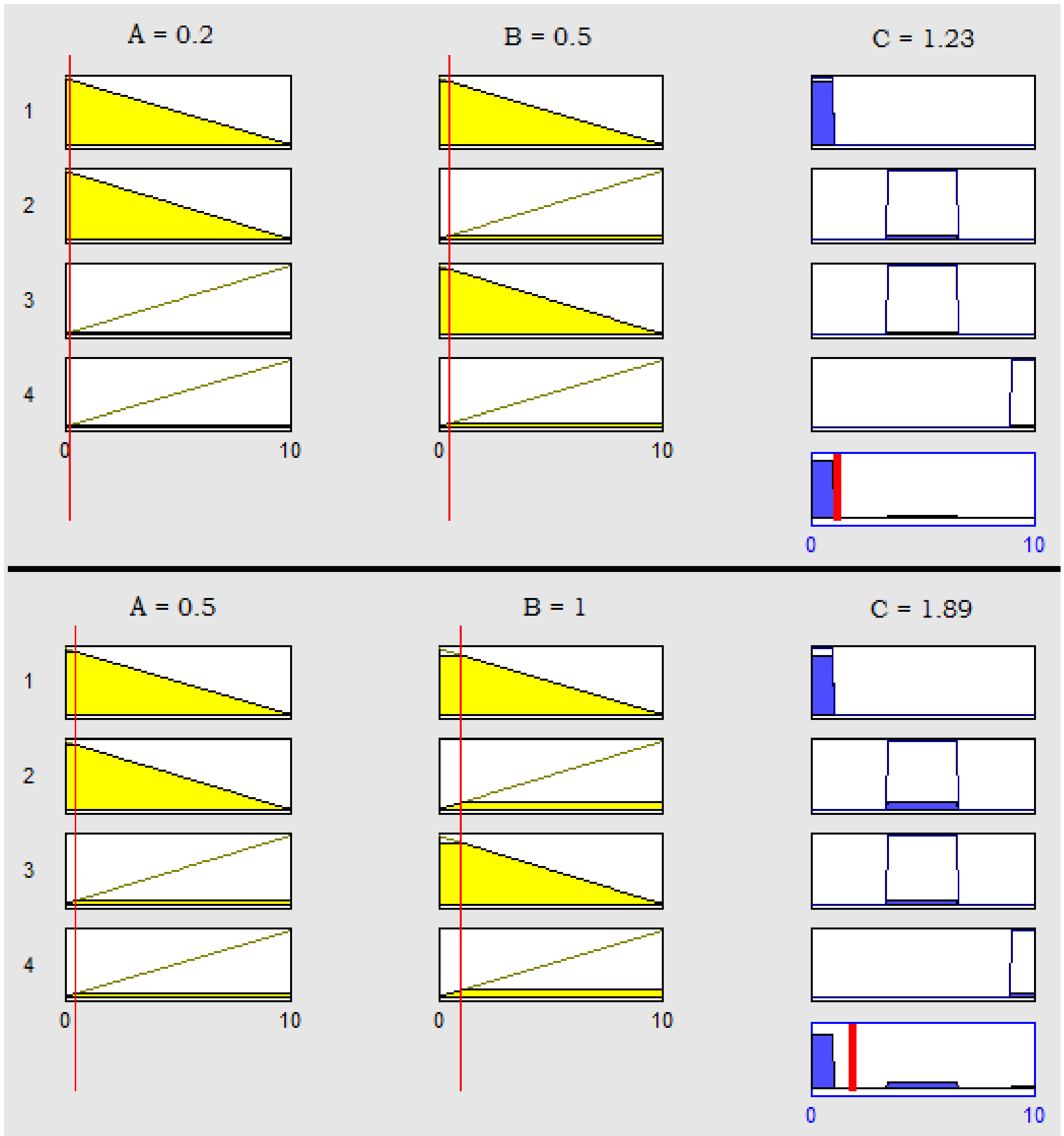


Figure 3.34 Implication and aggregation processes for Trial 3-24 at Position 1

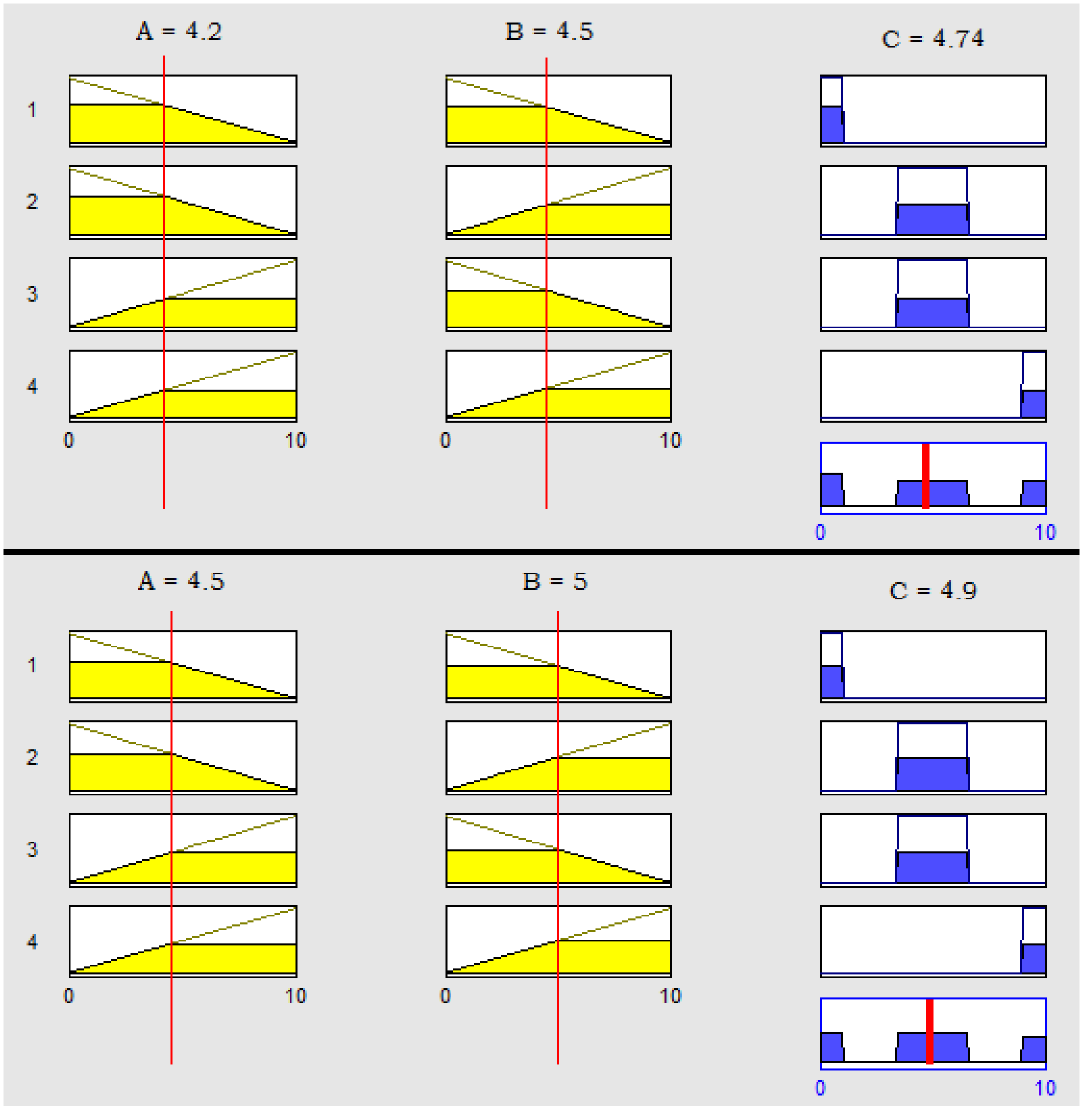


Figure 3.35 Implication and aggregation processes for Trial 3-24 at Position 2

**Trial 3-25:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

In Trial 3-25, oppositely, the support of output MF 'Fair' is contracted while the support of MFs 'Poor' and 'Good' are extended. Compared with the model in Trial 3-24, a sigmoidal input-output surface is anticipated for Trial 3-25. However, the surface does not show an input-output relation as we expected. Learn from Figure 3.37, when both values for inputs A and B approach toward a same endpoint of input scale, rule "If *A* is Low and *B* is Low, then *C* is Poor" or "If *A* is High and *B* is High, then *C* is Good" is activated to play dominant role on action spot (0, 0) or (10, 10). Because the matching output MFs for these rules are those with larger areas, the shift of the center of area will be negligible when each of the input value moves one small step, thus a low rate of change of output variable *C* will be demonstrated by sparse contour lines around action spots (0, 0) and (10, 10) on input space. On the other hand, when the values of inputs A and B move toward difference ends of input scale, as Figure 3.38 shows, the output MF 'Fair' with smaller area replaces the dominant position of MFs 'Poor' and 'Good' and works on action spot (0, 10) and (10, 0). In this case, the center of area will be more sensitive to small variation of input values, so a high rate of change of output variable *C* will be demonstrated by dense contour lines around action spots (0, 10) and (10, 0). Nevertheless, when both inputs A and B are located close to mid-value of input scale, all output MFs make contribution to aggregated area. Because the position of geometric center relies more on larger area, the rate of change of output *C* around middle region is still insensitive to the change of input values. Thus an ideal sigmoidal input-output surface is deformed in the middle region of input space.

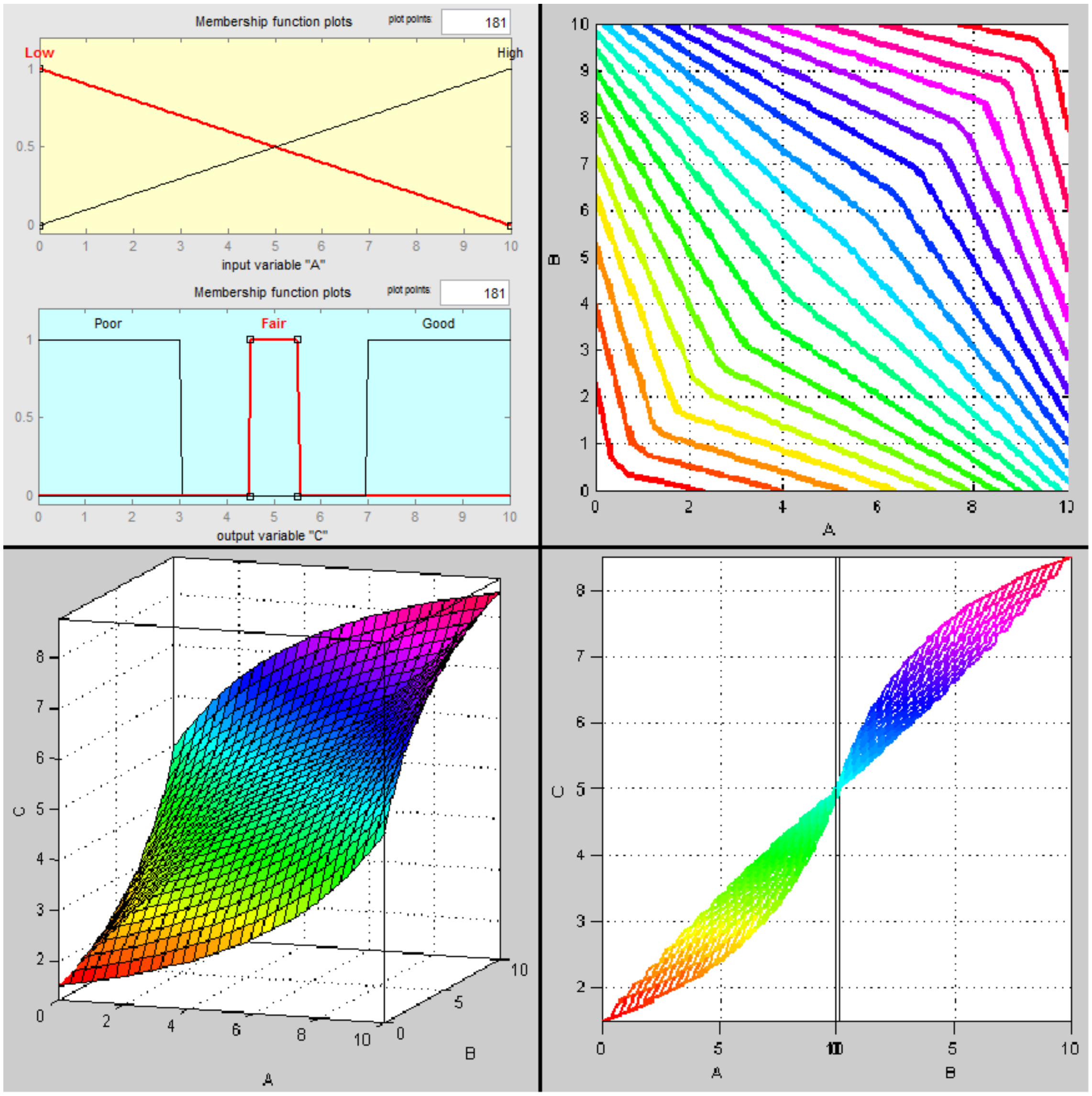


Figure 3.36 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-25

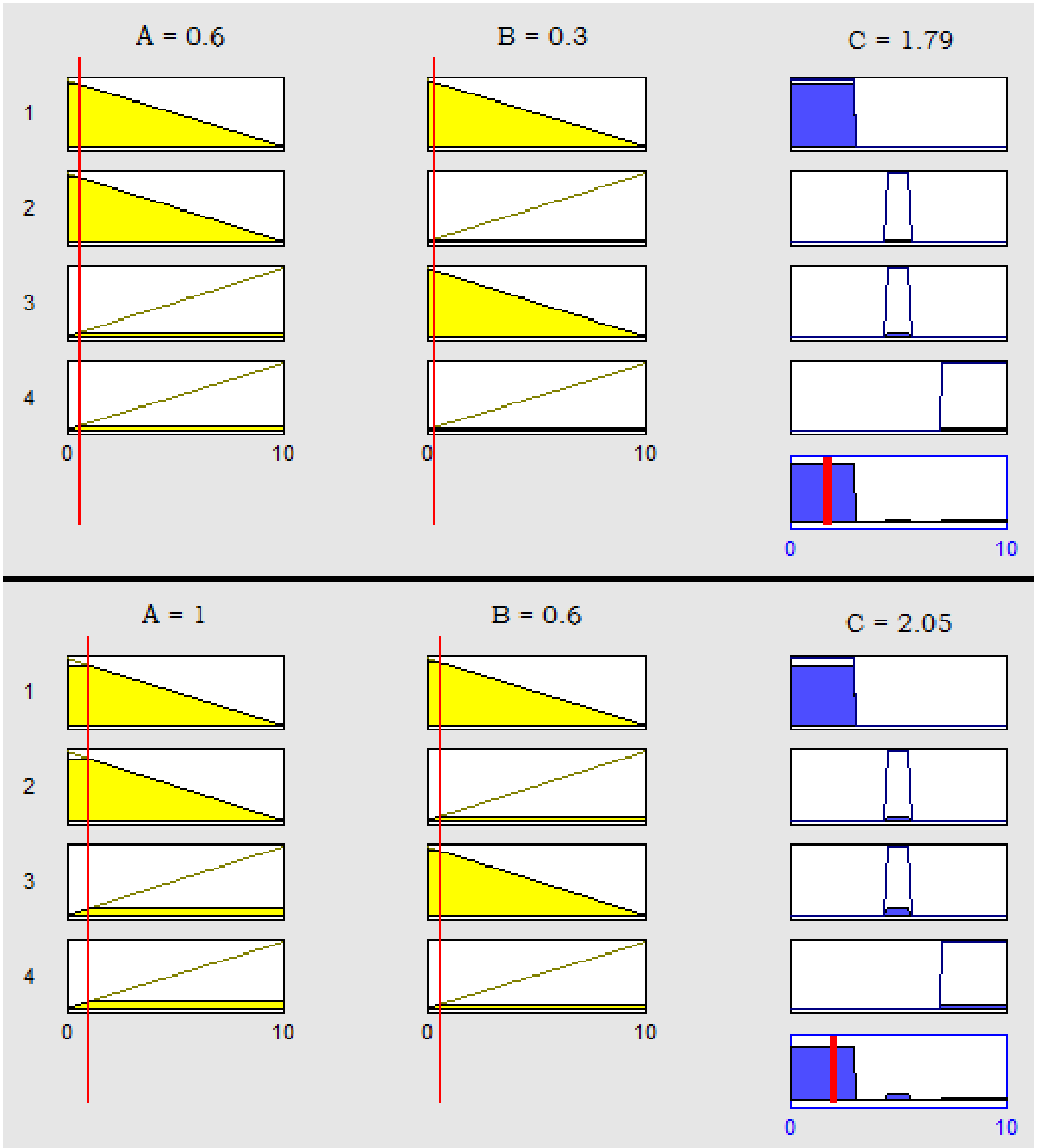


Figure 3.37 Implication and aggregation processes for Trial 3-25 at Position 1

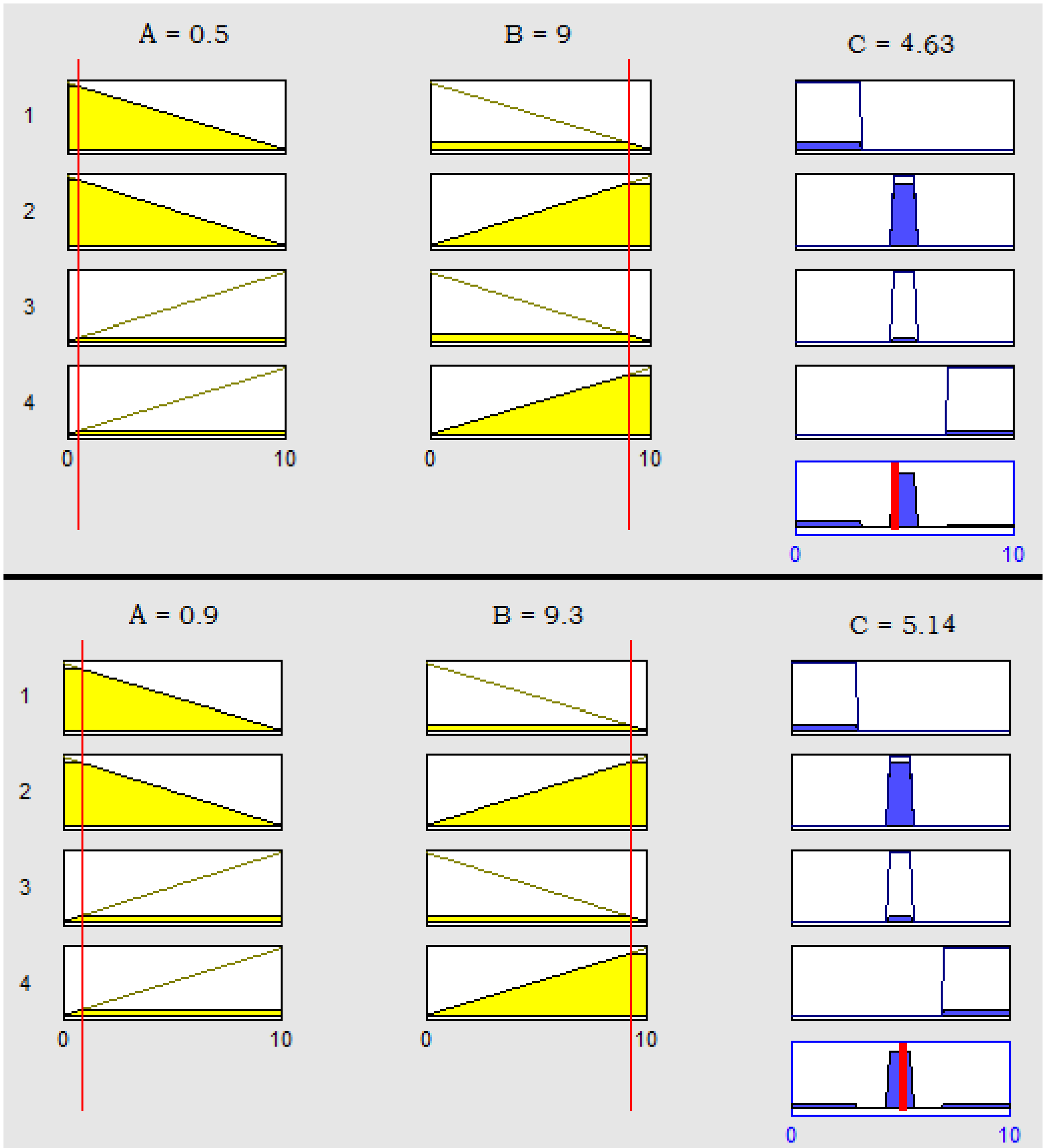


Figure 3.38 Implication and aggregation processes for Trial 3-25 at Position 2

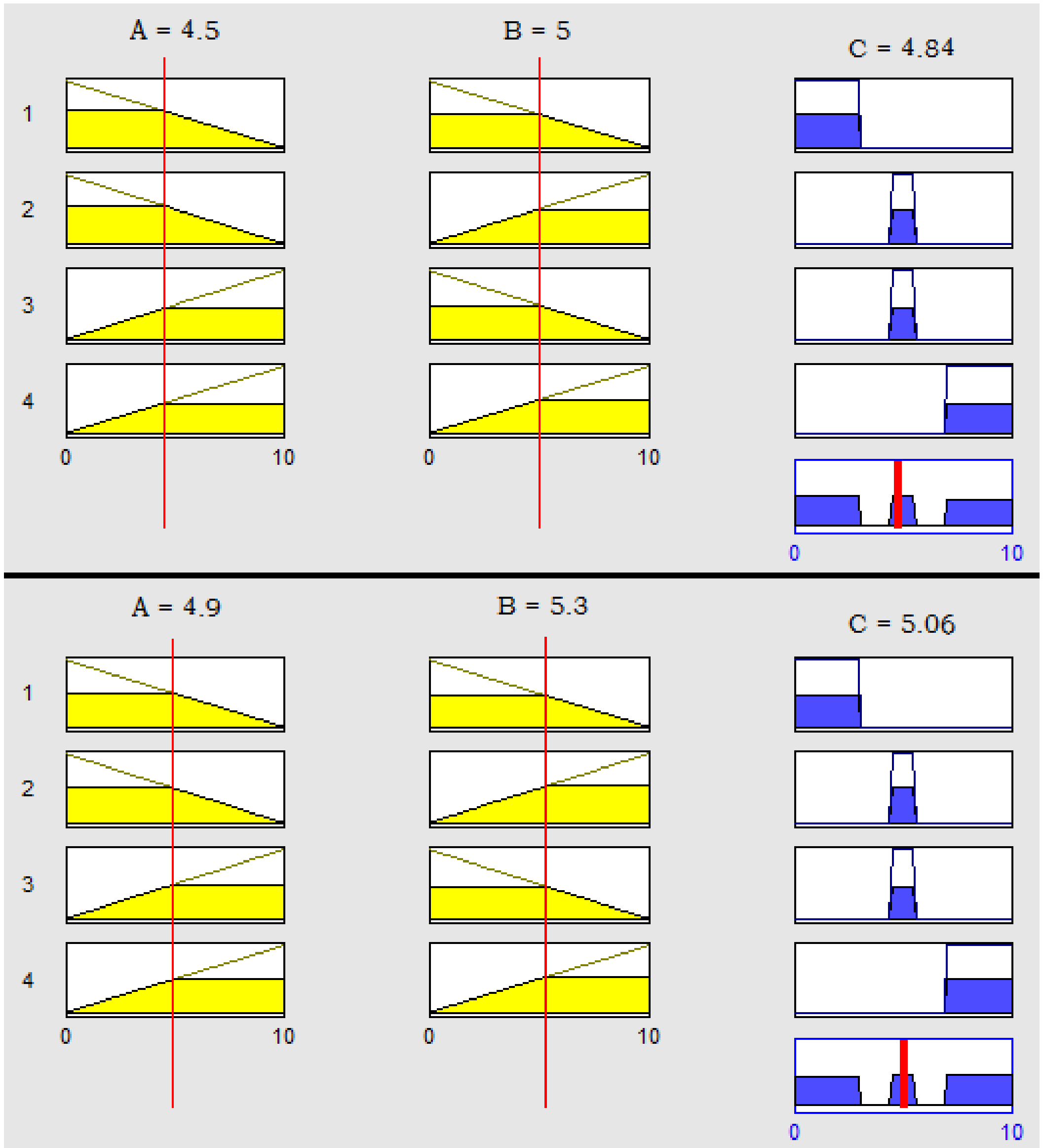


Figure 3.39 Implication and aggregation processes for Trial 3-25 at Position 3

**Trial 3-26:** Inputs *A&B*: 2 half-triangle MFs (100% OR)

Output *C*: 3 rectangular MFs (0% OR)

