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Civil and Environmental Engineering

1983

An elastic analysis of a cantilever slab panel subjected to an in-plane end shear, May 1983

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Diaphragm Behavior of Floor Systems and
Its Effect on Seismic Building Response

AN ELASTIC ANALYSIS OF A CANTILEVER SLAB PANEL SUBJECTED TO AN IN-PLANE END SHEAR

by

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This work has been carried out as part of an investigation sponsored by the National Science Foundation.

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Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

May, 1983
Fritz Engineering Laboratory Report No. 481.1

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ABSTRACT

An analytical solution is presented for the elastic response of a slab panel subjected to an in-plane end shear. The loading condition is separated into a pure shear component and an essentially bending component. Simplified approximations are provided and example applications are included.

I. INTRODUCTION

In a building structure containing several lateral load resisting systems, the floor systems act as diaphragms connecting these vertical systems and control the distributicn of lateral load among the several parallel systems. The basic behavior of a floor panel under such a condition may be taken to be that of a cantilever deep beam subjected to a distributed shear load on its free end as shown in Fig. 1a. Because of the geometry of the panel, where the planar dimensions B and H are typically of the same order of magnitude, and the thickness is much smaller, a satisfactory analysis cannot be achieved by the conventional methods of strength of materials. This report presents an analytical solution based on a separation of the shear and bending effects.

The distribution of the shear load T at the end of the floor slab panel is generally not known. In the proposed solution, a uniform distribution is initially assumed, considering that the loading is typically induced either by inertia (as in the case of earthquake) or relative displacement between the connected vertical systems. It is then possible to resolve the present problem into two component parts, as shown in Fig. 1b and 1c. The first component, shown in Fig. 1b, represents a pure shear condition. For a member of uniform thickness,

$$\tau_o = \frac{T}{Bt} \tag{1-1}$$

and

$$\Delta_s = \frac{\tau_o}{G} H = \frac{TH}{GBt} \tag{1-2}$$

where τ_o = Uniform shearing stress in the member

t = Thickness of slab

G = Shear modulus of elasticity

The second component of the load, shown in Fig. 1c, causes the slab panel to deform in an essentially "bending" mode. The free end displacement caused by this load, Δ_b, will be referred to as the "bending displacement" in this report. The solution for Δ_b is presented in the next section. It is clear that the total deflection Δ in Fig. 1a can be obtained by superposition.

$$\Delta = \Delta_s + \Delta_b \tag{1-3}$$

It should be pointed out that a conceptual difference exists between the shear and bending displacements defined here and those commonly found in literature on mechanics of materials. In the later case, the two components are caused by the same load, but are derived from the separate (but coexisting) strain components. Here, they refer to separate loading conditions.

II. SOLUTION FOR THE "BENDING DISPLACEMENT"

Fig. 2 shows the "bending component" problem in an idealized form. The in-plane dimensions are H and B in the x and y directions respectively. The thickness of the plate, in z direction, is t. The plate is fixed along the edge $x=0$, and free on the edges $x=H$ and $y=\pm B/2$. The loading consists of uniformly distributed shear stress τ_o (per unit area) on the edges $y=\pm B/2$. All other surface forces on the free edges are zero.

The problem as defined is a plane stress problem. It is well known that such a problem is solved by the biharmonic differential equation with appropriate boundary conditions.[2]

$$\nabla^2\nabla^2\phi = \frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = 0 \tag{2-1}$$

where ϕ = Airy stress function

which is related to the stress components by

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\phi}{\partial x^2} \quad \text{and} \quad \tau_{xy} = \frac{\partial^2\phi}{\partial x\partial y} \tag{2-2}$$

To facilitate a series solution for the stress function, the constant shearing stress τ_o on the edges $y=\pm B/2$ is first replaced by its Fourier series equivalent, using H as quarter period length for the fundamental mode.

$$\tau = \frac{4\tau_o}{\pi}\sum\frac{1}{m}\,\text{Sin}\,\frac{m\pi}{2H}x \qquad m = 1, 3, 5, \ldots$$

Or, defining

$$\alpha = \frac{m\pi}{2H}$$

$$\tau = \frac{2\tau_o}{H}\sum\frac{1}{\alpha}\,\text{Sin}\,\alpha x \qquad \frac{2H\alpha}{\pi} = 1, 3, 5, \ldots \tag{2-3}$$

The selection of H for quarter period length is necessary to satisfy the boundary conditions, as will be shown later.

It is now appropriate to suggest the following solution for the stress function:

$$\phi = \Sigma\phi_\alpha = \Sigma \frac{1}{\alpha} \text{Cos } \alpha x f_\alpha(y) \tag{2-4}$$

where each ϕ_α satisfies the biharmonic equation (2-1) separately. Solution of the biharmonic equation leads to:

$$f_\alpha(y) = C_1\text{Cosh}\alpha y + C_2\text{Sinh}\alpha y + C_3 y\text{Cosh}\alpha y + C_4 y\text{Sinh}\alpha y$$

From Equation (2-2)

$$\sigma_x = \Sigma \frac{1}{\alpha} \text{Cos}\alpha x f''_\alpha (y)$$

$$\sigma_y = - \Sigma\alpha \text{ Cos}\alpha x f_\alpha(y)$$

$$\tau_{xy} = \Sigma \text{ Sin}\alpha x f'_\alpha(y)$$

The stress field in the member is skew-symmetric with respect to the x-axis, i.e.,

$$\sigma_{x(y)} = -\sigma_x(-y)$$

$$\tau_{xy}(y) = \tau_{xy}(-y)$$

Consequently, $f''_\alpha (y)$ must be odd functions of y, and $f'_\alpha(y)$ must be even.

