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# Applications of the sway subassemblage methods, (73-10) original plans were scraped when AISI indicated that they intended to publish the information in their Bulletin so an APPENDIX DESIGN EXAMPLE was written and given 338.4 to agree with item 10.8 in the ASCE Manual No. 41, 1971.

J. H. Daniels

L. W. Lu

APPLICATIONS OF THE SWAY SUBASSEMBLAGE METHOD, original plans for 338.4 were scraped when AISI indicated that they intended to publish the information in a Bulletin so this APPENDIX was written

APPENDIX

DESIGN EXAMPLE



The load-deflection behavior of the one-story assemblage at Level 8 of the frame shown in Fig. 1 will be determined by the subassemblage method. The uniformly distributed factored gravity loads on the beams (0.321 kips per inch) and the axial loads in the columns are maintained constant. These loads are determined in accordance with the working loads shown in Fig. 1, using a load factor of 1.3 and the live load reduction factors suggested by ASA A58.1. The load-drift behavior is determined for the wind from left condition only. Although the analysis is more easily and quickly accomplished by computer, step-by-step manual calculations are presented in Plates I to VI to illustrate the procedure.

The first step is to isolate the one-story assemblage at Level 8 from the frame. The resulting one-story assemblage with known member sizes is shown in Plate I. Also shown are the distribution of bending moments under gravity loads alone ($\Delta/h = 0$), column and beam properties and the initial restraint coefficients.

The analysis of the one-story assemblage initially involves the calculation of the non-dimensional rotational restraint stiffnesses $M_r$ at each joint before and after the formation of each plastic hinge. In addition the non-dimensional restraining moments $M_r$ at each joint are calculated under the gravity loads alone and under the combined loads at the formation of each plastic hinge.

The comments which follow are intended to clarify the correspondingly lettered items in Plates I and III. Comments concerning calculations in Plate III will also be relevant to corresponding calculations in Plates II and IV.

Plate I.-

(a) The distribution of bending moments is determined by elastic analysis, assuming each column is laterally restrained at both ends at mid-height.

(b) The column axial forces are computed on the basis of a mechanism condition occurring in each story of the frame under the combined loads, assuming wind from the left.

(c) The reduced plastic moment capacity $M_{pc}$ of each column was computed from Eq. 5 of Part 1.

(d) The minimum plastic moment required to resist 1.3 times the working gravity loads is defined as $M_{pm}$.

$$M_{pm} = \frac{1.3wL^2}{16}$$

where L = beam span center-to-center of adjacent columns and

w = uniformly distributed working gravity load per unit of span length.

It is convenient to use this moment as a non-dimensionalizing factor when determining the total sway resistance of a beam.

(e) The initial restraint coefficients are computed from Eq. 15 of Part 1.

Plate III.-

(a) The analysis of interior subassemblage A-C begins by determining the total change in moment in the columns at joint B as sway $\Delta/h$ increases from the initial zero drift condition to the occurrence of the first plastic hinge in the subassemblage.

(b) The total change in moment in the columns at joint B is now required as the drift is further increased up to the formation of the second plastic hinge in the subassemblage.

(c) With the first two plastic hinges found to occur at the leeward ends of the two beams, the third or last plastic hinge can only occur somewhere in the windward half span of beam BC or in the columns at joint B.

(d) The initial moment in the columns under the gravity loads alone and zero drift is equal to the net moment from the beams or $M_e$ = 3514 - 2335 = 1179 k-in.

(e) The initial value of non-dimensional restraining moment $M_{r1}$ is now determined.

(f) The non-dimensional restraining moment at joint B when the first plastic hinge occurs is the sum of the initial restraining moment, 0.388 $M_{pcB}$ and the moment found in (d).

(g) For increased drift beyond the first plastic hinge, beams AB can no longer contribute to the rotational restraint stiffness at joint B. Thus the restraint stiffness in the interval between the first and second plastic hinges decreases to that provided by beam BC alone. The corresponding joint rotation increment $\delta\theta_B$ is 0.00238.

(h) The non-dimensional restraining moment at joint B when the second plastic hinge forms is again equal to the sum of the restraining moment at joint B when the first plastic hinge forms, 0.744 $M_{pcB}$ plus the increase in restraining moment up to the second plastic hinge.

(i) Since the second plastic hinge occurred at the leeward end of beams BC, $K_{BC}$ reduces from 5.949 to 3.0 when calculating the restraint stiffness $M_{r3}$ between the second and third plastic hinges.

(j) Since the third and last plastic hinge forms in the columns at joint B the total moment resisted by the two columns $M'_{r3}$ must be equal to twice the reduced plastic moment capacity $M_{pcB}$ of the restrained column.

