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Reservoir Sedimentation

A MATHEMATICAL MODEL OF DELTA FORMATION IN RESERVOIRS

by

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ABSTRACT

A mathematical model was developed to predict the rate and the pattern of bed load deposition in an arbitrary river-reservoir system where one-dimensional (unit width) flow phenomena predominate. Three different bed load equations, namely the modified (for deposition) Schoklitsch, the Meyer-Peter Muller, and the Einstein-1942 bed load equations were used. The calculations were made with an arbitrary set of input data with three different sediment sizes.

The most interesting result of this investigation is a qualitative one, namely the formation of a typical delta. In all cases, a delta is first built-up and then progresses in the downstream direction. The quantitative results are highly variable, largely due to the differences in bed load capacities predicted by the three bed load equations. Of the three equations, the Meyer-Peter Muller equation is the only one that consistently predicts the typical "steep-faced" delta.

Despite the simplicity of the present mathematical model, it is remarkable to observe that the predicted behavior of the delta formations are in good agreement with existing ones, such as in Lake Mead behind Hoover Dam.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

D	water depth at any section
D_{max}	water depth at the dam section
D_n	normal water depth (uniform flow) in the river
d_{50}	representative sediment size
g	gravitational acceleration
k, χ	sediment coefficients
L	distance from the dam section
n_M	Manning's roughness coefficient
q	water flow rate in volume per unit time per unit width
q_s	bed load rate in volume per unit time per unit width
S_b	slope of the channel bed
S_{br}	slope of the normal (uniform) flow in the river
S_e	slope of the energy grade line
s_s	specific gravity of the sediments
V	water velocity at any section
Z_b	elevation of the channel bed with respect to the bottom of the dam
δ_s	thickness of deposition
γ	specific weight of water
ρ	density of water

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1. INTRODUCTION

The sediment transported by a water course is forced to deposit as it proceeds into a deeper water body, such as a reservoir behind a dam, a lake or an ocean. This is due to the fact that the velocity, and thus the sediment transport capacity, of the flow is reduced as its depth is increased.

A water course may transport both cohesive and noncohesive sediments. Presently, the cohesive sediment transport is a problem without any plausible solution even in simplified cases [see GRAF (1971), Ch. 12]. There has been relatively more success in dealing with the transport of noncohesive (granular) sediments; the latter may be classified as the bed load, the suspended load, and the wash load [see GRAF (1971), Chs. 7, 8, and 9]. The bed load, the suspended load, and the wash load together make up the total load.

This study investigates the deposition of the bed load material, consisting of the relatively coarser sediments, as a river enters a reservoir. For this purpose, a mathematical model is developed as described in the following section.

2. MATHEMATICAL MODEL

2.1 Introductory Remarks

In this study a mathematical model was developed for the prediction of the rate and pattern of bed load deposition in a river-reservoir system. The deposition takes place in the form of a delta. The earlier developments of the model were described by YUCEL and GRAF (1973). The model considers an arbitrary river-reservoir system suitable for a one-dimensional (unit-width) analysis, as shown in Fig. 2.1. The characteristics of the model and the assumptions involved are described below.

2.2 One-Dimensional (Unit-Width) Model of a River-Reservoir System

As shown in Fig. 2.1, the model considers a reservoir formed by a dam constructed on the course of a river where one-dimensional flow phenomena are predominant. As a result of the retardation of the flow as it enters the reservoir, the sediment transported by the river is forced to deposit. If only the bed load is taken into account, such a deposition is usually considered to take place in two different ways:

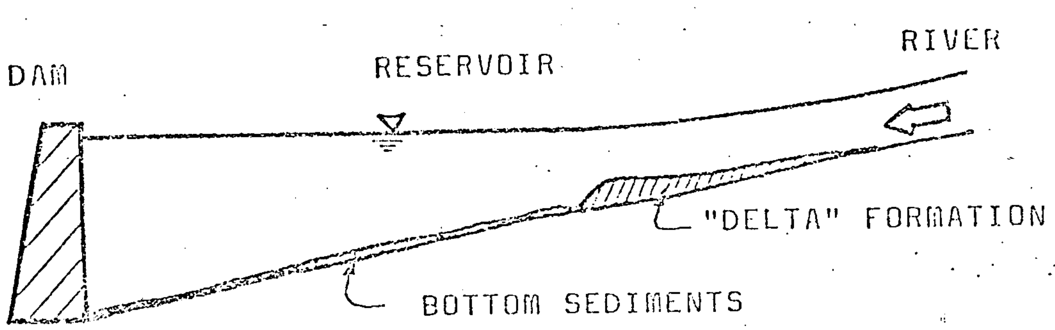


Fig. 2.1: River-Reservoir System

(i) The larger sized sediments are first deposited to develop a delta formation, which builds up at the upstream end (the mouth) of the reservoir and progresses downstream. (ii) The smaller sized sediments are carried further downstream to be deposited in relatively flat layers often referred to as the bottom sediments.

The objective of this model is to mathematically predict the deposition patterns during delta formation. The analysis is made in two parts: (a) The back water profile; and, (b) the sediment transport and deposition. These two parts of the analysis are made independently. Thus, a constant geometry of the river-reservoir system with no sediment transport is assumed in calculating the initial back water profile. Similarly, the back water profile is assumed to remain unchanged during each series of calculations made for the sediment deposition.

It is expected that any deposition in the reservoir which alters the bottom configuration will affect the back water profile. However if the quantity of deposition is small, the water surface profile will not be significantly changed. Therefore, in order to avoid unnecessary repetitions, the model warrants the calculation of a new back water profile only if a certain significant amount of deposition has taken place [see Sec. 2.2.2(f)]. A simplified logical scheme of the model is shown in the flowchart given by Fig. 2.2. The methods applied in calculating the back water profiles and the bed load depositions are described in the following sections. A detailed characteristic of the model is given in the Appendix.

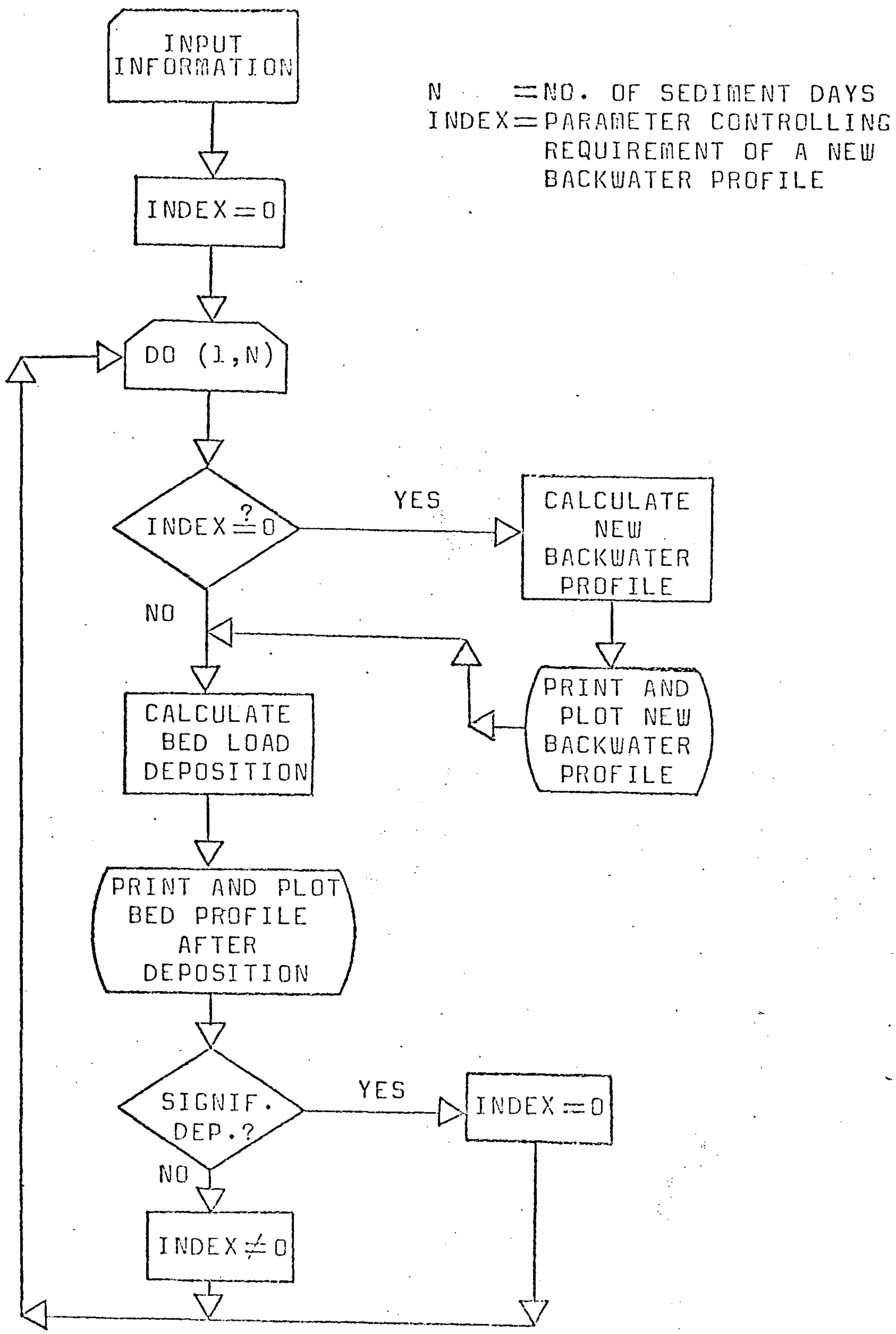


Figure 2.2 Flow Chart of the Computer Program

2.2.1 Back Water Profile

The back water profile in a river-reservoir system with a unit flow rate, q, and a fixed bed configuration (no sediment transport) can be calculated with the use of any one of the well-known methods [see CHOW (1959, Ch.10)]. The model developed in this study uses a standard "step-by-step method". As shown in Fig. 2.3, the calculations are started at the dam section where the water depth is maximum, i.e., $D = D_{max}$, and proceeded step-by-step in the upstream direction until the normal river flow conditions are reached. A typical cycle of calculations made for the back water profile can be described as follows:

(a) A typical section is considered where the water depth is known (or previously calculated) to be D_{i-1}.

(b) A water depth increment ΔD_i is assumed such that:

$$D_i = D_{i-1} + \Delta D_i \tag{2-1}$$

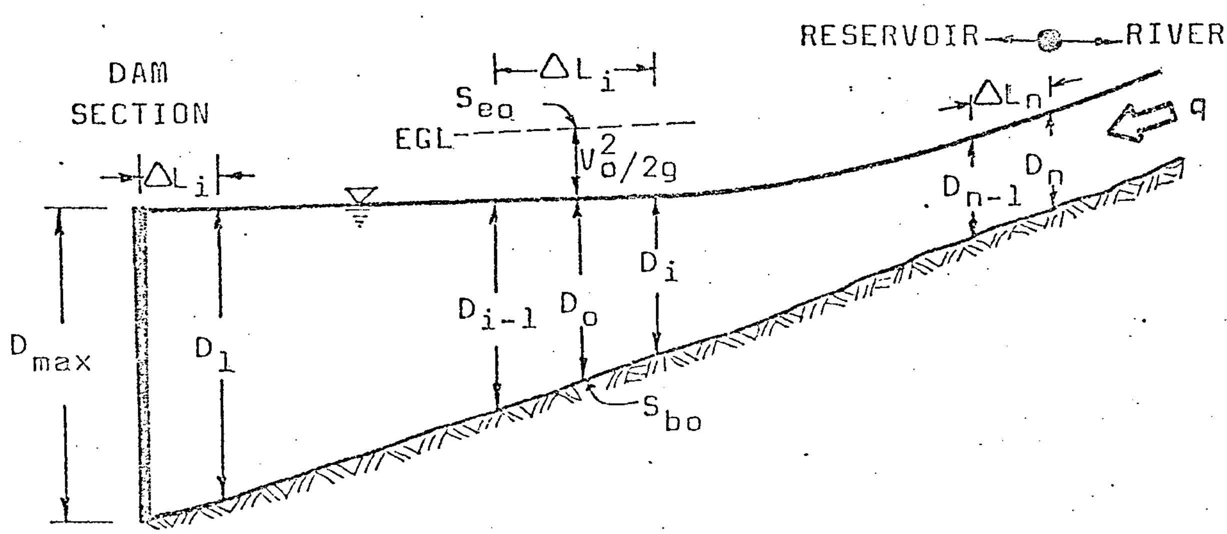


Fig. 2.3: Back Water Profile

where D_i is the water depth at a new section upstream of the previous one, where the water depth is D_{i-1}. Thus, a reach is formed between these two sections.

(c) The reach has a length of ΔL_i which is approximated by the following equation:

$$\Delta L_i = - \Delta D_i \frac{\left(1 - \frac{V_o^2}{gD_o}\right)}{\left(S_{bo} - S_{eo}\right)} \tag{2-2}$$

where V_o, D_o, S_{bo} and S_{eo} are the average velocity, water depth, the bottom slope, and the slope of the energy grade line, respectively, all calculated at the mid-section of the reach.

(d) Both the water depth increment ΔD_i assumed and the reach length ΔL_i calculated should be sufficiently small in order to justify the validity of Eq. (2-2). In this study, Eq. (2-2) is considered to be sufficiently adequate, if the mid-section parameters, S_{bo} and S_{eo}, are within 5% of those at the boundary sections of the reach, namely, S_{bi} and $S_{b(i-1)}$, and S_{ei} and $S_{e(i-1)}$, respectively.

(e) If the above conditions--described under (d)--are not satisfied, a new (smaller) water depth increment ΔD_i is assumed and the above procedure is repeated as given under (a) to (d) until the conditions described under (d) are satisfied.

(f) Special problems are encountered at two places during the calculations of the back water profile: (i) at the regions where there is considerable change in the channel bed slope, and (ii) at the regions where the normal river flow conditions are about to be reached. The procedures followed under these conditions are described in the Appendix, in the Subroutine Program WPROF.

2.2.2 Bed Load Deposition

Once the back water profile is determined for a particular geometry and the flow conditions known for the river-reservoir system, the bed load deposition calculations are made. As shown in Fig. 2.4, these calculations are started at the section approximating the normal river flow conditions and progressed downstream into the reservoir. The same sections, as determined in the back water profile calculations, are used for the bed load deposition calculations. A typical cycle of calculations made for the bed load deposition is described as follows:

(a) At some section within the river-reservoir system, where the water depth is D_i (the characteristics of the sections were determined during the back water profile calculations), the bed load transport capacity of the flow is designated by q_{si}. The latter can be determined with the use of a bed load equation (see Section 2.3).

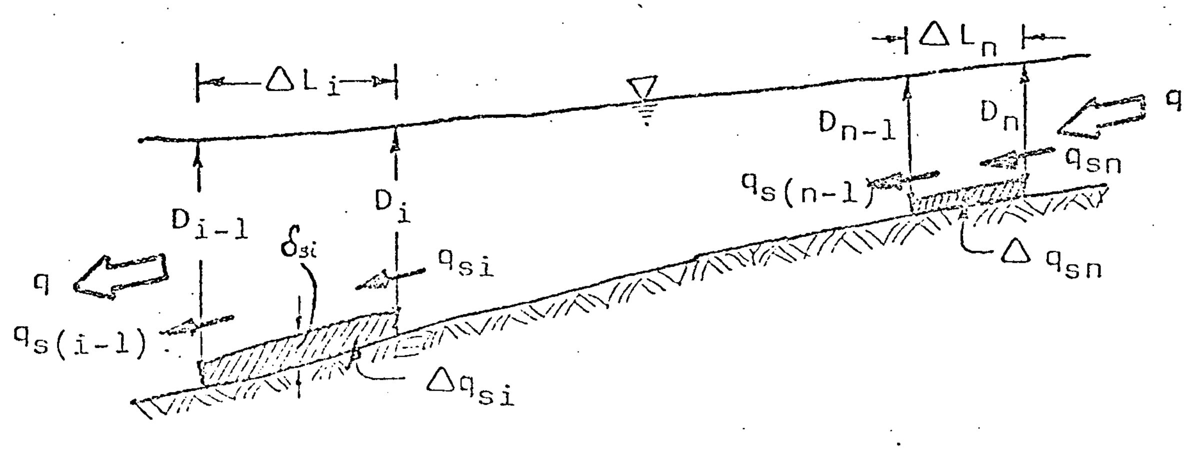


Figure 2.4 Sediment Transport and Deposition

(b) At the next downstream section, which is at a distance ΔL_i from the upstream one, the water depth is D_{i-1}. Since, in general, the water depth increases in the downstream direction, namely, $D_{i-1} > D_i$, the average flow velocity is decreased, or $V_{i-1} < V_i$ (the unit water flow rate is constant, i.e., q = const). As a result of smaller velocity, the bed load transport capacity at the downstream section, $q_{s(i-1)}$, will also be smaller than the one at the upstream section, $q_{s(i-1)} < q_{si}$.

(c) The difference between the bed load transport capacities at the upstream and the downstream sections is:

$$\Delta q_{si} = q_{si} - q_{s(i-1)} \tag{2-3}$$

This amount of bed load should be deposited between these two sections. If the length of the reach, ΔL_i, is sufficiently small, and if the change in the flow conditions between the two sections is gradual, then it may be assumed that the deposition of the bed load within the reach will be uniformly distributed. The average uniform thickness of the deposition, δ_{si}, per unit time (period) of deposition, T_d, is then:

$$\delta_{si} = \frac{\Delta q_{si}}{\Delta Li} \cdot T_d \tag{2-4}$$

(d) The calculations explained above are started at the section approximating the normal river flow conditions where the water depth is D_n, and progressed in the downstream direction. Each cycle of deposition calculations is ended if either of the following two conditions is approximately reached: (i) when the bed load transported by the water course is exhausted, or (ii) when the reservoir itself is exhausted, i.e., the dam section is reached.

(e) The deposition calculated according to the above procedure results in a change in the channel bottom elevation within each reach. Thus, a new channel bottom elevation is obtained at each section by adding the thickness of the deposition calculated to the original channel bottom elevation, or,

$$Z_{bi(new)} = z_{bi(original)} + \delta_{si} \qquad (2\text{-}5)$$

with the application of Eq. (2-5) at each section, a new channel bottom configuration is obtained.

(f) Any change in the channel bottom requires the determination of a new back water profile. However to avoid too lengthy calculations, a new back water profile is calculated only after deposition resulting in significant changes in the channel bottom profile. In this study significant deposition is assumed to have occurred only if any of the local thicknesses of deposition exceeds 2% (an arbitrary figure small enough such that bed load carrying capacities are not significantly changed) of the local water depth. Thus, a new back water profile is calculated only if,

$$\left(\frac{\delta_{si}}{D_i}\right)_{max} \geq 2\% \qquad (2\text{-}6)$$

(g) If the deposition obtained as a result of a cycle of calculations is not significant, or if $(\delta_{si}/D_i)_{max} < 2\%$, then another cycle is assumed to have taken place identical to the previous one, and the channel bottom elevations are adjusted accordingly.

The details of the above procedure are described in the Appendix in association with the Subroutine Program DPBL.

2.3 Bed Load Equations

The bed load deposition was calculated with the use of three different bed load equations: (1) the Schoklitsch equation (modified for deposition with the use of Hjulström's critical deposition velocity), the Meyer-Peter Muller equation, and (3) the Einstein-1942 bed load equation. GRAF (1971, Ch. 7) reviews these and other bed load equations in detail. A brief description of each of these equations is given below.

2.3.1 Modified Schoklitsch Equation

The Schoklitsch-type bed load equation [see GRAF (1971, pp. 130-131)] can be expressed in the following form:

$$q_s = \chi S^k (q - q_{cr}) \tag{2-7}$$

where q_s is the bed load transport rate in volume per unit time per unit width; S is the channel slope; q is the water flow rate in volume per unit time per unit width; q_{cr} is the critical water flow rate at which the bed material begins to move; and χ and k are empirical sediment coefficients.

In using Eq. (2-7), or any presently available bed load equation for that matter, for sedimentation (deposition) in reservoirs, two violations are unavoidable:

(i) All of the bed load equations are developed for uniform flow conditions, for which the slopes of the channel bed and of the energy grade line are identical. For flow in reservoirs, this is not the case as the two slopes are obviously different. In this study the slope of the energy grade line, S_e, is chosen since this is the slope that reflects the water velocity which in turn is responsible for the sediment transport.

