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BOND IN PRESTRESSED CONCRETE

PROGRESS REPORT NO. 3

END SUPPORT EFFECTS ON ULTIMATE FLEXURAL BOND
IN PRE-TENSIONED BEAMS

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ABSTRACT

This investigation is a pilot study of the bond characteristics of prestressed members pre-tensioned with multiple layers of strand. The primary objective was to develop information in regard to the additional flexural bond attained in strands found near the lower extremity of pre-tensioned members as a result of the pinching effect of the end reactions.

The results of tests conducted on three beams pre-tensioned with two layers of 1/2-in., 270K strand are presented and compared with similar flexural bond tests by Badaliane and VanHorn on beams with a single-layer strand pattern. The test results clearly indicate that additional flexural bond is developed as a result of the pinching effect of the end reaction on the prestressing steel near the soffit of the member. The study also demonstrated that bond slip occurred at loads which were considerably below both the actual and the computed ultimate flexural capacities of the members.

It is recommended that a more extensive study be made in the future, encompassing several variables such as vertical strand spacing and strand pattern.

I. INTRODUCTION

In a reinforced concrete member, forces are normally transmitted directly between the concrete and the reinforcing steel. The medium by which such a transmittal is accomplished is referred to as bond. In conventional concrete members reinforced with deformed steel bars, this bonding action is the result of a combination of adhesion and mechanical resistance between the steel and concrete. When considering pre-tensioned prestressed concrete members, the bonding action between the prestressing element and the concrete is a result of friction as well as adhesion and mechanical resistance.

In the fabrication of a pre-tensioned prestressed concrete member the prestressing strand is first tensioned to the desired stress level. Concrete is then cast about the strand and allowed to cure until it gains sufficient strength for release. The strands are then freed from the tensioning mechanism, causing a gradual build-up of force within the member. The stress in the prestressing tendon varies from zero at the end of the member to full prestress at some distance inside the concrete. This distance is known as the transfer length, and the bonding action responsible for this stress gradient is called transfer or anchorage bond.

The transfer length varies with the type, size, and surface condition of the strand, the concrete strength at release,

and the rate of release. For any one strand, in a particular concrete member, the transfer length will be greatest in beams that have been subjected to rapid release, and shortest in members where the prestress transfer was gradual.

When a beam is loaded the steel reinforcement helps the concrete resist the externally applied moment. To accomplish this in a pre-tensioned flexural member, the circumferential forces are transmitted from the concrete to the prestressing strand by bond. This type of bond is called flexural bond.

In discussing flexural bond in pre-tensioned prestressed members, two phases must be considered, before and after cracking of concrete. Before cracking occurs, the increase in the tensile steel stress and the resulting increase in flexural bond stress is relatively small. This flexural bond stress can be calculated through the use of a free body diagram of an uncracked concrete member, which yielded the expression

$$u_f = \frac{n e_s A_s V}{\sum_o I'_c}$$

When cracking occurs under flexure, the bond stress rises in the vicinity of the crack, and bond failure, resulting in slip between the strand and the concrete, occurs in the region adjacent to the crack. As the load is further increased, the high bond stress continues as a wave from the original crack toward the end

of the member. When the peak of this wave of high bond stress reaches the prestress transfer zone, the increase in steel stress resulting from bond slip decreases the diameter of the strand with the resulting decrease of frictional bond. It is at this point that strand movement at the ends of the beam can be detected. This phenomenon is known as bond slip.

In members prestressed with strands, the helical shape of the individual wires provides mechanical resistance so that the beam can support additional load even after the strand slips at the beam ends.

Of increasing interest in the characteristics of bond in pre-tensioned prestressed concrete members is the component resulting from friction. When the prestressing steel is tensioned to the desired tensile stress, the diameter of the strand contracts due to the effect of Poisson's ratio. After release the stress in the strand increases from zero at the end of the member to some constant level at the end of the transfer length. This variation in the tensile stress of the strand causes an expansion in the diameter of the strand in proportion to the reduction due to the initial tensile stress. This expansion is resisted by the concrete surrounding the strand, resulting in a radial force at the interface. This interface pressure between the strand and the concrete is for the most part responsible for the development of the frictional component of bond. However, at the member ends in the

area surrounding the reaction, the concrete is locally compressed due to the vertical force of the reaction. It is felt that this compressive force adds to the frictional component of the bond by increasing the radial pressure at the steel-concrete interface. This phenomenon may be termed the pinching effect of the reaction. In beams containing layers of strand, it is possible that bond slip may be more critical in the strands at the upper levels, since the pinching action is less effective at the higher level in the beam.

The nature of bond was reported in 1954 by Janney.⁹ Four sizes of prestressing wire and one size of prestressing strand (5/16-in.) were used in a study aimed at the evaluation of both transfer and anchorage bond characteristics. The principal variables considered were diameter, surface condition, and degree of initial pre-tensioning of the wire reinforcement. A variation in transfer length was noted for wires of various diameters, and it was found that the surface condition also played a major role. An elastic analysis of the deformations occurring when pre-tensioned steel is released, suggested that bond is largely a result of friction between concrete and steel.

In 1956, Thorsen¹⁴ showed that the bond forces in the end zones of a pre-tensioned member differs from the bond forces in the interior regions. It was further demonstrated that both types of bond can be determined by a curve indicating the maximum

tension which can be developed in a tendon, without slip, at various distances from the end of a member.

In 1957, Nordby and Venuti¹³ tested 27 beams cast from conventional and expanded shale aggregate concrete. The beams were tested both statically and in fatigue. The results indicated that the embedment length was the governing factor against failure rather than bond stress as computed from conventional equations. The equation

$$u_u = \frac{3f_s A_s}{4\pi D L_e}$$

was used to compute the average bond stress at the time of failure.

In 1958, a study by Dinsmore, Deutsch and Montemayor⁵ reported the results of an investigation of both transfer and flexural bond in test specimens pre-tensioned with 7/16-in. strands. It was concluded that friction played a major role in the development of both types of bond.

In 1959, Hanson and Kaar⁷ announced the results of a detailed investigation of flexural bond in beams pre-tensioned with seven-wire strand of 1/4, 3/8, and 1/2 in. diameter. The primary object was to find the effect of strand size and embedment length on the bonding action and strength of the member. It was found that bond slip occurs in pre-tensioned members when the wave of flexural bond stress reaches the prestress transfer length. A

series of curves were developed to predict the necessary embedment lengths for the various sizes of strand tested.

A study conducted at the University of Illinois was reported by Anderson, Rider, and Sozen³ in 1964. The investigation included a study of both anchorage and flexural bond in pre-tensioned prestressed members. Pull-out test specimens were fabricated to simulate both end block conditions and the tensile region of a beam. In addition several beams with non-prestressed strands were tested. The three types of strand used were: 7/16-in. round seven-wire strand, 7/16-in. rectangular seven-wire strand, and 1/4-in. rectangular three-wire strand. Embedment lengths and transfer lengths were determined, and it was emphasized that an additional axial stress may be developed in the strand after bond slip has occurred.

In 1965, Badalian and VanHorn⁴ at Lehigh University reported a study of the bond characteristics of 1/2-in. 270K seven-wire prestressing strand. The results of thirteen tests on twelve beams were evaluated to determine the embedment length necessary to produce bond slip. The critical observed embedment length for the strand tested was found to be 80 inches. An analytical concept was developed. However, because of lack of information on the development of friction, mechanical action and the coefficient of creep in concrete, a comparison between the experimental and analytical values was not possible.

To date, no investigation has been directed toward an attempt to isolate the added quantity of frictional bond brought about by the pinching effect of the reaction in pre-tensioned prestressed concrete members.

II. OBJECTIVE AND SCOPE

2.1 Objective

The principal objectives of this investigation were (1) the development of information on the effect of the pinching of the reaction on the flexural bond characteristics of prestressed concrete beams pre-tensioned with 1/2-in. 270K seven-wire strand, and (2) comparison of this information with the results obtained by Badaliane and VanHorn.⁴ A further objective was to generalize upon the mode of failure that can be expected in a pre-tensioned prestressed concrete flexural member, and also to demonstrate the post bond slip strength of such beams.

2.2 Scope

The bond characteristics of pre-tensioned prestressed members depend upon a number of characteristics such as:

- (1) size of the strand
- (2) ultimate strength of the strand
- (3) surface condition of the strand
- (4) concrete strength both at the time of release
and at the time of test
- (5) rate of release of the strands
- (6) spacing of the strands
- (7) steel percentage

In previous studies many of these variables have been investigated. However, since the principal objective in this investigation was the pinching effect of the end reaction in beams with more than one layer of strand, the vertical spacing of the strands and the embedment length were of maximum concern. Therefore, the test specimens had the following characteristics:

- (1) One size of 270K strand (1/2-in.) was used in all three specimens. The ratio of surface perimeter to cross-sectional area decreases as the size of the strand increases. Therefore, bond is most critical in members pre-tensioned with the largest size strand. The 1/2-in. size strand is the largest size currently used commercially. From a previous investigation by Hanson and Kaar⁷, it was found that rust or scale on the strand improves the bond characteristics of the strand. Therefore, a rust-free strand should represent the most critical case.
- (2) Five strands and one strand pattern were used in each of the test specimens. Of the five strands, analysis showed that bond would be critical in the four lower strands.

The four lower strands were placed in two layers, with one strand immediately above the other. The center-to-center vertical spacing was 2 inches, which is the minimum allowed by the current ACI Building Code.¹ The positioning of the strands was such that the difference in the strand stress between the lower two layers of strands at ultimate load of the test specimen was less than 0.5%.

- (3) The initial strand stress was constant for all specimens, that is 70% of the minimum ultimate strength which, for 270K strand, was 189 ksi.
- (4) The concrete strength was governed by two limits: (1) the release strength must be more than 4500 psi, and (2) the strength at time of test must not be more than 6000 psi.
- (5) From the results of a previous study by Badalian² and VanHorn⁴ it was shown that the critical embedment length was 80 inches. Therefore, an embedment length of 48 inches was chosen to ensure bond slip in the two

lower layers of strand, prior to a
flexural or shear failure.

The aforementioned characteristics were specified so as
to fulfill the requirements of the current specifications, and to
ensure a failure by bond slip in at least the lower level of strand.

III. TEST SPECIMENS

3.1 Description of Test Specimens

The three specimens were designed as under-reinforced beams such that a flexural failure would be initiated by yielding of the strand, followed by crushing of the concrete. In accordance with the objectives, the tests were designed to indicate (1) the value of the component of frictional bond that results from the pinching effect of the reaction, and (2) the amount of additional load that the beam may sustain after bond slip.

The specimens consisted of rectangular prestressed beams with a cast-in-place slab. The prestressed rectangular beam element was 7-1/2 inches wide and 12 inches deep, the slab was 6 inches deep and 20 inches wide. The beam width of 7-1/2 inches allowed a strand pattern that ensured the minimum cover requirements as specified by the current ACI Building Code.¹ The bottom layer of strand had a cover of 1-1/2 inches, the minimum allowed by the Code, so as to maximize the component of normal pressure at the interface of the strand and the concrete resulting from the pinching effect of the reaction. The depth of the beam section and the strand pattern were designed so that the maximum allowable prestress stress of the concrete was not exceeded. Five 1/2-in. 270K strands were used for prestressing the beam. The strands were located in three layers. Four of these strands were placed

near the bottom of the section, two strands in two layers with a vertical center-to-center spacing of 2 inches. The fifth strand was placed 2 inches from the top of the prestressed element and served to limit the prestress stress which was imposed upon the concrete. This strand pattern produced nearly equal stresses in the two lower levels of strand. The strands were initially tensioned to a stress of 70% of the specified ultimate stress which is 189,000 psi for 270K strand. Figure 1 includes a detailed sketch of the cross-section.

The test section was reviewed to determine to expected ultimate load and the associate failure characteristics. This series of calculations was carried out in accordance with the method presented by Janney, Hognestad, and McHenry.¹⁰ The ultimate moment was calculated using the factors, k_1 , k_2 , k_3 , and ϵ_u as presented by Hognestad, Hanson, and McHenry.⁸ The ultimate strength factors were expressed as a function of the concrete strength as follows:

$$k_1 k_3 = \frac{3900 + 0.35 f'_c}{3200 + f'_c}$$

$$k_2 = 0.50 - \frac{f'_c}{80,000}$$

$$\epsilon_u = 0.004 - \frac{f'_c}{6.5 \times 10^6}$$

Using these factors and a value of $F = 1.0$, the steel stress at ultimate flexural strength f_{su} was calculated by successive approximations. For beams containing more than one level of steel, the strain compatibility equations are:

$$\frac{c}{d} = \frac{F\epsilon_u \left(1 - \frac{a}{d}\right)}{\epsilon_{su_t} - \epsilon_{se} + F\epsilon_u} = \frac{F\epsilon_u \left(1 + \frac{a}{d}\right)}{\epsilon_{su_b} - \epsilon_{se} + F\epsilon_u}$$

The condition of equilibrium of forces leads to:

$$C = \frac{\sum \left[f_{su_n} A_{s_n} \right]}{k_1 k_3 f'_c b}$$

The flexural moment is given by the equation:

$$M_u = \sum \left[f_{su_n} A_{s_n} (d_n - k_2 c) \right]$$

Several trials were necessary to establish compatibility between the calculated f_{su} and the value of ϵ_{su} and f_{su} obtained from the stress-strain curve of the strand.

