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Welded Continuous Frames and Their Components

RESEARCH IN PLASTIC DESIGN OF MULTI-STORY FRAMES

by
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1. INTRODUCTION

The eighth National Engineering Conference, held at Lehigh University in 1956, was devoted in its entirety to the subject of plastic design(1). At that time there were no plastically designed structures in the United States, or at least there were none that were a matter of record. Undoubtedly there must have been a number of structures designed on the basis of what John Griffiths has described as a "plastic excuse for an elastic design"; and of course the provision in the former edition of the AISC Specification that permitted a twenty percent increase of stress at points of interior support could only be justified on the basis of the same concepts that lead to the current plastic design techniques.

The purpose of these remakrs is to review briefly the advances made since 1956 and to describe in some detail the research being conducted at Lehigh University on the plastic design of multi-story building frames. The latter discussion is divided into two parts: braced frames and unbraced frames. In addition the results of recent pilot tests on a three-story structure are presented.

What have been the developments in plastic design since the 1956 conference? The technique was first used in the United States the year following that conference, the authorization being based on the

Proceedings of that conference[1]. In 1958 the AISC issued a supplement to its basic specification, these "supplementary rules" being the first codification of the concept in the United States[2]. The AISC Manual "Plastic Design in Steel" followed very shortly in 1959[3]; it presented practical procedures for the plastic design of simple or continuous beams and one- or two-story rigid frames. By early 1960 plastic design was widely used throughout the United States, and had been adopted in most of the major building specifications. A detailed review of developments during this time of rapid expansion is available in Ref. 4.

The year 1961 saw the culmination of an effort, also begun in 1956, to give complete substantiation of this design technique. The ASCE issued its Manual No. 41, "Commentary on Plastic Design in Steel", giving the theoretical background for the method, secondary design considerations, experimental varification of the theory, and design guides[5]. Later this same year the "Supplementary Rules" became (in updated form) "Part 2" of the new AISC Specification[6]. In addition, many of the provisions of allowable stress design (part 1 of the Specification) were modernized based on knowledge of the plastic behavior of steel.

2. RESEARCH DEVELOPMENTS

What about the research work since 1956? Although a few specific details remained to be completed in connection with the earlier work, the principal effort at Lehigh University has been on two major extensions of the method:

1. Application to high strength steels
2. Application in a more general way to multi-story frames.

The work on high strength steels began in July of 1962 and is still underway. It constitutes, in the main, a major "checking" job (both theoretical and experimental) to see what modifications would be required in the application of plastic design to a steel of higher strength than A7 or A36. The results to date indicate that the same factors that were significant in A7 steel for plastic desing are also significant in A441--namely, that the stress-strain disgram should have a flat plateau or plastic range followed by a positive strain-hardening characteristic. Some further experimental work remains to be done, but it appears that design recommendations should be available shortly. The work not only extends the previous design guides to higher strength steels, but also provides new information on local buckling, the deformation capacity of beams and beam-columns, and the bracing requirements of inelastically deformed members[(7)].

Work on plastic design of multi-story building frames was underway to a modest extent at the time of the 1956 conference. In 1958 research began in earnest to provide a more complete application to tall buildings. Although the studies followed the lines of previous research, the problems and the approach were quite different. In the case of continuous beams and single story frames, the method of analysis (the formation of mechanisms) was available almost from the beginning. For such structures the major effort that was needed to bring plastic design to the point that it could be used was to determine the plastic behavior of isolated members (beams, columns, connections); to learn the influence in the inelastic range of shear, axial force, repeated load, and instability; and to establish experimental confirmation of the theories developed.

Not so with multi-story frames: The basic plastic analysis of the tall building had been explored for specific cases[8,9], but remained undeveloped as a practical design technique. So the emphasis in the new research was different. Building on the knowledge of the behavior of structural components, it was necessary to take an intermediate step and study the behavior of a "subassemblage", an element of a structure consisting of a group of columns with beams attached. Under the heading of "subassemblages" one of the first detailed studies was of restrained columns. Other studies were of frame stability, sway deflections, and bracing. Also, in contrast to the earlier work, much more effort has been required in developing plastic analysis techniques and design procedures.

Figure 1 gives two sketches symbolizing the principal types of frames being studied at Lehigh. These are frames of regular shape with relatively uniform column heights and bay spacings. The studies have been divided into two categories: "braced frames" and "unbraced frames."

The former include any of those structures for which sway due to instability under vertical loads or drift due to lateral loads is resisted with the aid of bracing in the form of diagonal X-bracing, K-bracing, knee-braces, or shear walls. All of these are symbolized in these figures by panels with X-diagonals. "Unbraced frames" are those in which all resistance to lateral drift and sidesway under vertical loads is provided by the rigid frame action of the structural frame work.

3. BRACED FRAMES

Studies of the problem of braced multi-story frames under vertical loads and under vertical load combined with horizontal load were considered first. The work on this aspect is nearing completion and should appear in the form of reports containing design guides and design charts. Indications of possible savings in steel are illustrated in Fig. 2, for a structure consisting of a ten-story five-bay frame with diagonal bracing. In the lower portion is tabulated the weights of four different designs, presented in bar chart form. The shaded portion shows the weight of beams and the open part the weight of the columns.

The four designs in Fig. 2 were as follows: Two were carried out according to allowable stress concepts and two according to plastic design concepts. The allowable stress designs were on the basis of simple beams and continuous beams, respectively. The difference in the plastic design is in the design of the columns, one method using allowable stress formulas, the other using maximum load techniques. The resulting weights of the four designs indicate that increased weight saving is possible as more and more utilization is made of plastic load-carrying capacity of the members. For this example, the weight saving for the complete plastic design is 22% compared with the allowable stress design using "simple beam" analysis.

The method for the design of braced frames starts with designing beams and girders to support their expected dead and live load multiplied by the appropriate load factor. As shown in Fig. 3, basic beam mechanisms control the selection of the girders. The formation of beam mechanisms in all of the girders leaves a series of continuous columns subjected to thrust and moment as shown in Fig. 4. In the earlier work on plastic design the maximum column thrust was limited to 0.6 P_y, and this limitation appears in the current AISC Specification, Part 2. However, there is no theoretical limitation on calculating the behavior with higher P/P_y values[10], and use of the design information from these solutions is responsible for some of the economy illustrated in Fig. 2.

The requirements for the performance of bracing have also been investigated in the research[11,12]. As shown in Fig. 5 the overturning moment in a given story caused by vertical loads displaced horizontally creates shear which add to the lateral shears caused by wind or earthquake. Diagonal braces provide a resisting shear force that is proportional to the sway of the story and elongation of the brace. The equation in Fig. 5 is one form of the solution for the area of the bracing member to resist shears due to lateral displacement of vertical column loads[11].

Sometimes the controling condition for the design of a column arises from checkerboard loading (Fig. 6), which causes single curvature bending in addition to high column thrust. In this condition some of the girders carry only dead load and therefore remain "elastic" at ultimate load, and these girders give added restraint to the columns. An important part of the research work has been an attempt to utilize this available restraint when proportioning columns. The design of a column can be improved by recognizing the help which it can receive from framing members which remain elastic.

One way to study such a restrained column is to consider a subassemblage formed of a column and the girders framing into it (Fig. 7). Moments equal in magnitude to the plastic hinge value in adjacent beams are applied to the stub ends at the same time that the thrust is applied to the column. These moments are the plastic hinge moments introduced as the result of full loading applied to the girders--moments that must be shared by the column and the restraining beams.

Figure 8 shows a photograph of a test setup that was developed to simulate the condition just described[13]. The vertical column (whitewashed) is shown in the center of the photograph. The tension jack and tension dynamometer through which the plastic moment is applied are shown in front of the column. The restraining beams extend away from the column to a support point.

Figure 9 shows symbolically the behavior to be expected of a typical restrained column in a subassemblage with external moment and thrust applied at the column tops. The figure shows the moment vs. rotation behavior of the beam, the moment vs. rotation of the column, and the moment vs. rotation of the entire joint or assemblage. The joint moment is the sum of the beam and column moments for the same rotation. It is particularly significant that the joint assembly can reach its maximum moment even after the column moment has started to decrease. At the instant at which the joint itself has reached the maximum moment, the column is somewhat below its point of maximum moment; however the beam is still on the increasing portion of its moment-rotation curve. This is one of the new concepts of structural behavior that has been solved and explained, and recently it has been confirmed experimentally[14].

Currently some tests of complete frames are being planned to help verify the procedures for the analysis of braced frames mentioned earlier. As shown in Fig. 10 the test setup involves 6-in. wide-flange columns and 12-in. beams in a two-bay, three-story structure. The testing scheme will involve four different loading conditions: full gravity load, checkerboard gravity load, gravity load plus wind load, and checkerborad gravity load plus wind load.

The planning of a suitable test setup for these frames has proven to be quite a challenge. In order to accommodate lateral motion and to avoid restraint by the loading system, what is really needed is a floating high capacity testing machine (or a way to carry 100 tons in a five pound bag!) After considering the use of dead weights, lever systems, block and tackle, hydraulic jacks, and testing machines, a system has been designed at Fritz Laboratory which makes use of a mechanism linkage which is able to apply vertical loads independently to each girder. It will permit sway of unbraced frames with no restraint. A pilot model of the "gravity load simulator" has been tested, and the results were so successful that the final arrangements for the first frame tests are now proceeding. Fig. 11 shows a diagrammatic sketch of the simulator with the mechanism in two different positions. Note that the direction of load application is always vertical and that the entire system can move freely as a unit in the plane of the test frame.

