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B. G. Johnston

E. H. Mount

DESIGNING WELDED FRAMES FOR CONTINUITY

by

Bruce Johnston* and E. H. Mount°

SYNOPSIS

This paper presents a procedure for the direct design of beams in welded building frames to take partial advantage of the reduction in maximum moment afforded by moment-resisting, semi-rigid, connections. The aim has been to develop a direct and simple design procedure which takes into consideration the behavior of the frame as well as the connection. The average saving in the weight of beams is fifteen per cent or more and the type of construction is applicable for floor loads specified for apartment house and office building construction with a maximum beam length of about twenty feet. The actual economy effected is less than the saving in weight because heavier welds are needed in the connection details to produce the required end restraint.

Previous research on the design of connection details is reviewed and design procedures and charts developed for each step in the design. The results of tests of two full-sized welded building frame bents are presented to illustrate the agreement between test results and the methods of theoretical analysis on which the proposed design methods are based.

* Assistant Director, in charge of research, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania
° Formerly American Welding Society Research Fellow, Lehigh University, now with Dravo Corporation, Pittsburgh, Pennsylvania

INTRODUCTION

The present investigation is one of several which have been sponsored by the Structural Steel Welding Committee of the American Welding Society at the Fritz Engineering Laboratory of Lehigh University. In September 1938, the Committee authorized the authors to proceed with a program involving the construction and test of two full-size model building frames using the type of beam-column connection previously developed by research at the Fritz Laboratory[1,4,5*]. Additional funds amounting to $600 were furnished by the Committee to cover the cost of constructing the frames and purchasing loading tanks. The Committee also requested that the authors review and correlate previous research on beam-column connections which had been made at the Fritz Laboratory under the direction of Dr. Inge Lyse, now Professor at the Norges Tekniske Høiskøle, Trondheim, Norway.

The following presents a brief review of recent investigations which are related to the present program. In February 1935, Inge Lyse and Norman G. Schreiner discussed welded seat angle design[1] and in June of the same year Heath Lawson[2] suggested a design procedure. These investigations considered the design of end connections to resist only the vertical reaction. In January 1936, Wilbur M. Wilson[3] presented the results of tests of beam-column connections designed for as nearly complete rigidity as possible. In

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* These numerals refer to references at the end of the article.

October of 1936 and 1937 Inge Lyse and Glenn Gibson[4,5], presented two successive reports which correlated test results with a design method applicable to beams with semi-rigid connections which were assumed to frame into fixed walls or non-bending columns. Inge Lyse and E. H. Mount[6] later presented a paper studying the stresses produced in the columns by this type of connection. The whole question of analysis and design of building frames, taking into account the existing continuity, has been studied by the Steel Structures Research Committee of Great Britain[7] in connection with tests of riveted building frames. The present authors in an as yet unpublished paper[8] have discussed in detail both the slope deflection and moment distribution methods of analysis as applied to frames with semi-rigid connections.

The present investigation is not presented as any final answer to the question of designing buildings to take account of partial continuity, but rather as a pilot investigation which shows that feasible methods for the design of beams in such frames may be developed. Much work remains to be done in the testing of other type joints in order to make design procedures similar to that proposed applicable to a wider range of beam sizes.

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THE GENERAL PROBLEM

In the past decade there has been rising interest among engineers toward the use of continuity in structures. Fully continuous steel girders and rigid frames, both riveted and welded, have been built in increasing numbers both in this country and abroad. Engineers who have designed beams in building frames have also been conscious of the fact that partial continuity exists to a greater or less degree in all building frames, whether taken account of in design or not.

If the beam-column connections of a building frame are designed to transmit bending moment without allowing any relative rotation between the end of the beam and column the connection may then be termed "rigid" and the resulting structure is fully continuous. The connections in such a case are said to afford one hundred per cent restraint. In the usual building frame the connections provide less than one hundred per cent restraint and there is a relative rotation between the beam and column when moment is transmitted.

Such a structure might be said to be "semi-rigid", "semi-continuous", or simply termed "a frame with moment resisting connections". The latter term would apply equally well to the so-called "rigid frame".

Rigid frame construction with one hundred per cent restraint in the connections does not afford the greatest possible economy in building construction for two reasons:

(1) The welded details to provide a connection with one hundred per sent restraint are more costly than those providing between fifty and seventy-five per cent restraint.

(2) Except for the case of a concentrated load in the center of a beam the potential economy in beam design is greater for a connection restraint of seventy-five per cent than for one hundred per cent.

Fig. 1 illustrates several different loading conditions, showing the center and end moments for 0, 50, 75, and 100 per cent restraint in connections and with columns assumed not to bend. It will be noted that the moment in the center of the beam is always critical for any degree of restraint between 50 and 75 per cent and for any type of loading. Above 75 per cent the moments at the ends of the beam become critical. It should also be noted that if the beam is designed for the center moment, with connections having 50 per cent restraint, the beam itself will not be overstressed for any variation of restraint between 50 and 100 per cent.

In an actual building frame the conditions illustrated in Fig. 1 do not exist because the columns are not rigid but are free to bend. The bending of the columns will reduce the end moments to some extent and increase the center moments. The problem of design of the beams depends therefore not only upon the degree of restraint provided by the connections but upon the general behavior of the building frame as well.

TEST OF THE MODEL BUILDING FRAME

In order to study the general behavior of welded building frames and to correlate this behavior with methods of analysis and design, two full-size model building frames were constructed. Methods of analysis had already been developed[7,8] and it was desired to corroborate these by experimental test results on welded building frames so that the analytical methods could then be used to develop design procedures. A similar procedure was used in tests of riveted building frames in Great Britain[7].

Frame No. 1 is illustrated by the line drawing in Fig. 2 and by Fig. 3, which shows a photograph of the frame with loading tanks in position. The 8 by 8-in. W.F. 31.0-lb. column sections were welded in a vertical position to 16 by

16 by 1-1/2 in. base plates. Cement grout was run between the base plates and piers and the anchor bolts were drawn tight after the grout had set up. The piers were flush with the floor, measured 1 ft. 6 in. by 1 ft. 6 in. at the top, and pyramided to a 2 ft. 6 in. by 2 ft. 6 in. size at the base. They were 2 ft. high and had four 1-in. diameter pipes cast in place to take removable 3/4-in. diameter anchor bolts 18 in. in length. The beams were 10-in. I's at 25.4 lb. placed upon 4 by 4 by 1/4 by 6-in. seat angles which had been welded to the columns with 1/4-in. fillet welds along each of their 4-in. legs prior to erection of the columns. The design method previously proposed by Lyse and Gibson[4,5] was used in proportioning the top angles which were cut 3 by 3 by 11/16 by 4-3/4 in. long from 3-1/2 by 3-1/2 by 11/16 in. original size. The top angles were welded to the top flange of the beam and to the column flanges with 11/16 in. fillet welds. The lower flanges of the beams were welded to the seat angles on each side with 1/4-in. fillet welds 3-1/2 in. long, designed to take the thrust developed by the moment at the end of the beams.

Frame No. 2 was similar in general dimensions and details of design except that the columns were turned 90° with respect to their position in Frame No. 1. The beams were thus connected to the column webs rather than to the column flanges.

Each frame was braced laterally near each joint by means of flexible ties welded between columns of the frame and columns of the laboratory. The lateral ties were 1-in. diameter bars with section reduced near each end to 1/4-in. diameter for a length of 1/2-in. The frame was in this way braced against movement out of its own plane but was at the same time free to bend or move in the plane of the frame.

Vertical loads were applied to the beams of the completed frame by means of water tanks which are shown in Fig. 3 and 5. Lateral load was applied in one test only by means of the outrigger device shown in Fig. 4.

The six loading tanks, which are shown in one loading position in Fig. 3, were of 3/16-in. welded plate construction. Each tank was 4 ft. 6 in. in diameter, 7 ft. 6 in. deep, and had a capacity of 7000 lb. of water. Both inlet and outlet was effected by means of 2-in. diameter valve and hose connections. A load increment of 6500 lb. was used in tests and a 1/2-in. round steel rod welded vertically in the center of each tank was marked at points of initial and final load. The tanks were calibrated by running the water into auxiliary weighing tanks placed on platform scales of 1000 lb. capacity. Each tank was hung in position on the beams of the frame by means of welded hangers shown in Fig. 5. The removable pins which may be seen in Fig. 5 allowed rapid change of the tanks to different loading positions.

Prior to building the frames the beam and column sections were tested individually in pure bending to determine experimentally their coefficient of bending stiffness, E x I. The size and thickness of material in beams and columns was also checked by micrometer calipers and was found to agree closely with handbook values.

The experimental determination of the moments in the beams and columns of the actual frame was made by measuring the absolute rotation of the ends of each beam, and of the columns at the joint centers at each floor level. The rotations were measured by the 20-in. level bar illustrated in Fig. 6, in position on one of the rotation bars. The level bar was sensitive to a rotation of $\pm \frac{1}{200,000}$ of a radian or $\pm$ 1 second, and consisted of a 10-second precision level bubble mounted on an aluminum bar. Two sharpened steel points supported the bar at one end and the other end was supported by a micrometer screw which was used to bring the bar to level position for each reading. The elevation of the micrometer end of the bar was read by a 1/10,000 Ames Dial. The dial movement divided by the gage length gave the relative rotation in radians between any two load conditions. For each measurement the level bar rested in an identical location upon the polished surface of rotation bars which were attached by arms to the gage stations of the frame. These bars are shown in position in Fig. 7a, 7b, and 7c for an interior joint, a column base, and for an exterior joint, respectively. Four 1/1000 Ames dials as shown in Fig. 7c, measured the lateral movement of the frame at each story level for the two exterior columns.

The moments at the ends of each beam and at the joint centers of each column were computed directly from the well known slope deflection equation:

$$M_{AB} = \frac{2EI}{l} \left(2\theta_A + \theta_B - \frac{\Delta}{l}\right) \pm M_{FR} \qquad (1)$$

θ_A and θ_B were the measured rotations at the near and far end, respectively, of any particular member. Δ was the relative lateral movement between the ends of a member, and was equal to zero for symmetrical loading conditions. The quantity EI was experimentally determined by the method previously explained. M_{FR} is the computed end moment in the loaded beams due to applied loads assuming rigid connections and fixed ends.

Direct strain readings were also made in the beams and columns by means of a 20-in. Whittemore strain gage. While these checked reasonably well with calculated strains the results were somewhat erratic because of the small increments of measured strain. The rotation readings were accurate, sensitive, and gave very consistent results.

As a basis for the theoretical analysis of the frame by the method of moment distribution it was necessary to evaluate the relation between moment and angle change for the beam-column connection used in this particular frame.

Three sample joints were therefore constructed identical with those used in the frames. Fig. 8a shows the test set-up for the beam-to-column flange connection used in Frame 1, and Fig. 8b shows the test set-up for the exterior beam-to-column web connections, in which case the column elements acted independently to allow free deformation of the web. A third joint for interior beam-to-column web connections was similar to Fig. 8a. The relative rotation between the ends of the beam and the center of the column at the joint were measured with the 20-in. level bar in the same manner as on the test frames. The relation between moment and angle change at the connection is denoted by the Greek letter "γ", and termed the "connection constant"[7].

$$\gamma = \frac{\phi}{M} \qquad (2)$$

In equation (2) ϕ is the relative angle change between the end of the beam and the column at the joint center. γ may be defined as "angle change for unit moment".

Another constant α, relates the constant γ with any particular beam section and length used in an actual frame:

$$\alpha = \frac{2EI\gamma}{l} \qquad (3)$$

In equation (3) I and l are the moment of inertia and length between connections respectively of any particular beam.

The method of theoretical analysis is developed elsewhere[7,8] but test results will be presented to show the agreement between theoretical and experimental values. Fig.9 shows the results of connection tests to determine γ, which is the inverse slope of the experimentally determined straight line curves. Because of the possibility of reversal of moment in the frame when loaded laterally the joints were loaded under both normal and reversed bending up to full design load. The dashed line in Fig. 9 represents a theoretical relationship in which all of the deformation is assumed to take place in the top angle. It is noted that the agreement is fairly good for the beam-to-column flange connections. The agreement is excellent for the double web connection on an interior column but the connection for an outside web connection tested as shown in Fig. 8b has much greater flexibility in normal bending than the other connection types. This is believed due to the distortion in the web and may be prevented by welding reinforcing plates on the outside columns opposite the connections.

Using the experimentally determined values of γ as determined from Fig. 9 the method of moment distribution was used to analyze the frames for the various loading conditions used in the test[7,8]. Width of member was taken account of in the analysis. Fig. 10 to 14 inclusive, show both the calculated and experimentally determined moments for several of the critical conditions of loading applied to Frames 1 and 2.

Fig. 10 shows the moments in Frame 1 with all of the first floor beams loaded and the second floor beams unloaded. The close agreement between experimental results and analysis based on an experimental test of a typical connection should be noted. Fig. 11 shows the test results for an unsymmetrical loading in which only one outside second story beam was loaded. Sidesway was present to a small degree but was neglected in the analysis. The agreement between theory and test is excellent and the rapidity with which the moments taper out away from the region of the loaded beam shows that only a local region of a building frame need be considered in analyzing any critical condition of loading. The analysis assuming completely rigid joints is also shown by the dash-dot line in this figure to show the divergence with experimental results and also to indicate the greater extent to which moments would be transferred in a rigid frame to sections remote from the loaded beam. Fig. 12 is presented to show the maximum probable divergence between theory and test for a critical condition of loading. In this test the order of loading was purposely unbalanced, but the final loading showed fair agreement between theory and test. A similar test with loading maintained balanced during application resulted in much better agreement, however.

Fig. 13 and 14 present the results of tests on Frame 2, with web connections. The difference in behavior between outside column connections and inside column connections was taken account of in the analysis and a very close agreement with test results is noted for each condition of loading.

All of these tests show that theoretical analyses which depend in part on a separate test of a sample joint agree well with experimental test results of a full size frame. The methods of analysis are therefore satisfactory as a basis for the development of design methods.

RELATION OF TEST RESULTS AND ANALYSIS TO DESIGN

The design procedures here developed are based on the results of both the present and former tests at the Fritz Laboratory[1,4,5,6]. The problem of design will be treated under the following separate headings.

1. Seat Angle Design.
2. Design of Top Angle Connections for Partial Restraint.
3. Design of Beams.
4. The Problem of Column Design.

The order of the preceding topics is the reverse of that to be followed in an actual design procedure.

SEAT ANGLE DESIGN

The proposed method for seat angle design is based upon the test results reported by Lyse and Schreiner[1] and is similar to that proposed by Heath Lawson[2]. The method places no dependence on top angle or other connecting medium and the seat angle alone is designed to carry the full reaction load of the beam.

Fig. 15 shows the details of a seat angle and the forces which it resists. The horizontal leg of the angle transmits the shear to the vertical legs which are welded to the column flange or column web. If the end connection is designed for end restraint the bottom flange of the beam must be welded to the seat angles with properly designed welds. The horizontal thrust induced along the top of the seat angle by end restraint tends to relieve the bending moments produced in the seat angle by the vertical loads but this will not be taken advantage of in the proposed design method. The end reaction of the beam will be assumed to act at a distance from the end of the beam equal to one-half the required bearing distance. The A.I.S.C. Specification for design of building construction requires the bearing length "N" to be:

$$N = \frac{R}{w\sigma_B} - k \tag{4}$$

R = end reaction of beam

w = web thickness

σ_B = allowable bearing stresses at root of beam fillet = 24 kips per sq in.

k = distance between outside of flange and root of fillet as listed in steel handbooks.

In the proposed design method "N" shall in no case be taken as less than k.

The research of Lyse and Schreiner showed the critical section for calculating bending stresses in the angle to be a short distance back of the root of the fillet. Theoretical formulas were derived on the basis of a curved cross-section[1], but a simple approximation on the side of safety will be made. The critical section will be taken in the fillet 1/4 in. from the exposed face of the angle, as shown in Fig. 15, and the cross-section of the angle in the straight portion will be used in calculations. The actual fillet radius of seat angles will nominally vary between 3/8 and 5/8 of an inch. The use of 1/4 in. is on the safe side and is convenient because it cancels with the 1/4 in. clearance usually allowed between column and beam in detailing.

The required section modulus for bending of the angle leg will be:

$$S_{reqd} = \frac{Wt^2}{6} = \frac{R}{\sigma_a}\left(\frac{N}{2} - t\right) \qquad (5)$$

S = section modulus

W = length of seat angle

t = thickness of seat angle

σ_a = allowable direct stress

= 20 kips per sq in.,

A.I.S.C. Specifications

The allowable reaction, from equation (5), is given by:

$$R = \frac{\sigma_a W t^2}{3N-6t} \qquad (6)$$

In rare cases the shear stress in the horizontal leg of the angle rather than the bending stress will determine the design, in which case the allowable reaction will be:

$$R = \tau_a \cdot W \cdot t \qquad (7)$$

τ_a = allowable average shear stress
= 13 kips per sq in. by
A.I.S.C. Specifications

For the unit stresses here quoted the shear stress will be critical when N is less than 2.51t.

The fillet welds attaching the vertical legs of the angle to the column flange or web will run the full length of the vertical leg. Lawson[2] recommends that the weld be run continuously around the corner at the top of the seat angle and extended 1/2 in. beyond as shown in Fig. 15. Most of the bending moment will be absorbed by the upper part of the vertical welds. Lyse and Schreiner[1] reported that the "neutral axis" in bending was considerably above the mid-height of the angle and this observation is in agreement with what should be expected theoretically. It will be assumed here that the neutral axis is at a distance 3t from the top of the angle. Using a rectangular distribution of stress the following formula uses the resultant of the horizontal and vertical shears as a criteria.

$$R = \frac{\tau_w A}{\sqrt{1 + (\frac{N}{3t})^2}} \qquad (8)$$

τ_w = allowable shear stress in weld metal = 13.6 kips per sq in. for Grade 2, 4, 10, or 15 Filler Metal by A.W.S. Code for Building Construction.

A = total area in both vertical welds through the throat.

In Equation (8) N should in no case be taken as less than k.

The allowable load on a seat angle is to be taken as the minimum of the values given by Equation (6), (7), or (8). The procedure is essentially the same as that proposed by Lawson[2] except that Equation (8) is altered in one essential detail and is found to give a fairly consistent factor of safety when compared with all of the test results of Lyse and Schreiner[1], which are summarized in Table I. The average factor of safety against yielding (Column 8) as determined by scaling of the whitewash is 2.5 with a minimum of 1.5 in 26 tests. The average factor of safety against initial fracture (Column 9) of weld metal is 3.6 with a minimum of 2.0. An allowable stress of 11.3 kips per sq in. was used in the welds since the filler metal was grade 40 in this investigation.

The foregoing procedure of seat angle design gives in every case a safe procedure of design. Design charts somewhat similar to those originally presented by Heath Lawson[2] are shown in Fig. 16a and 16b, which give the required angle thickness and weld size, respectively. The charts are based on Equations (6) and (8), using an allowable direct stress in the angle of 20 kips per sq in. and an allowable resultant shear stress through the throat of the weld of 13.6 kips per sq in. The procedure in using the charts is as follows:

1. Calculate N from Equation (4).
2. Select a trial length of seat angle, 1 to 2 in. longer than the flange width of the beam and calculate end reaction per inch length of seat angle.
3. Determine required seat angle thickness from Fig. 16a and select trial angle size.
4. Determine average end reaction per inch length of vertical weld and select weld fillet size from Fig. 16b.

The foregoing procedure does not include the design of the weld attaching the lower flange of the beam to the seat angle. Allowance for additional end reaction due to unbalanced end moment should also be made. These factors will be included in the section on "Design Procedure".

DESIGN OF TOP ANGLE CONNECTIONS FOR PARTIAL RESTRAINT

Test results and design procedures for top angle connections to give fifty per cent restraint have been presented in papers by Lyse and Gibson[4,5]. In the present paper a formula for calculating the end rigidity developed by the top angle connection has been derived which checks well with these and later[6] test results. The top angle must be strong enough to develop an end moment of half the fully-fixed end restraining moment with an ample factor of safety. It must also be flexible enough to allow partial rotation with an adequate margin of safety with respect to deformation.

