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PERFORMANCE DESIGN OF TALL BUILDINGS FOR WIND

by

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ABSTRACT

The report describes the design of tall buildings to satisfy certain performance criteria. Buildings subjected only to combined wind and gravity loads are considered.

The advantages of a performance code are discussed. Existing methods of calculating wind loads are reviewed and the development of the gust effect factor is sketched.

The four performance criteria which any tall building must satisfy are introduced. Each criterion is discussed and the corresponding acceptance criteria are given. These include the mean recurrence intervals for the design winds.

The report briefly discusses the nature of wind and structural response to wind loads. It is shown that wind can be considered to be made up of a mean wind and a fluctuating component. The mean wind causes quasi-static response and the fluctuating component causes horizontal vibration. The gust effect factor is the ratio of the maximum response to the mean response of a building.

Design procedures where each of the performance criteria is considered, are then presented. The basis of each design is the lateral load-deflection curve of a frame. In general curves determined for each of the several loading cases differ since the design gravity loads for the performance criteria are different.

The lateral load-deflection curves indicate the maximum strength of a building and also the drift caused by the various wind loads.

A comparison is made between a performance design and a traditional design. Areas requiring future work are discussed.

1. INTRODUCTION

At a conference held at Toronto in 1968, Davenport stressed the necessity for a new approach to the design of tall buildings against wind action.(1)* Two reasons were pointed out: 1) the customary static wind loads used in design are unrealistic and lead to either over-conservative or inadequate design, and 2) the new generation of buildings respond to wind in a manner different from their predecessors. The first reason was substiated by wind measurements on actual tall buildings.(2,3,4) This work showed that the concepts of smooth wind flow past a building and the customary distribution of total wind pressure between the leeward and windward faces were not realistic. The actual wind flow is turbulent and the measurements showed that practically all the wind pressure difference occurs across the windward face. In addition, small duration gusts are significant in loading a building.

The second reason was based on five recent developments in building technology and construction: 1) taller buildings with more amancipated forms are being built which have aerodynamic characteristics different from their predecessors, 2) the average density of tall buildings has decreased from 25 lb/cu.ft. to less than 10 lb/cu.ft., 3) the properties of structural materials have changed considerable-ie: yield stress levels for example have more than doubled, 4) the damping of

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*Numerals in parentheses refer to corresponding items in Appendix References.

structures has decreased significantly and 5) structural optimization is now possible through new methods of analysis and the computer.

The above reasons suggest that a tall building should be designed to meet certain performance criteria under wind loading. In other words, the structure should perform to certain specifications satisfactorily under the applied wind loads. This leads directly to the concept of performance design which is not a new design method but a new approach to the design of tall buildings for wind.

Whereas performance design is well established in some other fields of structural engineering, its application to the design of buildings has been slow to develop. This was mainly due to a lack of information on the real behavior of tall buildings under the applied loads and insufficient understanding of the nature of the loading itself. Recently, considerable research effort has been exerted in determining the magnitude of the applied loads and the dynamic effect they have on building structures. It is now possible to establish a basis for the design of tall buildings to meet certain performance criteria.

The advantages of a performance type of building code are being realized more and more.(5) For example, a typical performance specification for a tall building might be one that requires a building to withstand a wind with a mean recurrence interval of, say, 100 years. No method of construction or design is laid down as to how this performance is to be achieved. It therefore opens the way for the structural engineer to employ his knowledge and ingenuity to meet this performance requirement in his design. It inspires the search for innovation and can lead to more effective and economical structures.

To execute a performance design, it is evident that the engineer must have a sound knowledge, not only of the magnitude of the applied loads, but also the effect they have on a structure. The determination of wind forces on a structure is basically a dynamic problem.[6,7] However, it has been customary practice to apply wind as a static load. If the variation of wind with time is much slower than the natural period of vibration of the structure, then the structural response would be quasi-static and the approach acceptable. The implication is that small duration gusts should not significantly load a building. For some buildings this is not the case as the measurements reported in Refs. 3 and 4 indicate. It can therefore be expected that the dynamic effect of wind on a building will become an increasingly important design consideration for future tall buildings.

1.1 Magnitude of Wind Loads

With regard to the magnitude of wind loads, much information has been accumulated over the past decades. In 1960 Thom published isotach maps for the 48 states giving the annual extreme winds at 30 ft above the ground for mean recurrence intervals of 2, 50 and 100 years.[8] These were obtained by applying the Fisher-Tippett Type II function to an extreme value distribution of the annual fastest mile of wind recorded over many years.[9] The extreme-mile wind speed is defined as that 1-mile passage of wind with the highest speed recorded during any day. In 1968 new distributions of extreme winds were published with mean recurrence intervals of 2, 10, 25, 50 and 100 years.[10] From these the wind speeds V_Z at any height Z(ft) could be calculated using the power law formula

$$V_Z = V_{30} \quad (Z/30)^{1/n} \tag{1}$$

where V_{30} = extreme wind velocity at 30 ft.

n = coefficient depending on the terrain

The extreme wind speeds given by Eq. 1 are then multiplied by a gust factor to obtain the gust speed.[8] References 11 and 12 explain how gust factors can be evaluated. From the gust speed the design wind pressure p(psf) is then given by

$$p = 0.00256 \; C \; V^2 \tag{2}$$

where V = gust speed (mph)

C = shape factor

The shape factor C accounts for the distribution of wind pressure around a building. This design wind pressure can then be applied to a building as a static load in addition to the gravity loads to determine the total response of the building such as stresses, drift, etc.

1.2 Gust Effect Factor

To account for the dynamic effect of wind on buildings, Davenport in the early sixties began to accumulate information on all the factors which effect the response of a structure to wind load. The result was the gust effect factor in Ref. 13 which relates the total response (stresses, deflection etc.) of a structure to its mean response. Reference 14 by Davenport and Ref. 15 by Velozzi and Cohen contain the gust effect factor as applied to buildings in its final form. The gust effect factor plays a dominant role in evaluating the performance of a building under wind load and can perhaps be described as one of the most important developments in the field of tall building design in the last decade. In Chapter 3 the gust effect factor is more thoroughly described.

2. PERFORMANCE CRITERIA

The required performance of a tall building under gravity and wind loads can be stated in terms of four main criteria:[16,17]

1) Maximum Strength
2) Permanent drift
3) Comfort of occupants
4) Integrity of finishing material

2.1 Maximum Strength

Figure 1 shows the representative lateral load H versus drift Δ curve for a typical unbraced multi-story steel frame which is subjected to combined gravity loads W and lateral loads H. The maximum strength of the frame occurs at point A and the corresponding value of H is the stability limit load.[18] In a traditional maximum strength design, the frame is proportioned so that the design ultimate wind load is less than or equal to the stability limit load.

To ensure a satisfactory design of a building frame, the probability of the design ultimate wind load exceeding the stability limit load should be sufficiently small. How small this value should be is usually not important because maximum strength is seldom the governing design criterion for tall buildings. As a guide however the value 10^{-6} suggested in Ref. 16 might be used. This value corresponds to a mean recurrence interval R of 10^{6} years for the design ultimate wind load.

Table 1 shows the above choice of mean recurrence interval for a maximum strength performance criterion. The corresponding acceptance criterion is that the design ultimate wind load is less than or equal to the stability limit load.

2.2 Permanent Drift

Few buildings have actually failed under wind load except during the construction stages.[16] It is more likely that under a certain wind load some portions of the structure may be subjected to inelastic deformation such that excessive permanent drift is caused rendering the building unserviceable.

The occurrence of permanent drift is illustrated by the lateral load-deflection curve of Fig. 2. Assume the structure is loaded to point B. Upon removal of the lateral load, the structure returns along path BC which is essentially parallel to the initial tangent of the curve. The magnitude of permanent drift is represented by the distance OC.

The effect of permanent drift was strikingly illustrated by the response of the Great Plains Life Building to the Lubbock Tornado of May 11, 1970.[19] The building was not in the direct path of the tornado but was subjected to winds in the order of 90 mph at 30 ft. above ground. The building did not collapse but large inelastic deformations resulted in a permanent drift of approximately 12 in. The important fact is that the building was rendered unserviceable because of its appearance.

It is therefore necessary to limit the magnitude of permanent drift which a building might experience under certain extreme winds. It should be sufficiently small so as not to be readily noticeable. It will be a function of such factors as the relative stability of the building, the method and materials of construction and its use and occupancy.

Further research is necessary to determine acceptable limits for permanent drift. A mean occurrence rate per annum of 10^{-3} is suggested in Ref. 16 for the probability of permanent drift. The corresponding mean recurrence interval of 10^{3} years for the design wind load is also shown in Table 1.

2.3 Comfort of the Occupants

It is not possible to state all the factors which cause discomfort to occupants in tall buildings because no comprehensive study has yet been undertaken to determine them. However the factors which cause discomfort can probably be divided into three main classes namely physical experience, visual observation and noise. The following illustrates one possible way to include such performance criteria in design.

2.3.1 Physical Experience

The effect of the gust component of the wind on a building is to cause a horizontal vibration around a mean deflected position. Factors such as acceleration, change of acceleration and duration of vibration can be causes of human discomfort. Tests were conducted at the Boeing Co. to determine human reactions to low frequency vibration.[20] It was found that for most individuals an acceleration less than 0.5% of gravity (g) is not perceptible; from 0.5% to 1.5% the acceleration is perceptible and above 1.5% it becomes annoying. Figure 3, which was taken from Ref. 21 shows the variation of amplitude of vibration of a building with respect to its natural period for different values of acceleration. The lower boundary for perception namely 0.5% was more or less confirmed in Ref. 22 where measurements taken of the movement of a tall building are reported. It was found that an acceleration of 0.3% of gravity could not be felt.

It can be expected that future research will throw more light on this aspect of comfort and that exceptance criteria might be more relaxed. Change of acceleration and duration of vibration need also be investigated to determine their effects on occupants' comfort.

For purposes of this report the range of acceleration from 0.5% to 1.5% of gravity will be taken as the accepted limits. The occurrence rate per annum of 1 suggested in Ref. 16 will also be assumed. These values are shown in Table 1.

2.3.2 Visual Observation

An occupant staring out of a window may notice the movement of his building relative to an adjacent one. This relative movement could be either "head-on" or parallel. By "head-on" is meant the relative movement of one building towards another. Parallel movement is the relative motion of the adjacent buildings perpendicular to the "head-on" direction.

The number of occupants that notice this movement will probably depend on the type of building. People working in office buildings probably have less occasion to look out windows than the residents of apartment buildings.

