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Welded Continuous Frames and Their Components

PROGRESS REPORT NO. 28

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by

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Fritz Laboratory Report No. 205.61

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I. INTRODUCTION

In Ref. 1 a general method of solution was developed for the plastic design of rigid frame structures. The particular parts of that report that had to do with single and multiple span, pinned-base gable frames were presented in a separate report[2] which also included design curves and examples. This paper presents the same type of information for pinned-base "lean-to" type structures.

The assumptions that will be made are the same as those listed in the earlier papers. That is,

1. As moment approaches its fully plastic value, M_p, curvature increases indefinitely;
2. Equilibrium can be formulated in the undeformed position;
3. No instability occurs prior to the attainment of the fully plastic load;
4. The influence of shear and thrust is neglected;
5. There is a known amount of moment that can be transmitted through the connections;
6. All loads are increased proportionally, and
7. Failure corresponds to that condition where the structure is reduced to a mechanism through the development of yield (or plastic) hinges.

These assumptions correspond to those made in "simple plastic theory".

As shown in Ref. 3 and 4 the necessary and sufficient conditions for a plastic solution, according to the simple plastic theory, are:

1. The structure must be in equilibrium;
2. The moment at any section must be less than or equal to the fully plastic moment value, (i.e., $|M| \leq M_p$); and
3. A mechanism must be formed.

Several different approaches or methods of solution may be used which will ensure that these three conditions are met. The one that will be followed in this report is the Mechanism Method(5,6). In essence, this type of solution assumes that all possible failure configurations are examined and that the load corresponding to each is determined. The maximum load that the structure can sustain is then the one having the lowest critical load value.

II. DEVELOPMENT OF THE DESIGN CHARTS

1. Loading, Plastic Hinges and Mechanisms

The basic structure to be considered is the one shown in Fig. 1. It may exist either by itself or in combination with other pinned-base structures. For the single span frame, the assumption will be made that each of the members can deliver a resisting moment equal to M_p. When the frame is connected to other structures; that is, for multiple span frames; the interior columns will be chosen large enough to ensure development of hinges in the rafter. The size of these columns will be

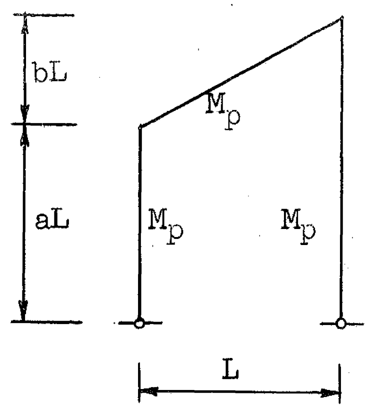

FIG. 1

determined from the moment diagram after the failure condition and corresponding load is ascertained.

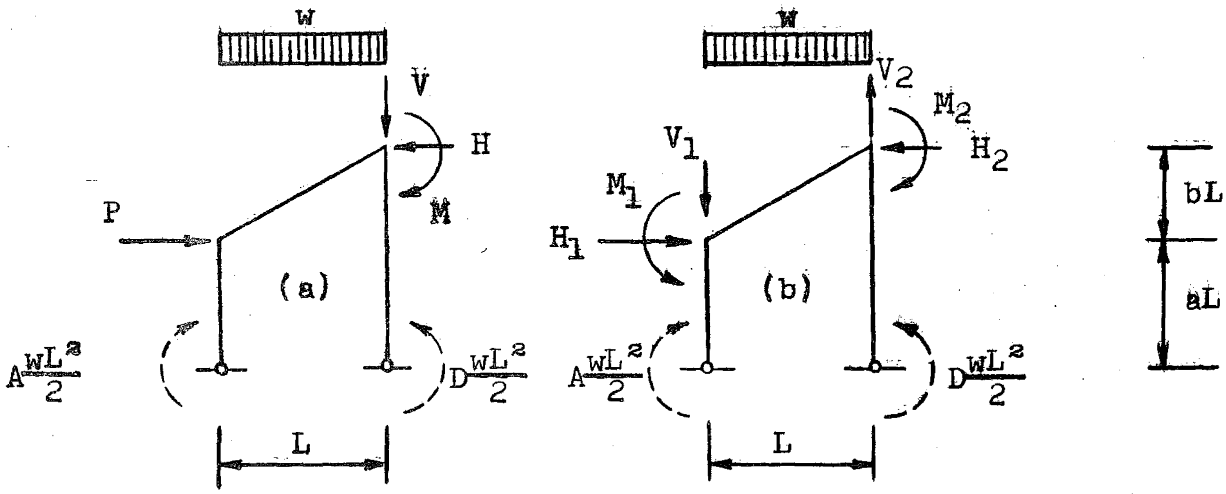

FIG. 2

The loading to which these structures may be subjected will be either of the two general cases illustrated in Fig. 2. In Fig. 2a it is assumed that an external load is applied to one side of the structure (could be either side) while the opposite side is connected to an adjacent span. Figure 2b is typical of an interior span. Since adjacent structures can transmit horizontal and vertical forces and bending moment at their points of connection, these effects must be considered when examining the behavior of any one span. However, since a mechanism method of solution will be used in solving the problem the only quantity of interest, as far as external load to the span in question is concerned, is the total work done by these forces and moments as the mechanism is formed. The total effect of these influences could therefore be replaced by hypothetical moments assumed to act about the base of the structure as is shown by the dashed moments in Fig. 2. These would be chosen such that they "apply" the same amount of external work to the span in question. This procedure is the same as that used in Ref. 1 and 2. Therein a more detailed discussion of the method is given.

For a concentrated, external, horizontal load assumed to act at the "lower eave", as shown in Fig. 2a, the corresponding hypothetical moment would be chosen equal to $\frac{1}{2}AwL^2$, where $A = (2a)(\frac{P}{wL})$. That is, the moments of the two systems about the base would be made equal,

$$P(aL) = \frac{1}{2}A(wL^2) \quad \ldots\ldots\ldots\ldots \quad (1)$$

This assumes that no hinge will occur in this column and such will always be the case for this loading.

When the structure is subjected to a uniformly distributed horizontal load of the type shown in Fig. 3 a conservative solution can be obtained by again making the two moments equal. This is proven in Ref. 1 and 2. That is,

$$A\frac{wL^2}{2} = \frac{\gamma w}{2} L^2 \ (a+b)^2$$

or

$$A = \gamma \ (a+b)^2 \quad \ldots\ldots \quad (2)$$

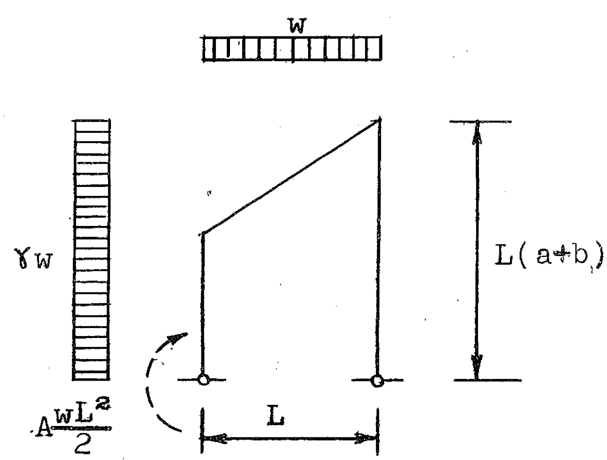

FIG. 3

Here again, it is assumed that the maximum moment in the windward column must occur at the point of junction between the column and the rafter.

As was pointed out in the preceding section, when for a given combination of applied loads the ultimate loading is reached, a sufficient number of hinges will develop to reduce the structure to a mechanism. For the structure and loading under consideration (see Fig. 2) three locations of possible plastic hinge formation exist: one at each of the junctions of the rafter and the columns, and one within the rafter. These are shown in Fig. 4. The exact location of hinge (2) in the rafter will depend on the loading and mechanism and will be chosen by minimizing such that the structure "fails" at its first opportunity.

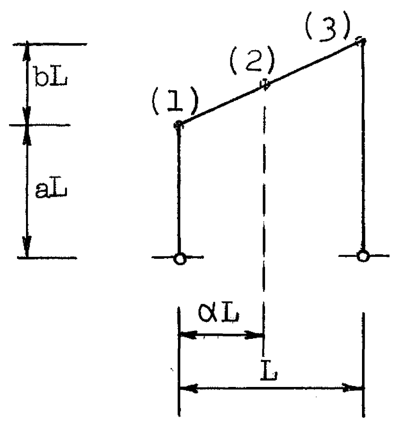

FIG. 4

For the structure in question it is in general necessary to develop two hinges to reduce the system to one of one degree of freedom. However, for certain combinations of the loading parameters A and D, it is also possible that three hinges will be developed. The hinge systems that must be examined are therefore as shown in Fig. 5. For a given loading situation the mechanism which requires the largest value of M_p will develop. This then will be the criterion of selection of the critical mechanism.

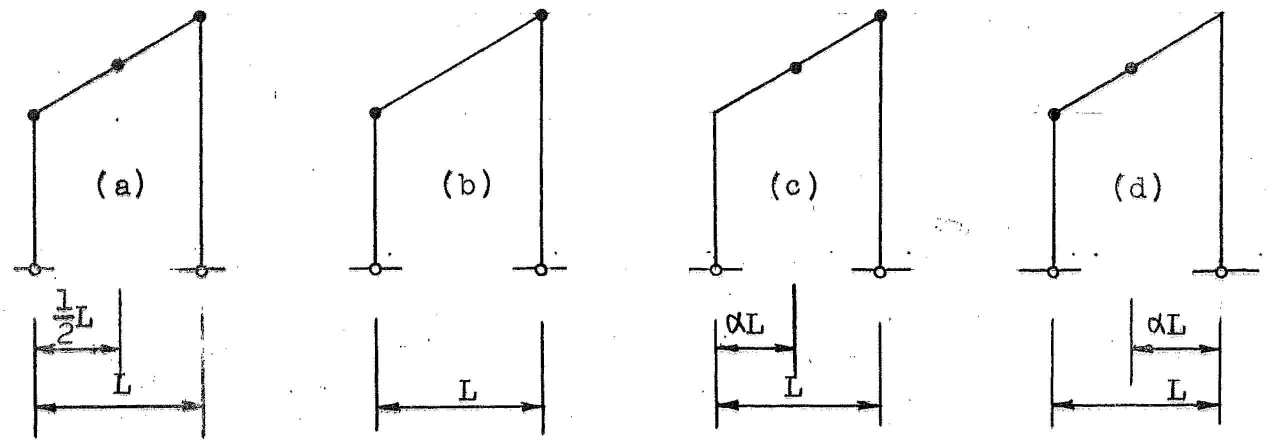

FIG. 5

2. Mechanisms for which the Structure Sways to the Higher Side

The solution corresponding to failure modes (a)* and (c) of Fig. 5 will be carried out to illustrate the mechanism method of solution.

(A) Mechanism (a)

With the three hinges forming as shown in Fig. 6, the rafter fails as if it were a beam of span length equal to L and consequently the applied hypothetical moments $A\frac{wL^2}{2}$ and $D\frac{wL^2}{2}$ do not enter into the solution. This assumes that the columns remain vertical and, as will be shown later, can occur

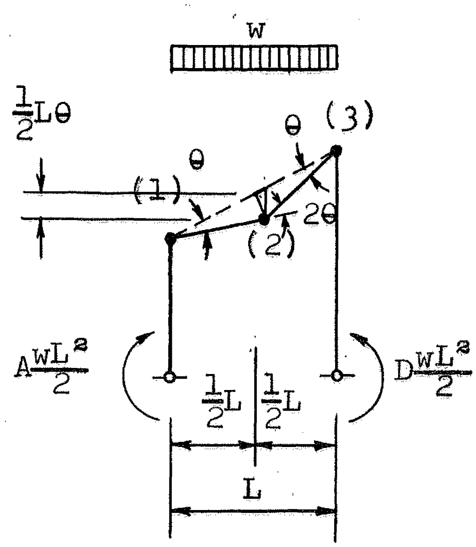

FIG. 6

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* It should be noted that for beam mechanism (a) of Fig. 5, the structure actually has two degrees of freedom. For this reason, it does not necessarily cause the structure to sway either to the higher or lower side.

only for certain particular combinations of the parameters A, D and $\frac{b}{a}$.

In determining the critical load that will result in this failure configuration a virtual displacement method will be used. It is assumed that the rafter is given a virtual rotation "θ" at the plastic hinge (1). Since hinge (2) is located in the center of the span, the vertical projection of its movement during this virtual disturbance equals $\theta(\frac{L}{2})$. From geometry, hinge (3) also rotates through a virtual angle θ; whereas, hinge (2) rotates through an angle of 2θ. Equating the internal and external work associated with this virtual disturbance of the assumed mechanism

$$W_{ext} = W_{int}$$

$$\sum (\text{Force})\ (\text{Average Distance Moved}) = \sum (M_p)(\theta)$$

$$wL(\tfrac{1}{2})(\tfrac{1}{2}\ L\theta) = M_p\ (\theta + 2\theta + \theta)$$

or

$$\boxed{\frac{M_p}{wL^2} = 0.0625} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(B) Mechanism (c)

Assuming hinges at locations (2) and (3), the mechanism will be that shown in Fig. 7. Bar (0-1-2) is constrained to rotation about the base (0). Bar (3-4) will rotate about point (4), and Bar (2-3) will rotate about its instantaneous center[8], which is denoted as I.C. in the figure. This point is determined geometrically in the same manner as for a "real" linkage system.

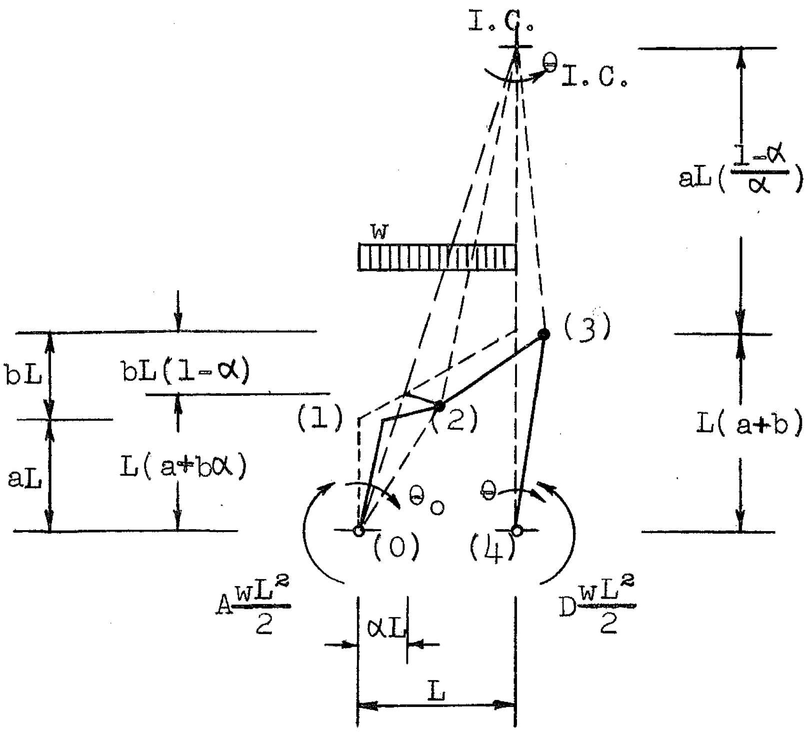


FIG. 7

Subjecting then, the right hand column to a virtual rotation equal to θ about point (4), hinge (3) will move horizontally to the right through a distance θL(a+b)*. For this movement to be possible Bar (2-3) must rotate about I.C. through an angle equal to

$$\theta_{I.C.} = \theta\left(\frac{a+b}{a}\right)\left(\frac{\alpha}{1-\alpha}\right) \quad \ldots \quad (4)$$

---

* It should be noted that in simple plastic theory only first order deformations are considered.

In like manner

$$\theta_o = \theta \left(\frac{a+b}{a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The total rotation at each of the plastic hinges is then given by the sum of the rotation at the two adjacent instantaneous centers[7]. That is,

$$\theta_{(2)} = \theta_o + \theta_{I.C.}$$

or

$$\theta_{(2)} = \theta \left(\frac{a+b}{a}\right)\left(\frac{1}{1-\alpha}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and

$$\theta_{(3)} = \theta_{I.C.} + \theta_{(4)}$$

or

$$\theta_{(3)} = \theta \left[\frac{a+\alpha b}{a(1-\alpha)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For the structure to be in equilibrium, the internal and external work associated with this virtual displacement must be equal. The applied moment $A\frac{wL^2}{2}$ (always assumed to do positive work) rotates through the angle θ_o. Similarly, $D\frac{wL^2}{2}$ (always assumed to do negative work) rotates through the angle θ. The external work is therefore

$$W_{ext} = A\frac{wL^2}{2}(\theta_o) + \frac{w(\alpha L)^2}{2}(\theta_o) + \frac{wL^2(1-\alpha)^2}{2}(\theta_{I.C.}) - D\frac{wL^2}{2}(\theta)$$

$$W_{ext} = A\frac{wL^2}{2}\,\theta\,(\frac{a+b}{a}) + \frac{w(\alpha L)^2}{2}\,\theta\,(\frac{a+b}{a})$$

$$+ \frac{wL^2(1-\alpha)^2}{2}\,\theta\,(\frac{a+b}{a})(\frac{\alpha}{1-\alpha}) - D\frac{wL^2}{2}\,\theta \quad \ldots\ldots (8)$$

Internally, work is done at each of the developed hinges, therefore,

$$W_{int} = M_p\,(\theta_2) + M_p\,(\theta_3)$$

$$W_{int} = M_p\theta\left[(\frac{a+b}{a})(\frac{1}{1-\alpha}) + (\frac{a+\alpha b}{a(1-\alpha)})\right] \quad \ldots\ldots\ldots\ldots (9)$$

Equating these values of internal and external work (Eq 8 and 9) and simplifying gives

$$\frac{M_p}{wL^2} = \frac{1}{2}\left[\frac{(1-\alpha)\left\{(1-\frac{b}{a})(A+\alpha) - D\right\}}{2+\frac{b}{a}(1+\alpha)}\right] \quad \ldots\ldots\ldots (10)$$

Since it is the value of α which results in the greatest value of M_p that is desired, the critical case will be determined from the condition

$$\frac{\partial M_p}{\partial \alpha} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Carrying out this operation, the following is obtained:

$$\alpha = \left[\frac{2+\frac{b}{a}}{\frac{b}{a}}\right]\left[\sqrt{1+\frac{b}{a}\left[\frac{\frac{b}{a}+2\left\{1+D-A(1+b/a)\right\}}{(2+\frac{b}{a})^2}\right]} - 1\right],$$

$$\text{for } \frac{b}{a} > 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and

$$\alpha = \left[\frac{1 - A + D}{2}\right] \quad , \quad \text{for } \frac{b}{a} = 0 \quad \ldots\ldots\ldots\ldots (13)$$

Of the four possible mechanisms shown in Fig. 5, mechanism (d) will result in negative external work being done as the structure moved to the higher side. The possibility of this mode of failure actually occurring is therefore comparatively remote.

The equations corresponding to each of the assumed failure mechanisms are summarized in the Appendix and the corresponding Design Curves are shown as Design Charts 1 thru 6.

3. Mechanisms that Result in a Sway to the Lower Side

The mechanisms that must be examined are those shown as (a), (b) and (d) of Fig. 5. It should be noted that Mechanisms (c) would result in negative external work for this direction of failure.

Equations corresponding to each of these assumed failure configurations are given in the Appendix. The corresponding Design Curves are shown as Design Charts 7 thru 12.

4. Transition Between Sway to the Higher Side and Sway to the Lower Side

Since the frame has a tendency to lean to the higher side even under its own weight, it is desirable to determine the relationship between A and D for which the transition between sway to the higher versus sway to the lower side is imminent.

It can be seen from the Design Curves of M_p/wL^2 (Design Charts 1 thru 6 and 7 thru 12) that the mechanisms which contain plastic hinges in the rafters and at the top of the leeward columns are the prevalent ones. Recalling that the solution which requires the greatest M_p value is the one necessary for design, it is possible to equate the M_p/wL^2 expressions for cases (c) and (d) of Fig. 5 and obtain the desired transition equations in terms of A, D and b/a. If this equation can be solved for A (or D) in the form

$$A = f(D, \frac{b}{a}) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)$$

then a set of curves can be plotted from this expression which will give the boundary between the ranges of application of the two conditions. Actually, because of the cumbersomeness of the expressions for α, these curves (given in Fig. 9) were defined by a direct consideration of the Design Curves.

