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Critical deposit velocities for low-concentration solid-liquid mixtures, M. S. thesis, 1971

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CRITICAL DEPOSIT VELOCITIES
FOR
LOW-CONCENTRATION SOLID-LIQUID MIXTURES

by

Millard P. Robinson, Jr.

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CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

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LIST OF SYMBOLS

a	correlation exponent, coefficient
A	cross-sectional area of the pipe
b	correlation exponent, coefficient
C	moving volumetric solids concentration
c	correlation exponent, coefficient
C_R	solids concentration in the "riser" pipe
C_D	solids concentration in the "downcomer" pipe
D	diameter of pipe (I.D.)
d	effective diameter of the sediment particles
d	correlation exponent
d_{50}	mean diameter of the sediment particles
d_{85}	solid's particle diameter (Sinclair)
d_{90}/d_{50}	non-uniformity coefficient of grain distribution
f	friction factor
f_m	mixture flow friction factor
f_ℓ	liquid flow friction factor
f, f_1, f_2, f_3	functions
F_L	modified Froude number (Durand)
f_s	function of correlation (Sinclair)
$F_r(I), \cdots F_r(IV)$	tested modified Froude numbers
g	gravitational acceleration
i_m or $\frac{\Delta h}{\Delta \ell}_m$	head loss of the total mixture
i_ℓ	head loss due to liquid component only
i_s	head loss due to solids component only
K_D	coefficient (Durand)

K	correlation parameter (Wilson)
k_1, k_2, k_3 k_3', k_4, k_4' k_5, k_5'	correlation coefficients
L	head loss length in both riser and downcomer pipes
Q_m	mixture flowrate
Q_s	volumetric solids delivery
Q_s'	optimum solids throughput
R_h	hydraulic radius
$\mathbb{R}e$	Reynolds number of the mean flow
s_s	specific gravity of solids
T^o	temperature
V	average velocity
V_C	critical deposit velocity
V_m	mixture velocity
v_{ss}	settling velocity of sediment
V_{max}	maximum limiting deposit velocity (Sinclair)
φ_D	dimensionless transport parameter (Durand)
ρ	carrying fluid density
ν	kinematic fluid viscosity
ε	pipe roughness
$\tan\theta$	pipe slope
ρ_s	solids particle density
Ψ_s	particle shapes; sphericity
Δh_R	head loss in the riser section
Δh_D	head loss in the downcomer section
Ψ_e	pressure gradient for mixture flowrate (Einstein)

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ABSTRACT

The present study deals with critical deposit velocity, "V_C", defined as the velocity at which particles begin to settle from the carrying medium and form a stationary (non-moving) deposit along the invert of the pipe. Newtonian suspensions of low solids concentrations ($C \leq 5\%$) are of particular interest, since the critical deposit velocity of low-concentration mixtures is presently not well defined.

An analysis of the significant parameters in this problem is presented and various forms of the modified Froude number are defined and tested. From a regression analysis of the experimental data, correlation of the tested parameters quantitatively defines the modified Froude number relationship.

Application of the Lehigh equations to some typical transport problems is examined and the economic advantages of such an application are discussed.

1. INTRODUCTION TO THE PROBLEM

The problem investigated in this study deals with an important aspect of solid-liquid transport technology in pipelines: The critical deposit velocity, "V_C". The critical deposit velocity in a closed conduit separates the "non-deposit" (deposit free) regime from the "deposit" regime. This velocity is sometimes also referred to as either the minimum transport velocity, the deposition velocity, or just the critical velocity.

The critical deposit velocity of low concentration mixtures ($C \leq 5\%$) is presently not well-defined, although it is sorely needed for application in pipeline design. Pressurized sewage collection lines, most often transporting low concentration loads, have been shown to be economically competitive with conventional means of sewage disposal but in need of additional design information. There exists an exhaustive list of Newtonian slurry transport applications, which can be found in the literature. Condolios et al. (1963) give the most thorough coverage, making readily apparent the economic advantages of pipeline transportation. Further, Shen et al. (1970), Robinson et al. (1971), and Graf (1971) report the most current state-of-the-art and economic significance of the critical deposit velocity determination.

There exist generally two prerequisites in properly designing a solid-liquid transport system: (1) Consideration of criteria that will ensure operation in a region of stability, and thus, provide for safe, uninterrupted transport of solids, and (2) Minimization of the power required to transport the solids, and optimization of system design parameters. The critical deposit velocity relates both of these requirements in designing a transport system which is both economic and safe to operate.

The present study continues the investigation of the critical deposit velocity problem through the use of a modified Froude number analysis. From a regression analysis of the Lehigh data, correlation of the tested parameters with different modified Froude numbers is evaluated, and equations quantifying the modified Froude number relationship are determined. The Lehigh data are subsequently compared with data reported in the literature. Application of the Lehigh equations to some typical transport problems is examined, and the economic advantages of such an application are discussed.

2. SOLIDS TRANSPORT IN PIPES

2.1 General Remarks on Solid-Liquid Mixture Flow

It is not within the scope of this paper to exhaustively present the general theory for flow of solid-liquid mixtures in pipelines. Shen et al. (1970a) and Graf (1971) have presented comprehensive surveys on the current state-of-the-art of sediment transport in pipes, and the interested reader is referred to these texts. However, some general comments are appropriate as an introduction to the critical deposit velocity problem.

Many fields of industry have become interested in the applicability of pipeline transport of solid materials along with a concern for the related problems of solid-liquid mixture flow. In all, transported solid-liquid mixtures may vary from suspensions in water of coal, sand, gravel, wood chips, chopped sugar cane, and ashes to slurries of sewage sludge, polymeric solutions, and concentrated suspensions. The economic advantages of hydraulic transport, the great variety of applications, and some concepts for designing a hydraulic transportation system are presented by Condolios et al. (1963a).

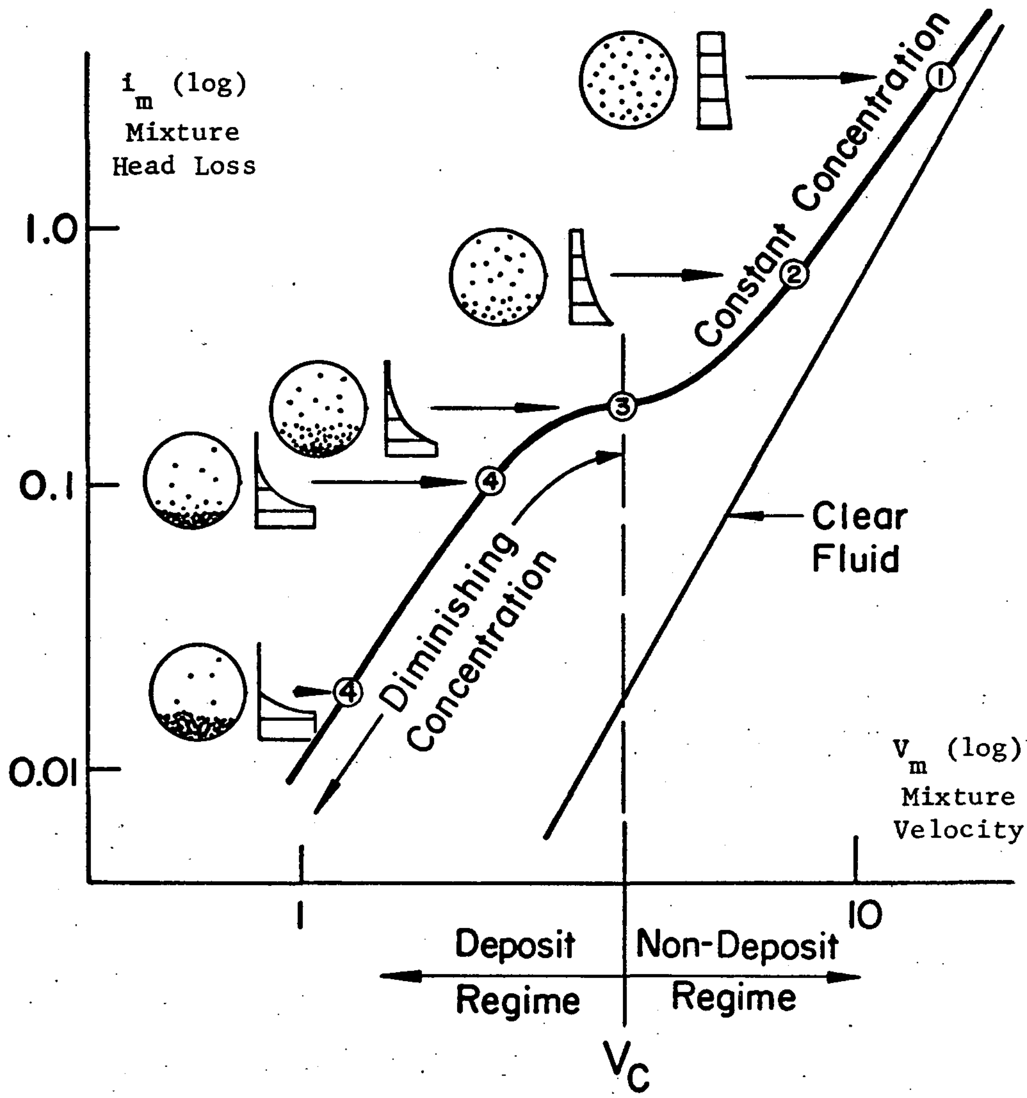
Solids suspensions are transported either as "Non-Settling" (homogeneous) mixtures or as "Settling" (heterogeneous) mixtures. The distinction between these two classifications has been presented by Durand (1953) and Govier et al. (1961). The present study is concerned with a "Settling" mixture, which exhibits Newtonian flow characteristics and is analyzed as a two-phase flow phenomenon. The suspension settling characteristics in a turbulent pipeflow are not discussed here, since the complex physics involved is beyond the scope of this study. Reference is made to Govier et al. (1961), Thomas (1962), Rose et al. (1969), or Carstens (1969,1971).

Regimes of Flow. The transport of "Settling" mixtures in pipes is qualitatively characterized by several different regimes of flow. Reference for an explanation of these different regimes is again made to Shen et al. (1970a) and Graf (1971).

The variety of flow regimes is diagramatically presented in Fig. 2.1, which is a typical curve of mixture head loss versus mixture velocity. An important distinction is made between the "Deposit" transport regime and the "Non-Deposit" transport regime. Within the non-deposit regime, several modes of transport prevail: 1 pseudo-homogeneous flow, 2a heterogeneous flow, and 2b heterogeneous flow with saltation. Flow in the deposit regime, 4, is described by bed and dune form irregularities. Separating the deposit and the non-deposit flow regimes, 3, is the transition region identified by the critical deposit velocity, "V_C".

The points of division between different flow regimes is somewhat arbitrary. Only a brief review of the flow regimes is presented herein.

Pseudo-homogeneous flow exists if suspensions of very fine particles, with fall velocities insignificant in relation to the fluid motion, are transported. Since homogeneity is not critically dependent



① -- PSEUDO-HOMOGENEOUS FLOW; concentration gradient is nearly uniform; suspended load transport

② -- HETEROGENEOUS FLOW; concentration gradient increases; transport by suspension and bed loads

③ -- TRANSITION REGION, "V_C"; beginning of bed formation; decrease in moving concentration

④ -- DEPOSIT REGIME FLOW; bed forms (plane and dunes); eventual clogging

Fig. 2.1: Regimes of Flow

on the flow conditions, f_m (mixture flow friction factor) = f_ℓ (liquid flow friction factor) may be assumed. Larger particle suspensions may behave similarly if transport velocities are extremely high. The pseudo-homogeneous flow regime is characterized by a nearly uniform vertical concentration gradient and a dimensionless transport parameter, φ_D (see Eq. ~~(2.1)~~), solely dependent on the relative density of the mixture. O'Brien et al. (1937) and Howard (1939) investigated flow of fine sand suspensions transported in this flow regime. Spells (1955) defines an "equivalent true fluid" with density equal to the two-phase mixture in the pseudo-homogeneous flow regime.

Heterogeneous flow occurs as the mixture flow velocity is decreased. Settling suspensions in this flow regime will exhibit a non-uniform concentration gradient and a noticeable increase in the mixture pressure gradient over the clear fluid head loss curve. Particles are transported both as bed load and suspended load now that the effect of gravity is felt by the solids. This regime of flow is normally shown to be the most important economically from the standpoint of total solids throughput. Wilson (1942) was one of the first investigators to present an expression for the total energy gradient for heterogeneous flow of mixtures. Durand (1953) and his co-workers at SOGREAH developed to date the most reliable theory of heterogeneous mixture flow transport.

Some investigators separate the heterogeneous flow regime into two: (1) transport of solids as suspended and bed loads, and (2) transport of solids mainly as bed load, sliding and saltating along the bottom of the pipe. Newitt et al. (1955) give the best account of the reasoning for this division. It should be noted here that the distinction between these two modes of heterogeneous flow is not to be mistaken as the separation between deposit and non-deposit regimes of flow or in no way related to the critical deposit velocity condition, as defined in this study.

The Deposit Regime of flow is entered as the sliding bed load of solid particles thickens and eventually becomes a non-moving bed on the invert of the pipe. The moving concentration diminishes, the clear flow area of the pipe decreases, and flow conditions are altered. The head loss component due to the solids is less effective, and the importance of flow-through geometry becomes a governing factor in head loss determination. Eventually, dunes will form as irregularities on the bed surface, and plugging flow becomes a serious concern. For the deposit regime of flow, two criteria may be employed. One is presented by Gibert (1960) as an adaption of the Durand-Condolios relationship for deposit flow conditions, and the other one is the transport-shear intensity relationship developed by Graf et al. (1968).

A Transition Region separates the deposit and non-deposit transport regimes. The head loss in this region flattens to a nearly constant value with further decrease in velocity; due to a complex deposit-scour feedback mechanism constantly altering the relative effects of the solid and liquid head loss components. The transition region is identified by a critical deposit velocity, "V_C", which is intricately dependent on fluid, solid, and flow parameters.

Investigation of the transition region flow conditions and the development of a relationship for quantitatively defining the critical deposit velocity has been the subject of many studies. Our task is to continue this effort.

Mixture Flow Head Loss. It has been always found seemingly appropriate to praise the technological advancements made through the efforts of investigators at the SOGREAH Laboratories in Grenoble, France, namely: Durand (1953), Gibert (1960), and Condolios et al. (1963a, b, & c). The solid-liquid flow theory developed at SOGREAH has been a long-standing criteria for determining mixture flow head loss of heterogeneous transport of solid suspensions through pipes. An early suggestion setforth by Blatch (1906), that the mixture head loss in a pipe is due to the clear flow head loss plus a head loss component due to the solids in transport, was further developed by Durand (1953) in defining a dimensionless transport parameter, φ_D:

$$\varphi_D = \frac{i_m - i_\ell}{C i_\ell} \qquad (2.1)$$

where i_m represents the total mixture head loss; i_ℓ the head loss due to just the liquid phase component; and C is the moving volumetric solids concentration. The excess pressure gradient in this case is often found to be proportional to the moving solids concentration.

The sediment transport parameter function is developed through a dimensional analysis, or:

$$\varphi_D = K_D \, f_1 (s_s - 1) \, f_2 \left(\frac{V^2}{gD}\right) f_3 \left(\frac{v_{ss}^2}{gd}\right) \tag{2.2}$$

where (s_s-1) represents the relative density of the mixture, and (V^2/gD) and (v_{ss}^2/gd) are, respectively, the flow and particle Froude numbers. The effect of both particle characteristics and flow parameters is evident, and the forms of K_D, f_1, f_2, f_3 are determined empirically from available data.

Further investigations of mixture flow theory and the associated economic implications were continued at SOGREAH. Later investigations have both praised and questioned the form of the so-called Durand-Condolios transport parameter, φ_D, but not one has yet touched on a better approach to the mixture flow problem.

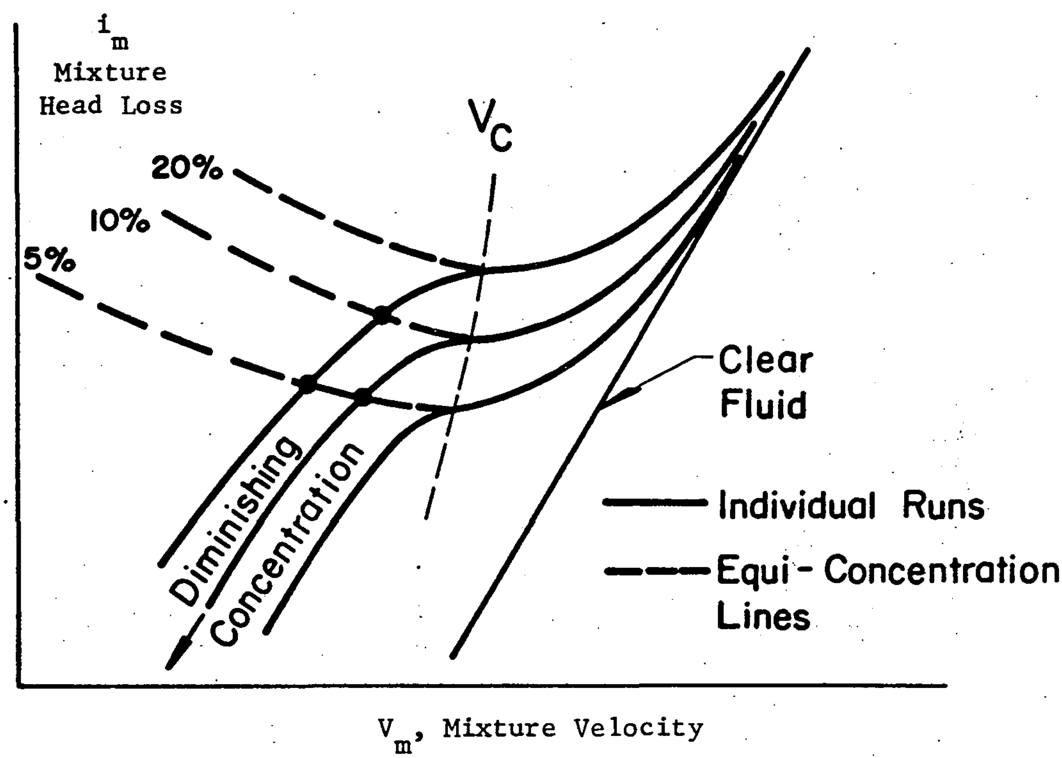


Fig. 2.2: Equi-Concentration Lines

The head loss plot of a typical mixture flow run from pseudo-homogeneous flow velocities down to deposit flow velocities was given in Fig. 2.1. Moving concentration decreases as flow enters the deposit regime. Determination of the minimum mixture head loss for a particular solids concentration flow is important in design. A rather typical plot of constant concentration lines is shown with Fig. 2.2. Note that the equi-concentration lines below the critical condition can only be plotted by connecting the points of the same moving concentrations from runs with different initial concentrations. Along these equi-concentration lines, the mixture head loss is seen to again increase in the deposit regime. The V_C dashed line shows the variation of critical velocity with change in solids concentration.

2.2 The Critical Deposit Velocity, "V_C"

2.2.1 Definition and Significance

The transition between deposit and non-deposit flow regimes is identified by a "critical condition". In the present investigation, "critical condition" is taken as the velocity at which particles being to settle from the flowing medium and form a stationary (non-moving) deposit along the invert of the pipe; this will be called the critical deposit velocity, "V_C".

At the "critical condition" a deposit-scour feedback mechanism transports solid particles in the form of a pulsating bed. Figure 2.3 shows typical bed motion at critical deposit velocity for plastic

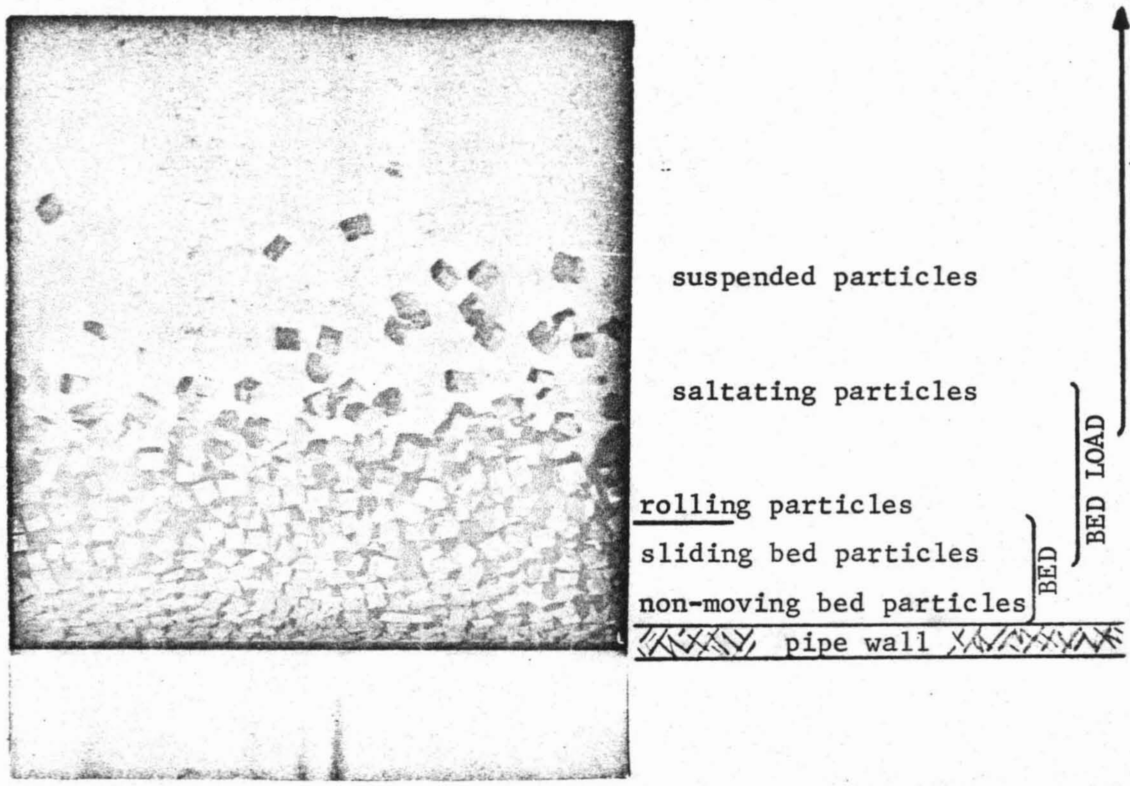


Fig. 2.3: Bed Motion of Plastic Pellets in a 6-inch Pipe at the Critical Deposit Velocity

pellets transported in a 6-inch pipe. Close to the pipe wall, the solid particles are stationary. When this condition is observed, the critical deposit velocity is recorded. Above this layer of stationary particles, the remainder of the bed is sliding. Other particles shove, roll, and saltate over the moving bed surface, and some will become completely suspended farther from the wall. The deposit of solids on the bottom of a pipe is a random phenomenon varying with local fluctuations of solid and liquid parameters. Within the same pump-pipe facility, duplication of results is not easily attainable.

The critical deposit velocity is sometimes referred to as the limit deposit velocity, by Durand (1953) and Sinclair (1962), the sediment limiting velocity, by Gibert (1960), the minimum transport velocity, by Rose et al. (1968), or the deposition velocity, by Wasp et al. (1970). It is imperative that a clearly defined "critical condition" becomes a primary concern in every solid-liquid transport investigation.

When using data from other "critical condition" studies, one must be cautious of the following: (1) Some investigators, such as, Blatch (1906), Wilson (1942), Bruce et al. (1952), Thomas (1962), Charles (1970), and Shen et al. (1970b), define a minimum or economic velocity which corresponds to the minimum head loss required for transporting a certain concentration of solids. Use of this criterion is in accordance with how one wishes to define "critical condition". It was found in the present and in other investigations that the critical deposit velocity is not in direct relationship with the minimum head loss criterion. Implementation of the assumption that these two criteria are identical is good only for preliminary evaluation. (2) The critical deposit velocity, approached from the non-deposit regime, is most often different from the critical scour velocity. To scour a deposited bed requires usually a greater shear force, thus a higher flow velocity, than when the same bed is deposited. (3) Some studies define a transition velocity between saltating and sliding bed load transport, which is at times mistaken for the critical deposit velocity.

The critical deposit velocity is an important design criterion both for safe operation and for system economics, but it is often vaguely defined in reports of solid-liquid transport research. Due to a lack of good definition and reproduceability of results, it is suggested that a conservative critical deposit velocity be used [see also Bonnington (1961)].

2.2.2 Previous Investigations

Interest in the "critical condition" of solid-liquid transport in pipes was initiated by Blatch (1906) and continued by O'Brien et al. (1937), Howard (1939), and others. However, Wilson (1942) developed the first relationship which quantitatively dealt with parameters related to the "critical condition". As a first approximation, the total energy gradient, i_m, consists of a liquid component, i_ℓ, and a solids component, i_s, or:

$$i_m = i_\ell + i_s \tag{2.3}$$

Wilson (1942) defined both terms and obtained the following:

$$i_m = f \frac{1}{D} \frac{V^2}{2g} + K C \left(\frac{v_{ss}}{V} \right) \tag{2.4}$$

where the terms on the right represent, respectively, a liquid head loss gradient derived from the Darcy-Weisbach equation, and a head loss gradient due to the solids dependent on solids concentration, C, particle settling velocity, v_{ss}, an average velocity, V, and correlation parameter, K.

Differentiating i_m with respect to V and minimizing, the resulting "critical condition" is given as:

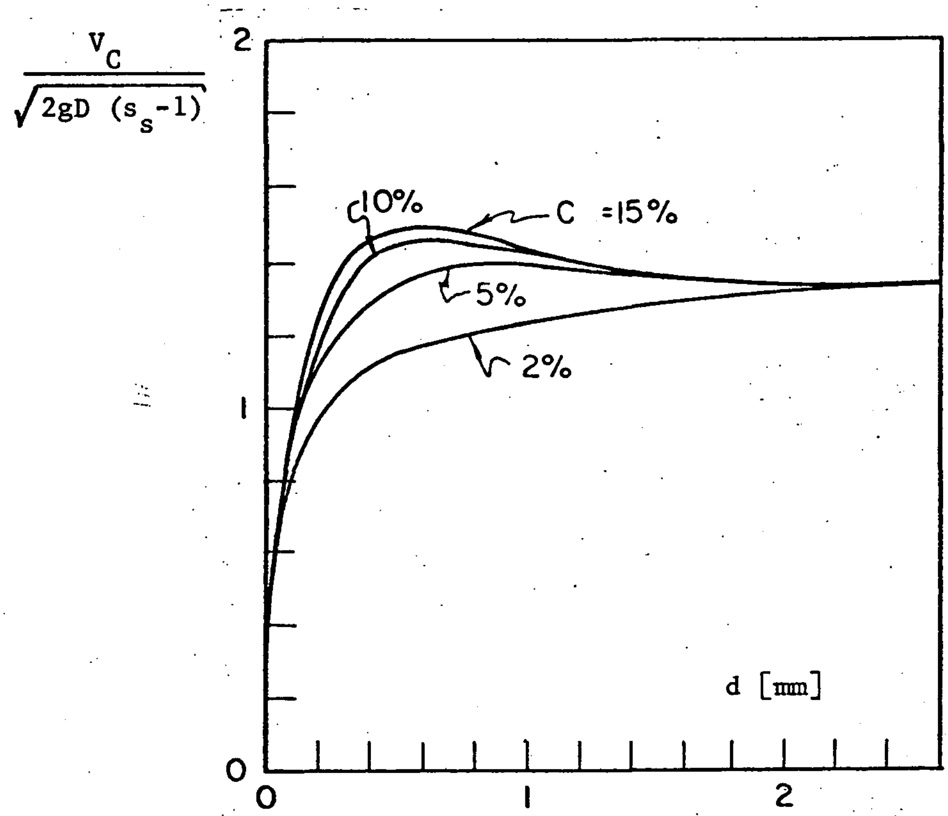
$$V_C = \sqrt[3]{\frac{K\,C\,v_{ss}\,g\,D}{f}} \qquad (2.5)$$

It should be noted that the flow velocity, V_C, at "critical condition" is defined here for minimum energy gradients. Nevertheless, the relationship given with Eq. (2.5) relates parameters which are of importance in the critical deposit velocity problem. These parameters are: C, the solids concentration; v_{ss}, the particle settling velocity; D, the pipe diameter; and f, the friction factor indicating flow resistance.

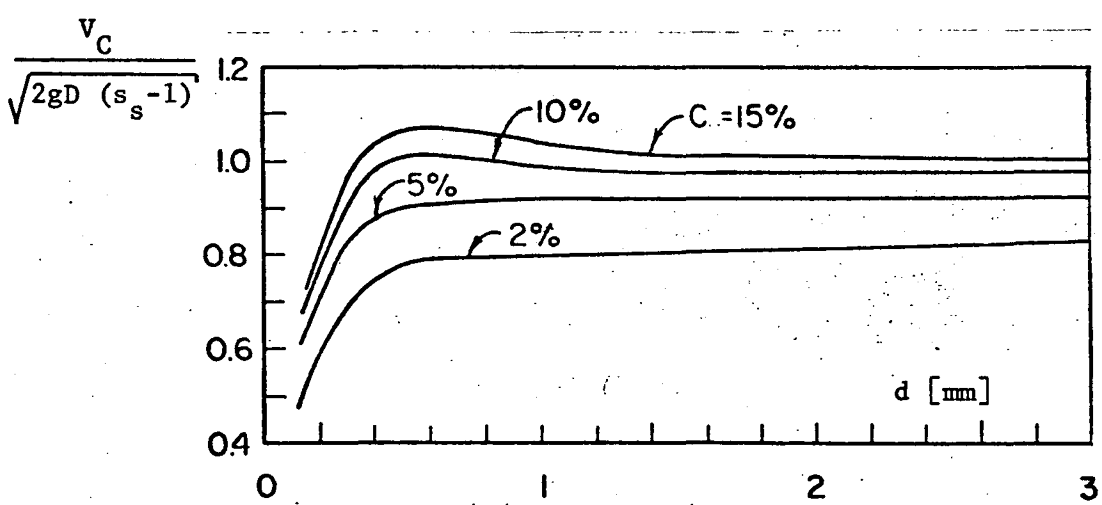
Durand (1953) used as the lower limit of his heterogeneous flow relationship an equation defining the limit deposit velocity, V_C, of sand mixtures which separates the zones of the regimes with and without deposit on the pipe bottom, or:

$$V_C = F_L\sqrt{2gD\,(s_s-1)} \qquad (2.6)$$

The parameter, F_L, known as a modified Froude number, varies with solids concentration, C, and particle diameter, d. This is given with Fig. 2.4a for uniformly graded material. Later, Durand et al. (1956) report findings for non-uniform material, which is shown with Fig. 2.4b. An appreciable difference is noted between Figs. 2.4a and 2.4b, and it becomes questionable that these discrepancies are accounted for solely



(a) Uniformly Graded Material
[after Durand (1953)]



(b) Non-Uniform Material
[after Durand and Condolios (1956)]

Fig. 2.4: The Modified Froude Number, F_L, versus Solids Concentration and Particle Diameter

by the difference in material distributions. Unfortunately, neither Durand et al. (1956) nor any of the later publications of the SOGREAH staff explain this difference.

Gibert (1960) reported on and analyzed the extensive SOGREAH data to obtain best-fit curves for Froude number, $V_C/\sqrt{gD}$, plotted against solids concentration, C. Subsequent to the study of Gibert (1960)*, Graf et al. (1970) included the effect of relative density, given by $\sqrt{2(s_s-1)}$, - as was similarly done by Durand (1953) - and Gibert's best-fit curves were replotted and are given with Fig. 2.5. This figure shows the general trend of results to be remarkably invariant for sand and gravel of particle sizes $d \geq 0.37$ mm. The curve for this larger material can be thought of as being a maximum envelope of F_L-values. For finer materials, in the range of d = 0.20 mm and less, there are distinctive variations in the curves. Condolios et al. (1963b) report a figure similar to Fig. 2.5 but only include an envelope curve for graded and mixed sands of $d \geq 0.44$ mm. Figure 2.6 is a replot of Fig. 2.5. It should be noted that Fig. 2.6 conforms closely to the non-uniform material results reported by Durand et al. (1956) in Fig. 2.4b. It is expected (!) that both Gibert (1960) and Durand et al. (1956) used the same set of SOGREAH data. Furthermore, it is believed that Figs. 2.4b and 2.6 supersede Fig. 2.4a; the latter is a result of earlier SOGREAH studies.

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*Translation and evaluation of Gibert (1960) was undertaken by Öner Yücel, Lehigh University.

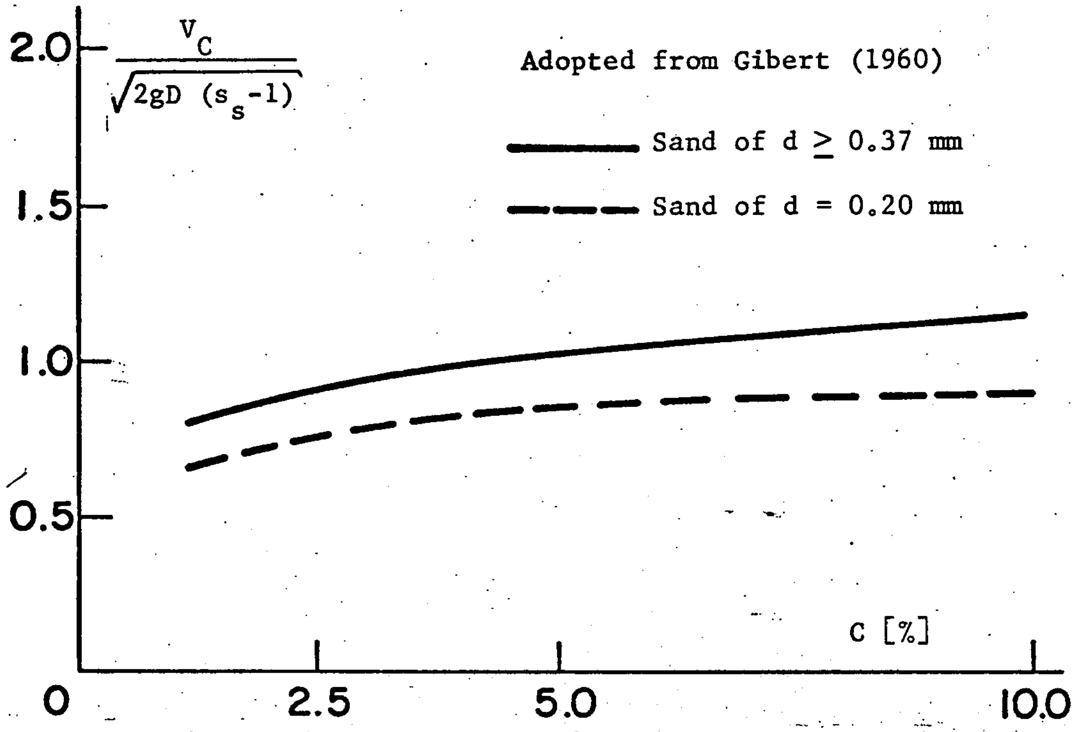

Fig. 2.5: Modified Froude Number versus Concentration; Particle Diameter as Parameter

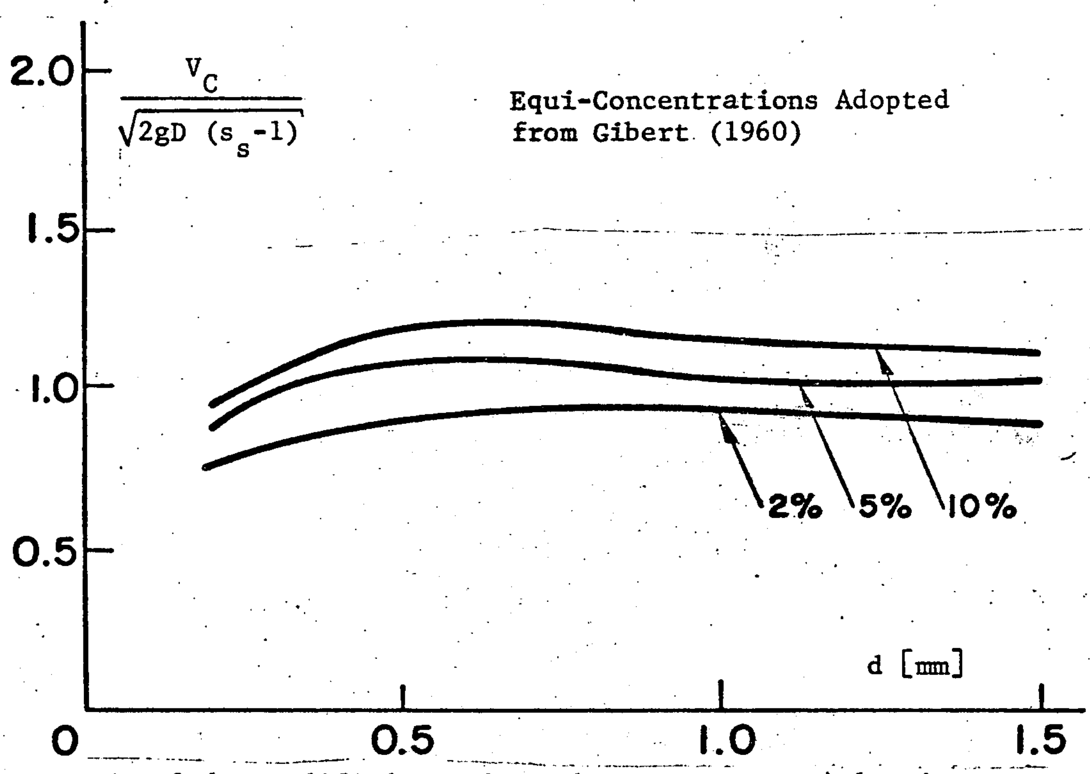

Fig. 2.6: Modified Froude Number versus Particle Diameter; Concentration as Parameter

General agreement with the relation, as defined in Eq. (2.6) and plotted in Figs. 2.6 and 2.4b, are found throughout the literature. Figure 2.4b is recommended by Graf (1971).

Gibert (1960) also discussed a theoretical approach to the critical deposit velocity problem, considering the "critical conditions" of flow in a conduit irregardless of flow-through geometry, to be related through the Froude Law of similitude. A discussion of Gibert's analysis is found in Robinson et al. (1971).

Sinclair (1962) conducted tests on sand-water, iron-kerosene, and coal-water mixtures at concentrations up to 20% flowing in 0.5-inch, 0.75-inch, and 1.00-inch pipe. Through a dimensional analysis of the variables expected to significantly influence the critical deposit velocity, Sinclair (1962) arrives at an equation, such as:

$$\frac{V_{max}}{\sqrt{gd_{85}\ (s_s-1)^{0.8}}} = f_s\left[\frac{d_{85}}{D}\right] \qquad (2.7)$$

where the modified Froude number is expressed with a solid's particle diameter, d_{85}. He observed that the critical deposit velocity reaches a maximum between 5 and 20% solids concentration, so that the effect of concentration could be eliminated by using V_{max} instead of V_C. Sinclair (1962) wrote Eq. (2.7), for d > 1.5 mm (when C does not enter the problem), as:

$$\frac{V_{max}}{\sqrt{2gD\ (s_s-1)^{0.8}}} = 1.30 \qquad (2.8)$$

This may be compared with Durand's results, similarly expressed by:

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}} \cong 1.32 \qquad (2.9)$$

For smaller particle sizes, Sinclair (1962) examines the relevance of boundary layer theory to the problem, and suggests that particle diameter, d_{85}, takes precedent over the pipe diameter, D, in their relative influence on the modified Froude number. It is within this smaller range of particle sizes that the present study is conducted.

Shen et al. (1970b) and others attempt to correlate critical deposit velocity with other important parameters in the form:

$$V_C = k_1\ d^a\ D^b\ C^c\ (s_s-1)^d \qquad (2.10)$$

The exponents, a, b, c, and d, and particularly the coefficient k_1, vary greatly, as could be expected, from one study to the next. The form of this function is questioned because of its inhomogeneity and is to be used only with extreme caution in data correlation.

