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Literature survey on: composite beams, plates, girders, connections, beam columns, CE 406, June 28, 1961

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List

It contains a Literature Survey on:

Composite Beams

Plate Girders

Connections

Beam Columns

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COMPOSITE BEAMS

The most common floor system for highway bridges and buildings consists of a reinforced concrete slab supported on steel beams. When the concrete slab is bonded to the top flanges of the beams by natural bond or by using connections to develop horizontal shear between the slab and the steel beam, a portion of the slab is forced to act with the beam, so that a steel-and-concrete T-beam results. This is called composite construction.

The advantages of composite construction, relative to non-composite construction are:

1. Stronger and stiffer
2. Savings in steel
3. Decrease in beam depth
4. Economical use of rolled sections for longer spans
5. Higher overload capacity and toughness.

These advantages of composite construction become especially pronounced if the non-composite design is governed by deflection limitations.

Theory and tests have shown that the transverse capacity of the floor slab is not impaired by the compressive stresses resulting from its action as a part of the beam.

In evaluating the savings in weight of beams, two types of construction must be considered: shored and unshored construction.

For highway bridges, the usual case is to use unshored construction where the formwork of the concrete slab is supported on the steel I-beams, so that the weight of the concrete (which is an appreciable item) is carried by the I-beam alone.

For buildings, the usual case is to use shored construction, so that the weight of the concrete is carried by composite action.

Since the section modulus of the composite beam is larger than that of the I-beam alone, larger savings in weight of steel may be realized by using temporary shores. However, shoring does not produce a cheaper structure unless the saving in cost of steel exceeds the cost of providing the shores. Shoring is a delicate operation, especially if settlement of the temporary supports is difficult to prevent, which is usually the case in bridge construction. Furthermore, this economy in materials results in a smaller overload capacity.

The strength and the stiffness of a composite beam are appreciably greater than the corresponding properties of the steel beam acting alone. The increase in strength resulting from the use of composite construction permits the use of a lighter beam section than would otherwise be required, and, in many instances, permits the use of a shallow beam. Moreover, even with a lighter steel section, the stiffness of the composite beam will be from two to three times as great as the stiffness of the original non-composite beam. In most cases, the increased stiffness is effective only for live load, since bridges of this type are commonly constructed without shoring beneath the beams, and dead load is carried by the steel beam acting alone. Since the size of the steel beam may be decreased in composite construction, there is likely to be some increase in dead load deflections for bridges of this type. However, since dead-load deflection can easily be provided for by cambering the beams, this increase is probably not significant. Of greater importance is the considerable decrease in live-load deflection resulting from the increased stiffness of the composite beam. Although present knowledge concerning impact on highway bridges is too meager to permit any definite statements, it seems reasonable to believe that from the standpoint of impact and vibrations, the stiffer structure is to be desired.

SAVINGS IN WEIGHT WITH COMPOSITE CONSTRUCTION

Studies have been made of the savings in weight resulting from the use of composite construction, utilizing several different types of beams. Comparative designs have been made for simple-span I-beam bridges having span lengths of 30 ft. to 90 ft., and beam spacings of 5 ft. to 7 ft., and designed for both H-15 and H-20 loading.

The results of the various studies are summarized in Table 1, in which the relative weights of the steel beams are compared with the various types of structures considered. An amount equal to the shear connectors has been added to the weights of the composite beams.

TABLE 1 - RELATIVE WEIGHTS OF STEEL BEAMS
IN COMPOSITE I-BEAM BRIDGES
(Maximum Weight Per Foot)

Non Composite Rolled Beams	Composite Beams: Symmetrical Rolled Beams, Without Cover Plate		Composite Beams: Symmetrical Rolled Beams, With Cover Plate*		Composite Beams: Unsymmetrical Beams, Welded Beams	
	Unshored	Shored	Unshored	Shored	Unshored	Shored
	92	77	76	64	69	40 to 60
*100	100	84	83	70	74	45 to 70
		100	99	83	88	53 to 75

* Average weight per foot based on cover plate 0.6 of the length of the beam.

Because the slab acts as a very heavy cover plate, the neutral axis of a composite beam is raised some distance above mid-depth of the steel beam, and the stresses on the top flange of a symmetrical beam are appreciably smaller than those on the bottom flange. This condition suggests that greater reductions in weight might be obtained with the use of beams having larger bottom flanges than top flanges.

If the most economical depth of beam cannot be used because of clearance requirements, the saving in weight arising from the use of composite construction may be more than twice as great as those quoted, and the shallower depth need not be accompanied by increased flexibility of the bridge insofar as live loads are concerned.

Design Considerations

A composite beam consisting of a slab and beam tied together by suitable shear connectors may be analyzed and designed on the basis of the transformed section. This conclusion was obtained by the analyses of several tests, which indicated that the effect of slip on the distribution of strain is a relatively localized effect, confined to the region extending a short distance on either side of the point of application of a concentrated load.

The effective width of slab to be assumed as the flange of the beam (except edge beams) must not exceed any of the following:

For Highway Bridges:

1. One-fourth of the span of the beam
2. The distance center-to-center of beams
3. Twelve times the least thickness of the slab

Edge Beams:

The flange of an edge beam is divided by the steel beam into two parts - inside and outside.

1. The effective width of either part must not exceed one-twelfth of the beam
2. Nor six times the slab thickness
3. The effective width of the inside part must not exceed one-half the distance center-to-center of the beams
4. The effective width of the outside part must not exceed the actual width. ([illegible]).

For Buildings: (T-Beams)

1. One-quarter the span of the beam
2. The distance center-to-center of beams
3. Sixteen times the least thickness of the slab

If the steel beam is fully encased, the effective width may be taken as sixteen times the least slab thickness, plus the stem width.

Ratio of Modulus of Elasticity for Steel to That of Concrete (n) to be Used in the Calculation of Stresses

Live Loads

Live loads are always carried by composite action, they are usually of short duration, no creep in concrete takes place, thus a modular ratio of n = 10 should be used to evaluate the properties of the composite section.

For live load of longer duration, such as in warehouses and from storage tanks, a multiplier of n should be used: k= 3. (Thus n = 30).

Dead Loads

All dead loads placed after the concrete slab has attained at least 75 per cent of its 28-day strength, may be assumed to be carried by the composite structure. Dead loads are permanent loads, causing the concrete to creep. This effect on the steel stresses may be accounted for by using a modulus ratio of n = 30, approximately.

Shrinkage

The shrinkage stresses in the slabs of simple beams and in the positive-moment regions of continuous beams are counteracted by the dead-and live-load stresses. In the negative-moment regions of continuous beams, the shrinkage stresses in the slab are unimportant, since the slab is considered ineffective in resisting tensile stresses.

Accordingly, the effect of shrinkage on the slab stresses may be ordinarily neglected in design.

