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AN INVESTIGATION OF STEEL RIGID FRAMES

by Inge Lyse* and W. E. Black°

SYNOPSIS

This paper presents the results of tests on two one-quarter size riveted steel rigid frames. In one frame, the knee sections were square, having a sharp reentrant angle at the inner corner. The other frame had a large circular fillet at the inside corner of the knee. The frames were tested chiefly as two-hinged structures under working loads.

In general, the structural behavior of the two rigid frames was in accordance with conventional theory. At the knees of both frames, however, the normal stress distribution departed markedly from the usual straight-line relationship. In the square knee, a concentration of stress existed at the inner corner but was found to be of minor importance. In the curved knee, compressive stresses in the flange of the curved fillet were greater than those computed by either the straight or curved beam theories. Further, a transverse variation of the stress in the curved flange added still more to the high compressive stresses in the curved knee. On the basis of the test results, recommendations for the analysis and design of each type of rigid frame have been made and are presented herewith.

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INTRODUCTION

During the last decade there has been a growing appreciation of the many structural and esthetic advantages of the rigid frame type of construction, particularly as applied to short span bridges. However, due to the lack of available information regarding the stress distribution in this type of structure, particularly at the knee section, the steel rigid frame is still viewed with some concern by many engineers.

In order to remedy this situation, the American Institute of Steel Construction initiated two investigations of the structural behavior of steel rigid frames. One was conducted by the National Bureau of Standards at Washington, D.C., the results of which have been published in ten reports.* The second investigation, reported in this paper, was made possible by the establishment at the Fritz Engineering Laboratory of a cooperative research fellowship by the A. I. S. C. and Lehigh University.

The program for this investigation was prepared by the Technical Research Committee of the Institute, consisting of Mr. Jonathan Jones, chairman; the late Mr. H. G. Balcom, and Messrs. F. H. Frankland, O. E. Hovey, H. D. Hussey, J. R. Lambert, and Aubrey Weymouth. Acknowledgment is due to the

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* Progress Reports No. 1 to 10 on STRESS DISTRIBUTION IN STEEL RIGID FRAMES, National Bureau of Standards, published by the American Institute of Steel Construction.

members of the committee for their active interest in the work and their advice and guidance; to Mr. E. L. Durkee, Mr. G. L. Gray, and Mr. W. B. McLean of Fabricated Steel Construction, Bethlehem Steel Company, for the design of the square knee model and for their valuable assistance in the theoretical analysis of the frames; to Mr. Hussey for the design of the curved knee model and to the members of the laboratory staff for their assistance in testing the models and the preparation of the report.

In the investigation conducted at the National Bureau of Standards, only knee specimens were tested, the primary purpose being to determine the stress distribution at this section. At Lehigh University, two complete rigid frames were tested to secure a check upon the stress distribution obtained by the testing of knee specimens alone, and also to permit the observation of other important data regarding the frame as a whole. The focal point of interest was the knee section, although such subjects as slippage of foundations, and accuracy of conventional methods of analysis and design were also studied.

TEST PROGRAM

The investigation was planned with the view of studying the behavior of the frames when hinged at the supports. The test program was laid out with the following objectives in mind.

A. Determination of Stress Distribution in the Knee Section
   1. Principal Stresses and Maximum Shears
   2. Normal Stresses and Shears on Arbitrary Sections

B. Effect of the Stress Distribution in the Knee upon the Behavior of the Frames as a Whole
   1. On Horizontal Reaction
   2. On Stresses in Frames
   3. On Deflections

C. Effect of Simulated Foundation Slippage
   1. On Horizontal Reaction
   2. On Stresses in Frames
   3. On Deflections

D. Determination of Restraint Provided by Flat-Plate Base

E. Comparison Between Experimental Data and Calculated Results

For the most part the program was carried out as outlined for both frames. Exceptions will be cited as they are discussed.

TEST SPECIMENS

In order to compare the results of this investigation with those obtained at the National Bureau of Standards, the shapes of the knee sections of the two model frames were made similar to two of the specimens tested at the Bureau.

The chief point of difference between the two models was the shape of the knee. One, which shall be referred to as the square knee frame, had a sharp reentrant angle of about 90° at the inside corner of the knee, while the other, designated as the curved knee frame, had a large circular fillet at the inside of the knee. The details of the two frames are shown in Fig. 1 and 2. The method of fabrication of the square knee, showing that the web does not extend through the vertical section at the inside corner of the knee, is illustrated in Fig.3. The size of the models was chosen to fit the testing machine available, and the applied loads were designed to produce working stresses in the models.

The square knee frame was considered to be reduced from an imaginary prototype with a clear span of 72 ft. and was designed as the middle one of three such frames spaced 15 ft. apart, with framed floorbeams and stringers supporting a 36-ft. roadway with H-20 loading. The linear dimensions of the model were one-fourth and the cross-sectional areas approximately one-sixteenth of those of the prototype. The curved knee was designed for the same conditions.

Due to the small size of the frames, it was considered advisable to have them fabricated at an ornamental iron works rather than at an ordinary structural shop. Bids were obtained from several such organizations in the vicinity of the laboratory; the square knee frame was fabricated by the Bethlehem Fence Works and the curved knee frame by the Allentown Iron Works.

In general the workmanship of the two frames was satisfactory. Overall dimensions and depths of the sections were sufficiently accurate to permit use of the nominal values in all but a few instances. One objectionable feature was found, however, in the square knee frame. At the intersection of the compression flanges at both knees, where tight bearing should be obtained, small gaps existed. It was considered advisable to fill these gaps with shims which were tack-welded in place, but which produced tight bearing only along the outstanding legs of the girder flange angles. Thus, a loose fit was eliminated at the expense of a concentration of stress at the bearing.

Tensile properties of the material in the two frames are given in Table I.

TEST METHODS

A. Loading Apparatus - The frames were tested in a 300,000-lb. Olsen machine having a 21-ft. beam which provided an excellent base on which to set the 18-ft. models. The load was transferred from the moveable head of the testing machine to the load points of the frames by a system of bars and loading beams as illustrated in Fig. 4.

The horizontal reaction was resisted by a 3/4-in. round bar extending between the two column bases in all tests. To allow adjustment of the reaction and the span length of the frame, the ends of the tie bar were threaded and fitted with nuts. Rollers under one of the column bases insured that only a negligible amount of friction might affect the horizontal reaction. The tie-bar attachment and rollers are shown in Fig. 4 and 5.

In order to prevent lateral buckling or twisting of the flexible horizontal girder, trussed frames were built up from the testing machine at midspan, and just inside the inner face of each column, as shown in Fig. 4. The frame had a tendency to bear against all of these lateral supports under load, but only at the center was there any appreciable deflection where friction might be developed. Comparative tests, with and without the center support, gave practically identical results, so the frictional restraint was considered negligible.

B. Base Details - 1. Hinged Base. The apparatus shown in Fig. 5 was developed to provide a point bearing for the frame supports and at the same time transmit the horizontal reaction to the tie-bar. The bearing area of the pin was always less than 1 in. wide, which was sufficiently small for the assumption of a point support.

2. Flat Base. It was considered worthwhile to investigate how much restraining moment was produced by the base when the frame rested upon flat bases which restrained horizontal movement, but not rotation. To provide this condition, the two sets of apparatus shown in Fig. 6 were developed. Since the two gave identical results, Base No. 2 was used in most tests.

C. Observations and Instruments - In all tests the total load was 1000 lb. for the zero condition, and 13,000 lb. for the working condition, giving a working range of 12,000 lb. Fig. 1 and 2 show the location of the load points. In some of the early tests, the working load was applied in two equal increments, but for the most part it was found convenient to apply the load in but one increment. For purposes of checking in the latter case, each loading was always repeated.

To determine the state of stress at each gage point on the web, three strain readings, horizontal, vertical, and inclined at 45°, were observed. Stresses were obtained from the observed strains by the graphical method developed by W. R. Osgood and R. G. Sturm* of the National Bureau of Standards. At each flange gage point, only the strains parallel to the longitudinal dimension of the flange were observed, as the transverse stresses in the flange may be considered negligible. Wherever possible, flange strains were observed at both heel and toe of the outstanding legs of the angles and on the edge of the web. At all gage points, strains were observed simultaneously on both sides of the frame in order to eliminate the the effect of lateral bending. These strain observations were made with Huggenberger tensometers having one-inch gage lengths. With these instruments, stresses could be obtained within an expected accuracy of 300 p.s.i. The instruments were held in position on the web plate by means of 1/8-in. bolts and tapped holes in the web. As it was desirable to keep the number of

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* THE DETERMINATION OF STRESSES FROM STRAINS ON THREE INTERSECTING GAGE LINES AND ITS APPLICATION TO ACTUAL TESTS, by Wm. R. Osgood and R. G. Sturm, Bureau of Standards Journal of Research, Vol. 10, 1933, Research Paper No. 559.

holes at a minimum, only three strain readings were observed at each point. Instead of the check usually obtained by the fourth reading, a repetition was made of each loading and observation. The tensometer attachments for web and flange strain measurements are shown in Fig. 4 and 7.

Huggenberger tensometers were also used for observing flange strains at midspan and at other points in the frames as required by the special problems which appeared in the testing of the frames.

The adjustment of the span length was measured by a 0.001-in. Ames dial bearing against one column base and fastened to a long light angle which was clamped at its opposite end to the other column base. Rollers supported the angle along its length. The two ends of this device are shown in Fig. 6. Readings at zero load were always taken at normal span length. By normal span length is meant the span length under no load. As the load was being applied, the span was adjusted by the nuts on the tie-bar, so that at maximum load, the span length was as desired, either normal, or plus or minus a given quantity.

The horizontal reaction was determined by observing the strain in the tie-bar with a ten-inch Whittemore strain gage and computing the stress and load therefrom. This instrument contains a 0.0001-in. Ames dial and indicates stress within about 300 p.s.i., or about 150 lb. load in the tie-bar.

The vertical deflection of the frame at midspan was measured by a 0.001-in. Ames dial between the top flange of the frame and a framework built up from the bases of the frame as shown in Fig. 8.

TEST DATA

1. Stress Distribution at Knee - Square Knee Frame. Three complete sets of stress-distribution data were obtained for the square-knee frame, one set for each knee of the frame at normal span length, and one for the East knee with the supports allowed to move outward 1/4-in. under load. In general, the results for the two knees were so similar that the data from only one are presented here. The variation of distribution due to movement of the supports is discussed in another section.

From the strains observed at each gage point on the web of the knee, principal stresses and maximum shears were determined graphically. On the backs of the flanges only longitudinal strains were observed, from which stresses were computed directly. The values of principal stresses and maximum shears are indicated by lines of equal stress, contour lines, for the normal span condition in Fig. 9. Directions and magnitudes of the principal stresses are also shown in Fig. 9.

In connection with the rigid frame investigation conducted at the Bureau of Standards, a theoretical analysis of a rectangular rigid frame knee was developed.* The analysis

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* STRENGTH OF A RIVETED STEEL RIGID FRAME HAVING STRAIGHT FLANGES, A. H. Stang, Martin Greenspan, and Wm. R. Osgood, Journal of Research of the National Bureau of Standards, Vol. 21, September 1938

involved an application of the theory of elasticity to a rectangular plate loaded at the edges with shears, thrusts and moments in its own plane as shown in Fig. 10. The resulting contours of principal stresses and maximum shears, which were worked up by Mr. H. D. Hussey of the American Bridge Company for a rigid frame knee of practically identical proportions as the square knee are also given in Fig. 10. It is noted that the general agreement between theoretical and observed results is very satisfactory.

The knee joint may be considered as a rigid beam and column connection in which the column extends to the top of the frame. With this idea in mind, normal stresses on sections approximately perpendicular to the axes of the column and the girder were determined from observations and are shown in Fig. 11. It will be noted that the neutral axis of the column deviates only a comparatively small amount from the centerline. The presence of direct stress in addition to bending was recognized in locating the neutral axis. Normal stresses on a plane at approximately 45° through the inside corner of the knee are also shown in Fig. 11. The stress on this plane at the inside corner was obtained from the flange stresses at the corner. Since these flange stresses could not be measured, they were obtained by extrapolation from observed values near the corner. The dotted curve on the vertical section in the girder nearest the column face indicates the stress distribution that would probably have existed had the vertical legs of the compression flange taken their proper share of the total compression in the flange.

Fig. 9 shows that the maximum shears are very nearly constant over most of the web in the knee and are all approximately horizontal and vertical, being at 45° with the principal stresses. Computations show that the average of the horizontal shears over the entire knee is 98.4 per cent of the average of the maximum shears. Thus, a correct determination of the horizontal shearing stresses would be adequate for design purposes. Fig. 12 presents a comparison between the horizontal shears determined from observations and those computed on the assumption that the web resists a horizontal shear equal to the tension in the top flange of the girder. The girder flanges have been assumed to take all the moment and thrust on the section at the column face as shown in Fig. 12.

A comparison of observed and computed extreme fiber stresses in the vicinity of the knee is presented in Fig. 13. Computed stresses are based on the conventional formulas for flexure and direct stress. In plotting the computed tension values it was assumed that the stresses reach a maximum at the sections which pass through the inside corner of the knee, and decrease uniformly along the flanges of the knee to zero at the outer corner. The stresses represented by triangular dots on the vertical section passing through the inside corner of the knee were computed on the assumption that the flange angles of the girder transmitted all of the moment and thrust in the girder to the column.

Curved-Knee Frame. In the testing of the curved knee frame, it was observed that stresses at the toes of the outstanding legs of the curved flange angles were 40 to 70 per cent less than stresses at corresponding points on the heels of the angles or on the edge of the web. The stresses at the heels of the angles and at the edge of the web agreed very closely. Fig. 14 shows flange stresses observed on the three gage lines, A, B, and C, at the knee. This transverse variation in stress in the outstanding legs did not exist on straight flanges, whether subjected to tension or compression.

Observations of flange strains transverse to the longitudinal axis of the curved flange indicated that the outstanding legs were bending back slightly away from the heels of the angle. Computations showed that in order for the observed stress variation to take place, the toes of the outstanding legs would have to deflect 0.01-in. with respect to the edge of the web. The slight transverse bending observed was not sufficient, however, to produce this deflection, indicating that a rotation was taking place about the rivet line. The stresses on all three gage lines increased approximately in proportion to the load.

Normal stresses on planes radial to the curved flange are shown in Fig. 15. Straight line stress distribution does not exist on these planes within the knee section. A comparison between average observed extreme fiber stresses at the knee and those computed by the conventional formulas for flexure and direct stress is shown in Fig. 16. The average stress values were obtained by assuming that the extreme fiber stress was constant across the backs of the angle at each section, that the neutral axis remained stationary, and that the moment of the compression area about the neutral axis remained constant. From the observed stresses and the observed position of the neutral axis, the average stress value was then computed. A comparison between these average values and the actual observed maximum stresses at the extreme compressive fibers is indicated in Fig. 15.

The observed principal stresses and maximum shears for the curved knee frame are shown in Fig. 17. It is noted that the greatest maximum shears in the knee lie approximately within a square at the exterior corner of the knee with two of its boundaries coinciding with the straight flanges of the knee and with its interior corner on the observed neutral axis (see Fig. 15). This square is shown in detail in Fig. 18. From Fig. 17 it is also noted that the maximum shears within the square are very nearly all horizontal and vertical. The forces acting on this section of the web are illustrated in Fig. 18, the shears introduced by the flange tensions being very great in comparison to the bending loads due to the web. Therefore, the horizontal and vertical shear in the square was considered to be approximately equal to the shear introduced by the flanges. The stresses in the angles which must be unloaded into the web along the two exterior sides of the square can be computed by the conventional formulas for flexure and direct stress. Assuming that the square was isolated from the remainder of the knee and loaded with horizontal and vertical shears equal to the average of the tensions in the two flanges, which were nearly equal, the resulting average unit shear agreed closely with the observed maximum shears. The comparison between observed and computed results is shown in Fig. 18.