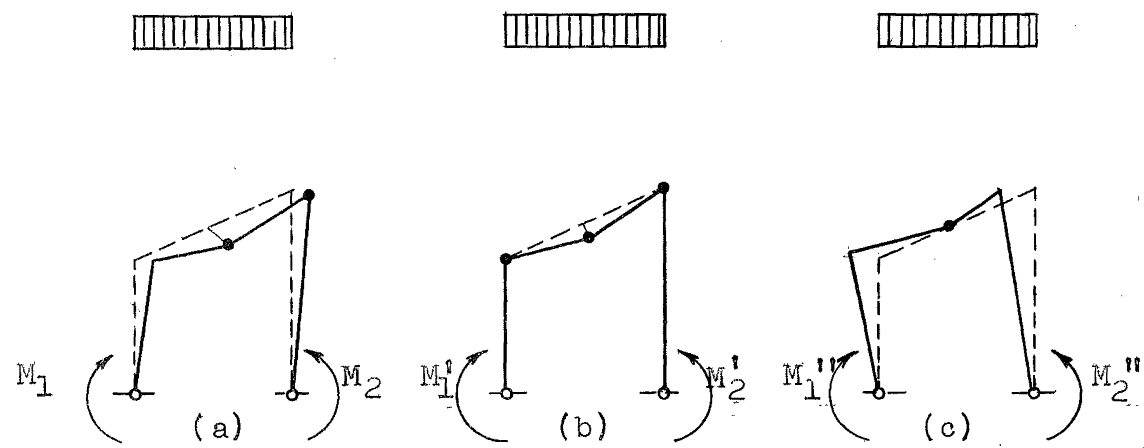


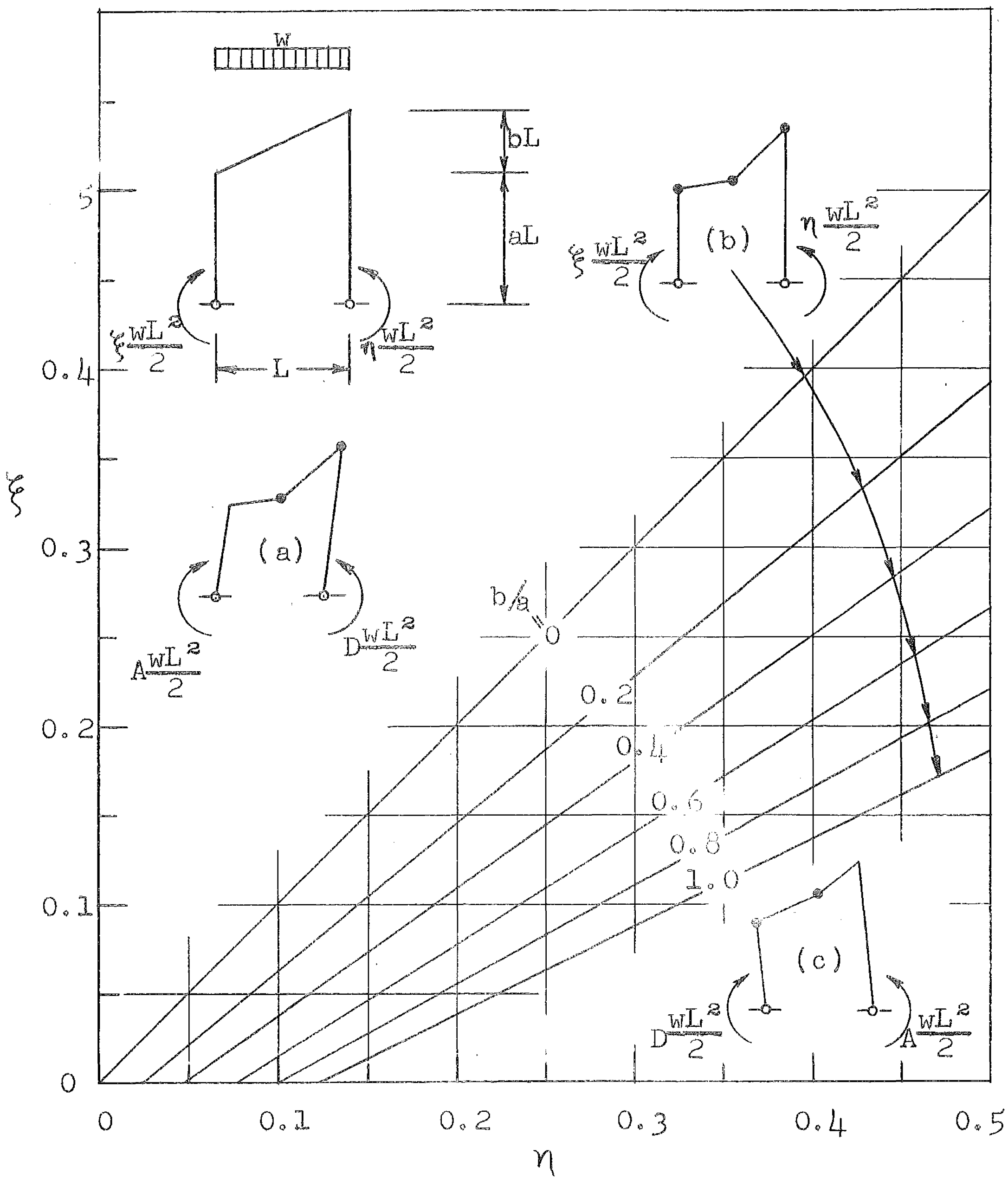
FIG. 8

Physically, the problem is the following: If moments are acting at the base of the two columns, then keeping one constant and increasing (or decreasing) the other, it should be possible to have the frame sway from one side to the opposite (Fig. 8a and 8c). There must therefore exist certain values of the moments that will result in a failure mode in which the columns remain vertical and for which hinges develop as shown in Fig. 8b. For this situation a small change in either one of the moment values (M_1' or M_2') will cause the structure to sway. In Fig. 9 are shown the curves of A versus D for which this transition occurs. Above each of the lines failure occurs by tilting to the higher side; whereas, below the lines the structure tilts to the lower side.

It should be emphasized that in deriving the equations shown in the Appendix it was assumed that the nondimensional loading (moment) parameter "A" always did positive work during the virtual displacement. That is, when the frame tilts to the higher side, "A" acts on the side of the shorter of the two columns. When the structure tilts to the lower side, "A" is assumed to act on the side of the taller of the two columns. This notation and choice of variables is consistent with that given in Ref. 1 and 2.

5. Design Curves for Single Span Structures

For a single span structure subjected to vertical load alone, failure will always occur with the structure swaying to the higher side. Furthermore, if this structure is to be subjected to vertical load plus the same wind from either side (note that D = 0), the case where the wind load is applied to the shorter side will be critical. From the previously defined Design Curves for multiple span structures, it is therefore possible to present curves for the design of single span frames. These are shown as Design Chart 13.



Note: For (a), above lines, $\xi = A$ $\eta = D$
For (c), below lines, $\eta = A$ $\xi = D$

FIG. 9

III. DESIGN EXAMPLES

Two design examples will be carried out to illustrate the procedure.

1. Mill Building

For the first example, the three span symmetrical frame loaded as shown in Fig. 10 will be considered.

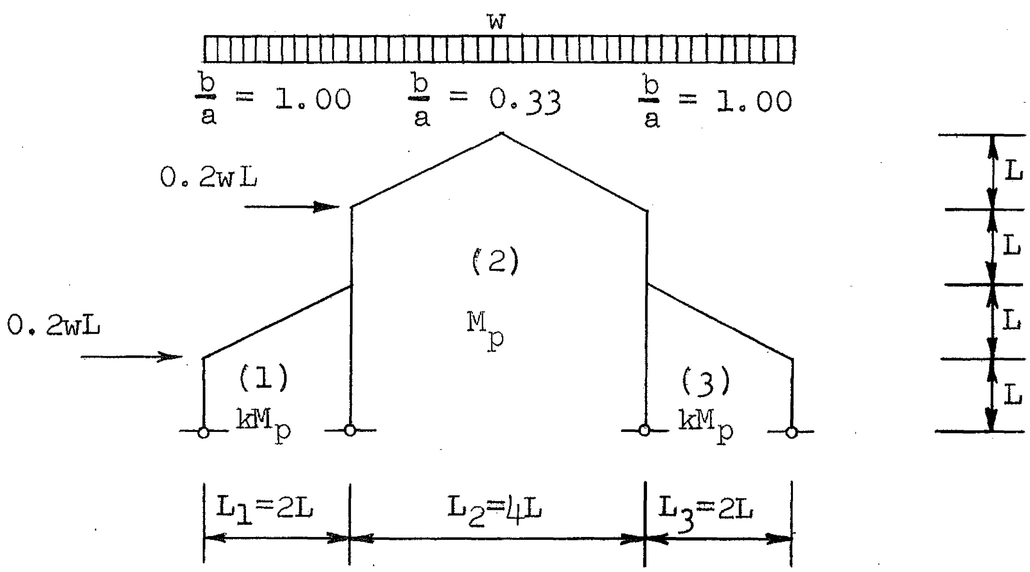


FIG. 10

$\frac{b}{a}$ Ratios:	Spans (1) and (3)	=	1.00
	Span (2)	=	0.33
Load Factors[8]:	Vertical Load only	=	1.88*
	Vertical Load plus Wind	=	1.41*

Design values for span (2); i.e., the pinned-base, gable frame, were obtained from Ref. 2.

---

* These values were chosen to be consistent with Ref. 1 and 2. Values of 1.85 and 1.40 have been suggested as more desirable[9] since they would not imply an accuracy of "three figures".

(A) With Vertical Load Only

Consider first the structure subjected to vertical load alone. The sub-structures that must be examined are those given in Fig. 11.

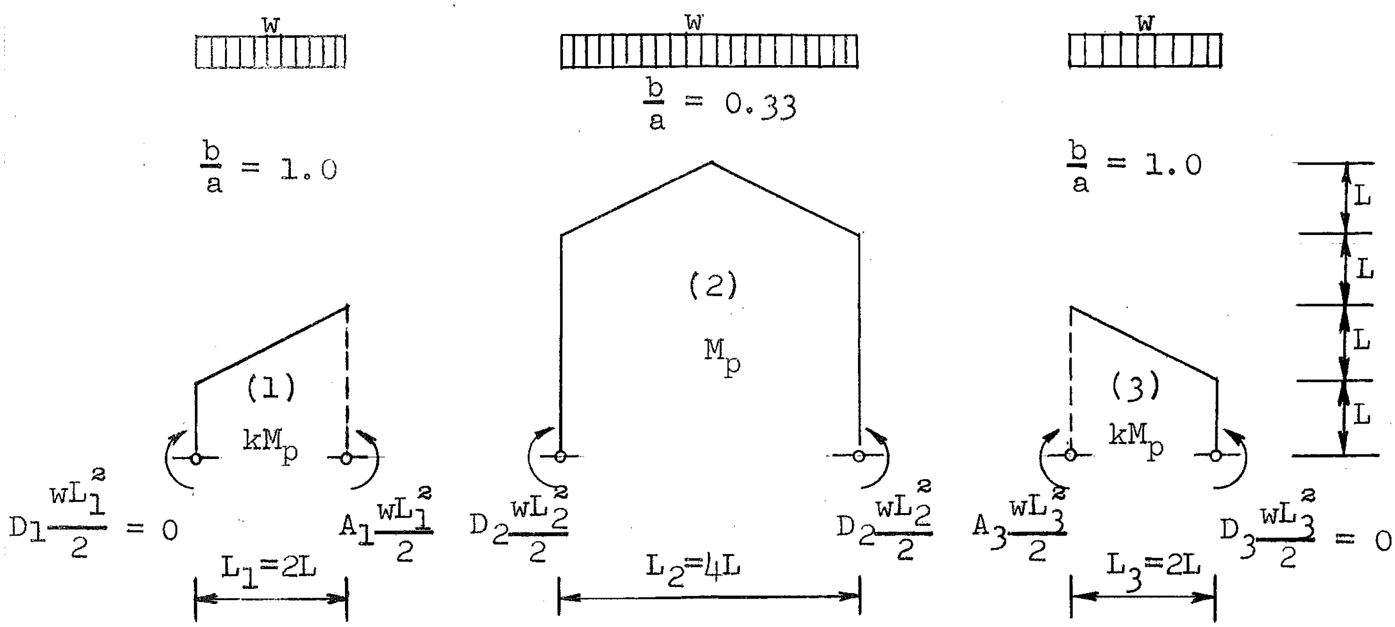


FIG. 11

The two outside columns and the rafters in the outside spans are assumed to have fully plastic values equal to kM_p while the rafters in the center span have plastic values equal to M_p. As to the size of the interior columns, these are presupposed to be sufficiently large that hinges are forced to form in the rafters; that is, the size of these columns will be determined from a moment diagram after the failure configuration and corresponding load have been ascertained.

Since the structure and loading are symmetrical about the center line, the two interior columns will tend to spread equal amounts and thus push the two side spans equally outward. For the various sub-structures, the following are then known (due to symmetry, only half of the structure need be considered):

For span (1); $\boxed{D_1 = 0}$ . . . .(15)

For span (2), the two base moments must be equal;

Between spans (1) and (2)

$$A_1 \frac{wL_1{}^2}{2} = D_2 \frac{wL_2{}^2}{2}$$

or

$$A_1 = 4D_2$$

Therefore, $\boxed{D_2 = \frac{1}{4}A_1}$ . . . .(16)

If A_1 is known, then corresponding values of M_p and kM_p can be determined from the design charts. From Fig. 9 with $D_1 = 0$, A_1 must be greater than 0.124 for the outside spans to tilt to the lower sides. That is,

$$A_1 \nless 0.124 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Assuming values of A_1 equal to or greater than 0.124 will result in possible designs for this given loading situation. Several of these are listed in Table I and each results in a straight forward determination of $\frac{M_p}{wL^2}$ and $\frac{kM_p}{wL^2}$ values. It should be noted that in Table I there has also been listed values of $\frac{M_p}{w_wL^2}$ and $\frac{kM_p}{w_wL^2}$ which include the load factor (1.88). For example, for the first solution tabulated

$$kM_p = 0.250(1.88w_w)L^2 = 0.470w_wL^2$$

or

$$\frac{kM_p}{w_w L^2} = 0.470 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

TABLE I

D_1	$\frac{kM_p}{wL_1^2}$	A_1	D_2	$\frac{M_p}{wL_2^2}$ +	$\frac{kM_p}{wL^2}$	$\frac{M_p}{wL^2}$	$\frac{kM_p}{w_w L^2}$	$\frac{M_p}{w_w L^2}$
0	0.0625	0.124	0.031	0.0517	0.250	0.8272	0.470	1.555
0	0.0702	0.20	0.05	0.0506	0.2808	0.8092	0.528	1.520
0	0.0922	0.40	0.10	0.0472	0.3688	0.7568	0.694	1.423
0	0.1167	0.60	0.15	0.0442	0.4668	0.7074	0.879	1.330
0	0.1432	0.80	0.20	0.0412	0.5728	0.6595	1.077	1.240

+ These values determined from Ref. 2

(B) With Both Wind Load and Vertical Load Acting

For the case where both vertical load and wind are assumed acting, the sub-structures and loading will be as shown in Fig. 12. Since the structure is symmetrical, only this one situation of wind loading need be considered.

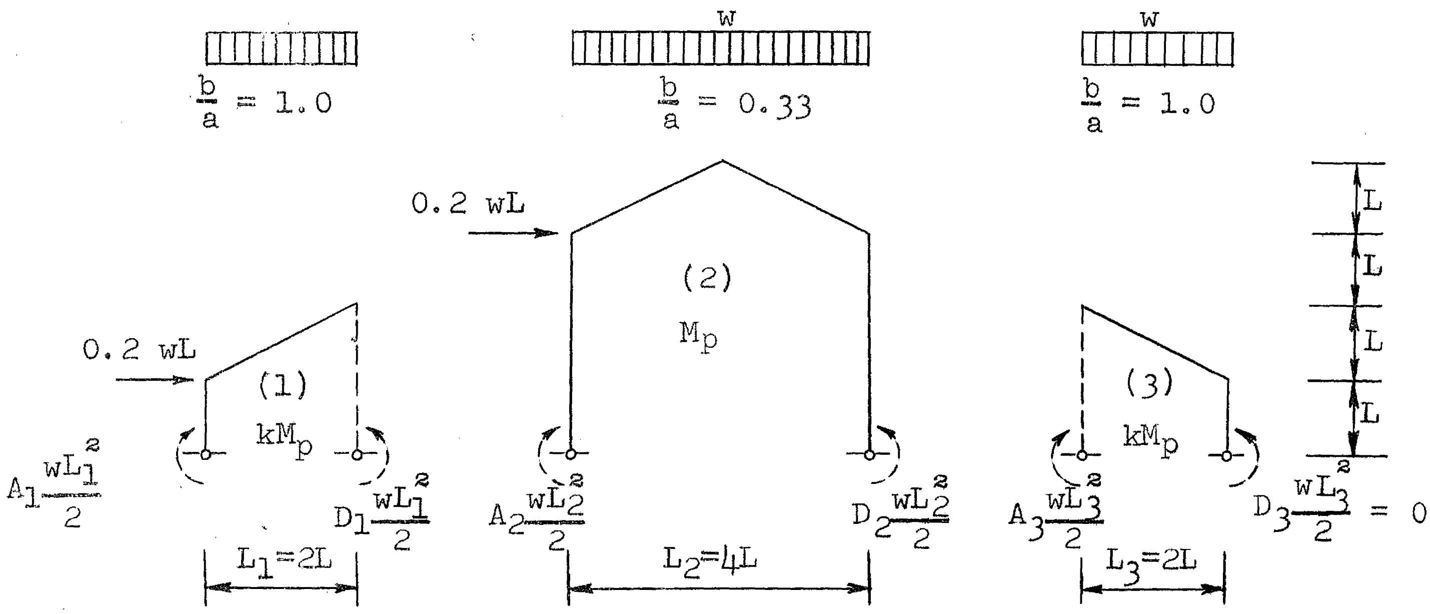


FIG. 12

For span (1)

$$A_1 \frac{wL_1^2}{2} = (0.2wL)(L)$$

or $\boxed{A_1 = 0.10}$ . . . .(19)

Between spans (1) and (2)

$$A_2 \frac{wL_2^2}{2} = D_1 \frac{wL_1^2}{2} + (0.2wL)(3L)$$

or $\boxed{A_2 = 0.25D_1 + 0.075}$ . . . .(20)

Between spans (2) and (3)

$$D_2\frac{wL_2^2}{2} = A_3\frac{wL_3^2}{2}$$

or $$\boxed{D_2 = 0.25A_3} \quad \text{. . . . .(21)}$$

For span (3)

$$\boxed{D_3 = 0} \quad \text{. . . . .(22)}$$

From the Design Charts for the "lean-to" structures (Design Charts 6a and 12a), it is noted that the smallest kM_p value that is possible occurs for a beam type failure. Furthermore, for a constant "A" (or "D") value and varying the other moment parameter, the change in required fully plastic moment value is a continuous function. This situation is also true for the pinned-base gable frame (where A = D gives the smallest value of M_p)[(2)]. For this "mill building" example, it is therefore possible to define two limiting cases; that is, a) where kM_p has its smallest value, and b) where M_p is as small as possible. These two situations confine the range of possible solution.

a) <u>Side spans smallest</u> (kM_p smallest)

From the governing design charts (Design Charts 6a and 12a),

span (1) $A_1 = 0.10$, $D_1 = 0.326$

$$\boxed{\frac{kM_p}{wL_1^2} = 0.0625} \quad \text{. . . . .(23)}$$

span (3) $D_3 = 0$, $A_3 = 0.124$

$$\boxed{\frac{kM_p}{wL_3^2} = 0.0625} \quad \text{. . . . .(24)}$$

span (2)

$$\boxed{\begin{aligned} A_2 &= 0.25D_1 + 0.075 = 0.1565 \\ D_2 &= 0.25A_3 \qquad\qquad = 0.031 \end{aligned}}$$

which gives (from the design charts of Ref. 2)

$$\boxed{\frac{M_p}{wL_2^2} = 0.0763} \quad \ldots\ldots\ldots\ldots (25)$$

Writing each of these in terms of the length parameter, L, and introducing the load factor for combined wind and vertical loading (load factor = 1.41)

$$\left.\begin{aligned} \frac{kM_p}{w_W L^2} &= 0.353 \\ \frac{M_p}{w_W L^2} &= 1.721 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (26)$$

b) <u>Center span smallest</u> (M_p smallest)

As was shown in Ref. 2, this requires that

$$\boxed{A_2 = D_2} \quad \ldots\ldots\ldots\ldots (27)$$

However,

$$0.25D_1 + 0.075 = 0.25A_3$$

Hence

$$\boxed{D_1 + 0.30 = A_3} \quad \ldots\ldots\ldots\ldots (28)$$

Furthermore, the two outside spans must give equal values of kM_p.

For the condition $A_1 = 0.10$ a curve of $\frac{kM_p}{wL_1^2}$ versus D_1 can be drawn as is shown by the solid curve of Fig. 13.

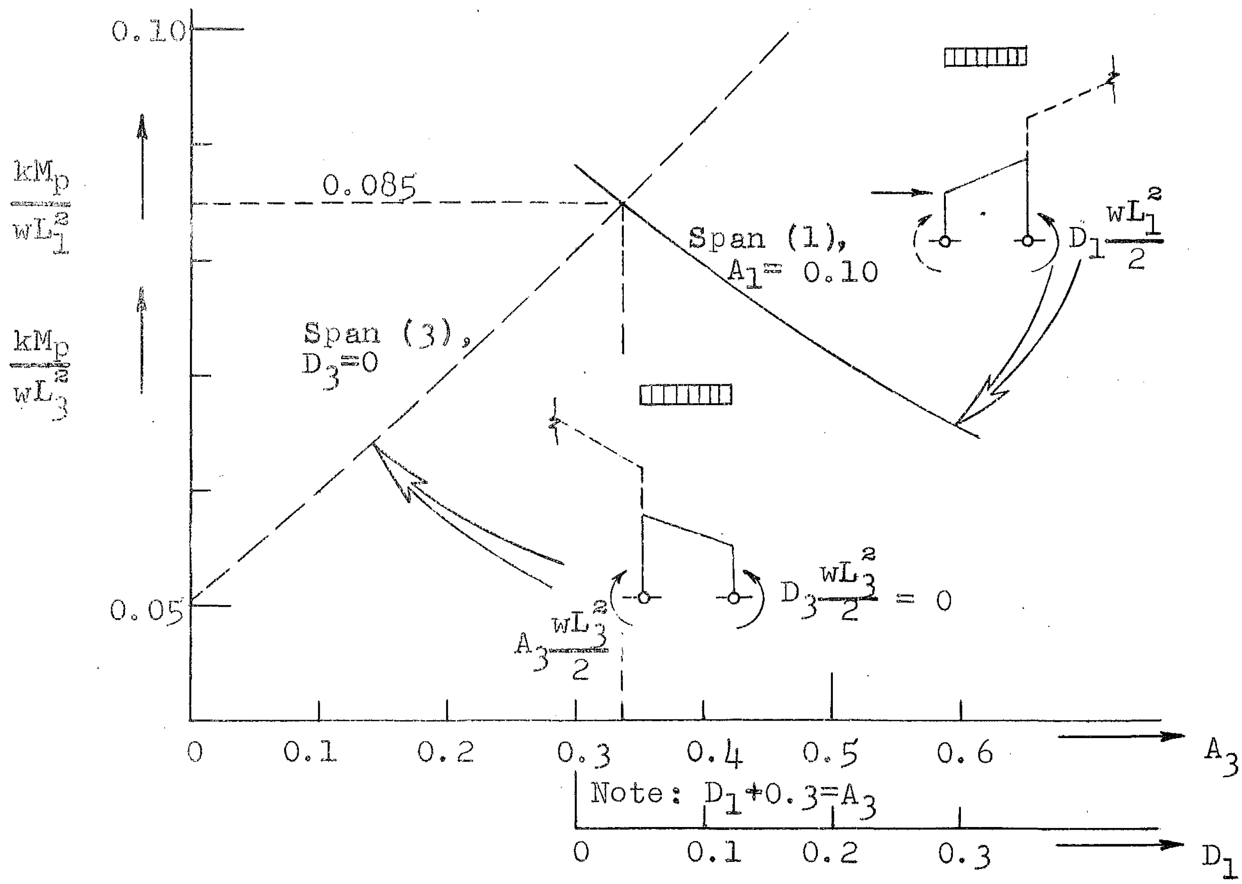


FIG. 13

Noting from Eq 28 that $D_1 + 0.3 = A_3$, it is also possible to superimpose on this diagram by displacing the coordinate system as shown in Fig. 13 the solution for the third span. This relationship has been shown by the dashed line.

The solution to this problem (determined by the intersection of these two curves) is

$$\left.\begin{aligned} D_1 &= 0.037 \\ A_3 &= 0.337 \\ \frac{kM_p}{wL_1^2} &= 0.085 \quad \text{or} \quad \frac{kM_p}{wL^2} = 0.340 \end{aligned}\right\} \qquad \ldots\ldots\ldots (29)$$

For these values of A and D for the outside spans,

$$\left.\begin{aligned} A_2 &= 0.25D_1 + 0.075 = 0.0843 \\ \text{and} \quad D_2 &= 0.25A_3 \qquad\qquad = 0.0843 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (30)$$

From Ref. 2 the corresponding size of the center span rafters is

$$\frac{M_p}{wL_2^2} = 0.0483 \quad \text{or} \quad \frac{M_p}{wL^2} = 0.773 \quad \ldots\ldots\ldots\ldots (31)$$

Including the load factor (1.41) in the determination of the sizes of the various members which make up this structure, the design values would be

$$\left.\begin{aligned} \frac{kM_p}{w_wL^2} &= (0.34)(1.41) = 0.479 \\ \frac{M_p}{w_wL^2} &= (0.773)(1.41) = 1.090 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (32)$$

Between the two extreme conditions that were considered: side spans as small as possible, and center span as small as possible, a continuous function of kM_p/wL^2 versus M_p/wL^2 can be defined. It could be defined by varying any one of the moment parameters; for example, D_1.