4. COLUMN DEFLECTION CURVES

A new concept which was developed in the multi-story frame research, is the "Column Deflection Curve." It is one of the important "building blocks" in current studies of both braced and unbraced frames[10]. Referring to Fig. 12, the column deflection curve is the shape that a compressed member will take when held in a bent position by an axial thrust. These curves are obtained by solving the equilibrium equation for the member, and there are no stability considerations involved. Each curve is defined by the load P and its end rotation θ_o, with the length L being a function of the values chosen for P and θ_o.

What is the significance and usefulness of the column deflection curve? The usual column in a building is one loaded with thrust and end bending moments. Consider, as shown in Fig. 13, a column loaded with equal and opposite moments and supporting axial load. This is the same condition as a member loaded with axial thrust and with equal end eccentricities. Such a member will assume the bent position shown in the third sketch as load is applied. The curve is not only the deflected shape of the column; since the bending moment is the product of applied load and distance to the deflected postion, this curve is also the shape of the moment diagram. As shown in the fourth sketch, this curve can be extended to the point at which the moment equals zero, and the only loading now necessary is the axial thrust. Such a curve is half of a column deflection curve. By drawing the mirror image, the full column deflection curve would be obtained (Fig. 12), the important characteristics being the length L, the load P, and the end slope θo.

Now, what can be obtained from the column deflection curve? Figure 14 shows some particular solutions of the equilibrium configuration of bent columns--solutions that are obtained from column deflection curves. For every loading condition to which a column could be subjected, there would be a segment of a column deflection curve to match, since the column deflection curve can be shifted along the member until a matching condition is obtained. Thus in Fig. 14 four cases are shown. At the top is a column with equal and opposite moment. Next is shown a member pinned at the base with moment applied at the top. In the third illustration a column deflection curve is fitted to a column with equal moments at the ends and in the same sense. And finally is shown a similar geometry in which the end moments are unequal. This latter case is the one that frequently would be encountered in building columns.

In Fig. 15 a column from a tall building (shown at the left) is sketched to a larger scale at the right. An appropriate column deflection curve would be fitted to the deformed shape of the member in the building. As indicated in Fig. 15, studies have shown that the load P_1 along the building column axis may be taken equal to P_2 (in the direction of the thrust line) as long as the thrust in the column is greater than 0.15 P_y [15]. The angles shown in Fig. 15 are very much exaggerated. If the drawing were to scale, it would be obvious that the angles are so small that the two loads are practically identical. The higher the load, the less the error.

5. UNBRACED FRAMES

Architectural requirements frequently require interior spaces which are free of diagonal bracing or shear walls. Thus the resistance to lateral loading and frame instability must be provided by rigid frame action. When compared with a building which is separately braced, this requirement places an economic penalty on the rigid frame structure, but nonetheless the best possible technique must be developed when conditions dictate use of an unbraced frame.

Figure 16 shows in a diagrammatic way the status of present studies on unbraced frames at Lehigh. In single-story frames the problem of stability under vertical load has been solved [16]. Work is nearing completion on the stability of single story frames subjected to combined vertical and lateral loads. Current work is well underway on the strength and stability of multi-story frames under vertical and combined vertical and lateral load.

An important consideration entering the picture for multi-story frames which did not need emphasis in single-story frames is the behavior of columns in a swayed position. In the plastic design of single-story structures, safe and adequate designs were assured when equilibrium of forces was calculated for the undeformed position of the structure. However, in multi-story frames, a correct solution is not always possible without considering equilibrium of the deformed structure.

In the studies of unbraced frames, restrained columns will again play a significant role. Additional information will be needed to use these curves. One of the questions now being studied is how to handle the boundary conditions which show the effect of other members in the structure upon the column under consideration. Procedures similar to plastic moment distribution, and slope-deflection are being examined. At the present time, computer programs are being developed which include the effects of residual stress and partial plasticity in the member.

6. FRAME TESTS

Preliminary pilot experiments have been conducted recently on a three-story unbraced frame. A photograph of the general setup is shown in Fig. 17. The structure was loaded with dead weights and a lever system. The test assembly was actually two parallel frames braced with diagonal bracing perpendicular to the plane of bending in order to prevent out-of-plane buckling. The girders of the frame were loaded with two concentrated loads distributed across the beam. Large concentrated loads would be involved in the lower columns--a situation which was simulated by the earlier tests on single story frames that preceded the current experiments[17]. The model was designed with "fence post" sections, with a depth of 2-5/8 in. and flange width of 1-5/8 in.

Figure 18 shows a sketch of the three story frame and the results of the first test in terms of deflection under load. To the left is shown the relationship between applied concentrated load on the beams vs. vertical deflection at the center of the top beam. The elastic slope approximates fairly well the predicted value shown by the dashed line. It is evident from the figure that the load approached very close to the predicted ultimate load. At this point the test was stopped because a beam mechanism was developing in the upper beam and it was evident that the whole assembly was being restrained. (The top beam would fail first because it is subjected to the least restraint by the columns.)

To the right in Fig. 18 is shown the lateral deflection at the top of the first floor column. This deflection, after an initial displacement, did not increase very rapidly--nor was it expected to increase until the point of frame instability had been reached. Even so, it was found that unexpected restraint was introduced as a result of misalignment of the loading system. It was because of the sensitivity of such experiments to very small restraints that the test was stopped and the loading system was rearranged.

In the revised setup, wire ropes were used instead of the solid round rods that had been employed in some parts of the loading system in the first test. The previously loaded frame was then realigned and subjected to a second test. In this test the restraints were very small indeed, and eventually an instability type of failure occurred.

Figure 19 shows the load deformation behavior of the "second test" on a basis similar to that described for the first test. The load vs. vertical deflection behavior in the "initial" region is similar to that obtained in the first test, the deflection being somewhat greater because a deformed frame was being tested.

The deflection of the structure horizontally under the action of loads applied in the second test also reflects the fact that the frame had been plastically deformed in the first experiment. Eventually the frame failed due to frame instability, but even so, the load increased significantly above an approximate critical load predicted by Merchants formula[18] and to a load that began to approach the ultimate load predicted by plastic theory. The upper portion of the curves are dotted because failure occurred while the loading system was being adjusted to preclude the development of restraints. The behavior of the structure indicates that some restraining force might have been present. Figure 20 shows a photograph of the second test of the three-story frame after failure. The bottom story had a "sudden" sway failure and it is the lower story which would be the most sensitive to such failure because of the proportionately higher axial loads present in the columns.

The results of the tests, although of a pilot nature, gave valuable information on experimental techniques. Even more important, they indicate that the load at which frame instability would occur may be much higher than previously considered possible. Although the evidence is incomplete this suggests that frame instability may not be as serious a limitation as had previously been supposed. Further tests on full-scale unbraced multi-story, multi-bay frames are needed to investigate this point. Such tests are being planned and will involve frames similar to that shown in Fig. 10, but with the bracing members removed. The gravity load simulator described earlier will also be used to apply load to the test frames.

One concept which appears to be quite promising for the plastic design of multi-story frames is the combined use of high strength steel columns with carbon steel girders proportioned according to restrained column theory. High strength steel columns that are short and stout can carry considerably more load than structural carbon steel columns. Frequently the carrying capacity of a column can be increased considerably by increasing the size of the girders framing into it. Especially when the girder remains elastic, the failure load of a restrained column is increased.

Recent studies of a two-story portion of a multi-story building have been made in which the sizes of A36 girders were increased to permit a reduction in the size of A441 columns. The net decrease in weight was 1.2%, and the net decrease in cost of steel was 2.5%; but the most dramatic result was the net decrease of 43% in sway of each story. The results of these and other studies show that the sway due to bending of the girders is about 80% of the total sway deflection. Increasing the girder sizes in less expensive structural carbon steel permits a reduction in the size of the more expensive high-strength steel columns. It also provides relative column and girder stiffnesses that are favorable for the resistance of lateral deflection and frame instability.

7. SUMMARY

In recent years it has been possible to make considerable progress in research leading to the extension of plastic design to high-strength steels and to multi-story frames. For the former, the analysis and design concepts are quite similar to those for structural carbon steels, and specific provisions should be available shortly.

In "braced" multi-story frames it has been shown that additional economies are possible in comparison with methods currently being used. Methods of proportioning the bracing members have been developed. By considering a "subassemblage", an elementary unit of a larger structural framework, it has been shown that a more complete utilization of the strength of columns and of the restraining influence of beams will be possible in plastic design of multi-story frames.

In "unbraced" frames, those which depend on the rigid frame itself to resist lateral loads and frame instability, the plastic analysis of the frame is a complex problem primarily because the formulation of equilibrium conditions must consider the deformed shape of the structure. Computer programs are being developed for "precise" solutions against which simpler approximate design procedures can be tested.

The results of pilot tests of three-story frames tend to indicate that the problem of frame instability may not be as serious as previously supposed. Further evidence is needed on this point, towards which tests are currently being planned. Also it has been shown that the substitution of high-strength steel for structural carbon steel in the columns, but using structural carbon steel girders (and with due consideration of economy) can decrease the drift in a story height by as much as 40% or more.

Additional tests of large multi-story frames will be made in the near future to correlate with current theoretical studies. These tests will be facilitated through the recent development of a "gravity" load simulator" which permits the application of jack-induced gravity loads without restraining the frame from lateral motion.

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as are the major contributions being made by members of the research project staff at Fritz Laboratory.