The forces acting on the end of a beam are assumed to be as shown in Fig. 17a and those acting on the top angle itself are illustrated in Fig. 17b. An analysis of the elastic behavior of the top angle has been made by the theory of least work. Bending, shearing, and direct deformations were taken account of in this analysis, the details of which are omitted. The results of the analysis are as follows:

(1) The point of inflection in the vertical leg of the top angle lies between 1/3 and 1/2 the distance from the top of the beam for usual design proportions.

(2) A formula is derived which satisfactorily approximates the minimum coefficient of connection rigidity, or "connection constant" to be counted upon at design load.

On the basis of (1) the horizontal pull on the top angle should be calculated by dividing the end moment by the sum of the beam depth and one-half the length of the vertical leg of the top angle. The following gives the total horizontal pull acting on the top angle as calculated on this basis.

$$P = \frac{M_F}{(d + \frac{a}{2})} \qquad (9)$$

P = load in kips

M_F = fifty per cent of end moment for one hundred per cent restraint, in inch-kips

a = length of the vertical angle leg

d = depth of beam

The critical stresses determining the strength of the top angle are a result of a maximum tensile stress due to the combination of direct load and bending moment in the throat of the top weld. A formula derived on the approximate assumption that the yield stresses are developed simultaneously in the throat of the weld and at the root of the angle fillet has been found to give a very consistent factor of safety with both the general yield and ultimate strength of the top angle as reported on previous tests[4,5,6]. The allowable horizontal pull on the top angle is given by:

$$P = \frac{18Wt^2}{\frac{3}{2}a - t} \qquad (10)$$

By solving Equations (9) and (10) for M_F the following gives directly the allowable end moment for any particular top angle and beam depth:

$$M_F = \frac{18Wt^2(d + \frac{a}{2})}{(\frac{3a}{2}) - t} \qquad (11)$$

Columns 5, 6, and 7 of Table 2 compare the design end moment calculated by Equation 11 with the end moment at general yield and ultimate load. Excluding Tests G1, M9, and M10, the average factor of safety with the general yield, shown in Column 8, is 1.50 and the minimum is 1.34. The average factor of safety with the ultimate for the same tests, shown in column 9, is 3.05 and the minimum is 2.43. A typical diagram of the relation between moment and relative angle change between beam and column is shown in Fig. 18. The method used for determining the general yield is illustrated in this figure and is one that has been found to give good results in these and other tests of steel structures. A straight line tangent to the initial load-deformation curve is extended to an ordinate equal to the ultimate load. The actual load at the deformation corresponding to this ordinate is taken as the general yield, or "useful limit" of the member.

The excellent agreement between Equation 11 and the test results of 3 by 3-in. top angle connections should allow its extension to 3-1/2 by 3-1/2 in. top angles. Fig. 19a and 19b give charts for the direct design of these two sizes of top angles for different angle sizes and depth of beam. The use of these charts will be explained in the succeeding section.

The top angle not only must be strong enough but must at the same time provide an end connection having a minimum of fifty per cent rigidity. In the discussion of tests of the building frame the experimental connection constant and its relation to frame analysis was discussed and the constants γ and α defined. In the application to design it is more convenient to use a new constant J, which is in direct relation with the joint rigidity and is given by:

$$J = \frac{1}{E\gamma} = \frac{2I}{\alpha l} \qquad (11)$$

The connection is designed for the restraining moment which would exist with no column rotation at the joints. In this case the actual end moment is given by:

$$M_F = \frac{M_{FR}}{1+\alpha} \qquad (12)$$

M_{FR} = fixed end moment for one hundred per cent rigid connections as calculated, or given in various tables.

$$\text{Percentage rigidity} = 100\frac{M_F}{M_{FR}} = \frac{100}{1+\alpha} = \frac{100}{1+\frac{2I}{Jl}} \qquad (13)$$

It follows from the preceding relations that for fifty per cent connection rigidity the minimum value of J is:

$$J = \frac{2I}{l}$$

for seventy-five per cent rigidity $J = \frac{6I}{l}$ and for eighty per cent $J = \frac{8I}{l}$. An eighty per cent connection rigidity may be taken as a satisfactory upper limit for design, although it is preferable to keep within the limits of fifty and seventy-five per cent.

The analysis of the top angle provides a formula for determining J. The assumption is made that the center of joint rotation is 0.2d above the seat angle, which is about the maximum reported in previous tests[4].

$$J^{*} = \frac{Wt^3(0.8ad^2+0.8bd^2+0.9atd+0.9btd+0.9b^2d)}{3.64b^3a+0.91b^4+4.9abt^2+3.9b^2t^2+a^2t^2}$$

$$+\frac{Wt^3(0.33b^3+0.5b^2t+0.34at^2+0.61bt^2)}{3.64b^3a+0.91b^4+4.9abt^2+3.9b^2t^2+a^2t^2} \qquad (14)$$

b = width of vertical leg

a = width of horizontal leg

For equal leg angles a = b and Equation 14 reduces to Equation 14a:

$$J = \frac{Wt^3(1.6d^2+1.8td+0.9ad+0.33a^2+0.5at+0.95t^2)}{4.55a^3+9.8at^2} \qquad (14a)$$

Table 2 provdes a comparison between previous test results[4,5,6] and values of J as determined by Equation 14a. Column 10 gives the value of J by Equation 14a and Columns 11 and 12 give the test values of J at design load and at the general yield. Columns 13 and 14 give the ratios of the test J, to J by Equation 14a at design moment and yield moment respectively.

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* The use of unequal leg angles might be economical in certain cases because 6 by 3-1/2 in. angles may be obtained from some mills up to one inch in thickness. The use of such an angle might eliminate the need for special cutting of thick angles to a 3 by 3 or 3-1/2 by 3-1/2 size. Design charts similar to Fig. 19 may be drawn for unequal leg angles but in the absence of test data such charts will be omitted in this report.

In the G and M series of tests the value of J at design is greater by test than by formula in all except two cases, M_{10} and M_{11}. In each of these cases unusually wide beam sections were used and the top angle extended the full width of the column flange. It is recommended that the length of the top angle be the same as the width of the beam flange but that it should in no case be greater than 0.8 of the depth of the column section. Top angle thickness should not be more than 7/8 for 3 by 3-in. angles nor more than 1 in. for 3-1/2 by 3-1/2 in. angles. Both fillet welds connecting the top angle to the beam and column are to be the same size as the top angle thickness.

It should be noted that the constant "J" is quite sensitive to small variations in dimension and that between the limits of fifty and seventy-five per cent connection restraint the numerical value of constant J varies three hundred per cent. Connections of the proposed type having adequate strength usually have excess stiffness. For example, in Table II the worst case is test M_{10} with J by test only 41 per cent of J by Equation 14. Assuming the beam to have a length of 10 ft. in this case the end connection has adequate strength and by Equations 13 and 14 should develop seventy-five per cent restraint. Using the test value of J the joint by Equation 13 actually developed fifty-five per cent restraint and hence would be satisfactory. Nevertheless, the size limitations in the foregoing paragraph would rule it out.

Tests F1, F2, and F3 were made with joints representative of those used in the actual building frame tested, in which case J by Equation 14 is 2.30. The comparison between theoretical and test results is shown in Table III.

TABLE III

	Theoretical Equations 13, 14	Test of Flange Connection	Test of Inside Web Connection	Test of Outside Web Connection
J	2.30	1.92	2.26	0.91
% Restraint	60.10	55.80	59.70	37.40

Table III shows that both the flange connection and inside web connection developed restraint reasonably close to the theoretical value. The outside web connection, however, developed only 62.6 per cent as much restraint as indicated by formula. The test set-up for an outside web connection, in which the beam frames into only one side of the column, is shown in Fig. 8b.

The test results of the actual frame with web connections as shown in Fig. 13 and 14 indicate that the interior column connections for such cases develop full moment equivalent to the column flange connections but that the outside column web connections do not. An analysis which assumes the rigidity of the outside connections equal to the inside is shown by the dot-dash lines for the outside loaded beams in Fig. 13 and 14 whereas the dashed lines are based on an analysis using the experimentally determined rigidities for both inside and outside column connections. The difference between actual maximum moment and the analysis is seen to be small in either case. The exterior column web connections may be reinforced by plates welded in position as shown in Fig. 20, in which case they should develop a rigidity fully equivalent to an interior connection.

Within the stated limitations the formula for J gives a satisfactory guide to minimum rigidity developed by a beam-column flange connection or an interior beam-column web connection. Charts for determining J for 3 by 3-in. and 3-1/2 by 3-1/2 in. angle sizes of different thickness and for varying beam depth are shown in Fig. 21a and 21b.

The following summary reviews the steps in the design of the top angle:

1. The beam and column sizes will have been selected. Assume a top angle length "W" equal to beam flange width.
2. Divide the end moment for fifty per cent rigidity by the top angle length to determine moment per inch ~~width~~ length and select top angle size and thickness from Fig. 19a or 19b.
3. Determine J from Fig. 21a or 21b for the top angle size selected. J must be more than $\frac{2I}{l}$ and preferably not more than $\frac{6I}{l}$. $\frac{8I}{l}$ may be taken as an upper limit.
4. Design the seat angle by the method outlined on page 19.

5. Determine the horizontal thrust P by Equation 9 and design fillet welds between lower beam flange and seat angle to take this thrust.

The foregoing steps will be illustrated later in the section on "Design Procedure".

DESIGN OF BEAMS

The preceding sections on the design of seat and top angles have been based on a predetermined beam size, the selection of which would be the first step in normal design procedure. The design of the beams is based on the assumption that the connections will have a minimum rigidity factor of fifty per cent at design load and not over seventy-five per cent at a load of one hundred and fifty per cent of the design load. The choice of fifty per cent as a connection rigidity factor has been made after a consideration of the economy of both beam and connection design and of the factor of safety in beam design. Too high a rigidity factor will require expensive connection details whereas fifty per cent connection rigidity takes advantage of a large measure of economy with the relatively simple seat and top angle connection.

The factor of safety requires special attention because of the non-linear relation between moment and angle change at the connection. It may be noted in Table 2 that the minimum factor of safety between design moment and yield moment is 1.34 for all the connections tested in the previous investigations. As a minimum criteria of behavior let it be assumed that the joint yields at 1.35 times the design moment and that it takes no additional moment whatever. The solid line in Fig. 22~~a~~ illustrates typical joint behavior and the dashed line represents the preceding assumptions. Now assume that the load on the beam increases until the stress in the center rises from a design stress of 20 kips per sq in. to the specified minimum yield point of 33 kips per sq in. The outside fibre stress at the partially restrained end could only rise from 10 kips per sq in. to 13.5 kips per sq in. after which it would maintain a constant value.

~~Fig. 22b shows the stress condition in a uniformly loaded beam for the design load and for a load producing a maximum stress of 33 kips per sq in.~~ The load ratio or factor of safety at the yield point will be a minimum of 1.55 as compared with the stress ratio of 1.65. This reduction is not serious even for the extreme assumption that the ultimate strength of the joint is no greater than the yield strength. If the joint actually develops more **rigidity** than the fifty per cent for which it is designed it may yield locally at design load until a condition of equilibrium is established. The load factor of safety with respect to beam fibre stress of 33,000 lb per sq in. will still be a minimum at 1.55.

It should be kept in mind that the total deformation is limited by the rotation of the end of the beam and that ultimate failure of the top angle can never occur in an actual structure prior to beam failure, provided the design is within the specified limitations. It should also be noted that even if the top angle is omitted the maximum stress in a beam designed for fifty per cent end restraint would be 30 kips per sq in. which is below the specified minimum yield-point stress of 33 kips per sq in. for structural grade steel. Reference may also be made to the original work of C. Batho [7], who has developed graphical procedures to give the actual end moments developed by any end connection.

The beam-column connections are designed to develop a full fifty per cent of the end moment in a fully-fixed beam, but in an actual building frame this moment can be developed only in the interior beams with all of the beams loaded as illustrated in Fig. 10 for the first story. For unsymmetrical conditions of loading or structure the columns will bend, relieving the end moments in the beams, and resulting in corresponding increase in the positive moment in the center of the beam. The maximum positive moment which may be expected from the most critical combination of live load will be used as a basis for design.

The condition of loading which produces the maximum moment in the center of any beam A-B is shown in Fig. 23a.

As a simplification of the problem the section of the structure shown in Fig. 23b will be considered. The moments at C, D, E, and F acting on the ends of the columns which frame with beam A-B will be assumed to act with a sense which produces greatest positive moment in the center of beam A-B and are assumed to have a magnitude equal to that at their juncture with the beam. The restraining effect of the beams on a level with A-B which frame to the opposite side of the columns will be neglected entirely, thus making the same procedure applicable to beams in both the exterior and interior bays of buildings.

The slope deflection equations for any beam A-B having end connections which develop fifty per cent restraint are:

$$\left.\begin{aligned} M_{AB} &= \frac{EI}{4l}\,(5\theta_A + \theta_B) - \frac{M_{FRA}}{2} \\ M_{BA} &= \frac{EI}{4l}\,(5\theta_B + \theta_A) + \frac{M_{FRB}}{2} \end{aligned}\right\} \qquad (15)$$

Using these equations for the beam and Equation 1 for the adjacent columns, which are assumed continuous, it may be shown that the end moment in the beam with fifty per cent end restraint is given by:

$$M_F = M_{FR}\left[\frac{1}{2 + \dfrac{K_B}{\Sigma K_c}}\right] \qquad (16)$$

In equation 16:

ΣK_c = the sum of $\frac{I}{l}$ for the column sections above and below one end of the beam

$K_B = \frac{I}{l}$ for the beam

M_{FR} = end moment in a fully fixed end beam with "rigid" connections

Equation 16 is exact for a symmetrically loaded beam in a symmetrical structure and is closely approximate for ordinary conditions of unsymmetry. It should be kept in mind that the additional restraint provided by other beams, floor slabs, walls, etc. has been neglected.

For symmetrical conditions the maximum center moment will be:

$$M_{\text{℄}} = M_S - M_F \tag{17}$$

M_F = minimum end moment by Equation 16

M_S = maximum moment in a simply supported beam

For symmetrical cases the design procedure may be simplified by calculating a reduction factor to be multiplied by the maximum simple beam moment assuming free supports.

$$\text{Reduction factor} = F = \frac{M_{\text{℄}}}{M_S} = 1 - \frac{M_{FR}}{M_S}\left[\frac{1}{2 + \frac{K_B}{\Sigma K_c}}\right] \tag{18}$$

The moment to be used in the design will then be:

$$M_{\text{℄}} = F \cdot M_S \tag{19}$$

In Equation 18 the ratio of $\frac{M_{FR}}{M_S}$ is $\frac{1}{2}$ for a concentrated load at the center, $\frac{2}{3}$ for uniform load or for two concentrated loads at third points, and $\frac{5}{8}$ for three concentrated loads at quarter points. Fig. 24 gives directly values of the reduction factor for various ratios of beam-to-column stiffness for these load conditions.

For unsymmetrical conditions the maximum center moment will be approximately equal to:

$$M_c = M_S - \text{Avg.}M_F \qquad (20)$$

Avg. M_F is the average of the end moments at each end as calculated by Equation 16. Equation 20 will give results within a few per cent except for unusual conditions of loading such as beams carrying offset columns near one end. Equations 16 and 20 may be used for all cases where lack of symmetry exists in the structure due only to the use of different size columns.

In the interior panels of structures of uniform beam length and uniform dead load, the full fifty per cent restraint could be counted on in calculating the dead load moment. The calculations will be simplified, on the safe side, and will give practically the same result in most cases, however, if the dead load is simply added to the live load and all treated alike.

A comparison of design moments calculated by the proposed method with the actual moments in the experimental building frame are presented in Fig. 25 and 26. The solid lines indicate the moments based on measured end moments superposed for all of the various load conditions and the dashed line indicates the design moment calculated by the proposed method. Fig. 25a and 25b are for the second and first story beams, respectively, of Frame 1 having beam-to-column flange connections, and Fig. 26a and 26b are for the corresponding beams in Frame 2 having beam-to-column web connections. It may be seen that there is an ample margin of safety between design moment and actual moment in every case for the variety of critical load conditions used in testing the frames. This is true even in the case of the outside web connections which did not develop fifty per cent restraint.

It has been shown that the proposed design procedure gives an ample margin of safety for a loaded frame consisting of bare beams and columns in one plane. In an actual structure additional stiffness and strength will be introduced by beams at right angles to the frame considered, by floor slabs, and by walls.

The procedure for designing symmetrically loaded beams may be reduced to the following.

1. Calculate M_s, the maximum moment in the beam assuming freely supported ends, and select a preliminary beam size designed to resist this moment. The required section modulus will be $\frac{M_s}{20}$ if the basic unit stress is 20 kips per sq in.
2. Calculate the ratio of $\frac{K_B}{\Sigma K_c}$ on the basis of the tentative selection, determine the design moment

M_c from Equation 19 or from Fig. 24, and select final beam size, the required section modulus being $\frac{M_c}{20}$.

3. Design seat and top angle connections by the methods previously outlined.

The selection of the tentative beam size, Step 1, need only be roughly approximate. To give a final design with the approximation on the safe side the tentative selection should have a moment of inertia greater than the final selection and such will always be the case by the procedure given.

The design procedure for unsymmetrical cases may be outlined as follows:

(1) Same as for the symmetrical case.

(2) Calculate the maximum moment by Equation 20 for moderate lack of symmetry or by graphical construction for doubtful cases and select required beam size.

(3) Same as for the symmetrical case.

It should be noted that the structure is to be considered unsymmetrical only if the columns into which it frames have different stiffness factors. Since no advantage is taken of adjoining beams the beam in an outside panel may be considered symmetrical if the columns are equal in size. The preceding design methods will be applied to illustrative cases in a succeeding section.

The proposed design method applies only to the beams or girders which frame directly into the columns of a frame. The seat and top angle type of connection usually is not used for connecting floor beams to main girder beams. Although there should be possibilities of economy in designing such beams for partial continuity such a treatment is outside the scope of the report.

DESIGN OF COLUMNS

An experimental and theoretical investigation on the subject of column design is now in progress at the Fritz Engineering Laboratory under the sponsorship of the American Institute of Steel Construction. It is expected that this work will be completed about a year from now. In view of these facts the scope of this paper will be limited to beam design. The information obtained in this investigation should be of value at a later date in connection with the problem of column design.

Designing the beams for partial continuity as herein proposed will increase moments in the columns by only a few per cent. Column design procedures which are now safe will have relatively the same degree of safety, therefore, when used in conjunction with the proposed method of beam design, provided the same type of connection is used. If a more rigid type of connection is used, such as the seat angle and top plate connection, the moments in the columns will be higher than by the proposed method even if no advantage is taken of continuity.

Columns are usually designed for vertical loads alone, whereas the tests of the frame show the presence of relatively large bending moments introduced by unbalanced end moments in the connecting beams. Such moments are present regardless of whether or not the beams are actually designed for end restraint or not. The British Steel Structures Research Committee[7] reports measured column stresses due to bending in actual buildings of riveted construction which exceed the yield point of the material. This Committee recommended procedures of design which take account of the moments introduced by continuity.

In view of these facts it may be concluded that changes in column design methods may possibly be advisable, but the consideration of such changes will be left to a later report.

DESIGN PROCEDURE

The procedures for designing the seat angle, top angle, and beams have already been presented under separate headings. The following will summarize briefly all of the steps in the design procedure.

1. Select preliminary beam size for the section modulus required by the maximum simple beam bending moment, M_s.

2. Calculate beam-column stiffness ratio $\frac{K_B}{\Sigma K_C}$ and select final beam size from moment obtained by applying a reduction factor from Fig. 24 to M_s.
3. Assume a top angle length not greater than the width of the beam flange nor 0.8 the column depth and determine the end moment per inch for fifty per cent connection restraint.
4. Select top angle size and thickness from Fig. 19a or 19b and check connection rigidity "J" by Fig. 21a or 21b. "J" should be more than $\frac{2I}{l}$ and preferably not more than $\frac{6I}{l}$. $\frac{8I}{l}$ may be taken as an upper limit.
5. Select a trial length of seat angle 1 to 2 in. longer than the width of beam flange and determine end reaction per inch length of seat angle. The end reaction should be taken as the simple beam reaction plus $\frac{M_F}{l}$ to allow for unbalanced moment.
6. Determine the required seat angle thickness from Fig. 16a and select trial seat angle size.