Little is known about the effect such movements may have on building occupants and no acceptance criteria have been included in Table 1. Reference 23 mentions that sway movements of more than 0.1% of the height or more than 1% of the width of a building may be noticed without a reference point by observers external to the building. Similar limits could be determined for the observers within a building.

2.3.3 Noise

The noise of the wind buffeting a building alerts the senses of perception. This may cause a hypersensitive state which leads to actual or imaginative feelings of motion. A similar situation occurs when partitions or curtain walls start to creak. Occupants who do not hear these noises are much less liable to be disturbed by the same wind than those who do.

An obvious solution appears to be to make the building more sound-proof. This would however have to be weighed against increased building construction cost which could lead to higher rents. The residents of an apartment building might be prepared to pay for this increased cost in return for additional comfort. People working in an office building will probably not be interested.

With regard to creaking of the partitions, a possible solution would be to place a limit on the racking of the frame. This limit will depend on the type of material used for the partitions and the method of attachment to the frame.

At this stage it is not possible to suggest acceptance criteria for this performance requirements and none are included in Table 1. Future research is necessary to determine the extent of the problem and how it should be solved.

2.4 Integrity of Finishing Material

Excessive racking of the elements of a building will cause windows, finishing material and partitions to crack. The amount of racking which a building can withstand without causing damage will depend on the type of material used and the method of attachment to the frame. Materials such as glass, brick, concrete and hollow clay tile which are frequently used for curtain walls or partitions, will be examined further and suitable racking limits suggested. Racking will be defined in terms of the ratio of the lateral shearing deflection of a rectangular element to the height of the element and will be called the racking index. This is shown in Fig. 4.

a) Glass

Reference 24 reports the results of tests that were conducted to examine the limitations on racking as laid down in the California Administrative Code Title 21. According to this code deflection in the plane of a window from the head to the sill is not to exceed one-sixteenth inch per foot of height of the window opening unless the glass is prevented from taking shear or distortion or wire glass is used. This amounts to a racking index of 0.0052. The tests results showed that if this value is to be used, then the mastic around the glass must be soft and usual clearances provided. In this report a maximum racking index of 0.005 will be assumed.

b) Brick

Reference 25 reports the results of one-story brick walls with surrounding frames that were subjected to racking forces. Reinforced concrete and steel frames were used. It was observed that the first crack appeared when the racking index was from 0.001 to 0.004. The lower value corresponds to a brick wall with a concrete frame and the higher value to a steel frame. The greater flexibility of the latter was ascribed to the normal lack of perfect fit between brick panels and steel frames as well as the greater deformations in the steel frame itself. For steel frames with brick infill the greater value of 0.004 may thus be used as the maximum racking index.

c) Concrete

Reference 26 reports the results of tests that were conducted on small steel panels with concrete infill. These panels were subjected to racking loads. No indication is given of the deflections at which the first cracks appeared but the deflections corresponding to 90% of the ultimate load is given. These correspond to a racking index of 0.003. Although it is possible that cracks appeared before this deflection was obtained, this value could probably be used as the allowable value for racking of steel frames with concrete infill.

d) Hollow Clay Tile

In Ref. 27 the results are given of encased steel frames with various wall-panel infills that were subjected to racking loads. Included in those tests were encased steel frames with 3" hollow clay tiles for infill. The deflections at which the first cracks appeared, are given. For the hollow clay tile this corresponded to a racking index of 0.003.

Reference 16 suggests a mean occurrence rate per annum of 10^{-2} to 10^{-1} for the performance criterion of integrity of the finishing material. This corresponds to mean recurrence intervals of 10 to 10^{2} years.

The performance and acceptance criterion are summarized in Table 1 and the racking limits in Table 2.

3. CHOICE OF MEAN RECURRENCE INTERVALS

The mean recurrence intervals suggested in Table 1 were calculated on the basis of life (from an ammortization viewpoint) of a building and the assumed risk of the design wind occurring during that life. Reference 28 suggests a method for deriving an equation between mean recurrence interval, life and risk which may be summarized as follows: Let R be equal to the mean recurrence interval in years, that is, the mean interval between occurrences of a wind velocity greater than V where V is the velocity corresponding to R. Let the life be L years. Assume that a small risk, r, of V being exceeded during those L years, is acceptable. If P is the probability that V is exceeded in any year, then (1-P) is the probability that it is not exceeded. The probability that V is not exceeded in L years, is $(1-P)^L$ and the probability, r, that V is exceeded during L years, then becomes

$$r = 1 - (1-P)^L$$

Noting that $P = 1/R$, the above equation can be solved for $1/R$ giving

$$1/R = 1 - (1-r)^{1/L}$$

This equation can be expanded into a series using the Binomial expansion giving

$$1/R = 1 - (1 - \frac{r}{L} + \frac{r^2}{2L} (\frac{1}{L} - 1) \text{-----})$$

If the risk r is small, then R is approximately given by

$$R = \frac{L}{r} \qquad (3)$$

Consider now some of the performance criteria

a) Maximum Strength

Let L = life of building

= 100 years

and $r = 10^{-4}$

$$\therefore R = \frac{100}{10^{-4}} = 10^{6} \text{ years}$$

b) Permanent Drift

Let L = life of building

= 100 years

and $r = 10^{-1}$

$$\therefore R = \frac{100}{10^{-1}} = 10^{3} \text{ years}$$

c) Integrity of Finishing Material

Let L = average period between redecoration of the building

= 1 to 10 years

$r = 10^{-1}$

$$\therefore R = \frac{1}{10^{-1}} \text{ to } \frac{10}{10^{-1}} \text{ years}$$

$$= 10 \text{ to } 100 \text{ years}$$

4. CALCULATION OF WIND LOADS

4.1 Extreme Wind

Because of the importance of gust action in wind sensitive buildings, the dynamic approach to the calculation of the wind pressure will be used as is recommended in Ref. 29. This approach includes the calculation of the gust effect factor which, as will be shown later, plays an important role in performance design. It is first necessary to calculate the extreme wind. The method described in Ref. 28 will be followed. Fundamentally, this is the same method through which the extreme winds in Ref. 9 were determined.

If a statistical sample consisting of the largest wind speed observed during each of several years is obtained, then the resulting distribution is termed an extreme value distribution. Using a Weibull, Rayleigh or exponential type of probability distribution, the annual probability P of an extreme wind V being exceeded may be written as

$$P = 1 - e^{-e^{-a(V-U)}} \qquad (4)$$

where U = modal wind speed

$1/a$ = scale wind speed

Through a double logarithmic transformation Eq. 4 leads to

$$V = U + \frac{1}{a} \left[-\log_e\{ -\log_e (1-P)\}\right] \qquad (5)$$

If V is plotted against the term in brackets, it leads to a straight line.

With

$$P = \frac{1}{R}$$

then for values of R greater than 5 years equation 5 may be written

$$V = V + \frac{1}{a} \log_e R \qquad (6)$$

Equation 6 appears in Ref. 28 and the method for determining U and $\frac{1}{a}$ is also described therein. Because of the importance of this equation in the next chapters the example given in Ref. 27 will be presented again here to illustrate how U and 1/a are calculated.

The maximum annual mean hourly wind speeds for London, Ontario Airport (anemometer height 40 ft. above ground) for the years 1939-1961 inclusive are sequentially as follows: 36, 37, 45, 50, 39, 33, 37, 35, 41, 52, 41, 58, 39, 46, 39, 42, 45, 36, 55, 32, 43, 34, 39. In Table 3 these data are ranked in magnitude, $m = 1 \rightarrow N$ where N is the number of years of observation. The best estimate of the probability of an extreme wind being less than any value in the ranked series is given by

$$p = \frac{m}{N + 1}$$

Note that p is equal to 1-P where P is the probability of a value being exceeded as in Eq. 4. The value of p is shown in column 4 of Table 3. In column 5 is given the values of $-\log_e(-\log_e p)$ which is the same as $-\log_e(-\log_e(1-P))$. The values of p or $-\log_e(-\log_e p)$ versus the velocity V is plotted in Fig. 5.

A straight line can now be fitted to the data in Fig. 5 by eye. Taking any two points on the straight line, the values of U and 1/a in Eq. 6 may be calculated. This gives values of 38 mph and 6.7 mph for U and 1/a respectively. Equation 6 now becomes

$$V = 38 + 6.7 \log_e R \tag{7}$$

Since this velocity applies only to a specific height above ground level (40 ft. in the example quoted), the variation of velocity with height is determined by using an exposure factor C_e. In Ref. 29 the velocity given by Eq. 7 is termed the reference velocity.

Because the values of U and 1/a, namely 38 and 6.7, were determined from an extreme value of distribution of the maximum hourly velocity, the reference velocity V has a period (60 min.) much greater than the natural periods of vibration of buildings. The response of a building to such a wind will therefore have a quasi-static appearance. Thus V corresponds to a mean velocity causing the mean response of a building.

The reference velocity given by Eq. 7 for any mean recurrence interval is less than the fastest mile of wind for the same recurrence interval as given in Ref. 10. However, it is possible to determine an approximate value of the hourly velocity from the fastest mile. Reference 30 gives a chart which relates the two. It was also found that the following empirical equation holds more or less:[31]

$$\text{Velocity of fastest mile} \approx 10 + \text{hourly velocity}$$

The isotach maps of Ref. 10 can thus be used to determine the maximum hourly velocity for recurrence intervals up to 100 years.

4.2 Wind Pressure

The reference velocity pressure is given by the formula

$$q = 0.00256\ V^2\ \text{psf} \tag{8}$$

where V = reference velocity (mph)

The maximum wind pressure is now obtained by multiplying q by three factors namely C_e, C_p and C_g

where C_e = exposure factor

C_p = pressure coefficient

C_g = gust effect factor.

The exposure factor C_e accounts for the variation of wind pressure with height and with the type of terrain and the pressure coefficient C_p allows for the effect of structural shape and wind direction. Dynamic behavior of the structure is given by the gust effect factor C_g. Reference 29 lists equations and graphs through which these factors may be evaluated.