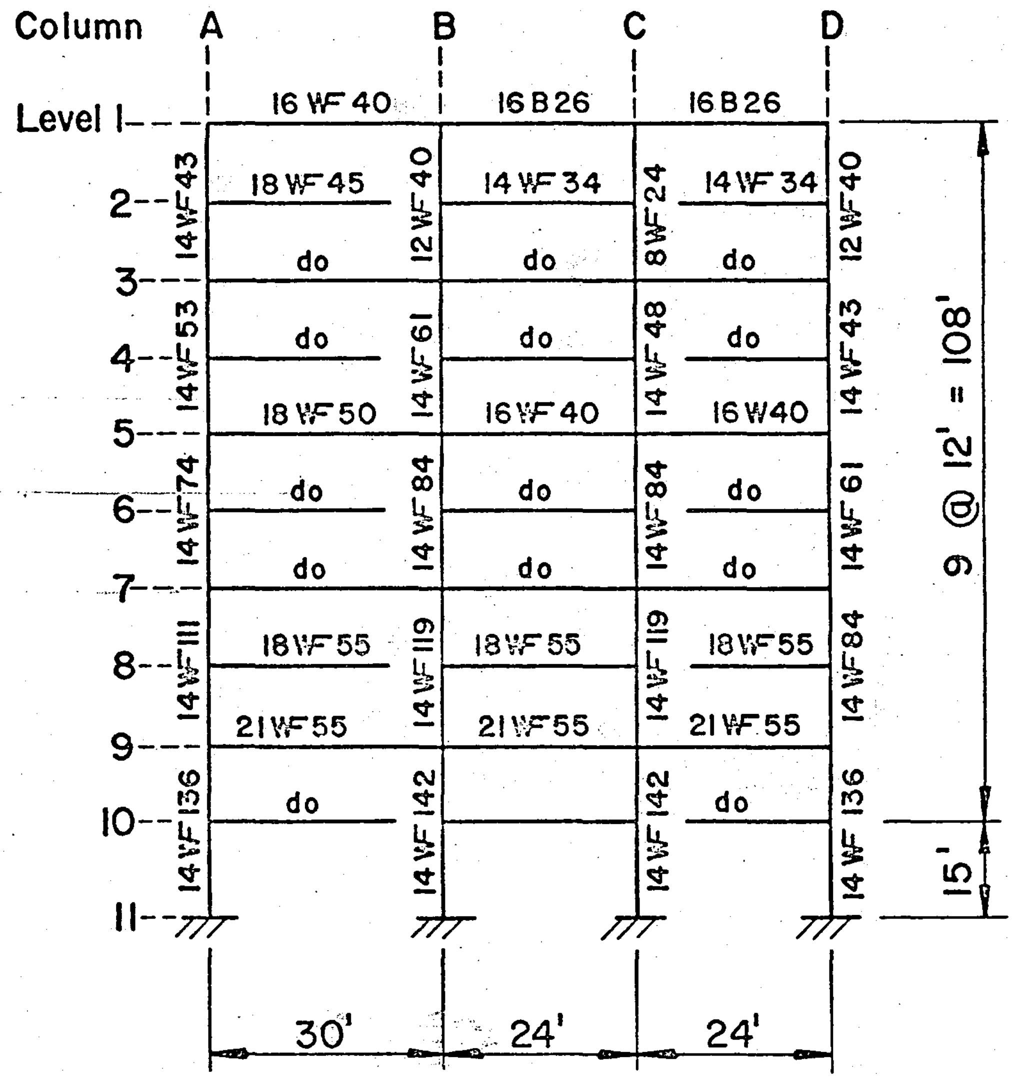
Load-Drift Behavior of the Four Subassemblages.-

The construction of the non-dimensional load-drift curve for sub-assemblage B-D, is shown in Fig. 2. The set of $M_r$ values calculated in Plate IV will determine the three complete restrained column curves o-a'-c, o-a"-c and o-b"'-d shown in the figure. These curves are given by Eq. 8 in Part 1. Similarly the set of $M_r'$ values will define the four sloping straight lines shown in the figure. The initial segment of the load-drift curve is i-a. This segment is parallel to o-a' of the load-drift curve corresponding to $M_{r1}$. The first plastic hinge occurs at point a, which lies on the intersection of curve i-a with the straight line corresponding to $M_{r1}'$. Similarly, the second segment, a-b, is parallel to segment a"-b", and the third segment, b-c, is parallel to segment b"'-c"'. The last plastic hinge occurs in the columns at point c on the load-deflection curve. The final segment, c-d is the second-order plastic mechanism curve for the subassemblage.

The non-dimensional load-drift relationships of the four subassemblages at Level 8 are shown in Fig. 3. In each case, the solid curves indicate the behavior determined in this analysis. The dashed curves were obtained using the computer analysis described in Part 1.