In the design calculation an estimate of the prestress losses was required. The components of losses which were included in the estimate were elastic shortening, shrinkage of the concrete, and creep in both the steel and concrete. The loss in steel stress

due to elastic shortening was determined by:

$$\Delta f_s = \frac{nF_i}{A_c + nA_s}$$

Lin¹² recommends that concrete shrinkage loss be calculated by:

$$\Delta f_{sc} = \epsilon_s E_s$$

where ϵ_s is equal to the unit shrinkage strain of the concrete. A value of $\epsilon_s = 0.0003$ was used in this study. The loss due to concrete creep was likewise calculated by the equation:

$$\Delta f_{cs} = (C_c - 1) n f_c$$

where the value of C_c , the creep coefficient, was assumed equal to 4.

The loss due to creep in the prestressing steel was similarly computed by:

$$\Delta f_{ss} = K_o F_i$$

where K_o is equal to three percent. Table 4 is a comparison between the measured and the calculated values of prestress losses.

3.2 Materials

Prestressing Steel

The prestressing steel used was 1/2-in., 270K, seven-wire, uncoated, stress relieved type strand. This type of strand is commercially manufactured and tested in accordance with ASTM designation A416-64. The physical properties of this strand are given in Fig. 3 and the load-elongation curves are shown in Fig. 4. Further information concerning the static and fatigue properties of this strand is given in a recent report by Tide and VanHorn.¹⁵ Although this type of strand is commercially available from several manufacturers, the strand used in this investigation was produced by John A. Roebling's Sons Division of the Colorado Fuel and Iron Corporation.

Shear Reinforcement

The shear reinforcement was fabricated from No. 3 deformed bars having a nominal yield stress $f_y = 50,000$.

Concrete

The concrete strength was not a variable in this investigation. The concrete used in the beam element was designed to yield an ultimate strength $f'_c = 6000$ psi, at an age of 21 days.

The mix chosen consisted of Type III (high-early strength) portland cement, sand, and crushed limestone coarse aggregate (3/4-in. maximum). The proportions of the mix, by weight, (cement-to-sand-to-coarse aggregate), were 1.00:2.64:2.98. The concrete was supplied by a local ready-mix plant and was delivered in 1-1/2 cubic yard batches. Although the concrete used in casting the slabs was of the same design as that used in the webs, its slump was 2-1/2 inches compared to a slump of 3-1/2 inches for the beams. This difference in water content yielded a higher 21-day compressive strength for the slabs than for the beams.

The three rectangular prestressed beams were all cast from one batch of concrete, and all of the slabs from another. Compression tests were conducted on 6 x 12 in. cylinders, which were cast along with each batch of concrete, to determine the compressive strength f'_c associated with the test beams at the time of prestress release and at the time of test. Strains were measured on selected cylinders with a compressometer to determine the shape of the stress-strain curve and the modulus of elasticity of the concrete at the time of test. As a measure of the tensile strength of the concrete splitting tensile tests were conducted on standard 6 x 12 in. cylinders. Strips of plywood approximately 1/8-in. thick, 1-in. wide, and 12 inches long were placed on the diametrical upper and lower bearing surfaces in the splitting test. The splitting cylinder tensile stress f'_{sp} was determined by the equation:

$$f'_{sp} = \frac{2P}{\pi d_c L}$$

The age and strength properties of the concrete described in the preceding paragraphs are presented in Table 2.

3.3 Fabrication

The beams were fabricated in a prestressing bed at the Fritz Engineering Laboratory. The sequence of operations was as follows: the strands were tensioned, strain gages were attached to the strands, lead wires were soldered into place, the shear reinforcement was positioned and wired, forms were erected, the beam concrete was placed and cured, the beam forms were removed, Whittemore targets were installed, the strands were released, the slab forms were set in place, the slab concrete was placed and cured, the slab forms were removed.

The prestressing bed has been described in previous Fritz Engineering Laboratory reports.¹⁶ The bulkheads of the prestressing bed were spaced 40 feet apart and bolted to the laboratory floor. The three beams were cast simultaneously using the same assemblage of strands and three individual steel forms. The prestressing strands were held in the chosen pattern with two one-inch thick anchorage plates, one at each end of the prestressing bed. The projecting ends of the strands were secured by strand chucks.

The tension was applied to the prestressing strands by jacking the moving bulkhead, using two 50-ton hydraulic jacks. The strand tension was measured for each strand individually by placing a load cell between the anchorage plate and the strand chuck at the stationary end of the prestressing bed. If required, the tension in individual strands was adjusted by means of a special hydraulic jacking arrangement. After pre-tensioning the strand, the strain gages were mounted on the strand.

The shear reinforcement was tied to the strand with 14 gage wire. In addition, wire ties were used between successive projecting elements of the stirrups in the compressive flange area, in order to prevent movement of the stirrups during the placement of the concrete.

Steel forms made of No. 7 gage steel plates were used to cast the test beams.

Dimensional checks, made after the forms had been removed, indicated that the cross-sectional dimensions were maintained to within 1/16 in. and consequently, the nominal dimensions of the cross-section were used in all calculations.

The concrete was brought from the ready-mix truck to the forms in steel buggies, and shoveled into the forms. The concrete was placed in two layers, and the tops of the beams were left unfinished. Ten standard concrete cylinders were cast with each beam. Waxed cardboard molds were used.

The concrete in the test specimen was internally vibrated, while the cylinders were rodded.

All specimens were covered with burlap and plastic sheeting for a period of two days, after which the forms were removed. After the surface of the test beams had become air dry, Whittemore targets were positioned on the beams. When cylinder tests indicated that the specimen's concrete compressive strength had reached a strength of 4500 psi, the prestress force was slowly released.

An oxy-acetylene torch was used to cut the strands. The beams were then removed from the casting bed, and stored in the laboratory.

Wood forms were used for fabrication of the slabs. The concrete was placed in one layer and allowed to cure under the sheets of plastic and burlap for one week. At that time, the forms were removed and the beams were air-cured until the time of test.

3.4 Instrumentation

In order to determine the behavior of the beams, data was recorded in the form of strains, strand movement and deflections.

3.4.1 Strains

Load-deformation data was measured in two fashions (1) strain gages mounted at various locations on the strands, and (2) strain gages mounted on the concrete surface.

Load-deformation data on the strand was measured with SR-4 electrical resistance strain gages, A-12 type, mounted on the lower four strands of all three beams. The gages were mounted upon completion of pre-tensioning, in order to increase the effective range of strain measurements. The gages were located on the instrumented strands at 20-in. intervals starting 20 inches from the ends of the members. One gage was attached to an individual wire at each gage location. Before the strain gage was mounted the strand was cleaned using emery cloth and acetone. The gage was glued to the strand using Duco-Cement. The waterproofing consisted of a coating of Armstrong adhesive A-16, followed by a layer of liquid rubber. The location of these strain gages can be seen in Fig. 5.

The primary objective in positioning strain gages on the prestressing strand, was to determine the change in strand force as load was applied to the beam. In addition, these strain gages were used to measure the elastic shortening of the strand at the time of release of the pre-tensioning force. The strain data received from the gages mounted on the strands was converted to force with a calibration curve. This plot of strain

vs. strand force was determined by averaging strains measured by three gages mounted on individual wires of a strand sample. Figure 4 compares the calibration curve to the load-deformation curve determined by Tide and VanHorn.¹⁴

The load-deformation data of the concrete surface was measured (1) with a 5-in. Whittemore gage, and (2) with SR-4 gages located around the cross-section at midspan of the test specimens.

- a. Whittemore strain gage data was used to determine concrete surface strains at a level midway between the lower two layers of prestressing strand. The Whittemore targets were fabricated from brass plugs, 7/32-in. in diameter and 1/16-in. in thickness. These targets were center-drilled and cemented at the prescribed level on both sides of the beam, after the forms were removed but before release of the pre-tensioning force. The purpose of the Whittemore data was to determine the transfer length, and the loss of prestress at various intervals up until the time of test. These gage readings were also used as an indicator of initial cracking of the concrete.

b. Twelve SR-4 gages, A-9 type, were mounted on the cross-section at midspan of the beam. Figure 5 shows the placement of these gages. The strain information from these gages was used to determine the location of the neutral axis as cracking became severe, and the maximum concrete strain at ultimate load.

3.4.2 Strand Slip

Strand slip was measured by the use of Ames dial gages with a least count of 1/10,000-inch. These dial gages were attached to a steel mounting bracket that clamped to the rectangular beam portion of the specimens. The gages indicated the movement of four small steel plates that were attached to the four lower strands by means of collars and set screws. (See Fig. 16)

3.4.3 Deflection Measurement

The midspan deflection was measured by level readings on scales graduated to the nearest 0.01 inch. These scales were located at each of the supports and at midspan.

IV. TESTS

4.1 Test Procedure

Three specimens were tested in the 300 kip hydraulic testing machine located in the Fritz Engineering Laboratory. The loading beam and associated apparatus were arranged to provide a symmetric two-point loading for all three specimens. The shear span was 42 inches and the overhang was 6 inches, resulting in a total embedment length of 48 inches. A sketch of the testing arrangement is shown in Fig. 2. The test specimens in all cases were initially loaded in increments of 10 kips, which was approximately 8 percent of the computed ultimate load. When cracking of the concrete became visible, the loading increment was reduced to 5 kips until failure occurred. The internal A-12 type, SR-4 gages mounted on the individual wires of the strand were used in conjunction with two different types of recording equipment. Six gages of Beam Z-1 were connected to channels of a Brush direct-writing recorder to provide a continuous record of the variation of the load in the strands. The remaining internal gages, as well as the A-9 type, SR-4 gages mounted on the surface of the concrete at the midspan cross-section, were connected to a Budd Datran digital strain indicator. All electric resistance strain gages were read following the application of each increment of load. A review of the test results from

Beam Z-1 revealed that little was gained from the use of the Brush recorder. Therefore, in the remaining tests the instrument was not used.

The Whittemore targets mounted on the surface of the concrete were read at selected load increments until cracking became severe. Midspan deflection readings were taken at each load increment until failure. The strand-slip dial gages were read continuously to detect initial bond slip. The development of the crack pattern was marked on the surface of the specimen after each increment of load had been applied. After failure the specimens were photographed.

4.2 Test Results and Discussion

4.2.1 Modes of Failure

In this investigation failure in all three test specimens has been associated with flexure. The tensile reinforcement initially yielded, followed by the eventual crushing of the concrete. However, prior to yielding of the reinforcement, bond slip was experienced in the four lower strands in all three tests. The failure mechanism associated with bond slip is closely related to the cracking of the concrete.

Cracking of the Concrete

In this study two basic types of cracking were exhibited, flexural and flexural-shear cracking.

Flexural cracking occurred in the high moment region of the test specimens when the stresses in the bottom fibers reached values which are normally associated with the tensile strength of the concrete. Flexural cracking was characterized by the initial development of vertical cracks to a level which varied between the lower and upper* strand levels. The longitudinal spacing of these flexural cracks was approximately 6 inches. However, cracks which formed closer together than approximately 2 inches would usually merge, or the further development of one of the two cracks would be prevented.

Flexural-shear cracking always followed flexural cracking in the test specimens. The flexural-shear crack was always initiated by a flexural crack that occurred in the region of the shear span. This flexural crack would initially develop to the level of the first or second layer of strand, and then become inclined toward the direction of increasing moment. A flexural-shear crack differs from a diagonal crack, in that a diagonal crack is caused by principal tensile stresses that developed in either the web of an I-beam or in the stem of a T-beam.

These diagonal tension cracks, as a general rule,

*In the remainder of this discussion the words upper strands shall refer to the strands found in the second layer from the bottom. The lower strands refer to the bottom level of strands.

develop near the c.g. of a section and propagate at some angle toward the extremities of the cross-section.

