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B. P. Parikh

J. H. Daniels

L. W. Lu

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DESIGN AIDS BOOKLET

by

B. P. Parikh

J. Hartley Daniels

Le-Wu Lu

This booklet is a supplement to the Lecture Notes on "PLASTIC DESIGN OF MULTI-STORY FRAMES".

Fritz Engineering Laboratory

Department of Civil Engineering

Lehigh University

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INTRODUCTION

This DESIGN AIDS BOOKLET was prepared as a supplement to the LECTURE NOTES on "PLASTIC DESIGN OF MULTI-STORY FRAMES." It contains tables and charts for use in the analysis and design of building columns. The information contained in the booklet has been used in the development of the design methods presented in Lectures 11 and 18 and in performing the designs for the three example frames given in Lectures 12, 14, 16 and 19. The development and preparation of the tables and charts are briefly described in the various parts. Examples are included to illustrate the applications to the solution of column problems.

TABLE OF CONTENTS

PART I - TABLE OF CRITICAL STRESSES OF AXIALLY LOADED COLUMNS

Critical stress values of axially loaded columns are given for A36 and A441 steels. These stresses have been computed from the Column Research Council "Basic Column Formula":

$$\frac{\sigma_{cr}}{\sigma_y} = 1 - \frac{\sigma_y}{4\pi^2 E} \left(\frac{Kh}{r}\right)^2$$

The following example will illustrate the use of this table.

Example: Determine the critical load of a centrally loaded 14WF158 column of A36 steel. The column has an unbraced length of 12 ft. and forms part of a braced multi-story frame. The column is assumed to be braced in the weak direction.

For the 14WF158 Section, $r_x = 6.40$ in and $A = 46.47$ in^2.

Also $K = 1$ (braced frame)

Thus

$$\frac{Kh}{r_x} = \frac{(1)\ (144)}{6.40} = 22.5$$

Entering the table and interpolating linearly

$$\sigma_{cr} = 35.42 \text{ ksi}$$

The critical load of the column is

$$P_{cr} = (35.42)(46.47) = 1645 \text{ kip}$$

Critical Stresses of Axially Loaded Columns

$\frac{Kh}{r}$	Material			
	A36	A441		
	σ_y = 36 ksi	σ_y = 50 ksi	σ_y = 46 ksi	σ_y = 42 ksi
0	36.00	50.00	46.00	42.00
5	35.97	49.95	45.95	41.96
10	35.89	49.78	45.82	41.84
15	35.75	49.55	45.59	41.65
20	35.55	49.13	45.26	41.38
25	35.29	48.64	44.85	41.04
30	34.98	48.04	44.33	40.61
35	34.61	47.33	43.74	40.11
40	34.19	46.51	43.04	39.53
45	33.71	45.58	42.26	38.88
50	33.17	44.54	41.38	38.15
55	32.58	43.40	40.41	37.34
60	31.93	42.14	39.35	36.45
65	31.22	40.78	38.19	35.49
70	30.45	39.30	36.94	34.45
75	29.63	37.72	35.60	33.33
80	28.76	36.03	34.17	32.14

TABLE I—1

PART II - REDUCED PLASTIC MOMENT TABLES

The tables give the reduced plastic moment (M_{pc}) values of column sections for various axial forces. The values are tabulated separately for A36 steel (Table II-1) and A441 steel (Table II-2). Only sections qualified as compact sections are included in the tabulation.

The axial force (P) is nondimensionalized with respect to the yield load of the section ($P_y = A\sigma_y$). The reduced plastic moment values are given at intervals of 0.1 P/P_y, from $P/P_y = 0.1$ to $P/P_y = 0.9$. The last column in each table gives the average difference between $P/P_y = 0.2$ and $P/P_y = 0.9$ to facilitate the interpolation for intermediate values.

The tabulated values were computed for bending about the strong axis and were based on the following equations: (Lecture 4, Ref. 4.19)

$$M_{pc} = M_p - \frac{P^2}{4\sigma_y w}$$

$$\left[0 \leq P \leq \sigma_y \, w \, (d-2t)\right]$$

and

$$M_{pc} = \frac{\sigma_y}{2}\left[d \, (A - P/\sigma_y) - \frac{1}{2b} (A - P/\sigma_y)^2\right]$$

$$\left[\sigma_y \, w \, (d - 2t) \leq P \leq P_y\right]$$

Example: An axial force of 755 kip is applied to a 14WF127 column of A36 steel. Determine the reduced plastic moment capacity of the column.

From Table II-1:

$P_y = 1344$ kip, $P/P_y = 755/1344 = 0.562$

$M_{pc} = 392$ kip-ft for $P/P_y = 0.5$

The average difference between intervals is 75.4 kip-ft.

By linear interpolation

$M_{pc} = 392 - (0.62)(75.4) = 345$ kip-ft.

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A36 Steel $\sigma_y = 36$ ksi

Section	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
							$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	in^2	kips	in	in	in.^{3}	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
14 WF 426	125.3	4509	7.26	4.34	869.3	2608	2545	2358	2113	1853	1580	1292	990.0	674.1	344.1	287.7
14 WF 398	117.0	4211	7.17	4.31	803.0	2409	2351	2174	1946	1705	1452	1186	908.2	617.8	315.1	265.5
14 WF 370	108.8	3916	7.08	4.27	737.3	2212	2158	1997	1785	1562	1329	1085	829.7	563.9	287.3	244.2
14 WF 342	100.6	3621	6.99	4.24	673.0	2019	1970	1823	1627	1423	1209	985.6	753.1	511.4	260.3	223.2
14 WF 320	94.12	3388	6.63	4.17	592.2	1777	1741	1636	1466	1281	1087	885.7	676.2	458.7	233.3	200.4
14 WF 314	92.30	3323	6.90	4.20	611.5	1835	1789	1652	1473	1286	1092	889.0	678.6	460.2	234.1	202.6
14 WF 287	84.37	3037	6.81	4.17	551.6	1655	1614	1490	1327	1157	981.0	798.0	608.4	412.2	209.4	182.9
14 WF 264	77.63	2795	6.74	4.14	502.4	1507	1470	1357	1207	1051	890.2	723.4	551.0	373.0	189.3	166.8
14 WF 246	72.33	2604	6.68	4.12	464.5	1394	1359	1253	1114	969.2	820.0	665.8	506.8	342.8	173.8	154.2
14 WF 237	69.69	2509	6.65	4.11	445.4	1336	1303	1202	1067	928.6	785.3	637.4	484.9	327.9	166.2	147.9
14 WF 228	67.06	2414	6.62	4.10	427.2	1282	1249	1151	1022	889.1	751.6	609.8	463.7	313.4	158.8	141.7
14 WF 219	64.36	2317	6.59	4.08	408.0	1224	1193	1100	976.3	848.6	717.0	581.4	442.0	298.6	151.2	135.5
14 WF 211	62.07	2235	6.56	4.07	391.7	1175	1146	1056	936.9	814.0	687.5	557.3	423.5	286.0	144.8	130.2
14 WF 202	59.39	2138	6.54	4.06	373.6	1121	1092	1006	892.4	775.0	654.2	530.1	402.6	271.8	137.6	124.1
14 WF 193	56.73	2042	6.51	4.05	355.1	1065	1038	956.9	848.0	736.1	621.1	503.0	381.9	257.6	130.4	118.1
14 WF 184	54.07	1947	6.49	4.04	337.5	1013	986.4	908.3	804.6	698.0	588.7	476.6	361.6	243.9	123.3	112.1
14 WF 176	51.73	1862	6.45	4.02	321.3	963.9	939.4	866.0	765.4	663.8	559.6	452.8	343.4	231.5	117.0	107.0
14 WF 167	49.09	1767	6.42	4.01	302.9	908.7	885.5	816.0	722.6	626.3	527.7	426.8	323.6	218.0	110.2	100.8
14 WF 158	46.47	1673	6.40	4.00	286.3	858.9	836.7	769.8	680.9	589.9	496.7	401.6	304.3	204.9	103.5	95.19
14 WF 150	44.08	1587	6.37	3.99	270.2	810.6	789.6	727.0	642.7	556.5	468.5	378.5	286.7	193.0	97.45	89.93
14 WF 142	41.85	1507	6.32	3.97	254.8	764.4	745.1	687.1	606.6	525.1	441.8	356.8	270.2	181.8	91.75	85.05
14 WF 136	39.98	1439	6.31	3.77	242.7	728.1	709.9	655.4	579.3	501.5	421.9	340.8	258.0	173.7	87.64	81.11
14 WF 127	37.33	1344	6.29	3.76	225.9	677.7	660.6	609.2	538.2	465.6	391.5	316.1	239.2	160.9	81.15	75.44
14 WF 119	34.99	1260	6.26	3.75	210.9	632.7	616.6	568.3	502.0	434.1	364.8	294.4	222.7	149.7	75.48	70.40
14 WF 111	32.65	1175	6.23	3.73	196.0	588.0	573.2	528.8	465.8	402.6	338.2	272.8	206.2	138.6	69.83	65.57
14 WF 84	24.71	889.6	6.13	3.02	145.4	436.2	426.0	395.6	349.2	301.6	253.3	204.1	154.2	103.6	52.18	49.06

TABLE II—1

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A36 Steel $\sigma_y = 36$ ksi

Section	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
							$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
14 WF 78	22.94	825.8	6.09	3.00	134.0	402.0	392.8	365.1	322.5	278.4	233.7	188.3	142.2	95.45	48.05	45.29
14 WF 74	21.76	783.4	6.05	2.48	125.6	376.8	368.9	345.2	306.9	265.2	222.8	179.6	135.8	91.22	45.96	42.75
14 WF 68	20.00	720.0	6.02	2.46	114.8	344.4	337.2	315.7	280.6	242.3	203.4	163.9	123.9	83.16	41.88	39.12
14 WF 61	17.94	645.8	5.98	2.45	102.4	307.2	300.8	281.7	250.2	215.9	181.1	145.9	110.1	73.90	37.19	34.93
14 WF 53	15.59	561.2	5.90	1.92	87.1	261.3	256.4	241.6	217.1	187.5	157.3	126.8	95.76	64.29	32.37	29.89
14 WF 48	14.11	508.0	5.86	1.91	78.5	235.5	231.1	217.9	195.9	168.7	141.5	113.9	80.01	57.71	29.04	26.98
14 WF 43	12.65	455.4	5.82	1.89	69.7	209.1	205.2	193.5	174.0	150.4	126.0	101.4	76.52	51.32	25.81	23.96
12 WF 190	55.86	2011	5.82	3.25	311.5	934.5	912.4	846.2	752.9	656.5	556.3	452.4	344.8	233.6	118.6	103.9
12 WF 161	47.38	1706	5.70	3.20	259.7	777.6	759.0	703.2	624.6	543.4	459.6	373.1	283.8	191.9	97.30	86.56
12 WF 133	39.11	1408	5.59	3.16	209.7	629.1	613.9	568.3	504.0	437.6	369.3	299.1	227.1	153.3	77.57	70.10
12 WF 120	35.31	1271	5.51	3.13	186.4	559.2	546.0	506.5	449.2	389.6	328.5	265.8	201.6	135.9	68.73	62.54
12 WF 106	31.19	1123	5.46	3.11	163.4	490.2	478.4	443.1	392.6	340.1	286.4	231.5	175.4	118.1	59.66	54.78
12 WF 99	29.09	1047	5.43	3.09	151.8	455.4	444.5	411.6	363.9	315.1	265.2	214.2	162.2	109.2	55.11	50.93
12 WF 92	27.06	974.2	5.40	3.08	140.2	420.6	410.5	380.3	336.4	291.1	244.8	197.7	149.6	100.6	50.77	47.08
12 WF 85	24.98	899.3	5.38	3.07	129.1	387.3	377.9	349.5	308.9	267.1	224.5	181.2	137.0	92.13	46.45	43.29
12 WF 79	23.22	835.9	5.34	3.05	119.3	357.9	349.3	323.5	285.4	246.7	207.2	167.1	126.4	84.90	42.78	40.10
12 WF 58	17.06	614.2	5.28	2.51	86.5	259.5	253.4	235.2	207.7	179.3	150.5	121.3	91.62	61.52	30.98	29.17
12 WF 53	15.59	561.2	5.23	2.48	78.2	234.6	229.3	213.5	188.5	162.7	136.5	109.9	82.97	55.68	28.02	26.50
12 WF 50	14.71	529.6	5.18	1.96	72.6	217.8	213.4	200.3	178.4	154.2	129.5	104.4	78.88	52.99	26.70	24.94
12 WF 45	13.24	476.6	5.15	1.94	64.9	194.7	190.8	179.1	159.7	137.8	115.7	93.19	70.38	47.25	23.79	22.19
12 WF 40	11.77	423.7	5.13	1.94	57.6	172.8	169.3	158.7	141.2	121.8	102.2	82.24	62.07	41.64	20.95	19.68
10 WF 112	32.92	1184	4.67	2.67	147.5	442.5	431.7	399.4	355.1	309.1	261.5	212.3	161.6	109.3	55.41	49.14

TABLE II—1

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A36 Steel $\sigma_y = 36$ ksi

Section	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
							$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
10 WF 100	29.43	1058	4.61	2.65	130.1	390.3	380.8	352.4	312.9	271.9	229.8	186.3	141.6	95.67	48.46	43.42
10 WF 89	26.19	942.8	4.55	2.63	114.4	343.2	334.8	309.7	274.7	238.4	201.2	163.0	123.7	83.48	42.24	38.21
10 WF 77	22.67	816.1	4.49	2.60	97.7	293.1	285.9	264.3	234.3	203.1	171.1	138.4	104.9	70.71	35.74	32.65
10 WF 72	21.18	762.5	4.46	2.59	90.7	272.1	265.5	245.7	217.3	188.2	158.5	128.1	97.10	65.39	33.03	30.38
10 WF 66	19.41	698.8	4.44	2.58	82.8	248.4	242.2	223.7	197.9	171.3	144.1	116.4	88.15	59.33	29.94	27.68
10 WF 60	17.66	635.8	4.41	2.57	75.1	225.3	219.7	202.8	178.7	154.6	130.0	104.9	79.37	53.38	26.92	25.13
10 WF 54	15.88	571.7	4.39	2.56	67.0	201.0	195.9	180.4	159.5	137.9	115.8	93.41	70.62	47.46	23.92	22.35
10 WF 45	13.24	476.6	4.33	2.00	55.0	165.0	161.2	150.0	132.7	114.7	96.39	77.77	58.82	39.54	19.93	18.58
10 WF 39	11.48	413.3	4.27	1.98	47.0	141.0	137.9	128.6	113.8	98.25	82.49	66.49	50.24	33.74	16.99	15.94
8 WF 67	19.70	709.2	3.71	2.12	70.1	210.3	205.2	190.1	169.0	146.9	124.2	100.8	76.62	51.79	26.24	23.41
8 WF 58	17.06	614.2	3.65	2.10	59.9	179.7	175.4	162.6	143.7	124.8	105.3	85.32	64.78	43.72	22.13	20.07
8 WF 48	14.11	508.0	3.61	2.08	49.0	147.0	143.3	132.3	116.9	101.3	85.35	69.02	52.32	35.25	17.81	16.36
8 WF 40	11.76	423.4	3.53	2.04	39.9	119.7	116.9	108.3	95.58	82.69	69.55	56.16	42.50	28.59	14.42	13.41
8 WF 35	10.30	370.8	3.50	2.03	34.7	104.1	101.6	94.00	82.96	71.70	60.25	48.60	36.74	24.70	12.45	11.65
8 WF 28	8.23	296.3	3.45	1.62	27.1	81.3	79.52	74.17	65.84	56.90	47.81	38.56	29.15	19.59	9.872	9.185
8 WF 24	7.06	254.2	3.42	1.61	23.1	69.3	67.77	63.20	55.97	48.32	40.55	32.67	24.68	16.57	8.340	7.837
8 WF 20	5.88	211.7	3.43	1.20	19.1	57.3	56.25	53.12	47.90	41.30	34.67	27.93	21.10	14.16	7.130	6.570
8 WF 17	5.00	180.0	3.36	1.16	15.8	47.4	46.58	44.14	40.06	34.71	29.11	23.43	17.68	11.86	5.964	5.454
8 M 34.3	10.09	363.2	3.40	1.87	32.7	98.1	96.06	89.96	80.08	69.21	58.15	46.90	35.46	23.83	12.01	11.14
8 M 32.6	9.59	345.2	3.45	1.90	31.6	94.8	92.00	85.99	70.30	65.92	55.37	44.64	33.74	22.67	11.42	10.65
8 M 28	8.23	296.3	3.31	1.47	25.6	76.8	75.50	71.59	65.08	56.51	47.47	38.28	28.94	19.45	9.800	8.827
8 M 24	7.06	254.2	3.45	1.53	23.3	69.9	68.34	63.67	56.49	48.76	40.92	32.97	24.90	16.71	8.414	7.894

TABLE II—1

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A 36 Steel σ_y = 36 ksi

Section	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
							$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
6 M 25	7.35	264.6	2.53	1.43	17.9	53.7	52.41	48.52	42.96	37.23	31.37	25.37	19.23	12.96	6.547	5.992
6 M 22.5	6.62	238.3	2.49	1.36	15.6	46.8	45.92	43.29	39.05	33.80	28.43	22.96	17.39	11.70	5.904	5.341
6 M 20	5.88	211.7	2.57	1.39	14.6	43.8	42.76	39.65	34.90	30.18	25.37	20.47	15.48	10.41	5.248	4.915
6 WF 25	7.37	265.3	2.69	1.52	19.0	57.0	55.73	51.91	46.01	39.84	33.54	27.10	20.52	13.82	6.975	6.419
6 WF 20	5.90	212.4	2.66	1.50	14.6	43.8	42.79	39.75	36.28	31.36	26.35	21.25	16.07	10.80	5.444	4.901
6 B 16	4.72	169.9	2.59	0.96	11.6	34.8	34.16	32.23	28.94	25.06	21.08	17.04	12.90	8.684	4.384	3.978
6 B 12	3.53	127.1	2.48	0.90	8.3	24.9	24.49	23.27	21.24	18.22	15.30	12.33	9.321	6.261	3.154	2.874
5 M 18.9	5.56	200.2	2.08	1.20	11.1	33.3	32.56	30.34	26.92	23.35	19.69	15.94	12.09	8.155	4.124	3.745
5 WF 18.5	5.45	196.2	2.16	1.28	11.4	34.2	33.36	30.84	27.13	23.52	19.82	16.03	12.16	8.194	4.141	3.814
5 WF 16	4.70	169.2	2.13	1.26	9.6	28.8	28.11	26.04	23.05	19.90	16.80	13.57	10.28	6.917	3.492	3.221
4 WF 13	3.82	137.5	1.72	0.99	6.2	18.9	18.51	17.34	15.36	13.33	11.24	9.103	6.908	4.660	2.357	2.140
4 M 13	3.82	137.5	1.65	0.94	6.1	18.3	17.86	16.55	14.68	12.75	10.77	8.723	6.626	4.473	2.264	2.041

TABLE II—1

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A441 Steel

Section	σ_y	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
								$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	ksi	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
14 WF 426	42	125.3	5261	7.26	4.34	869.3	3043	2969	2751	2465	2162	1843	1507	1155	786.4	401.4	335.7
14 WF 398	42	117.0	4913	7.17	4.31	803.0	2811	2743	2537	2270	1989	1694	1384	1060	720.8	367.6	309.9
14 WF 370	42	108.8	4569	7.08	4.27	737.3	2581	2518	2330	2083	1823	1550	1266	968.0	657.9	335.2	285.0
14 WF 342	42	100.6	4225	6.99	4.24	673.0	2356	2298	2127	1899	1660	1410	1150	878.6	596.6	303.7	260.4
14 WF 320	42	94.12	3953	6.63	4.17	592.2	2073	2032	1909	1711	1494	1268	1033	788.9	535.2	272.2	233.8
14 WF 314	42	92.30	3877	6.90	4.20	611.5	2140	2088	1927	1719	1501	1274	1037	791.7	537.0	273.1	236.3
14 WF 287	42	84.37	3544	6.81	4.17	551.6	1931	1883	1738	1548	1350	1144	931.0	709.8	480.9	244.3	213.4
14 WF 264	42	77.63	3260	6.74	4.14	502.4	1758	1715	1583	1408	1226	1039	844.0	642.9	435.2	220.9	194.5
14 WF 246	42	72.33	3038	6.68	4.12	464.5	1626	1585	1462	1299	1131	950.7	776.8	591.2	399.9	202.8	179.9
14 WF 237	42	69.69	2927	6.65	4.11	445.4	1559	1520	1402	1245	1083	916.2	743.6	565.7	382.5	193.9	172.6
14 WF 228	42	67.06	2817	6.62	4.10	427.2	1495	1458	1343	1193	1037	876.8	711.4	541.0	365.6	185.3	165.4
14 WF 219	42	64.36	2703	6.59	4.08	408.0	1428	1392	1283	1139	990.0	836.5	678.3	515.6	348.3	176.4	158.1
14 WF 211	46	62.07	2855	6.56	4.07	391.7	1502	1464	1349	1197	1040	878.4	712.1	541.1	365.4	185.0	166.3
14 WF 202	46	59.39	2732	6.54	4.06	373.6	1432	1396	1286	1140	990.2	835.9	677.3	514.4	347.3	175.8	158.6
14 WF 193	46	56.73	2610	6.51	4.05	355.1	1361	1327	1223	1084	940.5	793.6	642.7	487.9	329.2	166.6	150.9
14 WF 184	46	54.07	2487	6.49	4.04	337.5	1294	1260	1161	1028	891.9	752.2	608.9	462.1	311.6	157.6	143.3
14 WF 176	46	51.73	2380	6.45	4.02	321.3	1232	1200	1107	978.1	848.2	715.0	578.6	438.9	295.8	149.6	136.7
14 WF 167	46	49.09	2240	6.42	4.01	302.9	1161	1132	1043	923.3	800.3	674.3	545.4	413.4	278.6	140.8	128.8
14 WF 158	46	46.47	2138	6.40	4.00	286.3	1097	1069	983.6	870.0	753.7	634.7	513.1	388.8	261.9	132.3	121.6
14 WF 150	46	44.08	2028	6.37	3.99	270.2	1036	1009	928.9	821.2	711.1	598.6	483.7	366.3	246.6	124.5	114.9
14 WF 142	46	41.85	1925	6.32	3.97	254.8	976.7	952.1	878.0	775.1	670.9	564.5	455.9	345.2	232.3	117.2	108.7
14 WF 136	50	39.98	1999	6.31	3.77	242.7	1011	986.0	910.3	804.6	696.4	586.0	473.3	358.4	241.2	121.7	112.7
14 WF 127	50	37.33	1867	6.29	3.76	225.9	941.3	917.5	846.1	747.4	646.6	543.8	439.0	332.2	223.4	112.7	104.8
14 WF 74	50	21.76	1088	6.05	2.48	125.6	523.3	512.4	479.5	426.3	368.3	309.4	249.5	188.6	126.7	63.84	59.38
14 WF 68	50	20.00	1000	6.02	2.46	114.8	478.3	468.4	438.5	389.7	336.6	282.5	227.7	172.0	115.5	58.17	54.33
14 WF 53	50	15.59	779.5	5.90	1.92	87.1	362.9	356.1	335.5	301.5	260.4	218.5	176.1	133.0	89.30	44.96	41.51

TABLE II—2

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A441 Steel

Section	σ_y	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
								$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	ksi	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
14 WF 48	50	14.11	705.5	5.86	1.91	78.5	327.1	321.0	302.6	272.0	234.3	196.5	158.3	119.5	80.16	40.34	37.47
12 WF 190	46	55.86	2570	5.82	3.25	311.5	1194	1166	1081	962.1	838.8	710.8	578.1	440.6	298.5	151.6	132.8
12 WF 161	46	47.38	2180	5.70	3.20	259.2	993.6	969.8	898.5	798.1	694.4	587.3	476.7	362.7	245.2	124.3	110.6
12 WF 133	46	39.11	1799	5.59	3.16	209.7	803.9	784.4	726.2	644.0	559.1	471.9	382.2	290.2	195.9	99.11	89.58
12 WF 120	46	35.31	1624	5.51	3.13	186.4	714.5	697.7	647.2	574.0	497.8	419.7	339.7	257.7	173.7	87.82	79.91
12 WF 106	50	31.19	1560	5.46	3.11	163.4	680.8	664.5	615.5	545.3	472.3	397.8	321.5	243.6	164.1	82.86	76.08
12 WF 99	50	29.09	1455	5.43	3.09	151.8	632.5	617.3	571.7	505.5	437.6	368.3	297.5	225.3	151.6	76.55	70.74
12 WF 92	50	27.06	1353	5.40	3.08	140.2	584.2	570.2	528.2	467.3	404.3	340.0	274.5	207.8	139.8	70.52	65.38
12 WF 50	50	14.71	735.5	5.18	1.96	72.6	302.5	296.4	278.2	247.8	214.1	179.8	145.0	109.6	73.60	37.08	34.45
12 WF 45	50	13.24	662.0	5.15	1.94	64.9	270.4	265.0	248.7	221.7	191.4	160.7	129.4	97.75	65.62	33.04	30.81
10 WF 112	50	32.92	1646	4.67	2.67	147.5	614.6	599.6	554.8	493.2	429.3	363.1	294.8	224.4	151.8	76.96	68.26
10 WF 100	50	29.43	1472	4.61	2.65	130.1	542.1	528.9	489.4	434.5	377.7	319.1	258.8	196.7	132.9	67.31	60.30
10 WF 89	50	26.19	1310	4.55	2.63	114.4	476.7	465.0	430.2	381.5	331.1	279.4	226.3	171.8	115.9	58.67	53.08
10 WF 77	50	22.67	1134	4.49	2.60	97.7	407.1	397.1	367.1	325.3	282.0	237.7	192.2	145.7	98.21	49.63	45.35
10 WF 72	50	21.18	1059	4.46	2.59	90.7	377.9	368.8	341.3	301.8	261.4	220.2	178.0	134.9	90.82	45.87	42.20
10 WF 66	50	19.41	970.5	4.44	2.58	82.8	345.0	336.4	310.7	274.8	237.9	200.2	161.7	122.4	82.40	41.59	38.44
10 WF 45	50	13.24	662.0	4.33	2.00	55.0	229.2	223.9	208.3	184.2	159.3	133.9	108.0	81.69	54.92	27.69	25.80
8 WF 67	50	19.70	985.0	3.71	2.12	70.1	292.1	285.1	264.0	234.7	204.1	172.5	139.9	106.4	71.92	36.45	32.50
8 WF 58	50	17.06	853.0	3.65	2.10	59.9	249.6	243.6	225.8	199.6	173.3	146.3	118.5	89.98	60.72	30.73	27.87
8 WF 48	50	14.11	705.5	3.61	2.08	49.0	204.2	199.0	183.7	162.4	140.7	118.5	95.86	72.66	48.95	24.73	22.71

TABLE II—2

Reduced Plastic Moment Values For Various Axial Force Ratios
(For Sections Used as Columns)
A441 Steel

Section	σ_y	A	P_y	r_x	r_y	Z	M_p	M_{pc}									Aver. Increment
								$P/P_y=0.1$	$P/P_y=0.2$	$P/P_y=0.3$	$P/P_y=0.4$	$P/P_y=0.5$	$P/P_y=0.6$	$P/P_y=0.7$	$P/P_y=0.8$	$P/P_y=0.9$	
	ksi	in^2	kips	in	in	in^3	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
8 WF 40	50	11.76	588.0	3.53	2.04	39.9	166.3	162.3	150.5	132.7	114.9	96.60	78.00	59.03	39.71	20.03	18.63
8 WF 28	50	8.23	411.5	3.45	1.62	27.1	112.9	110.4	103.0	91.45	79.03	66.40	53.55	40.49	27.21	13.71	12.76
8 WF 20	50	5.88	294.0	3.43	1.20	19.1	79.58	78.13	73.77	66.51	57.37	48.15	38.79	29.30	19.67	9.903	9.124
6 M 25	50	7.35	367.5	2.53	1.43	17.9	74.58	72.79	67.39	59.67	51.71	43.57	35.23	26.71	18.00	9.093	8.328
6 WF 25	50	7.37	368.5	2.69	1.52	19.0	79.17	77.40	72.09	63.90	55.33	46.58	37.63	28.50	19.19	9.688	8.915
6 B 16	50	4.72	236.0	2.59	0.96	11.6	48.33	47.44	44.76	40.20	34.80	29.29	23.66	17.92	12.06	6.088	5.525
6 B 12	50	3.53	176.5	2.48	0.90	8.3	34.58	34.02	32.33	29.30	25.31	21.25	17.13	12.95	8.695	4.380	3.992
5 M 18.9	50	5.56	278.0	2.08	1.20	11.1	46.25	45.22	42.13	37.39	32.43	27.35	22.14	16.80	11.33	5.727	5.201
5 WF 18.5	50	5.45	272.5	2.16	1.28	11.40	47.50	46.33	42.83	37.68	32.66	27.53	22.27	16.89	11.38	5.752	5.297
5 WF 16	50	4.70	235.0	2.13	1.26	9.60	40.00	39.04	36.16	32.02	27.72	23.33	18.85	14.27	9.607	4.850	4.474
4 WF 13	50	3.82	191.0	1.72	0.99	6.30	26.25	25.71	24.09	21.34	18.52	15.62	12.64	9.595	6.472	3.273	2.972
4 M 13	50	3.82	191.0	1.65	0.94	6.10	25.42	24.81	22.98	20.39	17.71	14.95	12.12	9.203	6.212	3.145	2.834

TABLE II—2

PART III - MOMENT-ROTATION CURVES FOR BEAM-COLUMNS

For specified values of axial load ratio P/P_y and slenderness ratio h/r_x, these curves give the relationship between end moment and end rotation for beam-columns with the following three types of end conditions:

1. Moment applied at one end and pinned at the other end, $q = 0$. Charts III-1 to III-7.
2. Equal end moments causing symmetrical single curvature bending, $q = -1.0$. Charts III-8 to III-14.
3. Moment applied at one end and fixed at the other end. Charts III-15 to III-21.

The curves for the case of $q = 0$ can also be used for columns bent in symmetrical double curvature ($q = 1.0$). An equivalent slenderness ratio equal to one-half of the actual slenderness ratio should be used to enter the curves.

The basic information used in preparing the curves was the column deflection curves (CDC's), whose properties are described in Lecture 9. In calculating the needed CDC's, the moment-thrust-curvature relationships shown in Fig. 4.7 were used. These relationships include the effect of cooling residual stresses present in the members (Fig. 4.4). All the moment-rotation curves are for A36 steel columns ($\sigma_y = 36$ ksi) subjected to bending moments about the strong axis. The end moments are nondimensionalized with respect to the reduced plastic moments corresponding to the specified P/P_y ratios. The rotations are given directly in radians. All the curves terminate at the rotations corresponding to the occurrence of local buckling at the highly strained regions in the columns. (Lecture 4, Ref. 4.34)

The curves can also be used for columns having yield stresses different from 36 ksi. However, the slenderness ratios and rotations must be adjusted according to the following formulas:

$$\left(\frac{h}{r_x}\right)_{equ} \quad \left(\frac{h}{r_x}\right)_{\sigma_y} \times \sqrt{\frac{\sigma_y}{36}}$$

and

$$(\theta)_{\sigma_y} = (\theta)_{36} \times \sqrt{\frac{\sigma_y}{36}}$$

The required moment-rotation curves can be found from the appropriate charts for the equivalent slenderness ratios. The rotations may be determined directly, without the above adjustment, by using the second scale which gives the ratio $\theta/\sqrt{\epsilon_y}$. The desired end rotation is equal to the value read off from the scale multiplied by $\sqrt{\epsilon_y}$ or $\sqrt{\sigma_y/E}$

The moment-rotation curves have been used to obtain solutions to special restrained column problems (Lecture 10) and to develop design methods for columns in braced and unbraced frames (Lectures 11 and 18).

<u>Example</u>: A 10WF39 column (A36 steel) with $h/r_x = 35$ and $P/P_y = 0.40$ is subjected to equal end moments causing single curvature bending. Determine the maximum value of the end moment and the corresponding end rotation.

Entering Chart III-9 and locating the curve for $h/r_x = 35$, we find $M_m/M_{pc} = 0.80$. The M_{pc} value of the section is found from Table II-1 to be 98.3 kip-ft. Thus, $M_m = (0.80)\ (98.3) = 78.6$ kip-ft. The corresponding end rotation is $\theta_{peak} = 0.022$ radians.

Additional examples illustrating the applications of the curves to A441 columns are given in Lecture 4.

