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SYNOPSIS

The upper bound theorem of limit analysis is applied to analyze the stability of a slope when the slope is intercepted by a retaining structure. A closed-form solution is obtained in which the optimum location, penetration depth, and strength of the retaining structure could be determined. A straight-line failure plane is assumed in the analysis. Design charts developed from the limit analysis solution are presented for a useful range of friction angles and slope geometries.

INTRODUCTION

The problem of the stability of a slope intercepted by a retaining structure frequently appears in engineering practice but design data to assess the optimum location of installation, penetration depth, and strength of the retaining structure are very scant. This lack of detailed information is due largely to the difficult procedures in analysis encountered when the conventional limit equilibrium method is used. In particular, the friction condition between the soils and the wall poses special problems and the strength consideration of the wall can further complicate the computational procedures. However, as in previous works (Chen, et al. 1969, 1970, 1971; Fang and Hirst, 1970) on the stability of slopes, the upper bound theorem of the generalized theory of perfect plasticity (Drucker and Prager, 1952) can be used to obtain solutions for the critical height of the problem. It has been shown to yield reasonable answers to foundation engineering problems when compared with existing limit equilibrium solutions whose validity have been established on the basis of practical experience (Chen and Scawthorn, 1970; Fang and Hirst, 1970).

METHOD OF ANALYSIS

Failure mechanisms are shown in Fig. 1. The soil wedge is divided into two rigid bodies which are separated by the retaining structure. The frictional forces between the wall and the soil are obtained from equations of equilibrium which are shown in Fig. 2. The compatible velocity field is shown in Fig. 3. The rate of internal energy dissipation can be evaluated by the Coulomb yield condition and its associated flow rule. Equating both the external rate of work and internal rate of energy dissipation, the expressions for the stability factor can then be obtained. The maximized value of the stability factor gives the maximum properties of soil which is necessary to form a failure. Therefore, if the stability factor of the soil is greater than that maximized value, the slope is stable. Detailed steps are described in the following sections.

FAILURE MECHANISMS

The straight-line failure plane is assumed in the analysis. The yielding of the retaining structure is assumed to follow the maximum shear stress or Tresca criterion and Coulomb's yield criterion is assumed for the soil.

Four types of failure mechanisms are considered herein:

<u>Free-Sliding</u> - Sliding in the case of no wall. This consideration tells the necessity of a wall. It should be noted that for this case logarithmic spiral curve will give a better solution, however, the straight lines are used here for the purpose of consistency with other types of mechanism.
<u>Over-Sliding</u> - This consideration checks the sufficiency of the wall depth.
<u>On-Sliding</u> - A slip line starts from the bottom tip of the wall and another line starts from a different point on the wall. One part of the wall surface is separated from the soil during sliding. This consideration tells the necessary depth of the wall.

Through-Sliding - When the wall is long and weak, the slip-line may cut through the wall. This consideration tells the necessary strength of the wall.

Evaluation of the length of slip lines (L_1, L_2) and area of rigid bodies (A_1, A_2) is required for computation of the rate of energies. All these necessary dimensional properties are shown in Fig. 3. In order to handle the equations nondimensionally, the following notations are used.

$$\ell_1 = \frac{L_1}{H}, \quad \ell_2 = \frac{L_2}{H}, \quad a_1 = \frac{A_1}{H^2}, \quad a_2 = \frac{A_2}{H^2}, \quad x = \frac{X}{L}, \quad y = \frac{Y}{H}$$

$$d = \frac{D}{H}, \quad \ell_o = \frac{L_o}{H}$$

FRICTIONAL FORCES ON THE WALL

Frictional forces acting on the surfaces between the wall and the soil (see Fig. 2) must be evaluated for the computation of friction energy dissipation. Frictional force F is related to the normal force P with the friction coefficient μ as $F = \mu P$. The magnitude of the normal force P can be obtained from the equilibrium equations of the two rigid bodies.

Over-Sliding (Fig. 2-a)

There are eight-unknowns, i.e., R_1, T_1, R_2, T_2, R_o, T_0, F, P, and the eight equations are: four equilibrium equations, three yield conditions, and one equation of friction. Solving these equations for P,

$$P = p\ \gamma H^2 - qCH \tag{1}$$

where

$$p = \frac{1}{S}\{\gamma_2 a_1 M_1 - \gamma_1 a_2 M_2\}$$

$$q = \frac{\gamma_2}{S}\ (\ell_o+\ell_1\sin\theta_1)M_1+\ell_1\cos\theta_1 N_1\}$$

$$+\frac{\gamma_1}{S}\{(\ell_o+\ell_2\sin\theta_2)M_2+\ell_2\cos\theta_2 N_2\}$$

$$\gamma_1 = N_1+M_1\tan\phi \quad \gamma_2 = N_2-M_2\tan\phi$$

$$S = \gamma_1(M_2\mu-N_2)+\gamma_2(M_1\mu+N_1)$$

and

$$M_1 = \sin\theta_1-\tan\phi\cos\theta_1$$

$$M_2 = \sin\theta_2+\tan\phi\cos\theta_2$$

$$N_1 = \cos\theta_1+\tan\phi\sin\theta_1$$

$$N_2 = \cos\theta_2-\tan\phi\sin\theta_2$$

<u>On-Sliding and Through-Sliding</u> (Fig. 2-b,2-c)

There are eight-unknowns, i.e., R_1, T_1, R_2, T_2, P_1, F_1, P_2, F_2, and the eight equations are: four equilibrium equations, two yield conditions, and two equations of frictions. Solving for P_1 and P_2,

$$P_1 = p_1\ \gamma H^2 - q_1\ CH;\ P_2 = p_2\ \gamma H^2 - q_2\ CH \tag{2}$$

where

$$p_1 = \frac{a_1 M_1}{N_1+\mu M_1} \qquad q_1 = \ell_1 \frac{M_1 \sin\theta_1 + N_1 \cos\theta_1}{N_1+\mu M_1}$$

$$p_2 = \frac{a_2 M_2}{N_2-\mu M_2} \qquad q_2 = -\ell_2 \frac{M_2 \sin\theta_2 + N_2 \cos\theta_2}{N_2-\mu M_2}$$

In the above derivation, weight of the wall is neglected.