From the diagonal side view of Trial 3-26 in Figure 3.40, the input-output surface is modulated close to logarithmic curve. For action spots (0, 0) on input space, the matching output MF 'Poor' comes with smallest area, which causes a high change rate of output value and dense contour lines around this point. Along the diagonal direction from (0, 0) to (10, 10), the distance between adjacent contour lines gradually increases, until the maximum gap between neighboring lines is developed around action spot (10, 10) which corresponds to output MF 'Good' with largest area. The change rates of output *C* on action spots (0, 10), (10, 0) and around middle region of input space are almost the same, thus the transition process of input-output surface performs desirable smoothness.

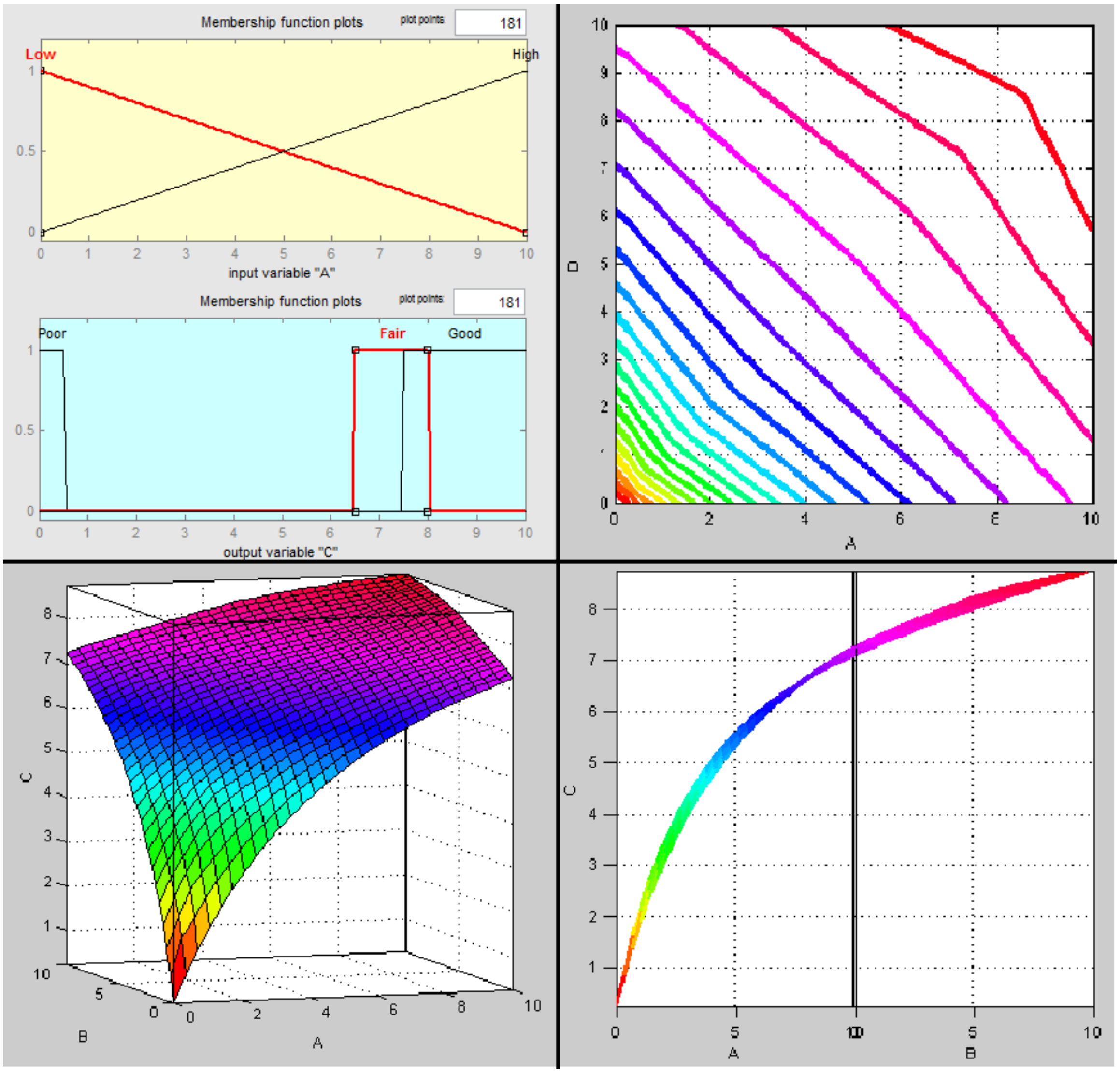


Figure 3.40 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-26

### 3.6.3 Input Variables with Three Membership Functions

Inputs *A* & *B*: Low / Medium / High

Output *C*: Very Poor / Poor/ Fair / Good / Very Good

Rule 1: If *A* is Low and *B* is Low, then *C* is Very Poor

Rule 2: If *A* is Low and *B* is Medium, then *C* is Poor

Rule 3: If *A* is Medium and *B* is Low, then *C* is Poor

Rule 4: If *A* is Low and *B* is High, then *C* is Fair

Rule 5: If *A* is High and *B* is Low, then *C* is Fair

Rule 6: If *A* is Medium and *B* is Medium, then *C* is Fair

Rule 7: If *A* is Medium and *B* is High, then *C* is Good

Rule 8: If *A* is High and *B* is Medium, then *C* is Good

Rule 9: If *A* is High and *B* is High, then *C* is Very Good

As Figure 3.41 shows, 9 action spots are required for 9 If-Then rules. The coordinates of action spots in input space are (0, 0), (5, 0), (10, 0), (0, 5), (5, 5), (10, 5), (0, 10), (5, 10) and (10, 10).

**Trial 3-27:** Inputs *A&B*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *C*: 5 rectangular MFs (0% OR)

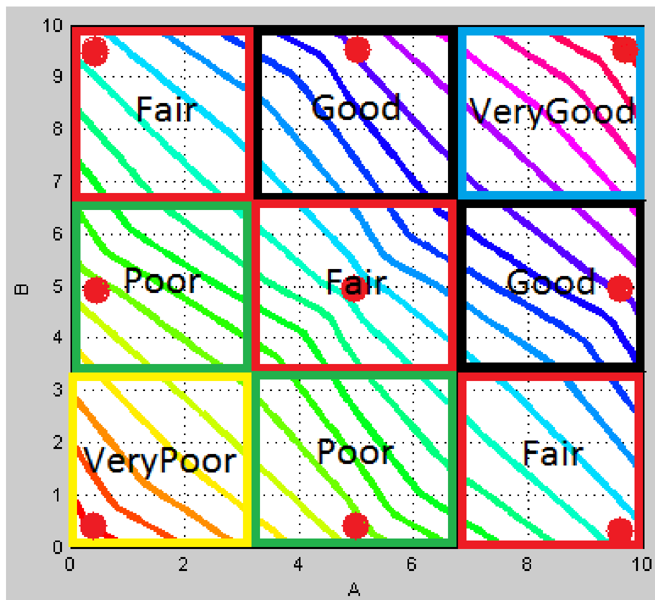


Figure 3.41 Action spots distribution for TISO models with 9 rules

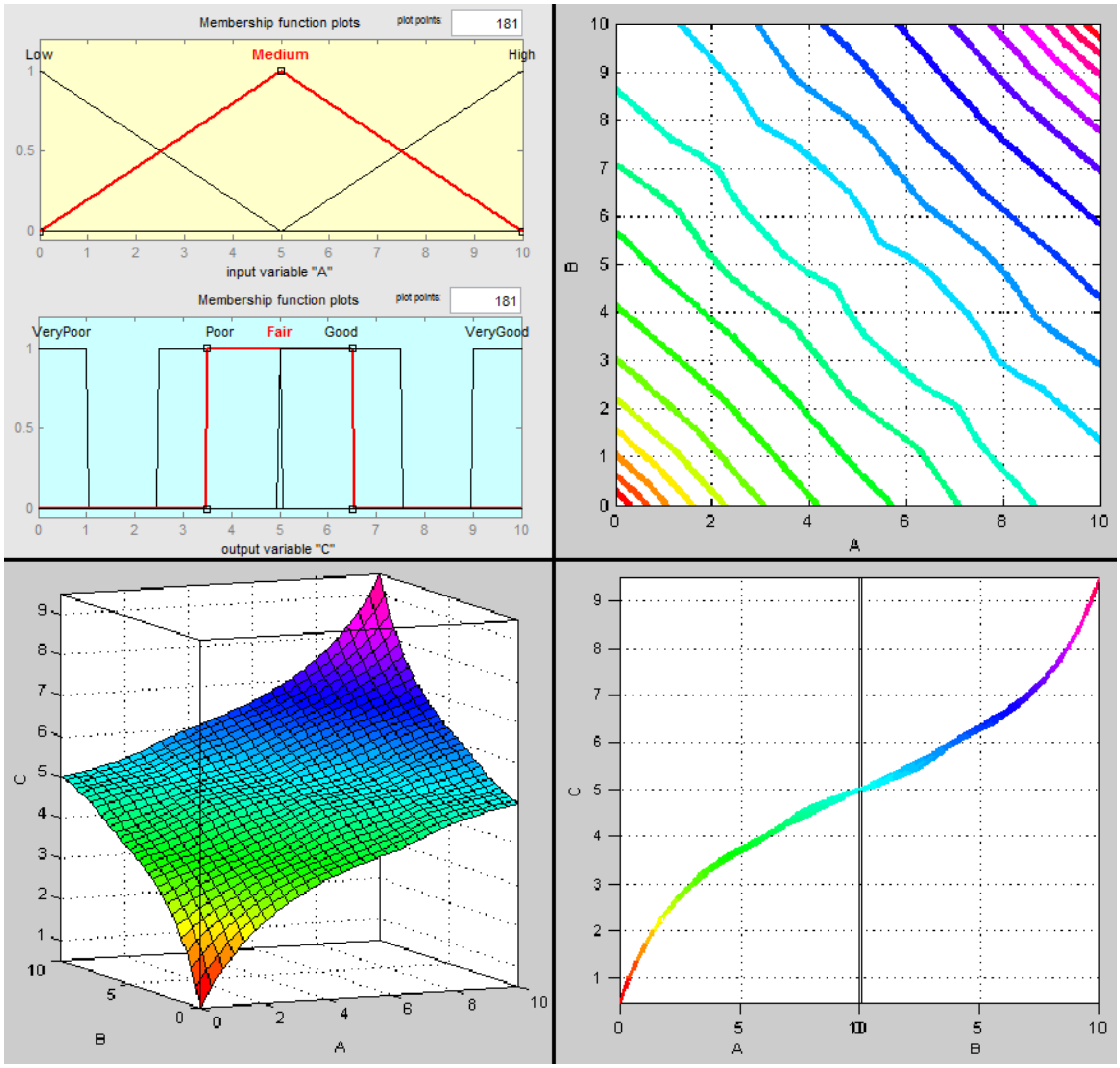


Figure 3.42 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-27

When one more fuzzy set ‘Medium’ is added into input variables *A* and *B* of Trial 3-24, a similar input-output surface with improved controllability is developed in Trial 3-27. The rise of quantity of input and output variables creates extra degree of freedom to finely adjust input-output surface, meanwhile it consumes more time to integrate output MFs, and damages the smoothness of input-output surface to a certain extent.

**Trial 3-28:** Inputs *A*&*B*: 3 triangle MFs (2 half MFs + 1 full MF, 100% OR)

Output *C*: 5 rectangular MFs (0% OR)

In Trial 3-28, the original model of Trial 3-25 is upgraded by introducing an extra fuzzy set ‘Medium’ for both inputs A and B as well. Based on rules mentioned above, action spots (0, 10), (5, 5) and (10, 0) represent a common output MF ‘Fair’. It is clear to observe that by matching MF ‘Fair’ with smallest area, the density of contour lines around three action spots is evidently higher than that in other regions. However the distributions of contour lines between action spots (0, 10), (5, 5) and (5, 5), (10, 0) are directly interfered by surrounding action spots (0, 5), (5, 10) and (5, 0), (10, 5) respectively. As a result, although the smoothness of input-output surface is slightly meliorated, there still exists noticeable error.

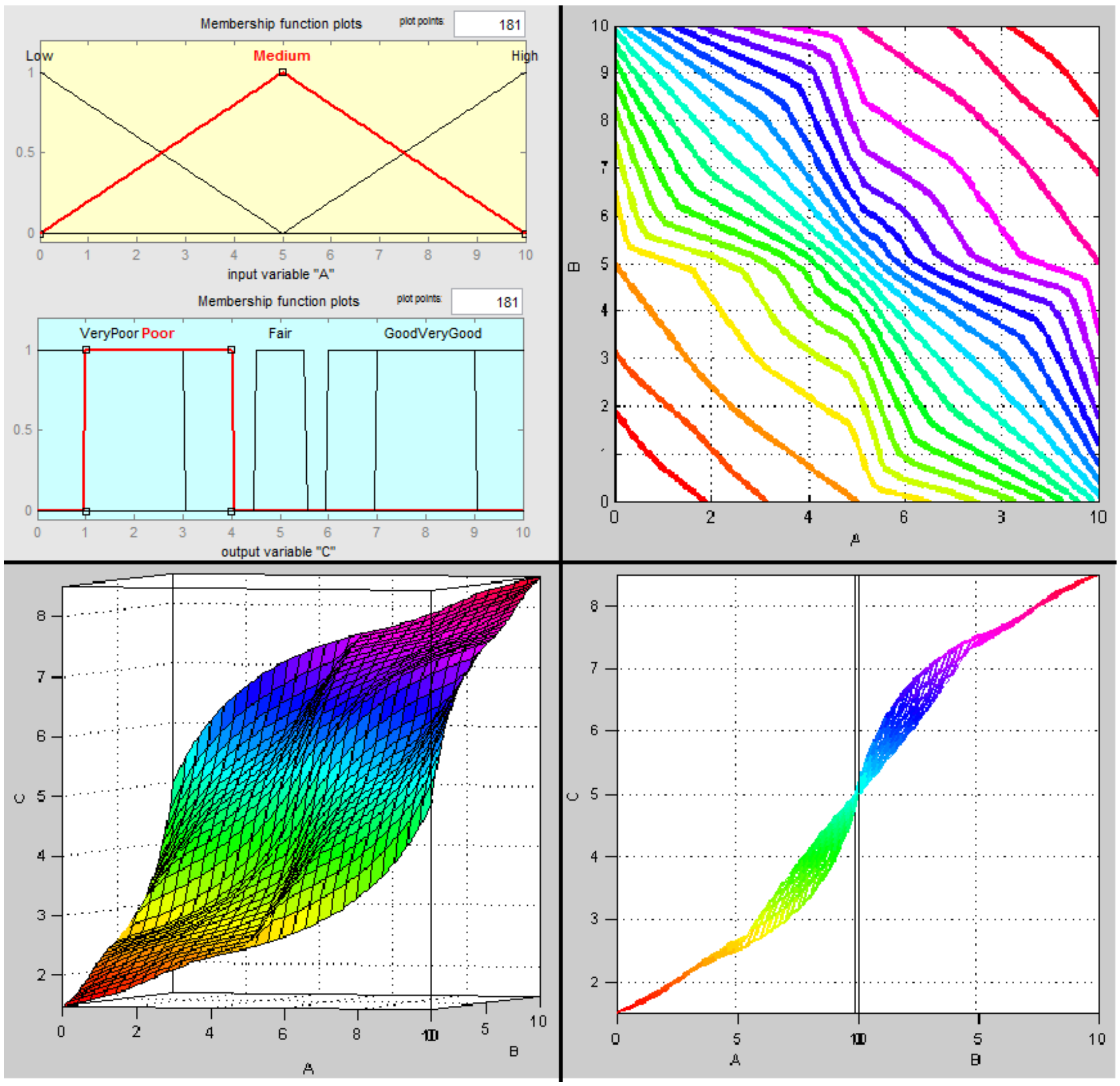


Figure 3.43 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-28

**Trial 3-29:** Inputs *A&B*: 3 triangle MFs (2 half MFs +1 full MF, 100% OR)

Output *C*: 5 rectangular MFs (0% OR)

In Trial 3-29, a similar logarithmic-shape surface from Trial 3-26 is repeated from a new model with three MFs for each input variable. It is imaginable that the new system with a larger number of MFs comes with improved flexibility for adjustment, and the loss of smoothness is negligible.

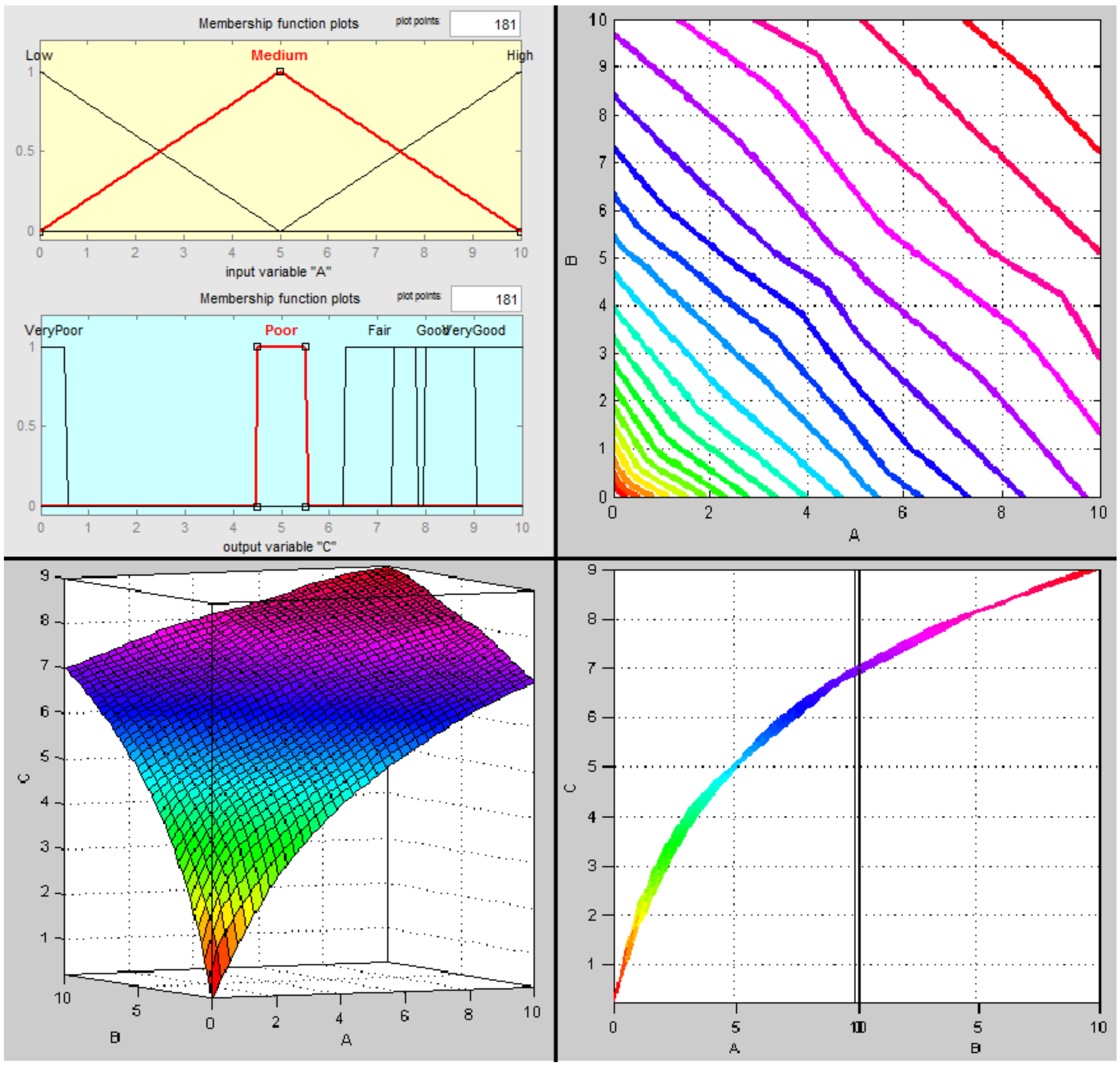


Figure 3.44 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-29

### 3.6.4 Input Variables with Four Membership Functions

Because in previous Trial 3-25 or 3-28, the input-output relation performs a rough sigmoidal surface which can lead significant difference between inferring result from fuzzy model and our expectation based on ideal sigmoidal surface, an improved model with four MFs for each input variable is tested in this section. Totally 16 If-Then rules are required, and 7 rectangular MFs are considered for output variable. For easy naming these 7 MFs, the default name 'mf+number' from MATLAB Fuzzy Logic Toolbox is remained.