Hence, $C_1 = C_4 = 0.$

The general solution of this problem is, therefore, as follows:

$$\phi = \Sigma \frac{1}{\alpha} \text{Cos}\alpha x \, (C_2 \text{Sinh}\alpha y + C_3 y \text{Cosh}\alpha y) \tag{2-5}$$

$$\sigma_x = \Sigma\alpha \, \text{Cos}\alpha x \, (C_2 \text{Sinh}\alpha y + C_3 y \text{Cosh}\alpha y + \frac{2}{\alpha} C_3 \text{Sinh}\alpha y) \tag{2-6}$$

$$\sigma_y = - \Sigma\alpha \, \text{Cos}\alpha x \, (C_2 \text{Sinh}\alpha y + C_3 y \text{Cosh}\alpha y) \tag{2-7}$$

$$\tau_{xy} = \Sigma\alpha \, \text{Sin}\alpha x \, (C_2 \text{Cosh}\alpha y + C_3 y \text{Sinh}\alpha y + \frac{1}{\alpha} C_3 \text{Cosh}\alpha y) \tag{2-8}$$

In equations (2-5) through (2-8), the summations are over the values of α such that

$$\frac{2H\alpha}{\pi} = m = 1, 3, 5, \ldots$$

The coefficients C_2 and C_3 are determined by the boundary stress conditions on $y = \pm B/2$, where

$$\sigma_y = 0 , \qquad \tau_{xy} = \tau$$

Substituting Equations (2-3), (2-7) and (2-8), and equating the corresponding terms of each series

$$C_2 \text{Sinh} \frac{\alpha B}{2} + C_3 \frac{B}{2} \text{Cosh} \frac{\alpha B}{2} = 0$$

$$C_2 \text{Cosh} \frac{\alpha B}{2} + C_3 \left\{ \frac{B}{2} \text{Sinh} \frac{\alpha B}{2} + \frac{1}{\alpha} \text{Cosh} \frac{\alpha B}{2} \right\} = \frac{2\tau_o}{\alpha^2 H}$$

Solving these equations for C_2 and C_3

$$C_2 = \frac{4\tau_o}{\alpha H} \frac{\frac{B}{2} \text{Cosh} \frac{\alpha B}{2}}{\alpha B - \text{Sinh}\alpha B}$$

$$C_3 = -\frac{4\tau_o}{\alpha H} \frac{\text{Sinh} \frac{\alpha B}{2}}{\alpha B - \text{Sinh}\alpha B}$$

Substituting into the general solutions (2-5) through (2-8)

$$\phi = \frac{4\tau_o}{H} \Sigma \frac{1}{\alpha^2} \text{Cos}\alpha x \frac{\frac{B}{2}\text{Cosh}\frac{\alpha B}{2}\text{Sinh}\alpha y - \text{Sinh}\frac{\alpha B}{2}y\text{ Cosh }\alpha y}{\alpha B - \text{Sinh}\alpha B} \tag{2-9}$$

$$\sigma_x = \frac{4\tau_o}{H} \Sigma \text{ Cos}\alpha x \frac{\frac{B}{2}\text{ Cosh}\frac{\alpha B}{2}\text{ Sinh}\alpha y - \text{Sinh}\frac{\alpha B}{2}y\text{ Cosh}\alpha y - \frac{2}{\alpha}\text{ Sinh}\frac{\alpha B}{2}\text{ Sinh}\alpha y}{\alpha B - \text{Sinh }\alpha B} \tag{2-10}$$

$$\sigma_x = -\frac{4\tau_o}{H} \Sigma \text{ Cos}\alpha x \frac{\frac{B}{2}\text{Cosh}\frac{\alpha B}{2}\text{ Sinh}\alpha y - \text{Sinh}\frac{\alpha B}{2}y\text{Cosh}\alpha y}{\alpha B - \text{Sinh }\alpha B} \tag{2-11}$$

$$\tau_{xy} = \frac{4\tau_o}{H} \Sigma \text{ Sin }\alpha x \frac{\frac{B}{2}\text{ Cosh}\frac{\alpha B}{2}\text{ Cosh}\alpha y - \text{Sinh}\frac{\alpha B}{2}y\text{ Sinh}\alpha y - \frac{1}{\alpha}\text{Sinh}\frac{\alpha B}{2}\text{ Cosh}\alpha y}{\alpha B - \text{Sinh }\alpha B} \tag{2-12}$$

Along the end boundary $x=H$, $\alpha x = \frac{m\pi}{2}$. Therefore, for $m = 1, 3, 5, \ldots,$ $\cos \alpha x = 0$, and $\sin \alpha x = \pm 1$. The boundary stress condition that $\sigma_x = 0$ is clearly satisfied, but τ_{xy} does not automatically vanish. The zero shear stress condition is only partially satisfied in that the total shear force is self-balanced over each half of the end width ($-\frac{B}{2} \leq y \leq 0$ and $0 \leq y \leq \frac{B}{2}$).

$$\int_0^{\frac{B}{2}} \tau_{xy}\alpha y = \frac{4\tau_o}{H} \Sigma \frac{1}{\alpha} \text{ Sin }\alpha x \left. \frac{\frac{B}{2}\text{ Cosh}\frac{\alpha B}{2}\text{ Sinh }\alpha y - \text{Sinh}\frac{\alpha B}{2}y\text{ Cosh }\alpha y}{\alpha B - \text{Sinh }\alpha B} \right|_0^{\frac{B}{2}}$$

$$= 0$$

It is interesting to note that the validity of this relationship is independent of the value of x. The total shear force is self-balanced within each half-width at any transverse section, not only at the free end boundary.