Behavior of Test Specimens

The value of prestress losses, effective prestress force, and prestress transfer length for each specimen is given in Table 3. The average transfer length for the 1/2-in. 270K strand was 27 inches. The average prestress force immediately after transfer was 25.45 kips, and the average effective prestress force at the time of test was 20.82 kips. These values are compared in Table 4 with the theoretically computed values. The individual values of effective prestress force for each specimen was used in conjunction with the ultimate strength of the concrete obtained from the cylinder tests given in Table 2, to compute the value of the ultimate flexural capacity of the specimens. The computed value of the ultimate moment, M_u , for each beam is given in Table 5.

During the testing period flexural cracking developed at a load of 65 kips in all three specimens. This corresponded to a computed tensile stress at the lower extremity of the cross-section of 630 psi. In comparison the average tensile stress calculated from the splitting cylinder tests was 554 psi. It should be noted that the cracking loads of the specimens were determined visually. It can safely be assumed that cracks not visible to the eye opened at a lower load and corresponding stress

than were reported. This flexural cracking was in all cases confined to the maximum moment region of the specimens. As the load was further increased, flexural cracks developed in the shear spans. These cracks developed to a level midway between the two lower levels of strand. Upon additional loading these flexural-shear cracks became inclined and progressed towards the maximum moment region of the beam. It is interesting to note that flexural-shear cracking progressed to within 40 inches of the member's end at the time of bond slip. Photographs of the crack patterns are shown in Figs. 9, 10, and 11.

Previous investigations by Badalian⁴ and VanHorn,⁴ and Hanson and Kaar⁷ have shown that when the wave of high bond stress, which results from cracking of the concrete, reaches the prestress transfer zone, bond slip occurs. Unfortunately, in this study, the strain gages that were mounted on the individual wires of the prestressing strands failed to yield consistent measurements after severe cracking occurred. In most cases the gage reading lost coherency prior to bond slip, this is attributed to the probable shearing of the gages when the wave of high strain reached a gage point.

The behavior of Beam Z-1 was very interesting. Bond slip initially occurred at a load of 92 kips in one of the strands in the upper layer, at the east end of the member. At a load of 100 kips bond slip was exhibited in both upper level strands; at

both ends of the specimen. When the load reached 105 kips all four strands had slipped. This sequence of strand slip occurred even though the strand stress and average bond stress in the lower level of strand was higher than in the upper level. It is probable that the additional bonding action displayed in the lower level strands is a result of the pinching effect of the reaction and was responsible for this bond slip sequence. The values of strand stress at bond slip and ultimate load were determined using the cross-sectional strain distribution as shown in Figs. 6, 7, and 8. In each case, the maximum concrete strain, as well as the location of the neutral axis of the cracked section was determined from the center line strain distribution as measured by the external electric strain gages. A straight line strain distribution was then assumed, allowing the calculation of the strand strains at the various levels within the beam. These strand strains were then converted into strand force using the load-elongation curve shown in Fig. 9.

Ultimate failure of Beam Z-1 occurred at a load of 126 kips. The mechanism responsible for failure was crushing of the concrete in the compression slab. At a load of 120 kips, diagonal shear cracks developed at approximately the middle of the shear span at the west end of the member. With increase of load, the crack elongated. At failure the crack extended from inside the west reaction to approximately 6 inches from the west

load point. Final failure occurred without additional load, and was the result of a crushing failure of a wedge of concrete adjacent to the west load point.

Behavior of Beams Z-2 and Z-3 were similar to that of Beam Z-1, except that there was no visible distinction between the load at which bond slip occurred in the lower and upper strand levels. In Beam Z-2 bond slip initially occurred in one of the lower-level strands at an applied load of 97 kips, and both of the upper-level strands on the east end of the member. At a load of 100 kips, all of the strands on the east end of the member slipped. Bond slip also occurred simultaneously in all strands at the west end of the member. In Beam Z-3 simultaneous slipping of all strands occurred at the east end of the member when a load of 92 kips was applied. When the load was increased to 99 kips, bond slip occurred in all of the strands at the west end. Again, it must be pointed out that even though both lower and upper strands displayed bond slip at the same load in specimens Z-2 and Z-3, the strand force in the lower-level strands was higher than in the upper-level strands. The values of strand stress at bond slip are given in Table 5. The high strand stress in lower-level strands yielded a higher average bond stress, which has been attributed to the pinching effect of the reaction.

Figure 12 compares the strand stresses at bond slip as determined in this investigation with similar stresses found in a

recent study by Badaliane and VanHorn.⁴ In this figure the strand stress is plotted against embedment length, all three specimens failed in a similar fashion, and the ratio of P_{ult}/P_u for specimens Z-1, Z-2, and Z-3 were 1.01, 1.04, and 0.93, respectively.

4.2.2 Load-Deflection Curves

The general characteristics of the test beams are shown by the load-deflection curves in Fig. 13. The three curves represent the midspan deflection from each beam as the load was applied. The small variation between the three curves is a further conformation of the consistency of the fabrication and testing procedure.

The initial portion of the curves up to approximately one-half of the ultimate load, are linear, corresponding to the uncracked loading range of the test beams. The sharpest change in slope in the load-deflection curves occurs just after flexural cracking, which marks the transition from the uncracked to the cracked loading range. Following the transition region, the load-deflection curve became quasi-linear to failure.

The load at which flexural cracking occurred was approximately 65 kips for all specimens, and marks the end of the linear region on the load-deflection curves.

Inclined cracking did not cause any abrupt change in the slope of the load-deflection curve. It is evident from the load-deflection curve that the presence of web reinforcement allowed the beams to withstand a greater deflection. This latter characteristic is particularly important because it is a measure of the ductility of the member.

4.2.3 Force in the Strand at Various Stages

As previously stated the strain gages mounted on the individual wires of the prestressing strands were intended to measure the strand force along the member at various loads. However, these gages failed to yield consistent results. Figure 14 is a representation of the expected strand force distribution, plotted along one-half of a member, as shown in a recent study by Badaliane and VanHorn.⁴ During the fabrication of a prestressed member, the strand is tensioned to some initial value of stress, f_{si} . After release the stress in the strand increases from zero at the end of the member to some constant level, $(f_{si} - \Delta f_s)$ at the end of the prestress transfer length, L_t . This curve is expressed by the dashed line in Fig. 14. During the time interval between release of the prestress force and testing of the specimen an additional loss of prestress takes place as a result of creep and shrinkage. The resulting curve ($P = 0$) shown in Fig. 14, parallels the $(f_{si} - \Delta f_s)$ curve at a lower stress level. When the specimen is loaded, but not stressed

sufficiently to cause cracking, the stress in the strand in the area adjacent to midspan of the specimen increases a small amount. This is represented by the (P Prior to Cracking) in Fig. 14. A further increase in load causes the section to crack, resulting in a high strand force near the center line. Additional loading causes the high strand force to propagate towards the end of the beam. This condition is represented by the (P at Ultimate) curve in Fig. 14. When the high strand force reaches the prestress transfer, length bond failure usually results.

It should be pointed out, from the experimental study by Badaliane and VanHorn,⁴ that for all specimens in which bond slip occurred, the stress in the strand at the end of the transfer length had reached the magnitude $(f_{si} - \Delta f_{cs})$. In specimens which failed in flexure prior to bond slip, the stress in the strand at the end of the anchorage length had not reached $(f_{si} - \Delta f_{cs})$. In conclusion, a simple check as to whether bond slip will occur prior to flexural failure would involve a computation of the stress in the strand at the end of the transfer length. If the computed value of stress is less than $(f_{si} - \Delta f_{cs})$, bond slip will not occur at that load. If the computed value of stress is greater than $(f_{si} - \Delta f_{cs})$, bond slip will occur.

4.2.4 Average Bond Stress

In pre-tensioned prestressed concrete, the stress in the steel is maintained by bond. Bond stresses are divided into two

parts: (a) anchorage bond that retains the prestress in the strand, and (b) bond due to flexure. The mechanism which is associated with bond failure has already been described. However, the calculation of bond stress is very important.

Both anchorage bond and flexural bond may be calculated from a series of curves similar to those shown in Fig. 14. According to Thorsen,¹⁴ the slope of the curve of strand force vs. length along the beam curve serves as an index of bond stress since

$$u_x = \frac{dF_s}{dx} \left(\frac{1}{a}\right)$$

where u_x = bond stress at a distance x from end of member
 a = circumference of reinforcement
 F_s = force in strand

This method is impractical since it requires that the appropriate curves be constructed for each individual case.

Flexural bond may be calculated, according to Nordby and Venuti,¹³ for an uncracked section by

$$u_f = \frac{n e_s A_s V}{\Sigma O I_c}$$

and a cracked section by

$$u_f = \frac{V}{\Sigma o_j d} \times \frac{\text{Increase in tensile stress}}{\text{Prestress} + \text{increase in tensile stress}}$$

Since distance jd cannot be computed, it must be found by actual measurement of the cracks on the sides of the member, or from cross-section strain data.

After many careful studies, the bond stress calculated in the above fashion yielded values which were extremely low in relation to those calculated for ordinary reinforced concrete members. Nordby and Venuti¹³ point out that bond stresses at bond slip have been found to be as low as 3.8 psi. In general, it seems that no limiting value of bond stress can be related to slip, therefore designing with a bond stress limitation in mind will not insure against a bond failure.

The best approach has been to specify an embedment length in order to make it possible to develop the ultimate strength of the strand at failure. When using the embedment length approach, it has been found that the calculation of an average bond stress is most convenient. The average bond is easily computed by

$$u_a = \frac{f_s A_s}{L_e a}$$

where f_s = the stress at the end of the embedment length at

the time of failure. The ratio a/A_s is given in Fig. 3, where a is the circumference of the strand. For 1/2-in. 270K strand, this ratio is equal to 13.95. When this ratio of 13.95 is substituted into the above equation we obtain $u_a = 0.0717 f_s/L_e$. The values of average bond stress for the three specimens tested are given in Table 5. It is clearly seen that the average bond stress is higher in the lower-level strands than in those of the upper load. This is further proof that the pinching of the reaction has contributed additional bond to the lower-level strands.

Figure 15 is a comparison of the average bond stresses found in this investigation with values determined in a study by Badaliane and VanHorn,⁴ for 1/2-in. 270K strand, plotted against embedment length. It is apparent from this plot that an embedment length of 80 inches is necessary to develop the ultimate strength (270,000 psi) of the strand.

4.2.5 Post Bond Slip Load Capacity

In prestress members pre-tensioned with strand, the mechanical component of bond is primarily responsible for the post bond-slip behavior. The results of this investigation are shown in Table 5, and indicate that bond slip occurred at an average load which was 76% of the ultimate load of the member. This points out that a bond failure in a member pre-tensioned with strand will not result in a sudden and catastrophic collapse of the beams. This reserve strength could possibly be included in future design criterion.

V. SUMMARY AND CONCLUSIONS

The objective of this investigation was to develop preliminary information on the pinching effect of the end reaction on the flexural bond characteristics of simply supported prestressed concrete beams pre-tensioned with 1/2-in. 270K, seven-wire strand.

Three specimens were fabricated and tested. The specimens were designed as under-reinforced beams which would experience bond slip prior to flexural failure. The specimens performed as predicted, and it was clearly indicated that additional flexural bond is developed as a result of the pinching effect of the reaction in the prestressing steel near the soffit of the member. The average bond stress at bond slip was distinctly higher in all cases for the lower layer of strand than for the upper.

A comparison was made between the average bond stress determined in this investigation and that determined in an earlier investigation by Badaliane and VanHorn.⁴ The similarity served to verify the results of the earlier investigations.

A further objective was to demonstrate the post bond slip strength of prestressed members pre-tensioned with strand. It was found that bond slip occurred at an average load which was

76% of the ultimate load. This points out that a bond failure in such a member will not necessarily result in a catastrophic collapse of the beam.

An attempt was made to generalize upon the various possible failure modes of pre-tensioned prestressed members. The purpose of this discussion was to serve as a reminder to the designer that all possibilities of failure must be given proper consideration.

In essence, this investigation was a pilot study in the respect that three identical specimens were tested. Since the scope of this study was limited, it was impossible to develop a valid method of accurately predicting the contribution of the pinching effect of the reaction toward the development of flexural bond. It is recommended that a more extensive study be made in the future, encompassing several variables such as vertical strand spacing, strand pattern, and unbonding of the strands.

VI. GENERAL DESIGN CONSIDERATIONS

In general, there are three primary modes of failure that are considered in evaluating the behavior of a pre-tensioned prestressed concrete flexural member:

1. Flexure - usually characterized by yielding of tensile reinforcement, and the eventual crushing of concrete.
2. Shear - characterized by either severe diagonal tension cracks, or by flexural-shear cracks.
3. Bond slip - the pulling in of the strands at the ends of the member.