(C) Governing Case

In parts (A) and (B) of this first design example, two sets of "required" plastic moments were determined. It is now necessary to compare them and see which is actually the critical case.

Figure 14 shows the M_p versus kM_p curves for case A), vertical load only, and case B), vertical load plus wind.

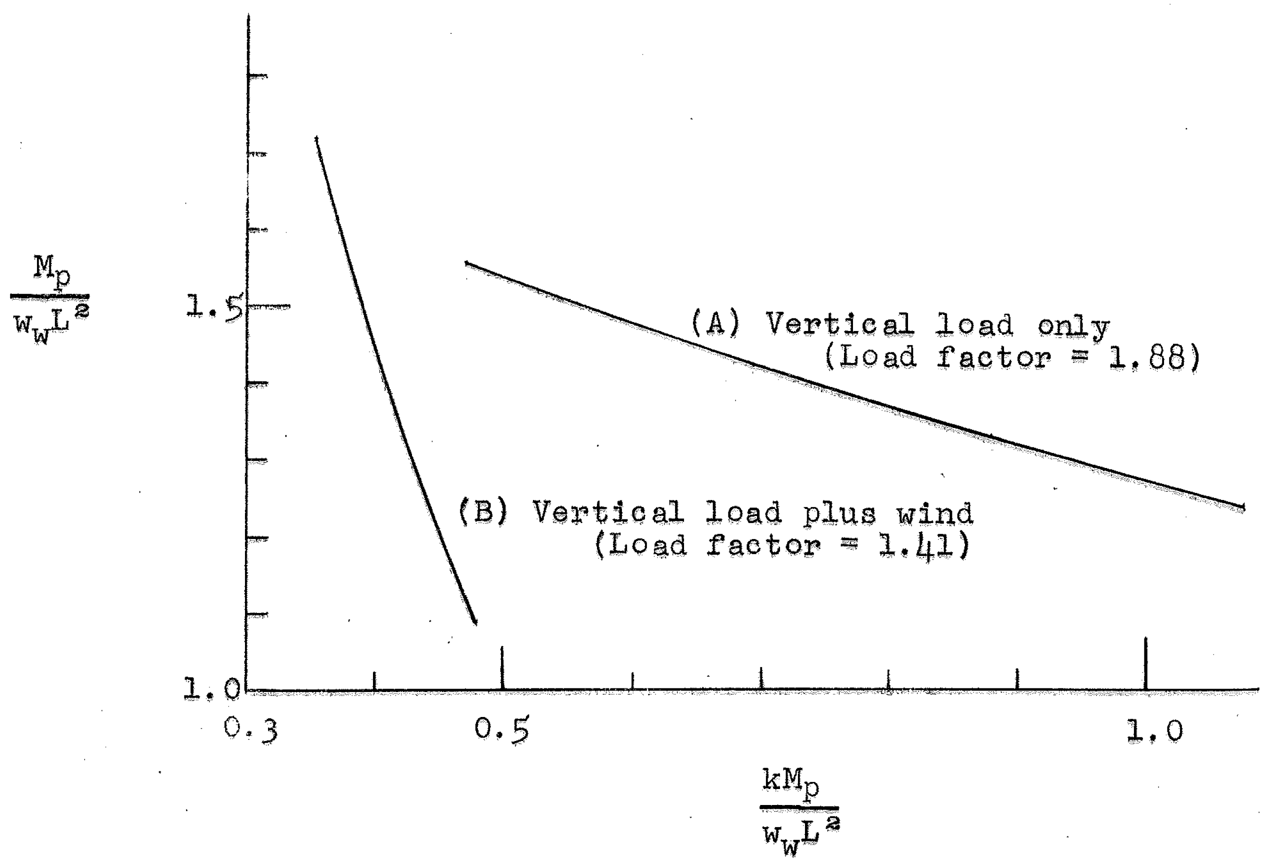


FIG. 14

The structure in question must provide the greatest M_p and kM_p value. Therefore the case where vertical load alone is acting will govern the design.

(D) Least Weight Design

As is shown in Table I and Figure 14, there are many possible design solutions for a given vertical loading. Each one will result in a different choice of relative member sizes.

Since economy of main member is usually desired, that particular combination of M_p and kM_p which results in the least total weight of material will probably be the more desirable solution.

Assuming that a linear relationship exists between the fully plastic moment value, M_p, and the unit weight, W; that is,

$$W = CM_p \quad \ldots\ldots(33)$$

the total weight of the frame can be determined from the relationship

$$\text{TOTAL WEIGHT} = \sum_{i=1}^{n} (WL_i) = C\sum_{i=1}^{n} (M_{pi}L_i) \quad \ldots\ldots(34)$$

Since it is only the relative total weight that is important in determining the combination of member sizes that result in a least weight solution, a weight function in terms of only M_{pi} and L_i can be used.

$$\text{WEIGHT FUNCTION} = \rho = \frac{\sum_{i=1}^{n} (WL_i)}{C} = \sum_{i=1}^{n} (M_{pi}L_i) \quad \ldots(35)$$

Assuming that the interior columns will have the same fully plastic values, M_p, as the rafters in the interior span, the following will be the weight function for this example:

$$\rho = (6.472L)(kM_p) + (10.472L)(M_p) \quad \ldots\ldots(36)$$

In the nondimensional form, this would be

$$\frac{\rho}{wL^3} = 6.472 \left(\frac{kM_p}{wL^2}\right) + 10.472 \left(\frac{M_p}{wL^2}\right) \quad \ldots\ldots(37)$$

The calculated results which correspond to the solutions tabulated in Table I are listed in Table II.

TABLE II

$\frac{kM_p}{wL^2}$	$\frac{M_p}{wL^2}$	$6.472\frac{kM_p}{wL^2}$	$10.472\frac{M_p}{wL^2}$	$\frac{\rho}{wL^3}$
0.25	0.8272	1.618	8.662	10.280
0.2808	0.8092	1.893	8.474	10.367
0.3688	0.7568	2.486	7.925	10.411
0.4668	0.7074	3.147	7.408	10.555
0.5728	0.6595	3.862	6.906	10.768

The least weight solution is the one which results in the smallest value of $\rho/_{wL^3}$ and is therefore the first one listed in the table. The design values for M_p and kM_p which include the load factor of 1.88 are therefore

$$\frac{M_p}{w_w L^2} = 1.555 \quad \text{and} \quad \frac{kM_p}{w_w L^2} = 0.470 \qquad (38)$$

(E) Plasticity Check

As was stated in the introduction, for a "simple plastic theory" solution to exist, it is necessary (and sufficient) that the following three conditions be met:

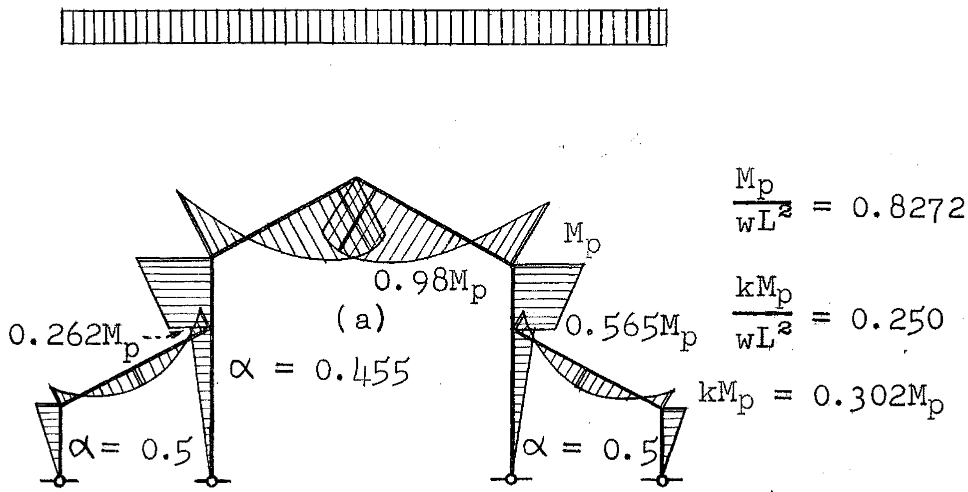
a) The structure must be in equilibrium,

b) $|M| \leq M_p$, and

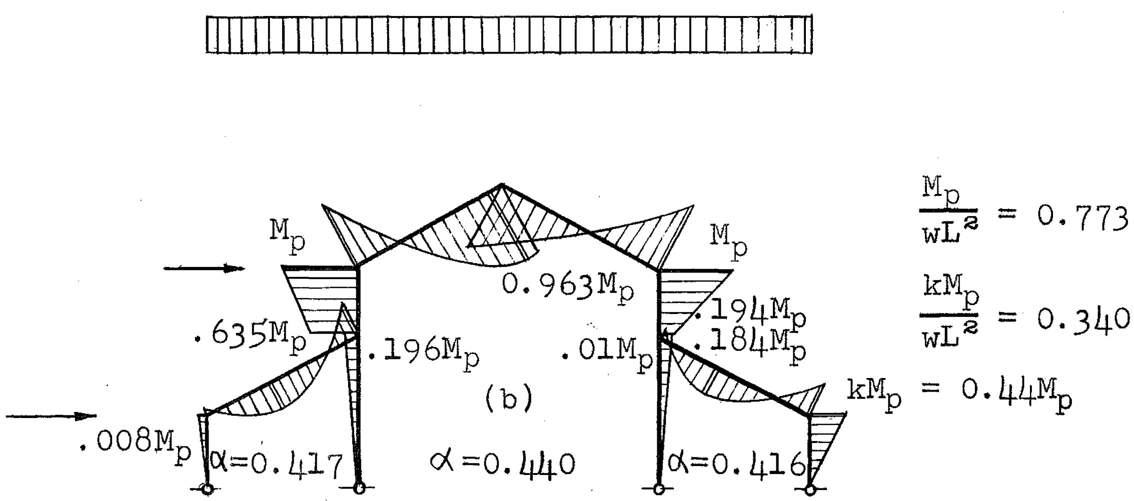
c) A mechanism must be formed.

In solving the preceding problem, mechanisms were assumed from the out-set; thus condition three was fulfilled. A virtual displacement procedure was used to determine the critical loads corresponding to each of the assumed mechanisms. The first requirement is therefore also satisfied. As to the second necessary condition, all of the mechanisms that were considered possible were examined and maximum required values of M_p were determined. Therefore, this condition is also fulfilled providing that all possible modes were considered. To be absolutely sure that this was the case a moment diagram should be drawn. If it nowhere exceeds the fully plastic moment of the section, then the solution is the correct one. (This moment diagram would also be useful in checking the lateral bracing of the main frame, a condition that is not considered in this report.)

Since the structure is statically determinate at failure, the moment diagram can be readily determined. The moment diagram corresponding to the least weight design is shown as Fig. 15a. Figure 15b is the moment diagram for one of the designs where the structure was subject to vertical load plus wind (that case corresponding to the smallest possible center span).



Vertical Load Only
Least Weight Solution



Vertical Load Plus Wind Load
Center Span Smallest

FIG. 15

2. "Saw-Tooth" Building

As the second design example consider the three span structure shown in Fig. 16a. For this problem, three loading conditions must be examined; A) Vertical load alone (Fig. 16b), B) Vertical load plus wind from the left (Fig. 16c) and C) Vertical load plus wind from the right (Fig. 16d).

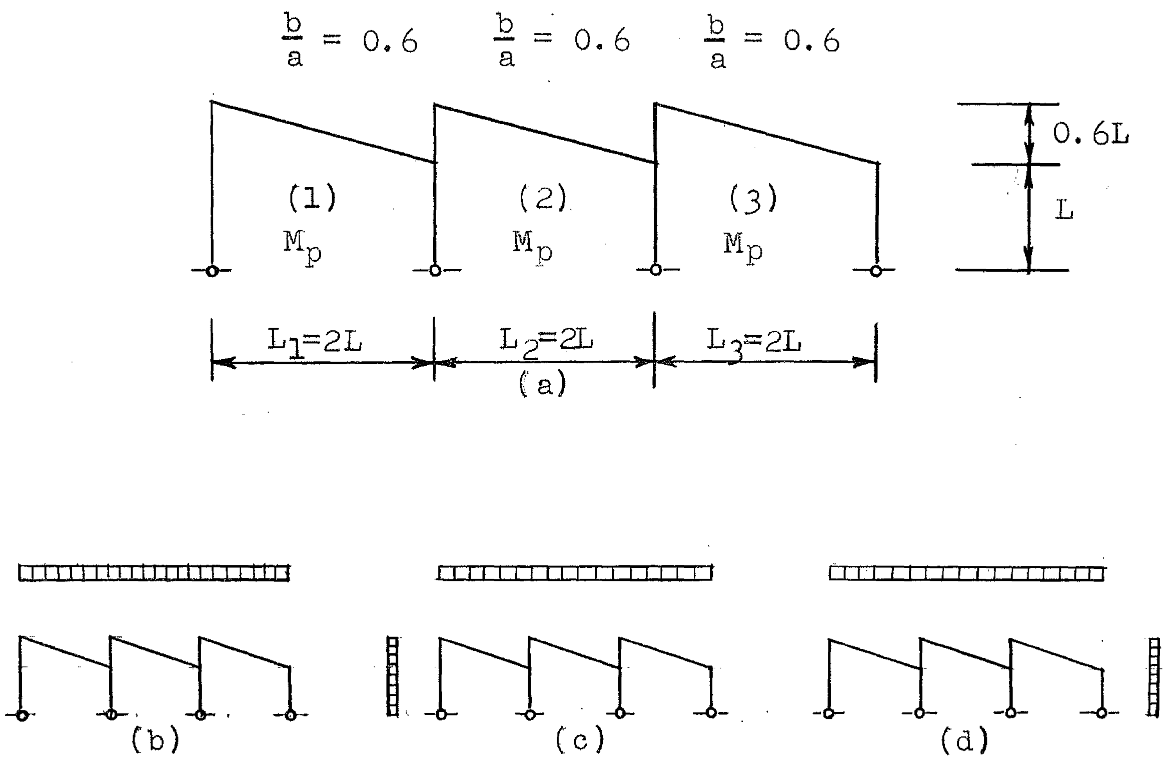


FIG. 16

A) Vertical Load Alone Acting

For the case of just vertical load acting, the sub-structures and loadings that must be considered are those shown in Fig. 17.

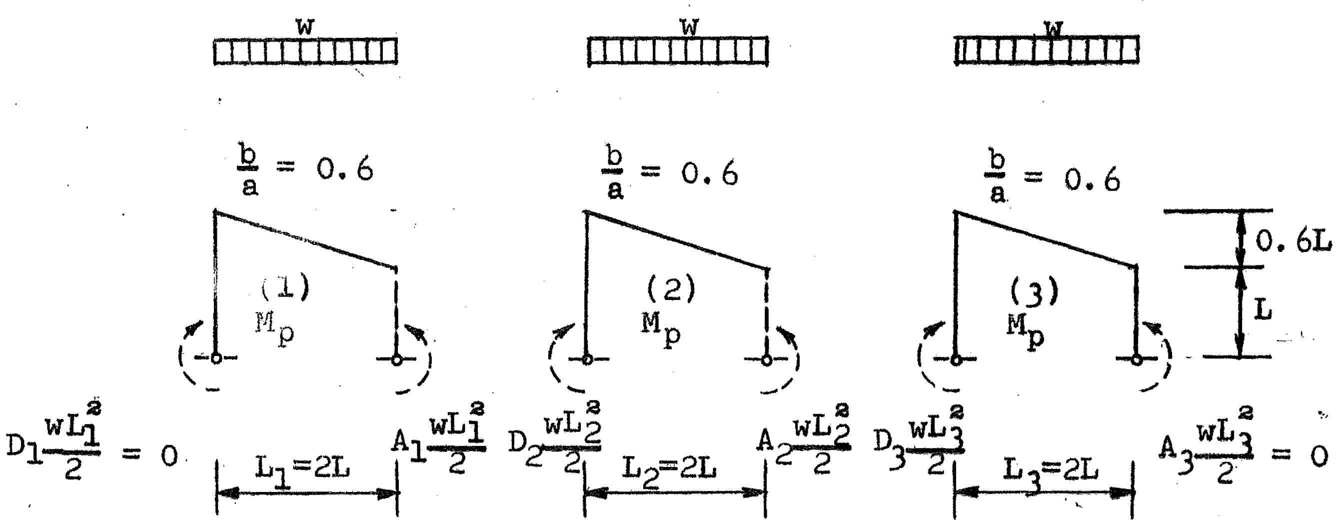


FIG. 17

Since each of the spans individually tends to sway to the higher side due to the vertical loading, all $(A_i \frac{wL_i^2}{2})$ moments occur on the "shorter column sides" of the structure. The $(D_i \frac{wL_i^2}{2})$ moments are then on the "longer column sides".

The known "outside" conditions are

$$\left.\begin{array}{l} \boxed{D_1 = 0} \\ \boxed{A_3 = 0} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Since the span lengths are equal, at the interior columns

$$\left.\begin{array}{l} \boxed{A_1 = D_2} \\ \boxed{A_2 = D_3} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

From the Design Curve for $\frac{b}{a} = 0.6$ (Design Chart 4a), it is noted that for spans (1) and (3) to have equal M_p values it is required that (see Fig. 18)

$$\left.\begin{array}{l} \boxed{A_1 = 0} \\ \boxed{D_3 = 0} \end{array}\right\} \qquad (41)$$

Thus,

$$\boxed{A_2 = D_2 = 0} \qquad (42)$$

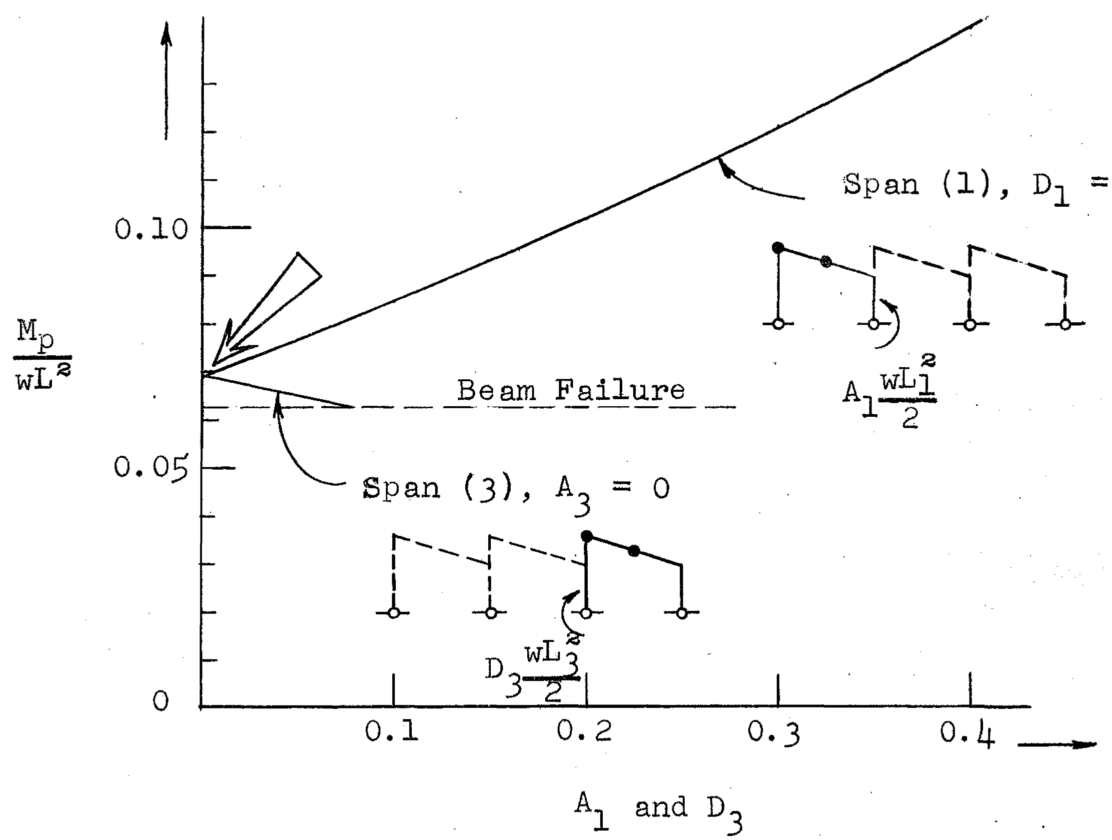


FIG. 18

The resulting solution is then

$$\frac{M_p}{wL_1^2} = 0.0692,$$

or in terms of the length parameter, L,

$$\frac{M_p}{wL^2} = 0.2768 \quad \ldots\ldots\ldots\ldots (43)$$

B) Vertical Load Plus Wind from the Left

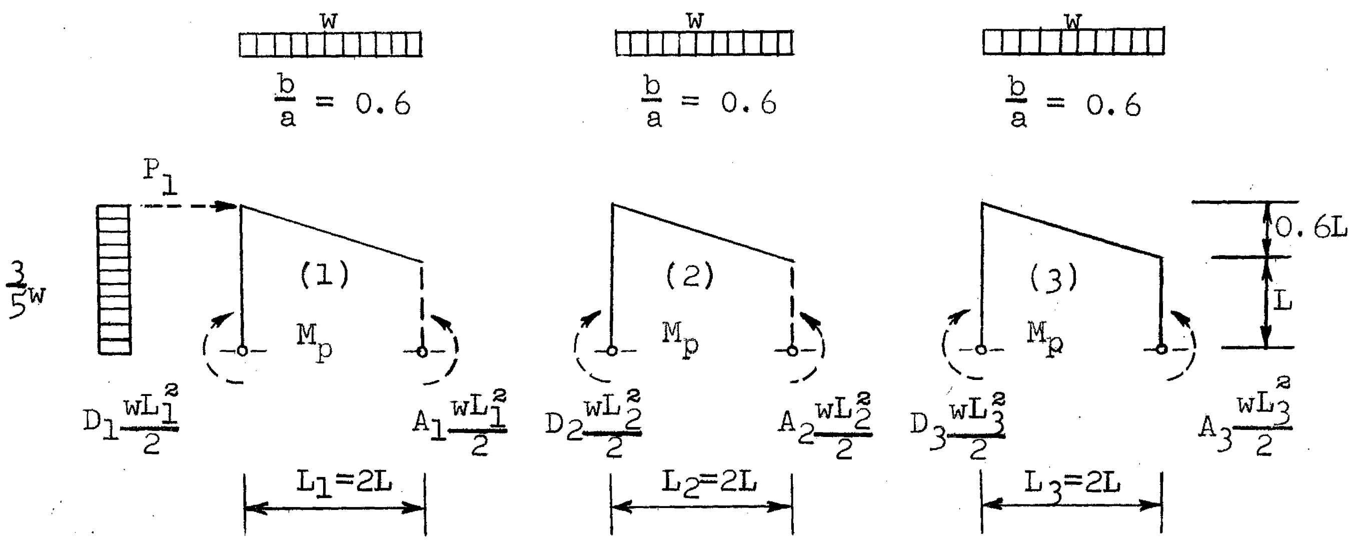


FIG. 19

Replacing the uniformly distributed horizontal load acting on the left hand structure by its equivalent* concentrated load, P_1, and in turn replacing this by its moment $D_1\frac{wL_1^2}{2}$, the loading

* Equivalent meaning—having the same overturning moment about the base of the structure.

parameter, D_1, becomes

$$P_1(1.6L) = D_1 \frac{wL_1^2}{2} = \frac{1}{2}\left(\frac{3}{5}w\right)(1.6L)^2$$

$$\text{or} \quad \boxed{D_1 = 0.384} \qquad (44)$$

Other known conditions are

$$\left.\begin{array}{l} \boxed{A_1 = D_2} \\ \boxed{A_2 = D_3} \\ \text{and} \quad \boxed{A_3 = 0} \end{array}\right\} \qquad (45)$$

As to the determination of to which side of the structure sway will occur and thereby the designation of the "A" and "D" moment terms, from Fig. 9 it is noted that if spans (1) and (3) sway to the left, then

$$A_1 > 0.193 \text{ and } D_3 < 0.076$$

This would require that for the center span

$$D_2 > 0.193 \text{ and } A_2 < 0.076 \qquad (46)$$

However for

$$D_2 = 0.193 \text{ and } A_2 = 0.076$$

span (2) also sways to the left. Hence the total structure must sway to the left and the choice of the A and D variables will be as shown in Fig. 19. The Design Curves shown as Design Chart 4a will therefore be used for this design.