7. Determine average end reaction per inch length of vertical weld and select weld fillet size from Fig. 16b.

8. Determine horizontal thrust on seat angle by Equation 9 and design horizontal welds connecting the beam flange to the seat angle.

In actual design practice involving a large number of beams the entire procedure may be carried out in tabular form on a single sheet and reduced to a fairly rapid procedure.

The following illustrative examples will be presented in somewhat greater detail than would be necessary after a familiarity with the design method is obtained.

<u>Case 1</u> - Beam has 20-ft. clear span between column flanges and carries a combined dead and live load of 2 kips per foot. Columns are 10-in. WF at 49 lb. and the distance between floors is 12 ft.

(Symmetrical structure and load)

Simple Beam Bending Moment $= \frac{2 \times 20^2 \times 12}{8} = 1200$ in-kips

Required Section Modulus for Simple Beam Design $S = \frac{1200}{20} = 60$

Selection as Simple Beam 16-in. WF at 40 lb.

$S = 64.4$

$I = 515.5$

Stiffness Ratio

$$K_B = \frac{I_B}{l} = \frac{515.5}{240} = 2.15$$

$$\Sigma K_c = \frac{\Sigma I_c}{l} = \frac{2 \times 272.9}{144} = 3.79$$

$$\frac{K_B}{\Sigma K_c} = \frac{2.15}{3.79} = 0.567$$

Reduction Factor from Fig. 24 is 0.740.

Required Section Modulus = 0.740 x 60 = 44.4

Beam Selection   14 WF at 34 lb

$S = 48.5$

$I = 339.2$

(Note: The most economical selection is a 15M at 33 but the narrow width of the flange is found to require too stiff a top angle)

Top Angle Length = 6.75 in. = flange width

End Moment per inch of Top Angle = $\frac{1200}{3 \times 6.75}$ = 59.3 in-kips per in.

(Note: End moment for fifty per cent restraint, beam uniformly loaded, is $M_s/3$)

Using Fig. 19b a top angle size of 3-1/2 by 3-1/2 by 1 in. is required.

Using Fig. 21b, the stiffness factor J of the connection is calculated as:

$$J = 1.71 \times 6.75 = 11.53$$

Upper Limit of J = $\frac{8I_B}{l} = \frac{8 \times 339.2}{240} = 11.30$

(Note: J is about 2 per cent over the upper limit for 80 per cent connection rigidity and the actual rigidity by Equation 13 is 80.4 per cent. The example is introduced purposely to show a case at the upper limit of span length, load, and end connection stiffness)

Choose Seat Angle Length as 6.75 + 1.25 = 8.0 in.

End reaction = $20 + \frac{400}{240}$ = 21.67 kips

End Reaction per inch of angle = $\frac{21.67}{8}$ = 2.71 kips per in.

Required Bearing Distance by Equation 4

$$N = \frac{21.67}{0.287 \times 24} - \frac{15}{16} = 2.21 \text{ in.}$$

Required Seat Angle Thickness from Fig. 16a is 5/8 in.

Choose a 6 by 4 by 5/8-in. angle.

Vertical Fillet Welds Between Column Flange and Seat Angle:

Avg.reaction per inch = $\frac{21.67}{12}$ = 1.81 kips per in.

$$\frac{N}{t} = \frac{2.21}{5/8} = 3.54$$

Fillet Weld Size from Fig. 16b is 5/16 in.

Horizontal Fillet Welds Between Beam Flange and Seat Angle:

Horizontal thrust from Equation 9.

$$P = \frac{400}{14 + 1.75} = 25.4 \text{ kips}$$

Assuming Welds Effective for 3-1/2 in.

Avg. shear = $\frac{25.4}{7}$ = 3.62 kips per in.

Fillet Weld Size Required:

$\frac{3.62}{9.60} = 0.377$ Use a 3/8-in. fillet weld 3-1/2 in. long on each side of beam flange.

<u>Case 2</u> (Unsymmetrical structure and load) - Beam A-B has an 18-ft. clear span and at A frames into the flange of a 10 by 10 WF 49-lb. column while at end B it frames into the web of the same size column. The beam carries concentrated loads at the

third points, the load nearest A being 21 kips and that nearest B being 15 kips. Distance between floor levels is 12 ft.

Simple Beam Reactions $V_A = \frac{2}{3}(21) + \frac{1}{3}(15) = 19$ kips ~~per in.~~

$V_B = \frac{1}{3}(21) + \frac{2}{3}(15) = 17$ kips ~~per in.~~

Maximum Simple Beam Moment $M_s = 19 \times 6 \times 12 = 1368$ in-kips

Required Section Modulus $= \frac{1368}{20} = 68.4$

Simple Beam Selection = 16 WF 45 lb.

$S = 72.4$

$I = 583.3$

Fixed End Moments for one hundred per cent rigidity (see steel handbooks for equations):

$$M_{FRA} = \left[\frac{21 \times 6 \times 12^2}{18^2} + \frac{15 \times 12 \times 6^2}{18^2}\right] \times 12 = 912 \text{ in-kips}$$

$$M_{FRB} = \left[\frac{21 \times 12 \times 6^2}{18^2} + \frac{15 \times 6 \times 12^2}{18^2}\right] \times 12 = 818 \text{ in-kips}$$

Stiffness Ratios

Beam: $K_b = \frac{583.3}{216} = 2.70$

Column at A: $\Sigma K_c = \frac{2 \times 272.9}{144} = 3.79$

at B: $\Sigma K_c = \frac{2 \times 93.0}{144} = 1.29$

Minimum End Moments by Equation 16

$$M_{FA} = 912 \left[\frac{1}{2 + \frac{2.70}{3.79}}\right] = 336 \text{ in-kips}$$

$$M_{FB} = 818 \left[\frac{1}{2 + \frac{2.70}{1.29}} \right] = 200 \text{ in-kips}$$

Maximum Moment = 1368 - 291 = 1077 in-kips

Required Section Modulus = $\frac{1077}{20}$ = 53.9

Use a 16-in. WF at 36 lb.

S = 56.3

I = 446.3

Design of top and seat angles follows the same procedure as given for Case 1. The connections at A and B are designed for fifty per cent of the full fixed end moments, or for 456 and 409 in-kips, respectively. The seat angles at A and B should be designed for 21.11 kips and 18.9 kips, respectively.

ECONOMY EFFECTED BY PROPOSED DESIGN PROCEDURE

In the two cases presented in the preceding section the saving in beam weight effected by the proposed method in comparison with simple beam design amounts to fifteen per cent for Case 1 and twenty per cent for Case 2. In order to obtain a better average of the saving which might be expected trial designs were made for beams of 17, 18, 19, and 20 ft. in length and for total floor loads corresponding to 80, 100, and 120 lb per sq ft. Each beam was assumed to frame into 10 by 10 WF 49-lb. columns and was assumed to carry a full panel load uniformly distributed. The results of these designs are summarized in Table IV. The average saving in weight of beams for these twelve designs is 17.6 per cent and the minimum saving in any single beam is 15 per cent.

The net saving in cost produced by decreased beam weight may be offset to some extent by the additional welding required in the end connections. If fairly rigid end connections are desired to assist in resisting lateral loads, the full saving may be realized, but if the top angles are added only to provide stability they may be made very light and flexible. The cost of field welds is uncertain and variable and no definite conclusions along these lines will be attempted in this report.

SUMMARY AND CONCLUSIONS

1. Results of tests of all-welded building frames are presented to show that methods of analysis which take account of the partial rigidity of the connections are a satisfactory guide to the behavior of the frame.

2. The rigidity of the joints in a frame may be determined satisfactorily by a test of a representative sample joint.

3. A simplified design method is presented for the beams and beam-column connections of a building frame under the action of vertical loads.

4. The application of the seat and top angle connection herein described is limited to a maximum span length of about 20 ft. and a maximum total load for this span length of about 50 kips.

5. The bending of the columns and partial rigidity of the connections under critical conditions of loading are taken account of in developing the method of design.

6. The proposed design method is more closely allied to the actual behavior of the frame than the usual procedure of designing beams as simply supported.

7. Approximations made in developing the design method are on the side of safety.

8. A saving in weight of beams of between fifteen and twenty per cent is effected by the proposed design method, but this economy will be offset to some extent by increased cost of end connections.

9. Procedures similar to those presented may be developed for other types of welded joints.

10. Further tests to develop methods for determining strength and rigidity of other types of welded joints are desirable.

11. The present report does not cover column design, but column design procedures which are now safe will also be safe when used in conjunction with the proposed procedures for beam design.

12. The question of column design for moments as well as direct stress should also be the subject of further study.

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TABLE I

SPECIMEN No.	SIZE OF SEAT ANGLE	FILLET SIZE In.	DIST. BACK OF ANGLE TO LOAD	YIELD* KIPS	LOAD AT FAILURE† KIPS	ALLOWABLE LOAD BY EQ. 6,7 or 8‡	F.S. AGAINST YIELD	F.S. AGAINST FAILURE
①	②	③	④	⑤	⑥	⑦	⑧	⑨
A 443 - 1	4×4×1/2×8	3/8	1.2	28.6	29.3	14.8 B	1.9	2.0
A 443 - 2	4×4×1/2×8	3/8	2.0	16.8	22.2	5.3 B	3.1	4.2
A 443 - 3	4×4×1/2×8	3/8	3.0	10.4	13.5	3.0 B	3.5	4.6
A 444 - 1	4×4×1/2×8	1/2	1.2	32.0	39.0	14.8 B	2.2	2.6
A 444 - 2	4×4×1/2×8	1/2	2.0	18.5	24.0	5.3 B	3.5	4.5
A 444 - 3	4×4×1/2×8	1/2	3.0	10.0	11.2	3.0 B	3.5	3.8
A 463 - 1	4×4×3/4×8	3/8	1.2	71.0	100.0	18.3 W	3.9	5.5
A 464 - 1	4×4×3/4×8	1/2	1.2	73.0	101.0	24.4 W	3.0	4.1
A 466 - 1	4×4×3/4×8	3/4	1.2	67.5	176.5	36.7 W	1.8	4.8
A 643 - 1	6×4×1/2×8	3/8	1.2	33.8	38.6	14.8 B	2.3	2.6
A 644 - 1	6×4×1/2×8	1/2	1.2	37.0	46.0	14.8 B	2.5	3.1
X - 1	6×4×1/2×8	1/2	1.2	22.2	44.1	14.8 B	1.5	3.0
Y - 1	6×4×1/2×8	1/2	1.2	25.8	43.7	14.8 B	1.7	3.0
A 654 - 1	6×4×5/8×8	1/2	1.2	53.1	74.7	29.3 B	1.8	2.5
A 655 - 1	6×4×5/8×8	5/8	1.2	57.0	102.0	29.3 B	1.9	3.5
A 664 - 1	6×4×3/4×8	1/2	1.2	100.0	136.3	36.7 W	2.7	3.7
A 843 - 1	8×4×1/2×8	3/8	1.2	49.0	74.3	14.8 B	3.3	5.0
A 844 - 1	8×4×1/2×8	1/2	1.2	53.0	74.0	14.8 B	3.6	5.0
A 864 - 1	8×4×3/4×8	1/2	1.2	87.2	132.0	48.9 W	1.8	2.7
A 884 - 1	8×4×1×8	1/2	1.2	137.5	164.5	54.3 W	2.5	3.0
A 884 - 2	8×4×1×8	1/2	2.0	55.0	120.5	35.6 B	1.5	3.4
A 884 - 3	8×4×1×8	1/2	3.0	36.1	51.4	15.2 B	2.4	3.4
A 886 - 1	8×4×1×8	3/4	1.2	137.5	170.0	81.4 W	1.7	2.1
A 444C - 1	4×4×1/2×8	1/2	1.2	37.0	75.0	14.8 B	2.5	5.1
A 644C - 1	6×4×1/2×8	1/2	1.2	37.0	54.1	14.8 B	2.5	3.6
A 644D - 1	6×4×1/2×8	1/2	1.2	35.1	52.5	14.8 B	2.4	3.5

*Load at first reported scaling in weld or angle
†Load at first crack at top of angle
‡Determined by bending of angle (Eq. 6), (W, Determined by weld, (Eq. 8))

TABLE 2

TEST NO.	SIZE OF TOP ANGLE	BEAM SIZE	COLUMN SIZE	MOMENT IN INCH KIPS: DESIGN FORM.	MOMENT IN INCH KIPS: GENERAL YIELD	MOMENT IN INCH KIPS: MAXIMUM	FACTOR OF SAFETY: AT YIELD	FACTOR OF SAFETY: AT MAX.	CONNECTION CONST. J: BY FORMULA	CONNECTION CONST. J: TEST J at DESIGN LOAD	CONNECTION CONST. J: TEST J at GENERAL YIELD	RATIO J_D/J_F	RATIO J_Y/J_F
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
G 1	3×3×7/8×8*	12WF	—	419.0	570.0	855.0	1.36	2.04	10.46	15.4	9.42	1.47	0.90
G 2	3×3×1/2×6	12WF	—	91.2	123.0	222.0	1.35	2.43	1.60	2.85	1.83	1.78	1.14
G 3	3×3×3/4×6	12WF	—	218.0	315.0	596.0	1.44	2.73	5.14	8.08	4.20	1.57	0.82
G 4	3×3×3/8×6 1/2	12WF28	8WF48	53.8	78.0	191.0	1.45	3.55	0.74	1.79	0.92	2.42	1.24
G 5	3×3×5/8×6 1/2	12WF28	8WF48	159.0	240.0	456.0	1.51	2.87	3.32	3.57	2.20	1.08	0.66
M 1	3×3×1/4×5+	12I31.8	10WF49	18.0	31.4	88.1	1.74	4.88	0.17	1.28	0.48	7.53	2.82
M 2	3×3×1/2×5	12I31.8	10WF49	76.0	102.0	194.5	1.34	2.56	1.33	1.90	1.01	1.43	0.76
M 3	3×3×5/8×5	12I31.8	10WF49	112.5	160.0	317.0	1.42	2.82	2.55	3.29	1.88	1.29	0.74
M 4	3×3×3/4×5	12I31.8	10WF49	182.2	260.0	472.0	1.42	2.59	4.29	7.90	5.75	1.84	1.34
M 5	3×3×1/2×10+	10WF49	10WF49	129.3	193.0	389.0	1.49	3.00	1.92	2.37	1.47	1.23	0.77
M 6	3×3×1/2×8	10WF49	10WF49	103.4	167.0	300.0	1.61	2.90	1.51	2.05	1.30	1.36	0.86
M 7	3×3×1/2×6	10WF49	10WF49	77.6	119.0	256.0	1.54	3.30	1.15	2.22	1.26	1.93	1.09
M 8	3×3×1/2×5 1/2	15I42.9	10WF49	102.1	149.0	276.5	1.46	2.70	2.20	2.71	1.77	1.23	0.81
M 9	3×3×1/2×4	8I18.4	10WF49	42.7	59.0	132.8	1.38	3.11	0.57	2.69	1.00	4.72	1.75
M 10	3×3×1×10+	10WF42.9	10WF49	591.0	808.0	1320.0	1.36	2.23	13.60	5.56	3.81	0.41	0.28
M 11	3×3×3/4×10+	10WF42.9	10WF49	320.0	516.0	710.0	1.66	2.22	6.16	4.08	2.44	0.66	0.40
M 12	3×3×3/4×5 3/4	10WF42.9	10WF29	210.0	401.0	702.0	1.91	3.34	4.91	8.08	5.56	1.64	1.13
F 1	3×3×11/16×4 3/4	10I25.4	8WF31	108.4	—	—	—	—	2.30	1.92	—	0.84	—
F 2	3×3×11/16×4 3/4‡	10I25.4	8WF31	108.4	—	—	—	—	2.30	2.26	—	0.98	—
F 3	3×3×11/16×4 3/4‡	10I25.4	8WF31	108.4	—	—	—	—	2.30	0.91	—	0.40	—

*3/4 weld; + Outside design range; ‡ Connection made to column web, interior column;
‡ Connection made to column web, exterior column

TABLE 4

SPAN LENGTH FEET	TOTAL LOAD KIPS	SIMPLE BEAM MOMENT INCH KIPS	REQD. SECTION MODULUS	BEAM SELECTION AS SIMPLE BEAM	REDUCTION FACTOR "F"	REQD. SECTION MODULUS	BEAM SELECTION	PERCENT SAVING IN WEIGHT	TOP ANGLE SIZE	% END RESTRAINT
1	2	3	4	5	6	7	8	9	10	11
17	23.1	590	24.5	12B22	.696	17.1	12B16.5	25.0	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 4$	80+
17	28.9	737	36.9	14WF30	.718	26.5	12WF25	16.7	$3 \times 3 \times \frac{13}{16} \times 6\frac{1}{2}$	79
17	34.7	885	44.3	15M33	.730	32.3	12WF28	15.1	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 6\frac{1}{2}$	77
18	25.9	700	35.0	12WF28	.704	24.6	12B22	21.4	$3\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 4$	78
18	32.4	875	43.8	15M33	.728	31.9	12WF28	15.1	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 6\frac{1}{2}$	78
18	38.9	1050	52.5	16WF36	.738	38.7	14WF30	16.7	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 6\frac{3}{4}$	79
19	28.9	824	41.2	14WF30	.714	29.4	12WF25	16.7	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{7}{8} \times 6\frac{1}{2}$	78
19	36.1	1030	51.5	16WF36	.735	37.9	14WF30	16.7	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 6\frac{3}{4}$	79
19	43.3	1233	61.7	16WF40	.740	45.7	15M33	17.5	$3\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 5\frac{1}{2}$	77
20	32.0	960	48.0	15M33	.722	34.6	12WF28	15.1	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{15}{16} \times 6\frac{1}{2}$	80
20	40.0	1200	60.0	16WF40	.740	44.4	14WF34	15.0	$3\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 6\frac{3}{4}$	80+
20	48.0	1440	72.0	16WF45	.748	53.9	16WF36	20.0	$3\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 7$	80

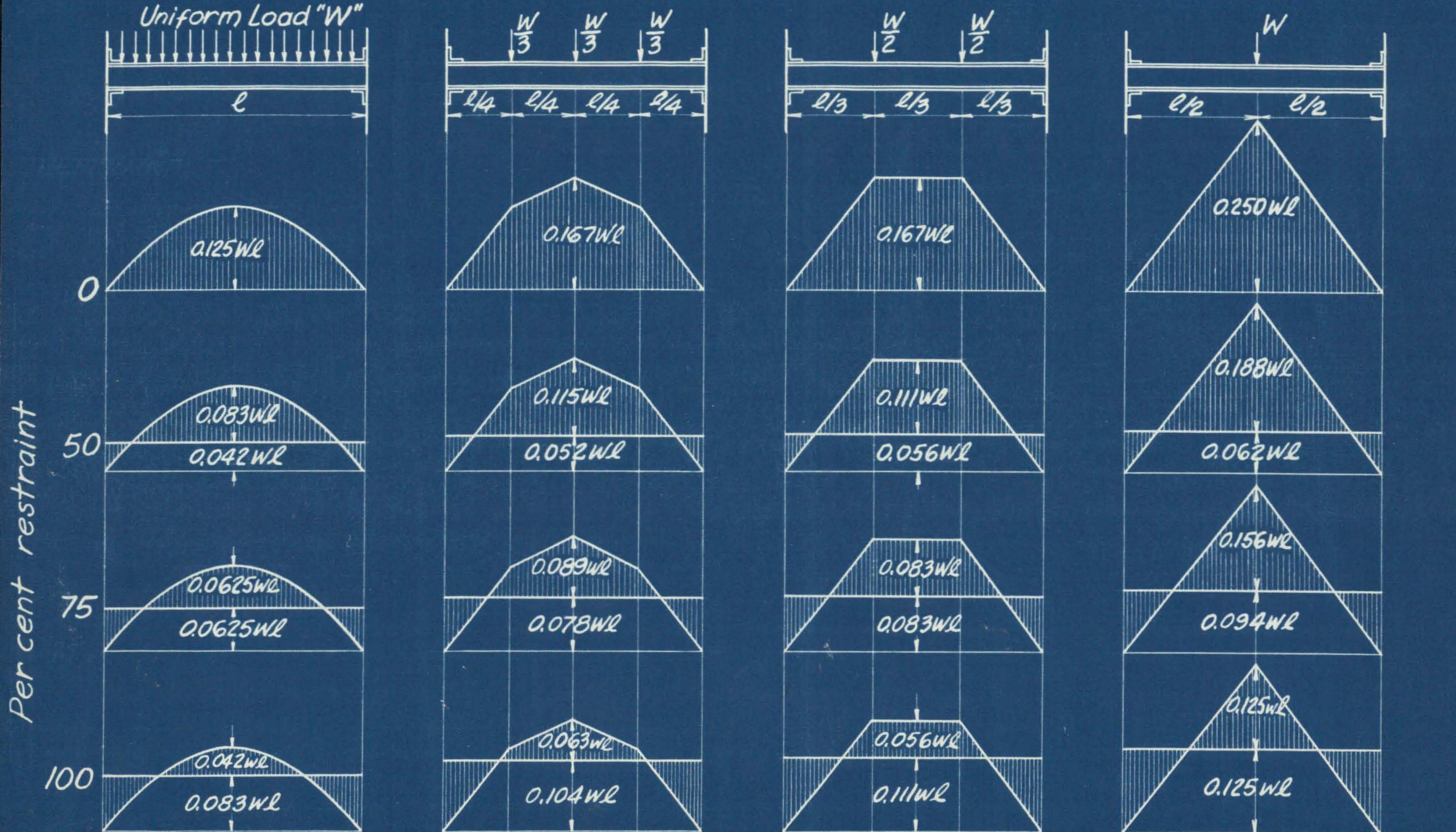


FIG. 1 CENTER AND END MOMENTS FOR BEAMS WITH VARIOUS AMOUNTS OF CONNECTION RESTRAINT FRAMING INTO FIXED WALLS.