The nature of wind is such that it may be considered as being composed of a mean wind with a period greater than one hour and a fluctuating or gust component with a period less than one hour. This is shown in Fig. 6a taken from Ref. 32. Figure 6b shows the corresponding structural response (deflections, stresses etc.). It can be seen that for periods greater than one hour the structural response is directly proportional to the wind velocity. On the other hand, the gusts cause large deflections especially when their periods approach the natural period of the building and resonance occurs.[7,32]

The maximum response may thus be considered as being composed of the mean response, caused by the maximum hourly wind, on which the dynamic response, caused by the gusts, is imposed. The mean and maximum response can be assumed to have Gaussian probability distributions each defined by its mean value and variance or standard deviation.[28] Figure 7 shows the distribution of structural response where

$\bar{u}$ = mean response

u = maximum response

σ = standard deviation

This distribution can be determined by wind tunnel testing and sometimes by theory. The maximum response may thus be written as

$$u = \bar{u} + g' \sigma \tag{8}$$

where g' is the number of standard deviations the average maximum response is in excess of the mean. It has been shown that g' varies between 3.5 and 4.5.[32] Dividing both sides of Eq. 8 by $\bar{u}$ and noting that $u/\bar{u}$ is equal to the gust effect factor, gives

$$C_g = 1 + g' \left(\frac{\sigma}{\bar{u}}\right) \tag{9}$$

which is the equation given in Ref. 29. The value of $\sigma/\bar{u}$ is given by[29]

$$\frac{\sigma}{\bar{u}} = \sqrt{\frac{K}{C_e} \left(B + \frac{sF}{\beta}\right)} \tag{10}$$

where K = factor related to the surface roughness coefficient of the terrain

C_e = exposure factor (as previously)

B = background turbulence factor

s = size factor

F = gust energy ratio

β = critical damping ratio (0.01 for steel structures)

The above coefficients can be calculated from the graphs of Ref. 29. The value of K depends on the terrain which Davenport classed into three zones namely Zone A, Zone B and Zone C. Zone A corresponds to open terrain, Zone B to suburban areas and Zone C to centres of large urban areas with large buildings. The corresponding values of K are:

$K = 0.08$ for Zone A

$K = 0.10$ for Zone B

$K = 0.14$ for Zone C

Under the method presented in Ref. 14 the gust effect factor is calcalculated only once for the whole building whereas in Ref. 15 it varies with height. In Ref. 33 the two methods for calculating the gust effect factor are compared and reasonable correlation exists.

The maximum wind pressure on a building, applied as a static load and accounting for the dynamic effect of wind, is thus given by

$$p = 0.00256\, C_e\, C_p\, C_g\, V^2 \qquad (11)$$

where the reference velocity V is calculated from Eq. 6.

5. PERFORMANCE DESIGN

As mentioned earlier, performance design constitutes a new approach to the design of tall buildings for wind and is not a new design method. The same basic procedures of any design method, namely performing a preliminary design, revising the member sizes and executing a final design will still have to be followed. Essentially the only differences are in obtaining the design wind and gravity loads for each performance criterion and in assessing the performance of the structure under those loads. The response of a structure to combined lateral and gravity loads can be represented by its lateral load-deflection curve.[34,35,36]

5.1 Maximum Strength

a) Design Ultimate Wind Load

Using Eqs. 6 and 11 the equivalent static wind pressure which accounts for the dynamic effect of wind is given by

$$p = 0.00256\ C_e\ C_p\ C_g\ [U + \frac{1}{a}\log_e R]^2 \text{ psf} \qquad (12)$$

If the value of R for maximum strength as in Table 1, namely 10^6, is used, then Eq. 12 gives the design ultimate load

$$p_{ult} = 0.00256\ C_e\ C_p\ C_g\ [U + \frac{13.8}{a}]^2 \text{ psf} \qquad (13)$$

b) Design Ultimate Gravity Load

The design ultimate gravity load must be evaluated for the same mean recurrence interval R as for the design ultimate wind load. This can be accomplished by establishing a relationship between the load factor and R. Figure 8 presents one possible solution. On the vertical axis the recurrence interval R is shown on a log-scale. On the horizontal axis the load factor is shown on an arithmetic scale. The working load (load factor = 1.0) will be defined herein as that load which will occur at least once during the life of the structure. If the life is assumed to be 10^2 years, then point A is established.

A load factor of 1.3 is presently assumed for the combined loading state to calculate the design ultimate gravity load.(18) If a corresponding value of 10^6 years is assumed for R, then point B in Fig. 8 is also established.

What form the curve connecting points A and B should assume is probably open to discussion. For the purpose of this report points A and B were arbitrarily connected by a straight line. In Fig. 8 this line was also extended to meet the horizontal axis.

Taking the value of the working gravity load from an appropriate building code and obtaining a load factor from Fig. 9 for any specific recurrence interval gives the design gravity load for any performance criterion. The load-deflection curve can then be obtained.

If the design ultimate wind load is less than or equal to the stability limit load, then the design will ensure satisfactory performance.

5.2 Permanent Drift

a) Design Wind Load

The design wind pressure is again given by Eq. 12. If a value of R as in Table 1, namely 10^3 years, is used for this performance criterion, then the design wind pressure is given by

$$p = 0.00256\, C_e C_p C_g \left[U + \frac{6.9}{a}\right]^2 \text{ psf} \qquad (14)$$

b) Design Gravity Load

From Fig. 8 the load factor can be determined. Using a value of R equal to 10^3 years as for the wind load, gives a load factor of approximately 1.08.

The load-deflection curve can now be constructed. Using the design wind load as obtained from Eq. 14, the permanent drift Δ_p must be less than some limiting value.

5.4 Comfort of the Occupants

As was mentioned in Chapter 2 a performance design for comfort of the occupants must be performed for three criteria namely a) Physical Experience, b) Visual Observation and c) Noise. Because of the lack of information on criteria b and c a performance design can at this stage only be executed for criterion a.

a) Physical Experience

The acceptance criterion here is that the acceleration of a building under a wind of 1 year mean recurrence interval should not exceed from 0.5 to 1.5 percent of gravity. Using the gust effect factor, it is possible to derive an equation for the acceleration of a building in terms of the various wind load parameters described in Chapter 4.

Reference 21 gives the following equation for the acceleration a' of a building:

$$a' = 4\pi^2 n_o{}^2 A \quad ft/sec^2 \tag{15}$$

where A = amplitude of horizontal vibration (ft.)

n_o = natural frequency of the building

Equation 15 is readily derived by considering the movement of a point at constant velocity along a circle of radius A. The maximum acceleration of the projection of this point on a diameter is given by Eq. 15.

The amplitude of horizontal vibration of a building is equal to the peak deflection minus the mean deflection. If u in Fig. 7 is assumed to be the deflection Δ, then the amplitude of vibration is given by

$$A = \Delta - \bar{\Delta} \tag{16}$$

where Δ = maximum deflection

$\bar{\Delta}$ = mean deflection

Since the ratio of Δ to $\bar{\Delta}$ is equal to the gust effect factor C_g, by rewriting Eq. 16 the amplitude is given by

$$A = \frac{(C_g - 1)}{C_g} \Delta \tag{17}$$

Substituting the value of C_g by Eq. 9 into Eq. 17 gives

$$A = \frac{g'}{C_g} \left(\frac{\sigma}{\bar{\Delta}}\right) \Delta \tag{18}$$

If. Eq. 18 is now substituted into Eq. 15 and using the value of

$(\sigma/\bar{\Delta})$ as given by Eq. 10, then the acceleration is given by

$$a' = \left[\frac{4\pi^2 n_o^2 g'}{C_g} \sqrt{\frac{K}{C_e} \left(B + \frac{sF}{\beta}\right)} \right] \Delta \tag{19}$$

The sample calculation in Ref. 29 shows that B is often considerably smaller than sF/β and may be dropped from the terms in brackets, thus giving the following approximate equation for the acceleration of a building

$$a' = \left[\frac{4\pi^2 n_o^2 g'}{C_g} \sqrt{\frac{K\ s\ F}{C_e \beta}} \right] \Delta \tag{20}$$

Reference 29 gives the following two empirical equations through which the natural periods of rectangular buildings may be calculated:

$$T = 0.05 \ \frac{H}{\sqrt{D}} \tag{21}$$

and

$$T = 0.1\ N \tag{22}$$

where H = total height of building

D = width in direction of wind

N = number of stories

For unbraced frames Eq. 22 only should be used. In Refs. 37 and 38 actual measured values of the natural periods of four buildings are compared with the values obtained from Eqs. 21 and 22 and good correlation is found.

A performance design would therefore consist of the following two steps: a) calculate design loads, b) determine the maximum acceleration.

a) Design Loads

The wind pressure for a recurrence interval of one year can be approximately determined from Eq. 12 as

$$p = 0.00256\ C_e C_p C_g\ U^2 \tag{23}$$

Using Fig. 8 the corresponding load factor for the gravity loads is 0.85 for R equal to 1.

b) Maximum acceleration

The maximum deflection Δ in Eq. 20 can now be determined by using the calculated design loads and determining a corresponding load-deflection curve. This is shown schematically in Fig. 10. Using the value of Δ in Fig. 10 enables calculation of the maximum acceleration.

5.4 Integrity of Finishing Material

The performance criterion for integrity of finishing material is that the racking limits of Table 2 should not be exceeded by a wind of 10 to 100 years recurrence interval. It is up to the designer to decide which recurrence interval in the given range he is going to use.

By again using Eq. 12 the design wind pressures corresponding to recurrence intervals of 10 and 100 years are given by p_{10} and p_{100} respectively where

$$p_{10} = 0.00256\ C_e\ C_p\ C_g\ [U + \frac{2.3}{a}]^2 \ \text{psf} \tag{24}$$

and

$$p_{100} = 0.00256\ C_e\ C_p\ C_g\ [U + \frac{4.6}{a}]^2 \ \text{psf} \tag{25}$$

The load factors corresponding to recurrence intervals of 10 and 100 years are obtained from Fig. 8 as 0.93 and 1.0 respectively. The load-deflection curve can now be constructed.

It should be remembered that the load-deflection curves used so far, refers to the maximum total drift at the top of a building. This drift results from the sum of the web plus chord deflections.[39,40] To check whether the racking limits of Table 2 are not exceeded, the relative deflections of each story are required. Since the racking of a story is represented by the web deflection only, the relative story displacement given by the difference between successive floor displacements are always greater than the racking of the story.

6. PERFORMANCE DESIGN VERSUS TRADITIONAL DESIGN

If a comparison is made between performance design as described herein and traditional design practice, then the following differences are apparent:

a) Performance design satisfies four performance criteria namely maximum strength, permanent drift, comfort of occupants and integrity of finishing material. Traditional design does not include designing for permanent drift.

b) Wind loads are determined on the basis of selected mean recurrence intervals with a different recurrence interval for each performance criterion. Each of these recurrence intervals are selected on the basis of risk. Traditional design is based on a working load for the wind which is the same for all performance criteria with the design ultimate load being obtained by multiplying the working load by a load factor. The same load factor is used for the comfort of occupants as for the integrity of finishing material.

c) Performance design recognizes the dynamic behavior of a building under wind loads and incorporates this into a design through the gust effect factor. Traditional design assumes that wind loads are static.

d) Comfort of the occupants is designed for on a rational basis by designing directly for the factors which can cause discomfort. Traditional design uses a deflection index to obtain satisfactory performance. The deflection index, which is the ratio of maximum deflection at the top of a building to total height, is selected on an arbitrary basis.