Input *A & B*: Low / Medium / High / Very High

Output *C*: mf1 / mf2 / mf3 / mf4 / mf5 / mf6 / mf7

Rule 1: If *A* is Low and *B* is Low, then *C* is mf1

Rule 2: If *A* is Low and *B* is Medium, then *C* is mf2

Rule 3: If *A* is Medium and *B* is Low, then *C* is mf2

Rule 4: If *A* is Low and *B* is High, then *C* is mf3

Rule 5: If *A* is High and *B* is Low, then *C* is mf3

Rule 6: If *A* is Medium and *B* is Medium, then *C* is mf3

Rule 7: If *A* is Low and *B* is Very High, then *C* is mf4

Rule 8: If *A* is Very High and *B* is Low, then *C* is mf4

Rule 9: If *A* is Medium and *B* is High, then *C* is mf4

Rule 10: If *A* is High and *B* is Medium, then *C* is mf4

Rule 11: If *A* is Medium and *B* is Very High, then *C* is mf5

Rule 12: If *A* is Very High and *B* is Medium, then *C* is mf5

Rule 13: If *A* is High and *B* is High, then *C* is mf5

Rule 14: If *A* is High and *B* is Very High, then *C* is mf6

Rule 15: If *A* is Very High and *B* is High, then *C* is mf6

Rule 16: If *A* is Very High and *B* is Very High, then *C* is mf7

16 If-Then rules define 16 action spots in all. The coordinates of action spots in input space are (0, 0), (3.3, 0), (6.7, 0), (10, 0), (0, 3.3), (3.3, 3.3), (6.7, 3.3), (10, 3.3), (0, 6.7), (3.3, 6.7), (6.7, 6.7), (10, 6.7), (0, 10), (3.3, 10), (6.7, 10) and (10, 10).

**Trial 3-30:** Inputs *A&B*: 4 triangle MFs (2 half MFs + 2 full MFs, 100% OR)

Output *C*: 7 rectangular MFs (0% OR)

In Trial 3-30, four action spots, (0, 10), (3.3, 6.7), (6.7, 3.3) and (10, 0), are located on the diagonal from (0, 10) to (10, 0). Output MF 'mf4' with smallest area is expressed by all of these four action spots, thus the highest density of contour lines repetitively appears four times along the diagonal from (0, 10) to (10, 0). Although surrounding action spots tend to distract the density concentration on these four action spots, this tendency does not distort the smoothness of input-output surface significantly, because a larger quantity of input MFs ensures finer subdividing input space and reduces interaction between adjacent action spots. The diagonal side view showed above displays a satisfactory input-output relation with smooth sigmoidal surface.

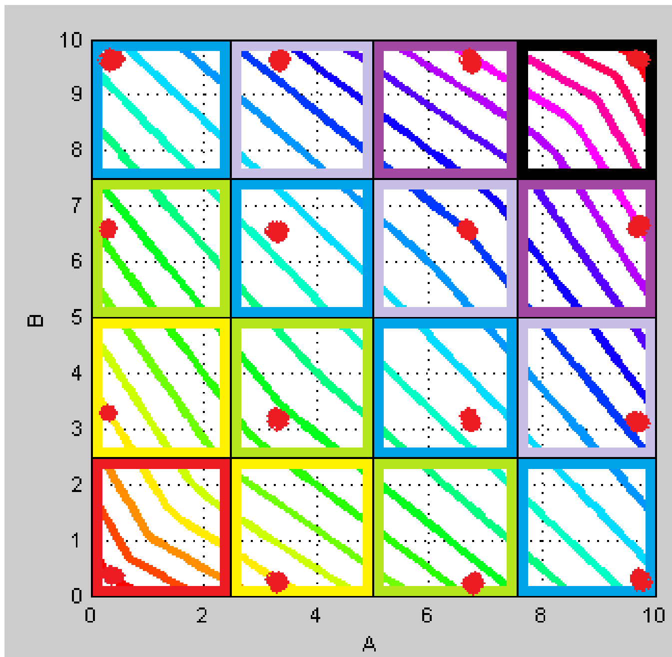


Figure 3.45 Action spots distribution for TISO models with 16 rules

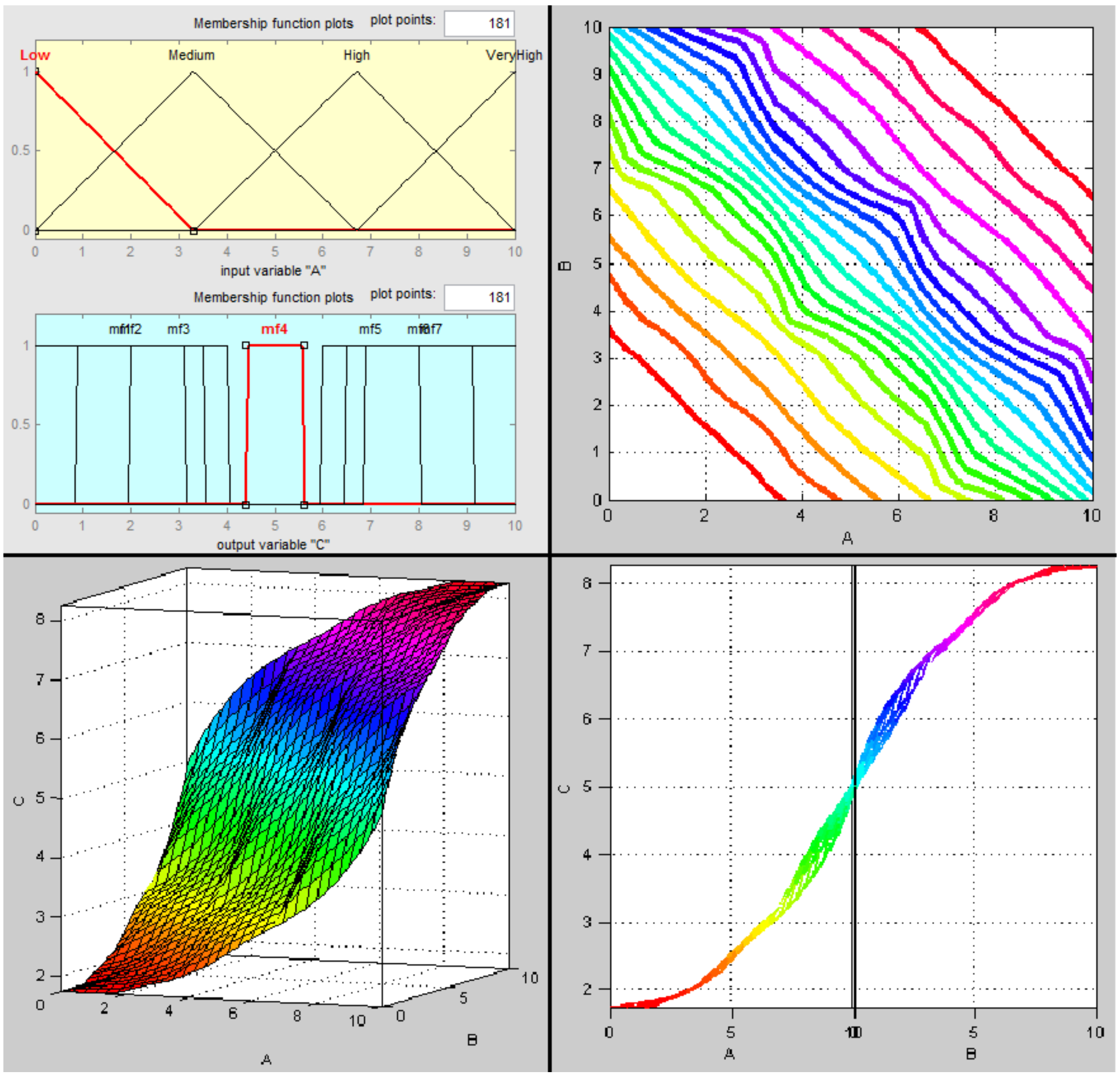


Figure 3.46 Membership functions, contour graph, input-output surface and side-view graph for Trial 3-30

## 3.7 Summary

Chapter 3 duplicates the experimental trials on TISO fuzzy inference models and proves the consistency of MF influences on both SISO and TISO systems. Thirty trials are performed and four features are concluded. From Trial 3-1 to 3-5, the effect of overlap ratio between adjacent MFs is retested on TISO models. Same as the conclusion from SISO systems, it is necessary to have input MFs to be completely and symmetrically overlapped by neighboring MFs, and distribute output MFs over the whole range of output space without overlap, in order to produce input-output relation with desirable continuity and monotonicity. From Trial 3-6 to 3-17, a total twelve possible combinations with 3 patterns of MFs for inputs *A* & *B* and 4 types of MFs for output *C* are chosen to learn the potential effect from the shape of MF on TISO models. When SISO and TISO systems are defined by exactly the same MFs, the input-output relations between each input variable and output variable from TISO model perform similar curve as SISO model does, and an integrated input-output relation presented by input-output surface from TISO model accords with that from SISO model as well. Thus similarly, the TISO inference model with triangle MFs for input and rectangular MFs for output produces input-output relation with optimal linearity and smoothness. Then Trial 3-18 to Trial 3-22 are introduced to discuss the different system performance caused by different quantities of MF. Same as the situation in SISO systems, the input-output surface becomes to repeat a certain curved surface periodically along with the increase of MF quantity for input variable. Also, the linearity of input-output surface is improved when the bouncing range of curved surface is diminished. Finally from Trial 3-23 to 3-30, based upon the optimal linear TISO inference system, the rectangular output

MFs are adjusted to constitute non-linear system performance. Similar with SISO inference systems, a TISO inference system with more MFs for each variable is more realizable to finely modulate input-output surface, but by doing that the smoothness cannot be assured because of manually adjusting more MFs.

# 4. Weight of Input Variables and A Method of Introducing Weight

In last chapter, all of the TISO fuzzy inference models are configured based upon a default assumption --- Input variables *A* and *B* have equal weight. However practically, it is more common to set up fuzzy inference system with several input variables for industrial purposes, and each of the input variables plays different weight toward a same output variable. This chapter is going to suggest a method of introducing weight into input variables, and maintain acceptable linear performance for input-output relation. This method will be tested through a Two Inputs Single Output fuzzy inference model and a Three Inputs Single Output fuzzy inference system.

## 4.1 Method Overview

Firstly, all input variables are equally defined with same number of triangle MFs. The corresponding fuzzy sets of each input variable are assigned different scores for representing linguistic levels. These scores start with 1. For instance, if an input variable is defined with three fuzzy sets, 'Low', 'Medium' and 'High', then the matching scores for these fuzzy sets are '1', '2' and '3' respectively. Every input variable is assigned a weight number, and the sum of all weight numbers from input variables is equal to 1.

Next, the score for antecedent-part of If-Then rule is added up by every product of fuzzy set score and corresponding input variable weight. If weight '0.8' and '0.2' are assigned to inputs A and B respectively, then score for antecedent “If *A* is Low and *B* is Medium” is calculated by 1*0.8 + 2*0.2 = 1.2. The score range of antecedent-part is determined by the maximum score of a single fuzzy set. If each input variable comes with three fuzzy sets, ‘Low’, ‘Medium’ and ‘High’, then ‘3’ will be maximum score with set ‘High’, and the score for antecedent-part will be constrained in interval [1, 3].

After working out the scores for all possibilities of antecedent, it is necessary to sort these antecedents from lowest score to highest score. In order to distribute values of output variable in a wide range, as the previous trials whose output spans roughly from 1 to 9, the interval of antecedent-score will be proportionally extended to [1, 9]. Finally, every different antecedent-score corresponds to a rectangular output MF. The antecedent-score after being extended represents the mid-point of support of rectangular MF, and the length of support for every rectangular output MF is set equal to 1. When excessive MFs are designed for a single output variable and integration of adjacent MFs is expected, those neighboring antecedents with small difference-value among their scores are appropriate to match with a common MF. The mean value of the neighboring antecedent-scores locates the mid-point of support of the common MF, and the length of support for this common MF remains 1.

In following sections of this chapter, this method of introducing weight to input variables is verified through TISO and MISO fuzzy inference models.

## 4.2 Weight of Input Variables in TISO Fuzzy Inference System

In this section, a fuzzy inference model with two input variables, *A* and *B*, and one output variable, *C*, is constructed for testing the method of introducing weight into input variables. Each input variable is defined by three fuzzy sets, 'Low', 'Medium' and 'High', corresponding to three full-overlapped triangle MFs. 9 If-Then rules are expected, and the setting of output rectangular MFs is presented in the following Antecedent Table. The weights for inputs A and B are 0.3 and 0.7 respectively.

In the following Antecedent Table, 9 rectangular MFs are used to define output variable *C*. All output MFs are constructed with equal length of support, and the matched extended antecedent-score locates the mid-point of support of output MF. The contour graph shows a collection of approximately parallel contour lines with similar distance between adjacent lines. The side-view graph produces an input-output surface with distinct linear performance. Although this surface is not desirably even and smooth, the variance which is relative to ideal flat is acceptable for fuzzy inference system with linear input-output relation. Most important of all, the input-output surface on the lower-left corner of above figure explicitly displays the different weights of input variables. The rate of change of output variable *C* with respect to input variable *B* is roughly 2.2 times as fast as the change rate of output *C* with respect to input *A*. Since the weights for inputs A and B are 0.3 and 0.7, it is reasonable for input B to play dominant role when influencing the increase of output value. Because 0.7 is 2.3 times as big as 0.3, it is achievable for this method to accurately express weight of input variable in TISO fuzzy inference system.

| Rule Num. | **Input A W = 0.3** | Fuzzy set Score | **Input B W = 0.7** | Fuzzy Set Score | Antec. Score | Extended Score | Output MF |
|---|---|---|---|---|---|---|---|
| 1 | Low | 1 | Low | 1 | 1 | 1 | MF1 |
| 2 | Medium | 2 | Low | 1 | 1.3 | 2.2 | MF2 |
| 3 | High | 3 | Low | 1 | 1.6 | 3.4 | MF3 |
| 4 | Low | 1 | Medium | 2 | 1.7 | 3.8 | MF4 |
| 5 | Medium | 2 | Medium | 2 | 2 | 5 | MF5 |
| 6 | High | 3 | Medium | 2 | 2.3 | 6.2 | MF6 |
| 7 | Low | 1 | High | 3 | 2.4 | 6.6 | MF7 |
| 8 | Medium | 2 | High | 3 | 2.7 | 7.8 | MF8 |
| 9 | High | 3 | High | 3 | 3 | 9 | MF9 |

Table 4.1 Antecedent Table for TISO fuzzy inference test model

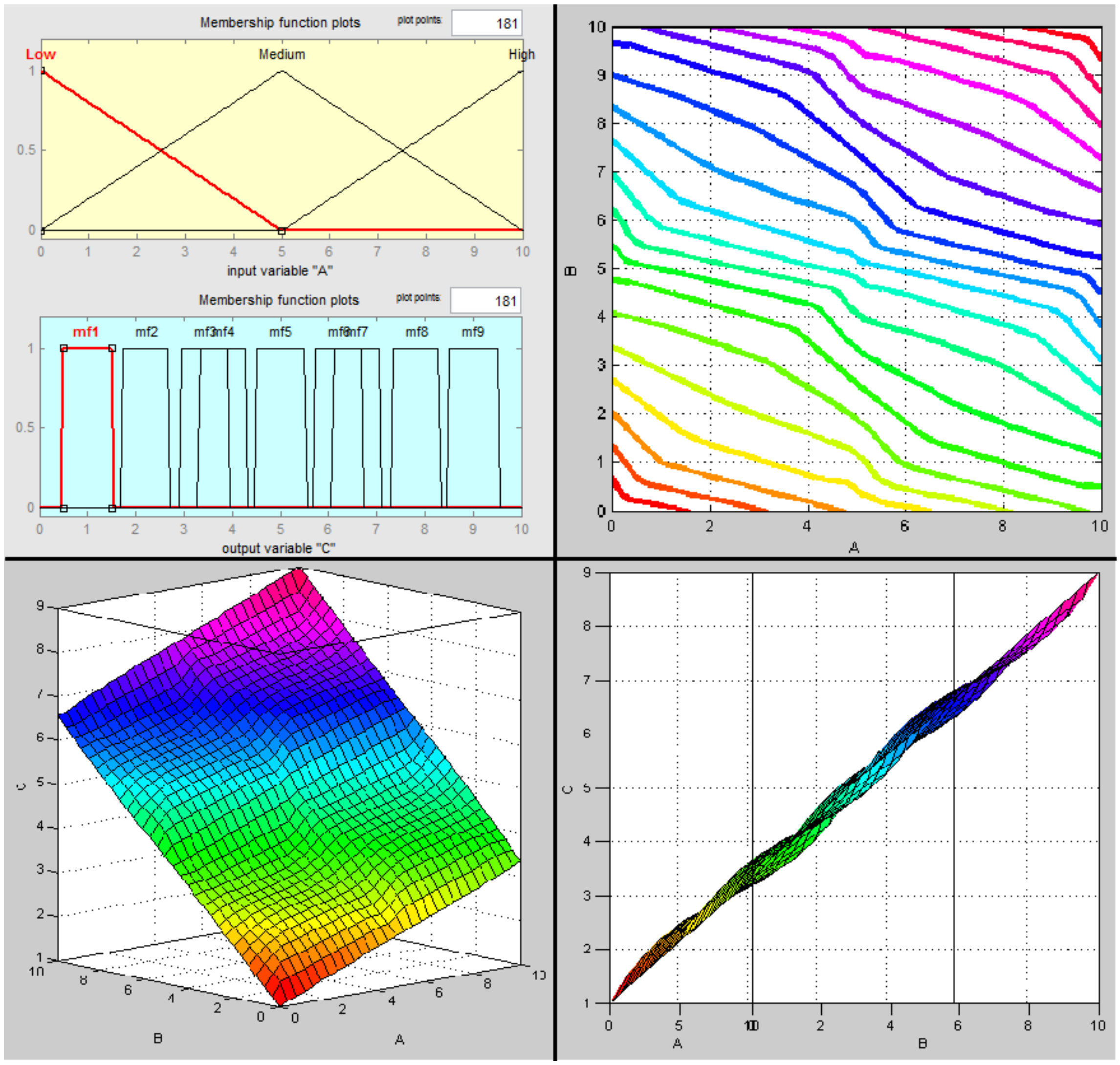


Figure 4.1 Membership functions, contour graph, input-output surface and side-view graph for TISO fuzzy inference test model

## 4.3 Weight of Input Variables in MISO Fuzzy Inference System

In order to further testify the availability of the method mentioned above, a Three-Inputs Single-Output fuzzy inference model is set up in this section to test its performance in MISO fuzzy inference environment. Similarly, input variables, *A*, *B* and *C*, are defined by three fuzzy sets, 'Low', 'Medium' and 'High', corresponding to three full-overlapped triangle MFs, and totally 27 If-Then rules are expected. The weights for inputs *A*, *B* and *C* are 0.2, 0.3 and 0.5 respectively. The neighboring antecedents with small difference among antecedent-scores will be matched with a common output MF, for cutting down the quantity of MF for output variable *D*.

Figure 4.2, 4.3 and 4.4 display input-output relations between each two of input variables and output variable *D* when the rest input variable is set equal to 5. Nineteen rectangular MFs are introduced for output variable, because antecedents with same score are assigned to identical MF. Clearly, the graphs of input-output surfaces exactly describe the relations between each of the input variables based on respective weight. Even though the contour lines showed on contour graphs are not perfect linear and parallel, the side-view graphs still reveal approximately flat surfaces without significant deviation.