Flow and particle Reynolds numbers have been investigated for their applicability as criterion in the critical deposit velocity problem. Spells (1955), Charles (1970), and studies by Cairns et al., as reported by Sinclair (1962), correlate the Reynolds number with a modified Froude number relationship. Correlation in these studies, however, is related to the minimum energy gradient criterion.

A modified Froude number relationship apparently presents a rather good criterion for evaluation of solid-liquid mixture flow through pipes. Its relationship to other parameters significant in the critical deposit velocity problem will be re-examined in the present study, and experimental findings checked against the SOGREAH data.

2.2.3 A Modified Froude Number Analysis

When transporting a solid-liquid mixture through a closed conduit, one may expect the following variables to be of importance:

(1) Flow Parameters -

V, mixture flow velocity
g, gravitational acceleration
v_{ss}, particle settling velocity

(2) Fluid Parameters -

ρ, carrying fluid density
ν, kinematic fluid viscosity

(3) Pipe Parameters -

D, pipe diameter
ϵ, pipe roughness
$\tan\theta$, pipe slope

(4) Sediment Parameters -

ρ_s, solids particle density
d, mean particle diameter
Ψ_s, particle shape; sphericity
$\left(\frac{d_{90}}{d_{50}}\right)$, non-uniformity coefficient of grain distribution
C, moving volumetric solids concentration

Proper grouping of variables into dimensionless parameters was reported in Graf et al. (1970) and is re-examined here:

$$f\left[\frac{V}{\sqrt{gD}},\ (s_s-1),\ \frac{VD}{\nu},\ \frac{d}{D\Psi_s},\ \frac{\epsilon}{D},\ \tan\theta,\ \frac{d_{90}}{d_{50}},\ C\right] = 0 \quad (2.11)$$

The relative density, (s_s-1), comes from $(\rho_s-\rho)/\rho$ where $s_s = \rho_s/\rho$.

It is expected that the flow Reynolds number, VD/ν, does not play a significant role in this problem, and it is omitted from the analysis without loss of generality. The mixture flow velocity, V, and pipe diameter, D, are accounted for by the remaining parameters in the relation, Eq. (2.11). The kinematic viscosity, ν, which depends on temperature, for all practical purposes varies insignificantly. Further, a Reynolds number near the critical deposit velocity is very unstable, because the flow-through geometry, $D = 4R_h$, varies continuously with fluctuating solids concentration, along with changing clear flow-through velocity.

Replacing the general flow velocity, V, with the critical deposit velocity, V_C, and considering the particle shape factor to be unity for natural quartz grains or already included in the adjustment of non-spherical particle sizes, Eq. (2.11) is rearranged and given by:

$$f\left[\frac{V_C}{\sqrt{2gD\,(s_s-1)}},\ \frac{d}{D},\ \frac{\varepsilon}{D},\ \tan\theta,\ \frac{d_{90}}{d_{50}},\ C\right] = 0 \qquad (2.12)$$

Note that the flow Froude number, $V/\sqrt{gD}$, and the relative density, (s_s-1), both given in Eq. (2.11), were combined in a densimetric or <u>modified Froude number</u>, $V_C/\sqrt{2gD\,(s_s-1)}$. Equation (2.12) is somewhat similar to relations proposed by Durand (1953), Sinclair (1962), and Barr et al. (1968).

For a certain relative pipe material roughness, ϵ/D, and solids grain size distribution, d_{90}/d_{50}, the applicability of Eq. (2.12) will be tested in the form of:

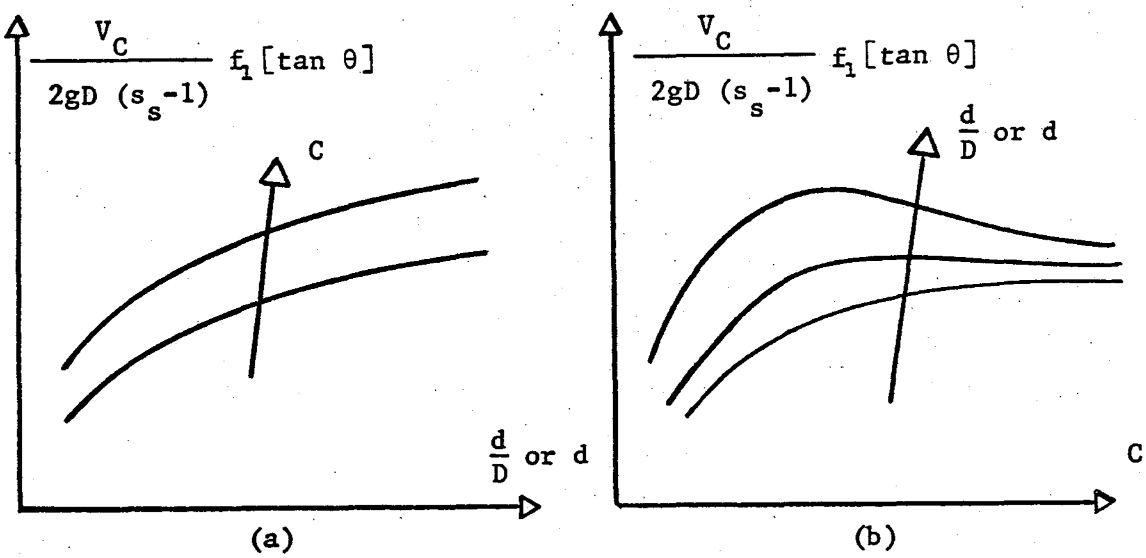

Fig. 2.7: Plot of Equation (2.13); the Modified Froude Number Relationship

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ f_1\ [\tan\theta] = f_2\left[\frac{d}{D}, C\right] \tag{2.13}$$

Equation (2.13) is displayed on plots such as given in Figs. (2.7a) and (2.7b). The effect of pipe slope, $\tan\theta$, is not a major concern in this study. The left side of Eq. (2.13) will absorb the $\tan\theta$ argument, and the best trigonometric relationship will be determined after fitting data against both:

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta]$$

and,

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ \frac{1}{\sqrt{[1 + \tan\theta]}}$$

The left side of Eq. (2.13) is a modified Froude number. The form of this parameter, raising both D and (s_s-1) to the 1/2 power, has been tested and shown to be a reliable criterion.

It is felt that without loss of generality, it may become frequently important to replace the relative particle to pipe diameter, d/D, by the particle diameter, d, itself. In this instance, the significance of D is considered to be wholly described in the Froude number. Sinclair (1962) remarks that when the particle is such a size that it is wholly immersed in the region where viscous forces predominate, as our sand particles are, d/D does not enter the correlation.

Investigators, like Bruce et al. (1952), Govier et al. (1961), Thomas (1962), and Rose et al. (1969), consider slip between the solid and liquid phases, v_{ss}/V or V_s/V (referred to as "hold-up"), to be a parameter of major importance. This concept requires a thorough treatment of particle dynamics, beyond the scope of the present study. It is therefore considered that near the critical deposit velocity, particles have already settled into a sliding bed; consequently, only the size and moving concentration of particles are significant.

In the subsequent discussion, data will be presented and compared in the way suggested with Fig. 2.7a.

3. LEHIGH EXPERIMENTS

3.1 Facilities

A three-story, pressurized and self contained solid-liquid transport system was constructed, modified from an open-tank recirculating system. The frequent use of victaulic couplings hastened erection and provided flexibility throughout the pipe system.

The experimental facility consists of: (1) a vari-drive motor-pump assemblage, (2) an adequately flexible pipeline arrangement, (3) a sediment feed and removal system, and (4) the necessary measuring and regulatory devices. Figure 3.1 schematically illustrates the general scale of the overall system. Detailed features of the sediment handling equipment are provided in Fig. 3.4.

Vari-Drive Motor-Pump. The hydraulic horsepower was supplied from a vari-drive motor-pump assemblage, functioning as the heart of the system. The pump, furnished by Ellicott, is a single suction centrifugal type with cast bronze casing and impeller. The suction pipe is 5-1/2 inch I.D., discharge pipe is 4-1/2 inch I.D., and the impeller diameter is 13-5/8 inch O.D. During the operation of the pump, cooling water is added continuously to the seal on the motor side of the pump, also providing a lubricating interface.

The drive unit is a Westinghouse - 3 phase 60 cycle 125 Hp - "Magna Flow" motor and is regulated by a vari-drive control. The driving unit is of the integral type, is water cooled, and has an adjustable speed range from 100 to 2153 rpm. Along with the motor,

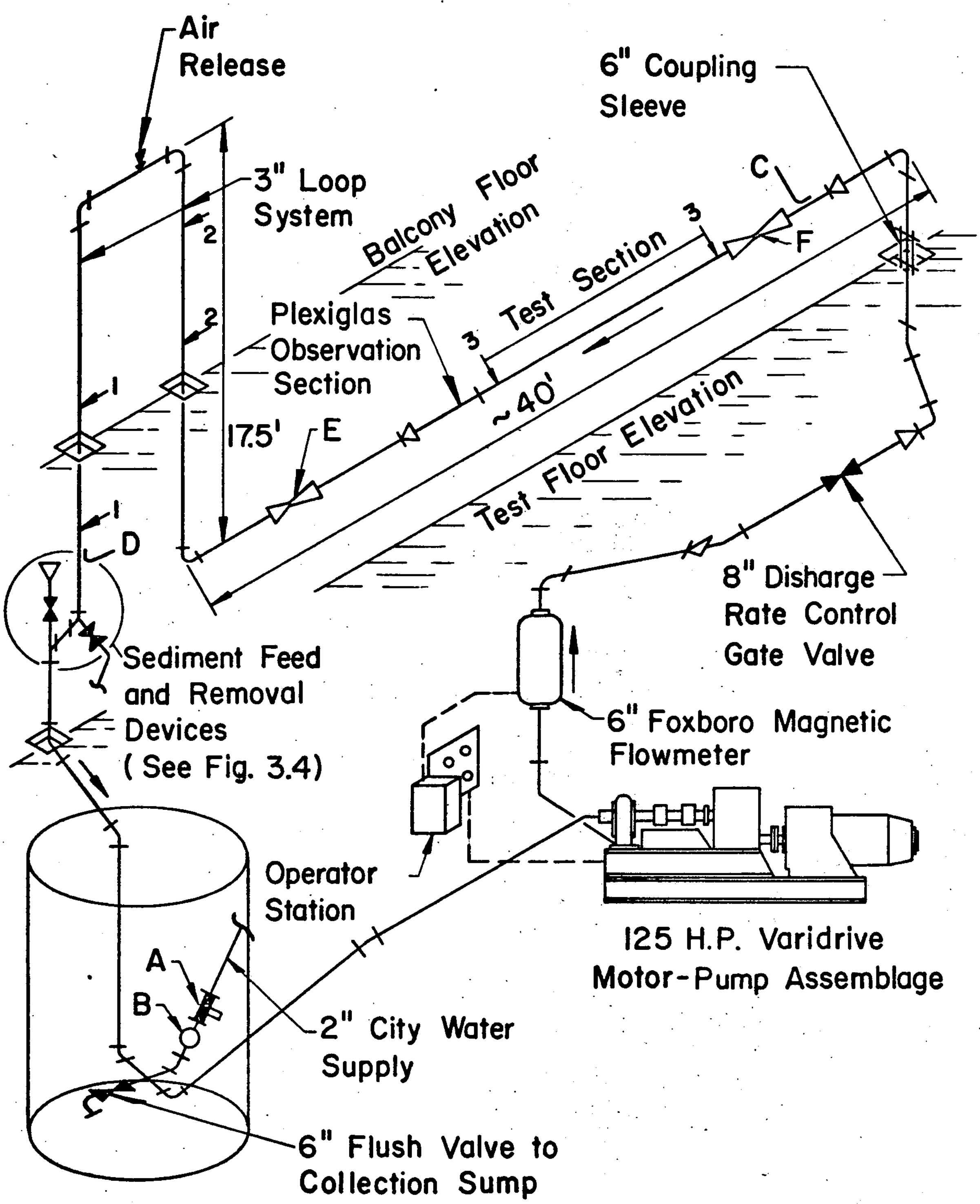

Fig. 3.1: Solid-Liquid Transport Test System

there is an operator's station, excitation unit, and a type 5L Autostarter. The entire system operates on 208 volts AC.

The pump and vari-drive motor assembly survived 18 months of testing. Pumping efficiency and impeller capacity were not noticeably altered throughout the testing period. Sand mixtures presented no pumping difficulty, however, the 3.63 mm diameter plastic pellets were extruded apparently along the surface between the impeller and encasing seal. Resulting conglomerations of plastic strands within the pump would put a strain on the motor at low flowrates, causing sudden velocity fluctuations. This complication is explained further in Section 3.3.2.

Pipeline. From the pump, mixture flow is discharged through a 6-inch Foxboro Magnetic Flowmeter leading to a horizontal reach of 8-inch pipe. An 8-inch gate valve regulates pump discharge below flowrates of 200 gpm. Often times the partially closed valve would cause difficulty in establishing stable flow conditions when critical flowrates occurred in this lower flow range. The solid-liquid mixture is then lifted to the test-floor elevation in 6-inch pipe.

Along the test length of approximately 40 ft, measurements are obtained, pipe slope is adjustable, and mixture flow phenomena are visually observed. A 4-inch pipe was installed together with its Plexiglas observation section; subsequently, a 6-inch pipe and Plexiglas section were installed. A strobotac set at a high frequency response aided the observation of solids flowing through the Plexiglas section, such that an accurate description of flow regime was obtainable. For example, Fig. 3.3 pictures the progressive dune transport of sand particles in the deposit regime, as seen through the 6-inch observation section. Both pipe sizes and slopes were altered throughout the testing program in accordance with the investigation of variable parameter affects. Figure 3.2 shows the horizontal 4-inch diameter pipe setup.

A "Loop System" follows which is employed as a device for simultaneously measuring mixture flowrate and solids concentration. Located atop the balcony-floor elevation between the 3-inch vertical pipe sections, commonly referred to as the "Riser" and "Downcomer", is the main air-release for the system.

The flow, upon leaving the "Loop System", bypasses a closed 3-inch sediment flush valve and enters a 6-inch vertical pipe, where sediment is gravitationally fed when an increase in concentration is desired. Flow continues downward to where a 6-inch gate valve empties the system and a 2-inch pipeline connects the city water supply. The system pressure was maintained and water supply assured through use of a constant pressure control valve (A in Fig. 3.1) set at 20 psi on the 2-inch supply line. A 2-inch check valve (B in Fig. 3.1) prevented backflow to the city supply under excessive system pressures.

The circuit is completed with 5-1/2 inch pipe leading to the suction side of the pump.

The pipeline, secured both laterally and from hanging steel supports, could safely handle flowrates up to 1000 gpm. Wear on the

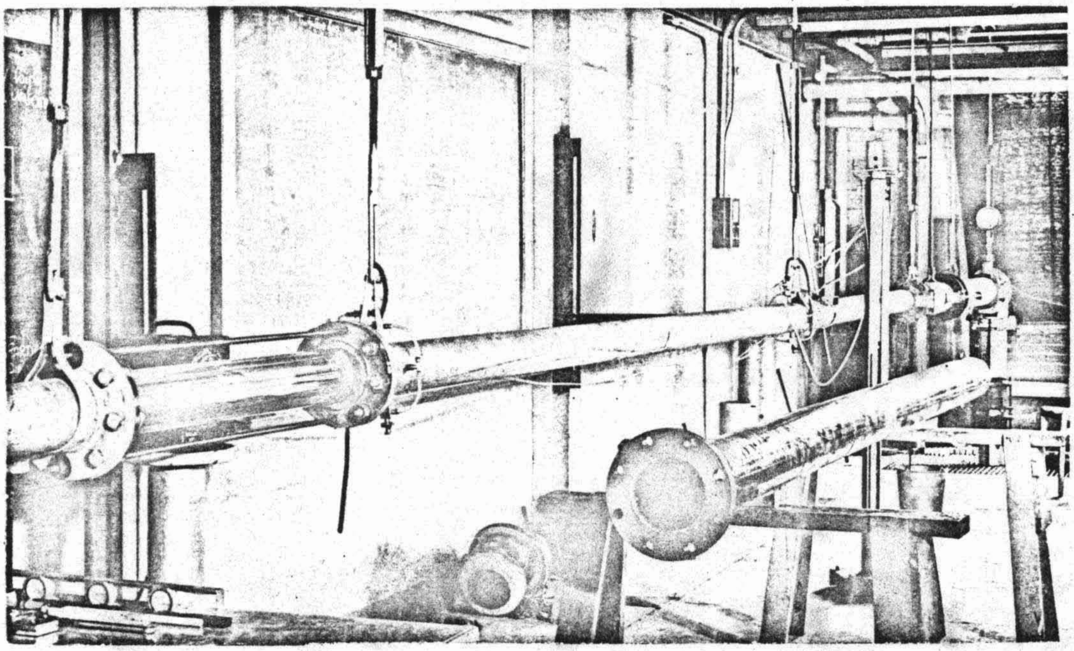

Fig. 3.2: Setup for Tests in a Horizontal 4-inch Diameter Galvanized Pipe

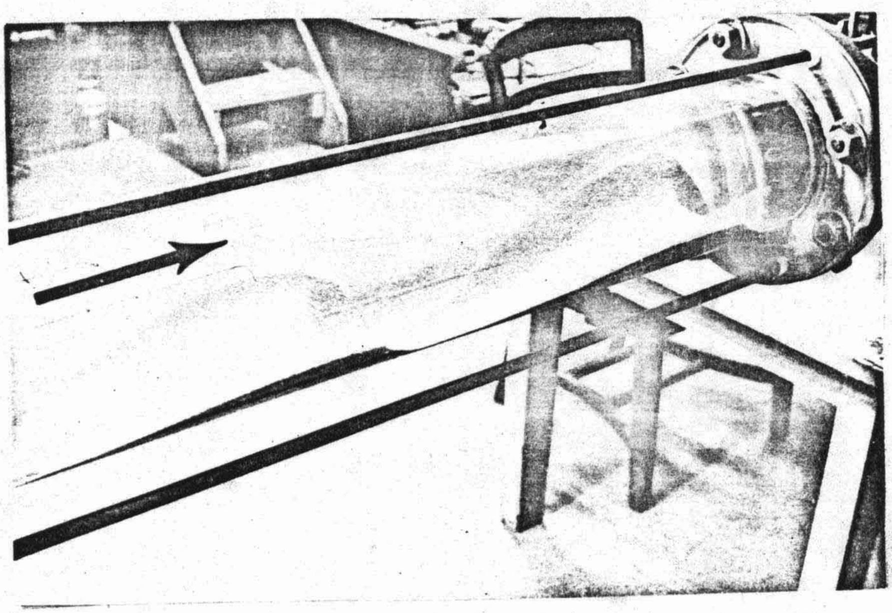

Fig. 3.3: Low Flow Dune Transport of Coarse Sand Particles in the Deposit Regime

inside pipe finish was apparent, however, not of serious consequence. Due to old pipe sections, iron oxide coloration eventually became a persistant recurrence causing only some difficulty in flow visualization. The system water was flushed clean when flowrates were lowered to a range ensuring no sediment transport. Transitions were attacked by the sand, but the use of tee fittings in the critical location of 90° elbows saved the necessity of replacement. The most persistent problem was caused by sand particles jamming the gate valves. Other valves on the market would have gauranteed greater success.

Pipe lengths and fittings were supplied by the Bethlehem division of Hajoca Corporation, and the Fritz Laboratory machine shop handled material alterations.

Sediment Feed and Removal System. The sediment feeding apparatus underwent several adaptions, until the technique, as explained here and illustrated in Fig. 3.4, was successfully applied. Supply valve 2 and overflow valve 3 are opened as the mixing chamber, isolated from the system by the closed mixing valve 1, is filled with solids material. Water is displaced through the overflow line as the mixing chamber is filled. Valves 2 and 3 are then closed and valve 1 is opened, fluidizing the solids and gradually feeding the particles into the flowing medium.

Also illustrated in Fig. 3.4 is a sediment removal facility (employed as a time-saving technique) for removing the solids or undesirable foreign material from the system and preventing discharge

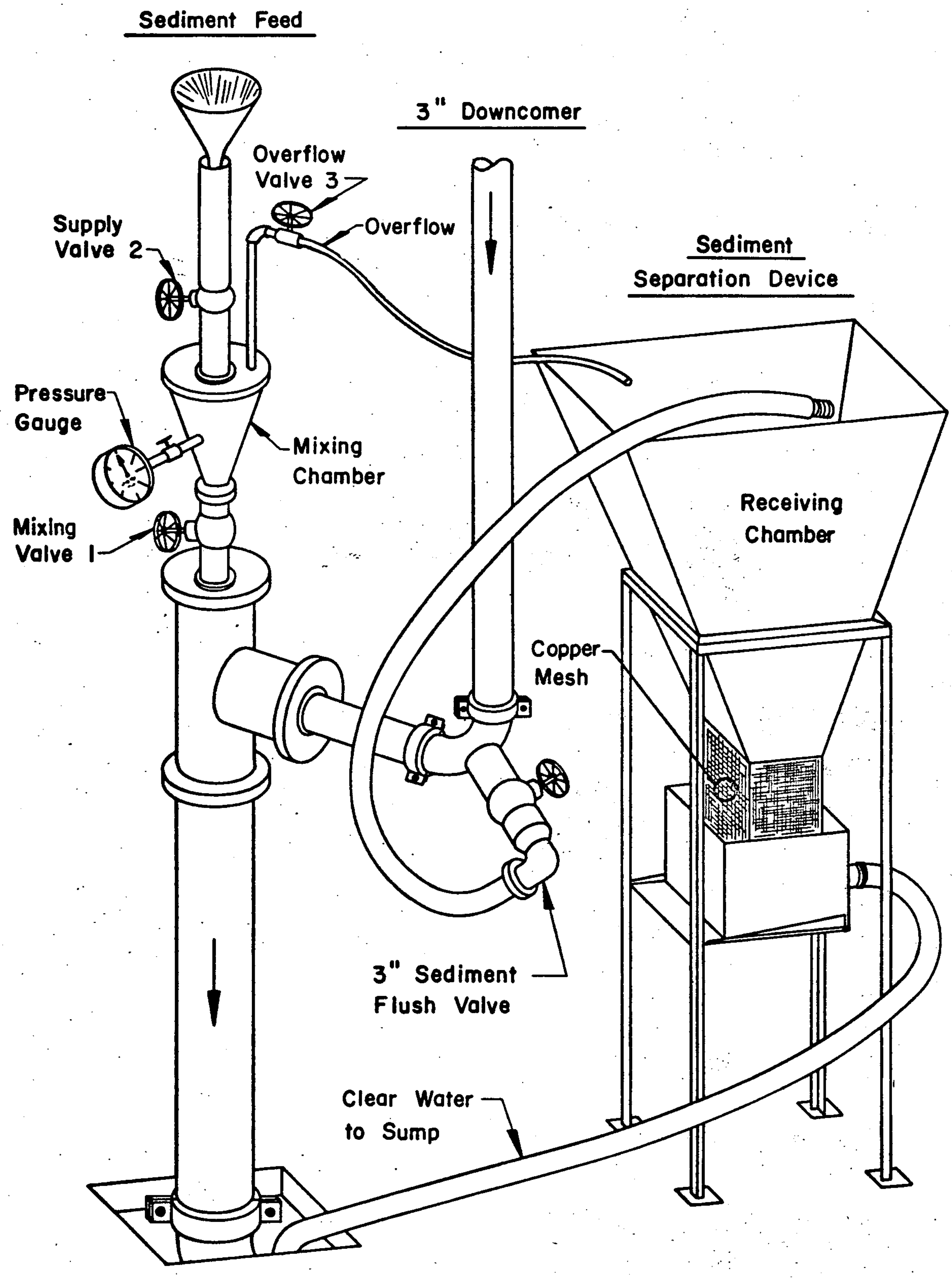

Fig. 3.4: Sediment Feed and Removal Facility

of polluted water to the collection sump. The 3-inch sediment flush valve was opened enough to maintain positive pressure in the system and divert the mixture flow into the receiving chamber of the sediment separation device. Two square feet of No. 60 cooper mesh screening prevented flow through of solids material. The screened clear water was removed to the sump.

Sediment feeding was the more troublesome of the two operations. Both the mixing and supply valves were replaced because of jamming, which caused unexpected backup of sand slurry from the mixing chamber.

Measurement and Flow Regulation. The volumetric concentrations of solids and the mixture flowrates were determined from "Loop System" head loss readings. Arrows 1 and 2 on Figure 3.1 indicate the respective locations of "Downcomer" and "Riser" pressure taps, both with 1.50 m (=59.1 in.) head loss lengths.

Loop readings were repeatedly checked against flow recordings from a Foxboro Magnetic Flowmeter by means of a Dynalog Receiver measuring accuracy to within 1 percent of full scale, throughout the scale (approximately $\pm$25 gpm). A Prandtl tube (C in Fig. 3.1) was employed to verify both the "Loop System" and flowmeter measurements of mixture velocities. A Pitot tube sediment-sampling device (D in Fig. 3.1) checked the "Loop System" indication of solids concentrations. Further discussion on determining concentrations and flowrates is found in Section 3.2.

Two Venturimeters were investigated for their applicability as mixture flow measuring devices, the results of which are reported by Robinson et al. (1970). A new 3 x 2 inch Venturimeter (E in Fig. 3.1) and an antiquated 4 x 2 inch device (F in Fig. 3.1) were tested and later used in checking flow conditions for this particular study.

The mixture head loss length for the test section was 3.60 m (=141.8 in.), as located at the arrows marked 3. At each pressure tap location, four holes, 3/32 inch in diameter, were drilled diagonally opposite about the circumference of the pipe. Brass fittings were assembled and connected with poly-flo tubing for transmitting the hydraulic pressure. Manometer fluids were selected according to the required range of readings. Most often air-water readings were adequate, however, a 2.95 fluid-water medium was needed at extreme flow conditions. The 50.0 in. manometer scales were graduated in tenths of an inch, readings to a hundredth of an inch were estimated, and each reading was converted to feet of water column. Minor manometer fluctuations always existed, partly due to the uneven distribution of sediment concentration through the large system and also due to the effect that concentrated slugs of sediment had on the pump's capacity for maintaining a constant mixture flowrate.

Flowrates between 200 and 1000 gpm were regulated by a varidrive rheostat control, located at the operator's station. The 8-inch discharge valve controlled lower range flowrates. Sediment feed rates were not rigorously monitored, except for an attempt to evenly distribute the sediment throughout the system.

3.2 Measuring Techniques

Clear-water calibration of the system was the initial course of action. The "Loop System" head loss readings were then evaluated and checked against flowmeter, Prandtl tube, and Pitot tube measurements.

3.2.1 Clear-Water Tests

Tests of clear-water flow were conducted to determine material roughness characteristics of the 3-inch "Loop System" pipes and the 4- and 6-inch diameter test lengths. Friction factors, f, were calculated from the Darcy-Weisbach equation, evaluating manometer head loss readings and Prandtl tube indication of velocities over the ranges of Reynolds number indicated in Table 3.1. Also summarized are the

Pipe Specification	ϵ/D	ϵ (ft)	Reynolds Nos.
Loop System:			
3 in. ∅ commercial steel	0.00004	0.00001	2.48×10^5 to 4.77×10^5
Test Length:			
4 in. ∅ galvanized	0.00009	0.00003	1.97×10^5 to 3.58×10^5
6 in. ∅ black steel	0.00032	0.00016	1.39×10^5 to 3.76×10^5

Table 3.1: Relative Roughness and Material Roughness Values for the Three Pipe Sizes

respective relative roughness values, ϵ/D, and material values, ϵ, determined from the Moody-Stanton Diagram of friction factors for commercial pipe. The friction factors for all three pipes fall in the transition regime. For further determination of friction factors at any mixture flow Reynolds number, an explicit solution of the Colebrook-White equation was used. Evaluation of extensive "Loop System" data required this type of solution for f.

3.2.2 The Loop System

The "Loop System" developed by Einstein et al. (1966) was used to simultaneously determine the mixture flowrate, Q_m, and the solid phase concentration, C. The device consists of two identical vertical pipe sections with opposite flow direction. Pressure head differences are obtained over these vertical pipe sections, namely, the "Riser" and the "Downcomer" section. The head loss in the riser section is

$$\Delta h_R = L C_R (s_s - 1) + f \frac{L}{D} \frac{\left(\frac{Q_m}{A}\right)^2}{2g} [1 + C_R (s_s - 1)] \qquad (3.1)$$

and in the downcomer

$$\Delta h_D = -L C_D (s_s - 1) + f \frac{L}{D} \frac{\left(\frac{Q_m}{A}\right)^2}{2g} [1 + C_D (s_s - 1)] \qquad (3.2)$$

where L represents the head loss length in either section, C_R and C_D are the solids concentrations in the riser and downcomer pipes, and Q_m is the total mixture flowrate.

If the summation and the difference of Eqs. (3.1) and (3.2) are respectively computed, the resulting equations are

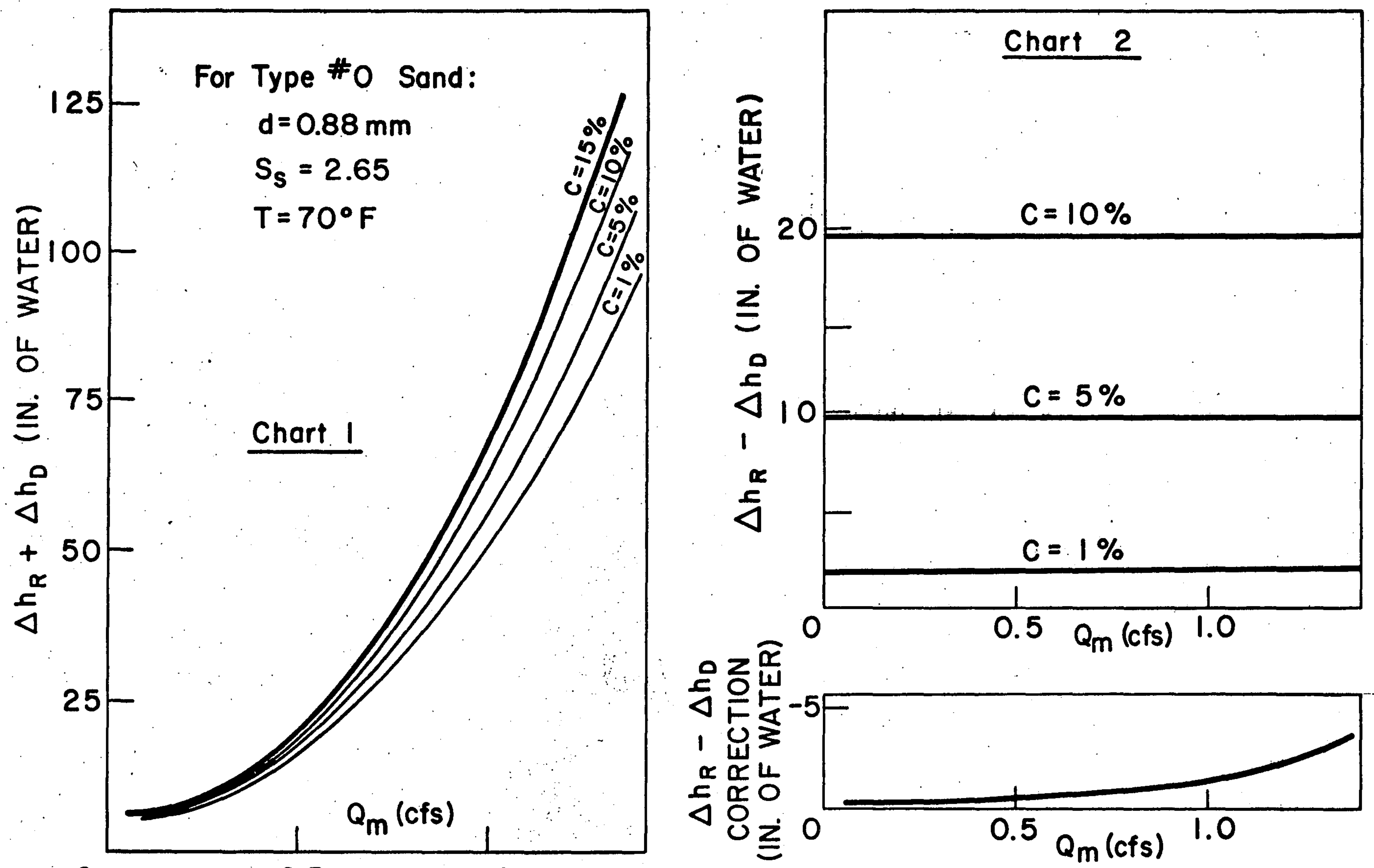

Fig. 3.5: Loop System Charts

$$\frac{\Delta h_R + \Delta h_D}{2L} = (s_s - 1)\ \frac{v_{ss}\ A}{Q_f}\ C(1-C)^2 + \Psi_e\ [1 + (s_s - 1)C] \tag{3.3}$$

$$\frac{\Delta h_R - \Delta h_D}{2L} = (s_s - 1)\left[C + \frac{v_{ss}\ A}{Q_f}\ C(1-C)^2\ \Psi_e\right] \tag{3.4}$$

The fluid flowrate, Q_f, in Eqs. (3.3) and (3.4) had replaced the total flowrate, Q_m, to distinquish between solid and liquid phase flowrates, or $Q_f = Q_m/(1-C)$. C is the average volumetric concentration of solids if flowing through a horizontal section. The symbol Ψ_e represents a pressure gradient for mixture flowrate, as

$$\Psi_e = \frac{f}{D}\left(\frac{Q_m}{A}\right)^2 \frac{1}{2g} \tag{3.5}$$

It is seen that knowing riser and downcomer head loss readings for a solid-liquid mixture flow, solids concentration, C, and mixture flowrate, Q_m, may be obtained from Eqs. (3.3) and (3.4).

To expedient the determination of Q_m and C from loop head loss readings obtained while testing, a program was developed and executed on the University's CDC 6400 Computer to print out data for plotting two charts. Plotted output for coarse sand particles at 70°F is illustrated in Charts 1 and 2 of Fig. 3.5. A $(\Delta h_R - \Delta h_D)$ correction curve shown below Chart 2 was determined from clear-water evaluation of the riser and downcomer readings. A set of charts were plotted for each of the three types of particles investigated, using readings determined from two different system temperatures of 70°F and 90°F. The program calculated relative values of Δh_R and Δh_D in functional relationship with various input combinations of Q_m and C. Q_m and C were generated in 0.10 cfs and 1% increments, respectively, and up to 2.15 cfs and 20%. The friction factors for each Reynolds flow number were explicitly determined from an equation developed by Wood (1966):

$$f = a + b\mathbb{R}_e^{-c} \tag{3.6}$$

which is a best fit solution to the Colebrook-White relationship. a, b and c are simple power functions of ϵ/D, ϵ/D determined to be 0.00004 for the 3-inch loop pipes.

Appendix A illustrates, by means of an example, how concentration and mixture flowrate for a particular test run are readily determined from location of head loss readings on <u>Charts 1</u> and <u>2</u>. Application of the clear-water correction data is also examined.

Loop indications of C and Q_m were checked against Prandtl tube and Pitot tube measurements and adjustment of the loop data recommended. However, it was found that adjustment is only necessary for data in the heterogeneous flow regime. The method of evaluating the loop data with respect to Prandtl tube and Pitot tube findings is explained in Appendix A.

3.3 Description of Experiments

3.3.1 Range of Parameters Tested

The important parameters in the critical deposit velocity problem were identified in Section 2. To understand the interrelationships involved, it is paramount to study the different effects due to independent variation of each parameter. Herein is described the attempt at satisfying that requirement and a qualification of the extensive data compilation.

A 4-inch and a 6-inch diameter pipe, each one having a different pipe roughness, as shown in Table 3.1, were evaluated for their relative effects on V_C. Each was tested separately at different slopes, assuring always a sufficient upstream flow transition length. Most of the data were obtained with the test section placed in a horizontal position. Some data were also obtained for both a positively and negatively sloped alignment, in the hope of showing some indiction of the $\tan\theta$ variable effect on critical velocity determination. The positive slope tested was $\tan\theta = +0.027$, and the negative slope, $\tan\theta = -0.060$ (geometrically speaking).

Three types of solid particles, wholly described in Table 3.2 and pictured in Fig. 3.6, were tested in various combinations with D and $\tan\theta$ variables, as are listed in Table 3.3. The mean sand particle diameters and non-uniformity coeffieients, d_{50} and d_{90}/d_{50}*

* d_{90}/d_{50} was selected for indication of non-uniform grain distribution to expedient the compilation of data similarly reported by other investigators. In a normal Gaussian distribution, it is often shown that a 95% confidence interval is represented by the d_{90} and d_{10} particle sizes. This adequately characterizes the particle aggradation.

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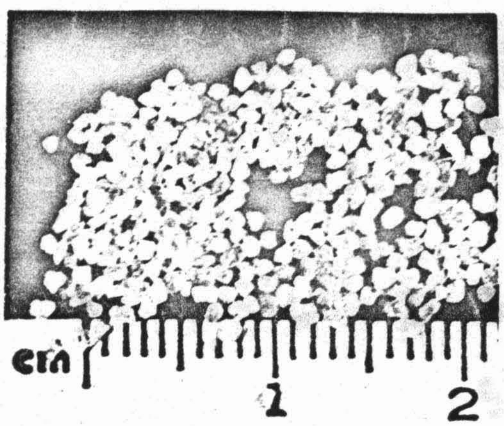
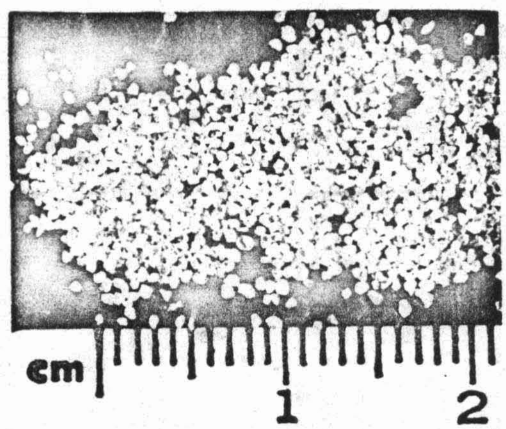
Solids Material	d_{50} (mm)	d_{90}/d_{50}	s_s	v_{ss} (ft/sec)
<u>Quartz Sand</u>:				
#0	0.88	1.21	2.65	0.312
#00	0.45	1.07	2.65	0.189
<u>Plastic Pellets</u>:				
PP	3.63	--	1.38	0.697

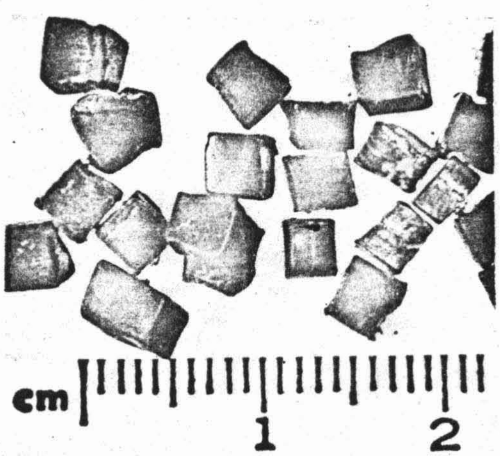
Table 3.2: Solid Particles Specification



(a) Sand #0



(b) Sand #00



(c) Plastic Pellets
$\Psi_s = 0.795$

Fig. 3.6: Photographic Representation of the Three Types of Solid Particles Investigated; (a) Coarse Sand Particles, (b) Fine Sand Particles, and (c) Plastic Pellets

respectively, were determined from a standard sieving analysis and remained constant throughout the testing period. The highly-silica,

Pipe Diameter, D in. (Material Roughness, ϵ ft)		Mean Particle Diameter, d_{50} (Specific-Gravity, s_s)			Pipe Slope, $\tan\theta$		
4 (0.00003)	6 (0.00016)	0.88 (2.65)	0.45 (2.65)	3.63 (1.38)	0	-0.060	0.027
*		*			*		
*		*				*	
*			*		*		
*			*			*	
	*	*			*		
	*	*					*
	*		*		*		
	*		*				*
	*			*	*		

Table 3.3: Tested Combinations of Pipe Diameter, Solid's Particle Diameter, and Slope

quite uniform, quartz sand was supplied by Whitehead Brothers, Co. in New Jersey, and the plastic pellets were manufactured by B. F. Goodrich Co. in Ohio.