Calculations for typical composite beams have shown that the shrinkage stresses in the bottom flange are always substantially less than the AASHO allowable 25 per cent overstress for group loadings. The maximum top flange stress in unsymmetrical steel sections may, in some exceptional cases, exceed slightly the allowable 25 per cent overstress.

Vibration and Impact

For structures subjected to vibration or impact, such as the effect of machinery or moving loads, the live loads should be increased to account for the dynamic effects.

Slab

To take advantage of the increased strength of the composite structure, the concrete slab must be adequately reinforced in longitudinal direction to prevent serious openings due to shrinkage or temperature effects. Where the continuity of the slab is interrupted too greatly, the upper flange of the composite section is materially weakened. Consequently, it is recommended to use an amount of steel about 0.5 per cent of the effective concrete area.

The effect of composite action is to increase appreciably the strength of the concrete slab. There is a reduction in the stresses of the transverse reinforcement, possibly due to the increased torsional resistance of the composite I-beams. There is also an increase in the loads required to produce failure of the slab by punching.

The full thickness of the slab is considered to be effective, unless the neutral axis turns out to lie above the bottom surface, in which case the portion of the concrete that would be in tension is neglected.

Shear Connectors

Composite action of the slab with the beams may exist to a limited extent, or even almost completely, although no provision for such action is made in the design. This is due to bond between the concrete slab and steel beam.

In the design of buildings of light occupancy with no shock or vibration and not subjected to overloading or large changes of temperature, failure of bond is unlikely to occur if the shearing stresses on the contact surfaces are low. If, however, bond is destroyed at one location, progressive failure can be expected in time, and the result will be a complete, or at least extensive, spread of bond failure. On the basis of test information, it is recommended that the design bond stress be limited to 25 psi for unpainted hot-rolled steel surfaces. If bond exceeds this value at any location, the full horizontal shear throughout the beam should be assigned to mechanical shear connectors.

In the design of bridges, it is not possible to depend on composite action unless positive provision is made for adequate shear connectors, since temperature changes, shrinkage, vibrations, and the effect of wheel loads may sooner or later destroy the adhesive bond between the slab and the beams.

Therefore, it must be assumed in the design computations, that all shears caused by forces acting on the composite structure are transmitted by the shear connectors.

Two requirements must be met by the shear connectors. First, they must effectively prevent slip between the slab and the beam; and, second, they must be strong enough to withstand the shearing forces with an adequate factor of safety. A third requirement which is desirable, although not necessarily essential, is that they be able to resist uplift forces tending to pull the slab vertically away from the beam. This tendency of separation may be the result of loading, or it may be caused by warping of the slab, resulting from shrinkage, unequal expansion due to radiant heat, or other effects.

Shear connectors in many bridges are subjected to repeated loading under conditions favorable to failure in fatigue. At the center of the span a complete reversal of stress occurs for moving loads. At the ends of the span, the stress varies from zero to a maximum with each passage of load. This large range of stress, plus the possible presence of stress raisers at or near welds or fillets, makes it desirable to consider the possibility of failure under repeated loading.

Tests of channel shear connectors subjected to repeated loading, with a load cycle diagram with a maximum of 2,000,000 cycles, showed that the final failure of the beam was by fracture of the shear connectors at the junction of the web and bottom flangs of the channel. There is doubt whether this fracture was always the primary cause of failure. If the concrete adjacent to the connector fails by crushing, there will be a redistribution of pressure on the connector in such a manner as to raise the center of pressure, and consequently to increase the moment on the channel. This increased moment, in turn, may produce a fatigue failure of the connector. Thus consideration must also be given to the stresses produced in the concrete slab in bearing against the shear connectors, for static or repeated loads. The effect of variations in the compressive strength of the concrete and static load, increases considerably the effectiveness of the shear connectors. This increase, however, is proportionately less for the higher strengths than for the lower ones.

For repeated loads, there is an increase in the ultimate load with concretes with higher compressive strengths. It is almost a linear function of the compressive strength between $f'_c = 1$ k/in^2 to $f'_c = 6$ k/in^2 with a gain of 50 per cent for $f'_c = 6$ k/in^2 over that for $f'_c = 1$ k/in^2.

The effect of creep is to relieve stresses in the concrete slab and thus to decrease the forces transmitted by the shear connectors. Creep is a time-dependant phenonenon; it progresses several months before the stress relief that it causes is fully materialized. Therefore, the effects of creep should be disregarded in the design of shear connectors.

Shrinkage takes place during the first few months after construction. In simple beams and in the positive-moment regions of continuous beams, the shrinkage loads on shear connectors act in a direction opposite the direction of maximum horizontal shear caused by the dead and live loads. In the negative-moment regions of continuous beams the concrete slab is stressed in tension, and in the design, considered ineffective; thus a possible overstress of the shear connectors in these regions is relatively unimportant.

In exceptional cases, expansion of concrete and differential temperature changes are examples of other effects which have to be considered.

The load to be carried by the shear connectors may be computed by the ordinary formulas of elementary mechanics from the shear on the composite T-beam, that is: $v = \frac{VQ}{I_{comp}}$ (unit horizontal shear stress).

<u>Load at One Shear Connector</u>

$$Q = \frac{vp}{N}$$

p = spacing of connectors in the direction of the beam axis
N = number of connectors at one transverse beam cross section
v = horizontal shear stress

<u>Design Load for One Shear Connector</u>

$$Q = \frac{Q_{uc}}{SF}$$

Q_{uc} = ultimate capacity of one shear connector

SF = safety factor: for highway bridges, an upper limit of FS = 4.0 is recommended; for buildings, a FS of 2.4 is recommended when shored construction is used

<u>Ultimate Capacity of Shear Connectors</u>

<u>Stud Shear Connectors:</u>

for h_s/d_s 4.2 $\quad Q_{uc} = 330\ d_s^2 \sqrt{f'_c}$ lb

for h_s/d_s 4.2 $\quad Q_{uc} = 80\ h_s d_s \sqrt{f'_c}$ lb

d_s = stud diameter (in.); h_s = stud height (in.)

(psi) f'_c = 28-day compressive strength of 6 by 12-in. concrete cylinder

Channel Shear Connectors:

$$Q_{uc} = 180\,(h + 0.5t)\,w\,\sqrt{f'_c}$$

One turn of spiral shear connector

$$Q_{uc} = 3840\,dsp\,\sqrt[4]{f'_c}$$

where h = maximum thickness of channel flange (in.)
t = thickness of channel web (in.)
w = width of channel connector (in.)
dsp = diameter of the spiral bar (in.)

Detailing of Shear Connectors

1. The spacing p of connectors should not be greater than 24 in., or two to three times the thickness of the slab.

2. The clear distance between the edge of the beam and the edge of the shear connectors should not be less than 1 in.

3. The clear depth of concrete cover over the top of the shear connector should not be less than 1 in.

4. The connectors should extend at least 2 in. above the bottom of the main body of the slab.

Design of Connecting Welds

For granular-flux-filled welded studs, no design of welds is required because the stud welding process always furnishes a weld having the minimum cross-sectional area equal to the cross section of the stud. Fatigue tests of bare studs have shown an excellent performance of both the weld and the stud under full reversal of loading.