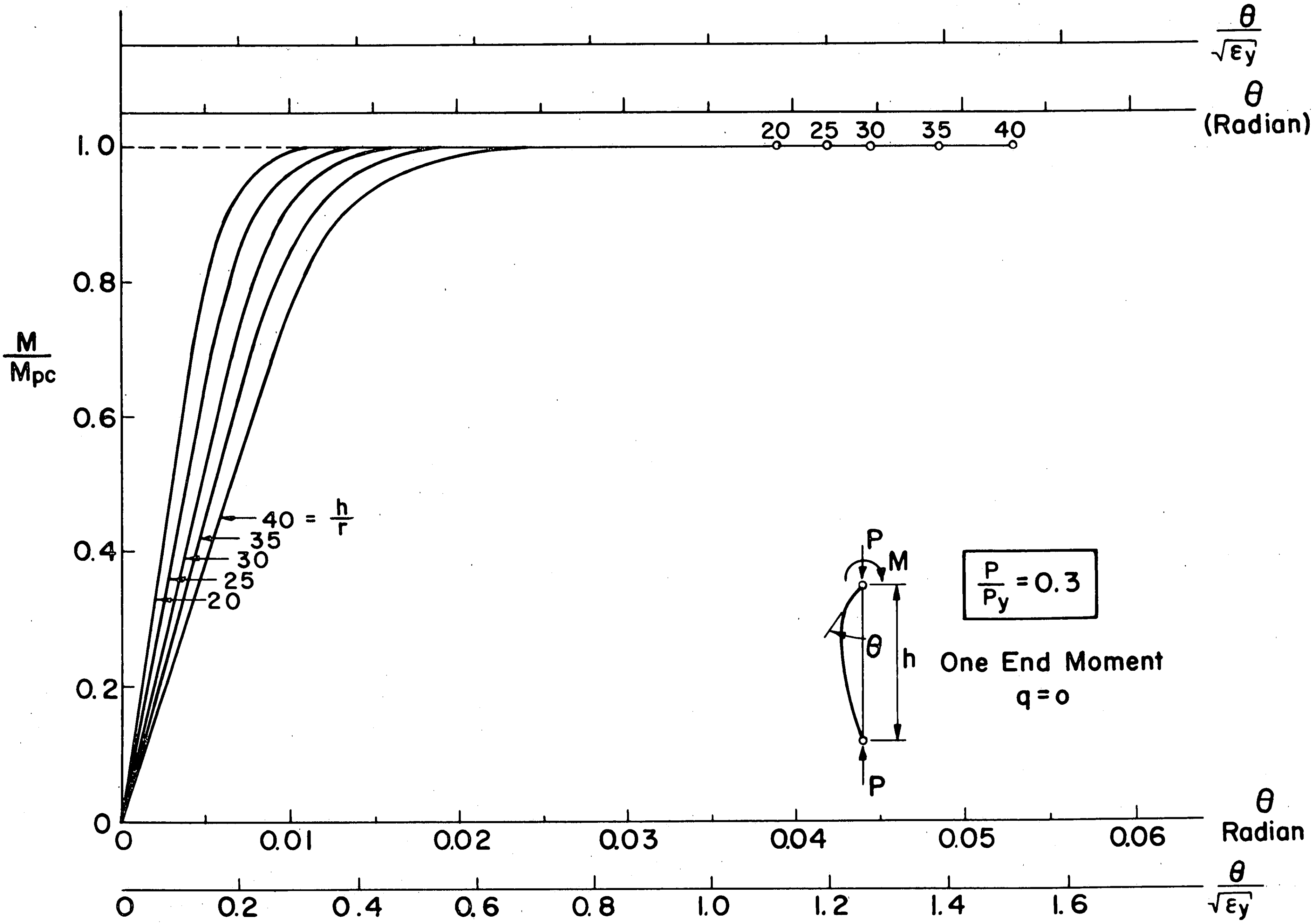
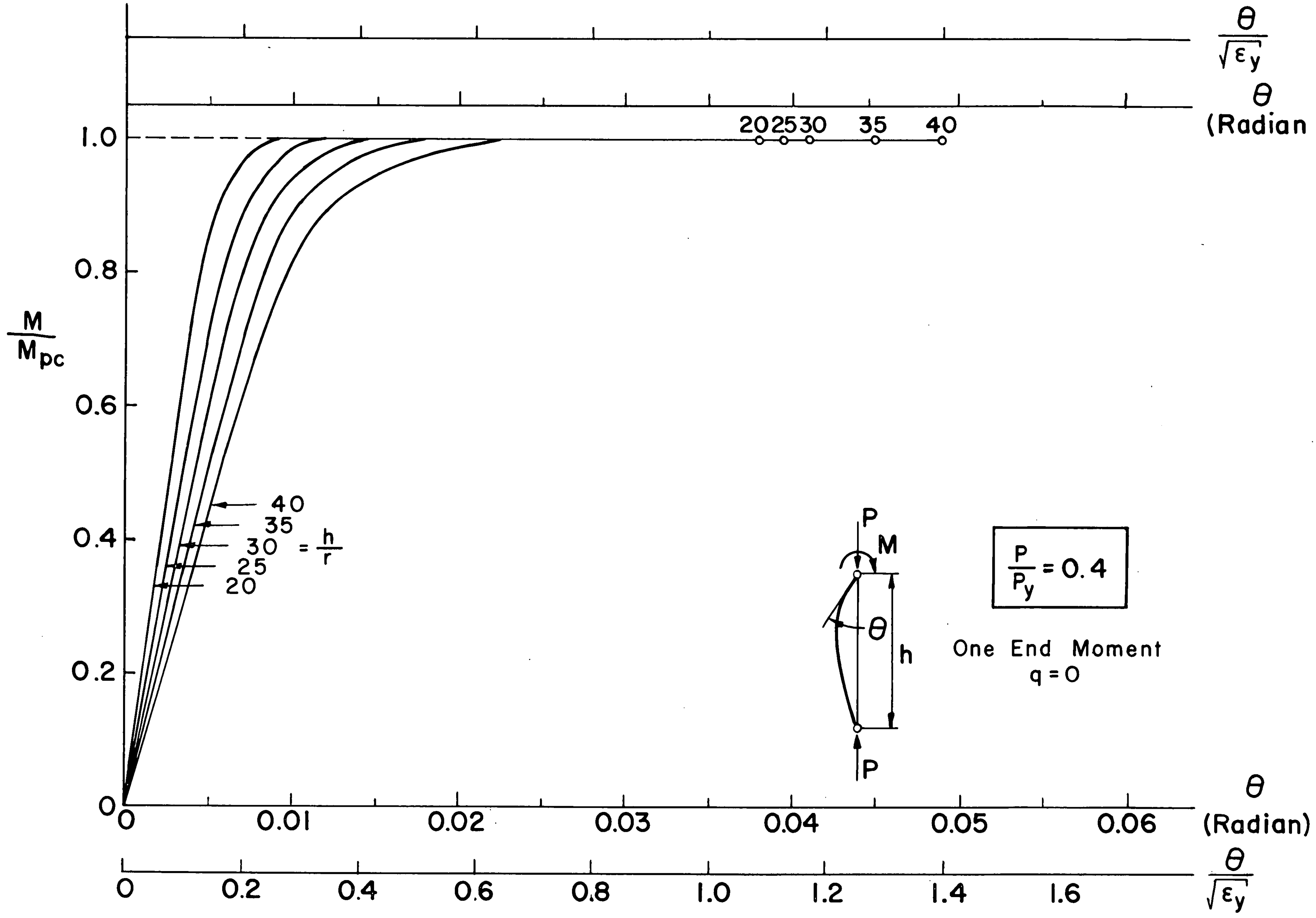
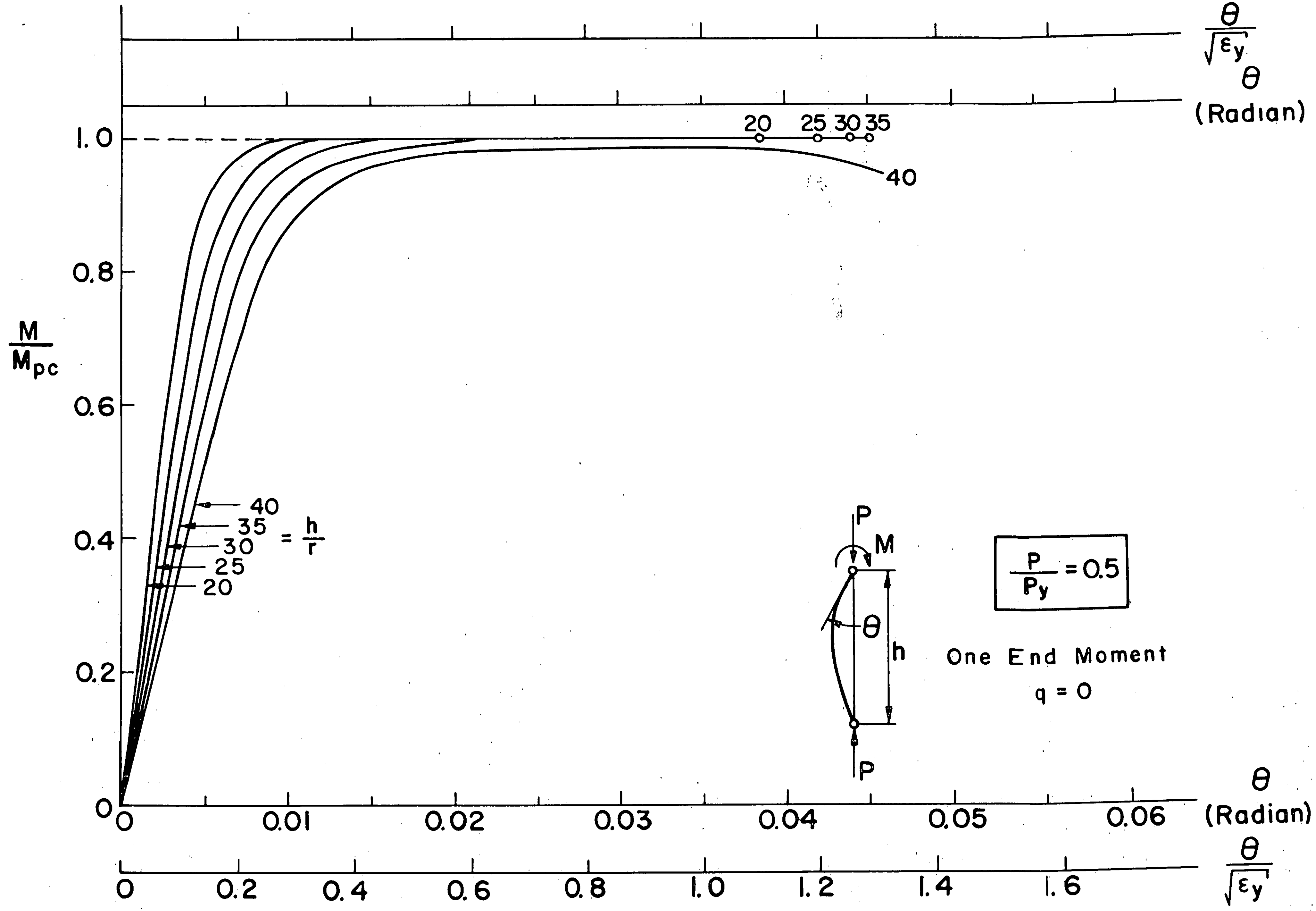

θ/√εy
θ (Radian)
20 25 30 35 40
1.0
0.8
0.6
0.4
0.2
0
M/Mpc
40 = h/r
35
30
25
20
P
M
θ
h
P
P/Py = 0.3
One End Moment
q = o
0 0.01 0.02 0.03 0.04 0.05 0.06
θ Radian
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
θ/√εy

CHART III—1



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
20 25 30 35 40
40
35
30 $= \frac{h}{r}$
25
20
P
M
θ
h
P
$\frac{P}{P_y} = 0.4$
One End Moment
q = 0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—2



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
20
25
30
35
40
40
35
30
25
20
$= \frac{h}{r}$
P
M
θ
h
P
$\frac{P}{P_y} = 0.5$
One End Moment
q = 0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—3

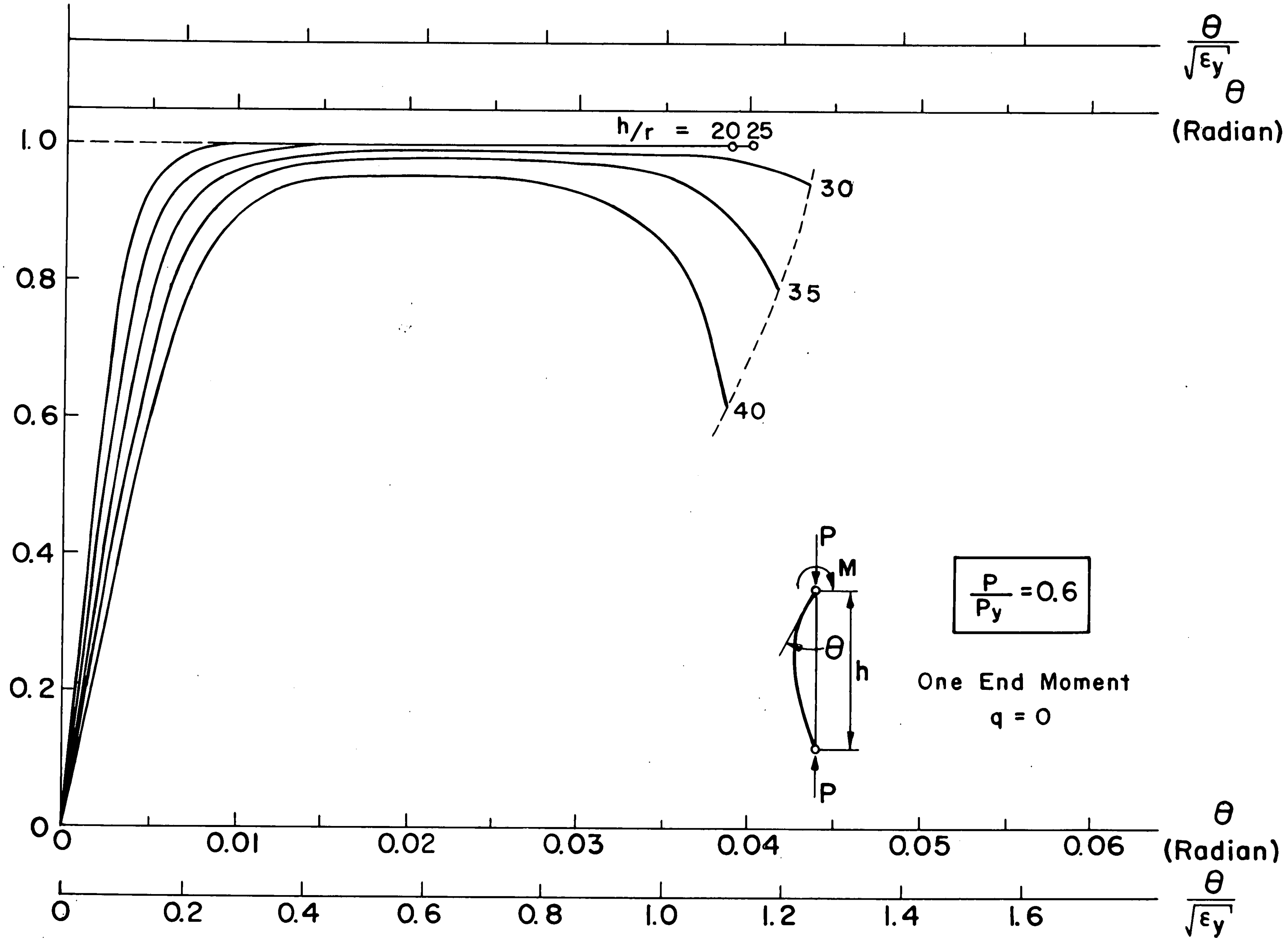

$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
h/r = 20 25
30
35
40
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
P
M
θ
h
P
$\frac{P}{P_y} = 0.6$
One End Moment
q = 0
0 0.01 0.02 0.03 0.04 0.05 0.06
θ (Radian)
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—4

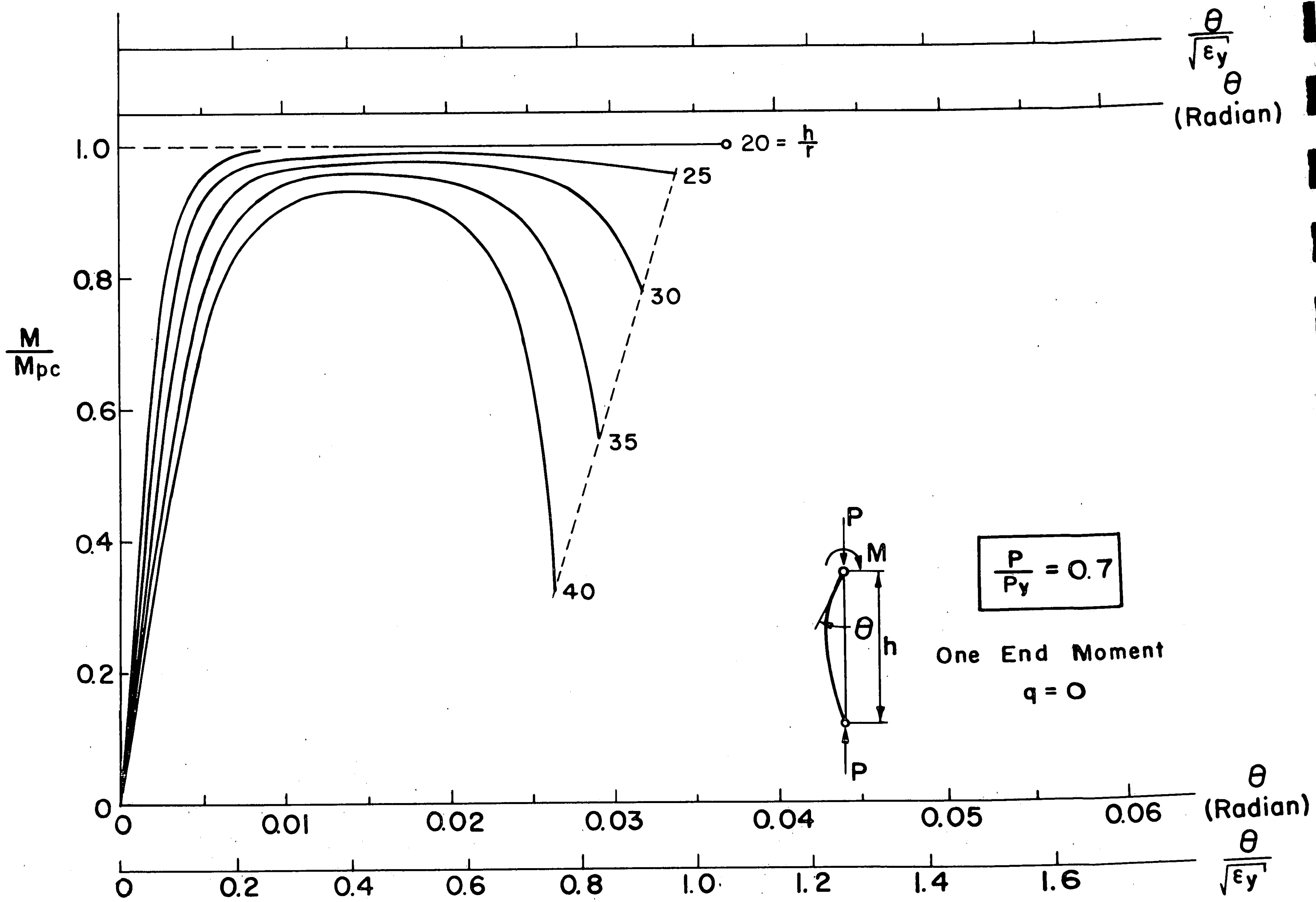
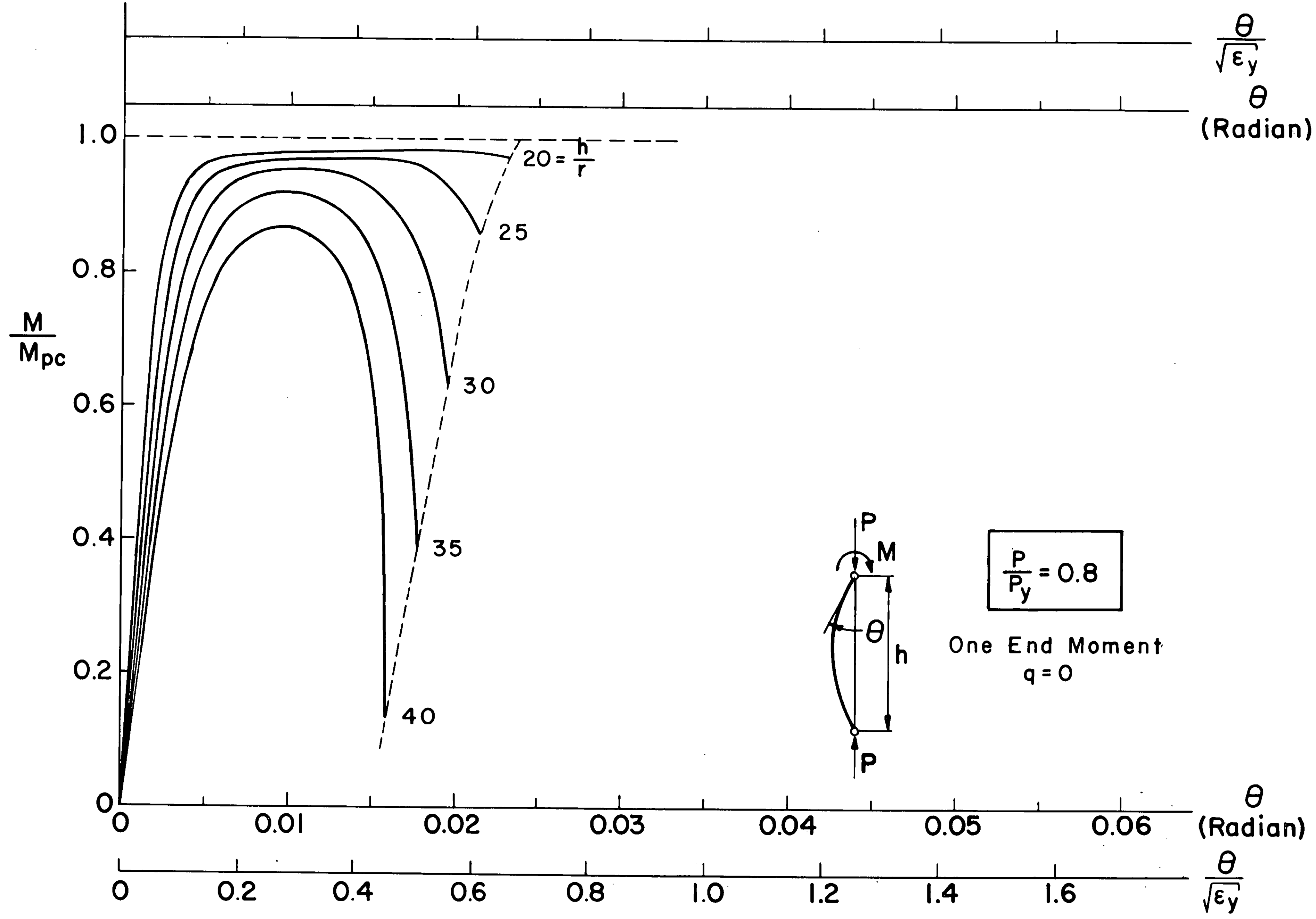
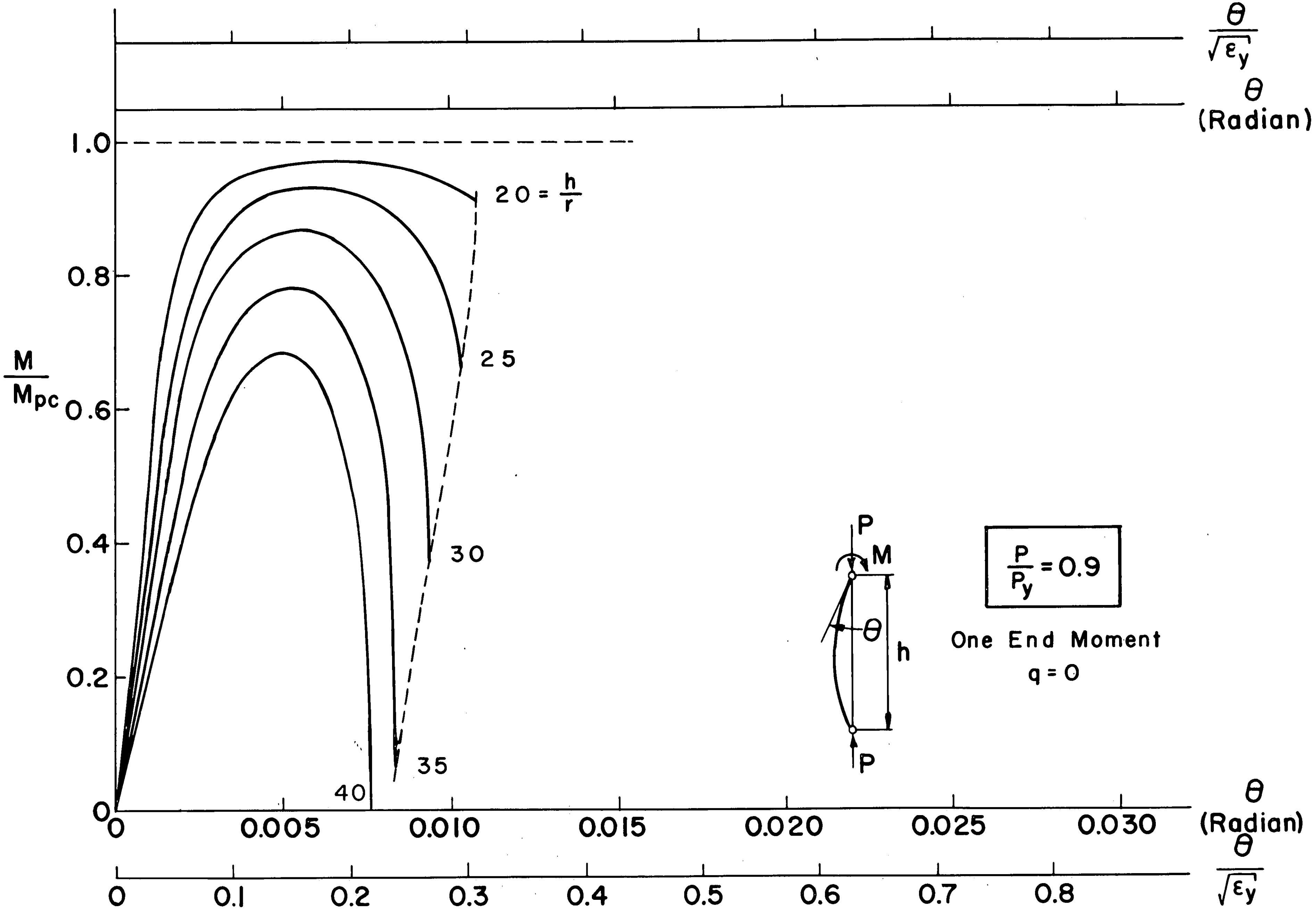

$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$20 = \frac{h}{r}$
25
30
35
40
P
M
θ
h
P
$\frac{P}{P_y} = 0.7$
One End Moment
q = 0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—5



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$20 = \frac{h}{r}$
25
30
35
40
P
M
θ
h
P
$\frac{P}{P_y} = 0.8$
One End Moment
q = 0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—6



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$20 = \frac{h}{r}$
25
30
35
40
P
M
θ
h
P
$\frac{P}{P_y} = 0.9$
One End Moment
q = 0
0
0.005
0.010
0.015
0.020
0.025
0.030
θ (Radian)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—7

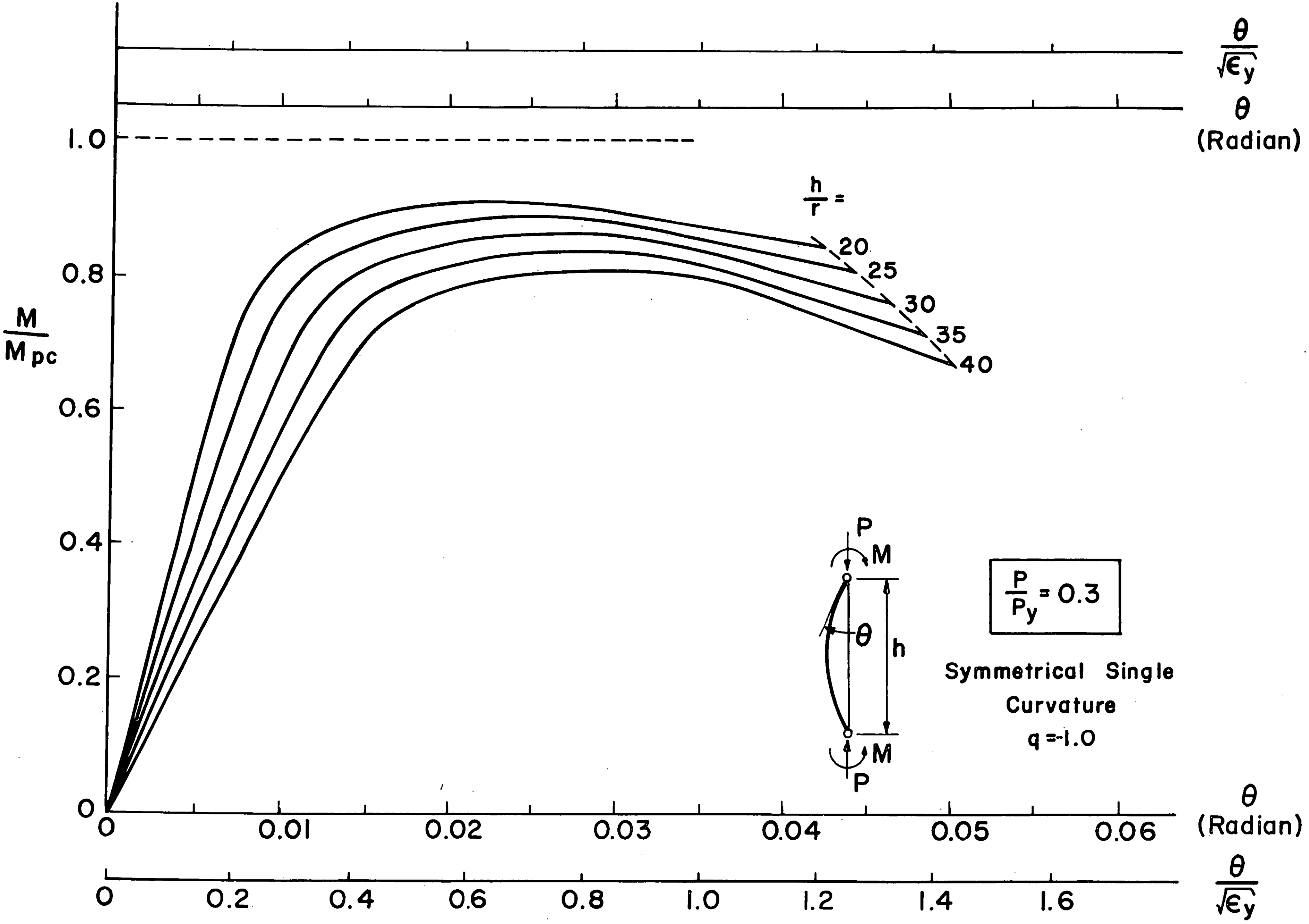

θ/√ε_y
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
h/r =
20
25
30
35
40
P
M
θ
h
P/P_y = 0.3
Symmetrical Single Curvature
q = -1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
θ/√ε_y

CHART III—8

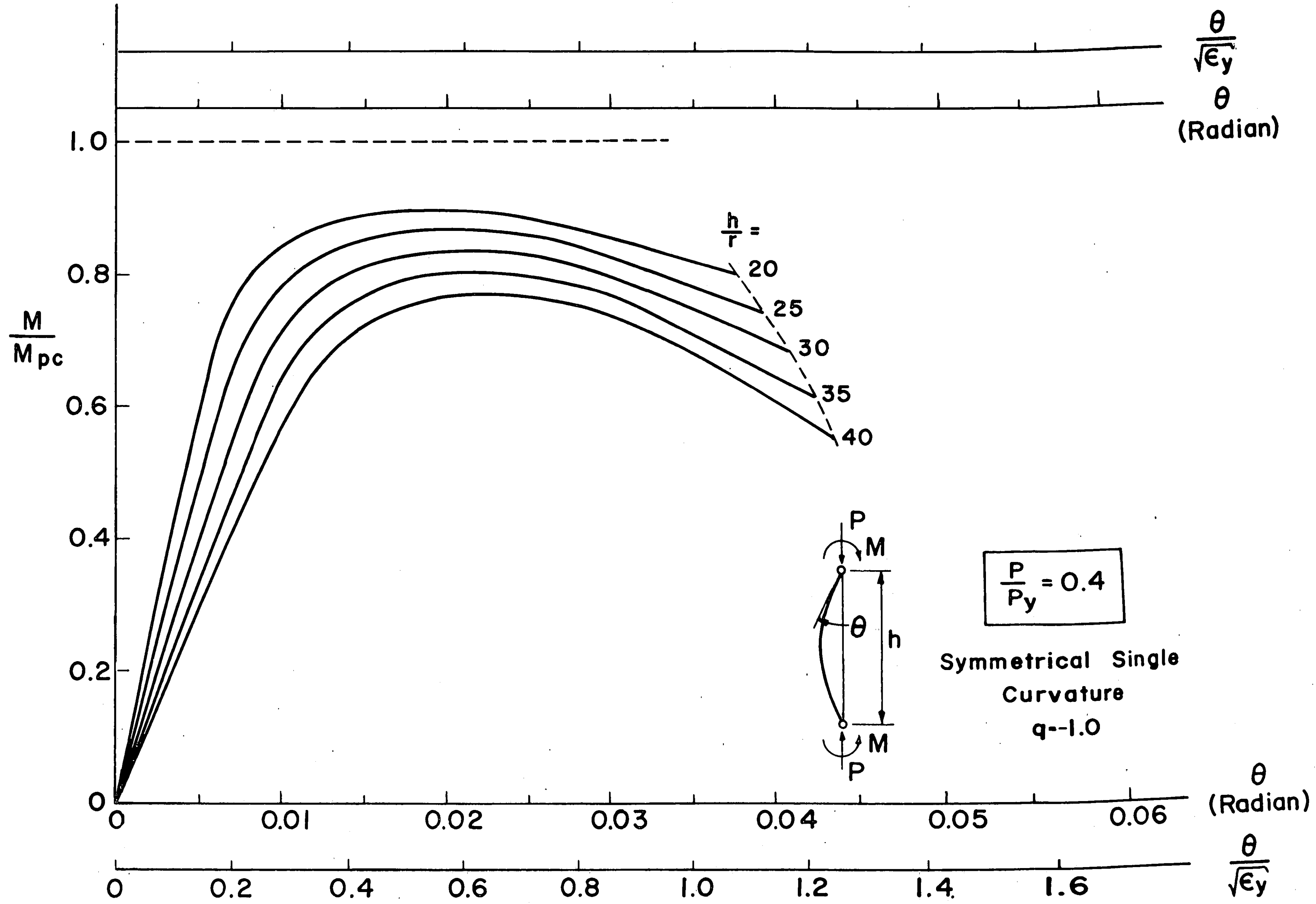
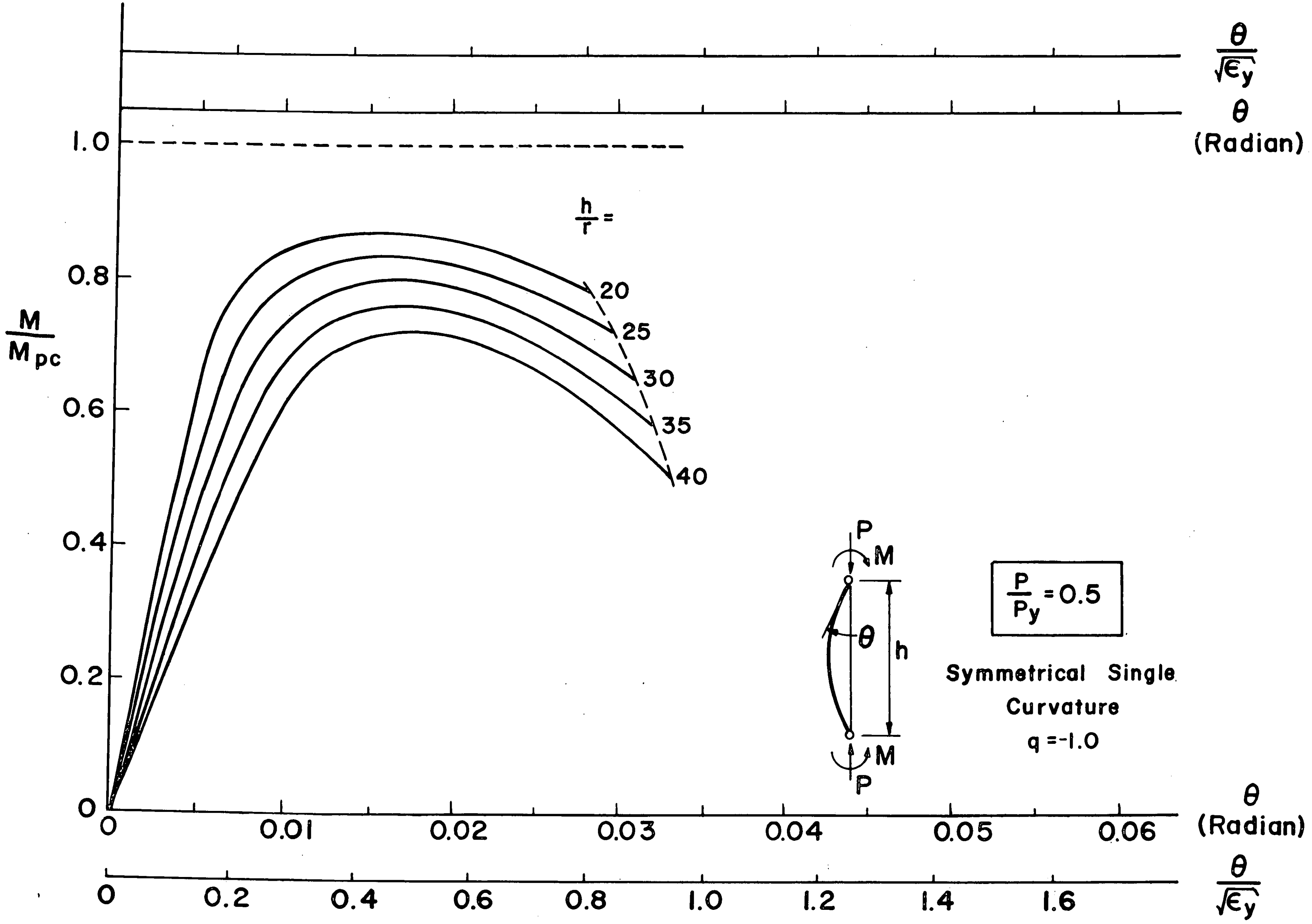

θ / √εy
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M / Mpc
h/r =
20
25
30
35
40
P
M
θ
h
M
P
P/Py = 0.4
Symmetrical Single Curvature
q=-1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
θ / √εy

CHART III—9



$\frac{\theta}{\sqrt{\epsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$\frac{h}{r}$ =
20
25
30
35
40
P
M
θ
h
P
M
$\frac{P}{P_y}$ = 0.5
Symmetrical Single Curvature
q = -1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\epsilon_y}}$