COMPATIBILITY OF VELOCITIES

Velocities V_1 and V_2 of rigid bodies I and II make a friction angle ϕ with the slip lines L_1 and L_2, respectively, as shown in Fig. 3. The wall is considered to move with either of the two rigid bodies, mostly with body II. Since the friction angle between wall and soil is ϕ_o, compatibility of velocities is obtained from the horizontal components of velocities, that is

$$V_2 \cos(\theta_2+\phi) - V_1 \cos(\theta_1-\phi) = [V_2 \sin(\theta_2+\phi) + V_1 \sin(\theta_1-\phi)] \tan\phi_o \tag{3}$$

or rearranged

$$V_1 \cos(\theta_1-\phi-\phi_o) = V_2 \cos(\theta_2+\phi+\phi_o) \tag{4}$$

Denoting the constant value

$$V_o = \cos(\theta_1-\phi-\phi_o)\cos(\theta_2+\phi+\phi_o)$$

as the reference velocity, the non-dimensional velocities

$$v_1 = \frac{V_1}{V_o} = \cos(\theta_2+\phi+\phi_o); \quad v_2 = \frac{V_2}{V_o} = \cos(\theta_1-\phi-\phi_o) \tag{5}$$

will satisfy Eq. 4 identically.

RATE OF ENERGIES

Rate of External Work $\dot{W}_E$

This is a product of the weight and the downward velocity of the rigid bodies (See Figs. 2 and 3).

$$\dot{W}_E = A_1\gamma V_1\sin(\theta_1-\phi)-A_2\gamma V_2\sin(\theta_2+\phi) \tag{6}$$

Rate of Internal Energy Dissipation $\dot{D}_I$

Energy dissipation occurs along all the slip lines of soil, on the friction surface between wall and soil and along the cut-line of the wall. The rate of energy dissipation $\dot{D}_I$ is considered along each line as:

(1) along a slip line of soil

$\dot{D}_I$ = (cohesion)x(length of slip line) x(relative tangential velocity)

$$\dot{D}_I = CL_1V_1\cos\phi \qquad \text{on line } L_1$$

$$\dot{D}_I = CL_2V_2\cos\phi \qquad \text{on ling } L_2 \tag{7}$$

$$\dot{D}_I = CL_o[V_1\sin(\theta_1-\phi)+V_2\sin(\theta_2+\phi)] \qquad \text{on ling } L_o$$

(2) on the friction surface between wall and soil

$\dot{D}_I$ = (frictional force)x(relative tangental velocity)

$$\dot{D}_I = \mu P[V_1\sin(\theta_1-\phi)+V_2\sin(\theta_2+\phi)] \tag{8}$$

(3) along a cut line of wall

$\dot{D}_I$ = (yield shear stress)x(length of the cut line) x(relative tangental velocity)

$$\dot{D}_I = kBV_1\cos(\theta_1-\phi) \tag{9}$$

where the wall was assumed to be cut in horizontal direction.

STABILITY FUNCTION

Stability function λ is defined as $\lambda = \frac{C}{\gamma H}$ and it represents a non-dimensional strength of the resistance of slope against sliding. By equating the rate of external work $\dot{W}_E$ to the total rate of internal energy dissipation $\dot{D}_I$, stability function λ is obtained in terms of ten parameters: β, ϕ, μ, ϕ_o, x, d, k, y, θ_1, θ_2; where β,ϕ = slope parameters, μ,ϕ_o = mutual parameters, x,d,k = wall parameters, and y,θ_1, θ_2 = mechanism parameters.

If λ-value of actual slope is less than λ_c, it is possible for the slope to slide in this specific mechanism. The maximum value of stability function λ_c must be searched by changing the assumed dimensional parameters. A slope which has a λ-value greater than λ_c can not form any mechanism, i.e., the slope is stable. The purpose of this paper is to evaluate this critical value of stability function λ_c. The derivation of the stability factor follows in each sliding case.

(1) <u>Free Sliding</u>

$$\dot{W}_E = A_1\gamma V_1\sin(\theta_1-\phi)-A_2\gamma V_2\sin(\theta_2+\phi) \tag{10}$$

$$\dot{D}_I = CL_1V_1\cos\phi+CL_2V_2\cos\phi+CL_o[V_1\sin(\theta_1-\phi)+V_2\sin(\theta_2+\phi)] \quad (11)$$

From $\dot{W}_E = \dot{D}_I$ and dividing by $H^2\gamma V_o$

$$a_1v_1\sin(\theta_1-\phi)-a_2v_2\sin(\theta_2+\phi)$$

$$= \frac{C}{\gamma H}[(\ell_1v_1+\ell_2v_2)\cos\phi+\ell_ov_1\sin(\theta_1-\phi)+\ell_ov_2\sin(\theta_2+\phi)]$$

$$\text{or } \lambda_1 = \frac{C}{\gamma H} = \frac{g_1}{g_2+\ell_og_3} \quad (12)$$

where $g_1 = a_1v_1\sin(\theta_1-\phi)-a_2v_2\sin(\theta_2+\phi)$; $g_2 = (\ell_1v_1+\ell_2v_2)\cos\phi$

$$g_3 = v_1\sin(\theta_1-\phi)+v_2\sin(\theta_2+\phi)$$

and v_1 and v_2 are defined in Eq. (5).

