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Computer programs(fortran iv) for m-p-0, CDC, and M-G relationships of WF beam columns bent about strong or weak axis, Sept. 1969

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Design of Laterally Unsupported Columns

COMPUTER PROGRAMS (FORTRAN IV) FOR M-P-Ø, CDC, AND M-θ RELATIONSHIPS OF W BEAM COLUMNS BENT ABOUT STRONG OR WEAK AXIS

by

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1. INTRODUCTION

This report contains a complete listing of two computer programs which have been developed to output the moment-rotation relationships of any wide-flange beam-column bent about either its strong or weak axis. These moment - rotation relationships (hereafter referred as M-θ relationships) are computed based on the concept of the Column Deflection Curve or CDC. The column deflection curves are possible equilibrium shapes of deformed beam-columns. A detailed presentation of the CDC theory can be found in Refs. 1-3.

The computer programs documented in this report were written in Fortran IV language and have been tested on the CDC 6400 computer located at the Lehigh University Computer Center in Packard Laboratory. It is believed that this program can also be run by other computers having the same characteristics as CDC 6400.

2. ASSUMPTIONS

The following assumptions are made in regard to the mechanical and geometrical properties of the wide-flange beam-column

1. Stress-strain relationship for steel is bilinear as shown in Fig. 1.

2. The yield stress level is identical σ_y for the tension flange, the compression flange, and the web.

3. The cross-section is assumed H shape. The fillet contribution is neglected.

4. The column is initially straight and free of crookedness.

5. Plane sections remain plane.

6. The beam-column is assumed to be under a constant axial thrust so that end moments and curvatures are the only variables.

7. Deflections are small.

8. There is no strain regression.

3. THE MOMENT-THRUST-CURVATURE RELATIONSHIPS

The moment-thrust-curvature relationships are developed by the ~~following~~ numerical technique described ~~in~~ below.

1. Section subjected to strong axis bending

   Each of the two flanges is cut into 200 finite elements; ten sections parallel to the x-axis ~~horizontally~~ and 20 sections parallel to the y-axis ~~vertically~~ as shown in Fig. 2. The web is divided into 20 ~~horizontal~~ strips parallel to the x-axis.

2. Section subjected to weak axis bending

   Each flange is sliced ~~slided~~ into 80 ~~vertical~~ strips parallel to the y-axis and the web is cut into 200 finite elements, 50 sections parallel to the x-axis and four sections parallel to the y-axis. (See Fig. 3).

~~A curvature is first assumed.~~ The combined ~~stress (axial stress + residual stress + bending stress)~~ in ~~at~~ every finite element is then computed. A summation is made on the normal forces acting on these elements. Usually this resultant ~~resulting~~ non-dimensionalized force $\sum_i \sigma_i A_i / P_y$ is not equal to the specified value of P/P_y and thus adjustments are made by trial and error such that after a few ~~adjustments~~ iterations,

$$\frac{\sum_i \sigma_i A_i}{P_y} = \frac{P}{P_y} \tag{1}$$

Once this is achieved, the computer will proceed to compute the moment on the section:

$$\frac{M}{M_{pc}} = \frac{\sum_i \sigma_i A_i y_i}{M_{pc}} \tag{2}$$

The computer will now pick up the next curvature value and repeat the process to find M/M_{pc}. This procedure is continued until a curvature ratio $\phi/\phi_{pc} = 200$ is reached. By now the computer will have a complete set of M-P-ϕ relationships for a particular section with a particular type of residual stresses, stored up in its memory.

4. COLUMN DEFLECTION CURVES

The column deflection curves are constructed using a numerical integration procedure that is valid for both the elastic and the inelastic portions of the CDC as long as no plastic hinge is formed. The M-P-ϕ relationships obtained by the method discussed earlier are the essential parameters for the construction of the CDC's.

A portion of a CDC near z = o is shown in Fig. 4. The P/P_y ratio and the initial end slope τ_o are specified. Segments of length ρ along the thrust line are selected. (These segments may not have equal length. For good accuracy, $\rho = R_x$ is recommended for the construction of the initial portion of the CDC. At locations close to the 1/4 wave point, smaller segments are desired, and $\rho = 0.1\ R_x$ has been found satisfactory). It is assumed that the deflection shape of each segment is circular. The distance y_1 and the slope τ_1 at the end of the first segment are:

$$y_1 = \rho_1 \tau_o - \frac{\rho_1^2 \phi_1}{2} \qquad (3)$$

$$\tau_1 = \tau_o - \rho_1 \phi \qquad (4)$$

At the end of the second segment:

$$y_2 = y_1 + \rho_2 \tau_1 - \frac{\rho_2^2 \phi}{2} \qquad (5)$$

$$\tau_2 = \tau_1 - \rho_2 \phi_2 \qquad (6)$$

In general, at the end of segment i:

$$y_i = y_{(i-1)} \tau_{(i-1)} \rho_i - \frac{\rho_i^2 \phi_i}{2} \tag{7}$$

$$\tau_i = \tau_{(i-1)} - \rho_i \phi_i \tag{8}$$

The integration is continued until $\tau_i \leq 0$ at which point, at least a 1/4 wave length has been established. The exact length of a 1/4 wave can be determined by interpolation. It is ~~denoted by~~ designated X_{max} XMAX in the programs.

The value of curvature ϕ_i in the Eqs. 3-8 is closely approximated by the curvature in the segment which is obtained from the M-P-Ø curve corresponding to the mean value of the moment in the segment. This mean moment is:

$$M_1 = \frac{P\alpha_o \rho_1}{2} \tag{9}$$

$$M_2 = P\, y_1 + \frac{P\tau_1 \rho_2}{2} \tag{10}$$

$$\vdots$$

$$M_i = Py_{(i-1)} + \frac{P\tau_{(i-1)} \rho_i}{2} \tag{11}$$

In the programs the moment M_i is non-dimensionalized by dividing it by M_{pc}.

5. MOMENT-ROTATION RELATIONSHIPS

This section of the program consists of five parts, one of which is used to interpret the M-θ relationship for a beam-column of a particular end-moment ratio (MRATIO):

Part 1 for $-1.0 < \text{MRATIO} < 0$

Part 2 for $0 < \text{MRATIO} < 1.0$

Part 3 for MRATIO = 0

Part 4 for MRATIO = 1.0

Part 5 for MRATIO = - 1.0

Parts 1 and 2 are actually sufficient for interpreting the M-θ relationships for beam-columns of any end-moment ratios (that is, $-1 \leq \text{MRATIO} \leq 1$). However each of these two parts uses more computer time than Part 3, 4 or 5 to generate M-θ relationships for the case of MRATIO = 0, 1.0, or - 1.0. These end-moment ratios of 0, 1 and -1 are very common in engineering usage and thus ~~justify~~ the inclusion of Parts 3-5 in the programs is justified.