The effect of particle shape or true sphericity, Ψ_s, is considered in adjusting the apparent mean particle size of the plastic pellets by the equation:

$$(d_{50})_{effective} = \frac{(d_{50})_{apparent}}{\Psi_s} \tag{3.7}$$

Ψ_s is defined as the ratio of the surface area of the equivalent-volume sphere to the actual surface area. It is an isoperimetric property of particles, and its hydrodynamic influence on settling velocity is developed by Graf et al. (1966).

The cube-shaped plastic pellets, with average dimensions of 1/8 in. x 1/8 in. x 3/32 in., indicate an "apparent" particle diameter, d_{50} = 2.89 mm. Upon application of the cube-shape sphericity factor, Ψ_s = 0.795, Eq. (3.7) defines an "effective" particle diameter, d_{50} = 3.63 mm. Irregular pellet shapes were removed, but a distribution was not determined.

The respective settling velocities were found from a graph and equation presented after Budryck by Durand (1953, p. 100). Budryck's graph and equation cover the entire range of settling velocities for "quartz grains" of 2.65 specific gravity in a quiescent medium. The consideration of sand particle sphericity was not necessary. Plastic pellet settling velocity, however, was determined from the "effective" particle diameter.

The specific weights of the solids, s_s, were provided by the material suppliers and are listed in Table 3.2.

Volumetric concentrations of $0.1\% < C < 17\%$ were handled at flowrates ranging from 0.1 cfs (~50 gpm) $< Q_m <$ 1.8 cfs (~800 gpm). The system temperature was recorded for each test run and sometimes varied from $60^\circ F < T^\circ < 100^\circ F$. The effect of temperature on the loop readings was accounted for, as explained in Section 3.2.2.

3.3.2 Testing Procedure

Preparation for a Series test run involves selection of a pipe diameter, D, (with determined material roughness, ϵ); the adjustment of the pipe slope, $\tan\theta$; and the feed of solid particles, d_{50}, (represented by solid's specific gravity, s_s, and a non-uniformity coefficient, d_{90}/d_{50}) into the system.

For a particular test series, the solids are circulated in a nearly pseudohomogeneous flow condition which ensures uniform distribution of the particles throughout the system. Once conditions were stabilized, the flowrate, the moving solids concentration, and the test section head loss readings were recorded; these are compiled in Appendix B. A qualitative description of the mixture flow, as observed through the Plexiglas section, is thereon commented. Flowrates are then decreased to the heterogeneous flow regime, and there becomes noticeable a not so unexpected development. The moving solids concentration diminishes, due to the premature settling of particles in the larger 8-inch pipe, located upstream from the test section, exhibiting a transport flow capacity less than that within the 4-inch or 6-inch test sections.

Further decrease in flowrate produces heavy bedload transport in which most particles are either rapidly sliding along the invert or saltating into the clear flow area of the pipe. Subsequent flowrate changes are more finely incremented. Lowering the flowrate to a velocity at which the bedload begins pulsating between deposit and non-deposit flow conditions, the sliding bed thickness builds and there exists no measureable transport of the bedload particles. In this study, this is the definition of the critical deposit velocity, V_C. The solids concentration corresponding to that particular V_C is recorded just prior to the critical condition, when all particles are in transit.

Readings are also recorded in the deposit regime to complete the data required for plotting the associated head loss curves. Dune formation and dune transportation are an ever fascinating phenomenon at these low flow ranges. Clogging of the system was never encountered.

In the early stages of this study, runs were repeated to check the consistency of data measurement. Once satisfactory agreement was obtained, solids were added or removed to change the concentration. At critical conditions, the concentrations never exceeded 7% by volume.

Inconsistencies are experienced in any sediment transport study, but low concentrations in this study presented an unusual problem. The necessity of almost fully closing the 8-inch flow discharge valve for reaching low critical velocities induced local scouring of the already well-deposited bed in the 8-inch pipe. Sudden slugs of sediment would then deposit in the test section at one moment, and completely scour clean the next, under the same flow conditions. The transport of plastic pellets posed an additional difficulty. Low flow conditions did not sufficiently entrain the pellets to flow freely through the pump. Rather, particles slid down between the seal and the impeller, straining the motor and causing sudden variation in flowrates.

After several runs were made at a variety of concentrations, the data were plotted on a typical mixture head loss versus mixture velocity graph, as explained in Appendix B, and one of the parameters changed for subsequent tests.

4. EVALUATION OF EXPERIMENTAL DATA

4.1 Analysis of Lehigh Results

Nine series of tests were conducted to determine the critical deposit velocities for varied concentrations of sand and plastic pellets transported with water in a pipeline. Most data were recorded from sand-water tests in a horizontal pipe over a range of low solids concentration ($C < 7\%$). It is expected that within this lower range of solids concentration, both the particle diameter, d, and solids concentration, C, effect the critical deposit velocity value.

By testing various combinations of solids concentrations, C, particle diameter, d, specific weight of solids, s_s, pipe diameter, D, and pipe slope, tan θ, different critical deposit velocities were recorded and compared. All experimental data are first tabulated and then plotted as mixture head loss against mixture velocity (see Appendix B).

Critical Deposit Velocities. The critical deposit velocity data are summarized in Table 4.1 with indication of run numbers for each series of tests, the volumetric solids concentrations, the critical deposit velocities, and four modified Froude numbers. These four modified Froude numbers are defined in Table 4.1 and were computed for each critical deposit velocity. Froude number (I) is the modified form, after Durand (1953), for critical deposit velocities in horizontal pipeflow. Subsequently, both Froude numbers (II) and (III) are introduced to evaluate critical deposit velocities in sloping pipes as well. Froude number (IV) is suggested by Wasp et al. (1970).

MODIFIED FROUDE NUMBERS EVALUATED...

$$F_r\ (I) = \frac{V_c}{\sqrt{2gD\ (s_s-1)}}$$

$$F_r\ (II) = \frac{V_c}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta]$$

$$F_r\ (III) = \frac{V_c}{\sqrt{2gD\ (s_s-1)\ [1 + \tan\theta]}}$$

$$F_r\ (IV) = \frac{V_c}{\sqrt{2gD\ (s_s-1)}\ (d/D)^{1/6}}\ [1 - \tan\theta]$$

RUN	VOLUMETRIC SOLIDS CONCENTRATION (PERCENT)	CRITICAL DEPOSIT VELOCITY (FT/SEC)	MODIFIED FROUDE NUMBER (I)	(II)	(III)	(IV)
Series G-01: PARTICLE DIAMETER = .88 MM; PIPE DIAMETER = 4.00 IN.; PIPE SLOPE = 0.000						
6	.12	3.90	.655	.656	.656	1.447
7	.15	4.65	.782	.782	.782	1.725
8	.20	5.10	.857	.857	.857	1.892
9	.50	5.35	.899	.899	.899	1.985
1	.50	5.00	.841	.841	.841	1.855
10	.60	5.80	.975	.975	.975	2.151
11	1.00	6.40	1.076	1.076	1.076	2.374
2	1.00	5.50	.925	.925	.925	2.040
3	1.75	5.75	.967	.967	.967	2.133
4	2.00	5.75	.967	.967	.967	2.133
5	5.00	5.95	1.000	1.000	1.000	2.207
Series G-02: PARTICLE DIAMETER = .88 MM; PIPE DIAMETER = 4.00 IN.; PIPE SLOPE = -.060						
1	.50	4.80	.807	.855	.832	1.887
2	1.00	5.10	.857	.909	.884	2.005
3	3.00	5.35	.899	.953	.928	2.104
4	7.00	5.00	.841	.891	.867	1.966

Table 4.1: Critical Deposit Velocity Data

RUN	VOLUMETRIC SOLIDS CONCENTRATION	CRITICAL DEPOSIT VELOCITY	MODIFIED FROUDE NUMBER (I)	(II)	(III)	(IV)
	(PERCENT)	(FT/SEC)				
Series BS-01: PARTICLE DIAMETER = .88 MM; PIPE DIAMETER = 6.00 IN.; PIPE SLOPE = 0.000						
1	.80	6.40	.878	.878	.878	2.074
2	1.10	6.70	.920	.920	.920	2.171
3	3.00	7.25	.995	.995	.995	2.349
4	5.00	7.40	1.016	1.016	1.016	2.398
Series BS-03: PARTICLE DIAMETER = .88 MM; PIPE DIAMETER = 6.00 IN.; PIPE SLOPE = .027						
1	1.00	6.40	.878	.855	.867	2.018
2	2.30	7.60	1.043	1.015	1.029	2.396
3	4.80	7.85	1.077	1.048	1.063	2.475
Series G-001: PARTICLE DIAMETER = .45 MM; PIPE DIAMETER = 4.00 IN.; PIPE SLOPE = 0.000						
5	.05	2.75	.462	.462	.462	1.141
6	.10	4.10	.689	.689	.689	1.701
7	.20	4.80	.807	.807	.807	1.991
8	.30	5.45	.916	.916	.916	2.261
1	.65	5.10	.857	.857	.857	2.116
9	1.00	5.70	.958	.958	.958	2.364
10	1.20	5.85	.983	.983	.983	2.427
2	1.50	5.60	.941	.941	.941	2.323
3	3.00	6.25	1.051	1.051	1.051	2.593
4	7.00	6.50	1.093	1.093	1.093	2.696

Table 4.1: (Continued)

RUN	VOLUMETRIC SOLIDS CONCENTRATION	CRITICAL DEPOSIT VELOCITY	MODIFIED FROUDE NUMBER (I)	(II)	(III)	(IV)
	(PERCENT)	(FT/SEC)				
Series G-002	PARTICLE DIAMETER = .45 MM; PIPE DIAMETER = 4.00 IN.; PIPE SLOPE = -.060					
1	.05	3.70	.622	.659	.642	1.627
2	.10	3.90	.655	.695	.676	1.715
3	.25	4.50	.756	.802	.780	1.979
4	.55	5.10	.857	.909	.884	2.243
5	2.25	5.50	.925	.980	.954	2.418
6	2.50	5.70	.958	1.016	.988	2.506
Series BS-001	PARTICLE DIAMETER = .45 MM; PIPE DIAMETER = 6.00 IN.; PIPE SLOPE = 0.000					
1	.75	5.85	.803	.803	.803	2.120
2	1.90	6.95	.954	.954	.954	2.519
3	2.50	7.45	1.023	1.023	1.023	2.700
4	5.40	7.95	1.091	1.091	1.091	2.881
Series BS-003	PARTICLE DIAMETER = .45 MM; PIPE DIAMETER = 6.00 IN.; PIPE SLOPE = .027					
1	.75	6.15	.844	.821	.833	2.168
2	2.00	7.10	.975	.948	.962	2.503
3	3.70	7.50	1.029	1.002	1.016	2.644
4	5.00	7.75	1.064	1.035	1.050	2.733
Series BS-PP1	PARTICLE DIAMETER =3.63 MM; PIPE DIAMETER = 6.00 IN.; PIPE SLOPE = 0.000					
1	1.30	3.40	.972	.972	.972	1.813
2	1.90	3.85	1.101	1.101	1.101	2.053
3	3.00	4.45	1.273	1.273	1.273	2.373
4	3.80	4.60	1.316	1.316	1.316	2.453

Table 4.1: (Continued)

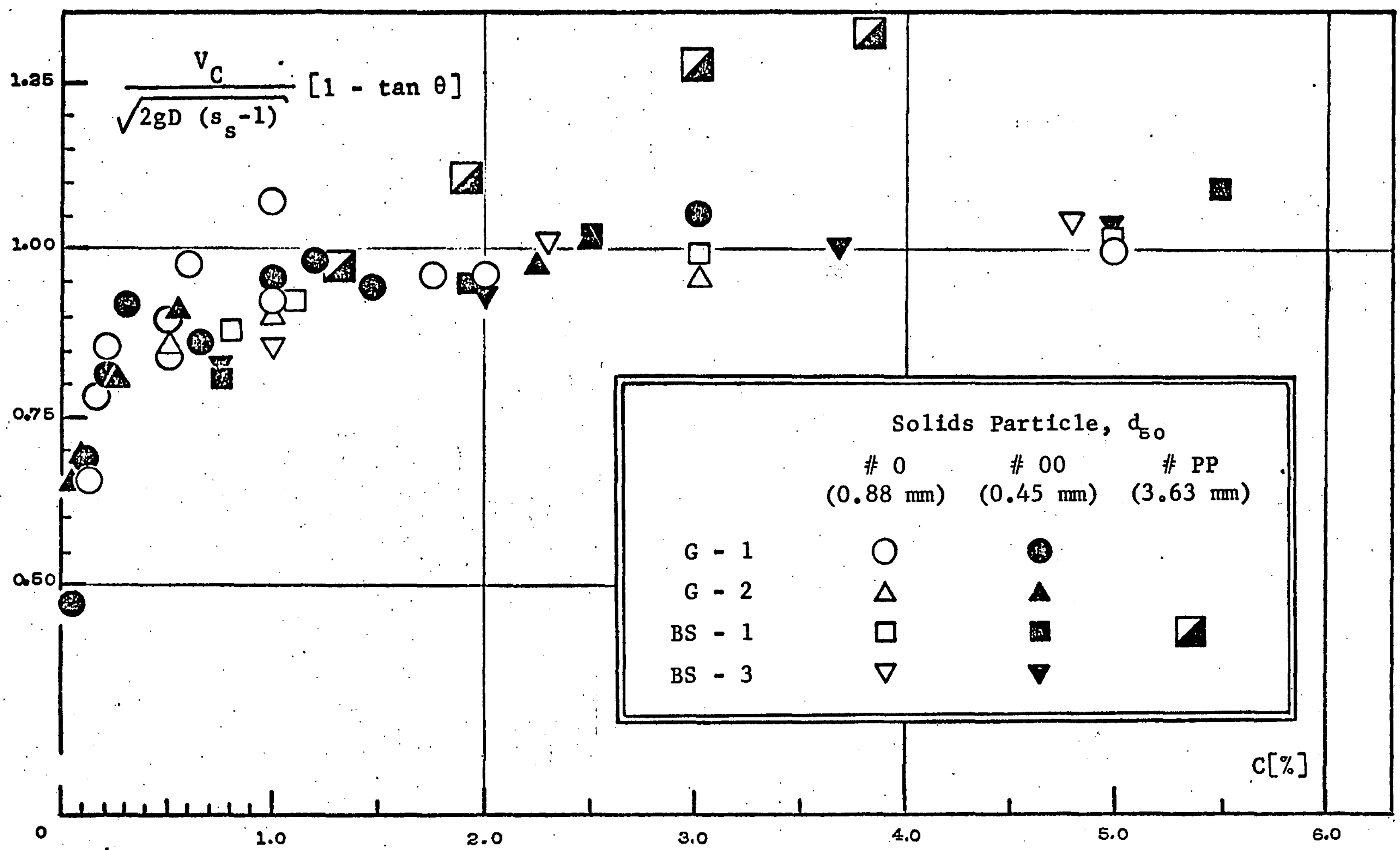


Fig. 4.1: Experimental Data from Lehigh Sand-Water and Plastic Pellet-Water Studies; Modified Froude Number versus Concentration, Particle Diameter as Parameter

From a preliminary study, plotting Froude numbers (I), (II), and (III) against solids concentration, C, it was found that Froude number (II) best correlates the data, including both horizontal and sloping flow values. Further, Froude number (IV) plotted against concentration, C, indicated no improvement in demonstrating the trend of results, and only increased the scatter of data. Lehigh values of d/D raised to the 1/6 power are very small and have little influence on the correlation.

It is therefore that Froude numbers (I), (III), and (IV) are no longer considered; the data are analyzed with Froude number (II) and presented in Fig. 4.1.

<u>Correlation of Data</u>. A regression analysis was made to correlate modified Froude number (II)* with the following parameters: concentration, C; concentration, C, and particle diameter, d; and concentration, C, and relative particle size, d/D.

The regression functions take two forms: (1) A least squares fit of modified Froude number, F_r, with concentration, C, written as:

$$F_r = k_1 \; C^{k_2} \tag{4.1}$$

where k_1 and k_2 are evaluated from logarithmic values of the data over five different particle size ranges, and (2) a least squares multiple

*The same was done for modified Froude numbers (I) and (III) and is given in Appendix C.

regression, using Gaussian iteration to fit modified Froude number, F_r, to both concentration, C, and particle size, either as d, or the dimensionless form, as d/D. These two regression functions are given by:

$$F_r = k_3 \; C^{k_4} \; d^{k_5} \tag{4.2a}$$

$$F_r = k_3' \; C^{k_4'} \; d/D^{k_5'} \tag{4.2b}$$

The exponents, k_4 and k_5, and coefficient, k_3 are determined for the sand-water data and also for the total range of data, including plastic pellet-water results.

An explanation of the multiple regression analysis and a statistical interpretation of the resulting equations are given in Appendix C. It should be noted at this point that plastic pellets data were eliminated from the analysis. The influence of 4 out of 50 data points is somewhat negligible and their imposition on the general trend of results, dictated by the 46 sand-water data points, was felt to be of little value. The regression analyses reported in Appendix C justify this reasoning.

Two regression equations are found to fit the Lehigh data particularly well: (1) Using only sand data, and assuming solids concentration, C, to be the only important independent variable, the best-fit equation is given as:

$$F_r = \frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta] = 0.901\ C^{0.106} \qquad (4.3)$$

The coefficient of correlation is 0.870. (2) Including the influence of particle diameter, d, the following equation was developed for sand alone:

$$F_r = \frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta] = 0.928\ C^{0.105}\ d^{0.056} \qquad (4.4)$$

where the particle diameter, d, is in mm. The coefficient of correlation is 0.877.

Note that the value for exponent $k_2 = 0.106$, given with Eq. (4.3), is very close to exponent $k_4 = 0.105$, given with Eq. (4.4). Further, coefficient $k_3 = 0.928$ in Eq. (4.4) differs only slightly from coefficient $k_1 = 0.901$ in Eq. (4.3). This similarity between the coefficients and exponents in Eqs. (4.3) and (4.4) is due to the almost negligible effect of particle diameter, d. Equations (4.3) and (4.4) are shown graphically in Fig. 4.2.

The regression analysis for the relation given by Eq. (4.2b) is presented in Appendix C and shows that the relative particle size, d/D, has very little influence on improving the correlation given with either Eq. (4.3) or Eq. (4.4).

It should be again noted that the form of the modified Froude number, including a tan θ argument, has been suggested to better correlate the Lehigh data. It is recommended that either Eq. (4.3) or

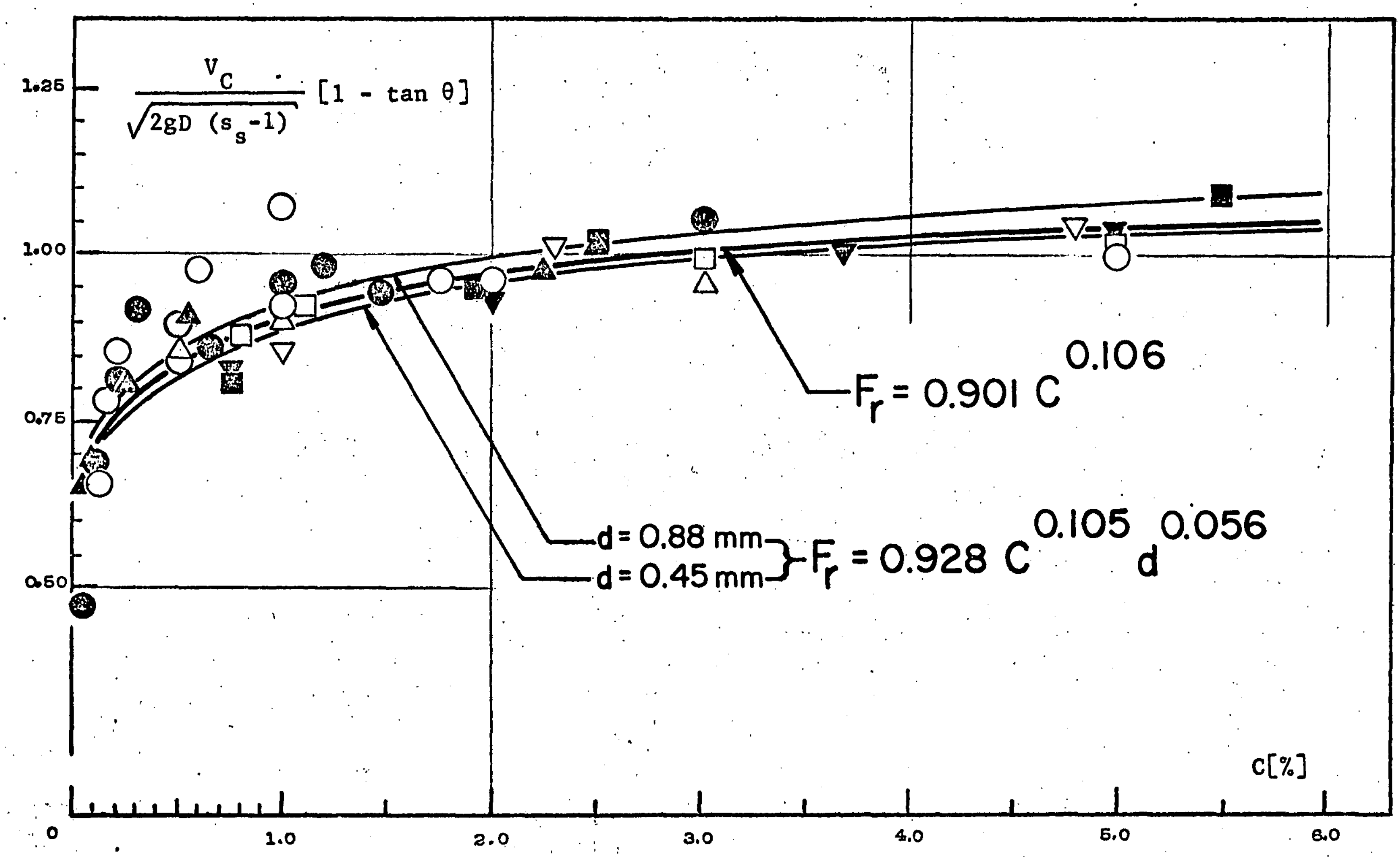


Fig. 4.2: Best-Fit Equations for Lehigh's Sand-Water Data Only; Modified Froude Number versus Concentration, Particle Diameter as Parameter

Eq. (4.4) be reliably applied in the design of sand-water transport systems with galvanized or black steel pipes on a slope: $-0.10 < \tan\theta < 0.05$. Either equation is certainly good within the range of particle diameters tested at Lehigh: $0.45 < d < 0.88$ mm, with $d_{90}/d_{50} \leq 1.21$.

Relative Influence of Tested Parameters. Needless to say, not all ranges of the parameters, D, d, s_s, C, $\tan\theta$, d_{90}/d_{50}, and ϵ/D, have been completely investigated and never will be. However, the resulting regression equations, Eqs. (4.1) and (4.2), offer insight to the relative influence of some of the tested parameters on the critical deposit velocity.

The influence of solids concentration, C, on the critical deposit velocity is found in this study to be of primary significance, particularly within a low-concentration range of $C < 7\%$. For concentrations above 5 to 10%, both Sinclair (1962) and Wilson (1965) find that critical deposit velocities decrease with concentration. A similar observation was made in the present study when concentrations exceeded 5%.

The particle diameter, d, has no direct effect on the critical deposit velocity value within the range of particle diameters tested in the present study, $0.45 < d < 0.88$ mm. However, with suspensions of fine particles in the range $d < 20$ mm, it is expected that solids settling is sufficiently delayed to decrease the critical deposit velocity. This is reported by Worster et al. (1955) and Gibert (1960).

While the Lehigh data provide insufficient evidence that relative density, s_s-1, expressed as $(s_s-1)^{0.5}$, is proportional to the critical deposit velocity, other studies have made this verification. Sinclair (1962), however, reports that $(s_s-1)^{0.4}$ better correlates his data for iron-kerosene, sand-water, and coal-water mixtures. Furthermore, Ellis et al. (1963b) conducted experiments with nickel shots in water, finding that critical deposit velocities were reduced for these solids of high density. They reasoned that this was due to both the "elastic rebounding" of the particles, which have large momentum as they strike the bottom of the pipe, and the increased lift forces imposed by the liquid as the particles come to a sudden rest at the boundaries. It appears reasonable to question the form $(s_s-1)^{0.5}$ if it is used to determine critical deposit velocities for solid-liquid mixtures other than sand-water. However, for any suspension of quartz particles, $(s_s-1)^{0.5}$ has been well founded to best correlate the critical deposit velocity parameters.

The grain size distribution, d_{90}/d_{50}, was also a parameter felt to be unimportant in the present study. In addition, the Lehigh sand samples were quite uniform and the effect of such a parameter could not be tested. The problem of mixed sized samples is complicated in that fine particles often create a supporting suspension for the coarser particles. It is realistic, when designing for the transport of a non-uniformly distributed material, to select an "effective" mean particle size, slightly greater than d_{50}, to account for the settling of the larger particles.

The relative material roughness, ϵ/D, was assumed to be an insignificant parameter in this study. Inclusion of this parameter in the correlation enters in the liquid head loss, and apparently does not influence the movement of the solids phase. The present study showed that for pipes of black steel and galvanized iron, material roughness is of negligent concern in critical deposit velocity determination. This is similar to what Durand (1953) observed with steel and cast iron pipes. Only with very fine particles and pipe roughness protrusions, which would disrupt the laminar boundary layer, might one find it necessary to include the effect of ϵ/D on critical deposit velocity.

4.2 Comparison with Other Data

Particularly important in the present study is the applicability of the modified Froude number relationship, given with Eq. (2.13), for low-concentration mixtures, $C < 7\%$. The strength of the Lehigh data is in the range with $0.10 < C < 2.0\%$. The low-concentration data are mainly responsible for the final form of the modified Froude number relationship, as given with Eqs. (4.3) and (4.4). In what follows we shall try to investigate as to how other experimental data compare with the present findings.

Sand-Water Mixtures. Many researchers have reported on sand-water mixture studies, but from all of these, only the studies by Gibert (1960), Führböter (1961), Sinclair (1962), and Durand, Smith, and Yotsurura, as reported by Wasp et al. (1970), rendered useful data for the present investigation. The ranges of parameters investigated in these studies are listed in Table 4.2, and the data are plotted in Fig. 4.3 for comparison with the Lehigh sand-water data given with:

$$F_r = 0.901\ C^{0.106} \qquad (4.3)$$

Data were retrieved from only those studies which investigated a "critical condition" identical to the critical deposit velocity, as defined in the present study. However, it must be pointed out that a certain amount of inaccuracy is inherent with any sediment transport study and results will vary within the same testing system, let alone from one system to another. In general, it is felt that the trend established by Gibert's (1960) data, for $d \geq 0.37$ mm, is rather well reflected in the Lehigh sand-water data. It is recalled that Gibert (1960) reports an exhaustive investigation obtaining 310 data points. Of interest is also that the Sincalir (1962) and Durand (1953) data are in reasonable agreement with the Lehigh findings. Further, it is noted that the Yotsurura data, reported by Wasp et al. (1970), reflect trends similar to the Gibert (1960) curve for fine particles.

Figure 4.3 together with the Lehigh sand-water data, represented with Eq. (4.3), suggest the following trends in the range where $C < 5\%$: (1) The critical deposit velocity, V_C, increases with solids concentration, C; the increase becomes less evident as the concentration rises to 5%. (2) For particle sizes, $d \geq 0.37$ mm, the critical deposit velocity remains practically unchanged with increase in d.

		Sediment Size	Pipe Size	Sediment Conc.	Specific Gravity	Remarks
		d [mm]	D	C	ρ_s/ρ	
Durand (1952)*	○ ●	0.44 2.04	5.90 in.	up to 15%	2.65 sand/ water	Extensive range of parameters tested
Smith (1955)*	⬡	0.18	3.00 in.	up to 26%	2.65 sand/ water	V_C obtained from V_C vs. C plot
Gibert (1960)	— — –o–	≥ 0.37 0.20	40.2 to 150.0 mm	up to 15%	2.65 sand/ water	Best-fit curves on $V_C/\sqrt{gD}$ vs. C plot
Führböter (1961)	□ ■	0.27 0.53, 0.88	0.30 mm	up to 25%	2.64 sand/ water	V_C is reported
Yotsurura (1961)*	▽ ▼	0.23 0.59, 1.15	4.25 in.	up to 25%	2.65 sand/ water	V_C is reported
Sinclair (1962)	△ ▲	0.35 0.68	0.50, 0.75, 1.00 in.	up to 20%	2.61 sand/ water	V_C obtained from V_C vs. C plot

*Reported in Wasp et al. (1970)

Table 4.2: Range of Parameters of the Data Reported by Other Investigators for Sand/Water Mixtures; Data are Plotted in Fig. 4.3

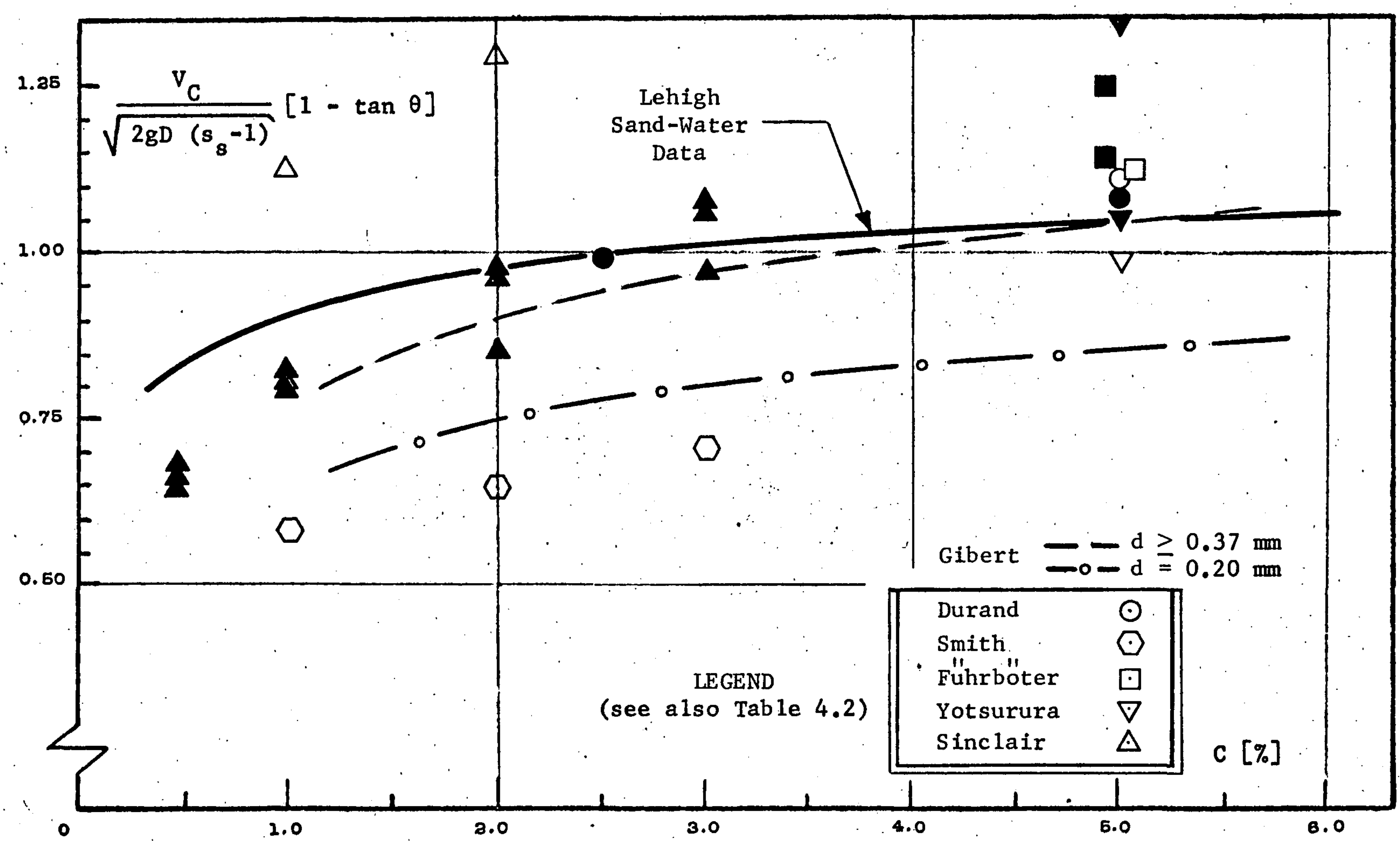


Fig. 4.3: Modified Froude Number versus Solids Concentration, Particle Diameter as Parameter (Data from Sand-Water Mixture Studies)

The Lehigh data exhibit this trend showing particularly good agreement with the other data, and will give conservative design values. (3) For particle sizes smaller than d = 0.37 mm, the critical deposit velocity, V_C, decreases with decreasing d. It is expected that this decrease in V_C levels off for very fine particles, but the data reported give inconclusive verification of this.

Neither particle size distribution nor the pipe material roughness were considered to be of importance in this comparison.

Solid-Liquid Mixtures other than Sand-Water. To show the general usefulness of the modified Froude number, data from other solids-liquid mixtures were studied. Wasp et al. (1970) report data from Wicks and Moye on the investigation of sand-kerosene and sand-oil mixtures, Sinclair (1962) reported on iron-kerosene mixtures, and Wilson (1965) on nylon-water mixtures. Again, the data are compared with the Lehigh sand-water data, as shown in Fig. 4.4; the ranges of parameters are listed in Rable 4.3.

Whether the density parameter, given as $(s_s-1)^{0.5}$, best correlates solid-liquid mixtures other than sand-water is difficult to assess from the reported data. Higher relative density mixtures tend to decrease the critical deposit velocity value as demonstrated by the Sinclair (1962) and Wasp et al. (1970) data, and as explained in Section 4.1, after Ellis (1963b). Whereas, the lower density suspensions reported by Wilson (1965), and shown with the present study, fall significantly above the Lehigh sand-water data.

	Sediment Size	Pipe Size	Sediment Conc.	Specific Gravity	Remarks
	d [mm]	D	C	ρ_s/ρ	
Sinclair (1962) ○ ●	0.12 0.09	0.50, 0.75, 1.00 in.	up to 20%	10.37 Iron/ 0.78 Kerosene	V_C is reported on V_C vs. C plots
Wilson (1965) ⬡	3.88	3.48 in., 3.69 in.	up to 20%	1.14 Nylon/ Water	V_C is shown on head loss curves
Wicks and* Moye (1968) △ ▲	0.25	1.05 in., 5.50 in.	1.0%	△ 2.65 Sand/ 0.91 Oil ▲ 2.65 Sand/ 0.81 Kerosene	V_C is reported
Lehigh (1971) ◩	3.63	6.00 in.	up to 5%	1.38 Plastic/ Water	V_C is reported

*Reported by Wasp et al. (1970)

Table 4.3: Range of Parameters of the Data Reported by Other Investigators for Solid/Liquid Mixtures other than Sand/Water; Data Plotted in Fig. 4.4

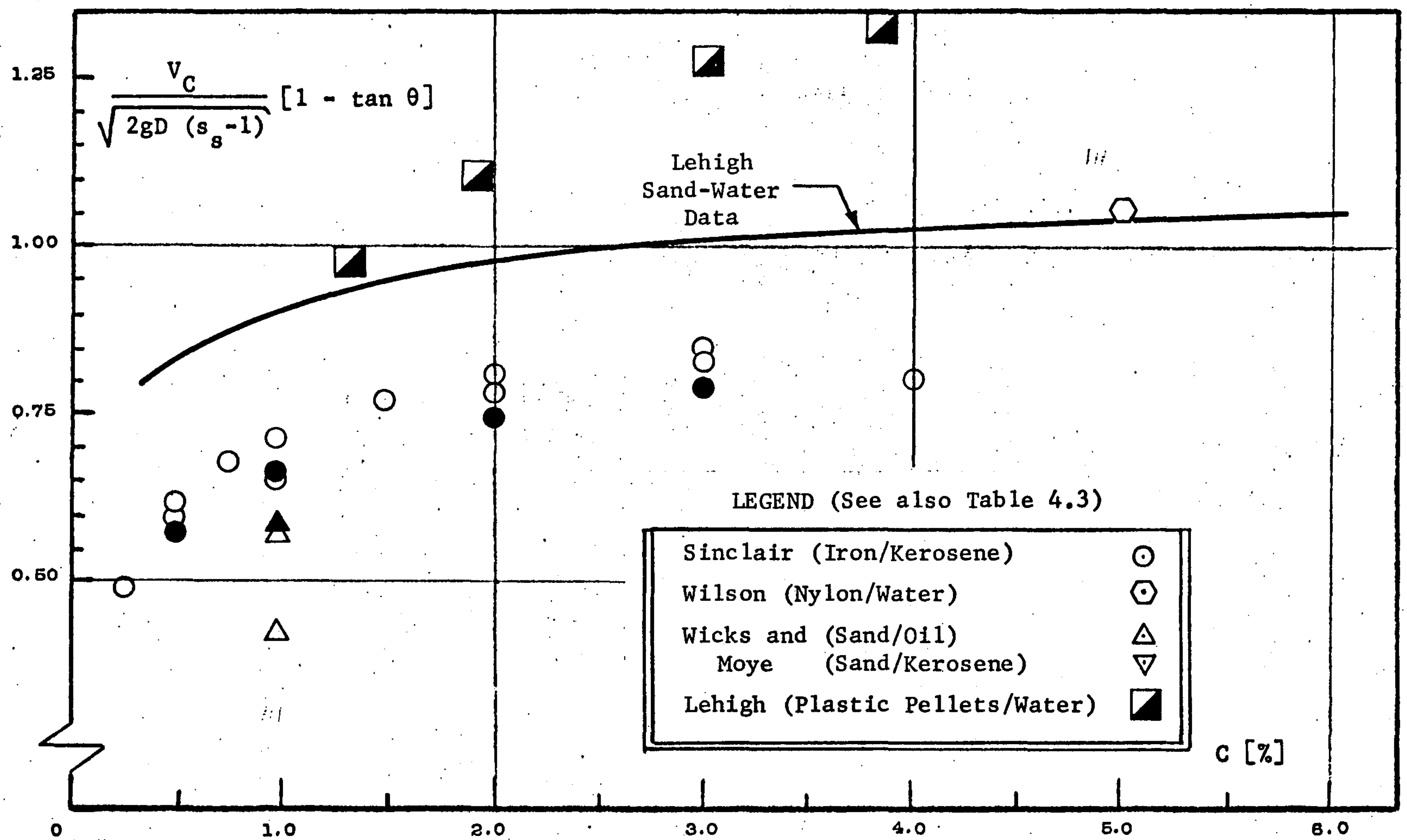


Fig. 4.4: Modified Froude Number versus Solids Concentration, Particle Diameter as Parameter (Data from Studies of other than Sand-Water Mixtures)

Although these results are inconclusive, it is suggested to use the modified Froude number relationship, in the form given with Eq. (2.13), till further data on non-sand-water mixtures become available.

4.3 Engineering Application

An engineer, confronted with the task of designing a solids transport system, finds that a theoretical application of critical condition transport has many limitations. In another instance, he may be unable to apply one particular approach, because its validity has not yet been tested for the type of mixture slurry he is considering. Furthermore, he is usually provided with little or no information on the economic factors to be considered in installation and operation of the system. The basic problem in design is one of safe operation and minimization of the costs to transport the mixture.

The critical deposit velocity relationship, as defined in the present study with either Eq. (4.3) or Eq. (4.4), provides the designer with a useful tool with which he may define the optimal operating conditions of the system. To ensure safe, uninterrupted transport of the mixture, the designer must also properly select pump, pipe material and instrumentation, after consideration of basic hydraulic parameters and power requirements. Condolios (1963b & c) and Graf (1971) treat the subject of solids pipeline operation with considerable proficiency.