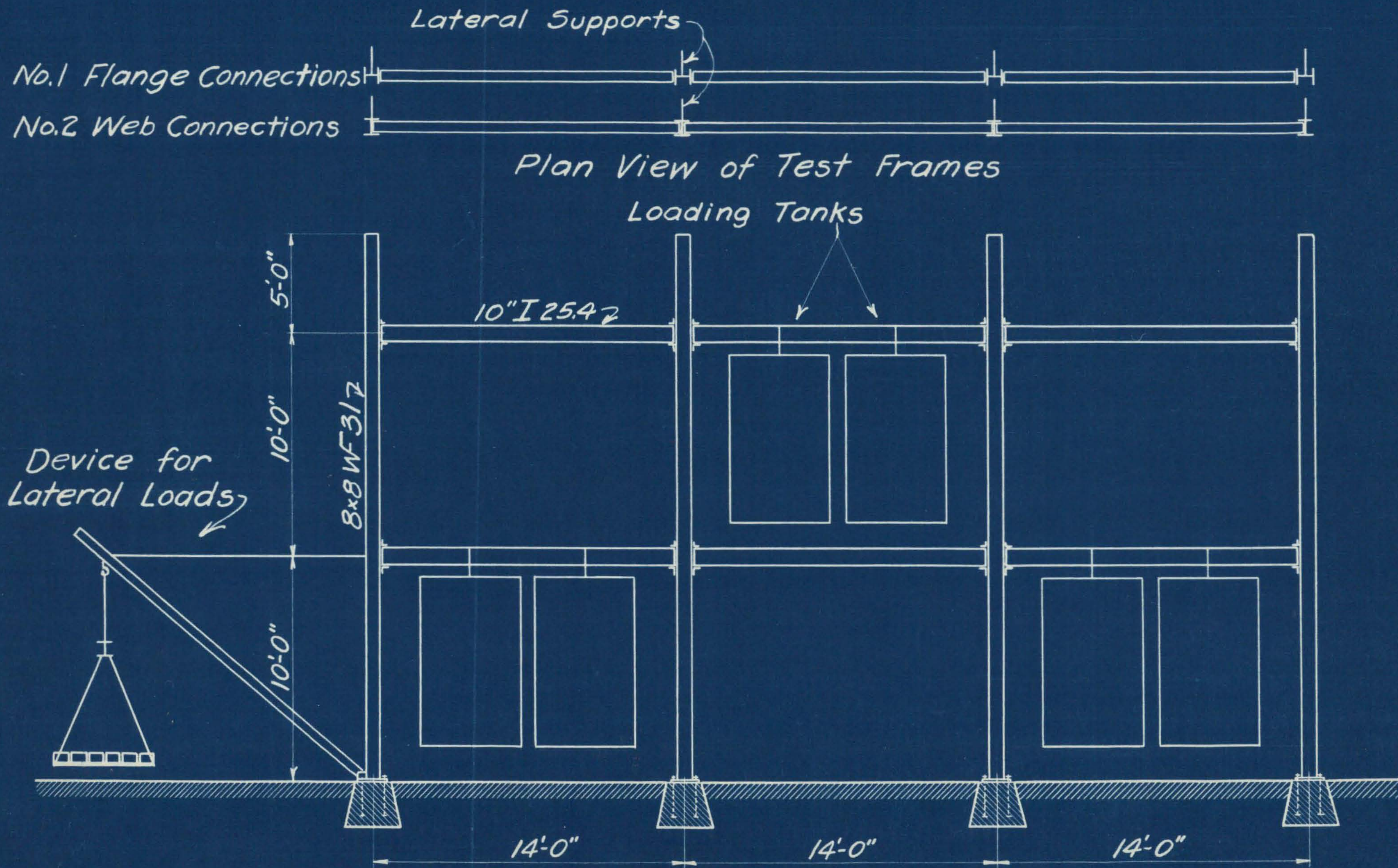


FIG. 2 GENERAL DIMENSIONS OF TEST FRAMES

Fig. 4

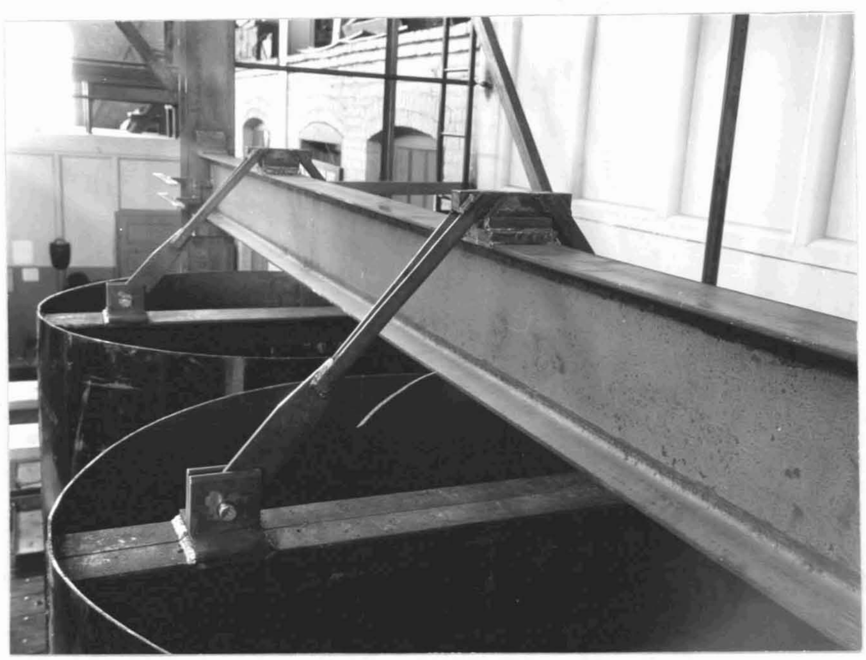


Fig. 5

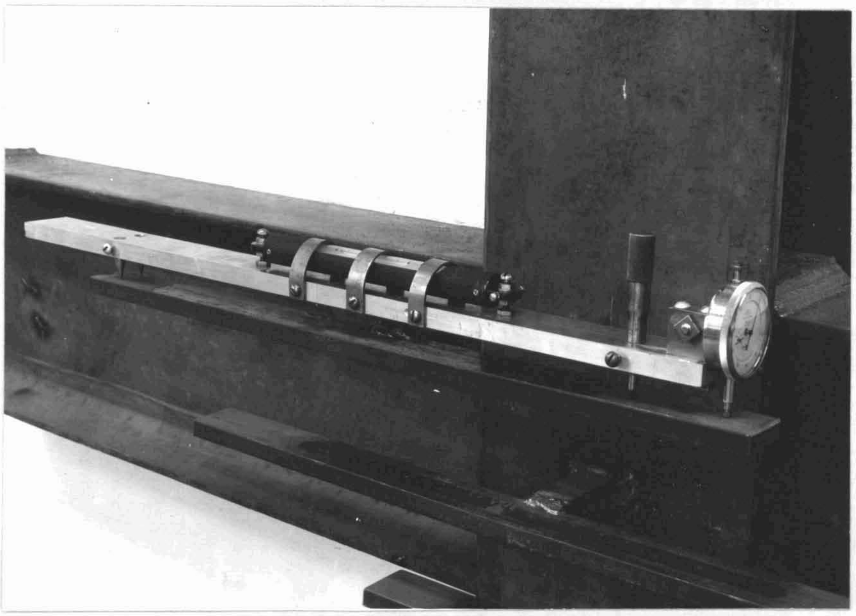


Fig. 6

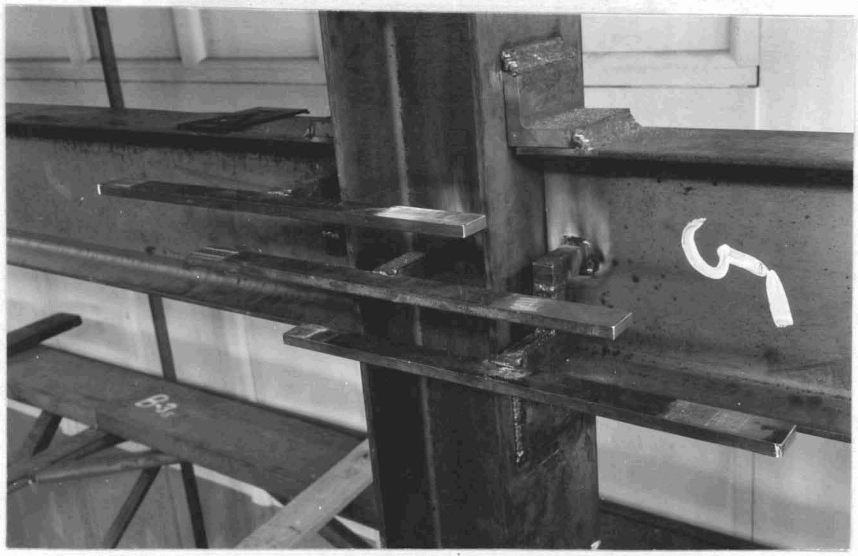


Fig. 7a

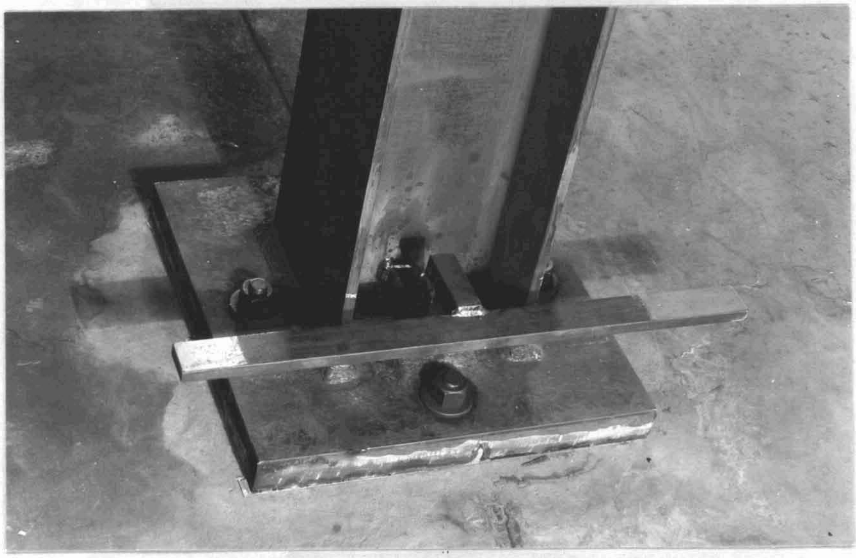


Fig. 7b

Fig. 7c

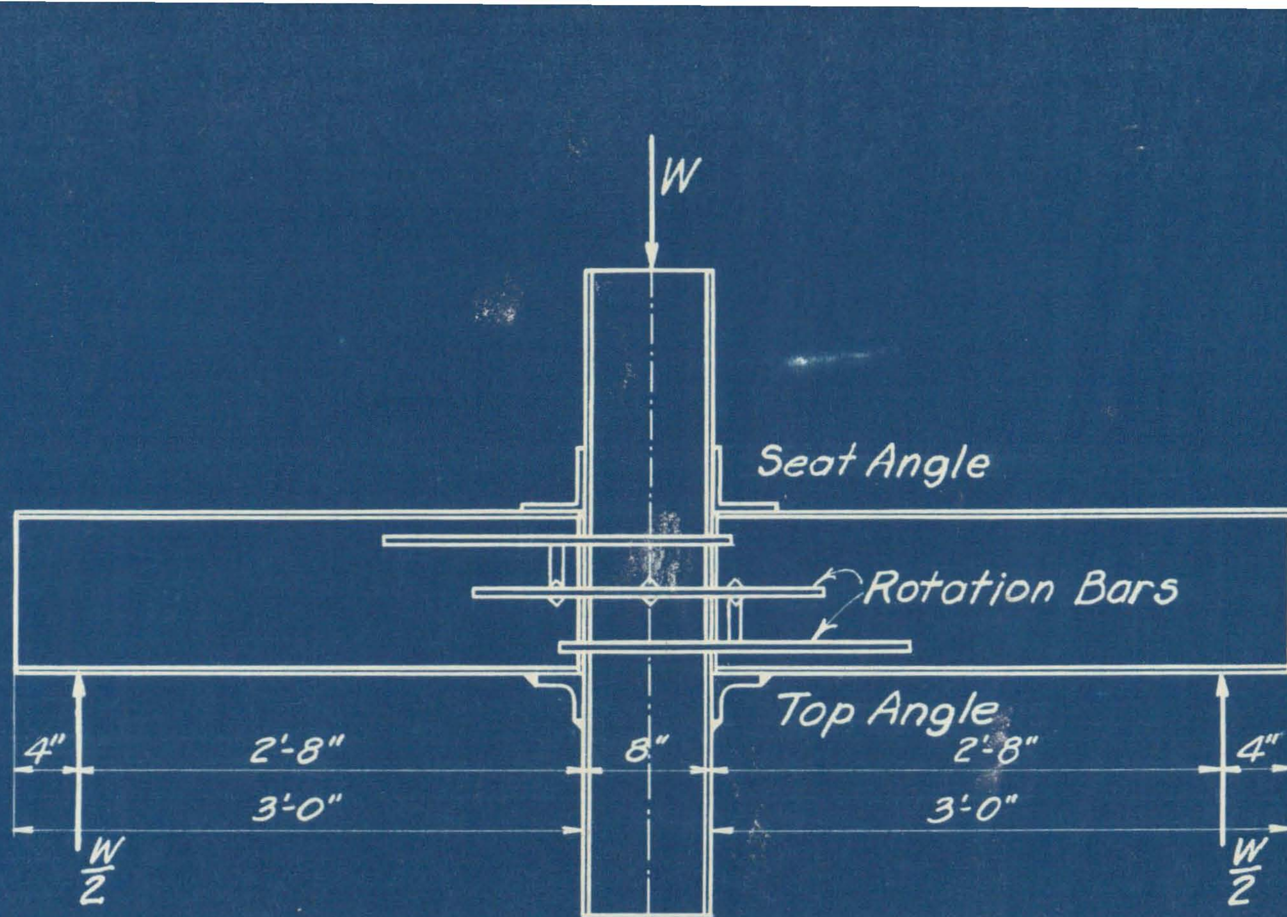


FIG. 8a TEST ARRANGEMENT TO DETERMINE BEAM-COLUMN FLANGE CONNECTION CONSTANTS

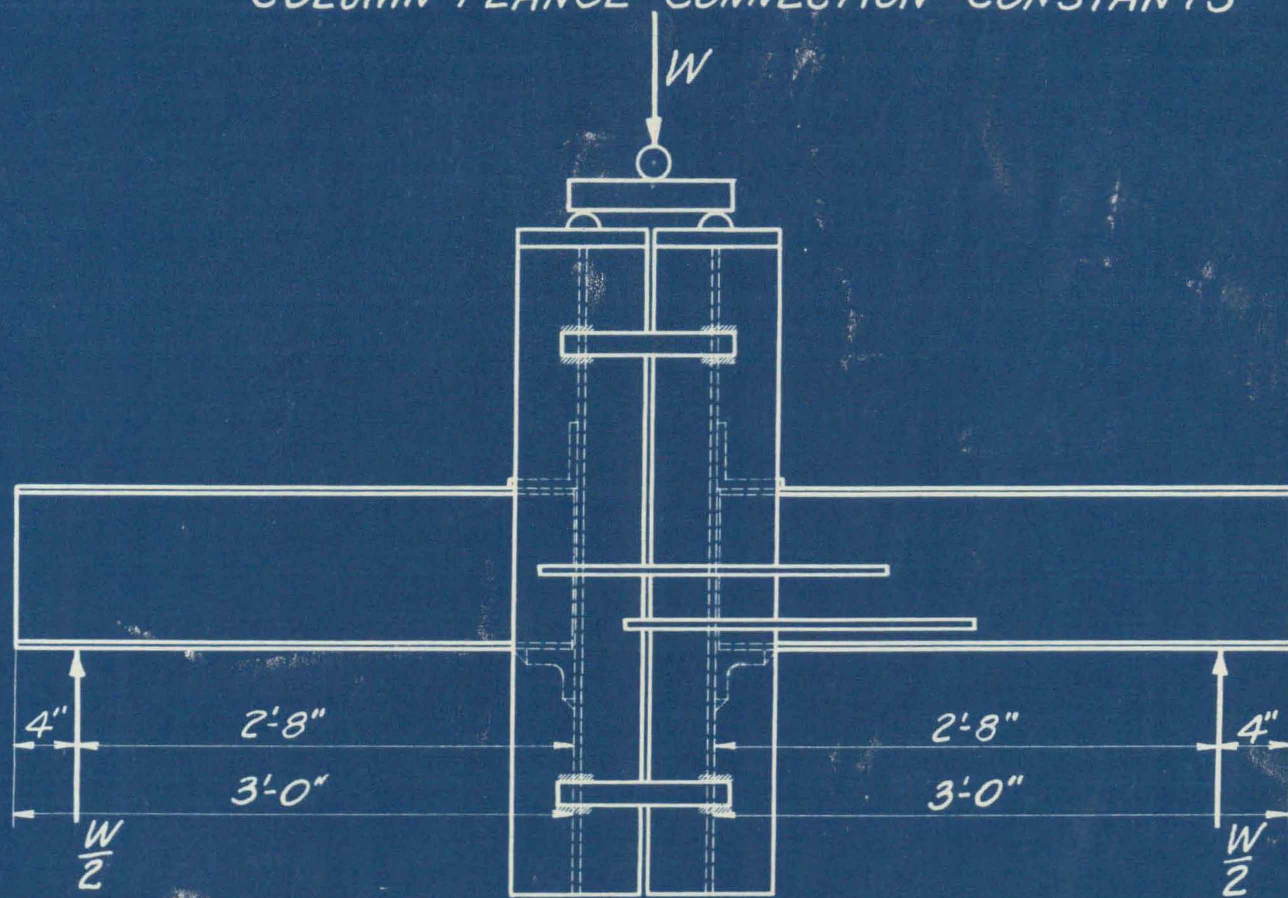


FIG. 8b TEST ARRANGEMENT TO DETERMINE BEAM-EXTERIOR COLUMN WEB CONNECTION CONSTANTS