In conclusion, the method of introducing weight into input variables suggested in this chapter performs good availability for MISO fuzzy inference system, and the linear system performance is satisfactorily remained.

| Rule Num. | **Input A W = 0.2** | F. set Score | **Input B W = 0.3** | F. set Score | **Input C W = 0.5** | F. set Score | Antec. Score | Extended Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Low | 1 | Low | 1 | Low | 1 | 1 | 1 | MF1 |
| 2 | Medium | 2 | Low | 1 | Low | 1 | 1.2 | 1.8 | MF2 |
| 3 | Low | 1 | Medium | 2 | Low | 1 | 1.3 | 2.2 | MF3 |
| 4 | High | 3 | Low | 1 | Low | 1 | 1.4 | 2.6 | MF4 |
| 5 | Medium | 2 | Medium | 2 | Low | 1 | 1.5 | 3 | MF5 |
| 6 | Low | 1 | Low | 1 | Medium | 2 | 1.5 | 3 | MF5 |
| 7 | Low | 1 | High | 3 | Low | 1 | 1.6 | 3.4 | MF6 |
| 8 | Medium | 2 | Low | 1 | Medium | 2 | 1.7 | 3.8 | MF7 |
| 9 | High | 3 | Medium | 2 | Low | 1 | 1.7 | 3.8 | MF7 |
| 10 | Medium | 2 | High | 3 | Low | 1 | 1.8 | 4.2 | MF8 |
| 11 | Low | 1 | Medium | 2 | Medium | 2 | 1.8 | 4.2 | MF8 |
| 12 | High | 3 | Low | 1 | Medium | 2 | 1.9 | 4.6 | MF9 |
| 13 | High | 3 | High | 3 | Low | 1 | 2 | 5 | MF10 |
| 14 | Medium | 2 | Medium | 2 | Medium | 2 | 2 | 5 | MF10 |
| 15 | Low | 1 | Low | 1 | High | 3 | 2 | 5 | MF10 |
| 16 | Low | 1 | High | 3 | Medium | 2 | 2.1 | 5.4 | MF11 |
| 17 | High | 3 | Medium | 2 | Medium | 2 | 2.2 | 5.8 | MF12 |
| 18 | Medium | 2 | Low | 1 | High | 3 | 2.2 | 5.8 | MF12 |
| 19 | Medium | 2 | High | 3 | Medium | 2 | 2.3 | 6.2 | MF13 |
| 20 | Low | 1 | Medium | 2 | High | 3 | 2.3 | 6.2 | MF13 |
| 21 | High | 3 | Low | 1 | High | 3 | 2.4 | 6.6 | MF14 |
| 22 | High | 3 | High | 3 | Medium | 2 | 2.5 | 7 | MF15 |
| 23 | Medium | 2 | Medium | 2 | High | 3 | 2.5 | 7 | MF15 |
| 24 | Low | 1 | High | 3 | High | 3 | 2.6 | 7.4 | MF16 |
| 25 | High | 3 | Medium | 2 | High | 3 | 2.7 | 7.8 | MF17 |
| 26 | Medium | 2 | High | 3 | High | 3 | 2.8 | 8.2 | MF18 |
| 27 | High | 3 | High | 3 | High | 3 | 3 | 9 | MF19 |

Table 4.2 Antecedent Table for MISO fuzzy inference test model

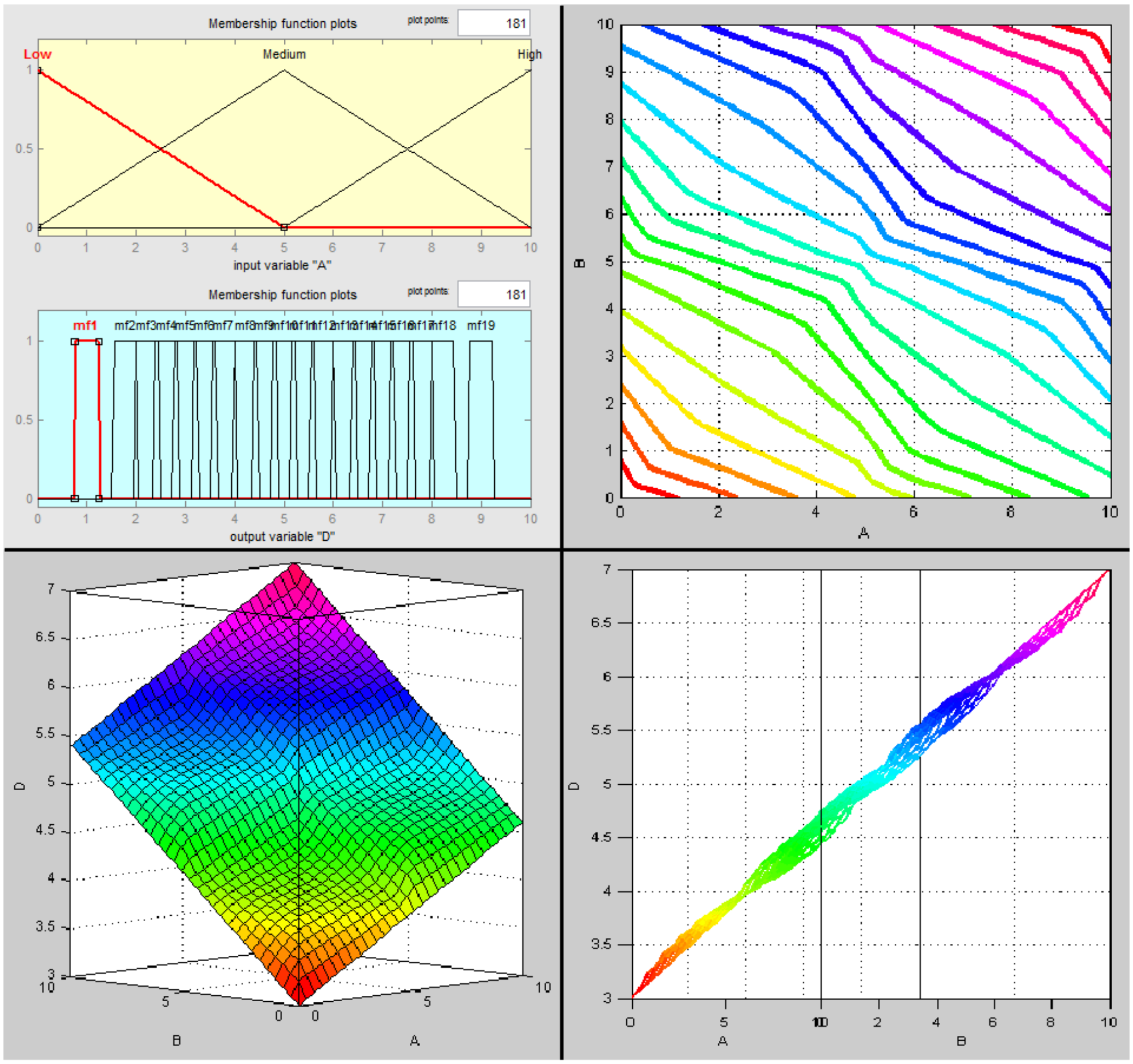


Figure 4.2 Membership functions, contour graph, input-output surface and side-view graph for of MISO test model with input A & B (C = 5)

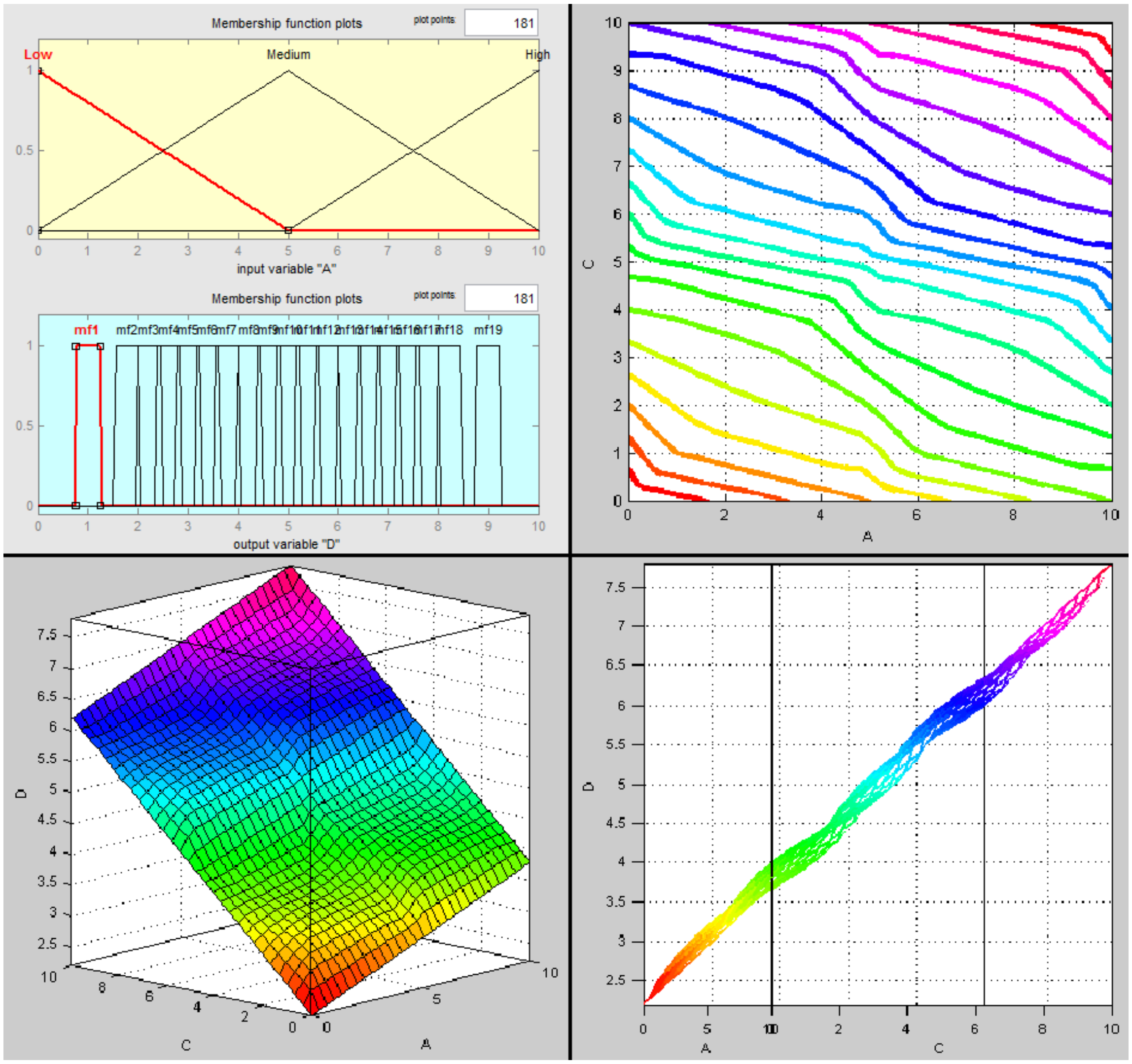


Figure 4.3 Membership functions, contour graph, input-output surface and side-view graph for of MISO test model with input A & C (B = 5)

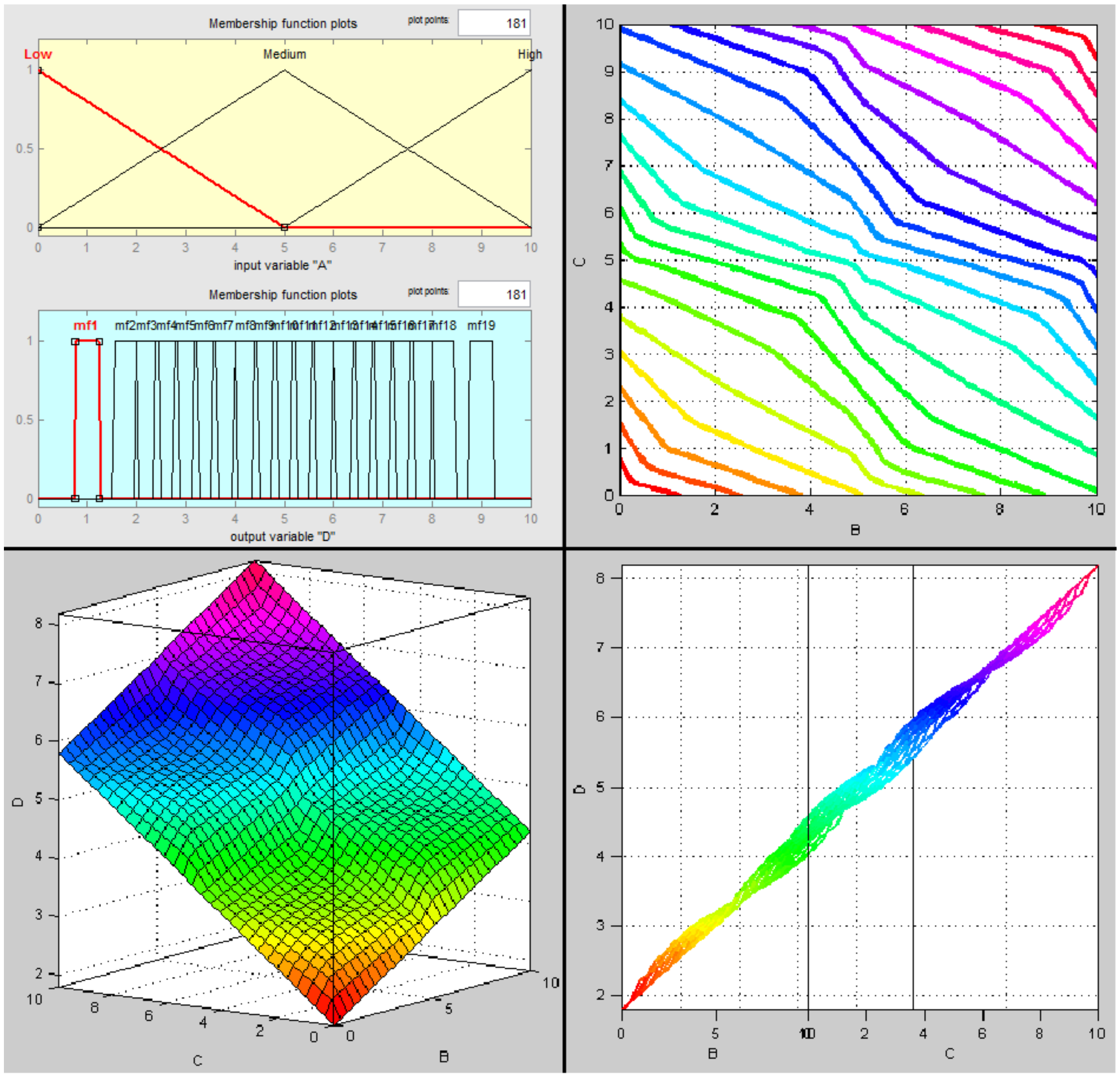


Figure 4.4 Membership functions, contour graph, input-output surface and side-view graph for of MISO test model with input B & C (A = 5)

# 5. Case Study: Decision-making Problem of Designing Timing System for Automotive Engine

In this chapter, the conclusions obtained from previous chapters are integrated and used for a case study, which is aimed at constructing Multi Inputs Single Output fuzzy inference systems with linear input-output relation for solving a decision-making problem related to timing system design for small passenger car engine.

## 5.1 Case Study Overview

### 5.1.1 Background and Problem Statement

Timing system is an extreme significant part in internal combustion engine. It synchronizes the rotation of crankshaft and the camshafts in order to ensure the engine's valves open and close at the proper times during each cylinder's intake and exhaust strokes. Generally, three methods are utilized in modern automobile engines for this transmission: Timing Belt, Timing Chain and Timing Gear. Each of these transmission mechanisms comes with respective advantages and disadvantages, and the preferences of automobile manufacturers are always keeping changing as time goes on.

Technically, either belt, chain or gear can well meet the basic mechanical requirements for timing system on small passenger vehicles, then other determinant is necessary for dominating priority. Actually, when considering which timing mechanism should be adopted for a newly designed engine, developing engineers must take several factors into account at the same time, including transmission efficiency, working noise and service life, etc. The interactions among these factors are complicated, and it is very difficult to compute an integrated performance for different timing mechanism depending on precise equations with respect to all of the factors.

In this case, Mamdani fuzzy inference method turns to be an ideal approach to quicken the decision-making process via converting complex equations to linguistic rules. All design factors compose input variables, and the corresponding weights and input values are produced from professional knowledge, industrial experience and product requirements. After all input variables going through the fuzzy inference mechanism, a single recommendation score will be generated as output. According to the recommendation score of every designing scheme, developing engineers will have theoretical support to decide whether timing belt or timing chain is more preferable.

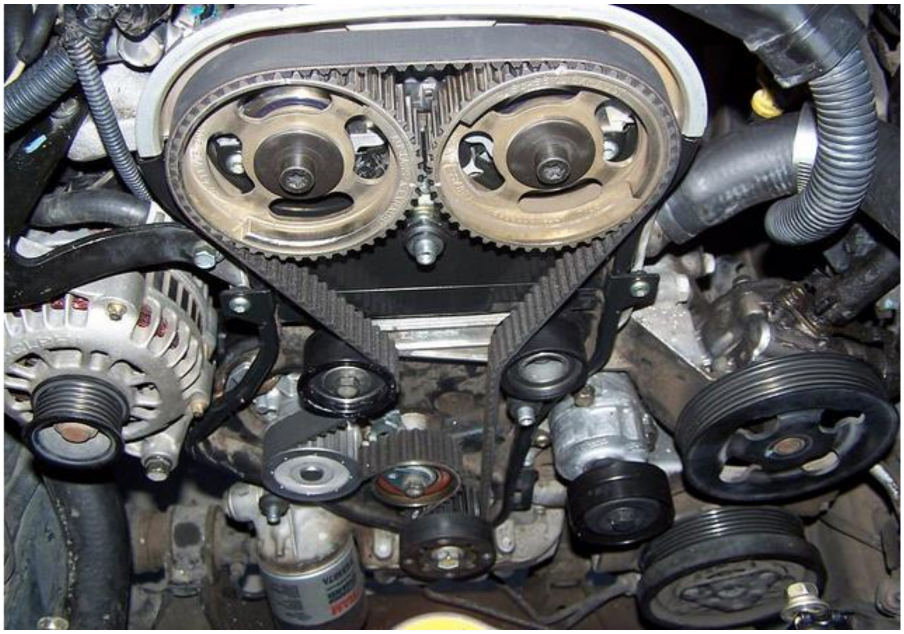

Figure 5.1 Engine with timing belt

Figure 5.2 Engine with timing chain

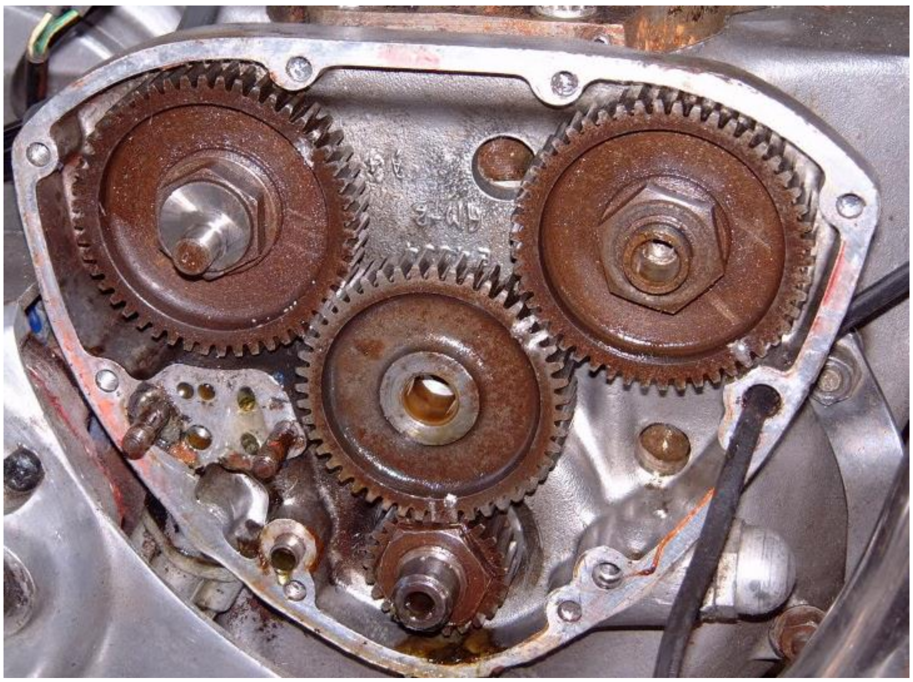

Figure 5.3 Engine with timing gear

### 5.1.2 Case Study Design

The MISO fuzzy inference models in this case study is built based on the approach of constructing linear system and the method of introducing weights developed in previous chapters, for rethinking a practical decision-making issue of timing system design of automobile engine. Firstly, the MISO fuzzy inference models will be set up depending on personal experience of mechanical expertise and general developing trend of current automotive industry, then the priority of three designing schemes is concluded according to the recommendation score from system output. Next, sampling investigation is implemented to research the utilization percentage of timing belt, timing chain and timing gear among current auto models from various manufacturers. Finally, comparison between recommended priority from MISO inference model and practical utilization on current auto models will demonstrate the reliability of MISO fuzzy inference system on decision-making issues.

In recent years, the design preference of timing system becomes different with that during the decade of the 1990s. To adapt changed customer requirements, some design factors which are negligible before the year 2000 tend to be significant for today and call engineers' attention again. In the MISO fuzzy inference models for this case study, two sets of weights for input variables will be introduced for simulating the different design considerations of 2000 and 2014. Later, the data about timing system design for models of 2000 and models of 2014 are separately collected. The samples of data collection are concentrated on small passenger vehicles with engine displacement from 1.0 liter to 4.0

liter.