4.3.1 Economics of Solid-Liquid Transport Systems

A rather attractive feature of the solid-liquid transport pipeline is the minimal cost required for operation and maintenance, as compared with the conventional means of transporting solids. In addition to the revealing economic advantages, pipelines are ammenable to automation, are dependable, and can overcome both natural and man-made obstacles.

Operating costs are minimized when the power required for transport is held to a minimum, however, certain precautions must be taken. The minimum power input and the minimum mixture head loss, i_m, are coincident and identify a region in which the system may become unstable. This leads inevitably to plugging of the system. Operation in this region is unsafe, and slightly higher flow velocities should be maintained to avoid system instability. Condolios et al. (1963b), Ellis et al. (1963a), and Wilson (1965) discuss application of the minimum power requirement in design.

The critical deposit velocity, V_C, is often found within the region of instability. It has been observed by Condolios et al. (1963b), Wilson (1965), and within the present study that the relationship between critical deposit velocity and the velocity corresponding to the minimum head loss is as given with Fig. 4.5. V_C is higher than the velocity associated with the minimum head loss at low concentrations - however, the opposite is true for $C > 5\%$. An explanation for this occurrence is reported by Wilson (1965). The heavy line in Fig. 4.5 represents a recommended envelope for determining the stable operating flow velocity.

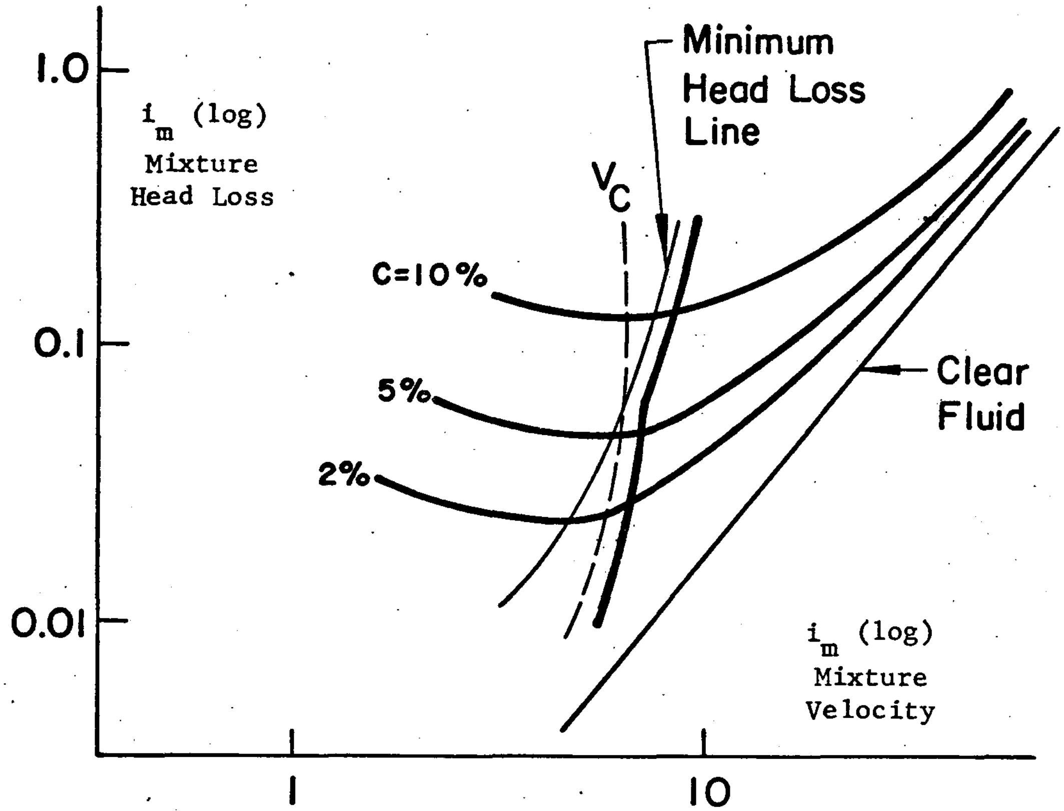


Fig. 4.5: Critical Velocity and the Velocity Corresponding to the Minimum Head Loss

Condolios et al. (1963c) report on instability of the pump characteristic curve, due to the fluctuations of solids concentration during operation. The designer must consider the characteristic stage-discharge curves of the pump in comparison with the mixture head loss curves for the pipeflow to ensure stable design.

A method for optimizing solids concentration, C, and pipe size, D, was reported by Hunt et al. (1968). Although some preliminary economic considerations of solids pipelining have been reported by Wasp et al. (1967), the relationship between hydraulic and economic decision variables had not been presented analytically. Hunt et al. (1968)

minimize a function containing seven cost groups and hydraulic parameters, with respect to C and D. The response surface generated by this cost function yields various combinations of C and D and the most suitable are selected for design.

The engineer, in designing a solid-liquid transport system, must concern himself with some basic considerations:

Installation:

(1) Physical characteristics of the mixture
(2) Adequate pumping facility
(3) Flushing and drainage
(4) Pipeline wear and corrosion

Operation:

(1) Physical characteristics of the mixture
(2) Stability of pipeflow
(3) Stability of pump operation
(4) Optimum delivery of solids

Lowenstein (1959), Ellis et al. (1963a), and Roberts (1967) present different methods for designing economically practical transport systems. Use of the Lehigh findings as a basic criterion in the design procedure is presented now.

4.3.2 Application of the Lehigh Findings to Design

The "critical condition" has seldom been used as a criterion for designing economic transport systems. The apparent reason is that relationships for the critical deposit velocity have been vague in conclusive evidence and thus, engineers have retained little confidence in their application. The Lehigh findings provide the designer with that criterion which will minimize the cost of operation and ensure safe, uninterrupted flow conditions.

For designing a system to transport sand with particle diameters, $0.45 < d < 0.88$ mm, in water, Eq. (4.3) is recommended, and is rewritten here as:

$$V_C = 0.901\ C^{0.106} \sqrt{2gD\ (s_s - 1)}\ \frac{1}{[1 - \tan\theta]} \tag{4.3'}$$

If the sand particle sizes are larger, $d > 0.88$ mm, Eq. (4.4) is recommended and can be rewritten as:

$$V_C = 0.928\ C^{0.105}\ d^{0.056} \sqrt{2gD\ (s_s - 1)}\ \frac{1}{[1 - \tan\theta]} \tag{4.4'}$$

Equation (4.4') will give more conservative values for V_C than Equation (4.3'), as particle size, d, increases in size over 0.88 mm. For particle sizes smaller than 0.45 mm, neither Eq. (4.3') nor Eq. (4.4') are recommended. One is then referred to Gibert (1960). Roberts (1967) presents a general method for extrapolating data to regions outside of the tested bounds, application of which would enable more extensive use of the Lehigh equations.

To illustrate general application of the Lehigh critical deposit velocity equations, Eqs. (4.3') and (4.4'), and Fig. 4.2, two typical design problems are examined.

Example (1). Suppose a long distance minerals-water mixture transport system is to be designed for a certain delivery rate of solids, Q_s (defined as tons/mile/hr), and given with diameter, d, and specific gravity, s_s. What parameters must the designer consider to minimize costs?

Delivery rate, Q_s, is defined with the following relationship:

$$Q_s = Q_m C = V_m A C \tag{4.5}$$

where Q_m is the mixture flowrate. It is recommended that the critical deposit velocity criterion, resulting from the present study, be employed. Equation (4.5) is therefore considered to be minimized with respect to unit costs by replacing V_m with V_C and rearranging:

$$Q_s' = \frac{\pi}{4} V_C C D^2 \tag{4.6}$$

where Q_s' now represents optimum solids throughput.

If particle diameter, d, as an example, is slightly larger than the range of particle sizes tested in this study; i.e., $d \sim 0.10$ mm, we can substitute Eq. (4.4) into Eq. (4.6) and obtain:

$$Q_s' = \frac{\pi}{4} 0.928 C^{0.11} d^{0.06} \sqrt{2gD (s_s-1)} \frac{1}{[1 - \tan \theta]} C D^2 \tag{4.7}$$

rearranging:

$$Q_s' = 5.85 C^{1.11} d^{0.06} D^{2.5} (s_s-1)^{0.5} (1 - \tan \theta)^{-1} \tag{4.8}$$

Note that this equation is similar in form to the relationship given by Eq. (2.10), but it is pointed out that the exponents and coefficient of Eq. (4.8) are constant over the entire range of Lehigh data, and the relation can be extrapolated in many instances to include parameters outside these tested ranges.

The pipe slope, tan θ, is identified, through a topographic survey, as to where it will be a maximum. From Eq. (4.8) the most equitable combination of concentration, C, and pipe size, D, can be determined through trial and error. If concentration is larger than 5%, extrapolation of the Lehigh data must be undertaken with caution. If the particle diameter, d, of the slurry to be transported is $0.45 < d < 0.88$ mm, Fig. 4.2 can be used directly and optimum modified Froude numbers located readily.

Example (2). Consider the design of a pressurized solid-waste disposal system. A difficulty encountered with the hydraulic transport of solid wastes is the identification of slurry characteristics. Non-Newtonian suspensions cause a problem which is not considered within the scale of this study, however, real concern is for the settling and possible clogging due to grit and sand in the mixture slurry.

If a system is designed to handle a specified concentration of settleable solids from domestic disposal units, will the 'working' operating velocity become a critical deposit velocity, or more seriously, a sub-critical, unstable flow velocity, if solids concentration is suddenly increased? The characteristics of the grit concentration, given with d and (s_s-1), dictate which Lehigh design equation is to be used. From either Eqs. (4.3), (4.4), or Fig. 4.2, the variation in modified Froude number, with increase in concentration, C, is observed. Subsequently, a new value for V_C is defined and compared to the original conservative operating velocity.

The application of the Lehigh equations can be extensive, considering that extrapolation is performed with caution, and one understands clearly the definition and relative influence of each parameter.

5. CONCLUSIONS

The critical deposit velocity, V_C, tested in the form of a modified Froude number, is correlated with other parameters, which is significant in the solid-liquid transport problem, over the following ranges:

$$
\begin{aligned}
0.01 &\le C \le 7.00\ \% \\
0.45 &\le d \le 0.88\ \text{mm} \\
4.00 &\le D \le 6.00\ \text{in.} \\
-0.060 &\le \tan\theta \le 0.027 \\
1.07 &\le d_{90}/d_{50} \le 1.21 \\
0.00009 &\le \varepsilon/D \le 0.00032
\end{aligned}
$$

From a dimensional analysis of these parameters, a modified Froude number relationship is developed, as given with Eq. (2.13). The relationship is tested for sand-water and plastic pellets-water transport. Data from the sand-water tests exhibit the following:

(1) Agreement with the Gibert (1960) curves for particle diameters, $d \ge 0.37$ mm.

(2) The increase in critical deposit velocity, V_C, becomes less evident as solids concentration, C, rises to 5%; above 5%, V_C tends either to remain constant or decrease with increase in C. [This was also observed by Sinclair (1962) and Wilson (1965)].

(3) For particle sizes, $d \ge 0.37$ mm, the critical deposit velocity remains practically unchanged with increase in d.

(4) The critical deposit velocity is higher than the

velocity associated with the minimum head loss at low concentrations; however, the opposite is true for $C > 5\%$.

Findings from the plastic pellet-water test data were inconclusive.

A regression analysis, made to correlate the Lehigh data, shows that the modified Froude number is highly dependent on concentration, C, slightly affected by particle diameters, $d \geq 0.37$ mm, and hardly influenced by relative particle size, d/D. The regression equations which best fit the data and are in reasonable agreement with data from other sand-water studies, are given with:

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\left[1 - \tan\theta\right] = 0.901\ C^{0.106} \tag{4.3}$$

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\left[1 - \tan\theta\right] = 0.928\ C^{0.105}\ d^{0.056} \tag{4.4}$$

Although the reliable application of these equations for solid-liquid mixtures other than sand-water has been inconclusively resolved, it is suggested to use Eqs. (4.3) and (4.4) in their present form till further data on non-sand-water mixtures become available.

The Lehigh critical deposit velocity equations give conservative values, and are presently the only relations available for predicting critical deposit velocities for low-concentration solid-liquid mixtures. It is recommended that either Eq. (4.3) or Eq. (4.4) be used as a critical deposit velocity design criterion, certainly within the range of parameters tested in the present study, and cautiously in ranges of parameters extending outside of the tested bounds.

APPENDIX A: EVALUATION OF LOOP READINGS FROM PROGRAMMED OUTPUT

Determination of Q_m and C

The "Loop System" became a useful tool for quickly determining the mixture flowrate, Q_m, and solids concentration, C, once the programmed output was plotted. Enlarged sections of Chart 1 and Chart 2, from Fig. 3.5, are shown in Figs. A.1 and A.2, respectively. With reference to these two charts, the determination of Q_m and C from loop head loss readings will be examined.

System water temperatures during a test run sometimes increased from 60°F, at the beginning of the run, to 100°F, after high flowrate testing of a large solids concentration mixture. The loop indication of mixture flowrate is appreciably affected by temperature changes, and since it could not be easily controlled, readings at temperatures of both 70°F and 90°F were plotted on Chart 1. Water temperatures were recorded during the progress of a test and employed in the evaluation of Q_m and C, but they are not reported in the data of Appendix B.

Recording for one test, Δh_R, the riser pressure drop, and, Δh_D, the downcomer pressure drop, the concentration, C, would normally be determined immediately from locating $(\Delta h_R - \Delta h_D)$ on Chart 2, since this relationship is hardly a function of flowrate, Q_m. Proceeding then to Chart 1 and knowing C, $(\Delta h_R + \Delta h_D)$, and temperature, Q_m, would be located.

However, through repeated clear-water calibration of the loop system, riser readings were observed to be consistently greater than those of the downcomer and generally increasing with mixture flowrate. These differences were attributed to insufficient transition length, incompletely dissipating the local turbulence effects following the elbow bends. The trend of deviation is shown in the "correction curve" below Chart 2 in Figs. 3.5 and A.2*. The difference was assumed to be equally shared by the two vertical sections, such that the $(\Delta h_R + \Delta h_D)$ reading needed no correction. The $(\Delta h_R - \Delta h_D)$ reading acquired the full correction directly. To better illustrate the additional implications and convergence on Q_m and C values, an example is presented.

In Series G-02-3 of Appendix B (tests of coarse sand transport through a downward sloping, 4-inch galvanized pipe), the first set of loop readings recorded are:

*For later investigations of plastic pellet and additional low concentration sand flows, the transition length before the loop pressure taps was extended 3 ft. This greatly reduced the correction curve to a nearly constant - 0.2 values over the entire range of flowrates.

$$\Delta h_R = 33.00 \text{ in.}$$

$$\Delta h_D = 11.05 \text{ in.}$$

Consequently, resulting in:

$$\Delta h_R + \Delta h_D = 44.05 \text{ in.}$$

$$\Delta h_R - \Delta h_D = 21.95 \text{ in.}$$

The system temperature for this particular run was recorded at 82°F.

A first approximation of concentration, C, obtained from Chart 2, would be 10%. On Chart 1, Fig. A.1, an 80°F recording for 10% mixture concentration would fall at point b in correspondence to the summed head loss value at a. Interpolated to an 82°F reading, point b shifts to c, locating Q_m = 410 gpm. In Fig. A.2, the correction value at d, corresponding to Q_m = 410 gpm, is -1.35 in. Applied to the head loss differential at point e on Chart 2, an adjusted differential head loss, of 21.95-1.35 = 20.60 in., is located at f. The resulting C = 10.5% was considered close enough to the original assumption of C = 10% to warrant acceptance of the values:

$$Q_m = 410 \text{ gpm}$$

$$C = 10.5\%$$

Further iteration of this procedure was seldom required, if an approximate correction value was considered in the first attempt.

When both the flowrates, Q_m, and volumetric concentrations of solids, C, were in their upper ranges, discrepancy of loop readings from Prandtl and Pitot tube observations was often detected. Adjustment of these readings is now discussed.

Adjustment of Q_m and C in the Heterogeneous Flow Regime

It was observed that the magnetic flowmeter readings were systematically higher than the velocity readings given by the loop. Further, visual observation of the flowing mixture indicated an apparently greater volumetric concentration of solids than determined by the loop. These discrepancies were particularly noticeable at

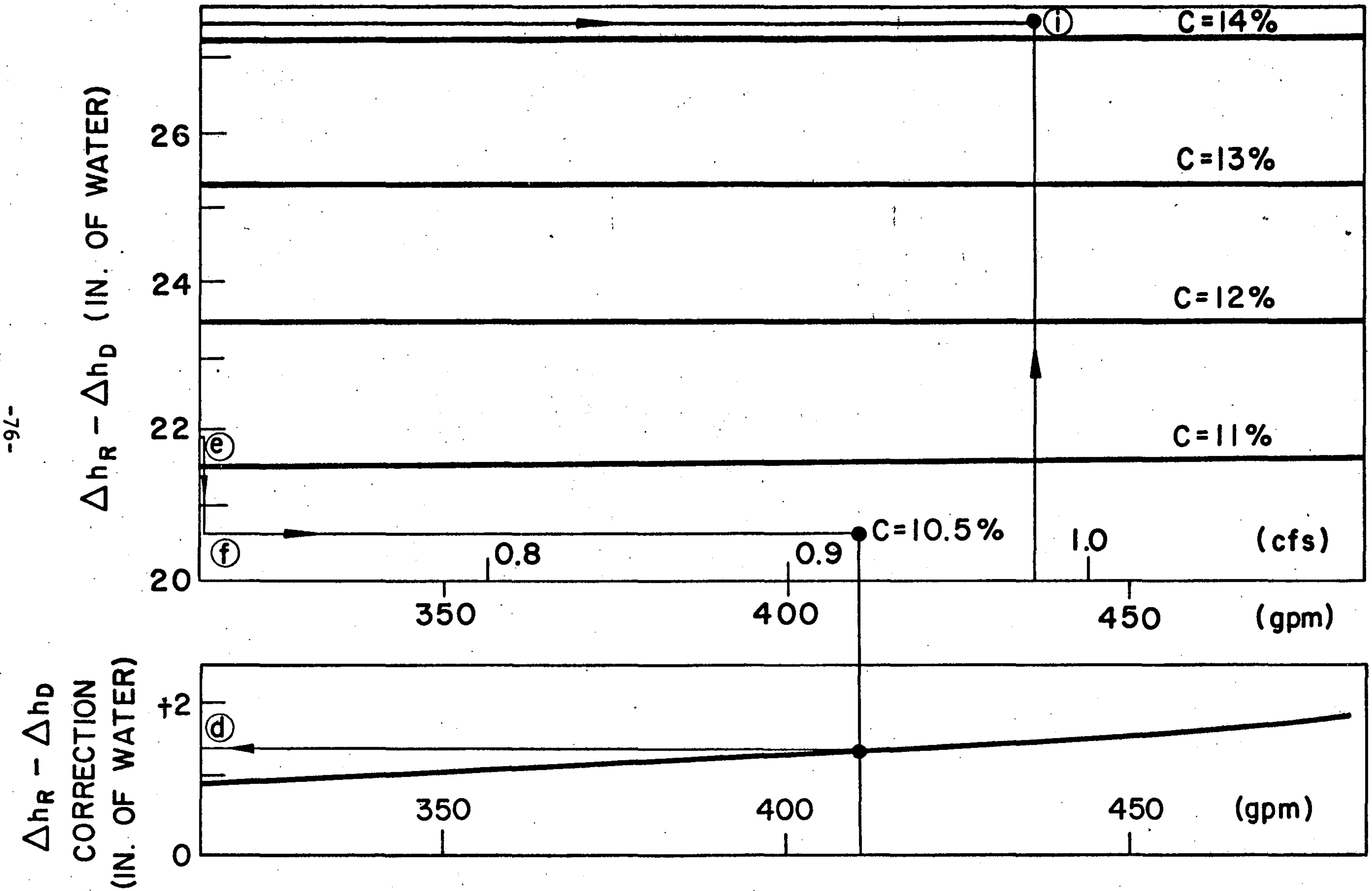


Fig. A.2: Enlargement of Chart 2 and Correction Curve in Fig. 3.5

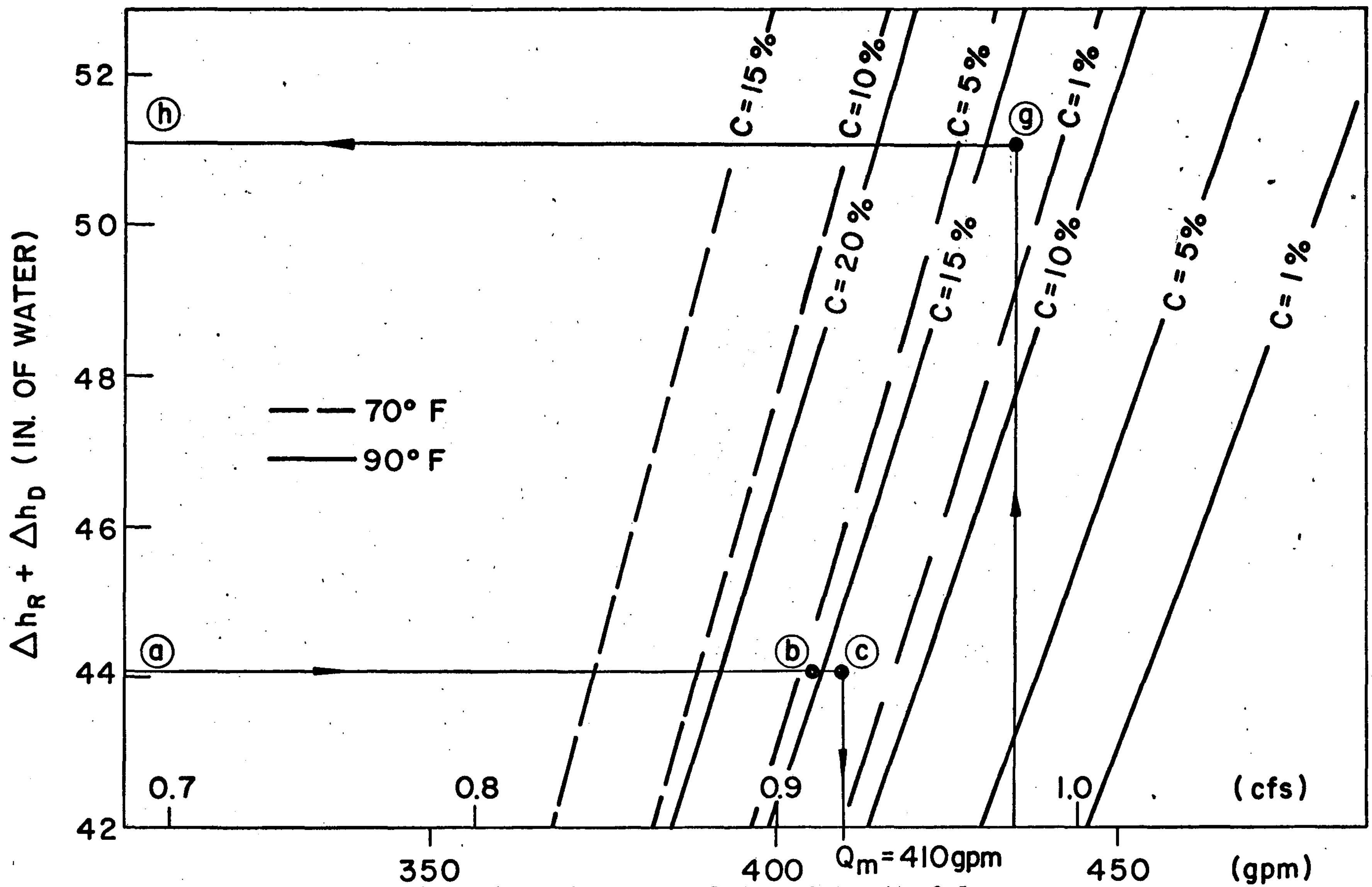


Fig. A.1: Enlargement of <u>Chart 1</u> in Fig. 3.5

flowrates and concentrations above the critical condition, well into the heterogeneous flow regime.

To assure confidence in the "Loop System" recordings of mixture flowrate, Prandtl tube traverses for clear-water flow were run over a range of flowrates between 160 and 600 gpm. Reliability was placed in the Prandtl tube results and were used to calibrate the Foxboro Magnetic Flowmeter. Within the range of flowrates tested, the flowmeter was found to be consistently indicating flowrates 12.5% in excess of the actual flow conditions. It was felt that the magnetic flux method of determining flowrate would be accurate in measuring mixture flow upon the entrainment of solids in the system, such that loop readings could be evaluated from flowmeter recordings using the 12.5% correction. Flowmeter indications of Q_m were indeed found to be greater than the loop, and the discrepancy increased with larger flowrates and larger concentrations, although never exceeding 8%.

A Pitot tube sediment-sampling device was employed to evaluate loop indications of solids concentration. The copper sampler was unable to withstand the sand-blast effect of the larger particles, however, samples were obtained for the finer sand. The difficulty of velocity flow equalization within the system and sampler was apparent, but an insignificant deterent for establishing some degree of reliability in the sampling results. It was discovered that the concentrations evaluated using the sediment-sampling device were also larger than those given by the loop. The discrepancy increased with flowrate and solids concentration to magnitudes of up to 50%.

Explanation of these unexpected discrepancies implicates a study in itself, and within the scope of this study, only a method of adjustment can be determined. The method recommended for adjusting the heterogeneous flow regime data is explained in what follows.

Considering the same set of data just examined, a flowmeter reading and Pitot tube sample might have respectively indicated:

$$Q_F = 490 \text{ gpm} \quad (\text{actual } Q_F \Rightarrow 490 \times 0.89 = 435)$$

$$C_p = 14\%^*$$

Digression from the loop readings is markedly significant and is represented as:

*The sediment-sampling device was clogged and damaged when testing the coarser sand so that the method of correction used for fine sand could only be assumed applicable to the coarser sand concentrations.

$$Q_F - Q_m = 435 - 410 = 25 \text{ gpm} \quad (6\% \text{ discrepancy})$$

$$C_p - C = 14 - 10.5 = 3.5\% \quad (33\% \text{ discrepancy})$$

The sum of the "Riser" and "Downcomer" head readings was first adjusted by locating on Chart 1, as illustrated in Fig. A.1, point (g) indicating a corrected value at (h):

$$(\Delta h_R + \Delta h_D)_{corr} \approx 51.0''$$

The deviation between flowmeter and loop reading is denoted as:

$$(\Delta h_R + \Delta h_D)_{corr} - (\Delta h_R + \Delta h_D) \approx 7.0''$$

It is then observed that the identical adjustment of head difference most completely corrects the concentration reading. This is shown on Chart 2, of Fig. A.2, where C of 14% is located at (i), following the appropriate adjustment of both $(\Delta h_R - \Delta h_D)$ and Q_m.

These findings were consistent at all concentration and flowrate combinations and became an integral part of a venturimeter investigation, Robinson et al. (1970). It was noted that at low flowrates and low concentrations, both the magnitude of deviation and percentage correction were no longer significant to warrant serious concern. Since the primary interest in the present study was in the critical velocity range for low concentrations, the minor adjustment, as discussed in this section, was deemed unnecessary. However, when applying heterogeneous flow data, from Appendix B, there should be consideration of appropriate adjustments, as just illustrated.

Figure A.3 is a useful tool for approximating the necessary corrections for any combination of Q_m and C up to 600 gpm and 15%, respectively. Q_I and C_I represent the recommended percentage increase over the loop values.

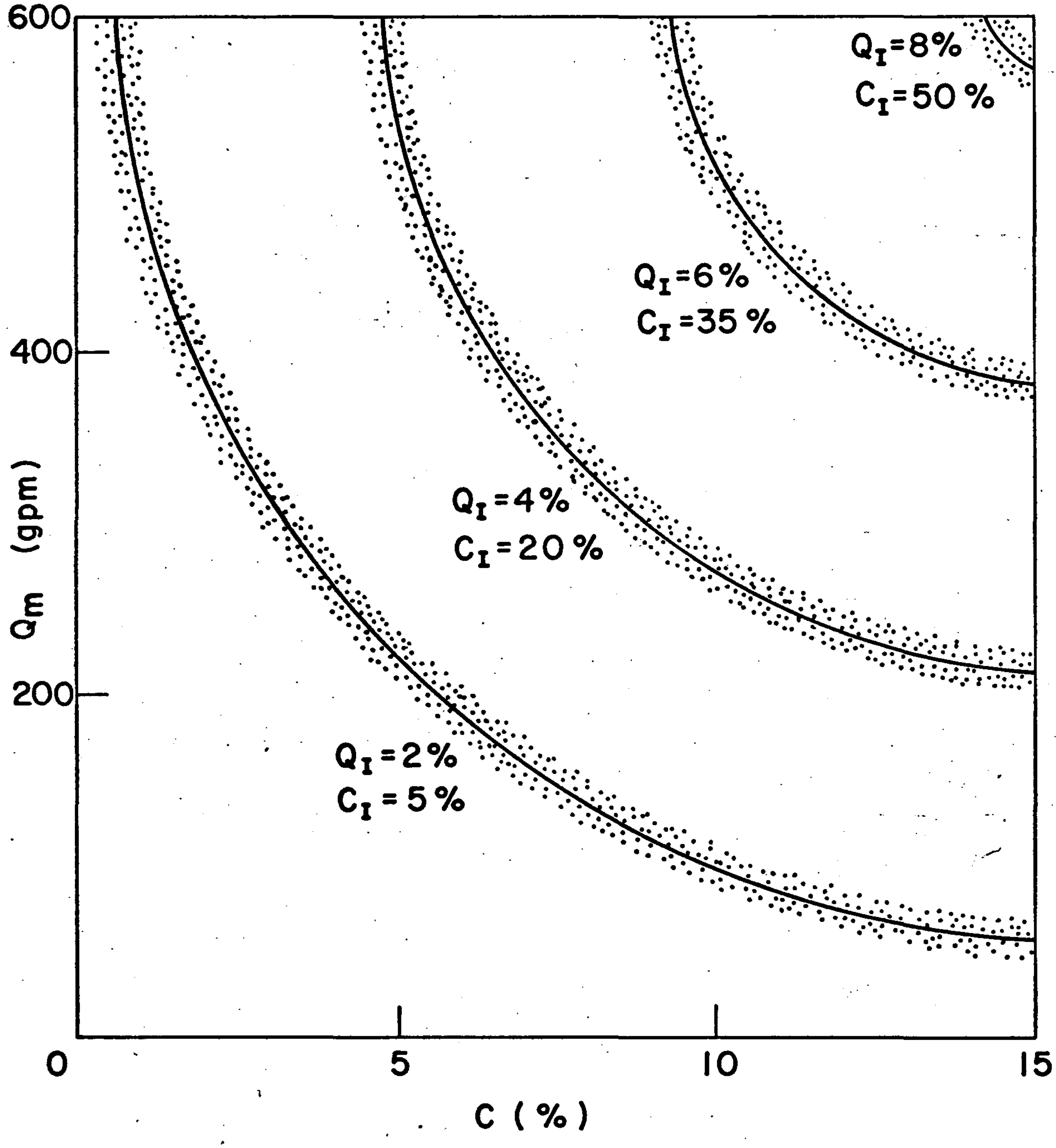


Fig. A.3: Percentage Increase Corrections of Both Flowrate and Concentration for all Combinations of the Two Parameters

APPENDIX B: TEST DATA COMPILATION

Parameters of primary significance in their effect on the critical deposit velocity are: The inside pipe diameter, D, the pipe material roughness, ϵ, the slope of the pipe, S, the mean sediment particle size, d_{50}, with consideration of the non-uniformity coefficient, d_{90}/d_{50}, and the specific weight of the solids, s_s. These parameters have been varied to determine how each enters into the modified Froude number relationship, defined in the text of this paper. The series of tests are coded with the following convention:

G - 01 - No.

G - 02 - No.

G - 001 - No.

G - 002 - No.

BS - 01 - No.

BS - 03 - No.

BS - 001 - No.

BS - 003 - No.

BS - PP1 - No.

RUN NUMBER (Indicates change in concentration

PIPE SLOPE
- 1 - Horizontal
- 2 - Downward, -0.060
- 3 - Upward, +0.027

SOLID PARTICLES
- 0 - d_{50} = 0.88 mm, $\frac{d_{90}}{d_{50}} = 1.21$, $s_s = 2.65$
- 00 - d_{50} = 0.45 mm, $\frac{d_{90}}{d_{50}} = 1.07$, $s_s = 2.65$
- PP - d_{50} = 3.63 mm, $s_s = 1.38$

PIPE
- G - 4 in. Ø Galvanized, ϵ = 0.00001 ft.
- BS - 6 in. Ø Black Steel, ϵ = 0.00016 ft.

Explanation of the Table Headings

Test Section: (Over a $\Delta\ell$ = 3.60 m (= 141.8 in.) test section the head loss was determined; U-tube manometers were used).

$\Delta h_{1.95}$: Measured mixture head loss (in inches of a liquid with a specific gravity of s_s = 2.95).

or

Δh_{H_2O} : (in inches of water)

$(\frac{\Delta h}{\Delta \ell})_m$: Mixture head loss gradient (calculated from $\Delta h_{1.95}$).

Loop Readings: (The "Loop System" developed by Einstein and Graf (1966) was used to simultaneously determine the mixture flowrate, Q_m, and the solid phase concentration, C.

Δh_R; Δh_D: Head losses in the Riser and Downcomer sections (3-inch pipe, 1.50 m (=59.1 in.) long; U-tube manometers are used).

$\Delta h_R + \Delta h_D$: Sum of the head losses.

Q_m : Mixture flowrate, according to theory of Einstein and Graf (1966), from the sum of the head losses.

V_m : Mixture velocity in test section determined with continuity relation.

$\Delta h_R - \Delta h_D$: Difference of the head losses.

$\Delta h_R - \Delta h_D$ (corr.) : Correction of above from predetermined clear-water test correction curve.

C : Concentration, determined according to theory of Einstein and Graf (1966), from the difference of the head losses.

Comments: Commentary of observations in Plexiglas section on the conditions of sediment transport and deposit.

Each table is summarized indicating the critical condition; this is the critical velocity, V_C, for a specific concentration, C.

Some Remarks to the Figures

Plotting of the data follows on mixture head loss versus mixture velocity graphs. These graphs show the variation of critical velocity, V_C, with a change in solids concentration. Constant concentration lines are fitted to the data, and the critical velocity for a particular concentration, subjectively observed as the velocity at which a non-moving bed forms on the bottom of the pipe, is located. At velocities below the critical, equi-concentration (constant "moving" concentration) lines are dashed (---), while the diminishing concentration line for a particular run is drawn solid (—).

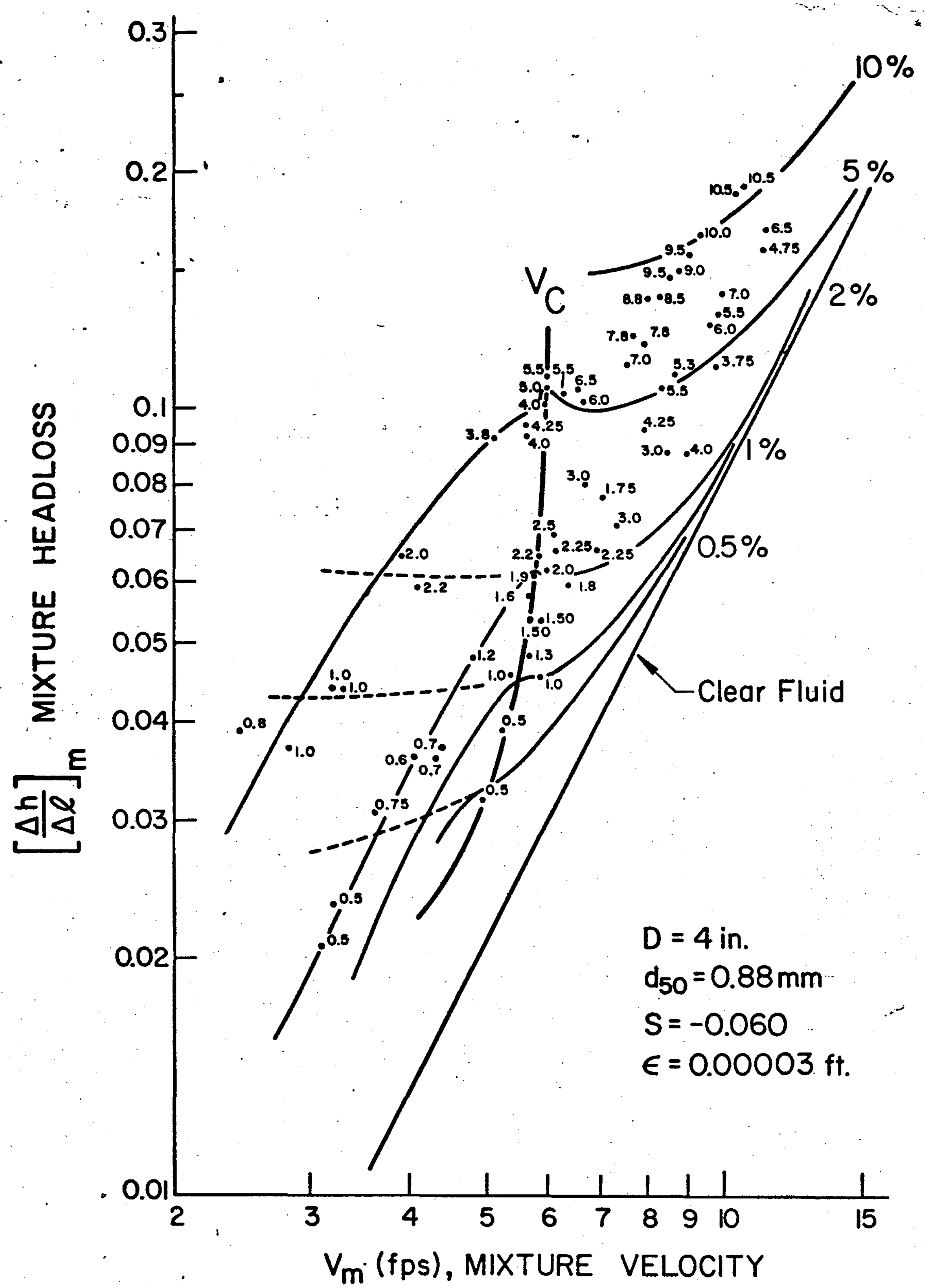
The relationship between critical velocity and the minimum head loss condition can be qualitatively examined.

Some Remarks to the Data

It was explained in Appendix A that some of the data recorded at high flowrates and high solids concentrations require adjustment according to observed Prandtl and Pitot tube corrections, as shown in Fig. A.3. These adjustments were found to be insignificant in the critical velocity ranges, hence, the data remain as recorded from the loop readings.

It is also to be noted that some drafting errata in pipe roughness values, ϵ, have been corrected since the first reporting of this data, Graf et al. (1970). Except for the inclusion of test data from plastic pellet and additional low concentration sand mixture flows, the original data remains unaltered.

These more recently obtained data were not included on the head loss figures, but are of extreme significance in the final evaluation of this problem. They are tabled under Series G-01-6 to G-01-11, G-001-5 to G-001-10, and BS-PP1-1 to BS-PP1-4 inclusively. It has been noted in the text that for these studies, an improved clear-water correction value was applied.