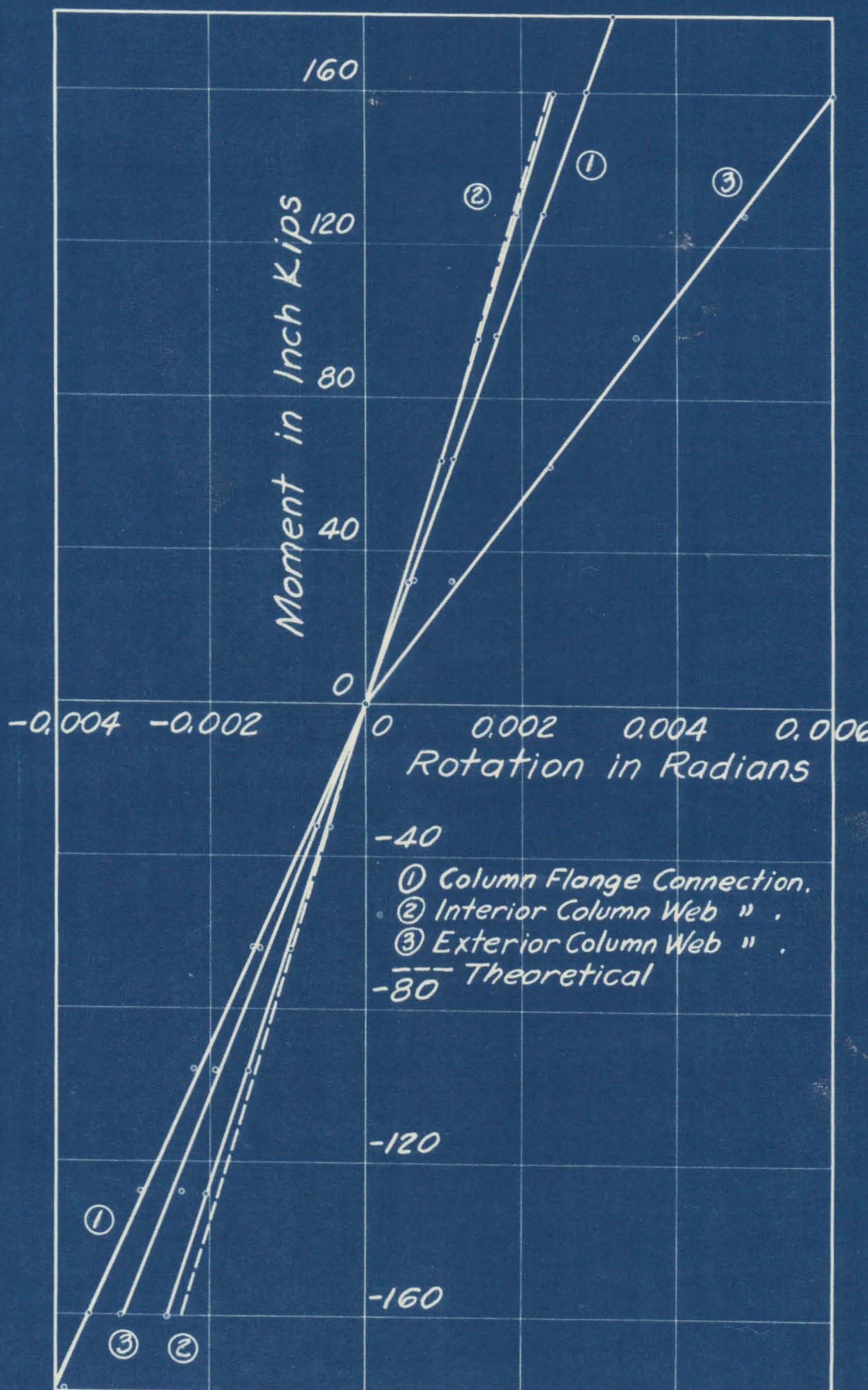


FIG. 9 TEST RESULTS OF CONNECTIONS USED IN WELDED BUILDING FRAMES

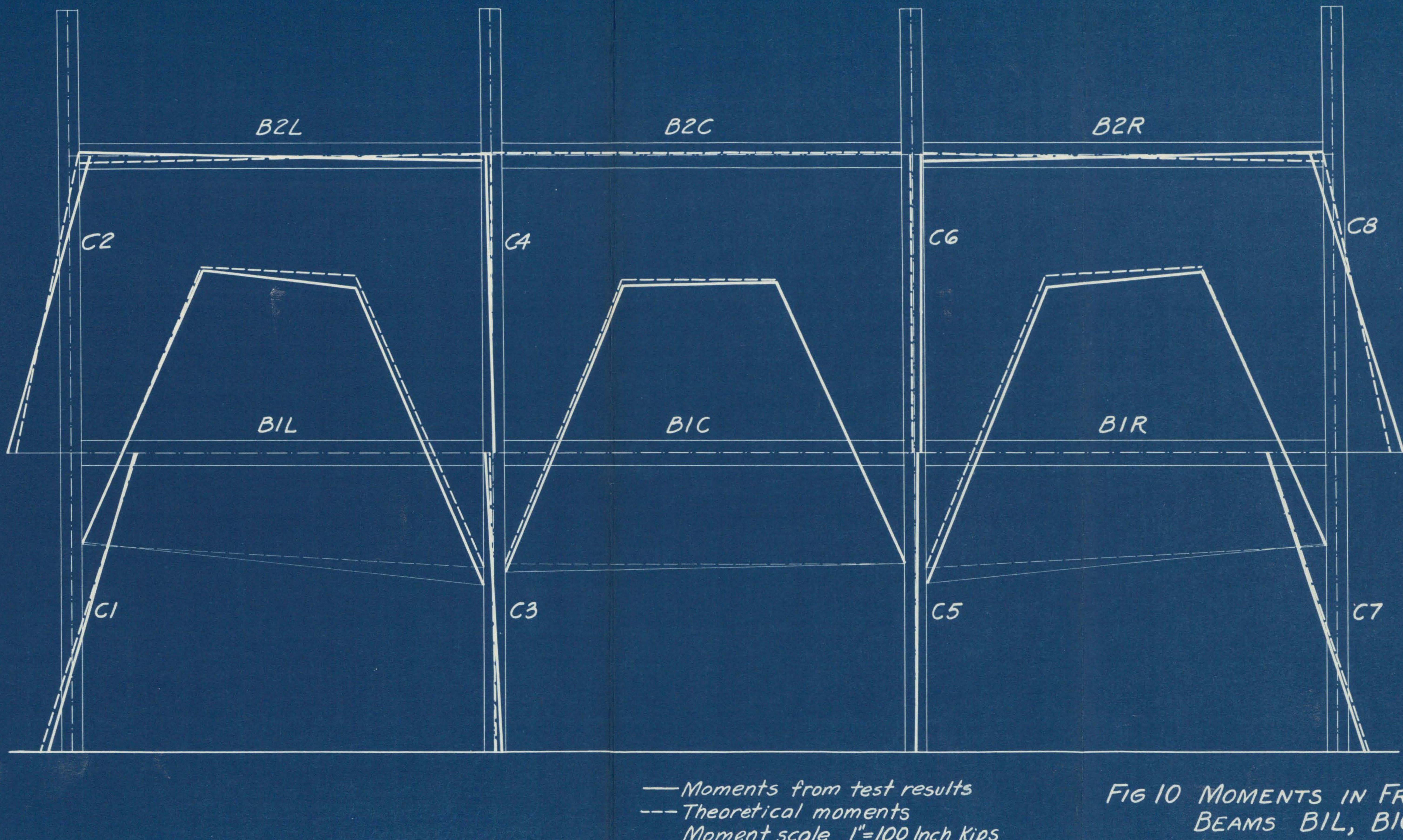
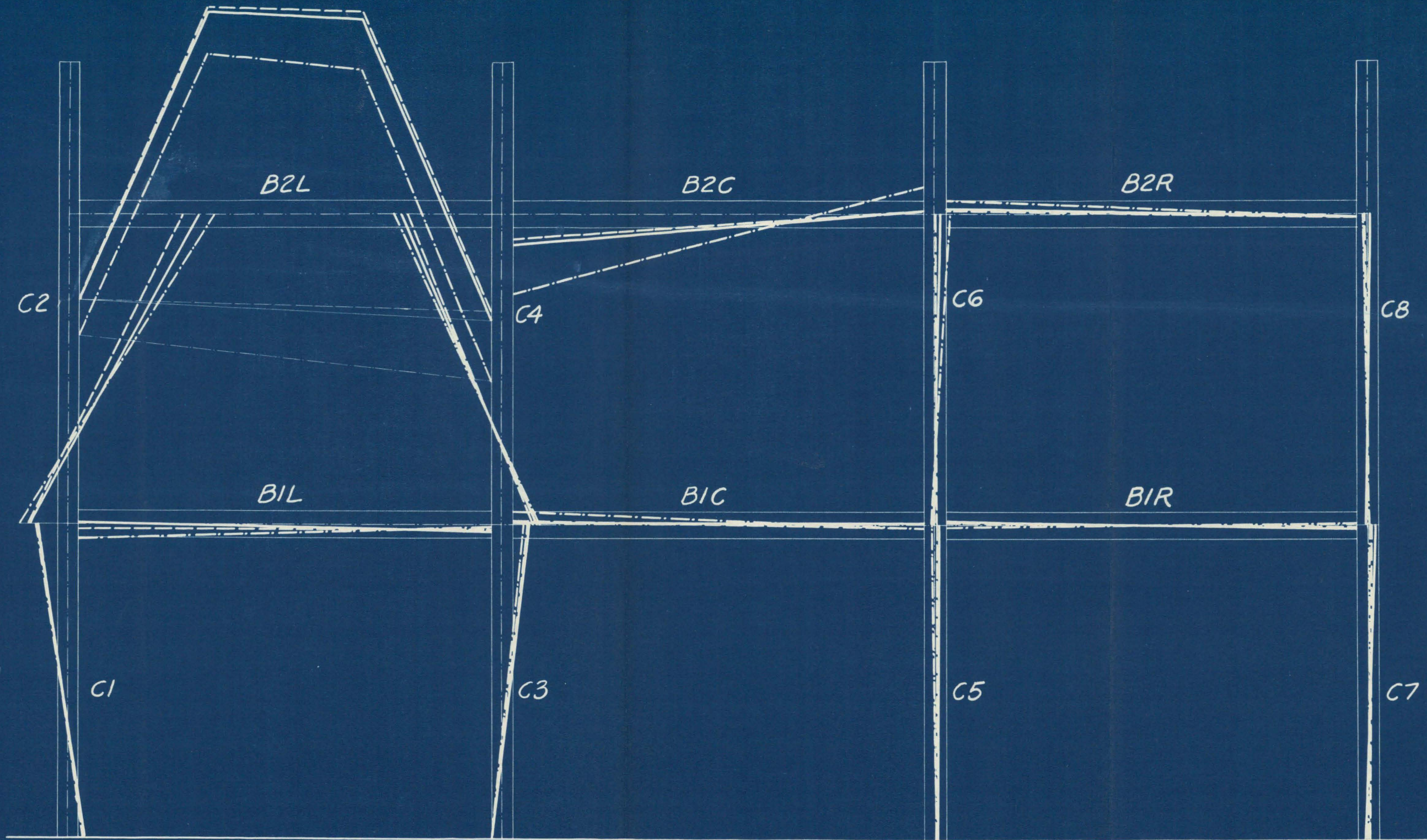
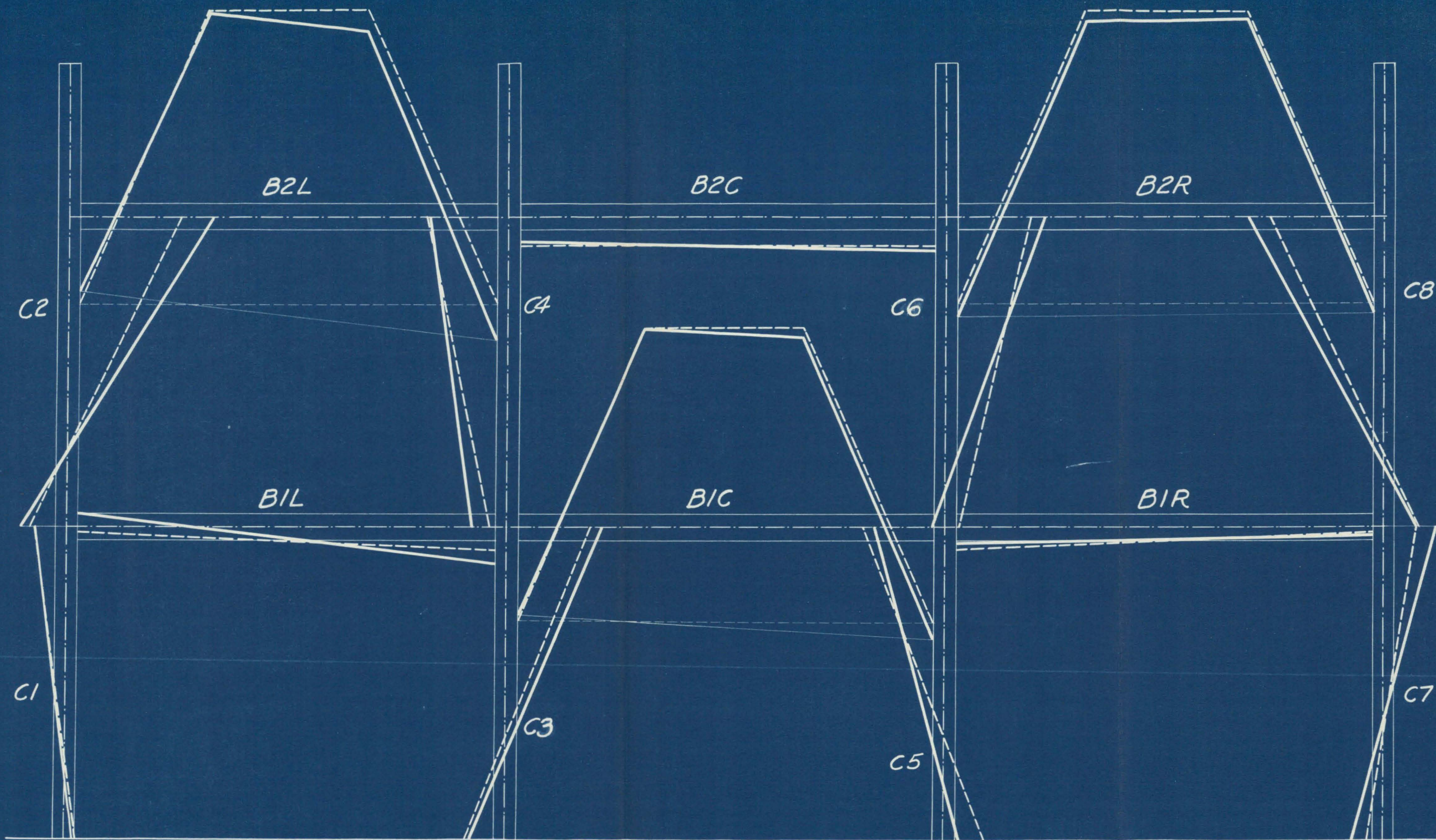


FIG 10 MOMENTS IN FRAME No. 1
BEAMS B1L, B1C, AND B1R
LOADED



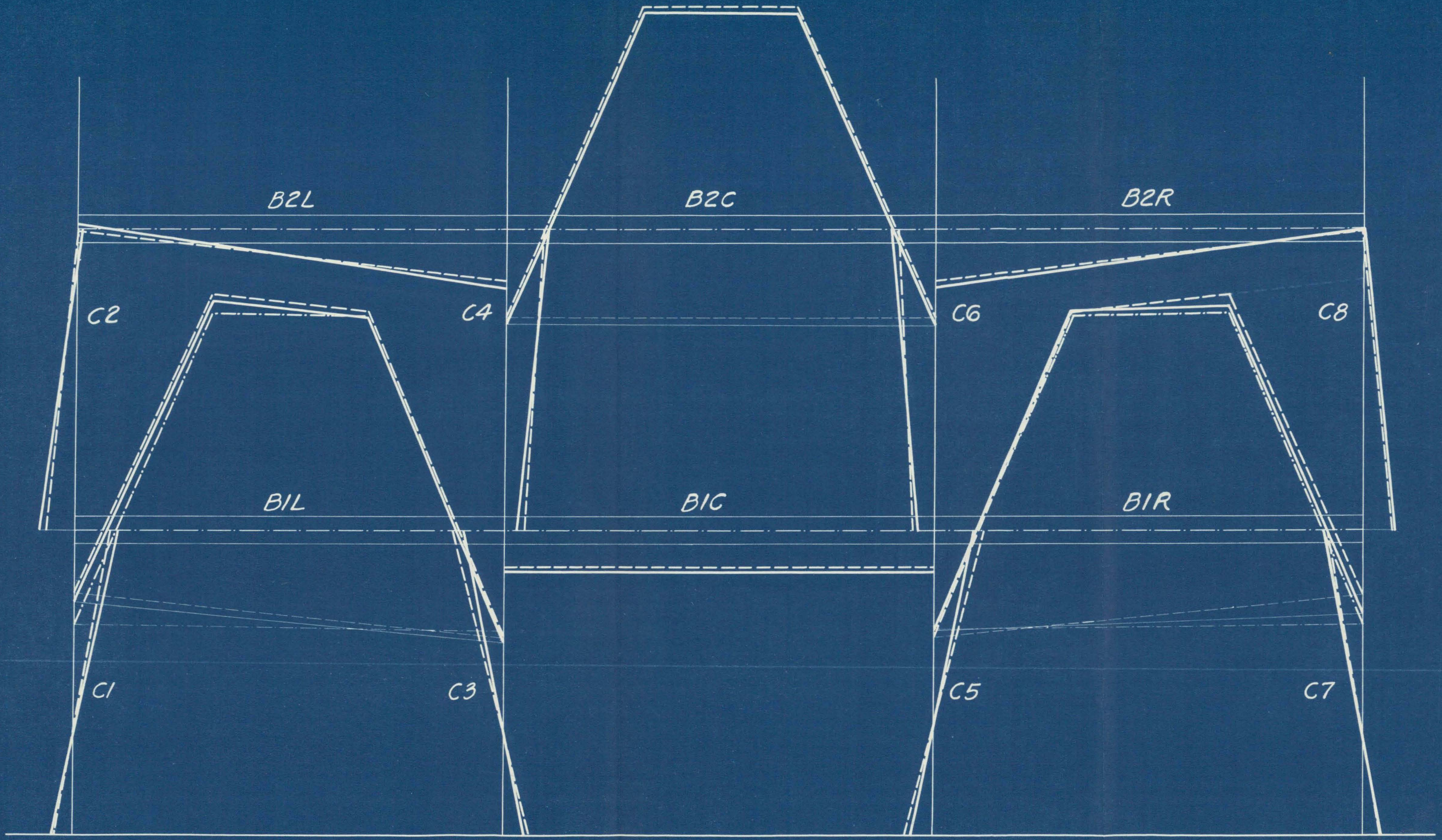
—— Moments from test results
- - - - Theoretical moments
—·—· Theoretical moments, assuming 100% connection rigidity
Moment scale 1" = 100 Inch Kips