In the final review of the member, the designer should be well acquainted with the various possible modes of failure.

According to Libby¹¹ and others, prestressed flexural members which are stronger in shear and bond than in bending, may fail in one of the following modes when loaded to ultimate:

1. Failure at cracking moment: In very lightly prestressed members, the cracking moment may be greater than the moment which the member can withstand in the cracked condition. Therefore, the cracking moment is the ultimate moment, and brittle failure will result. This form

of failure is possible in members which have a very small steel percentage, for example, concentrically prestressed members.

2. Failure due to rupture of the steel: Lightly prestressed members loaded to ultimate may fail as a result of the steel reaching its ultimate strength before the concrete has attained a highly plastic state.
3. Failure due to strain: The usual prestressed structures which are encountered are proportioned such that, if loaded to ultimate, the steel would be stressed well past the yield stress and the beam would experience a large deflection. The resulting strain in the concrete would eventually reach a limit whereby the final failure would be crushing of the concrete. A member which behaves in this manner is said to be under-reinforced.
4. Failure due to crushing of the concrete: An over-reinforced flexural member is classified as one which contains a relatively large amount of prestressing steel, and fails as a result of crushing of concrete prior to yielding of the tensile reinforcement. This type of failure is catastrophic in nature, since it occurs without excessive deflections.

It must be emphasized that there is no clear distinction between the different classes of flexural failures listed above. For convenience of design, certain ratios, such as the percentage of steel and the steel index, are used to distinguish between the different failure mode possibilities. For a rectangular beam, the steel percentage is expressed as $p = A_s/bd$, and the steel index as

$$q' = \frac{A_s f_{su}}{b d f'_c} = p \frac{f_{su}}{f'_c}$$

The limitation set by the ACI Building Code,¹ and corresponding Commentary,² on the reinforcing steel index is

$$p \frac{f_{su}}{f'_c} = 0.3$$

This value is taken as the dividing line between under-reinforced and over-reinforced members. This provision is typical, and appears in similar form in most current design specifications.

Shear is not a problem in prestressed concrete beams until inclined cracking occurs. When inclined cracking does occur, the behavior of the member is completely changed. Additional load carrying capacity is then dependent upon the amount of web reinforcement provided. If shear is critical, inclined cracking leads to a shear failure which may occur in several different ways.

It has been stated in a recent report by Hanson and Hulsbos¹⁶ that the type of shear cracking is primarily dependent upon the distance of the cracking from the load point, and the ratio of shear span to effective depth. In the study, diagonal tension cracking occurred in specimens where the a/d ratio was less than 3.5. Flexural-shear cracking occurred in test members

where the a/d ratio was greater than 5.0. Specimens where the a/d ratio was between 3.5 and 5.0, the inclined cracking had characteristics which, in different tests, were associated with either diagonal tension or flexural-shear cracking. The maximum principal tensile stress responsible for inclined cracking occurred close to the c.g. of the section, and the slope of the path of the crack was closely associated with the slope of the compressive stress trajectory at the c.g. For short spans, less than approximately twice the total depth of the beams, the magnitude of the normal stresses in the vertical direction at the c.g. influenced the state of stress. These vertical stresses delayed the formation of diagonal tension cracks.

The ultimate shear that can be carried by a beam at any cross-section is the sum of the portion of shear carried by the concrete plus the portion sustained by the web reinforcement. The ACI Building Code¹ and Commentary² states that the ultimate shear capacity of a prestressed flexural member can be taken as:

$$V_u = \phi \left[V_c + \frac{A_v f_y d}{S} \right]$$

where V_c = shear carried by the concrete

ϕ = strength reduction coefficient = 0.85

The value of V_c which must be determined depends upon the type of cracking. If flexural shear cracking occurs, then

then for normal weight concrete,

$$V_c = 0.6 b'd \sqrt{f'_c} + \frac{M_{cr}}{\frac{V}{d} - \frac{d}{2}} + V_d$$

but not less than $1.7 b'd \sqrt{f'_c}$

where

$$M_{cr} = \frac{I}{y} (6 \sqrt{f'_c} + f_{pe} - f_d)$$

If diagonal tension cracking occurs, then

$$V_c = b'd (3.5 \sqrt{f'_c} + 0.3 f_{pc}) + V_p$$

It must be noted that since the type of inclined cracking that will occur cannot be assured in all cases, the Code specifies that the smallest value of V_c be chosen from the aforementioned equations.

The characteristics and mechanism associated with bond failure have already been discussed in detail. The subject of bond is now discussed from two points of view (a) analysis, and (b) design.

- a. Analysis: When a specimen is tested in the Laboratory, the ultimate load

and mode of failure will depend upon the position of the load points. For example, consider a pre-tensioned prestressed member which is loaded with a concentrated load placed in the center of the span. Assume that the embedment length for the described loading is sufficient to enable the strand to develop its ultimate strength. The ultimate load of the member will depend upon the quantity of prestressing steel and the geometry of the cross-section. The resulting failure will then be either a result of a fracture of the prestressing steel, or a crushing of the concrete. However, if a similar specimen is loaded such that the concentrated load is within the embedment length necessary to develop the ultimate strength of the strand, the effective embedment length is reduced to a value equal to the distance from the member's end to the load point. The resulting failure will there result from bond slip, and the ultimate load will be limited by the force attained in the prestressing steel prior to bond slip.

b. Design: In the design procedure for a pre-tensioned flexural member, the working loads are initially calculated and then multiplied by an appropriate load factor to determine the ultimate moment which the section must be able to resist. From the moment diagram, the designer is able to determine the strand stresses required at ultimate moment at various locations along the length of the member. Of particular importance are the strand stresses that are to exist within the embedment length. If these strand stresses fall below the curve shown in Fig. 12 for the corresponding embedment length, the member can be considered safe against a failure by bond slip. If the stresses are above the curve, then bond slip will occur before the required ultimate moment is reached.

Another factor which should be considered stems from the results of this investigation. For members with multiple layers of strand, the embedment length necessary to develop a specific stress in strands placed some distance above the soffit of the beam will probably increase due to a decrease in the

component of frictional bond caused by the pinching of the reaction. Consequently, it is felt that additional research work should be aimed at a quantitative evaluation of the increases in embedment length as affected by the height of the strand above the end reaction.

VII. ACKNOWLEDGMENTS

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VIII. NOTATION

a	circumference of the strand
A_c	cross-sectional area of the beam
A_s	cross-sectional area of the tensile reinforcing steel
A_{s_n}	cross-sectional area of the tensile reinforcing steel in the "n" layer of strand
A_v	area of vertical reinforcing steel
b	width of slab
b'	width of web
c	depth of compression zone
c.g.	center of gravity of beam cross-section
C	horizontal component of the resultant compressive force in the concrete
C_c	creep coefficient
d	effective depth
d_c	diameter of concrete test cylinder
d_n	depth to the "n" layer of tensile reinforcing steel
D	nominal diameter of strand
e_s	eccentricity of a particular strand with respect to the transformed cross-section
E_c	modulus of elasticity of concrete
E_s	modulus of elasticity of steel

f_c	stress in the concrete
f'_c	ultimate compressive strength of concrete
f_d	stress due to dead load
f_{pc}	compressive stress in concrete due to prestress, after all losses, at the centroid of the cross-section
f_{pe}	compressive stress in concrete due to prestress, after all losses, at the extreme fiber of a section at which tension stresses are caused by applied loads.
f_s	steel stress
f_{si}	initial prestress in the strand
f'_{sp}	splitting tensile strength of concrete
f_{su_n}	stress in the "n" layer of tensile reinforcing steel at ultimate moment
f_y	nominal yield stress of shear reinforcement
F	$\frac{\epsilon_{su} - \epsilon_{se}}{\epsilon_{cu}}$
F_i	total initial prestress before release
I	moment of inertia of section resisting applied moment
I'_c	moment of inertia of the transformed section
jd	internal couple moment arm in a beam section
k_1, k_2, k_3	ultimate strength factors
L	length of concrete test cylinder
L_e	embedment length
L_s	length of shear span

M	moment due to applied loads
M_{cr}	moment due to applied loads when flexural cracking occurs
M_{slip}	moment due to applied loads at bond slip
M_u	calculated ultimate flexural moment
n	modular ratio
P	steel percentage
P_{slip}	measured load at bond slip
P_u	computed ultimate load
P_{ult}	measured ultimate load
q'	steel index
S	spacing of stirrups
u_a	average bond stress
u_f	bond stress due to flexure (uncracked section)
u_u	total bond stress at failure
V	total shear
V_c	shear carried by concrete
V_d	shear due to dead load
V_p	vertical component of effective prestress force at section considered
V_u	shear force due to specified ultimate load
y	distance from the centroidal axis of the section resisting the applied loads to the extreme fiber in tension

Δf_{cs}	loss of prestress due to creep of concrete
Δf_{sc}	loss of prestress due to shrinkage
Δf_s	loss due to elastic deformation at release
Δf_{ss}	loss of prestress due to creep of prestressing steel
ϵ_{cu}	tensile strain in the concrete level of the strand at ultimate moment
ϵ_{se}	tensile strain in the strand due to effective prestress
ϵ_{su_b}	tensile strain in the strand at ultimate moment (bottom layer of strand)
ϵ_{su_t}	tensile strain in the strand at ultimate moment (top layer of strand)
Σo	sum of the perimeters of strand in a beam based on a periphery of $4\pi D/3$ for each strand
ϕ	capacity reduction factor

IX. TABLES

Table 1 Computed Ultimate Moment Capacities of Test Specimens

Beam No.	f'_c Beam psi	f'_c Slab psi	$k_1 k_3$	k_2	$\epsilon_u \times 10^6$	f_{se} ksi	$\epsilon_{se} \times 10^{-3}$	f_{su} ksi	M_u k-in.
Z-1	6150	6740	0.63	0.416	2.96	137.6	4.66	271.9	2639.8
Z-2	5970	6750	0.63	0.416	2.96	137.6	4.66	271.8	2642.9
Z-3	5960	6980	0.62	0.413	2.93	133.1	4.51	271.8	2652.6

Table 2 Concrete Properties

Beam No.	Prior to release		At time of test								
			Beam					Slab			
	Age, days	f' _c psi	Age, Days Beam Test	Age, Days Cylinders	f' _c psi	f' _{sp} psi	E _c * ksi ₃ x10 ³	Age, Days Beam Test	Age, Days Cylinders	f' _c psi	f' _{sp} psi
Z-1	3	4630	36	35	6150	580	4.19	19	18	6740	602
Z-2	3	4590	45	44	5970	500	4.19	29	28	6750	579
Z-3	3	4520	59	57	5960	584	3.92	42	40	6980	652
Avg.		4580			6030	554	4.10			6820	601

*Initial Slope of Load-Deflection Curve

Table 3 Measured Prestress Losses

Beam No.	*Initial Stress, kips	Elastic loss at release, kips	Stress after release, kips	Percent loss after release	Add. loss prior to test, kips	Total Loss, kips	Total Loss, percent	Effective Stress, kips	Transfer Length, inches
Z-1	28.90	3.38	25.52	11.7	4.47	7.85	27.2	21.05	27
Z-2	28.90	3.52	25.38	12.2	4.33	7.85	27.2	21.05	27
Z-3	28.90	3.38	25.52	11.7	5.15	8.53	29.5	20.37	25

*Prior to placing of concrete

Table 4 Losses in Prestressing Strand

Losses	Elast. Shortening kips	Shrinkage kips	Creep, Steel and Concrete kips	Total Loss kips	Initial Stress kips	Stress after Release kips	Stress at Test kips
Theoretical	1.53	1.26	5.02	7.81	28.9	27.4	21.11
Measured Average	3.43	4.36		8.08	28.9	25.45	20.82

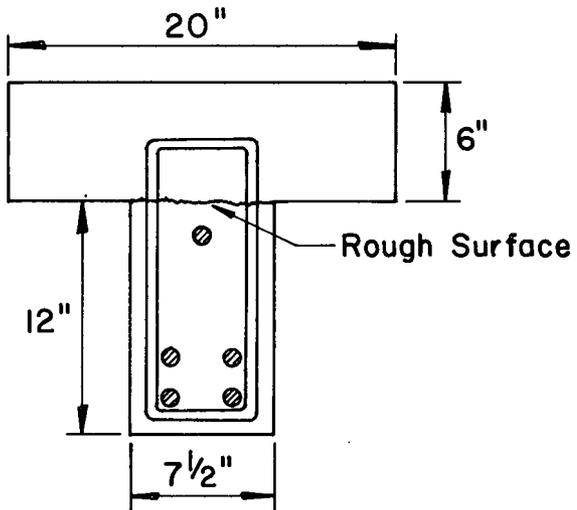
Table 5 Summary of Test Results

Beam No.	P_{ult} at crushing of concrete kips	P_{slip} at Bond Slip kips		M_{slip} at Bond Slip k-in.		M_u k-in.	Average Bond Stress at Bond Slip psi		Maximum Strand Stress at Bond Slip ksi	
		Upper Strands	Lower Strands	Upper Strands	Lower Strands		Upper Strands	Lower Strands	Upper Strands	Lower Strands
Z-1	126.5	95	100	1995	2100	2640	308	341	207.2	228.8
Z-2	131.0	97	97	2037	2037	2643	327	343	219.6	230.1
Z-3	117.5	92	92	1932	1932	2652	302	318	202.6	213.7