Load-Drift Behavior of the One-Story Assemblage

Transforming the ordinates to the curves in Fig. 3 from $Qh/2\,M_{pc}$ to Q and summing, results in the load-drift curves for the one-story assemblage at Level 8 as shown in Fig. 4. Also shown are the corresponding curves for the one-story assemblages at Levels 6 and 10 as computed manually (solid) and by computer (dashed). The sequence of formation of the plastic hinges in the one-story assemblages are also shown in Fig. 4.




Bent Spacing = 24' ASTM A36 Steel

Working Loads

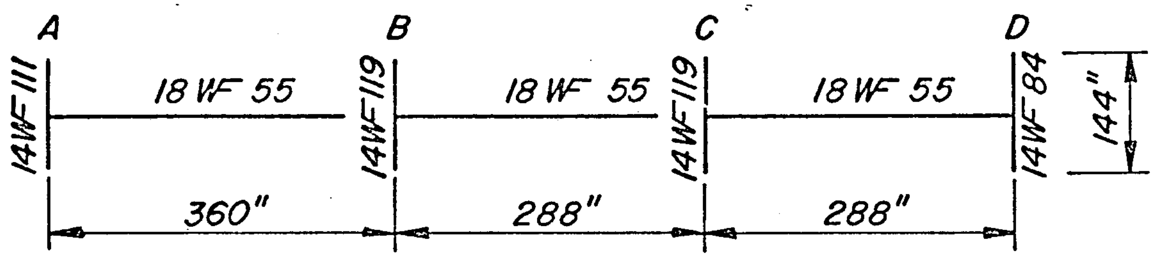
| | | |
|---|---|---|
| Level 1 | $w_D$ = 60 psf | $w_L$ = 30 psf |
| Levels 2-10 | $w_D$ = 80 psf | $w_L$ = 80 psf |
| Exterior Wall | $w_D$ = 45 psf | |
| Wind | $w_W$ = 20 psf | |

Percent Live Load Reduction by ASA A58.1

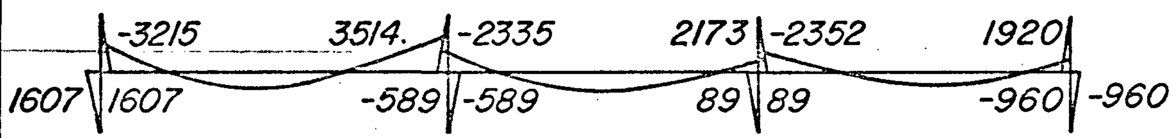
Fig. 1 Preliminary Frame Design

ANALYSIS OF LEVEL 8 - FRAME B — PLATE I

## ONE-STORY ASSEMBLAGE AT LEVEL 8




## BENDING MOMENTS - Gravity Loads Only and $\Delta/h = 0$ (a)




## RESTRAINED COLUMN PROPERTIES - Wind From Left Only

| Restrained Column | Section | $P_y$ | $h/r_x$ | Wind From Left | | |
|---|---|---|---|---|---|---|
| | | | | P (b) | $P/P_y$ | $M_{pc}$ (c) |
| Units | | Kips | | Kips | | K-in. |
| A8 - A9 | 14 WF 111 | 1175 | 23.1 | 505 | 0.430 | 4740 |
| B8 - B9 | 14 WF 119 | 1260 | 23.0 | 833 | 0.661 | 3040 |
| C8 - C9 | 14 WF 119 | 1260 | 23.0 | 745 | 0.591 | 3660 |
| D8 - D9 | 14 WF 84 | 889.6 | 23.5 | 597 | 0.671 | 2040 |

## BEAM PROPERTIES AT LEVEL 8

| Girder | Section | $M_p$ | $M_{pm}$ (d) | I | L |
|---|---|---|---|---|---|
| Units | | K-in. | K-in. | in.$^4$ | in.$^4$ |
| A8 - B8 | 18 WF 55 | 4010 | 2600 | 889.9 | 360 |
| B8 - C8 | 18 WF 55 | 4010 | 1670 | 889.9 | 288 |
| C8 - D8 | 18 WF 55 | 4010 | 1670 | 889.9 | 288 |

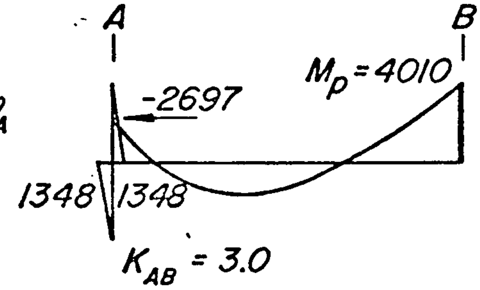
## INITIAL RESTRAINT COEFFICIENTS (e)

| Joint | $K_{right}$ | $K_{left}$ |
|---|---|---|
| A8 | $K_{AB} = 5.750$ | |
| B8 | $K_{BC}$ 5.949 | $K_{BA} = 6.286$ |
| C8 | $K_{CD}$ 6.154 | $K_{CB} = 6.053$ |
| D8 | | $K_{DC} = 5.857$ |