CHART III—10

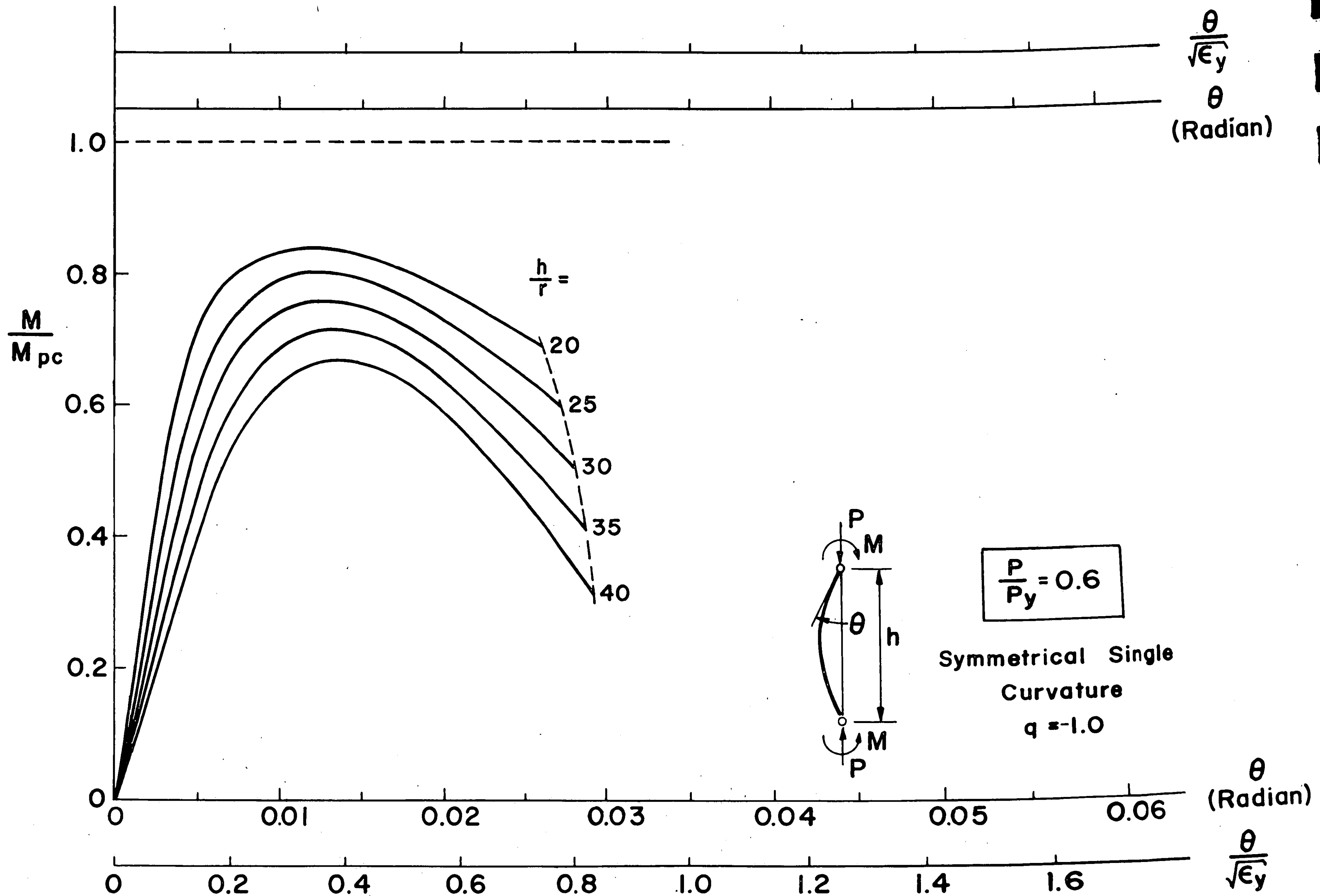
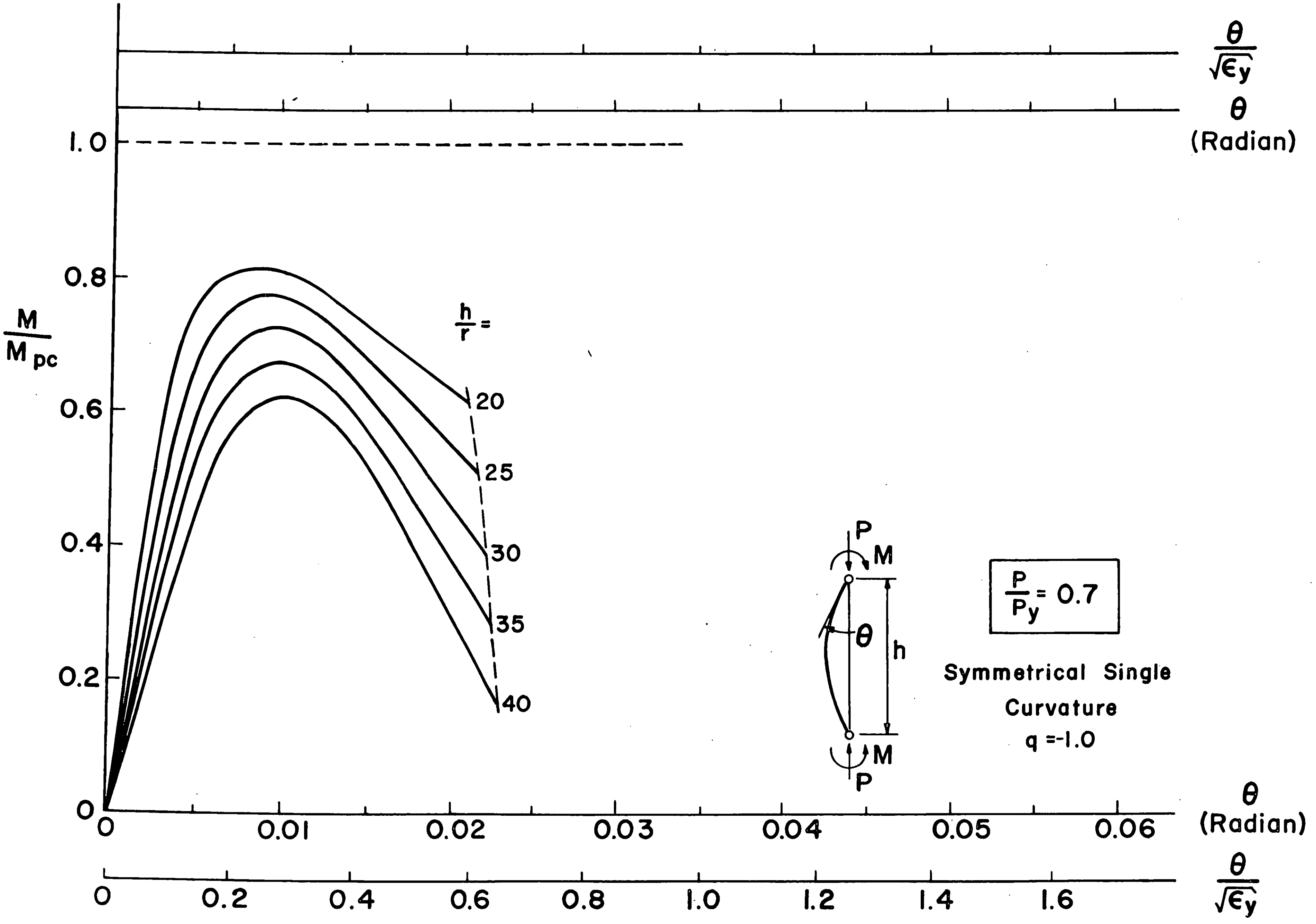

θ/√ε_y
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
h/r =
20
25
30
35
40
P
M
θ
h
M
P
P/Py = 0.6
Symmetrical Single Curvature
q = -1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
θ/√ε_y

CHART III—11



$\frac{\theta}{\sqrt{\epsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$\frac{h}{r}$ =
20
25
30
35
40
P
M
θ
h
M
P
$\frac{P}{P_y}$ = 0.7
Symmetrical Single Curvature
q =-1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\epsilon_y}}$

CHART III—12

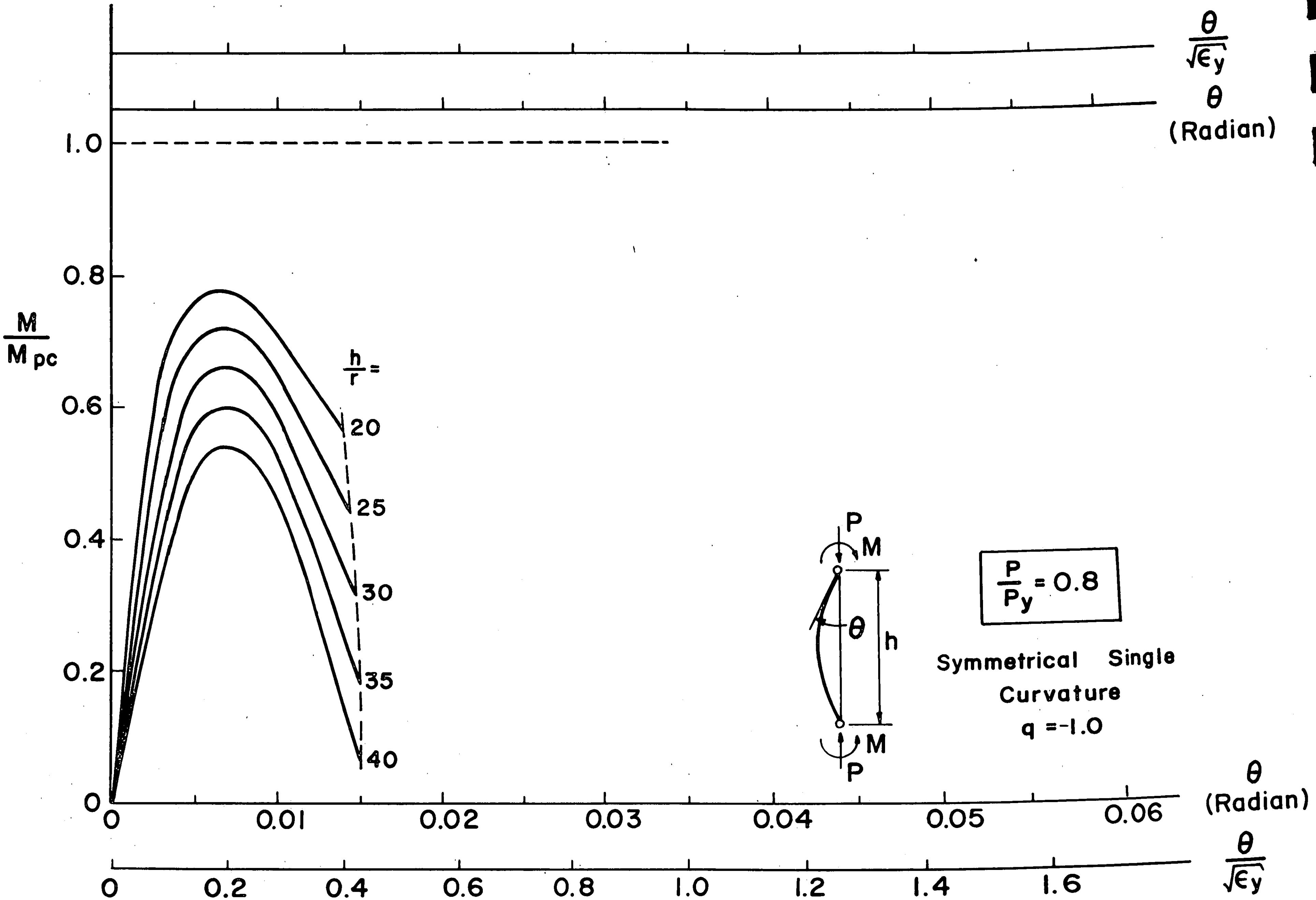
θ/√ε_y
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
h/r =
20
25
30
35
40
P
M
θ
h
M
P
P/Py = 0.8
Symmetrical Single Curvature
q = -1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
θ/√ε_y

CHART III—13

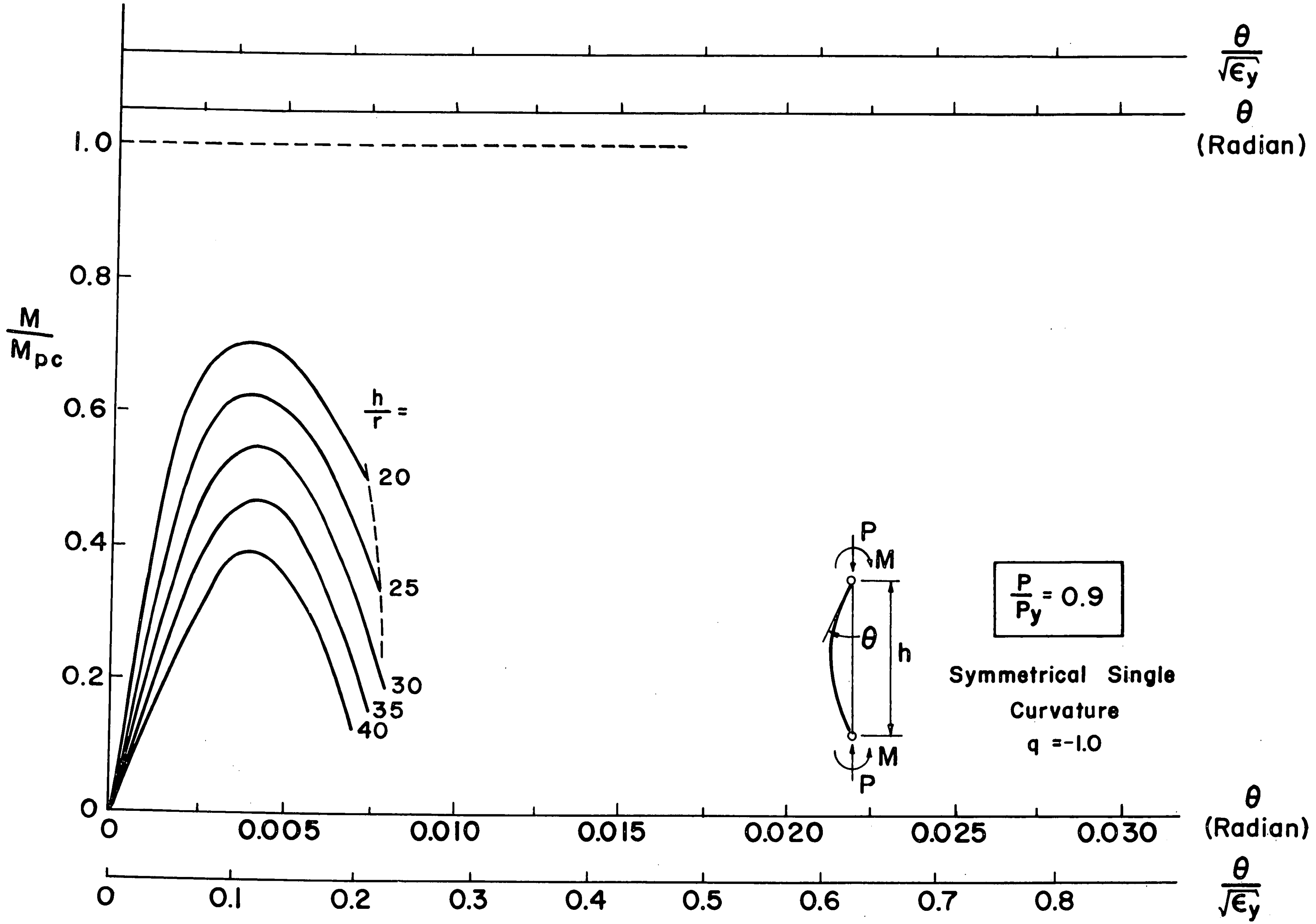
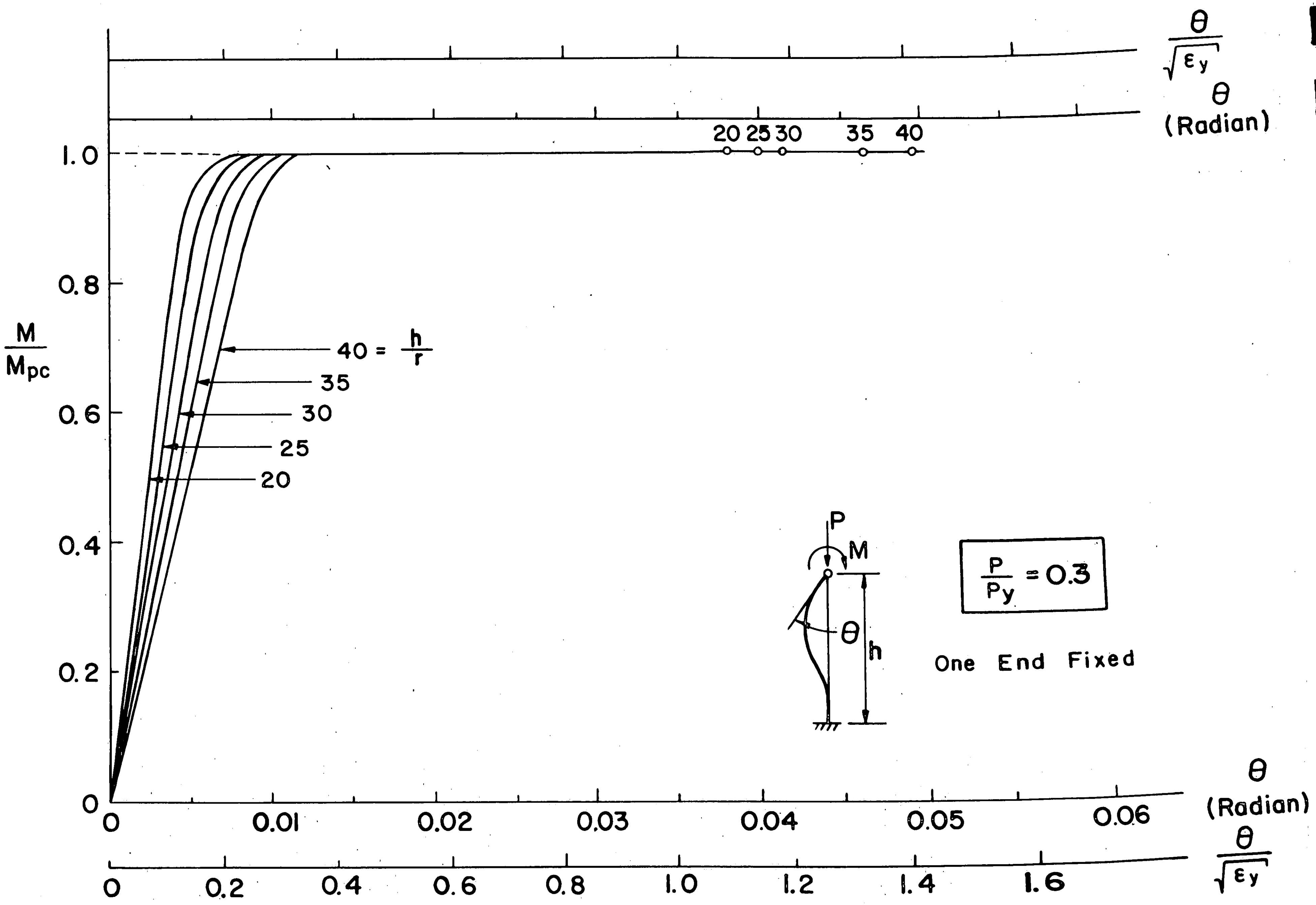
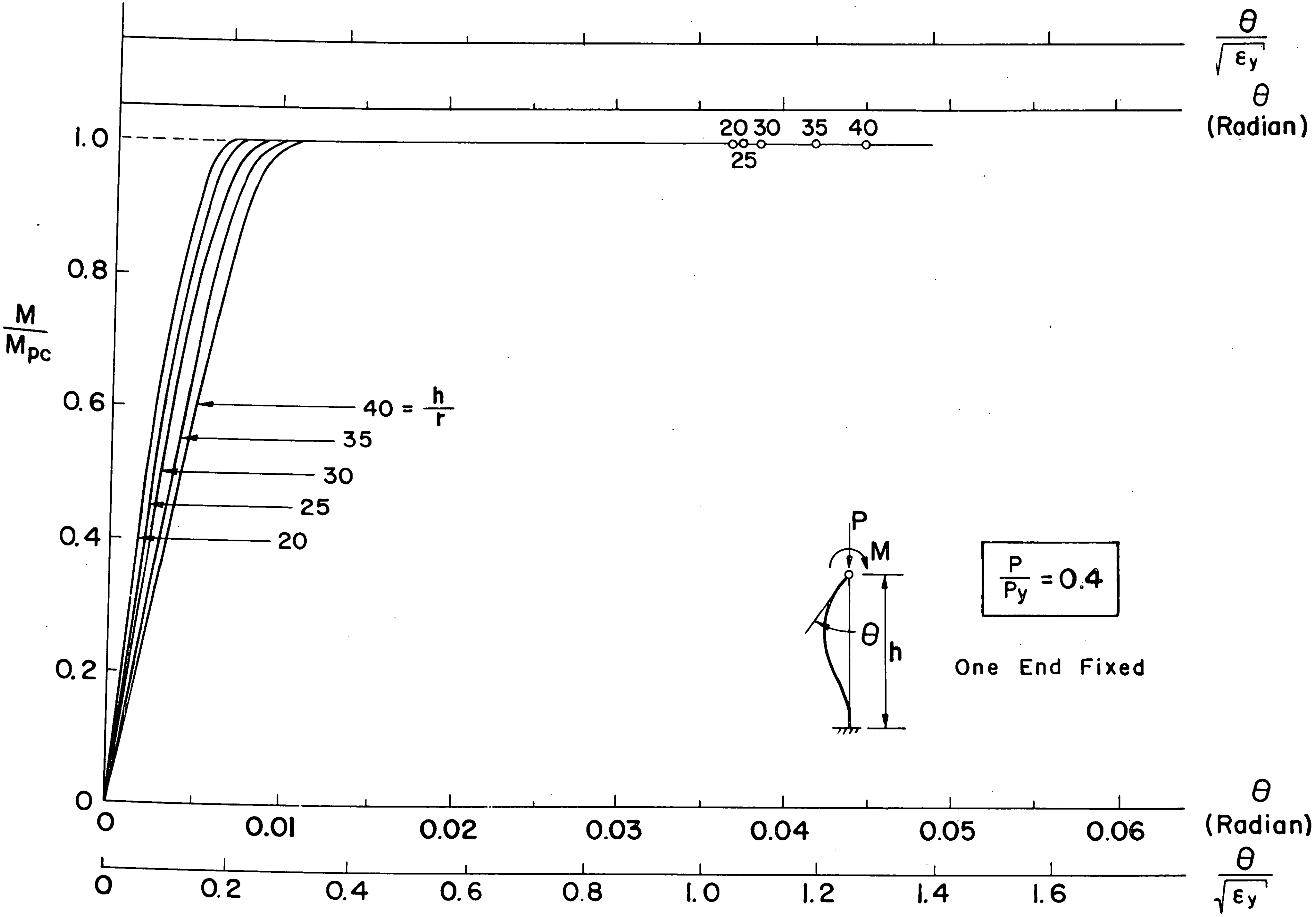

θ/√ε_y
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M/M_pc
h/r =
20
25
30
35
40
P
M
θ
h
M
P
P/Py = 0.9
Symmetrical Single Curvature
q = -1.0
0
0.005
0.010
0.015
0.020
0.025
0.030
θ (Radian)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
θ/√ε_y

CHART III—14



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
20 25 30 35 40
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$40 = \frac{h}{r}$
35
30
25
20
P
M
θ
h
$\frac{P}{P_y} = 0.3$
One End Fixed
0 0.01 0.02 0.03 0.04 0.05 0.06
θ (Radian)
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—15



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
20 30 35 40
25
$40 = \frac{h}{r}$
35
30
25
20
P
M
θ
h
$\frac{P}{P_y} = 0.4$
One End Fixed
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—16

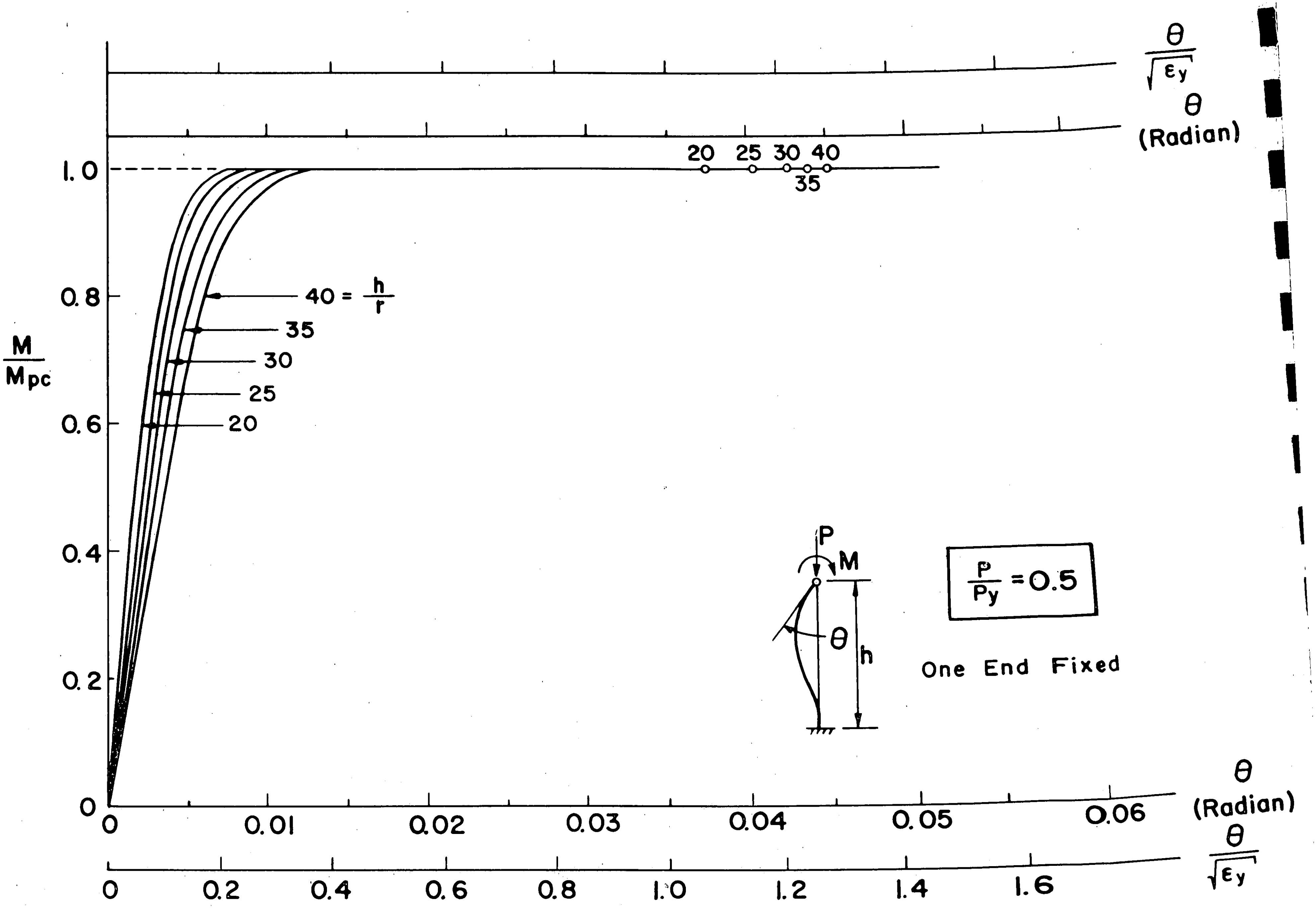
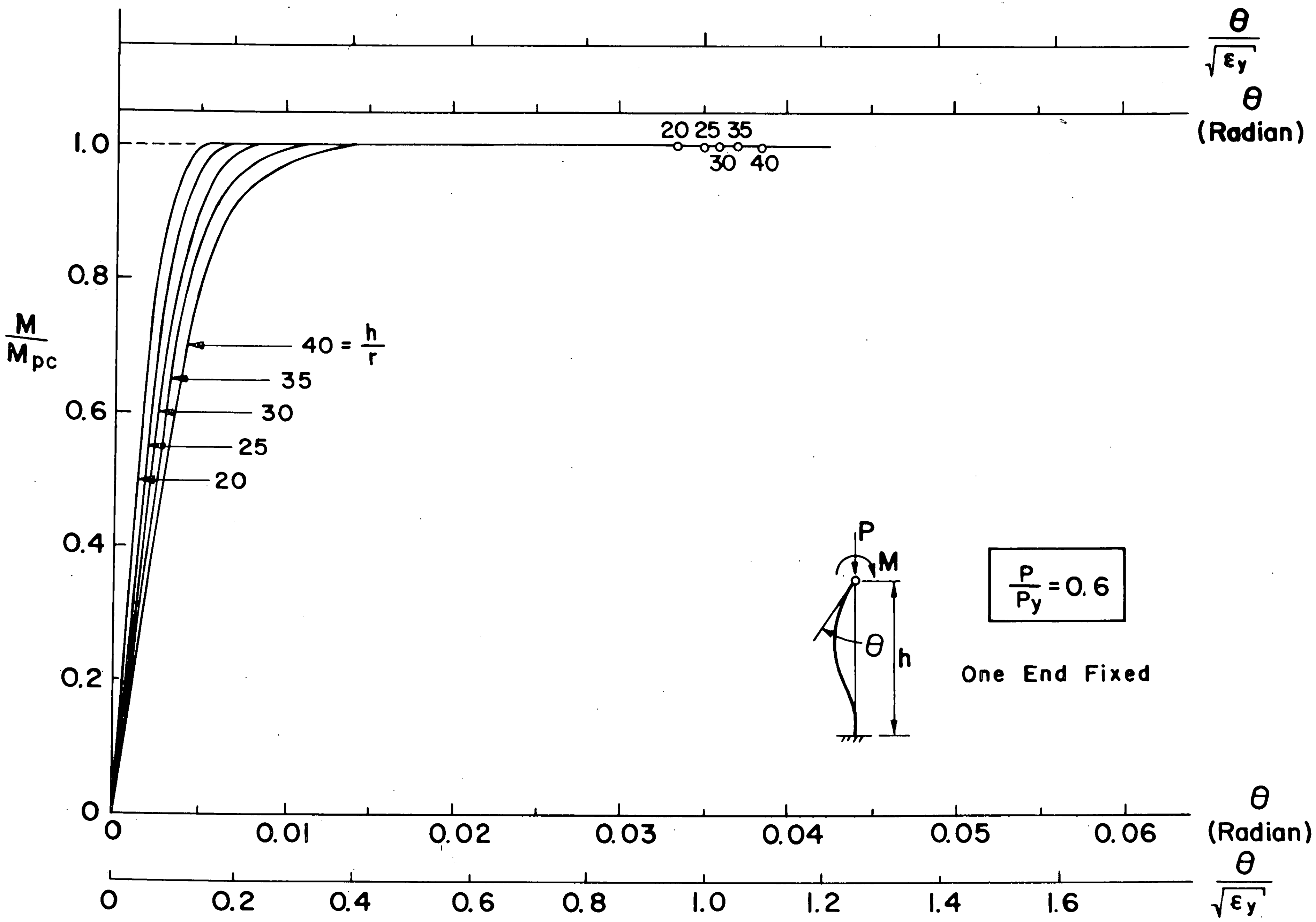
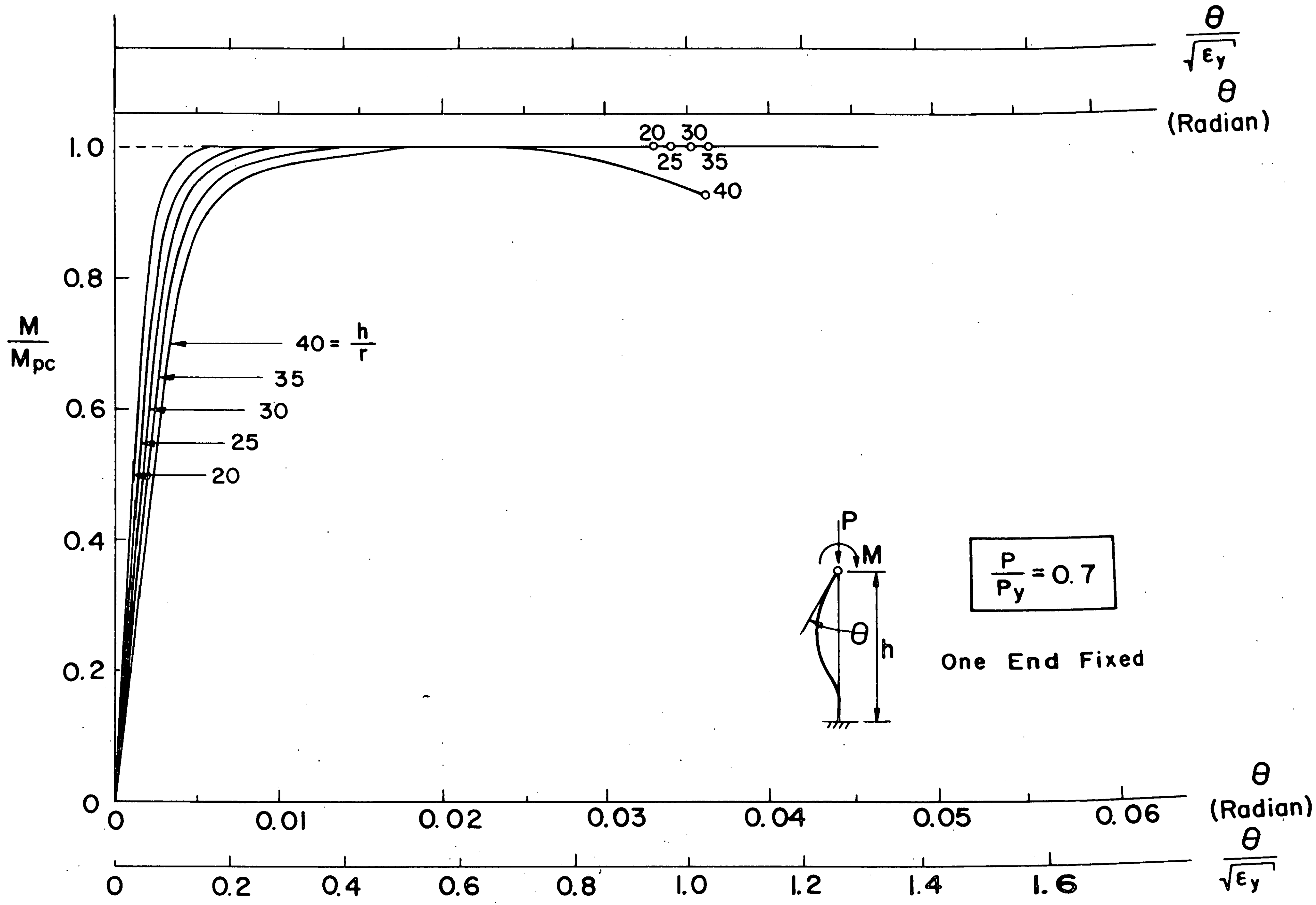

θ/√εy
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
M/Mpc
20 25 30 40
35
40 = h/r
35
30
25
20
P
M
θ
h
P/Py = 0.5
One End Fixed
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
θ/√εy

CHART III—17



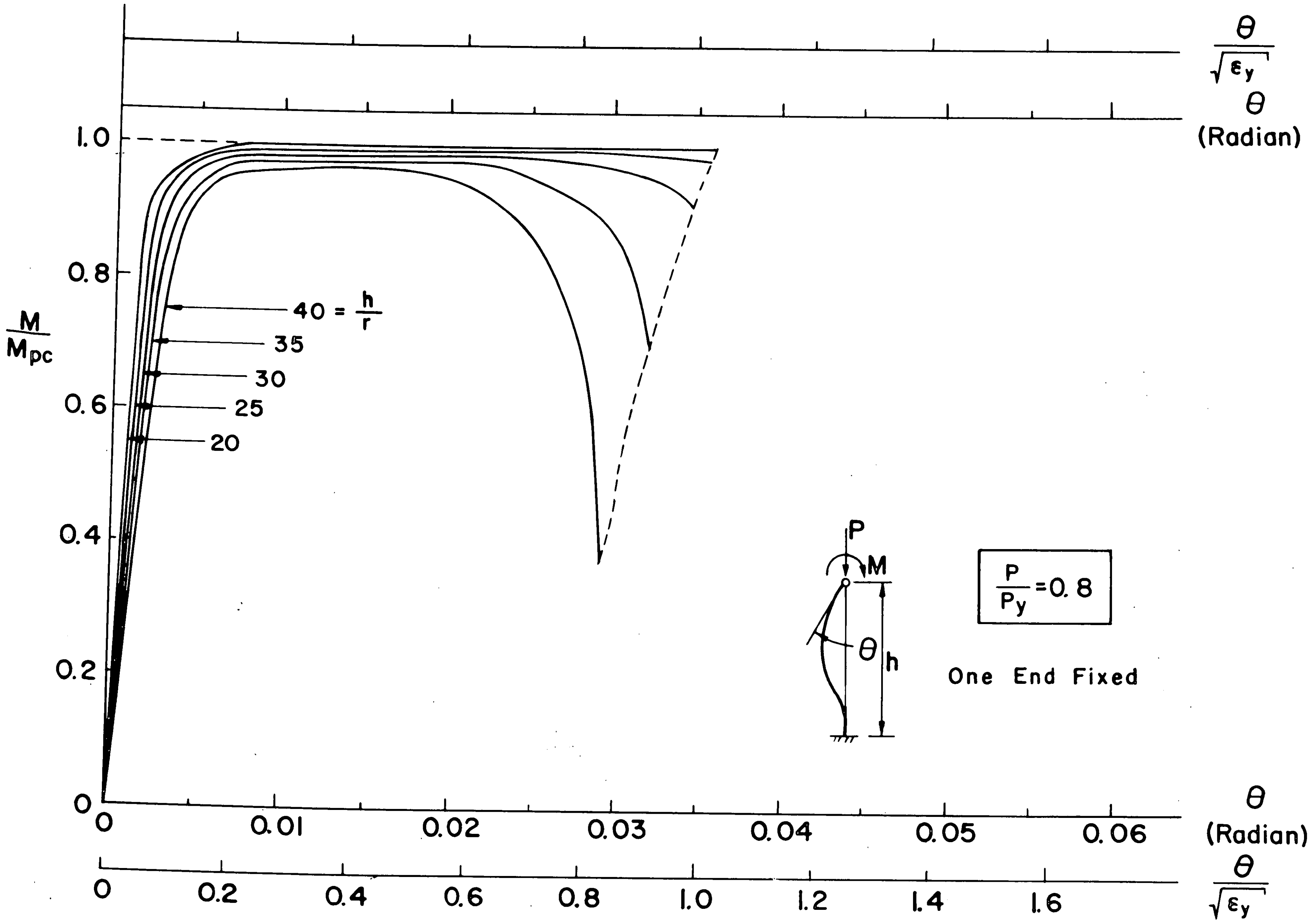
$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
20 25 35
30 40
40 = $\frac{h}{r}$
35
30
25
20
P
M
θ
h
One End Fixed
$\frac{P}{P_y} = 0.6$
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—18