Assuming various values of D_3, the following table of possible solutions can be defined.

TABLE III

(1) A_3	(2) $D_3 = A_2$	(3) $\frac{M_p}{wL^2}$	(4) $D_2(\overset{?}{=}A_1)$	(5) D_1	(6) A_1
0	0.076	0.0625	0.196	0.384	0.193
0	0.075	0.0626	0.194	0.384	0.194
0	0.050	0.0648	0.126	0.384	0.208
0	0	0.0692	0	0.384	0.240

Values of A_1 as shown in column (6), are determined from a knowledge of M_p/wL^2 (column 3) and D_1 (column 5). Comparing columns (4) and (6), it is noted that $A_1 = D_2$ for

$$\frac{M_p}{wL_1^2} = 0.0626$$

or in terms of the length parameter L,

$$\frac{M_p}{wL^2} = 0.2504 \quad \ldots\ldots\ldots\ldots (47)$$

This then is the solution for this loading condition. Including the load factor of 1.41

$$\frac{M_p}{w_w L^2} = 0.3531 \quad \ldots\ldots\ldots\ldots (48)$$

C) Vertical Load Plus Wind from the Right

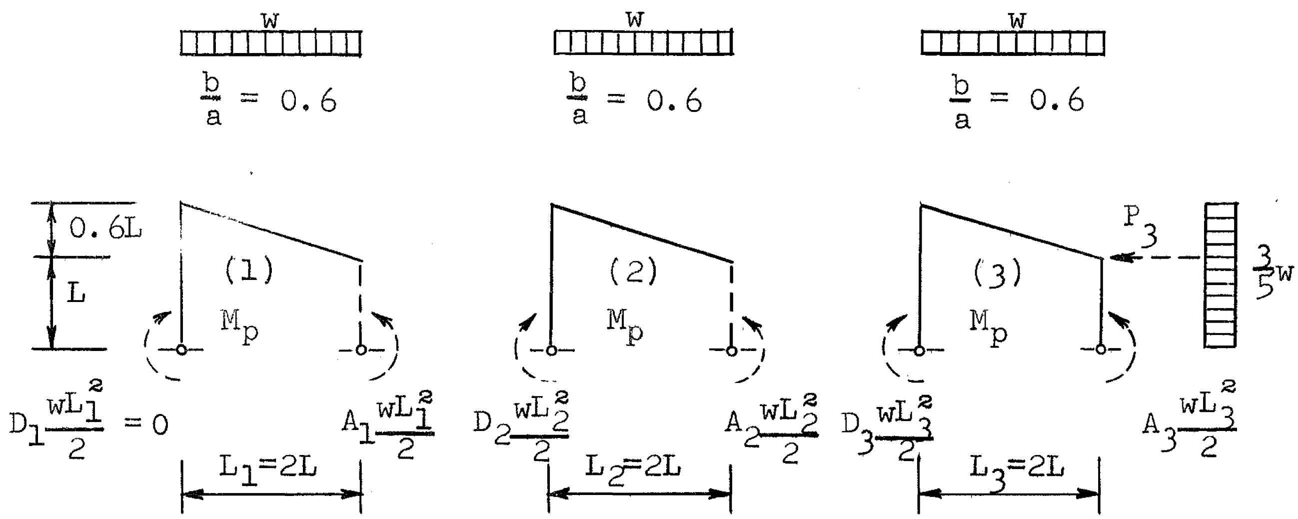


FIG. 20 Note: $L_1=L_2=L_3$

The known conditions are

$$P_3(L) = A_3\frac{wL_1^2}{2} = \frac{1}{2}(\frac{3}{5}w)(1.6L)^2$$

or $\boxed{A_3 = 0.384}$

Also, $\boxed{D_3 = A_2}$

$\boxed{D_2 = A_1}$

$\boxed{D_1 = 0}$ . . . . (49)

For this problem there is no question but that the frame will tilt to the left. A table similar to that given as Table III can therefore be worked out for this loading condition. It is given as Table IV.

TABLE IV

(1)	(2)	(3)	(4)	(5)	(6)
A_3	$D_3 = A_2$	$\frac{M_p}{wL_1^2}$	$D_2 (\overset{?}{=} A_1)$	A_1	D_1
0.384	0.20	0.1132	(—)	0.260	0
0.384	0.30	0.1008	0.173	0.195	0
0.384	0.309	0.1000	0.190	0.190	0
0.384	0.40	0.0902	0.425	0.136	0

From this tabulation it is noted that $A_1 = D_2 = 0.190$ for $\frac{M_p}{wL_1^2} = 0.1000$. Therefore, the solution to the problem is

$$\frac{M_p}{wL_1^2} = 0.1000$$

or

$$\frac{M_p}{wL^2} = 0.4000 \qquad \ldots\ldots (50)$$

Including the load factor of 1.41

$$\boxed{\frac{M_p}{w_wL^2} = 0.5640} \qquad \ldots\ldots (51)$$

D) Comparison of Solutions Including Load Factors

In summary, the solutions to the design of this structure for each of the conditions of loading are as follows:

$$\left.\begin{array}{ll} \text{Case A)} \quad \frac{M_p}{w_wL^2} = 0.520 & \text{(Load Factor = 1.88)} \\ \text{Case B)} \quad \frac{M_p}{w_wL^2} = 0.353 & \text{(Load Factor = 1.41)} \\ \text{Case C)} \quad \boxed{\frac{M_p}{w_wL^2} = 0.564} & \text{(Load Factor = 1.41)} \end{array}\right\} \qquad \ldots\ldots (52)$$

The critical condition of loading is the one that requires the most of the structure and is therefore case (C); vertical load plus wind from the right.

E) Plasticity Check

Assuming that the solution defined above (that is, $M_p/_{wL^2} = 0.564$) is the correct solution; the locations of the developed plastic hinges in each of the rafters can be determined from Design Chart 4b. They are found to be equal to

$$\alpha_1 = \alpha_2 = \alpha_3 = 0.367 \quad \ldots\ldots\ldots\ldots\ldots (53)$$

The corresponding moment diagram is given in Fig. 21. Thus the correctness of the solution is verified.

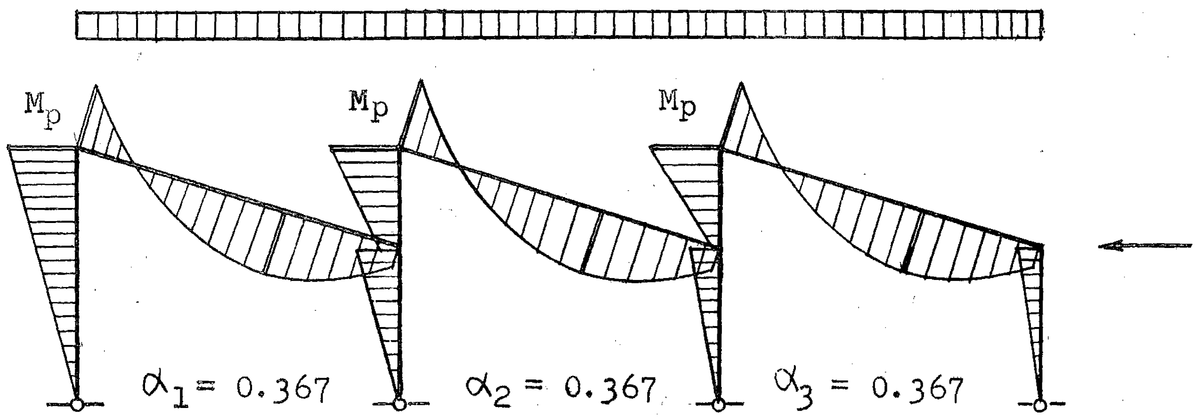


FIG. 21

Moment Diagram for Critical Loading
Design Example # 2

IV. SUMMARY

In this paper a method has been presented for the plastic design of "lean-to" type rigid frames. Both single and multiple span structures were considered and design charts were developed to facilitate solution.

When designing such a structure, it is first necessary to determine the direction in which the structure will sway at failure. Figure 9 was presented to aid in this selection. Having determined this, it is then possible to proceed with the design in the same manner as in Ref. 2.

For each of the design examples, a mill building and a "saw-tooth" type multiple span frame, the full range of possible design situations (i.e., relative member sizes) were considered. That particular solution which corresponds to the least total weight of structure was then determined.

V. ACKNOWLEDGEMENTS

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VI. NOMENCLATURE

a	non-dimensional parameter, relating the height of the shorter column to the span length
b	non-dimensional parameter, relating the rise of rafter to the span length
f	function value
k	non-dimensional parameter, relating the fully plastic moment values of two spans.
i, n	numbers denoting members
w	distributed vertical load per unit length
w_w	anticipated distributed vertical load per unit length
$A(A_1,A_2)$	non-dimensional parameter, relating the horizontal force acting on a structure (or the hypothetical "overturning" moment of one part of a structure on the adjacent part) to the vertical loads. It is assumed that "A" results in positive work being done as the structure fails.
C	constant
$D(D_1,D_2)$	non-dimensional parameter, relating the horizontal resisting force or hypothetical "over-turning" moment acting on a structure to its vertical loading. It is assumed that "D" results in negative work being done as the structure fails.
H	horizontal reaction
$L(L_1,L_2)$	span length; length measurement
M	bending moment
M_p	fully plastic moment value
P	concentrated load
V	vertical reaction
W	weight per unit length of a structural member
W_{ext}	external work associated with a virtual displacement of an assumed mechanism

VI. Nomenclature (contd)

W_{int}	internal work associated with a virtual displacement of an assumed mechanism
$\alpha\ (\alpha_1, \alpha_2)$	non-dimensional parameters, defining the distance to the plastic hinge in the rafter of a structure
γ	non-dimensional parameter, relating the distributed horizontal load per unit length to the distributed vertical load per unit length
$\theta(\theta_1, \theta_2)$	virtual rotations
ρ	weight function
ξ, η	non-dimensional parameter, relating the horizontal force or hypothetical "over-turning" moment acting on a structure to its vertical loading

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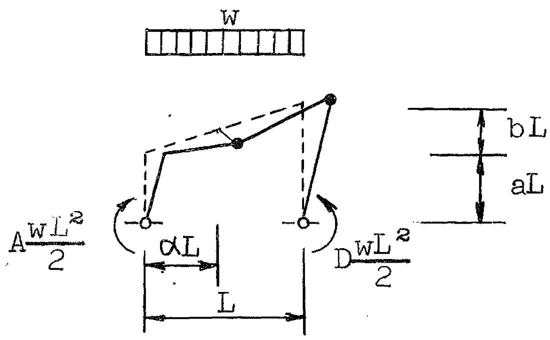
VIII. APPENDIX: SUMMARY OF IMPORTANT EQUATIONS

1.

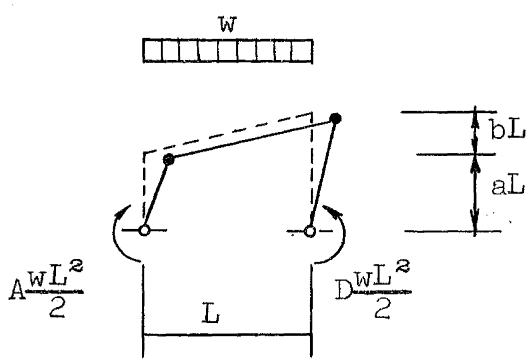


$$\frac{M_p}{wL^2} = \frac{1}{2}\left[\frac{(1-\alpha)\left\{(1+\frac{b}{a})(A+\alpha)-D\right\}}{2+(\frac{b}{a})(1+\alpha)}\right]$$

$$\alpha = \left[\frac{2+\frac{b}{a}}{\frac{b}{a}}\right]\left[\sqrt{1+\frac{b}{a}\left[\frac{\frac{b}{a}+2\left\{1+D-A(1+\frac{b}{a})\right\}}{(2+\frac{b}{a})^2}\right]}-1\right] \quad \ldots\text{for } \frac{b}{a}>0$$

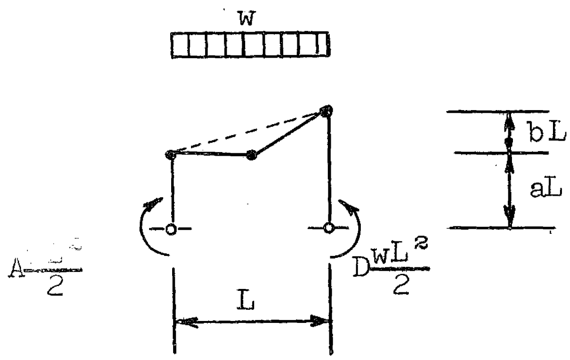
$$\alpha = \left[\frac{1-A+D}{2}\right] \quad \ldots\ldots\ldots\ldots\ldots\text{for } \frac{b}{a}=0$$

2.



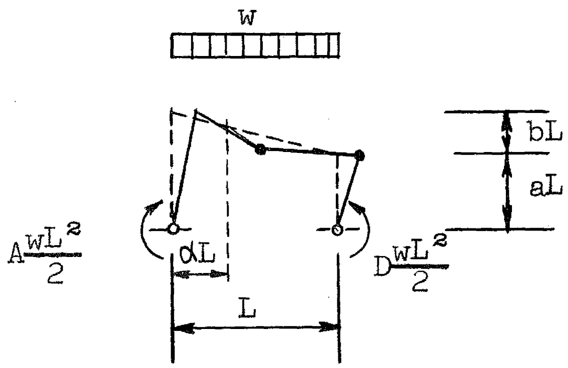
$$\frac{M_p}{wL^2} = \frac{1}{2}\left[\frac{A(1+\frac{b}{a})-D}{2+\frac{b}{a}}\right]$$

3.



$$\frac{M_p}{wL^2} = 0.0625$$

4.

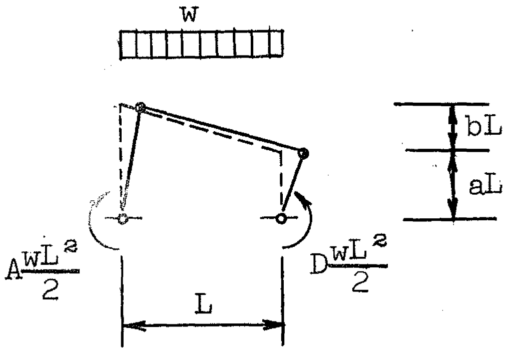


$$\frac{M_p}{wL^2} = \frac{1}{2}\left[\frac{(1-\alpha)\left\{A-D(1+\frac{b}{a})+\alpha\right\}}{2+\frac{b}{a}(1-\alpha)}\right]$$

$$\alpha = 1-\frac{2}{\frac{b}{a}}\left[\sqrt{1+\frac{\frac{b}{a}}{2}\left[A-D(1+\frac{b}{a})+1\right]}-1\right] \quad \text{. . . . . .for } \frac{b}{a} > 0$$

$$\alpha = \left[\frac{1-A+D}{2}\right] \quad \text{. . . . . .for } \frac{b}{a} = 0$$

5.



$$\frac{M_p}{wL^2} = \frac{1}{2}\left[\frac{A - D\left(1+\frac{b}{a}\right)}{2+\frac{b}{a}}\right]$$