The displacement boundary conditions at the fixed edge (x=0) will now be examined. From the stress solutions Equations (2-10), (2-11) and (2-12), the general expressions for the strain components are as follows:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma y \right)$$

$$= \frac{4\tau_o}{EH} \Sigma \, \mathrm{Cos}\alpha x \, \frac{(1+\nu) \left(\frac{B}{2}\,\mathrm{Cosh}\,\frac{\alpha B}{2}\,\mathrm{Sinh}\alpha y - \mathrm{Sinh}\frac{\alpha B}{2} y \mathrm{Cosh}\alpha y\right) - \frac{2}{\alpha}\mathrm{Sinh}\frac{\alpha B}{2}\mathrm{Sinh}\alpha y}{\alpha B - \mathrm{Sinh}\alpha B}$$

(2-13)

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$

$$= -\frac{4\tau_o}{EH} \Sigma \, \mathrm{Cos}\alpha x \, \frac{(1+\nu) \left(\frac{B}{2}\mathrm{Cosh}\,\frac{\alpha B}{2}\mathrm{Sinh}\alpha y - \mathrm{Sinh}\,\frac{\alpha B}{2} y \mathrm{Cosh}\alpha y\right) - \frac{2\nu}{\alpha}\mathrm{Sinh}\frac{\alpha B}{2}\mathrm{Sinh}\alpha y}{\alpha B - \mathrm{Sinh}\alpha B}$$

(2-14)

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

$$= \frac{8\tau_o(1+\nu)}{EH} \Sigma \, \mathrm{Sin}\alpha x \, \frac{\frac{B}{2}\mathrm{Cosh}\frac{\alpha B}{2}\mathrm{Cosh}\alpha y - \mathrm{Sinh}\frac{\alpha B}{2} y \mathrm{Sinh}\alpha y - \frac{1}{\alpha}\mathrm{Sinh}\frac{\alpha B}{2}\mathrm{Cosh}\alpha y}{\alpha B - \mathrm{Sinh}\alpha B}$$

(2-15)

Integrating,

$$u = \int \varepsilon_x dx$$

$$= \frac{4\tau_o}{EH} \Sigma \frac{\mathrm{Sin}\alpha x}{\alpha} \, \frac{(1+\nu) \left(\frac{B}{2}\mathrm{Cosh}\,\frac{\alpha B}{2}\mathrm{Sinh}\alpha y - \mathrm{Sinh}\frac{\alpha B}{2} y \mathrm{Cosh}\alpha y\right) - \frac{2}{\alpha}\mathrm{Sinh}\frac{\alpha B}{2}\mathrm{Sinh}\alpha y}{\alpha B - \mathrm{Sinh}\alpha B} + g_1(y)$$

(2-16)

$v = \int \varepsilon_y \, dy$

$$= -\frac{4\tau_o}{EH} \Sigma \frac{\text{Cos}\alpha x}{\alpha} \frac{(1+\nu)\left(\frac{B}{2}\text{Cosh}\frac{\alpha B}{2}\ \text{Cosh}\alpha y - \text{Sinh}\frac{\alpha B}{2} y\text{Sinh}\alpha y\right) + \frac{1-\nu}{\alpha}\text{Sinh}\frac{\alpha B}{2}\text{Cosh}\alpha y}{\alpha B - \text{Sinh}\alpha B} + g_2(x)$$

(2-17)

The strain-displacement relationship $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ then requires

$$g_1'(y) + g_1'(x) = 0$$

Observing that $g_1(y)$ is independent of x, and $g_2(x)$ is independent of y,

$$g_1'(y) = -g_2'(x) = \text{constant}$$

And

$$g_1(y) = A_1 y + A_2$$

$$g_2(x) = -A_1 x + A_3$$

The coefficients A_1, A_2 and A_3 will be determined by the given fixed boundary conditions at the origin. Let $u = v = \frac{\partial u}{\partial x} = 0$ at the origin, $x = y = 0$.

$$A_2 = 0$$

$$-\frac{4\tau_o}{EH} \Sigma \frac{1}{\alpha} \frac{(1+\nu)\left(\frac{B}{2}\text{Cosh}\frac{\alpha B}{2}\right) + \frac{1-\nu}{\alpha}\text{Sinh}\frac{\alpha B}{2}}{\alpha B - \text{Sinh}\alpha B} + A_3 = 0$$

$$-A_1 = 0$$

Therefore, $\quad g_1(y) = 0 \qquad$ (2-18)

$$g_2(x) = A_3 = \frac{4\tau_o}{EH} \Sigma \frac{1}{\alpha^2} \frac{(1+\nu)\left(\frac{\alpha B}{2}\text{Cosh}\ \frac{\alpha B}{2}\right) + (1-\nu)\text{Sinh}\frac{\alpha B}{2}}{\alpha B - \text{Sinh}\alpha B} \qquad (2\text{-}19)$$

The "bending displacement" Δ_b being sought for is the v-displacement at (x = H, y = 0). Noting that $\text{Cos}\alpha x = 0$ for x = H,

$$\Delta_b = -v = -A_3 = -\frac{4\tau_o}{EH} \Sigma \frac{1}{\alpha^2} \frac{(1+\nu)\frac{\alpha B}{2} \text{Cosh} \frac{\alpha B}{2} + (1-\nu)\text{Sinh}\frac{\alpha B}{2}}{\alpha B - \text{Sinh}\alpha B}$$

