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Different(ciation): An Invetigation into the Essence of Problems and Solutions

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The groundlessness of any solution to a philosophical problem has caught the focus of potentially every philosopher who has considered the essence of an all-unknowing state of problemcity. As is the nature of philosophy, some may choose to address the 'problem' of groundless-solution by posing new and intentionally enlightened insights into the methods by which we should seek to overcome this obstacle—that is, some who identify this barrier seek to circumvent it, or tunnel through it in hopes of having 'solved' this problem. Other philosophers, as is the case with Gilles Deleuze, seek not to destroy this problem, but to map its topology—to metaphysically determine not the *essence* of this problem of groundless solution, but rather its *existence* and thus its conceptual structure.

In setting his eye to the task of charting the landscape that is groundlessness (particularly in the realm of the traditional Cartesian *Cogito*, hybridized with Leibnizian, Platonic, and Kantian *Idea(s)*, and then later revealing that realm as only partially-determinable insofar as each conception is concerned) Deleuze finds that this realm is intrinsically entangled with differential distinctions as well as differential (in)distinctions. Addressing Kantian Ideas (as Deleuze conceptualizes them from *Critique of Pure Reason*) he notes:

> The understanding alone would obtain answers or results here and there, but these would never constitute a 'solution'. For every solution presupposes a problem – in other words, the constitution of a unitary and systematic field which orientates

> and subsumes the researches or investigations in such a manner that the answers, in turn, form precisely cases of solution. Kant even refers to Ideas as problems 'to which there is no solution'. By that he does not mean that Ideas are necessarily false problems and thus insoluble but, on the contrary, that true problems are Ideas, and that these Ideas do not disappear with 'their' solutions, since they are the indispensable condition without which no solution would ever exist. (Deleuze 168)

The intention here is to draw attention to the fact that the problems with which philosophers traditionally concern themselves are exactly the problems of Ideas, which are intrinsically *insoluble* in at least some sense of the word. Further, Deleuze identifies a Cartesian notion that solution and problem are intimately related—that is, one cannot come in contact with a problem which does not already presuppose the essential characteristic of there existing some sense of 'solution'. Conversely, it is true that when one encounters a solution, there is implied within its concept that it is the solution to something—a problem. The interplay between these two inseparable notions is anything but obvious much of the time; and Deleuze used the Differential and Integral Calculuses to exemplify the same optimistic entanglement as previously mentioned.

"Ideas, therefore present three moments," starts Deleuze, "undetermined with regard to their object, determinable with regard to objects of experience, and bearing the ideal of an infinite determination with regard to concepts of the understanding" (Deleuze 169). Already there is a pregnant tripartite distinction. It is important at this juncture to note that this tripartite distinction bears a bijective relation to the differential calculus which we shall address shortly. Similarly, this notion bears close resemblance to the Bergsonian conceptualization of Time as 'pure duration'. The resemblance is shown outwardly as Deleuze discusses the three 'moments' of Ideas in Kant's formulation of the *Cogito*:

> It is apparent that Ideas here repeat the three aspects of the Cogito: the *I am* as an indeterminate existence, *time* as the form under which this existence is determinable, and the *I think* as a determination. Ideas are exactly the thoughts of the Cogito, the differentials of thought. (Deleuze 169)

Evidently, Deleuze takes the *I* of the Cogito to be a 'fractured I', "an I split from end to end by the form of time which runs through it...Ideas swarm in the fracture, constantly emerging on its edges, ceaselessly coming out and going back, being composed in a thousand different manners" (Deleuze 169). According to Deleuze, Kant's transcendental Cogito mimics exactly the Bergsonian essence of Pure Duration. Determination (I think) as such, cannot directly act upon the undetermined (I am)—that is, there is nothing in the thinking which allows it to enact the action of determin*ing*. Determination is neither determined, nor undetermined—it is *becoming* in the Bergsonian sense. It is then reasonable to question (to pose the problem) of how the undetermined *becomes* determined—what specifically are the conditions by which the undetermined *are not*, but *become* determined?

> The entire Kantian critique amounts to objecting against Descartes that it is impossible for determination to bear directly upon the undetermined. The determination ('I think') obviously implies something undetermined ('I am'), but nothing so far tells us how it is that this undetermined is determinable by the 'I think.' (Deleuze 86)

Deleuze shows that the condition by which undetermined becomes determined is a Kantian approach as set-forth in *Critique on Pure Reason*, championing an *a priori* form of Pure Duration which internalizes the *difference* between thinking and Being (Deleuze 86). It is in this way that Pure Duration is the method by which one is to recognize the 'fractured I' becoming fractured—the 'swarming of Ideas, constantly emerging on its edges'.