IX. DESIGN CHARTS

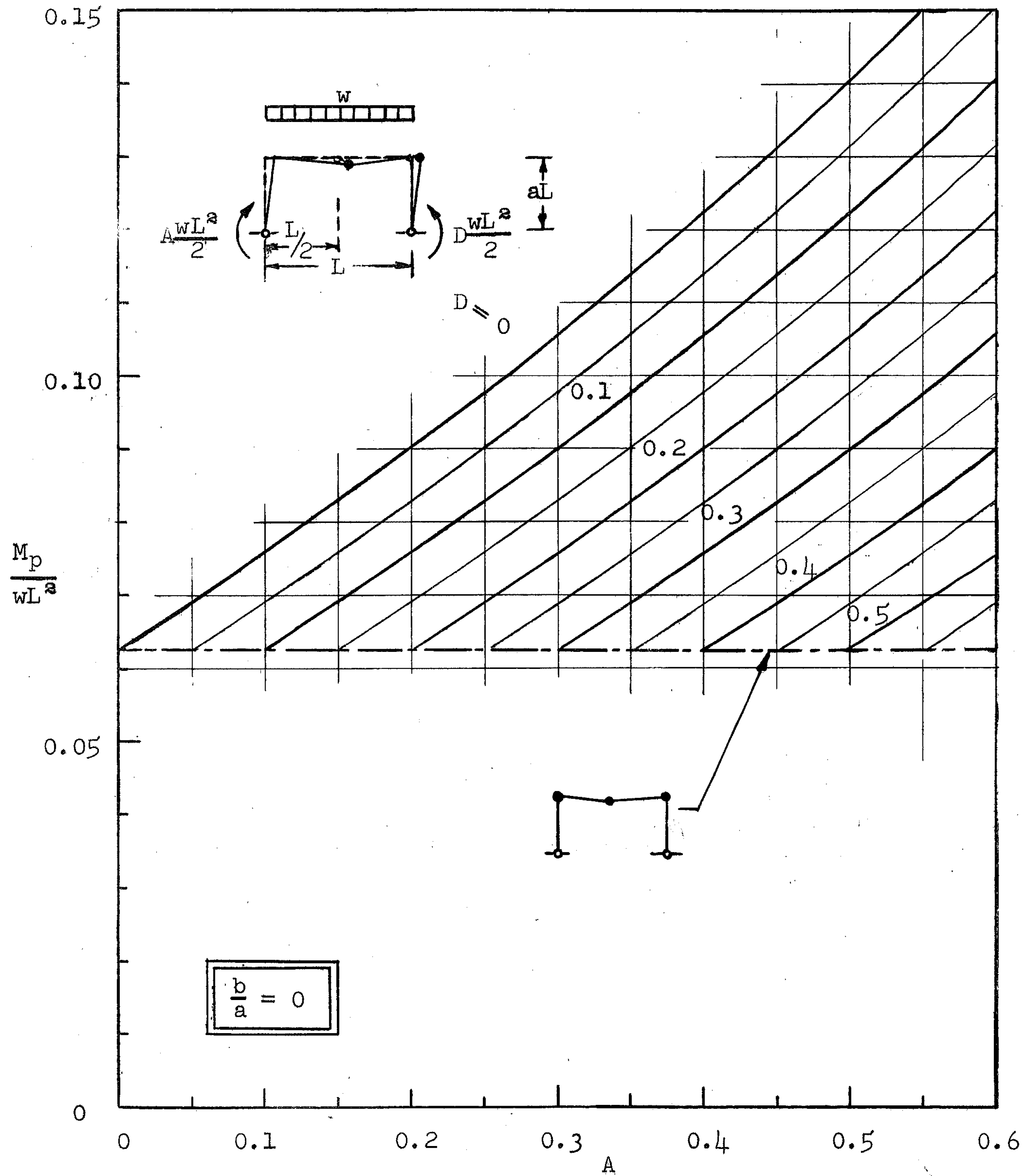


Chart 1a

FIG. 22 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

DETERMINATION OF MEMBER SIZE

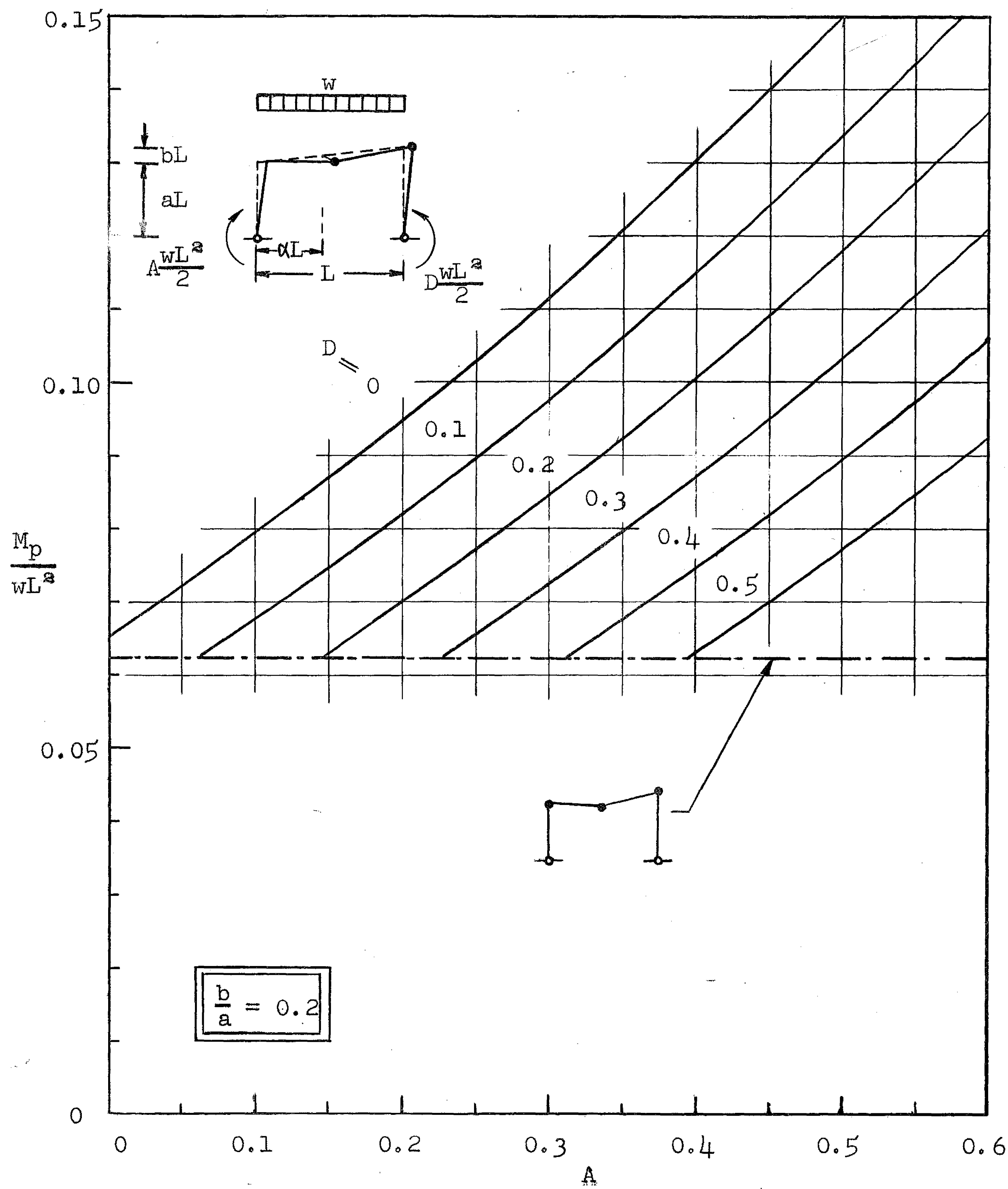


Chart 2a

FIG. 23 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

DETERMINATION OF MEMBER SIZE

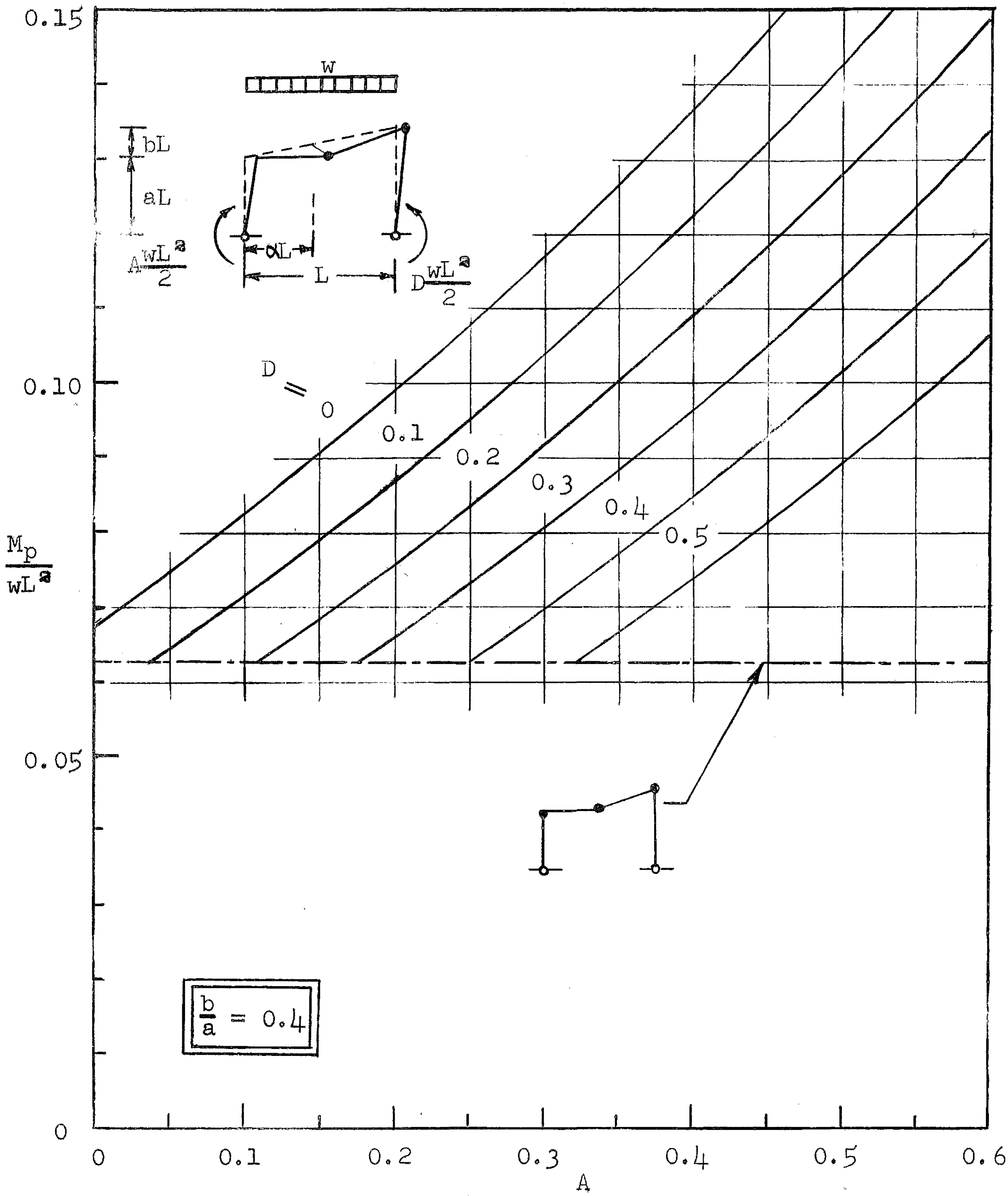


Chart 3a

FIG. 24 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

DETERMINATION OF MEMBER SIZE

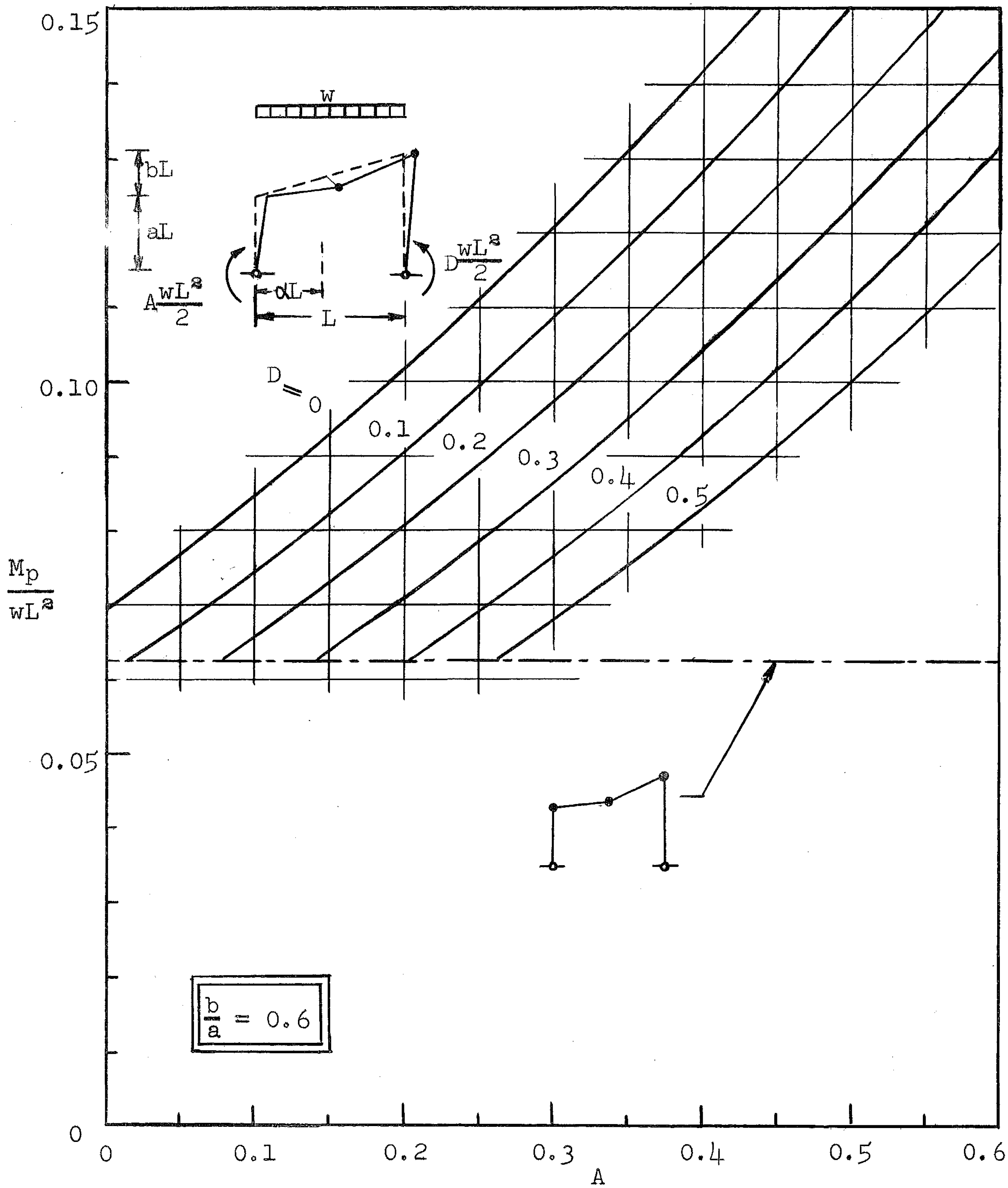


Chart 4a

FIG. 25 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

DETERMINATION OF MEMBER SIZE

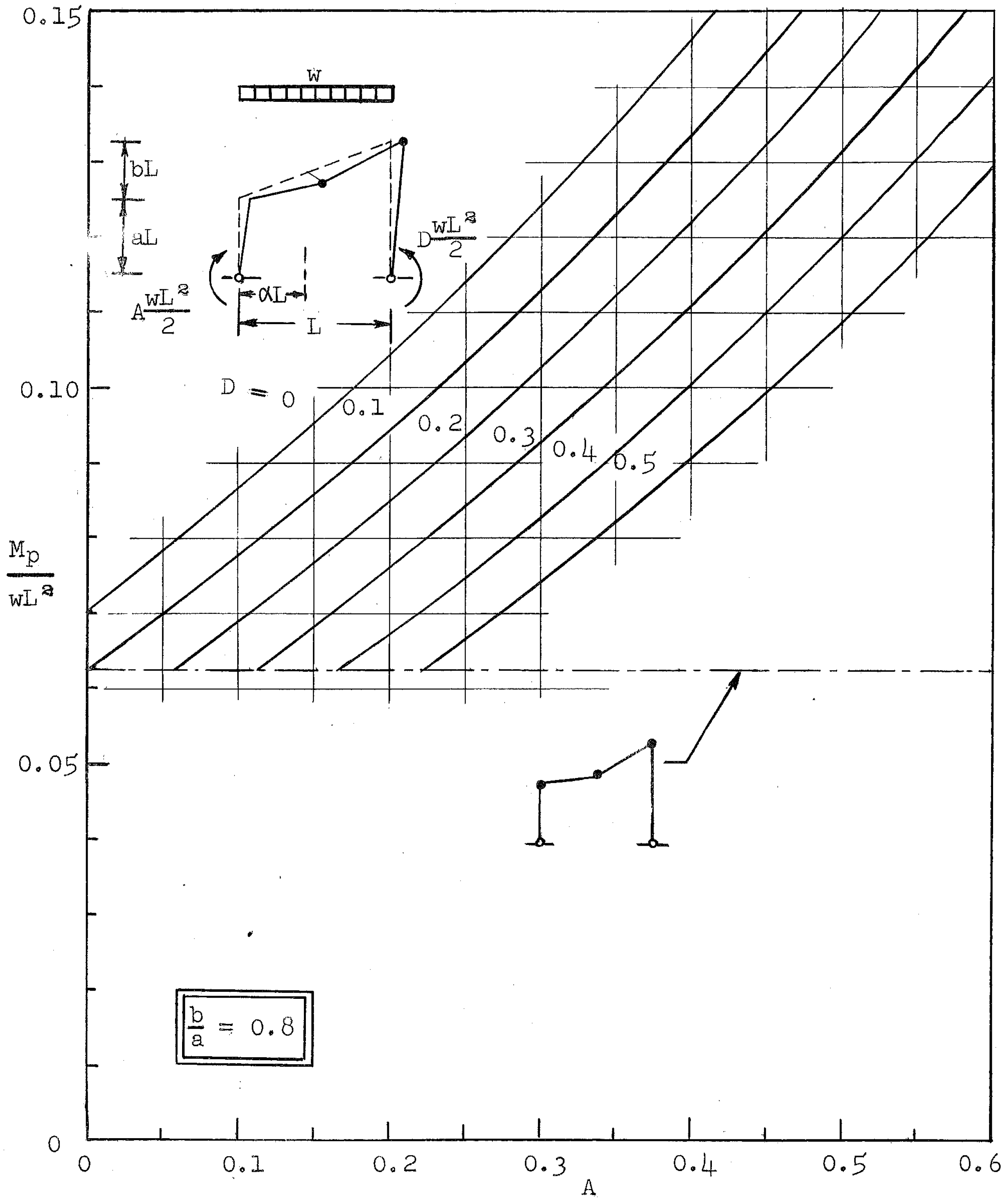


Chart 5a

FIG. 26 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE DETERMINATION OF MEMBER SIZE