(ii) All of the bed load equations are developed for "erosion" or "scour" and not for "deposition". One remedy to this situation is to adapt the "erosion" equations for "deposition", where the bed load equation is suitable for such a modification. The Schoklitsch-type bed load equations are suitable for such a purpose, since they involve a term such as q_{cr}, the critical "erosion" flow rate. In this study it is proposed to use Hjulström's critical "deposition" velocity, V_{cr}, to evaluate the critical flow rate, q_{cr} see GRAF (1971, p. 88) . Furthermore, having no better information, it is assumed that the empirical coefficients, χ and k, remain the same for both "erosion" and "deposition". Thus, Eq. (2-7) is modified for "deposition" and rewritten in the following form:

$$q_s = \chi S^k (q - D V_{cr}) \tag{2-9}$$

where D is the depth of flow and V_{cr} is the critical "deposition" velocity given by Hjulström.

2.3.2 Meyer-Peter Muller Equation

The second equation used in this study in calculating the bed load deposition is the Meyer-Peter Muller bed load equation [see GRAF (1971, pp. 136-139)] which can be written as

$$q_s = \frac{s_s \gamma}{(\gamma_s - \gamma)} \left(\left[\frac{\gamma D S}{d_{50}} - 0.047 (\gamma_s - \gamma) \right] \frac{d_{50}}{0.25 \sqrt[3]{\rho}} \right)^{3/2} \quad (2\text{-}10)$$

where q_s is the bed load transport rate in volume per unit time per unit width; s_s is the specific gravity of the sediment material; γ is the unit weight of water; D is the water depth; S is the slope of the energy grade line; d_{50} is the representative (50% passing) sediment size; and ρ is the density of water.

A simple modification for "deposition" is not plausible in this case, since there is no explicit dependence of Eq. (2-10) on any sort of a critical velocity. Thus, it should be kept in mind that the bed load deposition is calculated, in this case, based on the "erosion" concept and not the "deposition".

2.3.3 Einstein-1942 Bed Load Equation

The third equation used in calculating the bed load deposition is the Einstein bed load equation [see GRAF (1971, pp. 139-150)] which can be written as:

$$q_s = \frac{\sqrt{(s_s - 1)\, g\, d_{50}^3}}{0.465}\, e^{-\left[\frac{0.391 (s_s - 1) d_{50}}{D S}\right]} \quad (2\text{-}11)$$

where g is the gravitational acceleration. Here again a modification for "deposition" is not plausible due to the lack of an explicit critical velocity term.

2.3.4 Behavior of the Bed Load Equations for Uniform Flow

The research conditions preceeding the development of the Schoklitsch, the Meyer-Peter Muller, and the Einstein-1942 bed load equations varied significantly. The studies involved different sediment and stream characteristics.

Table 1 contains the ranges of particle diameters for which the equations are applicable.

Table 1

Particle Size for Which the Bed Load Equations are Applicable

Equation	Particle Diameter (mm)
Schoklitsch-Hjulstrom	>6
Meyer-Peter Muller	5 to 28
Einstein-1942	0.8 to 28

The bed load rates predicted by the three equations under a given set of parameters differ significantly, often by an order of magnitude. A comparison was made of how the equations react to varying parameters. The control set of parameters, or the base from which the parameters were varied, was the following:

Flow rate, q = 2.0 m^3/sec/m
Manning's n = 0.025
Bed slope, S_b = 0.001
Particle size, d_{50} = 0.010 m

Figures 2.1, 2.2, 2.3 and 2.4 show the bed load rate plotted against the flow rate, bed slope, particle size, and Manning roughness, respectively. These plots will be used to help explain the delta formation predictions in the following sections.

It should be noted here that the positions of the curves relative to each other will change if a different set of base conditions is chosen.

2.4 Characteristics of the Model River-Reservoir System

The following variable values were used as the initial characteristics of a model river-reservoir system:

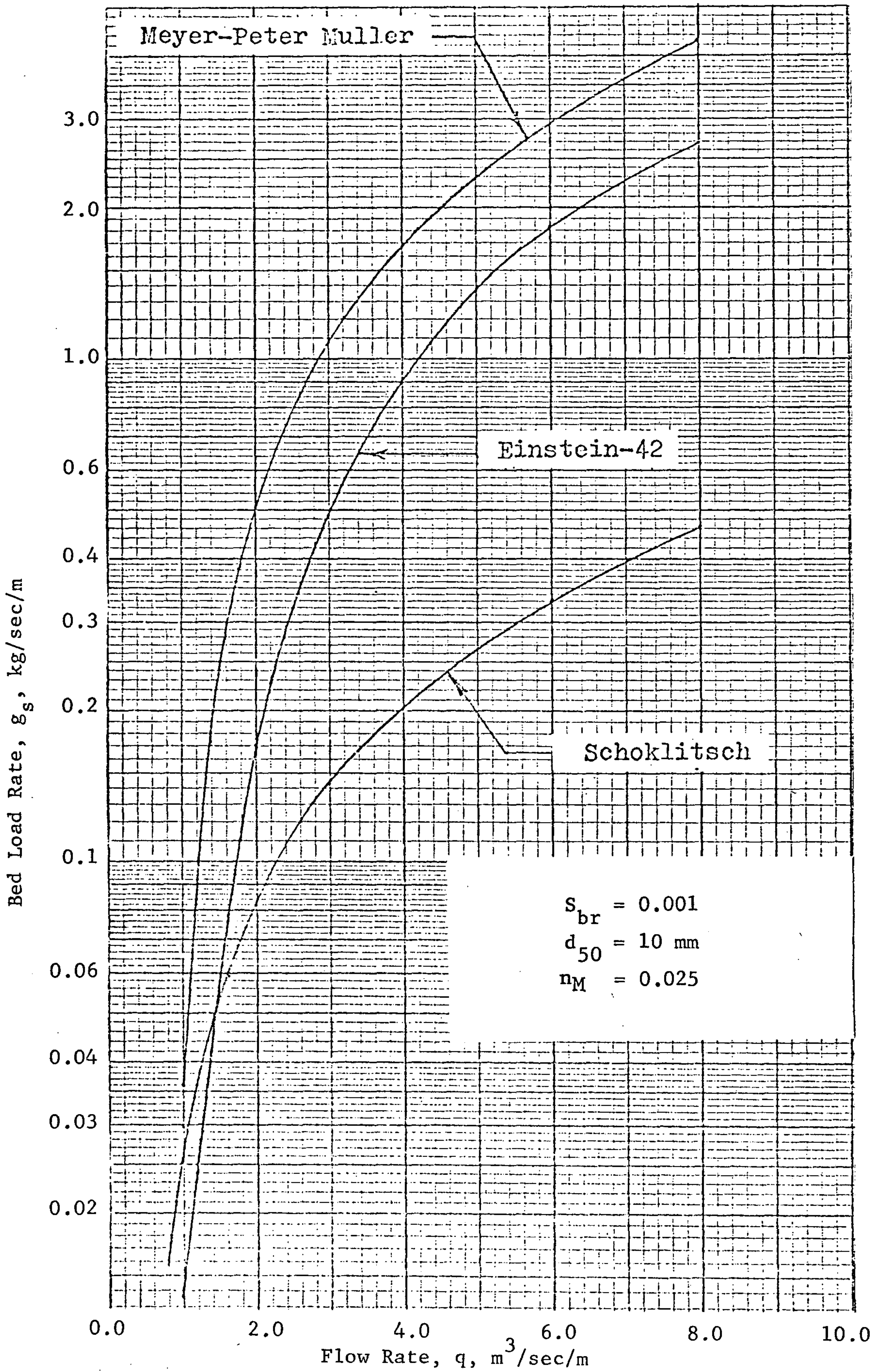


Figure 2.1 Bed Load Variation with Discharge

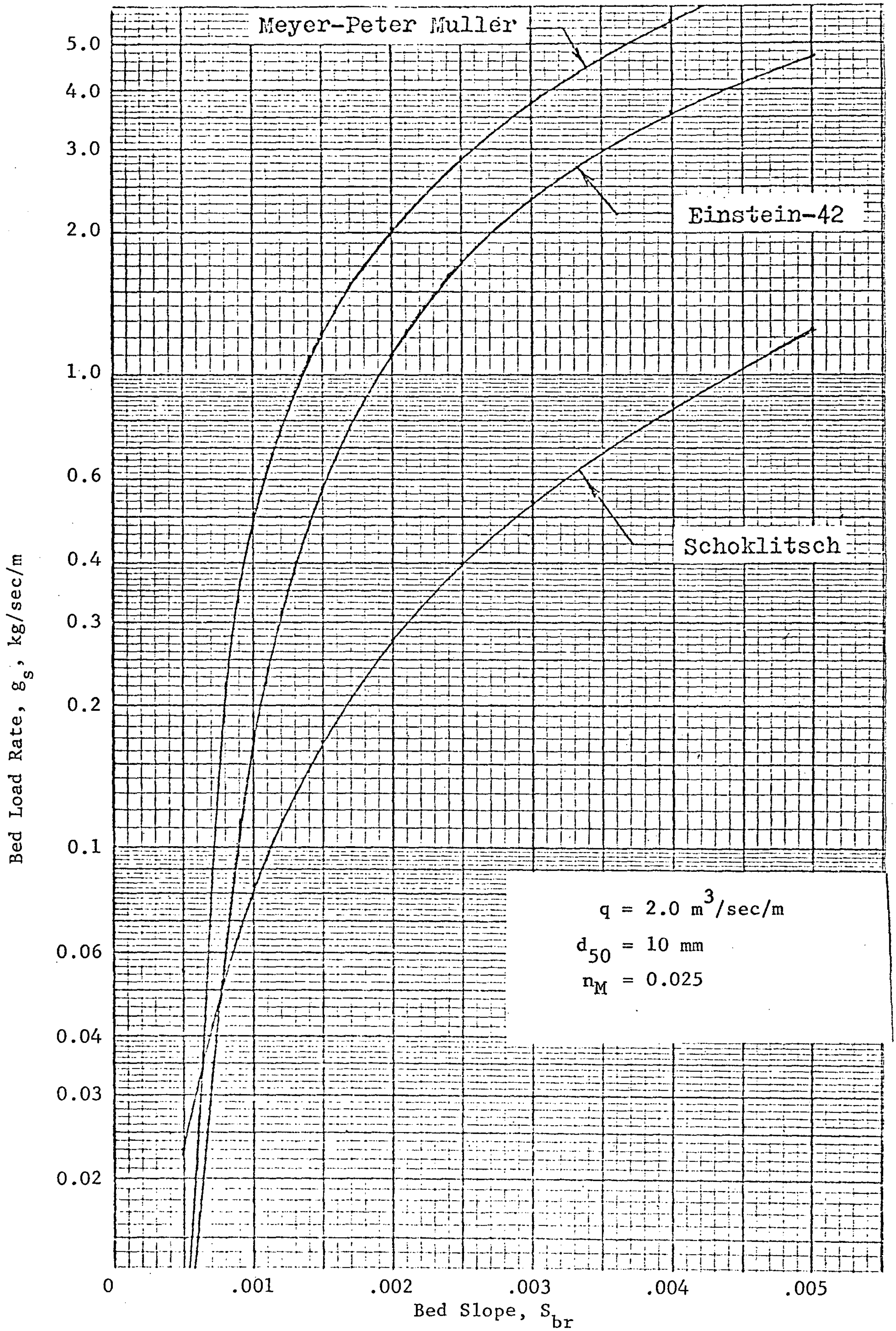


Figure 2.2 Bed Load Variation with Bed Slope

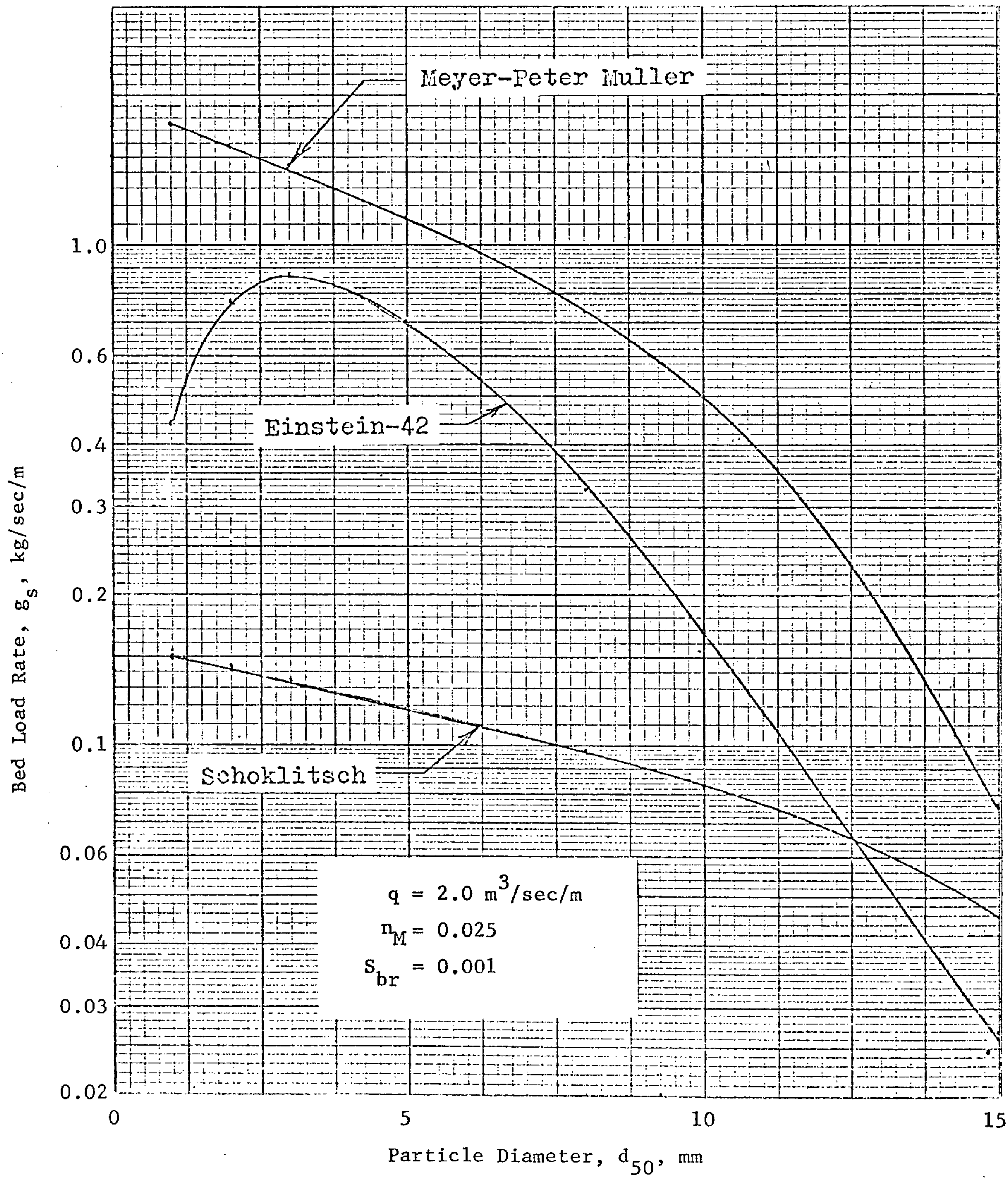


Figure 2.3 Bedload Variation with Particle Diameter

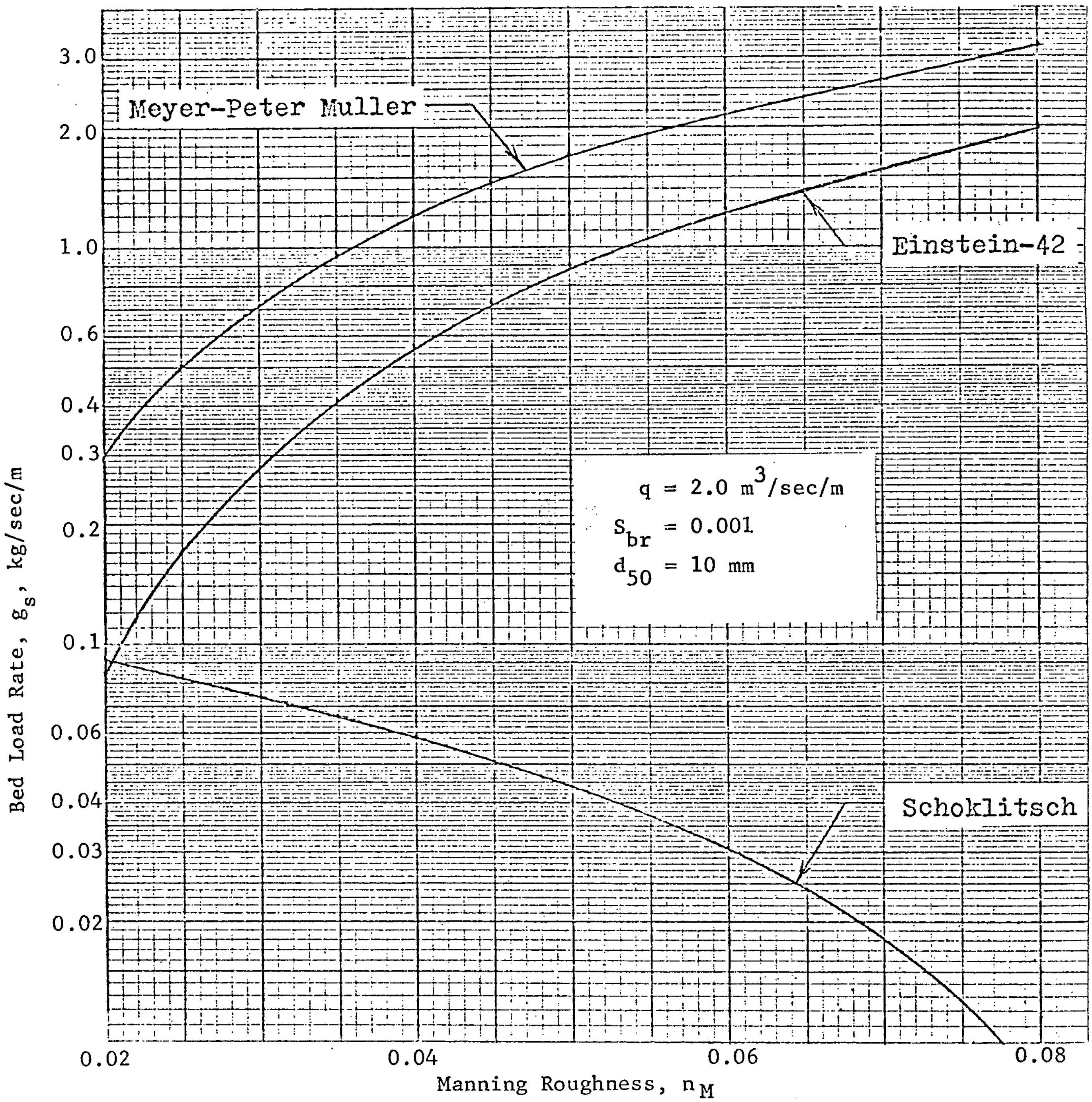


Figure 2.4 Bed Load Variation with Roughness

(a) A constant water flow rate per unit channel width, $q = 1.81\ m^3/sec/m$ (19.5 $ft^3/sec/ft$);

(b) The river bed slope, $S_{br} = 1.75x10^{-4}$

(c) The maximum water depth (at the dam section), $D_{max} = 23.5$ m (77 ft);

(d) A constant Manning's roughness coefficient, $n_M = 0.0234$;

(e) The specific gravity of the sediment particles, $s_s = 2.65$ (quartz);

(f) The representative sediment sizes, $d_{50} = 0.5$, 1.0, 2.0 mm (0.0017, 0.0033, 0.0066 ft).

These data represent very roughly the characteristics of the Missouri River-Ft. Randall Reservoir system as reported by LIVSEY (1955).

Subsequently, the effect of varying the input parameters was investigated. The river bed slope was increased to $S_{br} = 1.0x10^{-3}$ and the river flow rate was set at $q = 2.0\ m^3/sec/m$. Maintaining these two parameters constant, the following were investigated:

(a) Effect of Manning roughness, $n_M = 0.025$ and $n_M = 0.035$ for sediment sizes, $d_{50} = 0.5$ mm & 10 mm.

(b) Effect of sediment size variation from $d_{50} = 1$ mm to $d_{50} = 10$ mm for $n_M = 0.025$.

3. EVALUATION AND DISCUSSION OF RESULTS

3.1 Introductory Remarks

Three different bed load equations were used in calculating the rate and the pattern of the bed load depostion in a given river-reservoir system. For the bed slope, S_{br}, of 1.75×10^{-4} the calculations were carried out for periods of sediment months; a sediment month was assumed to be a period of 30 days during which the average flow rate was equal to an arbitrarily chosen constant value of $q = 1.81\ m^3/sec/m$ ($19.5\ ft^3/sec/ft$). Although the choice of a 30-day period was arbitrary, it was preferred over a shorter period, such as a sediment day, in order to avoid unnecessary calculations. However, for a bed slope of 1×10^{-3}, with $q = 2.0\ m^3/sec/m$ ($21.55\ ft^3/sec/ft$) the sediment period was chosen as one day because the sediment load carried by the river is much greater for a steeper slope (see Fig. 2.2).