FIG. 11 MOMENTS IN FRAME NO. 1
BEAM B2L LOADED



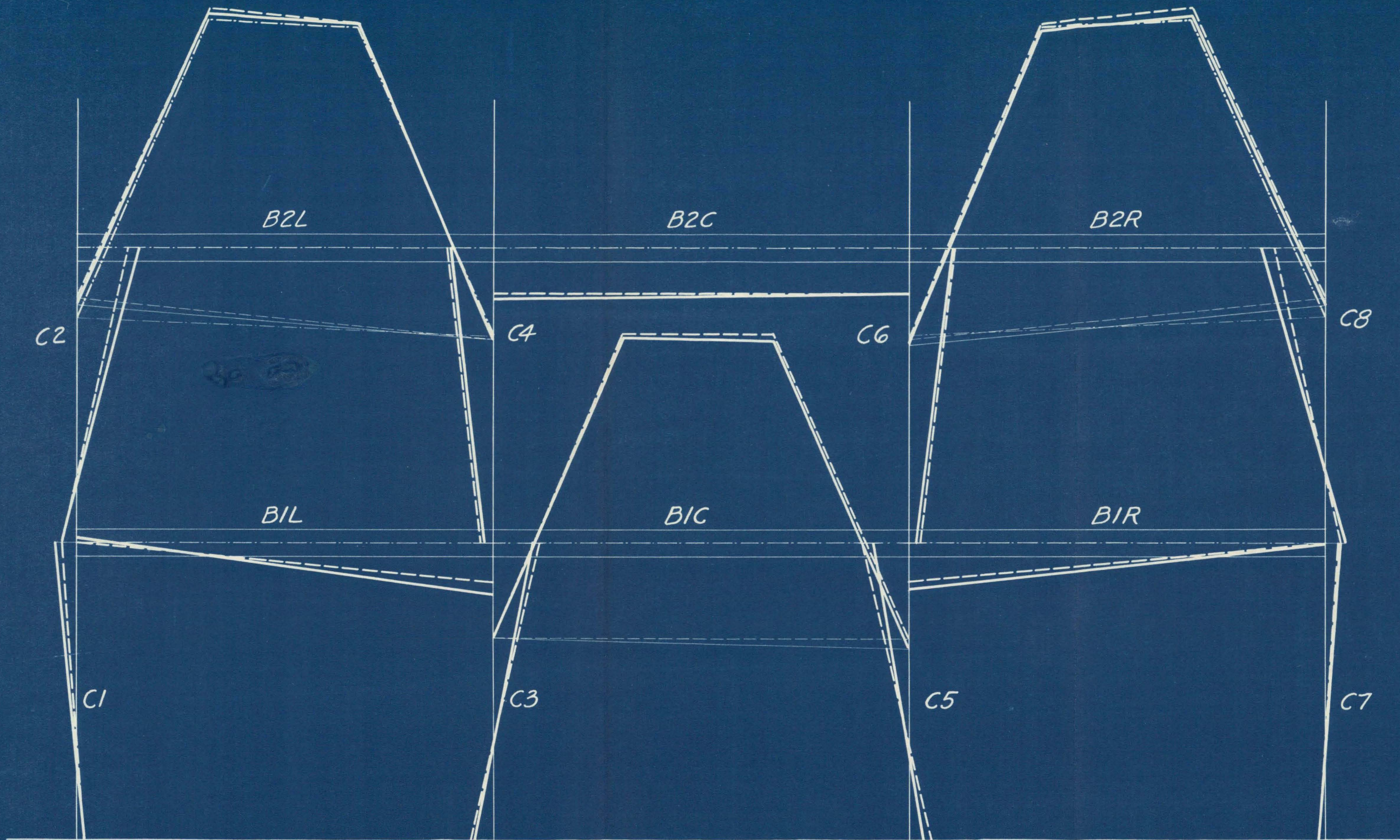
—— Moments from test results
---- Theoretical moments
Moment scale 1" = 100 Inch Kips

FIG 12 MOMENTS IN FRAME No. 1
BEAMS B2L, B1C, AND B2R
LOADED



——Moments from test results
----Theoretical moments
—·—·—Theoretical moments, exterior joints assumed equal to interior in rigidity
Moment scale 1" = 100 Inch Kips

FIG 13 MOMENTS IN FRAME No. 2
BEAMS B1L, B2C, AND B1R
LOADED



Moments from test results
Theoretical moments
Theoretical moments, exterior joints assumed equal to interior in rigidity
Moment scale 1" = 100 Inch Kips