7. FUTURE WORK

There are mainly seven areas on which future work could concentrate:

a) Choice of appropriate mean recurrence intervals

The mean recurrence intervals presented in this paper, should not be considered rigid as if applicable to all buildings under all circumstances. Factors such as economy, life and purpose of a building should have much greater bearing on the selection of an appropriate mean recurrence interval.

b) Permanent Drift

No value for the allowable permanent drift under extreme winds is available. Proposed values should be such that the appearance of a building under this permanent drift does not render it unserviceable. The values should also consider the economy, purpose and method and materials of construction of a building.

c) Comfort of Occupants

There is a need for a comprehensive study of this performance criterion. The effects that physical experience, visual observation and noise may have on the comfort of occupants need be investigated. Studies should be conducted on wind sensitive buildings to determine the actual experiences of occupants.

d) Effects of partitions, walls and slabs

The performance of a building is effected by such factors in the partitions, walls and slabs. Their effects should be incorporated into a performance design. A study is at present being conducted at Lehigh University and elsewhere to determine the above effects.

e) Fatigue and Brittle Fracture

It is necessary to know when a performance design should include designing for fatigue and brittle fracture. With the continuing research on the effects of wind on buildings and the fatigue and brittle fracture of such building components as beams, welds and connections, it can be expected that the near future will provide answers to the problem.

f) Performance specification for tall buildings

A performance specification for the design of tall buildings must be set up.

g) Performance design manual

When most of the above problems have been solved, then the drawing up of a performance design manual for tall buildings can commence. Such a manual should cover all structural aspects of tall building design such as design of the framing system, curtain walls etc.

8. SUMMARY AND CONCLUSIONS

New developments in the field of tall buildings necessitated a well formulated performance design. By performance design is meant the design of a building to satisfy certain performance criteria. Four performance criteria are considered in this report. They are 1) maximum strength, 2) permanent drift, 3) comfort of occupants, and 4) integrity of finishing material.

The treatment of wind in current design practice is reviewed and some short-comings are pointed out. By using the gust effect factor the dynamic effect of wind on a building is considered. The gust effect factor is the ratio of maximum to mean response of a building.

Each of the four performance criteria are then discussed and appropriate acceptance criteria are listed. The maximum strength of a structure is represented by the stability limit load which can be obtained from the lateral load-deflection curve. For satisfactory performance the design ultimate wind load must be less than or equal to the stability limit load. A corresponding mean recurrence interval for the design wind velocity is listed.

The performance criterion of permanent drift stems from the fact that a building is likely to experience permanent drift under extremely high winds. To enable the building to remain serviceable under such conditions, it is necessary to limit the amount of permanent drift. Because of lack of information no value can be given for the maximum permanent drift. An appropriate mean recurrence interval is listed.

There are three factors which may cause discomfort to the occupants of tall buildings. They are physically experiencing vibration, visual observation of the movement of a building and noise caused by wind buffeting a building or partitions and curtain walls creaking. Because of lack of information it is only possible to list acceptance criteria for vibration.

If excessive racking occurs in a building, then some of the partitions, windows and curtain walls may crack. Racking limits for different materials are listed and an appropriate mean recurrence interval for the design wind is listed.

It is then shown how the choice of an appropriate mean recurrence interval should be made. A relationship between mean recurrence interval, life of a building and risk is given.

The report then proceeds to show how the wind loads should be calculated. Relationships between recurrence interval and extreme wind velocity and between wind pressure and extreme wind velocity are presented. A description of gust effect factors is included.

Performance design with respect to each of the performance criteria is then described. The basis of design for all of the performance criteria are the lateral load-deflection curves of a frame.

These curves are dependent on the design gravity loads.

The design wind load for each of the performance criteria is given in terms of an equation. Design gravity loads are obtained by multiplying the working load by a suitable load factor which corresponds to the appropriate mean recurrence interval. A graph of recurrence interval versus load factor is included for this purpose.

A maximum strength design is executed to ensure that the design ultimate wind load will be less than or equal to the stability limit load.

Designing for permanent drift necessitates obtaining the permanent drift that corresponds to the design wind load. It is shown how this drift can be obtained from the appropriate load-deflection curve. This drift must be less than an allowable value.

A design for comfort of the occupants with respect to vibration consists of determining the acceleration of a building. The maximum acceleration is shown to be proportional to the maximum drift which can be obtained from the appropriate load-deflection curve. The allowable acceleration lies within certain comfort limits and the designer must decide which value to use.

Designing for integrity of the finishing material necessitates checking the racking of a building under the appropriate design wind load. The relative story displacement is the sum of the web plus chord deflection. Only the web deflection causes racking.

A comparison is then made between a performance design as described in this report and a traditional design. The significant differences are pointed out.

The following conclusions may be drawn:

a) New developments in tall buildings have necessitated a well formulated performance design.

b) Performance design constitutes a new approach to the design of tall buildings for wind. It is not a new design method.

c) Performance design provides more scope for the designer to use his skill and ingenuity and, as such, could lead to more effective and economical designs.

d) A tall building can now be designed to satisfy certain performance criteria.

e) Performance design is concerned with the response of a building to gravtiy and wind loads.

f) The dynamic response of a building to wind loads is automatically incorporated into a performance design.

9. ACKNOWLEDGMENTS

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10. NOMENCLATURE

A	=	amplitude of horizontal vibration
B	=	background turbulence factor
C	=	shape factor (general)
C_e	=	exposure factor
C_g	=	gust effect factor
C_p	=	pressure coefficient
D	=	building width in direction of wind
F	=	gust energy ratio
H	=	total building height; wind force
K	=	factor related to surface roughness
L	=	life of structure
N	=	number of years of observation; number of stories
P	=	probability of wind speed being exceeded in a year
R	=	mean recurrence interval
T	=	natural period of building
U	=	modal wind speed
V	=	velocity (general); reference velocity
a'	=	maximum acceleration
$1/a$	=	scale wind speed
g	=	gravity acceleration
g'	=	constant
h	=	story height
m	=	rank of an extreme value in a series

n	=	coefficient depending on terrain
n_o	=	natural frequency of buildings
p	=	pressure (general); probability of extreme wind being less than a given value in a ranked series
q	=	reference velocity pressure
r	=	probability of wind speed being exceeded during life of a structure
s	=	size factor
u	=	maximum response
$\bar{u}$	=	mean response
z	=	height above ground level
β	=	critical damping ratio
Δ	=	drift
Δ_p	=	permanent drift
σ	=	standard deviation
δ	=	racking deflection.

PERFORMANCE CRITERIA		ACCEPTANCE CRITERIA	MEAN RECURRENCE INTERVAL R years
MAXIMUM STRENGTH		DESIGN ULTIMATE LOAD ≤ STABILITY LIMIT LOAD	10^6
PERMANENT DRIFT		PERMANENT DRIFT ≤ ALLOWABLE VALUE	10^3
COMFORT OF OCCUPANTS	PHYSICAL EXPERIENCE	MAXIMUM ACCELERATION = 0.5 - 1.5% g	1
	VISUAL OBSERVATION	NONE YET	
	NOISE	NONE YET	
INTEGRITY OF FINISHING MATERIAL		ALLOWABLE RACKING INDEX (Table 2)	$10\text{-}10^2$

TABLE 1 PERFORMANCE CRITERIA

MATERIAL	ALLOWABLE RACKING INDEX
GLASS	0.005
BRICK	0.004
CONCRETE	0.003
HOLLOW CLAY TILE	0.003

TABLE 2 RACKING LIMITS

TABLE 3 ANNUAL MAXIMUM HOURLY WIND SPEEDS

RANK m	SPEED mph	YEAR	$p = \frac{m}{N+1}$	$-\log_e(-\log_e p)$
1	32	1958	.042	-1.16
2	33	1944	.083	- .91
3	34	1960	.125	- .73
4	35	1946	.167	- .58
5	36	1939	.208	- .45
6	36	1956	.250	- .33
7	37	1940	.292	- .21
8	37	1945	.333	- .09
9	39	1943	.375	.02
10	39	1951	.417	.13
11	39	1953	.458	.25
12	39	1961	.500	.37
13	41	1947	.542	.49
14	41	1949	.583	.62
15	42	1954	.625	.75
16	43	1959	.667	.90
17	45	1941	.708	1.06
18	45	1955	.750	1.25
19	46	1952	.792	1.46
20	50	1942	.833	1.70
21	52	1948	.875	2.01
22	55	1957	.917	2.44
23	58	1950	.958	3.15