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
20
30
25
35
40
$40 = \frac{h}{r}$
35
30
25
20
P
M
θ
h
One End Fixed
$\frac{P}{P_y} = 0.7$
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—19



$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$40 = \frac{h}{r}$
35
30
25
20
P
M
θ
h
$\frac{P}{P_y} = 0.8$
One End Fixed
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—20

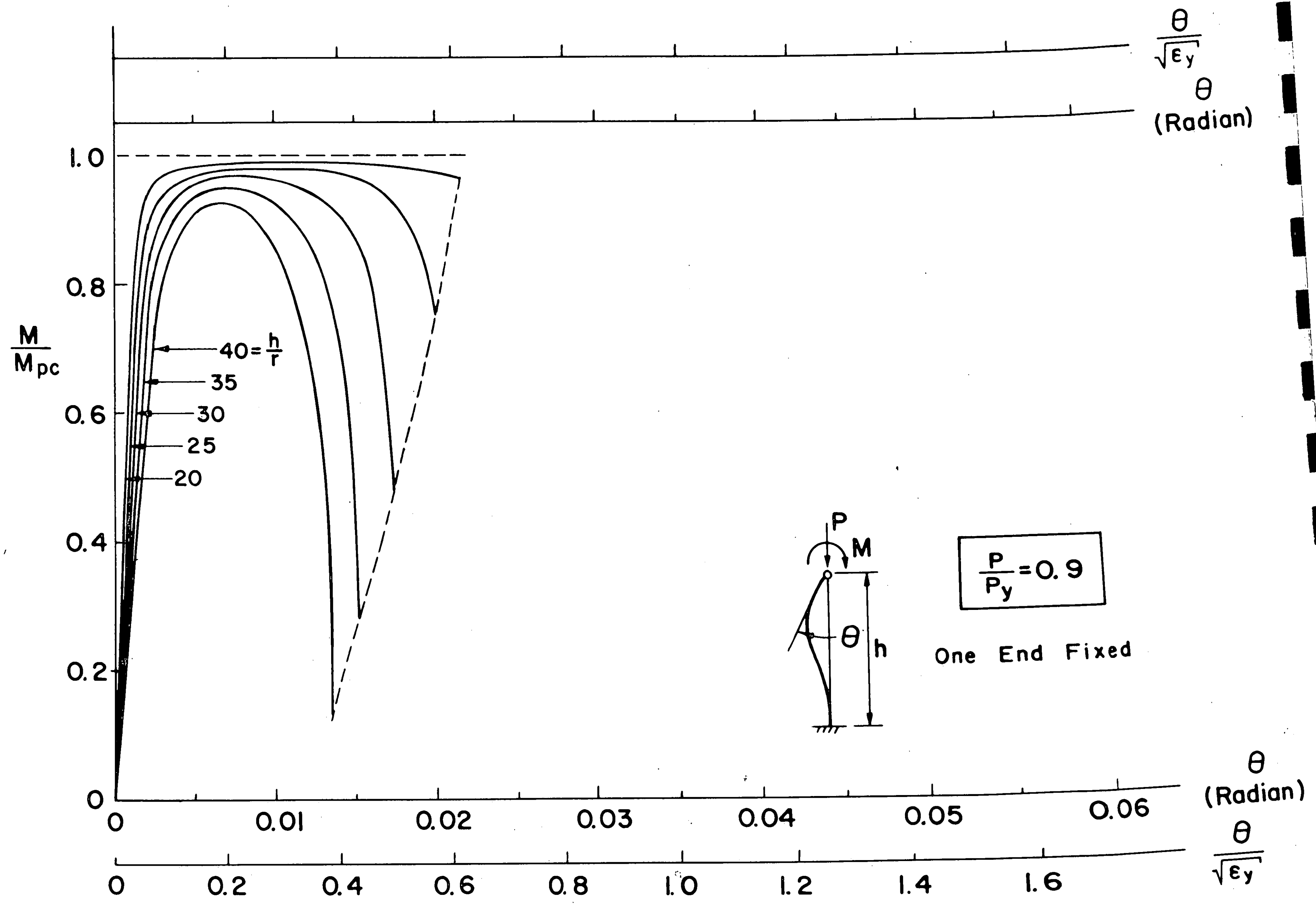
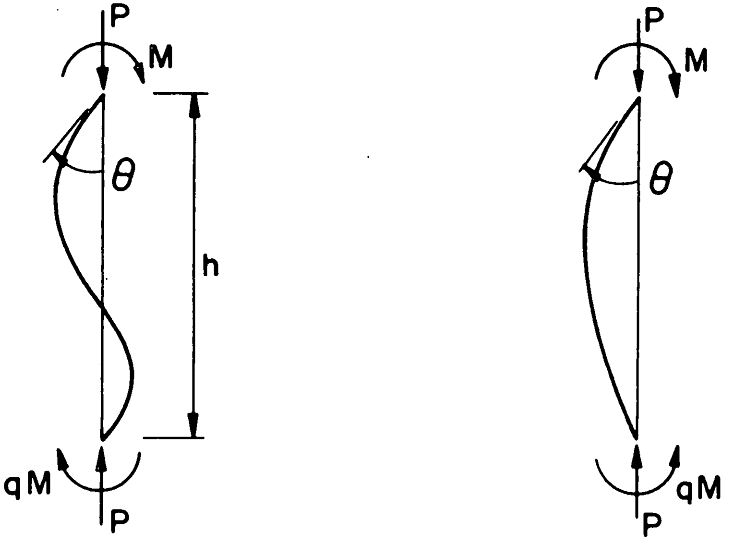

$\frac{\theta}{\sqrt{\varepsilon_y}}$
θ (Radian)
1.0
0.8
0.6
0.4
0.2
0
$\frac{M}{M_{pc}}$
$40 = \frac{h}{r}$
35
30
25
20
P
M
θ
h
$\frac{P}{P_y} = 0.9$
One End Fixed
0
0.01
0.02
0.03
0.04
0.05
0.06
θ (Radian)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\frac{\theta}{\sqrt{\varepsilon_y}}$

CHART III—21

PART IV - CHARTS FOR ANALYSIS OF RESTRAINED COLUMNS PREVENTED FROM SWAY

These charts give the moment-rotation relationships of beam-columns subjected to end moments of various combinations. The symbols used in the charts are defined in Fig. IV-1.



Positive q **Negative q**

SYMBOLS USED IN CHARTS IV-1 TO IV-12

Fig. IV-1

The moment ratio q is considered positive when the two moments act in the same direction and negative when they act in the opposite direction. The range of q values included in the charts varies from +5.0 to -5.0.

The charts were prepared for A36 steel columns (σ_y = 36 ksi) from the basic column-deflection-curve data which were used in developing the moment-rotation curves given in Part III of this booklet. The CDC data were rearranged and cross plotted to yield the necessary information for the charts (Lecture 9, Ref. 9.15). These charts can also be used for columns with different yield stress levels by following the procedure described in Part III.

The application of the charts to the analysis of restrained columns which are prevented from sway is outlined in Lecture 10. The charts can be used to find

1) The end moment ratio of a beam-column when the moment and rotation at one end are known, or
2) The rotation at one end when the moment at the same end and the end moment ratio are known.

Example: A 14WF264 column is subjected to an axial force of 1960 kip and two end moments, one equal to 385 kip-ft and acting in the clockwise direction, and the other equal to 193 kip-ft and acting in the counterclockwise direction. The column is made of A36 steel and has a length of 14 ft. Determine the rotation at the end where the larger moment is applied.

For the 14WF264 section, $P_y = 2795$ kip, $r_x = 6.74$ in, $h/r_x = 25$

$$\frac{P}{P_y} = \frac{1960}{2795} = 0.7, \qquad M_{pc} = 551 \text{ kip-ft}$$

(Table II-1)

$$\frac{M}{M_{pc}} = \frac{385}{551} = 0.7, \qquad q = -\frac{193}{385} = -0.5$$

The rotation θ is found from Chart IV-5 to be 0.004 radians or 0.0265 radians.

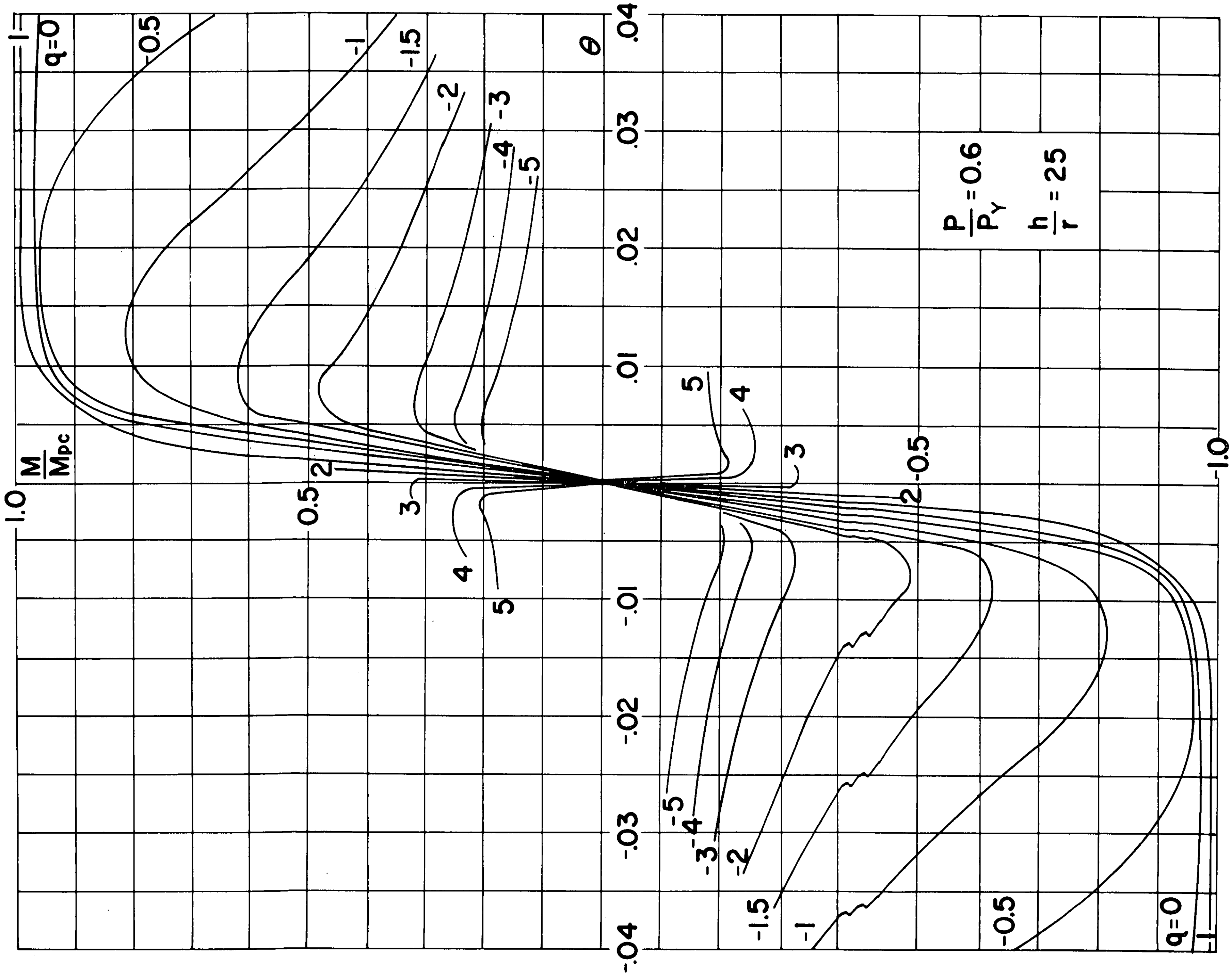

P/P_Y = 0.6
h/r = 25
M/Mpc
θ
1.0
0.5
-0.5
-1.0
.04
.03
.02
.01
-.01
-.02
-.03
-.04
q=0
-1
-0.5
-1.5
-2
-3
-4
-5
2
3
4
5

CHART IV—1

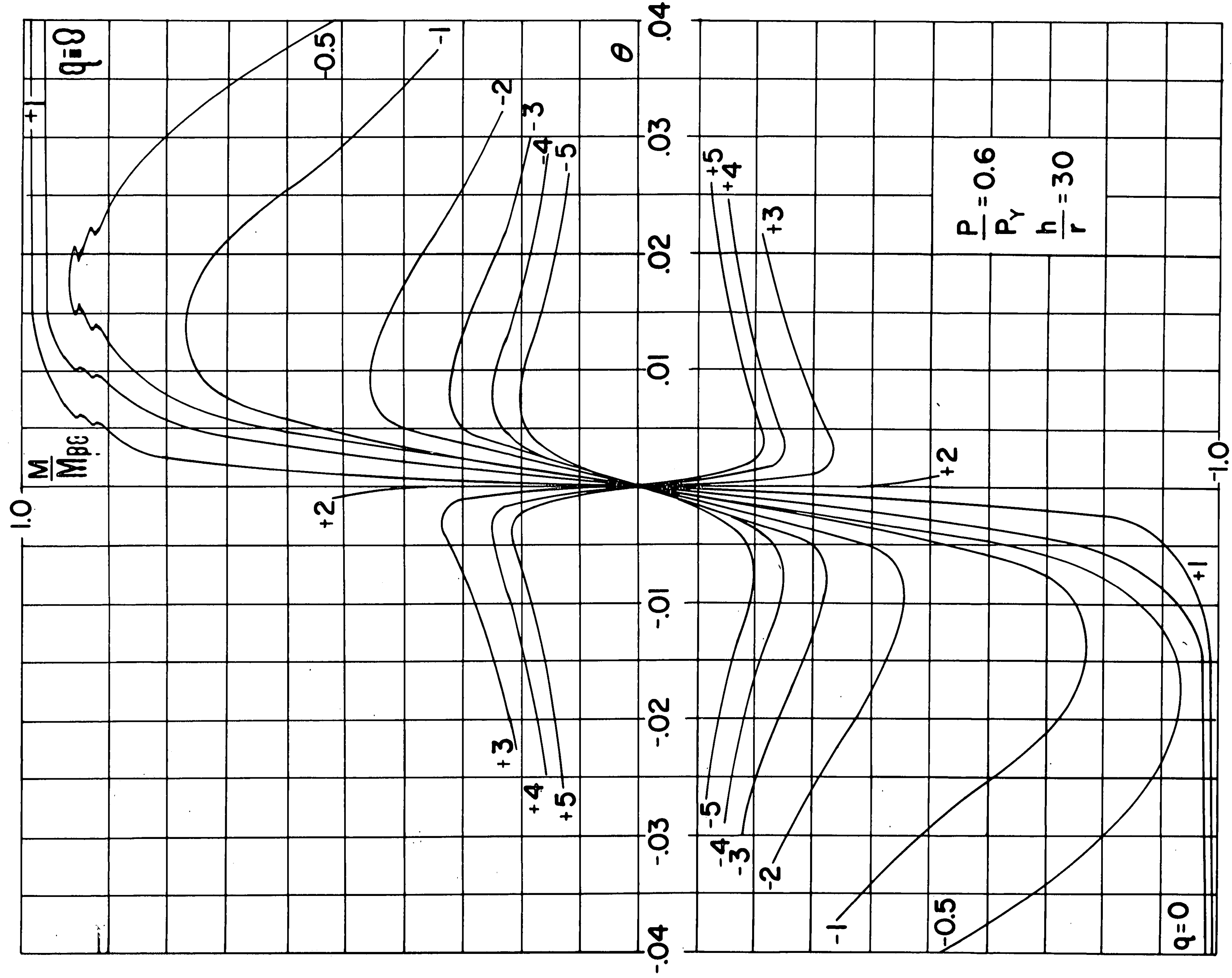
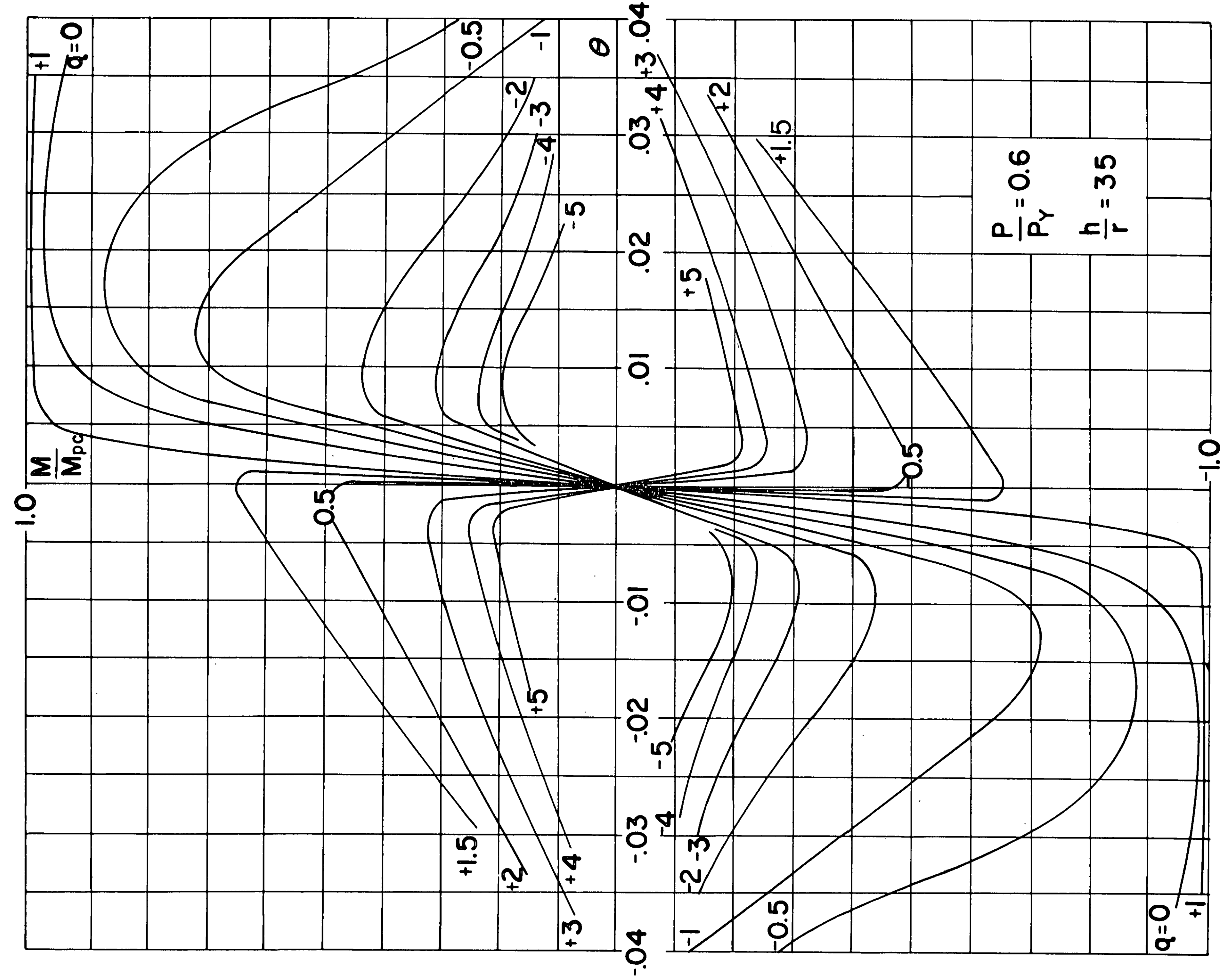

$\frac{M}{M_{pc}}$
θ
1.0
-1.0
.04
.03
.02
.01
-.01
-.02
-.03
-.04
$\frac{P}{P_Y} = 0.6$
$\frac{h}{r} = 30$
q=0
+1
-0.5
-1
-2
-3
-4
-5
+2
+3
+4
+5

CHART IV—2



$\frac{P}{P_Y} = 0.6$
$\frac{h}{r} = 35$
$\frac{M}{M_{pc}}$
1.0
0.5
-0.5
-1.0
θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
q=0
+1
-0.5
-1
-2
-3
-4
-5
+1.5
+2
+3
+4
+5

CHART IV—3

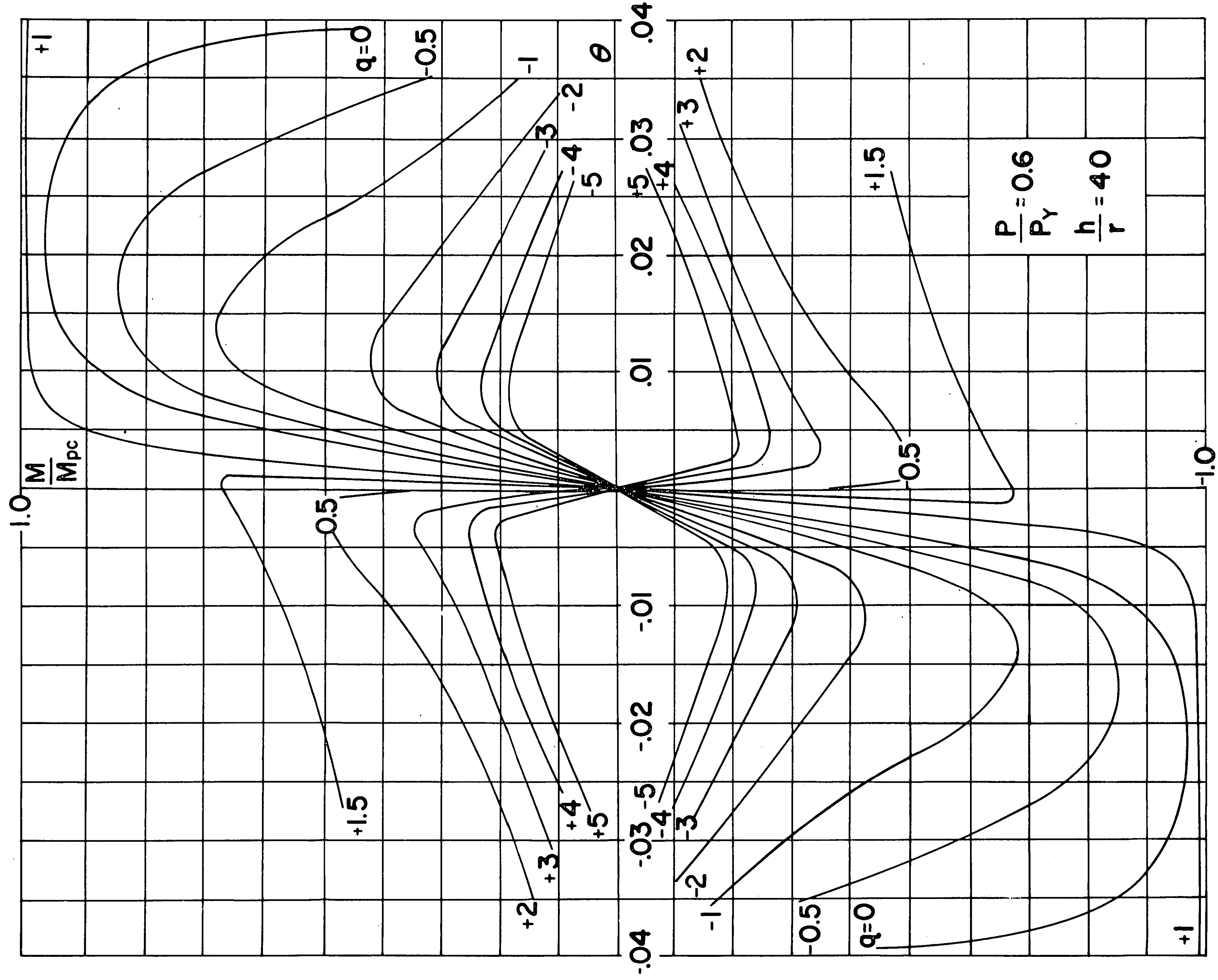

$\frac{P}{P_Y} = 0.6$
$\frac{h}{r} = 40$
θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
1.0
0.5
-0.5
-1.0
$\frac{M}{M_{pc}}$
q=0
-0.5
-1
-2
-3
-4
-5
+5
+4
+3
+2
+1.5
+1

CHART IV—4

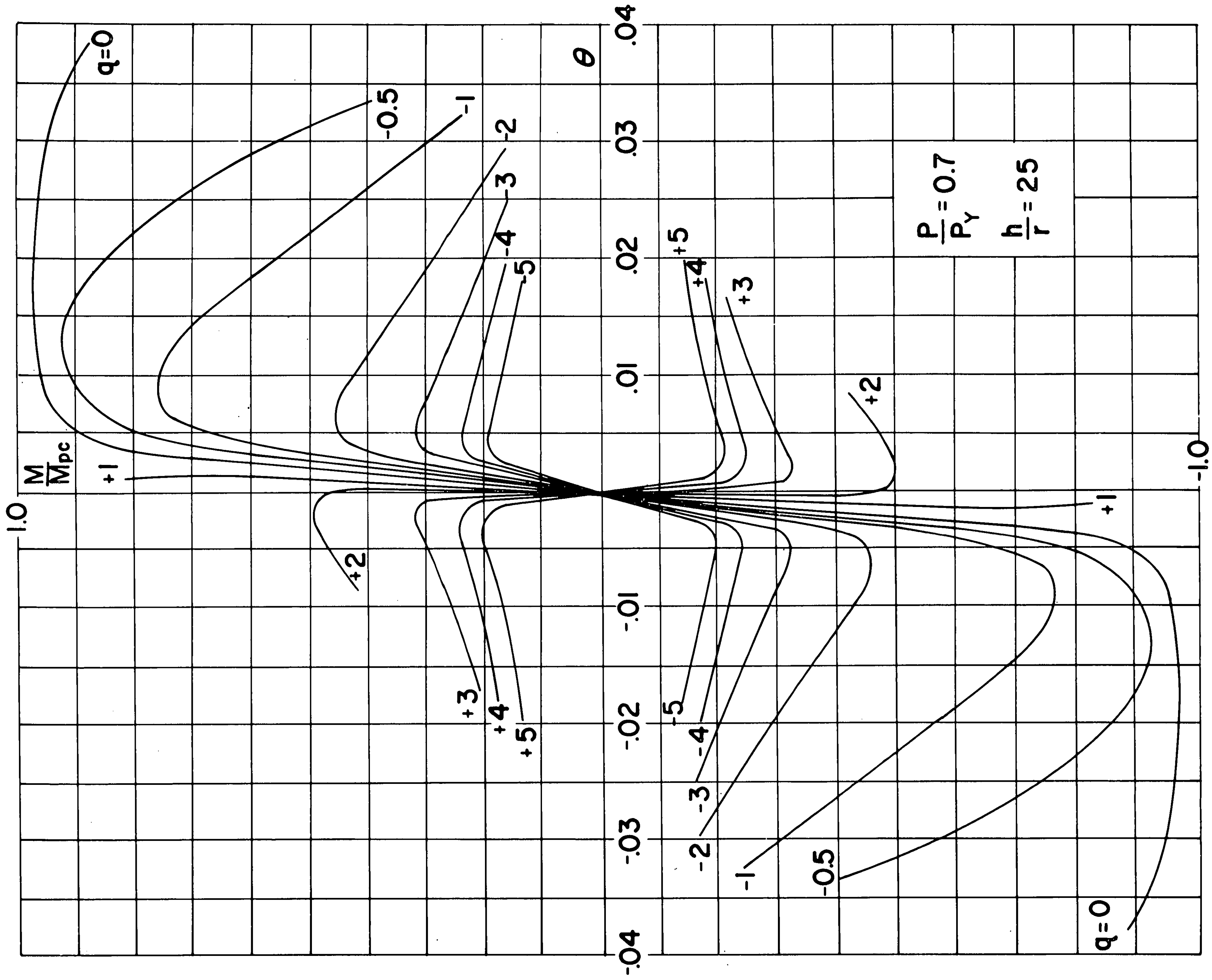

θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
$\frac{M}{M_{pc}}$
1.0
-1.0
$\frac{P}{P_Y} = 0.7$
$\frac{h}{r} = 25$
q=0
-0.5
-1
-2
-3
-4
-5
+1
+2
+3
+4
+5

CHART IV—5

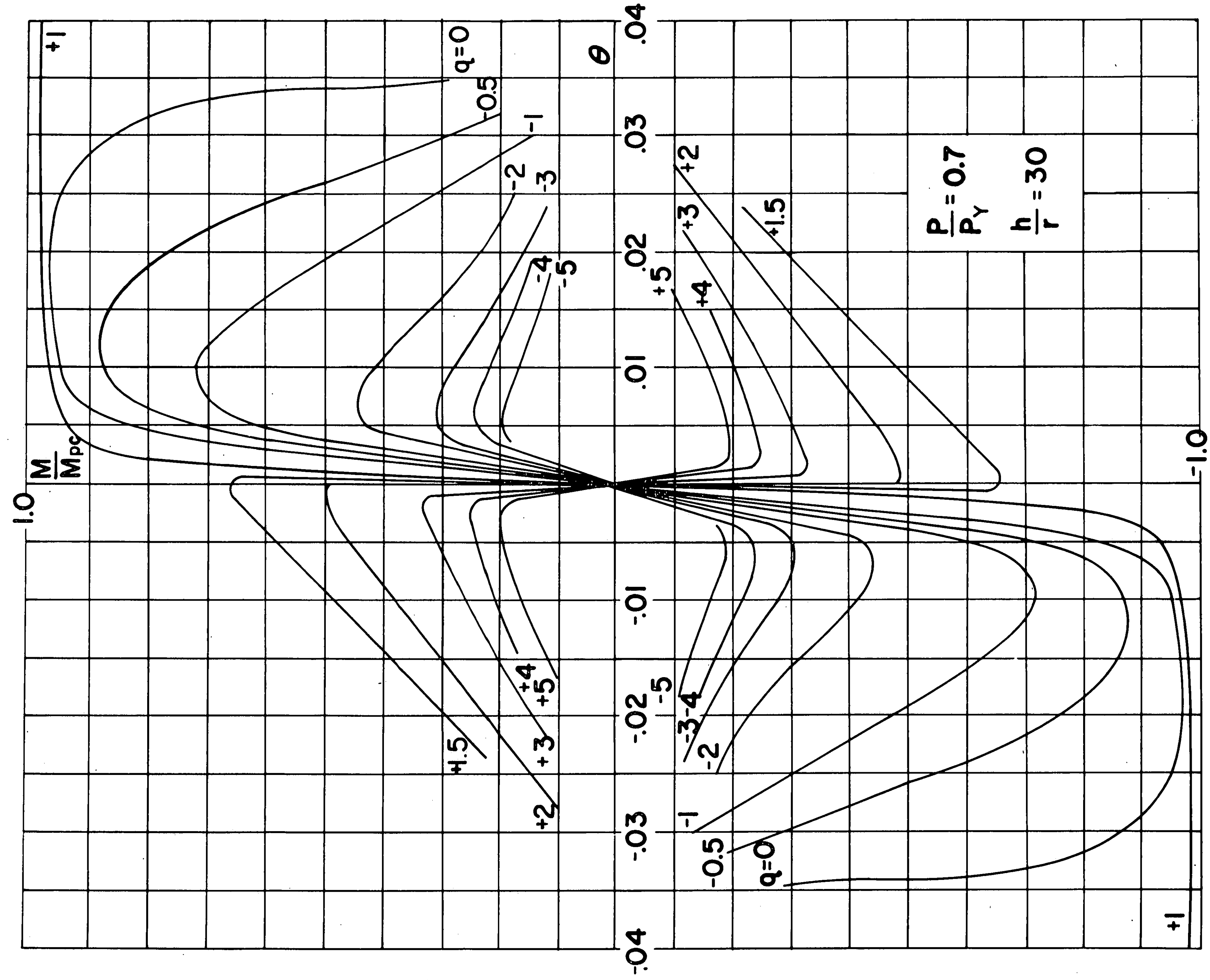

$\frac{P}{P_Y} = 0.7$
$\frac{h}{r} = 30$
$\frac{M}{M_{pc}}$
θ
1.0
-1.0
.04
.03
.02
.01
-.01
-.02
-.03
-.04
q=0
-0.5
-1
-2
-3
-4
-5
+5
+4
+3
+2
+1.5
+1

CHART IV—6

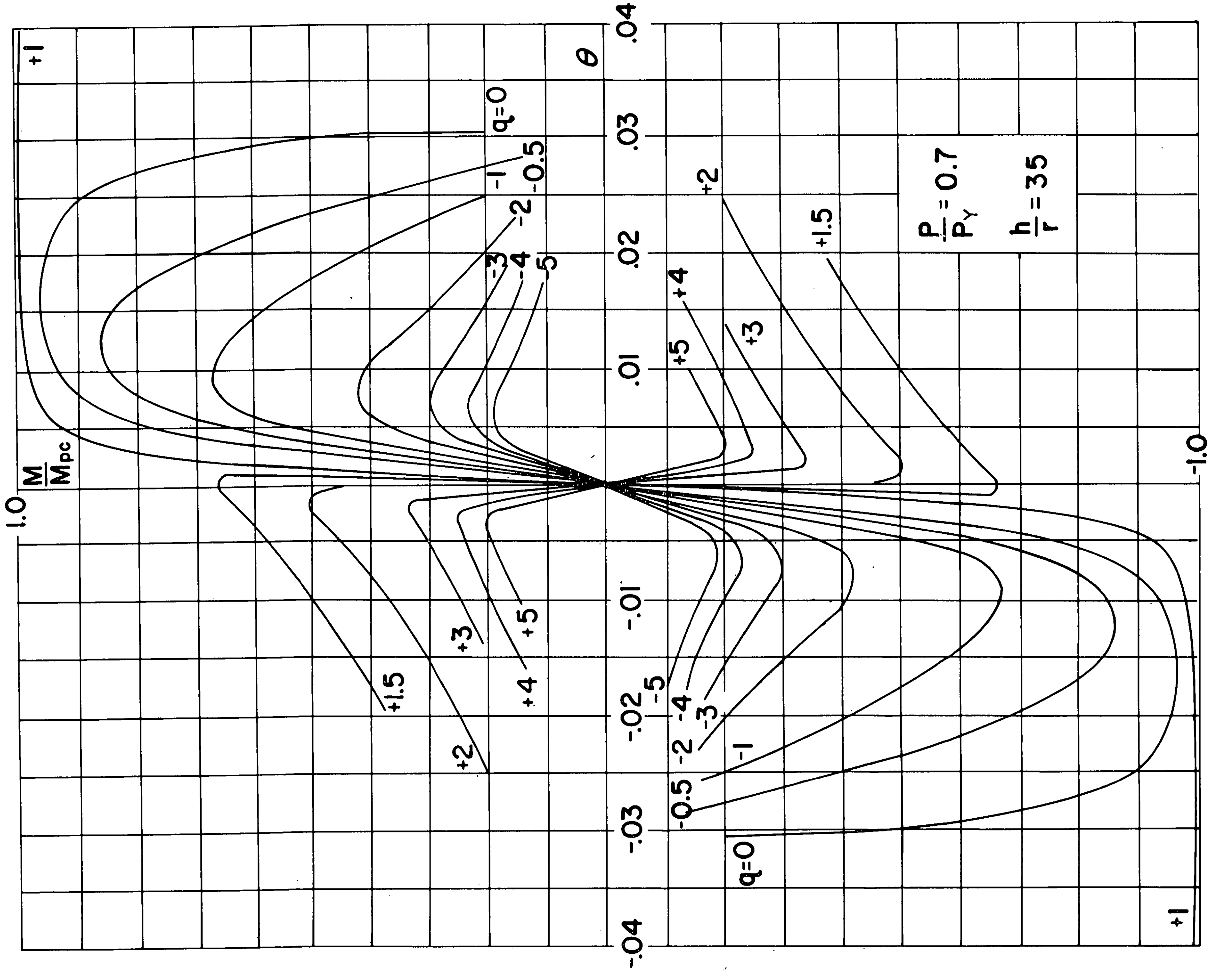

θ
$\frac{M}{M_{pc}}$
1.0
-1.0
.04
.03
.02
.01
-.01
-.02
-.03
-.04
q=0
-0.5
-1
-2
-3
-4
-5
+1
+1.5
+2
+3
+4
+5
$\frac{P}{P_Y} = 0.7$
$\frac{h}{r} = 35$