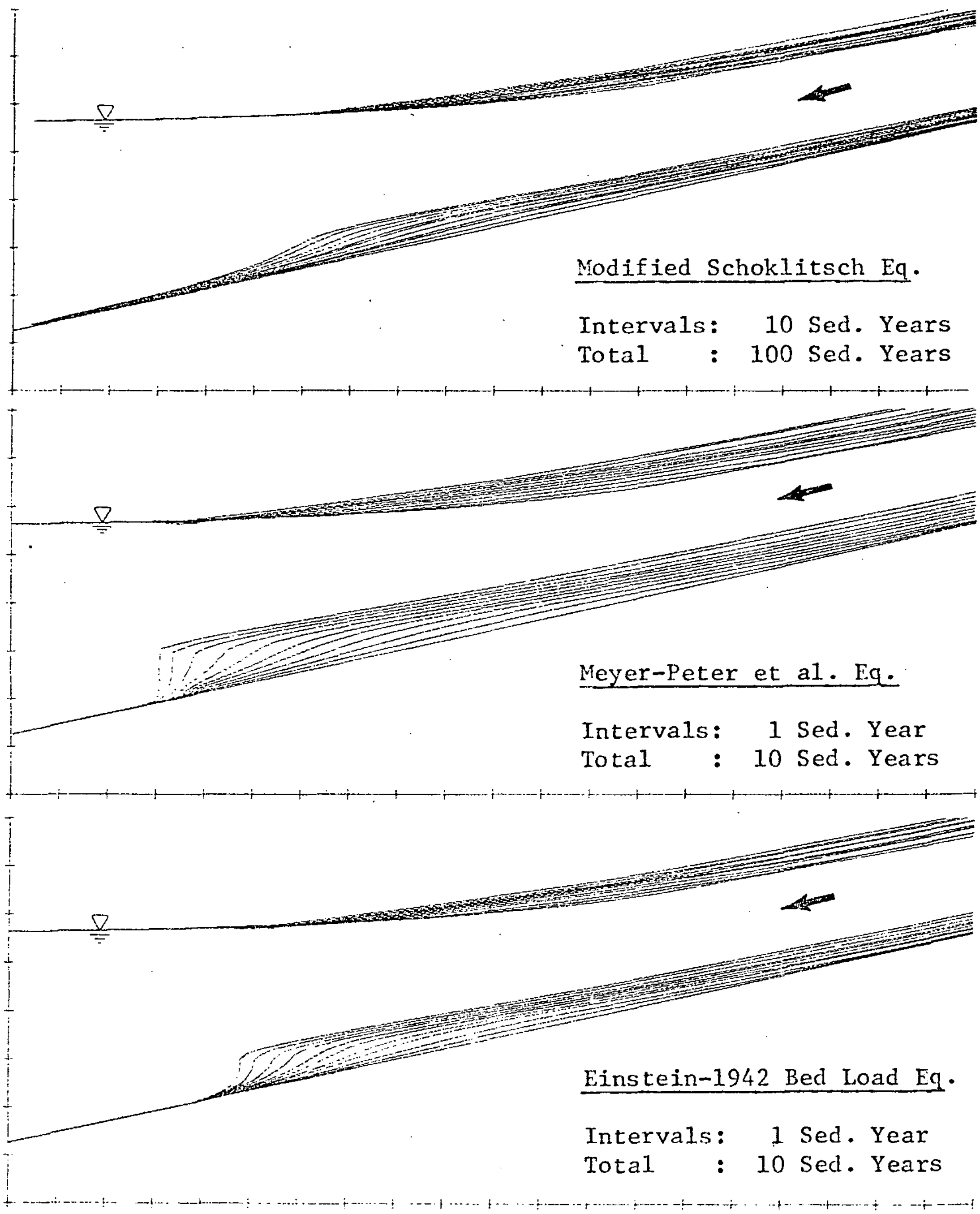
The deposition phenomena predicted by the present mathematical model can be discussed both qualitatively and quantitatively. Due to various limiting assumptions indicated earlier, the qualitative results are considered more important than the quantitative ones, such as the actual rates of deposition predicted by the model.

3.2 Rate of Bed Load Deposition

The rates of bed load deposition for the given river-reservoir system were different for the three different bed load equations used by the model. This is expected due to the fact that these bed load equations are essentially based on different methods of approach [see GRAF (1971), Ch. 7].

3.2.1 Results for $S_{br} = 1.75 \times 10^{-4}$ and $q = 1.81\ m^3/sec/m$.

Fig. 3.1 shows the bed load deposition pattern predicted by the three bed load equations for various intervals of time.



$q = 1.81\ m^3/sec/m\ (19.5\ ft^3/sec/ft)$; $D_{max} = 23.5\ m (77\ ft)$; $s_s = 2.65$

$d_{50} = 1.0\ mm\ (0.0033\ ft)$; $n_M = 0.0234$; $S_{br} = 1.75 \times 10^{-4}$

Figure 3.1 Rate of Bed Load Deposition Obtained with the Three Different Bed Load Equations

It is evident that the highest rate of deposition is predicted with the Meyer-Peter Muller equation, while the Einstein-1942 bed load equation predicts a slightly lower rate of deposition. The rate of deposition predicted by the modified Schoklitsch equation, however, is much lower than the others. In fact, approximately the same amount of deposition is obtained with the modified Schoklitsch equation in 100 sediment years, as compared to about 5 years for the Meyer-Peter Muller and Einstein-1942 bed load equations. This might be expected since the Schoklitsch bed load equation is known to yield rather low amounts of bed load [see GRAF (1971), pp. 156-159]. This can also be seen from Fig. 2.1. A comparison is also shown in Fig. 3.2 for the total bed load depositions resulting at the end of the above prescribed sediment periods.

Calculations were also made for different sediment sizes using both the modified Schoklitsch and the Einstein-1942 bed load equations. As shown in Fig. 3.2, a deposition period of 100 sediment years was obtained for the three sediment sizes, namely d_{50} = 0.5 mm (0.0017 ft), 1 mm (0.0033 ft), and 2 mm (0.0066 ft), with the use of the modified Schoklitsch equation. It is interesting to note that the total amount of the bed load deposited does not seem to be affected a great deal by the sediment size. It is observed from Fig. 3.3 that the total amount of the bed load deposition decreases only slightly as the sediment size is increased from d_{50} = 0.5 mm up to d_{50} = 2.0 mm. In contrast to the modified Schoklitsch equation, which appears to be insensitive to the sediment size, the Einstein-1942 bed load equation shows strong dependence on the sediment size. It can be observed from Fig. 3.4, that the total amount of sediment deposited decreases considerably as the sediment size is increased from d_{50} = 1.0 mm

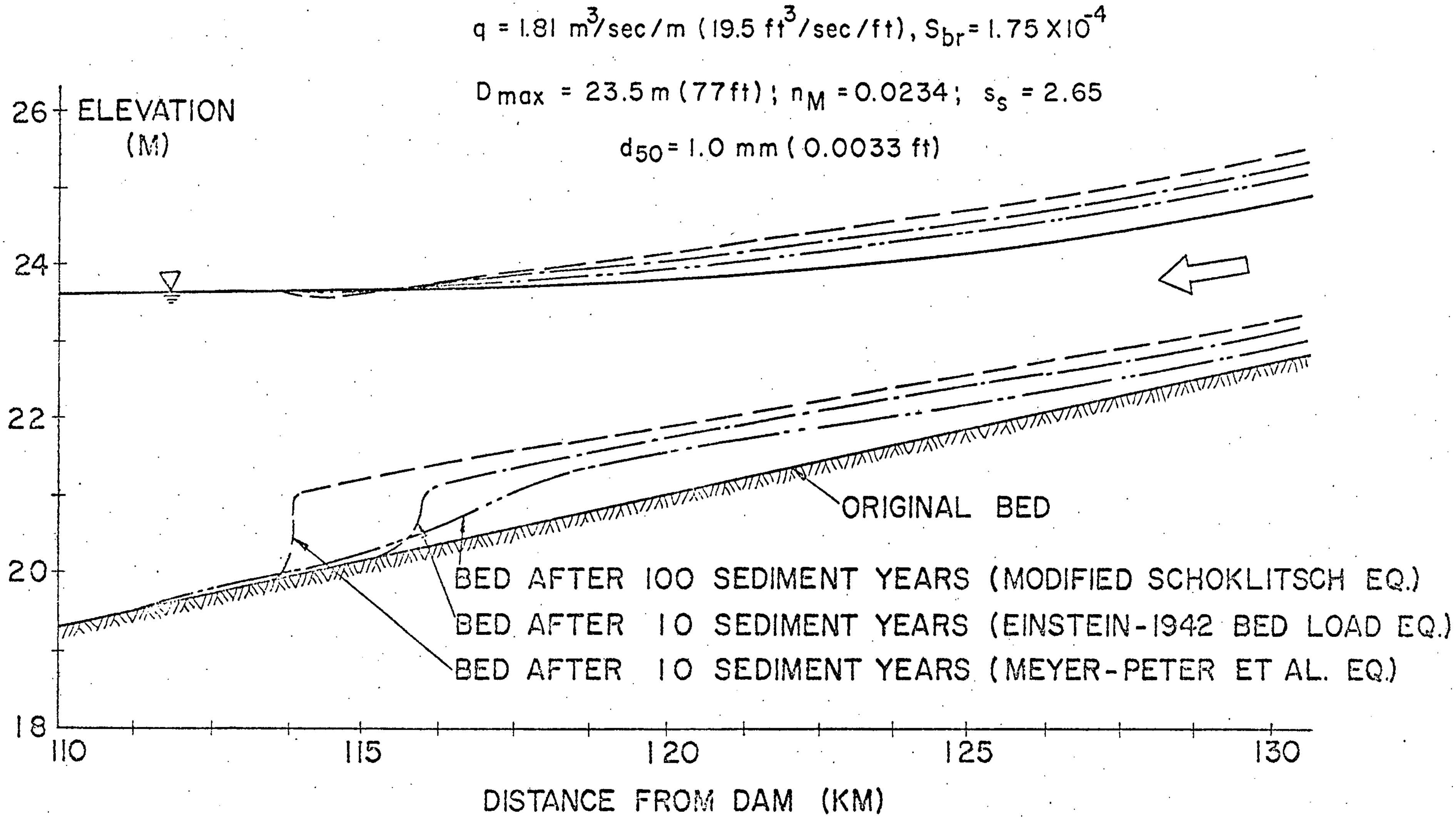
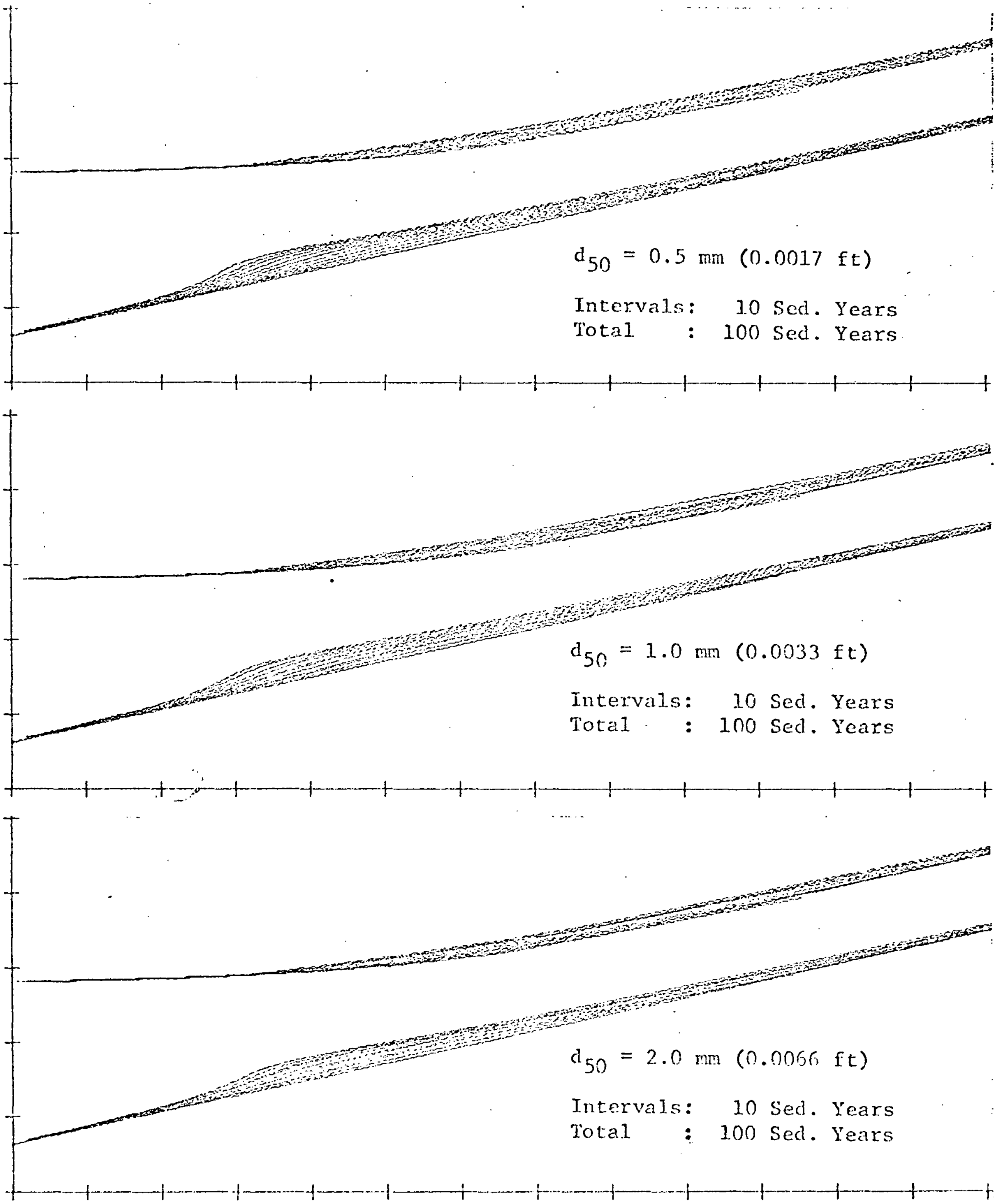


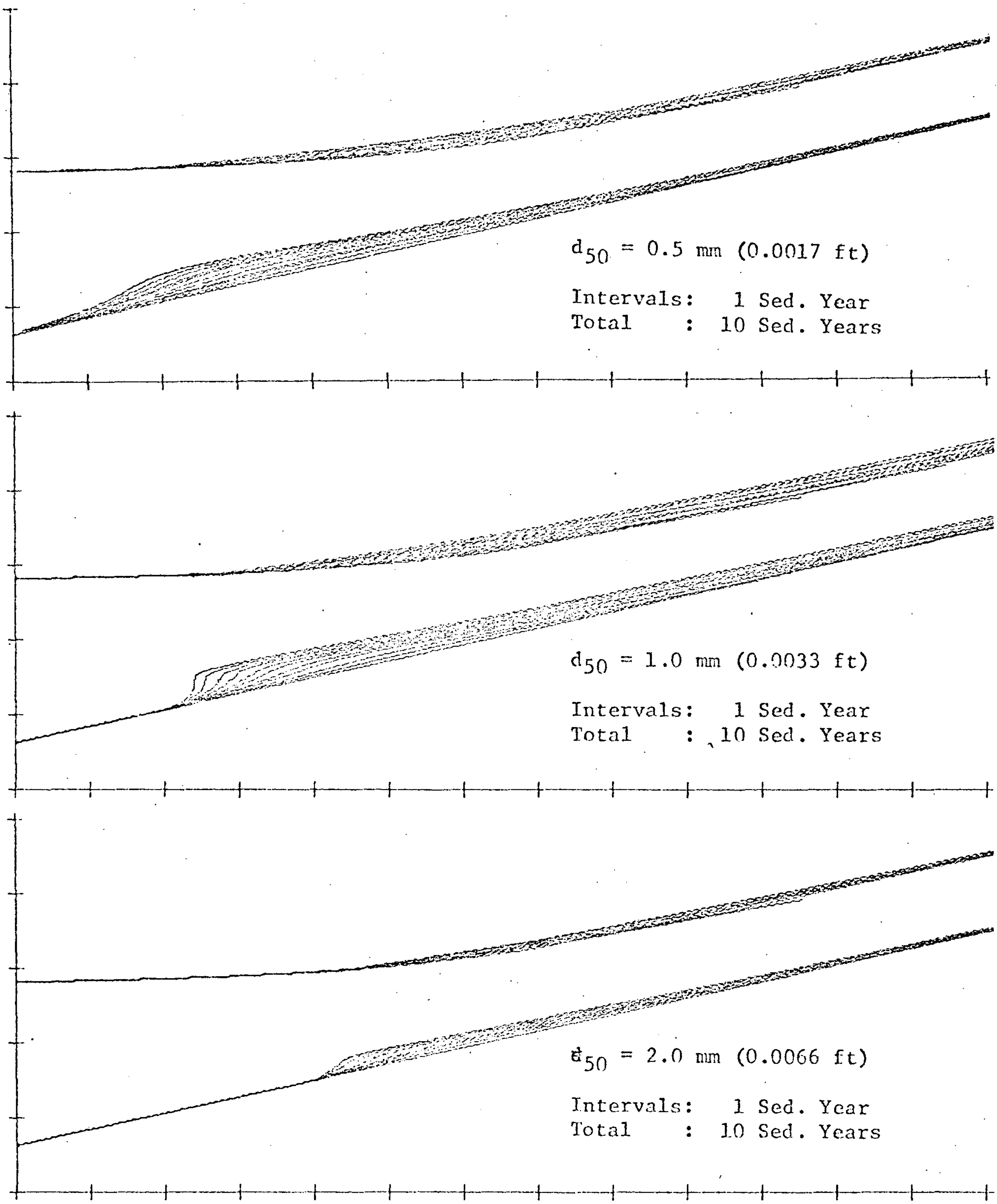
FIGURE 3.2 COMPARISON OF THE TOTAL BED LOAD DEPOSITION RATES OBTAINED WITH THE THREE DIFFERENT BED LOAD EQUATIONS



$q = 1.81\ m^3/sec/m\ (19.5\ ft^3/sec/ft);\quad n_M = 0.0234$

$D_{max} = 23.5\ m\ (77\ ft);\quad s_s = 2.65;\quad S_{br} = 1.75 \times 10^{-4}$

Figure 3.3 Bed Load Deposition with Three Different Sediment Sizes (Modified Schoklitsch Eq.)



$q = 1.81\ m^3/sec/m\ (19.5\ ft^3/sec/ft)$; $n_M = 0.0234$

$D_{max} = 23.5$ m (77 ft); $s_s = 2.65$; $S_{br} = 1.75 \times 10^{-4}$

Figure 3.4 Bed Load Deposition with Three Different Sediment Sizes (Einstein-1942 Bed Load Eq.)

up to d_{50} = 2.0 mm, within the same period of 10 sediment years. This trend could be inferred directly from computations based on the bed load equations for uniform flow (i.e., a plot similar to Fig. 2.3 with base conditions of $S_{br} = 1.75 \times 10^{-4}$, $q = 1.81\ m^3/sec/m$, and $n_M = 0.0234$). For instance the Einstein-1942 bed load equation yields the following uniform flow sediment carrying rates:

d_{50} (mm)	g_s (kg/sec/m)
0.5	0.102
1.0	0.121
2.0	0.058

The fact that there is less sediment inflow for d_{50} = 2.0 mm than for d_{50} = 1.0 mm is clearly shown in Fig. 3.4. For d_{50} = 0.5 mm, at first glance (Fig. 3.4), it appears that the delta formation is larger than for d_{50} = 1.0 mm. However upon closer examination it can be seen that there is actually a larger amount of deposition for d_{50} = 1.0 mm than for d_{50} = 0.5 mm (as the uniform flow equation indicates), but the distribution of sediment deposits is significantly different.

3.2.2 Results for $S_{br} = 1 \times 10^{=3}$ and $q = 2.0\ m^3/sec/m$

The effect of a steeper bed slope on delta formation as well as the effects of varying the Manning roughness coefficient, n_M, the sediment size, d_{50}, and the length of the sediment period are investigated in this section.

(a) Bed Slope, S_{br}. The effect of a steeper bed slope can be inferred directly from Fig. 2.2. It can be seen from this figure that the bed load is markedly affected by bed slope. In fact for a change in bed slope from 1.75×10^{-4} to 1×10^{-3} the magnitude of the bed load rate of sediment transport increases by a factor of 10 to 100 depending on the bed load equation used.

Consequently at the steeper bed slope, delta formation occurs much more rapidly. Hence for this steeper bed slope of 1×10^{-3} the sediment period used in the calculations is the sediment day rather than the sediment month used in section 3.2.1. The more rapid delta formation is apparent in the following figures where the total time for delta formation is expressed in days rather than months.