Notes

1. E = 29,000 ksi in the analysis, and
2. Letters in parentheses refer to items which are discussed in the text.

## WINDWARD SWAY SUBASSEMBLAGE A-B



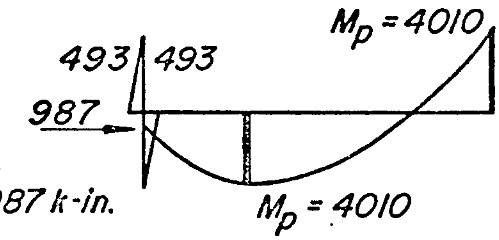

**First Plastic Hinge:**

$$\delta M_{BA} = 496 = 6.286 \times \frac{E \times 889.9}{360} \times \frac{5.750 - 4}{2} \delta\theta_A$$

$$= 13.6 E \,\delta\theta_A \; ; \qquad \therefore E\,\delta\theta_A = 36.5$$

$$\delta M_{AB} = 5.750 \times \frac{889.9}{360} \times 36.5 = 518 \text{ k-in.}$$

**Second Plastic Hinge:**




$$\frac{F_1}{F_2} = \frac{16 \times 4010}{0.321 \times 360^2} = 1.54 \; ;$$

$$\therefore \frac{M_{min}}{M_{pm}} = 0.38 \; ; \; M_{min} = 0.38 \times 2600 = 987 \text{ k-in.}$$

Check: $987 < 2 M_{pcA}$ (OK) $\delta M_{AB} = 987 - (-2697) = 3684$ k-in.

**Determine $M_r$ and $M_r'$ Values:**

**Initial** $(\Delta/h = 0)$: $M_r' = -\frac{3215}{4740} M_{pcA} = \underline{-0.678 M_{pcA}}$

$$M_{r1} = 5.75 \times \frac{29{,}000 \times 889.9}{360 \times 4740} \theta_A M_{pcA} = \underline{87.0 \,\theta_A M_{pcA}}$$

$$\delta M_A = 518 = 87.0 \times 4740 \,\delta\theta_A \qquad \therefore \delta\theta_A = \frac{518}{412{,}000} = 0.00126 \text{ rad.}$$

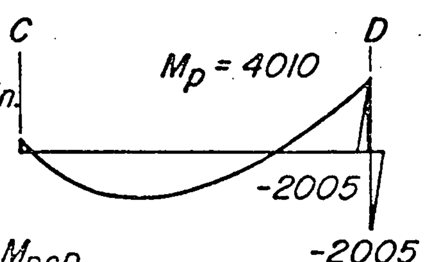
$$M_{r1}' = (87 \times 0.00126 - 0.678) M_{pcA} = \underline{-0.568 M_{pcA}}$$

$$M_{r2} = 3.0 \times \frac{29{,}000 \times 889.9}{360 \times 4740} \theta_A M_{pcA} = \underline{45.4 \,\theta_A M_{pcA}}$$

$$\delta M_A = 3684 = 45.4 \times 4740 \,\delta\theta_A \qquad \therefore \delta\theta_A = \frac{3684}{215{,}500} = 0.0171 \text{ rad.}$$

$$M_{r2}' = (45.4 \times 0.0171 - 0.568) M_{pcA} = \underline{0.209 M_{pcA}}$$

## LEEWARD SWAY SUBASSEMBLAGE C-D




**First Plastic Hinge:** $\delta M_{DC} = 4010 - 1920 = 2090$ k-in.

**Determine $M_r$ and $M_r'$ Values:**

**Initial** $(\Delta/h = 0)$: $M_r' = \frac{1920}{2040} M_{pcD} = \underline{0.941 M_{pcD}}$

$$M_{r1} = 5.857 \times \frac{29{,}000 \times 889.9}{288 \times 2040} \theta_D M_{pcD} = 257 \,\theta_D M_{pcD}$$

$$\delta M_D = 2090 = 257 \times 2040 \,\delta\theta_D \qquad \therefore \delta\theta_D = \frac{2090}{525{,}000} = 0.00398 \text{ rad.}$$

$$M_{r1}' = (257 \times 0.00398 + 0.941) M_{pcD} = \underline{1.963 M_{pcD}}$$

## INTERIOR SWAY SUBASSEMBLAGE A-C

**First Plastic Hinge:** (a)

$\delta M_{BA} = 496 = 6.286 \times \frac{E \times 889.9}{360} \delta\theta_B = 15.5 E\, \delta\theta_B$ ;  $\therefore E\delta\theta_B = 31.9$