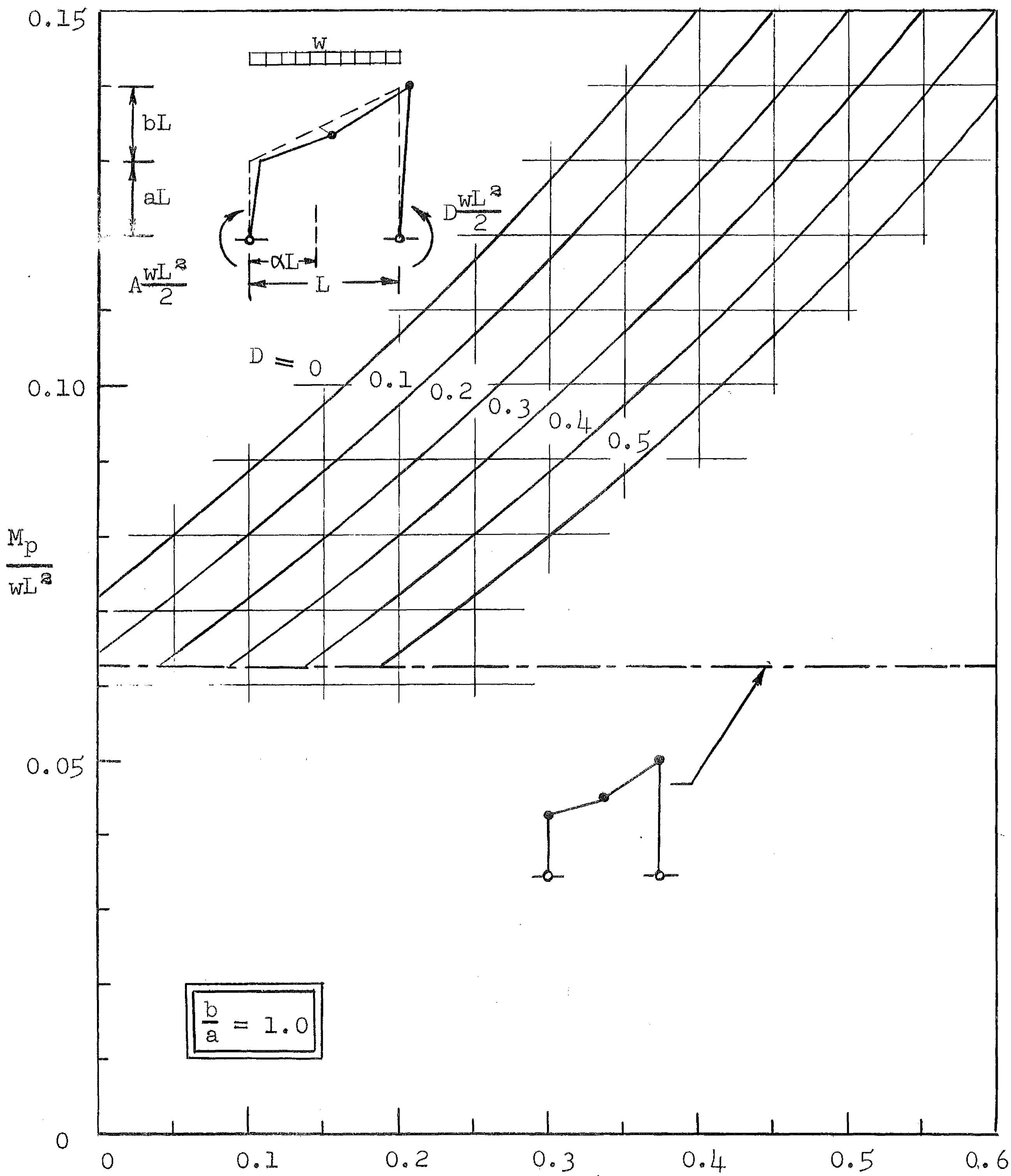


Chart 6a

FIG. 27 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

DETERMINATION OF MEMBER SIZE

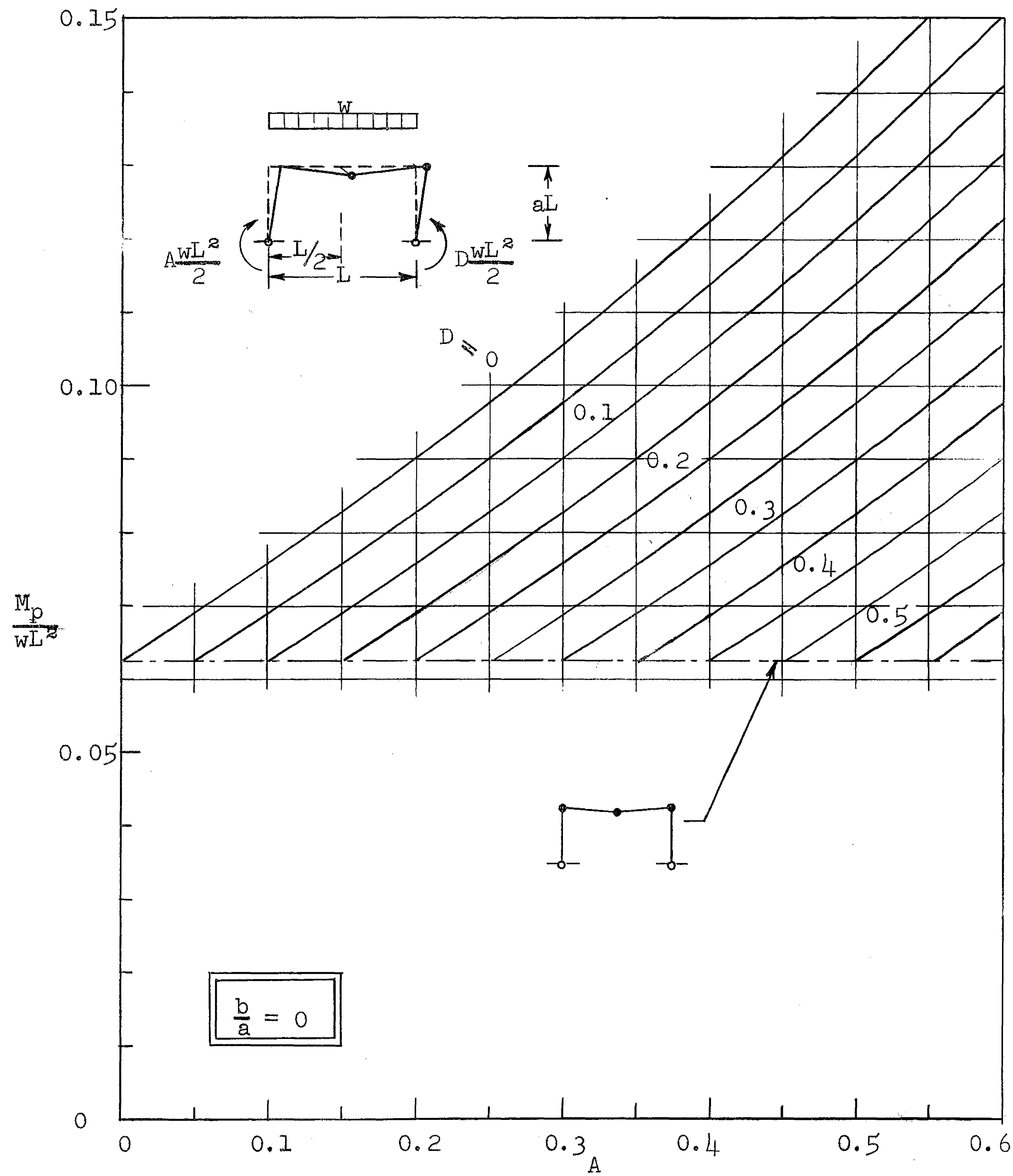


Chart 7a

FIG. 28 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

DETERMINATION OF MEMBER SIZE

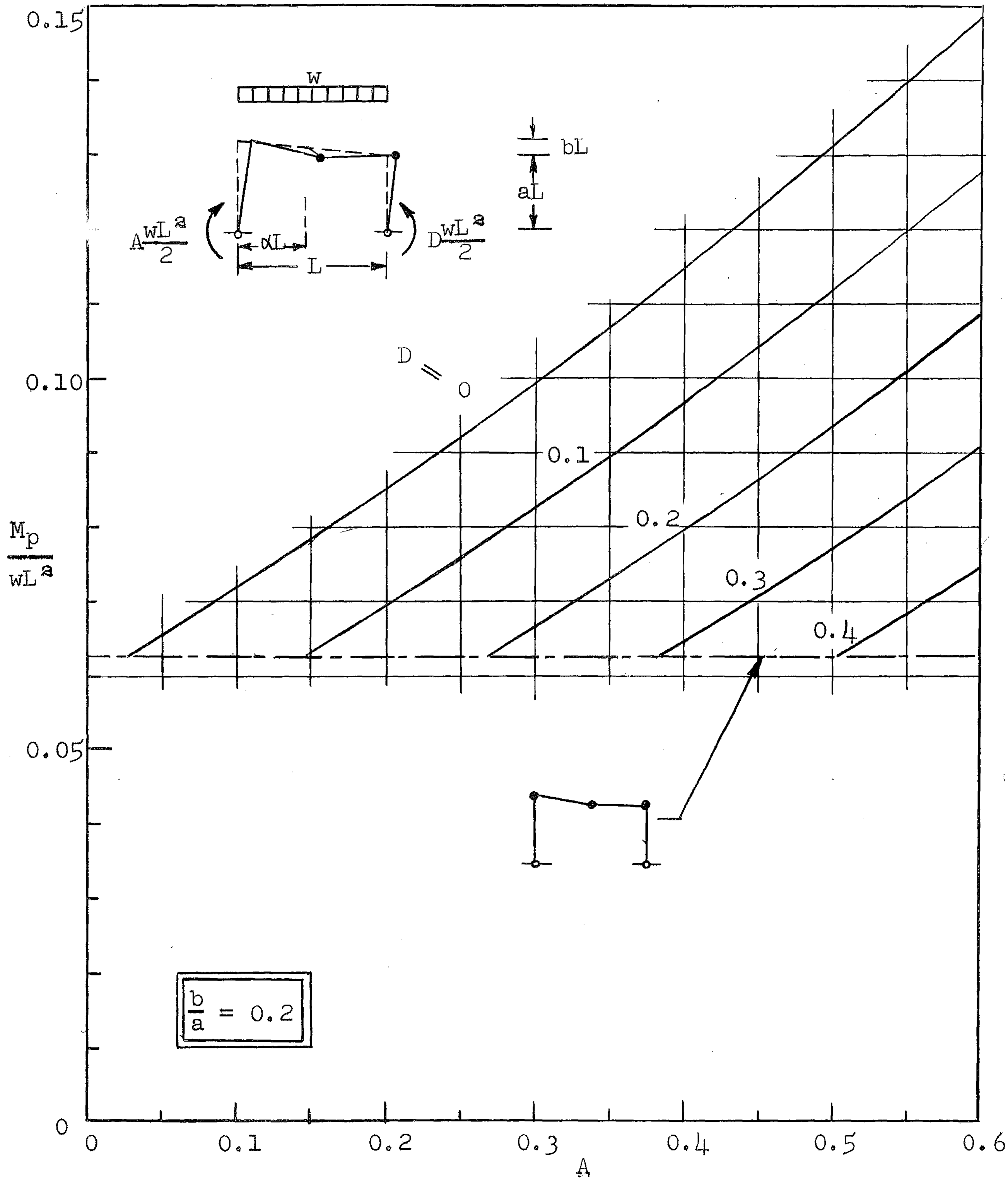


Chart 8a

FIG. 29 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

DETERMINATION OF MEMBER SIZE

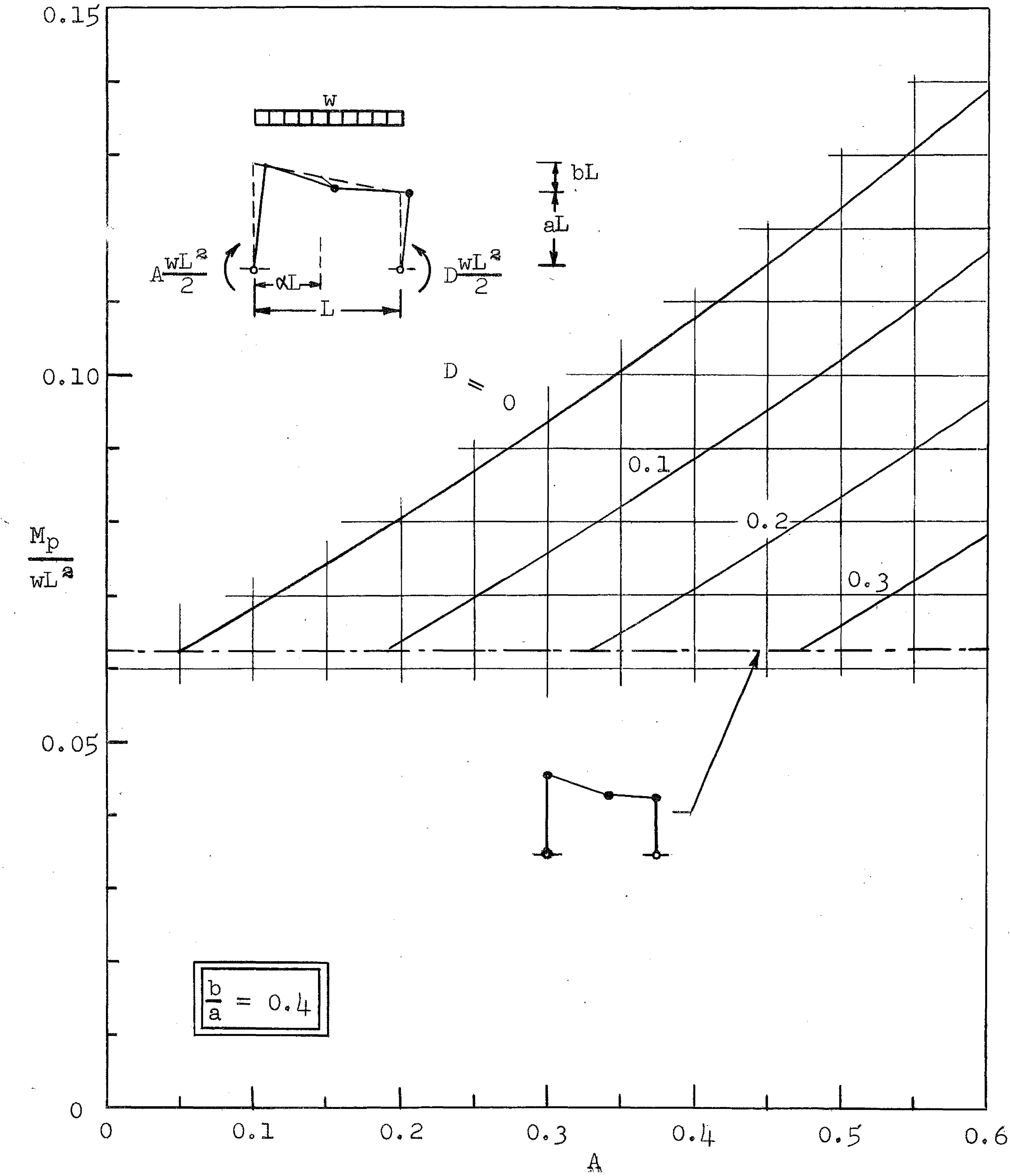


Chart 9a

FIG. 30 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

DETERMINATION OF MEMBER SIZE

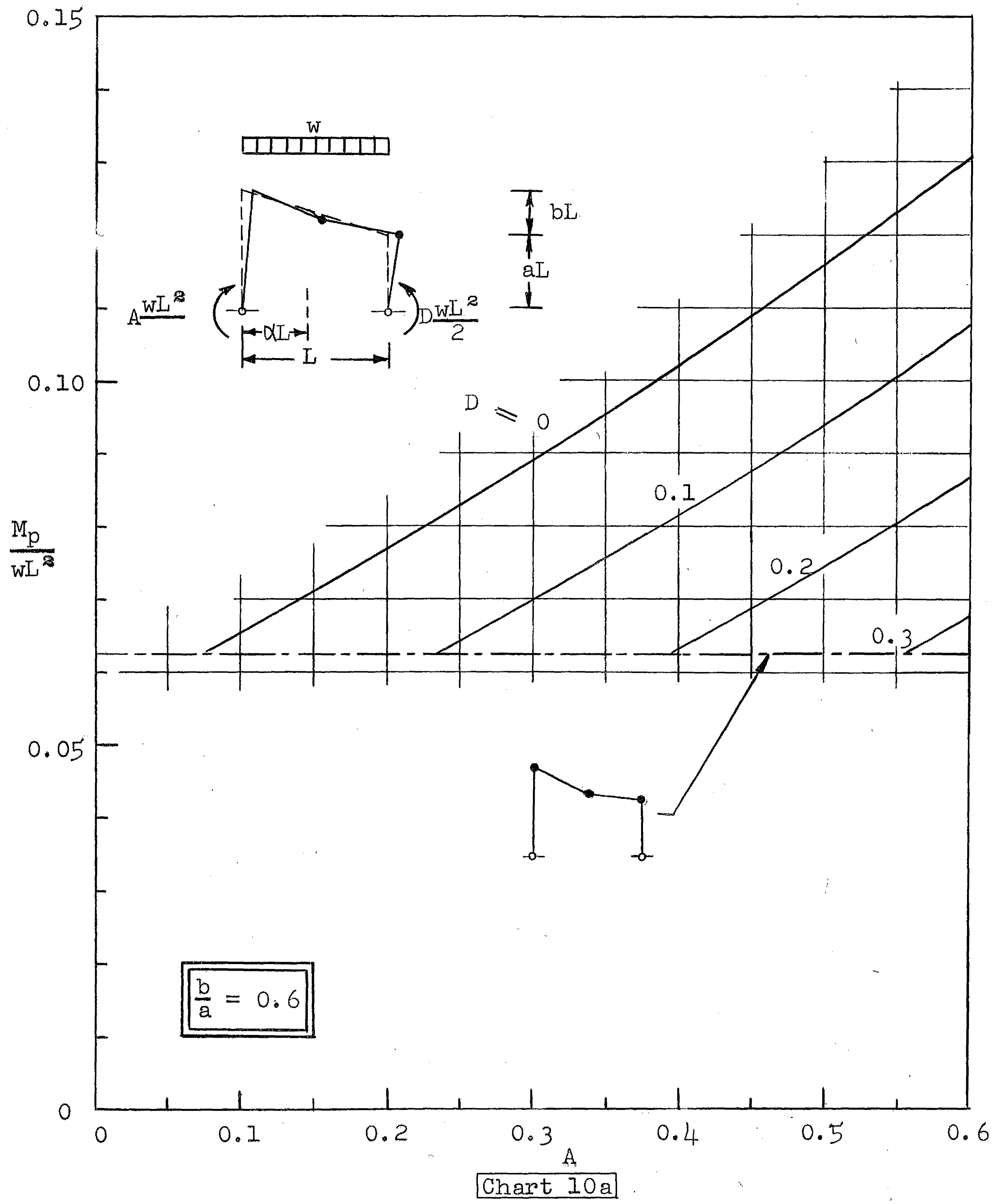


FIG. 31 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

DETERMINATION OF MEMBER SIZE

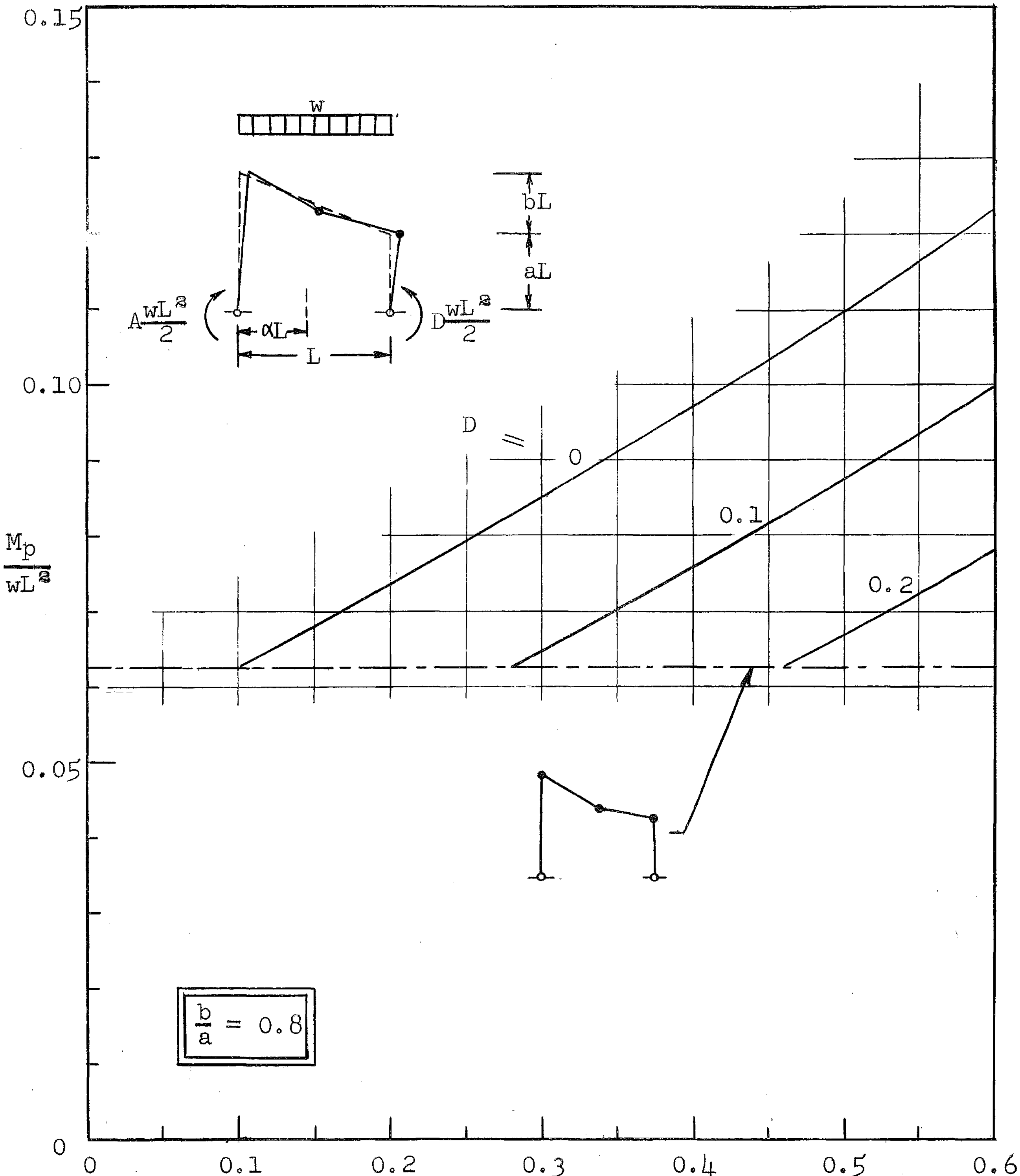


Chart 11a

FIG. 32 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

DETERMINATION OF MEMBER SIZE

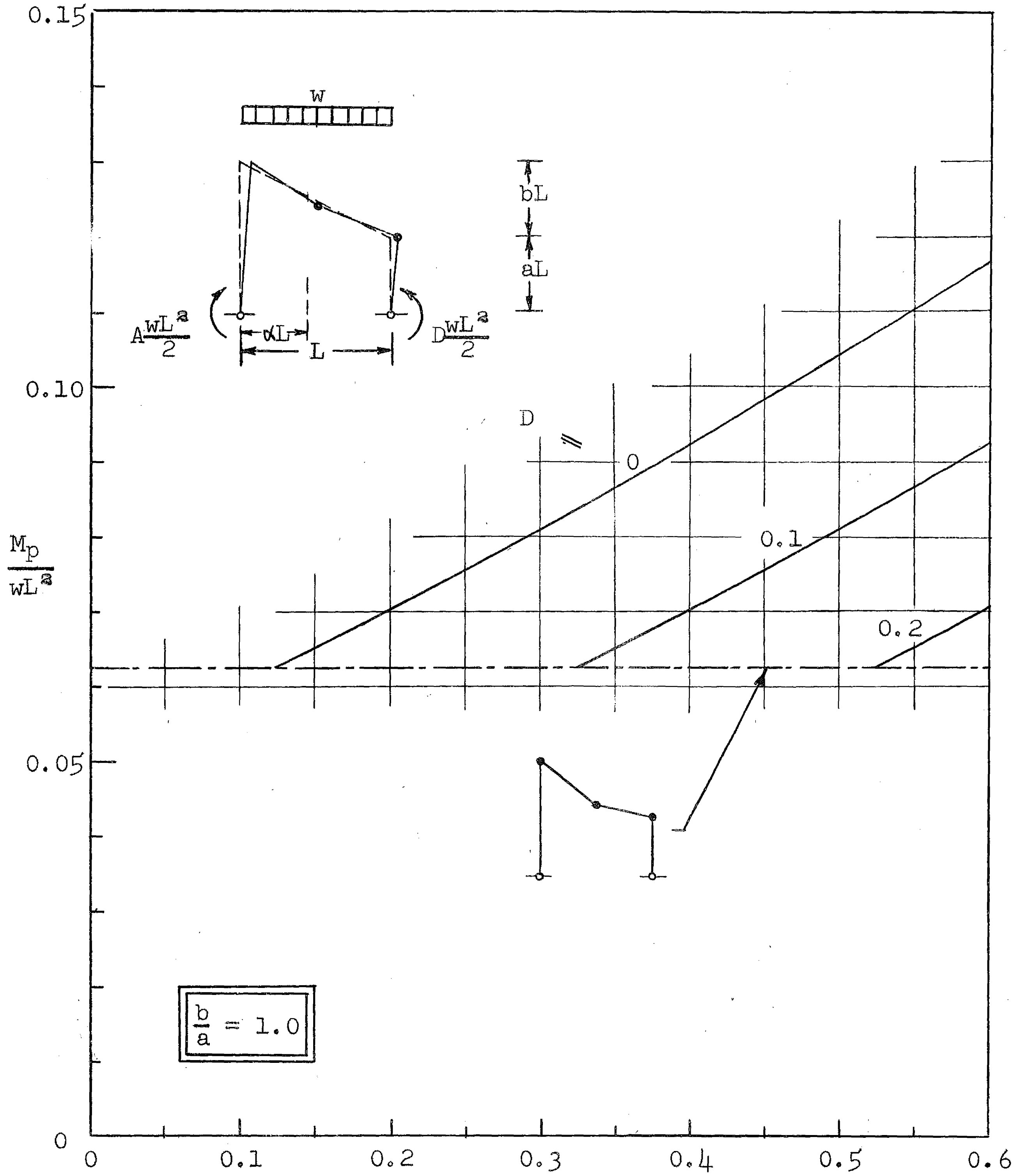


Chart 12a

FIG. 33 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE
DETERMINATION OF MEMBER SIZE