A comparison of the three bed load equations on a bed slope of 1×10^{-3} is shown in Fig. 3.5 for a Manning n of 0.025 and in Fig. 3.6 for a Manning n of 0.035. It can be clearly seen that the delta formation predicted by the Meyer-Peter Muller equation is about 10 times faster than either the modified Schoklitsch or the Einstein-1942 bed load equations, a result which is significantly different from that obtained in section 3.2.1. Once again this is due, at least in part, to the fact that, for these conditions, the Meyer-Peter Muller bed load equation predicts a larger sediment inflow.

(b) Manning Roughness, n_M. Computer runs were made with n_M values of 0.025 and 0.035 and sediment sizes of 0.5mm and 10mm. It should be noted that changing the roughness affects the solution in several ways. An increase in roughness, while maintaining a constant bottom slope, flow rate and sediment size, has the effect of increasing the normal depth and of decreasing the velocity. Figure 2.4 illustrates how the bed load capacity, as predicted by the three equations, varies with different values of Manning's n.

Figures 3.7, 3.8 and 3.9 show the delta formations for the modified Schoklitsch, Meyer-Peter Muller and Einstein-1942 bed load equations with a sediment size of 0.5mm. In all three cases, the higher n_M value of 0.035 causes the delta to form closer to the dam. In two of the figures, modified Schoklitsch equation (Fig. 3.7) and Einstein-1942 equation (Fig. 3.9), the

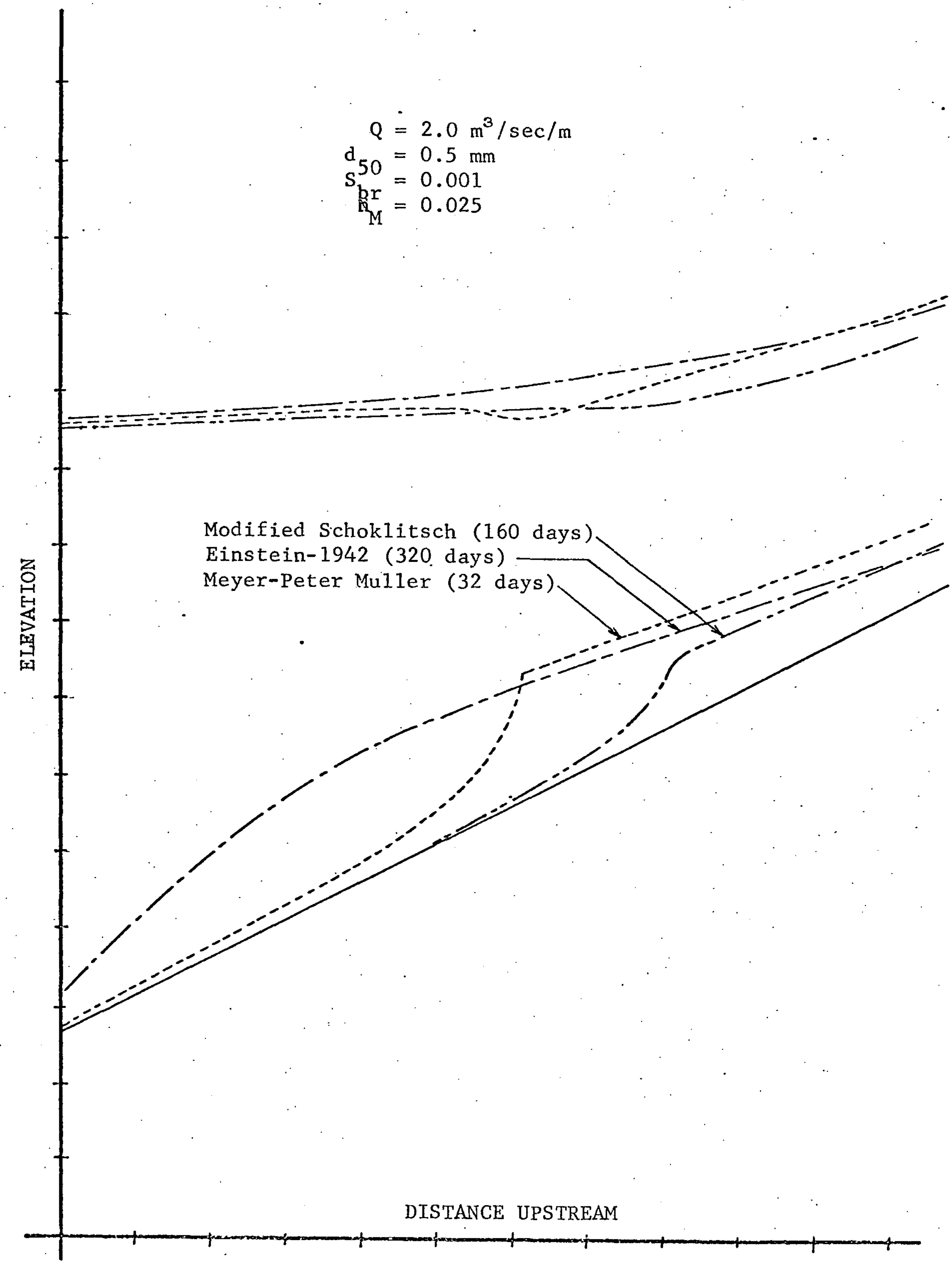


Figure 3.5 Comparison of Three Bed Load Equations

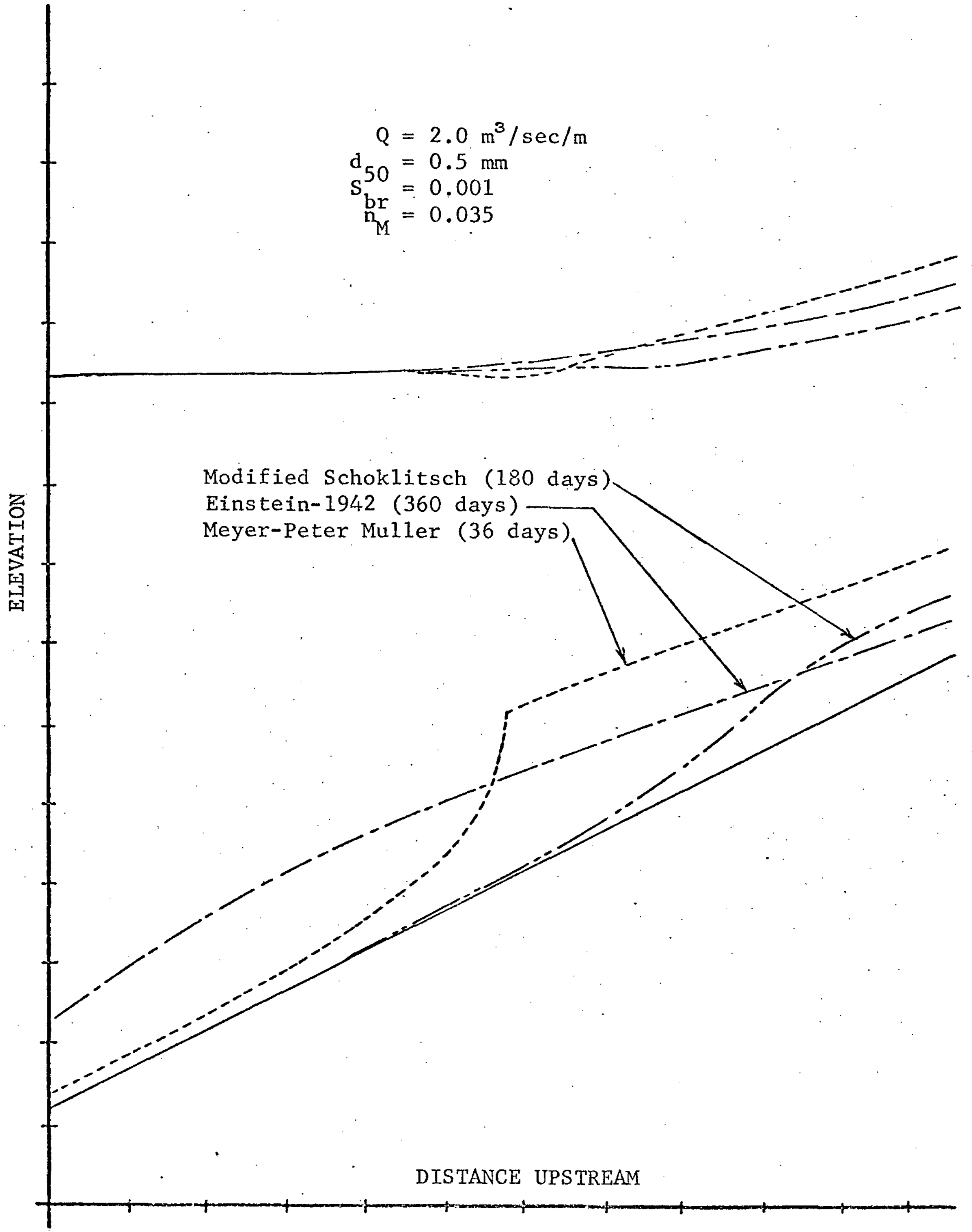


Figure 3.6 Comparison of Three Bed Load Equations

lower n_M value is responsible for forming a steeper-faced delta, whereas the opposite is true for the Meyer-Peter Muller equation (Fig. 3.8).

A similar study was made using a sediment size of 10 mm which is within the alleged applicable range of all three equations. The results are shown in Figs. 3.10 and 3.11 which correspond to the modified Schoklitsch and the Einstein-1942 bed load equations. (Due to a technical probelm not yet solved within the subroutine REACH, the computer program was not successful with this sediment size using the Meyer-Peter Muller equation). The qualitative results are the same as the ones discussed in the preceding paragraph.

The results for d_{50} = 10 mm which gave Figs. 3.10 and 3.11 are plotted in a different manner in Figs. 3.12 and 3.13 for comparison of the two bed load equations. Note that for a Manning n of 0.025, the rate of delta formation is about the same for the modified Schoklitsch and Einstein-1942 equations. However with a Manning n of 0.035 the rate of delta formation predicted by the Einstein-1942 equation is about 5 times faster than that predicted by the modified Schoklitsch equation.

(c) Sediment Size, d_{50}. Computer runs were made to determine the affect on delta formation due to a change in sediment size from 1 mm to 10 mm. Figure 2.3 shows that the bed load capacity, as predicted by both the Schoklitsch-Hjulstrom and the Einstein-1942 equations, decreases significantly with this change in sediment size. Figures 3.14 and 3.15 show that in both cases the deltas formed with the 10 mm particles are smaller and further upstream than 1 mm particle deltas. These two generalities are to be expected since the bed load capacity is smaller for the 10 mm sediment size and since