FIG 14 MOMENTS IN FRAME No. 2
BEAMS B2L, B1C, AND B2R
LOADED

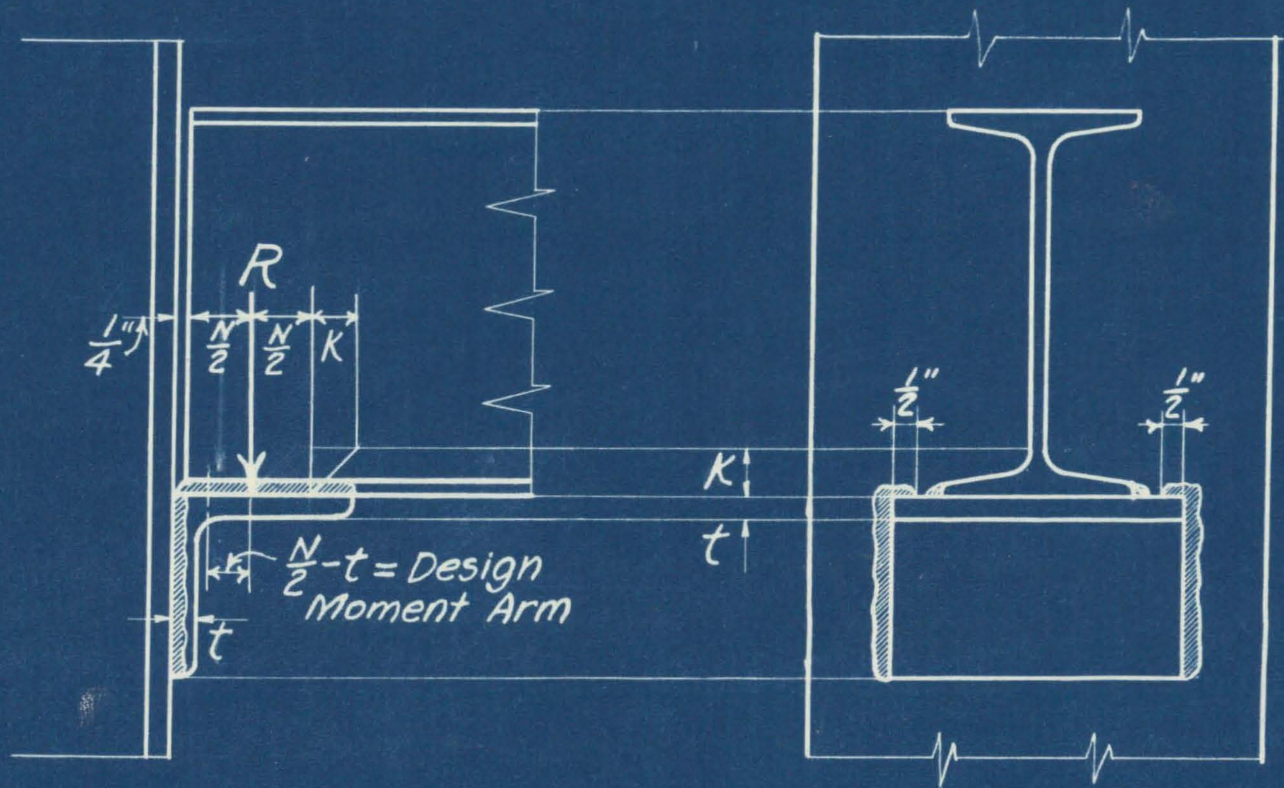


FIG 15 SEAT ANGLE

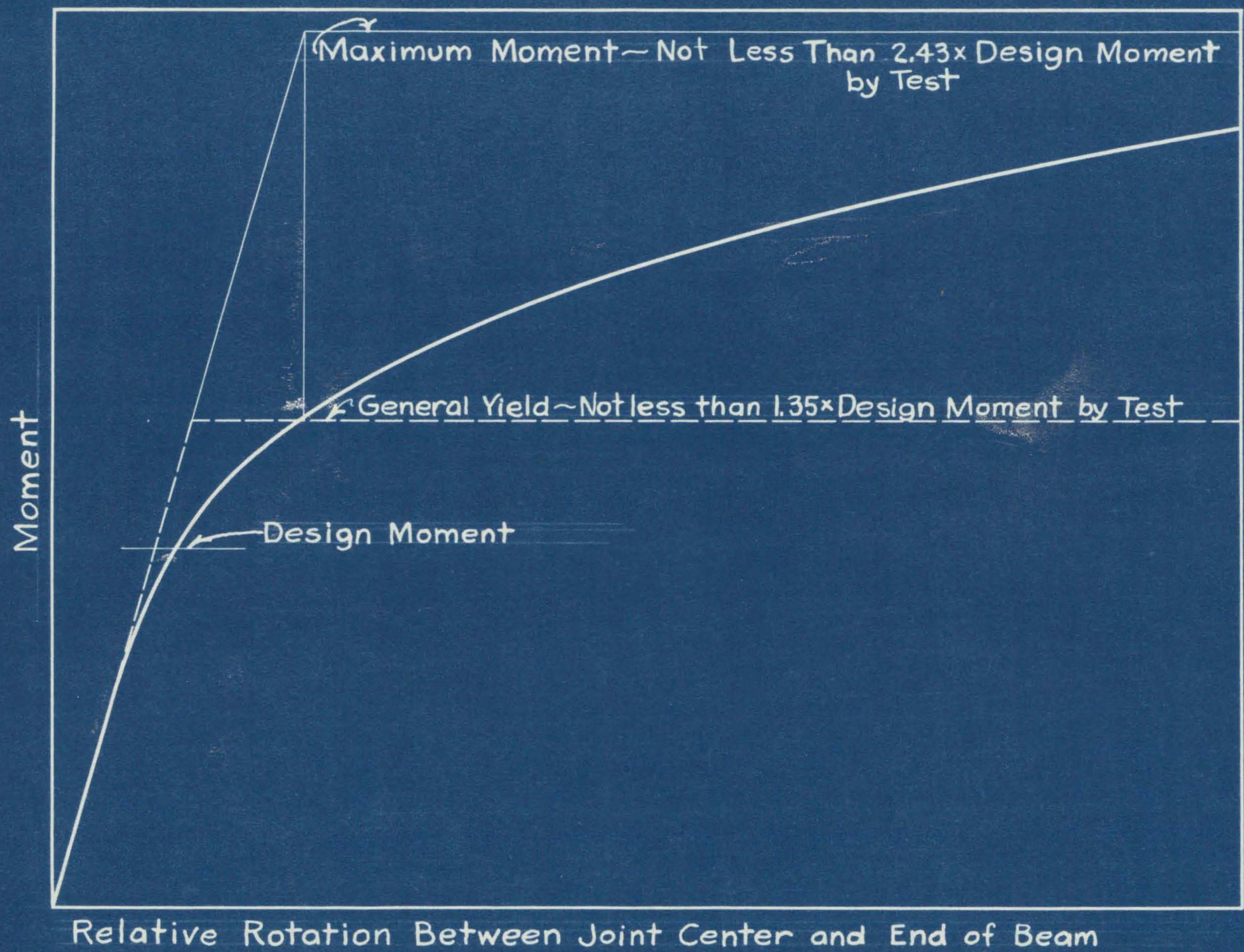


FIG. 22 MOMENT ~ ROTATION DIAGRAM, MINIMUM TEST VALUES

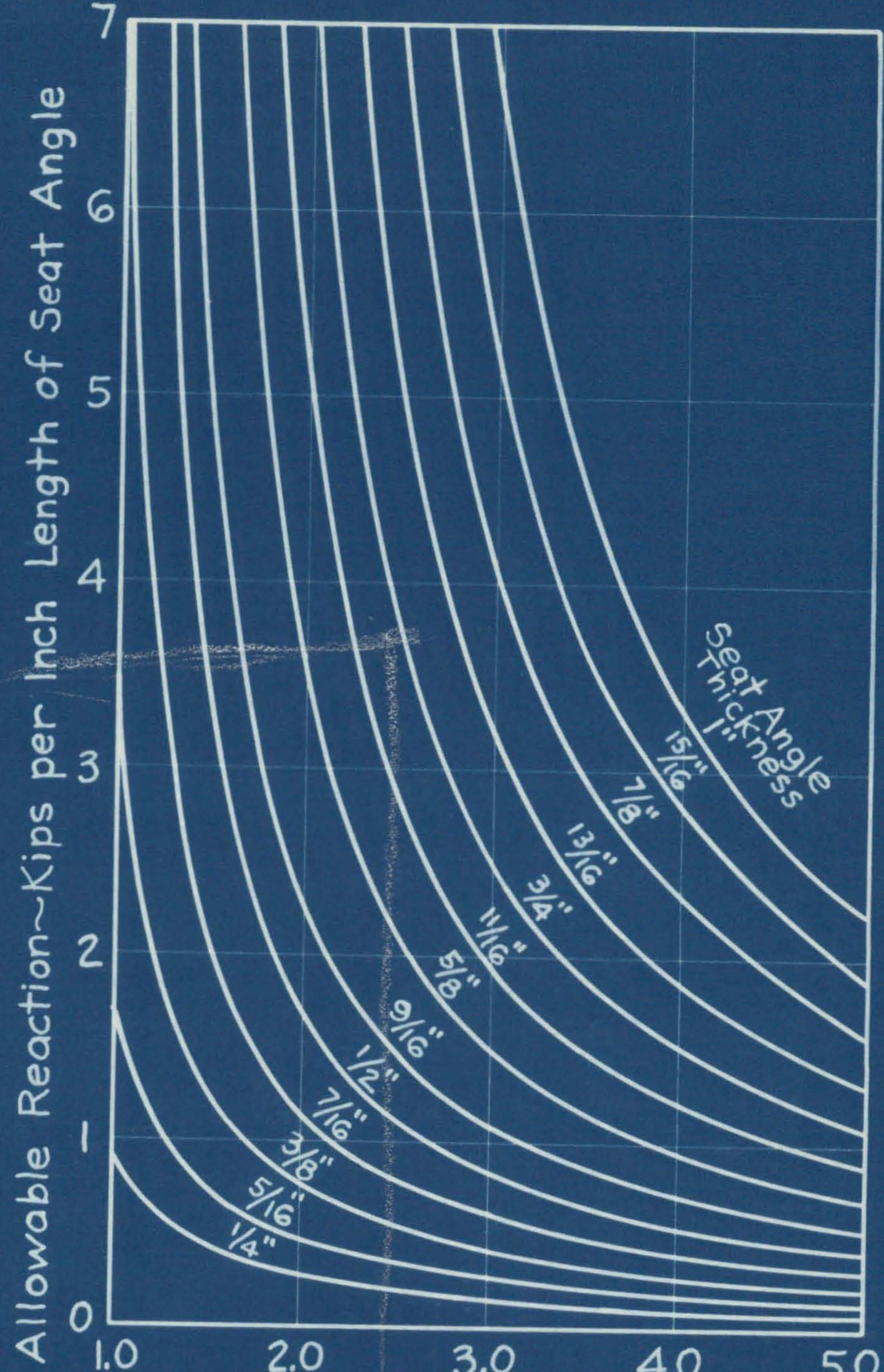


FIG 16a DESIGN CHART FOR SEAT ANGLE THICKNESS

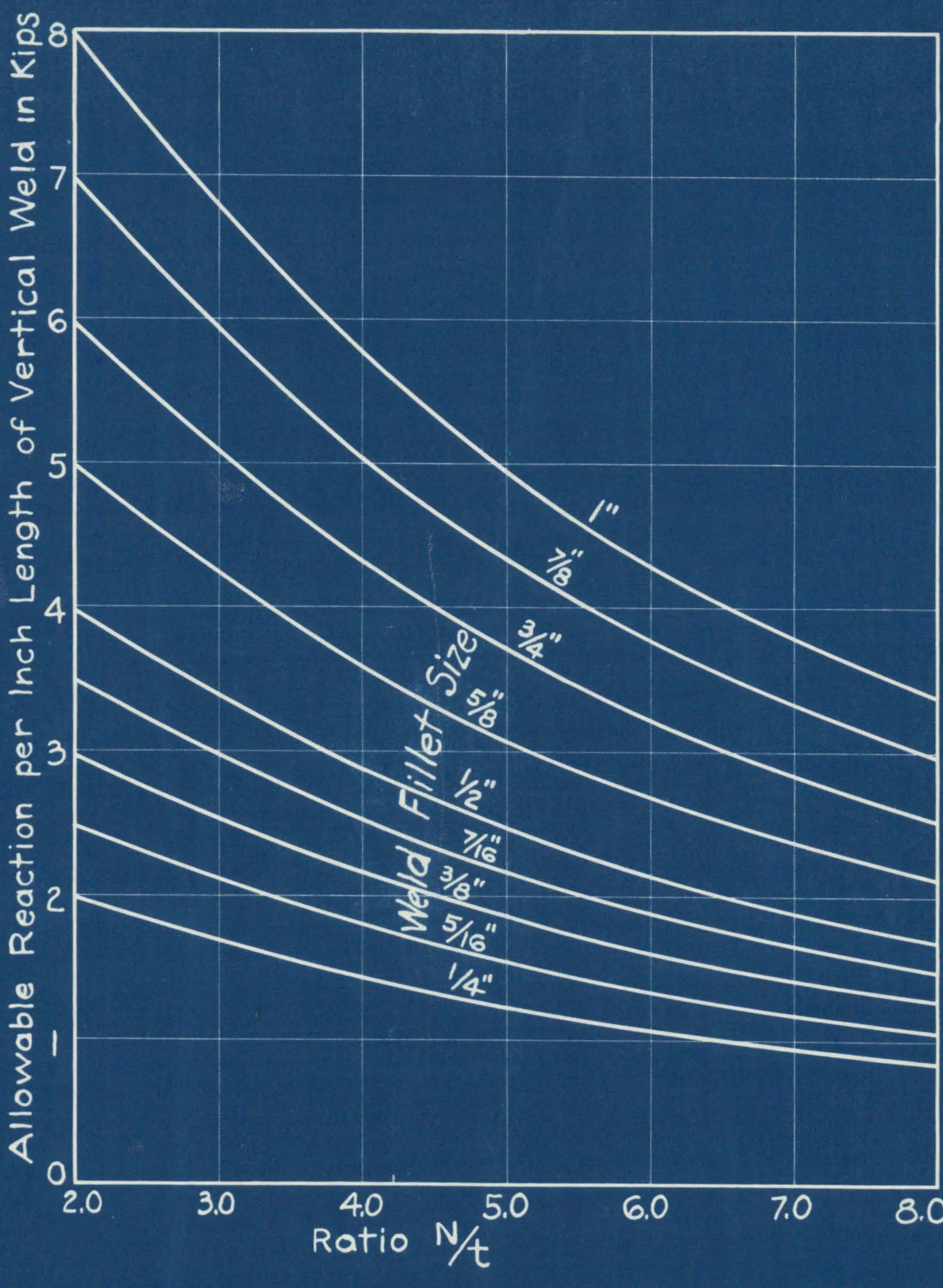


FIG. 16b DESIGN CHART FOR VERTICAL FILLET WELDS BETWEEN SEAT ANGLE AND COLUMN

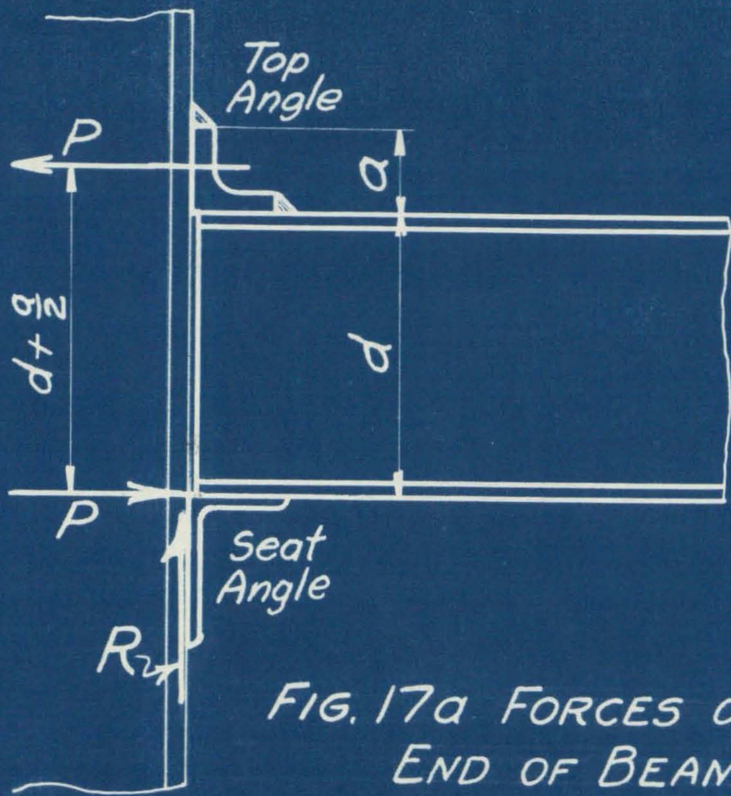


FIG. 17a FORCES ON END OF BEAM

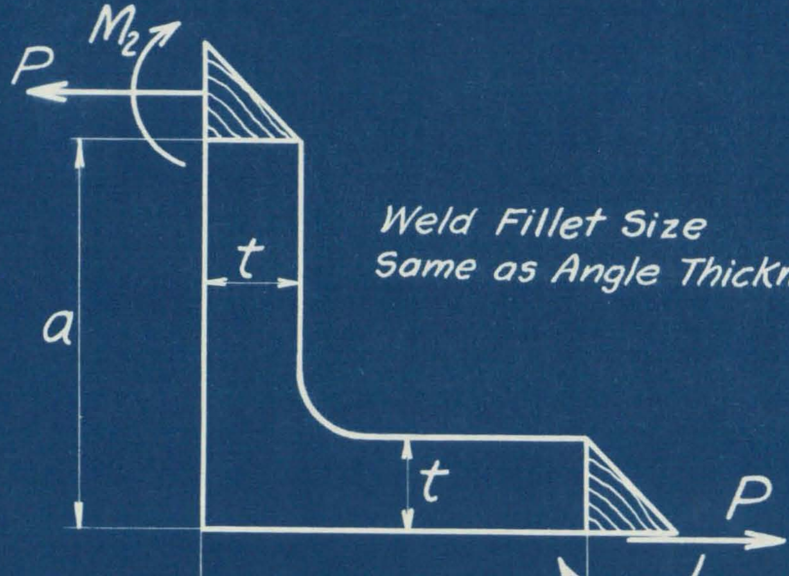


FIG. 17b FORCES ACTING ON TOP ANGLE

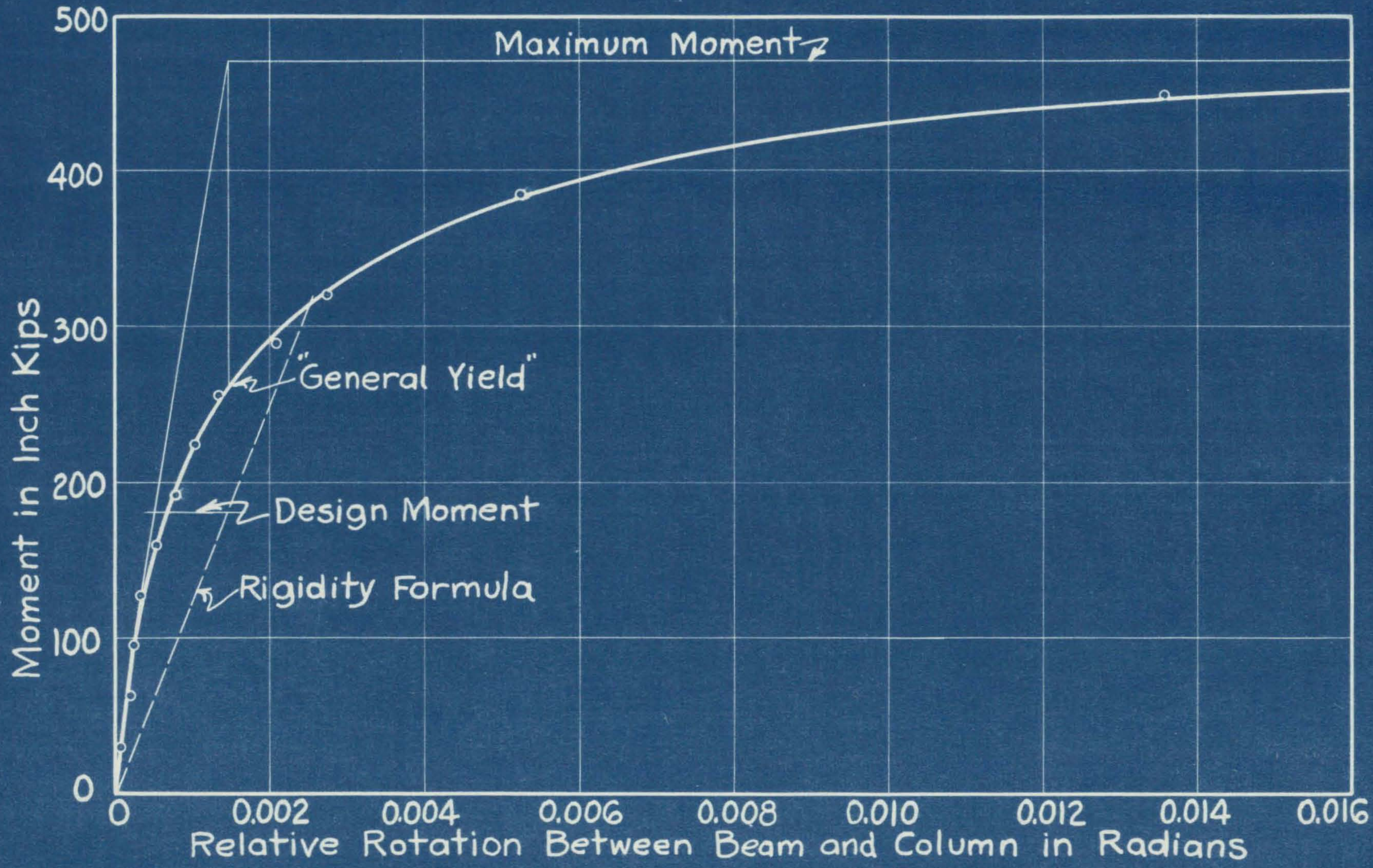


FIG 18 MOMENT~ANGLE CHANGE DIAGRAM FOR TYPICAL JOINT TEST

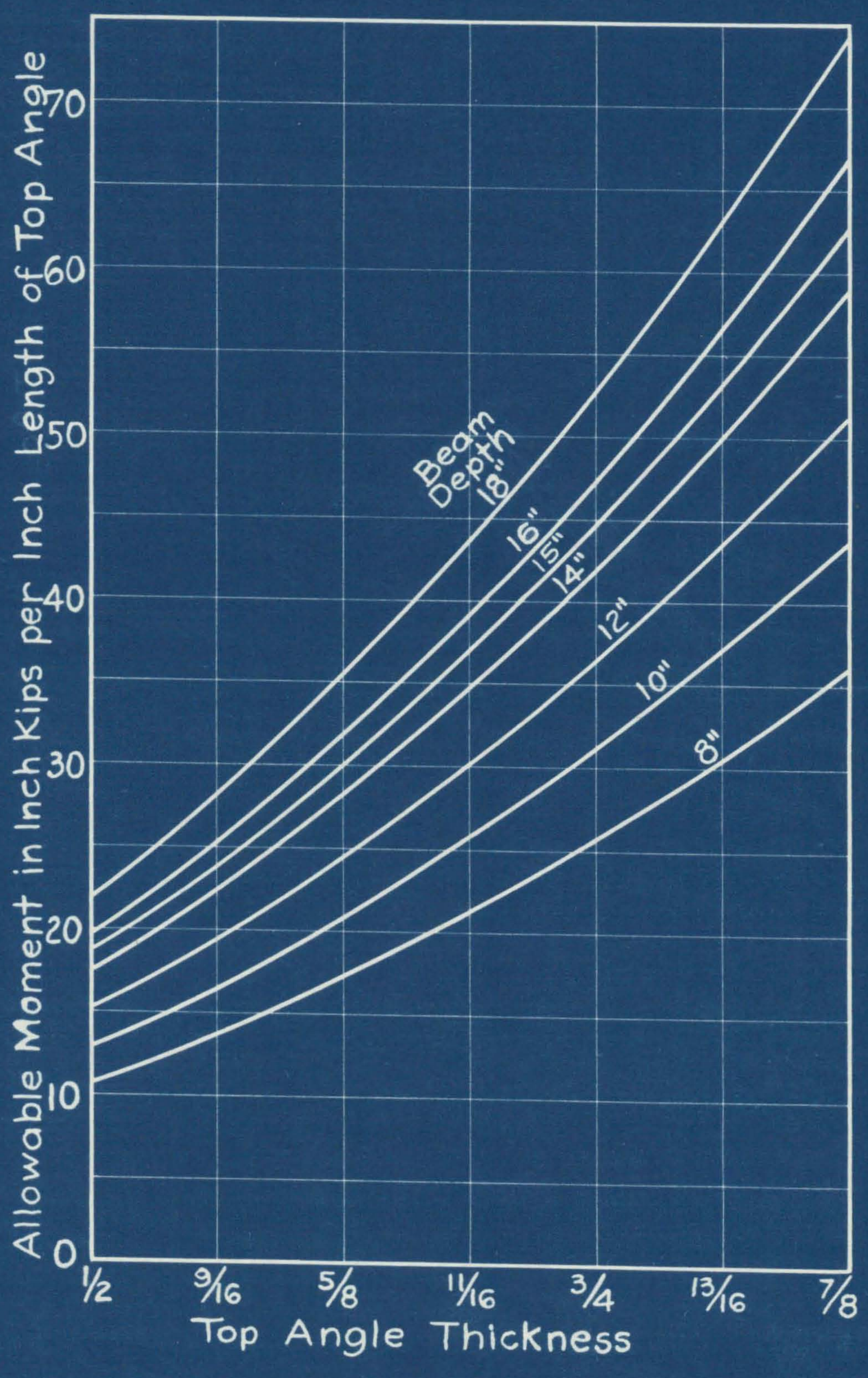


FIG. 19a DESIGN CHART FOR 3"×3" TOP ANGLES

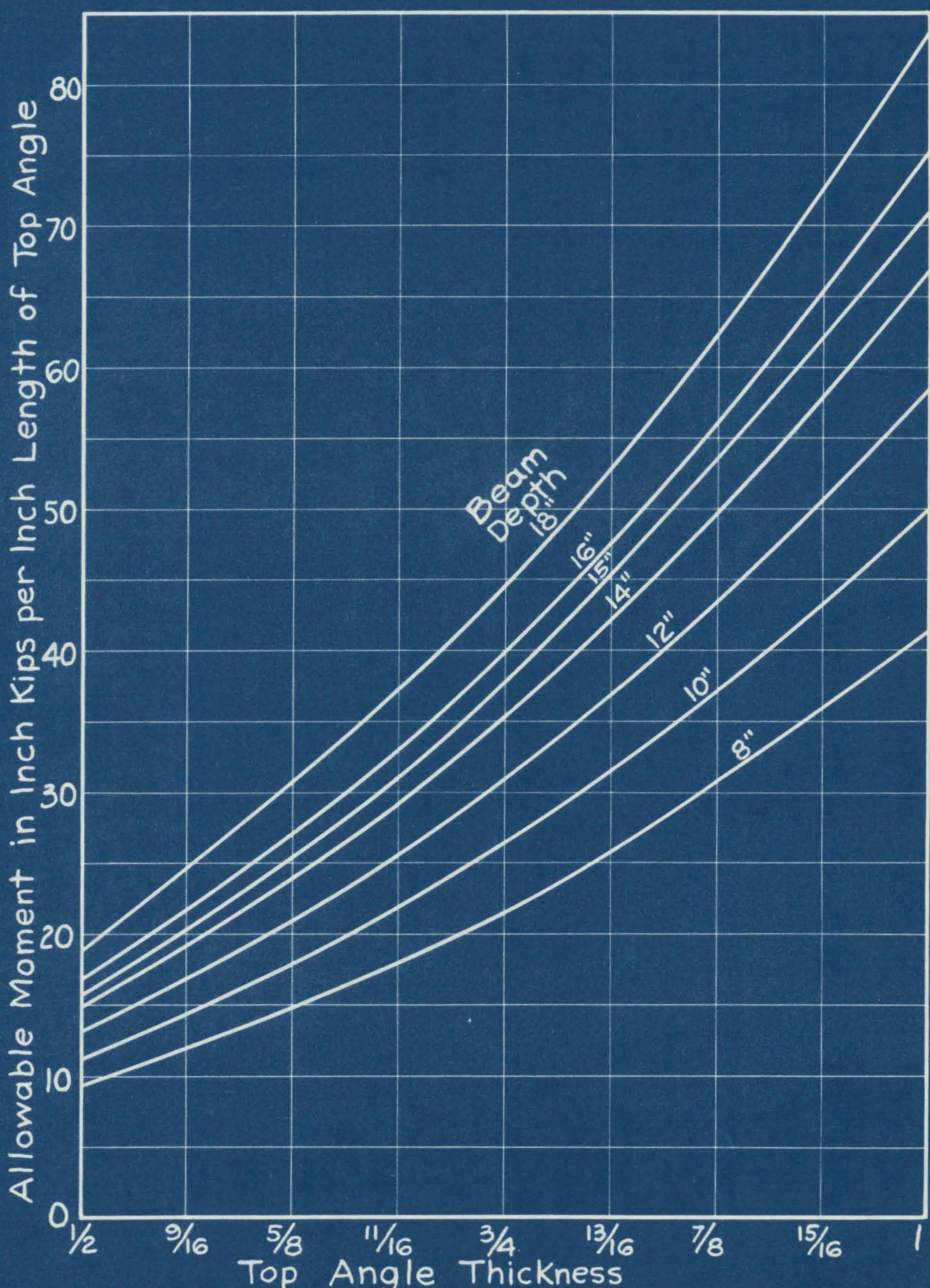


FIG. 19b DESIGN CHART FOR $3\frac{1}{2}" \times 3\frac{1}{2}"$ TOP ANGLES