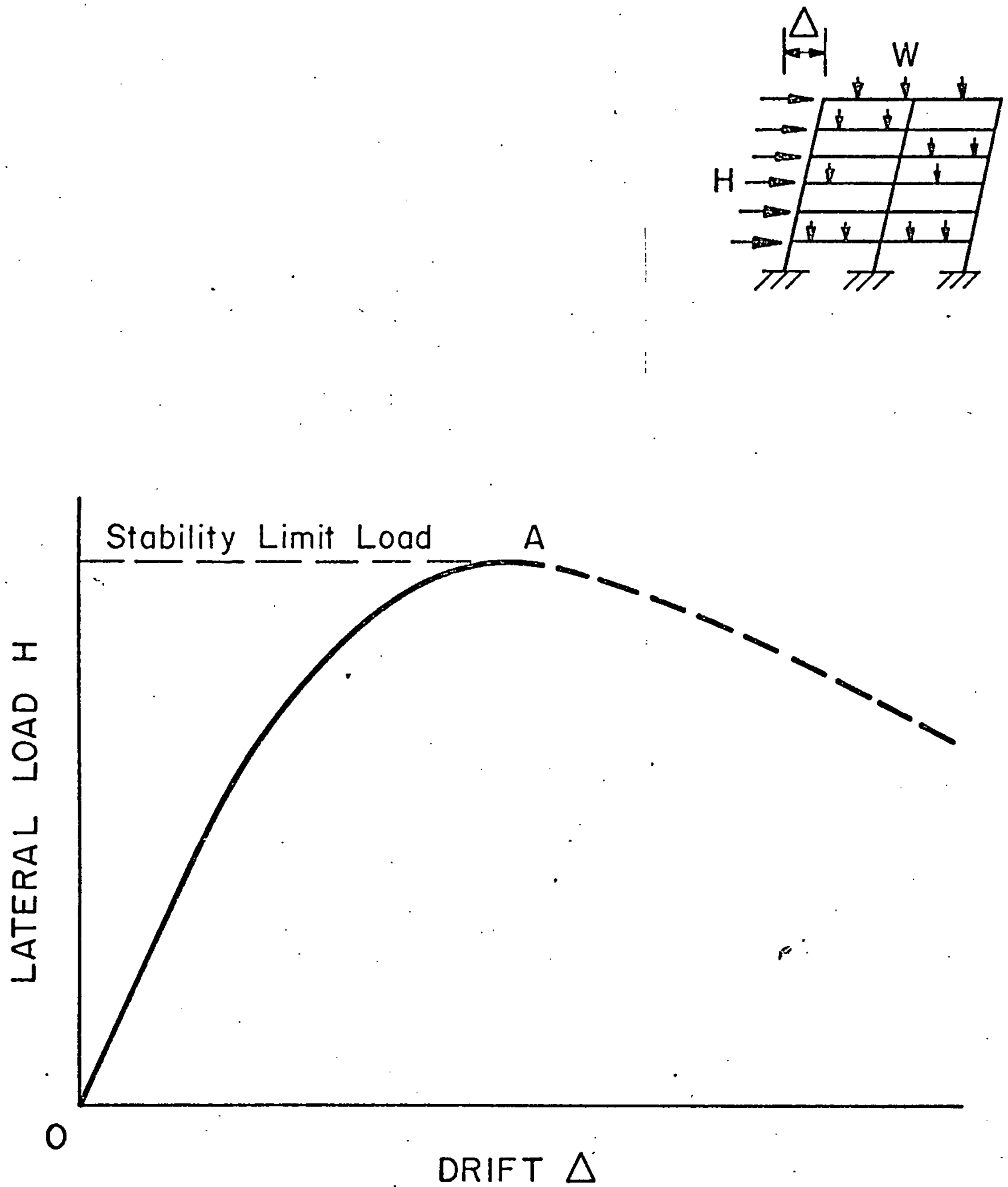


FIG. 1 LATERAL LOAD VERSUS DRIFT

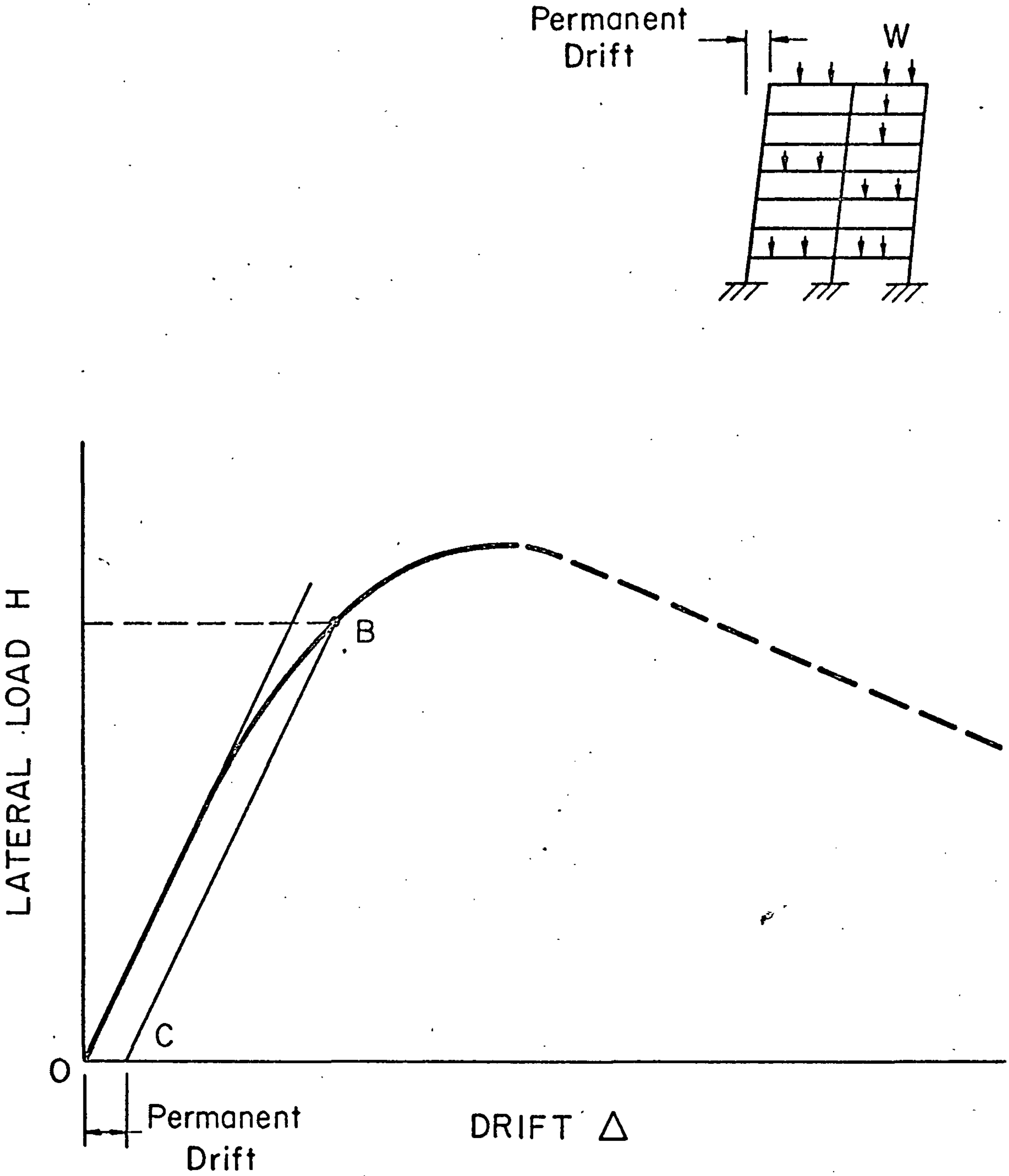


FIG. 2 PERMANENT DRIFT

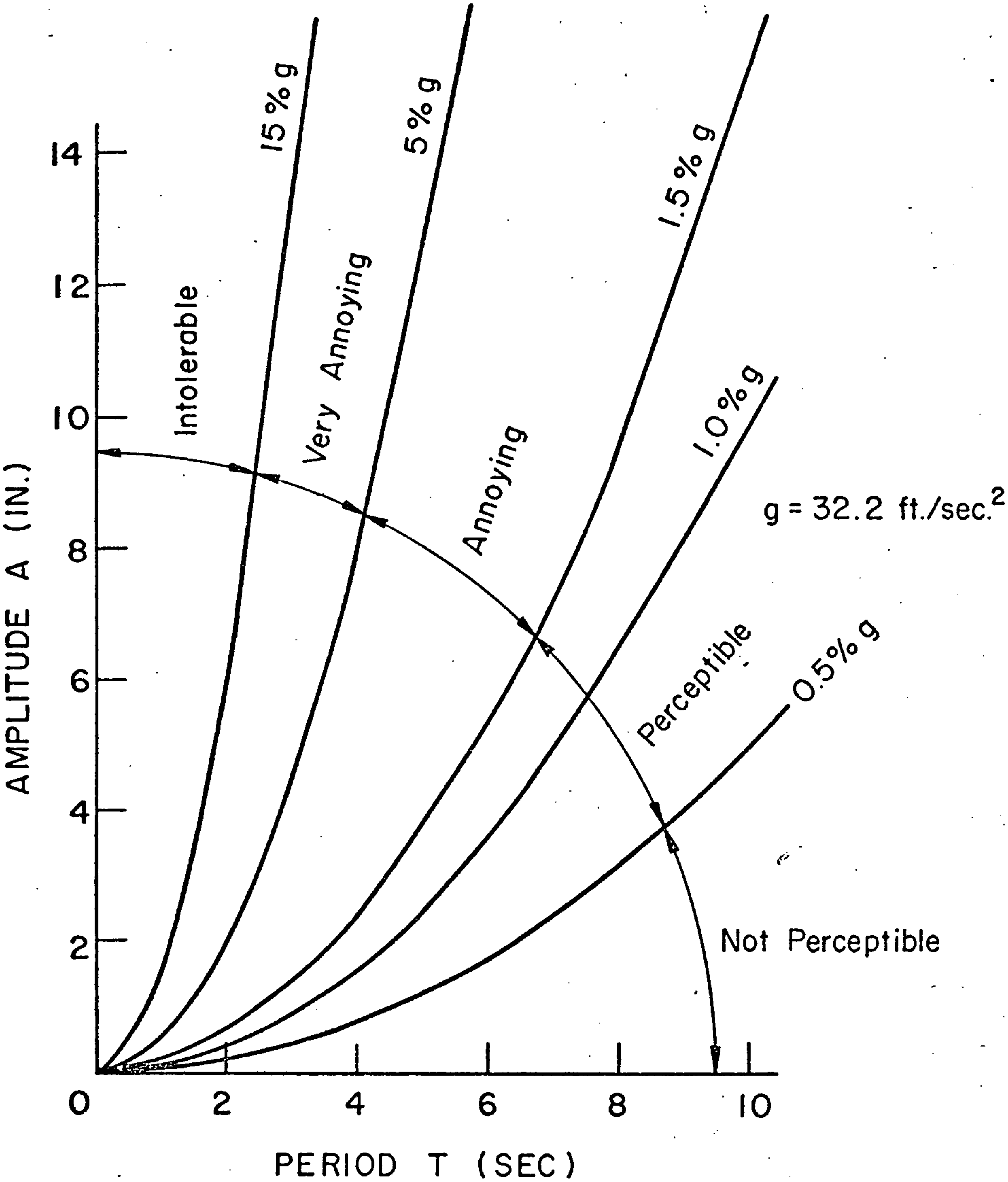
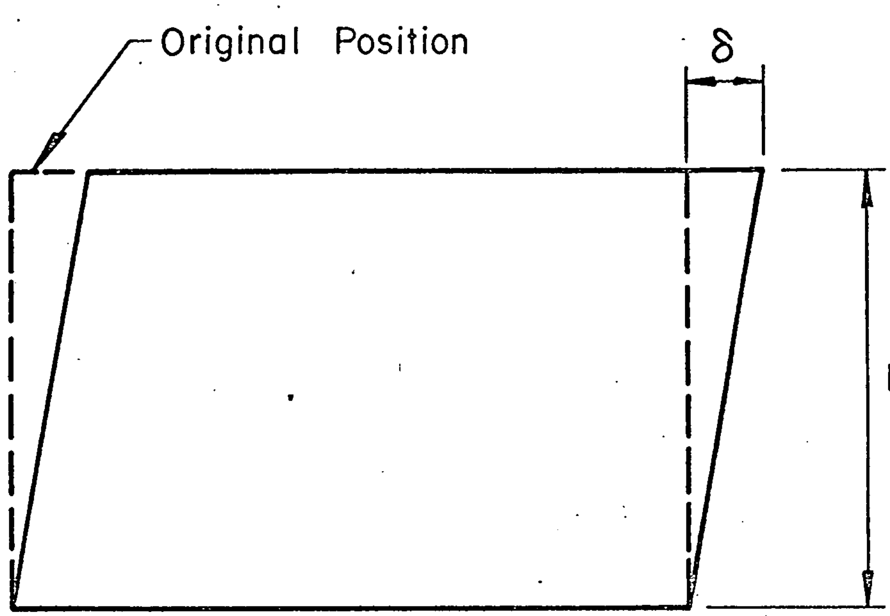