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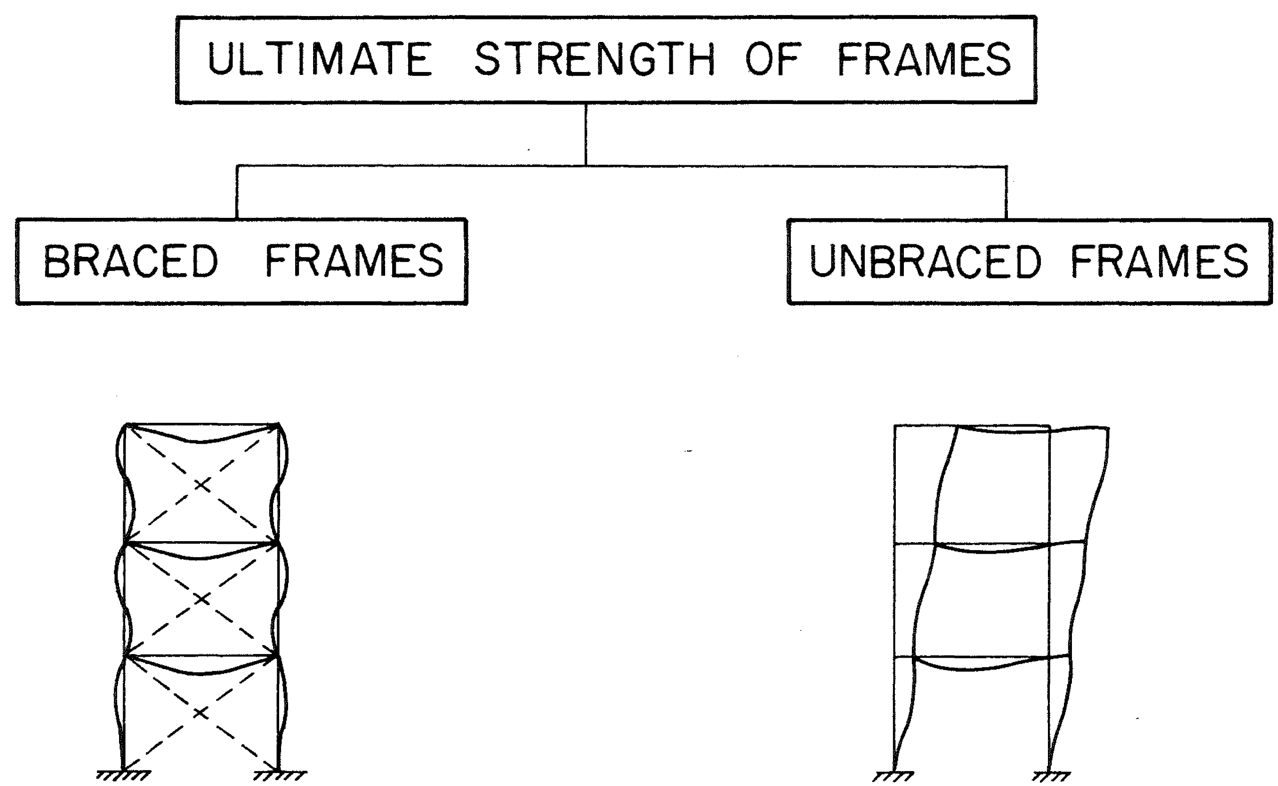

ULTIMATE STRENGTH OF FRAMES
BRACED FRAMES
UNBRACED FRAMES

FIG 1

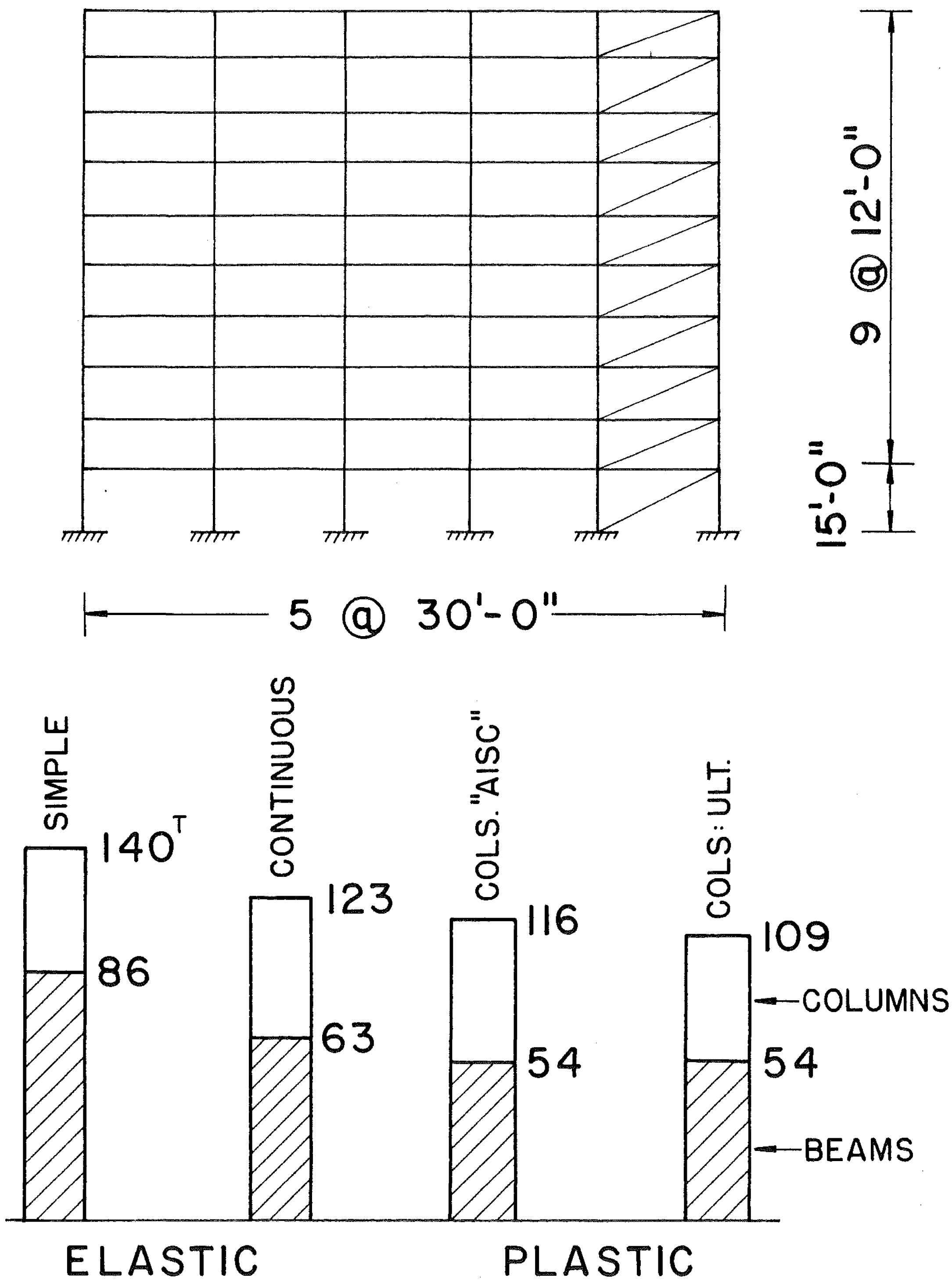

9 @ 12'-0"
15'-0"
5 @ 30'-0"
SIMPLE
140T
86
CONTINUOUS
123
63
COLS. "AISC"
116
54
COLS: ULT.
109
54
COLUMNS
BEAMS
ELASTIC
PLASTIC