The negative sign is introduced to conform with the coordinating directions shown in Figs. 1 and 2. Substituting $\alpha = \frac{m\pi}{2H}$,

$$\Delta_b = \frac{16\tau_o H}{E\pi^2} \Sigma \frac{(1+\nu) \frac{m\pi B}{4H} \text{Cosh}\frac{m\pi}{4} \frac{B}{H} + (1-\nu)\text{Sinh}\frac{m\pi B}{4H}}{m^2 \left(\text{Sinh} \frac{m\pi B}{2H} - \frac{m\pi B}{2H}\right)} \qquad m = 1, 3, 5, \ldots \tag{2-20}$$

In summary, the bending problem of Fig. 2 is completely solved by the stress function Equation (2-9) and the displacement functions (2-16) and (2-17) combined with the auxiliary functions (2-18) and (2-19). The solution satisfies the following stress and displacement boundary conditions:

Along the end boundary x = H:

$$\sigma_x = \sigma_y = 0 \qquad\qquad v = -\Delta_b$$

$$\int_0^{\pm\frac{B}{2}} \tau_{xy} dy = 0$$

Along the side boundaries $y = \pm \frac{B}{2}$:

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{2\tau_o}{H} \Sigma \frac{1}{\alpha} \text{Sin}\alpha\pi = \tau_o \qquad\qquad \frac{2H\alpha}{\pi} = 1, 3, 5, \ldots$$

Along the fixed boundary $x = 0$

$$u = 0, \qquad \frac{\partial u}{\partial y} = 0$$

At the center of the fixed boundary $x = 0, \quad y = 0$:

$$v = \frac{\partial v}{\partial x} = 0$$

It is interesting to note that at the end boundary $x = H$, v is independent of y, and the entire boundary undergoes uniform lateral displacement of Δ_b.

III. SIMPLIFICATION OF THE BENDING SOLUTION

The elastic solution for bending displacement Δ_b, presented in the preceding section, can be simplified considerably without introducing serious errors. In Equation (2-20), the terms under the summation sign diminishes in magnitude rapidly with increasing m, on account of the doubled argument of the hyperbolic sine function in the denominator, as well as the factor $1/m^2$. Table 1 shows numerical values of the first three terms of the series for several selected aspect ratios (B/H). It is clear that for the range of aspect ratio shown, the series under summation is strongly dominated by the leading term (m=1). Therefore, all other terms may be omitted without any significant effect.

$$\Delta_b \simeq \frac{16\tau_o H}{E\pi^2} \frac{(1+\nu)\frac{\pi B}{4H}\operatorname{Cosh}\frac{\pi B}{4H} + (1-\nu)\operatorname{Sinh}\frac{\pi B}{4H}}{\operatorname{Sinh}\frac{\pi B}{2H} - \frac{\pi B}{2H}} \tag{3-1}$$

Equation (3-1) can be further simplified by expanding the hyperbolic functions into the equivalent power series. Let $k = \frac{\pi B}{4H}$,

$$\Delta_b = \frac{16\tau_o H}{E\pi^2} \frac{(1+\nu)\ k\ (1+\frac{1}{2!}k^2 + \frac{1}{4!}k^4 + \ldots) + (1-\nu)(k+\frac{1}{3!}k^3 + \frac{1}{5!}k^5 + \ldots)}{\frac{1}{3!}(2k)^3 + \frac{1}{5!}(2k)^5 + \frac{1}{7!}(2k)^7 + \ldots}$$

$$= \frac{2\tau_o H}{\pi^2 E k^2} \frac{2 + \frac{1}{3!}(4+2\nu)k^2 + \frac{1}{5!}(6+4\nu)k^4 + \ldots}{\frac{1}{3!} + \frac{4}{5!}k^2 + \frac{16}{7!}k^4 + \ldots}$$

Both power series, in the numerator and the denominator, converge strongly, particularly for moderate values of k. For aspect ration B/H not more than 2.0 (k not exceeding approximately 1.5), two terms in each series would be quite adequate.

$$\Delta_b = \frac{2\tau_o H}{\pi^2 E k^2} \quad \frac{2 + \frac{4 + 2\nu}{6} k^2}{\frac{1}{6} + \frac{1}{30} k^2}$$

$$= \frac{384\tau_o H^3}{\pi^4 E B^2} \quad \frac{1 + 0.103\ (2+\nu)\left(\frac{B}{H}\right)^2}{1 + 0.123\ \left(\frac{B}{H}\right)^2} \tag{3-2}$$

Observing that π^4 is approximately equal to 96, the first factor on the right hand side of Equation (3-2) represents the end deflection (Fig. 1a) as computed by the conventional cantilever beam formula.

$$\Delta_{ba} = \frac{TH^3}{3EI} = \frac{\tau_o BtH^3}{3E\frac{1}{12}tB^3} = \frac{4\tau_o H^3}{EB^2} \simeq \frac{384\tau_o H^3}{\pi^4 EB^2} \tag{3-3}$$