(2) Over Sliding

The external rate of work is identical to that of Eq. (10). The total rate of internal dissipation of energy is obtained by adding the additional dissipation of energy due to wall friction to the previous calculated dissipation Eq. (11). Equating the external rate of work to the total dissipation shows that

$$\lambda_2 = \frac{C}{\gamma H} = \frac{g_1-\mu pg_3}{g_2+(\ell_o-\mu q)g_3} \quad (13)$$

where p and q are defined in Eq. (1).

(3) <u>On Sliding</u>

The external rate of work is again identical to that of Eq. 10 but the internal rate of dissipation is modified to

$$\dot{D}_I = CL_1V_1\cos\phi + CL_2V_2\cos\phi + \mu P_1V_1\sin(\theta_1-\phi) + \mu P_2V_2\sin(\theta_2+\phi) \quad (14)$$

$$\lambda_3 = \frac{C}{\gamma H} = \frac{g_1-\mu g_4}{g_2-\mu g_5} \quad (15)$$

where $g_4 = p_1v_1\sin(\theta_1-\phi)+p_2v_2\sin(\theta_2+\phi)$

$g_5 = q_1v_1\sin(\theta_1-\phi)+q_2v_2\sin(\theta_2+\phi)$

where p_1, q_1, and p_2, q_2 are defiend in Eq. (2).

(4) <u>Through Sliding</u>

The expressions for external rate of work and internal rate of dissipation are identical to the previous on-sliding case. The only term must be added to the internal dissipation of energy is the energy needed to cut-through the wall, that is

$$\lambda_4 = \frac{C}{\gamma H} = \frac{g_1-\mu g_4-\kappa g_6}{g_2-\mu g_5} \quad (16)$$

where $\kappa = \frac{kB}{\gamma H^2}$; $g_6 = v_1\cos(\theta_1-\phi)$

OPTIMIZATION OF STABILITY FUNCTION

For design purposes, the parameters β, ϕ, μ, k, and ϕ_o are considered to be given. Thus the stability function λ is a function of the five variables x, d, y, θ_1, θ_2. Since the three parameters x, d, and y are to be determined, the optimization of λ must be done with respect to θ_1 and θ_2.

Results are summarized in terms of the stability function λ. Equation (12) determines the necessity of the structure and its optimum location (Fig. 4). Equations (13) and (15) provide the depth of the structure (Fig. 5) and Eq. (16) determines the strength of the structure (Fig. 6). A computer program has been developed for this purpose, which gives the maximum or the minimum value of a function of n-variables in a given region for their variables.

NUMERICAL EXAMPLES

The optimized stability function (stability factor) $\lambda max = \lambda_c$ contains four constants $(\beta, \emptyset, \mu, \emptyset_o)$ and four variables (x, y, d, κ), thus $\lambda_c(\beta, \emptyset, \mu, \emptyset_o;\ x, y, d, \kappa)$.

Numerical computations were carried out for $\beta = 45°$, $\emptyset = 10°$, and $\mu = 0.2$. For all cases, the friction angle between the wall and the soil $\emptyset_o$ was assumed as: $\phi_o = 0$, (For on- and through-sliding cases), and $\phi_o = \phi$ (For free- and over-sliding cases).

Free-Sliding: $d = 0, \kappa = \infty$ (Fig. 4)

$x = -0.9$ to 0.9, $y = 0$ to 1.8

On- and Over-Sliding: $x =$ fixed, $\kappa = \infty$ (Fig. 5)

$d = 0$ to 1.8, $y = 0$ to 1.8

Through-Sliding: $x =$ fixed, $d = \infty$ (Fig. 6)

$\kappa = 0$ to 0.18, $y = 0$ to 1.8

Example: Given

$H = 30$ ft. $\beta = 45°$, $\gamma = 100$ pcf, $\phi = 10°$, $C = 180$ psf

Thus the stability factor of this slope is

$$\lambda_A = \frac{C}{\gamma H} = \frac{180}{(100)\ (30)} = 0.06$$

It is desirable to know if this slope is stable as it is. This answer is obtained from the Free-Sliding chart (see Fig. 4). From this chart, it is read that

λ free $= 0.085$

at $y = Y/H = 0.6$ and $x = X/L = 0$

Then is is known that

if $\lambda_A > 0.085$ the slope is stable

if $\lambda_A \leq 0.085$ a wall is required

When $\lambda_A = 0.085$, the only mechanism is possible with $x = 0.0$, $y = 0.6$, $\theta_1 = 42.0°$ and $\theta_2 = 12.0°$.

Since our present slope has $\lambda_A = 0.06$ (the dotted line in Fig. 4) many mechanisms are possible in the range $-0.6 \leq x \leq 0.5$. Therefore, a wall is certainly needed to retain the slope. Furthermore, it is known that the best (or the safest) location of the wall is at $x = 0$.