$\delta M_{BC} = 5.949 \times \frac{889.9}{360} \times 31.9 = 586$ k-in.; $\delta M_{CB} = 6.053 \times \frac{889.9}{360} \times 31.9 \times \frac{5.949-4}{2} = 581$ k-in

**Second Plastic Hinge:** (b)

$\delta M_{CB} = 1256 = 6.053 \times \frac{E \times 889.9}{288} \times \delta\theta_C = 18.7 E\, \delta\theta_C$ ;  $\therefore E\delta\theta_C = 67.1$

$\delta M_{BC} = 5.949 \times \frac{889.9}{288} \times 67.1 \times \frac{6.053-4}{2} = 1265$ k-in.

**Third Plastic Hinge:** (c)

$\frac{F_1}{F_2} = \frac{16 \times 4010}{0.321 \times 288^2} = 2.40$  $\therefore \frac{M_{min}}{M_{pm}} = 2.0$  $M_{min} = 2.0 \times 1670 = 3340$ k-in.

Check: $4010 + 3340 = 7350 > 2 M_{pcB}$ (NG)  $\therefore M_{min} = 2 \times 3040 - 4010 = 2070$ k-in.

$\delta M_{BC} = 2070 - (-484) = 2554$ k-in.

**Calculate $M_r$ and $M_r'$ Values:**

Initial $(\Delta/h = 0)$  $M_r' = \frac{1179}{3040} M_{pcB} = 0.388 M_{pcB}$ (d)

$M_{r1} = \left(6.286 \times \frac{29{,}000 \times 889.9}{360 \times 3040} + 5.949 \times \frac{29{,}000 \times 889.9}{288 \times 3040}\right) \theta_B M_{pcB}$ (e)

$= (148 + 175)\, \theta_B M_{pcB} = 323\, \theta_B M_{pcB}$

$\delta M_B = 1082 = (148 + 175) 3040\, \delta\theta_B$ ;  $\therefore \delta\theta_B = \frac{1082}{983{,}000} = 0.00110$ rad.

$M_{r1}' = (323 \times 0.00110 + 0.388) M_{pcB} = 0.744 M_{pcB}$ (f)

$M_{r2} = 175\, \theta_B M_{pcB}$ (g)

$\delta M_B = 1265 = 175 \times 3040\, \delta\theta_B$ ;  $\therefore \delta\theta_B = \frac{1265}{533{,}000} = 0.00238$ rad.

$M_{r2}' = (175 \times 0.00238 + 0.744) M_{pcB} = 1.161 M_{pcB}$ (h)

$M_{r3} = 3.0 \times \frac{29{,}000 \times 889.9}{288 \times 3040} \theta_B M_{pcB} = 88.5\, \theta_B M_{pcB}$ (i)

$\delta M_B = 2554 = 88.5 \times 3040\, \delta\theta_B$ ;  $\therefore \delta\theta_B = \frac{2554}{269{,}000} = 0.0095$ rad.

$M_{r3}' = (88.5 \times 0.0095 + 1.161) M_{pcB} = 2.000 M_{pcB}$ (Checks) (j)

## INTERIOR SWAY SUBASSEMBLAGE B-D

### First Plastic Hinge:

$$\delta M_{CB} = 1836 = 6.053 \times \frac{E \times 889.9}{288} \delta\theta_c = 18.7 E \delta\theta_c \qquad \therefore E\delta\theta_c = 98.3$$

$$\delta M_{CD} = 6.154 \times \frac{889.9}{288} \times 98.3 = 1870 \text{ k-in.}; \quad \delta M_{DC} = 5.857 \times \frac{889.9}{288} \times 98.3 \times \frac{6.154-4}{2} = 1918 \text{ k-in.}$$

### Second Plastic Hinge:

$$\delta M_{DC} = 172 = 5.857 \times \frac{E \times 889.9}{288} \delta\theta_D = 18.1 E\delta\theta_D \qquad \therefore E\delta\theta_D = 9.50$$

$$\delta M_{CD} = 6.154 \times \frac{889.9}{288} \times 9.50 \times \frac{5.857-4}{2} = 168 \text{ k-in.}$$

### Third Plastic Hinge:

$$\frac{F_1}{F_2} = \frac{16 \times 4010}{0.321 \times 288^2} = 2.40 \quad \therefore \frac{M_{min}}{M_{pm}} = 2.0 \quad M_{min} = 2.0 \times 1670 = 3340 \text{ k-in.}$$