The connecting welds for channel and spiral connectors should be designed in accordance with the AASHO requirements for the design of welds. The reversal of the live load stresses should be considered in the design of welds.

Behavior of Connectors

It is important to point out that the design of any shear connector must necessarily be based on experiment (experience). The action of the connectors is much too complicated to be accessible to an exact stress analysis. Even the loading of a connector is rather indeterminate, as a considerable amount of shear is transmitted by bond. In case the latter should be broken by slip, mechanical friction is still able to carry part of the shear.

Furthermore, the stress distribution in the connector itself is so highly complicated that any analysis must be regarded as an approximation.

Continuous Beams

There seems to be no particular advantage in providing for composite action in the region of negative beam moments in a continuous beam. Whether shear connectors are used or not, in the regions where the slab may be in tension at the upper surface, adequate reinforcing must be provided. In either case, the slab is subjected to deformation stresses rather than to load stresses.

Thus, from the standpoint of composite action, it makes little difference whether the shear connectors are provided or left out in the negative-moment regions. In this region, the curvature of the deflected structure is the reverse of that at midspan, so the slab tends to exert pressure on the I-beams instead of pulling away when loaded, thus the tie-down of the slab by the shear connectors is not necessary.

In the negative-moment regions of a composite, continuous I-beam bridge, the upper flange of the I-beam is subjected to tensile stresses. The presence of any stress raisers in this region might be detrimental from the stand point of repeated loading.

Thus it is preferable to provide shear connectors only in the positive-moment regions of continuous composite I-beam bridges.

Analysis and tests indicate that the beams can be designed for the same proportion of a wheel load as that proposed for use in connection with simple-span bridges, and that the slab in the regions of positive moment in the bridge may be similarly designed, using the expressions for moment recommended for use with simple-span bridges. In both cases, the span length of the simple-span bridge used in these calculations, should correspond to the portion of the continuous bridge between the points of contraflexure.

It is recommended that the amount of transverse reinforcement in the top of the slab should be increased in regions of negative moments.

It is possible to prestress the slab in the negative moment region so as to produce an initial compression of 80 kg/cm^2 to 100 kg/cm^2, or use a settlement of the intermediate support.

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PLATE GIRDERS

Plate girders are beams built up of steel plates and shapes, by riveting, welding, or bolting. Plate girders can be built to any desired proportion to suit the particular requirements. The saving in material due to this better proportioning, however, may be offset by the increase in cost of fabrication. Thus, speaking in general, for smaller beams where the saving in material is small compared with the increase in fabrication cost, rolled beams are cheaper. In heavier construction, where the available rolled beams are not sufficient to carry the load, plate girders have to be used.

For intermediate cases, say with section moduli between 500 and 1100 in^3, either rolled beams or plate girders, or rolled beams reinforced with plates may turn out to be most economical. For ordinary loading, rolled beams would be more economical for spans below 30 ft, and plate girders for spans above 70 ft.

Before the development of welded construction, plate girders were limited to spans not in excess of 150 ft. Since 1945, however, numerous plate girders spanning 200 to 300 ft have been built, and the girder bridge over the Save in Belgrade has a middle span of 850 ft and a depth over the support of 32 feet.

Considering the dimensions achieved it is wondered if there remains anything more to be known about the design and performance of such bridges. To answer this question, an investigation of plates girders is underway at Fritz Engineering Laboratory, Lehigh University, from whose experiments and reports these notes are taken. (References 23 & 24)

A literature survey in stability of plate girders was done by Konrad Basler and Bruno Thurlimann at Lehigh University. Comments are given on several references and finally in his conclusion he states:

"None of the discussed theories are able to predict the load carrying capacity of plate girders in general. The writers feel that no further effort should be made in checking the ordinary buckling theory by tests. To spend a major effort on an investigation on the web plate, using the large deflection theory, would disregard the essential problem of the stability of the framing made up by the flanges, and the horizontal and vertical stiffners. It appears that too much emphasis has been given in the past to the behavior of an isolated web of a plate girder. A study on the integral behavior emphasizing the stability of the framing is proposed".

For these reasons and as a literature survey was available, this report was focused in the study under work at Lehigh University.(23)(24)

Some references are listed on the transfer of stresses in welded cover plates. (41; 42)

INTRODUCTION

In the design of plate girders the tendency is to arrange as much material as possible in the extreme fibers. By keeping the web area as small as possible, the lever arm of the internal forces is maximized and with it the carrying capacity. It was assumed in the past that web buckling sets a clear limit to this tendency towards an optimum utilization of the material. Consequently, an enormous amount of effort has been spent in establishing web buckling values.

The conventional plate buckling theory predicts the load intensity under which a perfectly plane plate subjected to edge stresses deflects out of its plane. The formulation of the problem is the same as that for a column, and as a result the same word "buckling" is used to describe the phenonomen in a plate. Since the computed column buckling value gives an adequate measure of the strength of a column, it was natural to consider that a plate buckling value was of equal significance. Such an assumption is not true. The strength of a plate can go beyond this theoretical buckling limit, and this additional margin of strength is termed "post-buckling strength".

This has been pointed out ever since buckling values were computed, and as a consequence a somewhat smaller factor of safety was applied to web buckling than against primary column buckling. The experiments realized at Lehigh University demonstrated that the concept of expressing the post-buckling strength of a girder as a certain percentage of the web buckling strength is in error and should be replaced by a strength prediction which considers the influence of the flanges and transverse stiffners on the carrying capacity.

The assumptions made by the conventional theory as applied to the design of the web of plate girders may be summarized as follows:

1) The plate is assumed to be hinged at its boundaries

2) The plate is assumed to be perfectly plain.

3) It is assumed that a sudden lateral deflection will be produced when the buckling stress is reached, considering this stress as ultimate design stress.

4) If the edges were assumed to be built-in more than one wave would appear.

Experiments have shown that a perfectly plain plate is impossible to obtain; actually the plate contains initial inperfections due to welding sequences. Welding the stiffners first on one side introduces web deflections toward this side of the web; welding the stiffners on the other side may reduce the magnitude of the previously introduced deflections somewhat, but it still leaves a deflection pattern which is consistently unsymmetrical with respect to the plane of the web prior to welding.

Thus, stresses are introduced with unavoidable excentricities, such that lateral deflections occur from the very beginning of loading, and one wave appears no matter what the end conditions are. For this same reason there is no bifurcation of equilibrium at the critical load, and the plate will continue taking load as it deflects.