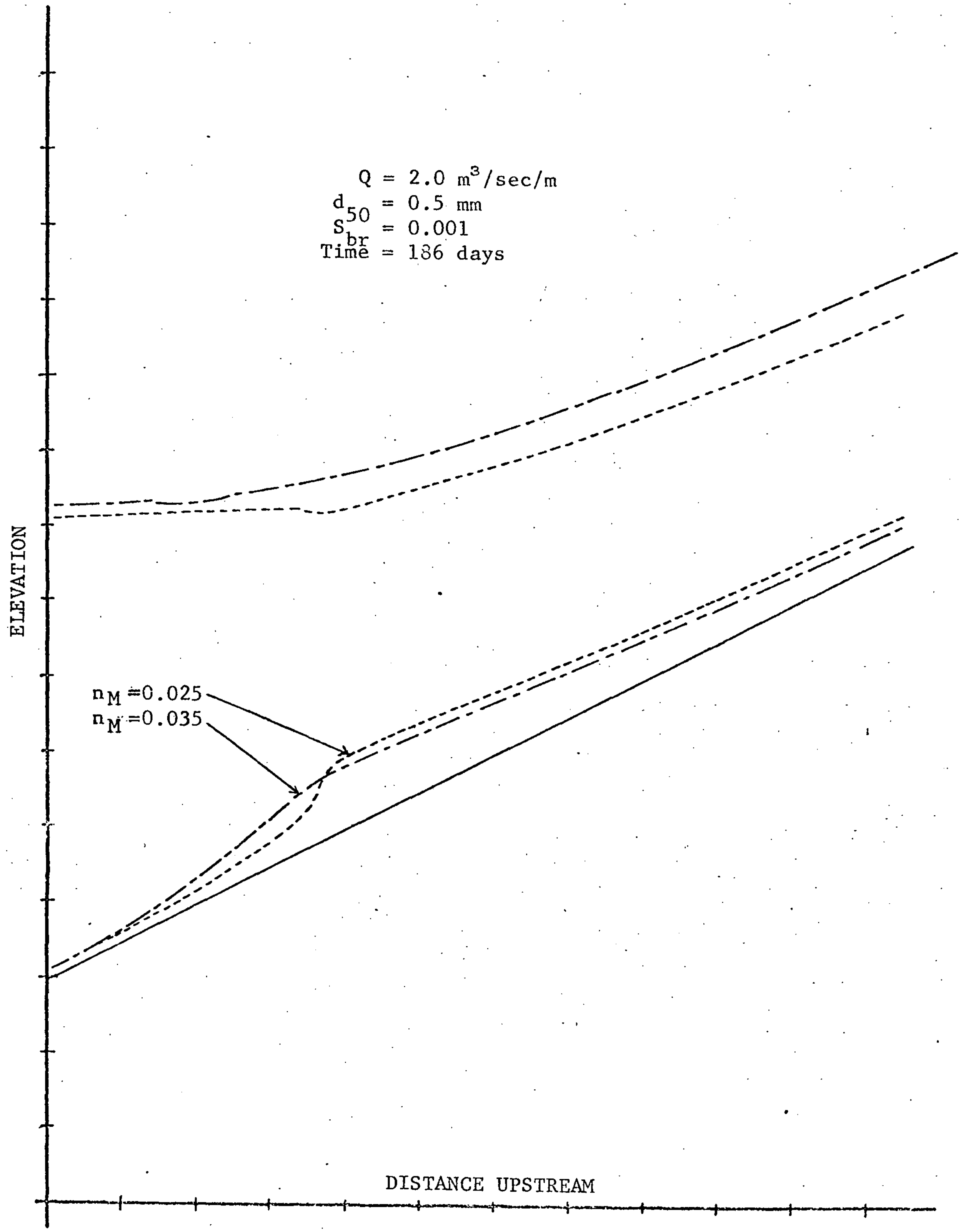


Figure 3.7 Effect of Manning n, Modified Schoklitsch Equation

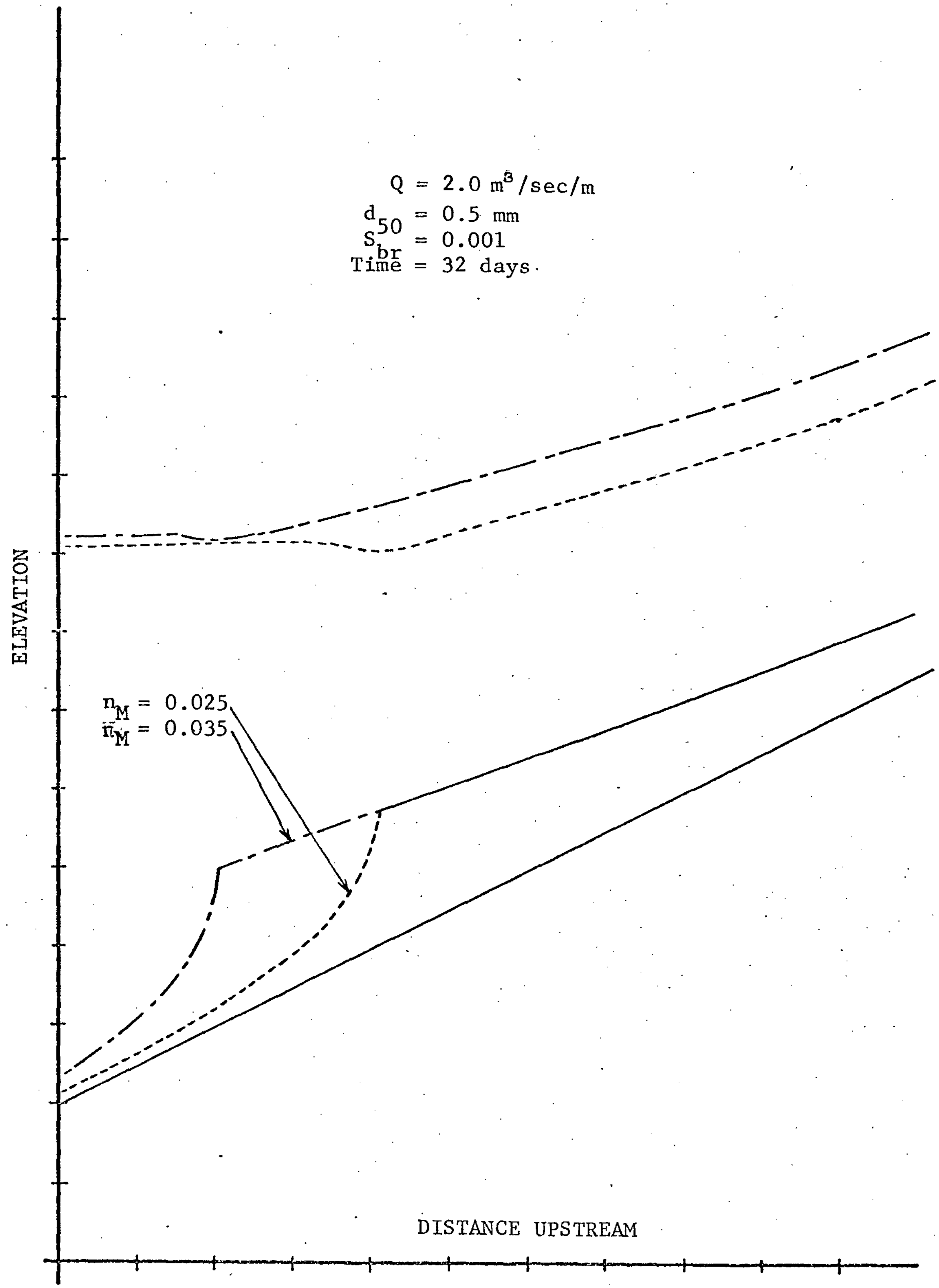


Figure 3.8 Effect of Manning n, Meyer-Peter Muller Equation

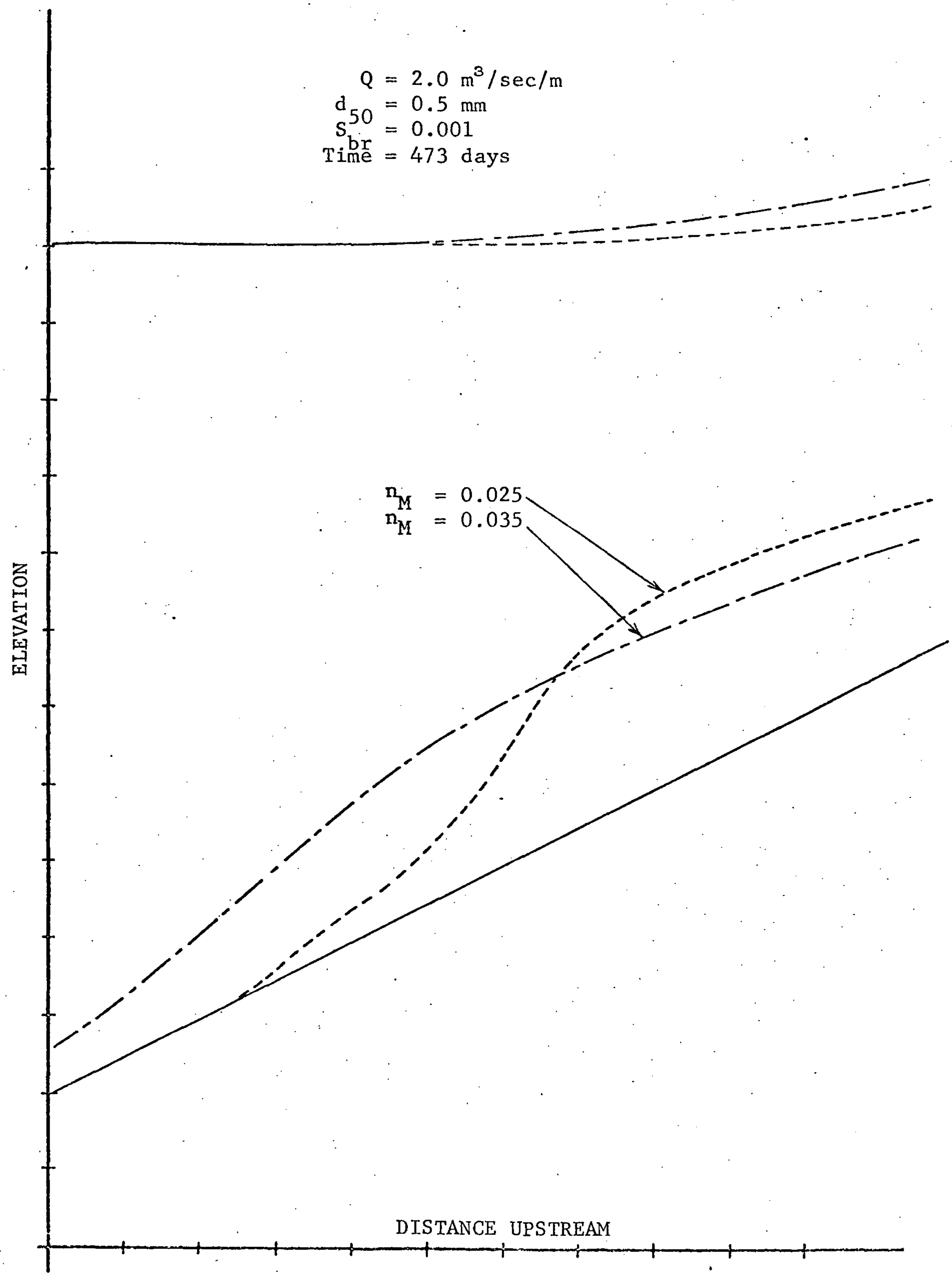


Figure 3.9 Effect of Manning n, Einstein-1942 Equation

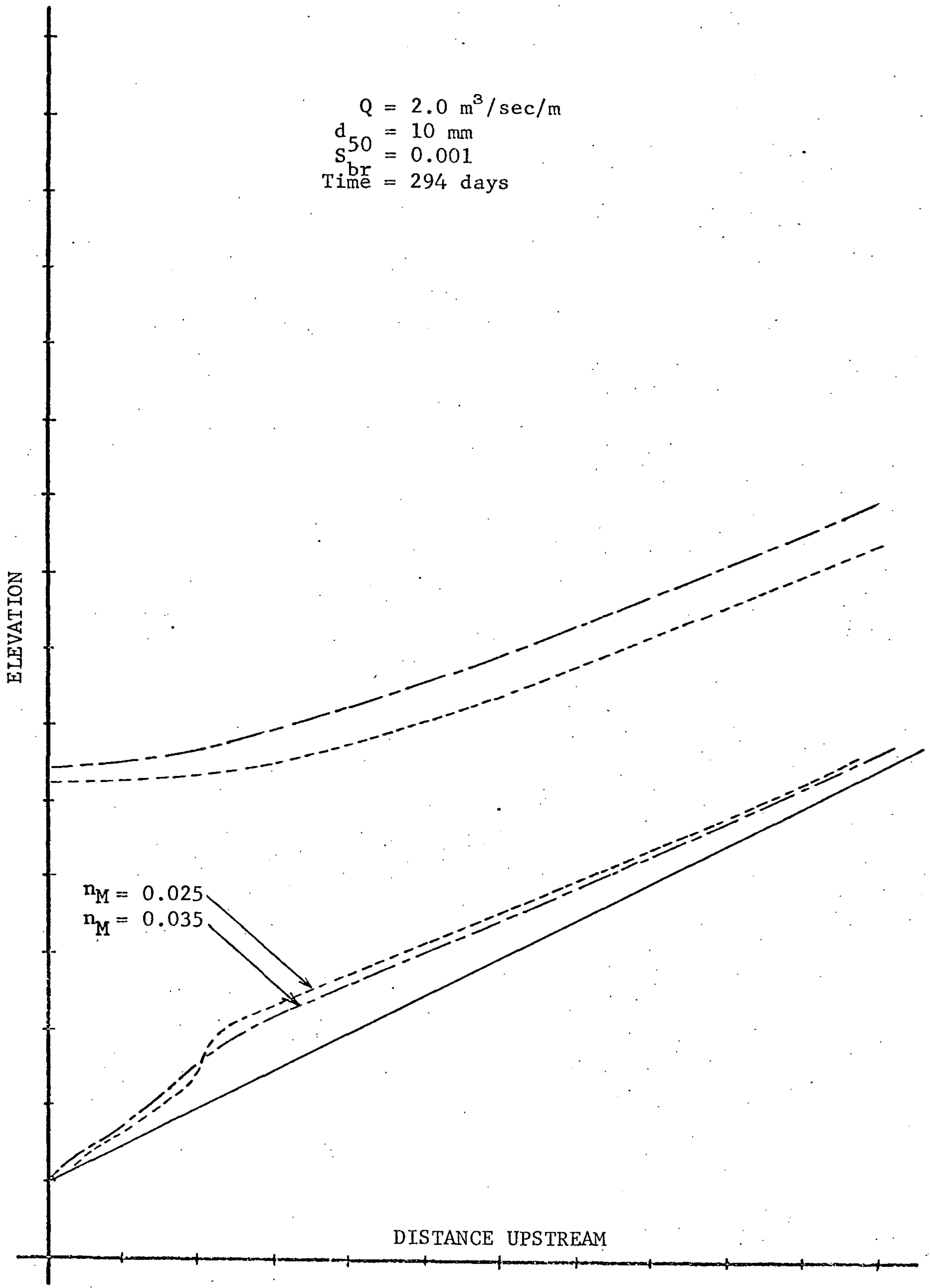


Figure 3.10 Effect of Manning n, Modified Schoklitsch Equation

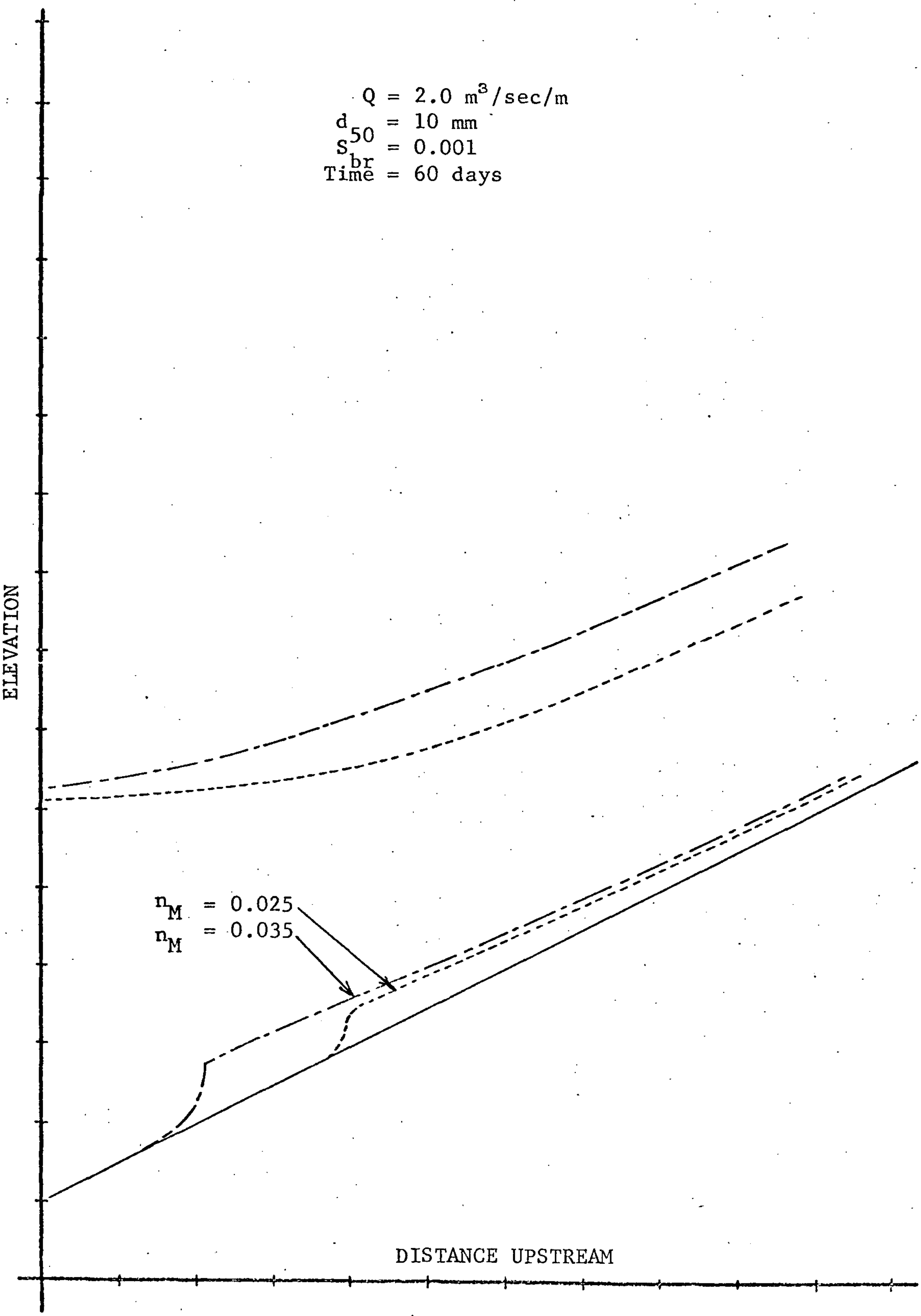


Figure 3.11 Effect of Manning n, Einstein-1942 Equation

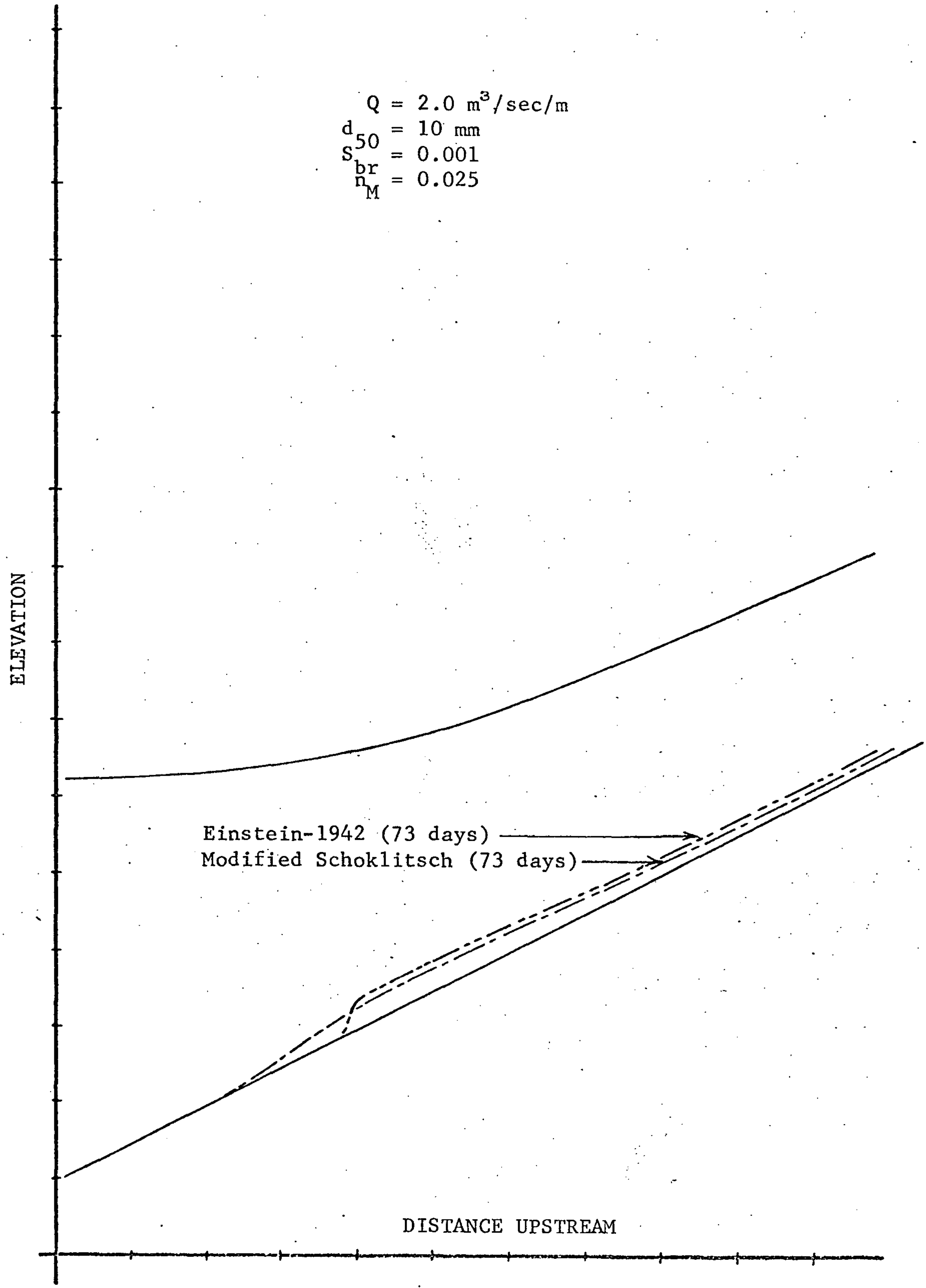


Figure 3.12 Comparison of Two Bed Load Equations for Large d_{50}

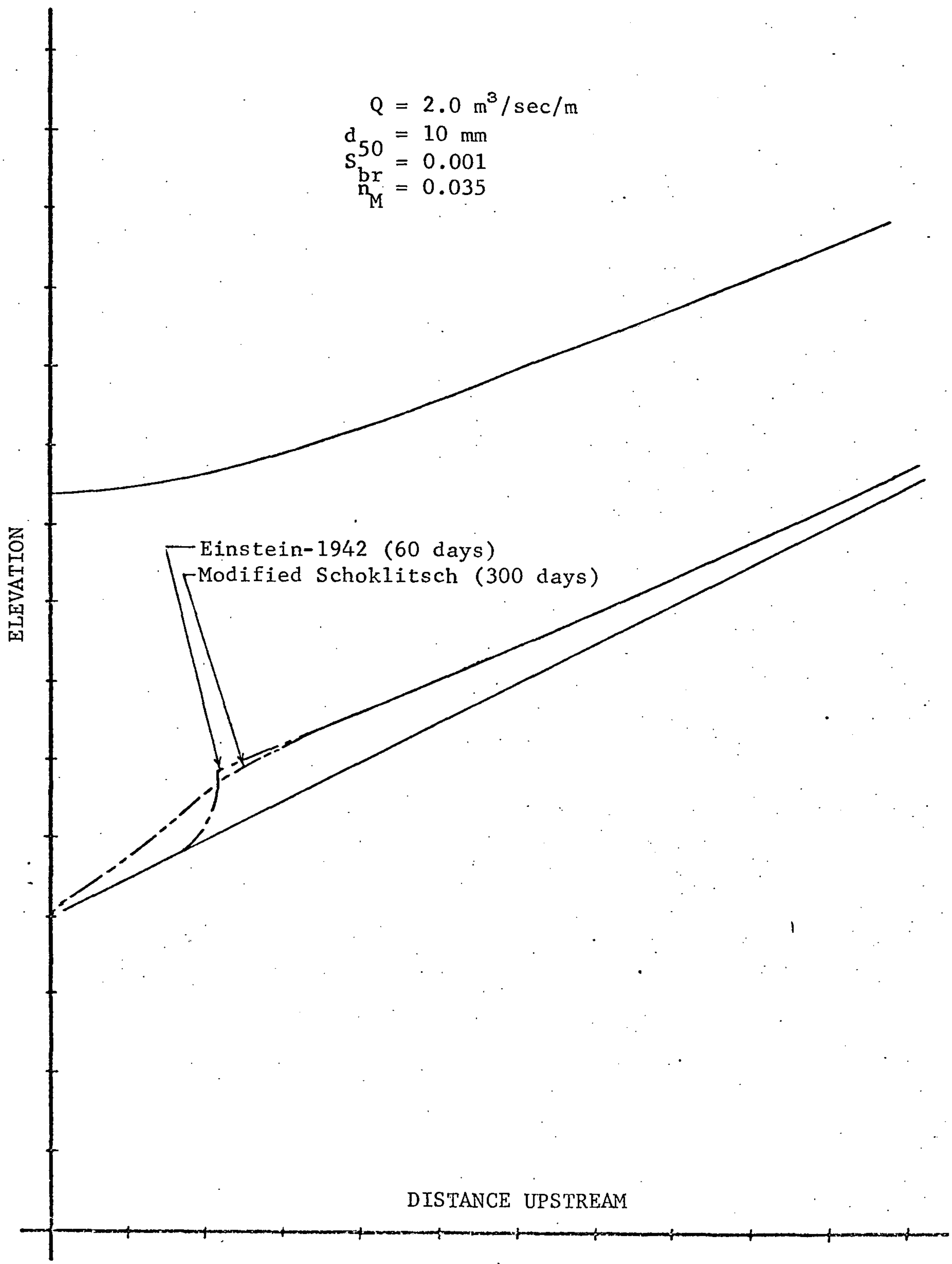


Figure 3.13 Comparison of Two Bed Load Equations for Large d_{50}

larger particles settle out faster and therefore further upstream than smaller particles. These observations differ somewhat from those made in section 3.2.1. It was noted there that the Schoklitsch-Hjulstrom delta formation rate did not depend much on sediment size. It should be emphasized that this generality applies only under certain conditions.

(d) Sediment Period. Computer runs were made to observe the effect of the sediment period length on the delta formation. The Meyer-Peter Muller and the Einstein-1942 equations were selected, because, for the flow parameters selected (d_{50} = 0.5 mm and n = 0.025), the former predicts a rapid delta formation while the latter predicts a slow one. Computations were made for sediment periods of six hours and one day.

The results of the Meyer-Peter Muller runs have shown that, for the six-hour sediment period, a smoother, more shallow and slightly larger delta is formed than that for the 24-hour sediment period. This is due to the fact that sufficient sediment is being carried during the 24-hour period to cause a deposition thickness of greater than 2% of the water depth. Hence, in this case, the amount of deposition within the sediment period is sufficient to cause a significant change in the backwater profile before the computer program warrants such a re-computation. Therefore, if the rate of sedimentation is rapid, then the specified period should be small. This precaution ensures a sufficient frequency of back water calculations. Another result of reducing the sediment period is, of course, an increase in computer time required.