Check: $4010 + 3340 = 7350 > 2 M_{pcc}$ (NG) $\therefore M_{min} = 2 \times 3660 - 4010 = 3310$ k-in.

$$\delta M_{CD} = 3310 - (-311) = 3621 \text{ k-in.}$$

### Calculate $M_r$ and $M_r'$ Values:

Initial $(\Delta/h = 0)$ $\quad M_r' = -\frac{179}{3660} M_{pcc} = -0.049 M_{pcc}$

$$M_{r1} = \left(6.053 \times \frac{29{,}000 \times 889.9}{288 \times 3660} + 6.154 \times \frac{29{,}000 \times 889.9}{288 \times 3660}\right) \theta_c M_{pcc}$$

$$= (148 + 150.5)\, \theta_c M_{pcc} = \underline{298.5\, \theta_c M_{pcc}}$$

$$\delta M_c = 3707 = (148 + 150.5) 3660\, \delta\theta_c \qquad \therefore \delta\theta_c = \frac{3704}{1{,}093{,}000}\ 0.00339 \text{ rad.}$$

$$M_{r1}' = (298.5 \times 0.00338 - 0.049) M_{pcc} = \underline{0.963 M_{pcc}}$$

$$M_{r2} = 150.5\, \theta_c M_{pcc}$$

$$\delta M_c = 168 = 150.5 \times 3660\, \delta\theta_c; \qquad \therefore \delta\theta_c = \frac{168}{551{,}000} = 0.000305 \text{ rad.}$$

$$M_{r2}' = (150.5 \times 0.000305 + 0.963) M_{pcc} = \underline{1.009 M_{pcc}}$$

$$M_{r3} = 3.0 \times \frac{29{,}000 \times 889.9}{288 \times 3660} \theta_c M_{pcc} = 73.5\, \theta_c M_{pcc}$$

$$\delta M_c = 3621 = 73.5 \times 3660\, \delta\theta_c; \qquad \therefore \delta\theta_c = \frac{3621}{269{,}000} = 0.01347 \text{ rad.}$$

$$M_{r3}' = (73.5 \times 0.01347 + 1.009) M_{pcc} = 2.000 M_{pcc} \qquad \text{(Checks)}$$