So how, it may be asked, does this tripartite distinction map bijectively into the co-domain of differential calculus? We shall first explore the traditional role of the differential and how it came to be that Deleuze rejected this in favor of this Kantian transcendental conception of the Cogito. He writes:

> Just as we oppose difference in itself to negativity, so we oppose *dx* to not-A, the symbol of difference [*Differenzphilosophie*] to that of contradiction. It is true that contradiction seeks its Idea on the side of the greatest difference, whereas the differential risks falling into the abyss of the infinitely small. This, however, is not the way to formulate the problem: it is a mistake to tie the value of the symbol *dx* to the existence of infinitesimals; but it is also a mistake to refuse it any ontological or gnoseological value in the name of a refusal of the latter. (Deleuze 170)

Instead, Deleuze summons the work of Salomon Maïmon, Hoëne Wronski, and Jean Bordas-Demoulin, whom he calls "a Leibniz, a Kant, and a Plato of the calculus" (Deleuze 171). As with the Kant-Bergson hybridized notion of Cogito as an entity internally playing in the transcendental construct of Pure Duration; Deleuze believed that the work (espoused by analytic philosophy and mathematics alike, which in-turn advocate a notion of differentiation (*dx*) as complying with rigorous contemporary scientific technique) grossly misrepresented and undercut a cohesive or rich understanding of the notion of differentiation. "The principle of a general differential philosophy," according to Deleuze "must be the object of a rigorous exposition, and must in no way depend upon the infinitely small" (Deleuze 171). As with the Cogito Deleuze likens the tripartite distinction to differential calculus as follows:

> The symbol *dx* appears as simultaneously undetermined, determinable and determination. Three principles which together form a sufficient reason correspond to these three aspects: a principle of determinability corresponds to the undetermined as such (*dx, dy*); a principle of reciprocal determination corresponds to the really determinable ($^{dy}/_{dx}$); a principle of complete determination corresponds to the effectively determined (values of $^{dy}/_{dx}$). In short, *dx* is the Idea – the Platonic, Leibnizian, or Kantian Idea, the 'problem' and its being. (Deleuze 171)

Here we see, all at once, the full force of Deleuze's intention; he wants to recreate a notion of *dx*, of the differential operand/operator, of the symbolization and notation which has a meaning (though we shall see that this meaning is substantially richer than the "infinitesimal magnitude" explanation of mathematico-philosophers) and whose meaning is crucial and integral to a more complete understanding of the Calculus, Idea, and difference in itself.

Both Deleuze and proponents for the traditional conception of calculus agree to start on the same footing: continuity. The notion of the differential (*dx*) is inexorably entangled with the sense of the continuous. The traditional notion of continuity depends on an iterative 'error-checking' method which proves, by mathematical induction, that sets on a logical structure (for example the respective sets of real, complex, rational, or irrational numbers; well-formed formulas, Ideals, or Cantor's Set) are sufficiently 'dense'. The notion of density however infers infinite count—that is to say that density requires a notion which allows one to conceptualize infinity in two key ways:

a. That one is able to understand what it means to iterate a process by mathematical induction *n* times, where *n* approaches 'infinity'; so one must understand what it means for a (natural) number *n* to "approach" something, and further that what it approaches is "infinity" and not another number. In this sense '*n* approaching infinity' is a purely iterative notion. Each time one would like to do the operation as designated by a function (in this case the function is Mathematical Induction) that desire is suppressed, the function is then left as undetermined, and the 'next' value of *n* is chosen so that its calculation is once again suppressed. In a Bergsonian sense the function demonstrates Pure Inerrability. The idea is that one recognizes that one may always choose the 'next' value of *n*—that there is no upper boundary to that choice. As such, iterative infinity is left indeterminate in the Deleuzian sense just as *dx, dy* or *I am.*

b. That each time one chooses an *n* to iterate, one must then identify that there exist an 'infinite' number of elements that fulfill a certain criterion (in this case that infinite elements remain in each iterated nested set, chosen by some iterative selection criteria). In this iteration the notion of infinity is somewhat more abstract than the above sense. Rather than being able to count towards the idea of infinity, reaching a notion of infinity by continually suppressing determination, a sense of sheer *amount* has been invoked. Amount not necessarily in the sense of 'count', but rather of 'magnitude', Infinity imagined as being of uncountable *size.*