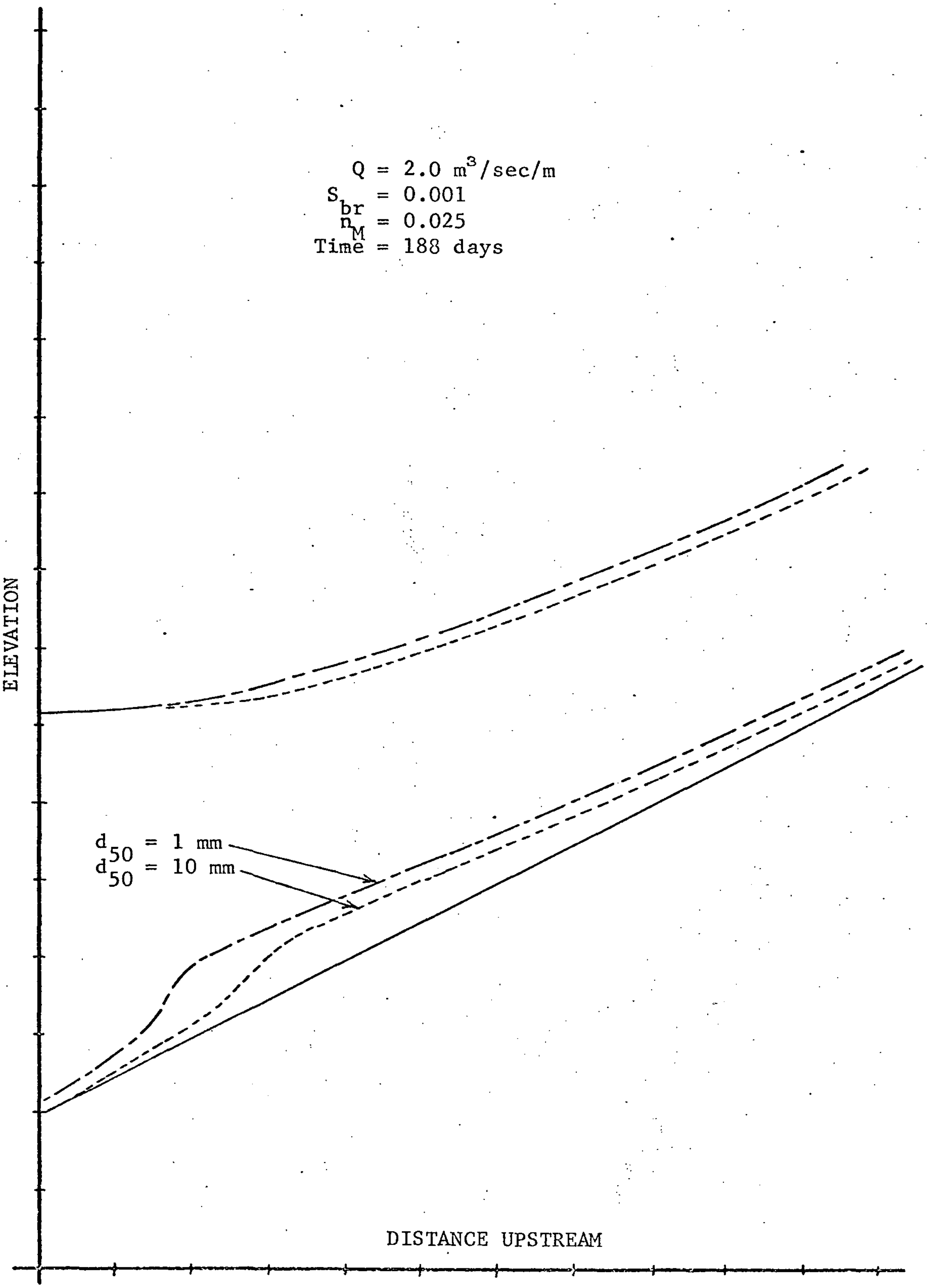


Figure 3.14 Effect of Sediment Size, Modified Schoklitsch Equation

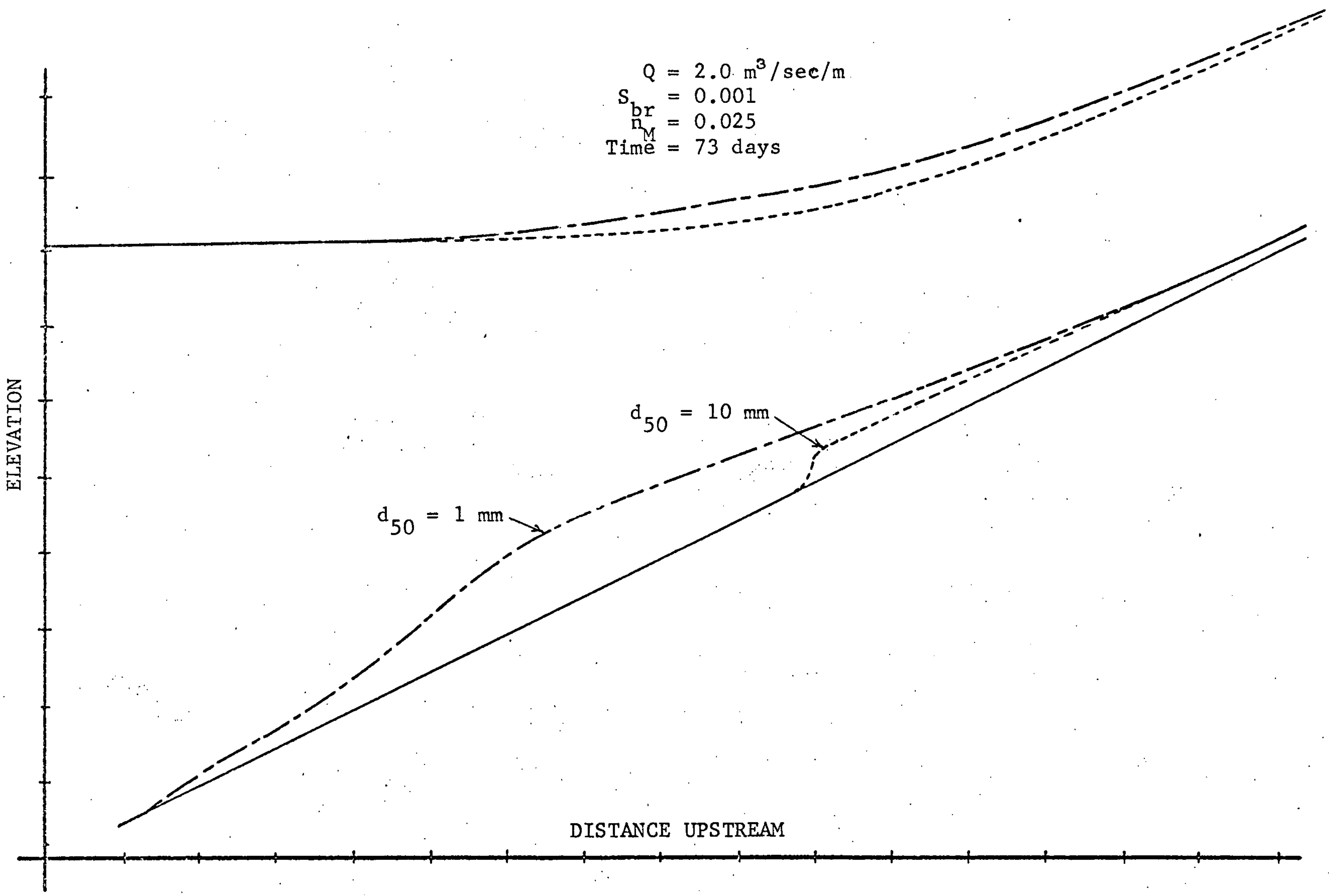


Figure 3.15 Effect of Sediment Size, Einstein-1942 Equation

The results of the Einstein-1942 equation runs have shown that the delta formations are almost identical. This was to be expected since the computer run with a 24-hour period had several deposition cycles between back water calculations. This means that the 24-hour period was sufficiently small and any further reduction would have no significant effect.

3.3 Delta Formation

The most interesting result obtained in this study was that a delta was being formed with features common to all cases in which the three different bed load equations were used. Such a typical delta formation is illustrated in Fig. 3.16 as plotted by the computer as a result of the calculations made with the use of one of the bed load equations. The following remarks can be made regarding the formation of the delta:

a) The deposition begins in the form of rather flat layers in the upstream regions of the reservoir. The thickness of these layers becomes gradually larger until a certain section is reached at which the rate of deposition is at a maximum. Downstream of this section the deposition layers tend to become thinner again. The repetition of this process results in a typical triangular shape of deposition, a delta. Thus, in the earlier stages of deposition, there is a process of build-up, and as such, a delta is formed.

b) Subsequently, the apex of this delta begins to advance in the downstream direction, such that the downstream side of the delta becomes shorter and steeper, while the upstream side becomes longer and flatter. Thus, the delta begins to advance towards the reservoir.

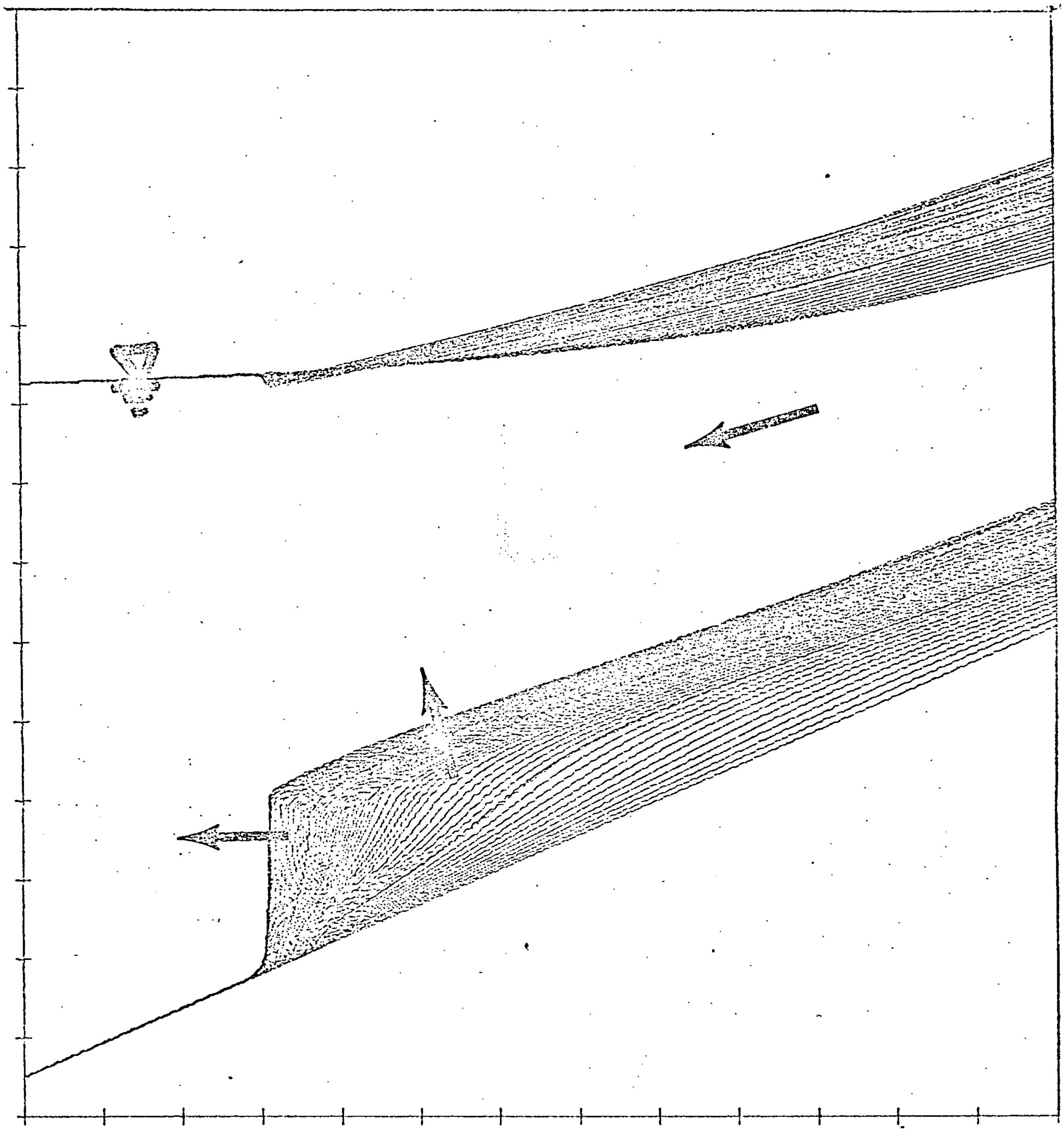


Figure 3.16 Build-up and Advancement of a Typical Delta Formation as a Result of Bed Load Deposition

The mathematical model is relatively simplistic in its present form. Yet, it is remarkable to observe, that the above basic features of the formation of a typical delta are in good agreement with the delta formations in existing reservoirs. A good example of such a reservoir would be Lake Mead behind Hoover Dam along the Colorado River. As shown in Fig. 3.17, the deposition pattern can be considered quite similar to the one predicted by the present model. It should immediately be noted, however, that this is an entirely qualitative observation, and not a quantitative one.

It is also interesting to note that the location of the delta formation seems to depend on the sediment size to a considerable extent. It has been clearly exhibited that the initial location of the delta appears at further downstream sections as the sediment size is decreased. This behavior, predicted by the model, is as expected and is observed to occur in existing reservoirs.

4. CONCLUSIONS

A mathematical model was constructed to predict the characteristics of bed load depositon in a reservoir. Three different bed load equations were used: (1) the modified (for deposition) Schoklitsch equation, (2) the Meyer-Peter Muller equation, and (3) the Einstein-1942 bed load equation. Several arbitrary sets of input information were chosen for the characteristics of the sediment and the river-reservoir system.

The following conclusions can be made:

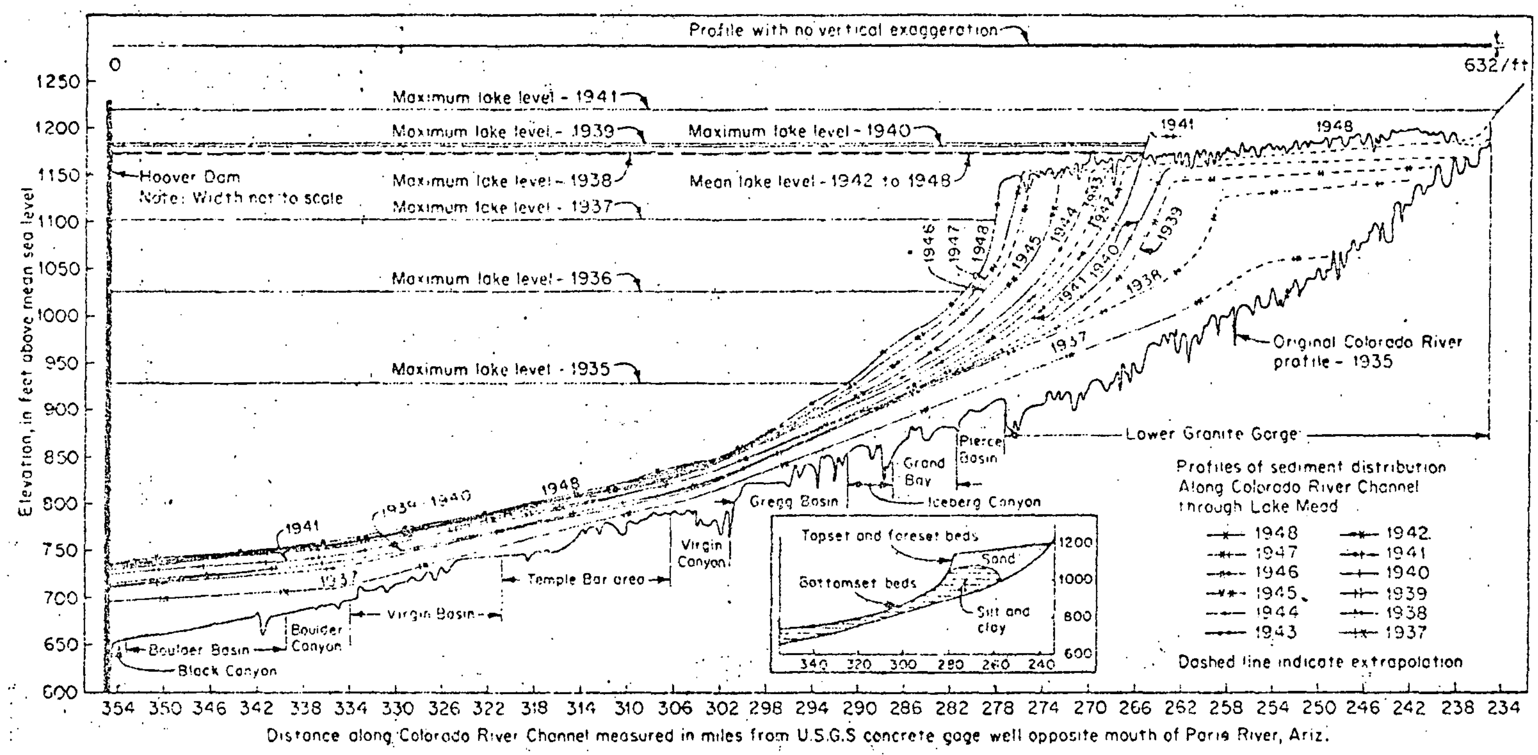


Figure 3.17 Deposition Pattern in Lake Mead behind Hoover Dam [after GRAF (1971)]

a) A delta is formed in the upstream regions of the reservoir, as a result of a build-up process. Subsequently, this delta begins to advance in the downstream direction maintaining its typical triangular shape which resembles actual delta formation in existing reservoirs.

b) Qualitatively, the shape and the method for formation of the delta seem to be quite similar to the ones that occur in existing reservoirs, such as Lake Mead behind Hoover Dam. This is particularly remarkable considering the fact that the present mathematical model is rather simplistic.

c) Delta formation rates as predicted by the three bed load equations differ markedly under certain circumstances. These differences can be largely attributed to the extremely different bed load capacities predicted by the equations for uniform flow.

d) A significant difference in bed load deposition distribution of sediments is noted. In general the Meyer-Peter Muller equation consistently predicts a steep-faced delta formation. The Modified Schoklitsch equation, on the other hand, tends to predict a more rounded delta face. The Einstein-1942 equation predicts a steep-faced delta for the larger diameter particles, but a smoother, more rounded delta face for the small diameter (d_{50} = 0.5 mm) investigated.

5. FUTURE WORK

In the present study, a mathematical model for predicting sedimentation in reservoirs was applied to one-dimensional (unit-width) river-reservoir systems, whose characteristics were chosen arbitrarily. Although results of the predicted delta formation are very encouraging, it is apparent that further study is needed. The computer program developed for the model is considered to be sufficiently flexible for improvement and for application to more complicated, yet more realistic, river-reservoir systems.

The following points are considered to be of interest for future investigations:

a) Other bed load equations should be studied possibly after being modified for deposition.

b) The model, in its present state, should be tested with other different values of the sediment size, water flow rate, river roughness, normal river slope and the sedimentation period. These values should be chosen so as to correspond to real river-reservoir systems for the purpose of comparing the predicted and the actual phenomena.

c) The size of the sediment transported by a river is hardly uniform. Rather, it is some mixture of various different sizes of sediments. This is not taken into account by this model in its present state. The simplest way of accounting for the mixture effects would be a mere superposition of the results obtained with the various fractional sediment sizes forming the mixture. The model would be further improved if the sedimentation periods are chosen to be rather small, and if during this period, the larger fractional sediment sizes are allowed to deposit before the smaller ones.

d) The present model assumes a constant water discharge throughout the system. In the actual river-reservoir systems, such is seldom the case; the water discharge is time-dependent. A hydrograph of the river water discharge would be used to improve the model to that effect. In such a case, the model would simply be executed over sedimentation periods for which the water discharge roughly remains a constant.

e) The sediments which are deposited are subject to a certain amount of compaction and consolidation. The model could be improved to take such phenomena into account. One way would be to assume and calculate only one rate of compaction and consolidation for every fractional sediment size.

f) The present mathematical model is designed for one-directional flow phenomena. The following steps could be considered for improvement of the model:

(i) The width of the river-reservoir system can be prescribed as a function of the distance from a control section, for example, the dam section.

(ii) Secondary flow and sediment phenomena can be considered for the given channel geometry. Velocity distributions in horizontal and vertical, flow patterns such as meanders and resulting sediment motions would ultimately have to be considered.

(g) The above points being considered, the model should next be extended to cover the suspended and total sediment transport, as well as the cohesive type of sediment transport, and the deposition resulting from these different modes of transport.

(h) It is clear that, with each step of improvement in the model the assumptions would become less severe, leading to the fact that the results predicted by the model can be considered more realistic and comparable with field data. The field data, on the other hand, are presently quite scarce. Consequently, efforts should also be concentrated on collecting field data with proper information on the sediment and river-reservoir characteristics. Only then would the mathematical model become really valuable in predicting the sedimentation phenomena in reservoirs.

6. REFERENCES

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APPENDIX - COMPUTER PROGRAM

A computer program was prepared for the mathematical model of the phenomena of sediment deposition in a one-dimensional (unit width) river-reservoir system. The program was written in Fortran IV and run with the CDC-6400 Computer and 620/F Calcomp Plotter facilities of the Lehigh University Computer Center.

Given a river with a normal (uniform) depth and slope, a unit discharge, a channel bed roughness, a representative sediment size (d_{50}), and a dam height, the computer program is designed to calculate the M1-type back water profile, the sediment transport and deposition within the reservoir, and recalculate the back water profile after significant deposition occurred, and so on. It also prints and plots the calculated data (see Fig. 2.2).

In the following, a detailed explanation of the computer program is presented. First given is a list of symbols used in the program. Subsequently, the flow charts for the main program and the individual subroutines are given along with some explanatory remarks wherever deemed necessary. Finally, a complete listing of the program and a typical output are presented.