2. General Behavior of Frames - Three observations on each frame were used as criteria for the general behavior of the frame, namely; the horizontal reaction, the internal moment at midspan, and the vertical deflection at midspan. By comparing the observed values with theoretical values and with semi-theoretical values obtained by using the observed horizontal reaction, a good indication of the efficiency of the knee-joint as well as of the frame as a whole was obtained. Such a comparison is given for each frame in Table II. All values refer to the normal span condition.

Since only six tensometers were available for use, a great many repetitions of loading were necessary to obtain all desired data. Strain observations on the tie-bar were made at regular intervals throughout the course of the testing, and no appreciable change was noticed at any time. Evidently such effects as permanent set due to high localized stresses and slippage of rivets and joints had a negligible effect upon the frame as a whole. However, a decrease of about 5 per cent in the high localized stresses at the knee of the square knee frame occurred during the course of the testing.

Load-reaction and load-deflection curves for each frame, shown in Fig. 19, illustrate that the frame as a whole behaved as an elastic structure, i.e., the observations varied in direct proportion to the load. Individual stress observations were generally made in but one load increment, but at several random points load-stress data were observed, a straight line relationship was obtained.

3. Horizontal Movement of Supports - In order to simulate one of the most important problems in rigid frame construction, that of foundation slippage, the bases of the frames were moved inward or outward a given distance as the load was being applied. Each frame was studied under five such conditions, the span length varying from the normal condition by the following amounts: -1/2 in., -1/4 in., 0-in., +1/4 in., +1/2 in. A 1/4-in. movement in the model corresponded to a 1-in. movement in the prototype.

Observed and computed values of the horizontal reactions, the center moments, and the center deflections for the various span lengths are shown in Fig. 20 and 21 for both frames. A linear variation was found for both theoretical and experimental values for all conditions. It is noted that the increments of the observed reactions, moments, and deflections for a variation in span length of any definite amount are slightly larger than those of the computed values.

For both frames the observed horizontal reactions were slightly less than the computed values, varying from 0 to 4 per cent for the square knee frame, and constant at about 4 per cent for the curved knee frame. The observed center moments and deflections were greater than the computed values, varying from 0 to 8 per cent for the square knee frame, and from 5 to 13 per cent for the curved knee frame. According to the laws of statics, observed reactions and center moments should always check each other. In general they agreed very well. Whenever the check was not satisfactory, the values of horizontal reaction and center moment were adjusted, equal weight being given to the two observations.

The stress distribution in the square knee ~~frame~~ for the span-variation of +1/4-in. differed but little from that for the normal span. In general the stresses observed were slightly less, as would be expected in view of the smaller horizontal reaction.

Flat Base Tests - Both frames were tested with the supports resting on the flat plate bases shown in Fig. 6 in order to determine the amount of rotational restraint produced by this type of base. Internal moments were determined from strain measurements at two locations in the frame. Since the horizontal reaction was also observed, it was possible to compute the location of the point of inflection in the column, if it existed. The results indicated that for both frames, the point of inflection lay so close to the base that in effect it could be considered a hinged support.

Table III presents ratios of observed to computed horizontal reactions, center moments, and center deflections for both hinged and flat base tests. It is noted that the two base conditions gave very similar results for both frames. In the computations it was assumed that a hinge existed at the flat base.

ANALYSIS

1. Moment of Inertia - A test was made to determine the correct moment of inertia for use in computations. With the curved knee frame inverted in the testing machine, the girder which was of uniform section, was tested as a simple beam with two equal and symmetrically situated loads. At a working load for the girder, maximum flange strains were observed. Stresses were computed from the observed strains, and the effective moment of inertia was computed from the maximum stresses, applied loads, and dimensions of the girder.

The effective moment of inertia was slightly less than the gross moment of inertia, and almost exactly equal to that obtained by assuming the total area of the rivet holes to be uniformly distributed along the length of the girder. This average moment of inertia was used in all computations. If the gross moment of inertia had been used, the error would have been about 3 per cent for this girder.

2. Horizontal Reaction - NOTATION

H = horizontal reaction

Δs = length of arbitrary section of frame

I = moment of inertia

A = cross-sectional area

A' = web area

E = modulus of elasticity in tension and compression

G = shearing modulus of elasticity

M = actual moments in the frame

V = actual shear in the frame

N = actual thrust in the frame

α = angular deformation of a section

M_1 = moment due to applied loads on simple frame, i.e., no restraint to horizontal movement of bases

V_1 = shear due to applied loads on simple frame

N_1 = direct thrust due to applied loads on simple frame

m_h = moment due to unit horizontal load applied at supports

v_h = shear due to unit horizontal load applied at supports

n_h = thrust due to unit horizontal load applied at supports

In order to compare the test results with computed values, analyses were made for both frames by the Maxwell-Mohr (unit dummy load) method as applied to a two-hinged arch. Since both frames were of non-uniform cross-section, the members of each frame were arbitrarily divided into short sections and the necessary integrations carried out as algebraic summations. The arbitrary sections are shown in Fig. 1 and 2.

The effects of shear and direct stress were included in the computations, since for comparison with test results a greater degree of accuracy than that ordinarily required in design was desired. The equation for the horizontal reaction, considering deformations due to bending, shear, and direct stress, can be written in this manner:

$$H = \frac{\Sigma \frac{M_1 m_h \Delta s}{EI} + \Sigma \frac{V_1 v_h \Delta s}{A'G} + \Sigma \frac{N_1 n_h \Delta s}{AE}}{\Sigma \frac{m_h^2 \Delta s}{EI} + \Sigma \frac{v_h^2 \Delta s}{A'G} + \Sigma \frac{n_h^2 \Delta s}{AE}} \qquad (1)$$

Actual evaluation of the different quantities indicated that for rigid frames of types similar to the two models, the last two terms of the numerator and the last term of the denominator may be neglected for all practical purposes, giving:

$$H = \frac{\Sigma \frac{M_1 m_h \Delta s}{EI}}{\Sigma \frac{m_h^2 \Delta s}{EI} + \Sigma \frac{v_h^2 \Delta s}{A'G}} \qquad (2)$$

Neglecting the remaining shear term would introduce an error of about 3 per cent for the square knee frame and about 1 per cent for the curved knee frame.

The application of this formula for H is a routine matter providing the structure behaves in accordance with the assumptions on which the theory is based. Test results indicate, however, that at the knees of both rigid frames, sections that are plane before bending do not necessarily remain plane after bending. As a result, the actual bending deformations $(\frac{M\Delta s}{EI})$ and shearing deformations $(\frac{V\Delta s}{A'G})$ in the sections at the knee will differ from those computed by the Maxwell-Mohr theory. Therefore, a study of the stress distribution at the knee of each frame was made for the purpose of determining the treatment of the corner sections which would most nearly approach actual conditions as interpreted from observed data.

The changes in horizontal reaction due to variations in span length were determined directly from equation (2). The term, $\Sigma \frac{M'm_h\Delta s}{EI}$ in the numerator of the expression for H represents the horizontal deflection of the supports due to the applied loads when the supports are free to move horizontally, i.e., the variation in span length for a 100 per cent change in reaction. Since the change in reaction is proportional to the variation in span length, the horizontal reactions for the various span lengths were obtained directly.

3. Deflections - Vertical deflections at the center of each frame were computed by the Maxwell-Mohr method, involving the application of a vertical unit load at the point at which the deflection was desired. In this determination, the following question arose: should the unit load be applied to the fully restrained frame, thus producing a horizontal reaction, or to the determinate frame with supports free to move horizontally? An investigation of the problem showed that the two procedures gave identical results. For convenience, it is therefore recommended that the moments due to the unit load be computed on the basis of the determinate frame, that is:

$$\delta = \Sigma \frac{Mm\Delta s}{EI} \qquad (3)$$

where: δ = deflection at any point

M = actual moments in structure due to applied load

m = moments due to unit load applied to determinate structure at the point and in the direction of the deflection desired

Δs, E, and I have their conventional significance

4. Corner Sections - Square Knee Frame. Of primary importance is the degree to which the sections within the knee may influence the determination of the horizontal reaction. Total neglect of knee sections in the computations for horizontal reaction, that is, considering the knee to be infinitely stiff,

increased the computed value by about 3 per cent.

The actual deformation of the square knee was obtained from the information in Fig. 9. Since the principal stresses within the knee were approximately circumferential and radial, the knee was divided into circumferential bands, as shown in Fig. 22a. Average radial and circumferential stresses were assigned to the bands in accordance with the observed stress distribution. From the average principal stresses the elongation of each band was computed, which when plotted together, gave the total angular change between the two internal faces of the square knee, also shown in Fig. 22a. This angular change was practically the same as that computed from the observed shears in the knee (Fig. 22b), indicating that bending contributed very little to the deformation of the knee.

The effect of shear in the knee is accurately represented by the analysis presented in Fig. 22c. This practice of treating the column as a free body acted upon by the reactions of the frame and by the flange forces and end shear of the girder is now used by some designers. When the knee is considered to be loaded in this manner, bending about the neutral axis of the column will contribute but little to the effective deformation of the knee, which was already indicated by test results in Fig. 22a and b.

Two conventional methods of treating the corner of a square rigid frame knee for mechanical integration are shown in Fig. 22d and e. The computed bending deformations at the knee for the two cases were approximately equal and considerably greater than that indicated by the analysis shown in Fig. 22a and b. If, However, the effect of shear is neglected throughout the frame, as would be done in design, the large bending deformations assigned to the knee tend to offset the absence of shear deformations in the frame as a whole. The results thus obtained differed but slightly from those in which shear was considered.

A comparison between the observed horizontal reaction and the values obtained by the several methods of analysis are given in Table IV.

A simple method for computing normal stresses on the 45° plane through the inside corner of the knee was developed from the test data shown in Fig. 11. In accordance with the stress distribution on the diagonal plane, the neutral axis for this section was assumed to be located at one-fifth the length of the diagonal from the inside corner, the tension stress area was assumed to be a second degree parabola, and the compression stress area a triangle. With these assumptions and the equilibrium equations for direct stress and for bending on the section, the maximum values of tension and compression were determined. Considering the compression flange, but not the tension flange of the girder nor the horizontal stiffening angles to be acting with the web, the maximum tension and compression stresses computed by the above method fell within 5 per cent of the observed values.

Curved Knee Frame - In the curved knee frame the horizontal reaction was 3-1/2 per cent below the computed value. Average compressive flange stresses at the knee varied from 15 to 50 per cent in excess of stresses computed by the conventional formulas for flexure and direct stress as shown in Fig. 16. Therefore, the bending deformations in sections within the knee were greater than those computed by conventional methods; i.e., the knee was not as stiff as assumed. To remedy this condition, a simple and arbitrary method for reducing the moments of inertia and section moduli of sections within the knee was developed from the test results. Using these modified values, computations gave a horizontal reaction and compressive stresses within the knee in close agreement with the observed data.

For knees of this type radial sections are the most convenient to use and were employed in all analyses involving mechanical integration. To obtain what will be called the "effective" section modulus (S') for each section, the moment was divided by the average observed compressive stress at the extreme inner fibers. The ratio of "effective" section modulus to actual section modulus ($\frac{S'}{S}$) was determined for each section and plotted against the central angle C as shown in Fig. 24. The approximately linear relationship between the ratio $\frac{S'}{S}$ and the angle C can be expressed by the linear equation:

$$\frac{S'}{S} = 1 - \frac{(C/45)}{2} \qquad (4)$$

or

$$S' = S\left(1 - \frac{C}{90}\right) \qquad (5)$$

Due to the origin of this equation, the compressive stresses based on the "effective" section-moduli agreed closely with the average observed compressive stresses. Further, the same principle used in the modification of the section modulus may be applied to the moment of inertia in order to correct for the discrepancy in observed and computed horizontal reactions. The procedure in an actual design, assuming that the stress in the extreme compressive fibers is constant across the width of the flange, would be to compute the "effective" moment of inertia (I'), from the relationship:

$$I' = S' \cdot c \qquad (6)$$

where c is one-half the actual depth of section. In the case of the curved knee frame however, the deformation of the knee depended not on the average flange stresses, but on the maximum compressive stresses which occurred at the edge of the web. By using the "effective" moments of inertia based on the maximum compressive stresses in the web, the value of the horizontal reaction (neglecting shear) was reduced from 4690 to 4480 lb., as compared with the observed value of 4530 lb.

Maximum tensile stresses at the knee occurred at the extreme fibers of sections near the points of tangency of the curved flange and are in fairly close agreement with values computed by the conventional formulas for flexure and direct stress. On the section through the exterior corner of the knee, however, the maximum tensile stress can not be computed in this manner. A simple solution for this section was derived from the test results in Fig. 15, identical in principle with the method applied to a corresponding section in the square knee frame. The assumptions made were that the neutral axis was located one-quarter the distance from the compression flange to the exterior corner, and that the stress distribution followed a second degree parabola on the tension side and a triangle on the compression side. Assuming that the tension flange angles did not act with the web, and applying the equations of equilibrium, the maximum observed tension and compression stresses on the section were checked within about 10 per cent. Since this section is not likely to be a critical section, the check was considered satisfactory.

5. Source of Computed Values Used in Comparisons - All computed values, unless otherwise specified, were based on horizontal reactions, computed by equation (2). For the purpose of comparison with test results the knee section of the square knee frame was analyzed as shown in Fig. 22*c*. Computations for sections in the curved knee were based on the conventional formulas for flexure and direct stress: $\alpha = \frac{M \Delta s}{EI}$; $s = \frac{N}{A} \pm \frac{MC}{I}$. The fact that the flanges were not parallel at the curved knee was disregarded in evaluating moments of inertia and section moduli.

DISCUSSION OF RESULTS

1. Stress Distribution - Square Knee Frame. In general, the portions of the frame outside of the knee behaved in accordance with conventional theory. On sections within a few inches of the boundaries of the knee, straight line distribution of normal stresses was obtained. However, immediately adjacent to the knee observed stresses in the compression flanges of both column and girder were considerably greater than stresses computed by the conventional beam formula for flexure and direct stress. Probably two principal causes contributed to this condition. First, the web of the frame was discontinuous through the vertical joint at the knee, which probably resulted in the flanges of the girder carrying more than their share of the moment and thrust on the section at the column face. This premise is upheld by the normal stress distribution shown in Fig. 11 on the girder section next to the knee. It is noted that on this section the web is understressed and the flange angles overstressed. Second, the presence of shims between the girder compression flange and the column caused practically all of the compression in the girder to be transmitted to the column through the outstanding legs of the flange angles. At a point on the girder 2-1/2 in. from the column face, the outstanding legs were carrying about three-fourths of the total load in the compression flange angles. The average stress in the compression flange agreed closely with the stress computed on the assumption that

the ~~outer~~ girder flanges carry all the moment and thrust in the girder at the knee joint. The slight excess stress noted in the compression flange of the column at the inside corner of the knee was probably caused by the extreme concentration of bearing at that point. It seems evident that any additional concentration of stress in the column or girder at the knee because of a sharp reentrant angle is of small magnitude. Whatever concentration does exist is directly dependent upon the bearing condition existing at the corner. An accurate fit between the compression flange angles will produce high stresses in the flanges, whereas a loose fit will produce lower stresses in the flanges but higher stresses in the web. Local variations of this nature are of general occurrence in steel structures, where these high localized stresses are usually disregarded as unimportant. Therefore, with regard to concentrations of stress at sections adjacent to the knee, the rigid frame may be treated as any other steel structure.

Within the knee the stress distribution can not be determined by any simple theoretical analysis. The application of the theory of elasticity is too complicated and tedious for practical use by designers. The test data indicated that the critical sections for normal stresses at the knee are the horizontal and vertical sections through the inside corner of the knee, which were discussed in the preceding paragraph.

Shear is apparently quite important in a rigid frame knee of this type. The unit shear within the knee was about twice the shear which existed just outside the knee, which is ordinarily considered to be the critical section with respect to shear. A study of the maximum shears showed that designing for horizontal and vertical shears in the knee on the basis of an external shear equal to the total tension in the top flange of the girder, was both adequate and correct.