CHART IV—7

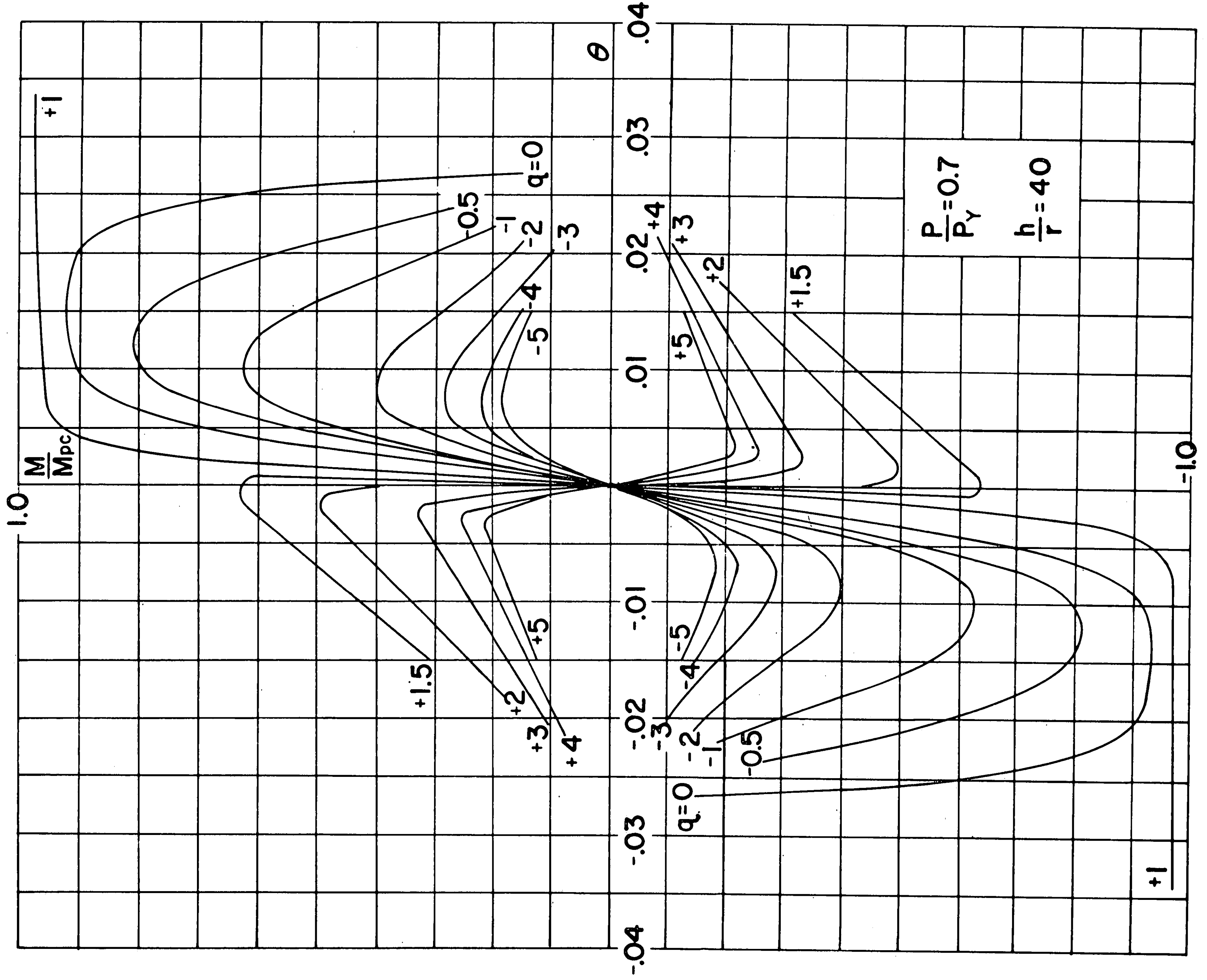

θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
1.0
$\frac{M}{M_{pc}}$
-1.0
+1
+1
q=0
-0.5
-1
-2
-3
-4
-5
+5
+4
+3
+2
+1.5
q=0
-0.5
-1
-2
-3
-4
-5
+5
+4
+3
+2
+1.5
$\frac{P}{P_Y}=0.7$
$\frac{h}{r}=40$

CHART IV—8

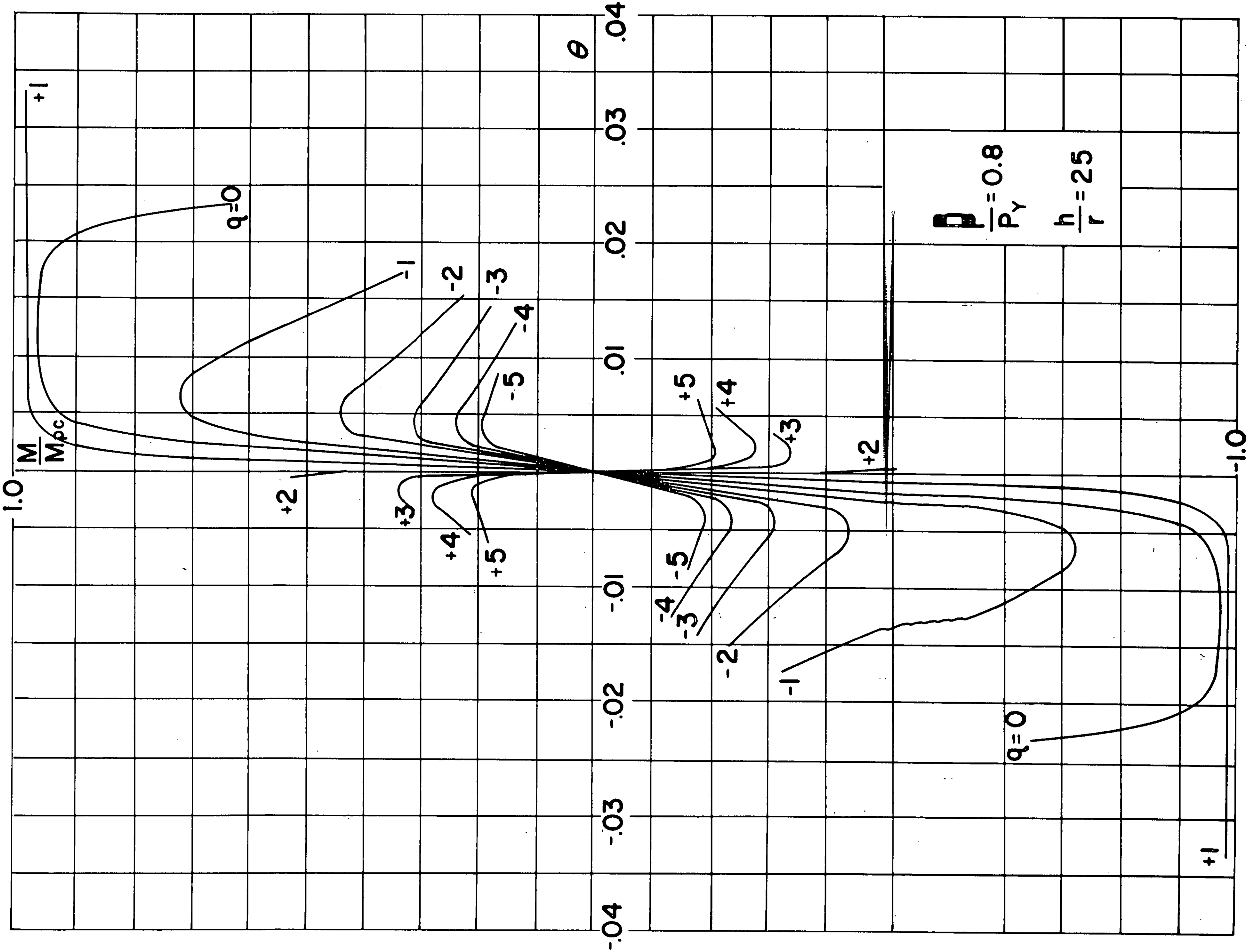


CHART IV—9

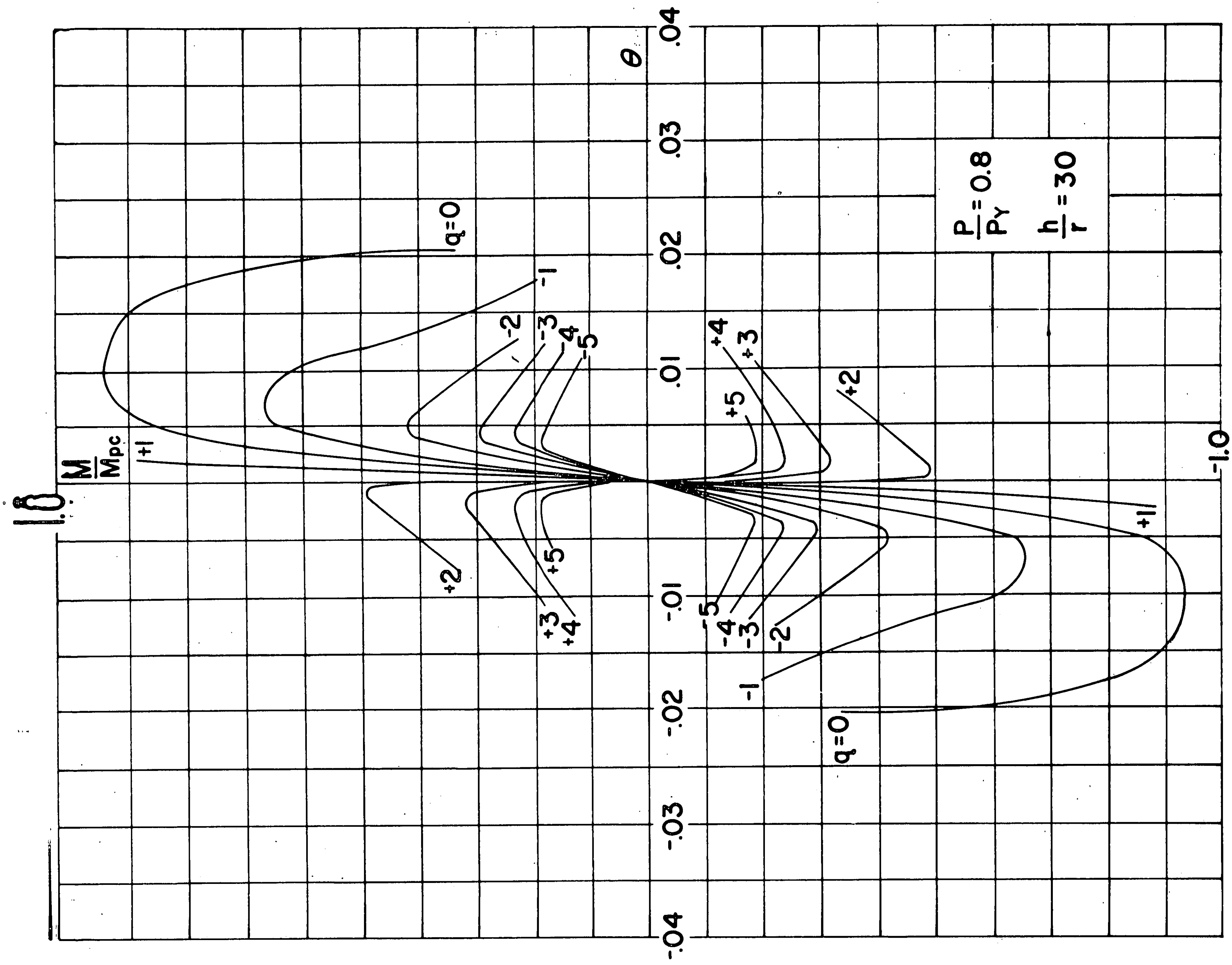

1.0
$\frac{M}{M_{pc}}$
-1.0
θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
$\frac{P}{P_Y} = 0.8$
$\frac{h}{r} = 30$
q=0
-1
-2
-3
-4
-5
+1
+2
+3
+4
+5

CHART IV—10

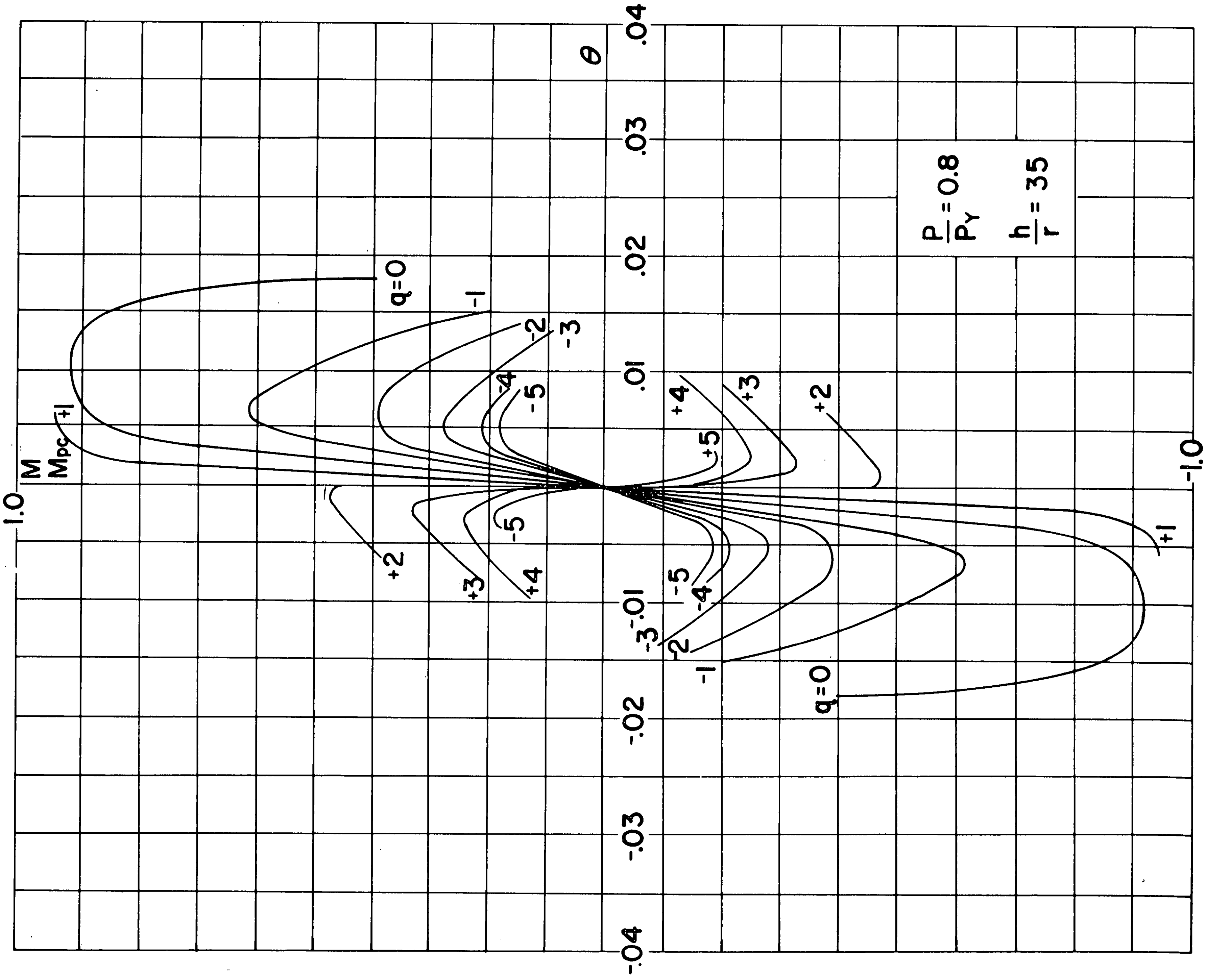
θ
.04
.03
.02
.01
-.01
-.02
-.03
-.04
1.0
-1.0
$\frac{M}{M_{pc}}$
$\frac{P}{P_Y} = 0.8$
$\frac{h}{r} = 35$
q=0
-1
-2
-3
-4
-5
+1
+2
+3
+4
+5

CHART IV—11

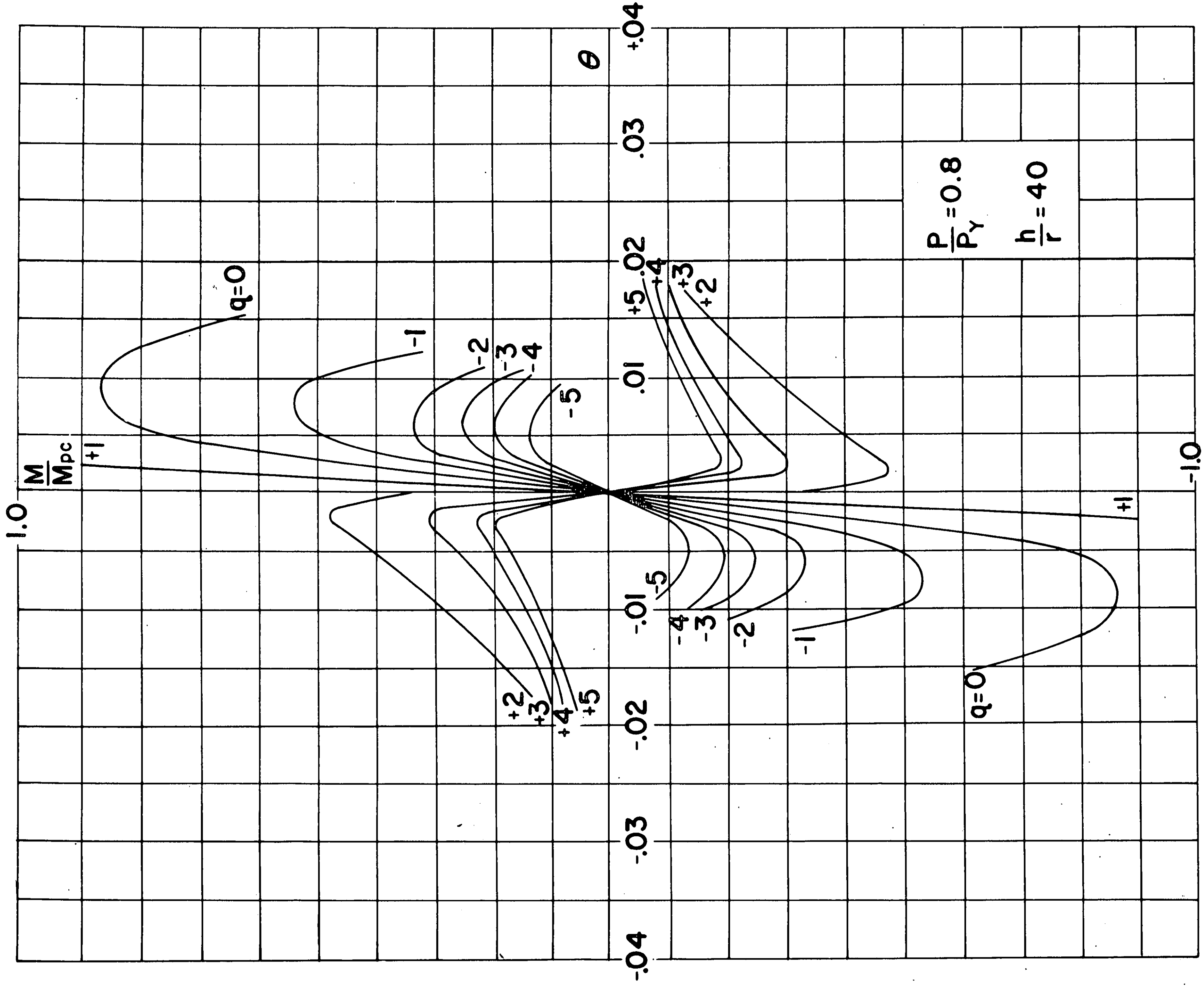


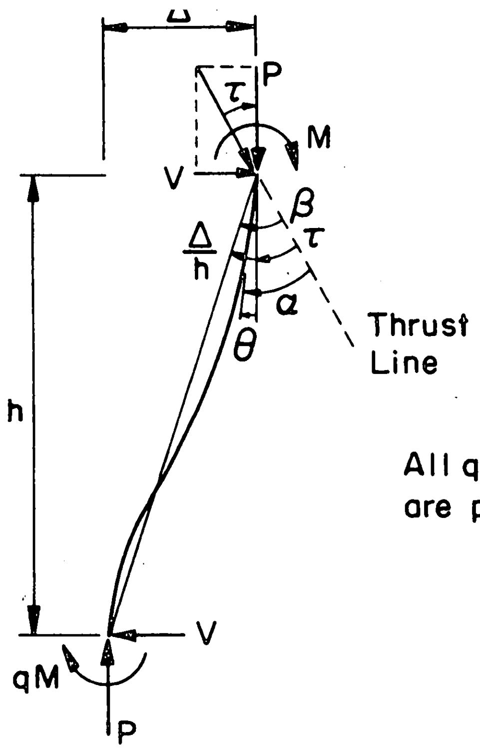
CHART IV—12

PART V - CHARTS FOR ANALYSIS OF RESTRAINED COLUMNS PERMITTED TO SWAY

These charts were developed for analyzing restrained columns which are permitted to sway in the plane of bending. The symbols used in the charts are defined in Fig. V-1. The column is subjected to combined axial and lateral forces, P and V, the resultant of which acts in the direction of the thrust line. The angles α and β are measured with respect to this line. The charts can be used to determine

1) The angle β when the moment M and the angle α at one end are known, or
2) The angle α when M and β are known.

The basic information used in the development of the charts was the column-deflection-curve data which were also used in preparing the charts contained in Parts III and IV of this booklet. The charts were prepared for A36 steel columns (σ_y = 36 ksi) according to the procedure outlined in Ref. 9.15. They can be used also for other steels if the modifications described in Part III are made. The application of these charts to the analysis of restrained columns with sway is illustrated in Lecture 17.



All quantities as shown are positive

SYMBOLS USED IN CHARTS V-1 TO V-12 AND IN ILLUSTRATIVE EXAMPLE

Fig. V-1

Example: A 14WF158 column, 16 ft. long, is displaced through a distance of $\Delta = 2.13$ in. when subjected to the following loads:

$P = 1000$ kip, $V = 2$ kip, $M = 73.5$ kip-ft

The column is made of A36 steel and is a part of an unbraced frame. Determine the slope θ of the deformed column at the upper end.

For the 14WF158 section, $P_y = 1673$ kip $r_x = 6.40$ in, $h/r_x = 30$.

$$\frac{P}{P_y} = \frac{1000}{1673} = 0.6, \qquad M_{pc} = 402 \text{ kip-ft} \text{ (Table II-1)}$$

$$\frac{M}{M_{pc}} = \frac{73.5}{402} = 0.183$$

The angle between the thrust line and the vertical is

$$\tau = \frac{2}{1000} = 0.002$$

and the chord rotation is

$$\frac{\Delta}{h} = \frac{2.13}{(12)(16)} = 0.0111$$

Combination of τ and Δ/h gives the angle $\beta = 0.0131$. Entering Chart V-2 with the computed values of M/M_{pc} and β, we find $\alpha = 0.0163$. The slope θ is therefore equal to $\alpha - \tau = 0.0143$.

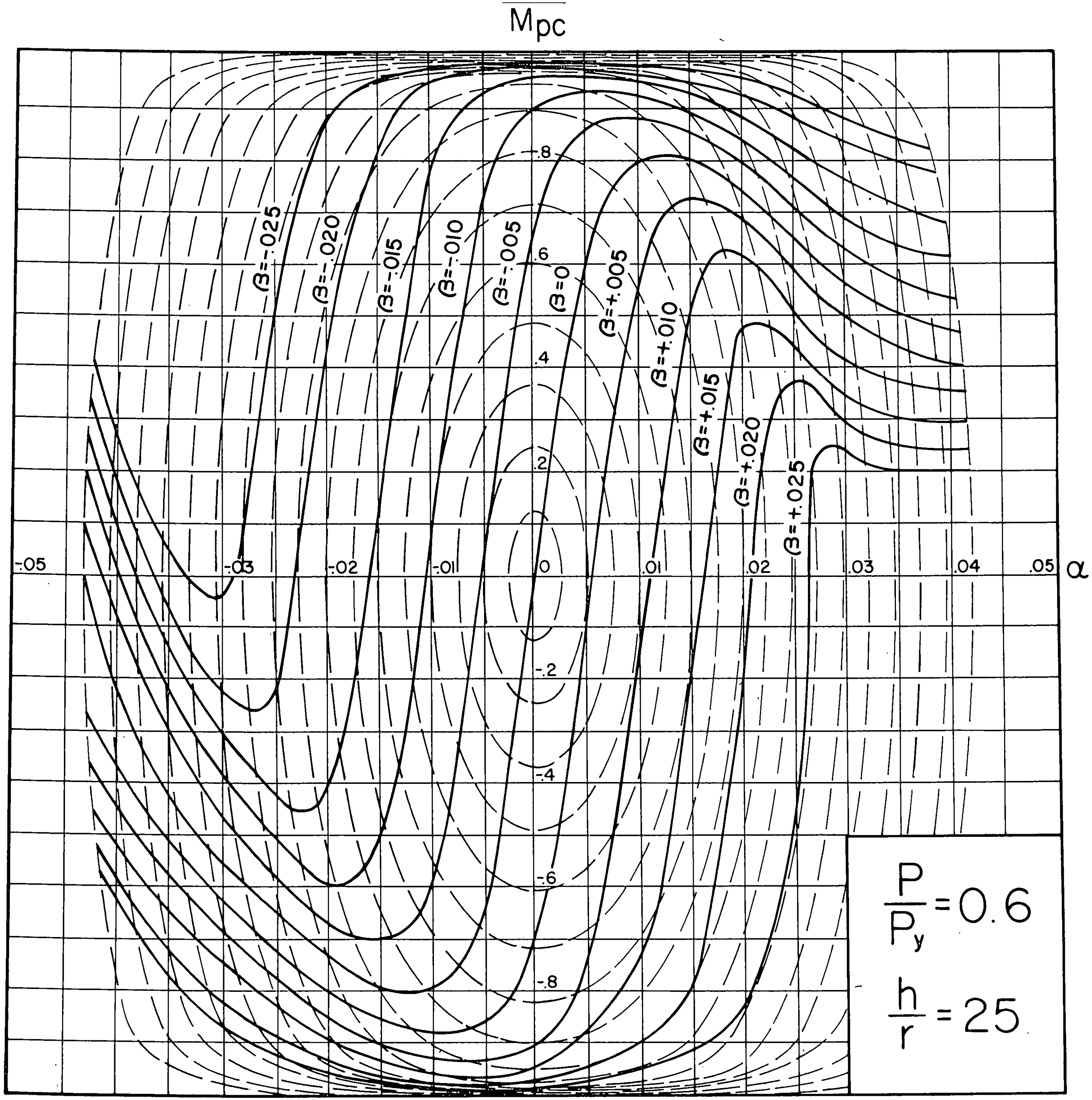

Mpc
α
-.05
-.03
-.02
-.01
0
.01
.02
.03
.04
.05
.8
.6
.4
.2
-.2
-.4
-.6
-.8
β=-.025
β=-.020
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
β=+.020
β=+.025
$\frac{P}{P_y}=0.6$
$\frac{h}{r}=25$

CHART V—1

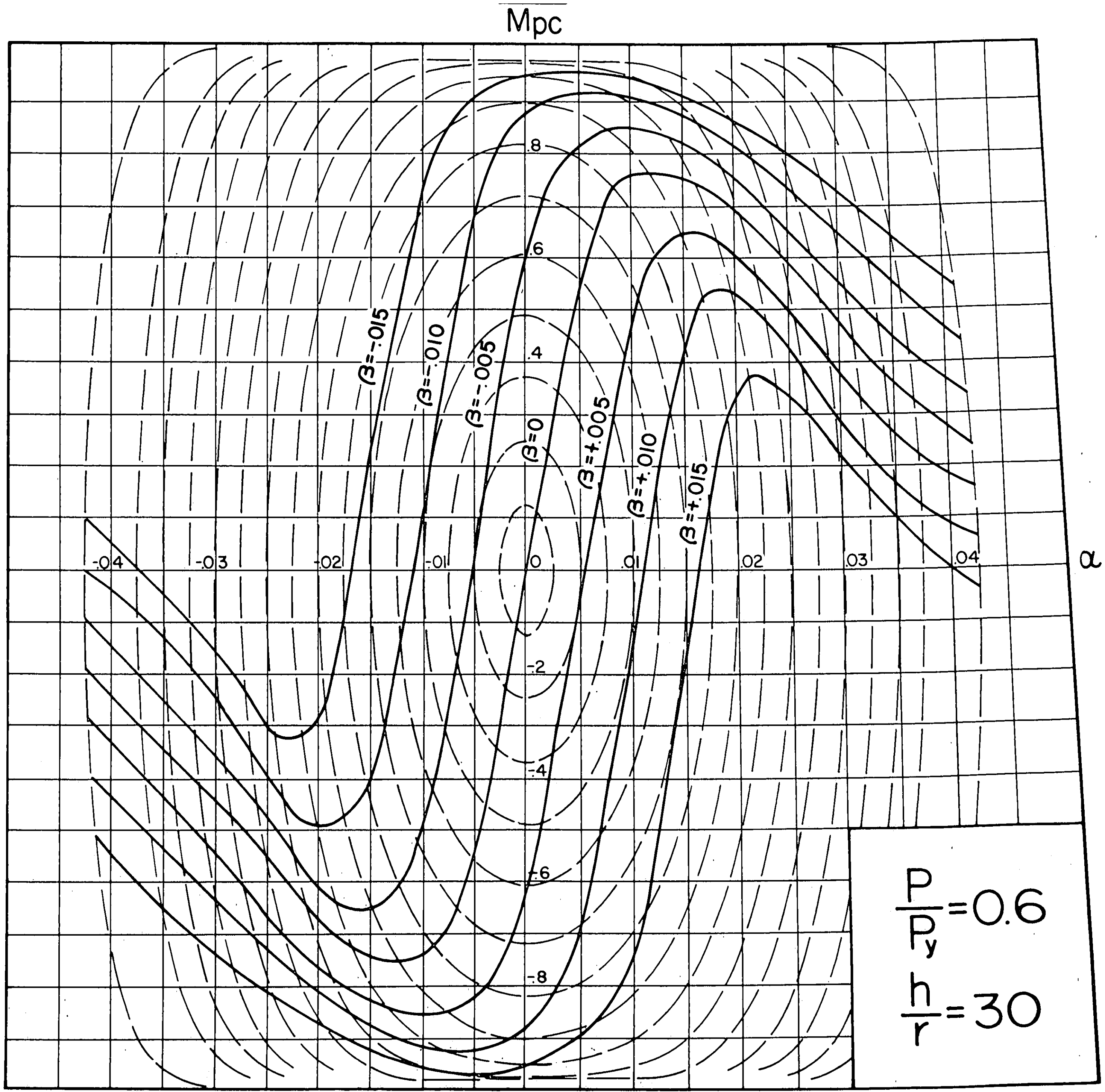

$\overline{M_{pc}}$
.8
.6
.4
0
-.2
-.4
-.6
-.8
-.04
-.03
-.02
-.01
.01
.02
.03
.04
α
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
$\frac{P}{P_y}$=0.6
$\frac{h}{r}$=30

CHART V—2

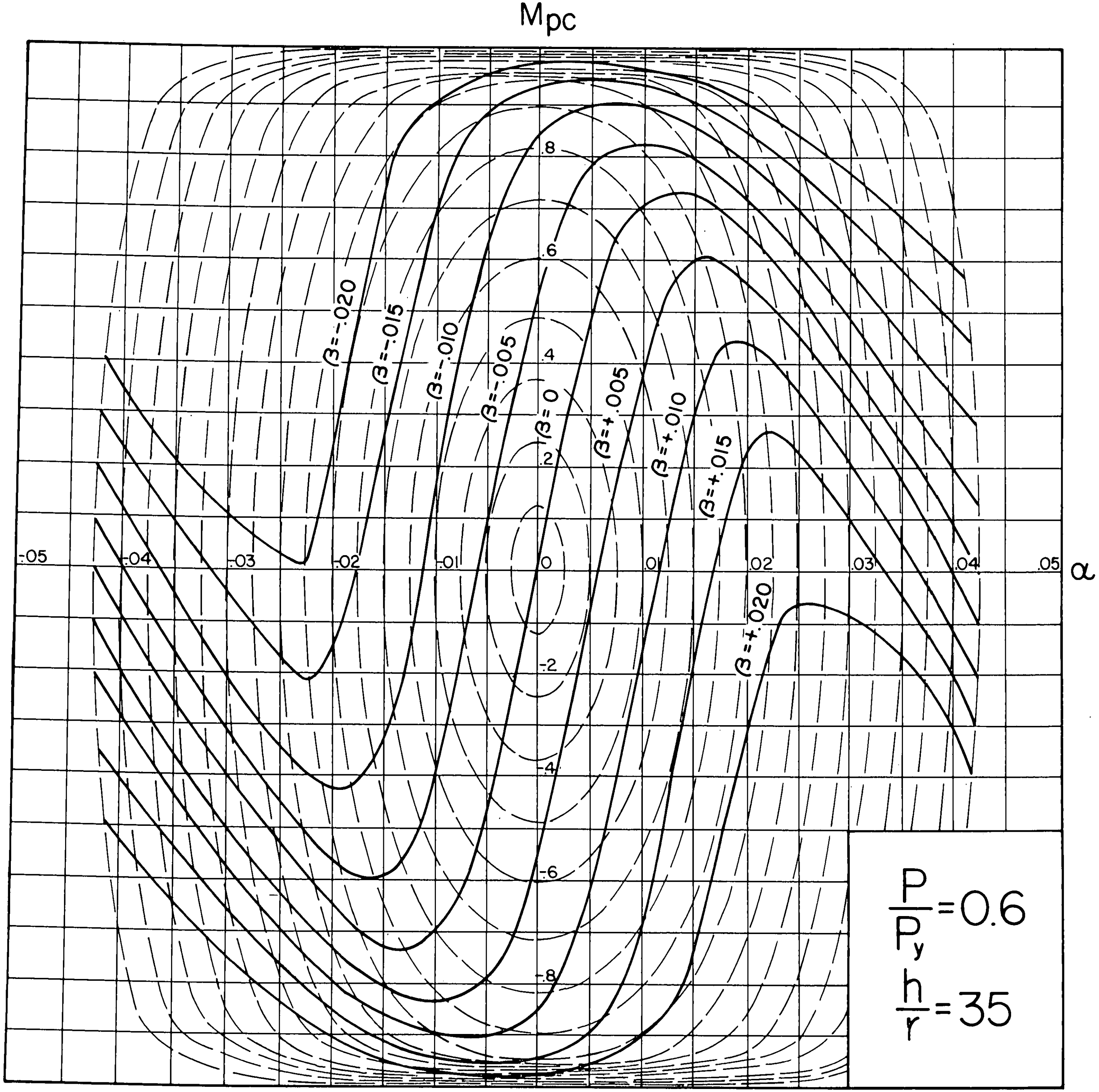

Mpc
α
-.05
-.04
-.03
-.02
-.01
0
.01
.02
.03
.04
.05
.8
.6
.4
.2
-.2
-.4
-.6
-.8
β=-.020
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
β=+.020
$\frac{P}{P_y}=0.6$
$\frac{h}{r}=35$