LIST OF RECURRING SYMBOLS IN THE COMPUTER PROGRAM

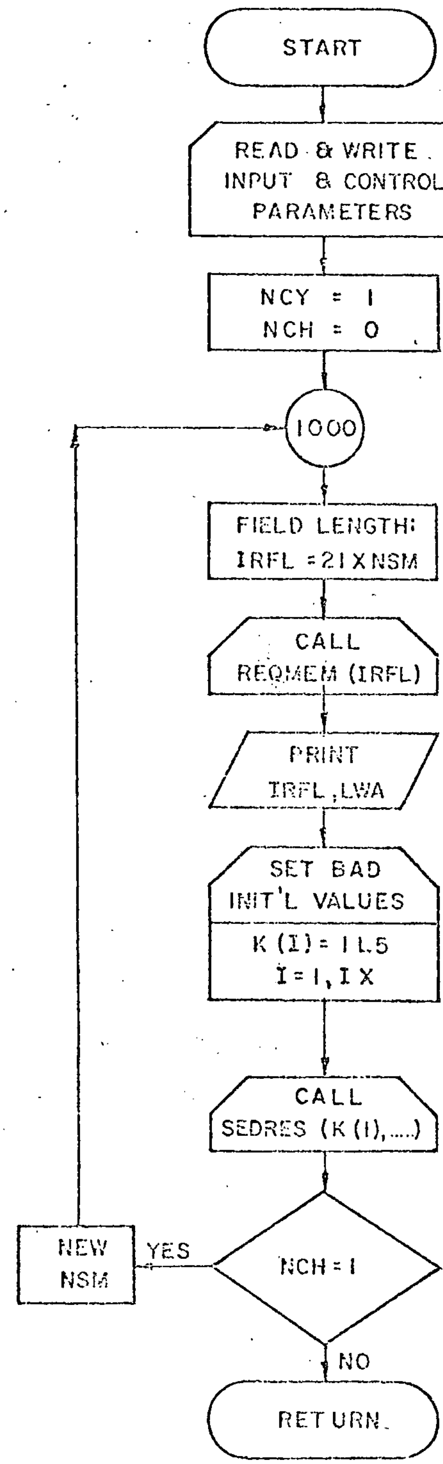
CDZBB	Cumulative increment in bed elevation at each section due to bed load deposition between two consecutive calculations of back water profile.
CDZBM	Maximum value of CDZBB
D	Water Depth
DD	Incremental water depth
DEPBL	Amount of bed elevation due to bed load deposition
DL	Incremental reach length
DMAX	Maximum water depth at dam section
DNORM	Normal depth of the river
DSB	Approximation parameter for bed slopes
DSE	Approximation parameter for energy slopes
DZBB	Increment in bed elevation due to bed load depostion
DO,VO,SBO...	Values of the variables at mid-section of the reach
D1,V1,SB1...	Values of the variables at entrance section of the reach
D2,V2,SB2...	Values of the variables at exit section of the reach
D50	Representative sediment size
FR	Froude number
GSB	Bed load rate in weight per width per unit time
IRFL	Field length required for dimensional variables
K,KD,KE	Iteration control parameters
K(I)	Dummy variable for blank common
KEY	Control parameter for significant deposition
L	Distance from the dam section

LWA	Last word address
LO	Distance of each section from the dam section at the end of each backwater curve calculation
NCASE	Computation case number
NCH	Control parameter for field length
NCM	Maximum cycle number
NCY	Cycle number
NEQ	Number of the bed load equation being used
NLAST	Cycle number of the last series of calculations
NM	Manning's roughness coefficient
NPLT	Control parameter for plot type
NS	Running section number
NSM	Maximum section number
QSB	Bed load rate in volume per width per unit time
QU	Water flow rate per unit width
QUCR	Critical (deposition) value of QU
SBA	Difference between the bed and the energy slopes
SBR	Normal slope of the river
SS	Specific gravity of solids
SBOT	Trial bed slope
TOTGSB	Total bed load rate in weight per width per cycle
TOTQSB	Total bed load rate in volume per width per cycle
VCR	Critical (deposition) velocity (after Hjulstrom)
ZB	Bed elevation at each section
ZBO	Bed elevation at each section at the end of each backwater curve calculation
ZE	Energy elevation at each section

MAIN PROGRAM

In the MAIN program, after the reading of the input and the control parameters, the required field length is determined based on the estimated maximum number of the sections, NSM, for the backwater profile calculations. Then, the subroutine SEDRES is called for the initiation of the actual calculations. If the estimated field length is not sufficient, then the related control parameter comes out to be NCH = 1, and a longer field length is determined based on an increased NSM, with this new field length the procedure outlined above is repeated to continue the calculations.

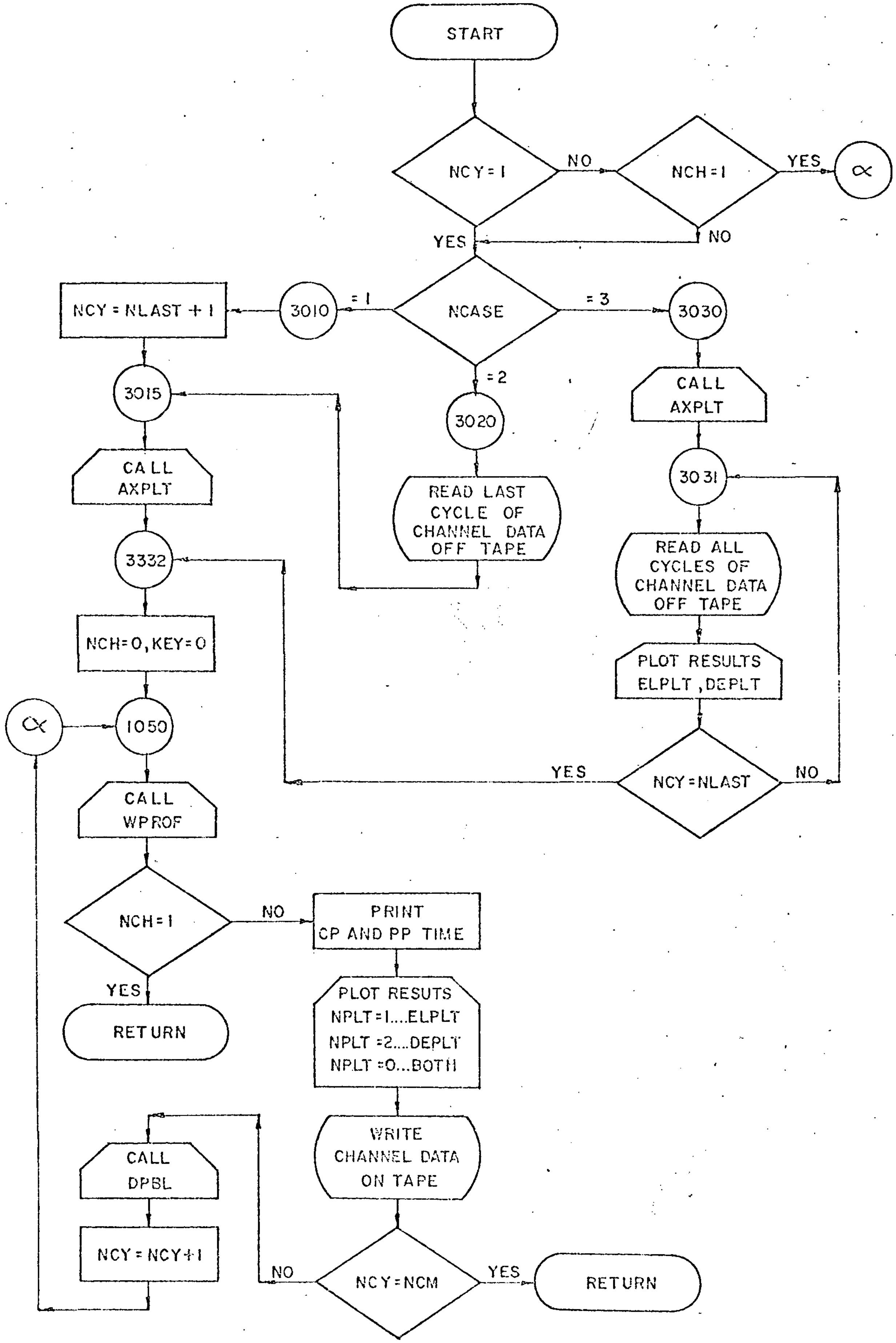
The MAIN program also makes sure that all the storage locations are filled in with "bad computer values", so that if a proper initialization is not made for any parameter, an error message should appear. A special command in the MAIN program also indicates the exact length of the dynamic part of the program as well as the "last word address". The main program has also a BLANK COMMON, and several regular COMMON blocks.



MAIN PROGRAM

SUBROUTINE SEDRES

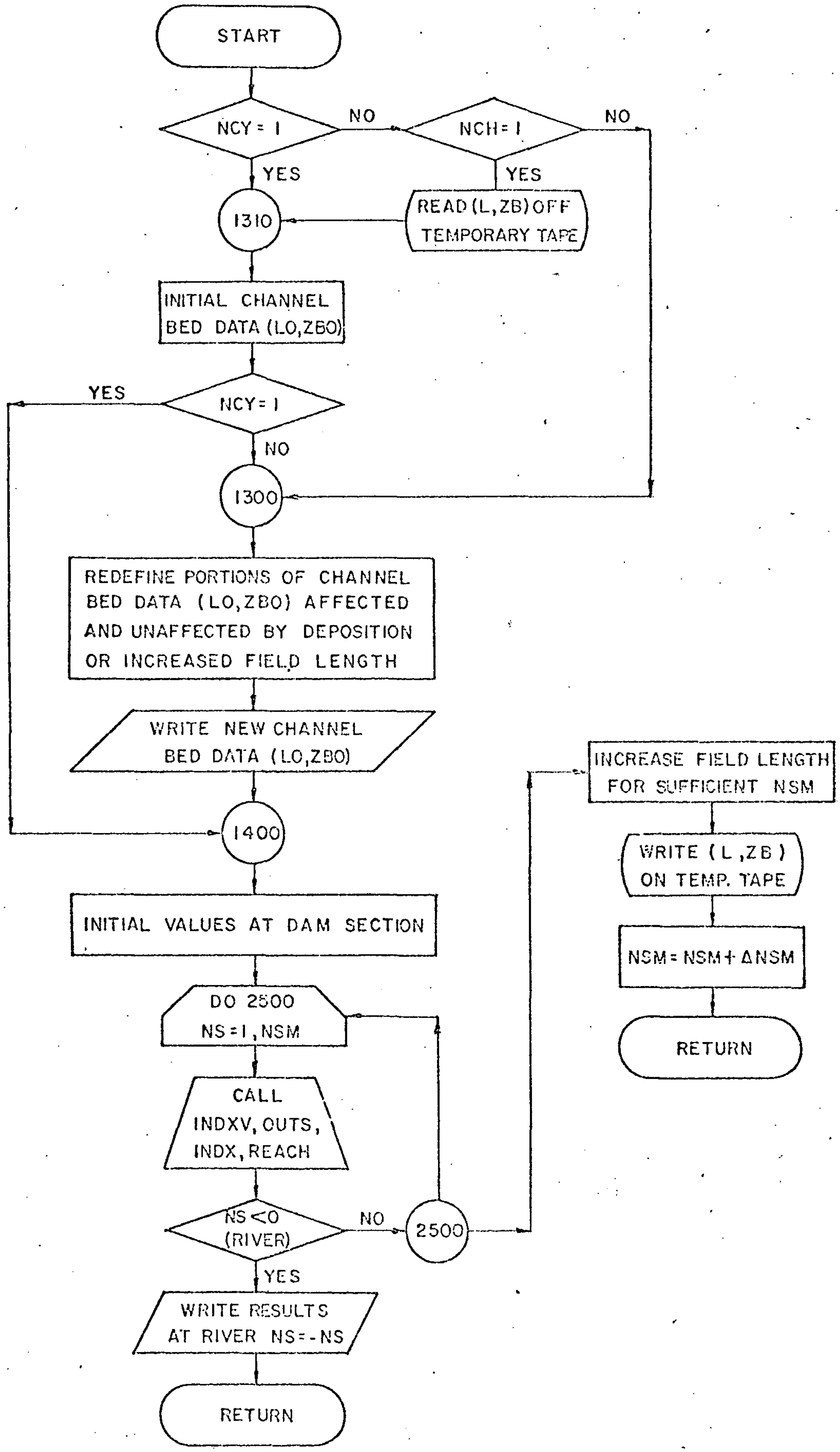
This subroutine is basically a dispatcher. If the calculations are just being initiated, then one has NCASE = 1, which is read in by the MAIN program and transferred through a COMMON block. If, on the other hand, there were previous calculations recorded on tape, then one has NCASE = 2 if only the last record of these previous calculations is to be read off the tape and further calculations are to be done. If one has NCASE = 3, all the records of the previous calculations are read off tape, plotted and branching is made to continue with the calculations. After branching off properly according to the value of NCASE, the subroutine WPROF is called for the backwater profile calculations. If the field length is not sufficient, then the related control parameter is NCH = 1, which returns the computer to the MAIN program to readjust the field length. If the field length is sufficient, the results of the backwater profile calculations are plotted and recorded on a tape. Then, the subroutine DPBL is called for the calculations of the bed load transport and deposition in the river-reservoir system. At this point one cycle of calculations is completed. The same procedure is repeated until a prescribed number of calculation cycles is attained.



SUBROUTINE SEDRES

SUBROUTINE WPROF

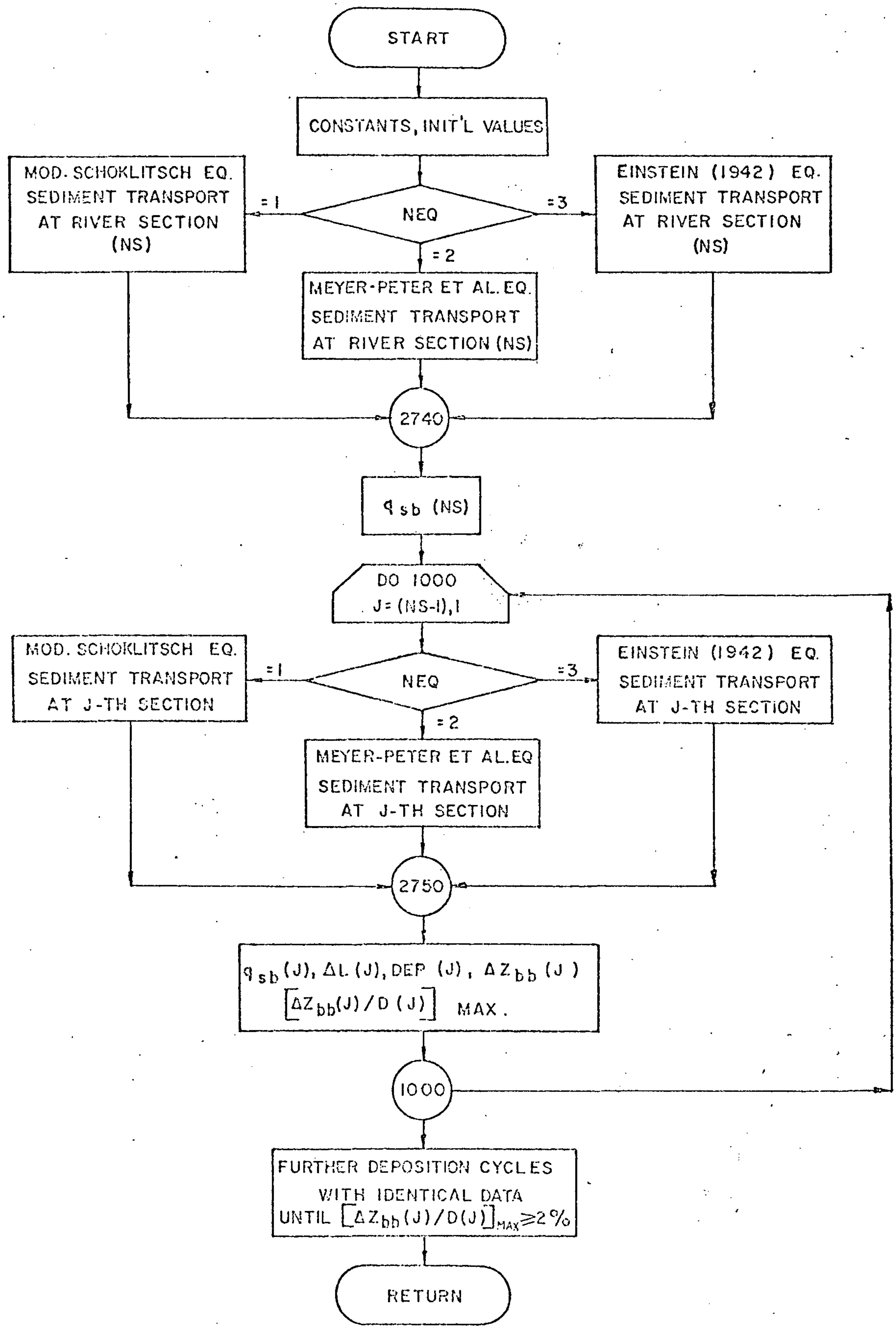
This subroutine makes the backwater profile calculations for a one-dimensional (unit-width) river-reservoir system by making use of a standard-step method. If the initial calculations are being started, in which case NCY = 1, the initial values are transferred through a common block as read in by the MAIN program. If NCY $\neq$ 1, then, the initial channel bed data for the next cycle of calculations are given by the last cycle of calculations of backwater profile and deposition. The calculations are started off at the dam section and continued upstream in a number of reaches until the normal conditions are reached. For the actual hydraulic calculations for each reach, the subroutine REACH is called. During these calculations, if the specified number of reaches is not sufficient, then NSM is increased by a certain percentage and a RETURN is made back to the MAIN program for restarting the calculations with increased field length. This subroutine also calls the output subroutine OUTS for printing out the results of the backwater profile calculations.



SUBROUTINE WPROF

SUBROUTINE DPBL

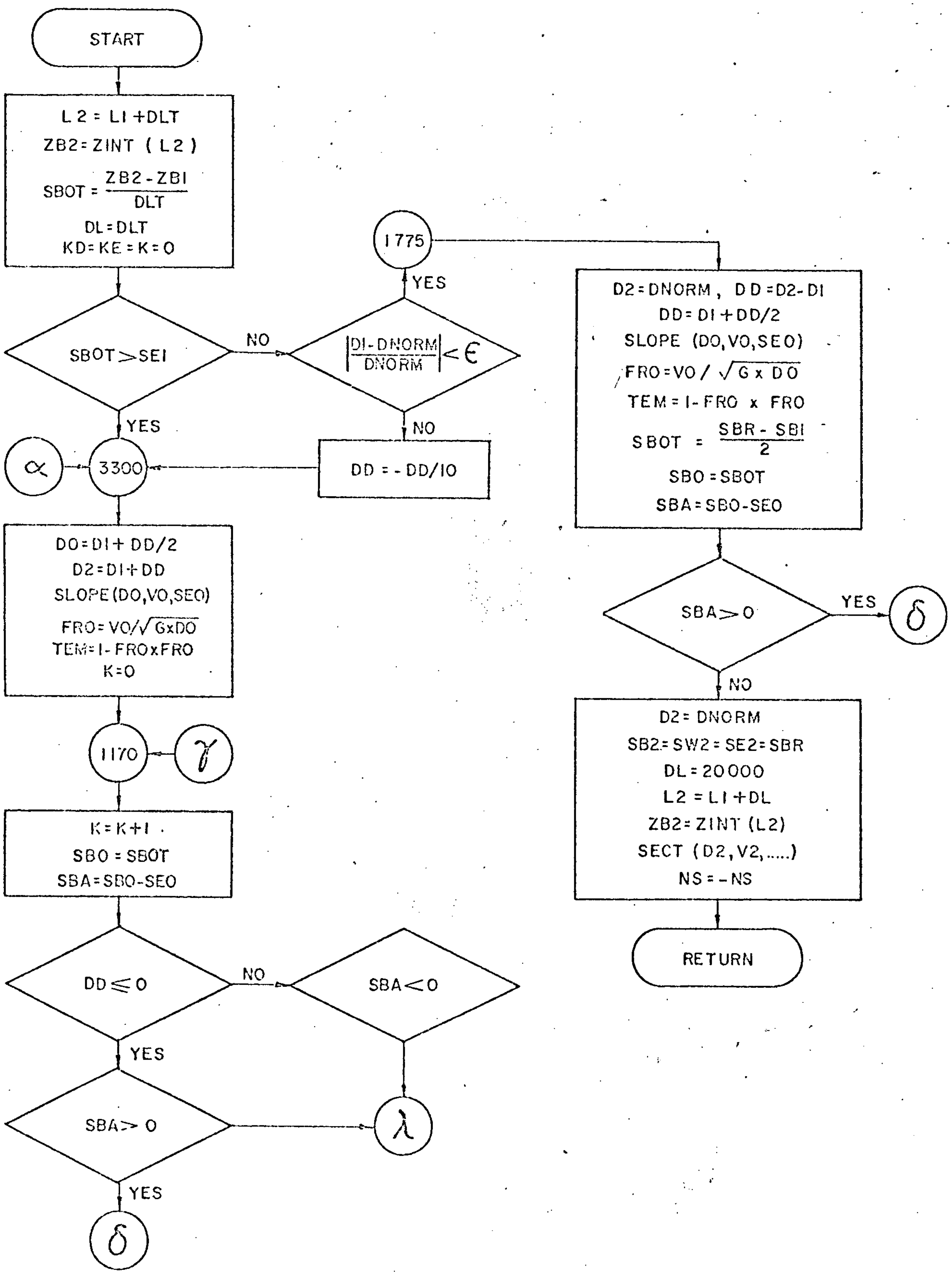
In this subroutine, the deposition calculations are made by making use of one of the bed load equations. In the present program, NEQ = 1 refers to the "Modified" Schoklitsch equation, NEQ = 2 to the Meyer-Peter et al. equation, and NEQ = 3 to the Einstein - 1942 bed load equation. Bed load deposition calculations are started at the "river" section and progressed in the downstream direction towards the dam. When the amount of deposition becomes too small, or the dam section is reached, one cycle of deposition calculations is completed. If the maximum thickness of deposition is less than a certain fraction (in the present case, 2%) of the local water depth, another identical cycle of deposition is assumed to have taken place, and the channel bed configuration is adjusted accordingly.