## 5.2 Constitution of Input Variables for MISO Fuzzy Inference Model

### 5.2.1 Definition of Input Variables

Based on personal experience and customer requirements, 9 factors are taken into account for design consideration of timing system of auto engine.

a. Transmission efficiency
b. Transmission accuracy
c. Working smoothness
d. Vibration absorbing
e. Working noise
f. Requirement of space for timing drive mechanism
g. Service life
h. Requirement of manufacturing and installation accuracy
i. Requirement of lubrication

For the simplest situation of MISO fuzzy inference system with 9 input variables, if only two fuzzy sets are used to describe each input variable, a completed inference mechanism will consist of 512 ($2^9 = 512$) If-Then rules. If so, the cost of time and complexity will be inconceivably high. Therefore, a simplification process of the 9 input variables is necessary. For developing a product welcomed by customer market, it is reasonable to convert these technical indexes into customer concerns. A suggested transform strategy is provided by Table 5.1.

After combining relevant input variables, four newly created input variables, "Fuel Economy", "Passenger Comfortability", "Durability" and "Manufacturing Cost", significantly simplify the original conceive with 9 input variables. Although both "Transmission efficiency" and "Transmission accuracy" are related to "Fuel Economy", these two technical factors make different contributions, thus appropriate weights are needed for distinguishing the degree of dominance. The sum of weights of factors for a single input variable is required to be 1. As Table 5.2 and Table 5.3 show, two types of weight allocations are considered for situation in the 2000 and the circumstance in 2014. The improved processing technology and different customer concerns lead the change of weights of a same design factor.

In "Fuel Economy", the weight of "Transmission efficiency" decreases from 0.7 in 2000 to 0.55 in 2014. Because of widely use of light-weight materials on auto engine, the effect from transmission efficiency of timing drive mechanism on fuel performance is weakened. On the contrary, the weight of "Transmission accuracy" increases from 0.3 in 2000 to 0.45 in 2014. Because of the development of Variable Valve Timing Technology, timing drive system with high transmission accuracy is expected to ensure the fuel-economic performance of VVT engine, thus transmission accuracy plays a more important role in 2014 than that in 2000.

| | |
|---|---|
| Transmission efficiency | Fuel Economy |
| Transmission accuracy | |
| Working smoothness | Passenger Comfortability |
| Vibration absorption | |
| Working noise | |
| Requirement of space for timing drive mechanism | |
| Service life | Durability |
| Requirement of manufacturing and installation accuracy | Manufacturing Cost |
| Requirement of lubrication | |

Table 5.1 Input variable conversion

| Weight Allocation for the year 2000 | Weight | Input Variables |
|---|---|---|
| Transmission efficiency | 0.7 | Fuel Economy |
| Transmission accuracy | 0.3 | |
| Working smoothness | 0.2 | Passenger Comfortability |
| Vibration absorption | 0.35 | |
| Working noise | 0.35 | |
| Requirement of space for timing drive mechanism | 0.1 | |
| Service life | 1 | Durability |
| Requirement of manufacturing and installation accuracy | 0.8 | Manufacturing Cost |
| Requirement of lubrication | 0.2 | |

Table 5.2 Weight allocation of design factors in 2000

| Weight Allocation for the year 2014 | Weight | Input Variables |
|---|---|---|
| Transmission efficiency | 0.55 | Fuel Economy |
| Transmission accuracy | 0.45 | |
| Working smoothness | 0.2 | Passenger Comfortability |
| Vibration absorption | 0.3 | |
| Working noise | 0.25 | |
| Requirement of space for timing drive mechanism | 0.25 | |
| Service life | 1 | Durability |
| Requirement of manufacturing and installation accuracy | 0.8 | Manufacturing Cost |
| Requirement of lubrication | 0.2 | |

Table 5.3 Weight allocation of design factors in 2014

In “Passenger Comfortability”, the weights of both “Vibration absorption” and “Working noise” are decreased from 2000 to 2014. Because the current noise insulation and shock absorption techniques on auto engine have made remarkable progress, the impact from timing system noise and vibration is diminished. On the other hand, the restriction of space for timing drive mechanism turns to be stricter. Based on a rising concern about vehicle’s inner space performance, the factor “Requirement of space for timing drive mechanism” gains more attention in recent years than before.

### 5.2.2 Values of Input Variables

For the 9 design factors mentioned above, a same value range from 0 to 10 is utilized to describe the degree of each design factor for timing belt, timing chain and timing gear. Within this value range, 0 represents the worst condition, while 10 defines the most desirable situation. For maintaining consistency, in the factors “Working noise”, “Requirement of space for timing drive mechanism”, “Requirement of manufacturing and installation accuracy” and “Requirement of lubrication”, the value expresses an opposite meaning as the name of factor does. For example, the value for factor “Working noise” of timing belt is evaluated as 9. Because timing belt always runs with negligible noise, which is very desirable, then ‘9’ is regarded as explaining a high degree of satisfaction for timing belt rather than define high-level working noise.

Showed in Table 5.4, the evaluations values for timing belt, chain and gear in each design factor are referred though Mechanical Design Handbook and personal expertise, and design

| Design Factors | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Transmission efficiency | 7 | 6 | 8 |
| Transmission accuracy | 6 | 7 | 9 |
| Working smoothness | 9 | 6 | 5 |
| Vibration absorption | 9 | 6 | 2 |
| Working noise | 9 | 7 | 5 |
| Requirement of space for timing drive mechanism | 3 | 7 | 4 |
| Service life | 4 | 7 | 9 |
| Requirement of manufacturing and installation accuracy | 8 | 6 | 4 |
| Requirement of lubrication | 9 | 6 | 5 |

Table 5.4 Evaluation value of design factors

| Input Values in 2000 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Fuel Economy (FE) | 6.7 | 6.3 | 8.3 |
| Passenger Comfortability (PC) | 8.4 | 6.45 | 3.85 |
| Durability (DU) | 4 | 7 | 9 |
| Manufacturing Cost (MC) | 8.2 | 6 | 4.2 |

Table 5.5 Values of input variables in 2000

| Input Values in 2014 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Fuel Economy (FE) | 6.55 | 6.45 | 8.45 |
| Passenger Comfortability (PC) | 7.5 | 6.5 | 3.85 |
| Durability (DU) | 4 | 7 | 9 |
| Manufacturing Cost (MC) | 8.2 | 6 | 4.2 |

Table 5.6 Values of input variables in 2014

circumstances in 2000 and 2014 share a common set of evaluation value.

The value of each input variable is equal to the sum of products between related evaluation values of design factors and matching weights. For example, for input variable "Fuel Economy", the weights of "Transmission efficiency" and "Transmission accuracy" in the year 2000 are 0.7 and 0.3 respectively, and the corresponding evaluation values for timing belt are 7 and 6. Then the input value of "Fuel Economy" for timing belt in 2000 is

$$0.7 \times 7 + 0.3 \times 6 = 6.7$$

While in the year 2014, the weights of "Transmission efficiency" and "Transmission accuracy" are changed to 0.55 and 0.45 respectively, but the corresponding evaluation values of these two design factors for timing belt are constants, then the input value of "Fuel Economy" for timing belt in 2014 turns to be

$$0.55 \times 7 + 0.45 \times 6 = 6.55$$

The completed input values of timing belt, timing chain and timing gear are presented in the tables below.

### 5.2.3 Weight Parameters of Input Variables

After simplification, "Fuel Economy", "Passenger Comfortability", "Durability" and "Manufacturing Cost" are considered as the indispensable input variables for the decision-making problem of timing system design. Although, all of the four input variables are expected to make contribution simultaneously for a final recommended decision, their degrees of importance must be different, and also the degree of importance of a same input variable in 2000 and 2014 may not be identical. Based upon the automobile market tendency and customer concerns, two sets of suggested weights for input variables in the year 2000 and 2014 are provided in Table 5.7.

Because of a rising tendency of gasoline price during the past decade, customers' concern about fuel economy is keeping increasing, so the weight of "Fuel Economy" in 2014 is higher than that in 2000. Oppositely, the weight of "Passenger Comfortability" in 2014 become much lower than that in 2000. The latest techniques of noise insulation and shock absorption on auto engine have significantly ameliorated passengers' feeling, so the noise and vibration from timing drive system do not severely annoy passengers anymore. Durability of timing drive system becomes a highly-focused technical index recently. More and more customers complain that they dislike spending large expenses to maintain the timing drive system by themselves during the service life of automobiles. For this reason, the weight of "Durability" dramatically increases in 2014. Finally, as it will always be, "Manufacturing Cost" possesses the most part of weight in both 2000 and 2014. But because of a more comprehensive market requirement, it is hard for automotive manufacturers to attract potential customers only by competitive price in nowadays. Thus the weight of "Manufacturing Cost" is declined in 2014.

| Weights of Input Variables | 2000 | 2014 |
| --- | --- | --- |
| Fuel Economy (FE) | 0.22 | 0.24 |
| Passenger Comfortability (PC) | 0.3 | 0.185 |
| Durability (DU) | 0.09 | 0.28 |
| Manufacturing Cost (MC) | 0.39 | 0.295 |

Table 5.7 Weights of Input Variables

## 5.3 Mamdani Fuzzy Inference Models for Timing Mechanism Design

### 5.3.1 Assumptions and System Modeling

In this section, MISO fuzzy inference models with linear input-output relations are constructed to solve the decision-making issue of timing system design of auto engine. The inference models are comprised by four input variable, "Fuel Economy (FE)", "Passenger Comfortability (PC)", "Durability (DU)", and "Manufacturing Cost (MC)", and one output variable "Recommendation Value (RV)". All inference models in this section observe the following assumptions.

(1). The universe of discourse for both input and output variables are normalized into interval [0, 10].

(2). All of the MFs are normal and convex. All MFs which define the same input or output variable are constructed with identical geometrical characteristics, and are translated to fill in the domain of discourse [0, 10].

(3). All input variables are identically defined with same quantity of fuzzy sets which are represented by full-overlapped triangle MFs.

(4). Output variable is defined by rectangular MFs. The number and layout of output MFs are depended on the method of introducing weight in Chapter 4.

(5). The If-Then rules in fuzzy inference models must be complete.

(6). The antecedent-part of each rule must have dependency on all input variables, and the consequent-part of rules are also depended on the method of introducing weight in Chapter 4.

(7). If necessary, combining adjacent output MFs to reduce the quantity of MFs for output variable.

Same as in above trials, function *min* and function *max* are used to implement implication process and aggregation process respectively, also *Centroid Method* is adopted for defuzzification process.

### 5.3.2 Fuzzy Inference Model with Two MFs for Input Variable

Firstly, the simpler MISO inference models whose input variable is defined by only two MFs are built. With four input variables, 16 If-Then rules are expected in all. The specific constructing process is developed by the Antecedent Table as below.

**Model 5-1: MISO fuzzy inference model with 16 rules for situation in 2000**

**Input Variables:**

Manufacturing Cost (MC): 2 half-triangle MFs (Low / High) with weight = 0.39

Passenger Comfortability (PC): 2 half-triangle MFs (Fair / Good) with weight = 0.3

Fuel Economy (FE): 2 half-triangle MFs (Fair / Good) with weight = 0.22

Durability (DU): 2 half-triangle MFs (Short / Long) with weight = 0.09

**Output Variable:**

Recommendation Value (RV): 14 rectangular MFs (MF1 ~ MF14)

| Rule Num. | MC | 0.39 | PC | 0.3 | FE | 0.22 | DU | 0.09 | Ante. Score | Extended Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | High | 1 | Fair | 1 | Fair | 1 | Short | 1 | 1 | 1 | MF1 |
| 2 | High | 1 | Fair | 1 | Fair | 1 | Long | 2 | 1.09 | 1.72 | MF2 |
| 3 | High | 1 | Fair | 1 | Good | 2 | Short | 1 | 1.22 | 2.76 | MF3 |
| 4 | High | 1 | Good | 2 | Fair | 1 | Short | 1 | 1.3 | 3.4 | MF4 |
| 5 | High | 1 | Fair | 1 | Good | 2 | Long | 2 | 1.31 | 3.48 | MF5 |
| 6 | High | 1 | Good | 2 | Fair | 1 | Long | 2 | 1.39 | 4.12 | MF6 |
| 7 | Low | 2 | Fair | 1 | Fair | 1 | Short | 1 | 1.39 | 4.12 | MF6 |
| 8 | Low | 2 | Fair | 1 | Fair | 1 | Long | 2 | 1.48 | 4.84 | MF7 |
| 9 | High | 1 | Good | 2 | Good | 2 | Short | 1 | 1.52 | 5.16 | MF8 |
| 10 | High | 1 | Good | 2 | Good | 2 | Long | 2 | 1.61 | 5.88 | MF9 |
| 11 | Low | 2 | Fair | 1 | Good | 2 | Short | 1 | 1.61 | 5.88 | MF9 |
| 12 | Low | 2 | Good | 2 | Fair | 1 | Short | 1 | 1.69 | 6.52 | MF10 |
| 13 | Low | 2 | Fair | 1 | Good | 2 | Long | 2 | 1.7 | 6.6 | MF11 |
| 14 | Low | 2 | Good | 2 | Fair | 1 | Long | 2 | 1.78 | 7.24 | MF12 |
| 15 | Low | 2 | Good | 2 | Good | 2 | Short | 1 | 1.91 | 8.28 | MF13 |
| 16 | Low | 2 | Good | 2 | Good | 2 | Long | 2 | 2 | 9 | MF14 |

Table 5.8 Antecedent Table of Model 5.1 with 16 rules for 2000

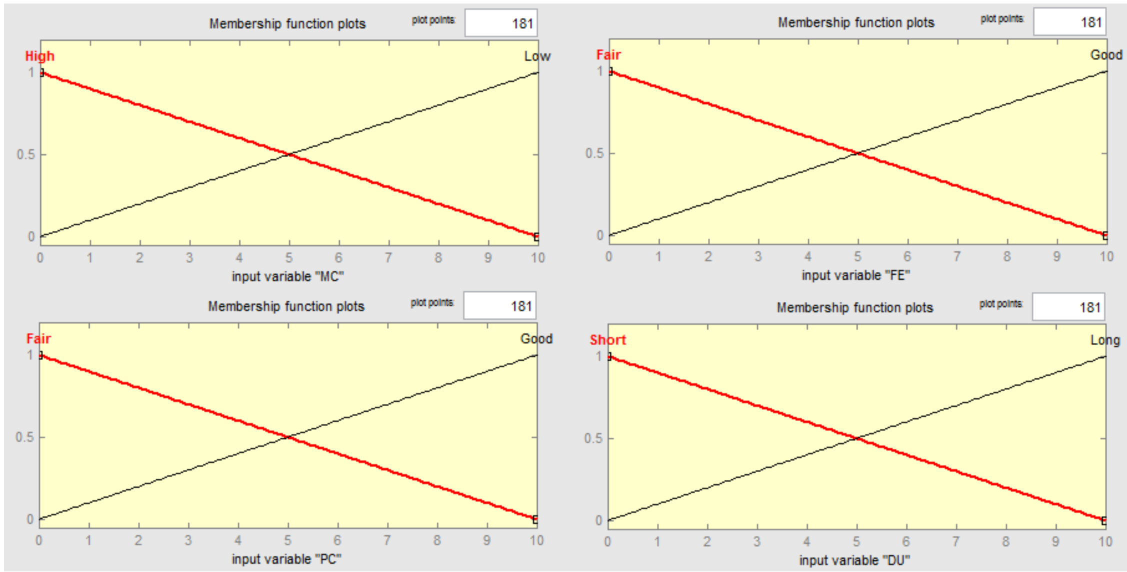


Figure 5.4 Membership function arrangement for input variables of Model 5-1

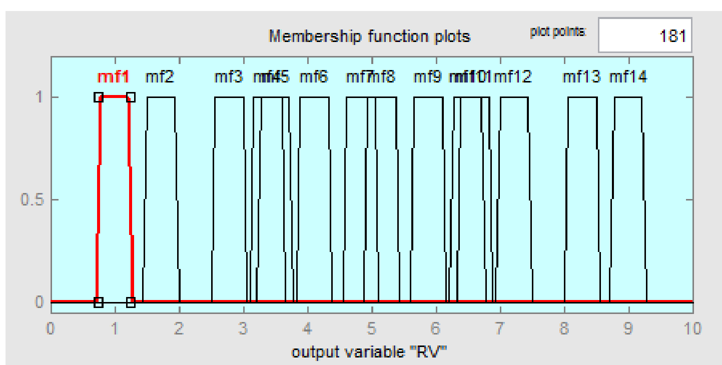


Figure 5.5 Membership function arrangement for output variable of Model 5-1

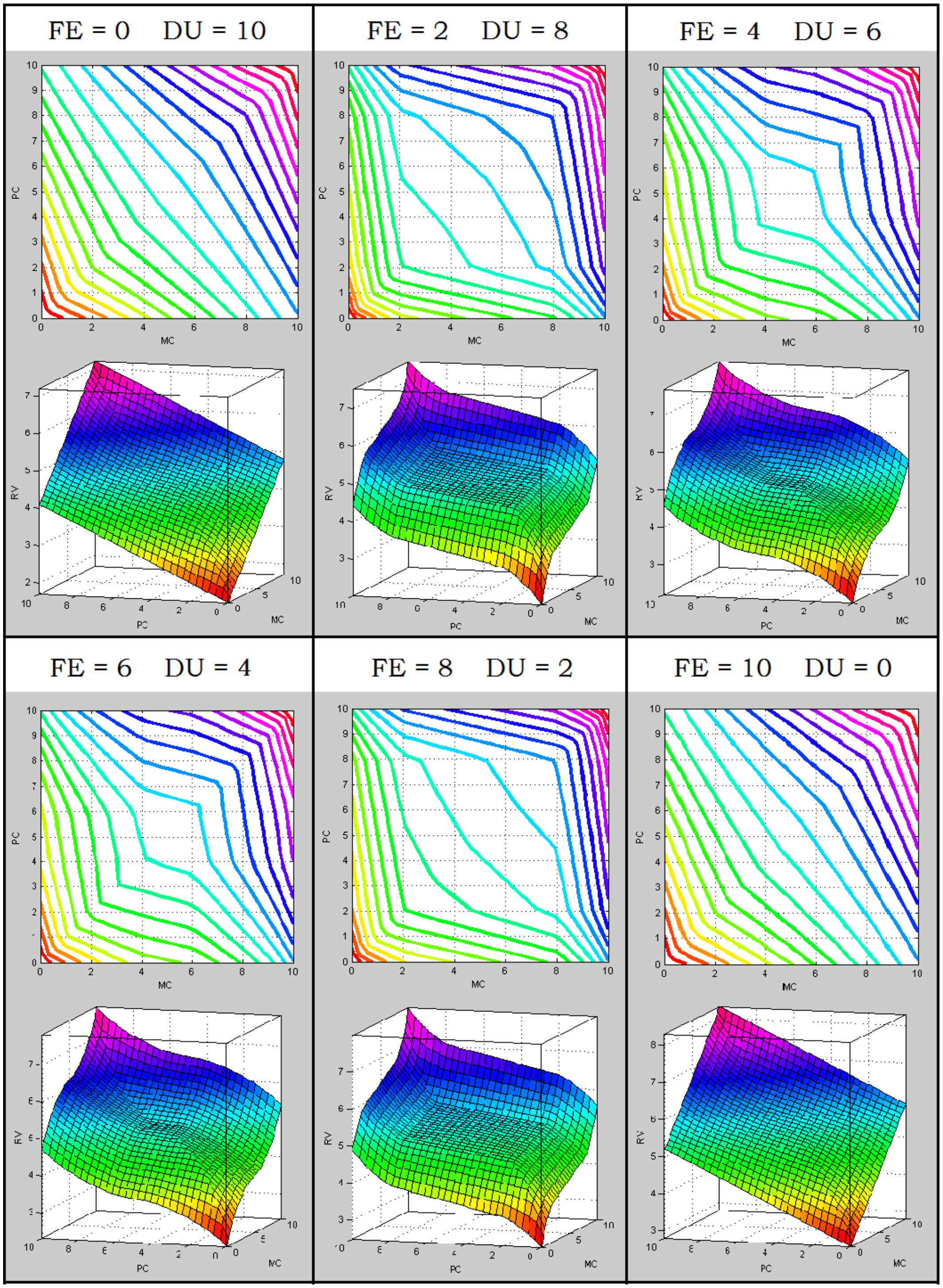


Figure 5.6 Contour graphs and input-output surfaces between MC & PC and RV of Model 5-1

**Model 5-2: MISO fuzzy inference model with 16 rules for situation in 2014**

**Input Variables:**

Manufacturing Cost (MC): 2 half-triangle MFs (Low / High) with weight = 0.295

Passenger Comfortability (PC): 2 half-triangle MFs (Fair / Good) with weight = 0.185

Fuel Economy (FE): 2 half-triangle MFs (Fair / Good) with weight = 0.24

Durability (DU): 2 half-triangle MFs (Short / Long) with weight = 0.28

**Output Variable:**

Recommendation Value (RV): 16 rectangular MFs (MF1 ~ MF16)

| Rule Num. | MC | 0.295 | PC | 0.185 | FE | 0.24 | DU | 0.28 | Ante. Score | Ext. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | High | 1 | Fair | 1 | Fair | 1 | Short | 1 | 1.00 | 1 | MF1 |
| 2 | High | 1 | Good | 2 | Fair | 1 | Short | 1 | 1.19 | 2.48 | MF2 |
| 3 | High | 1 | Fair | 1 | Good | 2 | Short | 1 | 1.24 | 2.92 | MF3 |
| 4 | High | 1 | Fair | 1 | Fair | 1 | Long | 2 | 1.28 | 3.24 | MF4 |
| 5 | Low | 2 | Fair | 1 | Fair | 1 | Short | 1 | 1.30 | 3.36 | MF5 |
| 6 | High | 1 | Good | 2 | Good | 2 | Short | 1 | 1.43 | 4.4 | MF6 |
| 7 | High | 1 | Good | 2 | Fair | 1 | Long | 2 | 1.47 | 4.72 | MF7 |
| 8 | Low | 2 | Good | 2 | Fair | 1 | Short | 1 | 1.48 | 4.84 | MF8 |
| 9 | High | 1 | Fair | 1 | Good | 2 | Long | 2 | 1.52 | 5.16 | MF9 |
| 10 | Low | 2 | Fair | 1 | Good | 2 | Short | 1 | 1.54 | 5.28 | MF10 |
| 11 | Low | 2 | Fair | 1 | Fair | 1 | Long | 2 | 1.58 | 5.6 | MF11 |
| 12 | High | 1 | Good | 2 | Good | 2 | Long | 2 | 1.71 | 6.64 | MF12 |
| 13 | Low | 2 | Good | 2 | Good | 2 | Short | 1 | 1.72 | 6.76 | MF13 |
| 14 | Low | 2 | Good | 2 | Fair | 1 | Long | 2 | 1.76 | 7.08 | MF14 |
| 15 | Low | 2 | Fair | 1 | Good | 2 | Long | 2 | 1.82 | 7.52 | MF15 |
| 16 | Low | 2 | Good | 2 | Good | 2 | Long | 2 | 2.00 | 9 | MF16 |