6. DESCRIPTION OF THE PROGRAMS

Each program consists of four major sections:

1. Computation of the M-P-Ø relationships
2. Calculation of CDC's
3. Interpretation of M-θ relationships from the CDC datas
4. Plotting of the M-θ curves

The program begins with a list of symbols and ends with the inclusion of three subroutines:

1. Subroutine RSMEA -- It permits the user to supply any residual stress pattern into the computer before the computation of M-P-Ø relationships. The residual stress pattern must be symmetrical about both axes ~~through~~ though the normal force on the section due to the residual stresses may not be balanced. This subroutine thus permits the "mean" measured residual stresses of any cross-section to be read into the computer.

2. Subroutine RS -- This subroutine interpolates the residual stress at every finite element from the residual stress data read into the computer by calling subroutine RSMEA. It then adjusts the unbalanced normal force by distributing the same magnitude of error to every measured point, and at the same time ensures stress compatibility at the flange-web junction.

3. Subroutine LMPLOT -- It instructs the computer to plot the results either in the term of moment vs. rotation or moment/reduced plastic moment (M/M_{pc}) vs. rotation.

A flowchart showing a general outline of the program is shown in Fig. 5. The program for the wide-flange beam-column bent about its strong axis is designated as BCS. That for the weak axis bending has been designated as BCW. Both of these two programs have identical flow history.

The initial readings to be read in are the shape size, flange width b, flange thickness t, web thickness w, yield stress σ_y, P/P_y ratios, moment M_1/M_2 ratios, and the initial slope values for the CDC's. The end-moment ratio M_1/M_2 is positive if the beam-column is bent into a simple curvature. That is, end moments is opposite rotation as shown in Fig. 6. The M_1/M_2 ratio is negative if the column is bent into double curvature by the end moments in same rotation.

The computer will then proceed with the first set of P/P_y and M_1/M_2 ratio to compute the M-P-Ø relationships using one of the following residual stress patterns:

1) Measured residual stresses -- This requires the execution of subroutine RSMEA and subroutine RS.

2) Standard residual stress pattern -- as shown in Fig. 7, this patterns is commonly assumed in wide-flange shapes. The design charts in Ref. 4 have been developed based on this residual stress pattern.

3) No residual stresses .

After the M-P-Ø relationships have been compiled, the program then proceeds to generate the CDC's. One CDC is generated at ~~the~~ time for a specified initial slope τ_o. After a quarter CDC has been generated, the computer interprets the M-θ These CDC results for the specified number of L/r ratios. Having executed this, the computer stores away these M-θ data and at the same time continues to generate the next CDC. This procedure is repeated until the last τ_o ~~no encountered. This τ_o has a negative value. Its purpose is to instruct the computer that there will be no more CDC that has to be generated for the specified P/P_y and M_1/M_2 ratios.~~

Next the computer will print the accumulated M-θ datas, then plot the same M-θ datas as moment vs. rotation or M/M_{pc} vs. rotation curves whichever case the user desires.

The computer will now pick the next I value. If this value I is less than the number VAL the computer will go to EXIT. However if I is equal to or less than VAL, the next computation will depend on whether the new P/P_y value is same as the previous one. If identical, the M-P-Ø computation is not repeated, and the computer will proceed to generate a ~~new set of~~ CDC's for the new end-moment ratios.

7. INPUT DATA

The symbols used in this section can be found in the computer programs listed in ~~the~~ Appendices A and B. Sample input data cards are given in Appendix C.

1. First Set of Cards

   1 card [FORMAT (I10, 5F10.5, F10.2, F10.4)]: ISEC, LBS, B, D, T, W, E, FY

   1 card [FORMAT (I5)]: VAL

   Set of VAL cards [FORMAT (2F10.5, 2I10)]: POPY (I), MRATIO(I), NP(I), NOPT(I)

   Set of VAL cards [FORMAT (I5)]: JREST (I)

   1 card [FORMAT(I10)]: ION

   Set of ION cards [FORMAT (F10.5)]: THO(NK)

2. Second Set of Cards

   IF JREST ≠ 1 :

   1 card [FORMAT (F10.5)]: FRC (FRC = 0.3 for standard residual stress pattern;
   = 0 for no residual stresses)

   IF JREST = 1 : (Subroutine RSMEA is called)

   1 card [FORMAT (2I10)]: JNUMF, JNUMW

   set of JNUMF cards [FORMAT (2F10.4)]: XSF(JL), FRF(JL)

   set of JNUMW cards [FORMAT (2F10.4)]: YSW(JL), FRW(JL)

   (see Fig. 8 for definitions of XSF, FRF, YSW, FRW. Symbols are listed in subroutine RSMEA)

Sufficient data must be provided to cover the full half flange width and full half web depth. In other words taking the flange as an example; if there is no measured data at the flange-web junction or at the tip of the flange, it is necessary to feed into the computer the extrapolated value from the measured data. It is obvious that there must be at least two sets of XSF and FRF readings for the flange and two sets of YSW and FRW readings for the web. The maximum number of sets of measured data for the flange or web is 20. However this number may be increased by increasing the field length in the DIMENSION declaration of XSF, FRF, YSW, FRW, FRFY, and FRWY. The spacing of XSF or YSW may not be equal. Compressive residual stresses are read in as positive values and tensile stresses as negative values. Only readings for a 1/4 flange and a 1/2 web are needed as the residual stress distribution has been assumed symmetrical about both axes.

3. Third Set of Cards

[FORMAT (I5)] 1 card: LOR

[FORMAT (F10.5)] Set of LOR cards: ROL(L)

(ROL = L/r_x in the BCS program;

= L/r_y in the BCW program)

4. Fourth Set of Cards

This set of cards will instruct the computer to set the ordinate and abscissa lines for plots of M-θ curves. It is applicable only if NP ≠ 1. Symbols are listed in Subroutine LMPLOT. (See Fig. 9 for clarification.)

[FORMAT (4F10.4)] 1 card: BJ, BK, BL, BM

[FORMAT (4F10.5)] 1 card: AY, OX, BX, CY

If NP = 1, the computer will plot M/M_{pc} vs. rotation in a standard format as shown in Fig. 10.

If VAL is greater than unity, it is necessary to read in (VAL-1) times the new input of second, third and fourth sets of data cards.

The curvature increments RA, RB, RC, and RD that have been selected for these programs are 0.05, 0.1, 1.0, and 10.0 respectively. ~~Those~~ The segment length ratios R1, R2, R3, and R4 that have been selected are 1.0, 0.1, 0.1, and 0.1 respectively. These quantities are declared in DATA statements ~~listed~~ in the BCS and the BCW programs immediately following the DIMENSION declaration. They have been found to give good accuracy in the computations of M-P-Ø and CDC's. Larger values may be used to save some computer time if good accuracy is ~~then~~ not the important factor. In this case, the user should alter the two DATA Statements ~~cards~~ of RA, RB, RC, RD, and R1, R2, R3, R4 in the main program.

8. OUTPUT

There are several pages of print out for every beam-column examples of which can be found in Appendix D.