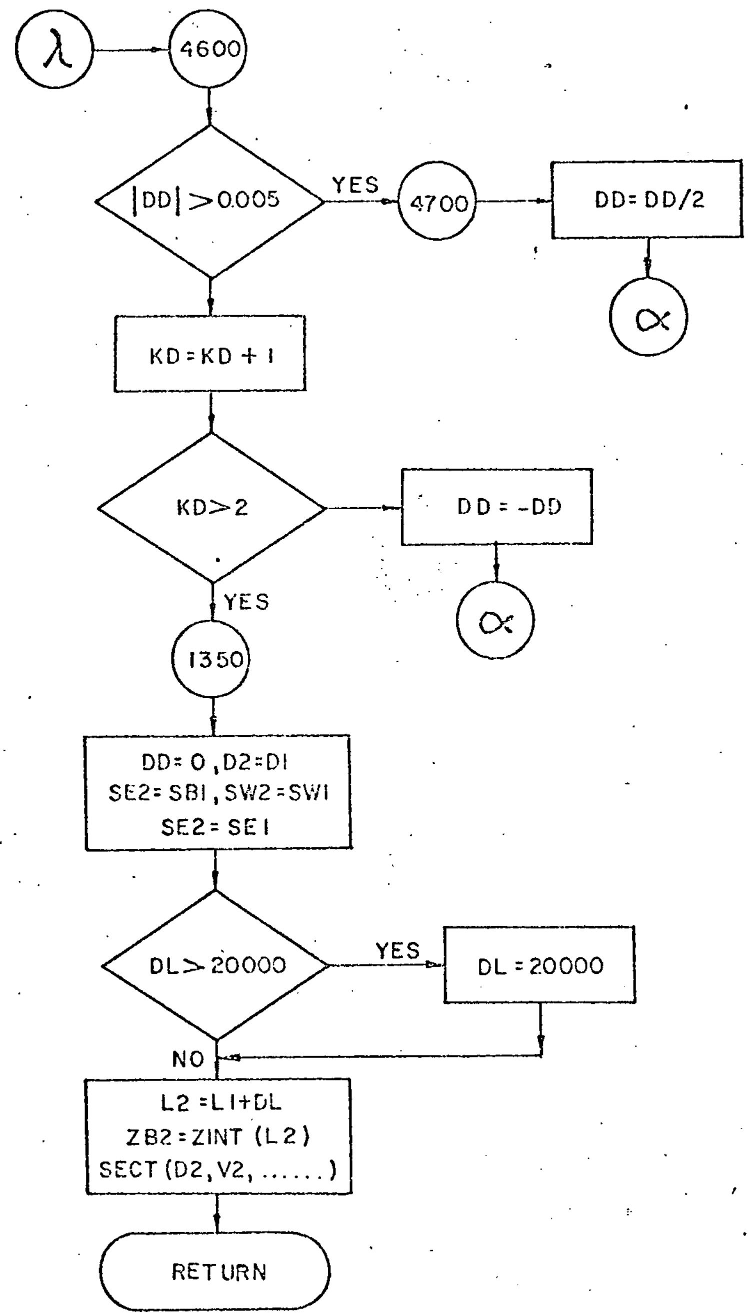
SUBROUTINE DPBL

SUBROUTINE REACH

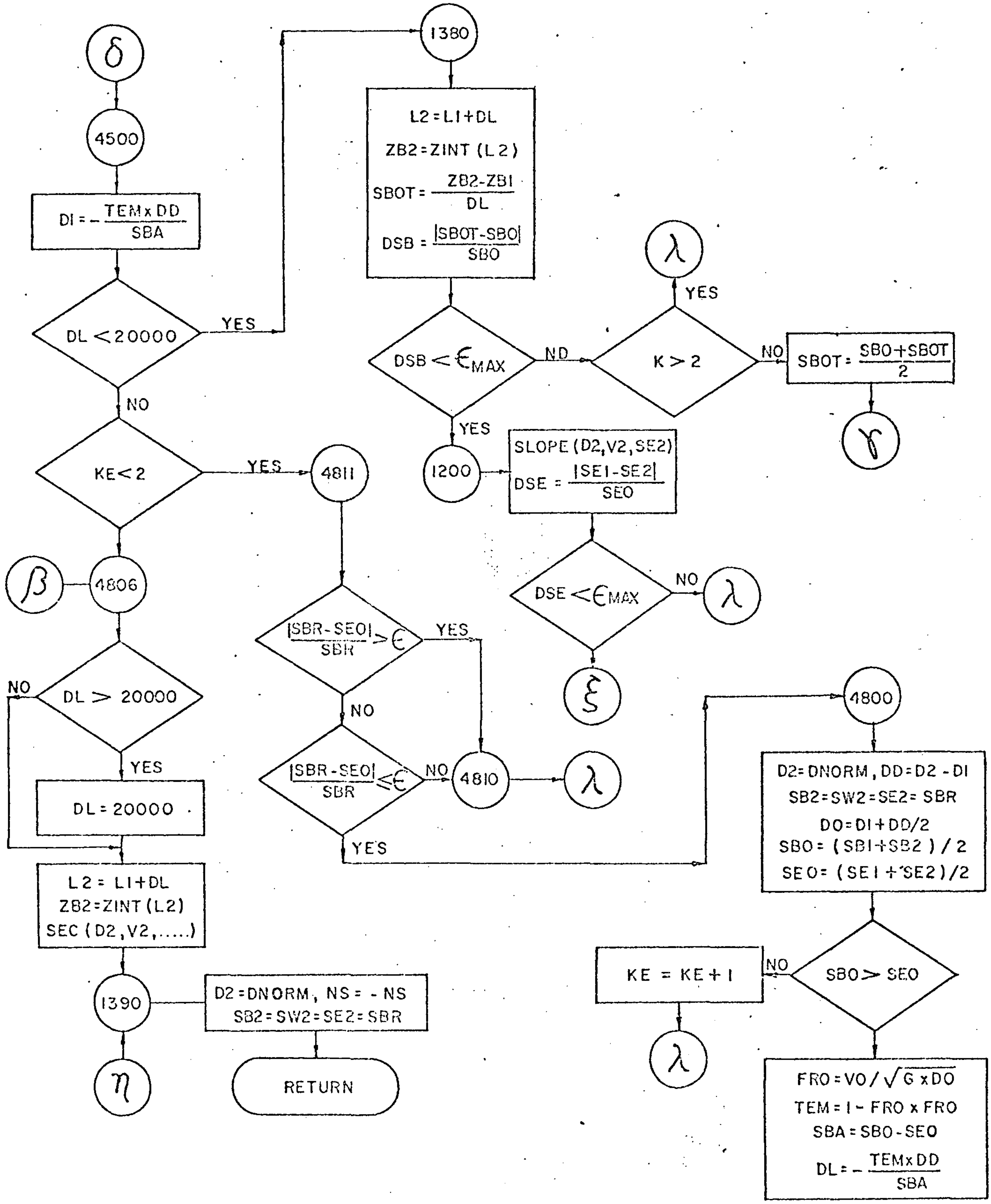
This subroutine is called by subroutine WPROF with all the hydraulic information given at one section, and it performs the necessary calculations to determine the flow conditions at the next upstream section. In these calculations, a trial-and-error procedure is applied. First, a trial-reach-length, DLT, is assumed by means of which a trial-bed-slope, SBOT, is obtained. Then, a trial-depth-increment, DD, is assumed, and with this information, the trial values of the flow characteristics are calculated, at the next section and at the mid-section of the reach. If these mid-section characteristics do not represent the whole reach with sufficient approximation, a new trial-depth-increment is assumed and calculations are repeated, and so on. In the present program, an error of $\varepsilon = 5\%$ is considered to represent sufficient approximation as far as the section characteristics (slopes of the bottom and the energy grade line) are concerned. The normal river conditions are assumed to be reached within the same approximation limits. The efficiency in the successive trial-and-error procedures is facilitated by various dynamic checking and control parameters and processes, the details of which are given in the flowchart of the subroutine itself in four parts.



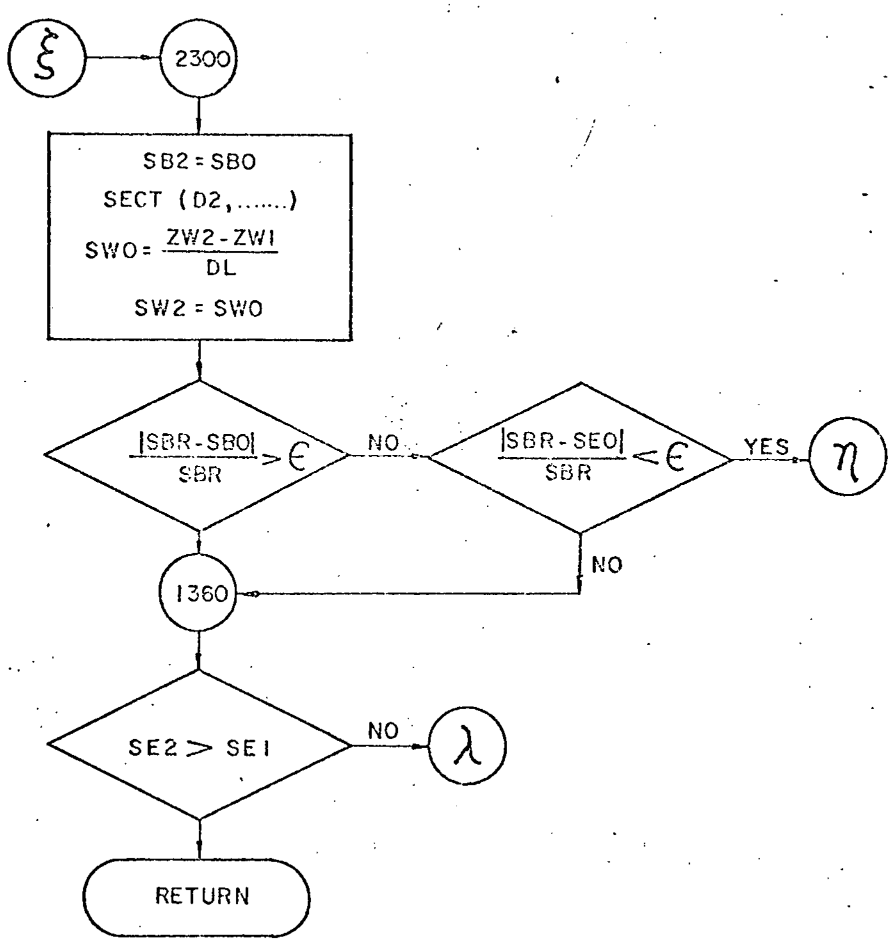
SUBROUTINE REACH (PART 1)



SUBROUTINE REACH (PART 2)



SUBROUTINE REACH (PART 3)



SUBROUTINE REACH (PART 4)

SECONDARY SUBROUTINES

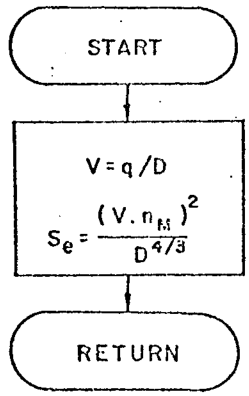
There are also some auxiliary subroutines in the program. Among these, the subroutine SLOPE calculates the slope of the energy grade line at any section, the subroutine SECT calculates all the flow characteristics at any section. Function ZINT makes use of a linear interpolation to calculate the imtermediate values of a function, in the present case the channel bed elevation as a function of the distance from the dam section. The subroutine INDX transfers the calculated parameters at a section to become the initial values of the next reach to be calculated. Finally, the subroutine INDXV transfers the constant variable values of the calculated parameters at any section to become the corresponding dimensional variables.

OUTPUT SUBROUTINES

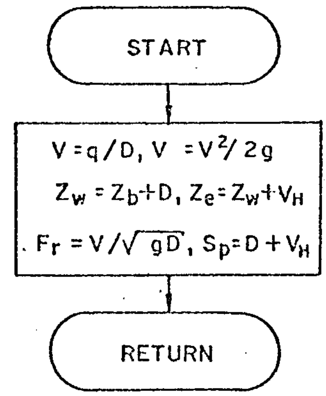
These subroutines are called for printing out the titles as well as the calculated data.

PLOT SUBROUTINES

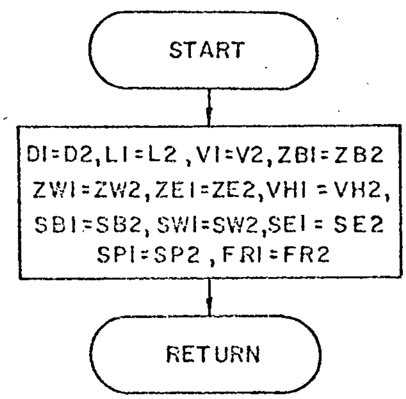
The subroutine AXPLT plots the axes and the relevant identifying information before the calculated data are actually plotted. NPLT = 1 causes the complete river-reservoir system to be plotted only, NPLT = 2 causes a detailed plotting of the delta only, and NPLT = 0 corresponds to both plots at the same time. The subroutine ELPLT plots the calculated data as a complete river-reservoir system, while the subroutine DEPLT plots a detailed delta. The latter two subroutines plot both the channel bed and the water surface elevations.



SUBROUTINE SLOPE



SUBROUTINE SECT



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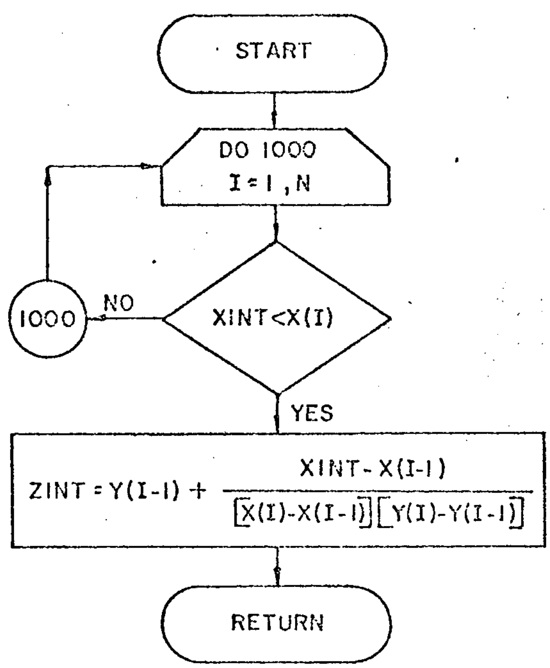
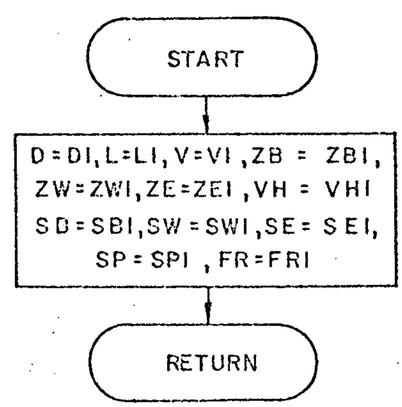
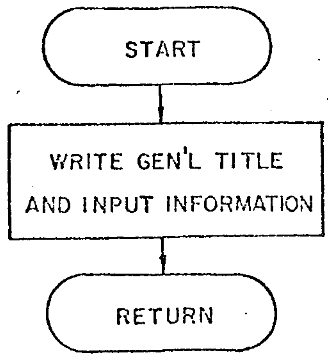


FUNCTION ZINT



SUBROUTINE INDXV

SECONDARY SUBROUTINES



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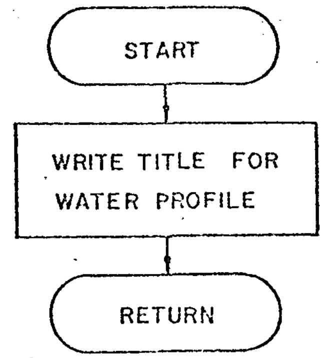

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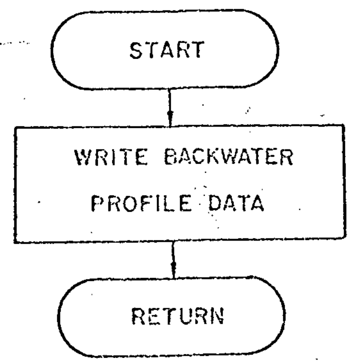
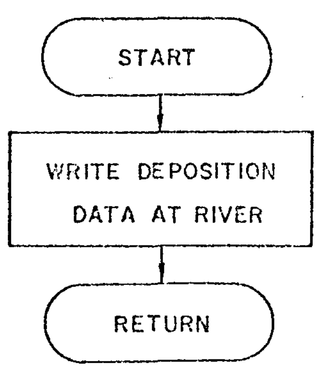


SUBROUTINE OUTS



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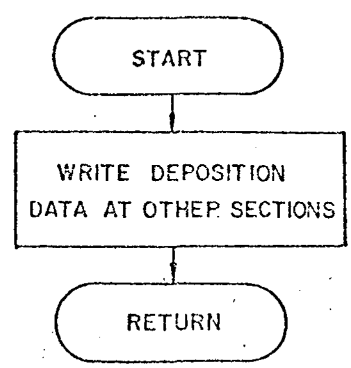
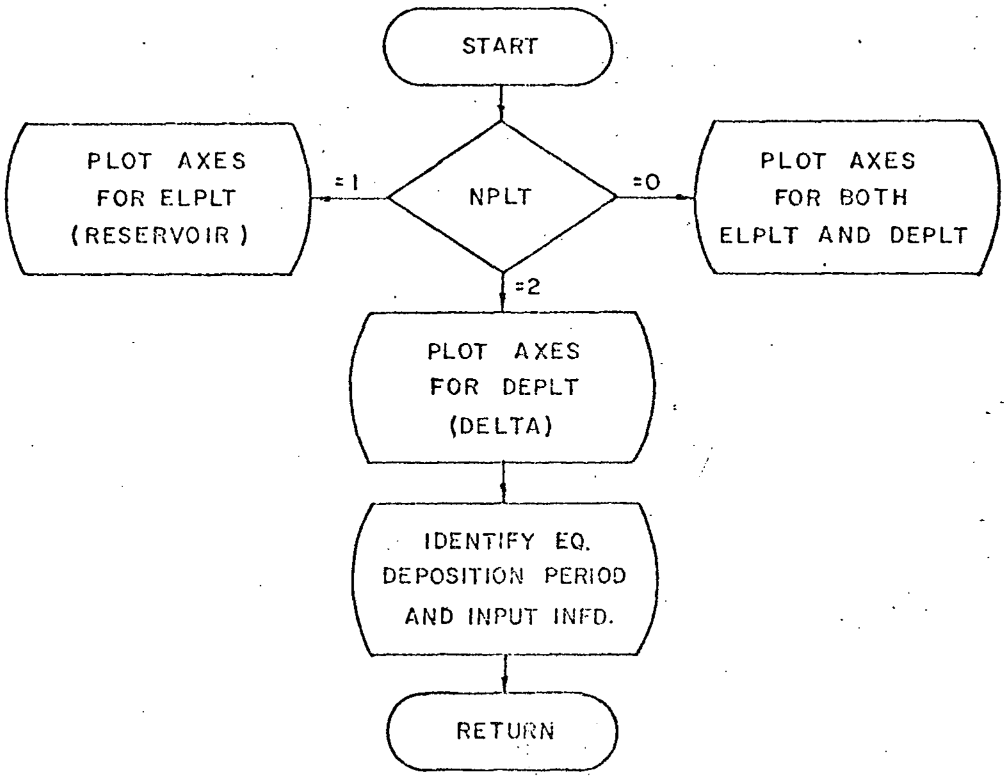


SUBROUTINE OUTBLI

OUTPUT SUBROUTINES



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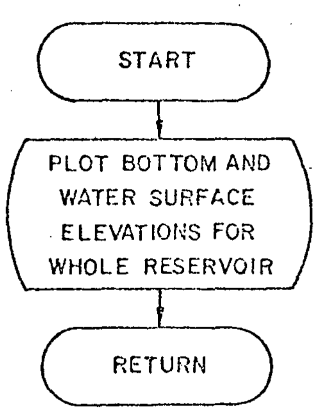
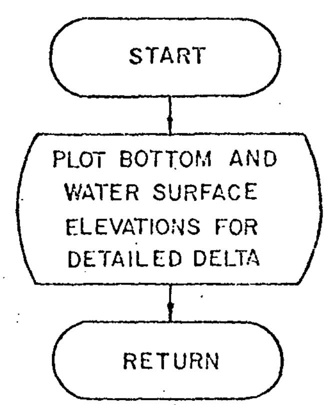


SUBROUTINE ELPLT



SUBROUTINE DEPLT

PLOT SUBROUTINES