Table 5.9 Antecedent Table of Model 5.2 with 16 rules for 2014

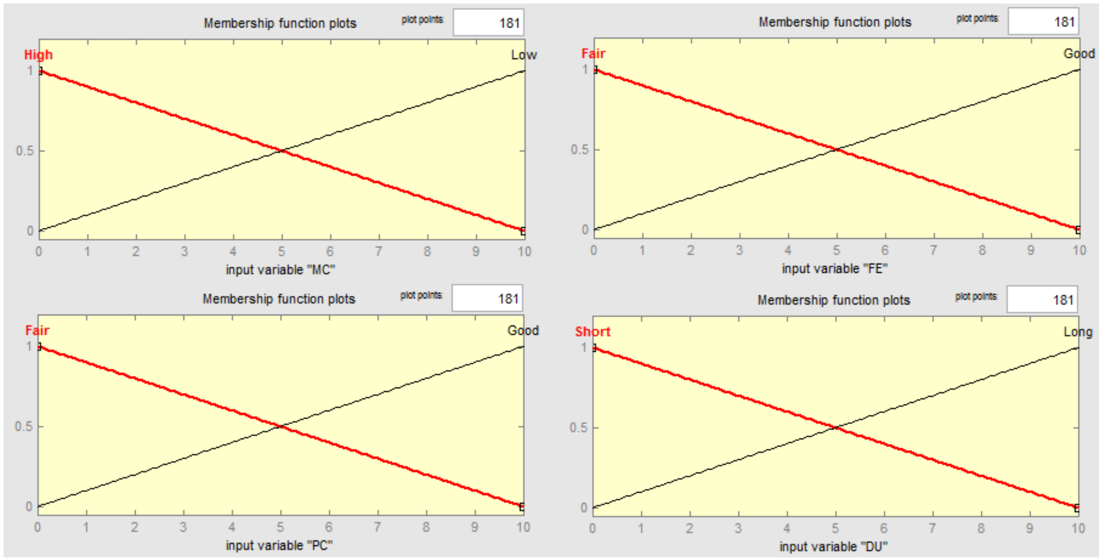


Figure 5.7 Membership function arrangement for input variables of Model 5-2

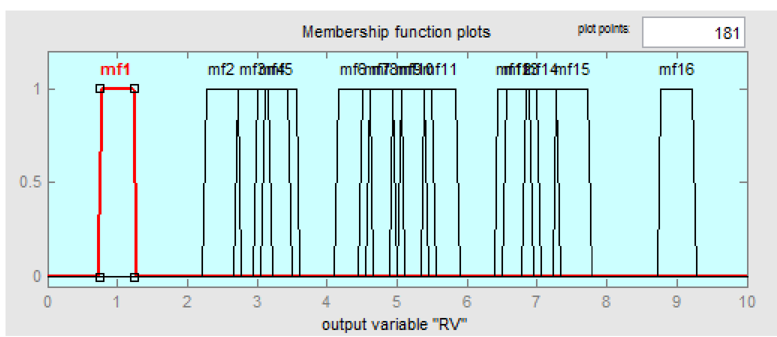


Figure 5.8 Membership function arrangement for output variable of Model 5-2

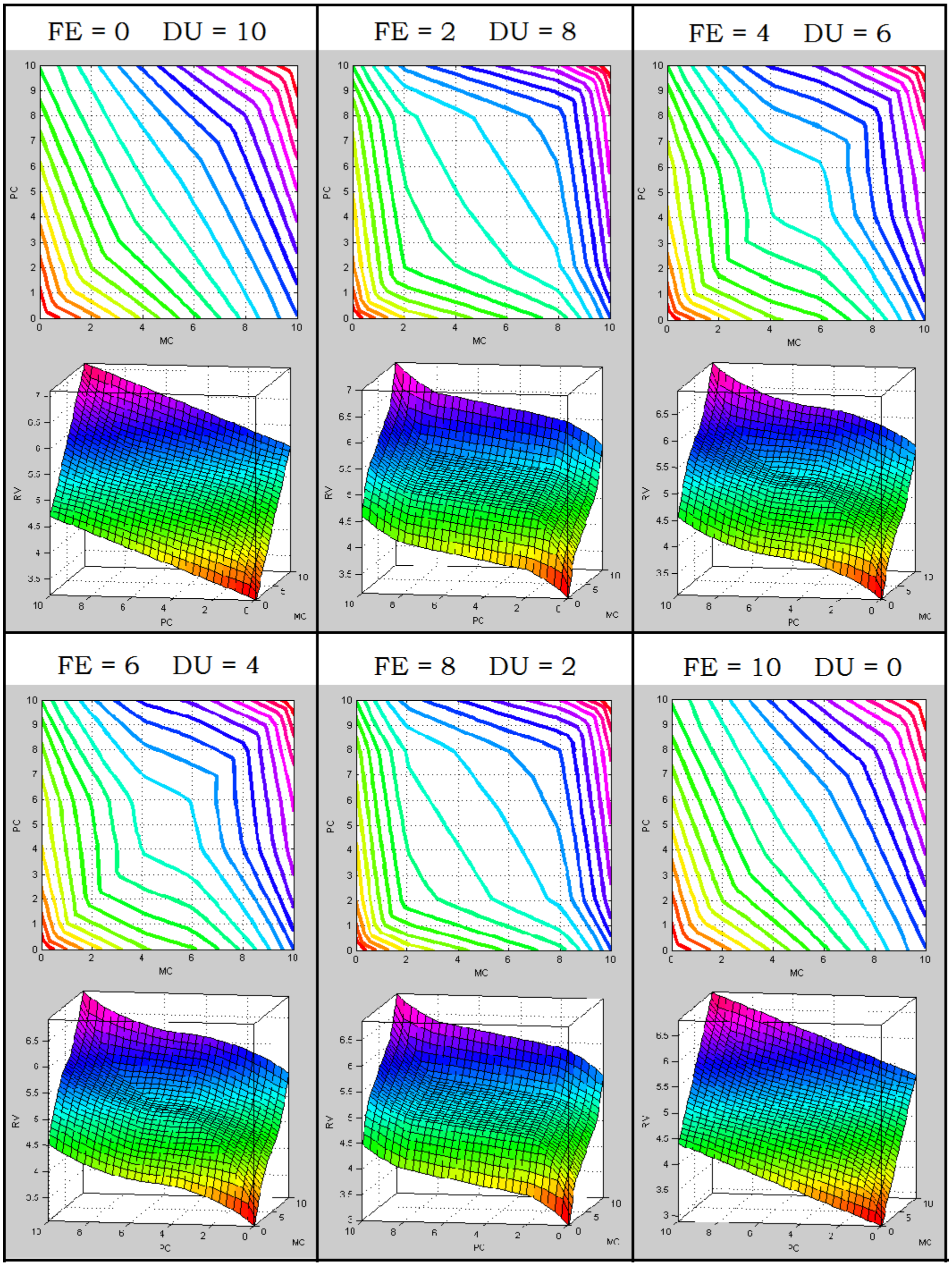


Figure 5.9 Contour graphs and input-output surfaces between MC & PC and RV of Model 5-2

| Output results in 2000 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Recommendation Value | 5.99 | 5.27 | 5.36 |

Table 5.10 Output results of Model 5.1 with 16 rules for 2000

| Output results in 2014 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Recommendation Value | 5.61 | 5.38 | 5.94 |

Table 5.11 Output results of Model 5.2 with 16 rules for 2014

From the contour graphs and input-output surfaces of Models 5-1 and 5-2, it is clear to observe that the MISO inference models with 16 rules represented good monotonicity and accurate weight of input variables. However, the input-output relations between MC & PC and RV in both models fail to perform consistent linear-performance. When input variables FE and DU take the extreme values of the input range, input-output surfaces present ideal linearity. But when FE and DU take any other values within the input range, it distorts the input-output surfaces and damages the linear relation. In this circumstance, Models 5-1 and 5-2 with 16 rules are less reliable and have high change to produce undependable recommendation.

### 5.3.3 Fuzzy Inference Model with Three MFs for Input Variable

In this section, based on the conclusion from previous chapters that increased quantity of MFs for input variables can improve the linear performance of fuzzy inference system, the Models 5-1 and 5-2 are modified by defining each input variables with three MFs. Thus as the Antecedent Table shows below, 81 If-Then rules are listed.

**Model 5-3: MISO fuzzy inference model with 81 rules for situation in 2000**

**Input Variables:**

Manufacturing Cost (MC): 2 half-triangle MFs and 1 full-triangle MF

(Low / Medium / High) with weight = 0.39

Passenger Comfortability (PC): 2 half-triangle MFs and 1 full-triangle MF

(Fair / Good / Excellent) with weight = 0.3

Fuel Economy (FE): 2 half-triangle MFs and 1 full-triangle MF

(Fair / Good / Excellent) with weight = 0.22

Durability (DU): 2 half-triangle MFs and 1 full-triangle MF

(Short / Medium / Long) with weight = 0.09

**Output Variable:**

Recommendation Value (RV): 29 rectangular MFs (MF1 ~ MF29)

| Rule Num. | MC W = 0.39 | | PC W = 0.3 | | FE W = 0.22 | | DU W = 0.09 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | High | 1 | Fair | 1 | Fair | 1 | Short | 1 | 1 | 1 | MF1 |
| 2 | High | 1 | Fair | 1 | Fair | 1 | Medium | 2 | 1.09 | 1.36 | MF2 |
| 3 | High | 1 | Fair | 1 | Fair | 1 | Long | 3 | 1.18 | 1.72 | MF3 (1.80) |
| 4 | High | 1 | Fair | 1 | Good | 2 | Short | 1 | 1.22 | 1.88 | |
| 5 | High | 1 | Good | 2 | Fair | 1 | Short | 1 | 1.3 | 2.2 | MF4 (2.22) |
| 6 | High | 1 | Fair | 1 | Good | 2 | Medium | 2 | 1.31 | 2.24 | |
| 7 | High | 1 | Good | 2 | Fair | 1 | Medium | 2 | 1.39 | 2.56 | MF5 (2.57) |
| 8 | Medium | 2 | Fair | 1 | Fair | 1 | Short | 1 | 1.39 | 2.56 | |
| 9 | High | 1 | Fair | 1 | Good | 2 | Long | 3 | 1.4 | 2.6 | |
| 10 | High | 1 | Fair | 1 | Excellent | 3 | Short | 1 | 1.44 | 2.76 | MF6 (2.87) |
| 11 | High | 1 | Good | 2 | Fair | 1 | Long | 3 | 1.48 | 2.92 | |
| 12 | Medium | 2 | Fair | 1 | Fair | 1 | Medium | 2 | 1.48 | 2.92 | |
| 13 | High | 1 | Good | 2 | Good | 2 | Short | 1 | 1.52 | 3.08 | MF7 (3.16) |
| 14 | High | 1 | Fair | 1 | Excellent | 3 | Medium | 2 | 1.53 | 3.12 | |
| 15 | Medium | 2 | Fair | 1 | Fair | 1 | Long | 3 | 1.57 | 3.28 | |
| 16 | High | 1 | Excellent | 3 | Fair | 1 | Short | 1 | 1.6 | 3.4 | MF8 (3.44) |
| 17 | High | 1 | Good | 2 | Good | 2 | Medium | 2 | 1.61 | 3.44 | |
| 18 | Medium | 2 | Fair | 1 | Good | 2 | Short | 1 | 1.61 | 3.44 | |
| 19 | High | 1 | Fair | 1 | Excellent | 3 | Long | 3 | 1.62 | 3.48 | |
| 20 | High | 1 | Excellent | 3 | Fair | 1 | Medium | 2 | 1.69 | 3.76 | MF9 (3.82) |
| 21 | Medium | 2 | Good | 2 | Fair | 1 | Short | 1 | 1.69 | 3.76 | |
| 22 | High | 1 | Good | 2 | Good | 2 | Long | 3 | 1.7 | 3.8 | |
| 23 | Medium | 2 | Fair | 1 | Good | 2 | Medium | 2 | 1.7 | 3.8 | |
| 24 | High | 1 | Good | 2 | Excellent | 3 | Short | 1 | 1.74 | 3.96 | |
| 25 | Medium | 2 | Good | 2 | Fair | 1 | Medium | 2 | 1.78 | 4.12 | MF10 (4.13) |
| 26 | High | 1 | Excellent | 3 | Fair | 1 | Long | 3 | 1.78 | 4.12 | |
| 27 | Low | 3 | Fair | 1 | Fair | 1 | Short | 1 | 1.78 | 4.12 | |
| 28 | Medium | 2 | Fair | 1 | Good | 2 | Long | 3 | 1.79 | 4.16 | |
| 29 | High | 1 | Excellent | 3 | Good | 2 | Short | 1 | 1.82 | 4.28 | MF11 (4.31) |
| 30 | High | 1 | Good | 2 | Excellent | 3 | Medium | 2 | 1.83 | 4.32 | |
| 31 | Medium | 2 | Fair | 1 | Excellent | 3 | Short | 1 | 1.83 | 4.32 | |
| 32 | Medium | 2 | Good | 2 | Fair | 1 | Long | 3 | 1.87 | 4.48 | MF12 (4.48) |
| 33 | Low | 3 | Fair | 1 | Fair | 1 | Medium | 2 | 1.87 | 4.48 | |
| 34 | High | 1 | Excellent | 3 | Good | 2 | Medium | 2 | 1.91 | 4.64 | MF13 (4.66) |
| 35 | Medium | 2 | Good | 2 | Good | 2 | Short | 1 | 1.91 | 4.64 | |
| 36 | High | 1 | Good | 2 | Excellent | 3 | Long | 3 | 1.92 | 4.68 | |

(*continued*)

| Rule Num. | MC W = 0.39 | | PC W = 0.3 | | FE W = 0.22 | | DU W = 0.09 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 37 | Medium | 2 | Fair | 1 | Excellent | 3 | Medium | 2 | 1.92 | 4.68 | MF13 |
| 38 | Low | 3 | Fair | 1 | Fair | 1 | Long | 3 | 1.96 | 4.84 | MF14 |
| 39 | Medium | 2 | Excellent | 3 | Fair | 1 | Short | 1 | 1.99 | 4.96 | MF15 (5.0) |
| 40 | Medium | 2 | Good | 2 | Good | 2 | Medium | 2 | 2 | 5 | |
| 41 | High | 1 | Excellent | 3 | Good | 2 | Long | 3 | 2 | 5 | |
| 42 | Low | 3 | Fair | 1 | Good | 2 | Short | 1 | 2 | 5 | |
| 43 | Medium | 2 | Fair | 1 | Excellent | 3 | Long | 3 | 2.01 | 5.04 | |
| 44 | High | 1 | Excellent | 3 | Excellent | 3 | Short | 1 | 2.04 | 5.16 | MF16 |
| 45 | Medium | 2 | Excellent | 3 | Fair | 1 | Medium | 2 | 2.08 | 5.32 | MF17 (5.34) |
| 46 | Low | 3 | Good | 2 | Fair | 1 | Short | 1 | 2.08 | 5.32 | |
| 47 | Medium | 2 | Good | 2 | Good | 2 | Long | 3 | 2.09 | 5.36 | |
| 48 | Low | 3 | Fair | 1 | Good | 2 | Medium | 2 | 2.09 | 5.36 | |
| 49 | Medium | 2 | Good | 2 | Excellent | 3 | Short | 1 | 2.13 | 5.52 | MF18 (5.52) |
| 50 | High | 1 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.13 | 5.52 | |
| 51 | Medium | 2 | Excellent | 3 | Fair | 1 | Long | 3 | 2.17 | 5.68 | MF19 (5.69) |
| 52 | Low | 3 | Good | 2 | Fair | 1 | Medium | 2 | 2.17 | 5.68 | |
| 53 | Low | 3 | Fair | 1 | Good | 2 | Long | 3 | 2.18 | 5.72 | |
| 54 | Medium | 2 | Excellent | 3 | Good | 2 | Short | 1 | 2.21 | 5.84 | MF20 (5.87) |
| 55 | Low | 3 | Fair | 1 | Excellent | 3 | Short | 1 | 2.22 | 5.88 | |
| 56 | High | 1 | Excellent | 3 | Excellent | 3 | Long | 3 | 2.22 | 5.88 | |
| 57 | Medium | 2 | Good | 2 | Excellent | 3 | Medium | 2 | 2.22 | 5.88 | |
| 58 | Low | 3 | Good | 2 | Fair | 1 | Long | 3 | 2.26 | 6.04 | MF21 (6.18) |
| 59 | Low | 3 | Good | 2 | Good | 2 | Short | 1 | 2.3 | 6.2 | |
| 60 | Medium | 2 | Excellent | 3 | Good | 2 | Medium | 2 | 2.3 | 6.2 | |
| 61 | Medium | 2 | Good | 2 | Excellent | 3 | Long | 3 | 2.31 | 6.24 | |
| 62 | Low | 3 | Fair | 1 | Excellent | 3 | Medium | 2 | 2.31 | 6.24 | |
| 63 | Low | 3 | Excellent | 3 | Fair | 1 | Short | 1 | 2.38 | 6.52 | MF22 (6.56) |
| 64 | Medium | 2 | Excellent | 3 | Good | 2 | Long | 3 | 2.39 | 6.56 | |
| 65 | Low | 3 | Good | 2 | Good | 2 | Medium | 2 | 2.39 | 6.56 | |
| 66 | Low | 3 | Fair | 1 | Excellent | 3 | Long | 3 | 2.4 | 6.6 | |
| 67 | Medium | 2 | Excellent | 3 | Excellent | 3 | Short | 1 | 2.43 | 6.72 | MF23 (6.84) |
| 68 | Low | 3 | Excellent | 3 | Fair | 1 | Medium | 2 | 2.47 | 6.88 | |
| 69 | Low | 3 | Good | 2 | Good | 2 | Long | 3 | 2.48 | 6.92 | |
| 70 | Medium | 2 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.52 | 7.08 | MF24 (7.13) |
| 71 | Low | 3 | Good | 2 | Excellent | 3 | Short | 1 | 2.52 | 7.08 | |
| 72 | Low | 3 | Excellent | 3 | Fair | 1 | Long | 3 | 2.56 | 7.24 | |

(*continued*)

| Rule Num. | MC W = 0.39 | | PC W = 0.3 | | FE W = 0.22 | | DU W = 0.09 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 73 | Low | 3 | Excellent | 3 | Good | 2 | Short | 1 | 2.6 | 7.4 | MF25 (7.43) |
| 74 | Medium | 2 | Excellent | 3 | Excellent | 3 | Long | 3 | 2.61 | 7.44 | |
| 75 | Low | 3 | Good | 2 | Excellent | 3 | Medium | 2 | 2.61 | 7.44 | |
| 76 | Low | 3 | Excellent | 3 | Good | 2 | Medium | 2 | 2.69 | 7.76 | MF26 (7.78) |
| 77 | Low | 3 | Good | 2 | Excellent | 3 | Long | 3 | 2.7 | 7.8 | |
| 78 | Low | 3 | Excellent | 3 | Good | 2 | Long | 3 | 2.78 | 8.12 | MF27 (8.2) |
| 79 | Low | 3 | Excellent | 3 | Excellent | 3 | Short | 1 | 2.82 | 8.28 | |
| 80 | Low | 3 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.91 | 8.64 | MF28 |
| 81 | Low | 3 | Excellent | 3 | Excellent | 3 | Long | 3 | 3 | 9 | MF29 |