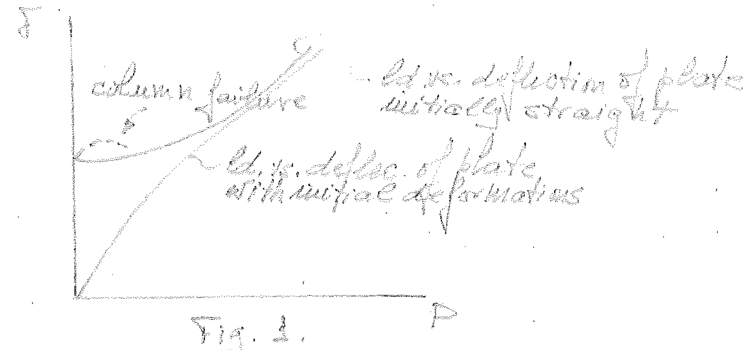


Fig. 1.

The reason for this increasing carrying capacity as compared with column behavior lies in the fact that the plate is able to establish a new stress distribution not envisioned in the "Linear Buckling Theory".

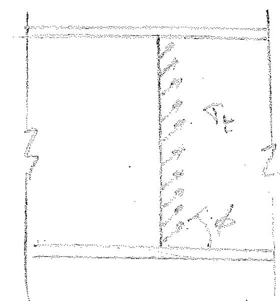


Fig 2

This is illustrated in Fig. 3. As the plate buckles a transverse strip GH starts to pull on the edges AD and BC. However, the stiffners of the plate along the edges resists this action such that tensile stresses are set up as indicated in the figure.

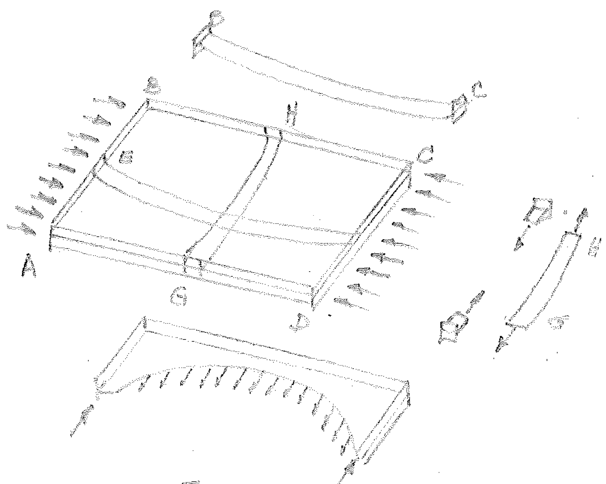
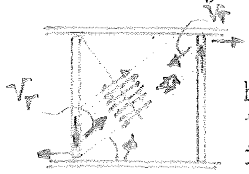


Fig 3.

These membrane stresses are neglected in the linear buckling theory.

It is evident that along the edges ABCD the conditions should be such that the plate is able to resist such membrane forces. Thus the carrying capacity of the plate is limited either by the yield strength of the material or the stability of the elements providing edge support.

Applied to the problem of plate girders, this means that the flanges and stiffners framing a panel should reinforce the plate edges sufficiently such that a state of membrane stresses can develop.



A flange of a conventionally built- welded plate girder has so little bending rigidity in the plane of the web that it cannot effectively resist vertical stresses at its junction with the web, therefore they do not serve as anchors for a tension stress field.

At the panel boundaries along the transverse stiffners the tension strip can transmit the stresses. Thus, only a part of the web contains a pronounced tension field which gives rise to a shear force.

In plate girders with slender webs neither a pure beam (V_{σ}) action nor a pure tension field action (V_{τ}) occurs alone, but is therefore the sum of both. Therefore the ultimate shear load V_u is: $V_u = V_{\sigma} + V_{\tau}$

This new approach to the design of the web plate of plate girders, will allow a greater slenderness ratio for the web than the present design practice. This will introduce new problems in their design, which will be discussed in the following pages.

For this purpose the presentation of the problem will be divided in three groups.

I. Plate girders in pure shear

II. Plate girders in pure bending

III. Plate girders under combined bending and shear.

Plate girders in pure shear

As explained before the ultimate shear capacity will be composed of two actions: Namely "beam action" and "tension Field action."

Formulas have been developed to obtain their values (23) as functions of the slenderness of the web, the ratio of distance between stiffners and depth of the web. In a graphical way the effect of the variables on the ultimate shear strength of a plate girder is clearly shown. Ref. 23.

The additional shear strength due to post-buckling allows the use of slender webs, and it is consequently necessary to set up limits to the slenderness ratio in order to avoid excessive distortions during fabrication and under load. These limits are set up (23) bearing in mind these considerations and others of practical significance.

Stiffners

Intermediate stiffners

In contrast to loading stiffners, intermediate stiffners are transverse elements through which no external forces are introduced into the girder. Their function will be two-fold.

Simple beam action will cause no axial load in the stiffner, it is only required to be rigid enough to force at its location, a nodal line in the lateral deflection mode of the web. Thus a minimum stiffness will be the only requirement due to beam action.

Tension field action will cause an axial force in the stiffner, to take the vertical component of the diagonal stresses out of the web at the end and transfer them to its other end. The maximum stiffner force and the strength of the connectors are obtained under the assumption that the stiffner should not fail before the ultimate shear strength of the adjacent panel is reached.

In girders subjected predominantly to bending, failure due to shear cannot occur and the required stiffner and connector forces may be reduced in proportion to σ/σ_{all}.

Bearing stiffners at the end supports

For the end panel of a girder the boundary conditions are different than in an intermediate panel. When web yielding sets in, there is no neighboring plate serving as anchor for a tension stress field. The bearing stiffner, at the end of a girder, together with and extending portion of the web, may offer partial restraint. Also a more or less pronounced gusset plate action in the upper corner of the end of the web, may help to develop a partial tension action. The degree of this contribution is uncertain. Since beam action does not depend on tension-resistant boundaries, the computed shear forces V_B and V_{B+T} are the limits between which the ultimate shear strength of an end panel lies.

In order to exclude the possibility of a premature end panel failure there are two possible approaches. The simplest and more economical solution is the choice of a stiffner spacing for the end panel, such that the possibility of development of a tension yield is avoided. An alternate solution is to make the end post resistant to membrane tension. This can be done by welding a plate to the end of the plate girder, thus forming a beam with the bearing stiffner, simply supported at the flanges of the plate girder. This beam will resist the diagonal stresses induced by tension field action.

PLATE GIRDERS SUBJECTED TO PURE MOMENT

Web Participation in resisting bending moments

As explained in the introduction, the web has initial deformations which are increased as loads are applied to the girder. As the web deforms laterally it avoids taking its share of longitudinal compression stress due to bending. This means that the longitudinal stresses in the web are transferred to the flanges, increasing the flange stresses over and above the nominal computed values. This is especially true for plate girders with very slender webs.

It was established that for very high web slenderness ratios $b/t > 360$ vertical buckling will take place before the yield moment is reached. As the web slenderness ratio is decreased the portion of the web taking longitudinal stresses will increase, reaching the value of M_p for $b/t = 53$. Between these two values a transition curve must be obtained to determine the ultimate capacity of the plate girder if lateral or torsional buckling is prevented.