Therefore, Equation (3-2) shows that the bending displacement Δ_b can be evaluated by applying a modifying factor to the conventional solution Δ_{ba}. Equation (3-2) is therefore rewritten in the form of Equation (3-3), with an additional simplification in the numerical coefficients as shown in Equation (3-4)

$$\Delta_b = \Delta_{ba}\mu \tag{3-4}$$

$$\mu = \frac{1 + 0.1(2+\nu)\ \left(\frac{B}{H}\right)^2}{1 + 0.12\ \left(\frac{B}{H}\right)^2} \tag{3-5}$$

Equations (3-4) and (3-5) make a very close approximation of the elastic solution (2-20). The two measures taken (the truncation of the summation series and the rationalization of the hyperbolic functions) induce errors opposite each other, resulting in very small total error. The closeness of the approximation is demonstrated in Table 2. For this comparison, Equation (2-20) is first rewritten with reference to Δ_{ba}

$$\Delta_b = \Delta_{ba}\mu_1$$

$$\mu_1 = \left(\frac{2B}{\pi H}\right)^2 \Sigma \frac{(1+\nu)\frac{m\pi B}{4H}\,\mathrm{Cosh}\,\frac{m\pi B}{4H} + (1-\nu)\,\mathrm{Sinh}\,\frac{m\pi B}{4H}}{m^2\left(\mathrm{Sinh}\,\frac{m\pi B}{2H} - \frac{m\pi B}{2H}\right)} \qquad m = 1, 3, 5, \ldots \qquad (3\text{-}6)$$

Table 2 shows values of μ and μ_1, calculated for ν=0.16 and a range of aspect ratios. For aspect ratio between 0.5 and 2.0, the two values do not differ by more than 0.5%. This degree of agreement is obviously acceptable.

It should be cautioned that the discussion in this section deals with the evaluation of displacement only. Whether the stress solutions, Equations (2-15), (2-16) and (2-17) could be similarly simplified was not examined in this study.

IV. EXAMPLES

Two examples are presented here to illustrate the application of the proposed method of displacement evaluation. The first example refers to a flat plate panel 61.33 inches long, 96 inches wide and 2.22 inches thick. (These dimensions were taken from a reduced scale specimen tested in a related study, Ref. 1) A shear force of 3,000 lbs. is applied at the free end. Material properties are $E = 3.1 \times 10^6$ psi, $\nu = 0.16$ and $G = 1.34 \times 10^6$ psi (Fig. 3)

$$\text{Aspect Ratio} = \frac{96}{61.33} = 1.565$$

$$I = \frac{1}{12}(2.22)\,(96)^3 = 163800 \text{ in}^4$$

$$\tau_o = \frac{3000}{96 \times 2.22} = 14.06 \text{ psi}$$

From Equation (1-2)

$$\Delta_s = \frac{14.06}{1.34 \times 10^6} \times 61.33 = 0.644 \times 10^{-3} \text{ in.}$$

From Equation (3-3)

$$\Delta_{ba} = \frac{3000(96)^3}{3(3.1 \times 10^6)(163800)} = 0.454 \times 10^{-3} \text{ in.}$$

From Equation (3-5)

$$\mu = \frac{1 + 0.1\ (2.16)(1.565)^2}{1 + 0.12\ (1.565)^2} = \frac{1.529}{1.194} = 1.18$$

Therefore,

$$\Delta_b = 0.454 \times 10^{-3} \times 1.18 = 0.536 \times 10^{-3} \text{ in.}$$

$$\Delta = \Delta_b + \Delta_s = 1.180 \times 10^{-3} \text{ in.}$$

In comparison, a finite element analysis of this flat plate panel yields an end displacement of 1.171×10^{-3} in., reflecting an error of less than one percent.

It is interesting to also compare the solution with that based on ordinary mechanics of materials theory, considering both flexual and shearing strains. The flexual displacement has been calculated above,

$$\Delta_{ba} = 0.454 \times 10^{-3} \text{ in.}$$

The shearing effect is

$$\Delta_{sa} = \frac{6}{5}\frac{TH}{GA} = \frac{6\ (3000)\ (61.33)}{5(1.34 \times 10^6)\ (2.22)\ (96)} = 0.774 \times 10^{-3} \text{in.}$$

$$\Delta_a = 1.228 \times 10^{-3} \text{ in.}$$

It is seen that the proposed solution agrees much better with the finite element solution than the conventional theory.

For a second example of application, a specimen waffle slab panel with dimensions shown in Fig. 4, under an end shear force of 900 lbs. is analyzed.

Although the derivations in Section 2 refer to a flat plate of uniform thickness, the Equations (3-3), (3-4) and (3-5) could be used for beam-supported floor panels as well. The effect of the beams (or ribs, in the case of a waffle slab) is included in the calculation of I in Equation (3-3), as illustrated in this example.

$$I_{slab} = \frac{1}{12}\ (o.67)\ (96)^3 = 49400 \text{ in.}^4$$

$$I_{ribs} = 2\ (2.66)\ (1.33)\ (10.67)^2\ (1^2+2^2+3^2+4^2) = 24100 \text{ in.}^4$$

$$I = I_{slab} + I_{ribs} = 73500 \text{ in.}^4$$

$$\Delta_{ba} = \frac{900\ (61.33)^3}{3(3.1 \times 10^6)\ (73500)} = 0.304 \times 10^{-3} \text{ in.}$$

$$\frac{B}{H} = 1.565$$

$$\mu = 1.18$$

$$\Delta_b = 0.304 \times 10^{-3} \times 1.18 = 0.358 \times 10^{-3} \text{ in.}$$

For the estimation of the pure shear displacement Δ_s, an equivalent slab thickness is used to account for the contribution of the closely spaced ribs. The development of the equivalent thickness approach is presented in a separate report.[3] For this example waffle slab, the equivalent thickness $\beta = 1.0434$,

$$\Delta_s = \frac{TH}{G\beta A} = \frac{900\ (61.33)}{1.34 \times 10^6\ (1.0434)\ (96)\ (0.67)} = 0.614 \times 10^{-3} \text{ in.}$$

$$\Delta = \Delta_s + \Delta_b = 0.972 \times 10^{-3} \text{ in.}$$

In comparison, a 2-D analysis by the finite element nethod yields an end displacement of 0.981×10^{-3} in. The discrepancy is again less than one percent.