1) First Set - It contains the raw datas that are fed into the computer initially.

2. Second Set - This set of printout contains the important properties of the column section such as: area, plastic modulus Z, plastic moment M_p and many others.

3. Third Set - A complete tabulation of the computed M-Ø relationships for a constant P/P_y ratio is given here.

4. Fourth Set - This page tabulates the loops that are being executed in the construction of CDC's. It provides the user the information as to which THO (NK) values the computer cannot handle because of the development of a plastic hinge somewhere in the CDC. It also provides the user a complete listing of all THO (NK) values so that he may decide whether smaller THO should be used to get smooth curve or to get more information in the vicinity of the peak moment.

5. Fifth Set - It contains a tabulation of the complete M-θ relationships for the beam-column under consideration: End rotations, M/M_{pc} and end moments are tabulated for the two ends of the beam-column.

In addition to the five sets of output discussed in avove, the computer will print out another set of information (between the second and the third sets) if subroutines RSMEA and RS are called. This additional set of output will have three pages of data as follows:

Page 1 : Raw data of XSF, FRF, YSW, and FRW.

Page 2 : Number of loops for balancing the resultant normal force on the section due to the residual stresses, normal force on the flanges PPYF, normal force on the web PPYW, and the PPYF/PPYW ratio.

Page 3 : List of adjusted XSF, FRF, YSW, and FRW to be used in the computations of M-P-Ø.

9. SUMMARY

Two computer programs in Fortran IV language have been developed to generate the M-P-Ø, CDC and M-θ relationships of any W beam-column bent about its strong or weak axis. These programs have the advantage of being capable of executing the complete computations of M-P-Ø, CDC, and M-θ in a run. The actual CP time per run depends on the type of computer, the number of operations to construct the M-P-Ø relationships and the CDC, and the type of interpretation of M-θ for a specified end-moment ratio. Experience on the ~~CDC completed~~ CDC6400 computer at Lehigh University has shown that the average CP time should be less than 30 sec. when using a binary deck, or alternatively, less than 40 sec. when using the original deck. The minimum field length for either program is $100,000_8$.

These computer programs can handle any residual stress pattern as long as the pattern is symmetrical about both axes. Therefore, they are useful for laboratory work in which measured residual stresses can be used to construct the M-P-θ relationships. The weak axis bending program BCW considers the whole column cross-section to resist the axial thrust and the applied moment. Previous research work on weak axis bending has neglected web contribution.(5,6)

The final results are plotted either in the form moment vs. rotation or M/M_{pc} vs. rotations.

Thus by using these computer programs, it is now possible to obtain more accurate M-θ curves for any W shape of any yield stress level instead of interpolating the values from the charts in Ref. 4 which were prepared for 8W31 shape of A36 steel.

10. ACKNOWLEDGMENTS

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The authors acknowledge the assistance of Irving Oppenheim.

11. FIGURES

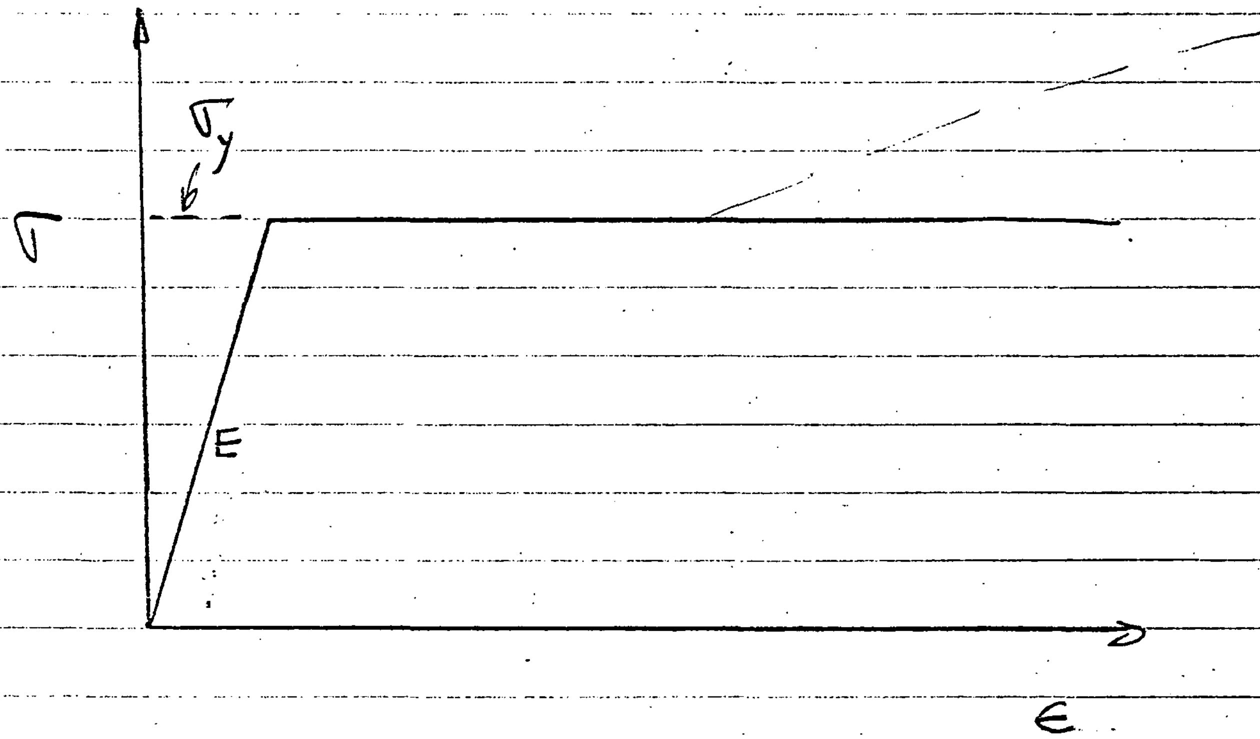
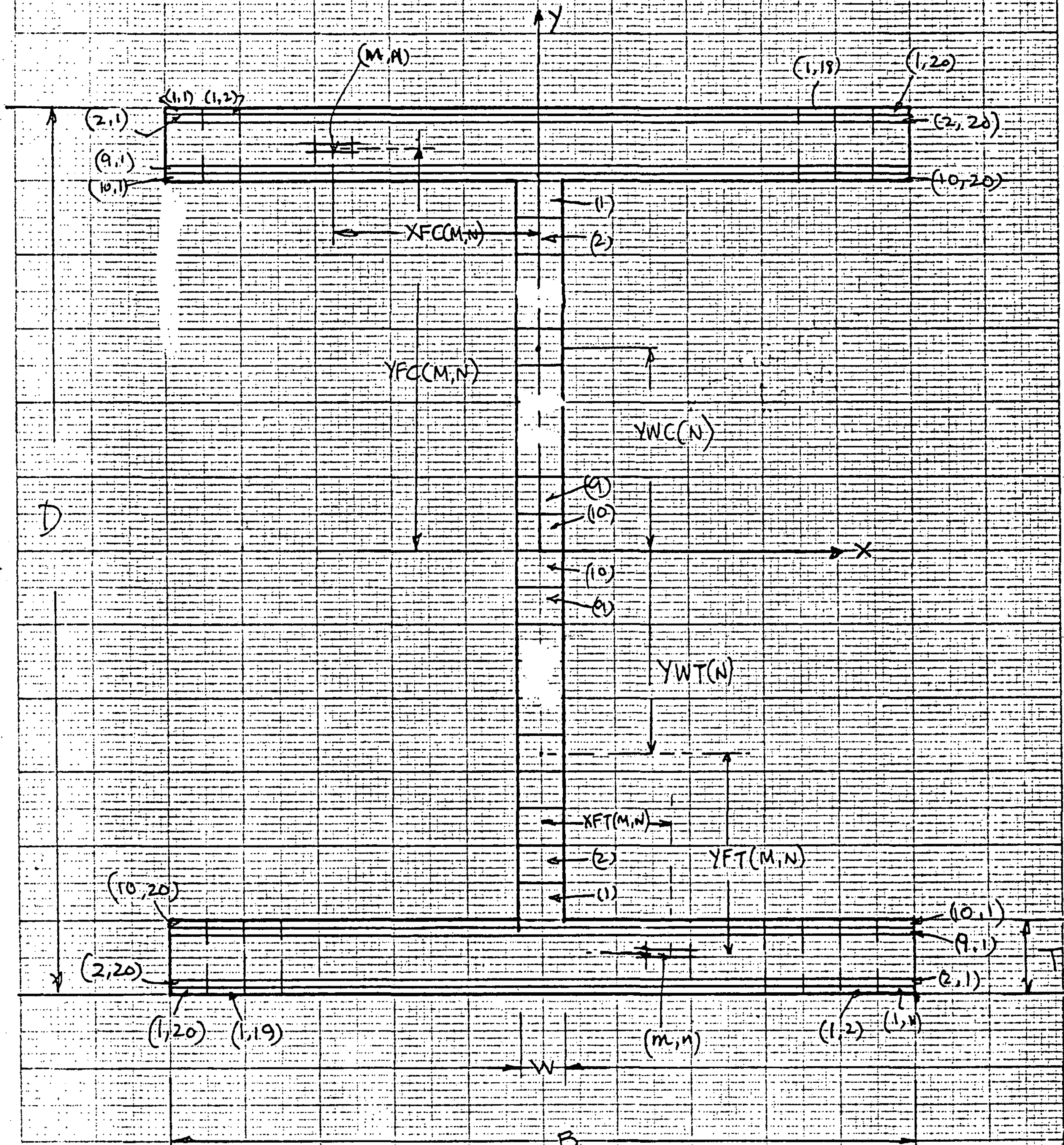