Table 5.12 Antecedent Table of Model 5.3 with 81 rules for 2000

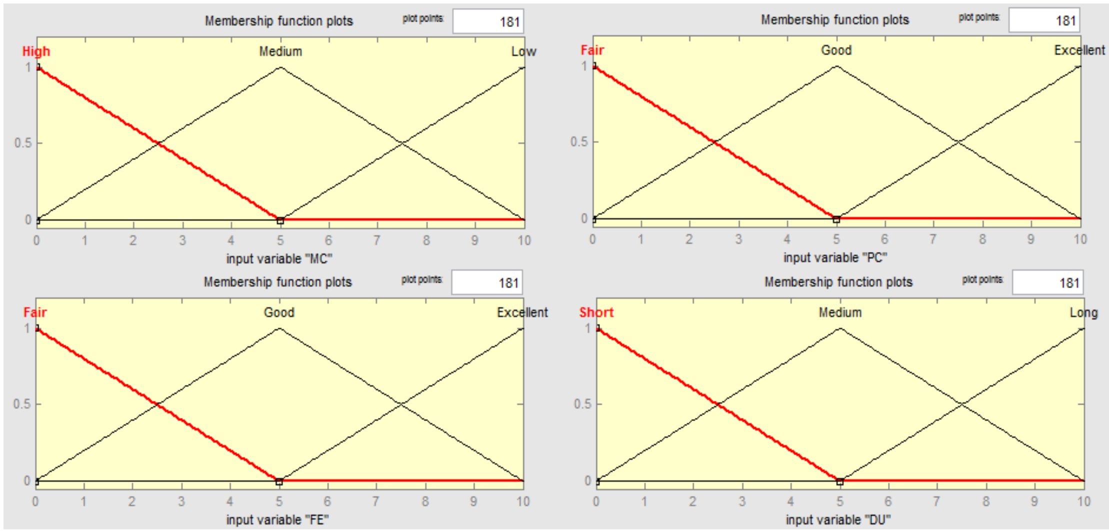


Figure 5.10 Membership function arrangement for input variables of Model 5-3

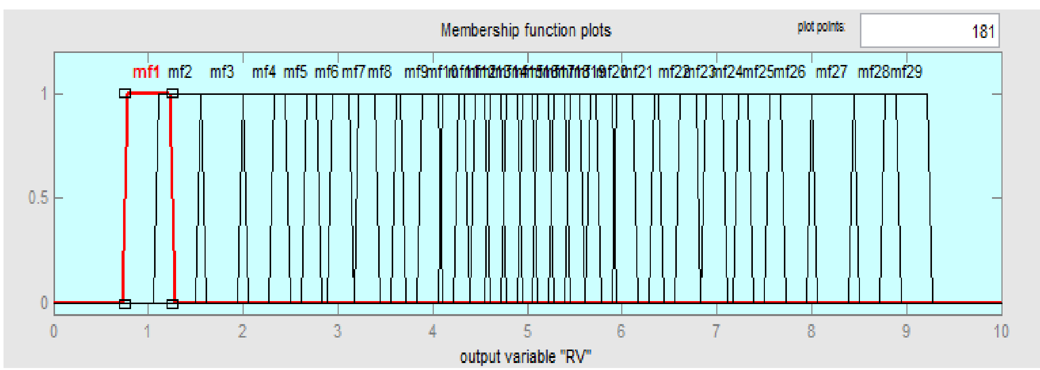


Figure 5.11 Membership function arrangement for output variable of Model 5-3

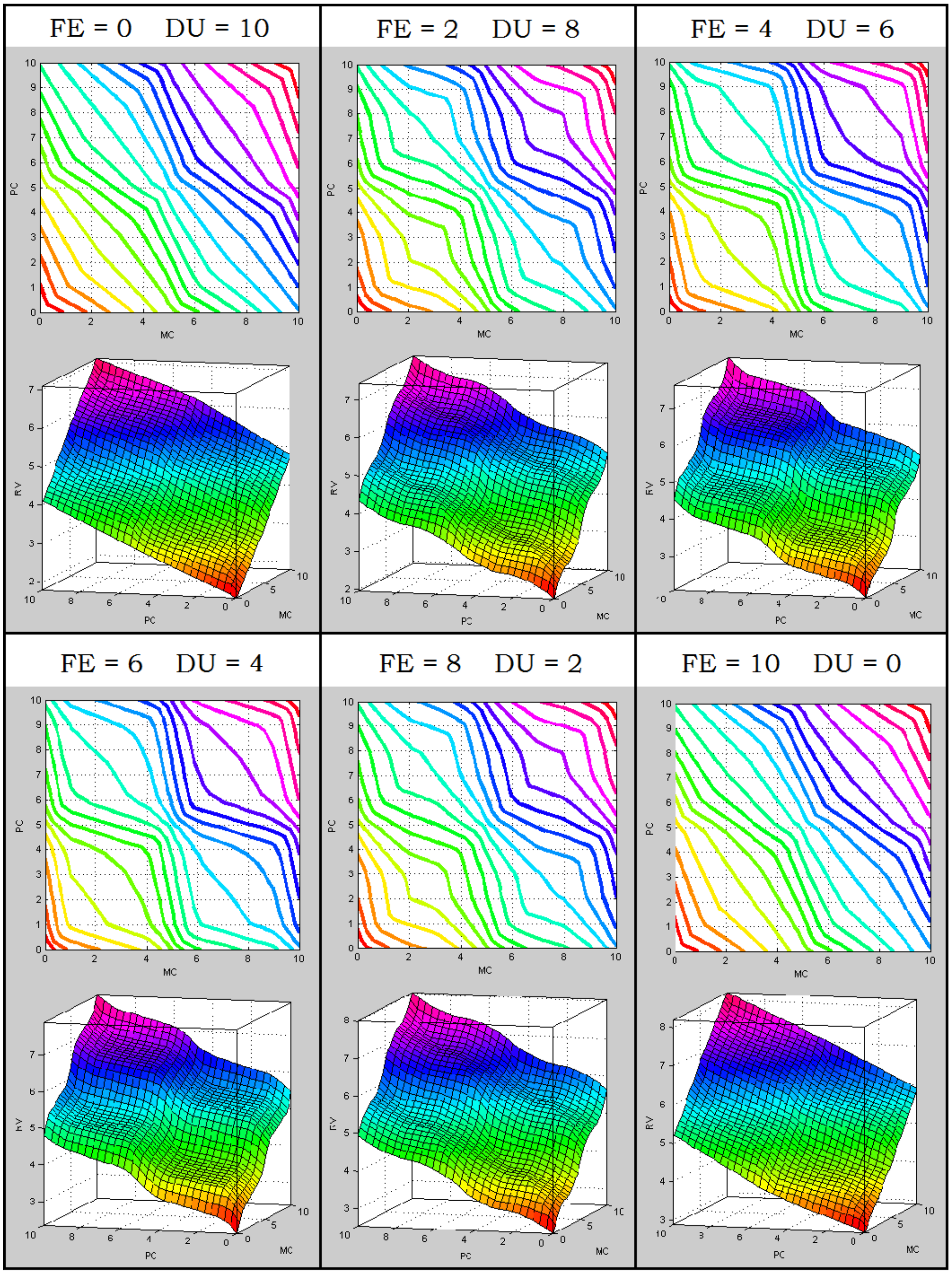


Figure 5.12 Contour graphs and input-output surfaces between MC & PC and RV of Model 5-3

**Model 5-4: MISO fuzzy inference model with 81 rules for situation in 2014**

Manufacturing Cost (MC): 2 half-triangle MFs and 1 full-triangle MF

(Low / Medium / High) with weight = 0.295

Passenger Comfortability (PC): 2 half-triangle MFs and 1 full-triangle MF

(Fair / Good / Excellent) with weight = 0.185

Fuel Economy (FE): 2 half-triangle MFs and 1 full-triangle MF

(Fair / Good / Excellent) with weight = 0.24

Durability (DU): 2 half-triangle MFs and 1 full-triangle MF

(Short / Medium / Long) with weight = 0.28

**Output Variable:**

Recommendation Value (RV): 31 rectangular MFs (MF1 ~ MF31)

| Rule Num. | MC W = 0.295 | | PC W = 0.185 | | FE W = 0.24 | | DU W = 0.28 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | High | 1 | Fair | 1 | Fair | 1 | Short | 1 | 1.00 | 1 | MF1 |
| 2 | High | 1 | Good | 2 | Fair | 1 | Short | 1 | 1.19 | 1.74 | MF2 |
| 3 | High | 1 | Fair | 1 | Good | 2 | Short | 1 | 1.24 | 1.96 | MF3 |
| 4 | High | 1 | Fair | 1 | Fair | 1 | Medium | 2 | 1.28 | 2.12 | MF4 (2.15) |
| 5 | Medium | 2 | Fair | 1 | Fair | 1 | Short | 1 | 1.30 | 2.18 | |
| 6 | High | 1 | Excellent | 3 | Fair | 1 | Short | 1 | 1.37 | 2.48 | MF5 |
| 7 | High | 1 | Good | 2 | Good | 2 | Short | 1 | 1.43 | 2.7 | MF6 (2.85) |
| 8 | High | 1 | Good | 2 | Fair | 1 | Medium | 2 | 1.47 | 2.86 | |
| 9 | High | 1 | Fair | 1 | Excellent | 3 | Short | 1 | 1.48 | 2.92 | |
| 10 | Medium | 2 | Good | 2 | Fair | 1 | Short | 1 | 1.48 | 2.92 | |
| 11 | High | 1 | Fair | 1 | Good | 2 | Medium | 2 | 1.52 | 3.08 | MF7 (3.15) |
| 12 | Medium | 2 | Fair | 1 | Good | 2 | Short | 1 | 1.54 | 3.14 | |
| 13 | High | 1 | Fair | 1 | Fair | 1 | Long | 3 | 1.56 | 3.24 | |
| 14 | Medium | 2 | Fair | 1 | Fair | 1 | Medium | 2 | 1.58 | 3.3 | MF8 (3.37) |
| 15 | Low | 3 | Fair | 1 | Fair | 1 | Short | 1 | 1.59 | 3.36 | |
| 16 | High | 1 | Excellent | 3 | Good | 2 | Short | 1 | 1.61 | 3.44 | |
| 17 | High | 1 | Excellent | 3 | Fair | 1 | Medium | 2 | 1.65 | 3.6 | MF9 (3.64) |
| 18 | High | 1 | Good | 2 | Excellent | 3 | Short | 1 | 1.67 | 3.66 | |
| 19 | Medium | 2 | Excellent | 3 | Fair | 1 | Short | 1 | 1.67 | 3.66 | |
| 20 | High | 1 | Good | 2 | Good | 2 | Medium | 2 | 1.71 | 3.82 | MF10 (3.89) |
| 21 | Medium | 2 | Good | 2 | Good | 2 | Short | 1 | 1.72 | 3.88 | |
| 22 | High | 1 | Good | 2 | Fair | 1 | Long | 3 | 1.75 | 3.98 | |
| 23 | High | 1 | Fair | 1 | Excellent | 3 | Medium | 2 | 1.76 | 4.04 | MF11 (4.1) |
| 24 | Medium | 2 | Good | 2 | Fair | 1 | Medium | 2 | 1.76 | 4.04 | |
| 25 | Medium | 2 | Fair | 1 | Excellent | 3 | Short | 1 | 1.78 | 4.1 | |
| 26 | Low | 3 | Good | 2 | Fair | 1 | Short | 1 | 1.78 | 4.1 | |
| 27 | High | 1 | Fair | 1 | Good | 2 | Long | 3 | 1.80 | 4.2 | |
| 28 | Medium | 2 | Fair | 1 | Good | 2 | Medium | 2 | 1.82 | 4.26 | MF12 (4.35) |
| 29 | Low | 3 | Fair | 1 | Good | 2 | Short | 1 | 1.83 | 4.32 | |
| 30 | High | 1 | Excellent | 3 | Excellent | 3 | Short | 1 | 1.85 | 4.4 | |
| 31 | Medium | 2 | Fair | 1 | Fair | 1 | Long | 3 | 1.86 | 4.42 | |
| 32 | Low | 3 | Fair | 1 | Fair | 1 | Medium | 2 | 1.87 | 4.48 | MF13 (4.52) |
| 33 | High | 1 | Excellent | 3 | Good | 2 | Medium | 2 | 1.89 | 4.56 | |
| 34 | Medium | 2 | Excellent | 3 | Good | 2 | Short | 1 | 1.91 | 4.62 | MF14 (4.67) |
| 35 | High | 1 | Excellent | 3 | Fair | 1 | Long | 3 | 1.93 | 4.72 | |
| 36 | High | 1 | Good | 2 | Excellent | 3 | Medium | 2 | 1.95 | 4.78 | MF15 |

(*continued*)

| Rule Num. | MC W = 0.295 | | PC W = 0.185 | | FE W = 0.24 | | DU W = 0.28 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 37 | Medium | 2 | Excellent | 3 | Fair | 1 | Medium | 2 | 1.95 | 4.78 | MF15 (4.81) |
| 38 | Medium | 2 | Good | 2 | Excellent | 3 | Short | 1 | 1.96 | 4.84 | |
| 39 | Low | 3 | Excellent | 3 | Fair | 1 | Short | 1 | 1.96 | 4.84 | |
| 40 | High | 1 | Good | 2 | Good | 2 | Long | 3 | 1.99 | 4.94 | MF16 (5.0) |
| 41 | Medium | 2 | Good | 2 | Good | 2 | Medium | 2 | 2.00 | 5 | |
| 42 | Low | 3 | Good | 2 | Good | 2 | Short | 1 | 2.02 | 5.06 | |
| 43 | High | 1 | Fair | 1 | Excellent | 3 | Long | 3 | 2.04 | 5.16 | MF17 (5.19) |
| 44 | Medium | 2 | Good | 2 | Fair | 1 | Long | 3 | 2.04 | 5.16 | |
| 45 | Medium | 2 | Fair | 1 | Excellent | 3 | Medium | 2 | 2.06 | 5.22 | |
| 46 | Low | 3 | Good | 2 | Fair | 1 | Medium | 2 | 2.06 | 5.22 | |
| 47 | Low | 3 | Fair | 1 | Excellent | 3 | Short | 1 | 2.07 | 5.28 | MF18 (5.33) |
| 48 | Medium | 2 | Fair | 1 | Good | 2 | Long | 3 | 2.10 | 5.38 | |
| 49 | Low | 3 | Fair | 1 | Good | 2 | Medium | 2 | 2.11 | 5.44 | MF19 (5.48) |
| 50 | High | 1 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.13 | 5.52 | |
| 51 | Medium | 2 | Excellent | 3 | Excellent | 3 | Short | 1 | 2.15 | 5.58 | MF20 (5.65) |
| 52 | Low | 3 | Fair | 1 | Fair | 1 | Long | 3 | 2.15 | 5.6 | |
| 53 | High | 1 | Excellent | 3 | Good | 2 | Long | 3 | 2.17 | 5.68 | |
| 54 | Medium | 2 | Excellent | 3 | Good | 2 | Medium | 2 | 2.19 | 5.74 | |
| 55 | Low | 3 | Excellent | 3 | Good | 2 | Short | 1 | 2.20 | 5.8 | MF21 (5.9) |
| 56 | High | 1 | Good | 2 | Excellent | 3 | Long | 3 | 2.23 | 5.9 | |
| 57 | Medium | 2 | Excellent | 3 | Fair | 1 | Long | 3 | 2.23 | 5.9 | |
| 58 | Medium | 2 | Good | 2 | Excellent | 3 | Medium | 2 | 2.24 | 5.96 | |
| 59 | Low | 3 | Excellent | 3 | Fair | 1 | Medium | 2 | 2.24 | 5.96 | |
| 60 | Low | 3 | Good | 2 | Excellent | 3 | Short | 1 | 2.26 | 6.02 | MF22 (6.11) |
| 61 | Medium | 2 | Good | 2 | Good | 2 | Long | 3 | 2.28 | 6.12 | |
| 62 | Low | 3 | Good | 2 | Good | 2 | Medium | 2 | 2.30 | 6.18 | |
| 63 | Medium | 2 | Fair | 1 | Excellent | 3 | Long | 3 | 2.34 | 6.34 | MF23 (6.36) |
| 64 | Low | 3 | Good | 2 | Fair | 1 | Long | 3 | 2.34 | 6.34 | |
| 65 | Low | 3 | Fair | 1 | Excellent | 3 | Medium | 2 | 2.35 | 6.4 | |
| 66 | Low | 3 | Fair | 1 | Good | 2 | Long | 3 | 2.39 | 6.56 | MF24 (6.63) |
| 67 | High | 1 | Excellent | 3 | Excellent | 3 | Long | 3 | 2.41 | 6.64 | |
| 68 | Medium | 2 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.43 | 6.7 | |
| 69 | Low | 3 | Excellent | 3 | Excellent | 3 | Short | 1 | 2.44 | 6.76 | MF25 (6.85) |
| 70 | Medium | 2 | Excellent | 3 | Good | 2 | Long | 3 | 2.47 | 6.86 | |
| 71 | Low | 3 | Excellent | 3 | Good | 2 | Medium | 2 | 2.48 | 6.92 | |
| 72 | Medium | 2 | Good | 2 | Excellent | 3 | Long | 3 | 2.52 | 7.08 | MF26 |

(*continued*)

| Rule Num. | MC W = 0.295 | | PC W = 0.185 | | FE W = 0.24 | | DU W = 0.28 | | Ante. Score | Ex. Score | Output MF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 73 | Low | 3 | Excellent | 3 | Fair | 1 | Long | 3 | 2.52 | 7.08 | MF26 (7.15) |
| 74 | Low | 3 | Good | 2 | Excellent | 3 | Medium | 2 | 2.54 | 7.14 | |
| 75 | Low | 3 | Good | 2 | Good | 2 | Long | 3 | 2.58 | 7.3 | |
| 76 | Low | 3 | Fair | 1 | Excellent | 3 | Long | 3 | 2.63 | 7.52 | MF27 |
| 77 | Medium | 2 | Excellent | 3 | Excellent | 3 | Long | 3 | 2.71 | 7.82 | MF28 (7.85) |
| 78 | Low | 3 | Excellent | 3 | Excellent | 3 | Medium | 2 | 2.72 | 7.88 | |
| 79 | Low | 3 | Excellent | 3 | Good | 2 | Long | 3 | 2.76 | 8.04 | MF29 |
| 80 | Low | 3 | Good | 2 | Excellent | 3 | Long | 3 | 2.82 | 8.26 | MF30 |
| 81 | Low | 3 | Excellent | 3 | Excellent | 3 | Long | 3 | 3.00 | 9 | MF31 |

Table 5.13 Antecedent Table of Model 5.4 with 81 rules for 2014

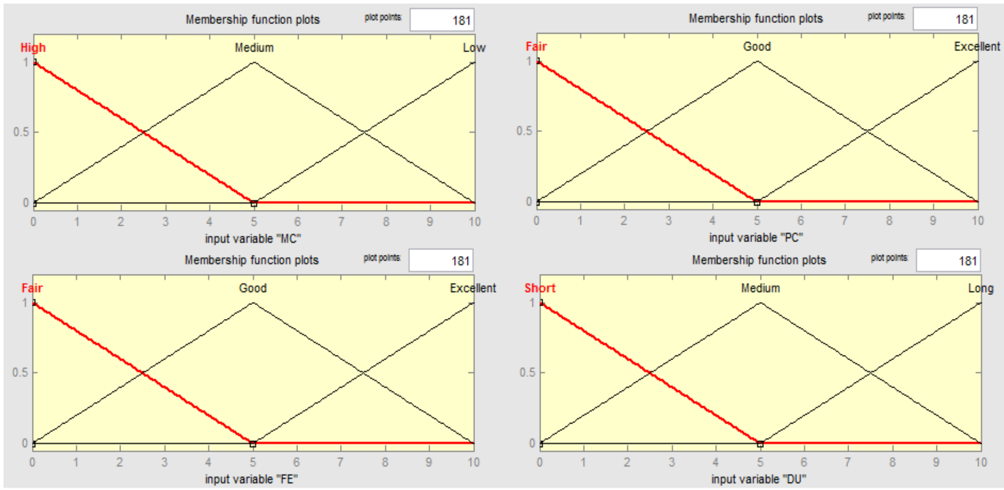


Figure 5.13 Membership function arrangement for input variables of Model 5-4

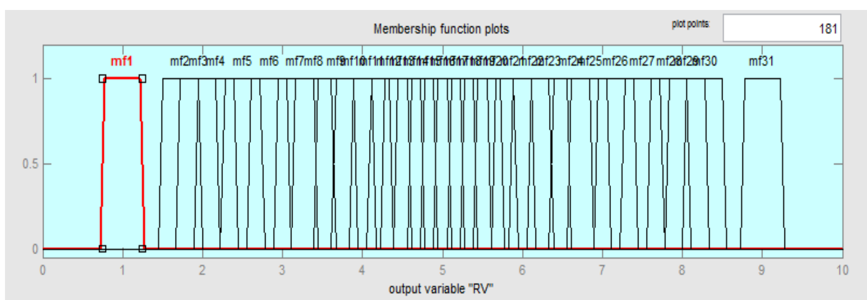


Figure 5.14 Membership function arrangement for output variable of Model 5-4

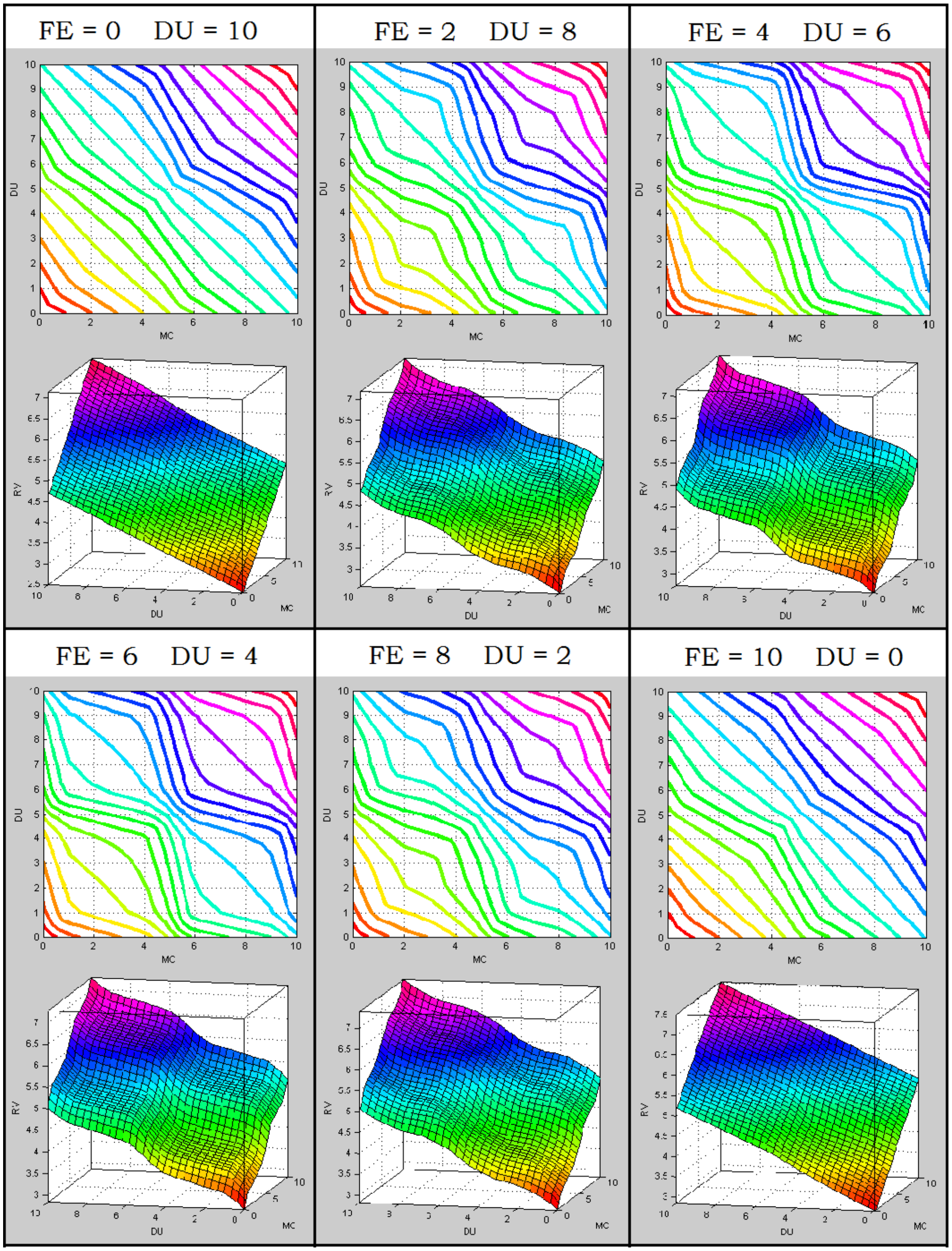


Figure 5.15 Contour graphs and input-output surfaces between MC & PC and RV of Model 5-4

| Output results in 2000 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Recommendation Value | 6.83 | 6.59 | 4.76 |

Table 5.14 Output results of Model 5.3 with 81 rules for 2000

| Output results in 2014 | Timing Belt | Timing Chain | Timing Gear |
|---|---|---|---|
| Recommendation Value | 6.15 | 6.57 | 5.81 |

Table 5.15 Output results of Model 5.4 with 81 rules for 2014

In Models 5-3 and 5-4 with 81 rules, the contour graphs and input-output surfaces also express distinct monotonicity and precise weights of input variables. As we predicted, by introducing one more fuzzy set to each input variable, the contour graphs of Models 5-3 and 5-4 express repetitive contour patterns which are similar to the corresponding contour graphs from Models 5-1 and 5-2. Because of repetition, the degree of oscillation of each contour line is diminished, thus the linear performance of Models 5-3 and 5-4 is improved, and input-output surface performs a better consistency under various restricted conditions. In this situation, the recommended decision from inference Models 5-3 and 5-4 is more reliable than that from Models 5-1 and 5-2.