Curved Knee Frame. As in the case of the square knee frame, the portions of the frame outside the knee gave results in accord with conventional theory. Within the knee, the stress distribution differed markedly from that obtained by the ordinary beam theory. Throughout the entire curved portion of the compression flange at the knee, the observed stresses were much greater than the computed values. Two factors probably contributed to the observed difference. First, the neutral axis has a pronounced curvature at the knee. However, an application of the curved beam theory accounted for only a small part of the differences between observed and computed stresses, particularly on the compression flange. The second and possibly the most important factor is the rapid change of section which takes place at the knee. This latter problem introduces complexities which may be reasonably solved only by a highly theoretical analysis, or by an arbitrary procedure based on test results. The arbitrary procedure presented in the section on analysis makes no differentiation between the separate effects of the two factors mentioned above. Such a differentiation would be of value only if the method of analysis were to be extended to knees having decidedly different degrees of curvature.

The stress distribution was complicated by a stress relief in the outstanding legs of the curved flange angles. The radial component of compression due to the curvature of the flange caused the outstanding legs to deflect away from the center of curvature and thus elongate relative to the edge of the web. If, however, there had been a cover plate on the backs of the curved flange angles, it is probable that no appreciable radial displacement of the outstanding legs could have taken place. Not only would the outstanding legs have been reinforced by an additional thickness of metal, but any rotation of the individual angles about the rivet line would have been effectively prevented. In the section on analysis it was assumed that cover plates would be present. Consequently the effect of stress relief in the outstanding legs was eliminated by using an average extreme fiber stress.

2. General Behavior - In both frames the horizontal reactions were slightly lower than the computed values, b-1/2 per cent for the square knee frame and 3-1/2 per cent for the curved knee frame. These relations refer to the normal span condition. Although this degree of accuracy appears to be satisfactory, the discrepancy is on the unsafe side. A decrease in the horizontal

reaction will produce an increase of much greater magnitude in the center moment. A greater degree of accuracy was obtained for the curved knee frame by arbitrarily reducing the moments of inertia within the knee in accordance with observed stresses.

In general, observed center moments and deflections agreed very well with corresponding values computed from the observed horizontal reactions. Therefore, the accuracy with which stresses and deflections can be computed is merely a reflection of the accuracy of the computations for the horizontal reaction. In other words, the knee of a two-hinged rigid frame affects the frame as a whole only insofar as it affects the horizontal reaction.

The observed normal span deflections of 0.46-in. and 0.67 in. for the square knee and curved knee frames, respectively, seem large when it is considered that the deflections in the prototypes are four times as great. In this connection, reference is made to deflection tests* made on three structural steel rigid frame bridges in Westchester County, New York. In these tests, the floor systems and cut-off walls, which were not included in the deflection calculations, evidently acted with the frame girders to a great extent. The observed deflections in some cases were only one-eighth of the computed values.

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* DEFLECTION TESTS SHOW RIGIDITY OF STEEL RIGID-FRAME BRIDGES, R. M. Hodges, Engineering News-Record, September 3, 1931.

3. Foundation Slippage - The importance of preventing horizontal movement of the supports of rigid frames is clearly illustrated by Fig. 20 and 21. For an increase in span length of 1/4-in., i.e., each support deflecting outward 1/8-in., the horizontal reaction for the square knee frame decreased 7 per cent and the center moment increased 20 per cent. Corresponding variations were not as great for the curved knee frame because of its greater flexibility.

The accuracy of computations for the effect of horizontal movement of the supports upon the frame as a whole depends chiefly on the accuracy with which the horizontal reaction is determined. Changes in horizontal reaction for known variations of span length did not check the computed values exactly, but the agreement was satisfactory for all practical purposes.

4. Flat Base Tests - It is of interest to note that practically no rotational restraint was developed at the supports by allowing the base plates of the rigid frame models to rest upon flat plates which resisted only horizontal movement. In fact, the horizontal reactions were less than the values computed assuming the supports to be hinged, while the presence of any reaction moment should have the effect of increasing the reaction. This might be due to a misalignment of the frames. If the bases were not truly horizontal, the reaction might easily have been located inside the centroid of the base. Such a condition would have produced a reduced horizontal reaction. However,

these test results definitely indicate that unless specific provisions are made to prevent rotation at the base of a rigid frame, the frame will act as a two-hinged structure.

RECOMMENDATIONS FOR DESIGN

For most practical purposes, horizontal reactions in two-hinged rigid frames of the types tested in this investigation, may be computed satisfactorily by any theoretically sound method of analysis. Fig. 20 and 21 present a criterion of the accuracy which can be expected.

If greater accuracy is desired for the curved knee frame, the following recommendations are made: arbitrarily reduce the moments of inertia of sections within the knee by means of the graph in Fig. 23. Shear is of secondary importance and need not be considered when radial sections at the knee are used.

Concerning the design of the knee in each frame, the following recommendations are made:

Square Knee: 1. The horizontal and vertical sections through the inside corner of the knee are critical sections with respect to normal stresses. Apply the usual formula for flexure and direct stress to the horizontal section. On the vertical section, assume that the flange angles carry all the moment and thrust in the girder.

2. Design the web to take a total horizontal shear equal to the tension in the top flange of the girder computed in the preceding step.

3. If the web plate is thin in comparison to that in the test specimen, it may be well to investigate the tension in the web on the 45° plane through the inside corner. For this section, the neutral axis will lay at about one-fifth the length of the diagonal from the inside corner, and the stress distribution may be considered to take the form of a second degree parabola on the tension side, and of a triangle on the compression side. Maximum stresses may then be computed from the equations of equilibrium for bending and direct stress on the section.

Curved Knee: 1. Critical sections for direct stress at the knee occur within 15° from the points of tangency of the curved flange. Maximum compression stresses may be determined by use of conventional formulas for flexure and direct stress in straight beams if the section moduli are reduced in accordance with Fig.23. Maximum tension stresses may be determined in the same manner, but using nominal section moduli.

2. The web should be designed for shear on the basis of the arbitrary method illustrated in Fig. 18. Shears existing outside the square will be smaller than those computed for the square.

3. If the web plate is thin in comparison to that in the test specimen, the 45° plane through the outside corner should be investigated using the same procedure that was recommended for a corresponding section in the square knee. The only difference is that in the curved knee the neutral axis is located at a distance from the compression flange about equal to one-fourth the whole diagonal distance.

SUMMARY

The important findings in this investigation may be summarized as follows:

1. The square knee did not act as a continuous homogeneous corner. It may therefore be analyzed and designed as a rigid girder and column connection, in which the reactions of the girder upon the column consist chiefly of a top flange tension, a bottom flange compression, and a shear.

2. Stress concentrations at the sharp reentrant angle of the square knee were due principally to imperfect bearing rather than to any inherent property of rigid frame knees.

3. Normal stresses upon radial sections of the curved knee did not exhibit a linear relationship. The neutral axis lay between the centroidal axis and the compression flange. On the radial section through the exterior corner the neutral axis was about one-fourth the distance from the compression flange to the exterior corner.

4. Maximum stresses in the curved knee occurred on sections just inside the points of tangency of the curved flange. Average extreme fiber stresses in the curved flange were from 10 to 30 per cent higher than stresses computed by the conventional formulas for flexure and direct stress in straight beams.

5. A simple method of reducing section moduli and moments of inertia within the curved knee produced close agreement between observed and computed values for compressive stresses in the curved flange and also for horizontal reactions.

6. The outstanding legs of the curved flange angles deflected back under the radial component of compression introduced by the curvature. As a result, the stresses observed near the toes of the outstanding legs at various points along the curved flange were only from 40 to 70 per cent of the stresses at corresponding points on the heels of the angles and on the edge of the web. Maximum stresses on the edge of the web were about 25 per cent greater than the average values used for comparing test results with computations.

7. Conventional methods of rigid frame analysis gave horizontal reactions slightly greater than those observed; 1-1/2 per cent for the square knee frame and 3-1/2 per cent for the curved knee frame at normal span.

8. Conventional methods gave center moments and deflections as much as ten per cent less than observed values for both frames.

9. Center moments and deflections generally agreed closely with values computed from observed reactions and applied loads.

10. Changes in horizontal reaction, center moment, and center deflection due to horizontal movement of supports also checked theoretical values fairly closely.

11. No appreciable rotational restraint was developed by setting the base plates of the rigid frames on flat plates which resisted horizontal movement of the supports, but not rotation. Rigid frames in which no special precaution for preventing base rotation is taken, will therefore act as two-hinged frames.

TABLE I - RESULTS OF TENSILE TESTS OF STEEL COUPONS

	Coupons Cut From:	Modulus of Elasticity	Yield Point	Tensile Strength	Elongation in 8 in.
		in 1000 p. s. i.			
Square Knee Frame	Angles	28,400	43.1	62.7	24.1
	Plates	28,570	45.8	54.4	22.0
Curved Knee Frame	Angles	29,370	39.5	59.3	30.0
	Plates	29,400	---*	63.6	15.7

*No definite yield point

TABLE II - COMPARISON BETWEEN TEST DATA AND COMPUTED VALUES FOR NORMAL SPAN TESTS

		Horizontal Reaction in pounds	Center Moment in 1000 in-lb.	Center Deflection in inches
Square Knee Frame	Observed	5280	98	0.465
	Computed	5340	93	.455
	Computed from Observed Horizontal Reaction	--	98	.485
Curved Knee Frame	Observed	4530	153	.678
	Computed	4690	143	.615
	Computed from Observed Horizontal Reaction	--	154	.700

TABLE III - COMPARISON BETWEEN HINGED BASE AND FLAT BASE RESULTS

	Type of Base	Ratio = $\frac{\text{Observed}}{\text{Computed}}$		
		Horizontal Reaction	Center Moment	Center Deflection
Square Knee Frame	Hinged	0.985	1.06	1.02
	Flat	.975	1.08	1.04
Curved Knee Frame	Hinged	.965	1.07	1.10
	Flat	.955	1.09	1.11

TABLE IV - HORIZONTAL REACTIONS OF SQUARE KNEE FRAME BY DIFFERENT ANALYSES

	Observed	Computed by Analysis in Fig. 22*c*		Computed by Analysis in Fig. 22*d* or *e*	
		Shear Included	Shear Neglected	Shear Included	Shear Neglected
Horizontal Reaction	5280	5340	5480	5320	5360