Fig 1 Idealized stress-strain relationship for steel.



FLANGE = 10 X 20 FINITE ELEMENTS

WEB = 20 ~~ELEM~~ FINITE ELEMENTS

Fig 2 (strong axis)

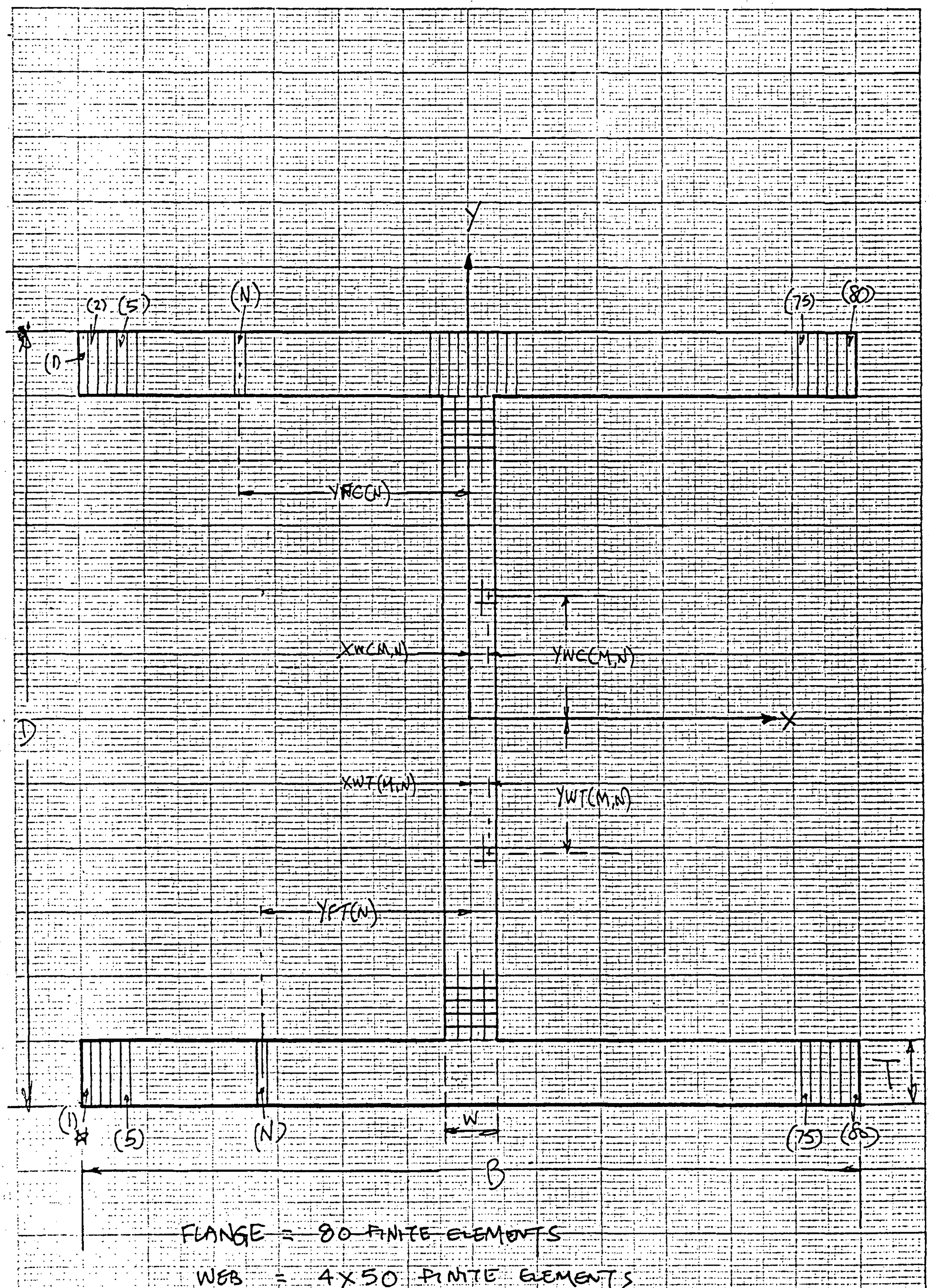


Fig 3 (with axis)