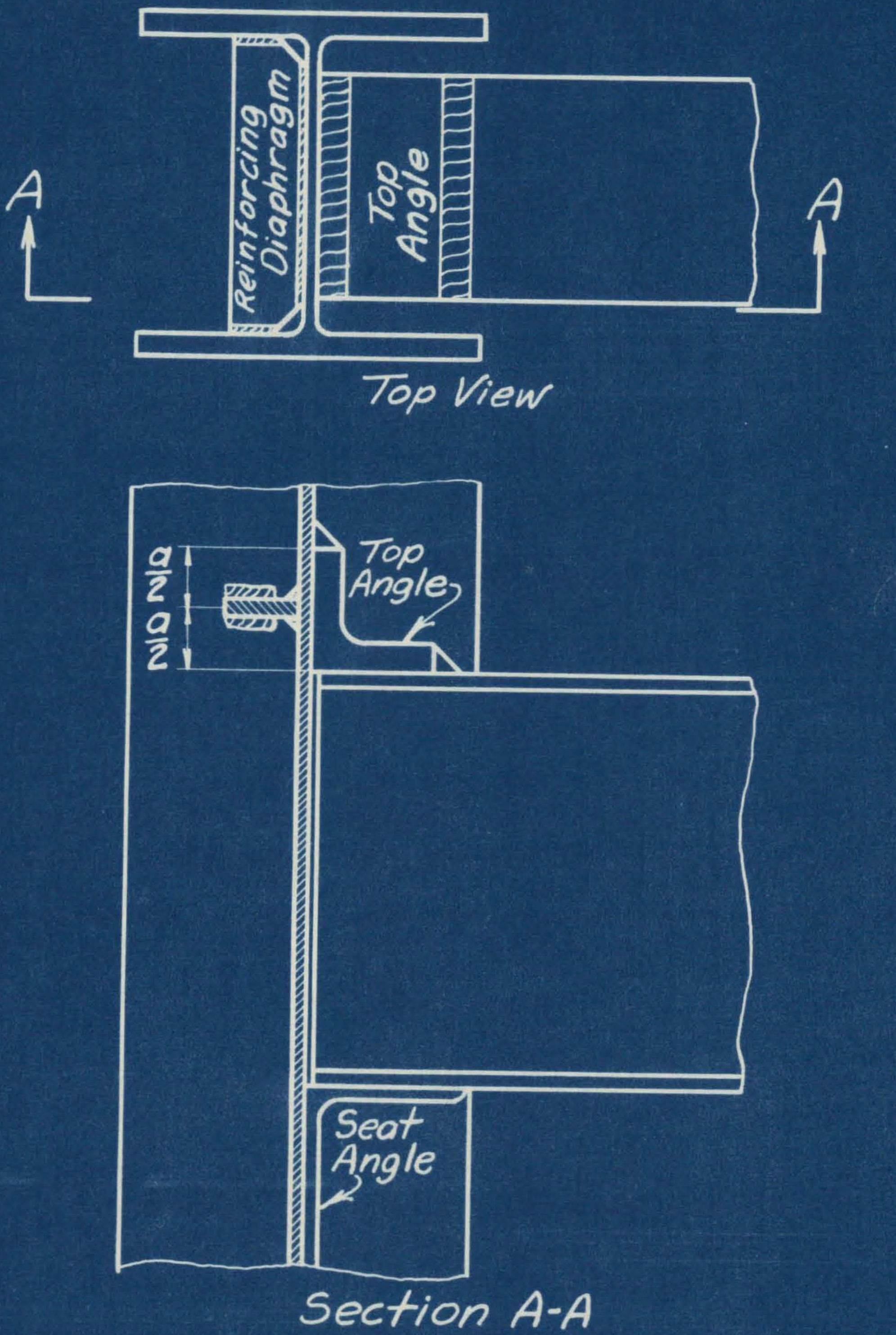


FIG 20 REINFORCING DIAPHRAGM FOR EXTERIOR COLUMN WEB CONNECTIONS

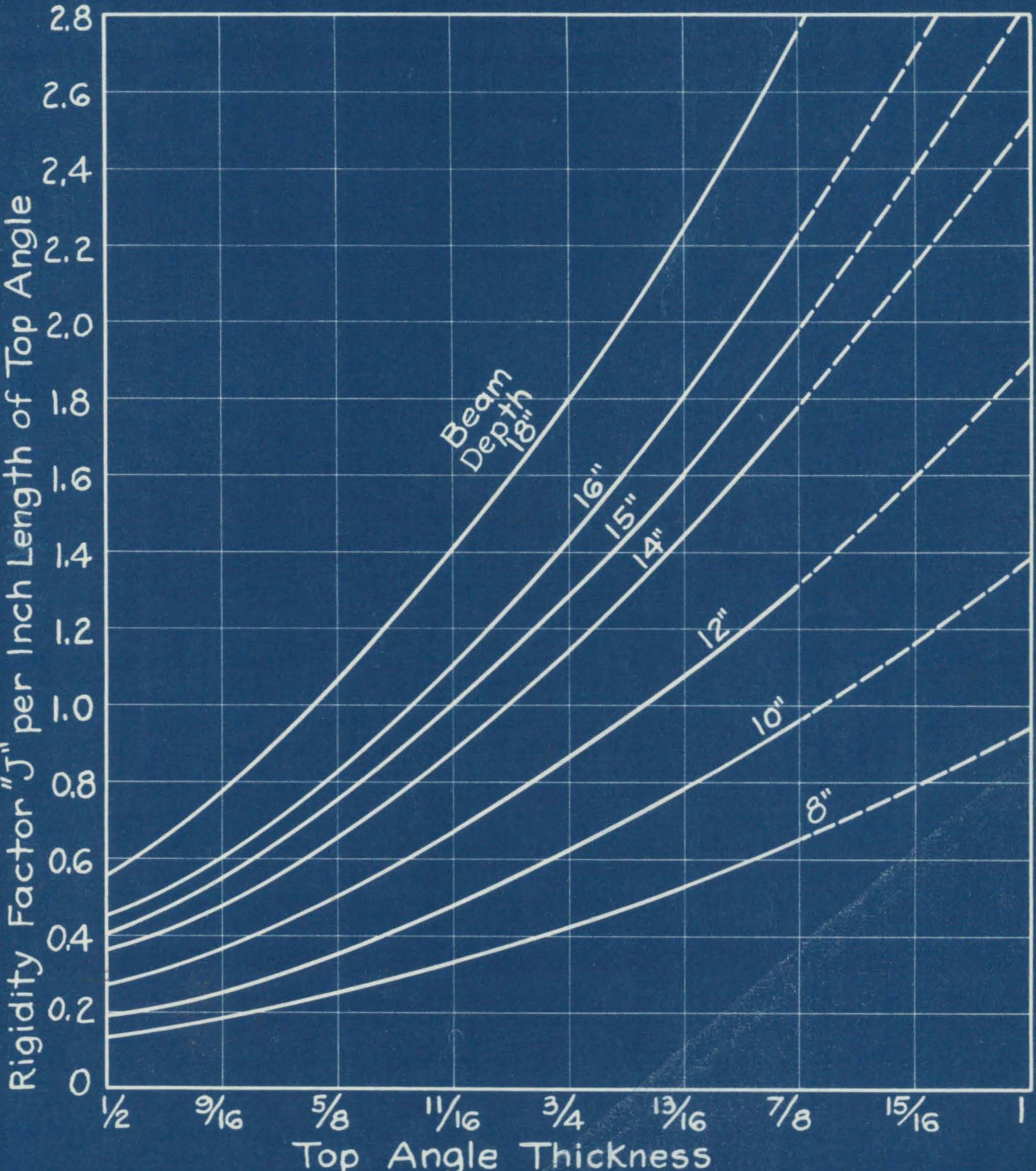


FIG. 21a CHART OF CONNECTION RIGIDITY FACTOR "J" FOR 3"×3" TOP ANGLES

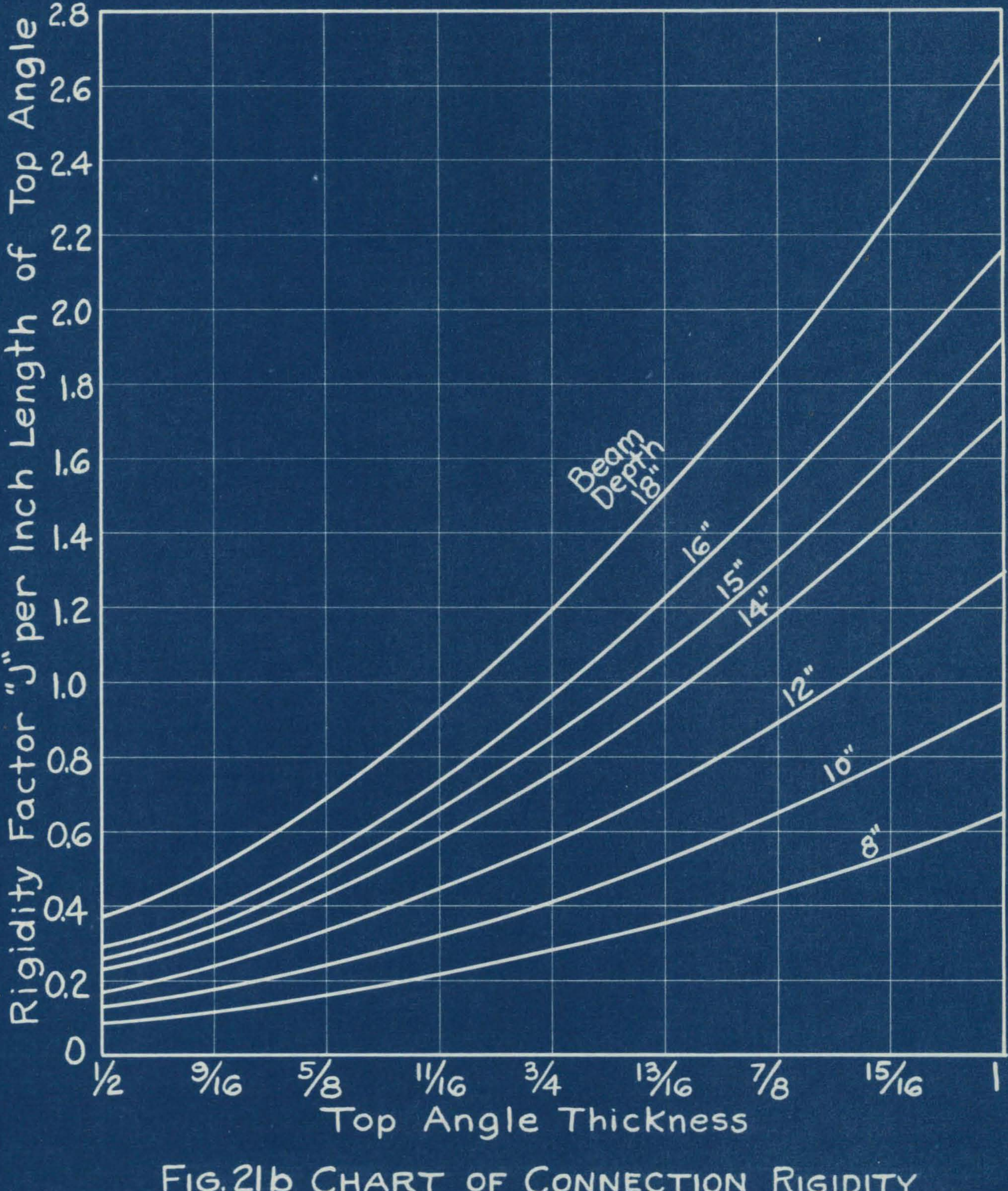


FIG. 21b CHART OF CONNECTION RIGIDITY FACTOR "J" FOR $3\frac{1}{2}" \times 3\frac{1}{2}"$ TOP ANGLES

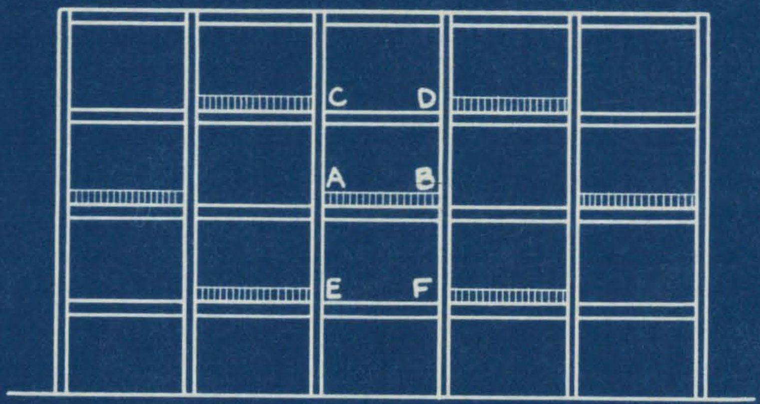


FIG 23a LOAD CONDITION FOR MAXIMUM CENTER MOMENT IN BEAM "A-B"

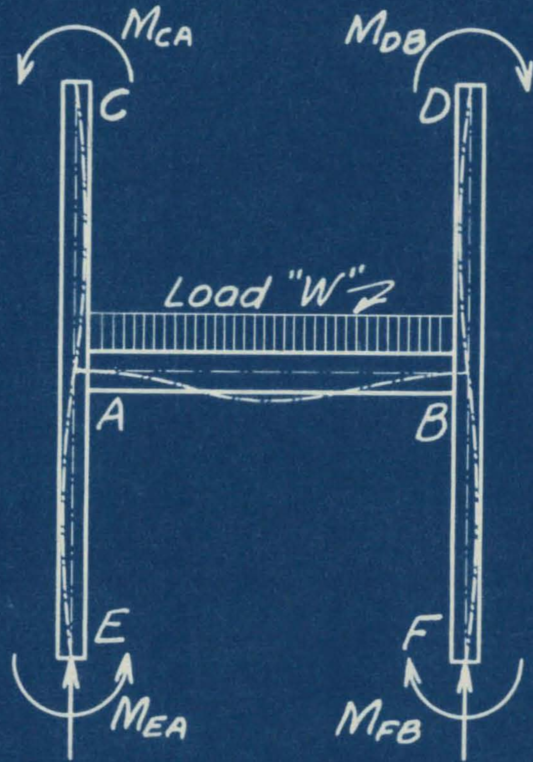


FIG 23b CONDITIONS ASSUMED FOR CALCULATING DESIGN MOMENT IN ANY BEAM "A-B"

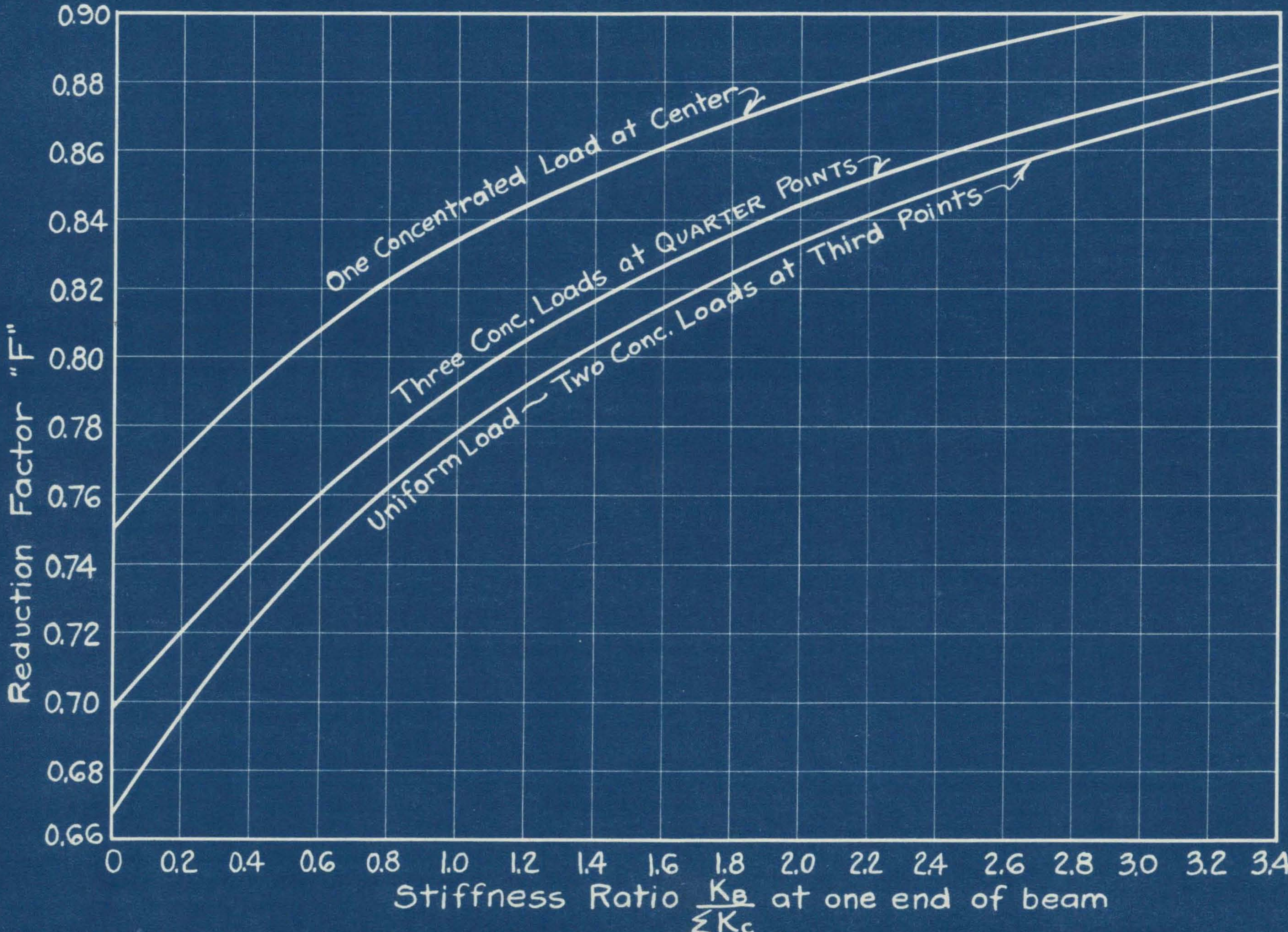


FIG. 24 ALLOWABLE REDUCTION IN SIMPLE BEAM MOMENT FOR SYMMETRICAL CONDITIONS

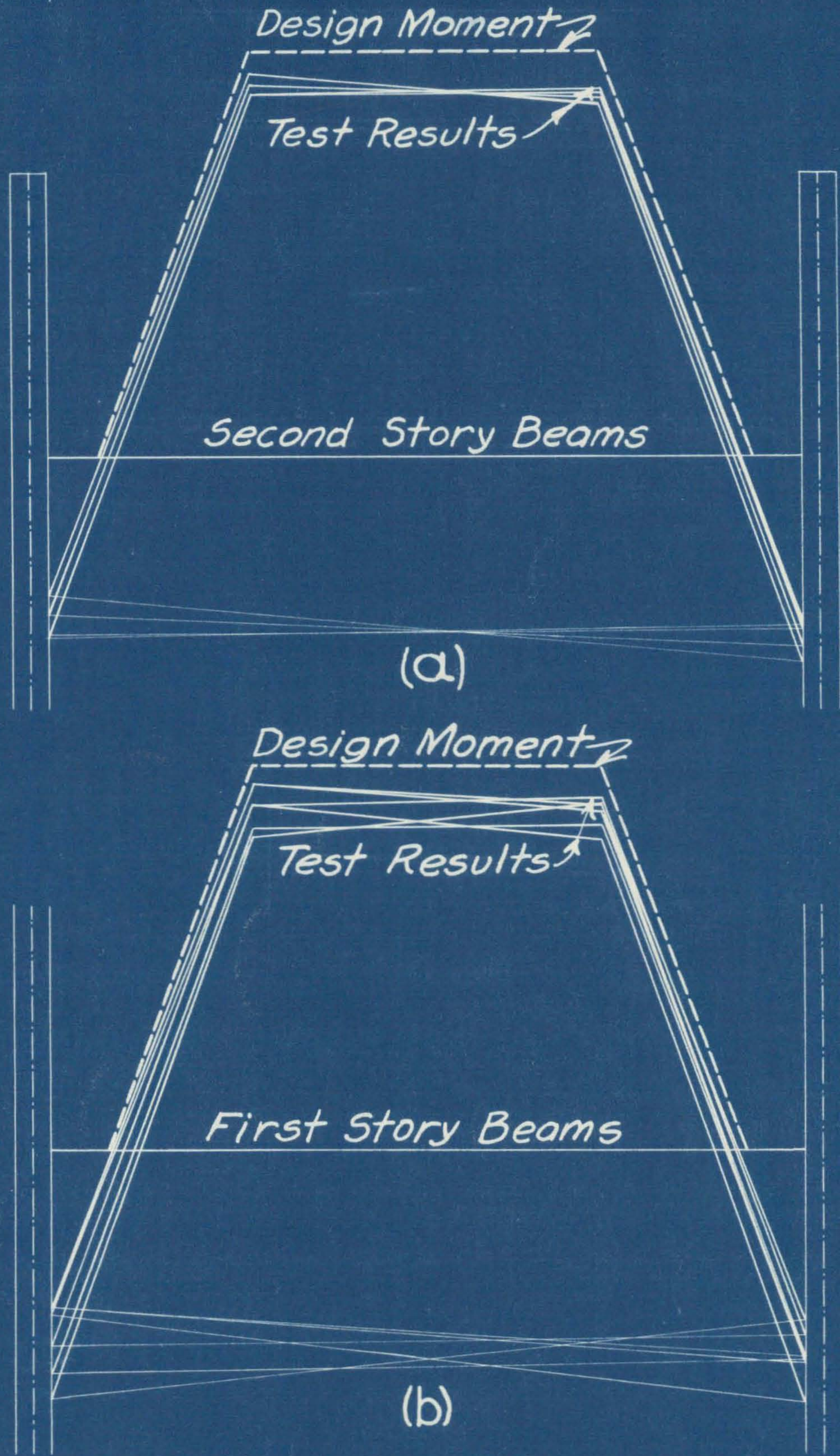


FIG 25 TEST RESULTS OF FRAME No. 1
COMPARED WITH DESIGN MOMENT

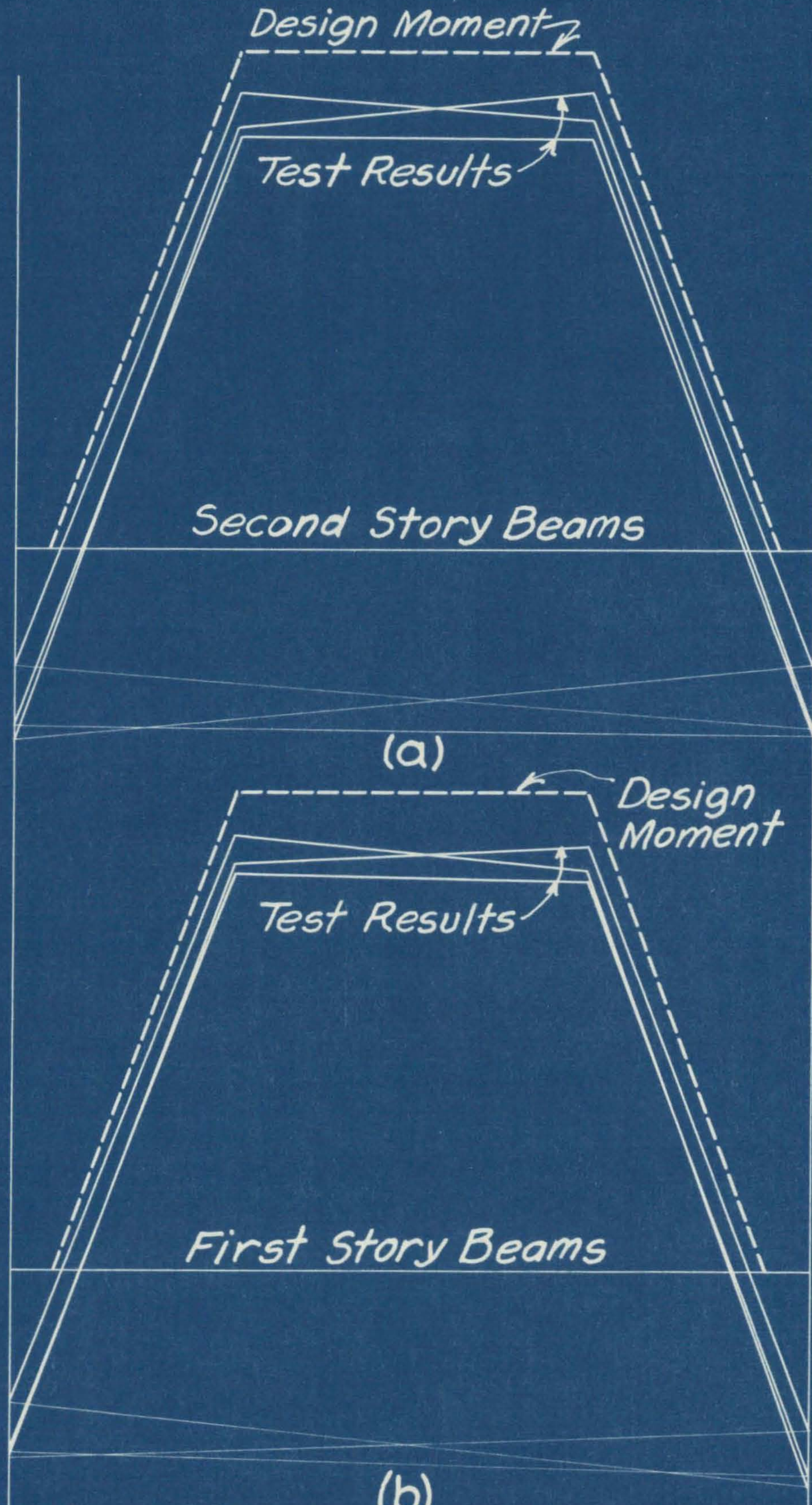


FIG 26 TEST RESULTS OF FRAME No.2
COMPARED WITH DESIGN MOMENT