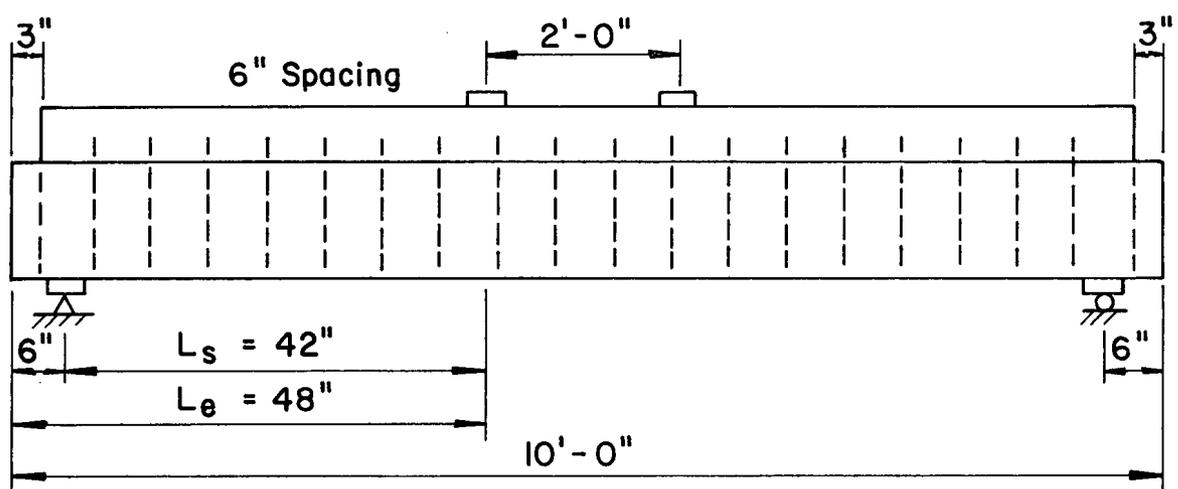
Note: Bond slip occurred prior to failure.

X. FIGURES

Prestress Stress Distribution



Strand: Type 270 k, 1/2" - Inch
 Shear Reinforcement: No. 3 Deformed Bars



Location of Stirrups

Fig. 1 Dimensions of Test Specimens

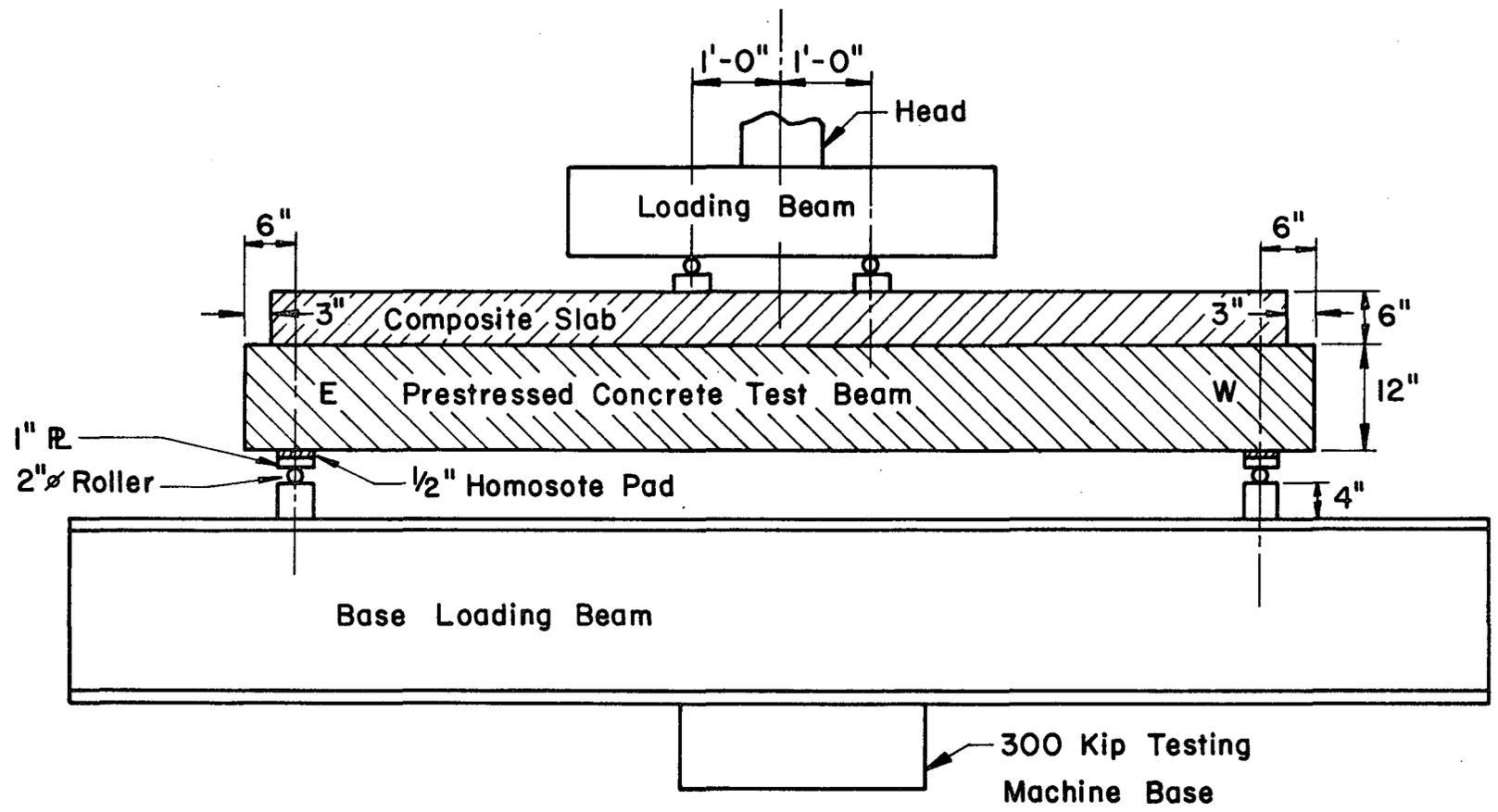
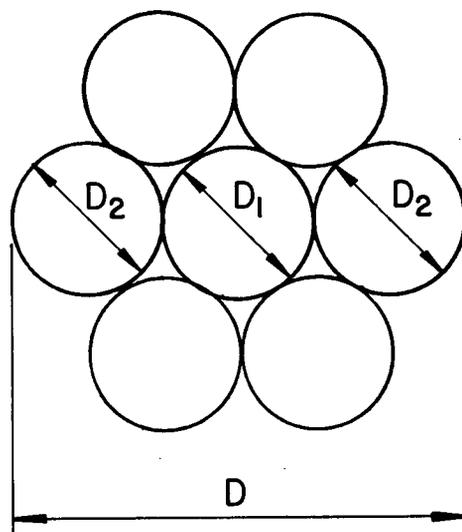


Fig. 2 Test Arrangement



$$D_1 = 0.174 \text{ in.}$$

$$D_2 = 0.167 \text{ in.}$$

$$D = D_1 + 2D_2 = 0.508 \text{ in.}$$

$$a = 2.139 \text{ in. (Outer Perimeter)}$$

$$A_s = 0.1531 \text{ in.}^2 \text{ (Total Area)}$$

$$\frac{a}{A_s} = 13.95 \text{ in./in.}^2$$

Fig. 3 Cross-Sectional Properties of the 1/2-in. 270K Strand

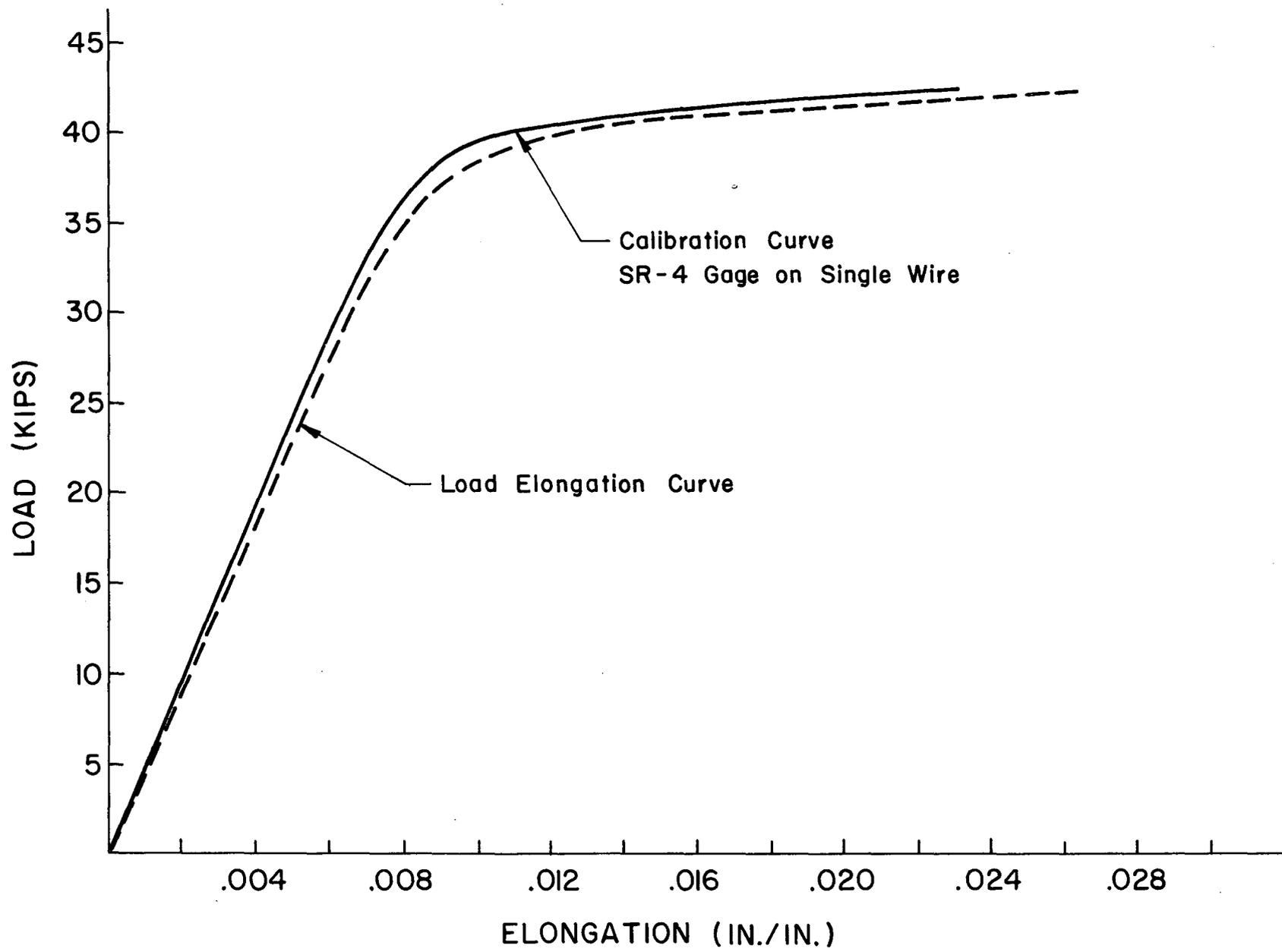
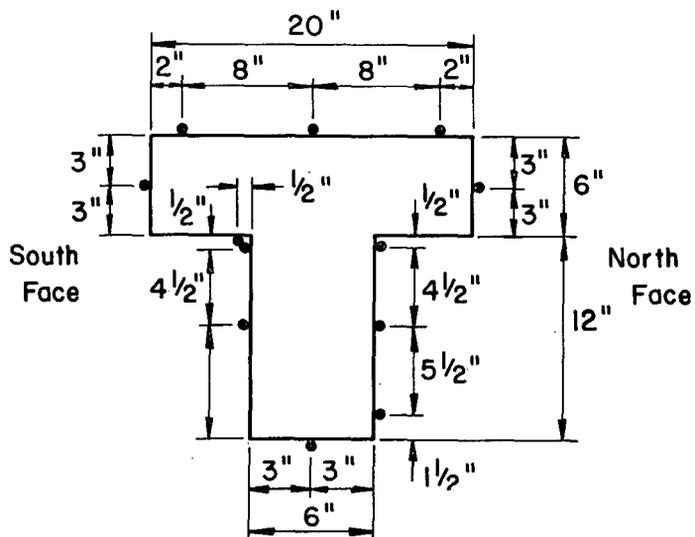
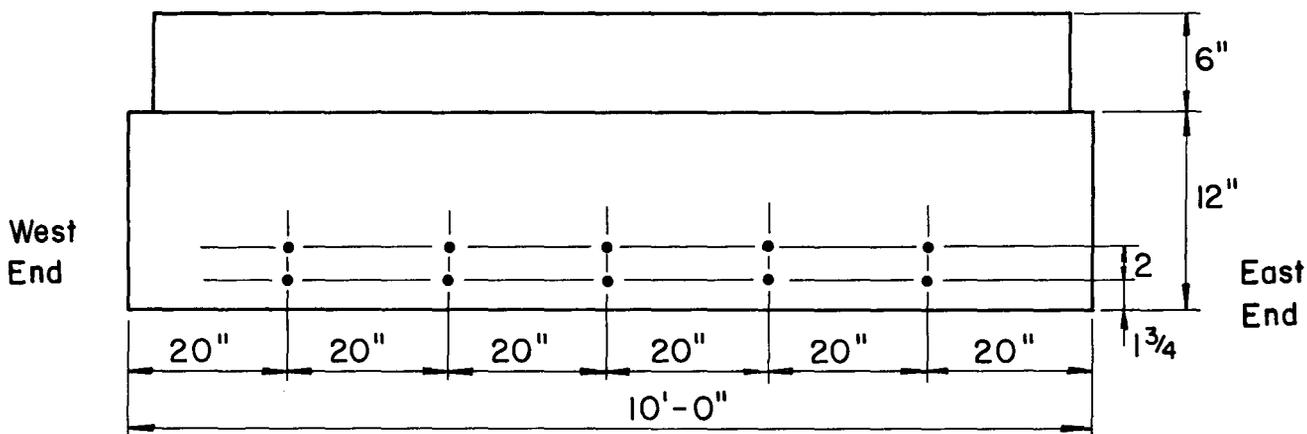