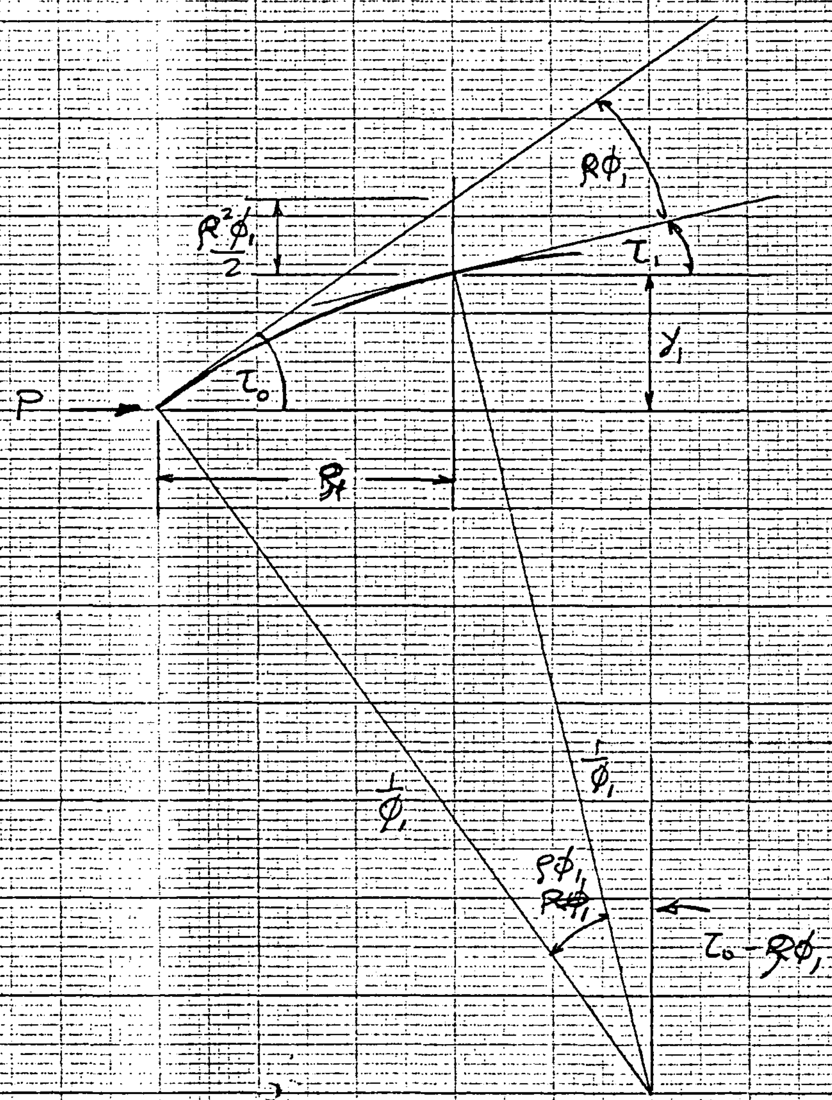


Fig. 4. First Segment on CDC Curve

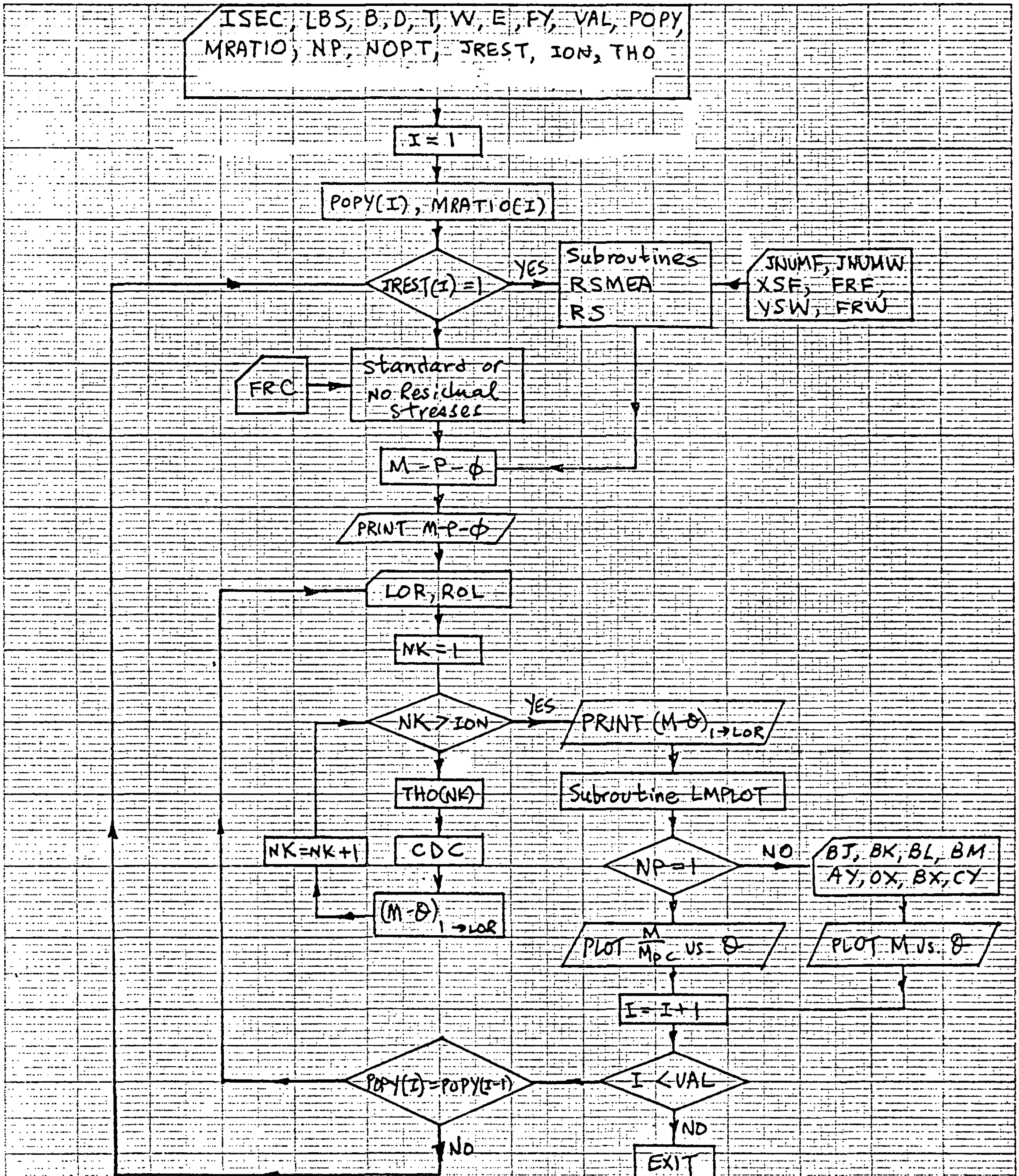
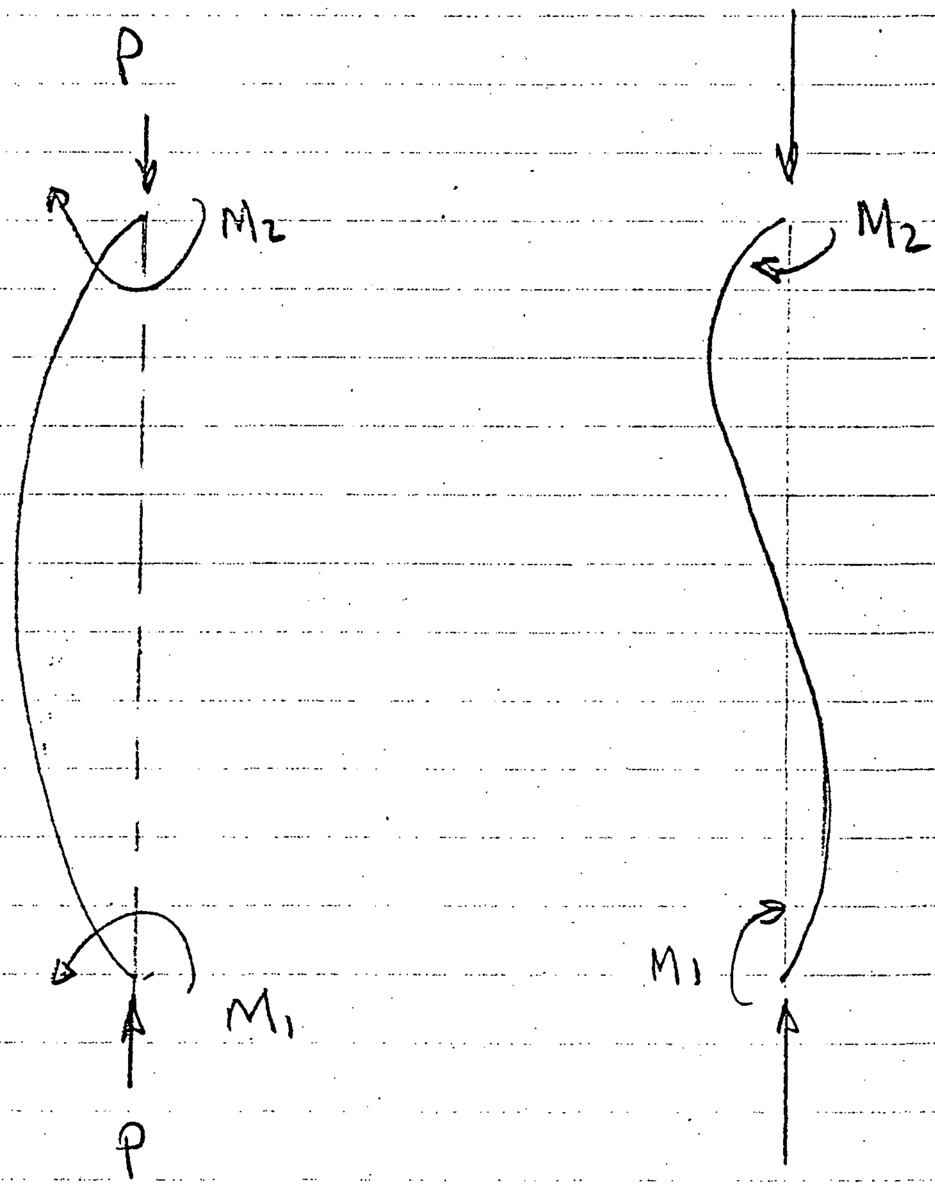