FIG 2

2/3.56

SLIDE 273.26

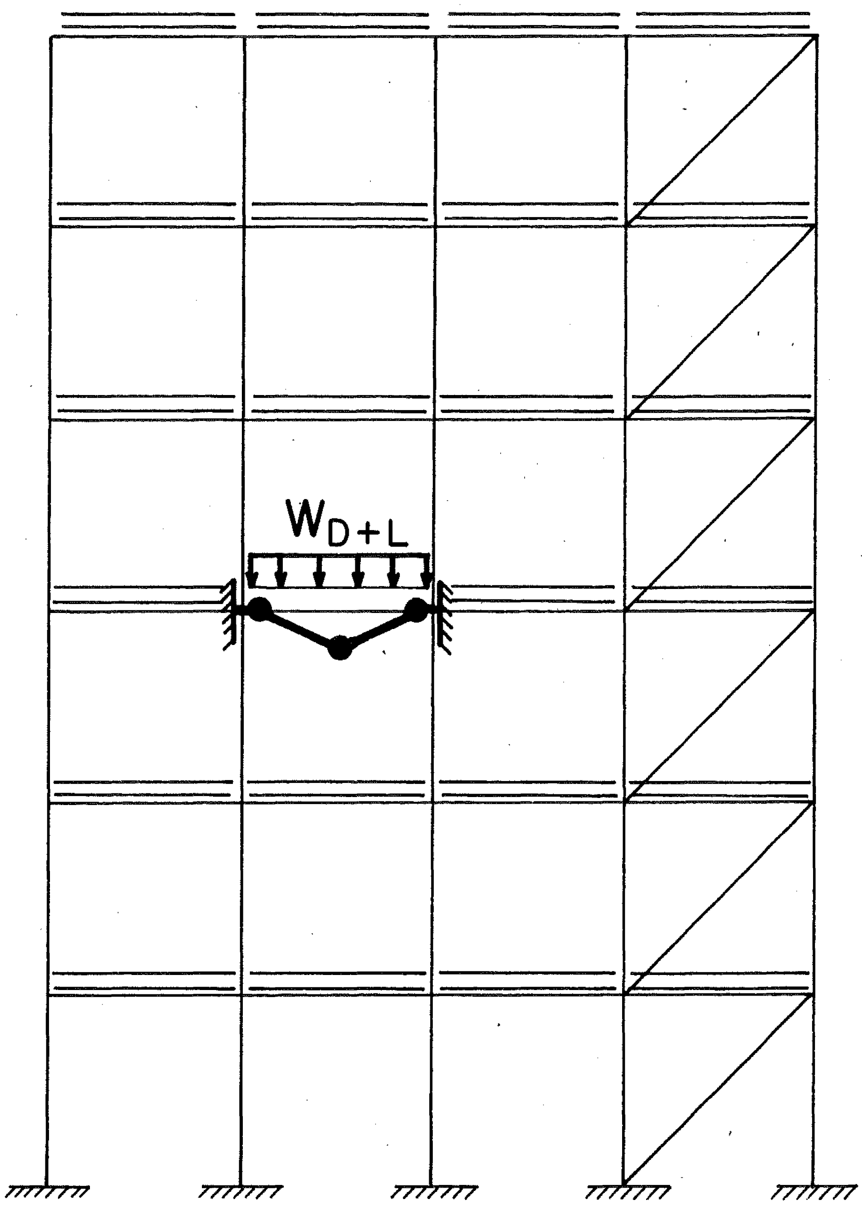


FIG 3

3
273
③

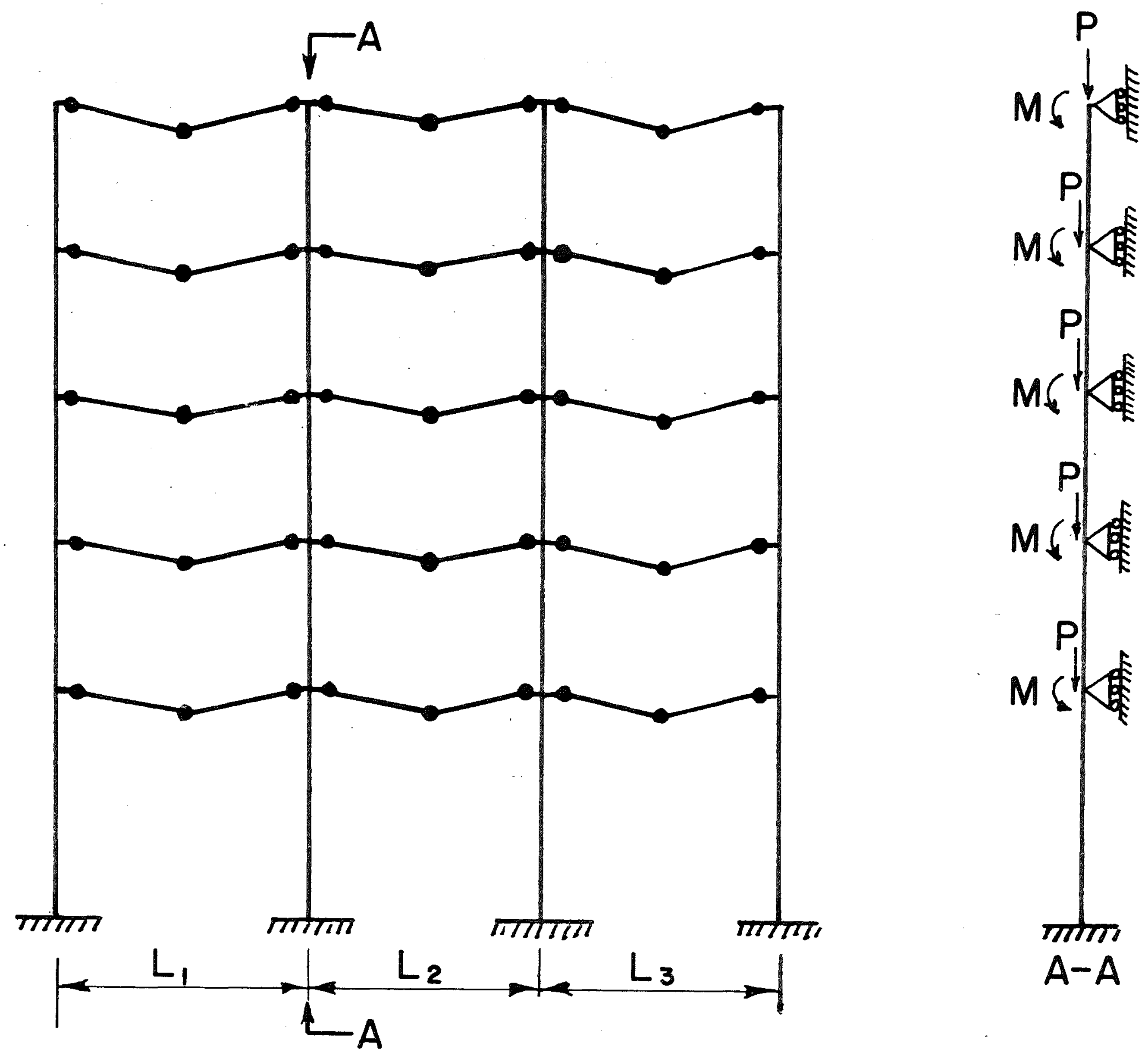
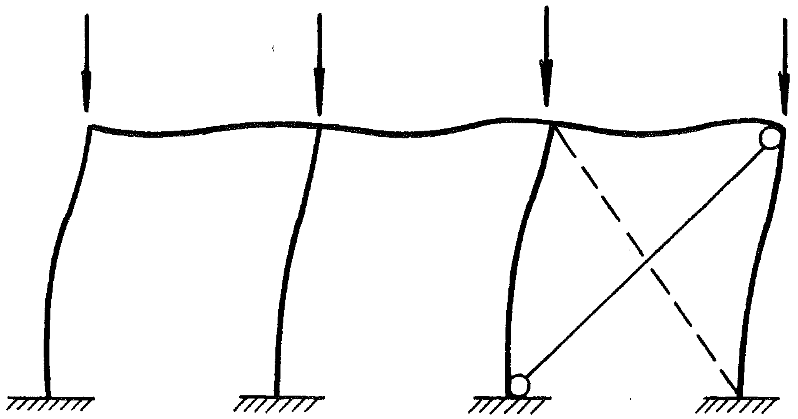