FIG. 3 AMPLITUDE VS. PERIOD OF VIBRATION FOR VARIOUS ACCELERATIONS



$$\text{Racking Index} = \frac{\delta}{h}$$

FIG. 4 RACKING OF AN ELEMENT

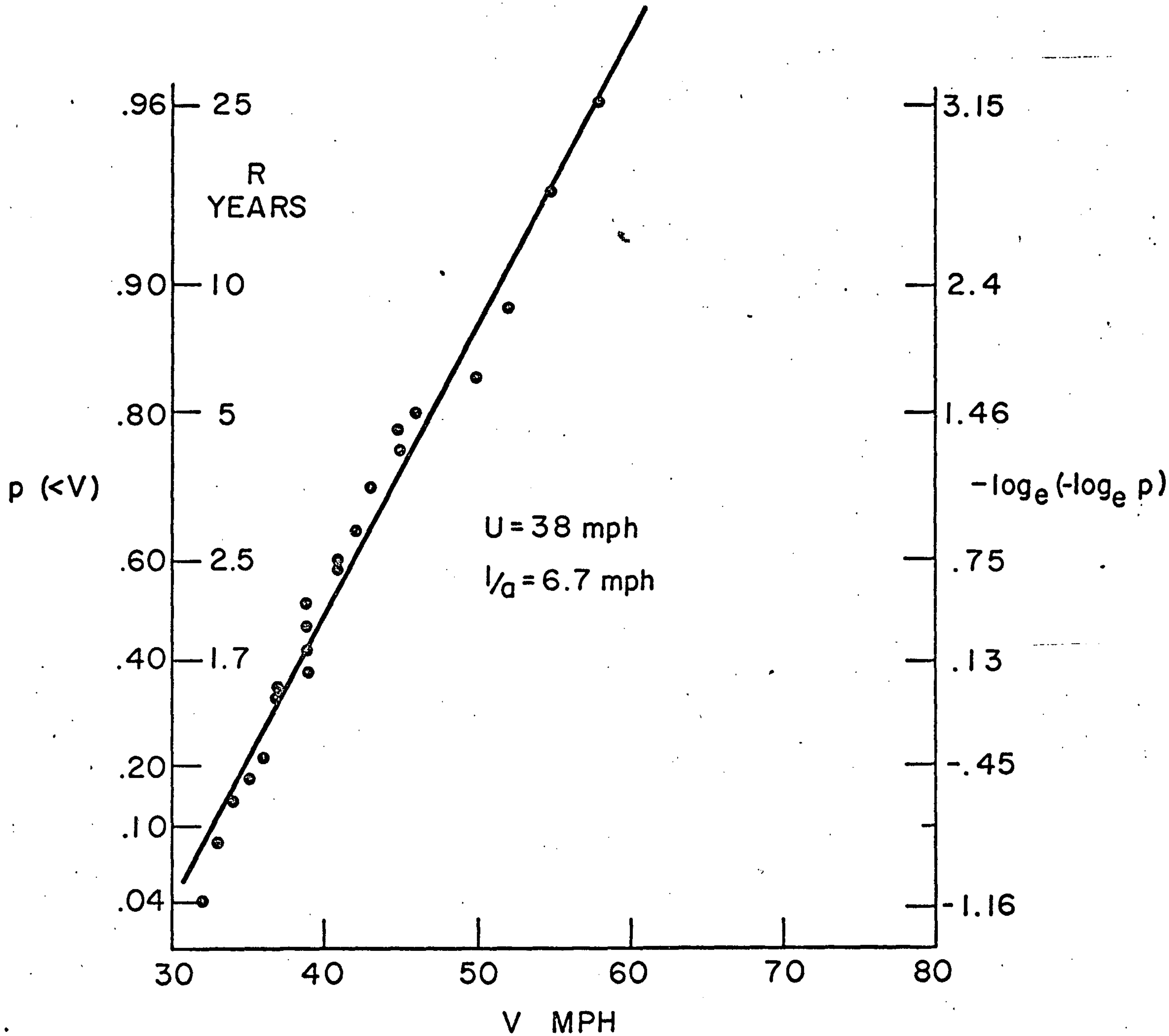


FIG. 5 PROBABILITY OF WIND SPEED LESS THAN V

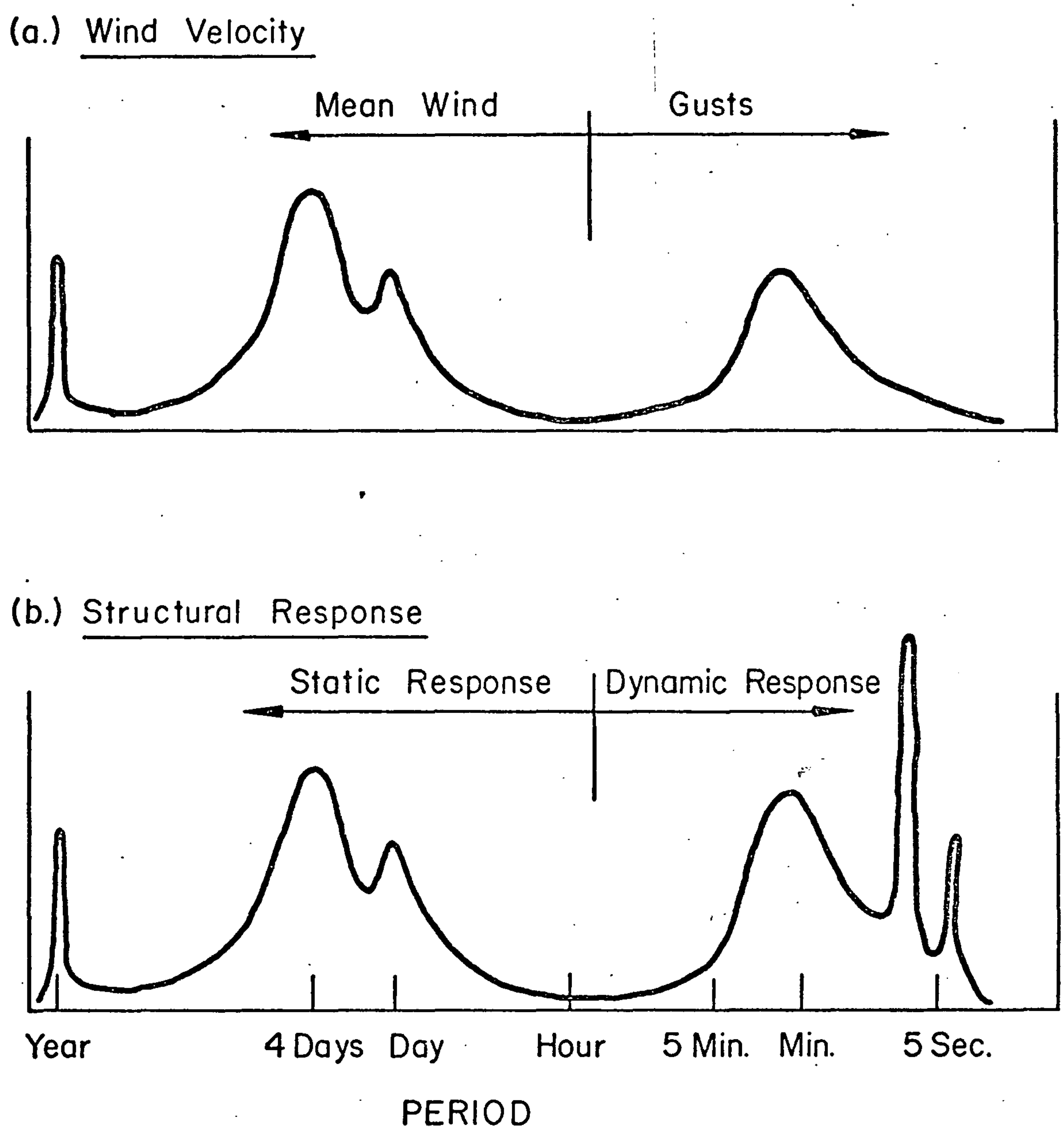


FIG. 6 WIND VELOCITY VS. STRUCTURAL RESPONSE

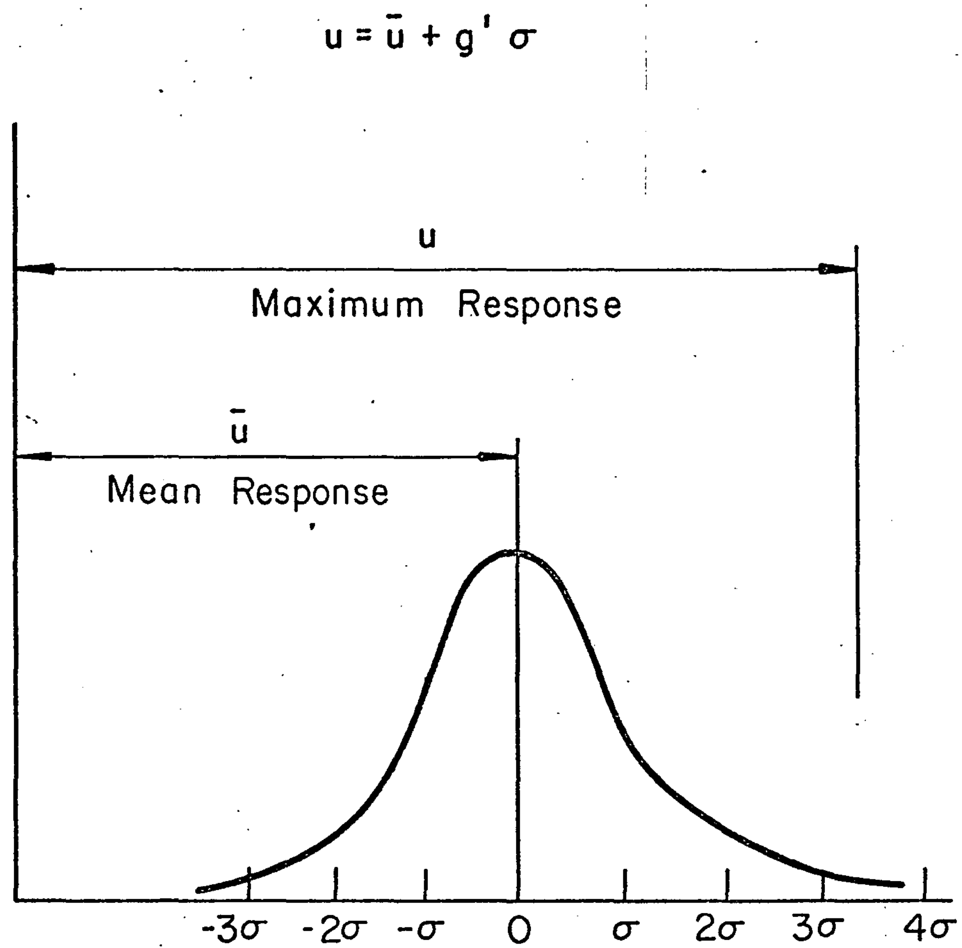


FIG. 7 PROBABILITY DISTRIBUTION OF STRUCTURAL RESPONSE

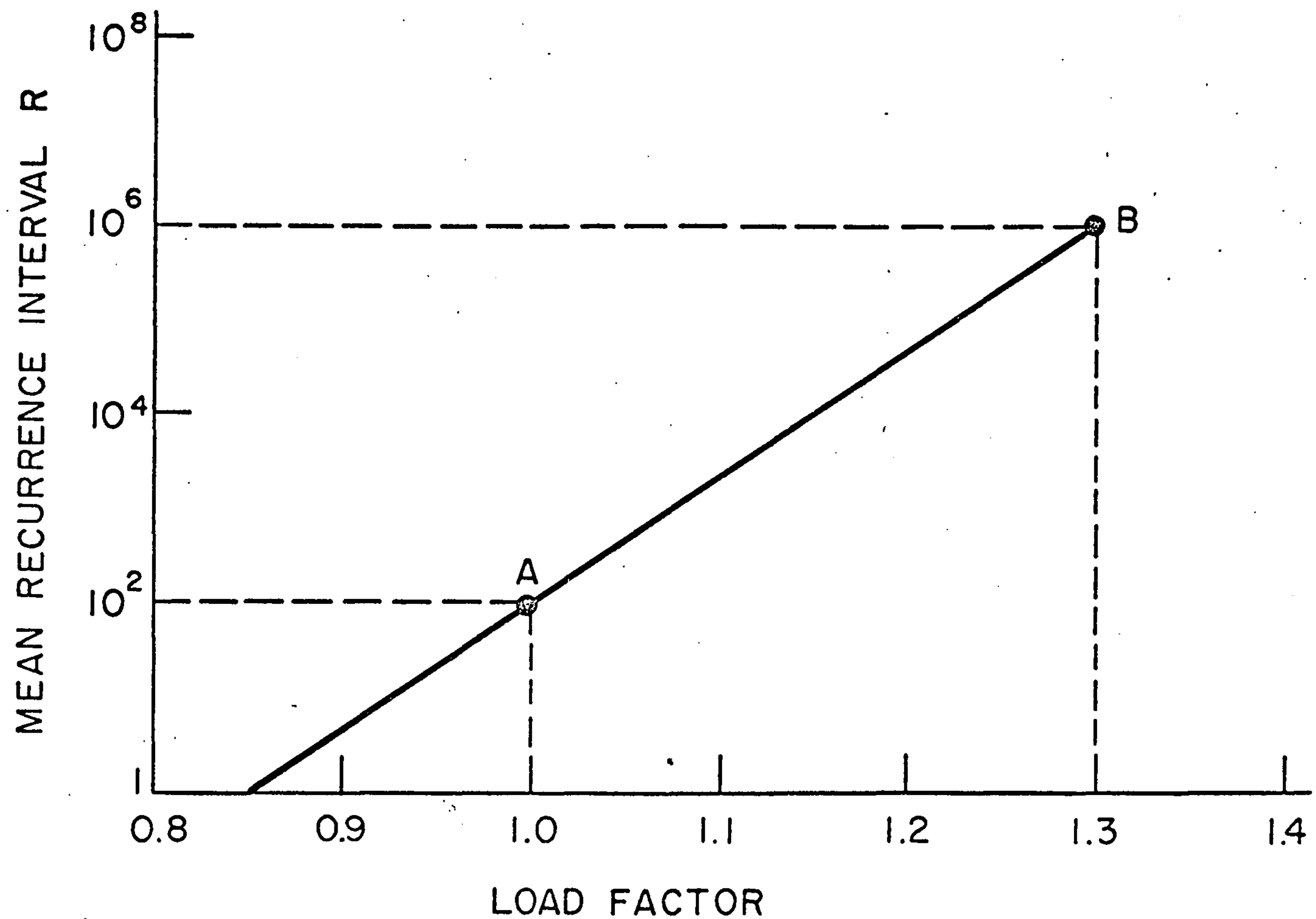


FIG. 8 MEAN RECURRENCE INTERVAL VS. LOAD FACTOR

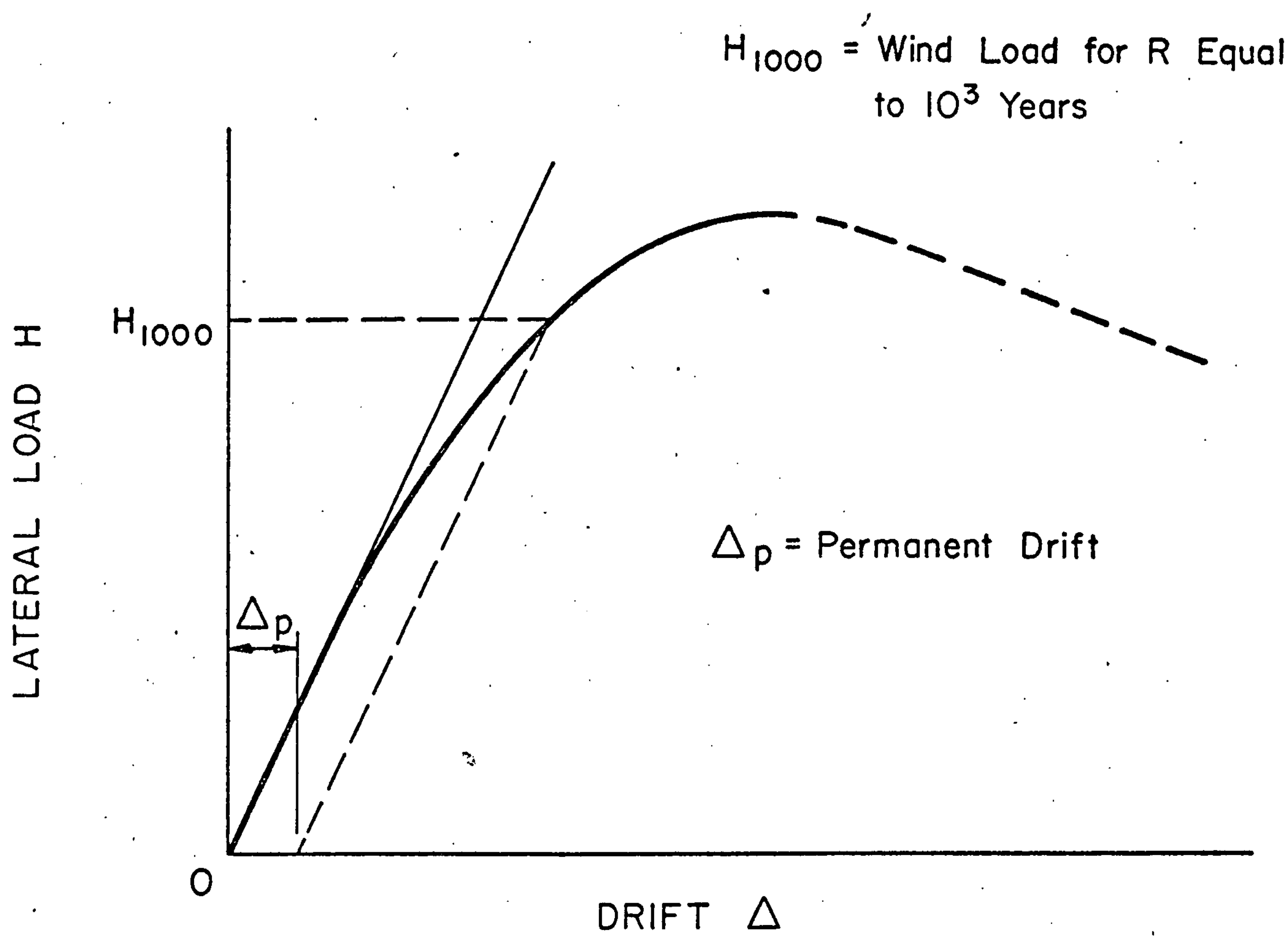


FIG. 9 PERMANENT DRIFT

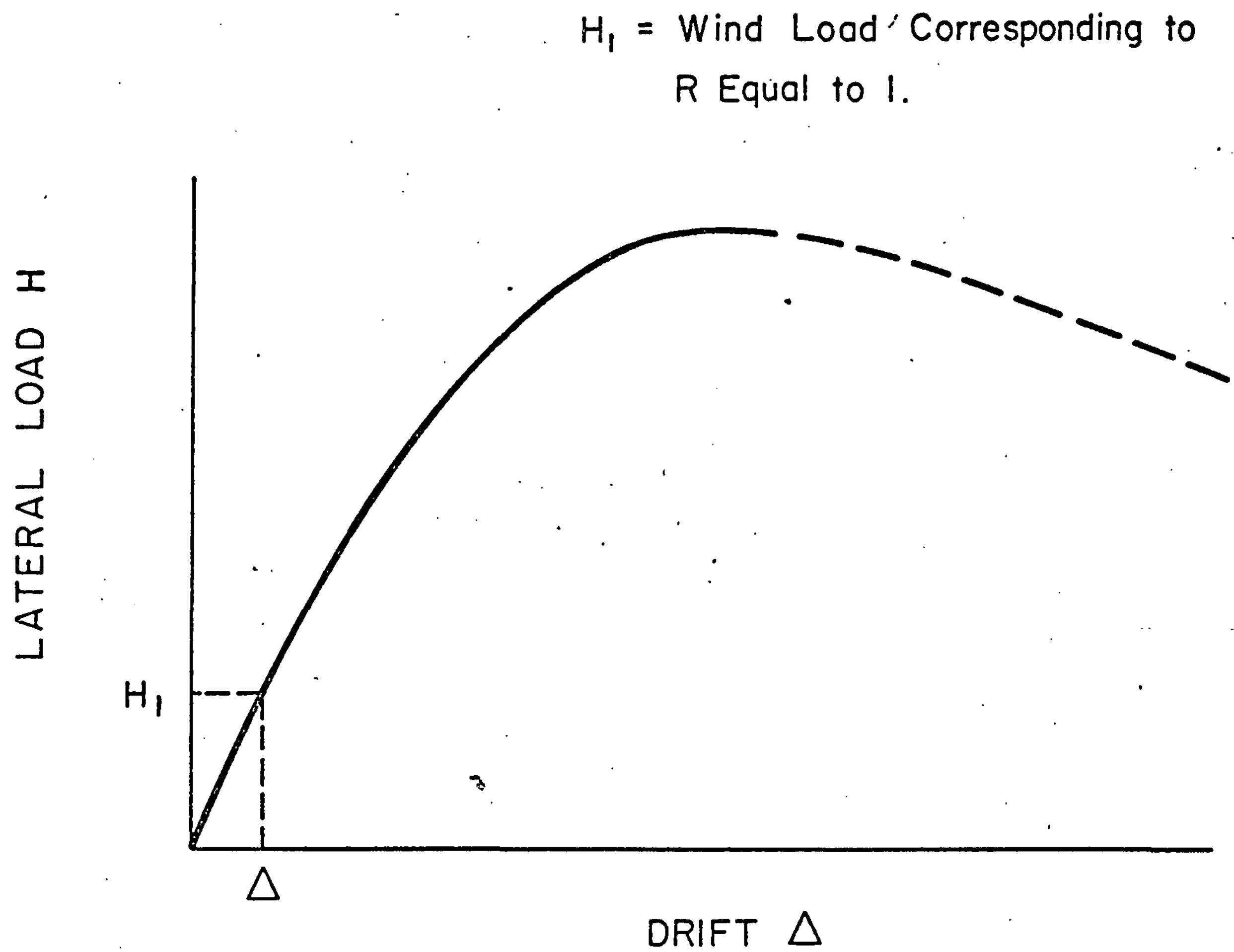


FIG. 10 DRIFT FOR CALCULATING ACCELERATION IN EQ. 20

13. REFERENCES

1. Davenport, A. G.
   NEW APPROACHES TO THE DESIGN OF STRUCTURES AGAINST WIND ACTION, Proceedings of the Canadian Structural Engineering Conference, Toronto, February 1968.

2. Schriever, W. R. and Dalgliesch, W. A.
   RECENT RESEARCH ON WIND FORCES ON TALL BUILDINGS, Proceedings of the Canadian Structural Engineering Conference, Toronto February 1968.

3. Newberry, C. W., Eaton, K. J. and Mayne, J. R.
   THE NATURE OF GUST LOADING ON TALL BUILDINGS, Building Research Station, England, Paper No. CP 66/68, August 1968.

4. Newberry, C. W., Eaton, K. J., and Mayne, J. R.
   WIND LOADING OF A TALL BUILDING IN AN URBAN ENVIRONMENT, Building Research Station, England, Paper No. CP 59/68, August 1968.

5. McGrew, E. J., Jr.
   PERFORMANCE TYPE OF BUILDING CODE GIVES SCOPE FOR ENGINEERING INGENUITY, Civil Engineering, August 1954.

6. WIND FORCES ON STRUCTURES, Transactions of the ASCE, Vol. 126, Part II, 1961, p. 1124.

7. RESPONSE OF BUILDINGS TO LATERAL FORCES, ACI, May 1970.

8. Thom, H. C. S.
   DISTRIBUTIONS OF EXTREME WINDS IN THE UNITED STATES, Journal of the Structural Division, Proceedings of the ASCE, Vol. 86, No. ST4, April 1960.

9. Thom, H. C. S.
   FREQUENCY OF MAXIMUM WIND SPEEDS, Journal of the Structural Division, Proceedings of the ASCE, Vol. 80, No. 539, Nov., 1954.

10. Thom, H. C. S.
    NEW DISTRIBUTIONS OF EXTREME WINDS IN THE UNITED STATES, Journal of the Structural Division, Proceedings of the ASCE, Vol. 94, No. ST7, July 1968.

11. Sherlock, R. H.
    GUST FACTORS FOR THE DESIGN OF BUILDINGS, A.I.B.S.E., Vol. 8, 1947, p. 207.