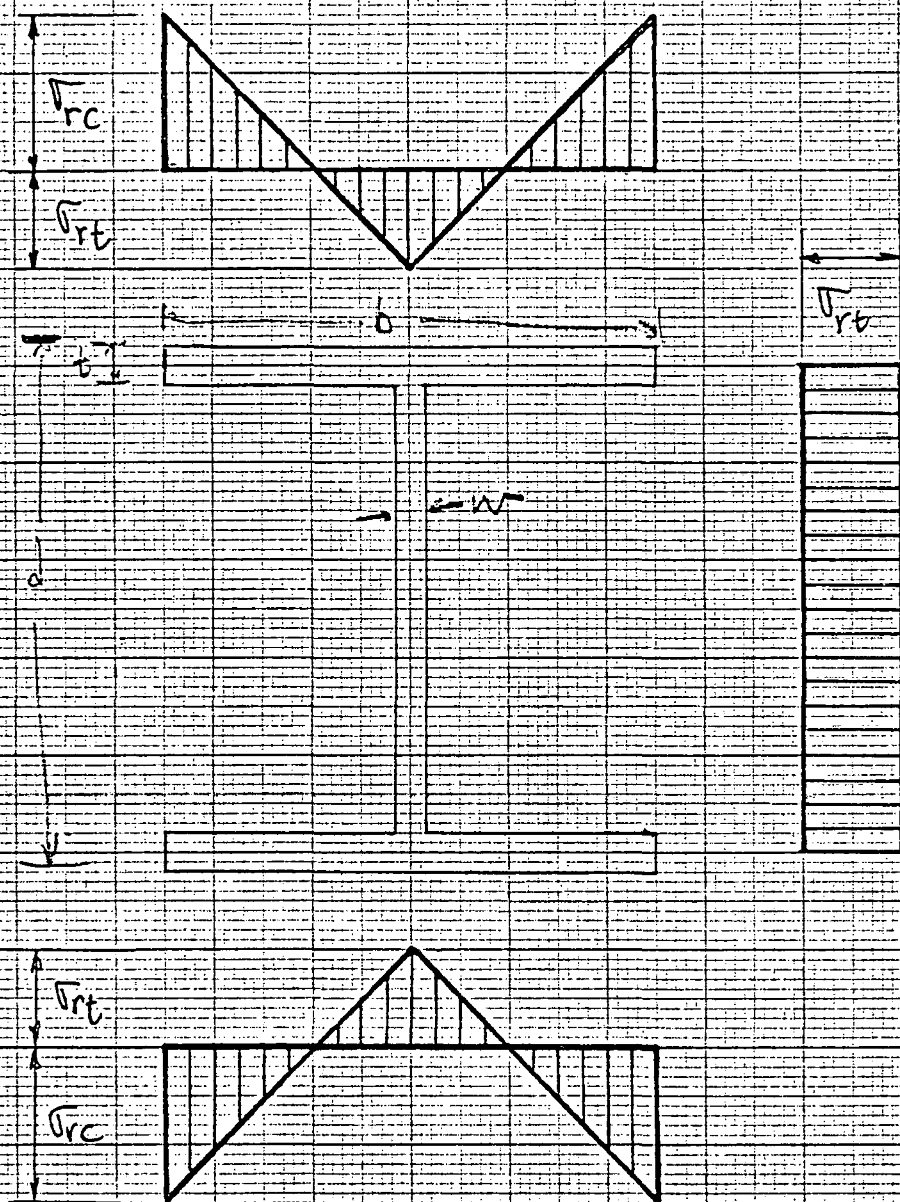
Fig 5 FLOW CHART FOR BCS AND BCW PROGRAMS