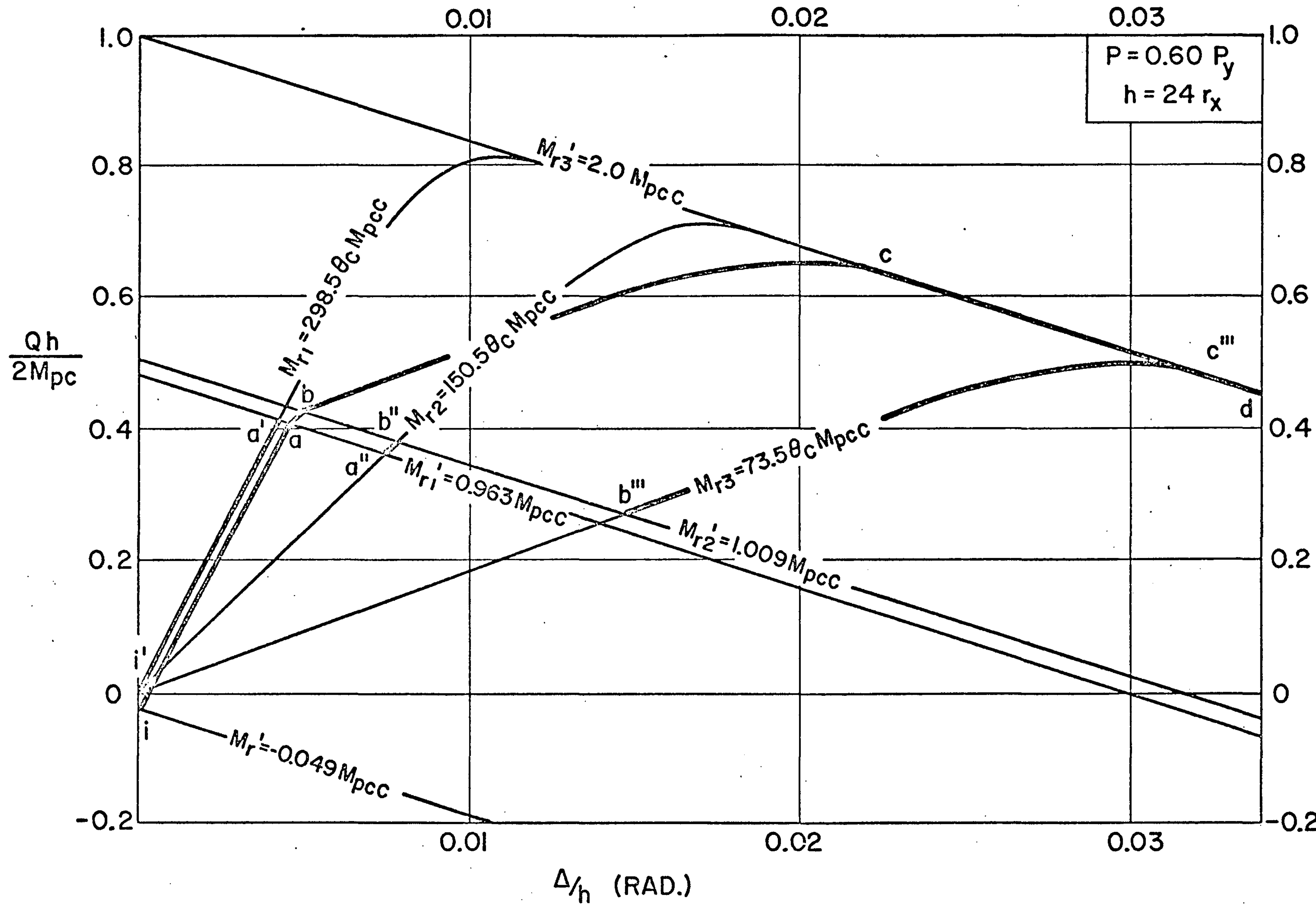



Fig. 2 Construction of Load-Drift Curve for Sway Subassemblage BD

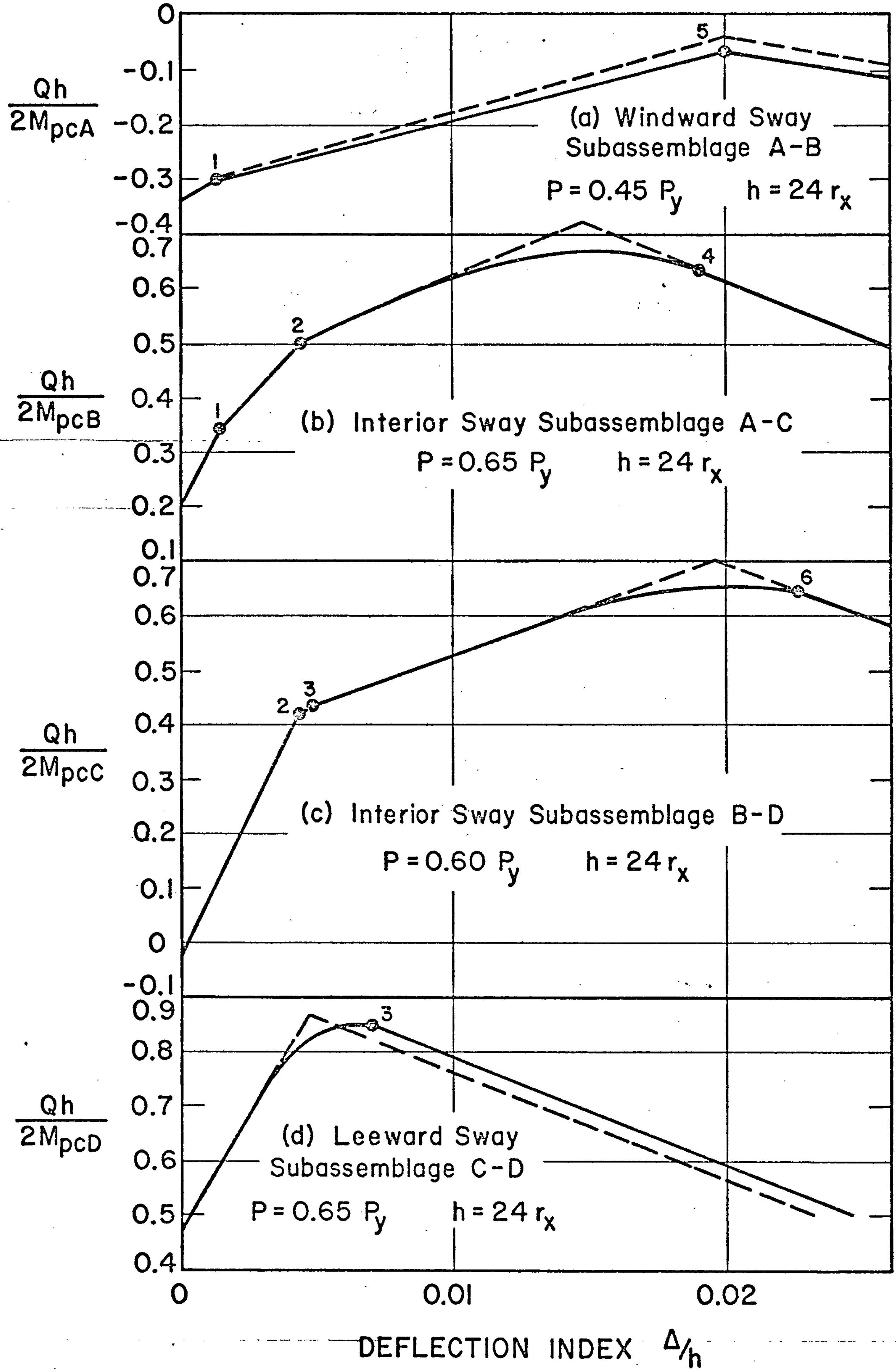