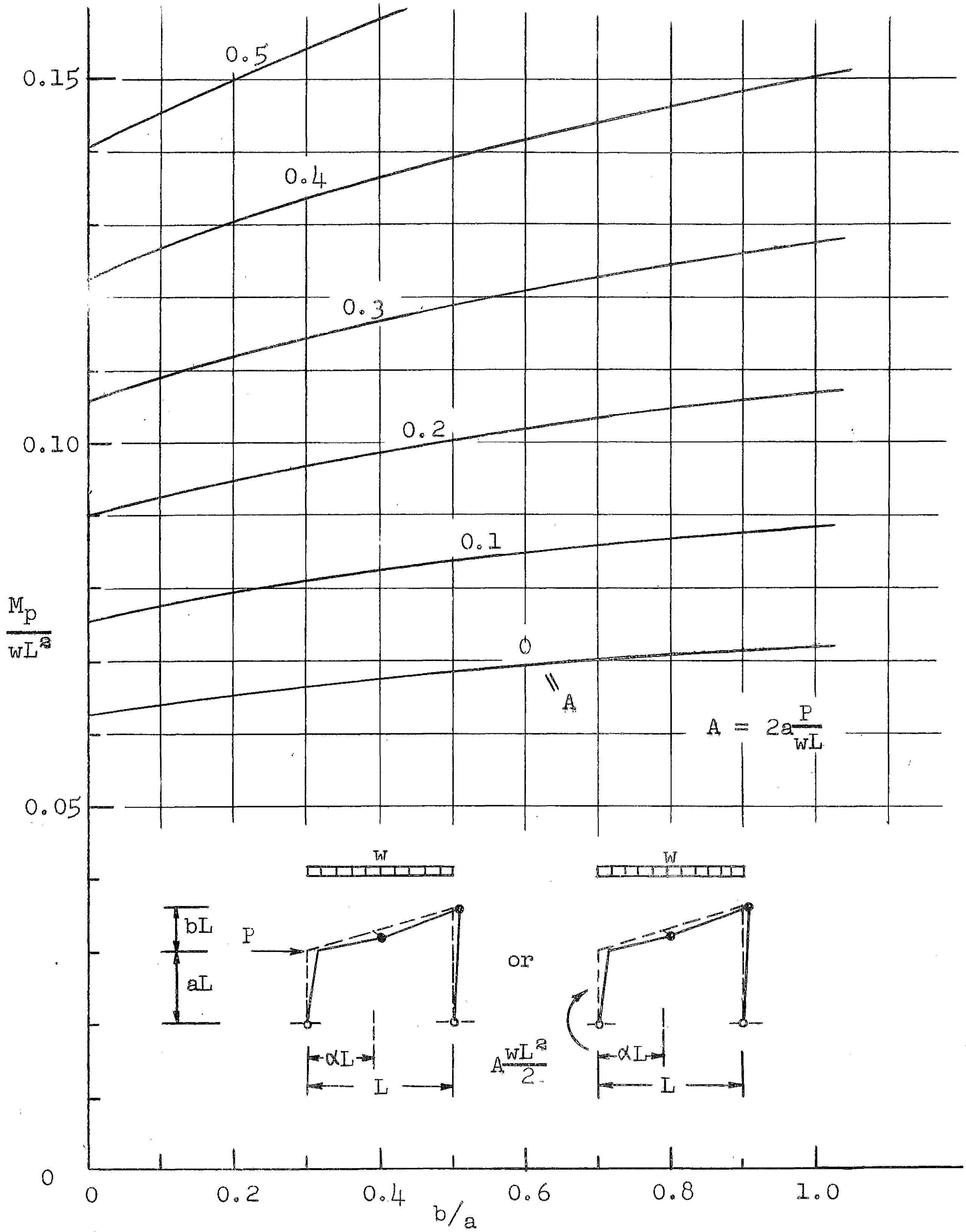


Chart 13a

FIG. 34 - DESIGN CURVES FOR SINGLE SPAN, PINNED-BASE, LEAN-TO FRAMES DETERMINATION OF MEMBER SIZE

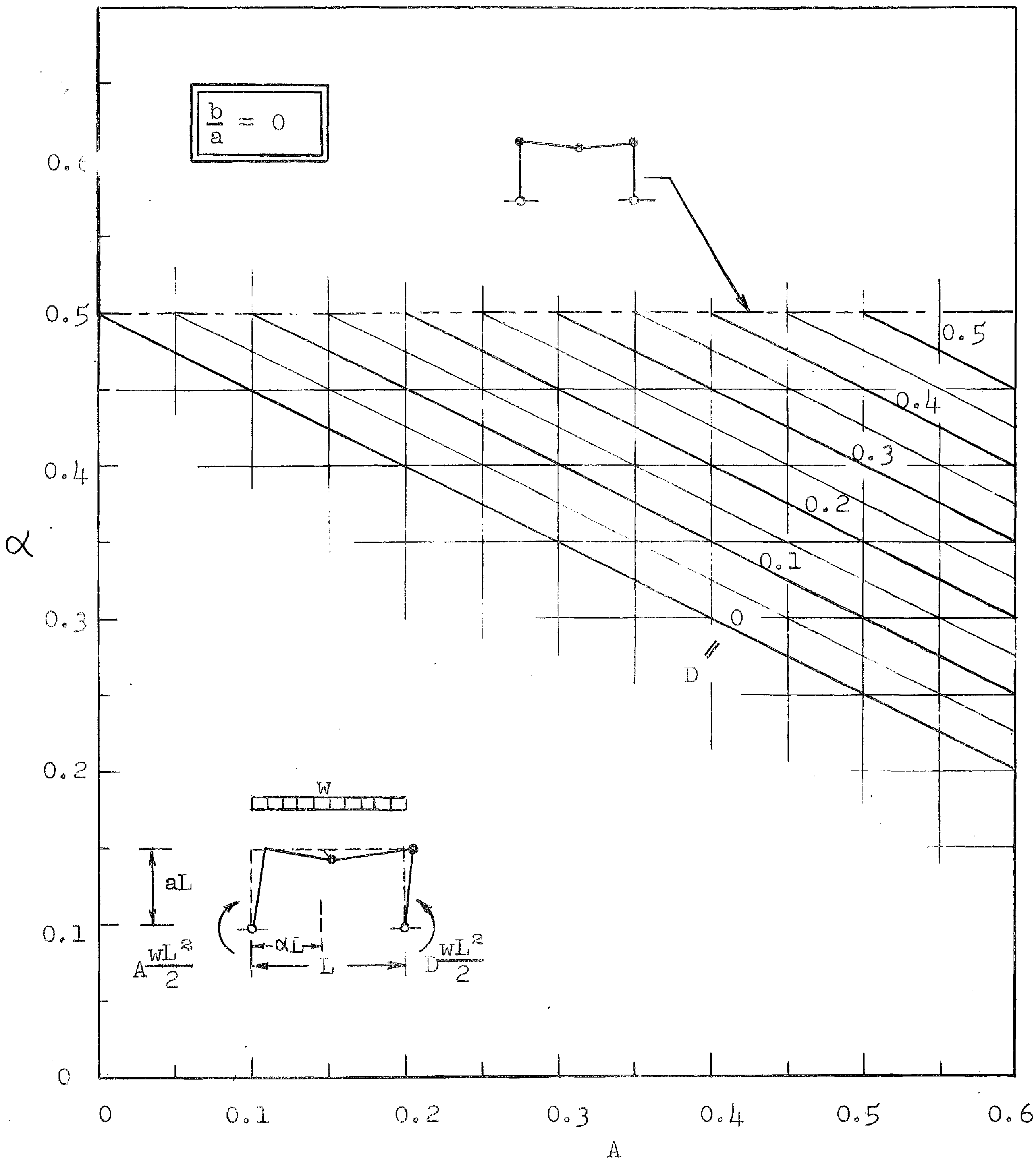


Chart 1b

FIG. 35 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

LOCATION OF PLASTIC HINGE

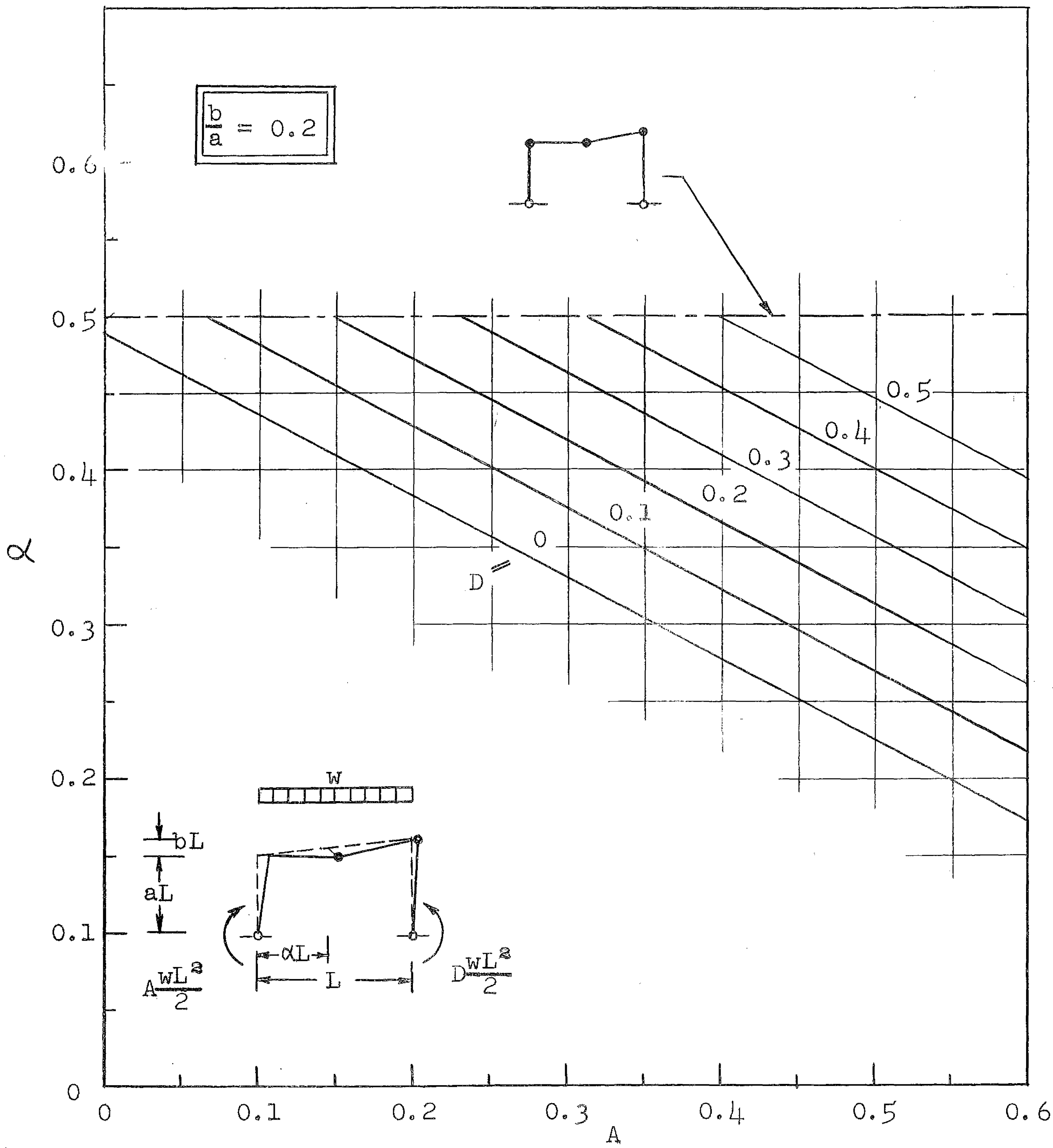


Chart 2b

FIG. 36 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

LOCATION OF PLASTIC HINGE

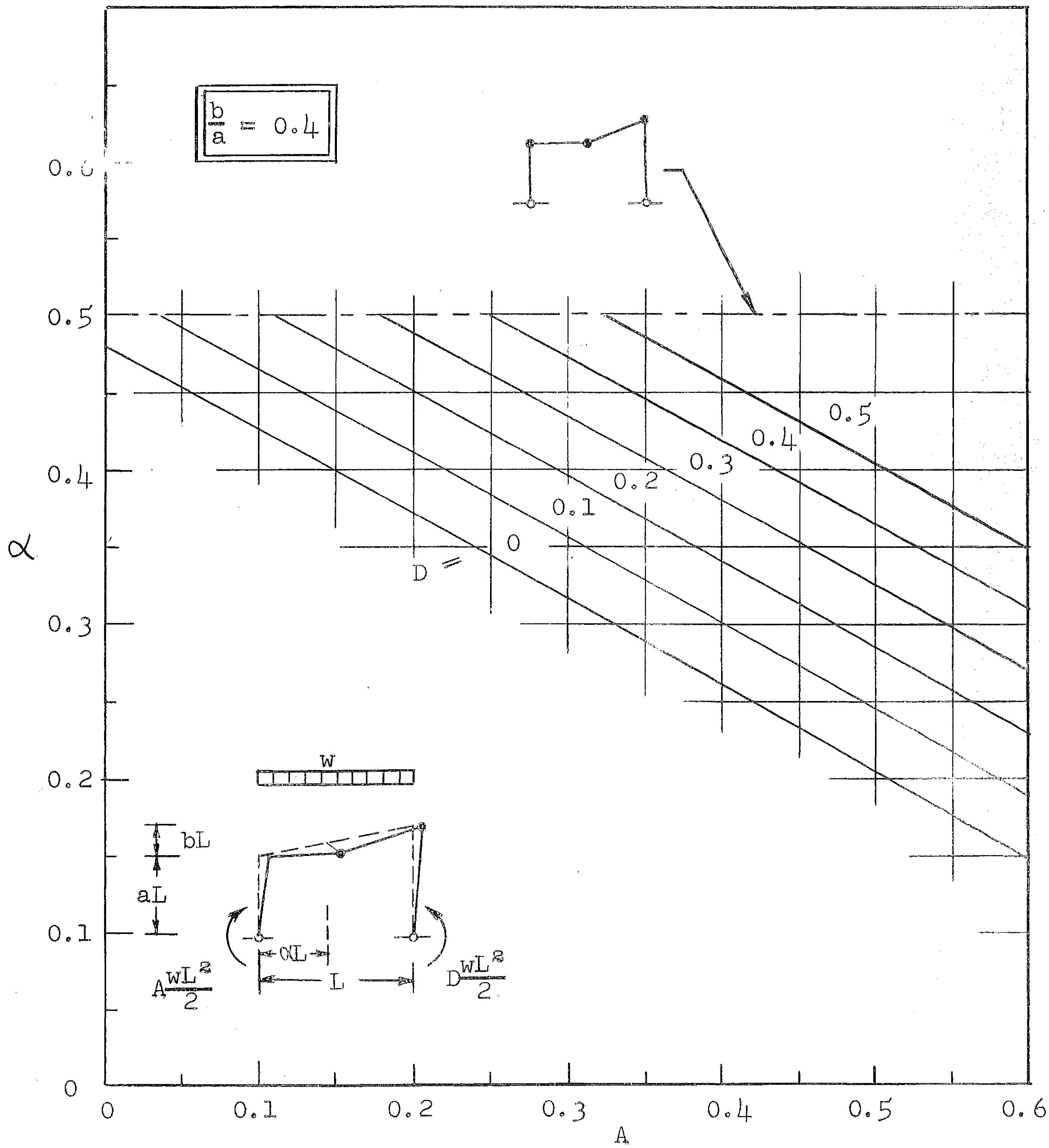


Chart 3b

FIG. 37 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE
LOCATION OF PLASTIC HINGE

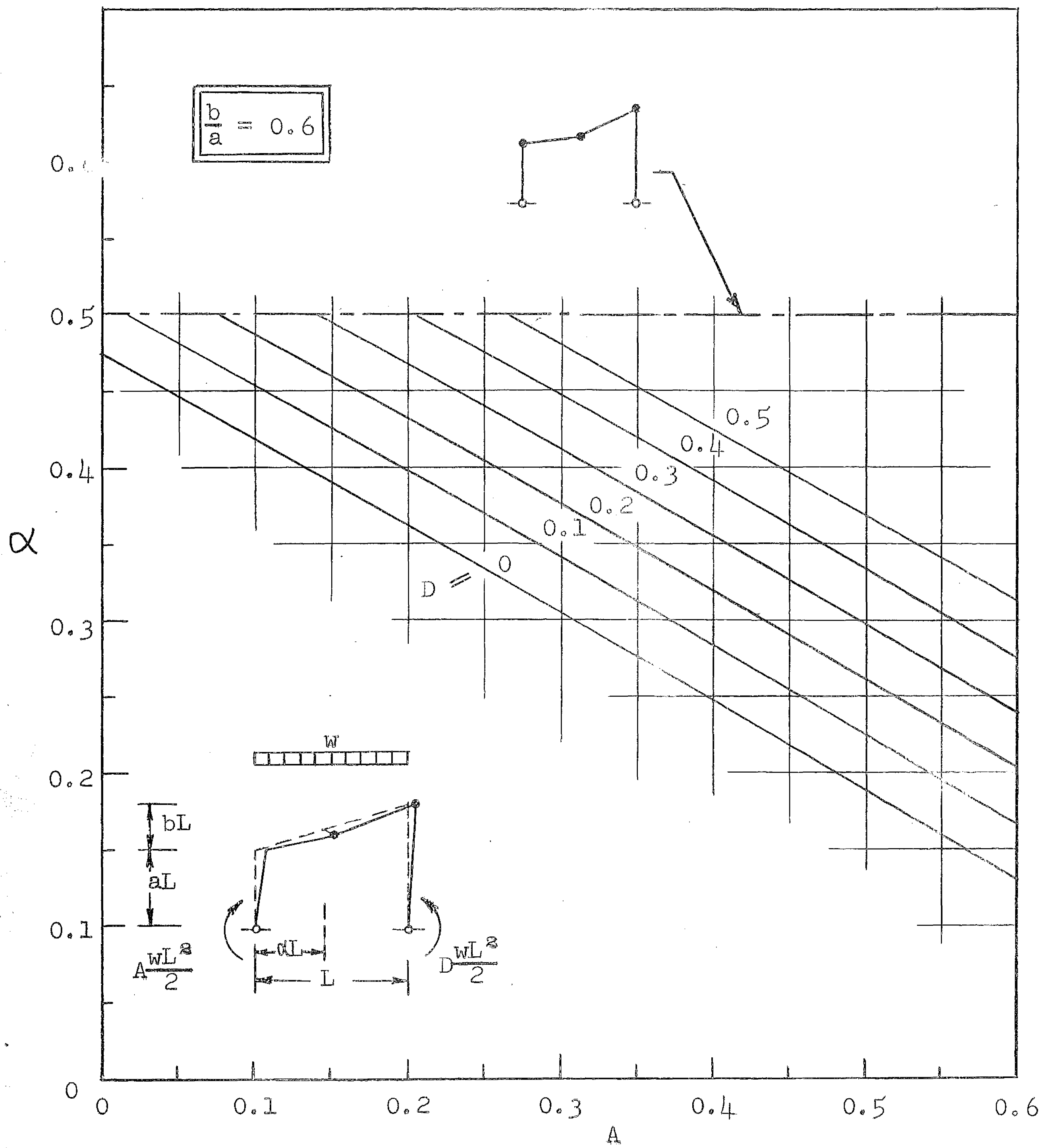


Chart 4b

FIG. 38 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

LOCATION OF PLASTIC HINGE

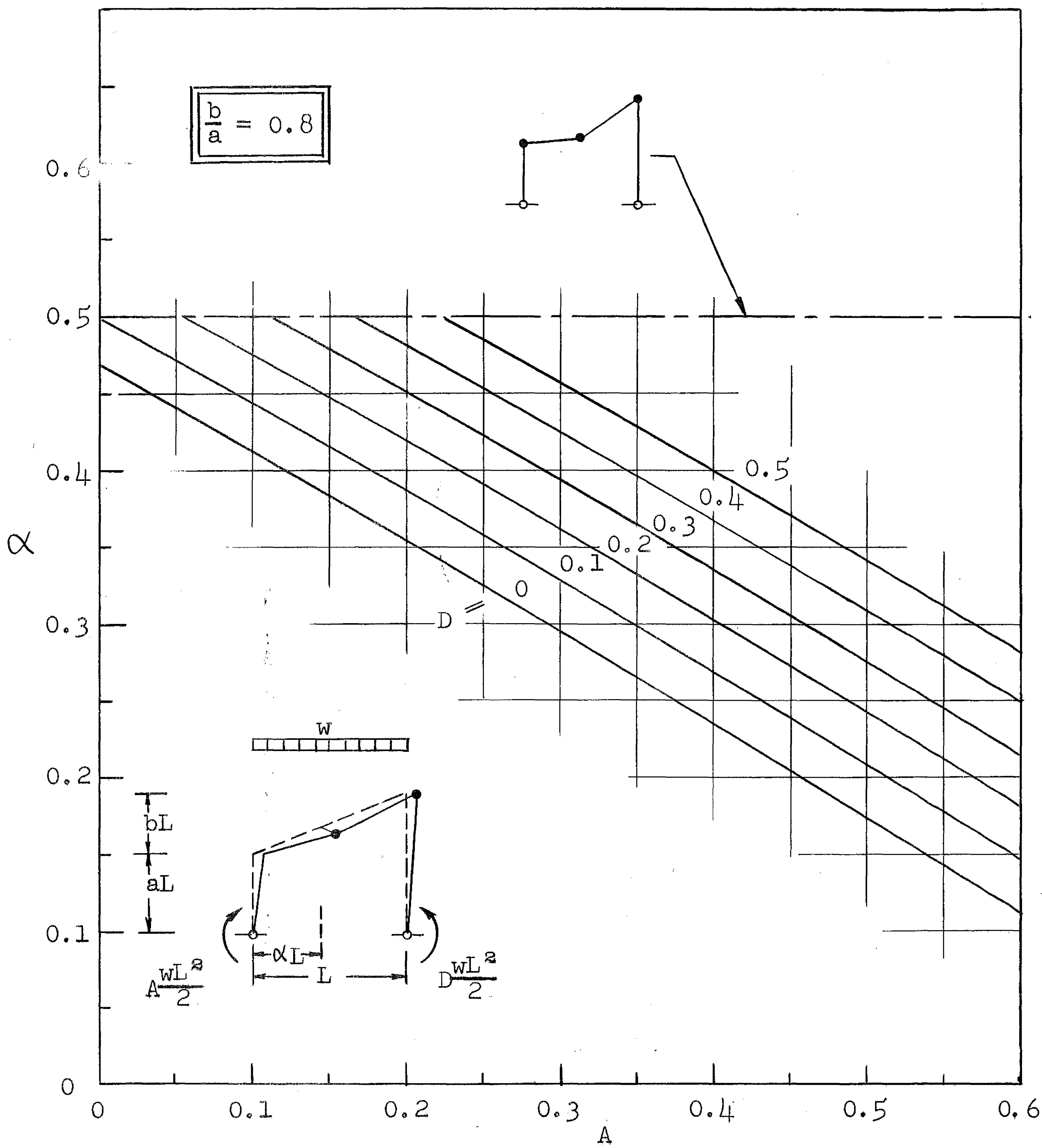


Chart 5b

FIG. 39 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE
LOCATION OF PLASTIC HINGE

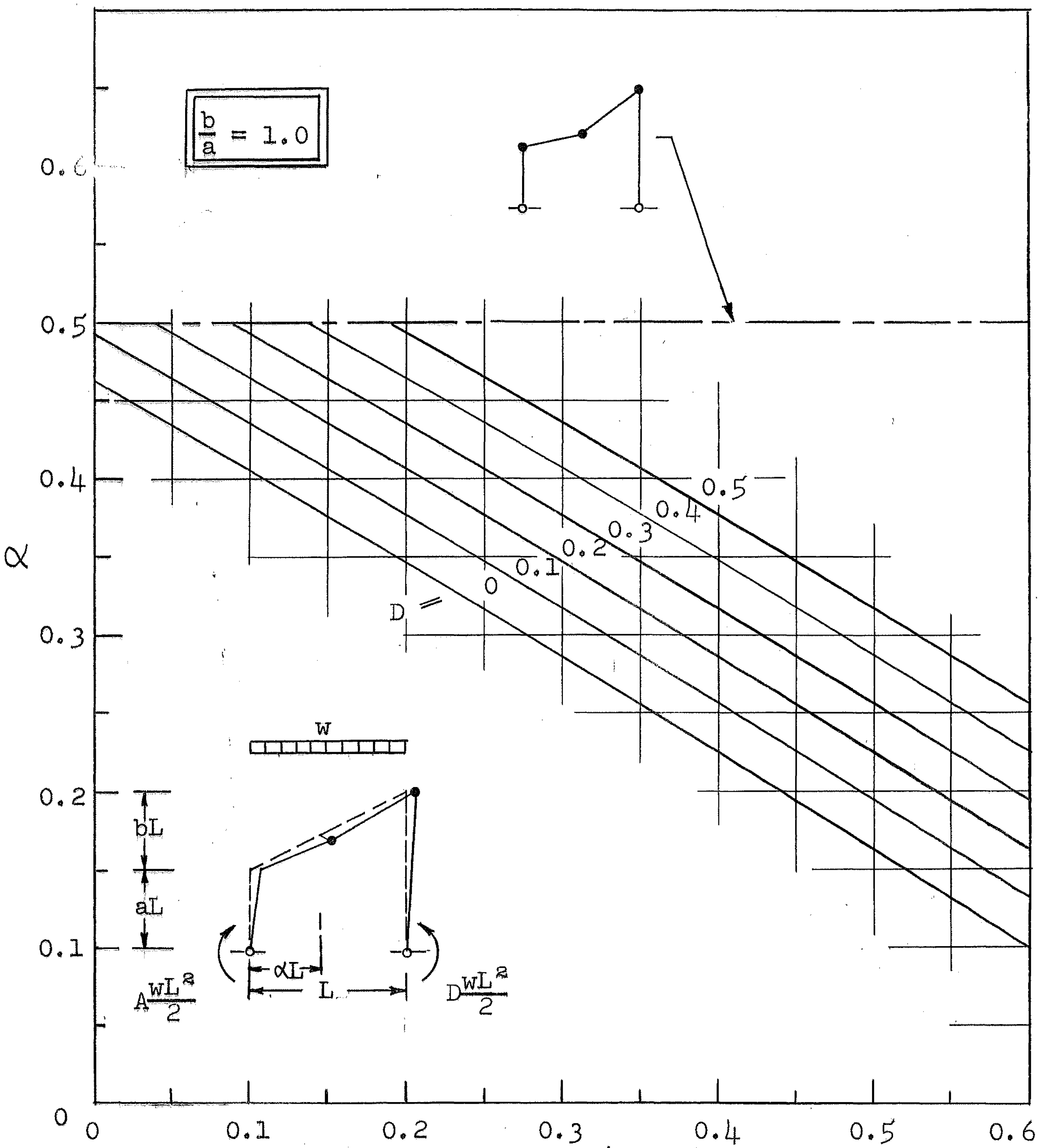


Chart 6b

FIG. 40 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE HIGHER SIDE

LOCATION OF PLASTIC HINGE

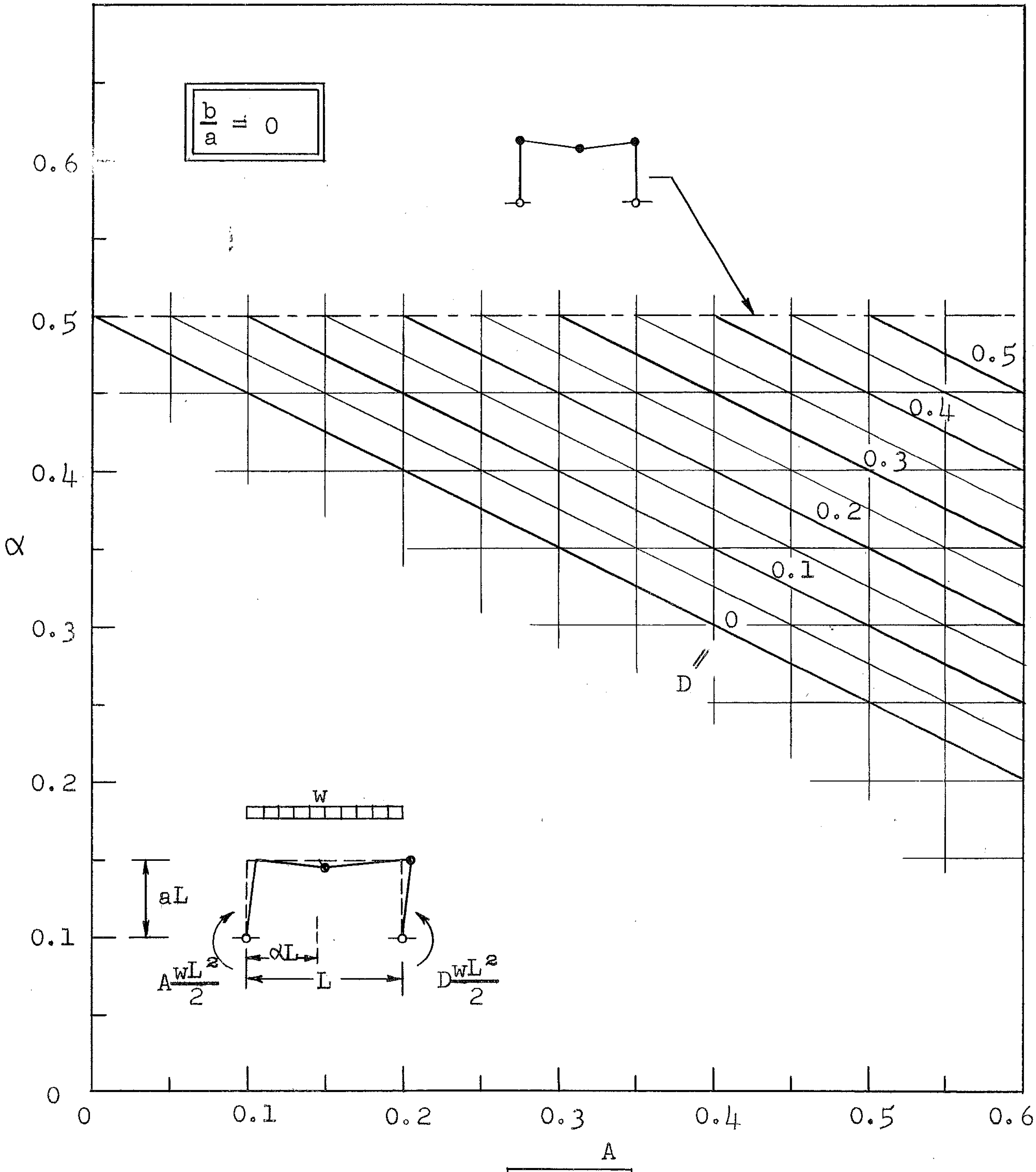


Chart 7b

FIG. 41 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE LOCATION OF PLASTIC HINGE