$$MRATIO = \frac{M_1}{M_2} = \text{positive}$$

$$MRATIO = \frac{M_1}{M_2} = \text{negative}$$

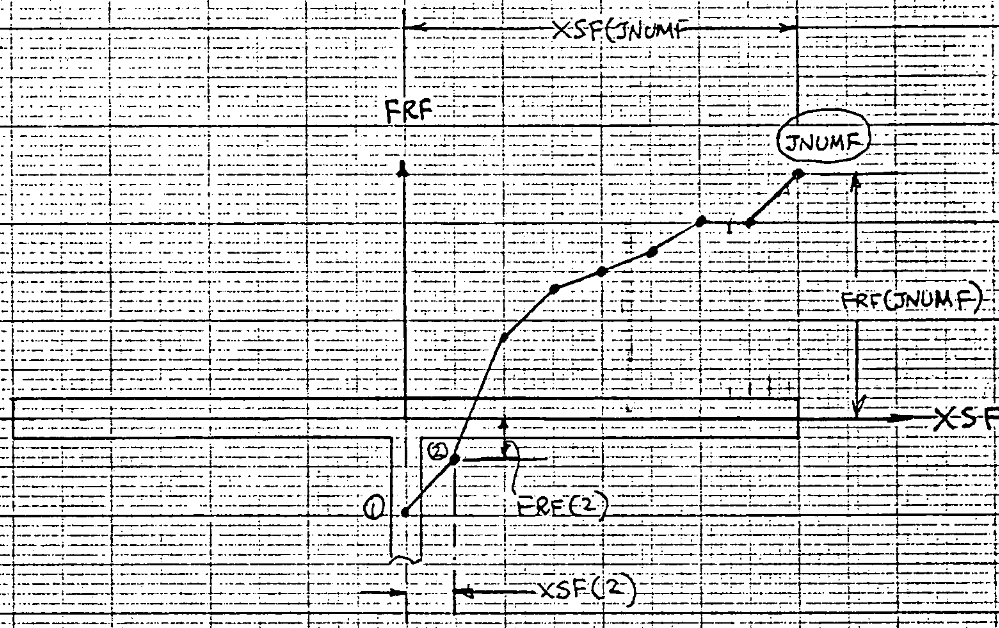
Fig 6 SINGLE OR DOUBLE CURVATURE



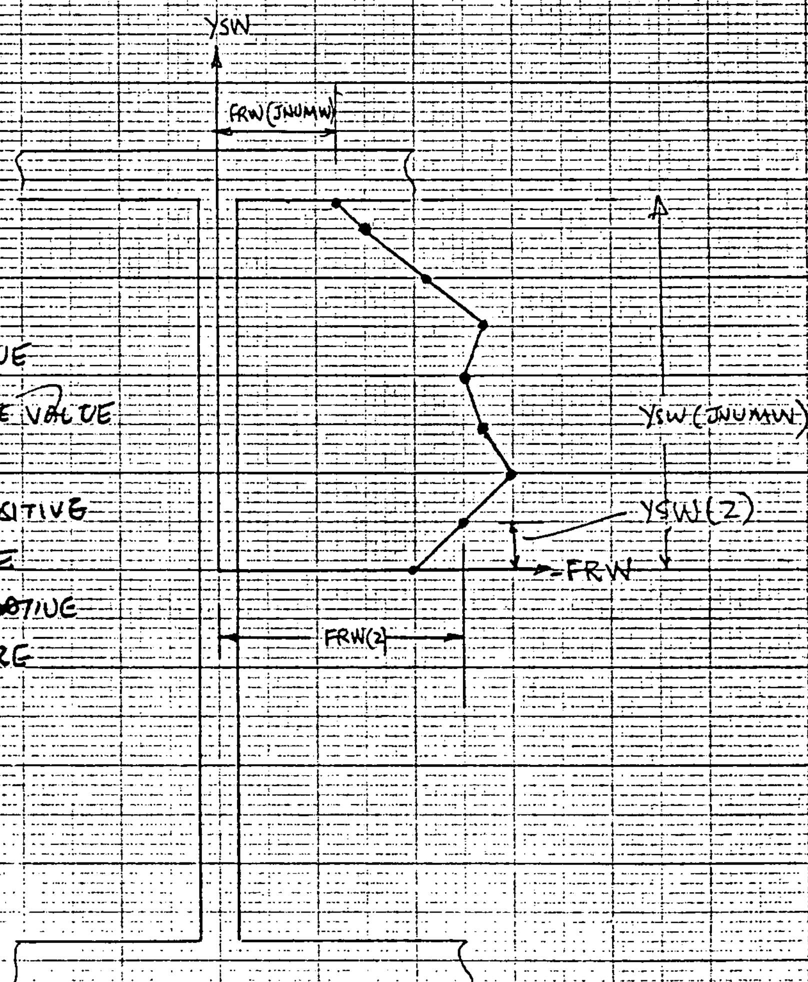
$$\sigma_{rc} = 0.3\,\sigma_y$$

$$\sigma_{rt} = \left[\frac{bt}{bt + w(d-2t)}\right]\sigma_{rc}$$

Fig 7 STANDARD RESIDUAL STRESS PATTERN



(a) FLANGE



NOTE:
FRF = +VE IF RESIDUAL STRESS IS COMPRESSIVE VALUE
= −VE IF IT IS TENSILE VALUE

NOTE: FRF AND FRW ARE POSITIVE IF THE RESIDUAL STRESSES ARE COMPRESSIVE. THEY ARE NEGATIVE IF THE RESIDUAL STRESSES ARE ~~POSITIVE~~ TENSILE