This transition curve was found to be dependent on the slenderness ratio of the web and the ratio of web area to flange area; assuming that an effective width equal to 30 t_w is considered as effective in taking longitudinal stresses due to bending.

This reduction of effective web area was proposed to be effective for web slenderness ratio over 170, that is in the post buckling range where a Navier-Bernoulli linear stress distribution would be an unconservative assumption.

In this way, the influence of the two parameters A_w/A_f and b/t would be restricted in the ordinary plate girder design to a range of high web slenderness ratio, where a reduction of the allowable compressive flange stress is indicated.

Lateral Buckling

The resistance of an I-beam against lateral buckling consists of two parts: namely, St. Venant torsion, and warping torsion. Salvadore (28) has convincingly demonstrated that for hot-rolled beam sections, St. Venant torsion is the governing factor, whereas in deeper beams such as plate girders, warping torsion is the governing factor and the design should be based on the following column concept.

It is considered that the comprssion flange behaves as a column with one-sixth of the web area, which buckles by bending in a plane perpendicular to the web of the plate girder.

It was found that for practical cases the differences between this procedure and the one considering both effects was on the safe side and very close for web slenderness ratio of 200 and even closer for higher slenderness ratio.

The tension flange has neither a detrimental nor a stablizing effect since it remains undeflected in the lateral direction.

Thus the phenomena of lateral buckling is simply one of lateral buckling of the compression flange and the buckling curves used in the inelastic range must be those of weak axis buckling of wide flange columns and given in the Guide to Design Criteria for Metal Compression Members, Ref. 12.

Torsional Buckling

In designing plate girders, the compression flange should be made as wide as possible to increase its lateral rigidity and with it its lateral buckling strength. But if this is done in excess, torsional buckling of the flange plate will replace lateral buckling at a lower ultimate stress. The relation between these two modes of failure is obtained by neglecting all restraint from the web, and the case is reduced to torsional buckling of a long, hinged plate under pure edge compression. Post buckling strength is ignored completely[20] to avoid excessive distortions.

In order to eliminate torsional buckling as a primary cause of failure, the critical stress of the flange plate due to torsional buckling should exceed that of lateral buckling.

In this way it is found that, a flange width to thickness ratio not exceeding 12 plus the ratio of lateral buckling length to flange width, would exclude the possibility of torsional buckling as a primary cause of failure for girder sections under uniform bending.

Vertical Flange Buckling

The compression flange of a plate girder of I-shape cross section is continuously braced against vertical buckling by its web. Since the required bracing stiffness is small, the danger of compression flange failure in the vertical direction is limited to high web slenderness ratios.

This problem is treated (24) by considering the transverse flange forces produced by the curvature of the plate girder in the process of bending. This uniform compression stress acts on the lower and upper edges of the web, and the web is considered as a column. In this way an upper limit to the web slenderness ratio is found ($b/t = 360$) for plate girders with no horizontal stiffners.

It is also pointed out that girders which are loaded not only at points of transverse stiffening, but at intermediate point as well, require webs sturdier than those found by the above method. This will be also the case in curved compression flanges, such as those sometimes used over interior supports of continuous girders.

GIRDERS SUBJECTED TO COMBINED BENDING AND SHEAR

In girders subjected to bending and shear, the web plate is stressed by forces due to these two kinds of loadings.

If the web is not able to carry its part of the moment, the flanges will take these stresses if they do not exceed their carrying capacity. In very slender webs the stress rearrangement is predominately due to web deflections, this is achieved without a loss in shear carrying capacity which is essentially contributed by a tension field action in this case. In girders with stockier webs, the bending moment which cannot be carried by the web, because of high concurrent shear is transferred to the flange through yielding.

Thus it is necessary to obtain an interaction curve which should be concerned with such a rearrangement of stresses.

This is found by considering the limits between which this rearrangement of stresses takes place and by obtaining a curve between these limits.

Finally it is obtained that for a factor of safety of 1.65 (AISC) an interaction check is required only if the shear stress exceeds about 60% of the allowable value; also an interaction limit in flange stress is not required below 15 ksi for $A_w/A_f < 2$. And the interaction equation is:

$$\tau < 27 - 12\ \sigma/\sigma_{all} \quad (AISC)$$

The Influence of Flange Instability

So far it has been assumed that failure would occur by shear exhaustion of ductility, or at least that the flanges could be strained up to the yield level without a premature instability failure due to lateral, torsional or vertical buckling of the compression flange.

The presence of shear has both a detrimental and beneficial aspect. The beneficial aspect is due to the fact that shear forces always imply a moment gradient, and therefore, only a short portion of the girder is affected by the maximum moment. The adverse factor is that a web which is exhausted by shear cannot simultaneously take its allotted bending moment and the flanges will have to compensate for it, resulting in a higher flange stress than computed by the section modulus concept.

Lateral Buckling

Since the resisting moment to lateral buckling is dependent on the lateral stiffners of the compression flange and which is slightly affected by a higher stress level, the adverse influence mentioned above can be neglected in an analysis of lateral buckling.

The beneficial aspects of a moment gradient can be evaluated in the way proposed by Clark and Hill (14), by including a coefficient C, to modify the critical stress which would result if the entire section were subjected to pure bending.

The effective lateral buckling length, can be introduced to account for restricting influences offered by neighboring sections. Since St. Venant tension is neglected the value of k is exactly the same as for columns subjected to identical axial stresses and end restraint as the compression flange. The relationship between stress raising coefficient C, and moment gradient is based on solutions by Salvadori (28), and may also be found in the Guide for Design Criteria for Metal Compression Members (13), CRC.

Torsional buckling

The beneficial effect of moment gradient in torsional buckling is small and may be neglected.

Vertical buckling

To prevent a premature failure due to vertical buckling of the flange plate, a limit for the web's depth to thickness ratio was obtained, $(b/t < 360)$ in the case of pure bending. When the girder web is slender, a prerequisite for vertical buckling, the shear is carried principally in tension field manner. The question was if whether or not the web's tension field would pull the flanges into the web. Interaction tests carried out on plate girders developed no detrimental effect prior to application of ultimate load, however, these did not cover the entire interaction range with slender webs.

Precautionary measures were taken in the derivation of the Interaction formula, by the use of a relatively large value of the ratio Aw/Af as representative of all girders.

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INTRODUCTIONS

Connections;

The single span rigid frame is a very popular structure due to its attractive appearance and to some of the advantages given below.

Steel rigid frames are adaptable to riveted and welded fabrication; they may be constructed of rolled beams, built-up members or any combination thereof. These frames may be efficiently constructed for flat, gabled or curved roofs, and when so built the economy and speed of erection is not surpassed by any other type of material or form of construction.