Fig. 4 Calibration Curve for Strain Gages Mounted on Individual Wires of the Strand



Locations of Type SR-4 A-9 Electric Resistance Strain Gages on Cross Section



Locations of Type SR-4 A-12 Electric Resistance Strain Gages Mounted on Prestressing Strands

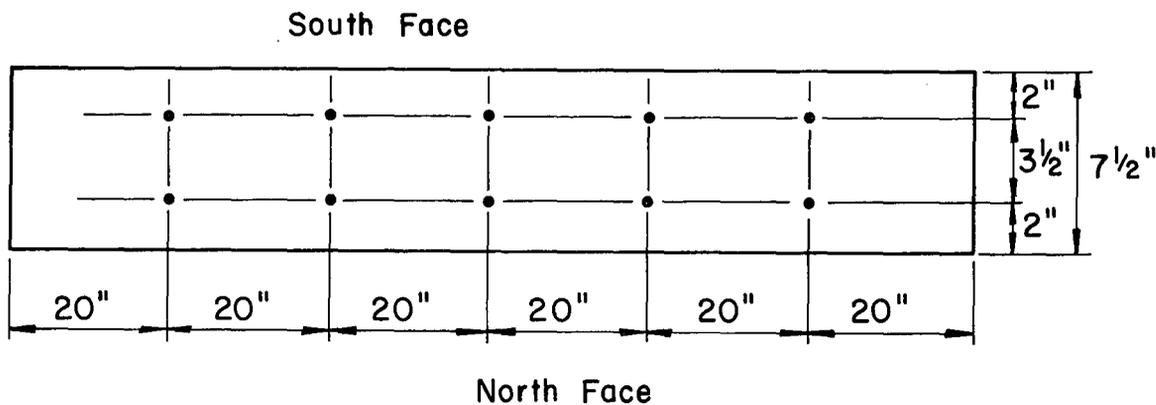


Fig. 5 Location of Electric Resistance Strain Gages

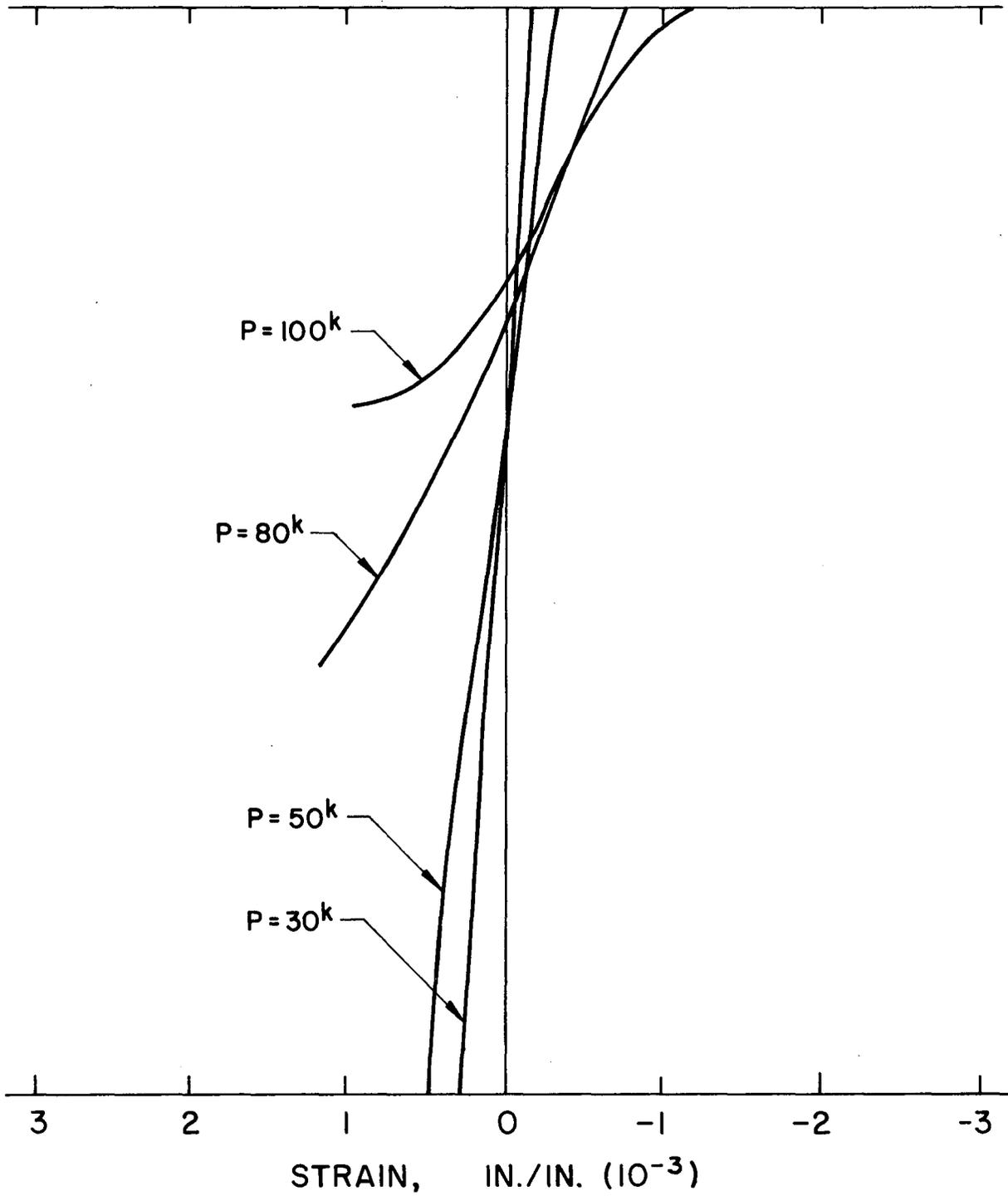


Fig. 6 Midspan Cross-Sectional Strain Distribution Beam Z-1

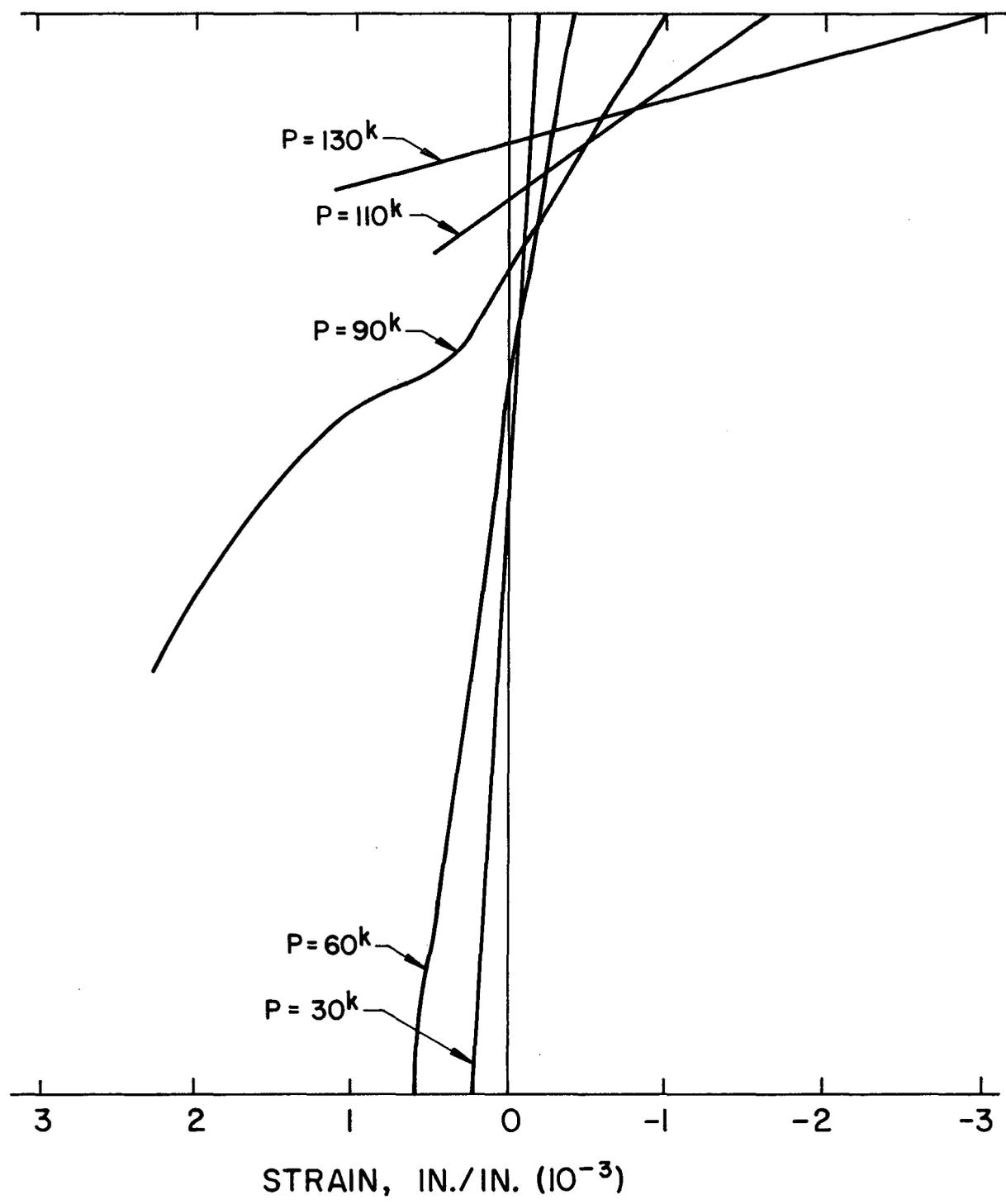