FIG. 4

BRACING



$$A_{br.} = \frac{1}{E} \frac{\ell^3}{hL^2} \Sigma P$$

FIG 5

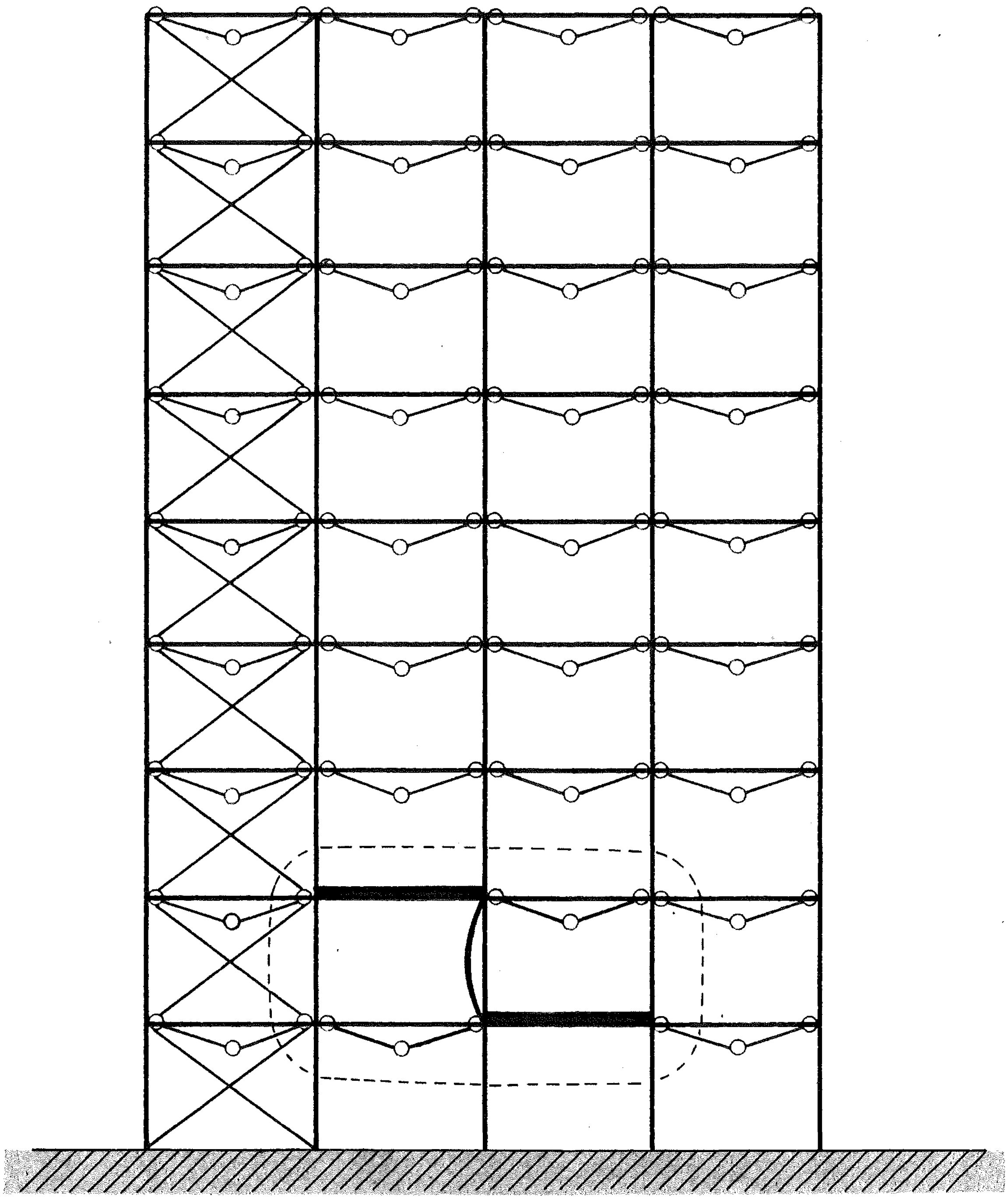
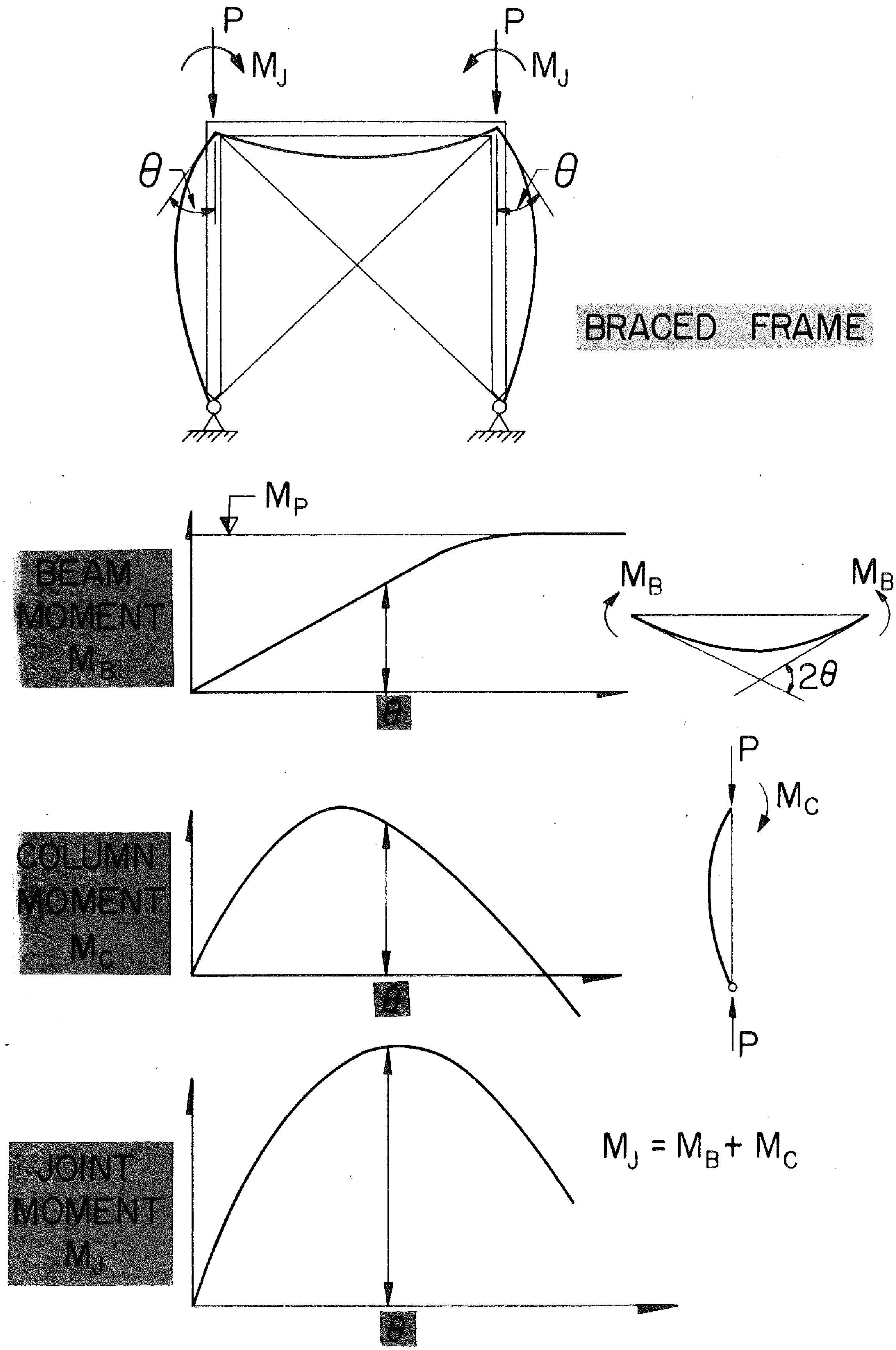


FIG 6



P
M_J
P
M_J
θ
θ
BRACED FRAME
M_P
BEAM MOMENT M_B