Now the location of the wall has been determined. The material of the wall must be selected. In our case, let us use a steel whose properties are

$$k = 20 \text{ ksi} = 2.88 \times 10^6 \text{ lb/ft}^2$$

$$\mu = 0.2$$

The next question is how deep the wall should be. This is answered by the On- and Over-Sliding chart (Fig. 5). Since our slope has $\lambda_A = 0.06$ (the dotted line in Fig. 6), it is read from the chart that the necessary depth of the wall is $d = 1.2$, i.e.,

$$D = 1.2H = 1.2 \times 30 = 36 \text{ ft}$$

because there are no deeper slip lines on this level ($\lambda_A = 0.06$) and the shorter ones are prevented by the wall (at this stage, the wall is assumed to be rigid).

As one can see here, the outermost curve (the envelope) governs the case in On- and Over-Sliding. This mostly happens when d = y which means the governing mechanism is one at the boundary between On-Sliding and Over-Sliding. This state is also obtained from Through-Sliding chart with $\kappa = 0$ as is seen later.

The thickness of the wall is determined from the Through-Sliding chart (Fig. 6). For our slope $\lambda_A = 0.06$ (the dotted line in Fig. 6), it is known that the required strength of the wall changes along the depth. The maximum value is

$$\kappa_{max} = \frac{kB_{max}}{\gamma H^2} = 0.11$$

$$\text{at } y = Y/H = 0.7$$

Then the maximum required thickness

$$\begin{aligned} B_{max} &= \frac{\kappa_{max}}{k} \gamma H^2 \\ &= \frac{0.11}{2.88\times10^6} \times 100 \times 30^2 \\ &= 0.00343 \text{ ft} = 0.0412 \text{ in.} \end{aligned}$$

In the Through-Sliding chart, the outermost curve is always $\kappa = 0$. It tells that the required thickness at the bottom of the wall is zero. It is also to be noted that the curve $\kappa = 0$ in the Through-Sliding chart (Fig. 6) is identical to the envelope of the curves in On- and Over-Sliding chart (Fig. 5).

Now all the information has been obtained that is necessary to design the retaining wall. In actual designs, a factor of safety (F.S) must be applied. Since the square of height (H^2) has the linear effect on the wall strength (κ), the depth of the wall should be multiplied by $\sqrt{F.S.}$ and the thickness by F.S.

In the present design, if one takes F.S. = 2.0, the design depth should be D = 36 ft x $\sqrt{2}$ = 50 ft. and the design thickness, B = 0.0412 x 2 = 0.0824 in. Then sheet piles of 1/8 in. thickness or circular tubes 1.66 x 1 1/4" spaced every 5 ft. suffice this condition.

CONCLUSION

Based on the upper bound theorem of limit analysis, a closed-form solution was obtained for which the optimum location, penetration depth, and strength of the structure can be determined. Equation (12) determines the necessity of the structure and its optimum location. Equations (13) and (15) provide the depth of the structure and Eq. (16) determines the strength of the structure. Typical design charts based on these equations are given and also numerical examples.

NOTATIONS

A_i, a_i = area of soil blocks ($a_i = A_i/H^2$)

B = thickness of wall

c = cohesion

$\dot{D}_I$ = rate of internal energy dissipation

D, d = depth of wall ($d = D/H$)

F = frictional force

g_i = parameters determine λ

H = height of slope

k = yield shear stress of wall material

L = horizontal length of slope

L_i, ℓ_i = length of slip lines ($\ell_i = L_i/H$)

P = normal force on wall

R_i = normal forces on slip lines

T_i = tangential force on slip lines

V_i, v_i = velocities of soil blocks ($v_i = V_i/V_o$)

$\dot{W}_E$ = rate of external work done

X, x = horizontal coordinate ($x = X/L$)

Y, y = vertical coordinate ($y = Y/H$)

β = slope angle

γ = unit weight of soil

θ_i = angles of slip lines

κ = strength parameter of wall ($\kappa = kB/\gamma H^2$)

λ = stability function ($\lambda = C/\gamma H$)

λ_c = stability factor (ultimized value of λ)

μ = friction coefficient between wall and soil

σ = normal stress

τ = shear stress

ϕ = frictional angle of soil

ϕ_o = frictional angle between soil and wall

REFERENCES

Chen, W. F., Giger, M., and Fang, H. Y. (1969). ON THE LIMIT ANALYSIS OF STABILITY OF SLOPES, Soils and Foundations, Vol. IX, No. 4, pp. 23-32.

Chen, W. F. and Scawthorn, C. R. (1970). LIMIT ANALYSIS AND LIMIT EQUILIBRIUM SOLUTIONS IN SOIL MECHANICS, Soils and Foundations, Vol. 10, No. 3, pp. 13-49.

Chen, W. F. and Giger, M. (1971). LIMIT ANALYSIS OF STABILITY OF SLOPES, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM1, pp. 19-26.

Drucker, D. C. and Prager, W. (1952). SOIL MECHANICS AND PLASTIC OR LIMIT DESIGN, Quarterly of Applied Mathematics, Vol. 10, pp. 157-165.

Fang, H. Y. and Hirst, T. J. (1970). APPLICATION OF PLASTICITY THEORY TO SLOPE STABILITY PROBLEMS, Highway Research Record No. 323, pp. 26-38.