Plot of Series G-01 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
10.50	0.158	29.55	18.90	48.45	440	11.15	10.65	9.05	4.75	Everything moving
8.30	0.114	23.10	14.65	37.75	385	9.7	8.45	7.35	3.75	Suspended and bed load
6.40	0.088	17.65	11.0	28.65	335	8.45	6.65	5.75	3.0	Suspended and bed load
5.60	0.077	11.60	7.70	19.30	275	7.0	3.90	3.30	1.75	Moving bed
3.30	0.0455	8.10	5.90	14.0	230	5.85	2.20	1.70	1.0	Pulsating, sliding bed
2.80	0.039	6.40	4.80	11.2	205	5.2	1.60	1.20	0.50	Pulsating, sliding bed
2.30	0.032	5.30	4.20	9.5	195	4.95	1.10	0.70	0.50	Slowly moving bed CRITICAL
2.30	0.032	5.30	4.15	9.45	195	4.95	1.15	0.65	0.50	Just below critical

Series G-01-1

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.50\% \\ V_C = 5.0 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
6.40	0.088	19.40	10.60	30.0	350	8.9	8.80	7.80	4.0	Suspended and bed load
5.20	0.0715	13.80	7.70	21.50	290	7.35	6.10	5.30	3.0	Everything moving
4.80	0.066	11.80	6.80	18.60	270	6.85	5.00	4.40	2.25	" "
4.30	0.0592	10.40	6.40	16.80	250	6.35	4.00	3.50	1.8	Moving slowly
3.90	0.0535	8.80	5.50	14.30	230	5.85	3.30	2.80	1.50	Moving bed, thickening layer
3.30	0.0455	7.10	4.75	11.85	210	5.35	2.35	2.00	1.0	Deposit bed CRITICAL
3.50	0.0481	7.60	4.95	12.55	220	5.65	2.65	2.30	1.3	Bottom limit of moving bed
2.60	0.0358	6.0	3.90	9.90	190	4.3	2.10	1.70	0.8	Below critical

Series G-01-2

$$\text{CRITICAL CONDITION} \begin{cases} C = 1.00\% \\ V_C = 5.5 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.30	0.128	23.95	11.10	35.05	375	9.55	12.85	11.75	6.0	Everything moving
8.10	0.111	20.35	9.0	29.35	340	8.65	11.35	10.35	5.3	" "
6.80	0.094	16.60	7.50	24.10	310	7.9	9.10	8.30	4.25	" "
5.80	0.080	12.00	5.75	17.75	265	6.7	6.25	5.65	3.0	Sliding bed
5.00	0.069	10.30	5.10	15.40	235	6.1	5.20	4.70	2.5	Pulsating bed
4.50	0.062	8.90	4.75	13.65	230	5.95	4.15	3.75	2.0	Bed slowly moving
										CRITICAL
4.20	0.058	8.25	4.50	12.75	220	5.6	3.75	3.35	1.75	Non-moving bed
2.60	0.036	4.50	2.95	7.45	160	4.1	1.55	1.25	0.6	Flat bed
1.70	0.0235	2.90	1.50	4.40	125	3.2	1.40	1.10	0.5	Long dunes
					2nd Run					
12.20	0.168	31.85	17.70	49.55	450	11.45	14.15	12.55	6.5	Everything moving
9.50	0.131	24.55	12.50	37.05	390	9.8	12.05	10.85	5.5	" "
6.70	0.092	16.35	8.35	24.70	310	7.9	8.00	7.25	3.75	" "
4.80	0.066	10.15	5.30	15.45	240	6.05	4.85	4.35	2.25	Sliding, pulsating bed
4.20	0.058	8.75	5.10	13.85	230	5.85	3.65	3.15	1.6	Bed slowly moving
4.10	0.0565	8.55	5.75	13.5	225	5.75	3.60	3.10	1.6	Critical
3.90	0.0535	7.70	5.95	12.45	220	5.65	2.95	2.65	1.5	Deposit
3.20	0.044	3.75	5.60	6.60	145	3.7	0.90	0.60	0.3	Flat bed

CRITICAL CONDITION: C = 1.75%, V_C = 5.75 fps

Series G-01-3

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
10.20	0.140	26.75	12.10	38.85	390	9.9	14.65	13.45	7.0	Suspended and bed load
7.70	0.106	20.10	8.70	28.80	330	8.35	11.40	10.50	5.5	Suspension, mostly bed load
5.90	0.081	11.85	5.50	17.35	260	6.6	6.35	5.75	3.0	Fast moving bed
4.70	0.0645	9.25	4.65	13.90	230	5.85	4.60	4.10	2.2	Sliding bed
4.40	0.0605	8.55	4.50	13.15	225	5.75	4.05	3.55	1.9	Just above V_C
4.20	0.0578	7.95	4.20	12.15	220	5.65	3.75	3.25	1.8	Non-moving bed CRITICAL
3.50	0.0481	6.65	3.85	10.50	190	4.8	2.80	2.45	1.2	Flat bed
2.20	0.0302	3.60	2.10	5.70	140	3.6	1.50	1.20	0.8	Flat bed--thinning (long dunes)
1.50	0.0206	2.40	1.50	3.90	120	3.1	0.90	0.60	0.5	6' long dunes at 2 intervals
4.40	0.0605	8.50	4.0	13.50	230	5.85	4.50	4.0	2.0	Scour (long impulse variations)

Series G-01-4

CRITICAL CONDITION $\begin{cases} C = 2.00\% \\ V_C = 5.75 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
12.10	0.167	28.55	7.95	36.50	365	9.25	20.60	19.50	10.0	Everything moving
10.70	0.148	24.85	5.80	30.65	335	8.45	19.05	18.15	9.5	" "
10.10	0.139	22.80	5.00	27.80	315	8.0	17.80	17.0	8.8	Heavy bed load
8.70	0.120	19.40	3.75	23.15	290	7.95	15.65	15.0	7.8	Quickly moving bed just above crit.
7.70	0.106	15.45	2.70	18.15	255	6.5	12.75	12.15	6.5	Deposit - and immediate scour
7.60	0.1045	14.65	2.50	17.15	245	6.2	12.15	12.15	6.5	Still squirming, pulsating bed
7.90	0.109	13.60	2.35	15.95	235	5.95	11.25	10.75	5.5	Above critical
										CRITICAL
7.30	0.101	10.75	2.60	13.35	230	5.95	8.15	7.65	4.0	Non-moving bed
6.70	0.092	9.35	1.50	10.85	200	5.1	7.85	7.45	3.8	Flat bed
4.70	0.065	5.40	1.35	6.75	150	3.9	4.05	3.75	2.0	Long flat dunes
2.70	0.037	2.90	1.10	4.00	110	2.8	1.80	1.80	1.0	Long flat dunes

Series G-01-5

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
						2nd Run				
13.90	0.191	32.90	10.70	43.60	410	10.45	22.20	20.90	10.5	All suspended
11.40	0.158	26.30	6.90	33.20	355	9.0	19.40	18.30	9.5	" "
10.70	0.139	22.25	5.30	27.55	325	8.25	16.95	16.15	8.5	Bed load
8.20	0.113	18.30	4.20	22.50	295	7.5	14.10	13.40	7.0	Slowly moving bed
7.50	0.103	15.40	2.90	18.30	260	6.6	12.50	11.90	6.0	Pulsating-sliding bed
7.80	0.1075	12.40	2.10	14.50	235	5.95	10.30	9.80	5.0	Bed just slightly moving
										CRITICAL
6.70	0.092	10.30	2.60	12.90	220	5.6	7.90	7.20	4.0	Non-moving bed just below critical
4.30	0.059	5.80	1.30	7.10	160	4.1	4.50	4.20	2.2	Flat bed, great saltation
3.20	0.044	3.50	1.20	4.70	130	3.3	2.30	2.00	1.0	Very little dune buildings
2.80	0.039	2.05	0.35	2.40	95	2.4	1.70	1.50	0.8	High dune formation

Series G-01-5

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
						3rd Run				
13.70	0.189	32.00	10.30	42.30	405	10.3	21.70	20.40	10.5	Everything moving
10.80	0.149	24.70	6.30	31.00	340	8.65	18.40	17.40	9.0	" "
9.00	0.124	20.05	4.40	24.45	300	7.6	15.65	14.950	7.75	Moving, sliding bed
7.70	0.106	13.90	2.75	16.65	250	6.35	11.15	10.55	5.5	Pulsating bed
7.90	0.109	12.10	2.35	14.45	235	5.95	9.75	9.25	5	Just slightly moving bed
										CRITICAL
6.90	0.095	10.80	2.15	12.95	220	5.6	8.65	8.25	4.25	Just below critical, non-moving bed
3.20	0.044	3.45	1.40	4.85	125	3.2	2.05	1.75	1.0	Flat bed

Series G-01-5

CRITICAL CONDITION $\begin{cases} C = 5.00\% \\ V_c = 5.95 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.40	0.0663	14.50	13.40	27.90	365	9.2	1.10	0.95	0.50	Complete suspension
6.90	0.0486	10.80	10.10	20.90	310	7.9	0.90	0.70	0.35	" "
5.00	0.0353	7.65	7.10	14.75	265	6.7	0.55	0.30	0.15	Heavy bed load
3.20	0.0226	4.85	4.50	9.40	220	5.6	0.35	0.20	0.10	Scour fluctuations
3.00	0.0211	{4.55 {4.60	4.15 4.10	8.70} 8.70}	200	5.1	{0.50 {0.40	0.30 0.20	0.15} 0.10}	Deposits for awhile then slides again
1.90	0.0134	2.95	2.85	5.80	165	4.15	0.10	--	--	Infrequent sand slugs; circulated system at high Q and made 2nd run
3.30	0.0233	5.05	4.25	9.30	215	5.45	0.80	0.60	0.30	Heavy bed load
1.65	0.0116	2.65	2.20	4.85	150	3.9	0.45	0.25	0.12	CRITICAL dune formation due to distribution

Series G-01-6

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.12\% \\ V_C = 3.90 \text{ fps} \end{cases}$$

test section: Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	loop readings: Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.90	0.0698	15.35	13.35	28.70	360	9.15	2.00	1.85	0.95	Complete suspension
7.10	0.0500	10.85	9.60	20.45	300	7.75	1.25	1.05	0.55	Bed load transport
3.40	0.0240	5.05	4.35	9.40	200	5.1	0.70	0.50	0.25	Pulsating bed
2.85	0.0201	4.05	3.60	7.65	180	4.8	0.45	0.35	0.15	Settling with immediate scour, just above crit.
										CRITICAL
2.10	0.0148	3.25	2.80	6.05	160	4.1	0.45	0.30	0.15	Dune formation

Series G-01-7

CRITICAL CONDITION $\begin{cases} C = 0.15\% \\ V_C = 4.65 \text{ fps} \end{cases}$

9.90	0.0698	15.05	12.45	27.50	355	9.00	2.60	2.45	1.25	Complete suspension
5.40	0.0381	8.30	7.00	15.30	265	6.70	1.30	1.10	0.55	Bed load transport
2.80	0.0198	4.35 4.45	3.75 4.00	8.10 8.45	200	5.10	0.60 0.45	0.40 0.30	0.20 0.15	CRITICAL

Series G-01-8

CRITICAL CONDITION $\begin{cases} C = 0.20\% \\ V_C = 5.10 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
8.35	0.0589	12.65	9.75	22.40	310	7.9	2.90	2.75	1.40	Suspended and bed load transport
5.30	0.0374	8.25	6.45	14.70	255	6.45	1.80	1.60	0.80	Pulsating bed motion and shear
4.85	0.0342	6.75	5.30	12.05	225	5.7	1.45	1.20	0.60	Just above the crit. condition
4.10	0.0289	5.90	4.65	10.55	210	5.35	1.25	1.00	0.50	CRITICAL
3.20	0.0226	5.00	4.25	9.25	190	4.8	0.75	0.60	0.30	Sporatic settling, long dunes
3.00	0.0212	3.75	2.95	6.70	160	4.1	0.80	0.60	0.30	Completely stationary bed

Series G-01-9

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.50\% \\ V_C = 5.35 \text{ fps} \end{cases}$$

test section Δh_{H_2O} [in.]	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	loop readings Δh_R [in.]	Δh_D [in.]	$\Delta h_R + \Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R - \Delta h_D$ [in.]	$\Delta h_R - \Delta h_D$ corrected [in.]	C [%]	COMMENTS
13.95	0.0985	21.45	14.90	36.35	400	9.55	6.55	6.30	3.20	Complete suspension
8.15	0.0575	12.35	9.70	22.05	310	7.9	2.65	2.45	1.25	" "
7.10	0.0500	10.60	8.30	18.90	290	7.35	2.30	2.10	1.05	" "
6.30	0.0443	9.10	7.10	16.20	265	6.7	2.00	1.80	0.90	Heavy bed load condition
5.60	0.0395	8.00	6.35	14.35	245	6.2	1.65	1.40	0.70	Particles sliding and becoming visible
4.80	0.0338	6.95	5.60	12.55	230	5.85	1.35	1.15	0.60	
4.20	0.0296	6.45	5.10	11.50	225	5.7	1.35	1.10	0.55	CRITICAL
4.50	0.0317	6.70	5.45	12.15	225	5.7	1.25	1.00	0.50	
3.50	0.0247	5.05	4.25	9.30	200	5.1	0.80	0.60	0.30	Bottom of deposit is non-moving
1.35	0.0095	1.95	1.50	3.45	110	3.3	0.45	0.30	0.15	Long dune deposit

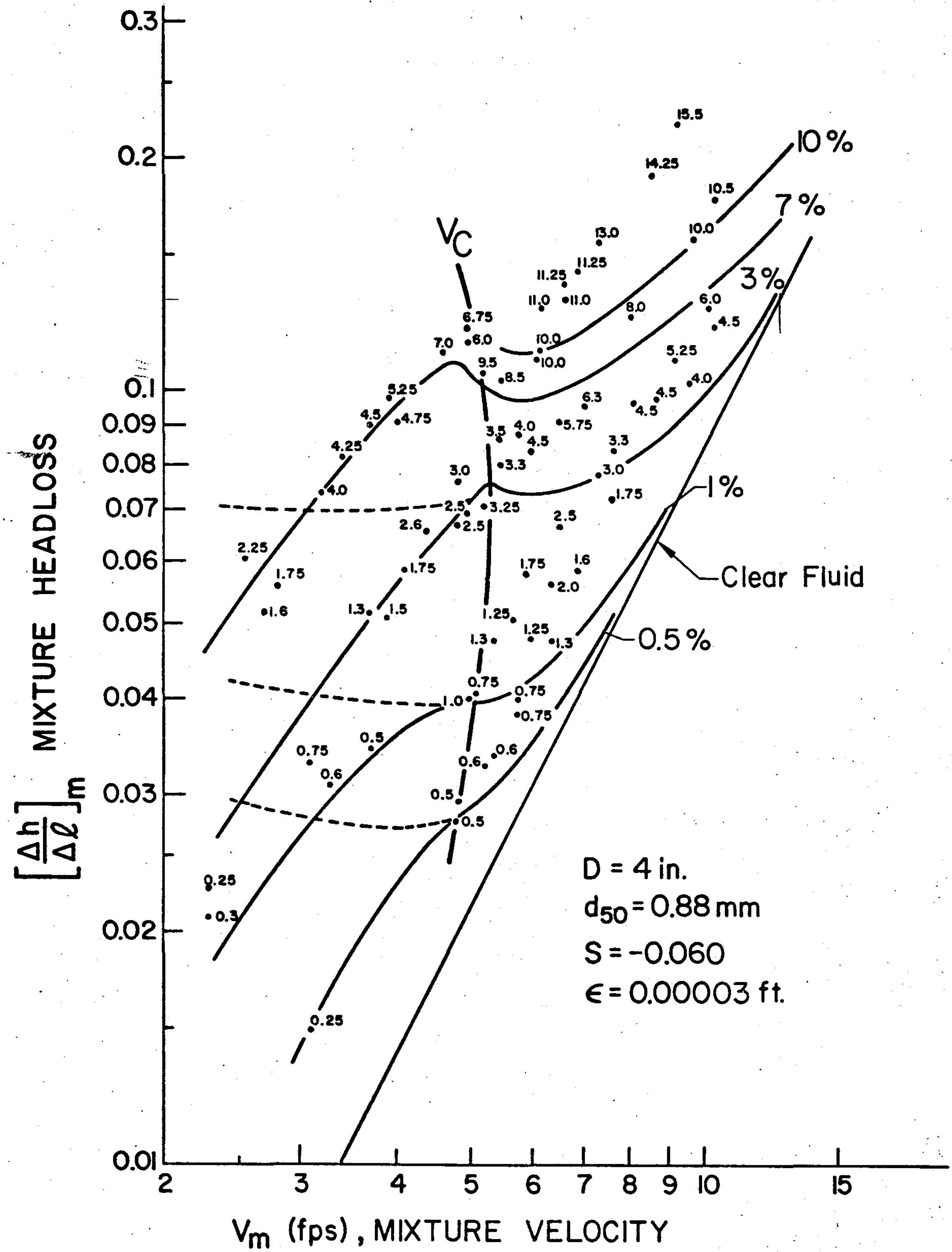
Series G-01-10

CRITICAL CONDITION $\begin{cases} C' = 0.60\% \\ V_C = 5.80 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
16.85	0.1190	25.15	16.10	41.25	440	11.15	9.05	8.80	4.40	Complete suspension
12.05	0.0849	17.25	11.10	28.35	360	9.1	6.15	5.95	3.00	Suspension and bed load transport
8.60	0.0606	12.05	8.20	20.25	300	7.65	3.85	3.70	1.85	Heavy bed load transport
6.15	0.0434	8.50	6.20	14.70	255	6.5	2.30	2.10	1.05	Just above the crit. condition
										CRITICAL
5.45	0.0384	7.65	5.70	13.35	235	5.95	1.95	1.70	0.85	Just into the deposit regime
3.30	0.0233	5.05	4.05	9.10	195	4.95	1.00	0.80	0.40	Dune deposit, fluctuating head loss readings due to sporatic duning

Series G-01-11

CRITICAL CONDITION $\begin{cases} C = 1.00\% \\ V_C = 6.40 \text{ fps} \end{cases}$



Plot of Series G-02 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
8.80	0.121	27.35	16.70	44.05	410	10.45	10.65	9.35	4.75	Everything moving
7.50	0.103	22.65	13.80	36.45	375	9.55	8.85	7.75	4.0	" "
5.20	0.072	13.65	9.40	23.05	300	7.6	4.25	3.55	1.75	" "
4.30	0.059	11.70	7.75	19.45	270	6.85	3.95	3.35	1.6	" "
3.50	0.048	8.90	6.25	15.15	235	5.95	2.65	2.15	1.25	Sliding bed
2.90	0.040	7.60	5.50	13.10	225	5.75	2.10	1.60	0.75	Pulsating bed
2.50	0.034	6.45	4.85	11.30	210	5.35	1.60	1.20	0.6	" "
2.00	0.028	5.55	4.10	9.65	190	4.8	1.45	1.05	0.5	Slowly pulsating
3.50	0.048	9.60	6.35	15.95	250	6.35	3.25	2.65	1.3	Everything moving
2.80	0.0385	7.30	5.45	12.75	225	5.75	1.85	1.35	0.75	" "
2.40	0.033	6.40	4.75	11.15	205	5.2	1.65	1.20	0.6	Pulsating
2.15	0.0295	5.45	4.20	9.65	190	4.8	1.25	0.85	0.5	CRITICAL

Series G-02-1

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.50\% \\ V_C = 4.8 \text{ fps} \end{cases}$$

test section		loop readings								COMMENTS
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.20	0.127	25.65	12.60	38.25	400	10.2	13.05	11.85	6.0	Everything moving
8.00	0.110	21.85	10.55	32.40	365	9.2	11.30	10.25	5.25	" "
7.10	0.098	19.25	9.40	28.65	340	8.65	9.85	8.95	4.5	" "
6.10	0.084	15.50	8.0	23.50	300	7.6	7.50	6.70	3.3	" "
5.70	0.078	13.90	7.05	20.95	290	7.35	6.85	6.15	3.0	" "
4.10	0.0565	9.85	5.50	15.35	250	6.35	4.35	3.75	2.0	Sliding bed
4.20	0.058	8.80	5.0	13.80	235	5.95	3.80	3.30	1.75	Just pulsating
3.70	0.051	7.25	4.55	11.75	220	5.65	2.70	2.20	1.25	Just above crit.
3.00	0.041	6.05	4.0	10.05	200	5.1	2.05	1.65	0.75	CRITICAL
2.50	0.0345	3.60	2.0	5.60	145	3.7	1.60	1.30	0.5	Deposit
					2nd Run					
7.00	0.0965	19.25	9.55	28.80	320	8.1	9.7	8.8	4.5	Everything moving
5.70	0.078	14.65	7.35	22.0	285	7.25	7.3	6.5	3.3	" "
4.90	0.067	11.40	5.85	17.25	255	6.5	5.55	4.95	2.5	" "
4.20	0.058	9.05	5.25	14.30	230	5.85	3.80	3.3	1.75	" "
3.50	0.048	7.30	4.30	11.60	210	5.35	3.0	2.6	1.3	Bed just moving
2.90	0.040	5.85	3.75	9.60	195	4.95	2.1	1.7	1	CRITICAL
1.10	0.015	2.25	1.85	4.10	120	3.1	0.4	0.2	0.25	Flat bed

CRITICAL CONDITION: $C = 1.00\%$
$V_C = 5.1$ fps

Series G-02-2

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
12.90	0.178	33.00	11.05	44.05	410	10.4	21.95	20.65	10.5	Everything moving
11.50	0.158	28.90	8.45	37.35	380	9.65	20.45	19.25	10.0	" "
9.10	0.125	21.60	5.20	26.80	315	8.0	16.40	15.60	8.0	" "
7.00	0.096	16.65	3.60	20.25	275	7.0	13.05	12.35	6.3	" "
6.70	0.092	14.60	2.80	17.40	255	6.5	11.80	11.20	5.75	Sliding bed
6.10	0.084	12.05	2.70	14.75	235	5.95	9.35	8.85	4.5	Qucikly pulsating
6.50	0.089	10.65	2.45	13.10	225	5.75	8.20	7.70	4.0	Slowly moving, just below crit.
5.80	0.080	9.15	2.30	11.45	215	5.45	6.85	6.35	3.3	CRITICAL
4.90	0.0675	7.40	2.0	9.40	190	4.8	5.40	5.0	2.5	Deposit
5.10	0.070	7.70	2.30	10.0	195	4.95	5.40	5.0	2.5	"
3.70	0.051	4.90	1.60	6.50	150	3.9	3.30	3.0	1.5	Flat bed
2.40	0.033	3.0	1.0	4.0	120	3.1	2.0	1.70	0.75	" "
1.70	0.023	1.45	1.0	2.45	90	2.3	0.45	0.30	0.25	Dunes
					2nd Run					
6.20	0.085	12.40	2.40	14.80	235	5.95	10.0	9.50	4.75	Quickly pulsating
6.30	0.087	9.65	2.40	12.05	215	5.45	7.25	6.85	3.5	Slowly pulsating

Continued

Series G-02-3

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
5.20	0.0715	8.60	2.05	10.65	205	5.2	6.55	6.15	3.25	CRITICAL
5.60	0.077	7.95	1.50	9.45	190	4.8	6.45	6.05	3.0	Flat bed
4.80	0.066	6.70	1.30	8.0	170	4.35	5.40	5.10	2.6	" "
4.30	0.059	5.70	1.70	7.40	160	4.1	4.0	3.60	1.75	" "
3.80	0.052	4.45	1.60	6.05	145	3.7	2.85	2.55	1.3	" "
2.40	0.033	2.95	1.40	4.35	120	3.1	1.55	1.25	0.6	" "
1.50	0.021	1.50	0.65	2.15	90	2.3	0.85	0.65	0.3	Dunes

Series G-02-3

CRITICAL CONDITION $\begin{cases} C = 3.00\% \\ V_C = 5.35 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
10.40	0.143	21.40	-1.05	20.35	270	6.85	22.45	21.85	11.25	Sometimes stopping
9.60	0.132	20.25	-1.65	18.60	260	6.6	21.90	21.30	11.0	" "
8.20	0.113	17.70	-2.0	15.70	240	6.1	19.70	19.20	10.0	" "
7.60	0.105	14.90	-2.20	12.70	215	5.45	17.10	16.70	8.5	Quickly pulsating
8.40	0.116	11.30	-0.95	10.35	200	4.95	12.25	11.85	6.0	CRITICAL
6.70	0.092	8.30	-1.50	6.80	155	4.0	9.80	9.40	4.75	Flat bed
6.60	0.091	7.85	-1.60	6.25	145	3.7	9.45	9.15	4.5	" "
6.00	0.0825	7.10	-1.70	5.40	135	3.4	8.80	8.50	4.25	" "
3.80	0.052	3.35	+0.05	3.40	105	2.7	3.30	3.10	1.6	" "
						2nd Run				
16.00	0.221	36.45	5.35	41.80	365	9.2	31.10	30.0	15.5	Everything moving
13.80	0.190	31.10	2.50	33.60	335	8.55	28.60	27.70	14.25	" "
11.30	0.156	25.65	-0.10	25.55	290	7.35	25.75	25.05	13.0	" "
10.00	0.138	21.55	-1.05	20.45	260	6.6	22.60	21.95	11.25	" "
8.00	0.110	18.35	-1.85	16.50	240	6.05	20.20	19.70	10.0	Slowly pulsating
7.70	0.106	15.10	-2.30	12.80	215	5.45	17.40	17.0	8.75	" "
8.80	0.121	12.0	-1.70	10.30	195	4.95	13.70	13.30	6.75	CRITICAL fluctuating with scour-deposit
4.40	0.061	4.10	-0.60	3.50	100	2.55	4.70	4.40	2.25	Flat bed

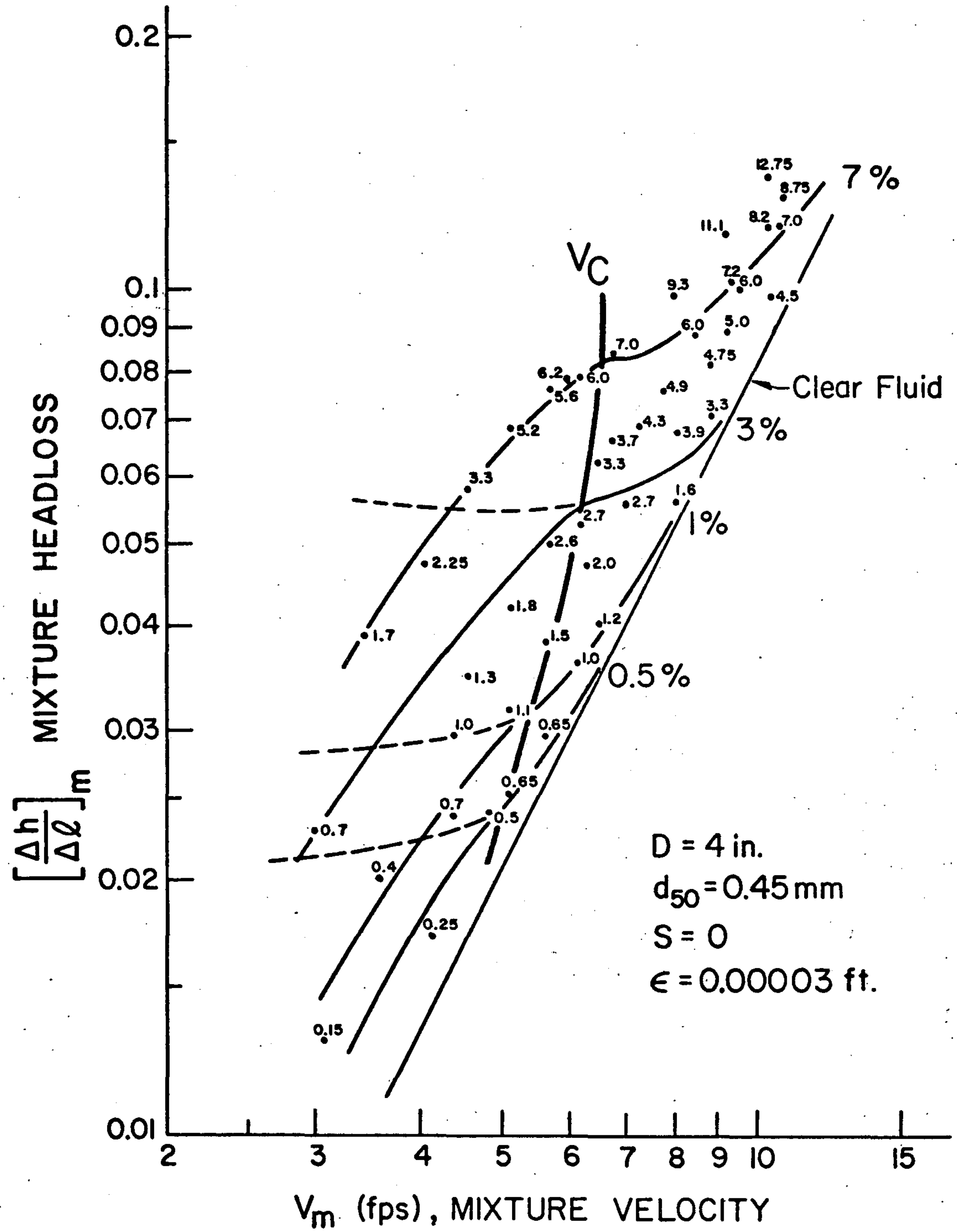
Continued

Series G-02-4

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
					3rd run					
9.30	0.128	19.95	-2.20	17.75	245	6.15	22.15	21.65	11.0	Deposit·scour
7.80	0.107	16.30	-2.90	13.40	205	5.2	19.20	18.80	9.5	" "
7.80	0.107	16.15	-2.75	13.40	205	5.2	18.90	18.50	9.5	" "
										CRITICAL
8.20	0.113	11.40	-2.10	9.30	180	4.6	13.50	13.10	7.0	Just deposited, thick bed
7.10	0.098	8.70	-1.90	6.80	150	3.9	10.60	10.30	5.25	Flat bed
5.40	0.074	6.50	-1.50	5.0	125	3.2	8.0	7.70	4.0	" "
4.00	0.055	3.60	+0.20	3.80	110	2.8	3.40	3.20	1.75	" "

Series G-02-4

CRITICAL CONDITION $\begin{cases} C = 7.00\% \\ V_C = 5.0 \text{ fps} \end{cases}$



Plot of Series G-001 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta l}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
7.15	0.098	24.45	14.35	38.80	415	10.55	10.10	8.70	4.45	Everything moving
5.20	0.0715	17.00	10.10	27.10	345	8.75	6.90	6.00	3.10	" "
4.10	0.0563	13.15	9.25	22.40	315	8.00	3.90	3.10	1.60	" "
2.95	0.0405	8.75	5.90	14.65	255	6.45	2.85	2.40	1.20	" "
2.65	0.0364	7.90	5.45	13.35	240	6.10	2.45	2.00	1.02	Bed particles visible
2.15	0.0296	6.75	4.70	11.45	220	5.60	2.05	1.65	0.85	Pulsating, almost deposited, just above critical
1.85	0.0254	5.60	3.95	9.55	200	5.10	1.65	1.25	0.65	CRITICAL
1.75	0.0240	5.10	3.70	8.80	190	4.80	1.40	1.00	0.50	Flat bed
1.15	0.0172	3.50	2.70	6.20	160	4.10	0.80	0.45	0.25	" "
0.95	0.0130	2.15	1.60	3.75	120	3.10	0.55	0.30	0.15	" "

Series G-001-1

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.65\% \\ V_C = 5.10 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
8.65	0.119	30.45	15.25	45.70	435	10.60	15.20	13.70	7.0	Everything moving
7.35	0.101	25.30	12.35	37.65	395 (400)	9.50	12.95	11.65	6.0	" "
6.45	0.089	21.20	10.30	31.50	360	9.15	10.90	9.90	5.0	Bed load
5.95	0.082	19.55	9.25	28.80	345	8.75	10.30	9.30	4.75	" "
4.95	0.068	16.05	7.55	23.60	315	8.05	8.50	7.70	3.9	Sliding bed
4.05	0.056	12.05	6.20	18.25	275	7.00	5.85	5.35	2.7	" "
3.45	0.0475	9.60	5.40	15.00	245 (250)	6.30	4.20	3.80	2.0	Pulsating bed
2.75	0.038	7.90	4.70	12.60	225	5.70	3.20	2.80	1.5	Just above critical
2.85	0.039	{7.85 {7.75	4.30 4.40	12.15} 12.15}	220	5.60	{3.55 {3.35	3.15 3.05	1.6} 1.5}	$\boxed{\text{CRITICAL}}$
2.35	0.032	6.20	3.80	10.00	200	5.10	2.40	2.10	1.1	Flat bed
1.75	0.024	4.55	3.00	7.55	170	4.40	1.55	1.25	0.7	" "

Series G-001-2

$$\text{CRITICAL CONDITION} \begin{cases} C = 1.50\% \\ V_C = 5.6 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.35	0.129	31.90	13.40	45.30	430	10.95	18.50	17.00	8.75	Everything moving
8.65	0.119	28.75	11.50	40.25	410	10.45	17.25	15.95	8.2	" "
7.45	0.103	23.85	8.80	32.65	365	9.30	15.05	13.95	7.2	Heavy bed load
6.45	0.089	20.00	7.40	27.40	330	8.40	12.60	11.80	6.0	" " "
5.55	0.0765	16.40	6.15	22.65	305	7.75	10.25	9.55	4.9	" " "
5.05	0.0695	14.35	5.35	19.70	285	7.25	9.00	8.40	4.3	Sliding bed
4.85	0.067	12.25	4.80	17.05	265	6.75	7.45	7.05	3.7	Quickly pulsating
4.55	0.0625	11.45	4.75	16.20	255 (260)	6.50	6.70	6.30	3.3	" "
3.85	0.053	9.75	4.25	14.00	245	6.20	5.50	5.20	2.7	$\boxed{\text{CRITICAL}}$
3.65	0.050	8.90	3.50	12.40	225	5.70	5.40	5.10	2.6	Flat bed
3.05	0.042	6.80	3.05	9.85	200	5.10	3.75	3.50	1.8	Thickening flat bed
2.55	0.035	5.45	2.70	8.15	180	4.55	2.75	2.55	1.3	Saltating bed load
2.15	0.0295	4.90	2.70	7.60	170	4.40	2.20	2.00	1.0	Saltating bed load
1.45	0.020	2.95	2.00	4.95	135	3.60	0.95	0.80	0.4	Thick bed, little moving

CRITICAL CONDITION: $C = 3.00\%$
$V_C = 6.25$ fps

Series G-001-3

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.85	0.136	34.25	8.40	42.65	400	10.20	26.15	24.85	12.75	Everything moving
8.55	0.1175	28.05	5.45	33.50	360	9.15	22.60	21.60	11.1	" "
7.15	0.0985	21.95	3.05	25.0	310	7.90	18.90	18.20	9.3	Mostly bed load
6.15	0.0845	16.00	1.70	17.7	265	6.75	14.30	13.80	7.0	Slowly moving bed, just above critical
										CRITICAL
5.75	0.079	13.80	1.70	15.5	245	6.20	12.10	11.70	6.0	Just below critical, thickening bed
6.45	0.089	16.05	1.80	17.85	270	6.85	14.25	13.75	7.0	Just below critical, thickening bed
5.75	0.079	13.50	1.05	14.55	235	5.95	12.45	12.05	6.2	Deep flat bed
5.55	0.0765	12.20	0.90	13.10	225	5.70	11.30	10.90	5.6	" " "
5.05	0.0695	10.05	0.70	10.75	200	5.10	10.35	10.05	5.2	" " "
4.25	0.0585	7.60	0.85	8.45	175	4.55	6.75	6.45	3.3	Still suspension load
3.45	0.0475	5.65	1.00	6.65	155	4.05	4.65	4.35	2.25	Saltation load
2.85	0.039	4.30	0.80	5.10	130	3.45	3.50	3.30	1.7	Flat bed
1.65	0.023	2.35	0.85	3.20	110	3.00	1.50	1.30	0.7	" "

CRITICAL CONDITION: $C = 7.00\%$
$V_C = 6.5$ fps

Series G-001-4

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.65	0.0681	14.90	13.80	28.70	375	9.55	1.10	1.00	0.50	Complete suspension
7.10	0.0501	10.90	10.30	21.20	310	7.9	0.60	0.45	0.22	" "
5.00	0.0353	7.80	7.30	15.10	265	6.7	0.50	0.30	0.15	Suspension and bed load transport
4.20	0.0296	6.55	6.20	12.75	245	6.2	0.35	0.15	0.07	Heavy bed load
3.15	0.0222	5.00	4.70	9.70	215	5.45	0.30	0.10	0.05	Sliding bed load
2.85	0.0201	4.45 4.55 4.35	4.10 4.05 4.10	8.55 8.60 8.45	195	4.95	0.35 0.55 0.25	0.15 0.35 0.10	0.07 0.15 0.05	Just above crit. with sporatic bed scour at partially closed valve
1.35	0.0095	2.20 2.15 2.15	2.00 1.85 1.95	4.20 4.00 4.10	135	3.6	0.20 0.30 0.20	0.10 0.15 0.10	0.05 0.07 0.05	Persistent scouring, sometimes critical CRITICAL
0.65	0.0046	1.10	0.90	2.00	100	2.55	0.20	0.10	0.03	Stationary deposit

Series G-001-5

CRITICAL CONDITION $\begin{cases} C = 0.05\% \\ V_C = 2.75 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
8.95	0.0632	14.00	12.50	26.50	355	9.0	1.50	1.30	0.65	Complete suspension
7.05	0.0497	11.20	10.00	21.20	310	7.9	1.20	1.05	0.55	" "
5.15	0.0363	8.20	7.40	15.60	265	6.7	0.80	0.60	0.30	Suspension and bed load transport
4.10	0.0289	6.50	5.90	12.40	230	5.85	0.60	0.40	0.20	Heavy bed load
3.20	0.0226	5.00	4.65	9.65	215	5.45	0.35	0.20	0.10	Sliding bed
3.10	0.0219	{4.90 {4.80	4.30 4.35	9.20} 9.15}	200	5.1	{0.60 {0.45	0.40 0.25	0.20} 0.12}	Sporadic scouring and deposit
1.80	0.0127	2.90	2.55	5.45	160	4.1	0.35	0.20	0.10	CRITICAL
1.75	0.0124	2.80	2.45	5.25	150	3.9	0.35	0.15	0.08	Just below crit.