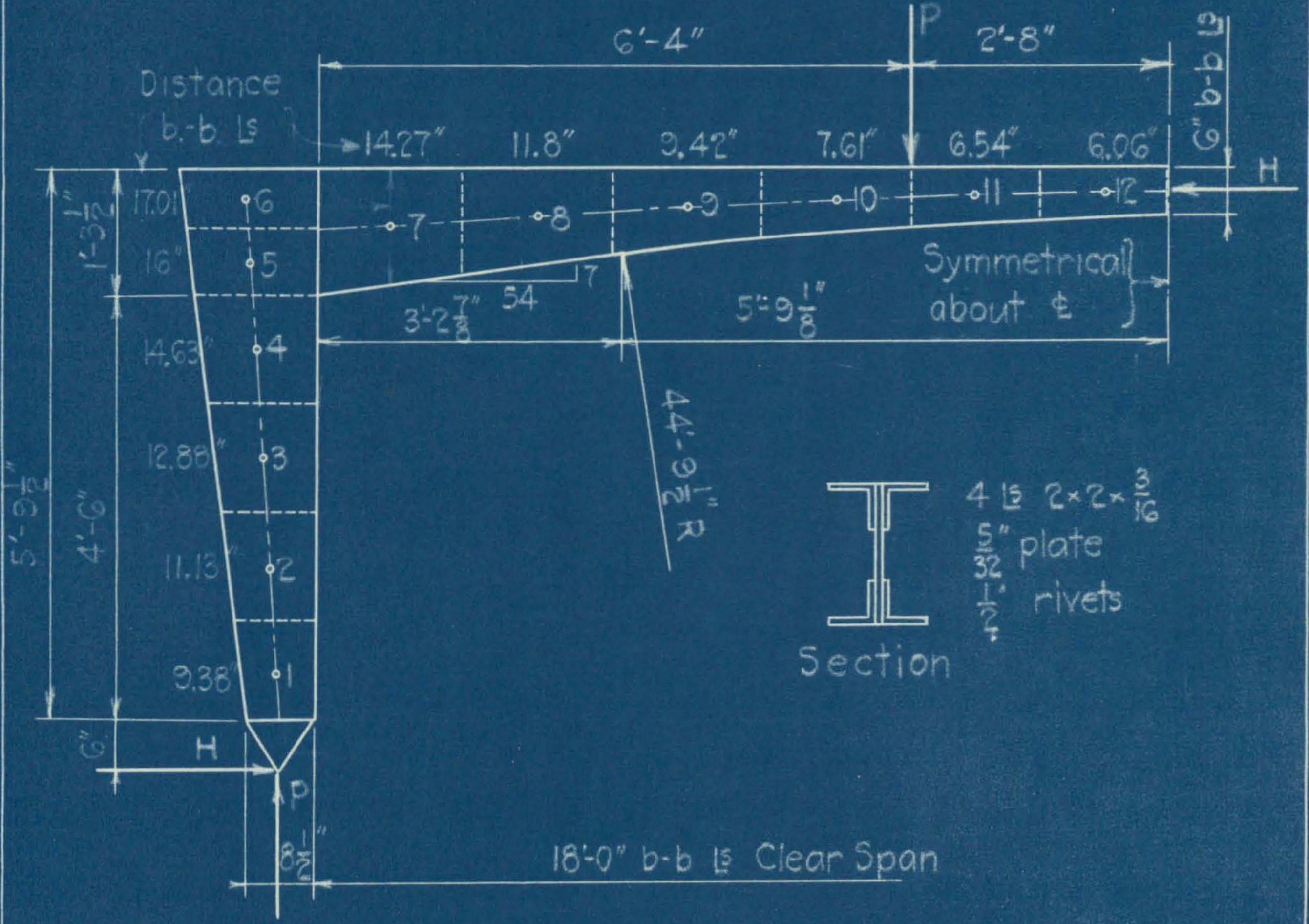


FIG. I- DETAILS OF SQUARE KNEE FRAME

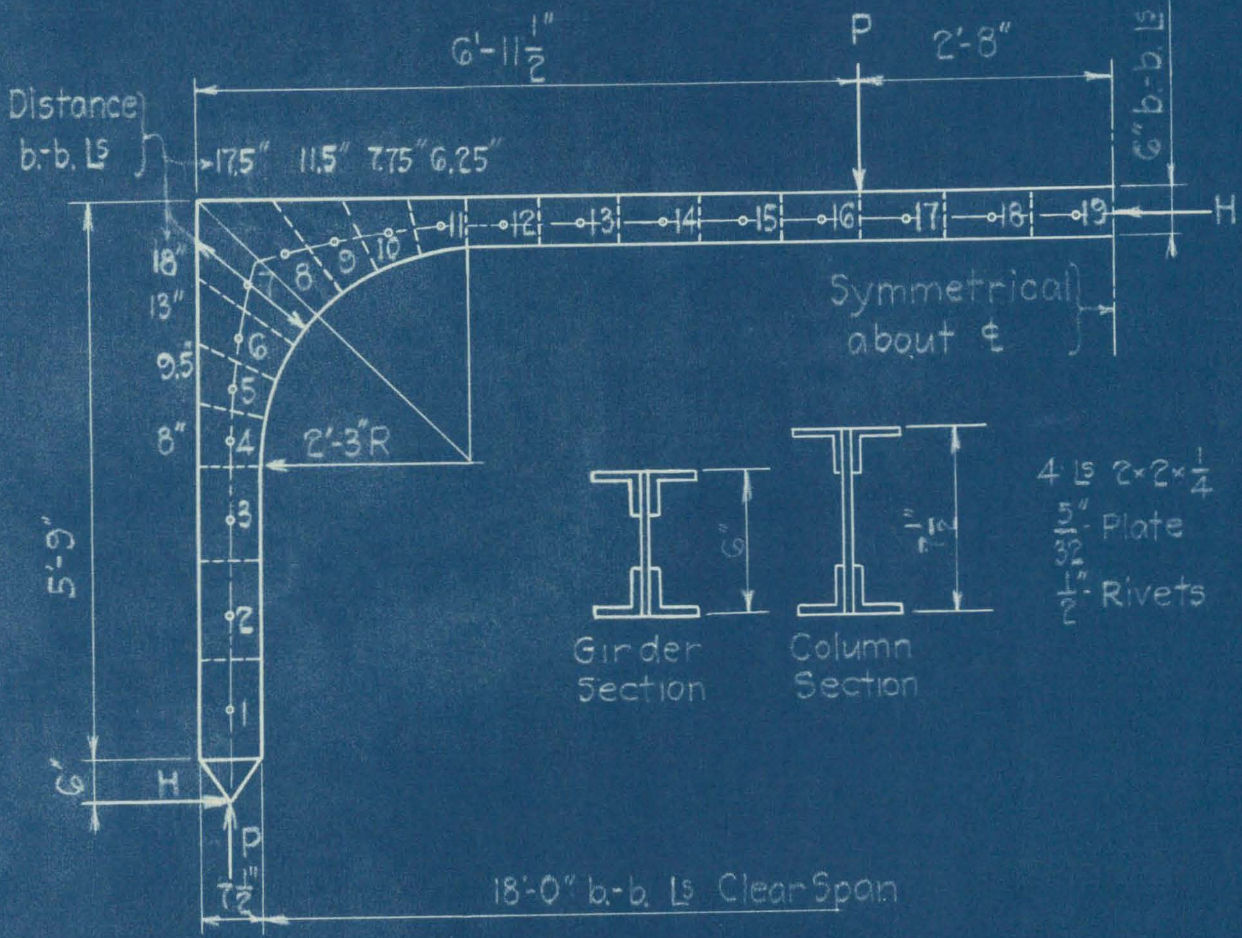
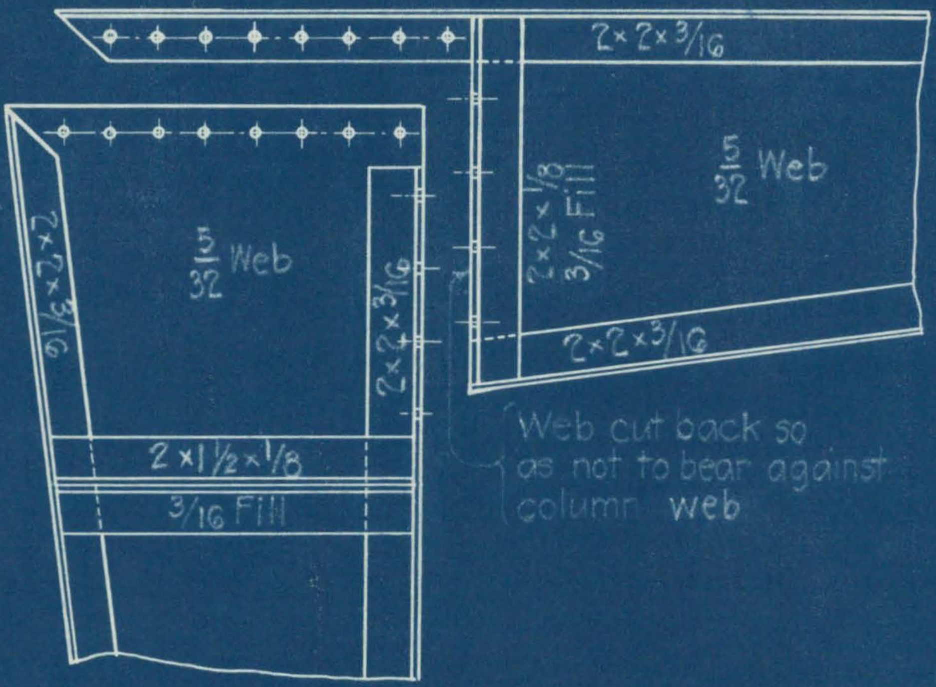


FIG. 2 - DETAILS OF CURVED KNEE FRAME



Web cut back so as not to bear against column web

FIG. 3 - FABRICATION DETAILS OF SQUARE KNEE FRAME

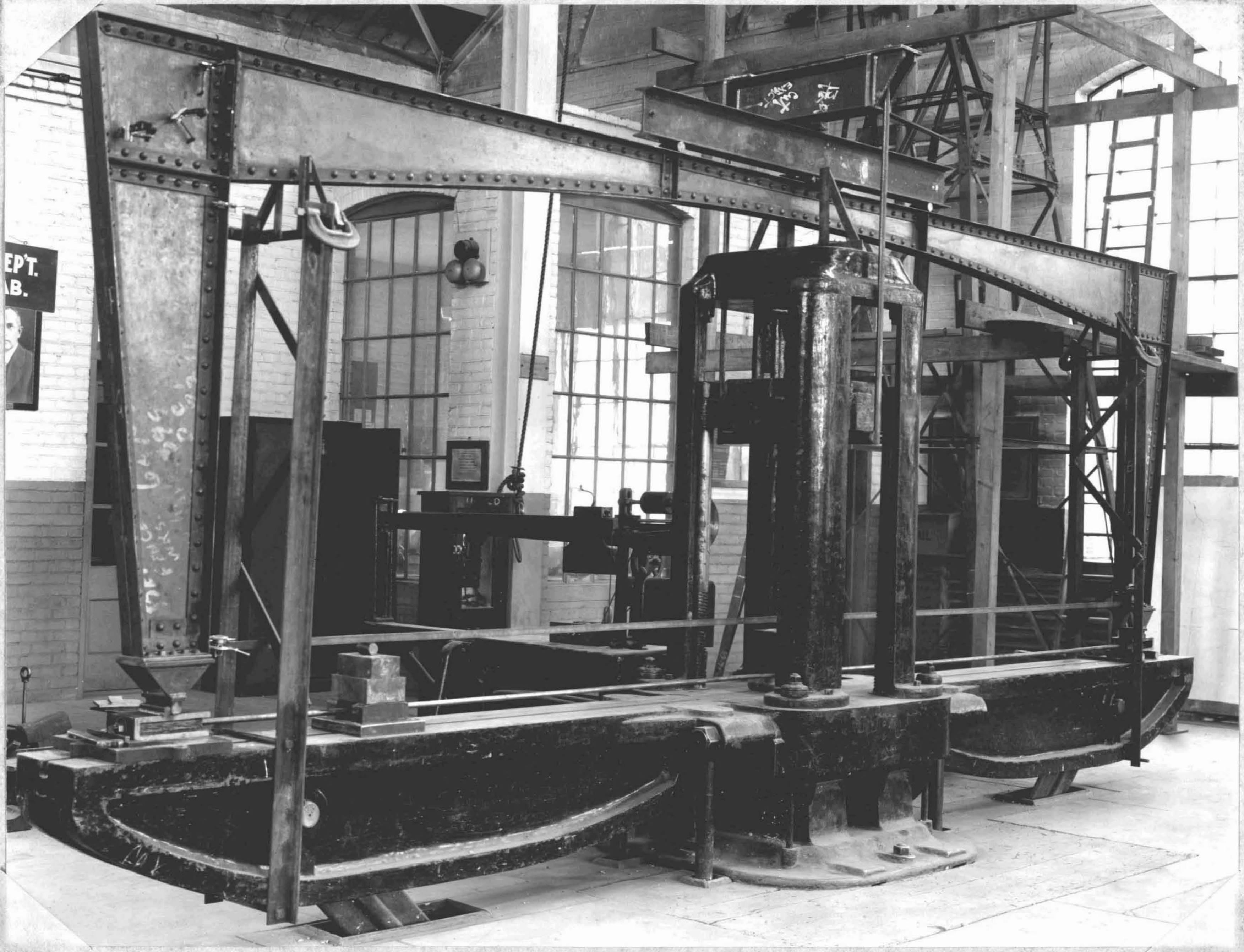


Fig. 4 - Square Knee Frame In Testing Machine

out

out

X

17

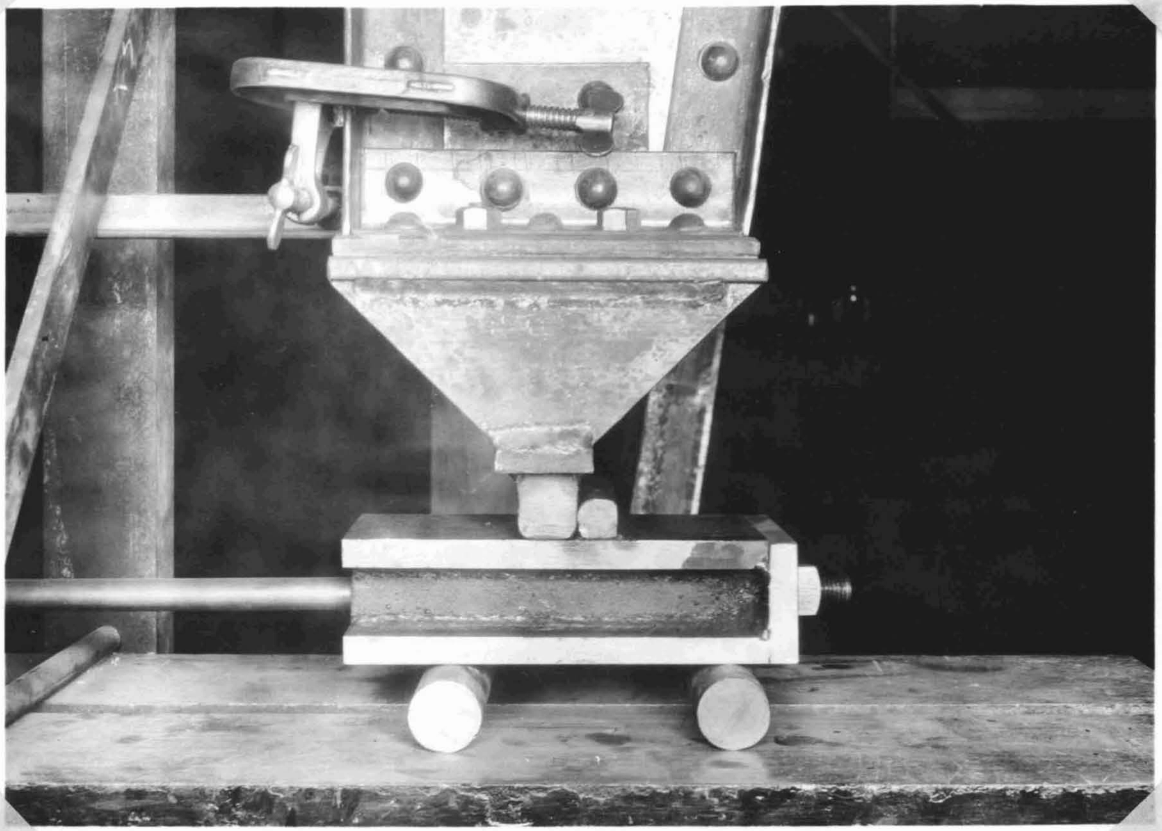
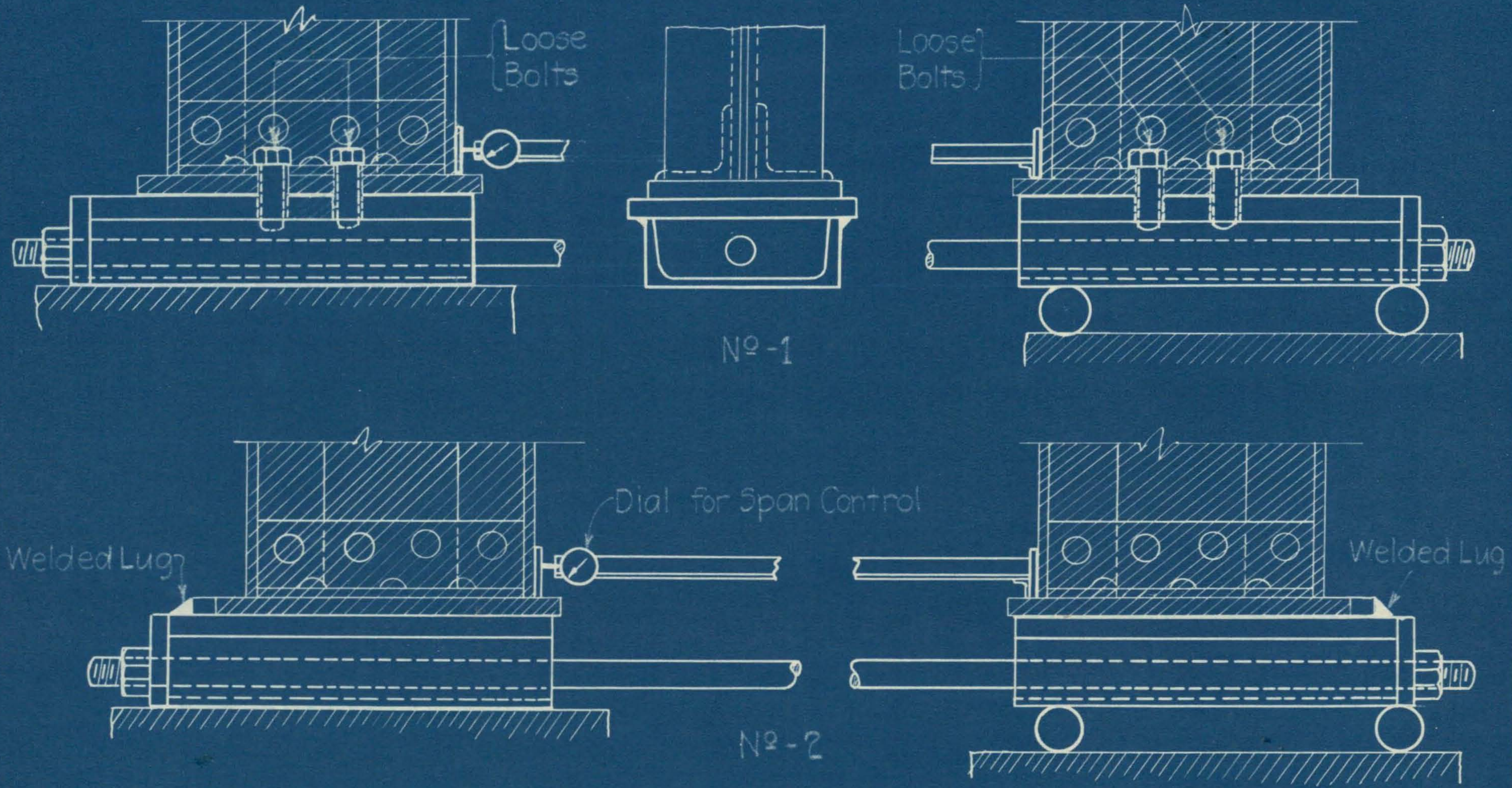