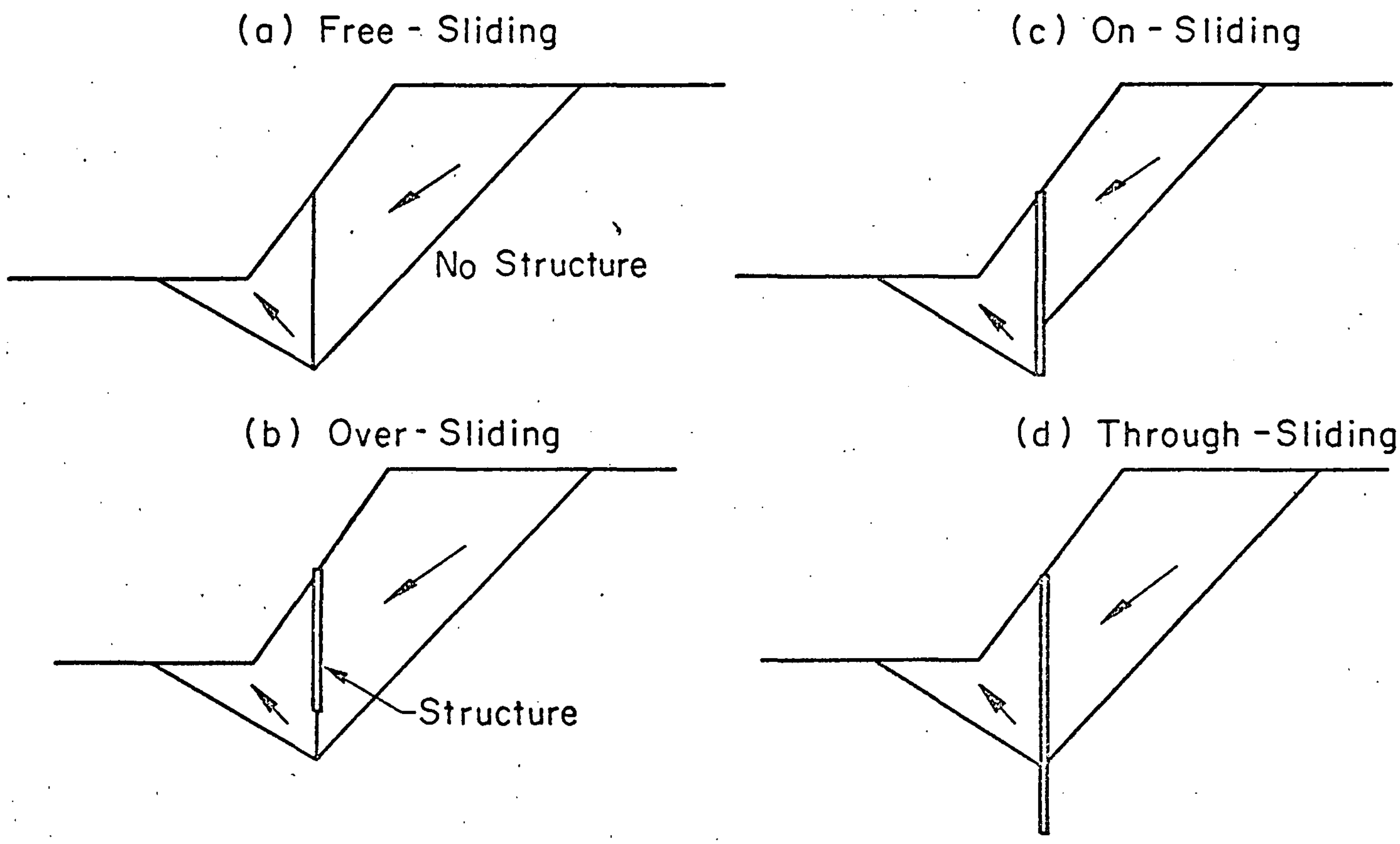

Fig. 1 Types of Mechanism.