Series G-001-6

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.10\% \\ V_C = 4.10 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.30	0.0656	14.65	12.60	27.25	365	9.2	2.05	1.95	1.00	Complete suspension
6.95	0.0490	10.80	9.50	20.30	310	7.9	1.30	1.20	0.60	" "
5.25	0.0370	8.25	7.30	15.55	265	6.7	0.95	0.80	0.40	Heavy bed load with saltation into less densely populated areas of the cross-section
4.10	0.0289	6.30	5.70	12.00	230	5.85	0.60	0.40	0.20	Thickening bed of sliding particles
3.20	0.0226	{4.95 4.90	4.15 4.30	9.10} 9.20}	200	5.1	{0.80 0.60	0.55 0.40	0.25} 0.20}	Pulsating conditions, just above critical
										CRITICAL
1.80	0.0127	2.70	2.45	5.15	155	3.95	0.35	0.20	0.10	Sufficiently below crit. settling

Series G-001-7

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.20\% \\ V_C = 4.80 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
9.40	0.0663	14.65	10.85	25.50	355	9.0	3.80	3.65	1.85	Complete suspension
7.10	0.0500	11.00	8.50	19.50	310	7.9	2.50	2.35	1.20	Suspension with noticeable bed load
5.35	0.0378	8.25	6.75	15.00	265	6.7	1.50	1.30	0.65	Heavy bed load
4.35	0.0307	6.60	5.60	12.20	240	6.1	1.00	0.80	0.40	Sliding bed, increasing deposit depth
3.60	0.0254	5.50	4.75	10.25	220	5.6	0.75	0.60	0.30	Just above crit. condition
3.20	0.0226	4.90	4.20	9.10	215	5.45	0.70	0.50	0.25	CRITICAL
3.10	0.0219	4.60	3.90	8.50	200	5.1	0.70	0.45	0.20	Deposit building
2.20	0.0155	3.40	3.00	6.40	180	4.6	0.40	0.25	0.15	" "

Series G-001-8

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.30\% \\ V_C = 5.45 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
11.20	0.0790	17.85	10.90	28.75	355	9.0	6.95	6.80	3.40	Total transport
9.00	0.0635	14.25	8.50	22.75	310	7.9	5.75	5.60	2.80	" "
6.25	0.0441	9.55	6.30	15.85	265	6.7	3.25	3.00	1.50	Heavy bed load
5.30	0.0374	7.95	5.70	13.65	235	5.95	2.25	2.05	1.05	Pulsating bed
4.85	0.0347	7.05	5.05	12.10	225	5.7	2.00	1.85	0.95	CRITICAL
3.55	0.0250	5.20	3.95	9.15	205	5.2	1.25	1.10	0.55	Stationary bed

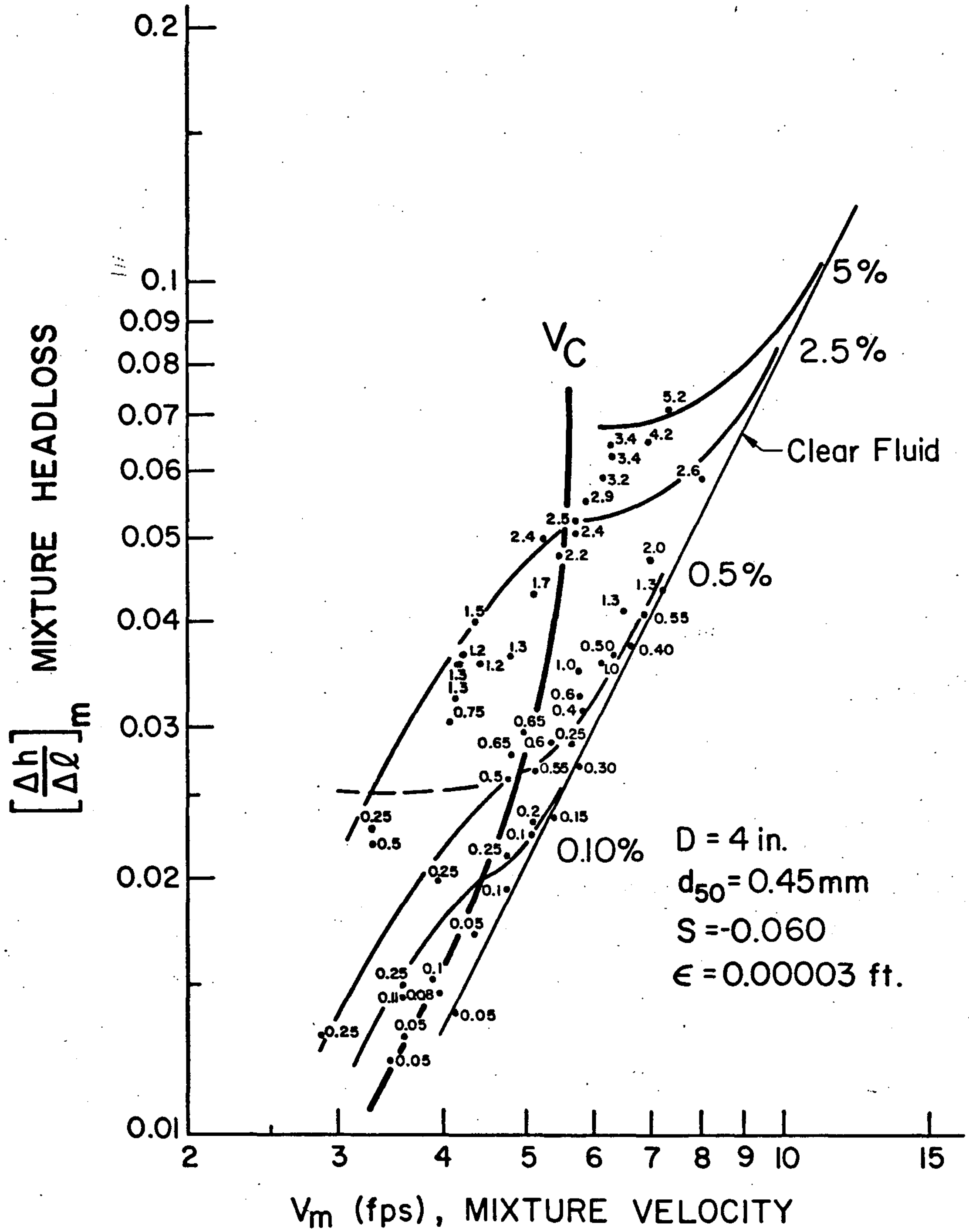
Series G-001-9

$$\text{CRITICAL CONDITION} \begin{cases} C = 1.00\% \\ V_{C'} = 5.70 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
12.10	0.0853	19.20	10.70	29.90	375	9.55	8.50	8.40	4.20	Full suspension
8.80	0.0621	13.70	7.65	21.35	310	7.9	6.05	5.90	2.95	" "
6.50	0.0459	9.95	5.95	15.90	265	6.7	4.00	3.80	1.90	Heavy bed load
6.25	0.0441	9.00	5.60	14.60	260	6.6	3.60	3.40	1.70	Sliding deposit
5.75	0.0405	8.45	5.45	13.90	250	6.35	3.00	2.80	1.40	Approaching crit.
5.35	0.0377	7.40	4.95	12.35	230	5.85	2.45	2.30	1.15	CRITICAL
3.55	0.0250	5.20	3.70	8.90	210	5.35	1.50	1.30	0.65	Stationary deposit

Series G-001-10

CRITICAL CONDITION $\begin{cases} C = 1.20\% \\ V_{C} = 5.85 \text{ fps} \end{cases}$



Plot of Series G-002 Data

test section		loop readings								COMMENTS
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
2.75	0.0378	8.75	7.50	16.25	265	6.7	1.25	0.75	0.40	Everything moving
1.95	0.0268	6.30	5.35	11.65	230	5.85	0.95	0.55	0.30	" "
1.65	0.0227	5.05	4.45	9.50	200	5.1	0.60	0.20	0.10	" "
1.25	0.0172	3.90	3.45	7.35	170	4.35	0.45	0.10	0.05	Rapid Pulses
0.95	0.0130	2.80	2.40	5.20	140	3.6	0.40	0.10	0.05	Deposit when enough sand CRITICAL
1.07	0.0147	3.20	2.70	5.90	155	3.95	0.50	0.15	0.08	Deposit
1.00	0.0137	3.20	2.75	5.95	160	4.1	0.45	0.10	0.05	Deposit mostly in larger pipe

Series G-002-1

CRITICAL CONDITION $\left\{ \begin{array}{l} C = 0.05\% \\ V_C = 3.7 \text{ fps} \end{array} \right.$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.00	0.0405	9.10	7.55	16.65	270	6.85	1.55	1.05	0.55	Everything moving
2.10	0.0288	6.25	5.30	11.55	220	5.65	0.95	0.50	0.25	" "
1.70	0.0233	5.00	4.45	9.45	200	5.1	0.55	0.10	0.05	" "
1.40	0.0192	4.25	3.60	7.85	180	4.8	0.65	0.30	0.15	Very slowly moving
1.10	0.0151	3.00	2.50	5.50	150	3.9	0.50	0.20	0.10	Deposit CRITICAL
0.90	0.0124	2.35	2.10	4.45	135	3.45	0.25	-	-	Flat bed, no moving concentration
0.20	0.00274	0.65	0.65	1.30	65	1.65	0.0	-	-	Small dunes
					2nd Run					
3.15	.0439	10.20	8.05	18.25	280	7.35	3.15	2.60	1.33	Everything moving
2.65	.0369	8.25	6.80	15.05	250	6.35	1.45	1.00	0.52	" "
2.25	.0313	7.10	5.90	13.00	230	5.85	1.20	0.80	0.42	" "
1.95	.0271	6.05	5.10	11.15	215	5.45	0.95	0.55	0.30	" "
1.70	.0237	5.30	4.50	9.80	200	5.1	0.80	0.40	0.20	Pulses
1.55	.0215	5.00	4.20	9.20	195	4.59	0.80	0.40	0.20	Particles visible
1.48	.0206	4.50	3.75	8.25	185	4.75	0.75	0.40	0.20	" "
0.95	.0132	3.10	2.60	5.70	150	3.9	0.50	0.20	0.10	Almost deposit CRITICAL
1.05	.0146	3.10	2.45	5.55	150	3.9	0.65	0.30	0.15	Deposit

Series G-002-2

Continued

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
1.05	.0146	3.15	2.50	5.65	150	3.9	0.65	0.30	0.15	Deposit, bed less thick
1.05	.0146	3.15	2.60	5.75	150	3.9	0.55	0.20	0.10	
0.70	.0097	2.00	1.65	3.65	115	2.95	0.35	0.05	0.02	Deposits a while then washes away
0.35	.00049	1.20	0.95	2.15	90	2.3	0.25	0.05	0.02	Single dunes

Series G-002-2

CRITICAL CONDITION $\begin{cases} C = 0.10\% \\ V_C = 3.9 \text{ fps} \end{cases}$

test section $\Delta h_{1.95}$ [in.]	test section $\left(\frac{\Delta h}{\Delta \ell}\right)_m$	loop readings Δh_R [in.]	Δh_D [in.]	$\Delta h_R+\Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R-\Delta h_D$ [in.]	$\Delta h_R-\Delta h_D$ corrected [in.]	C [%]	COMMENTS
4.35	0.0597	14.00	8.35	22.35	315	8.0	5.65	4.95	2.55	Everything moving
3.45	0.0474	11.05	6.60	17.65	275	7.0	4.45	3.95	2.00	" "
2.55	0.0350	7.55	5.20	12.75	225	5.75	2.35	1.95	1.00	Slowing down, bed particles visible
1.95	0.0268	5.50	4.05	9.55	200	5.1	1.45	1.10	0.55	Pulsating bed
2.15	0.0295	5.50	3.85	9.35	195	4.95	1.65	1.30	0.65	Pulsating slowly
2.05 1.95	0.0282 0.0268	5.30	3.70	9.00	190	4.8	1.60	1.30	0.65	Deposits, then slides
1.55	0.0212	4.20	3.35	7.55	175	4.75	0.85	0.50	0.25	Deposit
										CRITICAL
1.35 1.55 1.45	0.0185 0.0212 0.0199	3.45 3.45	2.65 2.60	6.10 6.05	155	3.95	0.80 0.85	0.50 0.55	0.25 0.30	Deposit, bed thickens
1.15 1.05	0.0158 0.0143	2.75	2.15	4.90	140	3.6	0.60	0.45	0.25	Deposit, first thinner, then thicker
0.95	0.0130	1.90	1.25	3.15	110	2.8	0.65	0.45	0.25	First flat bed, then dunes
0.40 0.60	0.0055 0.0082	1.20	0.95	2.15	90	2.3	0.25	0.05	0.03	1' long dunes

CRITICAL CONDITION: $C = 0.25\%$
$V_C = 4.5$ fps

Series G-002-3

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.00	0.0411	9.00	6.10	15.10	255	6.5	2.90	2.45	1.30	Everything moving
2.60	0.0357	7.75	5.50	13.25	240	6.1	2.25	1.85	0.95	" "
2.40	0.0327	6.95	5.05	12.00	225	5.75	1.90	1.50	0.80	Rapid pulses
2.10	0.0288	5.90	4.40	10.30	210	5.35	1.50	1.10	0.57	Slow pulses, bed particles visible
1.90	0.0261	5.50	4.20	9.70	205	5.2	1.30	0.90	0.47	Very slow pulses, almost deposit
2.10	0.0288	5.30	3.90	9.20	198	5.1	1.40	1.05	0.55	Deposit CRITICAL
2.10	0.0288	5.50	3.75	9.25	200	5.1	1.75	1.30	0.67	
1.90	0.0261	5.30	4.05	9.35	200	5.1	1.25	0.85	0.45	
1.87	0.0357	5.00	3.70	8.70	190	4.8	1.30	1.00	0.50	Deposit
1.90	0.0261	4.90	3.65	8.55	185	4.75	1.25	0.90	0.47	Deposit, pulsating
1.65	0.0226	4.45	3.40	7.85	175	4.75	1.05	0.75	0.40	Deposit
1.05	0.0144	2.60	2.10	4.70	140	3.6	0.50	0.20	0.10	"
0.55	0.0075	1.40	1.30	2.70	95	2.45	0.10	-	-	" long dunes forming

Series G-002-4

CRITICAL CONDITION $\begin{cases} C = 0.55\% \\ V_C = 5.1 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$ [in.]	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R [in.]	Δh_D [in.]	$\Delta h_R + \Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R - \Delta h_D$ [in.]	$\Delta h_R - \Delta h_D$ corrected [in.]	C [%]	COMMENTS
5.15	0.0707	15.50	4.90	20.40	290	7.35	10.60	10.00	5.15	Everything moving
4.80	0.0658	13.20	4.45	17.65	275	7.0	8.75	8.20	4.20	" "
4.70	0.0644	11.15	4.05	15.20	250	6.35	7.10	6.60	3.40	Particles visible
4.30	0.0590	10.45	3.85	14.30	240	6.1	6.60	6.25	3.20	Slow bed motion
3.70	0.0508	8.65	3.60	12.25	225	5.75	5.05	4.65	2.40	Slow pulsating, almost deposit
										$\boxed{\text{CRITICAL}}$
3.55	0.0487	8.00	3.35	11.35	215	5.45	4.65	4.30	2.20	Deposit
3.15	0.0431	6.70	3.10	9.80	200	5.1	3.60	3.20	1.65	Flat bed
2.65	0.0364	5.70	2.90	8.60	190	4.8	2.80	2.45	1.25	" "
2.75 2.55	0.0378 0.0350	5.25	2.50	7.75	165	4.2	2.75	2.40	1.22	" "
2.20	0.0302	4.10	2.35	6.45	160	4.1	1.75	1.50	0.75	" "
1.65	0.0226	2.70	1.90	4.60	130	3.3	0.80	0.50	0.25	" "
0.65	0.00895	0.70	0.60	1.30	65	1.65	0.10	-	-	1' long dunes, no moving concentrations

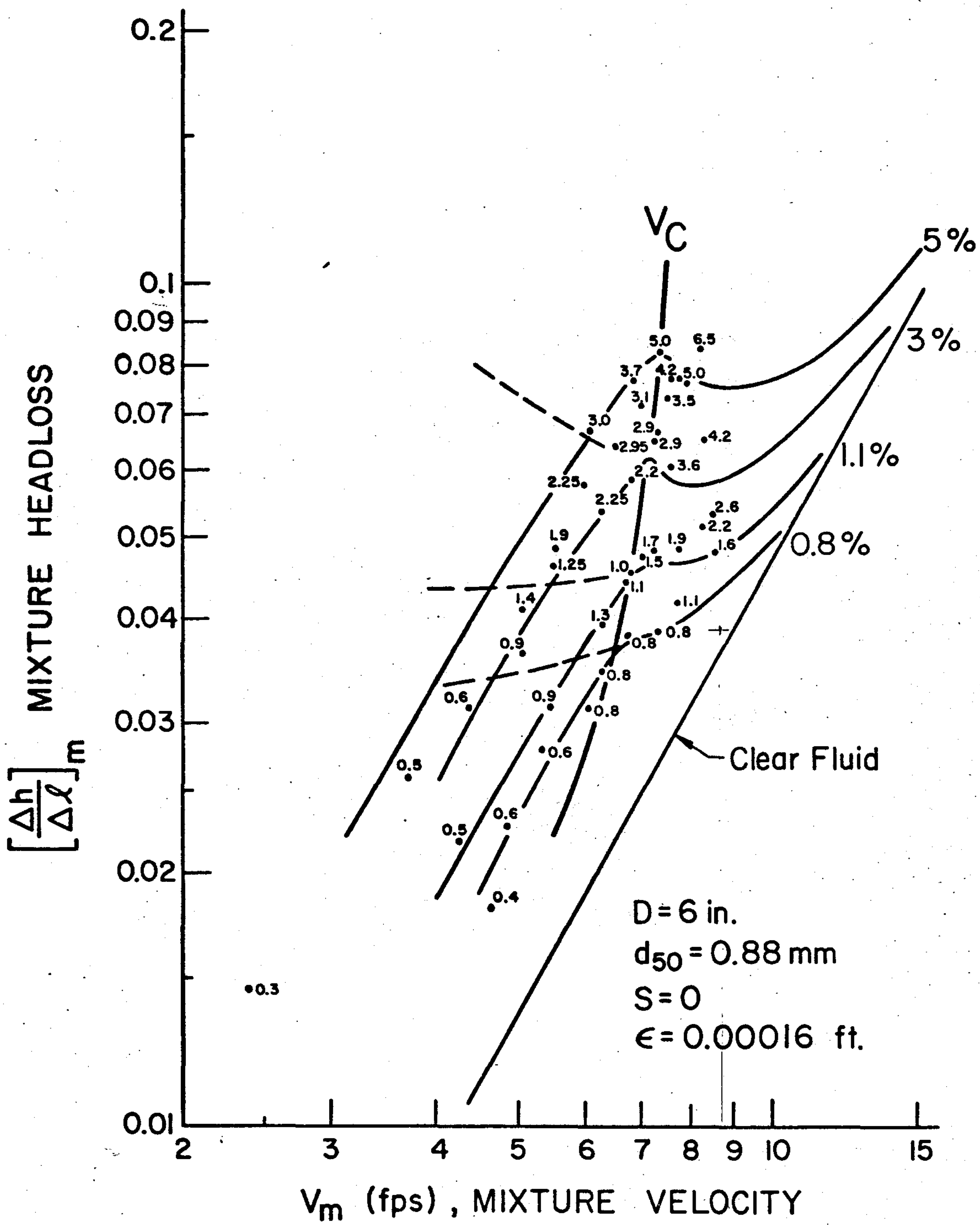
Series G-002-5

$$\text{CRITICAL CONDITION} \begin{cases} C = 2.25\% \\ V_C = 5.5 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.70	0.0645	13.60	4.45	18.05	270	6.85	9.15	8.60	4.40	Everything moving
4.50	.0626	11.00	3.90	14.90	250	6.35	7.10	6.65	3.45	Bed particles often visible, strong pulses
4.00	.0556	9.65	3.70	13.35	230	5.85	5.95	5.50	2.85	Particles visible slower pulses
3.80	.0530									
3.70	.0515	8.80	3.50	12.30	225	5.7	5.30	4.90	2.50	Deposit CRITICAL
3.90	.0542									
3.60	.0500	8.40	3.40	11.80	220	5.25	5.00	4.60	2.35	Flat bed
3.00	.0417	6.65	3.00	9.65	195	4.95	3.65	3.30	1.68	" "
3.10	.0432									
2.57	.0358	5.35	2.70	8.05	175	4.4	2.65	2.30	1.18	" "
3.00	.0417	5.50	2.30	7.80	170	4.35	3.20	2.85	1.45	
2.60	.0362	5.35	2.50	7.85	170	4.35	2.85	2.50	1.25	" "
2.80	.0390	5.45	2.40	7.95	175	4.4	3.05	2.80	1.42	
2.30	.0320	5.00	2.05	7.05	165	4.2	2.95	2.65	1.35	" "
2.40	.0334	4.90	2.00	6.90	160	4.1	2.90	2.60	1.32	
1.60	.0222	2.90	1.70	4.60	130	3.30	1.20	1.00	0.50	" "

CRITICAL CONDITION: C = 2.50%
V_C = 5.1 fps

Series G-002-6



Plot of Series BS-01 Data

test section		loop readings									
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.54	0.0480	64.2	55.0	119.2	750	8.50	9.2	3.1	1.6	Everything moving
3.05	0.0420	53.8	46.8	100.6	680	7.75	7.0	2.2	1.1	" "
2.82	0.0388	46.6	41.0	87.6	640	7.30	5.6	1.6	0.8	Heavy bed load
2.79	0.0385	40.6	35.5	76.1	595	6.75	5.1	1.6	0.8	Sliding bed
										CRITICAL
2.54	0.0349	35.3	30.9	66.2	550	6.25	4.4	1.6	0.8	Just below critical
2.36	0.0324	32.8	28.5	61.3	530	6.05	4.3	1.6	0.8	Deposit
2.05	0.0282	26.2	23.2	49.4	470	5.35	3.0	1.2	0.6	Flat bed
1.33	0.0183	19.2	17.0	36.2	410	4.65	2.2	0.8	0.4	" "

Series BS-01-1

CRITICAL CONDITION $\begin{cases} C = 0.80\% \\ V_C = 6.40 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$ [in.]	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R [in.]	Δh_D [in.]	$\Delta h_R+\Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R-\Delta h_D$ [in.]	$\Delta h_R-\Delta h_D$ corrected [in.]	C [%]	COMMENTS
3.90	0.0535	68.2	57.0	125.2	750	8.50	11.2	5.1	2.6	Everything moving
3.74	0.0515	62.4	52.6	115.0	725	8.25	9.8	4.2	2.2	" "
3.54	0.0487	55.2	46.7	101.9	680	7.75	8.5	3.7	1.9	Heavy bed load
3.54	0.0487	48.2	41.0	89.2	630	7.20	7.2	3.3	1.7	Sliding bed
3.46	0.0477	44.0	37.4	81.4	615	7.00	6.6	3.0	1.5	Just above critical
3.34	0.0459	42.4	37.0	79.4	600	6.80	5.4	2.0	1.0	Just above critical
3.26	0.0448	41.4	35.8	77.2	590	6.70	5.6	2.2	1.1	CRITICAL
2.87	0.0394	36.6	31.0	67.6	555	6.30	5.6	2.6	1.3	Thin bed
2.28	0.0314	28.4	24.6	53.0	480	5.45	3.8	1.8	0.9	Flat bed
1.67	0.0229	21.6	19.0	40.6	425	4.85	2.6	1.2	0.6	" "
1.59	0.0219	17.0	15.2	32.2	375	4.25	2.2	1.1	0.5	" "

Series BS-01-2

CRITICAL CONDITION $\begin{cases} C = 1.10\% \\ V_C = 6.70 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.75	0.0653	65.8	52.2	118.0	725	8.25	13.6	8.1	4.2	Everything moving
4.38	0.0604	56.8	45.3	102.1	670	7.60	11.5	7.0	3.6	Pulsating bed, just above critical
4.87	0.0670	52.4	42.6	95.0	650	7.35	9.8	5.6	2.9	Just above critical
4.76	0.0656	50.6	40.8	91.4	635	7.25	9.8	5.6	2.9	CRITICAL
4.28	0.0589	44.0	36.4	80.4	600	6.80	7.6	4.2	2.2	Thin bed
3.92	0.0540	38.6	31.4	70.0	550	6.25	7.2	4.3	2.25	Flat bed
3.36	0.0462	29.4	24.9	54.3	485	5.50	4.5	2.5	1.25	" "
2.66	0.0368	24.6	21.2	45.8	445	5.05	3.4	1.8	0.9	" "

Series BS-01-3

$$\text{CRITICAL CONDITION} \begin{cases} C = 3.00\% \\ V_C = 7.25 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
5.54	0.0762	59.0	45.2	104.2	700	7.95	14.8	9.8	5.0	Pulsating
5.56	0.0765	57.0	44.1	101.1	690	7.85	12.9	8.2	4.2	Just about critical
5.56	0.0765	54.1	41.4	95.5	670	7.60	12.7	8.5	4.35	CRITICAL
5.33 5.39	0.0738	50.7	39.7	90.4	655	7.45	11.0	6.8	3.5	Deposit
5.23	0.0720	45.7	35.9	81.6	615	7.00	9.8	6.1	3.1	Deposit
4.66	0.0642	39.7	31.0	70.7	565	6.45	8.7	5.7	2.95	"
4.15	0.0571	33.6	26.6	60.2	525	6.00	7.0	4.5	2.25	Flat bed
3.51	0.0484	28.4	22.6	51.0	480	5.55	5.8	3.8	1.9	" "
2.98	0.0410	24.1	19.8	43.9	445	5.05	4.3	2.7	1.4	" "
2.26	0.0310	17.6	15.2	32.8	385	4.40	2.4	1.2	0.60	Flat bed, saltating
1.64	0.0260	13.4	11.5	24.9	325	3.70	1.9	1.1	0.5	Flat bed, saltating

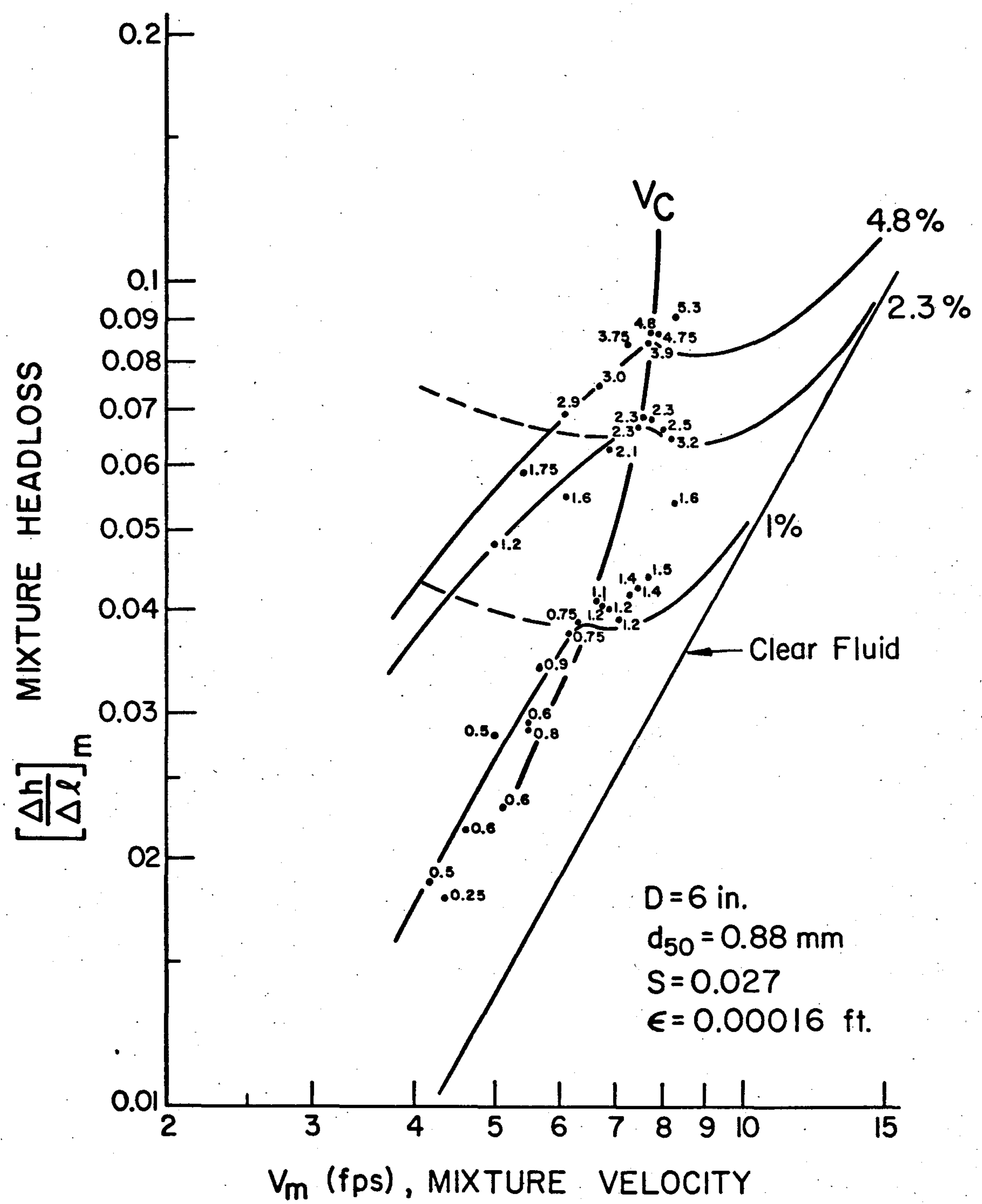
Continued

Series BS-01-4

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
						2nd Run				
6.05	0.0834	69.5	51.4	120.9	720	8.20	18.1	12.6	6.5	Heavy bed load
5.68	0.0783	64.2	47.4	111.6	685	7.80	16.8	12.0	6.1	Sliding bed
5.59	0.0769	62.0	46.3	108.3	680	7.75	15.7	11.0	5.7	" "
6.07	0.0835	55.4	41.6	97.0	650	7.40	13.8	9.7	5.0	CRITICAL
5.64	0.0775	47.4	36.8	84.2	600	6.85	10.6	7.2	3.7	Deposit bed
4.87	0.0670	35.8	27.5	63.3	530	6.05	8.3	5.8	3.0	Thick flat bed
1.05	0.0145	5.8	4.8	10.6	210	2.40	1.0	0.6	0.3	Dunes

Series BS-01-4

$$\text{CRITICAL CONDITION} \begin{cases} C = 5.00\% \\ V_C = 7.40 \text{ fps} \end{cases}$$



Plot of Series BS-03 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.22	0.0440	54.0	46.4	100.4	675	7.70	7.6	2.9	1.5	Everything moving
3.05	0.0420	49.4	42.5	91.9	645	7.35	6.9	2.8	1.4	" "
2.92	0.0401	43.7	37.9	81.6	610	6.95	5.8	2.3	1.2	Heavy bed load
3.00	0.0412	40.5	35.2	75.7	585	6.65	5.3	2.1	1.1	Pulsating, just above critical
2.82	0.0388	36.4	32.0	68.4	560	6.35	4.4	1.5	0.75	CRITICAL
2.74	0.0377	33.4	29.2	62.6	540	6.15	4.2	1.5	0.75	Deposit, thin bed
2.13	0.0292	27.0	23.9	50.9	485	5.50	3.1	1.1	0.6	Flat bed
2.05	0.0282	21.4	19.0	40.4	440	5.00	2.4	0.9	0.5	" "
1.31	0.01800	16.8	15.1	31.9	380	4.35	1.7	0.5	0.25	" "
					2nd Run					
3.95	0.0542	61.4	52.4	113.8	730	8.30	9.0	3.2	1.6	Everything moving
3.08	0.0422	50.8	43.6	94.4	660	7.50	7.2	2.8	1.4	" "
2.97	0.0394	45.2	39.1	84.3	625	7.10	6.1	2.3	1.2	Heavy bed load
2.95	0.0405	41.8	36.1	77.9	595	6.80	5.7	2.3	1.2	Pulsating, just above critical
2.82	0.0387	39.0	33.8	72.8	570	6.50	5.2	2.2	1.1	CRITICAL
2.54	0.0349	33.6	29.1	62.7	530	6.05	4.5	2.0	1.0	Deposit
2.49	0.0342	30.4	26.4	56.8	500	5.70	4.0	1.8	0.9	Thin bed

Continued

Series BS-03-1

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
2.10	0.0289	28.2	24.6	52.8	485	5.50	3.6	1.6	0.8	Flat bed
1.69	0.0232	24.0	21.2	45.2	455	5.15	2.8	1.2	0.6	" "
1.59	0.0218	20.4	17.6	38.0	410	4.65	2.8	1.3	0.6	" "
1.36	0.0187	16.85	14.7	31.55	370	4.20	2.15	1.0	0.5	" "

Series BS-03-1

CRITICAL CONDITION $\begin{cases} C = 1.00\% \\ V_C = 6.40 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.77	0.0655	59.5	47.9	107.4	720	8.20	11.6	6.1	3.2	Everything moving
4.83	0.0662	56.15	46.0	102.15	705	8.00	10.15	4.95	2.5	Heavy bed load
4.97	0.0683	53.4	43.1	96.5	685	7.80	9.3	4.5	2.3	Pulsating, just above critical
4.94	0.0680	51.0	41.9	92.9	670	7.60	9.1	4.5	2.3	CRITICAL
4.81	0.0662	49.8	40.8	90.6	660	7.50	9.0	4.6	2.3	Deposit, thin bed
4.56	0.0627	42.7	35.1	77.8	610	6.95	7.6	4.1	2.1	Flat bed
4.00	0.0550	33.6	28.0	61.6	540	6.15	5.6	3.1	1.6	" "
3.51	0.0483	27.8	13.80	41.6	440	5.00	4.0	2.3	1.2	" "

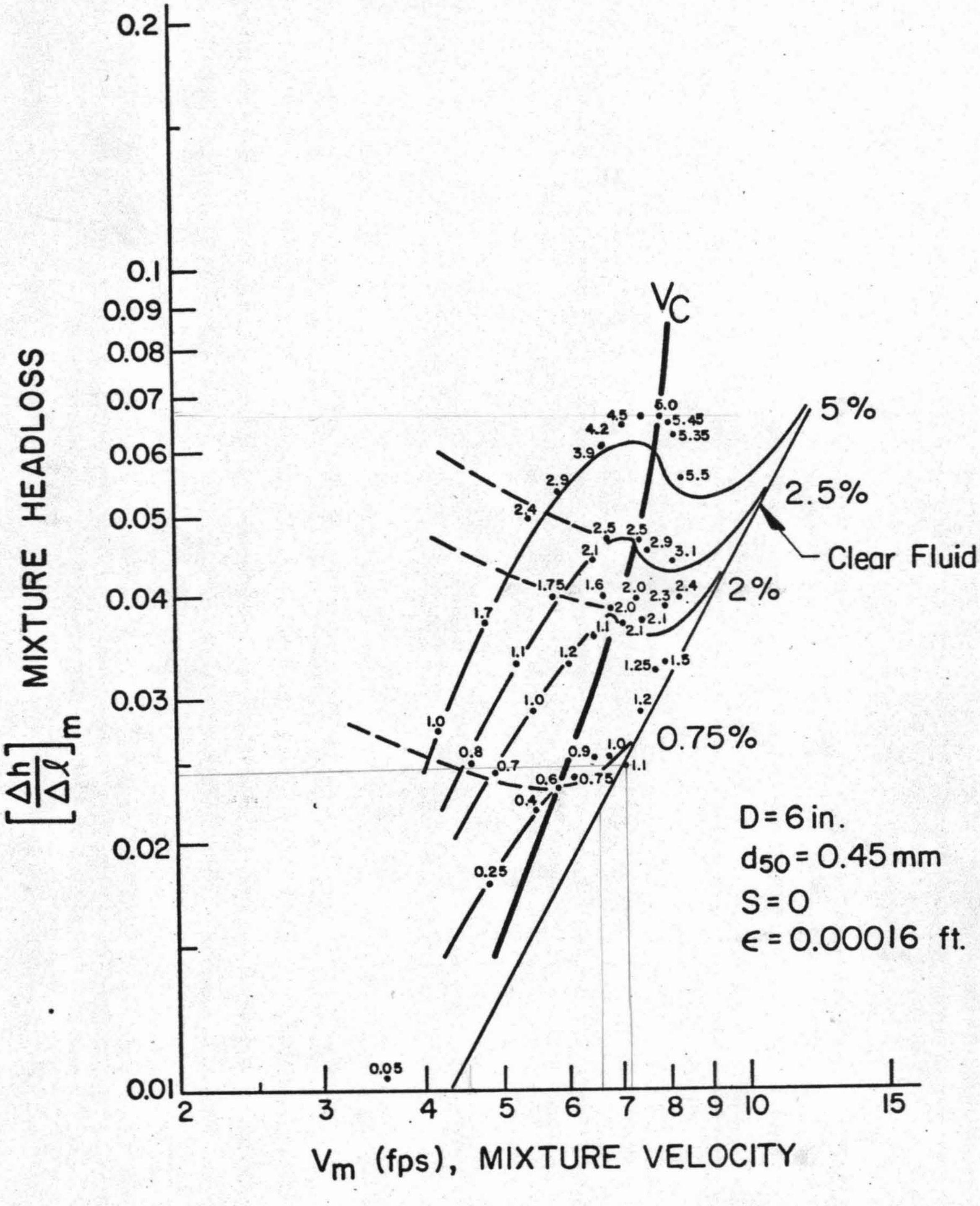
Series BS-03-2

CRITICAL CONDITION $\begin{cases} C = 2.30\% \\ V_C = 7.60 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$ [in.]	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R [in.]	Δh_D [in.]	$\Delta h_R+\Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R-\Delta h_D$ [in.]	$\Delta h_R-\Delta h_D$ corrected [in.]	C [%]	COMMENTS
6.56	0.0902	68.0	51.4	119.4	755	8.30	16.6	10.4	5.3	Everything moving
6.36	0.0874	58.7	44.5	103.2	700	7.95	14.2	9.1	4.75	Pulsating, just above critical
6.36	0.0874	56.8	42.7	99.5	690	7.85	14.1	9.2	4.8	CRITICAL
6.29	0.0864	53.7	41.3	95.0	675	7.75	12.4	7.7	3.9	Deposit
6.13	0.0842	48.3	37.0	85.3	640	7.30	11.3	7.3	3.75	Flat bed
5.44	0.0746	41.6	32.4	74.0	590	6.70	9.2	5.9	3.0	" "
5.07	0.0697	34.4	26.1	60.5	535	6.10	8.3	5.8	2.9	" "
4.26	0.0585	27.0	21.5	48.5	480	5.45	5.5	3.5	1.75	" "

Series BS-03-3

CRITICAL CONDITION $\begin{cases} C = 4.80\% \\ V_C = 7.85 \text{ fps} \end{cases}$



Plot of Series BS-001 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
2.40	0.0330	53.7	45.7	99.4	700	7.95	8.0	3.0	1.5	Everything moving
2.35	0.0324	49.8	42.8	92.6	675	7.7	7.0	2.5	1.25	Heavy bed load
2.10	0.0289	46.7	40.0	86.7	650	7.4	6.7	2.4	1.2	" " "
1.80	0.0248	43.1	37.1	80.2	625	7.1	6.0	2.1	1.1	Rapidly moving bed
1.85	0.0255	40.1	34.7	74.8	600	6.8	5.4	1.9	1.0	Pulsating bed
1.80	0.0255	36.0	31.2	67.2	570	6.5	4.8	1.8	0.9	" "
1.75	0.0241	32.3	28.0	60.3	540	6.15	4.3	1.5	0.75	Slowly pulsating bed
1.70	0.0234	29.7	26.2	55.9	515	5.85	3.5	1.2	0.6	CRITICAL
1.60	0.0220	25.8	23.0	48.8	485	5.5	2.8	0.8	0.4	Thin bed
1.30	0.0179	20.2	18.0	38.2	420	4.8	1.8	0.4	0.25	Flat bed
0.75	0.0103	11.5	10.6	22.2	315	3.6	0.9	0.1	0.05	" "

Series BS-001-1

$$\text{CRITICAL CONDITION}\begin{cases} C = 0.75\% \\ V_C = 5.85 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(-\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
2.90	0.0399	61.8	51.2	113.0	730	8.3	10.6	4.7	2.4	Everything moving
2.80	0.0386	56.4	46.6	103.0	700	7.95	9.8	4.5	2.3	Heavy bed load
2.70	0.0371	50.0	41.4	91.4	655	7.45	8.6	4.1	2.1	Pulsating, sliding bed
2.85	0.0392	47.0	39.0	86.0	645	7.35	8.0	3.8	1.95	Pulsating, sliding bed
2.70	0.0371	44.0	36.4	80.4	620	7.05	7.6	3.8	1.95	Just above critical
										$\boxed{\text{CRITICAL}}$
2.80	0.0386	41.4	34.8	76.2	600	6.85	6.6	3.1	1.60	Deposit
2.60	0.0358	35.8	30.7	66.5	570	6.5	5.1	2.1	1.10	Thin bed
2.40	0.0330	31.4	26.6	58.0	530	6.05	4.8	2.3	1.20	Flat bed
2.10	0.0289	26.9	23.0	49.9	485	5.5	3.9	1.9	1.0	" "
1.75	0.0241	21.7	18.8	40.5	430	4.9	2.9	1.4	0.7	" "
0.55	0.0076	6.0	5.2	11.2	220	2.5	0.8	0.4	0.2	Dunes

Series BS-001-2

$$\text{CRITICAL CONDITION} \begin{cases} C = 1.90\% \\ V_C = 6.95 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$ [in.]	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R [in.]	Δh_D [in.]	$\Delta h_R+\Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R-\Delta h_D$ [in.]	$\Delta h_R-\Delta h_D$ corrected [in.]	C [%]	COMMENTS
3.20	0.0440	59.9	48.2	108.1	715	8.15	11.7	6.1	3.1	Heavy bed load
3.30	0.0454	53.0	42.8	95.8	670	7.6	10.2	5.5	2.85	Quickly moving bed
3.40	0.0469	49.7	40.4	90.1	655	7.45	9.3	4.8	2.45	CRITICAL
3.40	0.0469	42.8	34.4	77.2	600	6.8	8.4	4.9	2.47	Thin bed
3.20	0.0440	38.0	30.9	68.9	570	6.5	7.1	4.1	2.10	Thickening bed
2.90	0.0399	31.1	25.4	56.5	510	5.8	5.7	3.4	1.75	" "
2.40	0.0330	24.85	21.0	45.85	460	5.25	3.85	2.15	1.10	" "
1.80	0.0248	19.4	16.5	35.9	410	4.65	2.9	1.50	0.80	Flat bed
0.60	0.0083	6.0	5.3	11.3	220	2.5	0.7	0.30	0.15	Very little saltation, dunes