(b) WEB

Fig 2 [illegible] SUBROUTINE RSMEAT

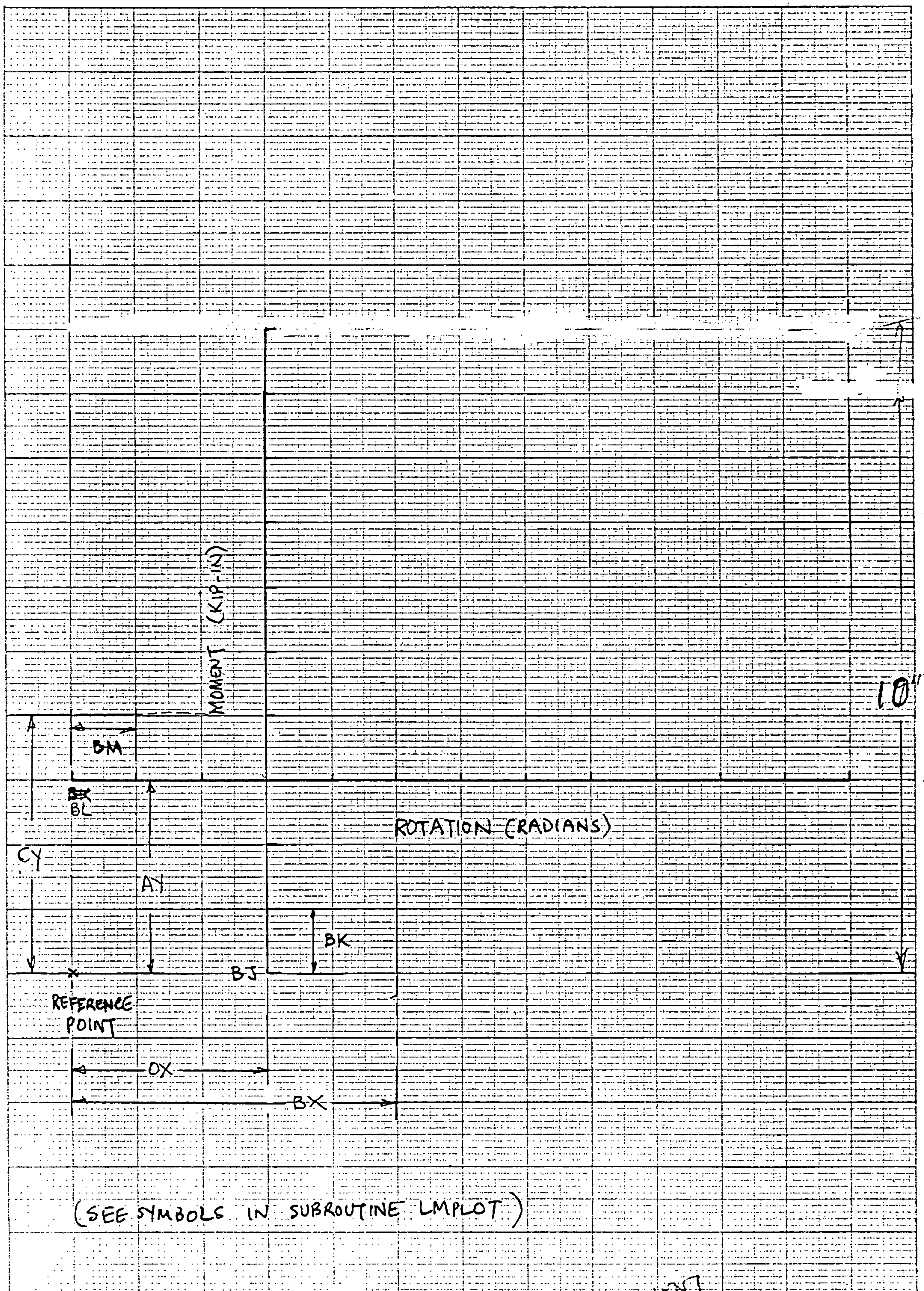


Fig 9 PLOT LAYOUT FOR MOMENT VS. ROTATION

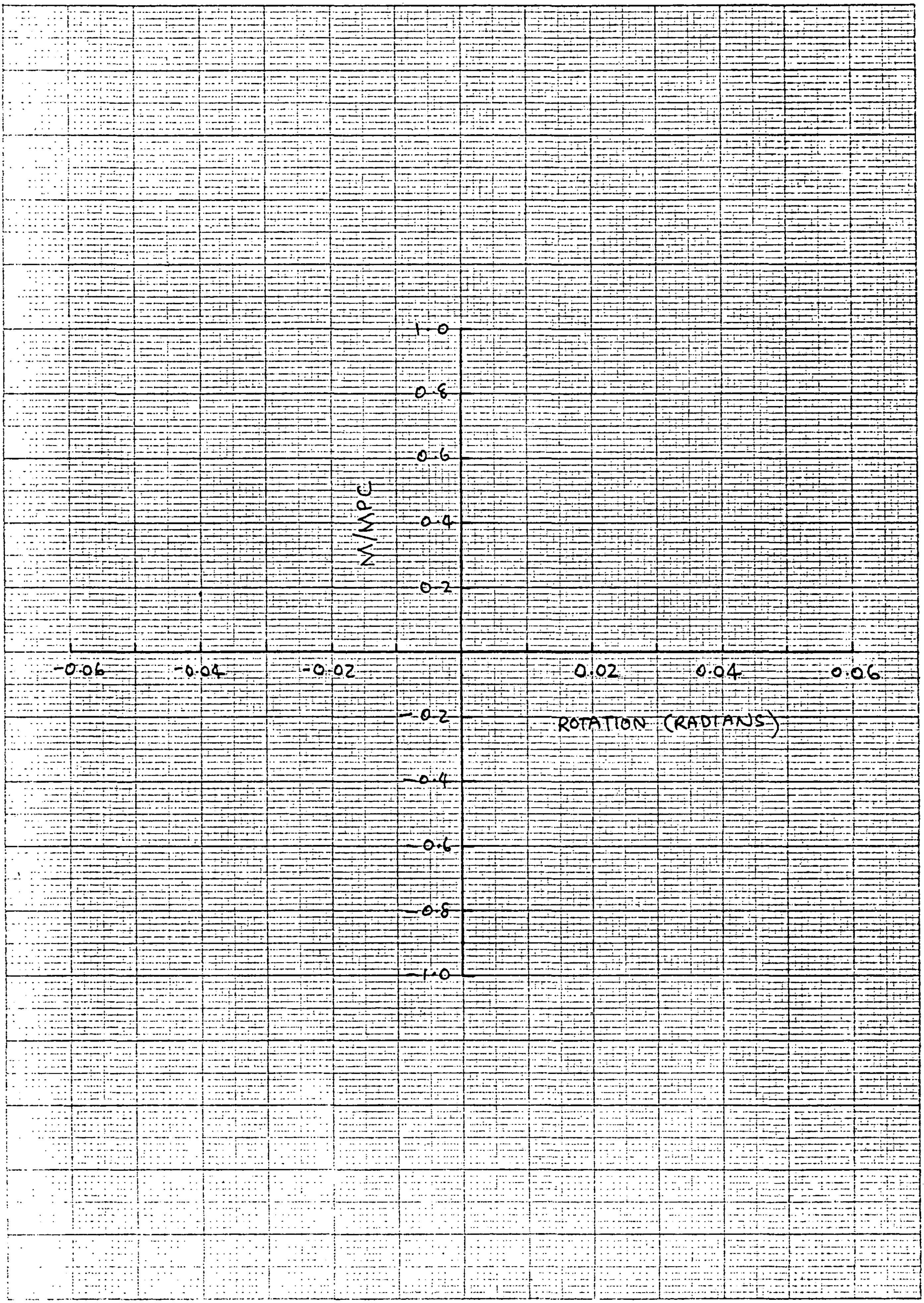


Fig 10 $\frac{M}{M_{PC}}$ vs ROTATION PLOT LAYOUT

12. REFERENCES

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