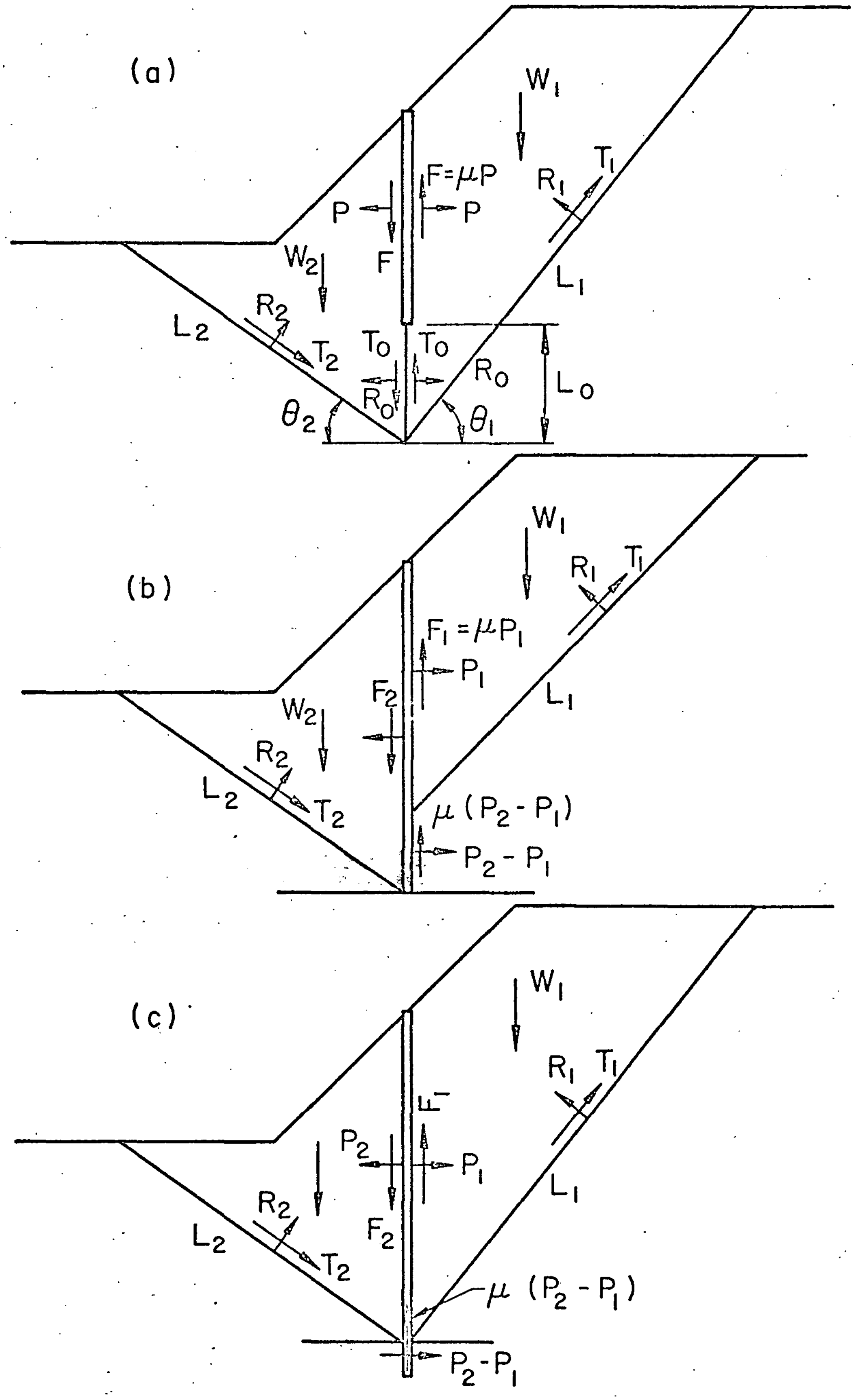

Fig. 2 Forces Acting on Slip-lines

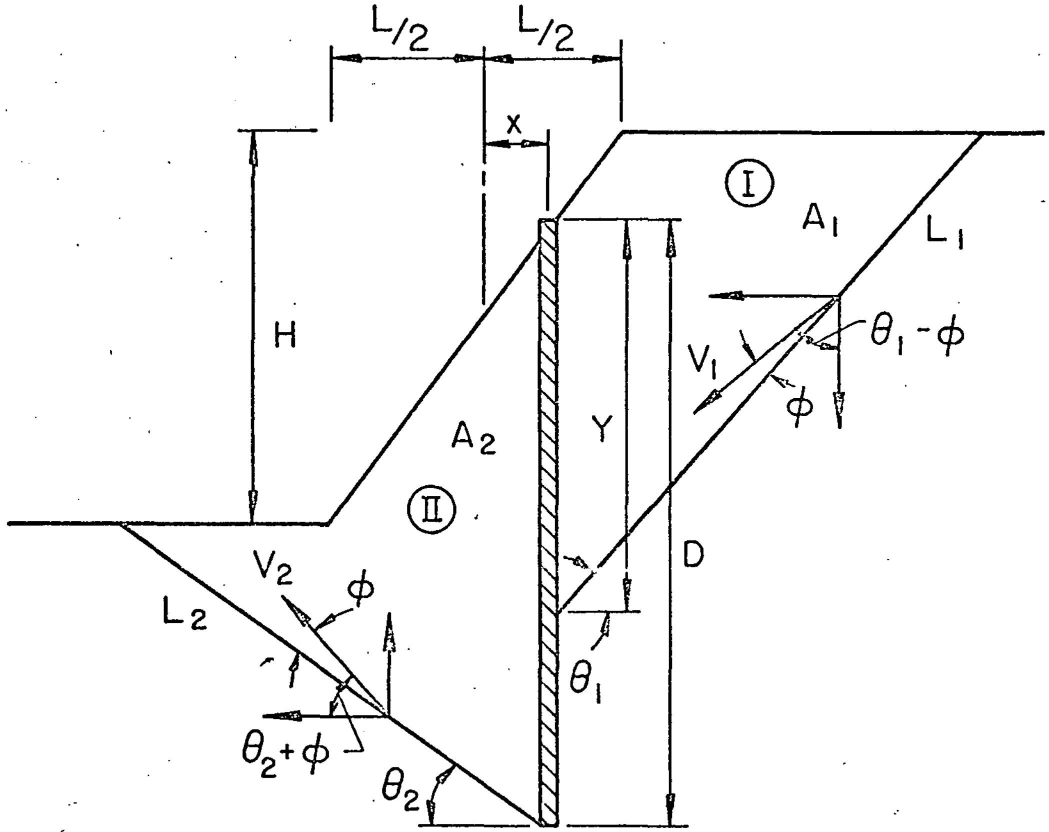

Fig. 3 Velocities of Rigid Bodies