## 5.4 Practical Data Collection

To validate the reliability of recommended priorities among timing belt, timing chain and timing gear from above models, sampling investigations about timing mechanism design in practical automotive industry are requisite for comparing with inference system results. For getting knowledge of practical situations in the year 2000 and 2014, two data collection processes are implemented to study the utilization of timing mechanism on small passenger vehicles produced in 2000 and 2014. Investigation samples are chose from different automotive manufacturers, and the engine displacement is limited from 1.0 liter to 4.0 liter.

| Make | Model | Engine Type | Emission | Timing System |
|---|---|---|---|---|
| Audi | A4 | 1.8L I4 20V MPFI DOHC Turbo | 1.8 | Belt |
| Audi | A4 | 2.8L V6 30V MPFI DOHC | 2.8 | Belt |
| Audi | A6 | 2.7L V6 30V MPFI DOHC Twin Turbo | 2.7 | Belt |
| Audi | A6 | 2.8L V6 30V MPFI DOHC | 2.8 | Belt |
| Audi | S4 | 2.7L V6 30V MPFI DOHC Twin Turbo | 2.7 | Belt |
| Audi | TT | 1.8L I4 20V MPFI DOHC Turbo | 1.8 | Belt |
| Chevy | Cavalier | 2.2L I4 8V MPFI OHV | 2.2 | Chain |
| Chevy | Impala | 3.4L V6 12V MPFI OHV | 3.4 | Chain |
| Chevy | Impala | 3.8L V6 12V MPFI OHV | 3.8 | Chain |
| Chevy | Malibu | 3.1L V6 12V MPFI OHV | 3.1 | Chain |
| Chrysler | 300M | 3.5L V6 24V MPFI SOHC | 3.5 | Belt |
| Chrysler | Cirrus | 2.4L I4 16V MPFI DOHC | 2.4 | Belt |
| Chrysler | Cirrus | 2.5L V6 24V MPFI SOHC | 2.5 | Belt |
| Chrysler | Sebring | 2.5L V6 24V MPFI SOHC | 2.5 | Belt |
| Chrysler | Town & Country | 3.3L V6 12V MPFI OHV | 3.3 | Chain |
| Chrysler | Town & Country | 3.8L V6 12V MPFI OHV | 3.8 | Chain |
| Dodge | Caravan | 2.4L I4 16V MPFI DOHC | 2.4 | Belt |
| Dodge | Caravan | 3.0L V6 12V MPFI SOHC | 3 | Belt |
| Dodge | Neon | 2.0L I4 16V MPFI SOHC | 2 | Belt |
| Dodge | Caravan | 3.3L V6 12V MPFI OHV | 3.3 | Chain |
| Dodge | Grand Caravan | 3.3L V6 12V MPFI OHV | 3.3 | Chain |
| Dodge | Grand Caravan | 3.8L V6 12V MPFI OHV | 3.8 | Chain |
| Ford | Focus | 2.0L I4 16V MPFI DOHC | 2 | Belt |
| Ford | Focus | 2.0L I4 8V MPFI SOHC | 2 | Belt |
| Ford | Escort | 2.0L I4 16V MPFI DOHC | 2 | Belt |
| Ford | Taurus | 3.0L V6 24V MPFI DOHC | 3 | Belt |
| Ford | Explorer | 4.0L V6 12V MPFI OHV | 4 | Chain |
| Ford | Mustang | 3.8L V6 12V MPFI OHV | 3.8 | Chain |
| Honda | Accord | 2.3L I4 16V MPFI SOHC | 2.3 | Belt |
| Honda | Accord | 3.0L V6 24V MPFI SOHC | 3 | Belt |
| Honda | Civic | 1.6L I4 16V MPFI SOHC | 1.6 | Belt |
| Honda | CR-V | 2.0L I4 16V MPFI DOHC | 2 | Belt |
| Honda | Odyssey | 3.5L V6 24V MPFI SOHC | 3.5 | Belt |
| Jeep | Cherokee | 4.0L I6 12V MPFI OHV | 4 | Chain |
| Jeep | Grand Cherokee | 4.0L I6 12V MPFI OHV | 4 | Chain |
| Jeep | Wrangler | 4.0L I6 12V MPFI OHV | 4 | Chain |

(*continued*)

| Make | Model | Engine Type | Emission | Timing System |
|---|---|---|---|---|
| Jeep | Wrangler | 2.5L I4 8V SPFI OHV | 2.5 | Chain |
| Nissan | Frontier | 3.3L V6 12V MPFI SOHC | 3.3 | Belt |
| Nissan | Pathfinder | 3.3L V6 12V MPFI SOHC | 3.3 | Belt |
| Nissan | Xterra | 3.3L V6 12V MPFI SOHC | 3.3 | Belt |
| Nissan | Altima | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Nissan | Frontier | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Nissan | Maxima | 3.0L V6 24V MPFI DOHC | 3 | Chain |
| Nissan | Sentra | 1.8L I4 16V MPFI DOHC | 1.8 | Chain |
| Nissan | Sentra | 2.0L I4 16V MPFI DOHC | 2 | Chain |
| Nissan | Sentra | 2.5L I4 16V MPFI DOHC | 2.5 | Chain |
| Toyota | 4Runner | 3.4L V6 24V MPFI DOHC | 3.4 | Belt |
| Toyota | Avalon | 3.0L V6 24V MPFI DOHC | 3 | Belt |
| Toyota | Camry | 3.0L V6 24V MPFI DOHC | 3 | Belt |
| Toyota | Camry | 2.2L I4 16V MPFI DOHC | 2.2 | Belt |
| Toyota | RAV4 | 2.0L I4 16V MPFI DOHC | 2 | Belt |
| Toyota | Sienna | 3.0L V6 24V MPFI DOHC | 3 | Belt |
| Toyota | Tundra | 3.4L V6 24V MPFI DOHC | 3.4 | Belt |
| Toyota | 4Runner | 2.7L I4 16V MPFI DOHC | 2.7 | Chain |
| Toyota | Corolla | 1.8L I4 16V MPFI DOHC | 1.8 | Chain |

Table 5.16 Utilization of timing drive mechanism on passenger vehicles in 2000

| Make | Model | Engine Type | Emission | Timing System |
|---|---|---|---|---|
| Audi | A4 | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Audi | A6 | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Audi | Q5 | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Audi | TT | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Audi | TTS | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Belt |
| Audi | A6 | 3.0L V6 24V GDI DOHC Supercharged | 3.0 | Chain |
| Audi | Q5 | 3.0L V6 24V GDI DOHC Supercharged | 3.0 | Chain |
| Audi | Q7 | 3.0L V6 24V GDI DOHC Supercharged | 3.0 | Chain |
| Cadillac | CTS | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Cadillac | ATS | 2.5L I4 16V GDI DOHC | 2.5 | Chain |
| Cadillac | ATS | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Cadillac | XTS | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Cadillac | CTS | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Chevy | Spark | 1.2L I4 16V MPFI DOHC | 1.2 | Chain |
| Chevy | Sonic | 1.4L I4 16V MPFI DOHC Turbo | 1.4 | Chain |
| Chevy | Sonic | 1.8L I4 16V MPFI DOHC | 1.8 | Belt |
| Chevy | Cruze | 1.8L I4 16V MPFI DOHC | 1.8 | Belt |
| Chevy | Malibu | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Chevy | Equinox | 2.4L I4 16V GDI DOHC | 2.4 | Chain |
| Chevy | Malibu | 2.5L I4 16V GDI DOHC | 2.5 | Chain |
| Chevy | Impala | 2.5L I4 16V GDI DOHC | 2.5 | Chain |
| Chevy | Impala | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Chevy | Equinox | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Chevy | Traverse | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Chrysler | 200 | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Chrysler | 200 | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Chrysler | 300 | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Dodge | Dart | 1.4L I4 16V MPFI SOHC Turbo | 1.4 | Belt |
| Dodge | Dart | 2.0L I4 16V MPFI DOHC | 2.0 | Chain |
| Dodge | Avenger | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Dodge | Dart | 2.4L I4 16V MPFI SOHC | 2.4 | Chain |
| Dodge | Avenger | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Dodge | Charger | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Dodge | Challenger | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Dodge | Grand Caravan | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Dodge | Durango | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |

(*continued*)

| Make | Model | Engine Type | Emission | Timing System |
|---|---|---|---|---|
| Ford | Escape | 1.6L I4 16V MPFI DOHC | 1.6 | Belt |
| Ford | Fiesta | 1.6L I4 16V MPFI DOHC | 1.6 | Belt |
| Ford | Escape | 2.0L I4 GTDI ECOBOOST ENG | 2.0 | Chain |
| Ford | Focus | 2.0L I4 16V GDI DOHC | 2.0 | Chain |
| Ford | Fusion | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| Ford | Escape | 2.5L I4 16V MPFI DOHC | 2.5 | Chain |
| Ford | Explorer | 3.5L V6 24V GDI DOHC Twin Turbo | 3.5 | Chain |
| Ford | Mustang | 3.7L V6 24V MPFI DOHC | 3.7 | Chain |
| GMC | Terrain | 2.4L I4 16V GDI DOHC | 2.4 | Chain |
| GMC | Terrain | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| GMC | Acadia | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| Honda | Civic | 1.8L I4 16V MPFI SOHC | 1.8 | Chain |
| Honda | Accord | 2.4L I4 16V GDI DOHC | 2.4 | Chain |
| Honda | CR-V | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Honda | Accord | 3.5L V6 24V MPFI SOHC | 3.5 | Belt |
| Honda | Pilot | 3.5L V6 24V MPFI SOHC | 3.5 | Belt |
| Honda | Odyssey | 3.5L V6 24V MPFI SOHC | 3.5 | Belt |
| Jeep | Compass | 2.0L I4 16V MPFI DOHC | 2.0 | Chain |
| Jeep | Compass | 2.4L I4 16V MPFI DOHC | 2.4 | Chain |
| Jeep | Cherokee | 2.4L I4 16V MPFI SOHC | 2.4 | Chain |
| Jeep | Cherokee | 3.2L V6 24V MPFI DOHC | 3.2 | Chain |
| Jeep | Grand Cherokee | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Jeep | Wrangler | 3.6L V6 24V MPFI DOHC | 3.6 | Chain |
| Nissan | Sentra | 1.8L I4 16V MPFI DOHC | 1.8 | Chain |
| Nissan | Altima | 2.5L I4 16V MPFI DOHC | 2.5 | Chain |
| Nissan | Altima | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Nissan | Maxima | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Nissan | Murano | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Nissan | Pathfinder | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Nissan | GT-R | 3.8L V6 24V MPFI DOHC Twin Turbo | 3.8 | Chain |
| Nissan | Frontier | 4.0L V6 24V MPFI DOHC | 4.0 | Chain |
| Nissan | Xterra | 4.0L V6 24V MPFI DOHC | 4.0 | Chain |
| Toyota | Yaris | 1.5L I4 16V MPFI DOHC | 1.5 | Chain |
| Toyota | Corolla | 1.8L I4 16V MPFI DOHC | 1.8 | Chain |
| Toyota | Pruis | 1.8L I4 16V MPFI DOHC Hybrid | 1.8 | Chain |
| Toyota | Camry | 2.5L I4 16V MPFI DOHC | 2.5 | Chain |

(*continued*)

| Make | Model | Engine Type | Emission | Timing System |
| --- | --- | --- | --- | --- |
| Toyota | RAV4 | 2.5L I4 16V MPFI DOHC | 2.5 | Chain |
| Toyota | Highlander | 2.7L I4 16V MPFI DOHC | 2.7 | Chain |
| Toyota | Camry | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Toyota | Avalon | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Toyota | RAV4 | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Toyota | Highlander | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| Toyota | Sienna | 3.5L V6 24V MPFI DOHC | 3.5 | Chain |
| VW | Jetta | 1.8L I4 16V GDI DOHC Turbo | 1.8 | Chain |
| VW | Passat | 1.8L I4 16V GDI DOHC Turbo | 1.8 | Chain |
| VW | Jetta | 2.0L I4 8V MPFI SOHC | 2.0 | Chain |
| VW | GTI | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| VW | Beetle | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| VW | Tiguan | 2.0L I4 16V GDI DOHC Turbo | 2.0 | Chain |
| VW | Golf | 2.5L I5 20V MPFI DOHC | 2.5 | Chain |
| VW | Beetle | 2.5L I5 20V MPFI DOHC | 2.5 | Chain |
| VW | Passat | 3.6L V6 24V GDI DOHC | 3.6 | Chain |
| VW | Touareg | 3.6L V6 24V GDI DOHC | 3.6 | Chain |

Table 5.17 Utilization of timing drive mechanism on passenger vehicles in 2014

In Table 5.16 showed above, 55 sample vehicles with engine displacement from 1.0 liter to 4.0 liter are studied. Among these models from the year 2000, 58% of samples are designed using timing belt, while 42% of the samples come with timing chain. No single sample adopted gear transmission as timing drive mechanism.

In Table 5.17, 89 sample vehicles with engine displacement from 1.0 liter to 4.0 liter in 2014 present an enormous preference on timing chain. Only 10% of samples are designed using timing belt, while the rest 90% of the samples all adopted timing chain. As before, no single vehicle chose timing gear drive.

## 5.5 Fuzzy Inference Model Verification

In this section, the recommended priority of timing belt, timing chain and timing gear from MISO fuzzy inference models are compared with the realistic utilizing situation on practical automotive products.

**Design recommendation for timing drive system in 2000**

- MISO fuzzy inference model with 16 rules for 2000 (model 5-1)

  Recommendation Value: 1$^{st}$ Belt (5.99) 2$^{nd}$ Gear (5.36) 3$^{rd}$ Chain (5.27)
- MISO fuzzy inference model with 81 rules for 2000 (model 5-3)

  Recommendation Value: 1$^{st}$ Belt (6.83) 2$^{nd}$ Chain (6.59) 3$^{rd}$ Gear (4.76)
- Practical situation of timing drive mechanisms on sample vehicles in 2000

  Utilizing percentage: 1$^{st}$ Belt (58%) 2$^{nd}$ Chain (42%) 3$^{rd}$ Gear (0%)

**Design recommendation for timing drive system in 2014**

- MISO fuzzy inference model with 16 rules for 2014 (model 5-2)

  Recommendation Value: $1^{st}$ Gear (5.94) $2^{nd}$ Belt (5.61) $3^{rd}$ Chain (5.38)
- MISO fuzzy inference model with 81 rules for 2014 (model 5-4)

  Recommendation Value: $1^{st}$ Chain (6.57) $2^{nd}$ Belt (6.15) $3^{rd}$ Gear (5.81)
- Practical situation of timing drive mechanisms on sample vehicles in 2000

  Utilizing percentage: $1^{st}$ Chain (90%) $2^{nd}$ Belt (10%) $3^{rd}$ Gear (0%)

Comparing the recommendation results from MISO fuzzy inference models and practical preference of automotive manufacturers, MISO fuzzy inference model with 81 rules and sampling investigation provide accordant priority among timing belt, timing chain and timing gear for the timing drive system design in both situations of 2000 and 2014. However, the MISO inference model with 16 rules offers different design suggestion.

As we detected from Figure 5.6 and Figure 5.9 as before, the contour graphs of inference models with 16 rules predict an inferior consistency on system linear performance under different input conditions. While with three membership functions for each input variable, Models 5-3 and 5-4 with 81 rules improve the stability of linear system performance. For this reason, the output results gained from MISO inference models with 2 MFs for each input variable deflect from practical truth, but the MISO inference models with 3 MFs for every input variable work out more reliable recommendation for reference.

The result of this case study also indicates that in Multi-Input Single-Output circumstance, a Mamdani fuzzy inference system with ideal linear input-output relation is difficult to be constructed. When weight of input variables are required to be considered, it becomes even harder to satisfy acceptable linear performance. As discussed in previous chapters, increasing the quantity of MFs for input variable is a feasible approach which can diminish the degree of oscillation and meliorate linearity. But this is not an efficient method, because new fuzzy set, or new membership function, will lead to rapid growth of rules' number, as a result much more time cost must be imaginable. Thus, it is necessary to proceed further discussion about control of MISO fuzzy inference system performance in future.

# 6. Conclusions

Throughout the five chapters of this thesis, the potential effects from membership function features on Mamdani fuzzy inference system are studied. Conclusions are drew based on 69 experimental trials. Besides, an approach of introducing input variables' weights is developed and testified through case study about timing drive system design on automotive engine. Meanwhile, the validity of conclusions from this thesis is proved by case study.

In Chapter 2, the guideline of discussion is trying to set up SISO Mamdani fuzzy inference model with ideal linear input-output relation via trial and error method, then take the basis of linear model to construct non-linear fuzzy inference models. Through 39 trials in Chapter 2, it is discovered that to ensure input-output relation with desirable continuity and monotonicity in SISO model, the support of membership function defining input variables is necessary to be fully overlapped by the support of adjacent membership function(s), while overlap between contiguous membership functions for output variable causes negative effect on displaying distinct monotonicity. When membership functions are symmetrically distributed along input scale and output scale, and completed If-Then rules are evenly built, the combination of triangular input MFs with rectangular output MFs will produce SISO fuzzy inference model with perfect linear performance. Through adjusting geometric features of rectangular membership functions, linear SISO model can be easily converted into various non-linear models. Finally, by increasing the quantity of fuzzy sets (MFs) for input variables, the linearity of input-output relation will be improved, and the controllability of non-linear fuzzy inference system will also be enhanced as well. All of these features of membership function are retested and verified on TISO Mamdani fuzzy inference models through Chapter 3.

In Chapter 4, the weight of input variables are taken into account and a method of integrating weight effects into MISO Mamdani fuzzy inference model is developed. Depending on this method, MISO fuzzy inference system with accurate response to weights and acceptable linear performance is achievable, but it is at the cost of more complex layout of membership functions for output variable. At last, chapter 5 tests and verifies the previous conclusions on a case study concentrating on a decision-making issue related to timing system design of automotive engine. Although triangular input MFs and rectangular output MFs are used, in MISO fuzzy inference model with weight effect from input variables, an input-output relation with consistent linearity is still difficult to be obtained. On a basis of membership function's feature learned before, by increasing the number of MFs for every input variable, the linear performance of MISO inference model is ameliorated, and reliable recommendations are produced.

In conclusion, this thesis is aimed at researching the process of Mamdani fuzzy inference and discovering constructing methods to make Mamdani fuzzy inference system to be more controllable and reliable. Conclusions and questions drew from this thesis are worthy to be pondered over, and further discussion is still necessary in future.

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# Vita

The author was born in Beijing, China on February 3, 1990 to Yuzhu Huang and Jian Wang. He earned his Bachelor of Science degree in Mechanical Engineering department from North China University of Technology. In January, 2013 he continued to pursue higher studies in Lehigh University. He will receive his Master of Science degree in Mechanical Engineering in January, 2015.