12. Sherlock, R. H.
VARIATION OF WIND VELOCITY AND GUSTS WITH HEIGHT, Proceedings of the ASCE, Vol. 78, No. 126, April 1952.

13. Davenport, A. G.
THE APPLICATION OF STATISTICAL CONCEPTS TO THE WIND LOADING OF STRUCTURES, Proceedings of the Institution of Civil Engineers, Vol. 19, 1961, p. 449.

14. Davenport, A. G.
GUST LOADING FACTORS, Journal of the Structural Division, Proceedings of the ASCE, Vol. 93, No. ST3, June 1967.

15. Vellozzi, J. and Cohen, E.
GUST RESPONSE FACTORS, Journal of the Structural Division, Proceedings of the ASCE, Vol. 94, No. ST6, June 1968.

16. Davenport, A. G.
WIND LOADING ON TALL BUILDINGS, Proceedings of the Symposium on "Wind Effects on Highrise Buildings", Northwestern University, Illinois, March 1970.

17. Robertson, L. E. and Chen, P. W.
THE TREATMENT OF WIND IN THE DESIGN OF VERY TALL BUILDINGS, Proceedings of the Technical Meeting Concerning Wind Loads on Buildings and Structures, N.B.S., Gaithersburg, January 1969.

18. ASCE-WRC Manual No. 41, "PLASTIC DESIGN IN STEEL" A Guide and Commentary, March 1971.

19. McDonald, J. R.
STRUCTURAL RESPONSE OF A TWENTY-STORY BUILDING TO THE LUBBOCK TORNADO, Texas Tech. Univ. Storm Report 01, Oct., 1970.

20. Park, D. L. and Snyder, F. W.
HUMAN REACTIONS TO LOW FREQUENCY VIBRATION, The Boeing Co., Wichita Div., Report D3-3512-1, July 24, 1961.

21. Chang, F. K.
WIND AND MOVEMENT IN TALL BUILDINGS, Civil Engineering, Aug. 1967, p. 70.

22. Wiss, J. F. and Cirth, O. F.
WIND DEFLECTIONS OF TALL CONCRETE FRAME BUILDINGS, Journal of the Structural Division, Proceedings of the ASCE, Vol. 96, No. ST7, July 1970.

23. Waller, R. A.
THE DESIGN OF TALL STRUCTURES WITH PARTICULAR REFERENCE TO VIBRATION, Proceedings of the Institute of Civil Engineers, Vol. 48, Feb. 1971.

24. Bouwkamp, J. G. and Meehan, J. F.
DRIFT LIMITATIONS IMPOSED BY GLASS, Proc. 2nd World Conference on Earthquake Engineering, Tokyo and Kyoto, Vol. 3, 1960, p. 1763.

25. Benjamin, J. R. and Williams, H. A.
THE BEHAVIOR OF ONE-STORY BRICK SHEAR WALLS, Journal of the Structural Division, Proceedings of the ASCE, Vol. 84, No. ST4, July 1958.

26. Holmes, M.
STEEL FRAMES WITH BRICKWORK AND CONCRETE INFILLING, Proceedings of the Institute of Civil Engineers, Vol. 19, 1961, p. 473.

27. Wood, R. H.