θ
M_B
M_B
2θ
P
M_C
COLUMN MOMENT M_C
θ
P
JOINT MOMENT M_J
$M_J = M_B + M_C$
θ

FIG 9

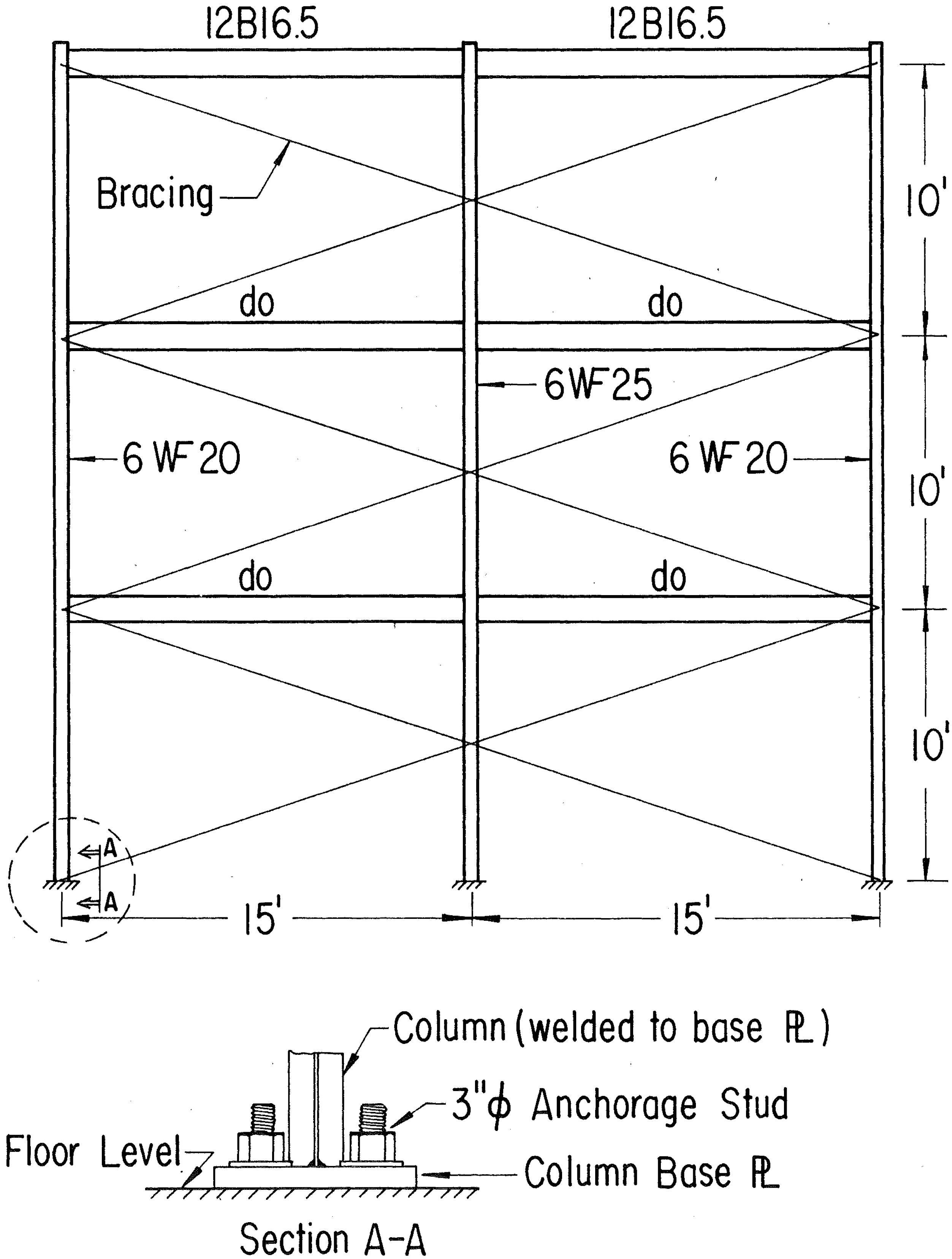
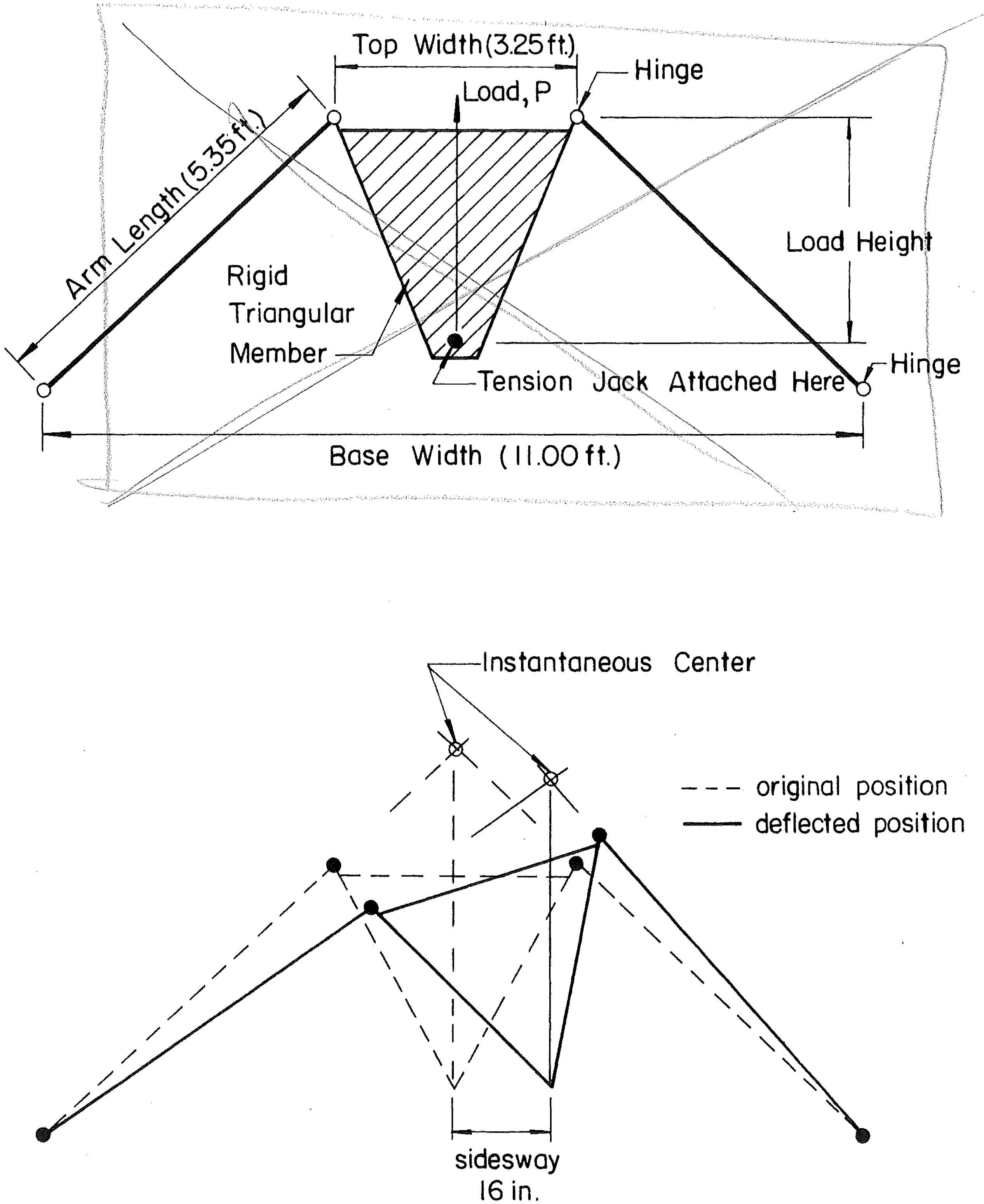
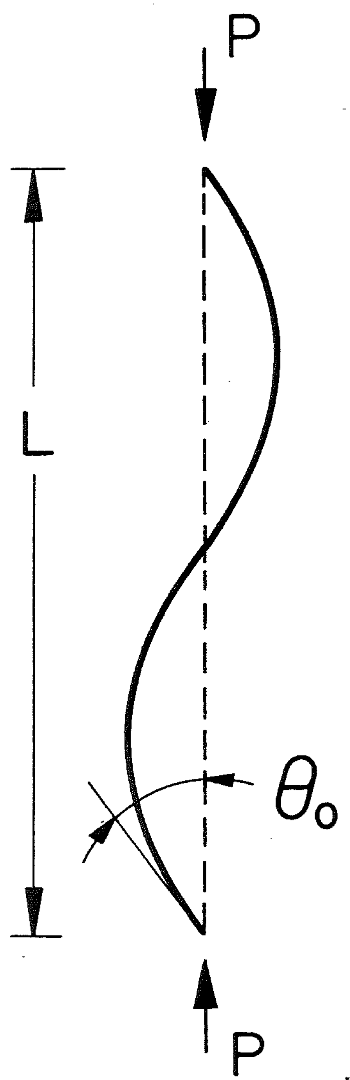