Thus, the conception of continuity as determined by traditional mathematico-philosophers employs two senses of large infinity and force-fits them into the infinitesimal. By building an idea of an infinitesimal, an account of the differential (*dx*) will be given Deleuzionally (the problem will be solved by first noticing the solution within the problem and recognizing the problem in the solution). Proponents for the infinitesimal explanation of (*dx*) construct their notion of (*dx*) circularly by assuming the interesting essences of pure differentiation, and nonsensically attributing them to a corrupt sense of condensed infinity. One must choose a set that might be 'dense', then find the boundaries of that set, and exceed them on either 'side' demarking the exceeded boundary as a newer supra-boundary. (If there is no orientation such that "side" makes sense, choose a different way to demark a new boundary. Though in this case a set of Real Numbers will be used, it is important to recognize that one is not limited to this selection.) Take the infinite set **A**=[0,1] where "[0,1]" represents a closed interval on the real numbers with lower and upper boundary points 0, and 1, respectively. This set is infinite by definition: **R** contains all the rational numbers **Q**, and all the irrational numbers **R**\{**Q**} defined as those number in the domain of the Real set, but not in the domain of the Rational set. So **R**=**Q**∪**R**\{**Q**}; the union of the rationales and the irrationals defined recursively (the 'rationality' of each number can be tested using the same 'error checking' technique as will be employed in determining continuity).

The reason for making this distinction is to show that the set **A** really is infinite. If one simply takes the set **Q** on [0,1] one can completely cover the entirety of the set by taking **Q**=$^{m}/_{n}$ where m<n, & m and n are relatively prime (i.e. m divided by n cannot yield a value that is an element of the Natural numbers={0,1,2,...p}). Notice here that the first/iterative sense of infinity as outlined in (i) has been invoked. In mapping all the rationales and restricting the domain to [0,1] the irrationals have also been negatively mapped as all elements in [0,1] which do not belong to the set as designated by **Q**. In doing so a countable/iterative sense of infinity has been smuggled into the numerical space

between 0 and 1. One should also recognize that for example, the numbers 2, or -1 have not been included; the upper limit to the set is 1 and the lower limit to the set is 0, no values less than 0 are included, nor are values greater than 1 included. This is an honest-to-goodness bounded infinite set, **A**.

One must now show that this set is 'infinitely dense', that no matter what interval, of arbitrary size chosen, there will be an infinite number of elements in **A**. Traditionally this is done by the method alluded to in (ii), by arbitrarily choosing an infinite number of iteratively smaller intervals (nested intervals) and showing that in each case there are always infinite members of the infinite set **A**. If this can be proven successfully, a method by which continuity can be defined by infinitesimals (where the infinitesimal is an interval of uniform size, smaller than the smallest possible arbitrarily small interval defining this continuity). This can be done because every interval contains infinitely many members of **A**.

Here (as Deleuze would have it) what has been attained by the infinitesimal *dx* is something "simultaneously undetermined, and determinable" though we've failed to acquire any sense of *determination*. Here is how Deleuze identifies what has gone wrong:

> While it is true that continuousness must be related to Ideas and to their problematic use, this is on condition that it be no longer defined by characteristics borrowed from sensible or even geometric intuition, as it still is when one speaks of the interpolation of intermediaries, of infinite intercalary series or parts which are never the smallest possible. Continuousness truly belongs to the realm of Ideas only to the extent that an ideal cause of continuity is determined. (Deleuze 171)

He then continues by distinguishing between the "fixed quantities of intuition [quantum] and…variable quantities in the form of concepts of the understanding [quantitas]" (Deleuze 171). This distinction is wildly important, as it shows why exactly the infinitesimal notion of *dx* is wholly inadequate. When *dx* is defined as infinitesimal the notions of *quantum* and *quantitas* are exchanged and interchange haphazardly, without ever using them simultaneously—the distinction between the *difference* and the *differents* is ignored:

> *dx* is strictly nothing in relation to *x*, as *dy* is in relation to *y*. The whole problem, however, lies in signification of these zeroes. Quanta as objects of intuition always have particular values; and even when they are united in a fractional relation, each maintains a value independently of the relation. As a concept of the understanding, *quantitas* has a general value; generally here referring to an infinity of possible particular values: as many as the variable can assume. (Deleuze 171)

What Deleuze intended here is to draw attention to the relational essences of differentials. The differential *dx* is a sort of undefined and immutable 'nothingness' with respect to *x*, it literarily carries no weight, and likewise *dy* to *y*. What is left is a superimposed notion of comparison between nothing (*dx*) and once again nothing (*dy*), in effect $^0/_0$—the systematic exclusion of nonsense. But with $^{dy}/_{dx}$, or $^0/_0$, it is precisely *nothing* that is expressed, indeed a sort of undefined-nothing has been tapped-into, simultaneously definition-differentiation and pure 'undifferenciatedness' incarnate.