CHART V—3

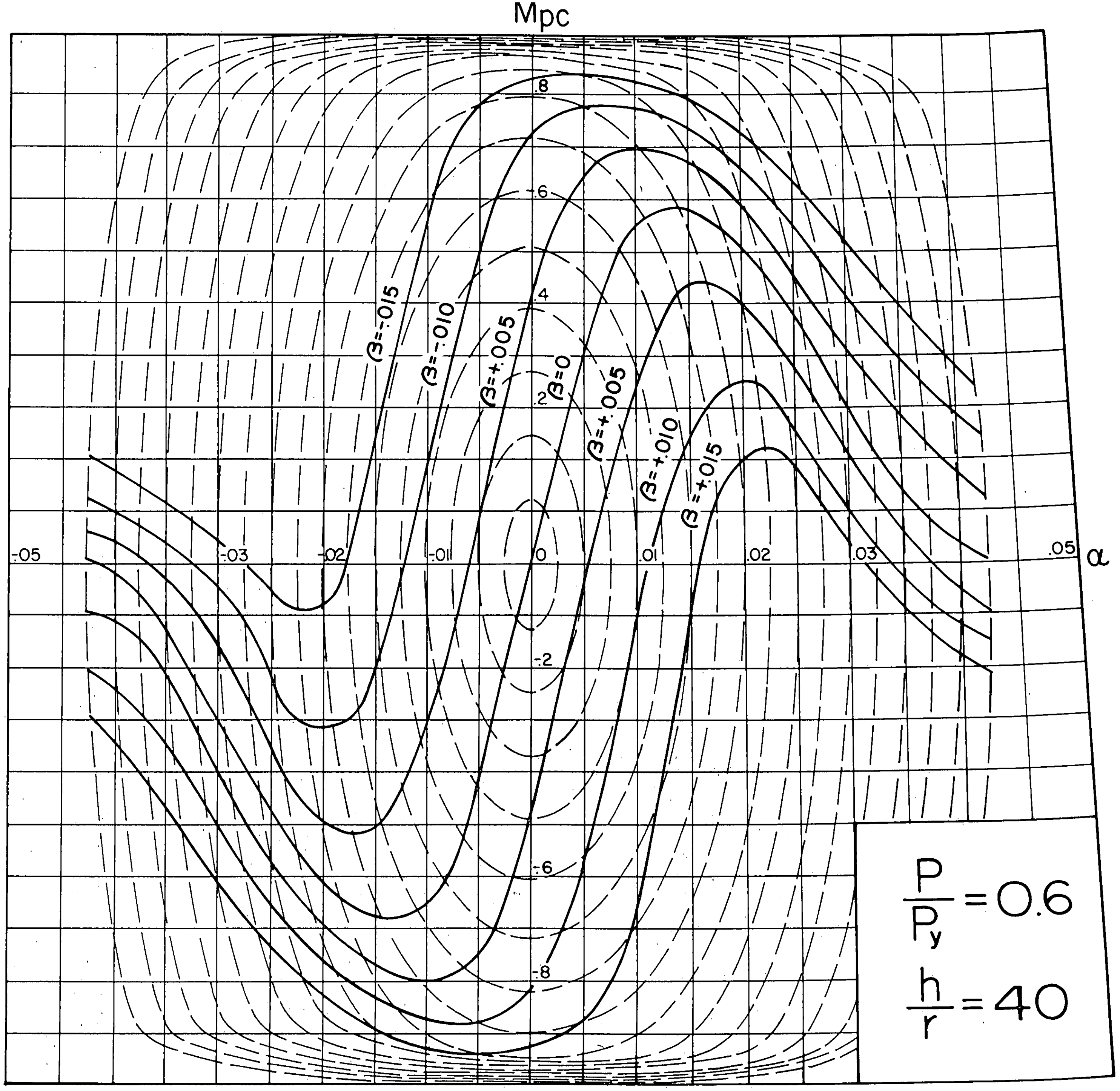

Mpc
α
-.05
-.03
-.02
-.01
0
.01
.02
.03
.05
.8
.6
.4
.2
-.2
-.6
-.8
β=-.015
β=-.010
β=+.005
β=0
β=+.005
β=+.010
β=+.015
P/Py = 0.6
h/r = 40

CHART V—4

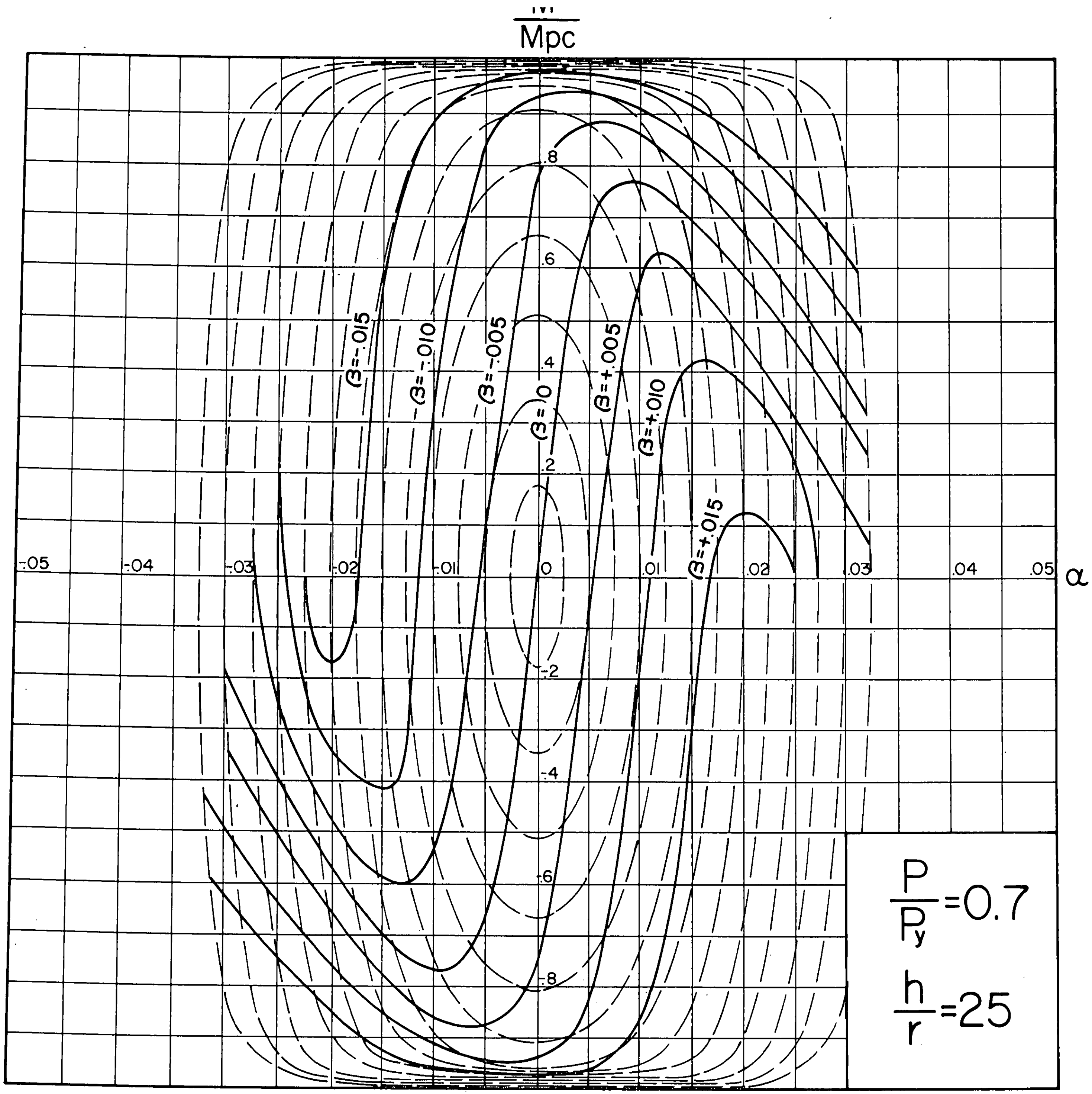
M/Mpc
.8
.6
.4
.2
-.2
-.4
-.6
-.8
-.05
-.04
-.03
-.02
-.01
0
.01
.02
.03
.04
.05
α
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
P/Py=0.7
h/r=25

CHART V—5

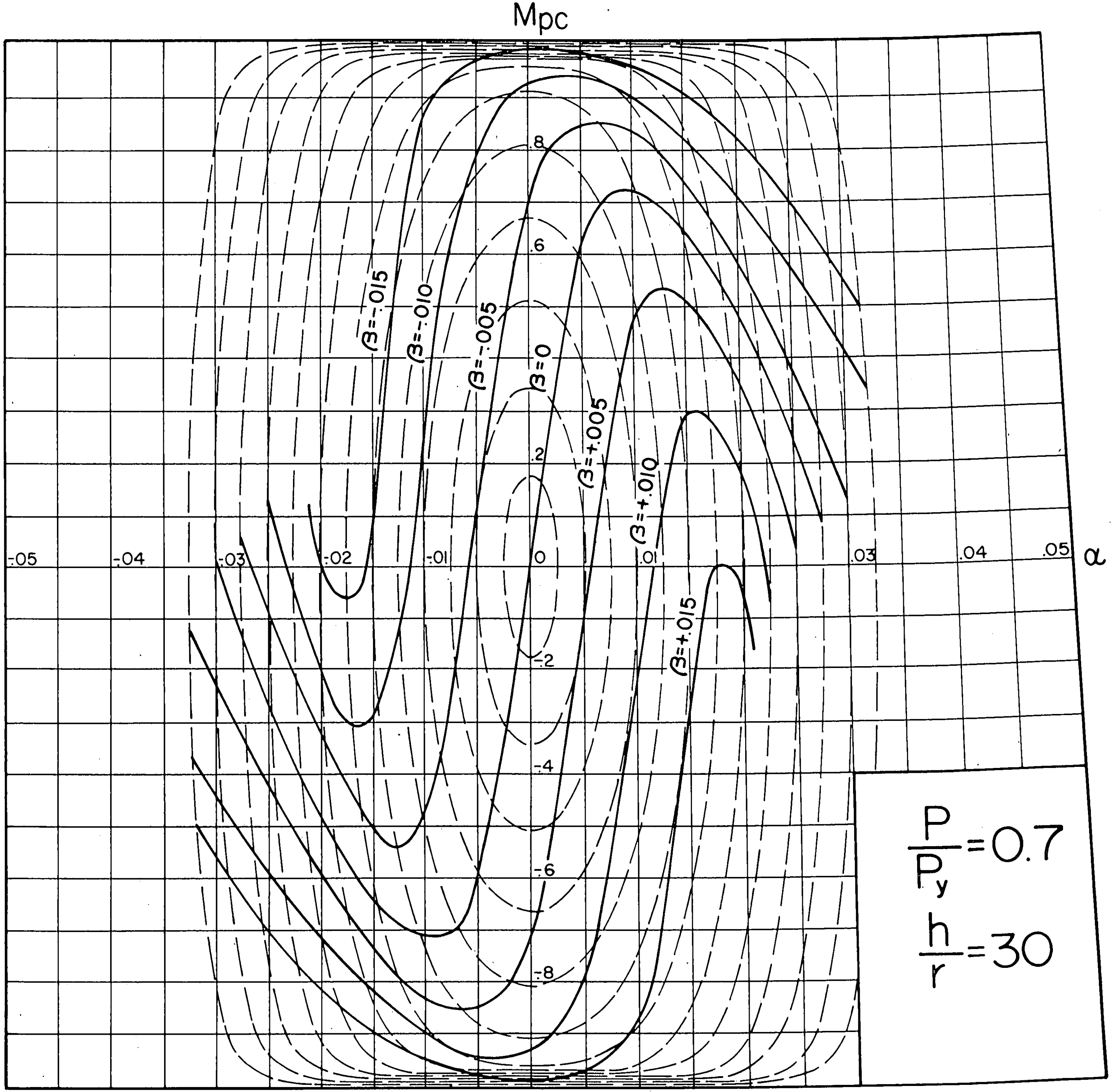

Mpc
.8
.6
.2
-.2
-.4
-.6
-.8
-.05
-.04
-.03
-.02
-.01
0
.01
.03
.04
.05
α
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
$\frac{P}{P_y}=0.7$
$\frac{h}{r}=30$

CHART V—6

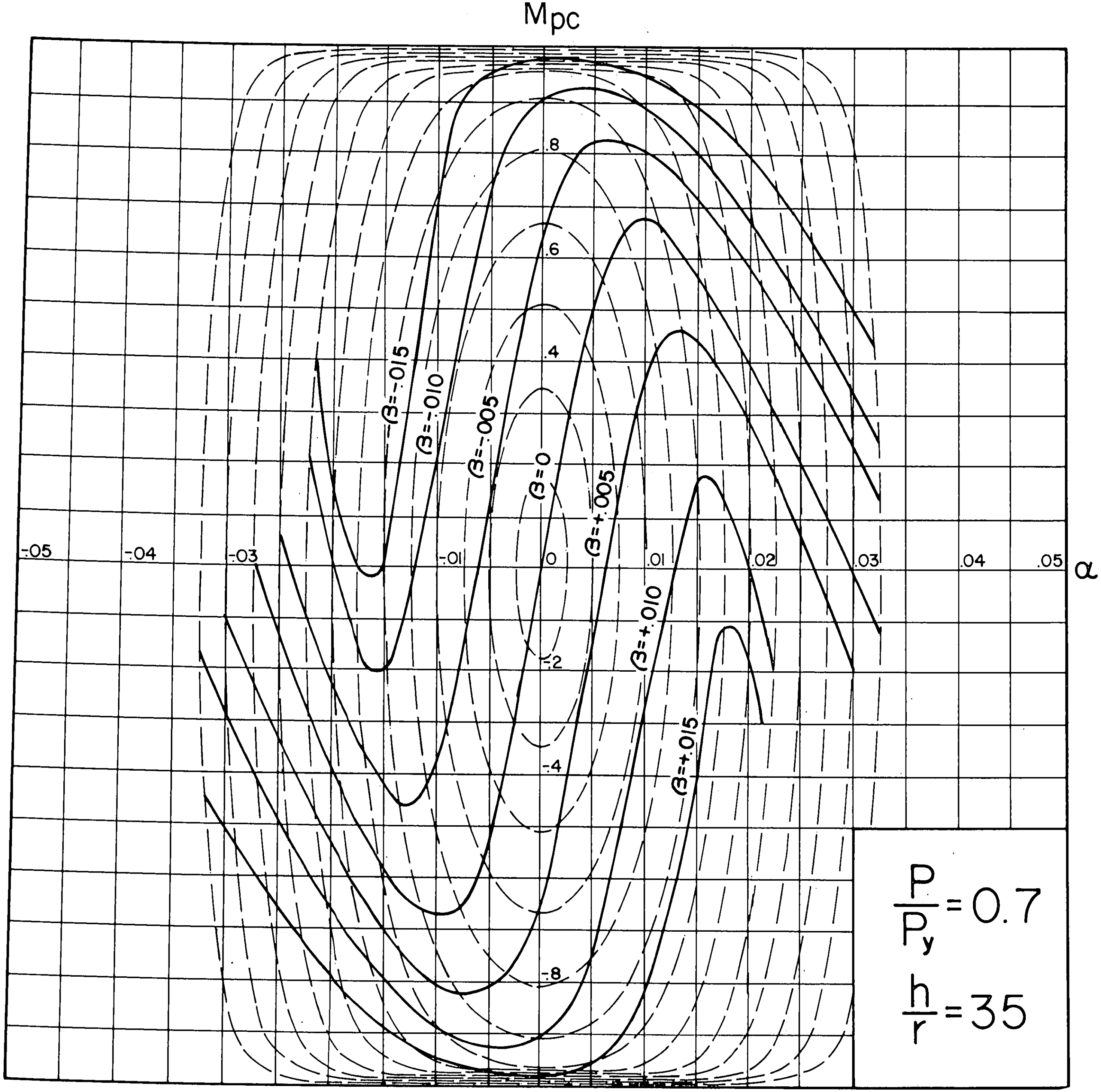

Mpc
.8
.6
.4
-.2
-.4
-.8
-.05
-.04
-.03
-.01
0
.01
.02
.03
.04
.05
α
β=-.015
β=-.010
β=-.005
β=0
β=+.005
β=+.010
β=+.015
P/Py = 0.7
h/r = 35

CHART V—7

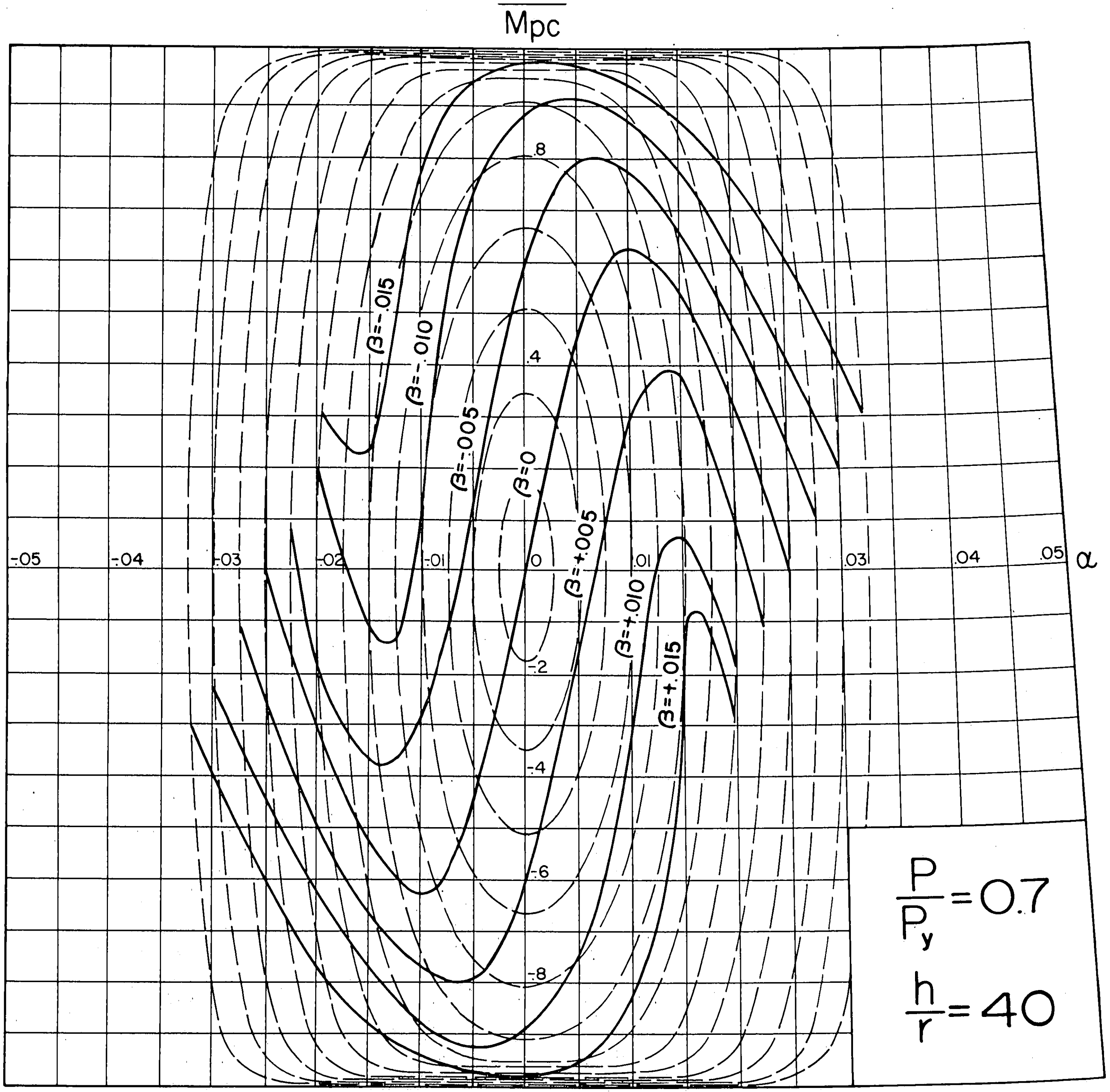

$\overline{M_{pc}}$
.8
.4
-.2
-.4
-.6
-.8
-.05
-.04
-.03
-.02
-.01
0
.01
.03
.04
.05
α
$\beta = -.015$
$\beta = -.010$
$\beta = -.005$
$\beta = 0$
$\beta = +.005$
$\beta = +.010$
$\beta = +.015$
$\frac{P}{P_y} = 0.7$
$\frac{h}{r} = 40$

CHART V-8

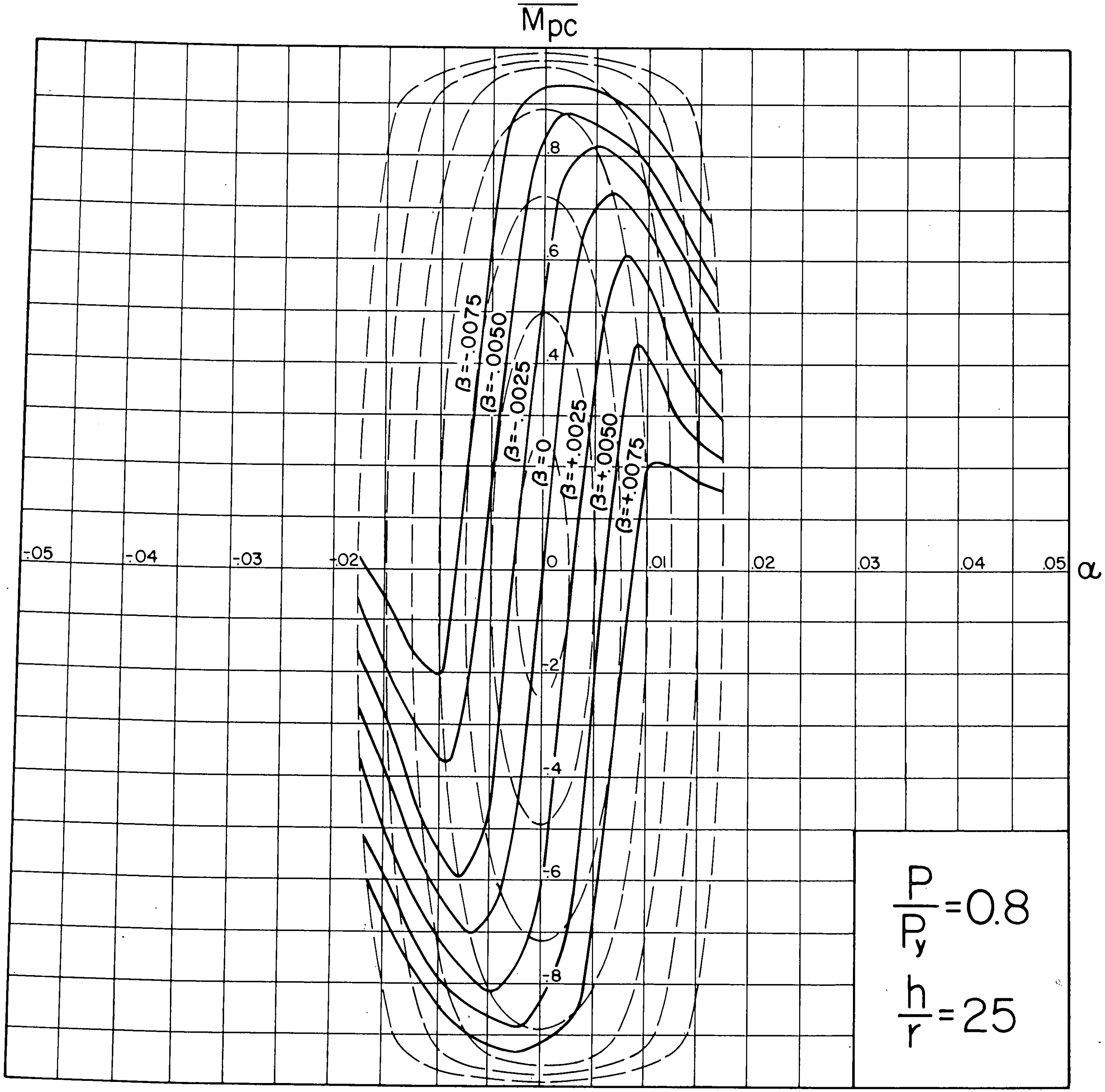

$\overline{M_{pc}}$
-.05
-.04
-.03
-.02
0
.01
.02
.03
.04
.05
α
.8
.6
.4
-.2
-.4
-.6
-.8
β=-.0075
β=-.0050
β=-.0025
β=0
β=+.0025
β=+.0050
β=+.0075
$\frac{P}{P_y}=0.8$
$\frac{h}{r}=25$

CHART V—9

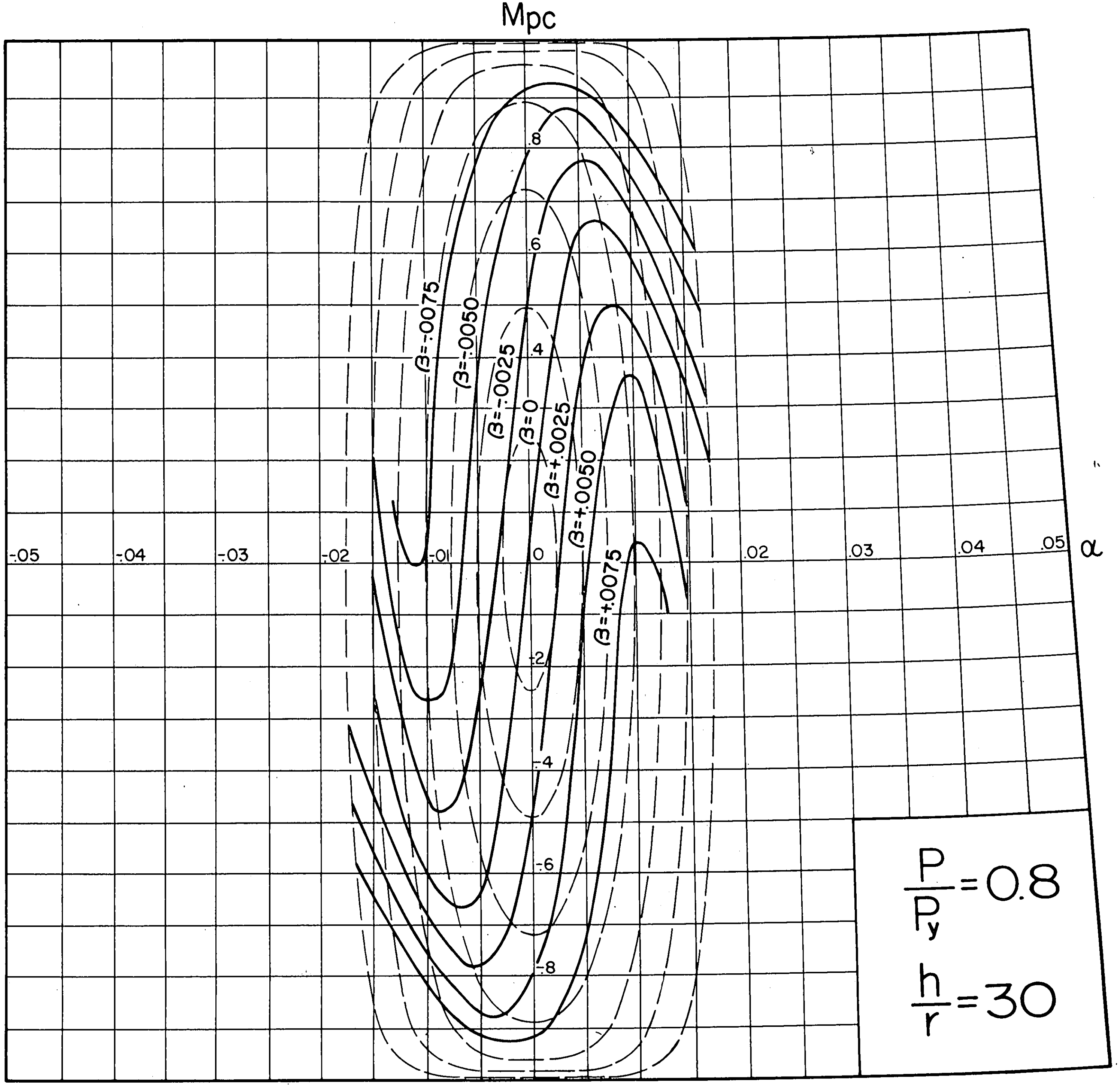

Mpc
.8
.6
.4
0
-.2
-.4
-.6
-.8
-.05
-.04
-.03
-.02
-.01
.02
.03
.04
.05
α
β=-.0075
β=-.0050
β=-.0025
β=0
β=+.0025
β=+.0050
β=+.0075
$\frac{P}{P_y}=0.8$
$\frac{h}{r}=30$

CHART V—10

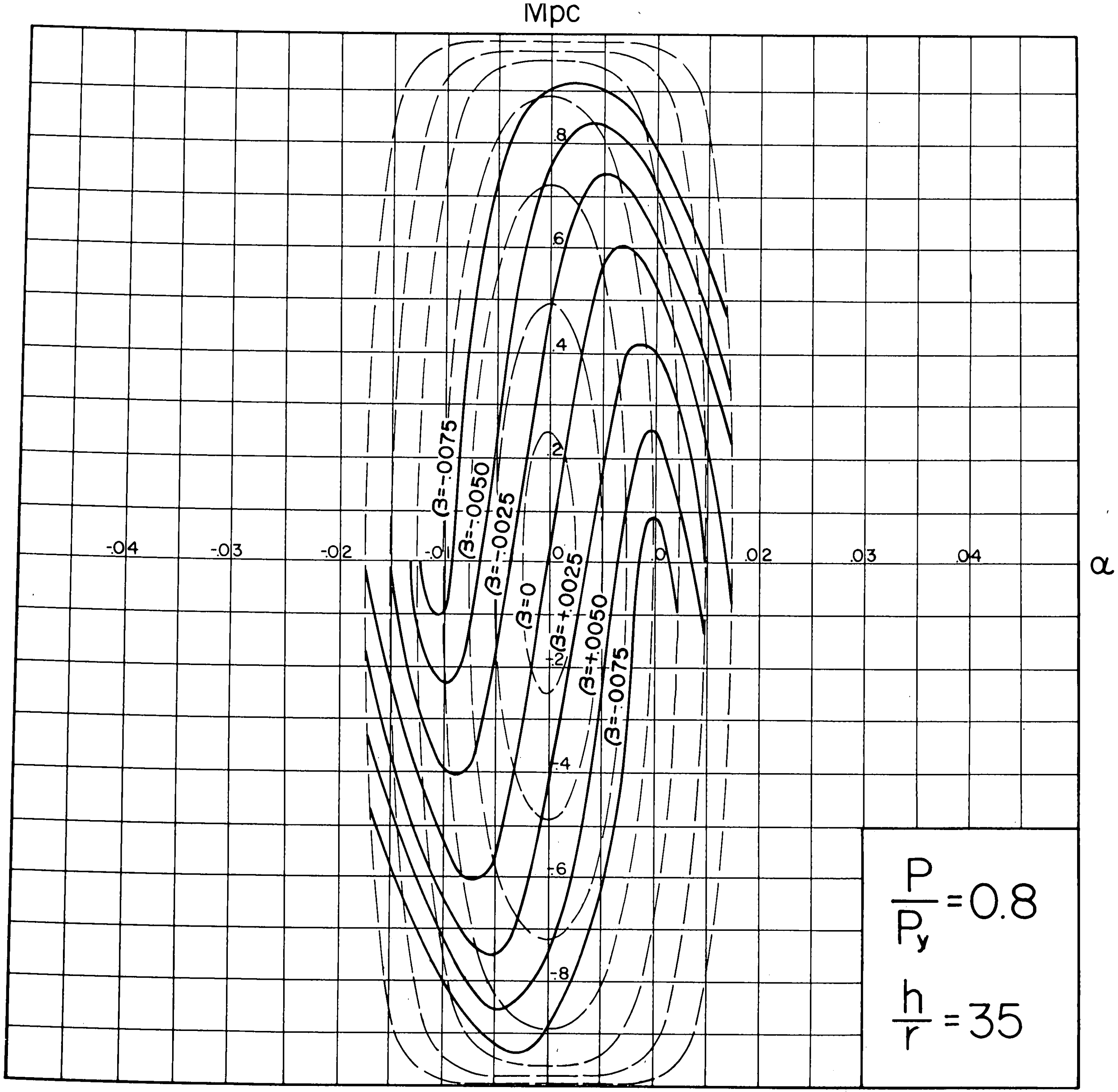

Mpc
α
-.04
-.03
-.02
-.01
0
.01
.02
.03
.04
.8
.6
.4
.2
-.2
-.4
-.6
-.8
β=-.0075
β=-.0050
β=-.0025
β=0
β=+.0025
β=+.0050
β=-.0075
P/Py = 0.8
h/r = 35

CHART V—11

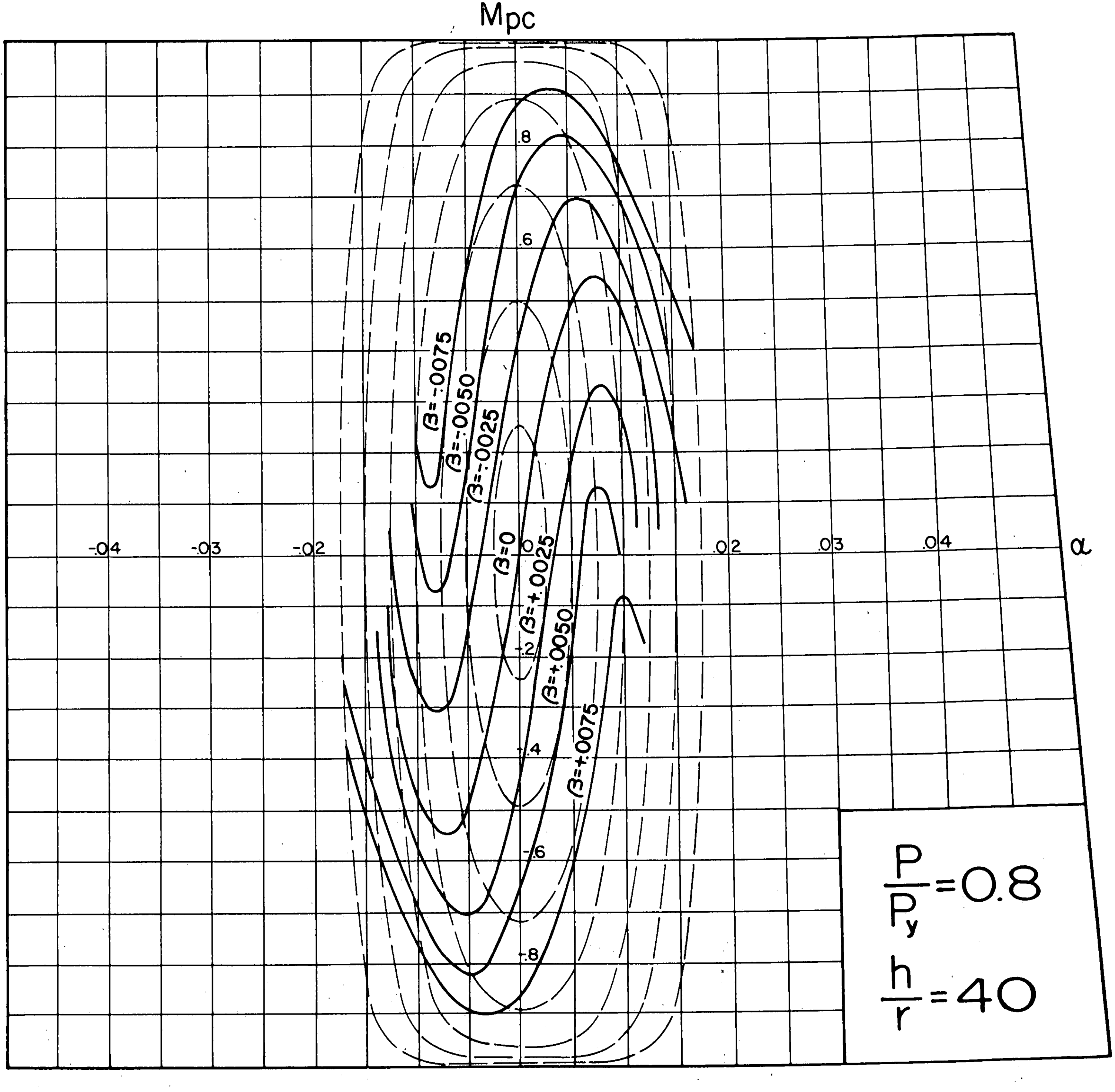

Mpc
.8
.6
.2
.4
.6
.8
-.04
-.03
-.02
.02
.03
.04
α
β=-.0075
β=-.0050
β=-.0025
β=0
β=+.0025
β=+.0050
β=+.0075
$\frac{P}{P_y}=0.8$
$\frac{h}{r}=40$

CHART V— 12

PART VI - CHARTS FOR DESIGN OF COLUMNS IN UNBRACED FRAMES

These charts give the lateral load vs. sway deflection relationships of restrained columns which are loaded by a constant axial load P and a gradually increasing lateral load Q. In addition to these loads, the columns are also subjected to a bending moment M_e defined by

$$M_e = -\left[Q\frac{h}{2} + P\frac{\Delta}{2}\right]$$

in which h/2 represents the column height and $\Delta/2$ the sway deflection. The reason for choosing such a moment is discussed in Lecture 18.

Each chart is for a given combination of P/P_y and h/r_x ratios of the column. The lateral load Q is nondimensionalized as $\frac{Qh}{2M_{pc}}$ with M_{pc} being the plastic moment of the column corresponding to the given P/P_y ratio. Each curve in the chart defines the relationship between $\frac{Qh}{2M_{pc}}$ and $\frac{\Delta}{h}$ for a constant restraining function. The sloping lines in the charts give the load vs. deflection relationships obtained from a rigid-plastic solution (Lecture 18). The symbol M_r represents the maximum restraining moment that can be provided by the restraining member.

The charts were prepared from the moment-rotation curves given in Part III of this booklet (Charts III-1 to III-2) according to the procedure described in Lecture 18. The application of these charts to the design of columns in unbraced frames is explained in Lectures 18 and 19. The following example will illustrate the use of these charts in solving restrained column problems.