For identical loading it is possible that a greater weight of steel may be required for a rigid frame than for a truss and column design of similar span length. However, when consideration is given to the over-all cost of the framing, the ease and simplicity of erection increased clear headroom of the finished structure and saving in wall heights, it will be found that even on a first cost basis, the rigid frame is highly competitive.

Rigid frames composed of rolled sections are commonly used for spans up to 100 feet in length; built-up members have been utilized on spans of 250 feet. Welded fabrication offers particular advantages with short spans; with variable depth members; and on parabolic shape roofs.

The analyses of rigid frames by elastic and plastic theory is a very well known problem. Manuals with solved cases and simplified procedures to determine the moment and forces in frames are available in a great number.

However, the design of knee joints and their behavior has been a great problem to engineers. The analysis and design of knee joints by the elastic method uses formulas that are very complex and difficult to use in the design office. Plastic design has simplified the design of knee joints considerably.

A great effort has been made to correlate theory and experiments and to develop simplified design procedures.

The object of this work is to summarize the work done until now. A list of references will be given with a brief commentary for each.

Connections

Connections for use in rigid frames can in general be divided into three types: square, tapered and curved.

1. - Square knees

The first theoretical analyses of square knees was made between

1938 and 1941 by Stang, Greenspau and Osgood (1) at the U. S. National Bureau of Standards.

A theoretical analysis of the stresses in the web of an unreinforced square knee was given and the results of this analysis were confirmed by the test of a full-scale built-up unreinforced riveted square knee; the test results and predictions were satisfactory.

It was also found that leg stresses, even close to the knee, were predicted satisfactorily by the ordinary beam formula.

They also found that this solution was in error in extreme cases where the flange area was very small. The solution given for this case is complex and need not be considered for practical cases.

To supplement the work at the Bureau of Standards, a complete quarter-scale built-up riveted portal frame with square knees was tested at Lehigh University by Lyse and Black (2) (1942). They found that Osgood's solution predicted stresses very satisfactorily.

Hendry (3) (4) published two papers in 1947 and 1950. In his first paper he found that the flange stresses in the legs adjacent to the knee could be estimated quite accurately by the ordinary flexural theory, but the stresses in the web of the knee were not estimated accurately by Osgood's theory. The reason for this may have been the absence of stiffners across the web, in line with the flanges, which would have served to distribute load across the web.

Photo-elastic models, geometrically similar to the steel frame tested showed the effect of web stiffners in knees. Stiffners placed diagonally across the knee were found to be most effective in reducing web shear stresses, stiffners placed in line with the inner flanges had no effect on the web shear stresses.

In his second paper, Hendry observed that in general web buckling and/or lateral deflection of the compression flange tended to occur after the web had yielded in shear.

In the conclusion of his paper, Hendry suggested that square knees should always have web reinforcement and be used in secondary structures.

F. Bleich in 1943, proposed an approximate method for the design of square knees. The assumptions made were: a) the flexural moment and total thrust are taken by the flanges; b) the shear forces are taken by the web.

Griffiths, in 1948 set up recommendations for design. They do not suggest a method of analysis, but are more a rule of thumb procedure based on the theories developed by Osgood and Bleich.

At this time the advances made in the plastic design of steel structures, made necessary the study of connections up to the inelastic range. For this purpose a program of investigation was set forth at Lehigh University.

In 1951 and 1952, Topractsoglou, Beedle and Johnston published three papers, containing the results and proposed theories for square, haunched and curved knees.

Part I of these reports gives the test results. To obtain information on the relative cost of fabrication, the welding and cutting operations were carefully timed. It was impractical to determine accurately the entire labor cost of fabrication of each knee. However, the data taken was believed to be an expression of the comparative costs of fabrication of the knees.

Part II, gives a theoretical analysis for straight knees which form the basis for comparing experimental results with theory. Elastic and plastic analysis was made for identical and dissimilar rolled sections joining at the knee. A rotational analysis for reinforced and unreinforced square knees is also given.

In Part III, the test results are discussed. Square knees were found to develop the plastic moment with sufficient rotation capacity. It was also found that yielding in shear of the web took place before flexural yielding in the flanges. An expression was derived for the web thickness, so as to have shear yielding in the web simultaneously with yielding due to flexure.

For reinforced webs in square knees, it was recommended that plastic design should be used, in which the yield capacity of the web in shear is simply added to that of the stiffners in compression.

No comparisons between test results and the theories for reinforced square knees was made.

Further experiments were made by Toppac, Beedle, Fisher, Driscoll and Schultz, at Lehigh University; the references are listed at the end of this paper.

Straight and curved haunched Connections

Stang, Greenspan and Osgood, between 1938 and 1941, presented an approximate analysis for knees with curved inner flanges. This theory was checked experimentally at the U.S. National Bureau of Standards.

Lyse and Black tested a complete quarter-scale built-up riveted portal frame with a curved inner flange knee. Osgood's solution was found to predict stresses very satisfactorily.

Hendry in 1947 tested two knees with curved inner flanges. It was found that Vierendel's solution estimated satisfactorily the stresses in the flange, whereas the effect of cross bending was estimated satisfactorily by Bleich's solution. Radial stresses in the web attached to the curved flange were predicted by Campus' formula.

Tests made by the photo-elastic method, showed the effectiveness of haunching in reducing flange stresses as well as web stresses.

The same conclusion for straight knees with regard to stiffners across the web were found to hold for haunched knees.

Hendry in 1950, tested six knees with curved inner flanges, and it was found that an unstiffened web offered less resistance to instability in the case of knees with curved inner flanges, than it did in the case of simple square knees. Thus, although stiffners may not be needed as web reinforcement in knees with curved inner flanges, they may be needed to provide stability.

Three complete portal frame models, with varying amounts of haunching at the knees, were tested, and it was observed that the load carrying capacity of a portal frame could be increased appreciably by haunching the knees.

Topractsoglou, Beedle and Johnston in 1951 and 1952 tested several welded knee connections,(square, haunched and with curved inner flanges).

These tests were carried well into the plastic range, and with adequate bracing and web reinforcements to prevent shear failure. All the knee types tested were strong enough to develop at the knee the full plastic moment capacity of the legs.

It was found that although the haunched knees and knees with curved inner flanges had a greater moment capacity than the square knees, they were not able to sustain large deflections at near maximum moments. The primary cause of these premature failures was the instability of the haunched portion of the connections. However, no theories were proposed and no checks were made between the experimental results obtained and theories proposed elsewhere for these types.

Until now the theories for the elastic analyses of haunched connections lead to methods generally too unwieldly to be used used in the design office. The method proposed by Osgood is rationally developed from the equations of compatability and equilibrium from the theory of the wedge. However, it is tedious to use.

F. Bleich developed a theory based on the relationships between stress and strain with regard to a curved beam. This theory unfolds rationally, but again terminates in unwieldly expressions.

The recommendations for design as set forth by Griffiths do not suggest a method of analysis, but are more a rule of thumb procedure based on the theories developed by Osgood and Bleich.