> In relation to *x*, *dx* is completely undetermined, as *dy* is to *y*, but they are perfectly determinable in relation to one another. For this reason, a principle of determinability corresponds to the undetermined as such. The universal is not a nothing since there are, in Bordas's expression, 'relations of the universal'. *dx* and *dy* are completely undifferenciated [*indifferenciés*], in the particular and in the general, but completely differentiated [*differentiés*] in and by the universal. (Deleuze 172)

Thus the symbolization $^{dy}/_{dx}$ is not the conglomeration of *dy* and *dx* in some functional relation to each other—they are not fractional as ½ is, but rather hold a new and independent sense of reciprocally. While $^{dy}/_{dx}$ is indeterminate (like the sky from lightning) there is a determination *occurring*—there exists a problem *becoming* solution, *becoming* determined. It is in this way that $^{dy}/_{dx}$ is the spacio-mathematical relation analogous to that of Kant's "*time* as the form under which this existence is determinable" in his transcendental conception of Cogito, as well as an analogue with Bergson's conception of *becoming* as the eternal temporo-differencial demarcation. $^{dy}/_{dx}$ simultaneously is, and is not.

The traditional matematico-philosophical party-line is that $^{dy}/_{dx}$ is the *instantaneous* rate of change—the change is not changing, nor is the rate of change changing at any given instant, so to speak of an *instantaneous rate of change* is to speak of Bergsonian *becoming* as though it is *being*. This is a confusion between what Deleuze and Bordas called the 'universal' relative to the quality of becoming and the specified particular determined values that are expressed by $^{dy}/_{dx}$:

> The universal in relation to a quality must not, therefore, be confused with the individual values it takes in relation to another quality. In its universal function it expresses not simply that other quality, but a pure element of qualitability. In this sense the Idea has the differential relation as its object: it then integrates variation, not as a variable determination of a supposedly constant relation ('variability') but, on the contrary, as a degree of variation of the relation itself ('variety') to which corresponds, for example, the qualified series of curves. (Deleuze 172)

Here Deleuze distinguishes to variety in the sense of multiplicity as repetition from variability in the sense of indeterminate but restricted iteration. With multiplicity as its aim, the integral variation takes on an infinity of potentials none of which are specifically selected. It is as though variation works for quality in differentiation as $^{dy}/_{dx}$ works for quantity in differenciation—neither quality in the former, nor quantity in the latter is explicitly determined, but rather alludes to determination in the same way.

Difference, as the indeterminate lightning storm carries a multiplicity of differents without difference—*Repetition* as the lightning distinguishes itself without being distinguished—*dy* as an indeterminate zero with respect to *y*—*dx* an infinitesimal iterated/iterable zero with respect to x— $^{dy}/_{dx}$ without independence; a hopelessly dependant relation constantly varying (in the sense of variety) never released, but always becoming—and finally, $^{difference}/_{repetition}$ the Idea intricately incorporating all the above into a "concrete universal," fully extended to incorporate all $^{dy}/_{dx}$*'s*. This is the "synthesis" to which Deleuze refers when he seeks to accurately describe the differential reciprocal relation. This "is what defines the universal synthesis of the Idea (Idea of the Idea, etc.): the reciprocal dependence of the degrees of the relation, and ultimately the reciprocal dependence of the relations themselves" (Deleuze 173).

It is under this interpretation of the calculus as it relates to Ideas that Deleuze suggests

> We should speak of a dialectics of the calculus rather than a metaphysics. By 'dialectic' we do not mean any kind of circulation of opposing representations which would make them coincide in the identity of a concept, but the problem element in so far as this may be distinguished from the properly mathematical element of solutions. Following Lautman's general theses, a problem has three aspects: its difference in kind from solutions; its transcendence in relation to the solutions that it engenders on the basis of its own determinant conditions; and its immanence in the solutions which cover it, the problem *being* the better resolved the more *it is* determined. (Deleuze 178-9)

With this Deleuze wishes to wed the same tripartite-distinction, previously employed, to the distinction between problems and solutions—this time with a glance back toward the distinction between differentiation and differenciation. The three aspects of which he spoke are of varying metaphysical scope.