Example: A 14WF219 column, 6 ft. and 10 in. long, is framed at its top to a 21WF62 beam having a span length equal to 21 ft. and 4 in. The lower end of the column and the far end of the beam are both pinned. The column is subjected to a constant axial force P = 1855 kip and a varying lateral force Q. In addition, a moment M_e is applied at the top of the column which is related to the forces P and Q according to the formula given above. Both the column and beam are made of A36 steel. Determine the maximum lateral force that can be resisted by the column.

For the 14WF219 section, $P_y = 2317$ kip, $r_x = 6.59$ in, $h/r_x = 25$.

$$\frac{P}{P_y} = \frac{1855}{2317} = 0.8, \qquad M_{pc} = 299 \text{ kip-ft}$$

For the 21WF62 section, $Z = 144.1 \text{ in}^3$ $M_p = 432$ kip-ft, $I_x = 1327 \text{ in}^4$.

Note that the plastic moment of the beam is larger than that of the column. A plastic hinge will therefore form at the top of the column.

Restraining function

$$= \frac{3EI}{LM_{pc}} \theta M_{pc} = \frac{(3)(29,000)(1327)}{(12)(21.3)(12)(299)} \theta M_{pc}$$

$$= 125 \ \theta M_{pc}$$

Entering Chart VI-17 and locating the curve for $M_r = 125 \ \theta M_{pc}$, we find the peak value of $\frac{Qh}{2M_{pc}}$ to be 0.193. Thus the maximum lateral force Q is

$$(Q)_{max} = \frac{(0.193)(299)}{6.83} = 8.44 \text{ kip}$$

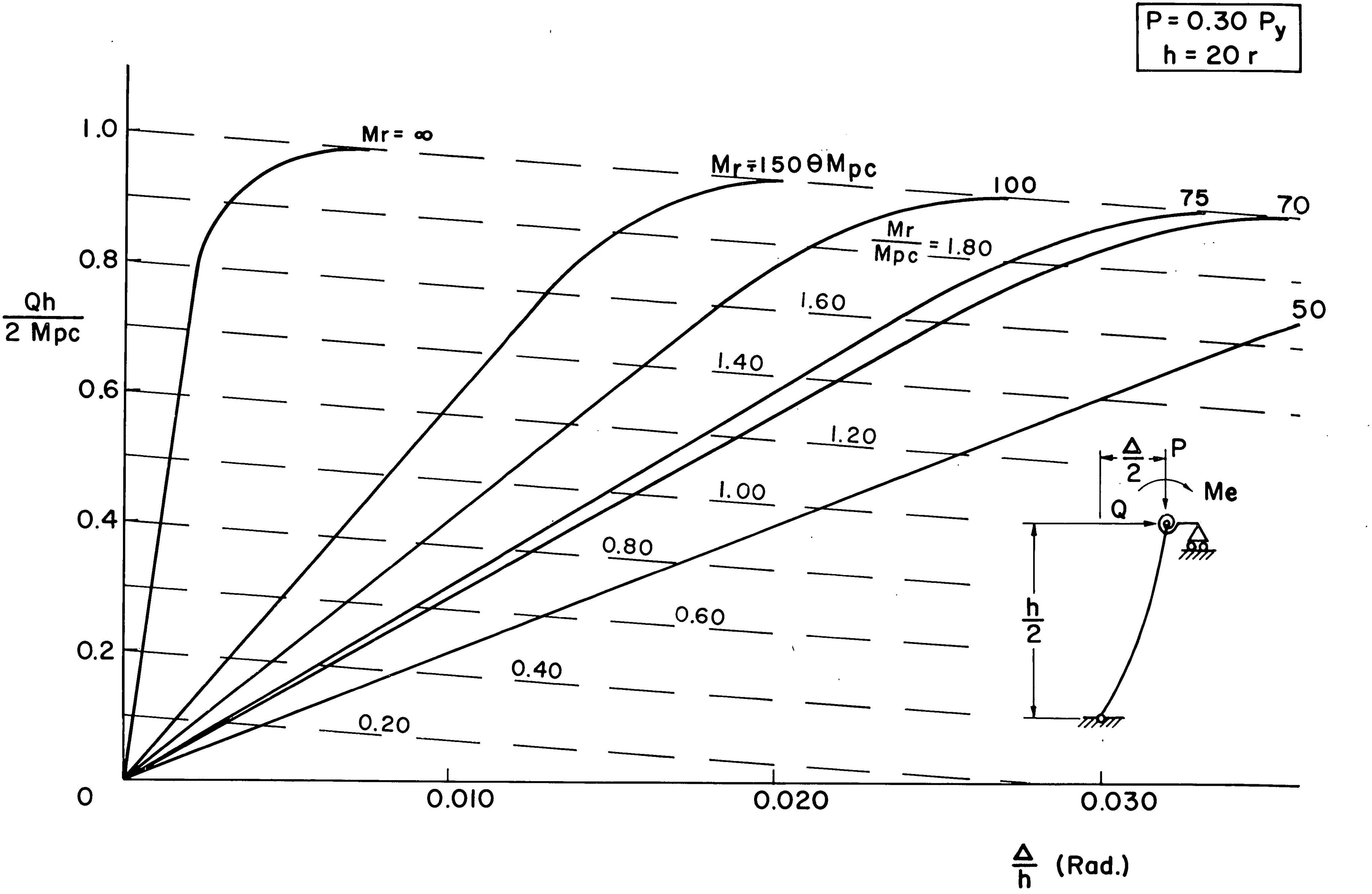
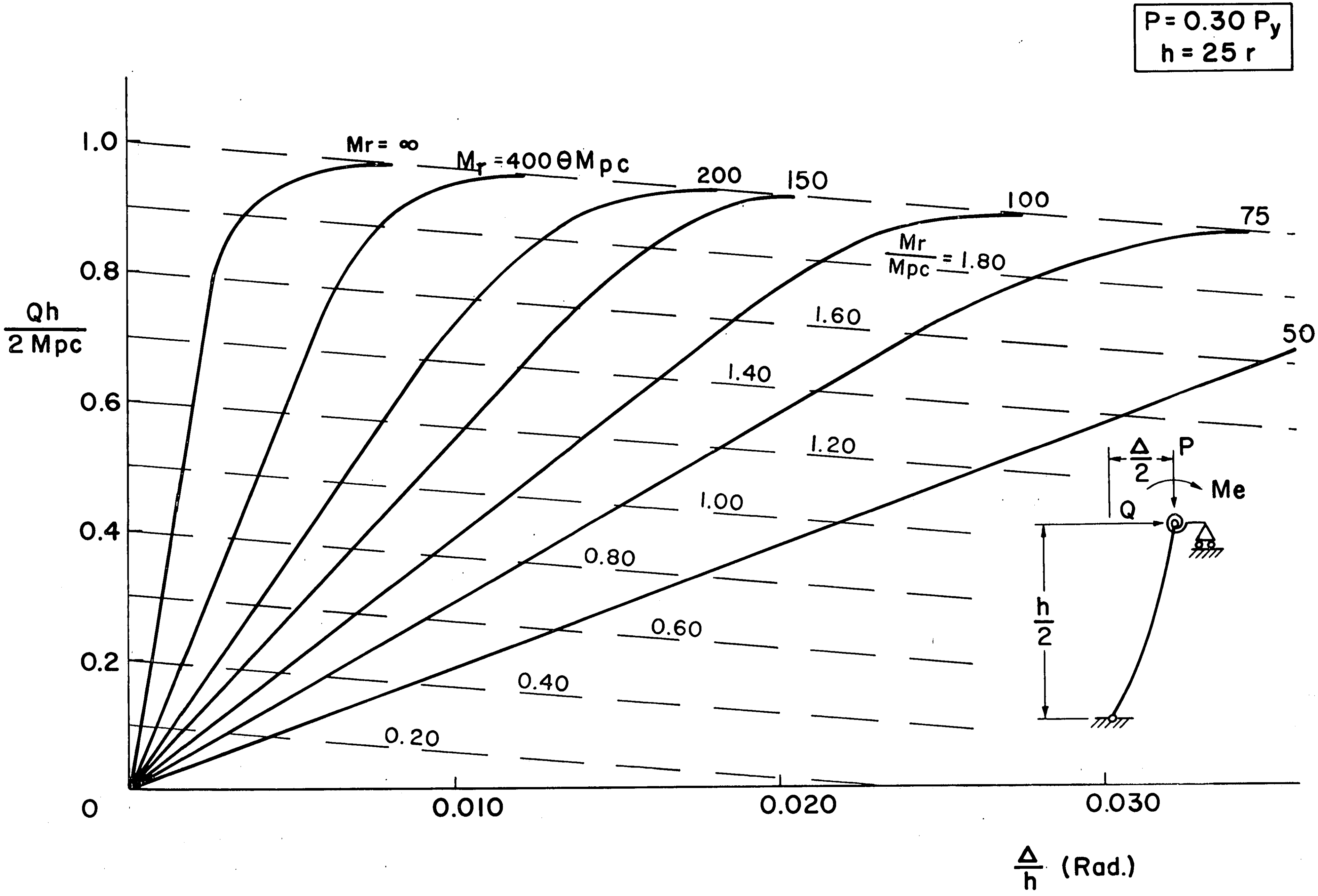

P = 0.30 P_y
h = 20 r
$\frac{Qh}{2\,M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
$M_r = 150\,\theta M_{pc}$
100
75
70
50
$\frac{M_r}{M_{pc}}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—1



P = 0.30 P_y
h = 25 r
$\frac{Qh}{2\,M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
M_r = 400 θ Mpc
200
150
100
75
50
$\frac{M_r}{M_{pc}}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—2

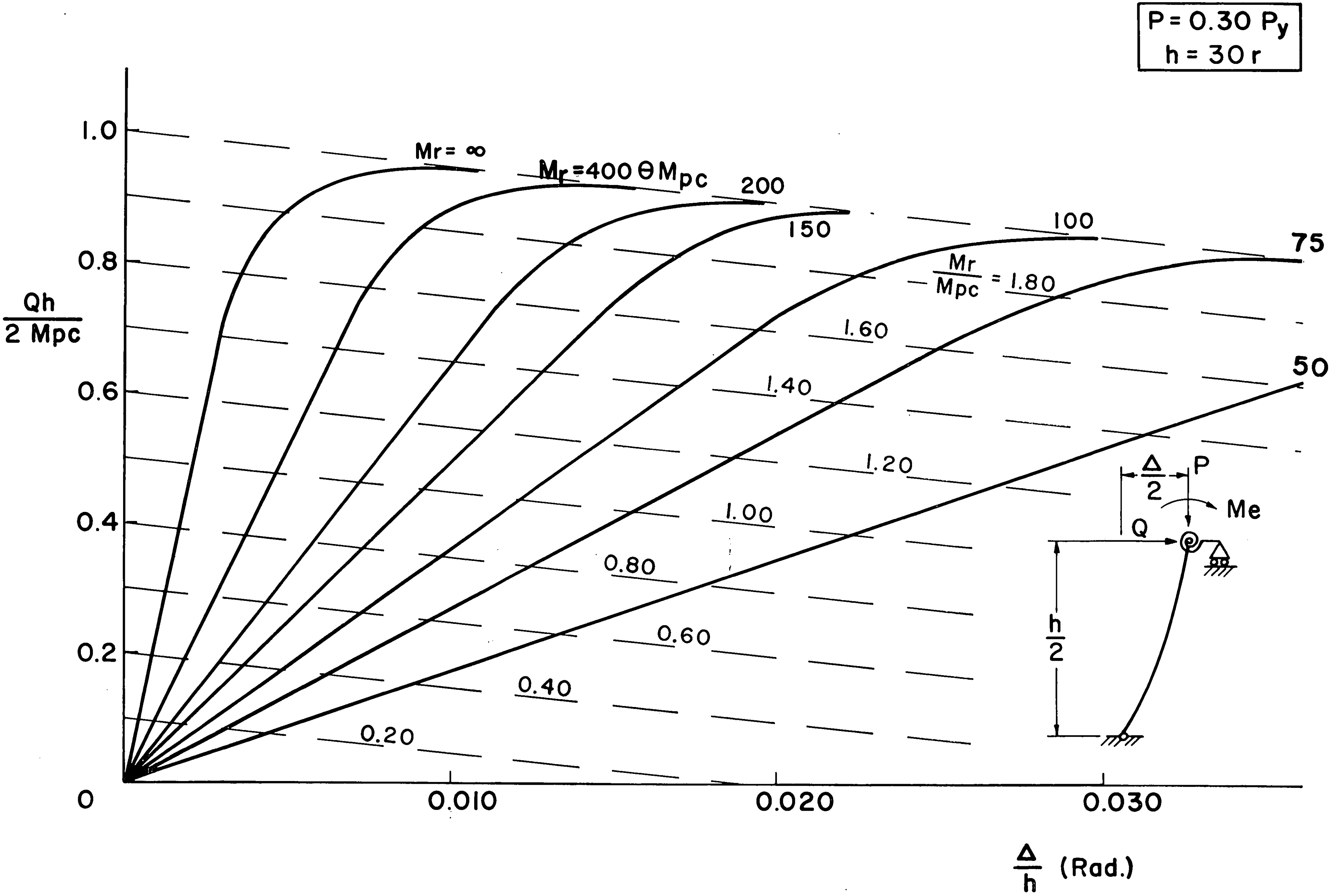

P = 0.30 P_y
h = 30 r
$\frac{Qh}{2 M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
$M_r = 400\,\theta M_{pc}$
200
150
100
75
50
$\frac{M_r}{M_{pc}}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—3

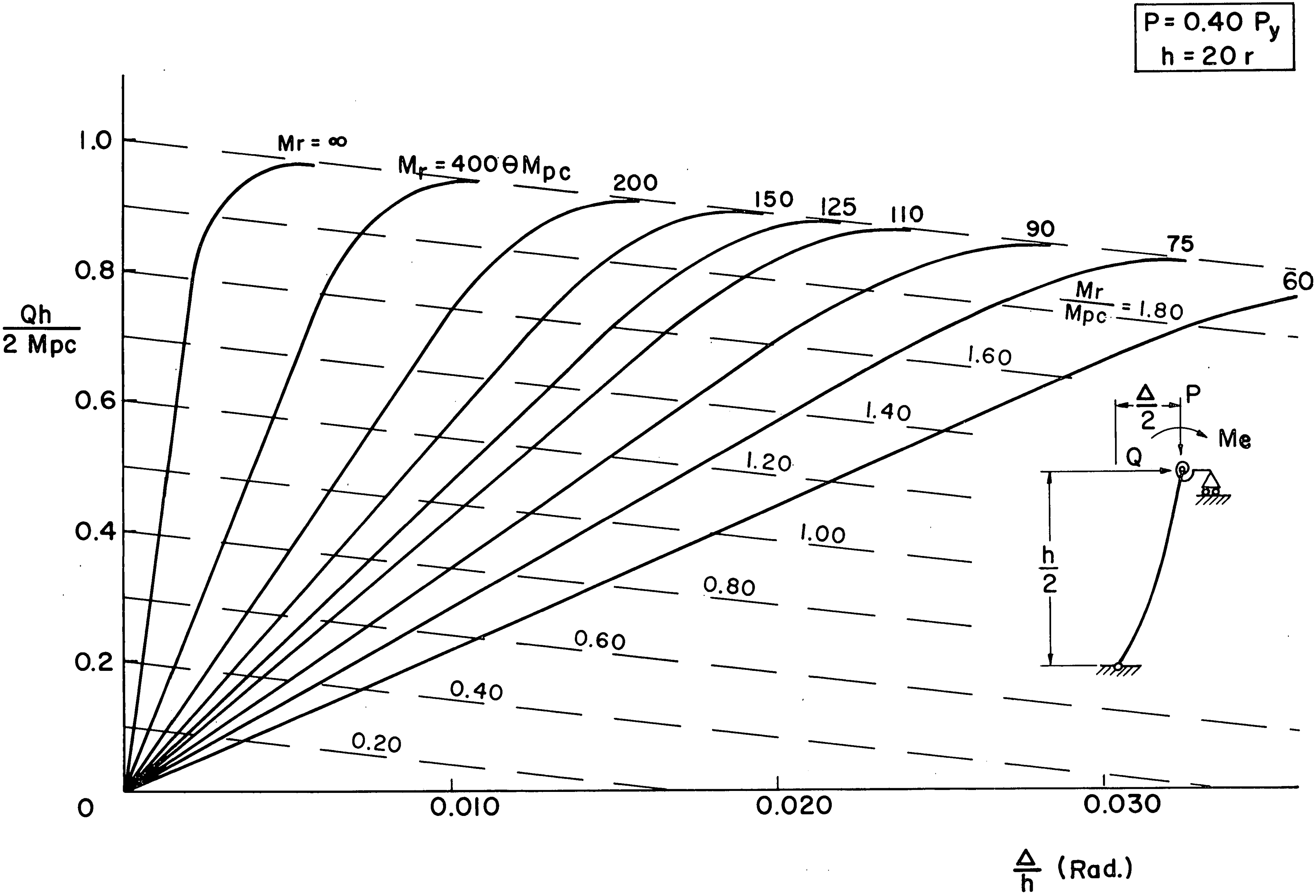
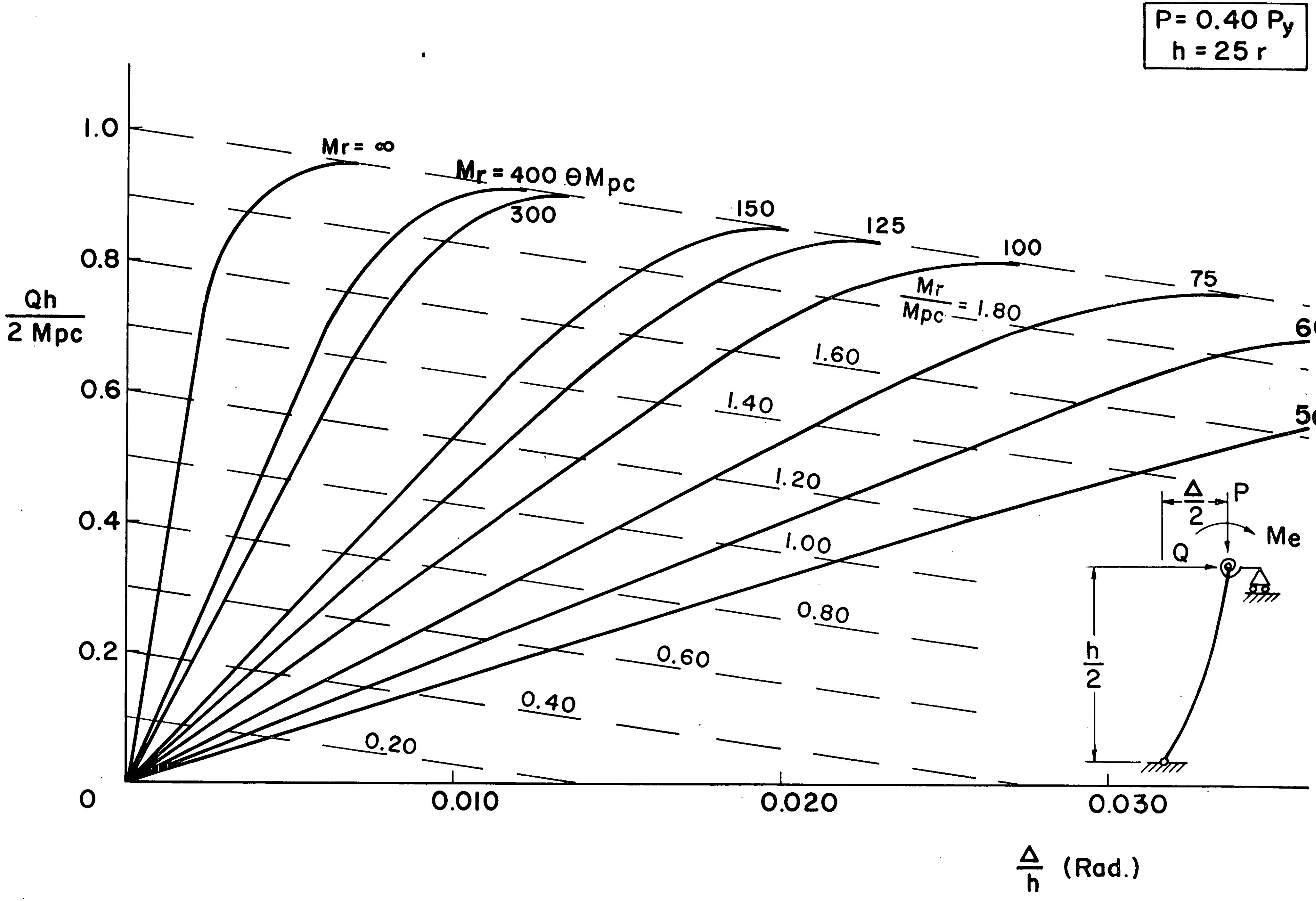

P = 0.40 P_y
h = 20 r
$\frac{Qh}{2\,Mpc}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
M_r = 400 θ M_{pc}
200
150
125
110
90
75
60
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—4



P = 0.40 P_y
h = 25 r
Mr = ∞
Mr = 400 θ Mpc
300
150
125
100
75
60
50
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
1.0
0.8
0.6
0.4
0.2
0
$\frac{Qh}{2\,Mpc}$
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—5

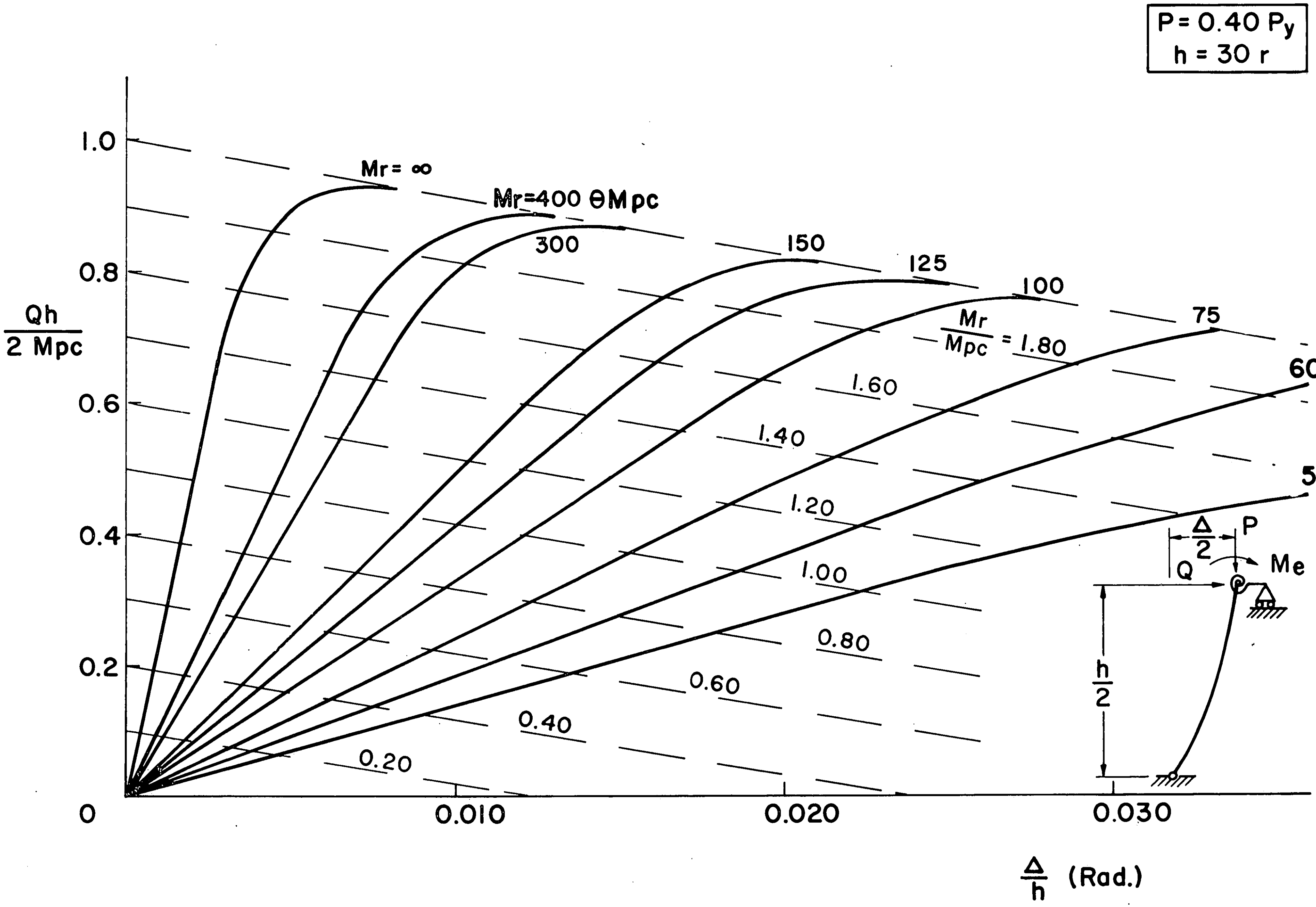

P = 0.40 P_y
h = 30 r
$\frac{Qh}{2\,M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
Mr=400 θMpc
300
150
125
100
75
60
50
$\frac{M_r}{M_{pc}}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
$\frac{\Delta}{2}$
P
Q
Me
$\frac{h}{2}$

CHART VI—6

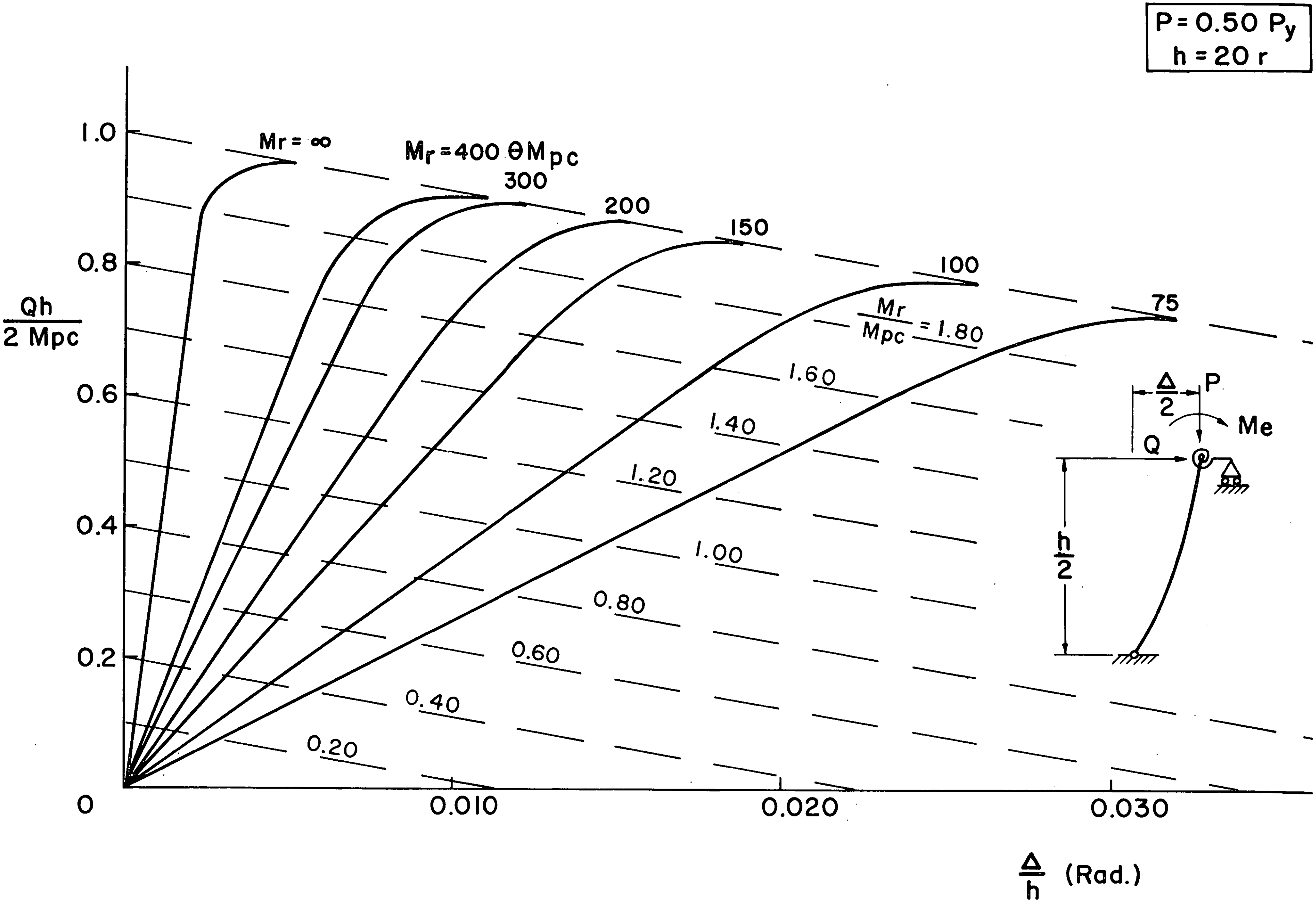

P = 0.50 P_y
h = 20 r
1.0
0.8
0.6
0.4
0.2
0
$\frac{Qh}{2\,Mpc}$
Mr = ∞
M_r = 400 θ M_{pc}
300
200
150
100
75
$\frac{M_r}{M_{pc}}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—7

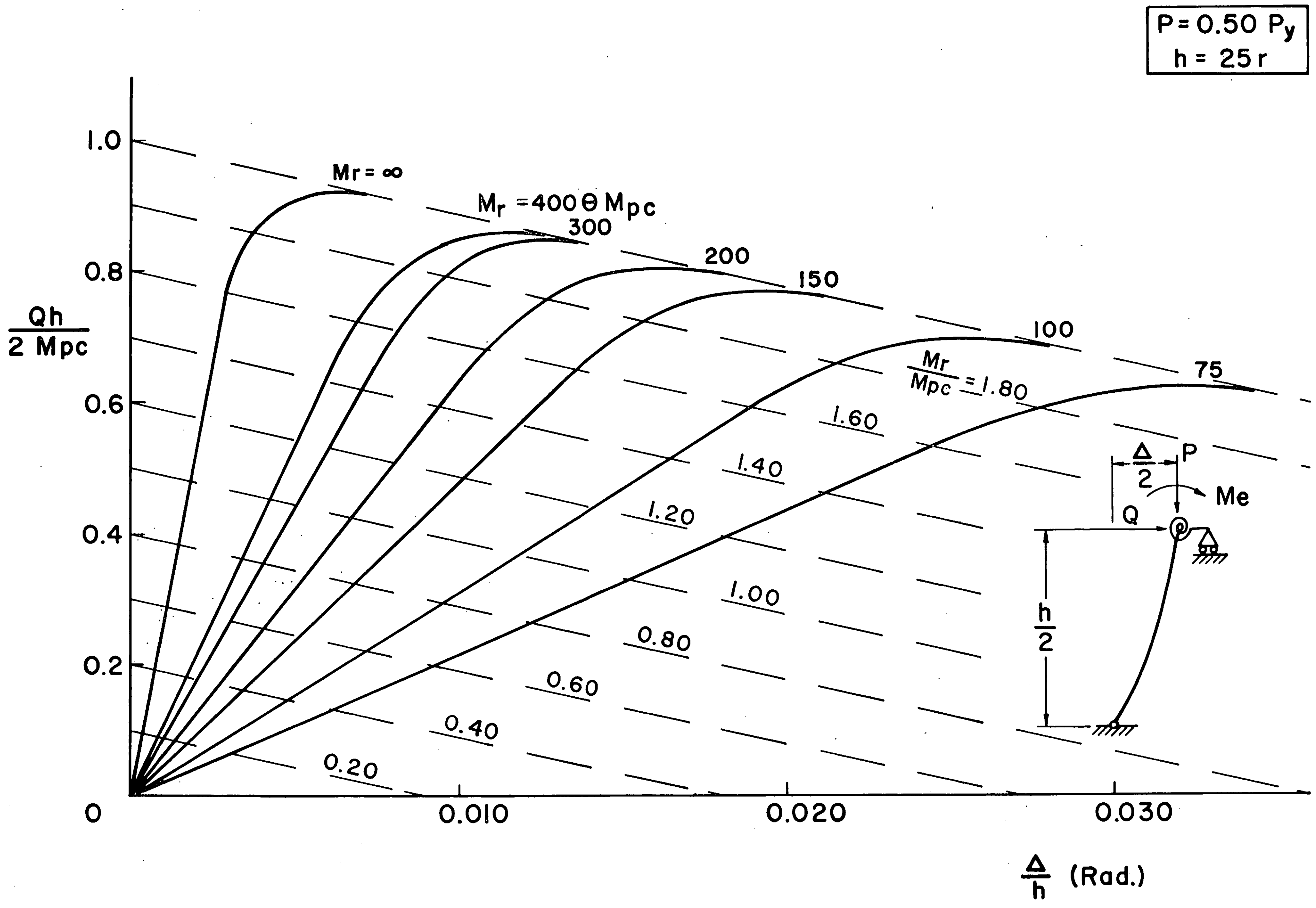

P = 0.50 P_y
h = 25 r
1.0
0.8
0.6
0.4
0.2
0
Qh / 2 Mpc
Mr = ∞
M_r = 400 θ Mpc
300
200
150
100
75
Mr / Mpc = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.010
0.020
0.030
Δ / h (Rad.)
Δ / 2
P
Me
Q
h / 2

CHART VI—8

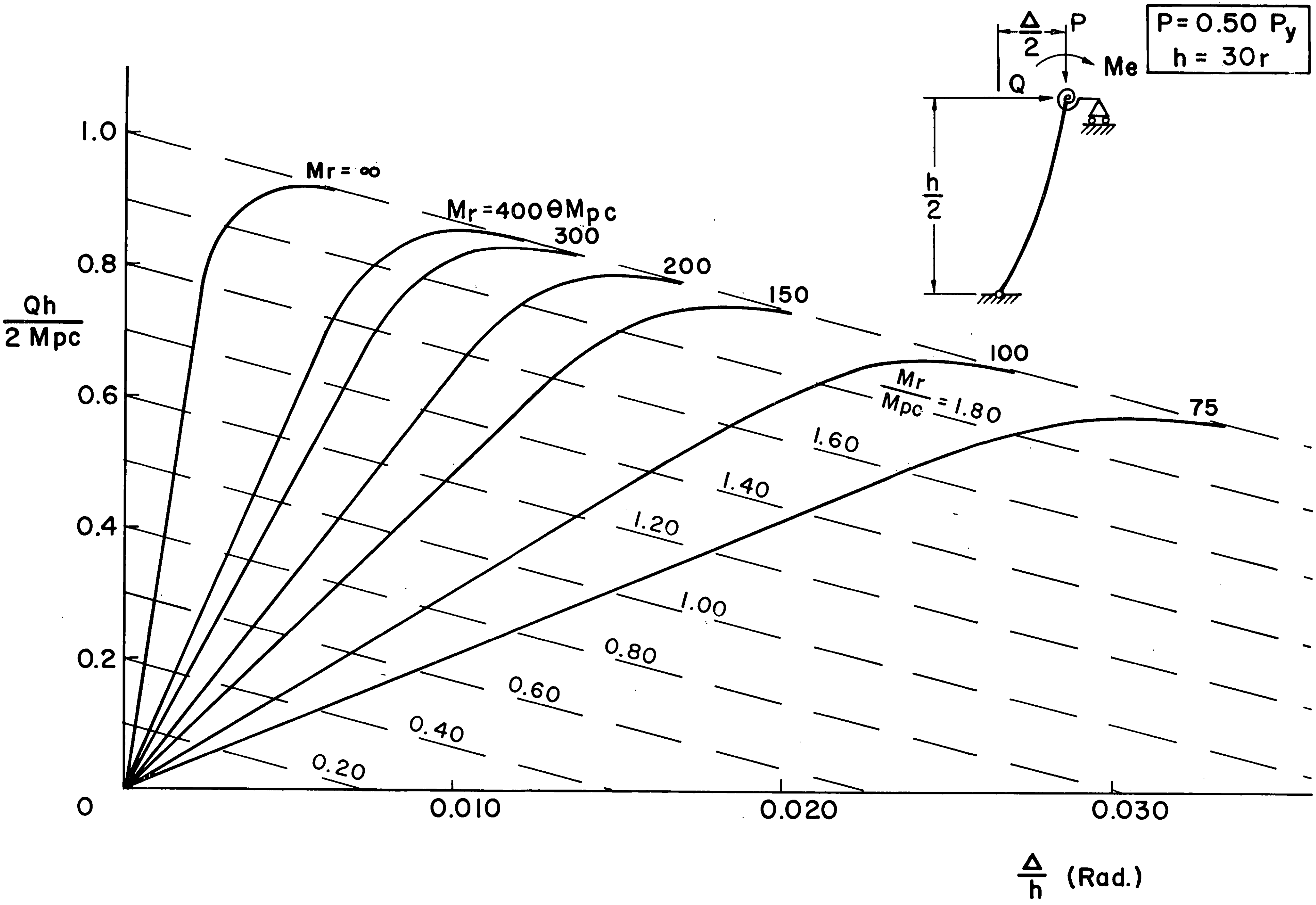

P = 0.50 P_y
h = 30r
Δ/2
P
Me
Q
h/2
Mr = ∞
Mr = 400θMpc
300
200
150
100
75
Mr/Mpc = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
Qh/2 Mpc
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
Δ/h (Rad.)