Olander has somewhat simplified the elastic approach by an approximation to the wedge theory utilizing the conventional beam formulas. This has not reduced the amount of work involved in the design, although the expressions involved are in a more familiar form.

Smith in 1956, at Lehigh University, made a comparison between the methods proposed by Osgood and Olander, and found that Olander's

Method was for all practical purposes as accurate as Osgood's method. A design example is presented indicating how this method is applied. In his work the knee was designed to remain in the elastic range, while the plastic hinge was forced to be produced at beam junction. This was made so due to the previous findings that haunched connection did not have adequate rotation capacity.

A complete test was made on a haunched connection whose girder intersected at an angle greater than ninety degrees with the column. This connection was proportioned by Olander's method. The connection had a very good performance and sustained large deflections at maximum load (plastic hinge in the beam). Very good agreement between Olander's method and tests results was found.

Fisher in 1958, made a theoretical analysis which led him to the conclusion that plastic hinges could form in the haunch and sustain large deflections at maximum load, if adequate provisions were made to prevent lateral buckling of the compression flange. For this purpose he derived the equation necessary to determine the maximum unsupported length of the compression flange for haunches with straight and curved inner flanges.

He made comparisons with previous experiments finding a very good correlation between theory and test results.

In 1959, Fisher, Lee and Driscoll continued the work on haunched connections - verifying the theory developed by Fisher.

Beam to Column Connections

Innumerable experiments and theoretical analysis have been made for these types of connections, as well as a very large number of books containing procedures for design are available (references are given).

Experiments carried into the plastic range have been conducted, and the summary of this work may be found in the report of Graham, Sherbourne and Khabbaz, 1960. Design rules are suggested, among others, the determination of column stiffener sizes required to prevent buckling, or excessive bending of the column web and flanges. Also are given several design examples which illustrate very clearly the design procedures.

* * * * *

SUMMARY

A summary of rules to design all these types of connections in the plastic range, was prepared by the Fritz Engineering Laboratory, Lehigh University, and published in the Proceedings of the American Society of Civil Engineers: "Commentary on Plastic Design in Steel" April 1960.

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B E A M   C O L U M N S

BEAM COLUMNS

Beam columns are members which are subjected to combined bending and compression. Bending may be produced by lateral loads, couples applied at any point on the beam, or end moments resulting from excentricity of the axial loads at one or both ends of the member.

Beam columns fail by buckling in the plane of the applied bending forces, with or without twisting.

Considering first the case of buckling without twisting, the ultimate strength of a beam-column has been described by von Karman (95) by the following reasoning:

"A beam-column collapses when, because of yielding caused by the combined bending and axial stresses, the stiffness of the member against additional bending is reduced to the point where instability occurs, that is, the ultimate axial load is the critical load of the partially yielded column."

Thus the determination of the ultimate load of a beam-column is essentially a non-linear stability problem in which the effects of inelastic action must be considered.

The method used is a step by step procedure (32) for which it is necessary to know the stress-strain diagram of the material, initial conditions of residual stress, crookedness and boundary conditions. With these elements the $M - \phi - P$ curves are found and together with Hooke's and Navier's laws the effect of yielding and the reduction of the stiffness of the member is found.

This involves laborious and complex computations not suited for design purposes.

F. Bleich (32) and S. Timoshenko (148) have analyzed the major contributions by von Karman (94) (95), Ross and Brunner (32), Chwalla (52), Weestergaard and Osgood (166), Jezek (86)(87) and others. Further developments and verifications were made by O. M. Sidebottom and M. E. Clark (145); R. L. Ketter, L. Kaminsky and L. S. Beedle (105); T. V. Galambos and R. L. Ketter (65); R. E. Mason, G. P. Fisher, P. P. Bijlaard and G. Winter (24); F. Campus and C. Massonnet (52) etc.

T. V. Galambos and R. L. Ketter (65), based on the theorie developed by von Karman, made computations of the ultimate strength of a 8 WF 31 section subjected to combined bending and compression. Residual stresses are taken into account, and no torsional buckling is considered. Interaction diagrams are given for different ldg. conditions and are assumed to apply to all sections.

A very popular procedure is to consider as the limit of the strength of a beam-column when the most stressed fiber reached the yield stress. This is called the limiting stress criteria.

Exact methods to determine the maximum bending moment and axial force, are summarized by S. Timoshenko (148). Several types of loadings are considered and in the appendix of his book several tables are given to facilitate the use of the methods treated.

Tabular and graphical solutions for several loading conditions have been developed by S. Timoshenko (148); D. H. Young (171) (172); S. Zavriev; O. G. Julian (92); H. K. Stephenson and K. Clominger (143); AASHO, etc.

In the case of equal end excentricities the secant formula is used to predict the initiation of yielding. In this formula two methods are used to take account of initial imperfections of the column, crookedness and residual stresses. One method is to introduce an equivalent end excentricity. The second method is to use the tangent modulus correction (3). Both give the same results if the initial excentricities are those recommended in the CRC Handbook (38).

Another popular method to determine the strength of a beam-column are the interaction equations. Interaction equations are based on an assumed interaction curve between axial load alone and bending moments alone. This curve was first considered to be a straight line but now is being modified to take into account the magnifiance effect of the axial load on the deflection of the beam-column. (136) (113) (3)

Interaction equations have been developed to take account of elastic and plastic or inelastic action. They also may take account of lateral torsional buckling which cannot be done by the secant formula procedure.

Interaction equations have been developed by the AISC (1); Shanley (142), J. Zickel and D. C. Drucker (174); H. N. Hill, E. C. Hartman and J. Clark (73); M. Salvadori (136); W. J. Austin (3); Massonnet (113), Horne (75); etc.

They have proven to agree well with experimental results; they are not always on the safe side, but the error in the unsafe zone doesn't exceed 10%.

RESTRAINED COLUMNS

Columns in framed structures are connected at their ends to other members that provide translational and rotational restraints.

The beams of an ordinary tier building transmit bending moments and axial forces to the columns because of the rigidity of the beam to column connections. These bending moments together with the axial loads in the columns, cause the columns to flex from the very beginning of loading. Eventually if the axial load in a column becomes large enough, the sign of the bending moments at the column ends may change and the beams take over the role of restraining the end rotations of the column and thereby limit its deflections. (Introduction from Ojalov's Paper) (126).

Various aspects of the restrained column problem have been investigated by E. Chwalla (51), P. P. Bijlaard, G. P. Fisher, and G. Winter (24); T. C. Kavanagh (97) (98) (99) (100) (101) (102) (103), Bijlaard (26); L. F. Baker, M. R. Horne, and J. W. Roderick (28); J. S. Ellis (59); M. Ojalvo (126) and others.

The phenomena of buckling of framed (restrained) columns is non-linear. This is due to two separate effects: first, there is the change in geometry of the structure particularly in the column; secondly, there is the change in stiffness of the material where the stresses have exceeded the proportional limit.