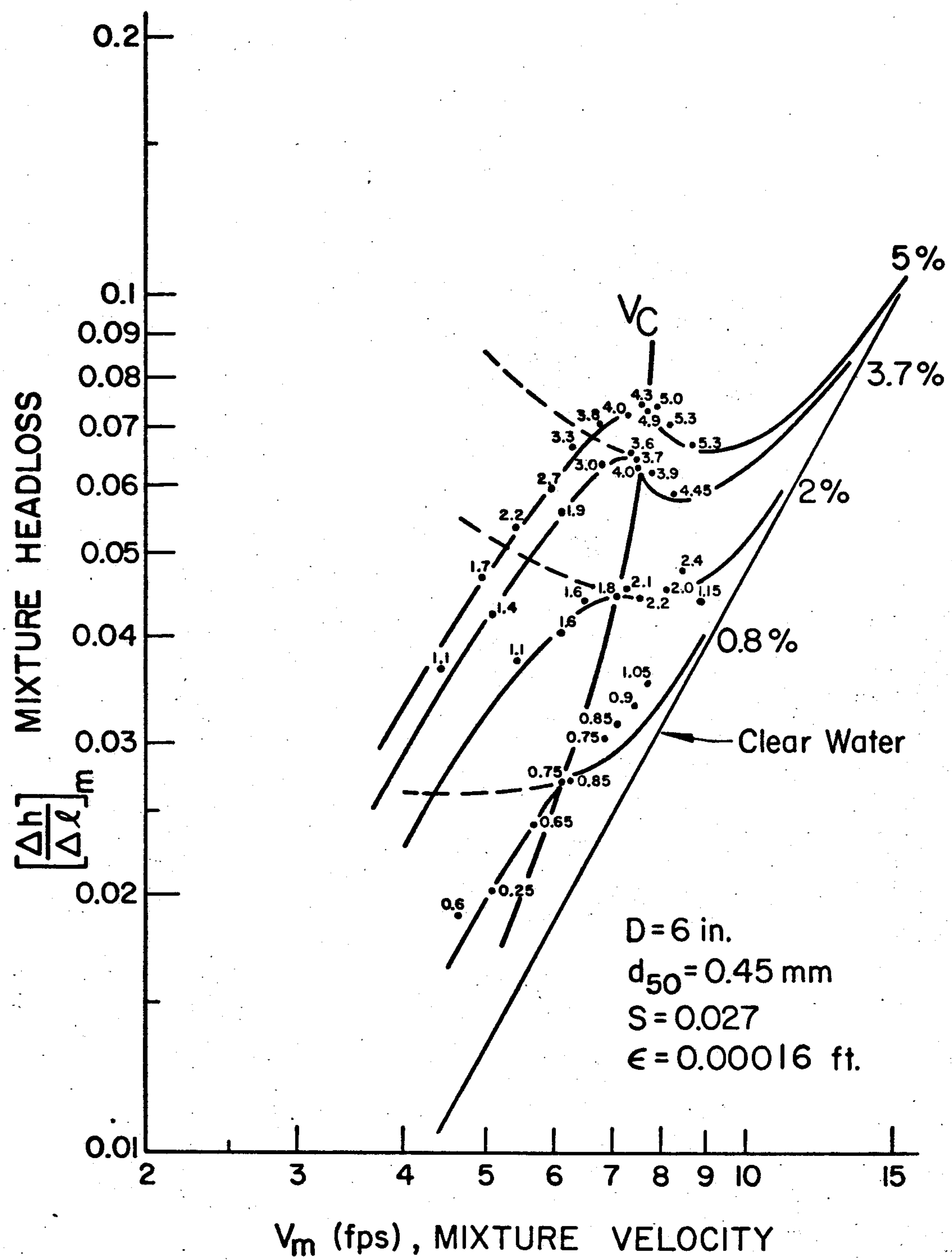
Series BS-001-3

$$\text{CRITICAL CONDITION} \begin{cases} C = 2.50\% \\ V_C = 7.45 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.10	0.0555	68.10	51.30	119.40	740	8.4	16.80	10.70	5.5	Above critical, slowly pulsating
4.60	0.0631	64.40	48.20	112.60	720	8.2	16.20	10.50	5.35	Just above critical
4.70	0.0645	62.10	45.90	108.00	710	8.1	16.20	10.70	5.45	Sliding
										CRITICAL
4.80	0.0660	58.40	43.40	101.80	690	7.85	15.00	9.90	5.05	Just below critical
4.80	0.0660	53.00	39.60	92.60	660	7.5	13.40	8.80	4.50	Flat bed
4.70	0.0645	48.80	36.80	85.60	620	7.05	12.00	8.3	4.2	" "
4.40	0.0605	42.80	32.00	74.80	585	6.65	10.80	7.5	3.85	" "
3.90	0.0536	33.40	25.30	58.70	520	5.9	8.10	5.70	2.90	" "
3.60	0.0495	28.90	22.25	51.15	480	5.45	6.65	4.65	2.40	" "
2.70	0.0371	21.90	17.30	39.20	420	4.8	4.60	3.20	1.65	" "
2.00	0.0275	16.50	13.50	30.00	370	4.2	3.00	1.90	1.00	Flat bed, little bed load
0.70	0.0096	5.90	5.10	11.00	215	2.45	0.80	0.40	0.20	Dunes

CRITICAL CONDITION: C = 5.40%
V_C = 7.95 fps

Series BS-001-4



Plot of Series BS-003 Data

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.20	0.0440	63.8	55.0	118.8	775	8.84	8.8	2.3	1.15	Heavy bed load
2.56	0.0352	49.2	42.5	91.7	675	7.70	6.7	2.1	1.05	" " "
2.44	0.0335	46.2	40.2	86.4	655	7.45	6.0	1.8	0.9	Sliding bed
2.31	0 0317	43.0	37.5	80.5	625	7.10	5.5	1.7	0.85	Pulsating bed
2.20	0.0304	40.0	35.0	75.0	605	6.87	5.0	1.5	0.75	" "
2.0	0.0274	33.0	28.6	61.6	550	6.25	4.4	1.7	0.85	Just above critical
1.97	0.0271	31.8	27.7	59.5	540	6.15	4.1	1.5	0.75	$\boxed{\text{CRITICAL}}$
1.77	0.0243	27.6	24.2	51.8	500	5.70	3.4	1.3	0.65	Deposit
1.49	0.0204	21.4	19.3	40.7	450	5.10	2.1	0.5	0.25	Thin bed

Series BS-003-1

$$\text{CRITICAL CONDITION} \begin{cases} C = 0.75\% \\ V_C = 6.15 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.49	0.0480	63.4	52.6	116.0	745	8.50	10.8	4.8	2.4	Heavy bed load
3.31	0.0455	57.4	48.2	105.6	710	8.10	9.2	3.9	2.0	" " "
3.26	0.0448	53.8	44.4	98.2	670	7.60	9.4	4.5	2.2	Sliding bed
3.31	0.0455	47.4	39.2	86.6	635	7.25	8.2	4.2	2.1	Pulsating bed
										CRITICAL
3.23	0.0444	44.2	37.0	81.0	620	7.05	7.2	3.5	1.8	Just below critical
3.21	0.0441	38.6	32.4	71.0	575	6.55	6.2	3.1	1.6	Thin bed
2.95	0.0405	34.0	28.2	62.2	540	6.15	5.8	3.1	1.6	Flat bed
2.72	0.0374	27.0	23.0	50.0	480	5.45	4.0	2.1	1.1	" "
2.61	0.0190	19.8	17.2	37.0	410	4.65	2.6	1.2	0.6	" "
0.66	0.0092	5.8	5.4	11.2	210	2.40	0.4	0.1	0.05	Dunes

Series BS-003-2

CRITICAL CONDITION $\begin{cases} C = 2.00\% \\ V_C = 7.10 \text{ fps} \end{cases}$

test section		loop readings								
$\Delta h_{1.95}$	$(\frac{\Delta h}{\Delta \ell})_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.23	0.058	65.4	51.0	116.4	730	8.30	14.4	8.7	4.45	Everything moving
4.51	0.062	57.8	45.3	103.1	685	7.80	12.5	7.6	3.9	Sliding bed
4.56	0.0627	55.3	43.0	98.3	665	7.55	12.3	7.8	4.0	Pulsating bed
4.62	0.0640	55.2	43.5	98.7	665	7.55	11.7	7.2	3.7	Just above critical
										CRITICAL
4.75	0.0652	53.0	41.8	94.8	655	7.45	11.2	7.0	3.6	Just below critical
4.59	0.0630	44.6	35.25	79.85	605	6.90	9.35	5.85	3.0	Flat bed
4.07	0.0560	34.6	28.30	62.90	540	6.15	6.3	3.7	1.9	" "
3.08	0.0422	24.0	19.60	43.60	450	5.10	4.4	2.7	1.4	" "

Series BS-003-3

$$\text{CRITICAL CONDITION} \begin{cases} C = 3.70\% \\ V_C = 7.50 \text{ fps} \end{cases}$$

test section		loop readings								
$\Delta h_{1.95}$	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R+\Delta h_D$	Q_m	V_m	$\Delta h_R-\Delta h_D$	$\Delta h_R-\Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.82	0.0663	69.2	52.7	121.9	760	8.65	16.5	10.3	5.3	Sliding bed
5.10	0.0702	64.0	48.2	112.2	720	8.20	15.8	10.3	5.3	" "
5.39	0.0741	60.4	45.7	106.1	700	7.95	14.7	9.6	5.0	Pulsating bed
5.35	0.0738	58.8	44.5	103.3	680	7.75	14.3	9.5	4.9	CRITICAL
5.46	0.0752	55.6	42.6	98.2	670	7.60	13.0	8.5	4.3	Deposit
5.46	0.0738	50.2	38.4	88.6	635	7.25	11.8	7.9	4.0	Thin bed
5.13	0.0706	44.8	34.0	78.8	600	6.80	10.8	7.4	3.8	Flat bed
4.82	0.0663	39.5	30.1	69.6	560	6.35	9.4	6.5	3.3	" "
4.34	0.0596	34.0	26.2	60.2	525	6.00	7.8	5.4	2.7	" "
3.90	0.0536	28.2	22.1	50.3	475	5.40	6.1	4.3	2.2	" "
3.41	0.0470	23.7	18.8	42.5	435	4.95	4.9	3.4	1.7	" "
2.64	0.0364	19.0	15.7	34.7	390	4.45	3.3	2.1	1.1	" "

Series BS-003-4

CRITICAL CONDITION $\begin{cases} C = 5.00\% \\ V_C = 7.75 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O} [in.]	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R [in.]	Δh_D [in.]	$\Delta h_R + \Delta h_D$ [in.]	Q_m [gpm]	V_m [fps]	$\Delta h_R - \Delta h_D$ [in.]	$\Delta h_R - \Delta h_D$ corrected [in.]	C [%]	COMMENTS
2.60	0.0183	22.70	21.65	44.35	445	5.05	1.05	0.95	2.10	Total transport, heavy bed load
2.50	0.0176	18.80	17.75	36.55	400	4.5	1.05	0.95	2.00	Heavy bed load
2.00	0.0141	15.15	14.30	29.45	355	4.05	0.85	0.75	1.70	Thickening bed sliding along invert
1.90	0.0134	13.25	12.40	25.65	330	3.75	0.85	0.70	1.50	Pulsating bed movement with sporadic settling
1.80	0.0127	12.50	11.70	24.20	320	3.6	0.80	0.65	1.30	Just above critical condition
1.70	0.0120	11.40	10.60	22.00	300	3.4	0.80	0.65	1.25	CRITICAL
1.40	0.0099	9.20	9.10	18.80	270	3.05	0.60	0.45	1.00	Sporadic dune behavior

Series BS-PP1-1

$$\text{CRITICAL CONDITION} \begin{cases} C = 1.30\% \\ V_C = 3.40 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.35	0.0236	28.20	26.80	55.00	500	5.7	1.40	1.25	2.80	Full bed load transport
3.00	0.0212	24.00	22.70	46.70	460	5.25	1.30	1.15	2.50	Slowly moving bed load
3.05	0.0216	20.30	19.15	39.45	420	4.8	1.15	1.05	2.35	Sliding bed
2.90	0.0204	18.55	17.45	36.00	395	4.55	1.10	1.00	2.20	" "
2.80	0.0197	16.65	15.75	32.40	365	4.15	0.90	0.85	1.95	Sporadic, pulsating transport conditions
		15.40	14.40	29.80	360	4.1	1.00	0.90	2.00	
2.70	0.0190	14.05	13.10	27.15	345	3.9	0.95	0.85	1.95	Just above crit. condition
2.50	0.0176	13.45	12.60	26.05	340	3.85	0.85	0.75	1.70	CRITICAL
2.10	0.0480	11.45	10.75	22.20	315	3.6	0.70	0.60	1.30	Infrequent duning concentrations

Series BS-PP1-2

CRITICAL CONDITION $\begin{cases} C = 1.90\% \\ V_C = 3.85 \text{ fps} \end{cases}$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
3.90	0.0275	25.80	24.15	49.95	485	5.5	1.65	1.50	3.30	Total transport basically bed load
3.95	0.0279	23.20	21.60	44.80	455	5.15	1.60	1.45	3.20	Heavy bed load
4.10	0.0289	21.10	19.60	40.70	440	5.00	1.50	1.40	3.15	Sliding, thickening bed load
4.10	0.0289	18.95	17.55	36.50	415	4.75	1.40	1.30	3.00	Pulsating just above crit. condition of bed stoppage
3.80	0.0268	17.70	16.20	33.90	395	4.55	1.40	1.30	3.00	Almost crit.
										CRITICAL
4.10	0.0289	14.45	13.30	27.75	360	4.10	1.15	1.05	2.3	Bed and long dune build-up

Series BS-PP1-3

$$\text{CRITICAL CONDITION} \begin{cases} C = 3.00\% \\ V_C = 4.45 \text{ fps} \end{cases}$$

test section		loop readings								
Δh_{H_2O}	$\left(\frac{\Delta h}{\Delta \ell}\right)_m$	Δh_R	Δh_D	$\Delta h_R + \Delta h_D$	Q_m	V_m	$\Delta h_R - \Delta h_D$	$\Delta h_R - \Delta h_D$ corrected	C	COMMENTS
[in.]		[in.]	[in.]	[in.]	[gpm]	[fps]	[in.]	[in.]	[%]	
4.70	0.0332	29.00	26.55	55.55	515	5.9	2.45	2.30	5.10	Most all transport in form of heavy bed load
4.70	0.0332	27.25	24.95	52.20	495	5.6	2.30	2.10	4.70	Bed load
4.50	0.0318	26.25	23.95	50.20	485	5.5	2.30	2.10	4.65	Slow moving thickening bed
4.80	0.0338	24.30	22.05	46.35	470	5.35	2.25	2.05	4.55	Effective scour mechanism
4.90	0.0345	23.05	21.00	44.05	450	5.10	2.05	1.95	4.30	Pulsating bed
4.55	0.0320	19.05	17.15	36.20	415	4.75	1.80	1.70	3.90	Slugs of varying concentration
										CRITICAL
3.90	0.0275	16.30	14.60	30.90	380	4.35	1.70	1.60	3.50	Deposit condition, impulsive dune motion

Series BS-PP1-4

$$\text{CRITICAL CONDITION} \begin{cases} C = 3.80\% \\ V_C = 4.60 \text{ fps} \end{cases}$$

APPENDIX C: REGRESSION ANALYSIS DATA

A regression analysis was made to correlate each of three modified Froude numbers (I), (II), and (III), as defined in Section 4.1 of the contents, with the following parameters: concentration C; concentration; C, and particle diameter, d; and concentration, C, and relative particle size, d/D. The results of this analysis are tabulated in Tables C.1(a), C.1(b), and C.1(c) for each Froude number.

The modified Froude numbers were calculated with solids concentration, C, over five different ranges of data. Correlation was also evaluated for regression of each modified Froude number with both solids concentration, C, and either particle diameter, d, or relative particle size, d/D, over two ranges of data. These ranges are specified in Tables C.1(a), C.1(b), and C.1(c) along with indications of "goodness of fit".

The regression analysis fits data to a geometric curve, correlating logarithmic values on a linear or arithmetic scale, as given with:

$$\text{Log } F_r = k_2 \text{ Log } C + \text{Log } k_1 \tag{C.1}$$

Reconverting to arithmetic scale gives the form:

$$F_r = k_1 \, C^{k_2} \tag{4.1}$$

Likewise, for a multiple regression analysis with modified Froude number, F_r, solids concentration, C, and either particle diameter, d, or relative particle size, d/D, the linear form for log-log data fitting is given with:

$$\text{Log } F_r = k_4 \text{ Log } C + k_5 \text{ Log } d + \text{Log } k_3 \tag{C.2}$$

and subsequently written as:

$$F_r = k_3 \, C^{k_4} \cdot d^{k_5} \tag{4.2a}$$

Standard deviation, S.D., coefficient of correlation, R, and standard error of estimate, S_y, are given for each analysis listed in Table C.2, defined respectively as:

$$\frac{V_C}{\sqrt{2gD\,(s_s-1)}} = f_2\ (C)$$

Range	No. of Data	Equation	S.D.	R	S_y
d = 0.88 mm	22	$F_r = 0.901\ C^{0.086}$	0.049	0.845	0.0264
d = 0.45 mm	24	$F_r = 0.892\ C^{0.131}$	0.088	0.935	0.0311
d = 3.63 mm	4	$F_r = 0.909\ C^{0.290}$	0.052	0.994	0.0059
d = 0.45 to 0.88 mm	46	$F_r = 0.893\ C^{0.114}$	0.073	0.886	0.0336
all d	50	$F_r = 0.905\ C^{0.122}$	0.078	0.872	0.0380

$$\frac{V_C}{\sqrt{2gD\,(s_s-1)}} = f_2\ (C,d)$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.921\ C^{0.109}\ d^{0.058}$	0.871
all d	50	$F_r = 0.927\ C^{0.110}\ d^{0.070}$	0.863

$$\frac{V_C}{\sqrt{2gd\,(s_s-1)}} = f_2\ (C,\frac{d}{D})$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.905\ C^{0.113}\ \frac{d}{D}^{0.002}$	0.879
all d	50	$F_r = 0.905\ C^{0.114}\ \frac{d}{D}^{0.002}$	0.818

Table C.1(a): Correlation with Modified Froude Number (I)

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta] = f_2\ (C)$$

Range	No. of Data	Equation	S.D.	R	S_y
d = 0.88 mm	22	$F_r = 0.908\ C^{0.080}$	0.047	0.831	0.0259
d = 0.45 mm	24	$F_r = 0.900\ C^{0.124}$	0.084	0.919	0.0332
d = 3.63 mm	4	$F_r = 0.909\ C^{0.290}$	0.052	0.994	0.0059
d = 0.45 to 0.88 mm	46	$F_r = 0.901\ C^{0.106}$	0.069	0.870	0.0343
all d	50	$F_r = 0.912\ C^{0.114}$	0.075	0.854	0.0387

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta] = f_2\ (C,d)$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.928\ C^{0.105}\ d^{0.056}$	0.877
all d	50	$F_r = 0.934\ C^{0.106}\ d^{0.068}$	0.866

$$\frac{V_C}{\sqrt{2gD\ (s_s-1)}}\ [1 - \tan\theta] = f_2\ (C,\frac{d}{D})$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.913\ C^{0.108}\ \frac{d}{D}^{0.002}$	0.884
all d	50	$F_r = 0.912\ C^{0.110}\ \frac{d}{D}^{0.002}$	0.820

Table C.2(b): Correlation with Modified Froude Number (II)

$$\frac{V_C}{\sqrt{2gD\,(s_s-1)\,[1+\tan\theta]}} = f_2\ (C)$$

Range	No. of Data	Equation	S.D.	R	S_y
d = 0.88 mm	22	$F_r = 0.905\ C^{0.082}$	0.048	0.839	0.0259
d = 0.45 mm	24	$F_r = 0.896\ C^{0.127}$	0.086	0.928	0.0319
d = 3.63 mm	4	$F_r = 0.909\ C^{0.290}$	0.052	0.994	0.0059
d = 0.45 to 0.88 mm	46	$F_r = 0.898\ C^{0.110}$	0.071	0.880	0.0336
all d	50	$F_r = 0.909\ C^{0.118}$	0.076	0.864	0.0381

$$\frac{V_C}{\sqrt{2gD\,(s_s-1)\,[1+\tan\theta]}} = f_2\ (C,d)$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.925\ C^{0.107}\ d^{0.057}$	0.878
all d	50	$F_r = 0.931\ C^{0.108}\ d^{0.069}$	0.867

$$\frac{V_C}{\sqrt{2gD\,(s_s-1)\,[1+\tan\theta]}} = f_2\ (C,\tfrac{d}{D})$$

Range	No. of Data	Equation	R
d = 0.45 to 0.88 mm	46	$F_r = 0.909\ C^{0.110}\ \left(\frac{d}{D}\right)^{0.002}$	0.885
all d	50	$F_r = 0.909\ C^{0.112}\ \left(\frac{d}{D}\right)^{0.002}$	0.822

Table C.3(c): Correlation with Modified Froude Number (III)

$$R = \pm \sqrt{\frac{\text{explained variation}}{\text{total variation}}} \tag{C.3}$$

or:

$$R = \pm \sqrt{\frac{\Sigma\ (F_{est} - \bar{F})^2}{\Sigma\ (F - \bar{F})^2}} \tag{C.4}$$

where $\Sigma\ (F_{est} - \bar{F})^2$ is the sum of the deviations of fitted (or estimated) values from the average, squared; and $\Sigma\ (F - \bar{F})^2$ is the sum of the deviations of actual data values from the average, squared.

$$S.D. = \sqrt{\frac{\Sigma\ (F - \bar{F})^2}{N}} \tag{C.5}$$

where N is the total number of data analyzed.

$$S_y = S.D. \sqrt{1 - R^2} \tag{C.6}$$

The standard error of estimate, S_y, includes both central tendency, related to standard deviation, S.D., and variability, described by the coefficient of correlation, R, in indicating "goodness of fit".

One is warned that the coefficient of correlation, R, determined on a log-log scale, as reported in this study, may give a misleading indication of "goodness of fit" that would be found on an arithmetic scale. Log-log data near to the origin have the strongest influence on the regression. Since most of the Lehigh data were obtained at low solids concentrations, $0.10 < C < 2.0\%$, log-log fitting works to our advantage. Correlation, on the other hand, weighs every data point equally, and an insignificant change in regression at a high solids concentration data point may mistakenly infer greatly improved correlation, or vica versa. For a closer look at the raw data which determined best-fit, the regression analysis data output is on file in Fritz Laboratory at Lehigh University.

Some resulting best-fit equations, from the Froude number (II) analysis, are presented in Figs. C.1 to C.3, inclusively. Figure C.1 shows the best-fit equation for modified Froude number, F_r, correlated with solids concentration, C, as evaluated for each of the three tested particle diameters, d. A relationship between sand and plastic pellet results is not immediately recognized. However, the similarity in form exhibited between the equations for sand is to be expected, subsequent to a study of Gibert (1960).

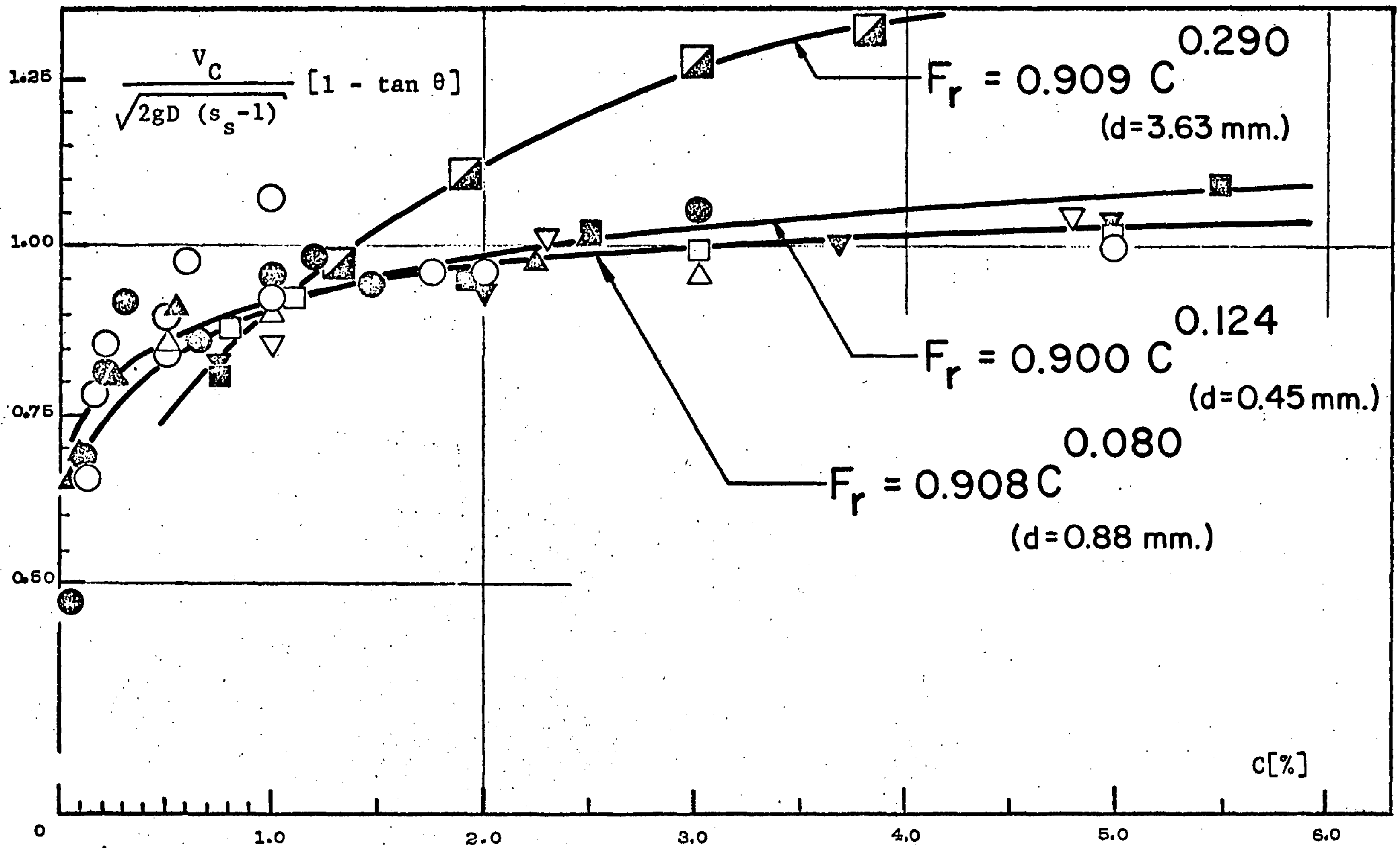


Fig. C.1: Equations Best Fitting Modified Froude Number with Solids Concentration; Particle Diameter (for two different sands and plastic pellets) as Parameter

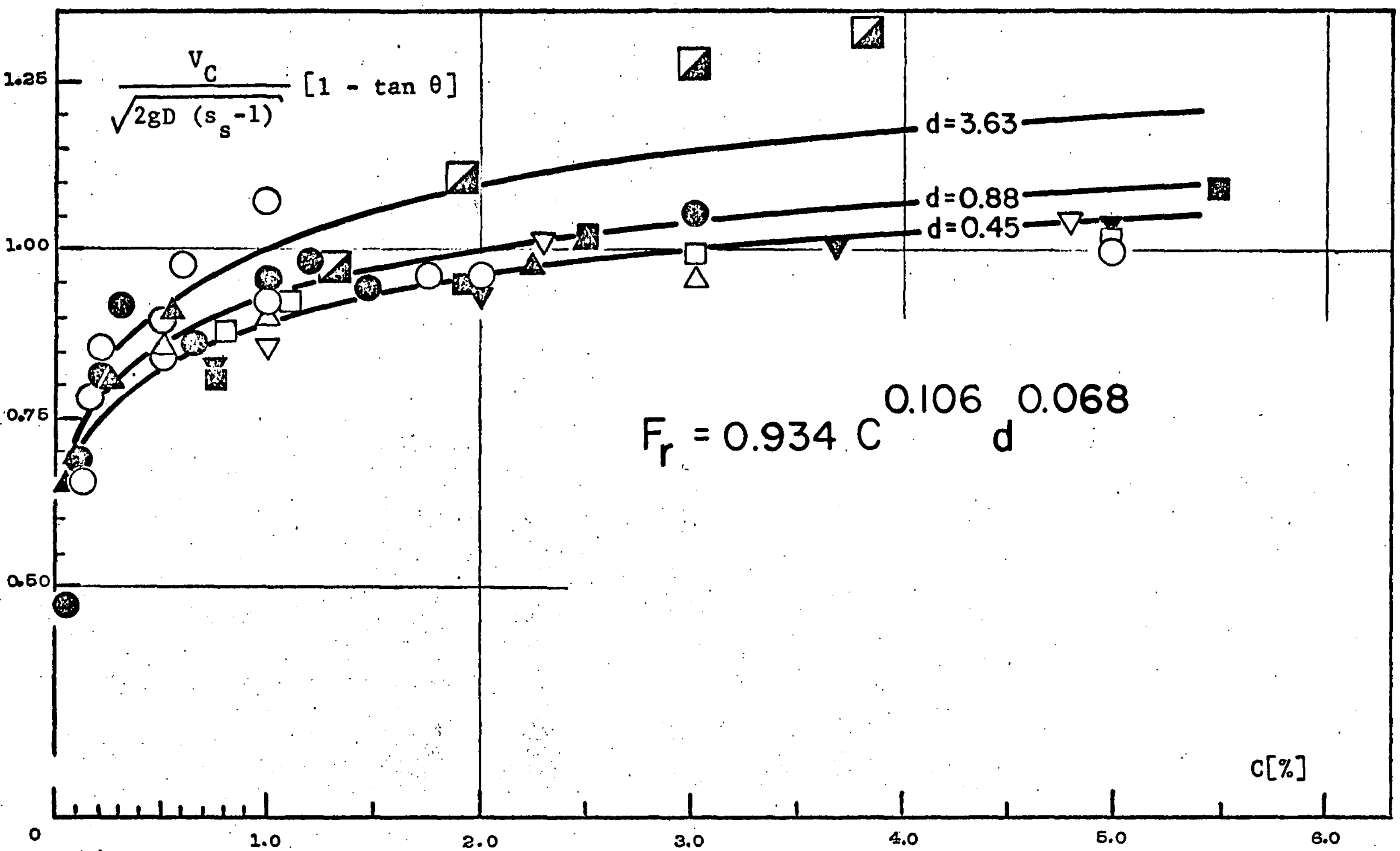


Fig. C.2: Equation Best Fitting Modified Froude Number with Solids Concentration and Particle Diameter, Evaluated for Both Sand-Water and Plastic Pellet-Water Data

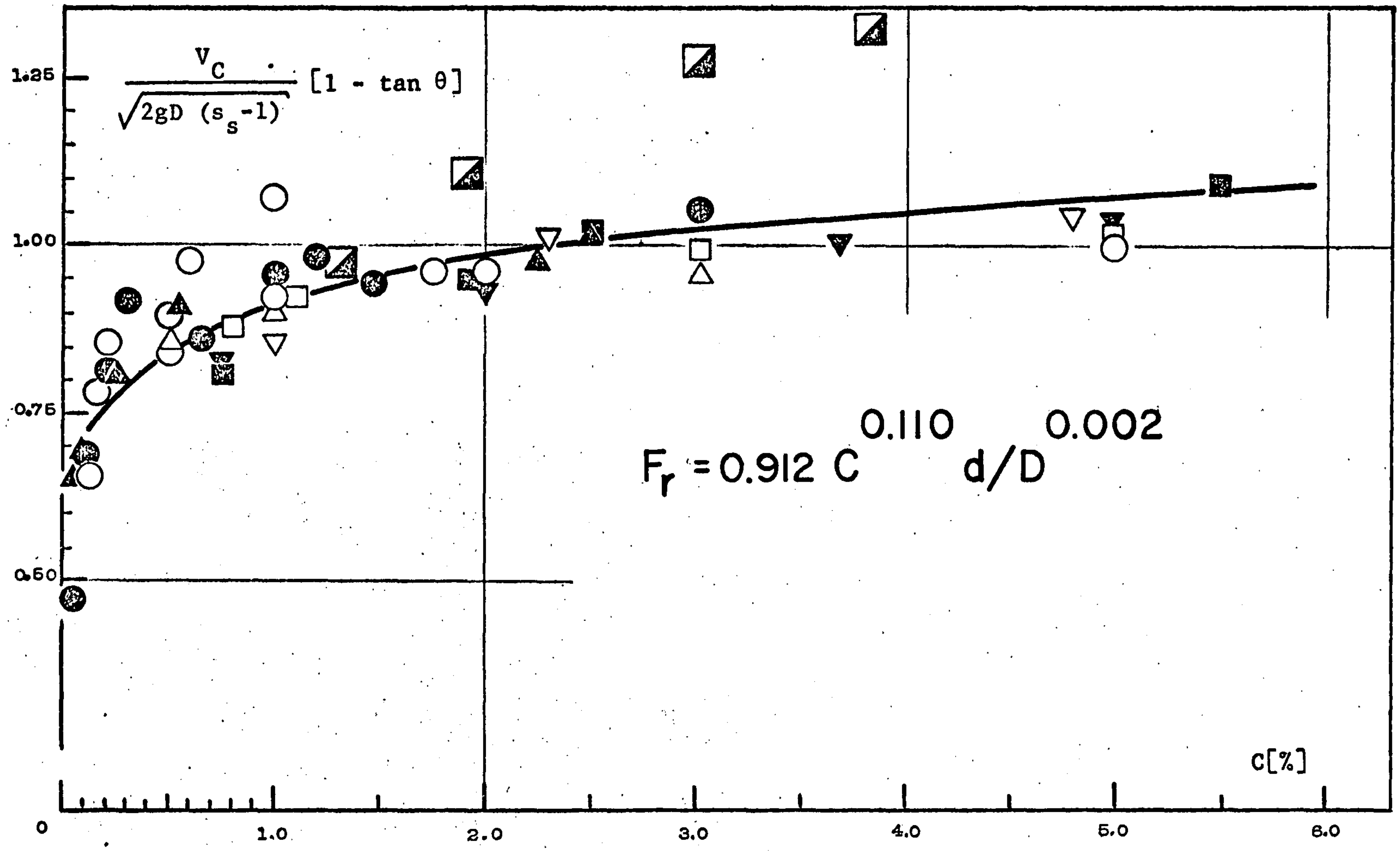


Fig. C.3: Equation Best Fitting Modified Froude Number with Solids Concentration and Relative Particle Size, Evaluated for Both Sand-Water and Plastic Pellet-Water Data

Figure C.2 illustrates the effect of including particle diameter, d, as an independent variable in correlating all of the data. Since there are relatively few data points for sufficiently expressing the trend of the plastic pellets data, the sand particles dictate the general form of the function. However, it should be noted that the plastic pellets significantly impinge upon the form of the sand particle curves at low concentrations. It is to this end that use of Fig. 2.3 and the associated relationship is discouraged.

Figure C.3 gives the relationship for Froude number (II) fitted with solids concentration, C, and relative particle size, d/D, over the entire range of data. The inclusion of d/D is relatively negligible, and the effect due to different particle diameters, d, is essentially eliminated. Further, the plastic pellet data impose a greater relative influence on the regression than indicated in other correlations of the total data. The relationship given with Fig. C.3 is also not recommended.

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VITA

The author, Millard P. Robinson, Jr., was born to Millard P. Robinson and Louise C. Robinson on November 24, 1947. At that time he was the first and to be the only brother of a ten year old sister, Joanne.

In 1959, the author resided in Baumholder, Germany for one year with his family and returned the next year to enroll at Springfield High School in Springfield, Pennsylvania. Upon graduation from Springfield in 1965, the author was enrolled as a Civil Engineering undergraduate at Lehigh University in Bethlehem, Pennsylvania. His academic endeavors continued in 1969 as a candidate for the Master's Degree in Civil Engineering, with a major interest in Hydraulics and Sanitary Engineering.

The author's engineering experience has been limited to summer work in water systems and environmental engineering, but he has maintained an active status as an associate member of ASCE and author of three technical publications. Employment within the Environmental Division of Gilbert Associates, Inc., located in Reading, Pennsylvania, will follow receipt of the MSCE.

The author's fiancee, Mireille A. Haudricourt, will become Mireille H. Robinson on June 26, 1971.

CASE NO.1 FROUDE NO. (1)

THE BEST-FIT EQUATION IS...

$$\log F = (.0860) * \log C + (-.0453)$$

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9010\ C^{.0860}}$$

STANDARD DEVIATION = .0494

CORRELATION COEFFICIENT = .8447

STANDARD ERROR OF ESTIMATE = .0264

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.75088
.15	.78168	.76542
.20	.85733	.78458
.50	.89936	.84887
.50	.84052	.84887
.60	.97500	.86228
1.00	1.07587	.90098
1.00	.92457	.90098
1.75	.96660	.94538
2.00	.96660	.95629
5.00	1.00022	1.03465
.50	.80690	.84887
1.00	.85733	.90098
3.00	.89936	.99020
7.00	.84052	1.06501
.80	.87844	.88387
1.10	.91962	.90839
3.00	.99511	.99020
5.00	1.01570	1.03465
1.00	.87844	.90098
2.30	1.04315	.96785
4.80	1.07746	1.03102

Table C.2a Best-Fit Data For Mod Fr No 1

CASE NO.2 FROUDE NO. (1)

THE BEST-FIT EQUATION IS...

LOGF = (.1309) * LOGC + (-.0497)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .8919\ C^{.1309}}$$

STANDARD DEVIATION = .0878

CORRELATION COEFFICIENT = .9350

STANDARD ERROR OF ESTIMATE = .0311

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.05	.46229	.60258
.10	.68923	.65980
.20	.80690	.72246
.30	.91617	.76184
.65	.85733	.84297
1.00	.95819	.89187
1.20	.98341	.91341
1.50	.94138	.94048
3.00	1.05065	1.02979
7.00	1.09268	1.15057
.05	.62198	.60258
.10	.65561	.65980
.25	.75647	.74387
.55	.85733	.82474
2.25	.92457	.99174
2.50	.95819	1.00551
.75	.80295	.85891
1.90	.95393	.97003
2.50	1.02256	1.00551
5.40	1.09119	1.11214
.75	.84413	.85891
2.00	.97452	.97656
3.70	1.02942	1.05845
5.00	1.06374	1.10100

CASE NO.3 FROUDE NO. (1)

THE BEST-FIT EQUATION IS...

LOGF = (.2898) * LOGC + (-.0416)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9086 \ C^{.2898}}$$

STANDARD DEVIATION = .0524

CORRELATION COEFFICIENT = .9937

STANDARD ERROR OF ESTIMATE = .0059

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
1.30	.97244	.98036
1.90	1.10114	1.09434
3.00	1.27275	1.24925
3.80	1.31565	1.33784

CASE NO.4 FROUDE NO. (1)

THE BEST-FIT EQUATION IS...

LOGF = (.1135) * LOGC + (-.0490)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .8934\ C^{.1135}}$$

STANDARD DEVIATION = .0726

CORRELATION COEFFICIENT = .8863

STANDARD ERROR OF ESTIMATE = .0336

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.70235
.15	.78168	.72036
.20	.85733	.74426
.50	.89936	.82580
.50	.84052	.82580
.60	.97500	.84306
1.00	1.07587	.89337
1.00	.92457	.89337
1.75	.96660	.95193
2.00	.96660	.96646
5.00	1.00022	1.07234
.50	.80690	.82580
1.00	.85733	.89337
3.00	.89936	1.01196
7.00	.84052	1.11407
.80	.87844	.87103
1.10	.91962	.90308
3.00	.99511	1.01196
5.00	1.01570	1.07234
1.00	.87844	.89337
2.30	1.04315	.98191
4.80	1.07746	1.06739
.05	.46229	.63594
.10	.68923	.68797
.20	.80690	.74426

CASE NO.4 FROUDE NO. (1)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.77930
.65	.85733	.85075
1.00	.95819	.89337
1.20	.98341	.91204
1.50	.94138	.93542
3.00	1.05065	1.01196
7.00	1.09268	1.11407
.05	.62198	.63594
.10	.65561	.68797
.25	.75647	.76334
.55	.85733	.83478
2.25	.92457	.97946
2.50	.95819	.99124
.75	.80295	.86468
1.90	.95393	.96085
2.50	1.02256	.99124
5.40	1.09119	1.08175
.75	.84413	.86468
2.00	.97452	.96646
3.70	1.02942	1.03633
5.00	1.06374	1.07234

CASE NO.5 FROUDE NO. (1)

THE BEST-FIT EQUATION IS...