Fig. 5 - Moveable Hinged Base



Note-Frame proper indicated by lightly shaded lines

FIG. 6- FLAT BASE DETAILS

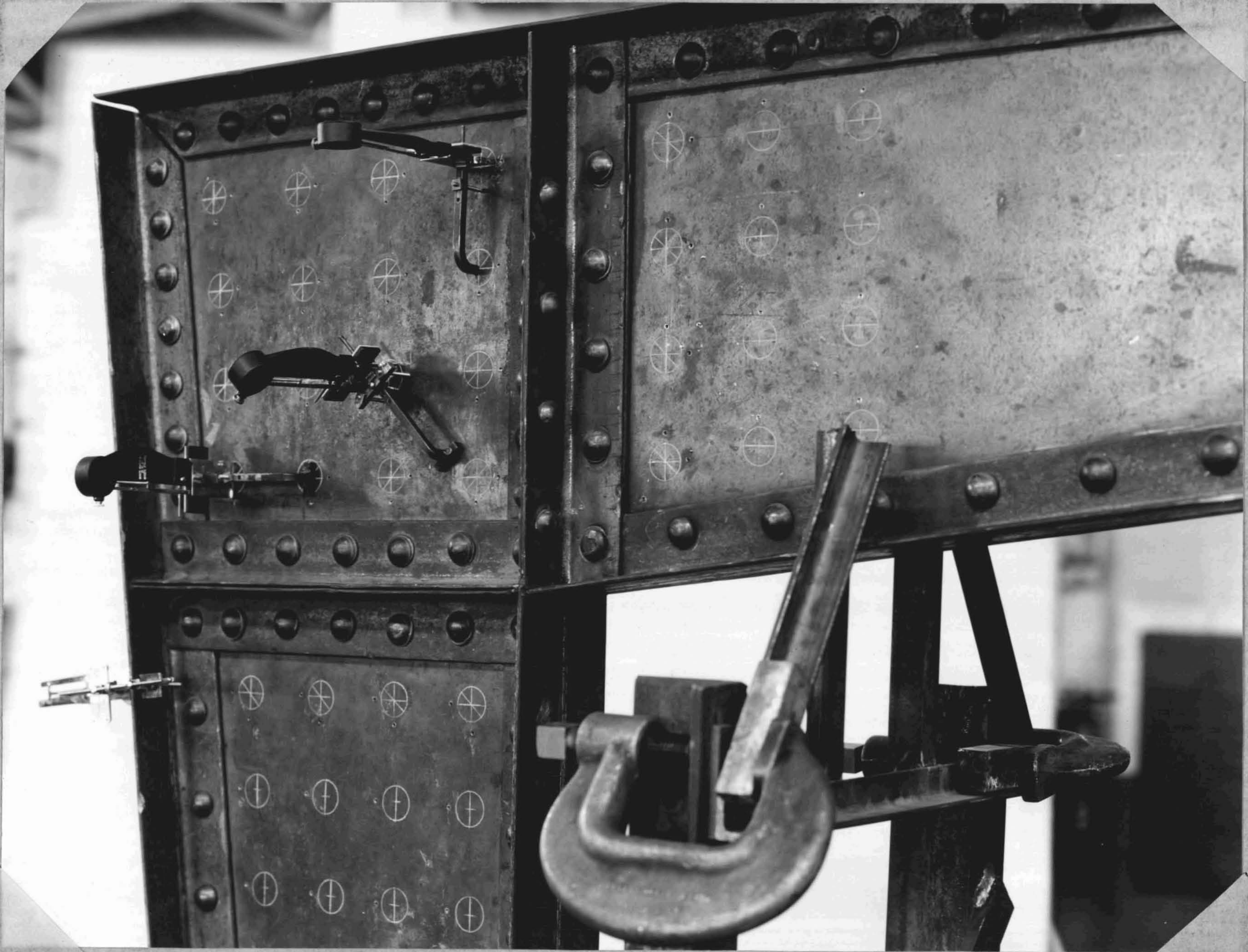


Fig. 7 - Huggenberger Tensometers on Square Knee

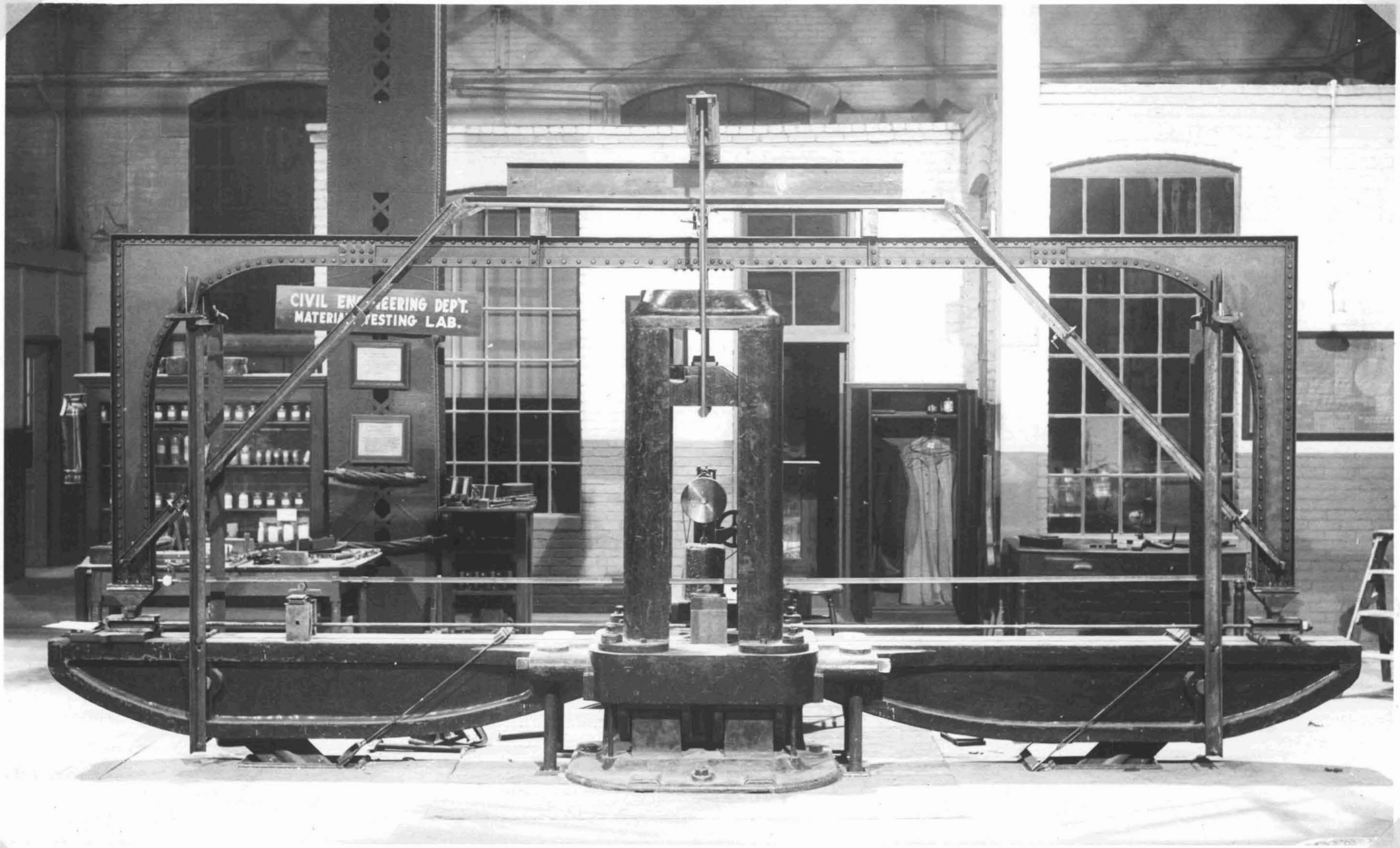
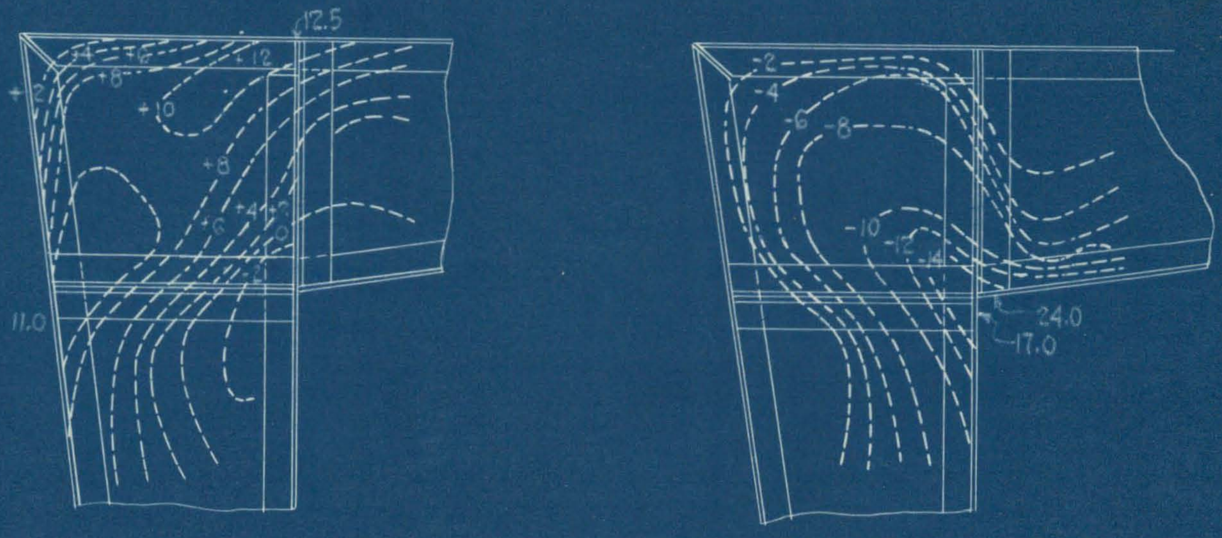


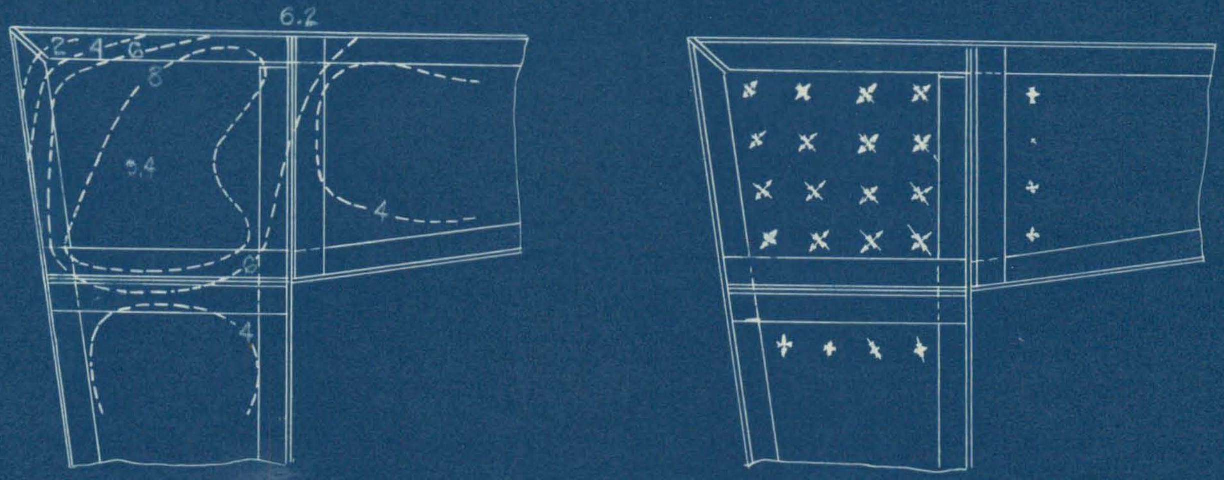
Fig. 8 - Curved Knee Frame and Deflection Apparatus

4 only



CONTOURS OF MAXIMUM STRESS

CONTOURS OF MINIMUM STRESS

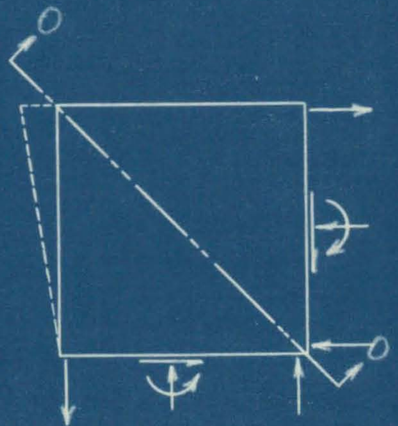


CONTOURS OF MAXIMUM SHEAR

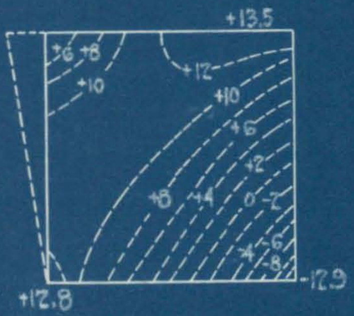
DIRECTION OF PRINCIPAL STRESSES

STRESSES IN 1000 P.S.I.