12B16.5
12B16.5
Bracing
do
do
6WF25
6 WF20
6 WF20
do
do
10'
10'
10'
A
A
15'
15'
Column (welded to base PL)
3"φ Anchorage Stud
Floor Level
Column Base PL
Section A-A

FIG 10



Top Width (3.25 ft.)
Hinge
Load, P
Arm Length (5.35 ft.)
Load Height
Rigid
Triangular
Member
Tension Jack Attached Here
Hinge
Base Width (11.00 ft.)
Instantaneous Center
original position
deflected position
sidesway
16 in.

FIG 11 Fig 11



COLUMN DEFLECTION CURVE

Deflected Shape for a given

LOAD	P
LENGTH	L
END SLOPE	θ_o
MEMBER	

FIG. 12

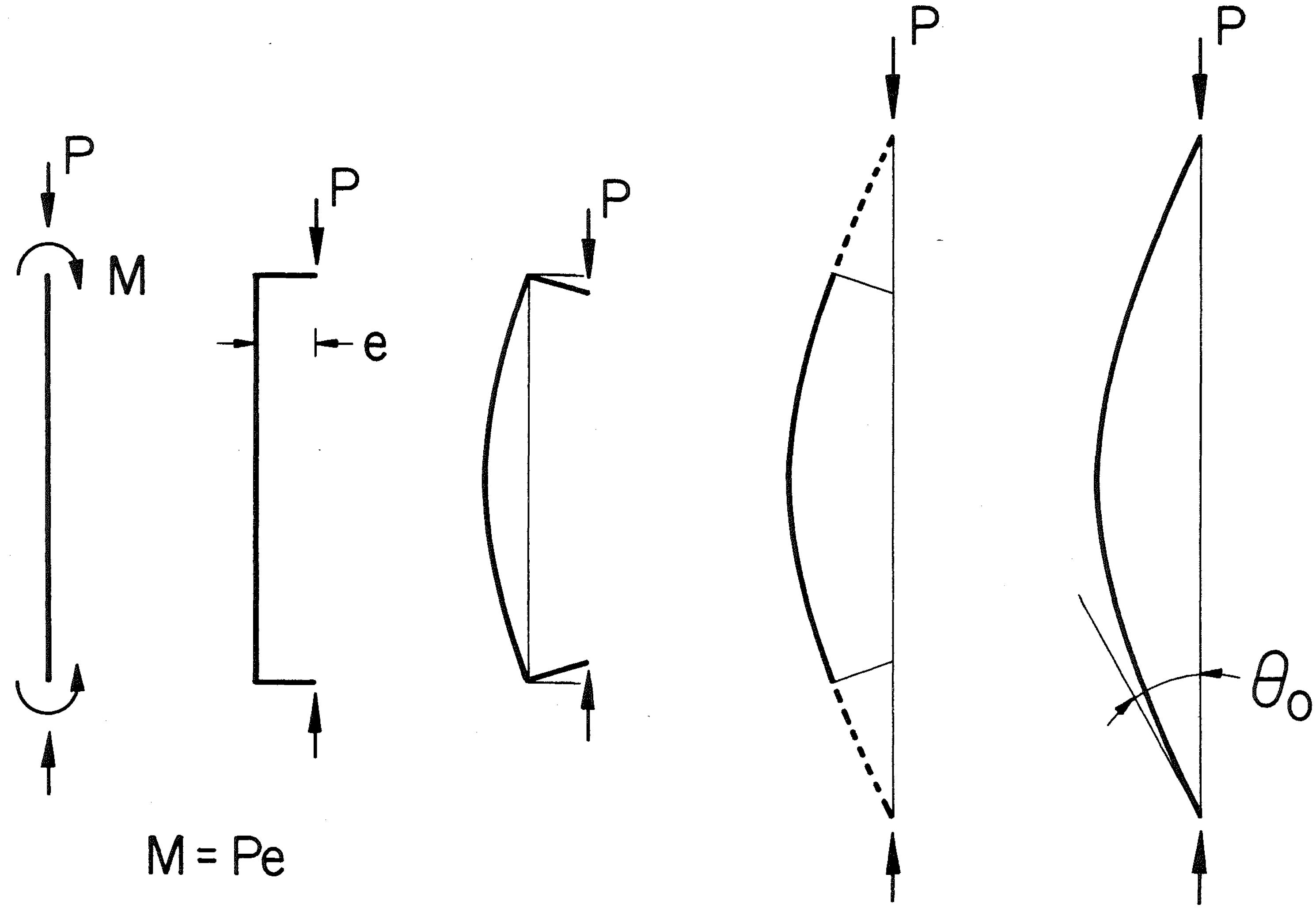


FIG 13

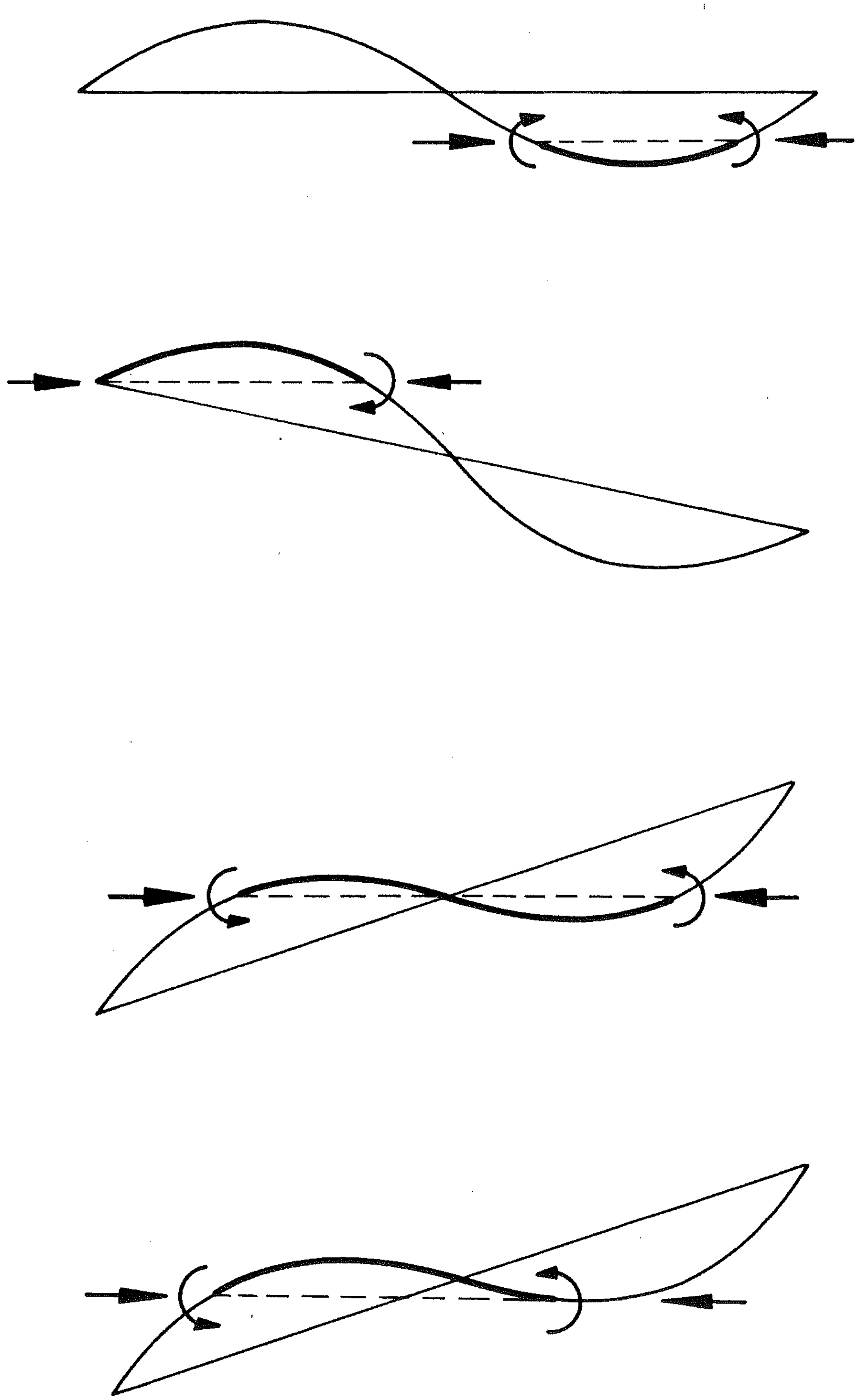