V. CONCLUSIONS

A solution has been presented for the estimation of displacement caused by an in-plane end shear load on a cantilever slab panel, based on the separation into a pure shear condition and a "bending" condition. The bending solution is simplified without incurring significant errors.

Examples show that the results obtained from Equations (1-3), (3-4) and (3-5) are very nearly the same as those obtained by finite element analyses.

The example on waffle slab further demonstrates the Equations (3-4) and (3-5) can be extended to non-flat slab panels. In these cases, the moment of inertia of the bending section is calculated to include contributions of the beams as flanges.

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TABLE 1

Convergence of Series in Equation (2-20)

Aspect Ratio	0.7	1.0	2.0
Term 1	5.1923	2.6509	0.7738
Term 2	0.0790	0.0413	0.0063
Term 3	0.0108	0.0043	0.0001

NOTE: Calculations based on $\nu = 0.16$

TABLE 2

Comparison of Complete and Approximate Solutions

Aspect Ratio (B/H)	μ	μ_1	Percentage Difference
0.50	1.024	1.025	0.1%
0.75	1.051	1.054	0.3
1.00	1.085	1.088	0.3
1.25	1.128	1.130	0.2
1.50	1.170	1.175	0.4
1.75	1.216	1.221	0.4
2.00	1.261	1.267	0.5