Fig. 7 Midspan Cross-Sectional Strain Distribution Beam Z-2

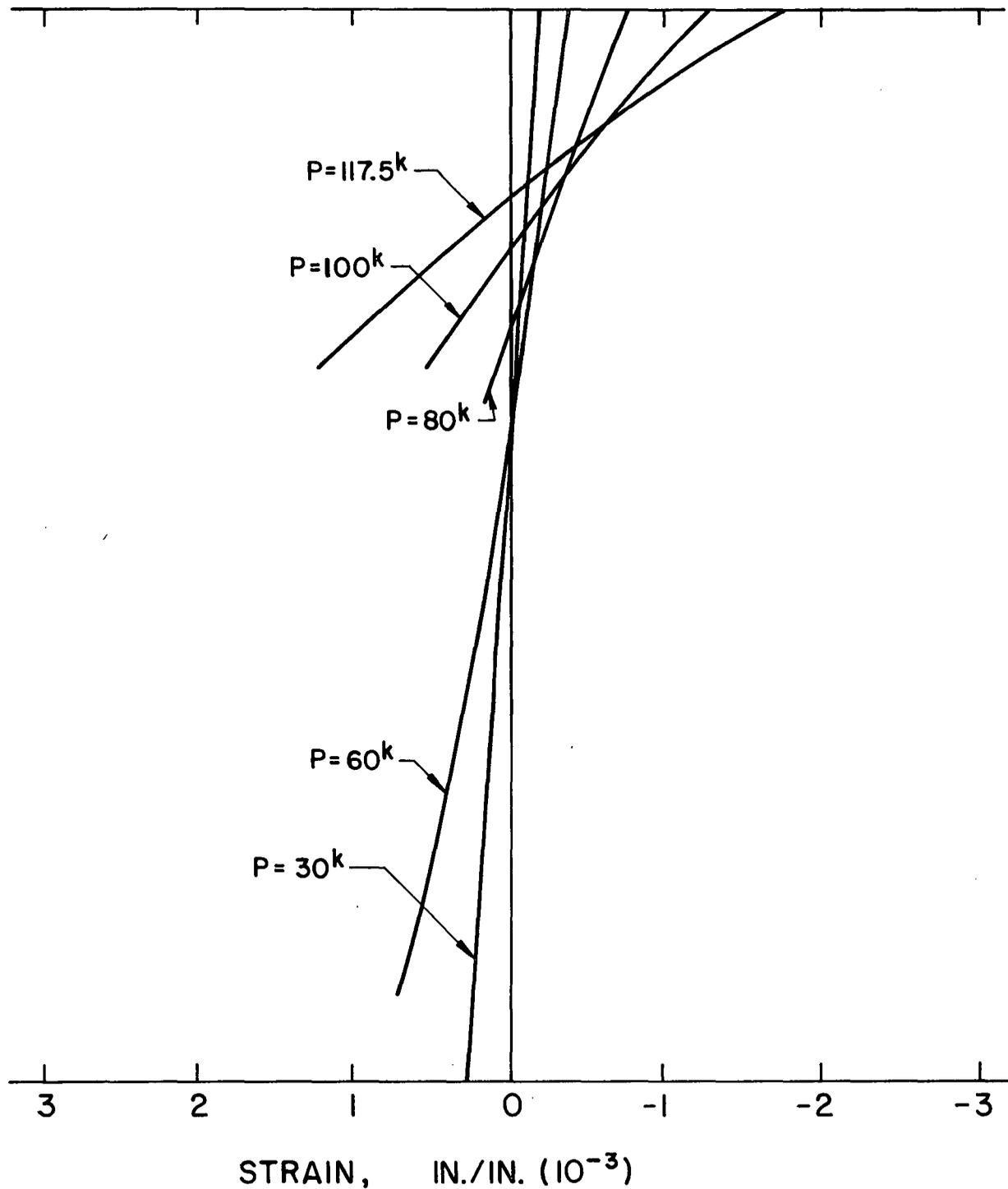


Fig. 8 Midspan Cross-Sectional Strain Distribution Beam Z-3



Fig. 9 Crack Pattern Beam Z-1

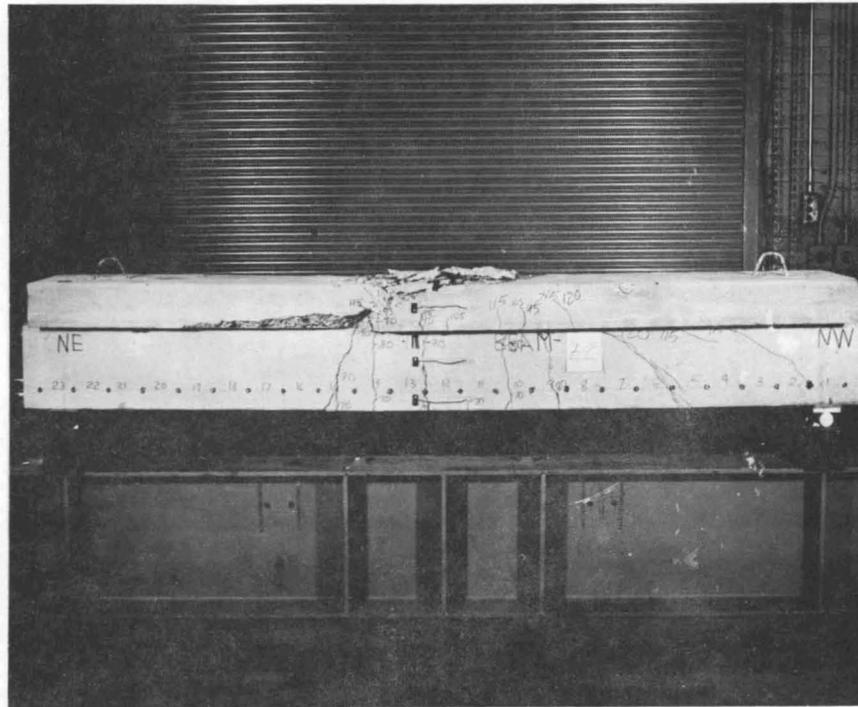


Fig. 10 Crack Pattern Beam Z-2

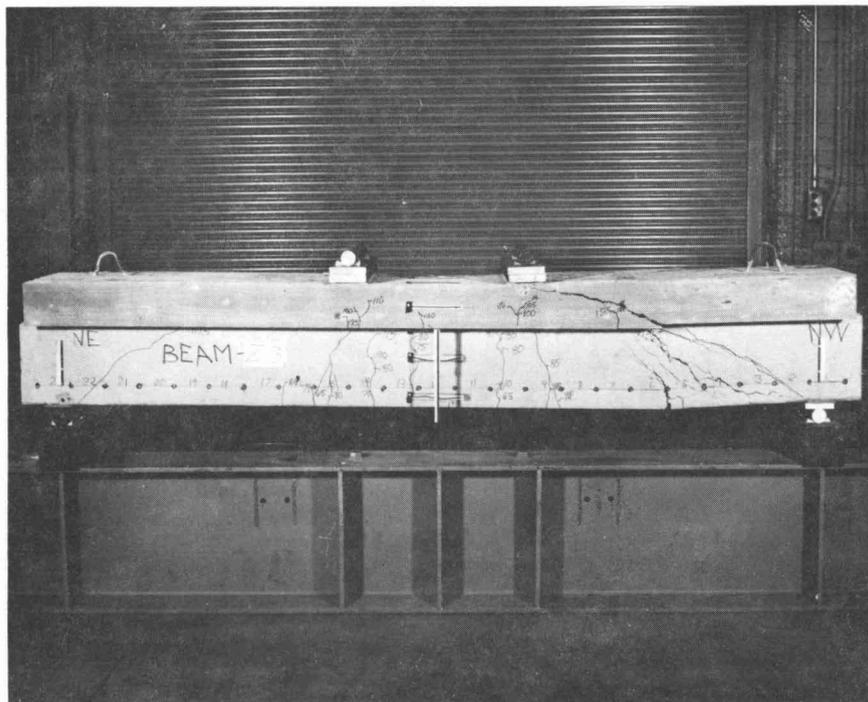


Fig. 11 Crack Pattern Beam Z-3

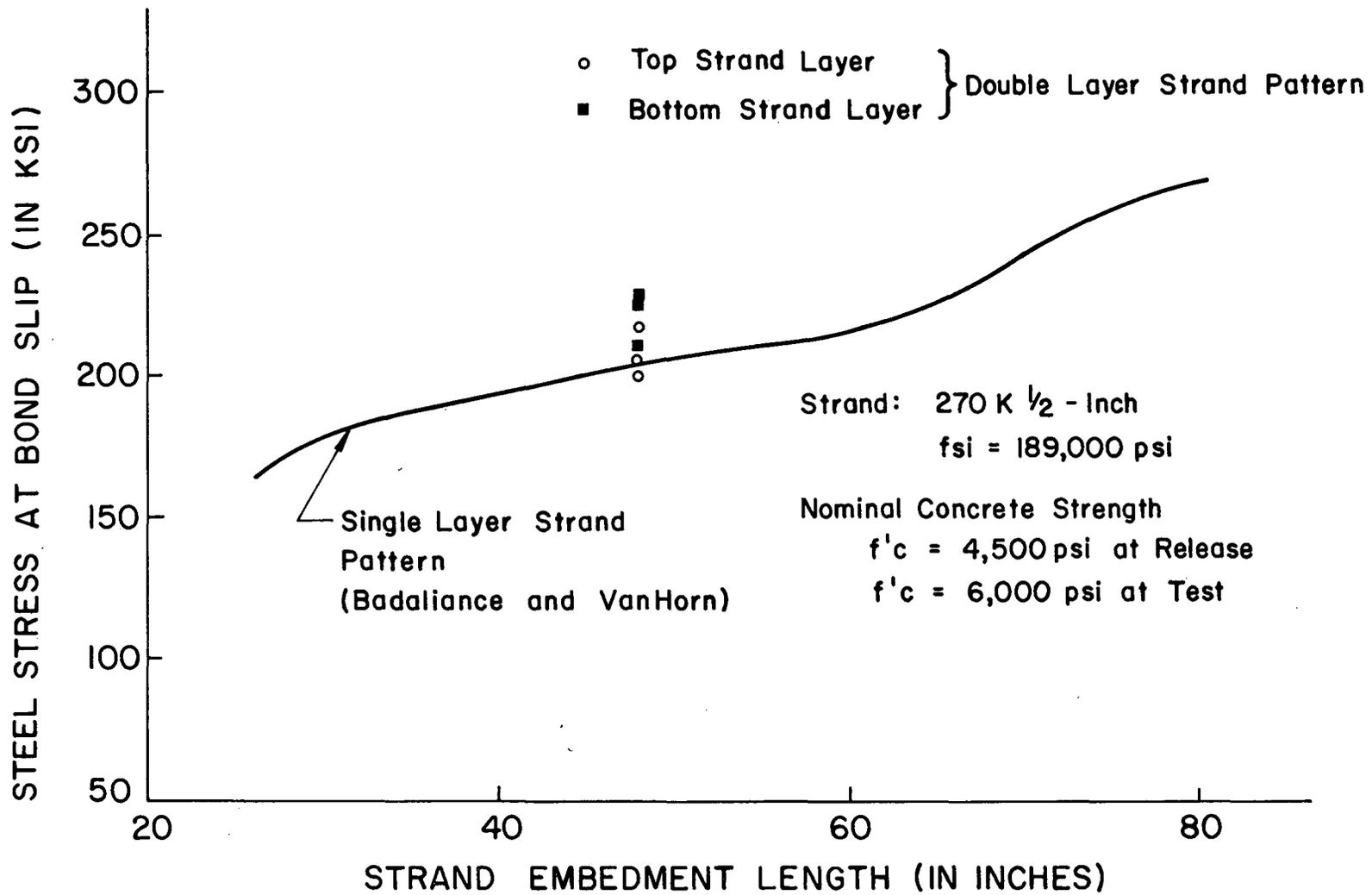


Fig. 12 Comparison of Steel Stress at Bond Slip for Single and Double Layer Strand Patterns