FIG 14

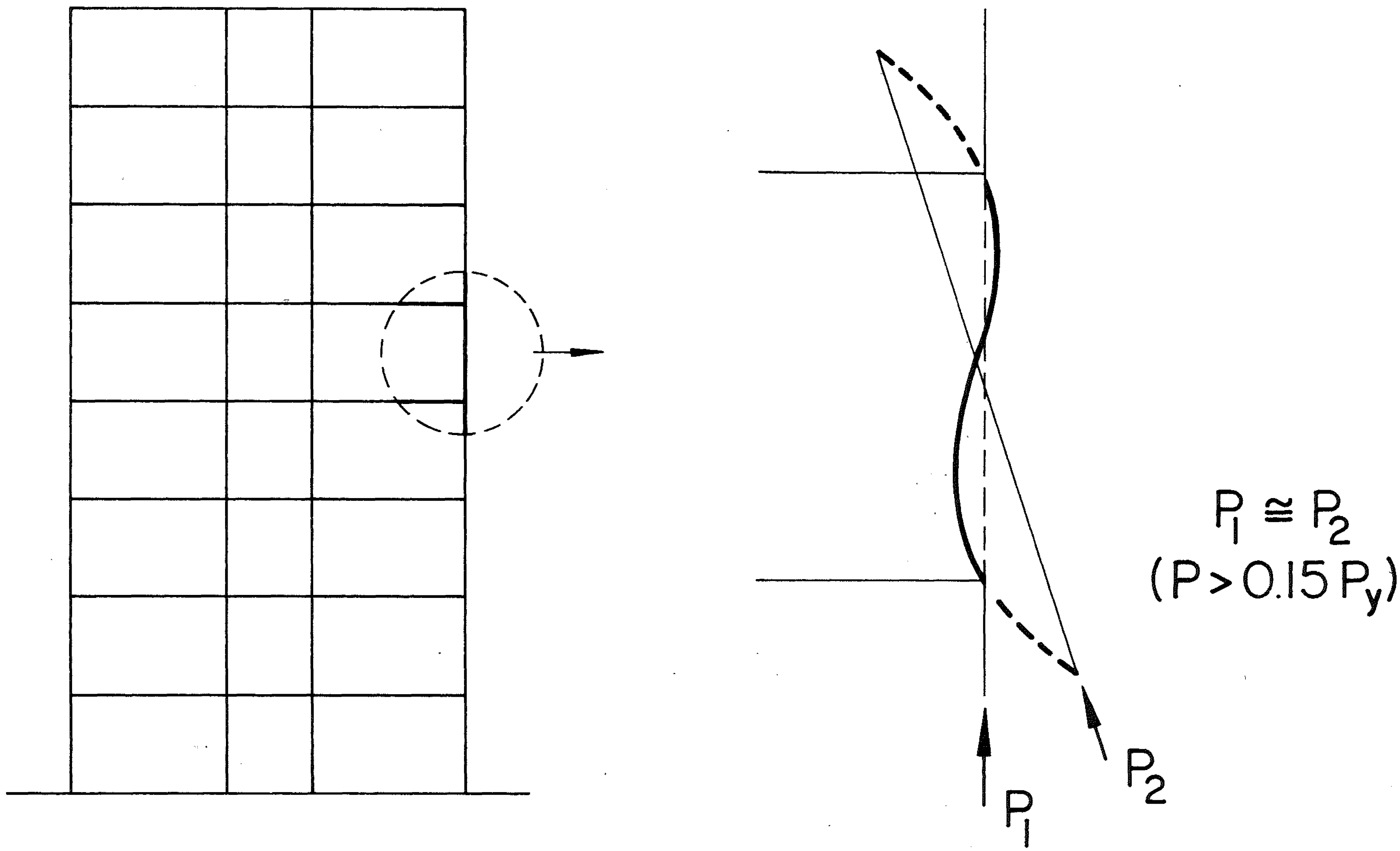


FIG 15

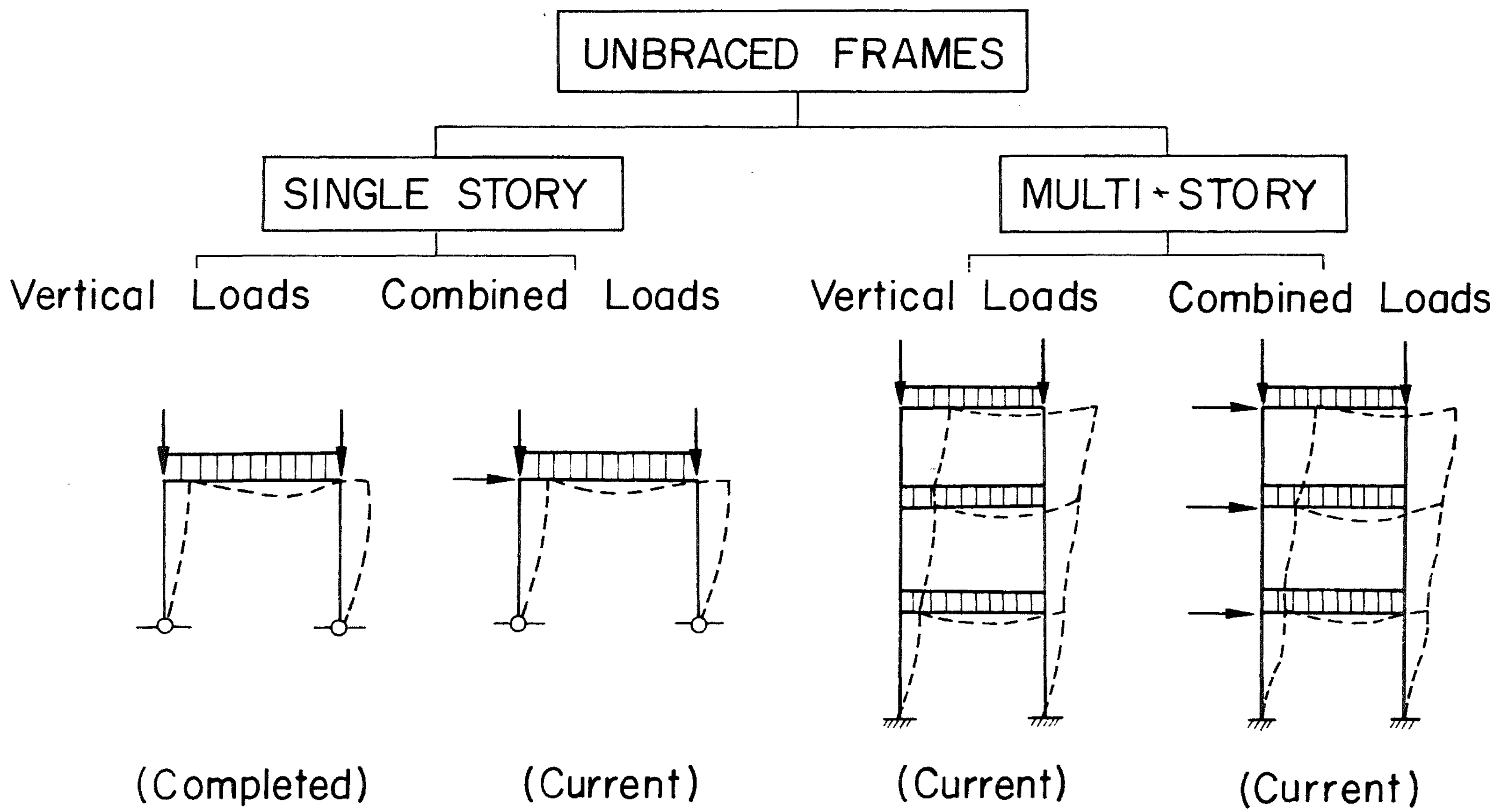
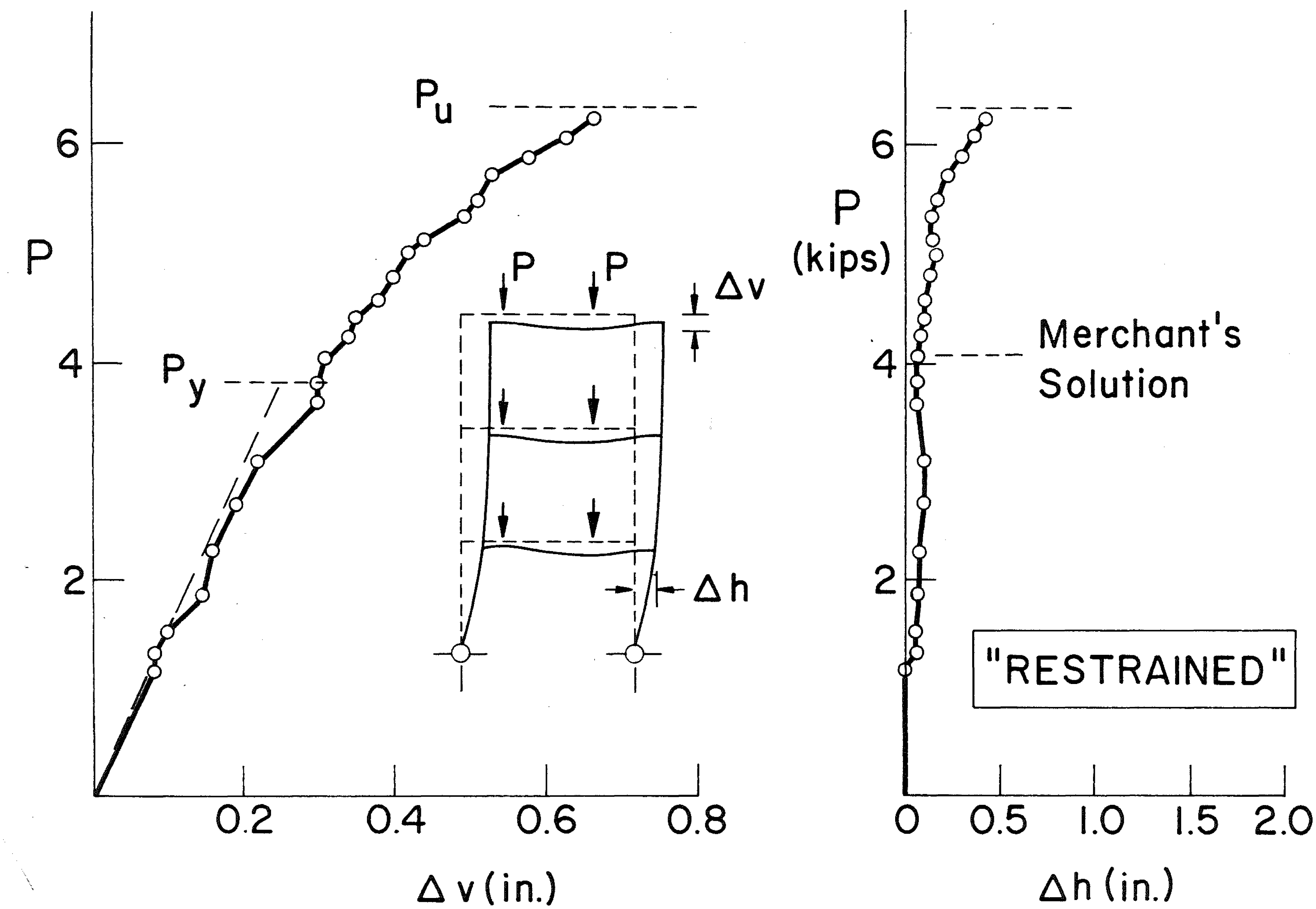
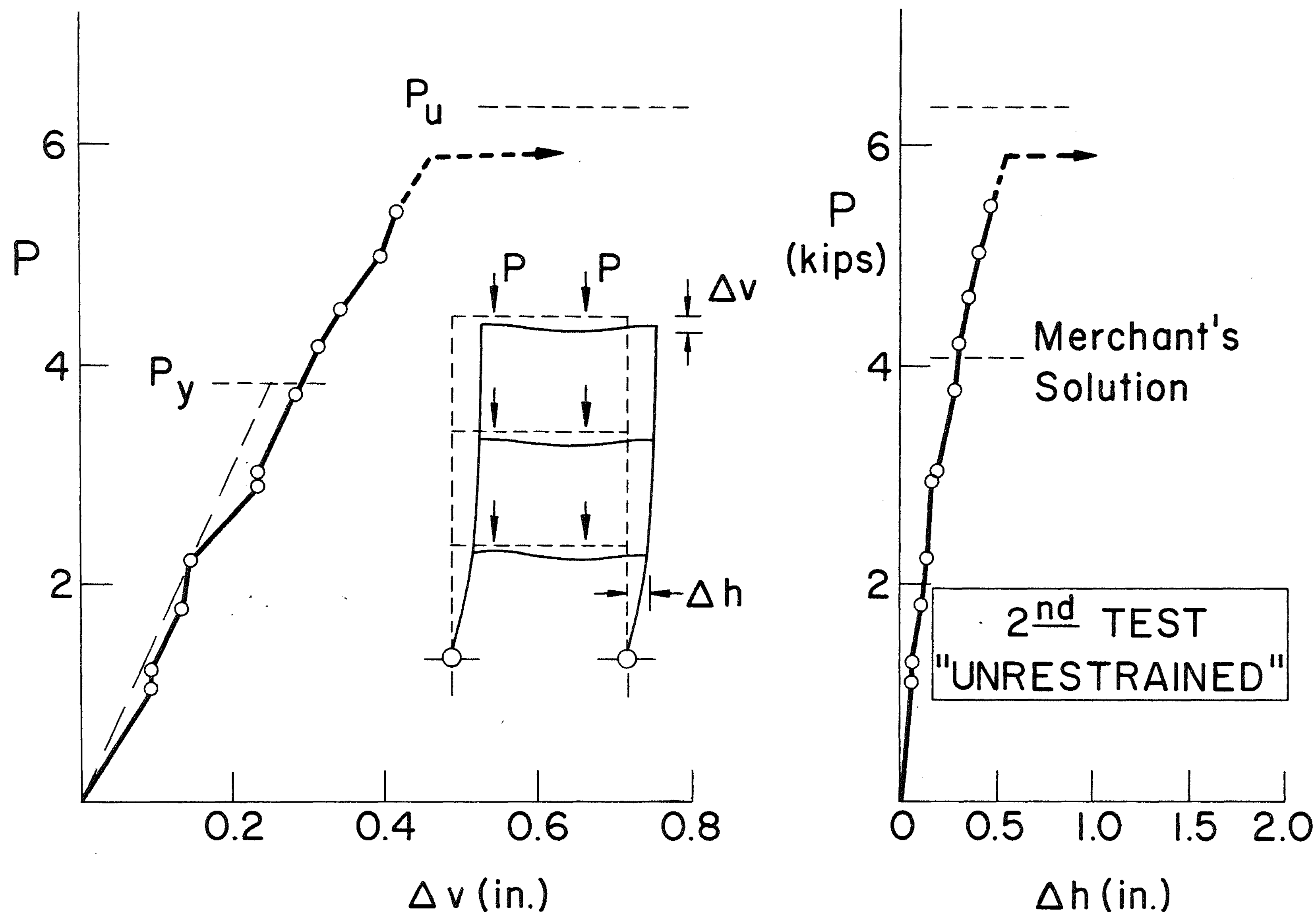

UNBRACED FRAMES
SINGLE STORY
MULTI-STORY
Vertical Loads
Combined Loads
Vertical Loads
Combined Loads
(Completed)
(Current)
(Current)
(Current)

FIG 16



P
6
4
2
P_u
P_y
P
P
Δv
Δh
0.2
0.4
0.6
0.8
Δv (in.)
P
(kips)
Merchant's
Solution
"RESTRAINED"
0
0.5
1.0
1.5
2.0
Δh (in.)

FIG 18



P
6
4
2
P_u
P_y
0.2
0.4
0.6
0.8
Δv (in.)
P
P
Δv
Δh
P
(kips)
Merchant's
Solution
2nd TEST
"UNRESTRAINED"
0
0.5
1.0
1.5
2.0
Δh (in.)

FIG 19