The first thesis represents in a primordial sense the most general type of difference: a difference in kind. The only further explanation one should be able to attempt, if one were to follow the Deleuzional conceptual construct would be to affirm this as difference in itself. Problems are different from Solutions in that they are not the same; they bear their kind as a difference, but also retain it as indifference—indeterminate, but *being determined.*

The second thesis draws awareness to the relation that binds the problem and solution: it is not that the undetermined *determines* but rather that the determination is borne of determining. Conversely, that determination determines the determined—that determination *is within* determining. This shows an interesting contrast to the first thesis: the indeterminate is within the determining, as is the determined; yet the two, indeterminate and determined are incapable of co-existing, even in determining! The trick, of course is recognizing the subtle difference between Being and Becoming. The first thesis draws a relation between the undetermined problem and the problem *being determined.* The second thesis relates the solution *becoming determined* with the determined solution.

The third thesis re-encapsulates the *Being* of the determined—the closer the problem to solution, the further from being a problem, less-distant from becoming solution (but farther nonetheless) being a solution defines that solution's determination. There is little difference here from what happens in the calculus with respect to x, dx, ${}^{dy}/_{dx}$, and $f(c)= {}^{df(c)}/_{dx}$, where c represents specific (though unarticulated) conditions which exude solution(s), x is a problem yet-undetermined, dx is nothing with respect to x—it *is* x, but naught, ${}^{dy}/_{dx}$ is no longer x, but x is *in* it—it is dx becoming dy , and $f(c)= {}^{df(c)}/_{dx}$ is a solution fully determined, but hitherto incomplete.

> We have seen how all three of these aspects were present in the differential calculus: the solutions are like the discontinuities compatible with differential equations, engendered on the basis of an ideal continuity in accordance with the conditions of the problem...*Problems are always dialectical*...What is mathematical...are the solutions. (Deleuze 179)

Yet one is not limited by this apparent restriction. Deleuze continues by affirming that there are 'solutions' in mathematics which, while they technically lay in the domain of 'solutions' they essentially assume the role of 'problem'. His explanation is that there are different orders of problems and solution, and different respects from which to understand them. Just as the mathematical notions of 'order', 'degree', and 'power' represent the same reaffirmation, repetition, or reiteration, a veritable multiplicity without recurrence; problem and solution mimic this structurization. Deleuze notes that:

> each dialectical problem is duplicated by a symbolic field in which it is expressed. That is why it must be said that there are mathematical, physical, biological, psychical, and sociological problems even though every problem is

> dialectical by nature and there are no non-dialectical problems. Mathematics, therefore, does not include only solutions to problems; it also includes the expression of problems relative to the field of solvability which they define, and define by virtue of their very dialectical order. (Deleuze 179)

This he gives as the reason by which differential calculus belongs exclusively to the field of mathematics since mathematics contains within it the models which describe solvability (reference Gödel's Incompleteness Theorems), which are not solutions within themselves, but rather expressions of "problems relative to the field of solvability which they define" (Deleuze 179). This structural characteristic is essential to mathematics and its application to any other field necessarily introduces a purely mathematical characteristic to that field. Instead of looking at the internal properties of an Idea or problem in order to determine solvability, Deleuze suggests that one look to the external structure of the problem or Idea (p 180). It is in this sense that Turing and Church designed their famous test with the goal of algorithmically deciding whether a truth-functional statement is structurally (syntactically & semantically) solvable. The field of mathematical logic is devoted specifically to the task of learning (being able to reach solution) through exclusively formal (structural) means. Deleuze concludes that:

> Calculus recognizes differentials of different orders. However, the notions of differential and order accord with the dialectic in a quite different manner. The problematic or dialectical Idea is a system of connections between differential elements, a system of differential relations between genetic elements. There are different orders of Ideas presupposed by one another according to the ideal nature of these relations and the elements considered (Ideas of Ideas, etc.). (Deleuze 181)

With this Deleuze has contented himself with his treatment of the Differential Calculus. He has contracted a recurrent tripartite distinction such that Bergsonian duration fluently weaves through and around Leibnizian differential calculus via the articulate flying shuttle of Kantian Ideas. His goal to pull apart the loose conceptual textile of the mathematico-philosophical notion of differential as solely infinitesimal succeeds astonishingly considering the philosophical setting in which Deleuze seeks to apply his handiwork. If the particular ambition was to map the topology of groundlessness—to metaphysically determine not the *essence* of this problem of groundless solution, but rather its *existence*, all Deleuze found was the middle-ground between light and shadow. In a characteristically Wittgensteinian sense, the problem did not 'exist' because the question was nonsense—the solution was not solution at all, but rather a reiteration of a higher-order problem.

Works Cited

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