FIG. 9 - OBSERVED PRINCIPAL STRESSES IN RECTANGULAR KNEE

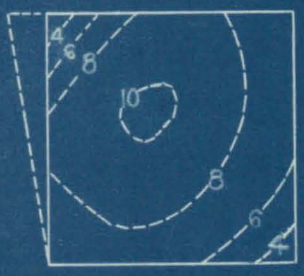


ASSUMED LOADING OF PLATE



CONTOURS OF MAXIMUM STRESS

CONTOURS OF MINIMUM STRESS

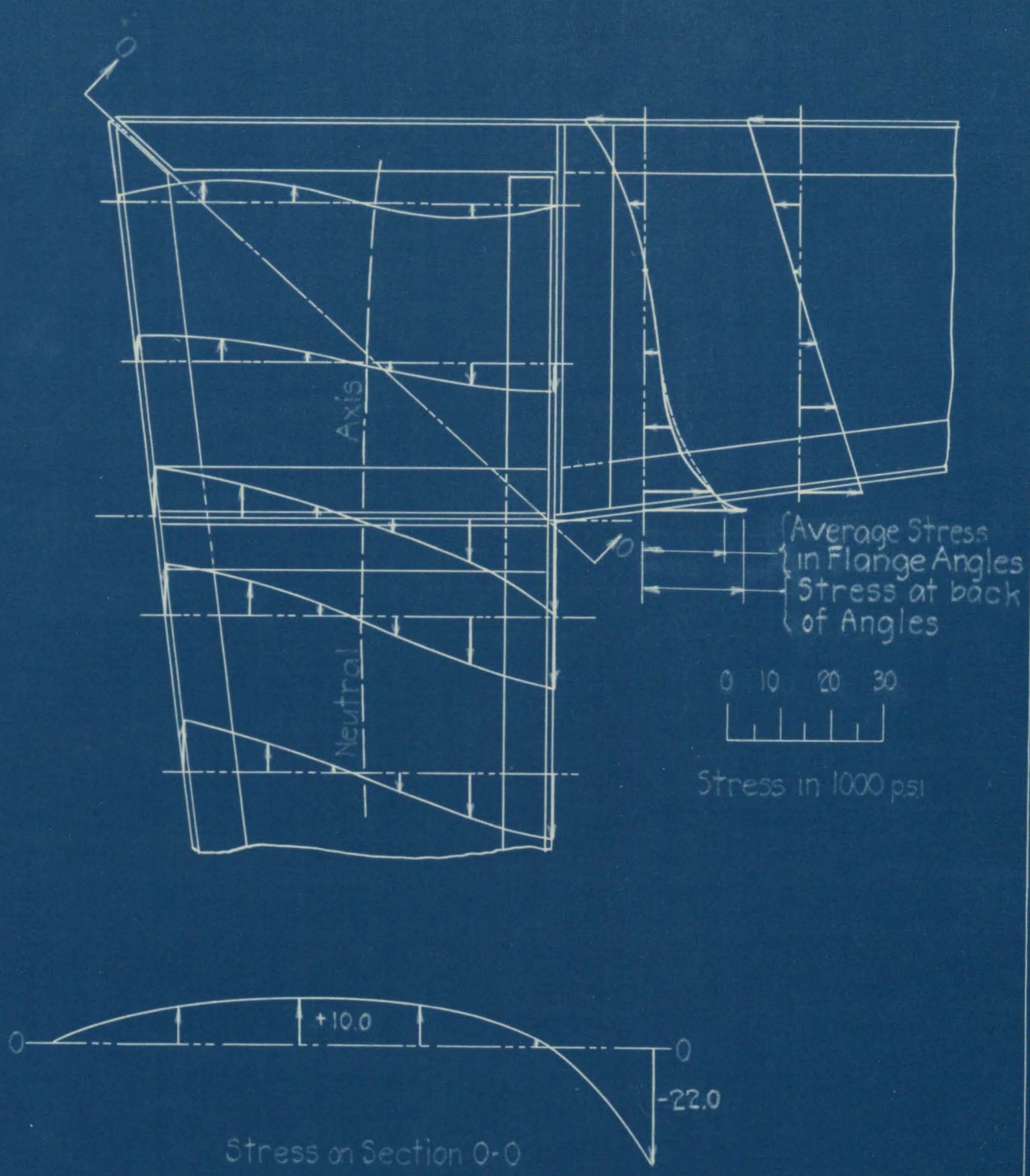


CONTOURS OF MAXIMUM SHEARS

NORMAL STRESS ON 45° PLANE

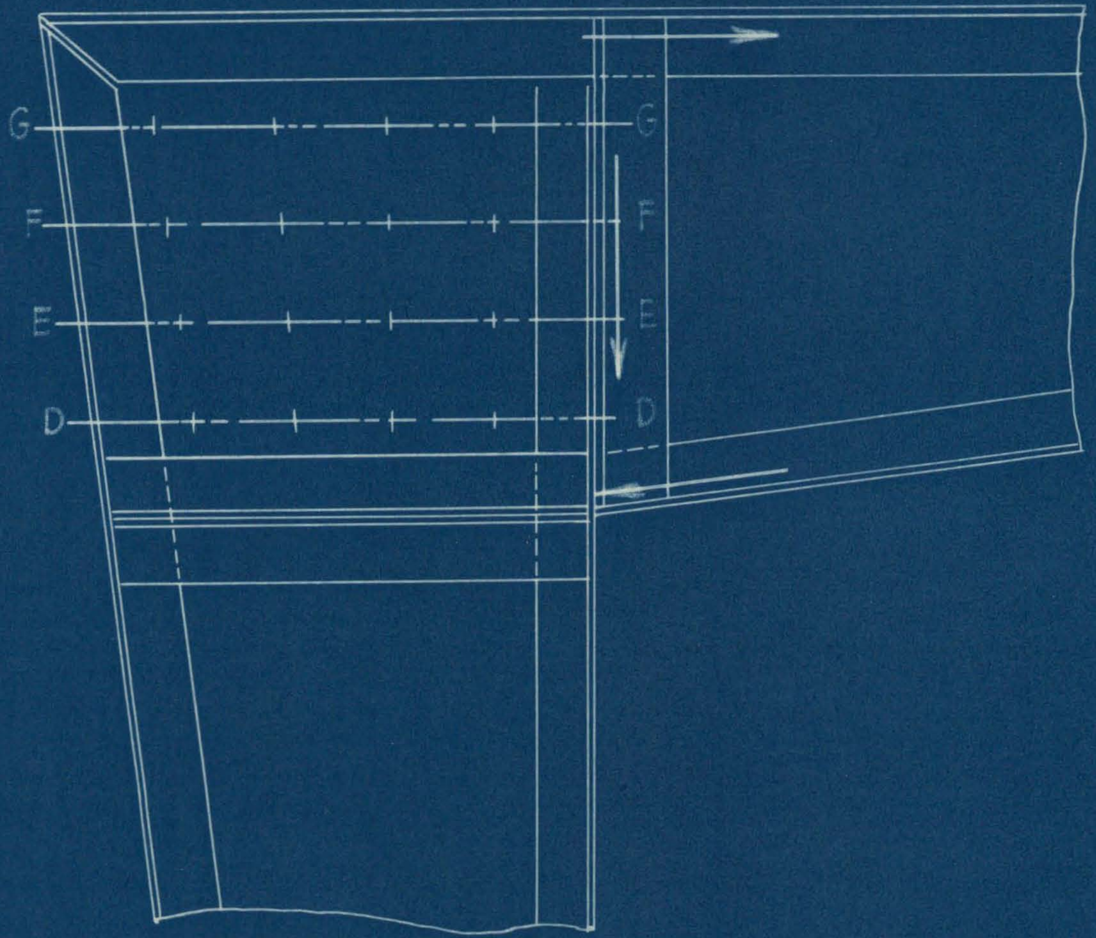
STRESSES IN 1000 P.S.I.

FIG. 10 - ANALYSIS OF RECTANGULAR KNEE BY THEORY OF ELASTICITY



Stress on Section 0-0

FIG. 11-NORMAL STRESSES ON ARBITRARY SECTIONS



SECTION	AVERAGE HORIZONTAL SHEARS	
	CALCULATED ON BASIS OF INDICATED LOADING	OBSERVED
D	8,240 P.S.I.	8,630 P.S.I
E	8,040 " "	8,120 "
F	7,860 " "	7,990 "
G	7,670 " "	7,580 "