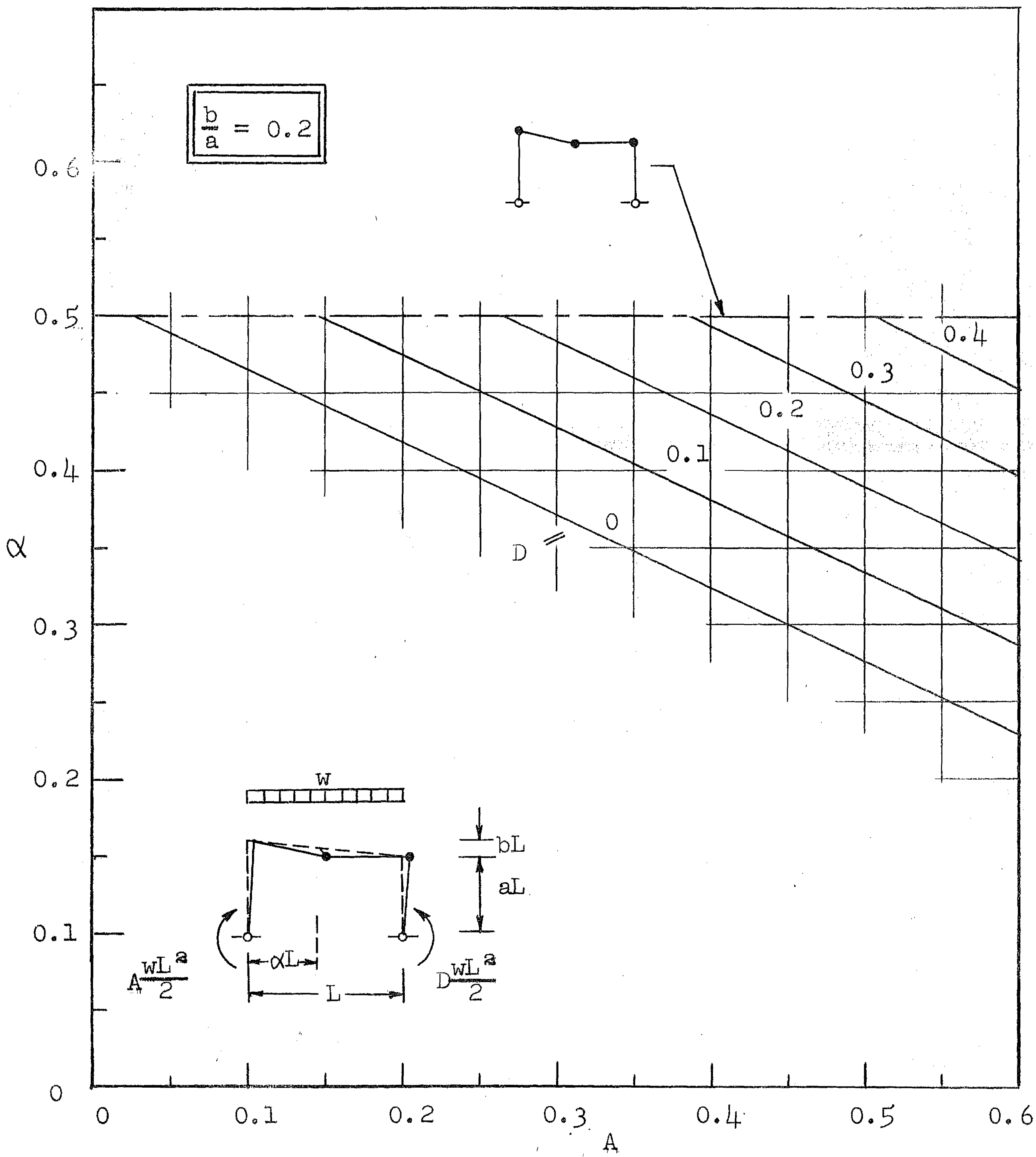


Chart 8b

FIG. 42 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE
LOCATION OF PLASTIC HINGE

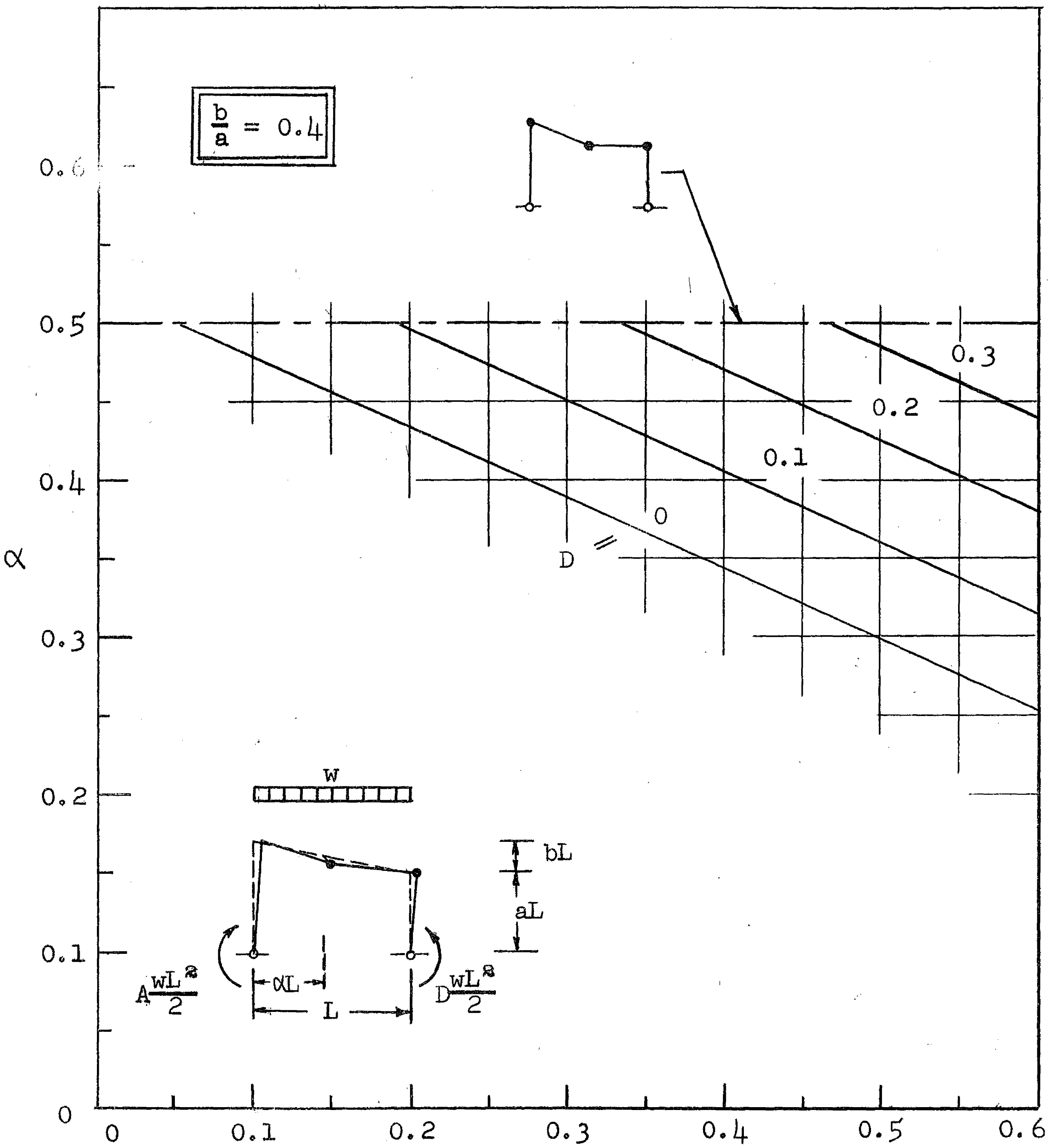


Chart 9b

FIG. 43 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE LOCATION OF PLASTIC HINGE

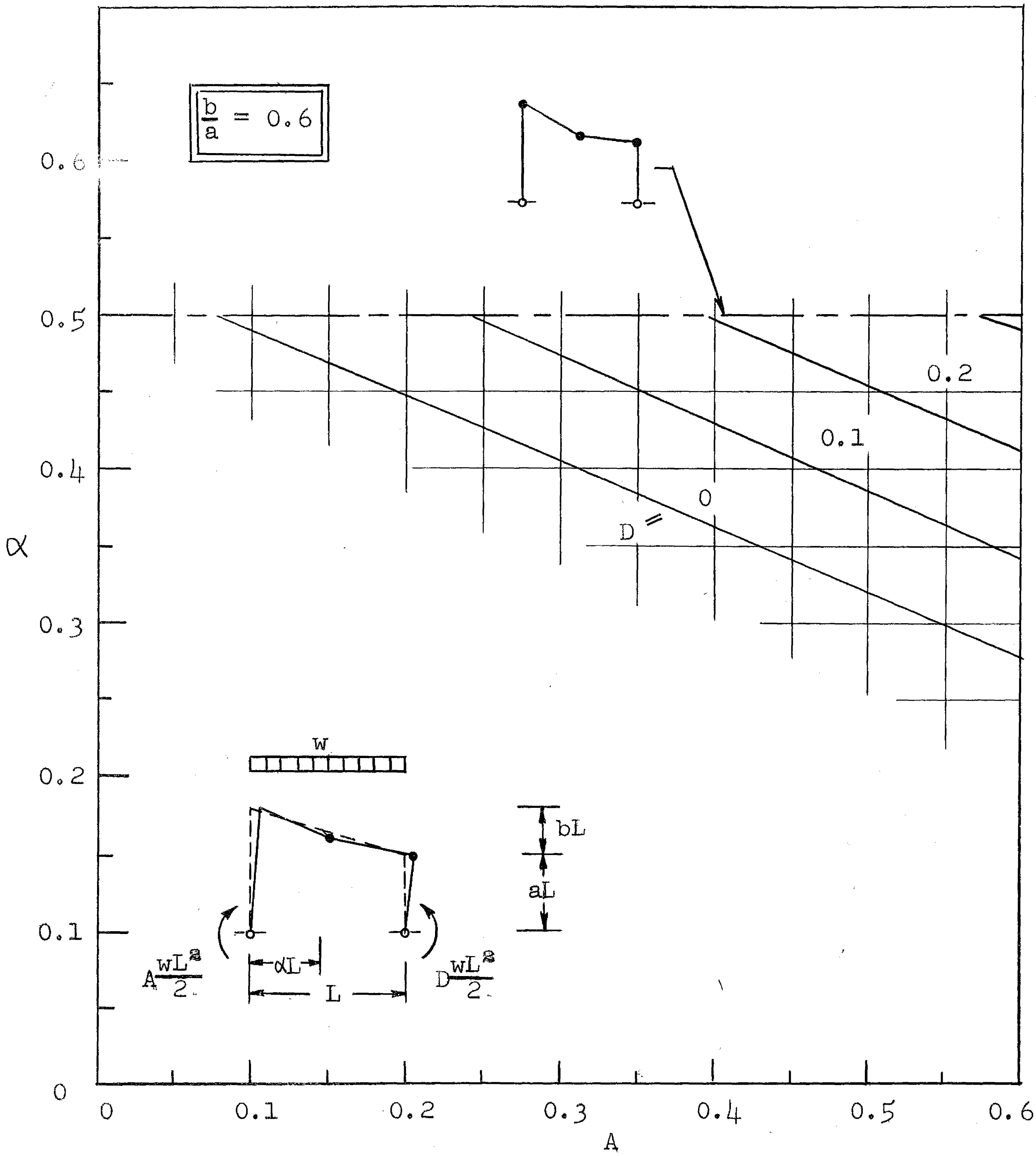


Chart 10b

FIG. 44 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

LOCATION OF PLASTIC HINGE

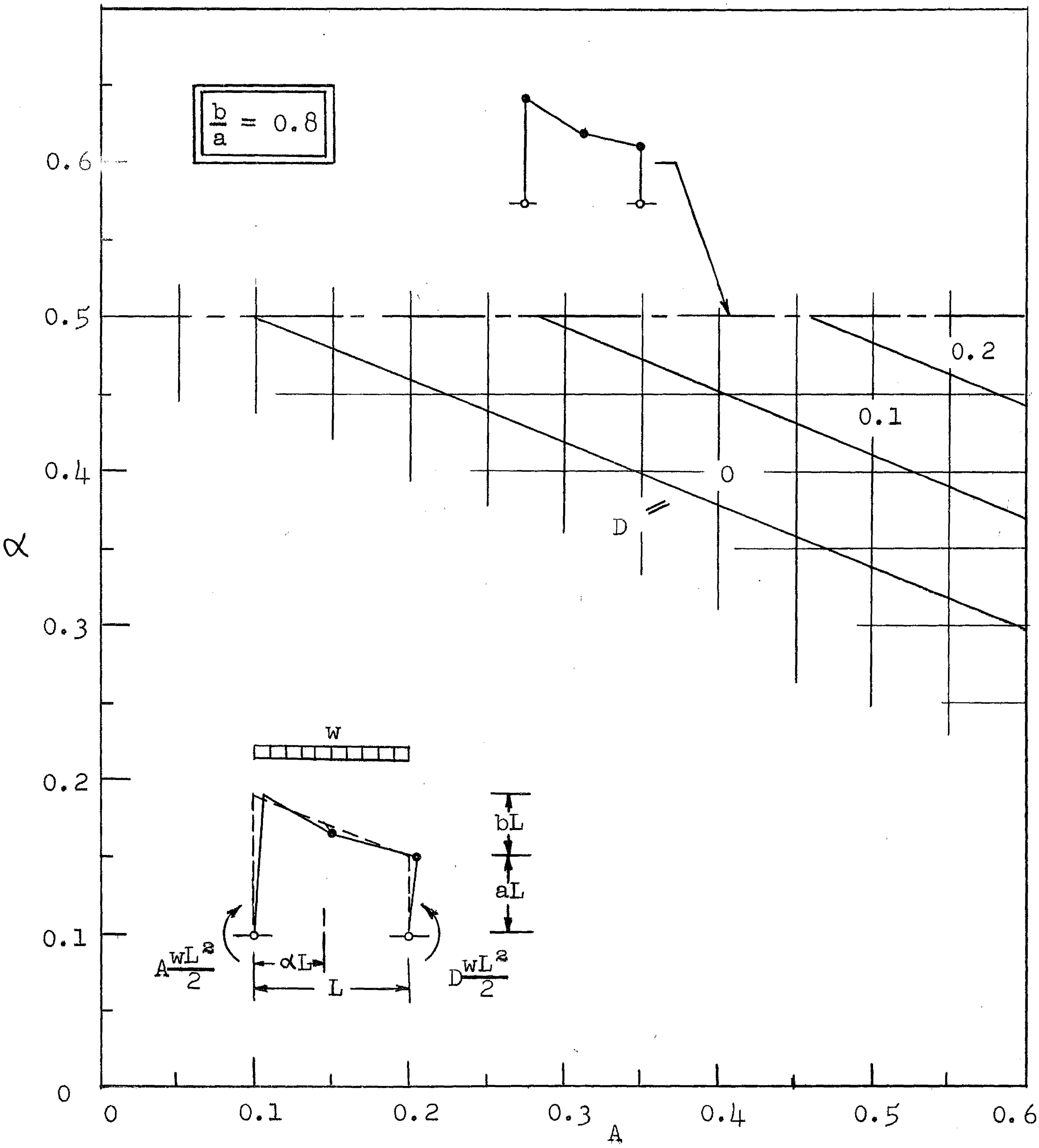


Chart 11b

FIG. 45 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE

LOCATION OF PLASTIC HINGE

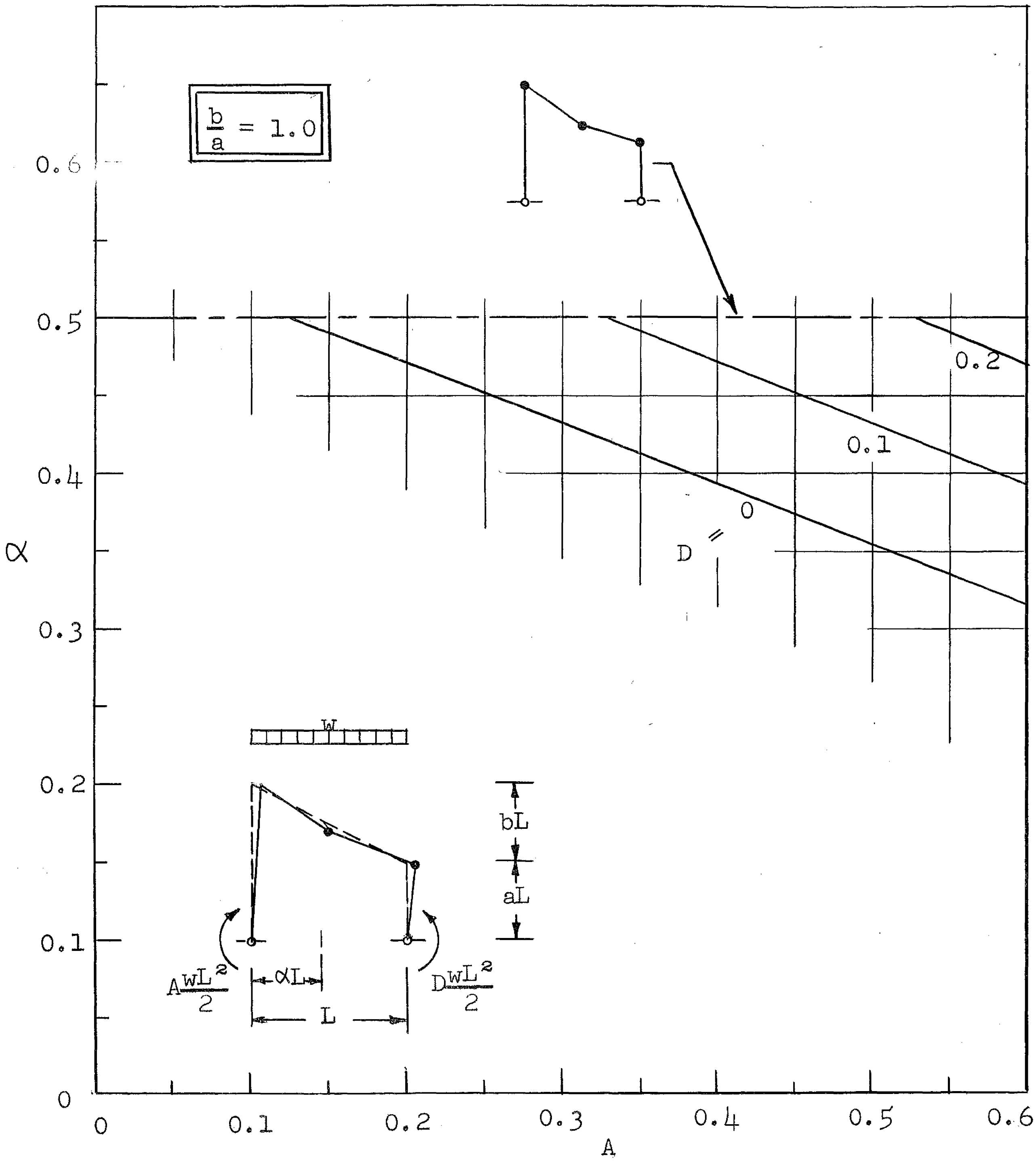


Chart 12b

FIG. 46 - DESIGN CURVES FOR PINNED-BASE, LEAN-TO FRAMES SWAYING TO THE LOWER SIDE
LOCATION OF PLASTIC HINGE

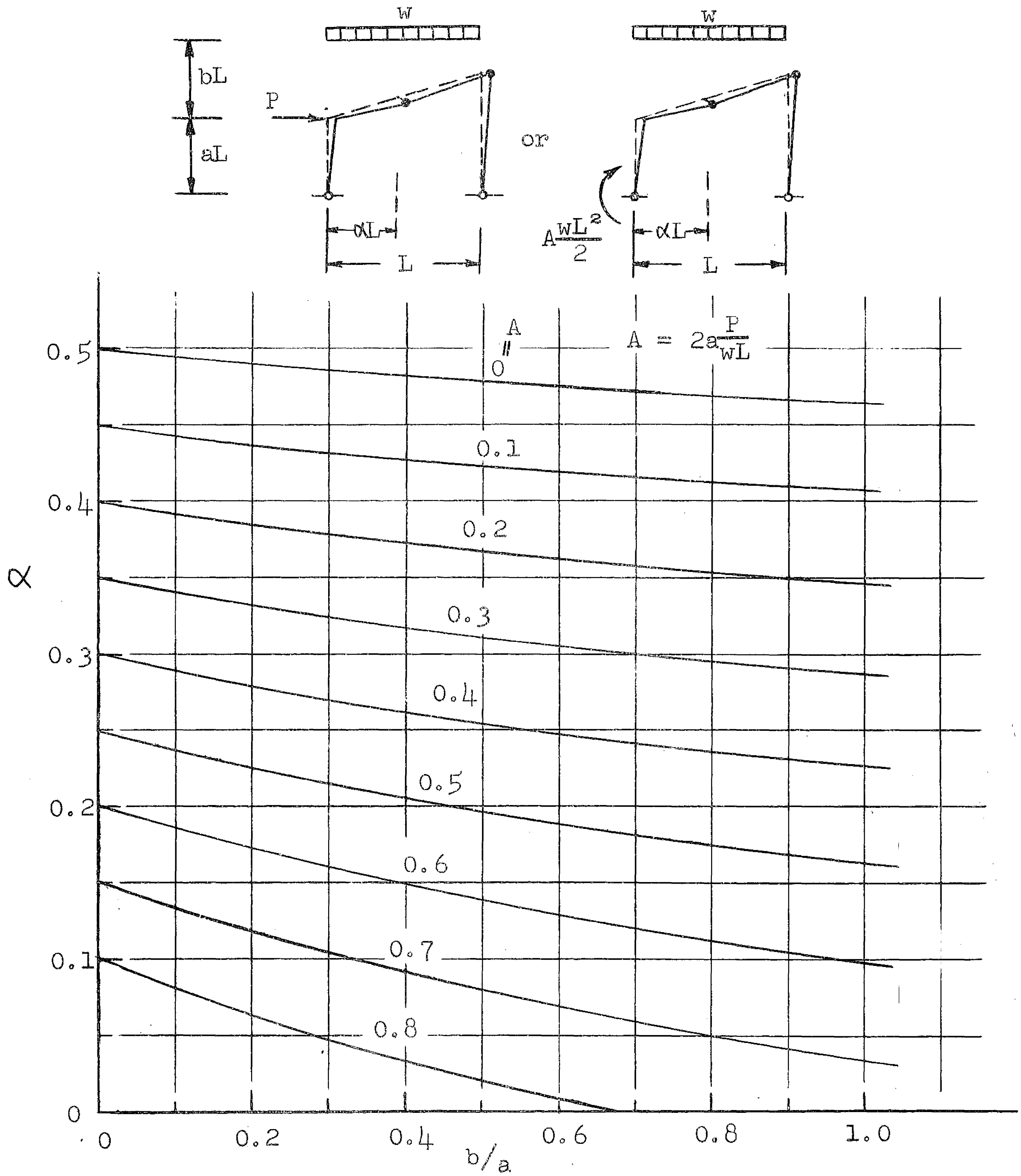


Chart 13b

FIG. 47 - DESIGN CURVES FOR SINGLE-SPAN, PINNED-BASE, LEAN-TO FRAMES LOCATION OF PLASTIC HINGE