Fig. 3 Four Sway Subassemblage Curves

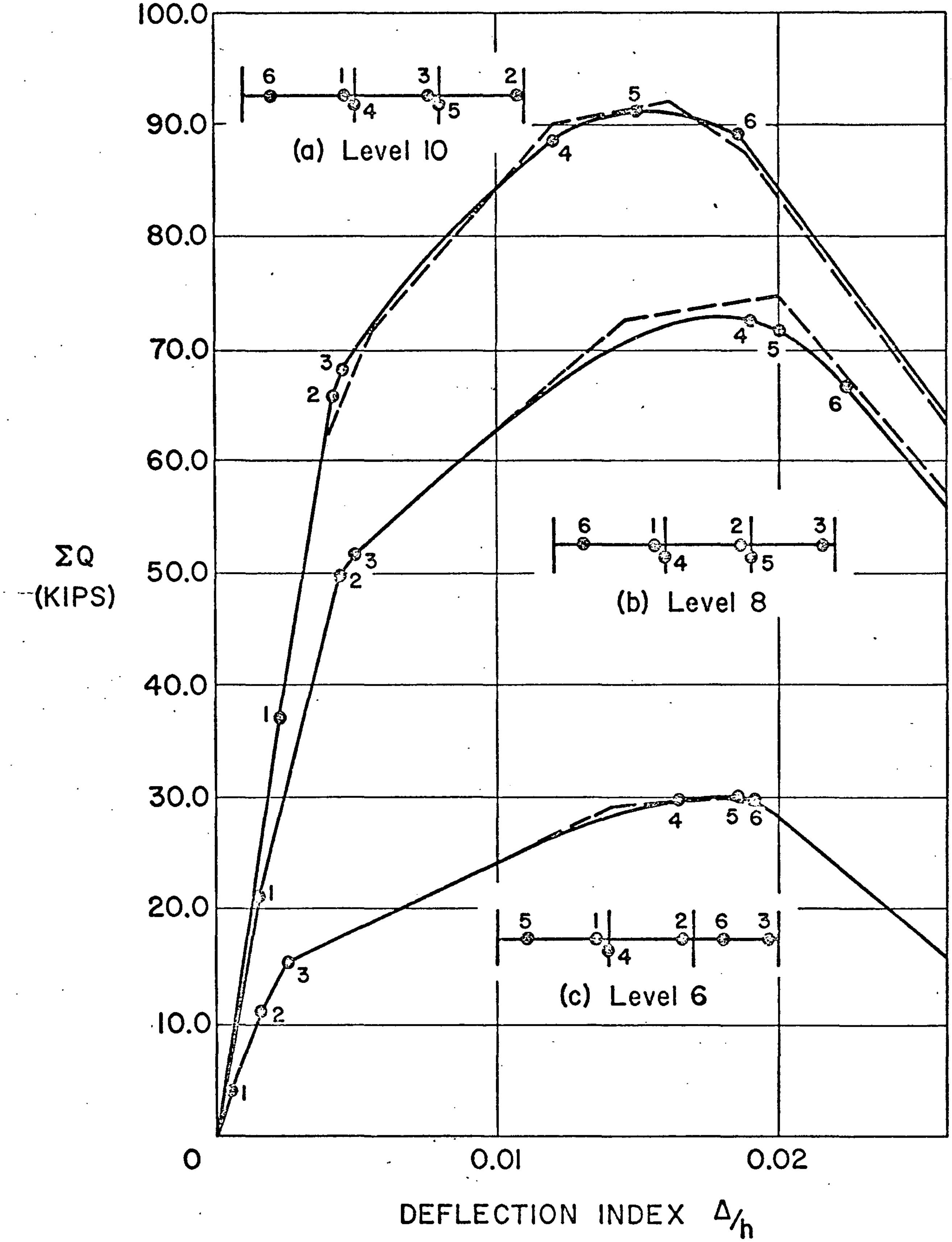



Fig. 4 One-Story Assemblage Curves for Levels 6, 8 and 10