FIG. 12 - SHEARING STRESSES IN SQUARE KNEE

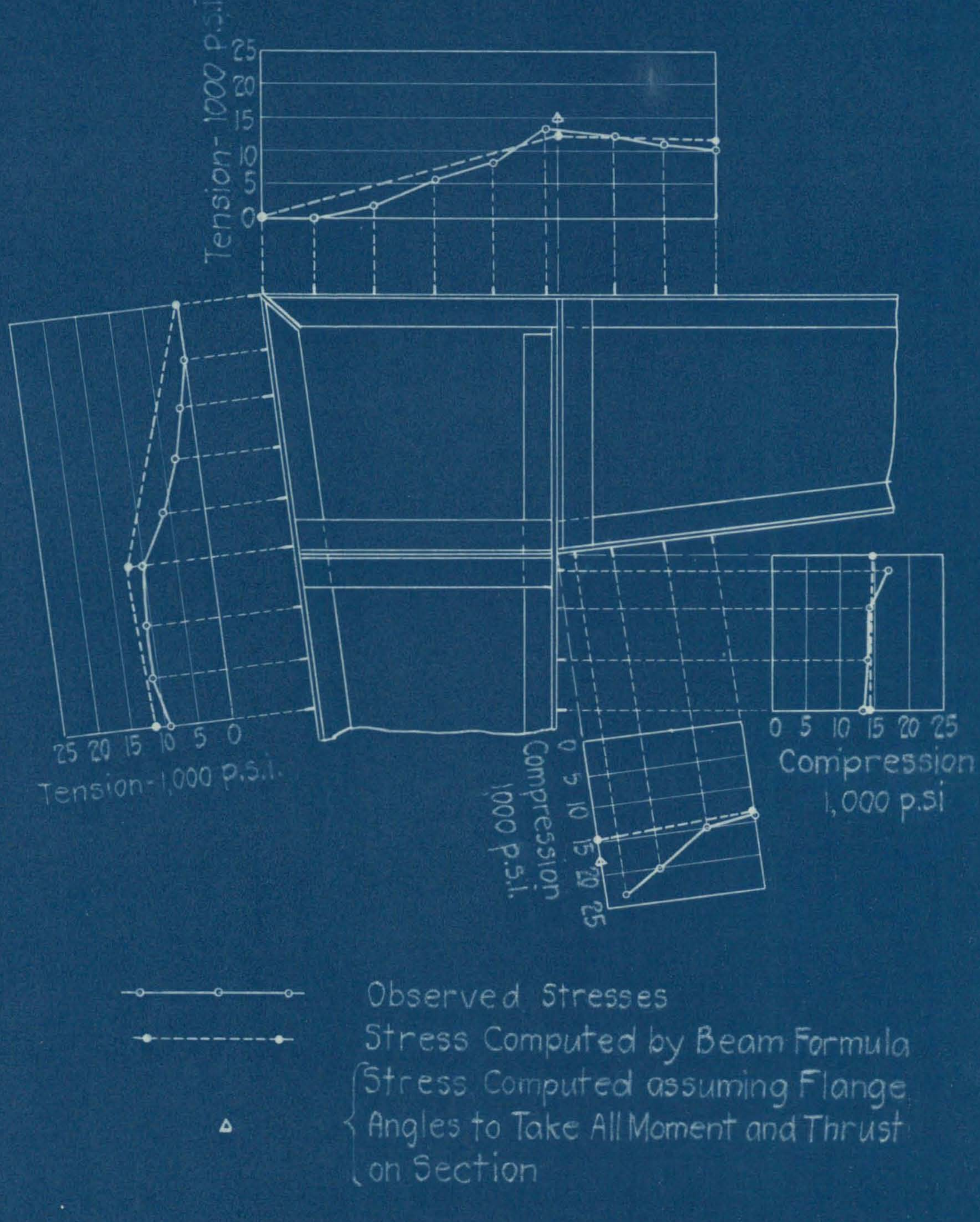


FIG. 13 - COMPARISON OF OBSERVED AND COMPUTED FLANGE STRESS

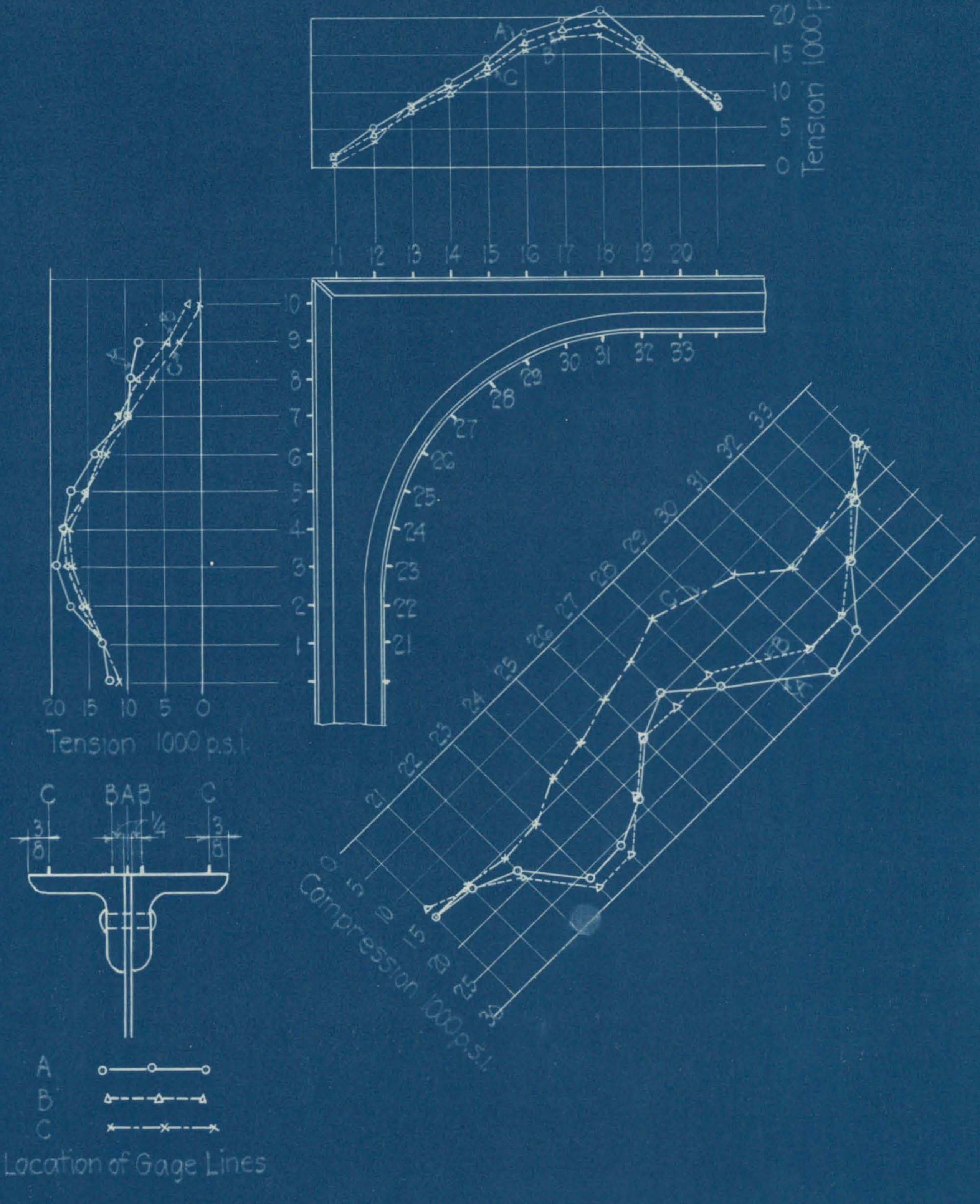


FIG. 14 - FLANGE STRESSES ON VARIOUS GAGE LINES

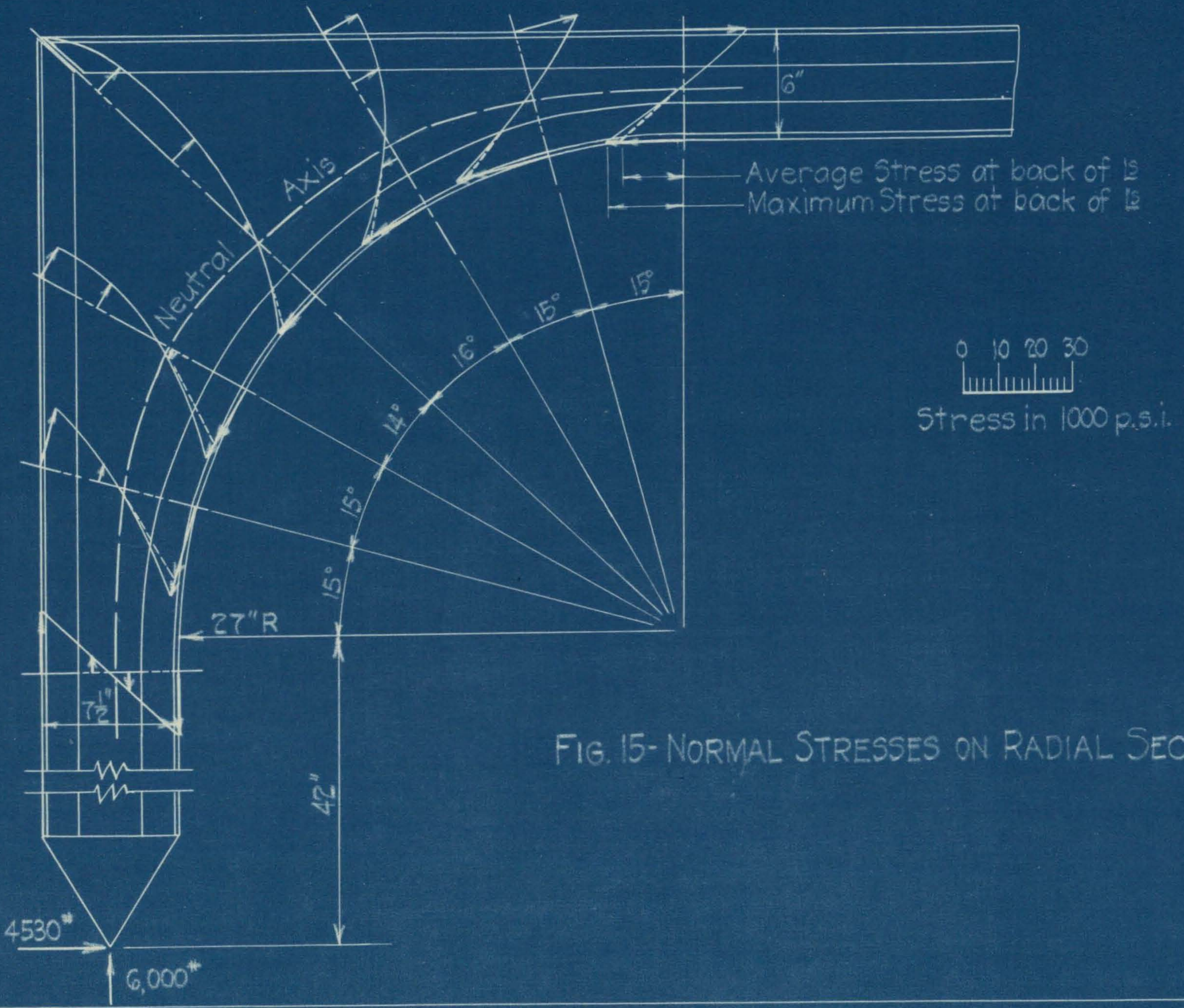


FIG. 15- NORMAL STRESSES ON RADIAL SECTIONS

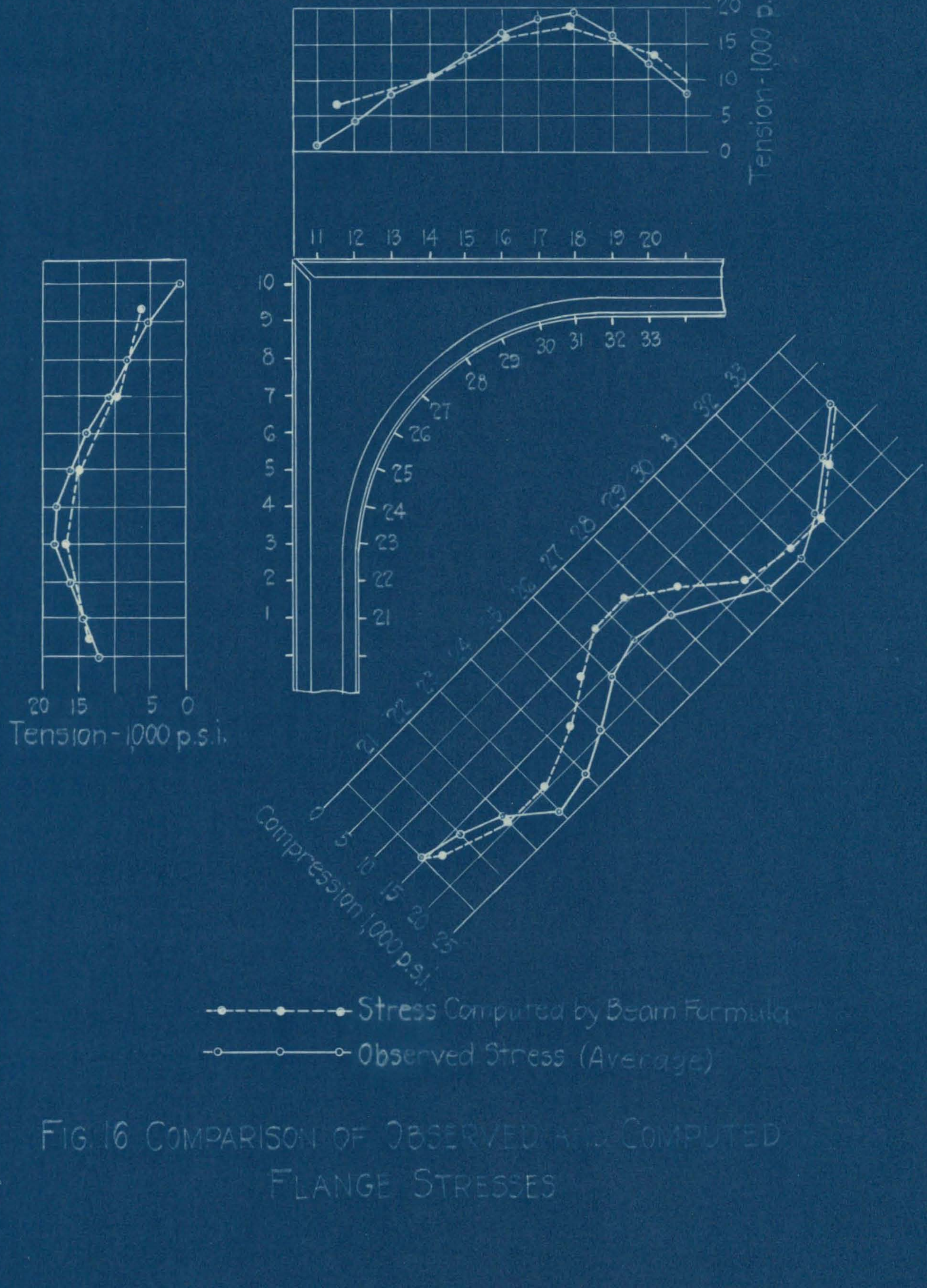
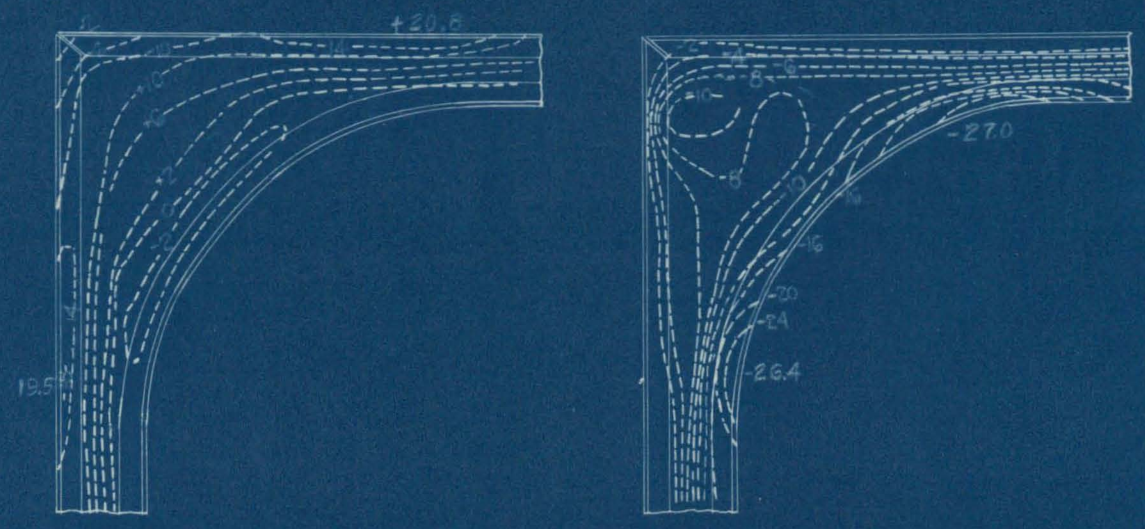
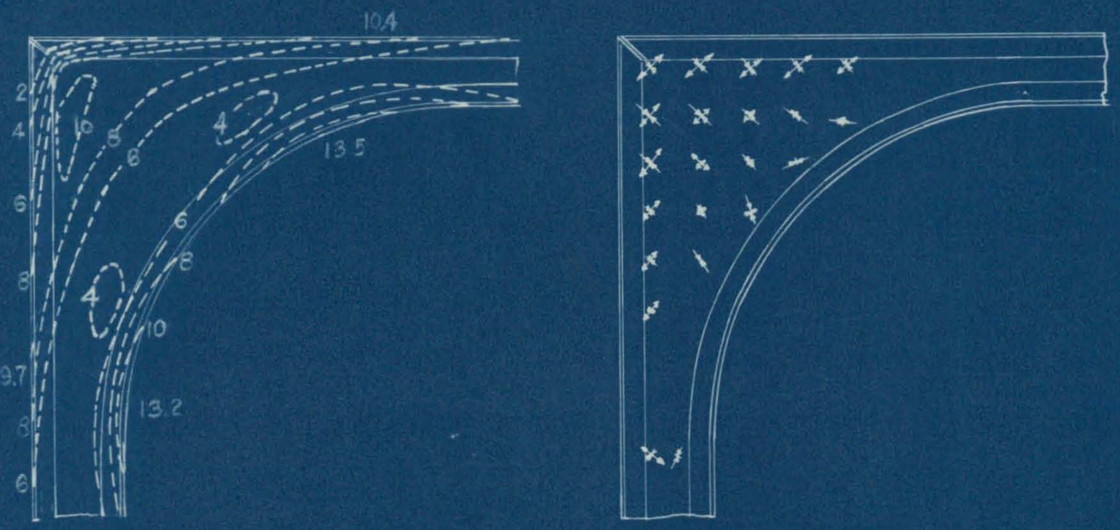


FIG. 16 COMPARISON OF OBSERVED AND COMPUTED FLANGE STRESSES



Maximum Stresses

Minimum Stresses

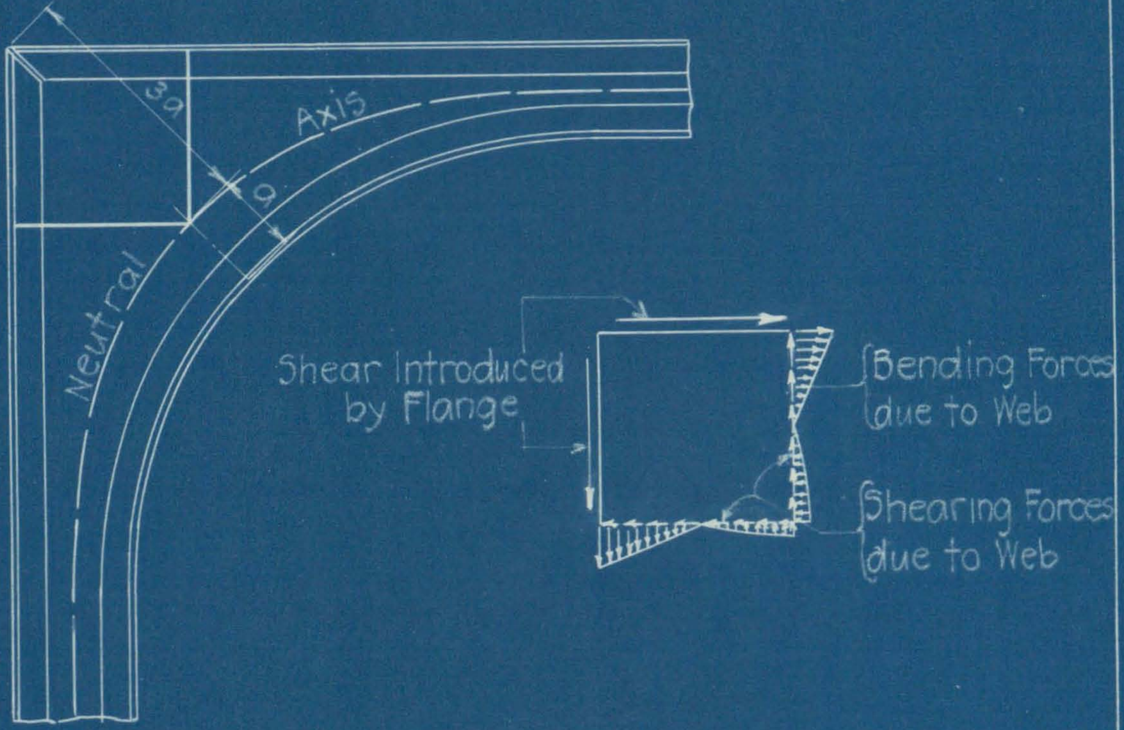


Maximum Shears

Direction of Principal Stresses

Stresses in 1000 p.s.i.

Fig. 17- Principal Stresses in Curved Knee



Maximum Observed Shear - 11,100 p.s.i.
Average Observed Shear - 8,300 "
Average Computed Shear - 10,500 "