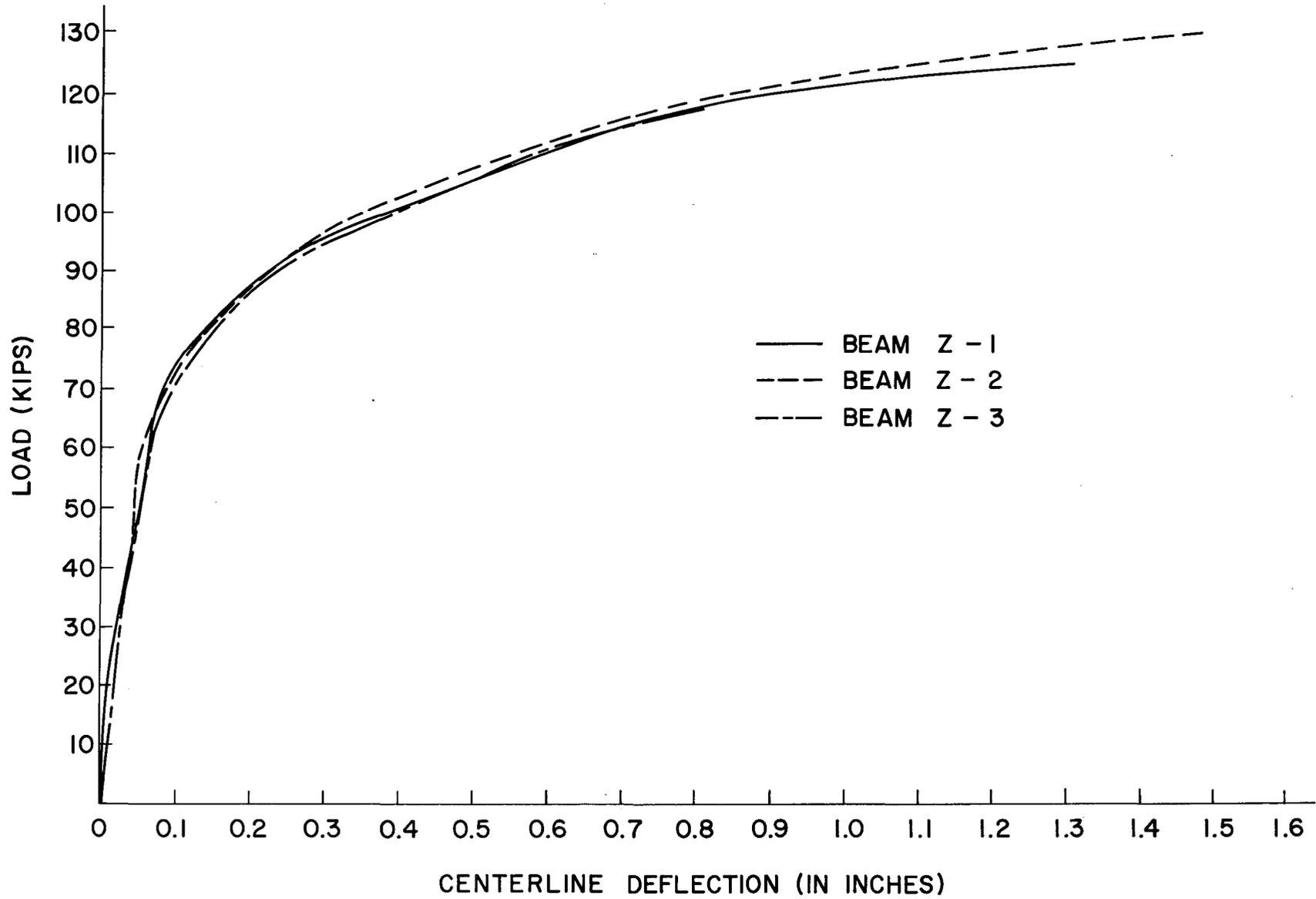


Fig. 13 Load Vs. Midspan Deflection Curves

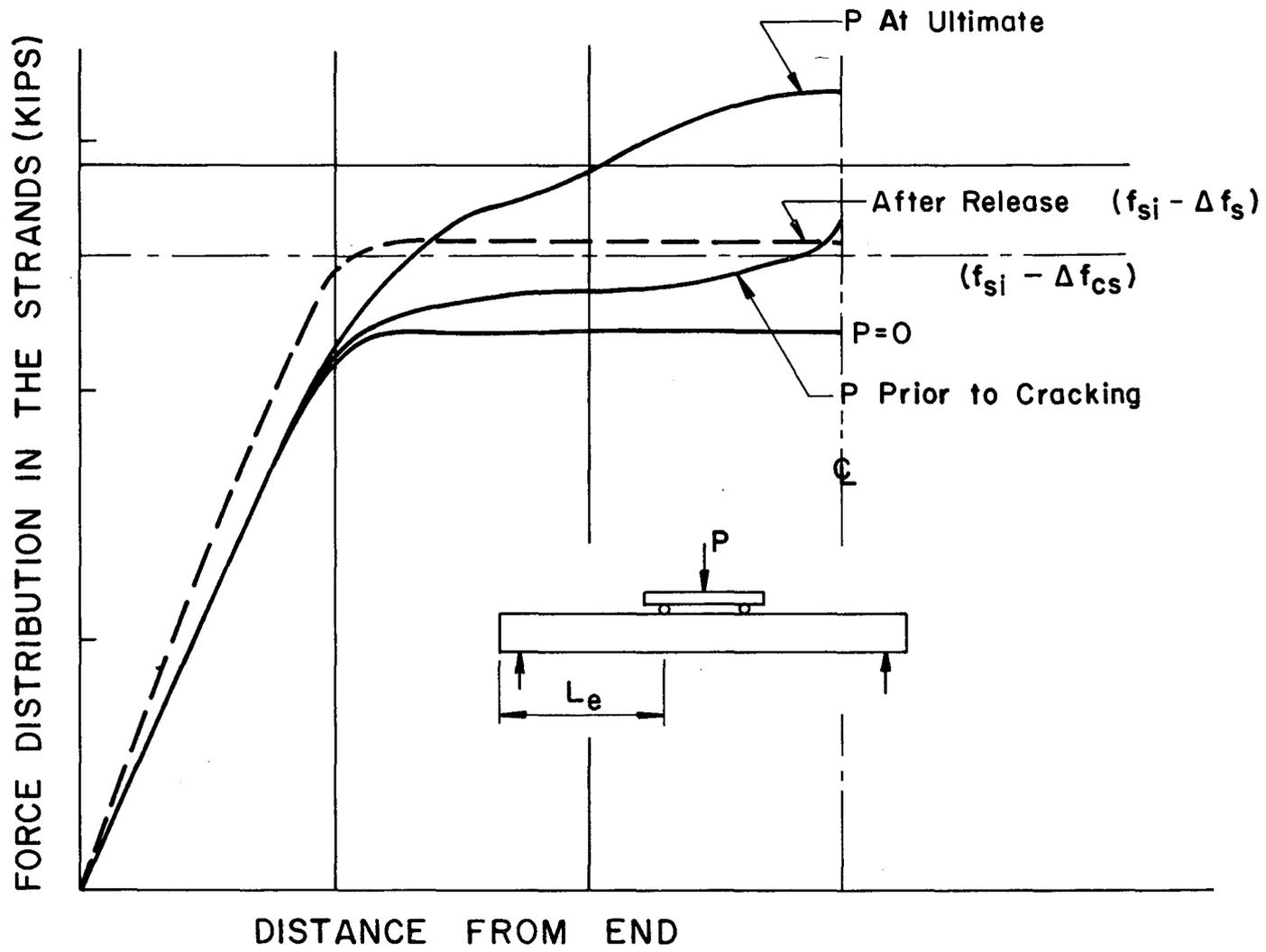


Fig. 14 Force Distribution in the Strand

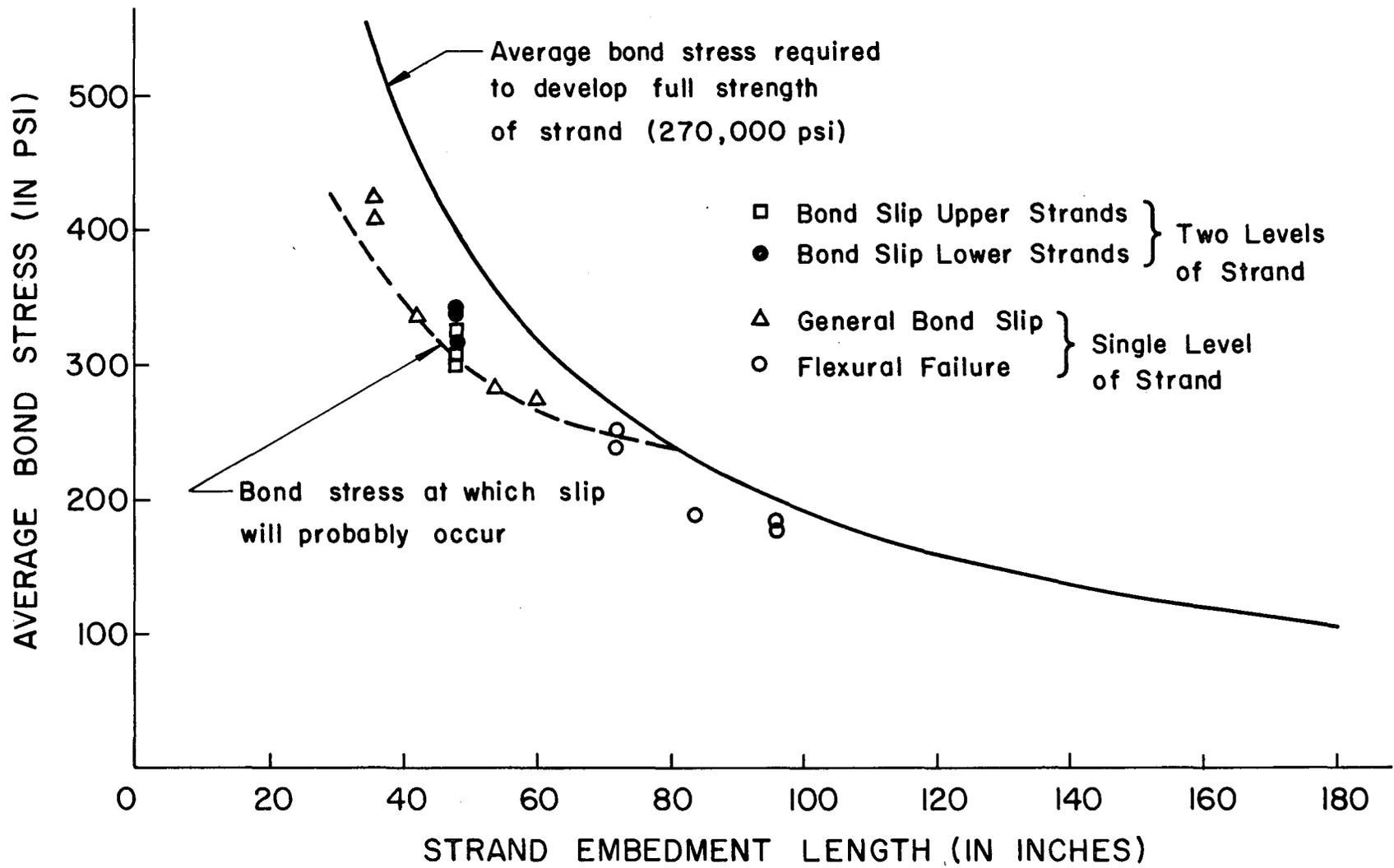


Fig. 15 Comparison of Embedment Length on Average Bond Stress for Single and Double Layer Strand Patterns

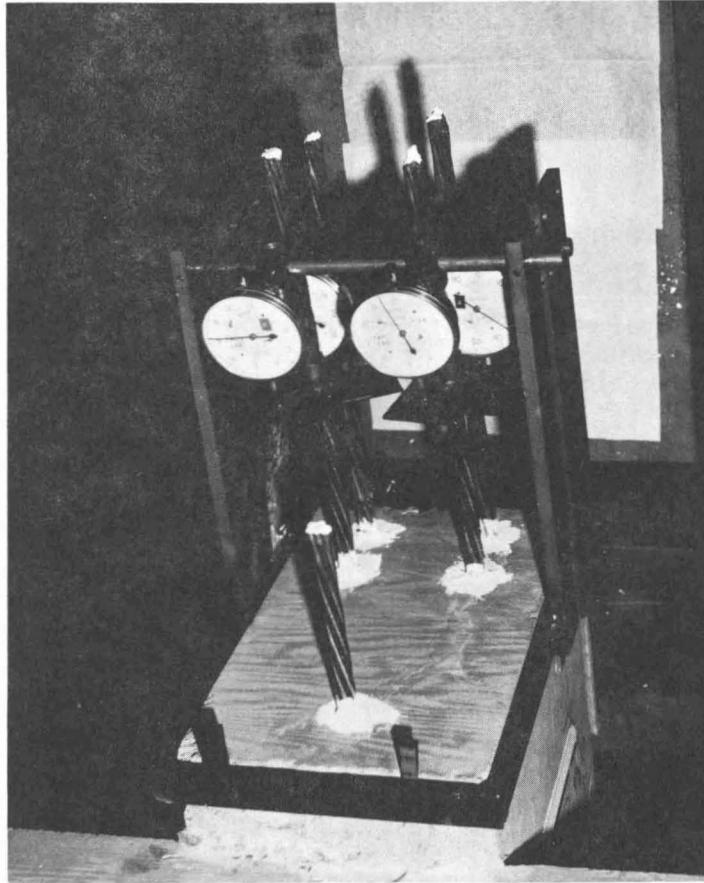


Fig. 16 Assembly of Strand-Slip Dial Gages

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