LOGF = (.1218) * LOGC + (-.0434)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9049\ C^{.1218}}$$

STANDARD DEVIATION = .0777

CORRELATION COEFFICIENT = .8720

STANDARD ERROR OF ESTIMATE = .0380

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.69899
.15	.78168	.71824
.20	.85733	.74385
.50	.89936	.83165
.50	.84052	.83165
.60	.97500	.85032
1.00	1.07587	.90489
1.00	.92457	.90489
1.75	.96660	.96871
2.00	.96660	.98459
5.00	1.00022	1.10080
.50	.80690	.83165
1.00	.85733	.90489
3.00	.89936	1.03442
7.00	.84052	1.14684
.80	.87844	.88064
1.10	.91962	.91546
3.00	.99511	1.03442
5.00	1.01570	1.10080
1.00	.87844	.90489
2.30	1.04315	1.00149
4.80	1.07746	1.09535
.05	.46229	.62831
.10	.68923	.68364
.20	.80690	.74385

CASE NO.5 FROUDE NO. (1)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.78150
.65	.85733	.85865
1.00	.95819	.90489
1.20	.98341	.92521
1.50	.94138	.95069
3.00	1.05065	1.03442
7.00	1.09268	1.14684
.05	.62198	.62831
.10	.65561	.68364
.25	.75647	.76434
.55	.85733	.84136
2.25	.92457	.99881
2.50	.95819	1.01171
.75	.80295	.87374
1.90	.95393	.97846
2.50	1.02256	1.01171
5.40	1.09119	1.11117
.75	.84413	.87374
2.00	.97452	.98459
3.70	1.02942	1.06117
5.00	1.06374	1.10080
1.30	.97244	.93427
1.90	1.10114	.97846
3.00	1.27275	1.03442
3.80	1.31565	1.06463

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .871

SOLUTION VECTOR IS...

.1088 .0588 -.0357

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9211\ C^{.1088}\ D50^{.0588}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.12	.88	.65561	.72586
.15	.88	.78168	.74371
.20	.88	.85733	.76736
.50	.88	.89936	.84782
.50	.88	.84052	.84782
.60	.88	.97500	.86481
1.00	.88	1.07587	.91425
1.00	.88	.92457	.91425
1.75	.88	.96660	.97165
2.00	.88	.96660	.98588
5.00	.88	1.00022	1.08925
.50	.88	.80690	.84782
1.00	.88	.85733	.91425
3.00	.88	.89936	1.03035
7.00	.88	.84052	1.12988
.80	.88	.87844	.89231
1.10	.88	.91962	.92378
3.00	.88	.99511	1.03035
5.00	.88	1.01570	1.08925
1.00	.88	.87844	.91425
2.30	.88	1.04315	1.00099
4.80	.88	1.07746	1.08442
.05	.45	.46229	.63437
.10	.45	.68923	.68408

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50 (MM)	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.20	.45	.80690	.73767
.30	.45	.91617	.77095
.65	.45	.85733	.83863
1.00	.45	.95819	.87888
1.20	.45	.98341	.89649
1.50	.45	.94138	.91853
3.00	.45	1.05065	.99049
7.00	.45	1.09268	1.08617
.05	.45	.62198	.63437
.10	.45	.65561	.68408
.25	.45	.75647	.75581
.55	.45	.85733	.82352
2.25	.45	.92457	.95996
2.50	.45	.95819	.97103
.75	.45	.80295	.85179
1.90	.45	.95393	.94246
2.50	.45	1.02256	.97103
5.40	.45	1.09119	1.05592
.75	.45	.84413	.85179
2.00	.45	.97452	.94774
3.70	.45	1.02942	1.01336
5.00	.45	1.06374	1.04711

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .863

SOLUTION VECTOR IS...

.1097 .0704 -.0329

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9270\ C^{.1097}\ D50^{.0704}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.12	.88	.65561	.72811
.15	.88	.78168	.74614
.20	.88	.85733	.77006
.50	.88	.89936	.85145
.50	.88	.84052	.85145
.60	.88	.97500	.86864
1.00	.88	1.07587	.91869
1.00	.88	.92457	.91869
1.75	.88	.96660	.97683
2.00	.88	.96660	.99124
5.00	.88	1.00022	1.09601
.50	.88	.80690	.85145
1.00	.88	.85733	.91869
3.00	.88	.89936	1.03631
7.00	.88	.84052	1.13721
.80	.88	.87844	.89648
1.10	.88	.91962	.92834
3.00	.88	.99511	1.03631
5.00	.88	1.01570	1.09601
1.00	.88	.87844	.91869
2.30	.88	1.04315	1.00655
4.80	.88	1.07746	1.09112
.05	.45	.46229	.63094
.10	.45	.68923	.68077

CASE NO.5
TOTAL NUMBER OF DATA = 50

C (PERCENT)	D50 (MM)	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.20	.45	.80690	.73453
.30	.45	.91617	.76792
.65	.45	.85733	.83587
1.00	.45	.95819	.87630
1.20	.45	.98341	.89400
1.50	.45	.94138	.91615
3.00	.45	1.05065	.98849
7.00	.45	1.09268	1.08474
.05	.45	.62198	.63094
.10	.45	.65561	.68077
.25	.45	.75647	.75272
.55	.45	.85733	.82070
2.25	.45	.92457	.95780
2.50	.45	.95819	.96893
.75	.45	.80295	.84909
1.90	.45	.95393	.94020
2.50	.45	1.02256	.96893
5.40	.45	1.09119	1.05431
.75	.45	.84413	.84909
2.00	.45	.97452	.94551
3.70	.45	1.02942	1.01149
5.00	.45	1.06374	1.04545
1.30	3.63	.97244	1.04474
1.90	3.63	1.10114	1.08913
3.00	3.63	1.27275	1.14507
3.80	3.63	1.31565	1.17514

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .879

SOLUTION VECTOR IS...

.1126 .0021 -.0434

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9048\ C^{.1126}\ D50/D^{.0021}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.12	.00866142	.6556	.7055
.15	.00866142	.7817	.7235
.20	.00866142	.8573	.7473
.50	.00866142	.8994	.8285
.50	.00866142	.8405	.8285
.60	.00866142	.9750	.8457
1.00	.00866142	1.0759	.8957
1.00	.00866142	.9246	.8957
1.75	.00866142	.9666	.9540
2.00	.00866142	.9666	.9684
5.00	.00866142	1.0002	1.0737
.50	.00866142	.8069	.8285
1.00	.00866142	.8573	.8957
3.00	.00866142	.8994	1.0137
7.00	.00866142	.8405	1.1151
.80	.00577428	.8784	.8728
1.10	.00577428	.9196	.9046
3.00	.00577428	.9951	1.0128
5.00	.00577428	1.0157	1.0727
1.00	.00577428	.8784	.8950
2.30	.00577428	1.0431	.9830
4.80	.00577428	1.0775	1.0678
.05	.00442913	.4623	.6384
.10	.00442913	.6892	.6902

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50/D	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.20	.00442913	.8069	.7462
.30	.00442913	.9162	.7811
.65	.00442913	.8573	.8521
1.00	.00442913	.9582	.8945
1.20	.00442913	.9834	.9130
1.50	.00442913	.9414	.9362
3.00	.00442913	1.0507	1.0122
7.00	.00442913	1.0927	1.1135
.05	.00442913	.6220	.6384
.10	.00442913	.6556	.6902
.25	.00442913	.7565	.7652
.55	.00442913	.8573	.8362
2.25	.00442913	.9246	.9800
2.50	.00442913	.9582	.9917
.75	.00295276	.8029	.8652
1.90	.00295276	.9539	.9607
2.50	.00295276	1.0226	.9908
5.40	.00295276	1.0912	1.0805
.75	.00295276	.8441	.8652
2.00	.00295276	.9745	.9662
3.70	.00295276	1.0294	1.0355
5.00	.00295276	1.0637	1.0712

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .818

SOLUTION VECTOR IS...

.1139 .0017 -.0436

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9045\ C^{.1139}\ D50/D^{.0017}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.12	.00866142	.6556	.7048
.15	.00866142	.7817	.7229
.20	.00866142	.8573	.7470
.50	.00866142	.8994	.8292
.50	.00866142	.8405	.8292
.60	.00866142	.9750	.8466
1.00	.00866142	1.0759	.8973
1.00	.00866142	.9246	.8973
1.75	.00866142	.9666	.9563
2.00	.00866142	.9666	.9710
5.00	.00866142	1.0002	1.0778
.50	.00866142	.8069	.8292
1.00	.00866142	.8573	.8973
3.00	.00866142	.8994	1.0169
7.00	.00866142	.8405	1.1199
.80	.00577428	.8784	.8742
1.10	.00577428	.9196	.9065
3.00	.00577428	.9951	1.0162
5.00	.00577428	1.0157	1.0771
1.00	.00577428	.8784	.8967
2.30	.00577428	1.0431	.9859
4.80	.00577428	1.0775	1.0721
.05	.00442913	.4623	.6372
.10	.00442913	.6892	.6895

CASE NO.5
TOTAL NUMBER OF DATA = 50

C (PERCENT)	D50/D	FROUDE NUMBER (I)	FITTED FROUDE NUMBER (I)
.20	.00442913	.8069	.7461
.30	.00442913	.9162	.7814
.65	.00442913	.8573	.8534
1.00	.00442913	.9582	.8963
1.20	.00442913	.9834	.9151
1.50	.00442913	.9414	.9386
3.00	.00442913	1.0507	1.0158
7.00	.00442913	1.0927	1.1187
.05	.00442913	.6220	.6372
.10	.00442913	.6556	.6895
.25	.00442913	.7565	.7654
.55	.00442913	.8573	.8373
2.25	.00442913	.9246	.9830
2.50	.00442913	.9582	.9949
.75	.00295276	.8029	.8668
1.90	.00295276	.9539	.9636
2.50	.00295276	1.0226	.9942
5.40	.00295276	1.0912	1.0854
.75	.00295276	.8441	.8668
2.00	.00295276	.9745	.9692
3.70	.00295276	1.0294	1.0396
5.00	.00295276	1.0637	1.0759
1.30	.02381890	.9724	.9261
1.90	.02381890	1.1011	.9670
3.00	.02381890	1.2727	1.0186
3.80	.02381890	1.3157	1.0464

CASE NO.1   FROUDE NO. (2)

THE BEST-FIT EQUATION IS...

LOGF = (.0797) * LOGC + (-.0420)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9079\ C^{.0797}}$$

STANDARD DEVIATION = .0465

CORRELATION COEFFICIENT = .8313

STANDARD ERROR OF ESTIMATE = .0259

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.76668
.15	.78168	.78044
.20	.85733	.79854
.50	.89936	.85906
.50	.84052	.85906
.60	.97500	.87164
1.00	1.07587	.90787
1.00	.92457	.90787
1.75	.96660	.94929
2.00	.96660	.95945
5.00	1.00022	1.03216
.50	.85531	.85906
1.00	.90877	.90787
3.00	.95332	.99097
7.00	.89095	1.06023
.80	.87844	.89186
1.10	.91962	.91480
3.00	.99511	.99097
5.00	1.01570	1.03216
1.00	.85472	.90787
2.30	1.01498	.97020
4.80	1.04837	1.02881

CASE NO.2   FROUDE NO. (2)

THE BEST-FIT EQUATION IS...

LOGF = (.1236) * LOGC + (-.0459)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .8997\ C^{.1236}}$$

STANDARD DEVIATION = .0843

CORRELATION COEFFICIENT = .9193

STANDARD ERROR OF ESTIMATE = .0332

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.05	.46229	.62134
.10	.68923	.67691
.20	.80690	.73744
.30	.91617	.77532
.65	.85733	.85305
1.00	.95819	.89969
1.20	.98341	.92018
1.50	.94138	.94591
3.00	1.05065	1.03049
7.00	1.09268	1.14423
.05	.65930	.62134
.10	.69494	.67691
.25	.80186	.75805
.55	.90877	.83562
2.25	.98005	.99451
2.50	1.01568	1.00754
.75	.80295	.86827
1.90	.95393	.97394
2.50	1.02256	1.00754
5.40	1.09119	1.10812
.75	.82134	.86827
2.00	.94821	.98014
3.70	1.00163	1.05755
5.00	1.03502	1.09763

CASE NO.3 FROUDE NO. (2)

THE BEST-FIT EQUATION IS...

LOGF = (.2898) * LOGC + (-.0416)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9086\ C^{.2898}}$$

STANDARD DEVIATION = .0524

CORRELATION COEFFICIENT = .9937

STANDARD ERROR OF ESTIMATE = .0059

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
1.30	.97244	.98036
1.90	1.10114	1.09434
3.00	1.27275	1.24925
3.80	1.31565	1.33784

CASE NO.4 FROUDE NO. (2)

THE BEST-FIT EQUATION IS...

$$LOGF = (.1064) * LOGC + (-.0454)$$

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9008\ C^{.1064}}$$

STANDARD DEVIATION = .0694

CORRELATION COEFFICIENT = .8697

STANDARD ERROR OF ESTIMATE = .0343

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.71884
.15	.78168	.73611
.20	.85733	.75899
.50	.89936	.83673
.50	.84052	.83673
.60	.97500	.85312
1.00	1.07587	.90078
1.00	.92457	.90078
1.75	.96660	.95606
2.00	.96660	.96974
5.00	1.00022	1.06906
.50	.85531	.83673
1.00	.90877	.90078
3.00	.95332	1.01250
7.00	.89095	1.10804
.80	.87844	.87965
1.10	.91962	.90997
3.00	.99511	1.01250
5.00	1.01570	1.06906
1.00	.85472	.90078
2.30	1.01498	.98427
4.80	1.04837	1.06443
.05	.46229	.65489
.10	.68923	.70502
.20	.80690	.75899

CASE NO.4 FROUDE NO. (2)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.79246
.65	.85733	.86042
1.00	.95819	.90078
1.20	.98341	.91843
1.50	.94138	.94050
3.00	1.05065	1.01250
7.00	1.09268	1.10804
.05	.65930	.65489
.10	.69494	.70502
.25	.80186	.77723
.55	.90877	.84526
2.25	.98005	.98197
2.50	1.01568	.99304
.75	.80295	.87363
1.90	.95393	.96446
2.50	1.02256	.99304
5.40	1.09119	1.07786
.75	.82134	.87363
2.00	.94821	.96974
3.70	1.00163	1.03535
5.00	1.03502	1.06906

CASE NO.5 FROUDE NO. (2)

THE BEST-FIT EQUATION IS...

LOGF = (.1144) * LOGC + (-.0399)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9122\ C^{.1144}}$$

STANDARD DEVIATION = .0745

CORRELATION COEFFICIENT = .8541

STANDARD ERROR OF ESTIMATE = .0387

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.71574
.15	.78168	.73424
.20	.85733	.75882
.50	.89936	.84269
.50	.84052	.84269
.60	.97500	.86045
1.00	1.07587	.91224
1.00	.92457	.91224
1.75	.96660	.97257
2.00	.96660	.98754
5.00	1.00022	1.09670
.50	.85531	.84269
1.00	.90877	.91224
3.00	.95332	1.03443
7.00	.89095	1.13974
.80	.87844	.88925
1.10	.91962	.92225
3.00	.99511	1.03443
5.00	1.01570	1.09670
1.00	.85472	.91224
2.30	1.01498	1.00346
4.80	1.04837	1.09159
.05	.46229	.64751
.10	.68923	.70096
.20	.80690	.75882

CASE NO.5 FROUDE NO. (2)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.79485
.65	.85733	.86837
1.00	.95819	.91224
1.20	.98341	.93147
1.50	.94138	.95556
3.00	1.05065	1.03443
7.00	1.09268	1.13974
.05	.65930	.64751
.10	.69494	.70096
.25	.80186	.77844
.55	.90877	.85193
2.25	.98005	1.00094
2.50	1.01568	1.01308
.75	.80295	.88271
1.90	.95393	.98176
2.50	1.02256	1.01308
5.40	1.09119	1.10640
.75	.82134	.88271
2.00	.94821	.98754
3.70	1.00163	1.05956
5.00	1.03502	1.09670
1.30	.97244	.94004
1.90	1.10114	.98176
3.00	1.27275	1.03443
3.80	1.31565	1.06279

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .877

SOLUTION VECTOR IS...

.1047 .0560 -.0326

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9276\ C^{.1047}\ D50^{.0560}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.12	.88	.65561	.73764
.15	.88	.78168	.75508
.20	.88	.85733	.77817
.50	.88	.89936	.85653
.50	.88	.84052	.85653
.60	.88	.97500	.87304
1.00	.88	1.07587	.92102
1.00	.88	.92457	.92102
1.75	.88	.96660	.97660
2.00	.88	.96660	.99035
5.00	.88	1.00022	1.09009
.50	.88	.85531	.85653
1.00	.88	.90877	.92102
3.00	.88	.95332	1.03331
7.00	.88	.89095	1.12918
.80	.88	.87844	.89975
1.10	.88	.91962	.93026
3.00	.88	.99511	1.03331
5.00	.88	1.01570	1.09009
1.00	.88	.85472	.92102
2.30	.88	1.01498	1.00495
4.80	.88	1.04837	1.08544
.05	.45	.46229	.64822
.10	.45	.68923	.69701

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50 (MM)	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.20	.45	.80690	.74949
.30	.45	.91617	.78200
.65	.45	.85733	.84794
1.00	.45	.95819	.88707
1.20	.45	.98341	.90417
1.50	.45	.94138	.92555
3.00	.45	1.05065	.99522
7.00	.45	1.09268	1.08756
.05	.45	.65930	.64822
.10	.45	.69494	.69701
.25	.45	.80186	.76721
.55	.45	.90877	.83324
2.25	.45	.98005	.96569
2.50	.45	1.01568	.97640
.75	.45	.80295	.86075
1.90	.45	.95393	.94874
2.50	.45	1.02256	.97640
5.40	.45	1.09119	1.05841
.75	.45	.82134	.86075
2.00	.45	.94821	.95385
3.70	.45	1.00163	1.01732
5.00	.45	1.03502	1.04991

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .866

SOLUTION VECTOR IS...

.1056 .0676 -.0299

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9335\ C^{.1056}\ D50^{.0676}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.12	.88	.65561	.73989
.15	.88	.78168	.75753
.20	.88	.85733	.78088
.50	.88	.89936	.86019
.50	.88	.84052	.86019
.60	.88	.97500	.87691
1.00	.88	1.07587	.92550
1.00	.88	.92457	.92550
1.75	.88	.96660	.98182
2.00	.88	.96660	.99576
5.00	.88	1.00022	1.09689
.50	.88	.85531	.86019
1.00	.88	.90877	.92550
3.00	.88	.95332	1.03930
7.00	.88	.89095	1.13655
.80	.88	.87844	.90395
1.10	.88	.91962	.93485
3.00	.88	.99511	1.03930
5.00	.88	1.01570	1.09689
1.00	.88	.85472	.92550
2.30	.88	1.01498	1.01056
4.80	.88	1.04837	1.09217
.05	.45	.46229	.64467
.10	.45	.68923	.69361

CASE NO.5
TOTAL NUMBER OF DATA = 50

C (PERCENT)	D50 (MM)	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.20	.45	.80690	.74627
.30	.45	.91617	.77891
.65	.45	.85733	.84515
1.00	.45	.95819	.88447
1.20	.45	.98341	.90166
1.50	.45	.94138	.92315
3.00	.45	1.05065	.99324
7.00	.45	1.09268	1.08617
.05	.45	.65930	.64467
.10	.45	.69494	.69361
.25	.45	.80186	.76406
.55	.45	.90877	.83038
2.25	.45	.98005	.96352
2.50	.45	1.01568	.97430
.75	.45	.80295	.85801
1.90	.45	.95393	.94648
2.50	.45	1.02256	.97430
5.40	.45	1.09119	1.05682
.75	.45	.82134	.85801
2.00	.45	.94821	.95162
3.70	.45	1.00163	1.01547
5.00	.45	1.03502	1.04827
1.30	3.63	.97244	1.04714
1.90	3.63	1.10114	1.08995
3.00	3.63	1.27275	1.14379
3.80	3.63	1.31565	1.17269

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .884

SOLUTION VECTOR IS...

.1083 .0022 -.0398

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9125\ C^{.1083}\ D50/D^{.0022}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.12	.00866142	.6556	.7177
.15	.00866142	.7817	.7353
.20	.00866142	.8573	.7585
.50	.00866142	.8994	.8376
.50	.00866142	.8405	.8376
.60	.00866142	.9750	.8543
1.00	.00866142	1.0759	.9029
1.00	.00866142	.9246	.9029
1.75	.00866142	.9666	.9593
2.00	.00866142	.9666	.9733
5.00	.00866142	1.0002	1.0748
.50	.00866142	.8553	.8376
1.00	.00866142	.9088	.9029
3.00	.00866142	.9533	1.0170
7.00	.00866142	.8910	1.1147
.80	.00577428	.8784	.8806
1.10	.00577428	.9196	.9115
3.00	.00577428	.9951	1.0161
5.00	.00577428	1.0157	1.0739
1.00	.00577428	.8547	.9021
2.30	.00577428	1.0150	.9873
4.80	.00577428	1.0484	1.0691
.05	.00442913	.4623	.6518
.10	.00442913	.6892	.7026

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50/D	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.20	.00442913	.8069	.7574
.30	.00442913	.9162	.7914
.65	.00442913	.8573	.8605
1.00	.00442913	.9582	.9016
1.20	.00442913	.9834	.9196
1.50	.00442913	.9414	.9421
3.00	.00442913	1.0507	1.0155
7.00	.00442913	1.0927	1.1131
.05	.00442913	.6593	.6518
.10	.00442913	.6949	.7026
.25	.00442913	.8019	.7759
.55	.00442913	.9088	.8451
2.25	.00442913	.9800	.9843
2.50	.00442913	1.0157	.9956
.75	.00295276	.8029	.8732
1.90	.00295276	.9539	.9656
2.50	.00295276	1.0226	.9947
5.40	.00295276	1.0912	1.0812
.75	.00295276	.8213	.8732
2.00	.00295276	.9482	.9710
3.70	.00295276	1.0016	1.0379
5.00	.00295276	1.0350	1.0723

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .820

SOLUTION VECTOR IS...

.1096 .0017 -.0400

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9119\ C^{.1096}\ D50/D^{.0017}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.12	.00866142	.6556	.7169
.15	.00866142	.7817	.7347
.20	.00866142	.8573	.7582
.50	.00866142	.8994	.8383
.50	.00866142	.8405	.8383
.60	.00866142	.9750	.8552
1.00	.00866142	1.0759	.9045
1.00	.00866142	.9246	.9045
1.75	.00866142	.9666	.9617
2.00	.00866142	.9666	.9759
5.00	.00866142	1.0002	1.0789
.50	.00866142	.8553	.8383
1.00	.00866142	.9088	.9045
3.00	.00866142	.9533	1.0202
7.00	.00866142	.8910	1.1195
.80	.00577428	.8784	.8820
1.10	.00577428	.9196	.9133
3.00	.00577428	.9951	1.0195
5.00	.00577428	1.0157	1.0782
1.00	.00577428	.8547	.9038
2.30	.00577428	1.0150	.9902
4.80	.00577428	1.0484	1.0734
.05	.00442913	.4623	.6506
.10	.00442913	.6892	.7019

CASE NO.5
TOTAL NUMBER OF DATA = 50

C (PERCENT)	D50/D	FROUDE NUMBER (II)	FITTED FROUDE NUMBER (II)
.20	.00442913	.8069	.7573
.30	.00442913	.9162	.7917
.65	.00442913	.8573	.8618
1.00	.00442913	.9582	.9034
1.20	.00442913	.9834	.9217
1.50	.00442913	.9414	.9445
3.00	.00442913	1.0507	1.0190
7.00	.00442913	1.0927	1.1182
.05	.00442913	.6593	.6506
.10	.00442913	.6949	.7019
.25	.00442913	.8019	.7761
.55	.00442913	.9088	.8461
2.25	.00442913	.9800	.9874
2.50	.00442913	1.0157	.9989
.75	.00295276	.8029	.8748
1.90	.00295276	.9539	.9686
2.50	.00295276	1.0226	.9982
5.40	.00295276	1.0912	1.0860
.75	.00295276	.8213	.8748
2.00	.00295276	.9482	.9740
3.70	.00295276	1.0016	1.0420
5.00	.00295276	1.0350	1.0769
1.30	.02381890	.9724	.9325
1.90	.02381890	1.1011	.9721
3.00	.02381890	1.2727	1.0220
3.80	.02381890	1.3157	1.0488

CASE NO.1 FROUDE NO. (3)

THE BEST-FIT EQUATION IS...

LOGF = (.0823) * LOGC + (-.0434)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9048\ C^{.0823}}$$

STANDARD DEVIATION = .0476

CORRELATION COEFFICIENT = .8393

STANDARD ERROR OF ESTIMATE = .0259

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.75988
.15	.78168	.77397
.20	.85733	.79252
.50	.89936	.85462
.50	.84052	.85462
.60	.97500	.86755
1.00	1.07587	.90482
1.00	.92457	.90482
1.75	.96660	.94748
2.00	.96660	.95796
5.00	1.00022	1.03303
.50	.83225	.85462
1.00	.88427	.90482
3.00	.92762	.99048
7.00	.86693	1.06204
.80	.87844	.88834
1.10	.91962	.91194
3.00	.99511	.99048
5.00	1.01570	1.03303
1.00	.86682	.90482
2.30	1.02934	.96904
4.80	1.06320	1.02956

CASE NO.2 FROUDE NO. (3)

THE BEST-FIT EQUATION IS...

LOGF = (.1270) * LOGC + (-.0476)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .8962\ C^{.1270}}$$

STANDARD DEVIATION = .0858

CORRELATION COEFFICIENT = .9284

STANDARD ERROR OF ESTIMATE = .0319

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.05	.46229	.61267
.10	.68923	.66903
.20	.80690	.73057
.30	.91617	.76916
.65	.85733	.84849
1.00	.95819	.89618
1.20	.98341	.91717
1.50	.94138	.94352
3.00	1.05065	1.03031
7.00	1.09268	1.14732
.05	.64153	.61267
.10	.67621	.66903
.25	.78024	.75156
.55	.88427	.83068
2.25	.95362	.99336
2.50	.98830	1.00674
.75	.80295	.86404
1.90	.95393	.97227
2.50	1.02256	1.00674
5.40	1.09119	1.11013
.75	.83296	.86404
2.00	.96162	.97862
3.70	1.01580	1.05811
5.00	1.04966	1.09934

CASE NO.3 FROUDE NO. (3)

THE BEST-FIT EQUATION IS...

LOGF = (.2898) * LOGC + (-.0416)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9086\ C^{.2898}}$$

STANDARD DEVIATION = .0524

CORRELATION COEFFICIENT = .9937

STANDARD ERROR OF ESTIMATE = .0059

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
1.30	.97244	.98036
1.90	1.10114	1.09434
3.00	1.27275	1.24925
3.80	1.31565	1.33784

CASE NO.4   FROUDE NO. (3)

THE BEST-FIT EQUATION IS...

LOGF = (.1097) * LOGC + (-.0470)

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .8975\ C^{.1097}}$$

STANDARD DEVIATION = .0707

CORRELATION COEFFICIENT = .8801

STANDARD ERROR OF ESTIMATE = .0336

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.71120
.15	.78168	.72883
.20	.85733	.75220
.50	.89936	.83174
.50	.84052	.83174
.60	.97500	.84855
1.00	1.07587	.89746
1.00	.92457	.89746
1.75	.96660	.95429
2.00	.96660	.96837
5.00	1.00022	1.07078
.50	.83225	.83174
1.00	.88427	.89746
3.00	.92762	1.01242
7.00	.86693	1.11105
.80	.87844	.87576
1.10	.91962	.90690
3.00	.99511	1.01242
5.00	1.01570	1.07078
1.00	.86682	.89746
2.30	1.02934	.98334
4.80	1.06320	1.06600
.05	.46229	.64607
.10	.68923	.69712
.20	.80690	.75220

CASE NO.4 FROUDE NO. (3)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.78641
.65	.85733	.85603
1.00	.95819	.89746
1.20	.98341	.91559
1.50	.94138	.93829
3.00	1.05065	1.01242
7.00	1.09268	1.11105
.05	.64153	.64607
.10	.67621	.69712
.25	.78024	.77084
.55	.88427	.84049
2.25	.95362	.98097
2.50	.98830	.99237
.75	.80295	.86958
1.90	.95393	.96294
2.50	1.02256	.99237
5.40	1.09119	1.07986
.75	.83296	.86958
2.00	.96162	.96837
3.70	1.01580	1.03599
5.00	1.04966	1.07078

CASE NO.5 FROUDE NO. (3)

THE BEST-FIT EQUATION IS...

$$LOGF = (.1178) * LOGC + (-.0415)$$

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9089\ C^{.1178}}$$

STANDARD DEVIATION = .0758

CORRELATION COEFFICIENT = .8644

STANDARD ERROR OF ESTIMATE = .0381

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.12	.65561	.70802
.15	.78168	.72689
.20	.85733	.75195
.50	.89936	.83767
.50	.84052	.83767
.60	.97500	.85585
1.00	1.07587	.90894
1.00	.92457	.90894
1.75	.96660	.97089
2.00	.96660	.98629
5.00	1.00022	1.09872
.50	.83225	.83767
1.00	.88427	.90894
3.00	.92762	1.03455
7.00	.86693	1.14316
.80	.87844	.88536
1.10	.91962	.91921
3.00	.99511	1.03455
5.00	1.01570	1.09872
1.00	.86682	.90894
2.30	1.02934	1.00266
4.80	1.06320	1.09345
.05	.46229	.63863
.10	.68923	.69298
.20	.80690	.75195

CASE NO.5 FROUDE NO. (3)

C (PERCENT)	FROUDE NO. (DIMENSIONLESS)	FITTED FROUDE NO. (DIMENSIONLESS)
.30	.91617	.78874
.65	.85733	.86396
1.00	.95819	.90894
1.20	.98341	.92868
1.50	.94138	.95342
3.00	1.05065	1.03455
7.00	1.09268	1.14316
.05	.64153	.63863
.10	.67621	.69298
.25	.78024	.77198
.55	.88427	.84713
2.25	.95362	1.00007
2.50	.98830	1.01256
.75	.80295	.87865
1.90	.95393	.98035
2.50	1.02256	1.01256
5.40	1.09119	1.10873
.75	.83296	.87865
2.00	.96162	.98629
3.70	1.01580	1.06043
5.00	1.04966	1.09872
1.30	.97244	.93748
1.90	1.10114	.98035
3.00	1.27275	1.03455
3.80	1.31565	1.06377

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .878

SOLUTION VECTOR IS...

.1067 .0572 -.0340

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9247\ C^{.1067}\ D50^{.0572}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.12	.88	.65561	.73205
.15	.88	.78168	.74969
.20	.88	.85733	.77307
.50	.88	.89936	.85250
.50	.88	.84052	.85250
.60	.88	.97500	.86925
1.00	.88	1.07587	.91797
1.00	.88	.92457	.91797
1.75	.88	.96660	.97447
2.00	.88	.96660	.98846
5.00	.88	1.00022	1.09001
.50	.88	.83225	.85250
1.00	.88	.88427	.91797
3.00	.88	.92762	1.03217
7.00	.88	.86693	1.12987
.80	.88	.87844	.89636
1.10	.88	.91962	.92735
3.00	.88	.99511	1.03217
5.00	.88	1.01570	1.09001
1.00	.88	.86682	.91797
2.30	.88	1.02934	1.00331
4.80	.88	1.06320	1.08528
.05	.45	.46229	.64165
.10	.45	.68923	.69092

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50 (MM)	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.20	.45	.80690	.74398
.30	.45	.91617	.77688
.65	.45	.85733	.84372
1.00	.45	.95819	.88342
1.20	.45	.98341	.90078
1.50	.45	.94138	.92249
3.00	.45	1.05065	.99333
7.00	.45	1.09268	1.08735
.05	.45	.64153	.64165
.10	.45	.67621	.69092
.25	.45	.78024	.76191
.55	.45	.88427	.82881
2.25	.45	.95362	.96329
2.50	.45	.98830	.97419
.75	.45	.80295	.85670
1.90	.45	.95393	.94606
2.50	.45	1.02256	.97419
5.40	.45	1.09119	1.05765
.75	.45	.83296	.85670
2.00	.45	.96162	.95126
3.70	.45	1.01580	1.01582
5.00	.45	1.04966	1.04899

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .867

SOLUTION VECTOR IS...

.1076 .0688 -.0312

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9306\ C^{.1076}\ D50^{.0688}}$$

C (PERCENT)	D50 (MM)	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.12	.88	.65561	.73430
.15	.88	.78168	.75214
.20	.88	.85733	.77578
.50	.88	.89936	.85614
.50	.88	.84052	.85614
.60	.88	.97500	.87310
1.00	.88	1.07587	.92242
1.00	.88	.92457	.92242
1.75	.88	.96660	.97966
2.00	.88	.96660	.99384
5.00	.88	1.00022	1.09679
.50	.88	.83225	.85614
1.00	.88	.88427	.92242
3.00	.88	.92762	1.03815
7.00	.88	.86693	1.13722
.80	.88	.87844	.90055
1.10	.88	.91962	.93193
3.00	.88	.99511	1.03815
5.00	.88	1.01570	1.09679
1.00	.88	.86682	.92242
2.30	.88	1.02934	1.00889
4.80	.88	1.06320	1.09199
.05	.45	.46229	.63816
.10	.45	.68923	.68757

CASE NO.5
TOTAL NUMBER OF DATA = 50

C (PERCENT)	D50 (MM)	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.20	.45	.80690	.74080
.30	.45	.91617	.77382
.65	.45	.85733	.84094
1.00	.45	.95819	.88083
1.20	.45	.98341	.89828
1.50	.45	.94138	.92010
3.00	.45	1.05065	.99134
7.00	.45	1.09268	1.08594
.05	.45	.64153	.63816
.10	.45	.67621	.68757
.25	.45	.78024	.75879
.55	.45	.88427	.82597
2.25	.45	.95362	.96113
2.50	.45	.98830	.97208
.75	.45	.80295	.85399
1.90	.45	.95393	.94380
2.50	.45	1.02256	.97208
5.40	.45	1.09119	1.05605
.75	.45	.83296	.85399
2.00	.45	.96162	.94902
3.70	.45	1.01580	1.01396
5.00	.45	1.04966	1.04734
1.30	3.63	.97244	1.04599
1.90	3.63	1.10114	1.08957
3.00	3.63	1.27275	1.14445
3.80	3.63	1.31565	1.17393

CASE NO.4
TOTAL NUMBER OF DATA = 46

CORRELATION COEFFICIENT = .885

SOLUTION VECTOR IS...

.1104 .0022 -.0414

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9090\ C^{.1104}\ D50/D^{.0022}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.12	.00866142	.6556	.7120
.15	.00866142	.7817	.7297
.20	.00866142	.8573	.7533
.50	.00866142	.8994	.8335
.50	.00866142	.8405	.8335
.60	.00866142	.9750	.8504
1.00	.00866142	1.0759	.8997
1.00	.00866142	.9246	.8997
1.75	.00866142	.9666	.9571
2.00	.00866142	.9666	.9713
5.00	.00866142	1.0002	1.0746
.50	.00866142	.8323	.8335
1.00	.00866142	.8843	.8997
3.00	.00866142	.9276	1.0157
7.00	.00866142	.8669	1.1153
.80	.00577428	.8784	.8771
1.10	.00577428	.9196	.9084
3.00	.00577428	.9951	1.0148
5.00	.00577428	1.0157	1.0737
1.00	.00577428	.8668	.8989
2.30	.00577428	1.0293	.9855
4.80	.00577428	1.0632	1.0689
.05	.00442913	.4623	.6455
.10	.00442913	.6892	.6968

CASE NO.4
TOTAL NUMBER OF DATA = 46

C (PERCENT)	D50/D	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.20	.00442913	.8069	.7522
.30	.00442913	.9162	.7866
.65	.00442913	.8573	.8567
1.00	.00442913	.9582	.8984
1.20	.00442913	.9834	.9167
1.50	.00442913	.9414	.9395
3.00	.00442913	1.0507	1.0142
7.00	.00442913	1.0927	1.1137
.05	.00442913	.6415	.6455
.10	.00442913	.6762	.6968
.25	.00442913	.7802	.7709
.55	.00442913	.8843	.8410
2.25	.00442913	.9536	.9825
2.50	.00442913	.9883	.9940
.75	.00295276	.8029	.8696
1.90	.00295276	.9539	.9635
2.50	.00295276	1.0226	.9932
5.40	.00295276	1.0912	1.0813
.75	.00295276	.8330	.8696
2.00	.00295276	.9616	.9690
3.70	.00295276	1.0158	1.0371
5.00	.00295276	1.0497	1.0721

CASE NO.5
TOTAL NUMBER OF DATA = 50

CORRELATION COEFFICIENT = .822

SOLUTION VECTOR IS...

.1117 .0017 -.0416

EQUATION ON ARITHMETIC SCALE IS...

$$\boxed{F = .9086\ C^{.1117}\ D50/D^{.0017}}$$

C (PERCENT)	D50/D	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.12	.00866142	.6556	.7112
.15	.00866142	.7817	.7292
.20	.00866142	.8573	.7530
.50	.00866142	.8994	.8341
.50	.00866142	.8405	.8341
.60	.00866142	.9750	.8513
1.00	.00866142	1.0759	.9013
1.00	.00866142	.9246	.9013
1.75	.00866142	.9666	.9594
2.00	.00866142	.9666	.9738
5.00	.00866142	1.0002	1.0788
.50	.00866142	.8323	.8341
1.00	.00866142	.8843	.9013
3.00	.00866142	.9276	1.0189
7.00	.00866142	.8669	1.1201
.80	.00577428	.8784	.8785
1.10	.00577428	.9196	.9103
3.00	.00577428	.9951	1.0182
5.00	.00577428	1.0157	1.0780
1.00	.00577428	.8668	.9006
2.30	.00577428	1.0293	.9885
4.80	.00577428	1.0632	1.0731
.05	.00442913	.4623	.6442
.10	.00442913	.6892	.6961

CASE NO.5
TOTAL NUMBER OF DATA = 30

C (PERCENT)	D50/D	FROUDE NUMBER (III)	FITTED FROUDE NUMBER (III)
.20	.00442913	.8069	.7521
.30	.00442913	.9162	.7870
.65	.00442913	.8573	.8579
1.00	.00442913	.9582	.9002
1.20	.00442913	.9834	.9188
1.50	.00442913	.9414	.9419
3.00	.00442913	1.0507	1.0178
7.00	.00442913	1.0927	1.1188
.05	.00442913	.6415	.6442
.10	.00442913	.6762	.6961
.25	.00442913	.7802	.7711
.55	.00442913	.8843	.8421
2.25	.00442913	.9536	.9856
2.50	.00442913	.9883	.9973
.75	.00295276	.8029	.8712
1.90	.00295276	.9539	.9665
2.50	.00295276	1.0226	.9966
5.40	.00295276	1.0912	1.0861
.75	.00295276	.8330	.8712
2.00	.00295276	.9616	.9720
3.70	.00295276	1.0158	1.0412
5.00	.00295276	1.0497	1.0768
1.30	.02381890	.9724	.9297
1.90	.02381890	1.1011	.9699
3.00	.02381890	1.2727	1.0207
3.80	.02381890	1.3157	1.0480