and Dimensional Parameters of Mechanism

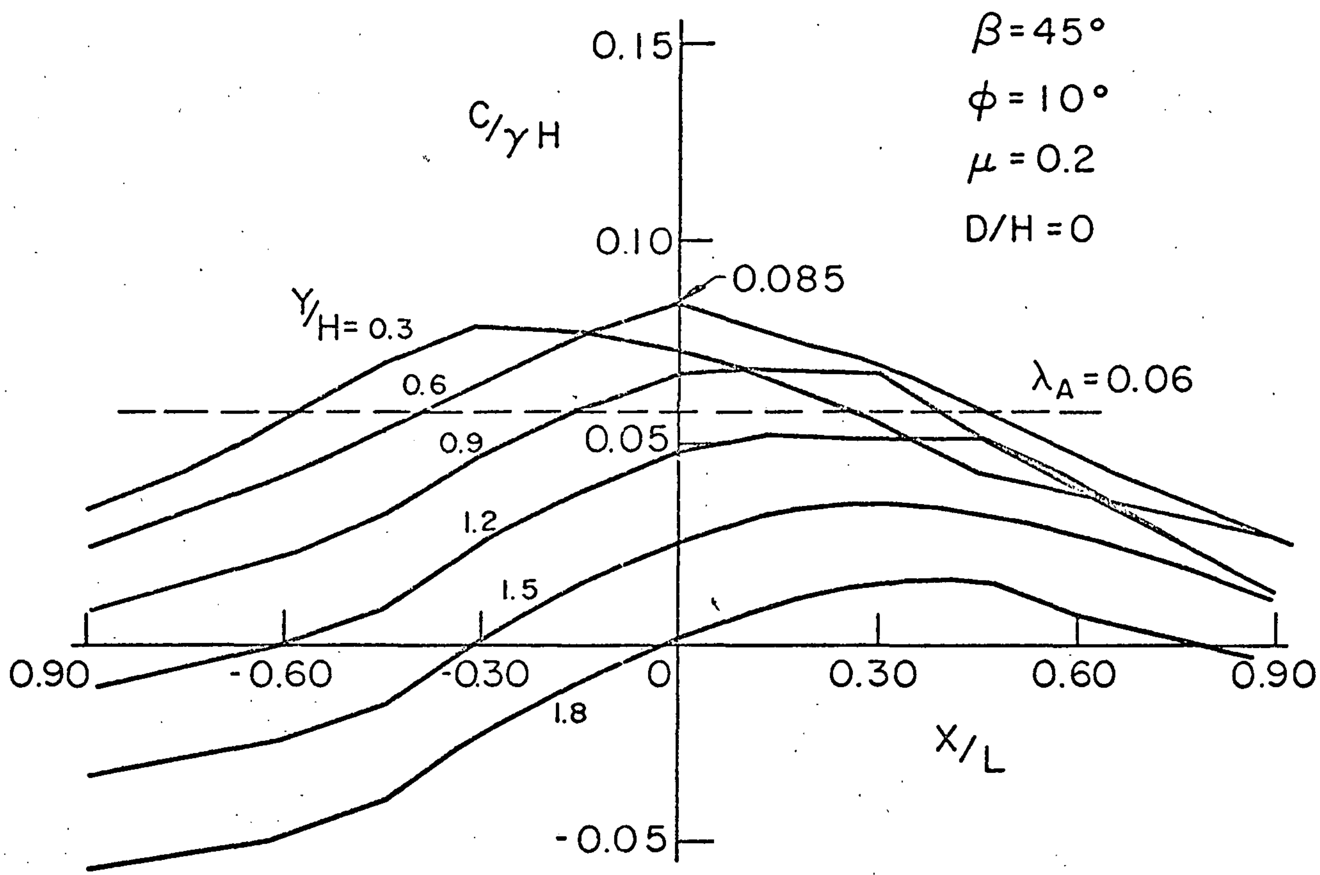

Fig. 4 Stability of Slopes with Free-sliding

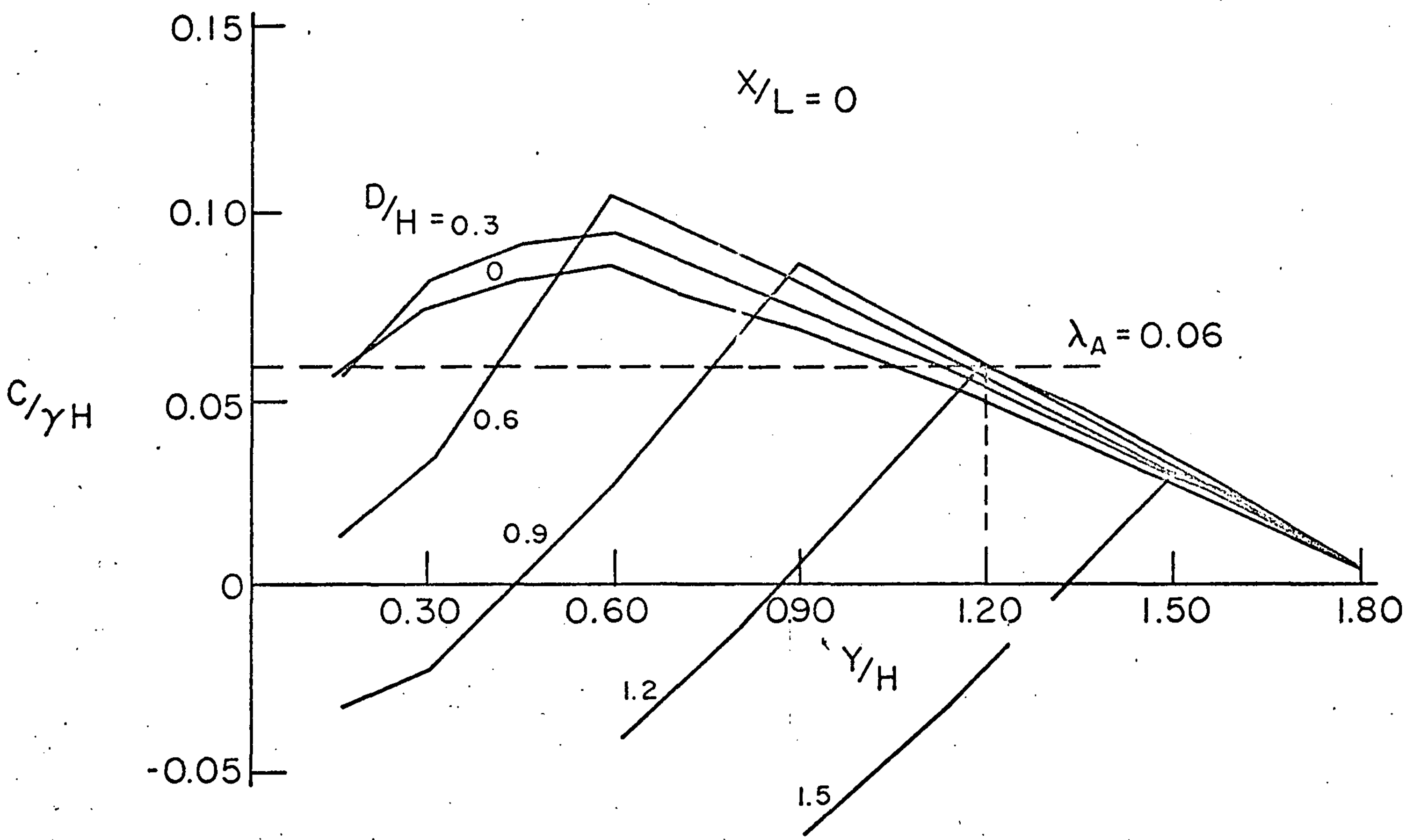