NOTE: Calculations based on $\nu = 0.16$

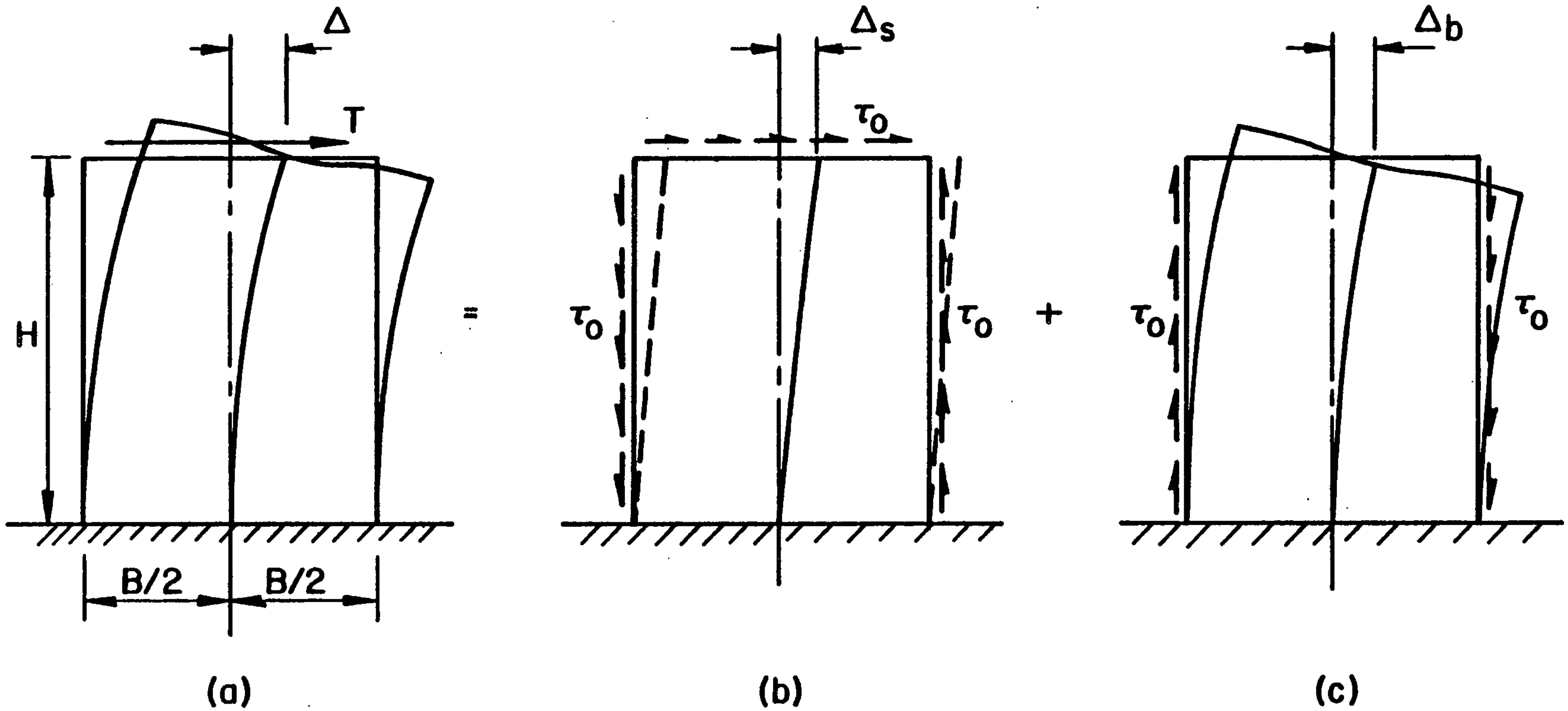

Fig. 1 Slab Panel Under End Shear

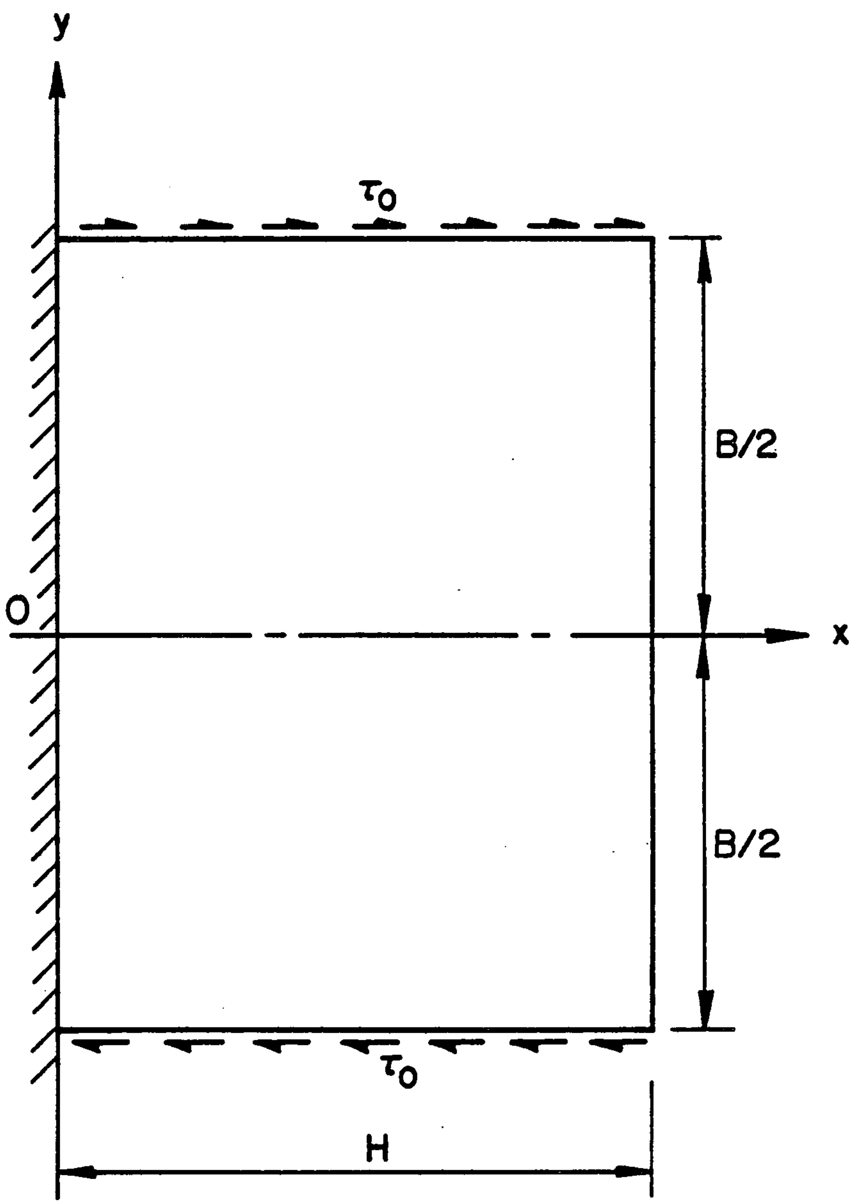

Fig. 2 Idealized "Bending" Problem

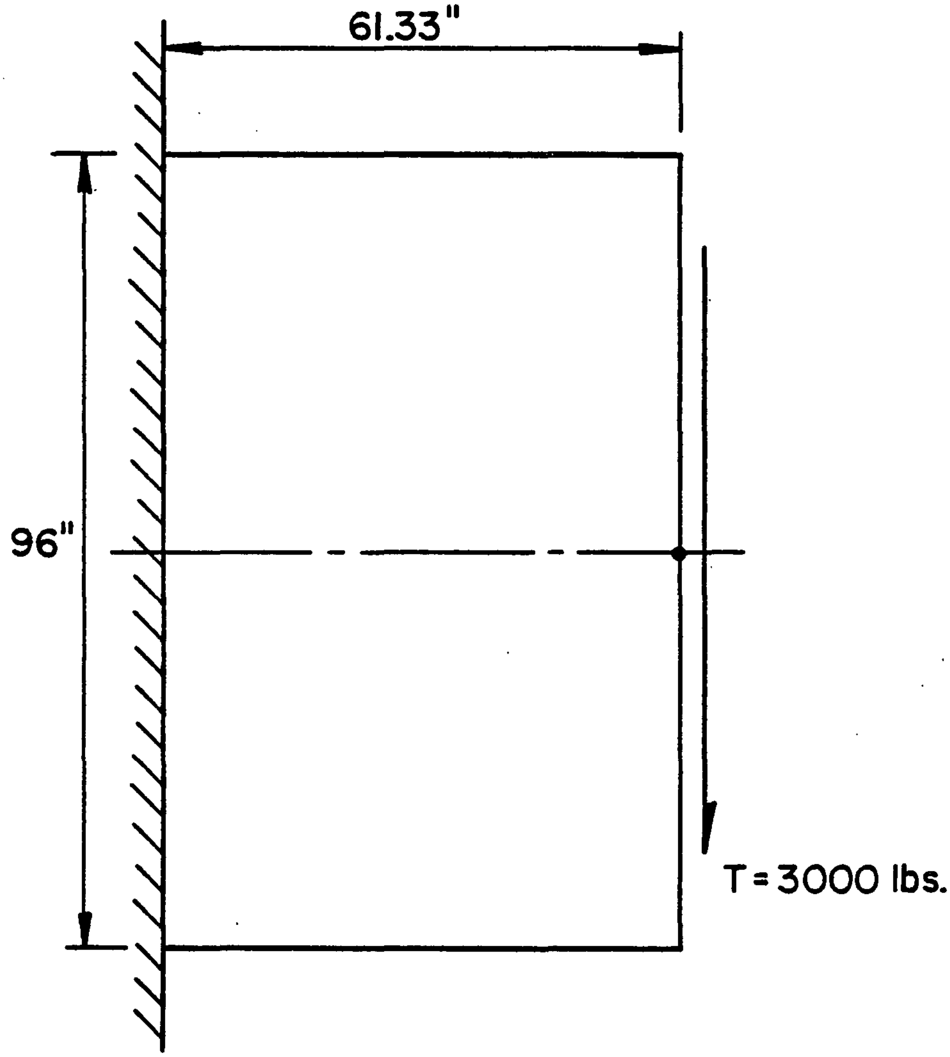

Fig. 3 Flat Plate Example

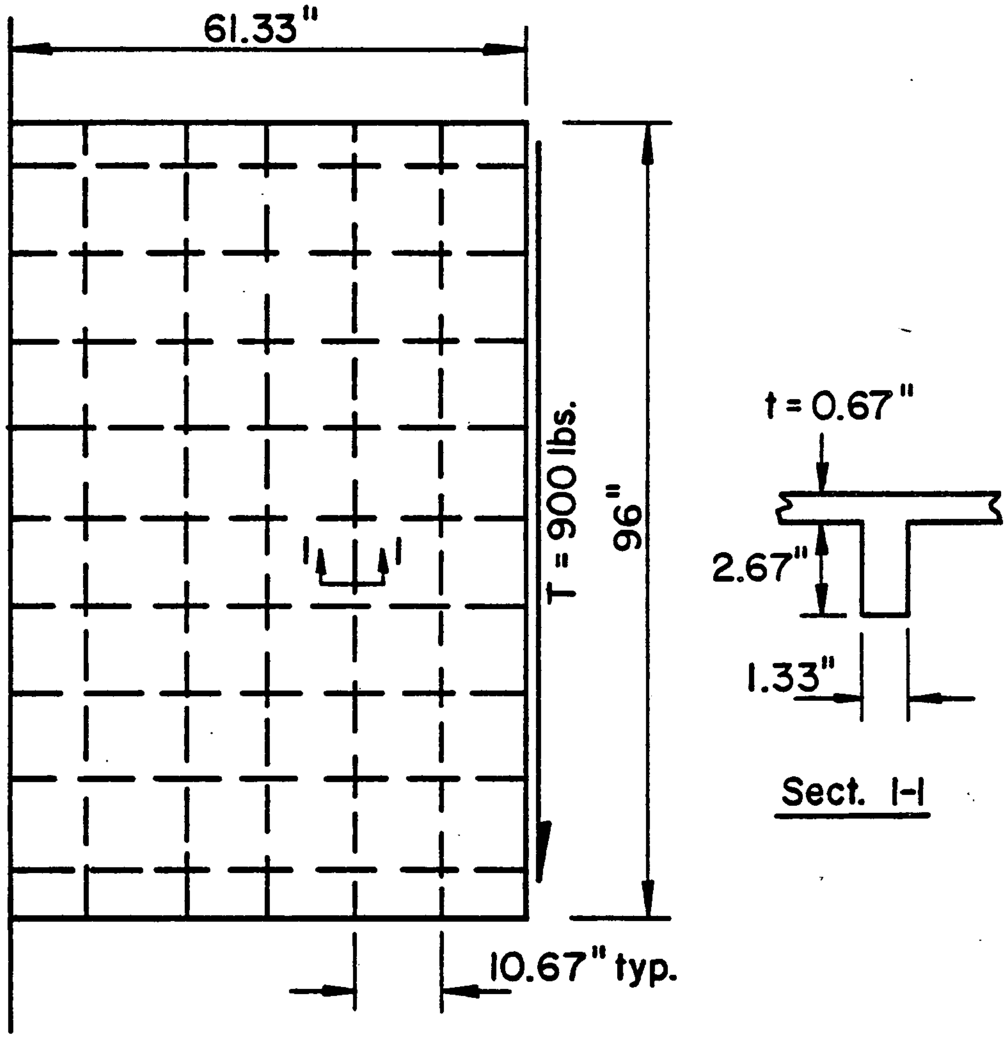

Fig. 4 Waffle Slab Example

ACKNOWLEDGMENTS

The work reported herein was conducted in Fritz Engineering Laboratory of Lehigh University. Dr. Lynn S. Beedle is the Director of the Laboratory. The work was part of a research project sponsored by The National Science Foundation (NSF Grant No. CEE-8120589). Dr. Michael Gaus is the Program Manager at NSF. The project is under the co-directorship of Dr. Le-Wu Lu and the second author.

Valuable suggestions were received from Mr. Xueren Ji, to whom the authors are grateful.

The manuscript was typed by Mrs. G. Clinchy. The graphical work was prepared by Mr. J. Gera.