CHART VI—9

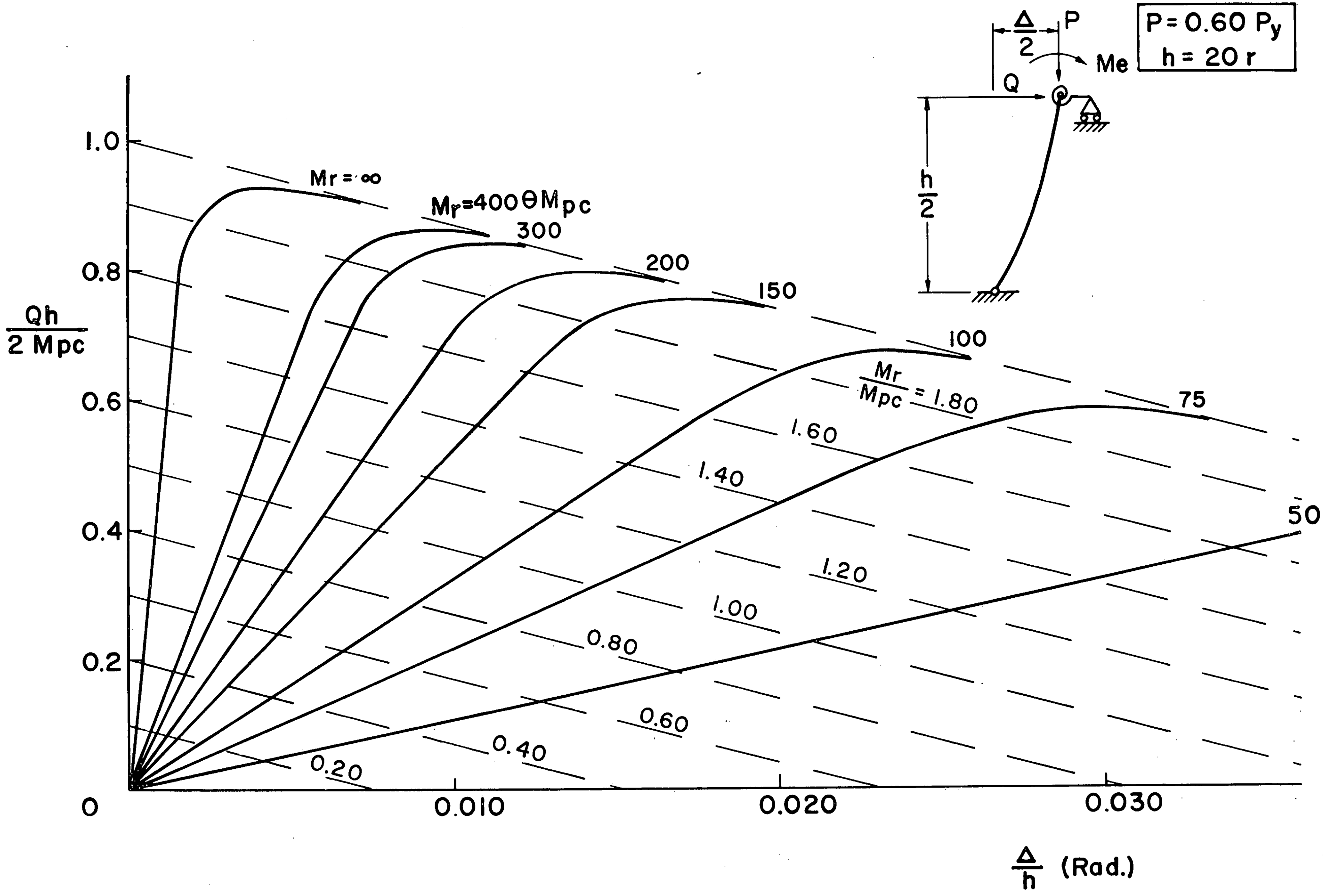

P = 0.60 P_y
h = 20 r
Δ/2
P
Me
Q
h/2
Qh / 2 Mpc
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
Δ/h (Rad.)
Mr = ∞
Mr = 400 θ Mpc
300
200
150
100
75
50
Mr/Mpc = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20

CHART VI—10

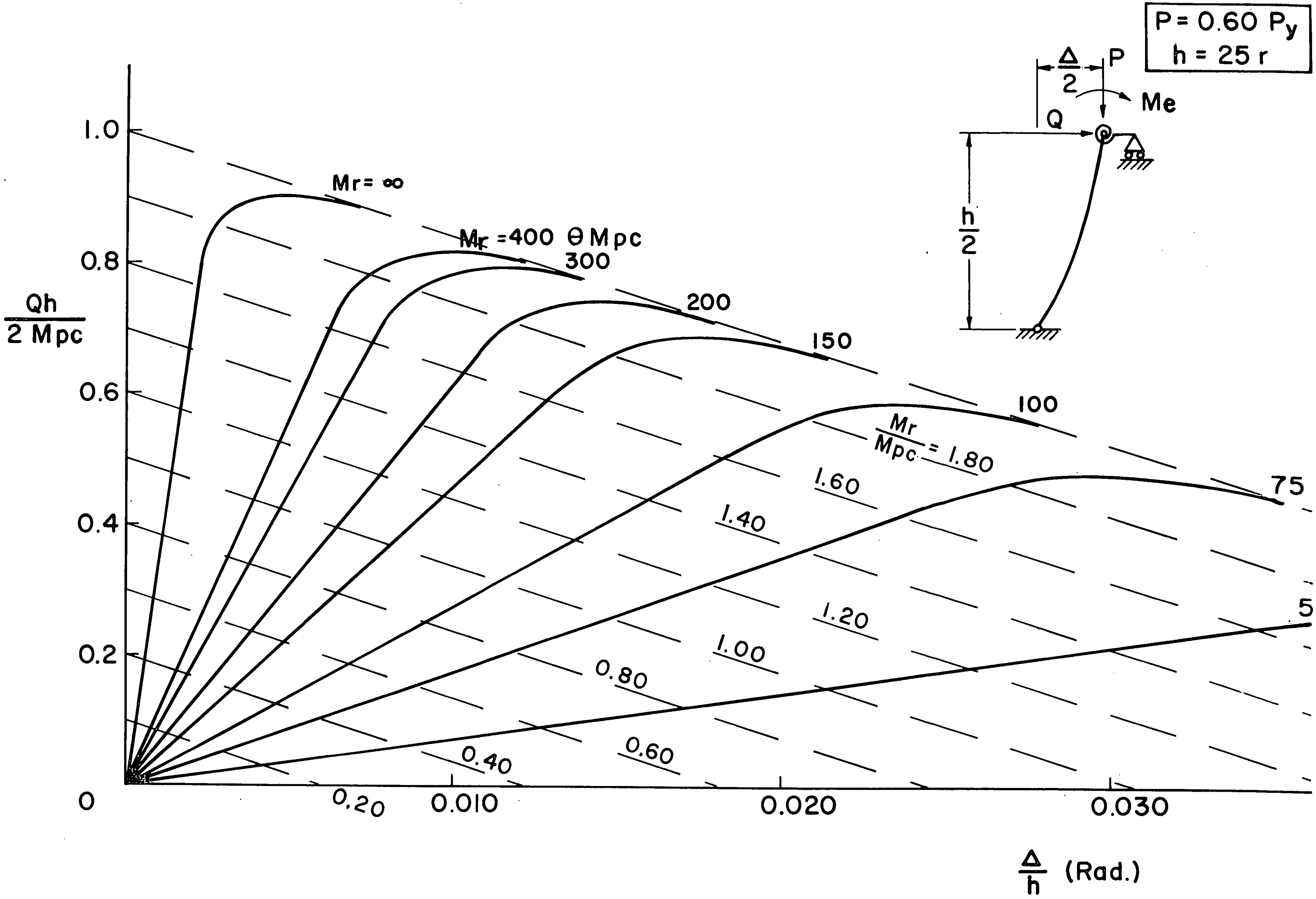

P = 0.60 P_y
h = 25 r
Δ/2
P
Me
Q
h/2
Qh / 2 Mpc
1.0
0.8
0.6
0.4
0.2
0
Mr = ∞
Mr = 400 θ Mpc
300
200
150
100
75
50
Mr/Mpc = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.010
0.020
0.030
Δ/h (Rad.)

CHART VI—11

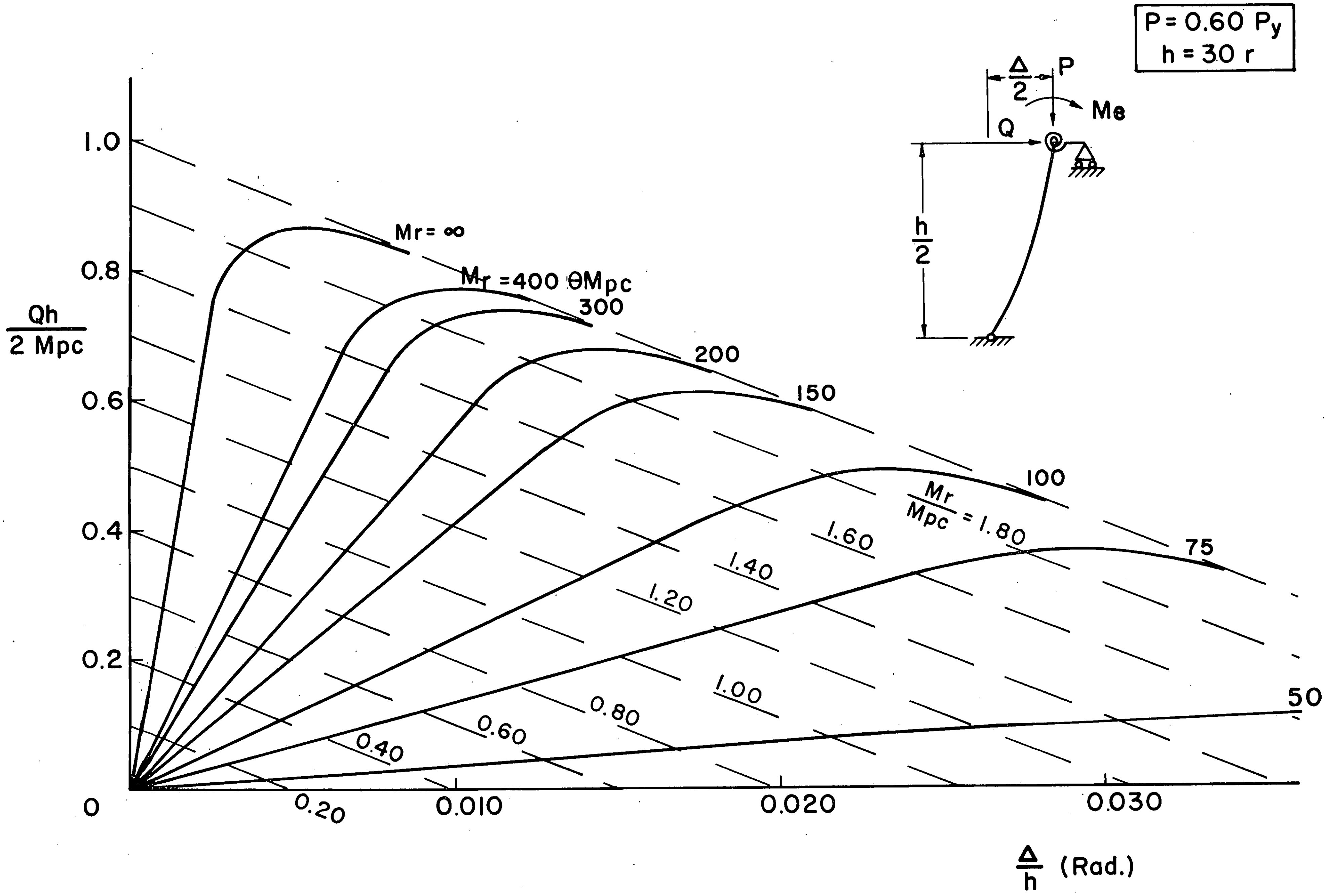

P = 0.60 P_y
h = 30 r
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$
$\frac{Qh}{2\,Mpc}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
M_r = 400 θMpc
300
200
150
100
75
50
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20

CHART VI—12

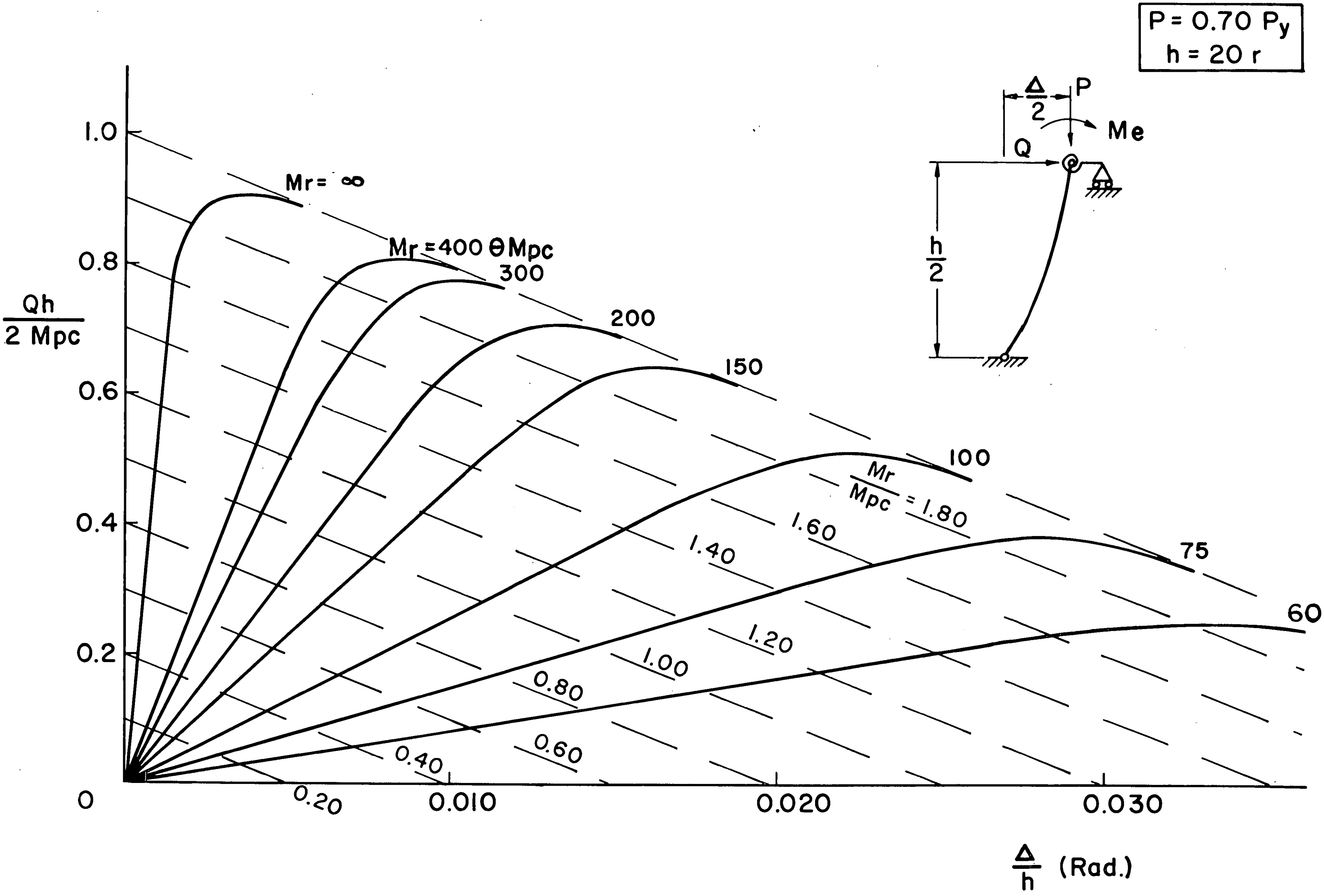

P = 0.70 P_y
h = 20 r
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$
$\frac{Qh}{2\,Mpc}$
1.0
0.8
0.6
0.4
0.2
0
0.20
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
Mr = 400 θ Mpc
300
200
150
100
75
60
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40

CHART VI—13

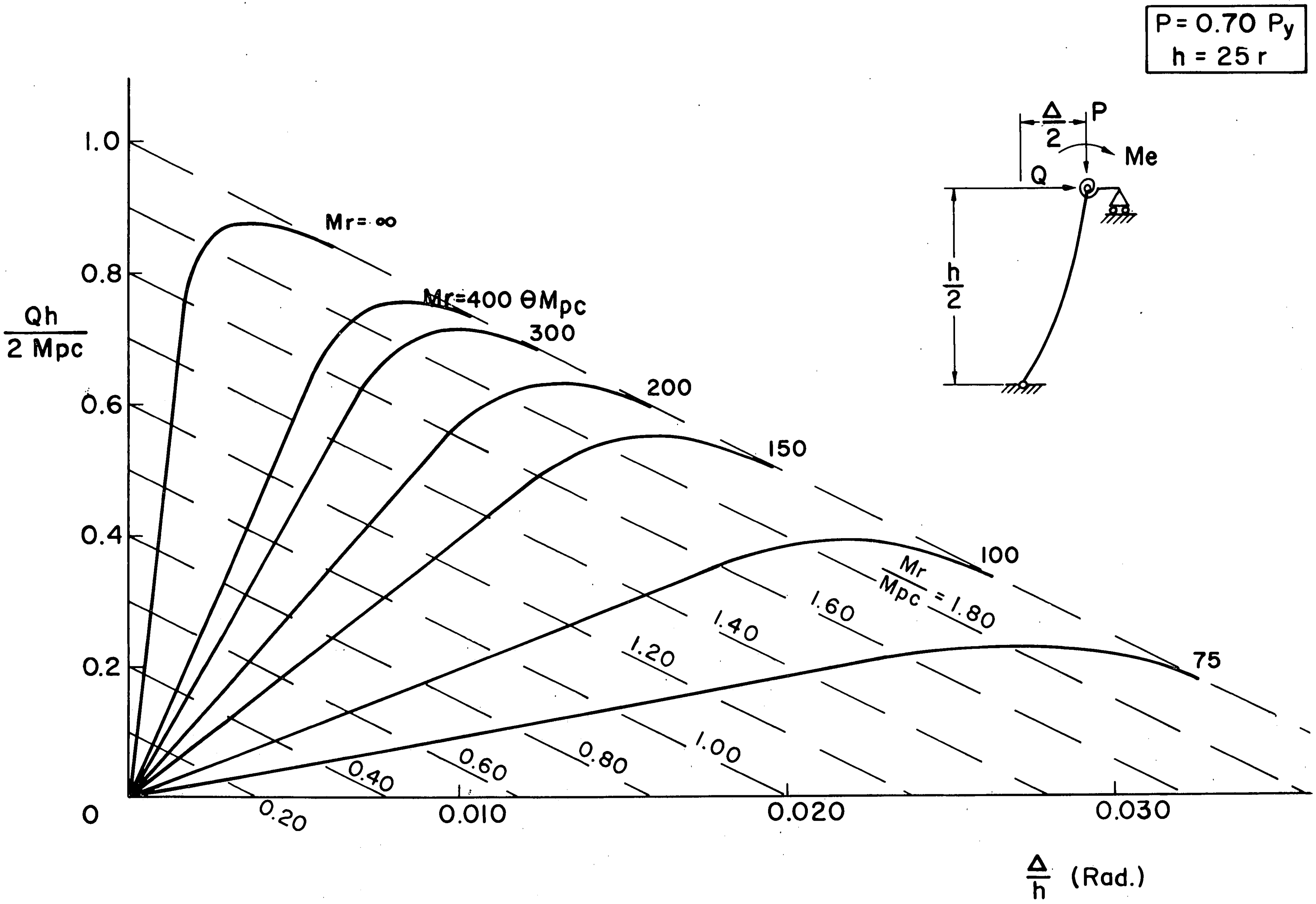

P = 0.70 P_y
h = 25 r
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$
$\frac{Qh}{2\ \text{Mpc}}$
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
Mr = ∞
Mr=400 θMpc
300
200
150
100
75
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20

CHART VI—14

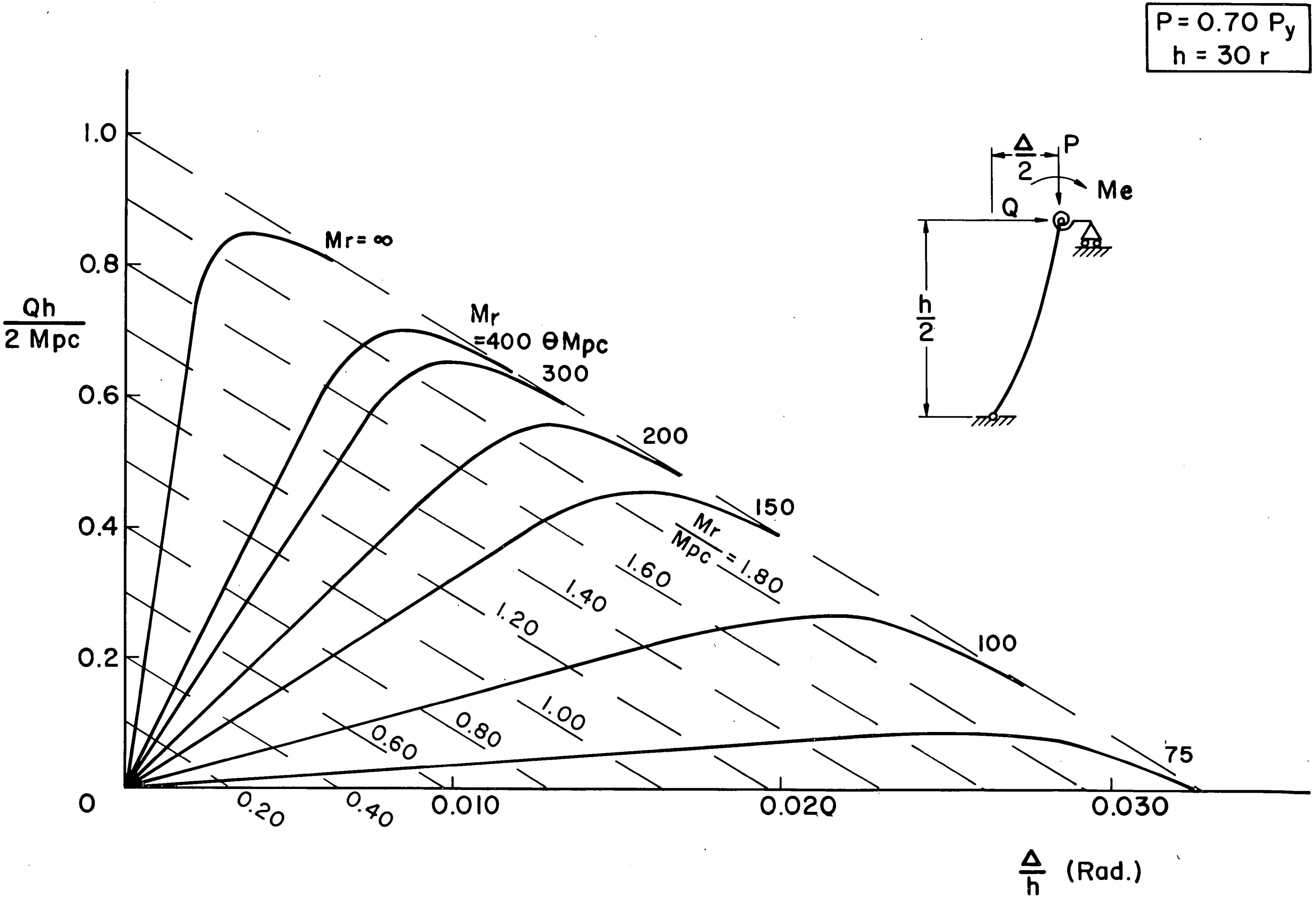

P = 0.70 P_y
h = 30 r
$\frac{Qh}{2\,Mpc}$
1.0
0.8
0.6
0.4
0.2
0
Mr = ∞
Mr
=400 θMpc
300
200
150
100
75
$\frac{Mr}{Mpc}$ = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—15

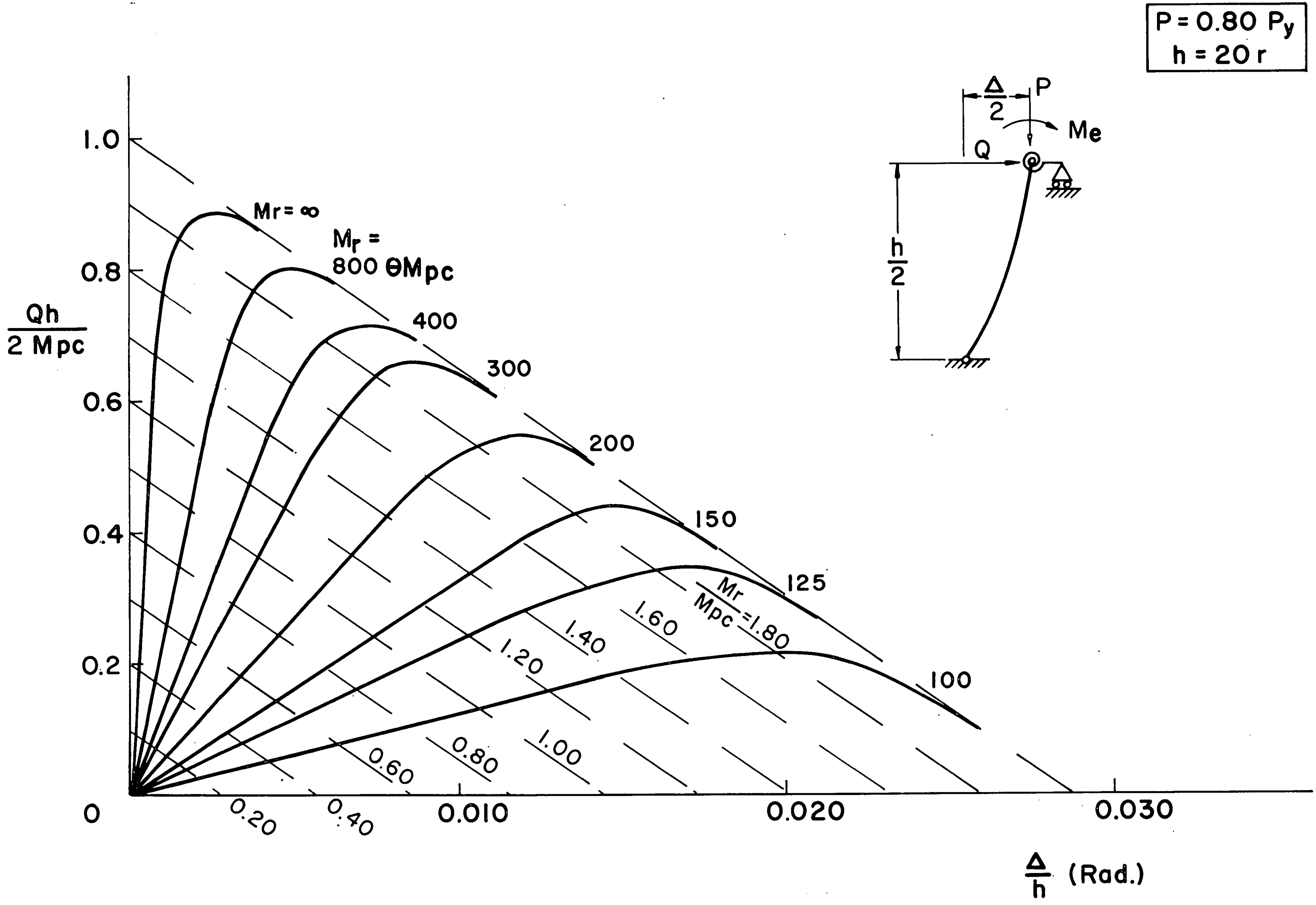

P = 0.80 P_y
h = 20 r
Δ/2
P
Me
Q
h/2
Qh/2 Mpc
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
Δ/h (Rad.)
Mr = ∞
M_r = 800 θMpc
400
300
200
150
125
100
Mr/Mpc = 1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20

CHART VI—16

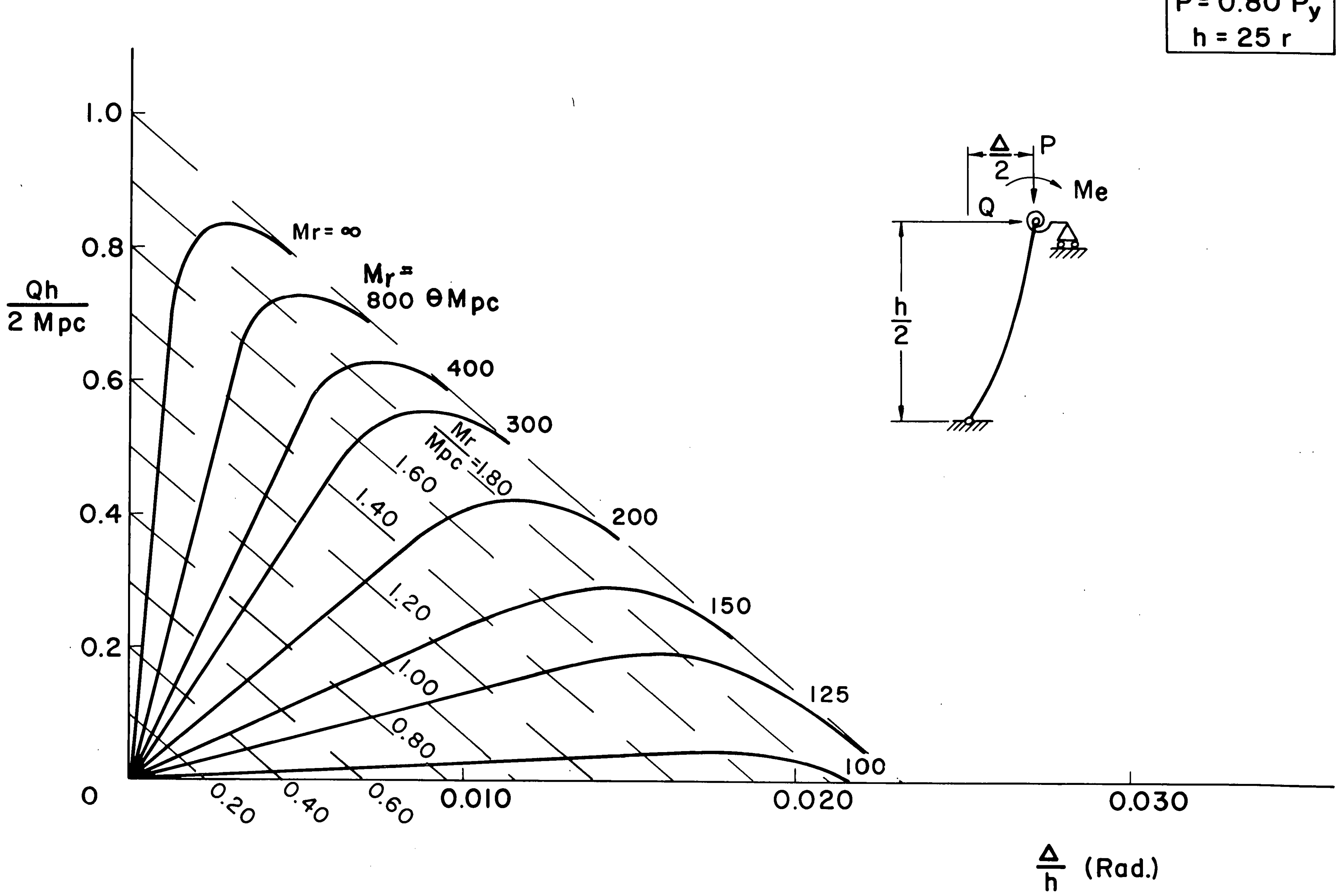

P = 0.80 P_y
h = 25 r
$\frac{Qh}{2\,M_{pc}}$
1.0
0.8
0.6
0.4
0.2
0
$M_r = \infty$
$M_r =$ 800 θM_{pc}
400
300
200
150
125
100
$\frac{M_r}{M_{pc}} = 1.80$
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.010
0.020
0.030
$\frac{\Delta}{h}$ (Rad.)
$\frac{\Delta}{2}$
P
Me
Q
$\frac{h}{2}$

CHART VI—17

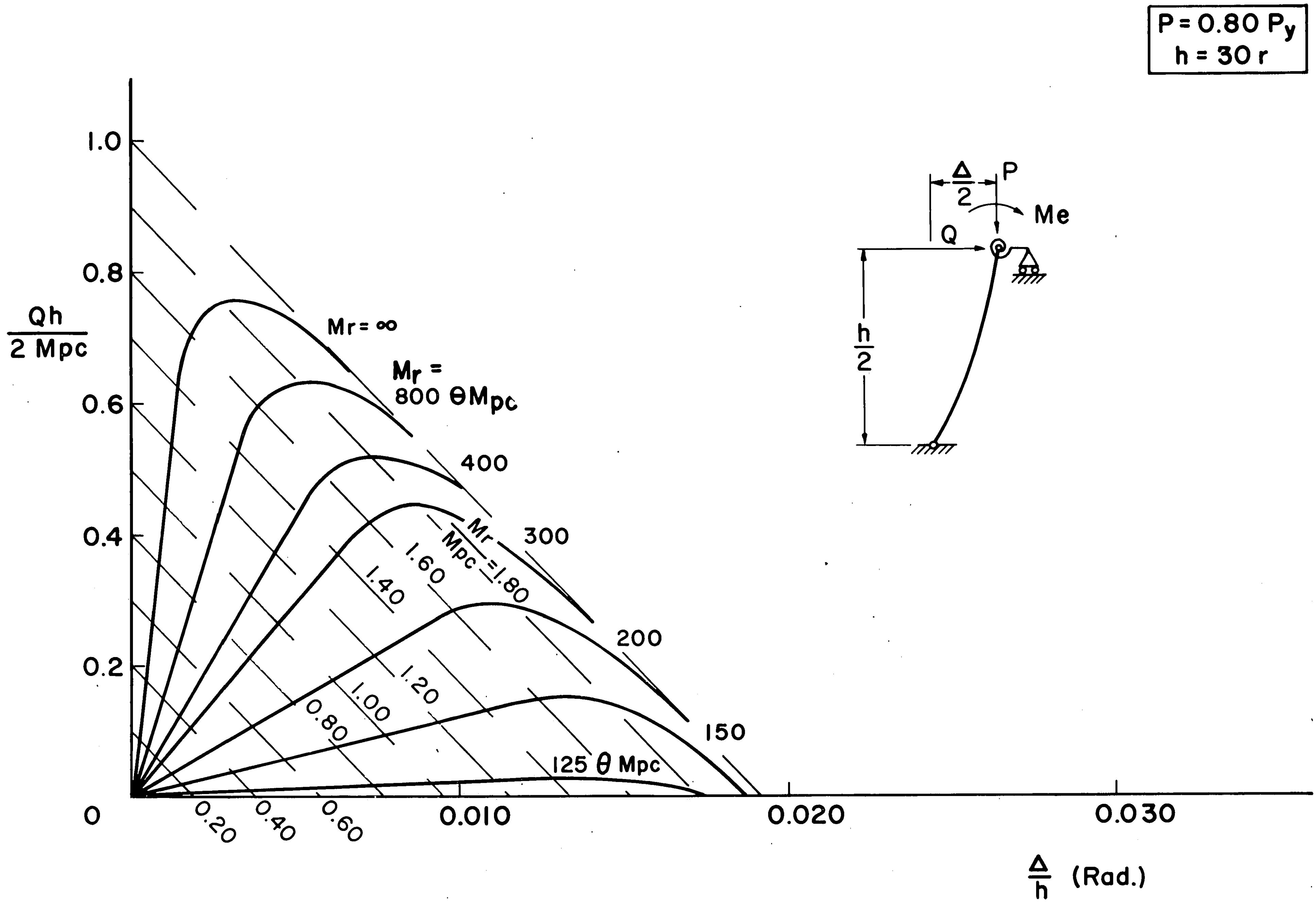

P = 0.80 P_y
h = 30 r
Qh / 2 Mpc
1.0
0.8
0.6
0.4
0.2
0
0.010
0.020
0.030
Δ / h (Rad.)
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
Mr / Mpc = 1.80
Mr = ∞
Mr = 800 θMpc
400
300
200
150
125 θ Mpc
Δ / 2
P
Me
Q
h / 2

CHART VI—18