THE STABILITY OF TALL BUILDINGS, Proceedings of the Institute of Civil Engineers, Vol. 11, 1958, p. 69.

28. METEOROLOGY AND WIND LOADING - II, Proceedings of the Conference on "New Approaches to Structural Design Against Wind and Earthquake Action, University of Western Ontario, London, Ontario, Canada

29. NATIONAL BUILDING CODE OF CANADA, 1970.

30. Hollister, S. C.
THE ENGINEERING INTERPRETATION OF WEATHER BUREAU RECORDS FOR WIND LOADING ON STRUCTURES, Proceedings of the Technical Meeting Concerning Wind Loads on Buildings and Structures, National Bureau of Standards, Gaithersburg, Jan. 1969.

31. Thom, H. C. S.
ENGINEERING CLIMATOLOGY OF DESIGN WIND SPEEDS WITH SPECIAL APPLICATION TO THE CHICAGO AREA, Proceedings of the Symposium on "Wind Effects on Highrise Buildings", Northwestern University, Illinois, March 1970.

32. Davenport, A. G.
STRUCTURAL SAFETY AND RELIABILITY UNDER WIND ACTION, Paper presented at the "International Conference on Structural Safety and Reliability, Smithsonian Institute, Washington, D. C., April 1969.

33. Vickery, B. J.
ON THE RELIABILITY OF GUST LOADING FACTORS, Proceedings of the Technical Meeting Concerning "Wind Loads on Buildings and Structures, National Bureau of Standards, Gaithersburg, Jan. 1969.

34. Jennings, A. and Majid, K.
AN ELASTIC-PLASTIC ANALYSIS BY COMPUTER FOR FRAMED STRUCTURES LOADED UP TO COLLAPSE, The Structural Engineer, Vol. 43, No. 12, December 1965.

35. Majid, K. I. and Anderson, D.
THE COMPUTER ANALYSIS OF LARGE MULTI-STORY FRAMED STRUCTURES, The Structural Engineer, Vol. 46, November 1968.

36. Parikh, B. P.
ELASTIC-PLASTIC ANALYSIS AND DESIGN OF UNBRACED MULTI-STORY STEEL FRAMES, Ph.D. Dissertation, Lehigh University, 1966, University Microfilms, Inc., Ann Arbor, Michigan.

37. Ward, H. S. and Crawford, R.
DETERMINATION OF THE NATURAL PERIODS OF BUILDINGS, Bulletin of Seismological Society of America, Vol. 54, No. 6, Dec. 1964, p. 1743.

38. Ward, H. S. and Crawford, R.
WIND INDUCED VIBRATIONS AND BUILDING MODES, Bulletin of the Seismological Society of America, Vol. 56, No. 4, Aug. 1966, p. 793

39. Spurr, H. V.
WIND BRACING, McGraw-Hill, 1930.

40. WIND BRACING IN STEEL BUILDINGS, Final report of Subcommittee No. 31, Transactions of the ASCE, Vol. 105, 1940, p. 1713.