Fig. 5 Stability of Slope with On-(or Over) Sliding

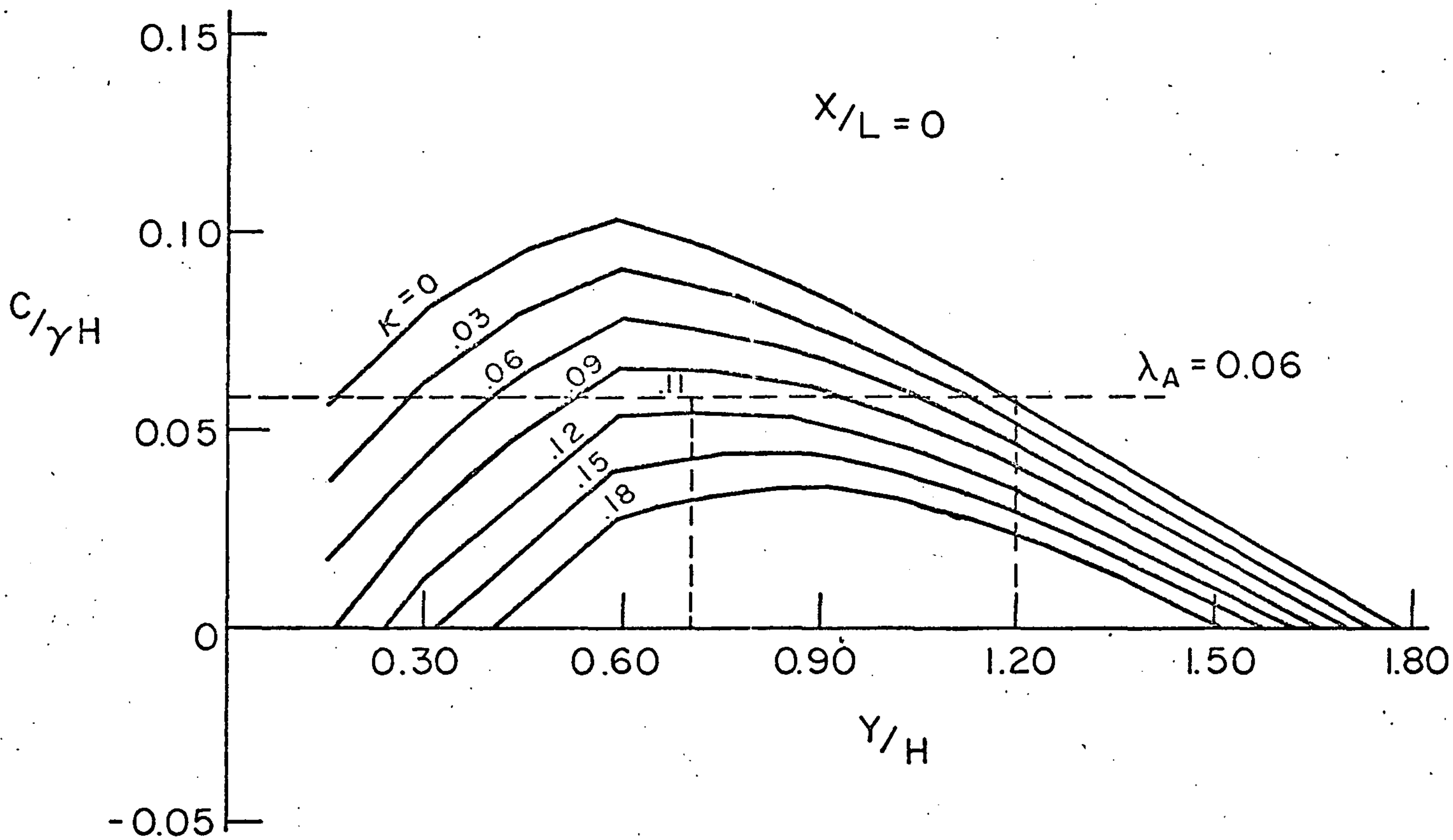

Fig. 6 Stability of Slope with Through Sliding