FIG. 18-SHEARING STRESSES IN CURVED KNEE

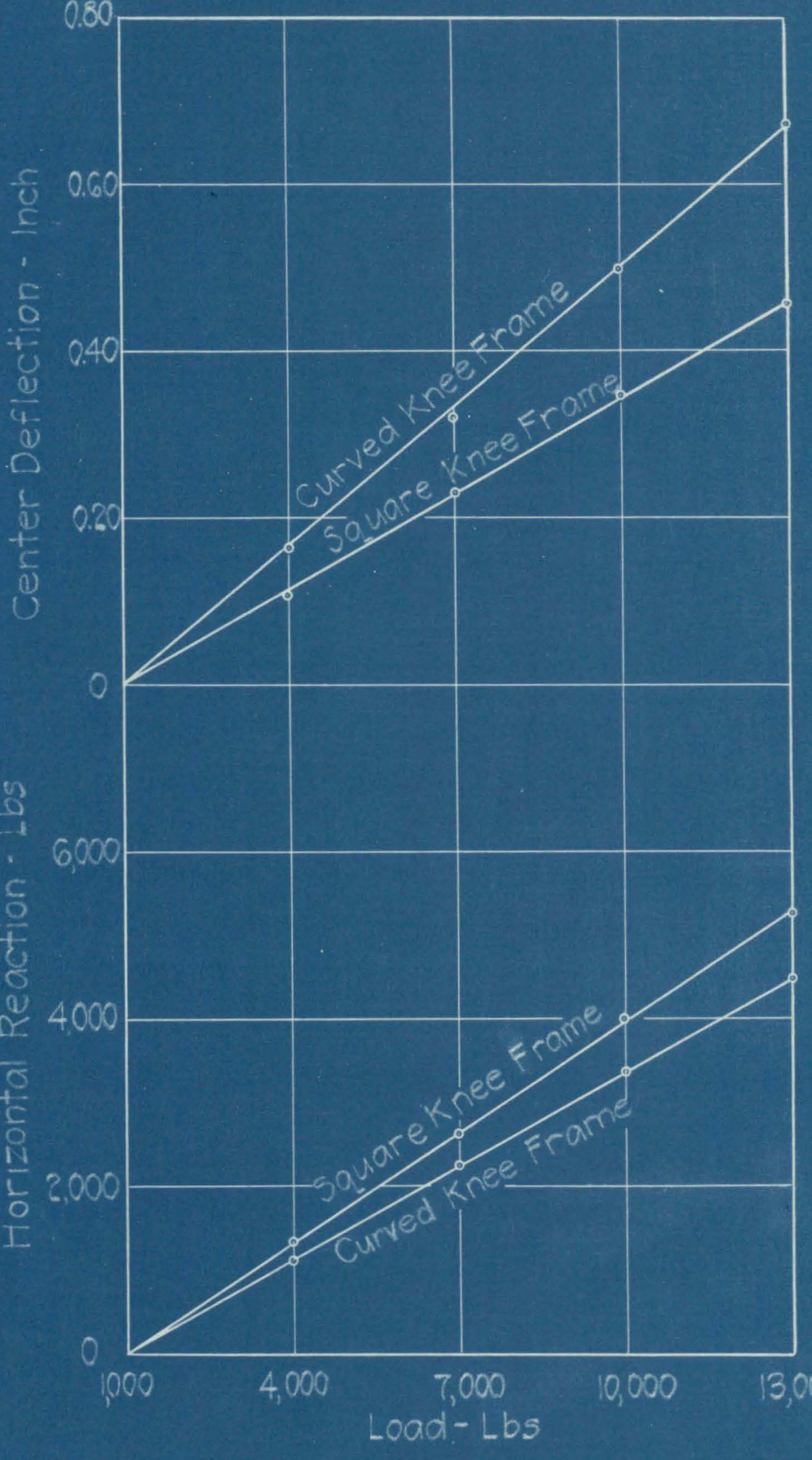


FIG. 19 - OBSERVED LOAD-DEFLECTION AND LOAD-REACTION CURVES FOR NORMAL SPAN

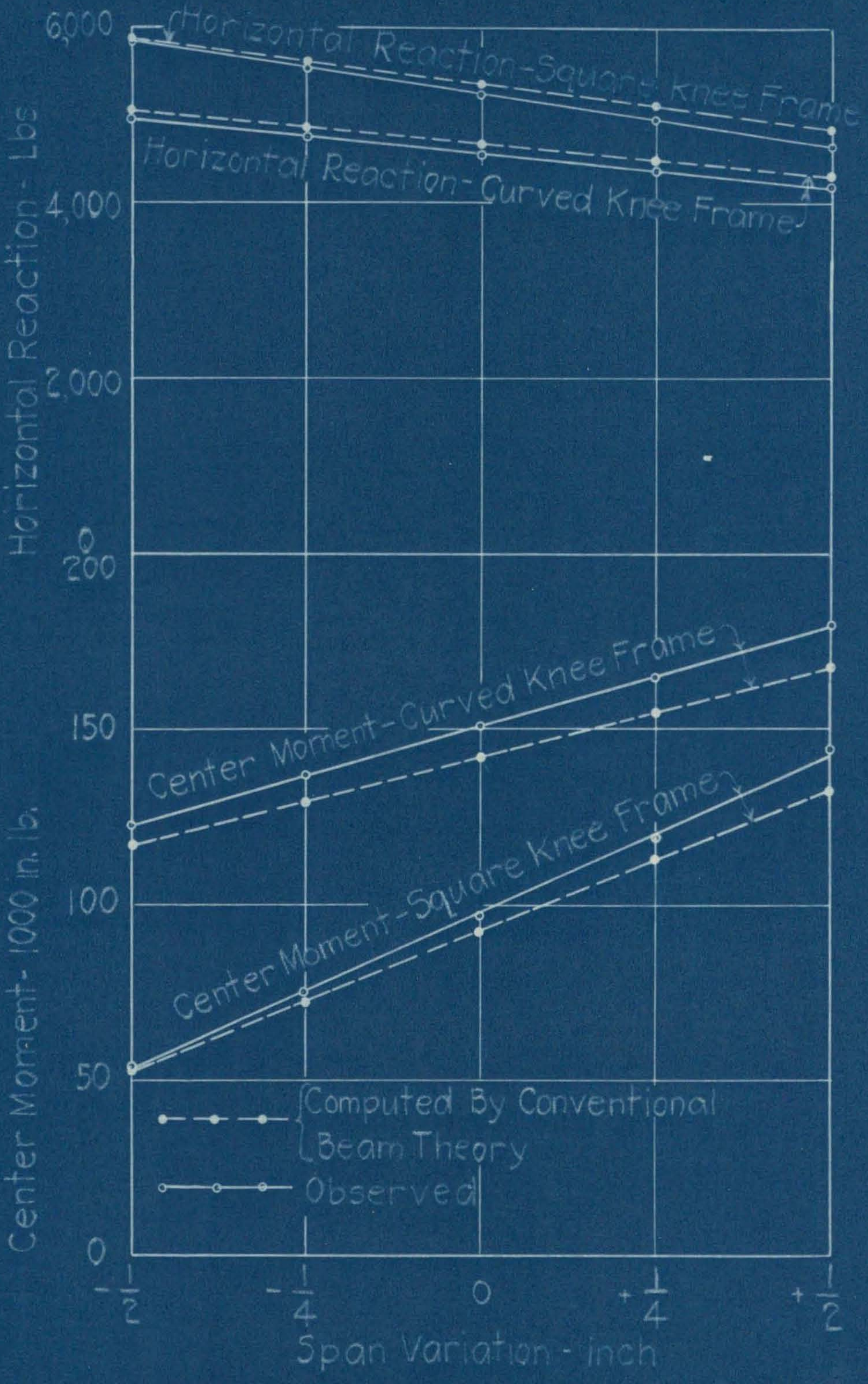


FIG-20 COMPARISON OF OBSERVED AND COMPUTED HORIZONTAL REACTIONS AND CENTER MOMENTS

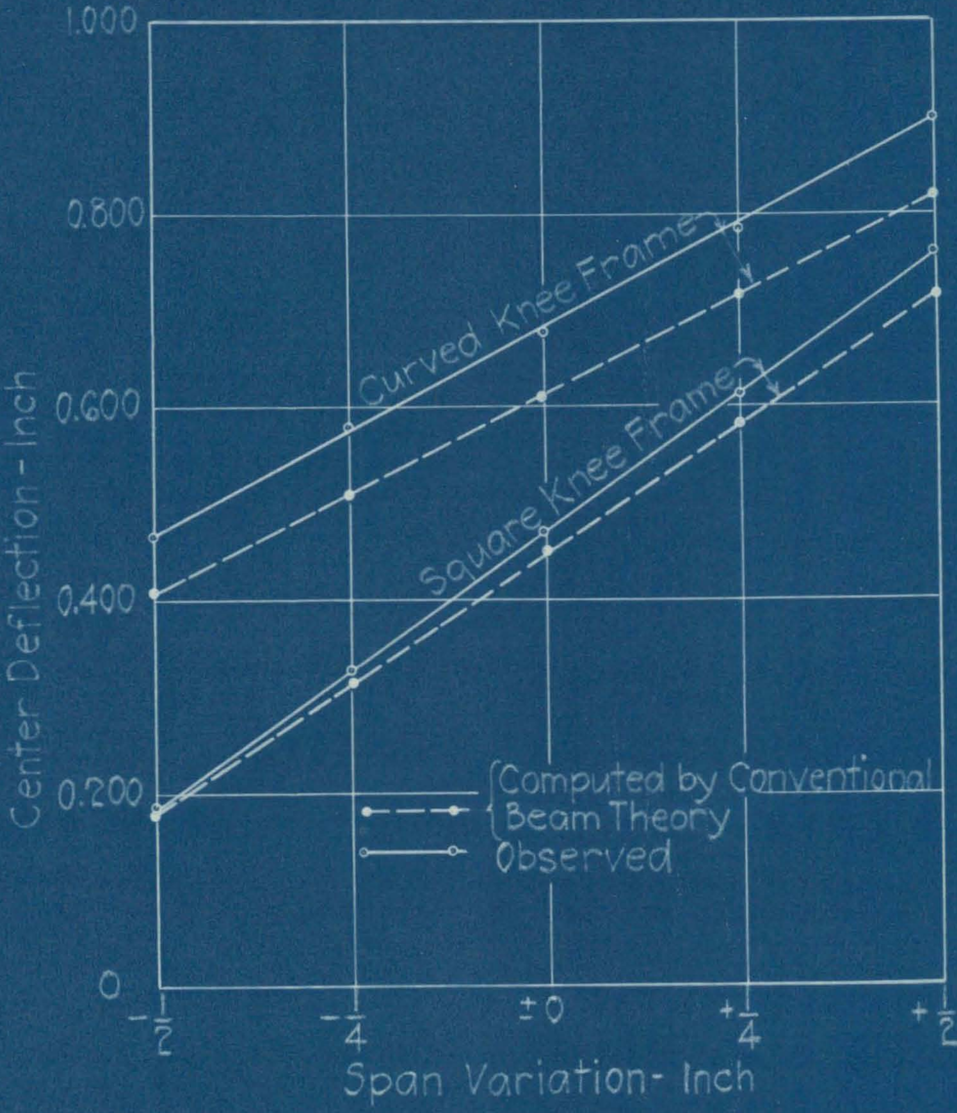
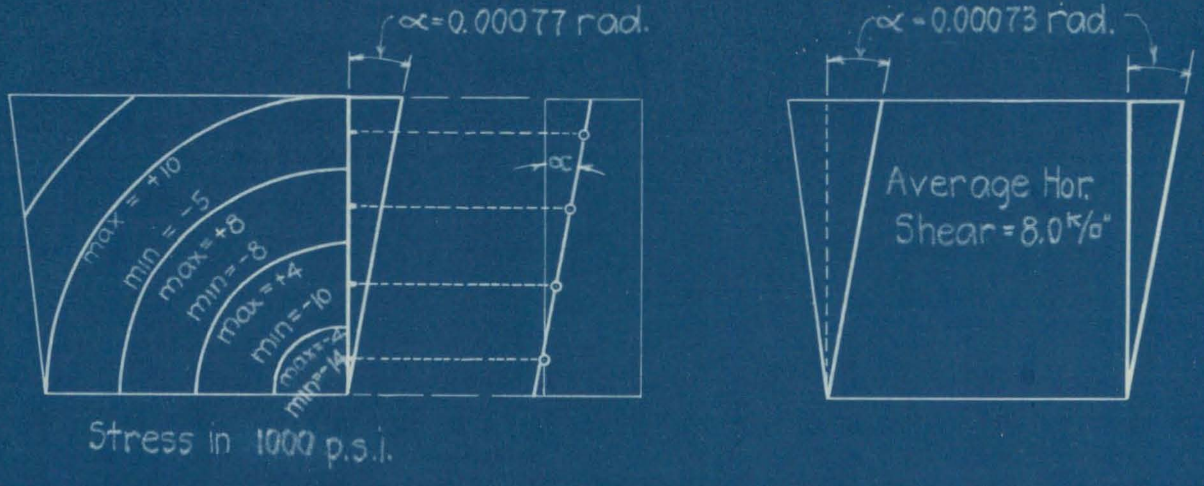
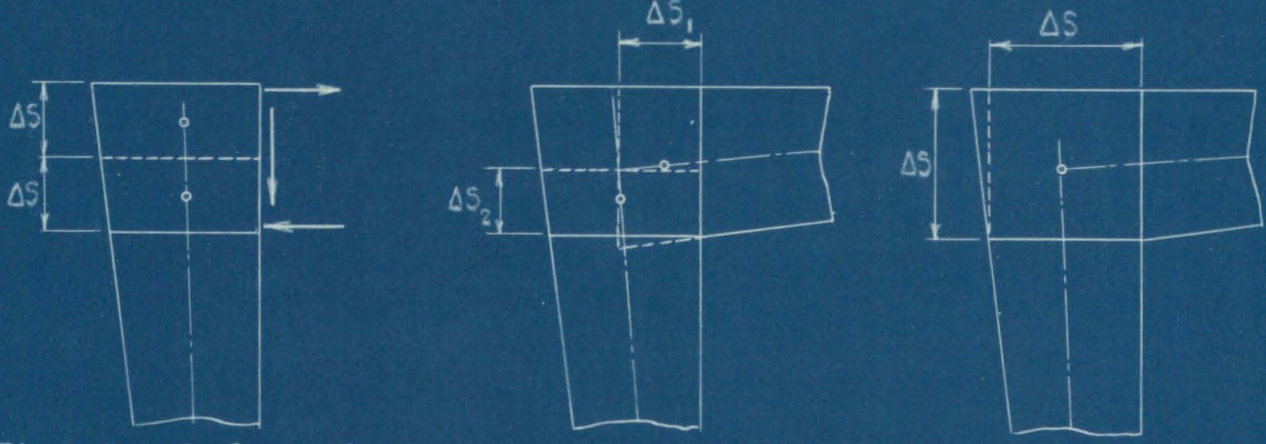


FIG.-21 COMPARISON OF OBSERVED AND COMPUTED DEFLECTIONS



FROM OBSERVED PRINCIPAL STRESSES
(a)

FROM OBSERVED HORIZONTAL SHEARS
(b)



Shearing Deformation
$\Sigma \frac{V\Delta s}{A'G} = 0.000.72$
(c)

Bending Deformation
$\Sigma \frac{M\Delta s}{EI} = 0.0018$
(d)

Bending Deformation
$\frac{M\Delta s}{EI} = 0.0017$
(e)

FIG. 22- DISTORTION OF SQUARE KNEE
AND
POSSIBLE CHOICES OF SECTIONS FOR
MECHANICAL INTEGRATION

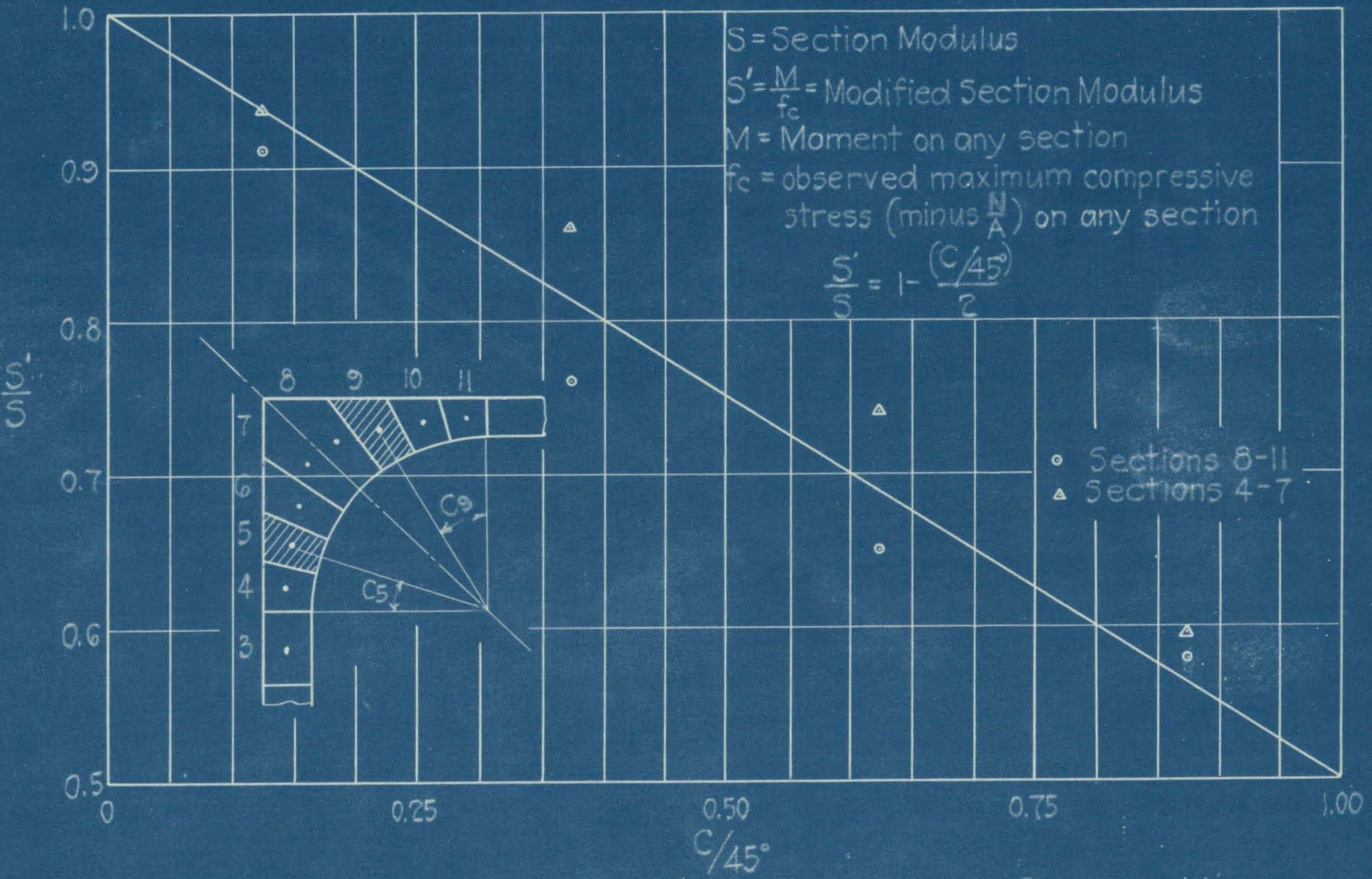


FIG. 23- GRAPH FOR DETERMINATION OF MODIFIED SECTION MODULUS

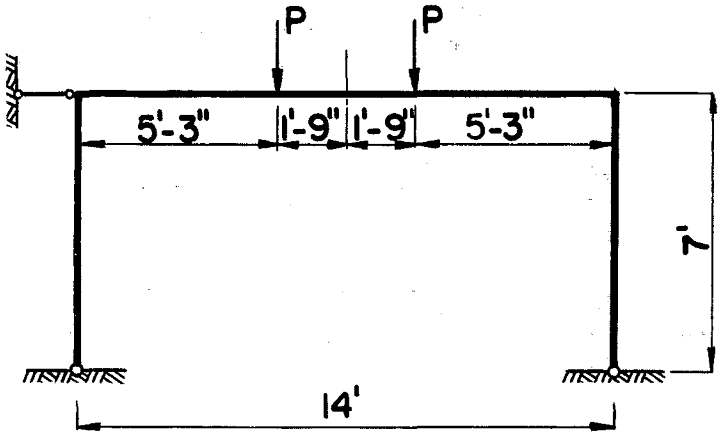


FIG. 1. TYPE OF FRAME AND LOADING

FIG. 2. OVER-ALL VIEW OF TEST APPARATUS WITH FRAME I TESTED TO COLLAPSE.