Three principal general theoretical approaches have been used to solve the problem of restrained columns. These are:

The method of von Karman (94) and Chwalla (51)
The method of Baker, Horne and Roderick (28)
The method of Bijlaard (26)

The theoretical solutions for isolated restrained columns that have been developed so far are only applicable to columns of rectangular or anular cross sections and to an I cross section for bending about the weak axis. With the exception of Bijlaard's solution the methods are also inadequate to cover the general case of unequal end restraints and unequal applied end moments.

M. Ojalvo (126) has developed a method based on Chwalla's approach, and extends its applicability to cases where the rotational restraints at the ends of the column are not elastic. The method is applicable for loads applied to the columns with equal or unequal excentricities and can be used for cases where the restraining moments are not linear functions of the end rotations of the column. It also takes account of the inelastic action of the column and thus leads to the true ultimate capacity of such members. Nomographs are made available, such that may be used in design office.

Simplified procedures have been proposed by Winter (159), Lee (109) and Austin. (3)

Rigorous methods of calculating the stability of complete frames are available and have been checked by test. These methods involve the application of both the classical and modern techniques of indeterminate analysis, i.e. moment distribution, modified to take account of axial loads and of plastic action on the stiffness or rigidity of the members. These procedures are not applicable to routine design problems.

A simplified procedure has been outlined by T. C. Kavanagh (102) and recommended in the CRC Guide (38).

Further references may be found in the summary given further.

BIAXIAL BENDING

No precise theoretical studies, based on the von Karman theory of the strength of beam-columns, which fail by bending about both principal axes, seem to have been made. Also, little test data is available.

The limiting stress criterion has been used to predict the buckling loads.

The secant formula, or the solution given by Young or the nonograph by Julian can be extended to include biaxial buckling.

The interaction equations for plane buckling can be modified to take account of bending about both principal axes. (AISC (1) Austin (3) etc.) Test of six columns loaded with an excentricity diagonal to the principal axes were reported by the ASCE Special Committee on Steel Column Research (42). The average stress at collapse was from 15% to 40% greater than the theoretical average stress at initial yielding, computed by the secant formula. Thus the initial yield criterion may provide a rather conservative estimate of the buckling load.

DIFFERENCE BETWEEN CONVENTIONAL DESIGN AND PLASTIC DESIGN WHEN APPLIED TO COLUMN DESIGN

In conventional design the ultimate strength of a member is considered to be reached when the **most** stressed fiber reaches the yield stress. In plastic design the ultimate strength is obtained when all the cross section of a member is fully yielded.

In the design of axially loaded columns, conventional design is based on the ultimate capacity of the member. Thus no difference exists between conventional and plastic design of axially loaded columns. In the design of beam-columns, conventional design is based on the limiting stress criteria for the combined effects of bending and axial forces. In plastic design the beam-column is allowed to be stressed beyond the yield stress, until the remaining elastic portion fails by instability due to direct compression.

In the interaction formulas, the yield moment used in conventional design is replaced by the plastic moment if lateral buckling is prevented. Exact computations for this case were made by T. V. Galambos and Ketter (64). Both procedures agree very well with test results, although the interaction equations in certain cases are somewhat unsafe. (10%).

If lateral buckling is not prevented, modifications to the conventional critical stress formula for lateral buckling must be made to consider inelastic failure. An exact solution for this case was made by T. V. Galambos (66), approximate methods are suggested in the CRC Guide to be used in the interaction formulas for the inelastic case.

More reference for this case may be found in the first pages of this report.

DIFFERENCES BETWEEN ROLLED, WELDED AND COLD FORMED COLUMNS

Welded columns as compared to rolled sections are weaker because of a more severe residual stress pattern due to welding. There are few tests reported on welded columns and very little information on the residual stress pattern due to welding.

Cold formed sections as compared to rolled sections of the same mechanical proptetics are stronger, because they have no residual stresses due to cooling as in rolled sections. Bending of the plates to form the sections modifies the stress strain diagram (Baushinger effect) of the material, but in a transverse direction. In the longitudinal direction there is no effect of the cold bending operation. For this reason cold bent sections of the same mechanical proterties, stress-strain diagrams, are stronger than rolled sections.

Tests are now being conducted at Cornell University under the direction of G. Winter to determine the stress-strain diagram from **a stub** column test of the entire cross section of a cold formed member.

SUMMARY OF REFERENCES

Exact methods	T. Von Karman (94)(95) Ros and Brunner (32) Weestergaard and Osgood (166) Chwalla (51) Yezek (86)(87) Jager (85)

SUMMARY BY BLEICH (32) and Timoshenko (148)

Computations based on exact solutions for excentrically loaded columns	Considering lateral torsional buckling	T. V. Galambos (66)
	Without considering lateral tors. buckling	T. V. Galambos and Ketter (64) D. M. Sidebottom and M. E. Clark (145) R. L. Ketter, L. Kiminsky and L. S. Beedle (105)
Interaction formulas	Horne (75) Salvadori (136) Hill, Hartman and Clark (73) Zickel and Drucker (174) D. T. Wright (161) W. J. Austin (3) Massonnet (113) CRC Guide (43) AISC (1) Shanley (142)	

BEAM COLUMNS ELASTICALLY RESTRAINED

Theoretical approaches	E. Chwalla (51) Bijlaard, Fisher and Winter (24) Kavanagh (97) Bijlaard (5) Baker, Horne and Roderick (28) Ellis (59) Ojalvo (126) Horne (76)
Simplified Analysis	Horne (75) Ojalvo (126) Salvadori (137) Stephenson (143)

COLUMNS WHICH FAIL BY COMBINED BENDING AND TORSION

Investigation of torsional flexural buckling	Wagner (157) Kappus (96) Goodier (62) (63) Timoshenko (149) Bleich (32) Campus & Massonnet (52)
Important solutions	Johnston (89) Goodier (62) (63) Hill and Clark (72) Salvadori (137) Nylander (120) Zickel (173) Thurlimann (151) Horne (75)

STRENGTH OF FRAMEWORKS

Strength analysis considering axial forces	S. Timoshenko (148) F. Bleich (32) Kavanagh (99) Puwein (128 a) Sahmel (137) Sievres (138)
Practical means to estimate the effective length in framed columns	Kavanagh (103) CRC (43) Julian and Lawrence (93) British Standard (34) Winter (A 24) A. Y. Lee (109) Austin (3) Lundquist & Kroll (110)

Campus and Massonnet (52)
Tests Mason Fisher and Winter (112)
University of Wisconsin, CRC (41) (42)
Johnston and Cheney (90)
W. Clark (46)
Ketter, Beedle and Johnston (104)
Ketter, Kaminsky and Beedle (105)
Hill, Hartman and Clark (73)
Massonnet (113) (114)

Baker, Horne and Heyman (29)
Baker, Horne and Roderick (28)
Bijlaard, Fisher and Winter (6)
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