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H. B. Harrison

Plastic Design in High-Strength Steel

THE ECONOMIC USE OF HIGH-STRENGTH STEEL IN RIGID FRAME BUILDINGS

by

H. B. Harrison

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SYNOPSIS

A study has been made of the economics of using steels of various yield strengths in the design of building frames. A four-story, single-bay frame has been used as an example to obtain quantitative information as to weights, strengths and costs when mixtures of steel grades are incorporated in a design. Two separate aspects of the problem have been investigated. Firstly, numerical studies have been made of the frame strengths and costs when various groups of members have been assigned different yield strengths in a basic frame of constant member sizes and hence constant weight and elastic properties. Next, the elastic-plastic behavior was studied of a group of frames of approximately the same plastic strength but with members of different sections as well as different yield strengths.

In general, it was observed that the strength/cost ratio in the first series of frames was highest when the highest grade of steel was used. The deformations at working load and at failure were correspondingly larger than for identical frames of mild steel.

In the second series of frames of constant strength, it was demonstrated that the use of mild steel will often produce a frame of superior stiffness at only a small cost increase over that for a frame of high-strength steel.

The conclusion is drawn that there is a slight advantage in using high-strength steels in building frames. This advantage is less pronounced than it is in the case of bridge structures where reductions in structural weight are more significant. The study involved an extensive use of computers to determine both the elastic critical loads and the plastic failure loads for the chosen frame examples.

CONTENTS

I. INTRODUCTION

In proportioning a steel frame of fixed geometry, the criterion for efficiency is simply adequate strength and stiffness at minimum cost. A considerable variety of steels is now available[(1),(2)] with properties of weldability, notch toughness and ductility not signficantly different from those of mild steel. Special steels have been available for many years but it is only quite recently that manufacturers have advertised strength increases that have not been more than compensated for by cost increases. Without this new development, the use of high-strength steels was confined to large structures such as bridges where the stresses caused by the structural weight were dominant. In building frames, the weight of the steel skeleton is generally not a very large proportion of the total load so that a small reduction in frame weight occasioned by the use of high-strength steel would not lessen the design loads very significantly. This situtation, which precluded the general use of high-strength steels in multistory buildings except for minor components has been changed because new steelmaking processes have produced steels for which the strength increase has exceeded the cost increase. If the increase in strength had been accompanied by a corresponding increase in elastic modulus, there would be no problem in deciding which type of steel to use in a structure, but unfortunately, this is not the case as is well known. Hence the simple solution of using that steel for which the yield strength per unit cost is a maximum will be seen to be fallacious for many types of structures.

Of the two criteria for the adequacy of a steel design, namely strength and stiffness, the limits to the former are more generally accepted than those for the latter. The strength criterion however is the more important and it is the margin above the working loads at which failure could occur. Attention will be largely confined to this aspect of design. Unfortunately, the ultimate strength of a steel frame is a function of both the yield stress level of the material and the stiffness of the structure which depends upon the elastic modulus and the form of the material. It is immediately obvious that an increase in the yield stress level will not always result in the same proportional increase in frame strength. The ultimate load behavior of a framework is in many ways similar to the behavior of a pin-ended steel column. At high values of slenderness ratio, the strength of a column is a function almost solely of the elastic modulus and section shape so that there would be no advantage in using a steel grade other than the cheapest available, usually mild steel. For very low values of slenderness, the strength is almost solely a function of the yield stress level so that a steel grade should be used for which the yield stress per unit cost is a maximum. This is the case with the currently available special steels. As the slenderness ratio increases, the benefits of the high-strength steels decrease and the values of slenderness ratio at which the next lower strength steel becomes attractive could be computed if accurate and local cost data were available. As the unit costs for the various grades of steel will vary with locality and country, it will be desirable to consider relative rather than absolute costs in any economic study.

There will exist in a building frame some structural elements where the strength is directly proportional to the yield stress of the steel and a material should be chosen for these members for which the yield stress-cost ratio is a maximum. Examples would be simply supported beams, fully restrained laterally, rivets, bolts and most tension members. However, even in the case of the design of tension members, requirements of stiffness can sometimes be more severe than those of strength. This can be the situation in much of the diagonal bracing in multi-story frameworks. Quite apart from handling requirements which impose a minimum slenderness ratio, the forces in diagonal braces arise only in part from the wind loading on a building but they depend also upon the amount of sway in the frame. If the bracing truss is designed to resist the overturning moments associated with the vertical loads when sway occurs, the tension forces developed in the braces will be a function of their axial stiffness. At the upper levels in a tall building, the stiffness requirement could dominate so that it is likely that mild steel would be the most efficient material to use. Further down the frame, the strength requirements will increase at a higher rate than the stiffness requirements and so there could be a progressive substitution of steels of higher yield strengths with the most expensive material used in the lower stories. The judicious use of the available range of steel gradies would result in the minimizing of the cost of the entire bracing system.

II. ECONOMIC FACTORS IN FRAME DESIGN

It is well known that a linear-elastic analysis of a rigid frame, while providing reasonably accurate information on deformations at working load, will not provide any accurate assessment of the ultimate frame strength. However, if design is based upon such a method of analysis, high-strength steels could be used at the positions of high stress but the economic advantages would result largely from the saving in design time as a consequence of the elimination of the need for further analysis, rather than in the reduction of frame cost.

Plastic analysis will produce a more accurate estimate of the strength of a frame provided axial stresses in members are not significant and hence it can be used in its simplest form for multistory frames of only a few stories in height. Much attention has been given to the problem of minimum weight design using the principles of plastic analysis.[(3),(4)] As Neal has stated [(5)], "the design which involves the use of the least possible weight of material has a fair claim to be regarded as the best possible design, but to assert that minimum weight is the only important criterion in design is to disregard the numerous other economic factors which must always be considered." In a building frame, no special advantages can be claimed for a minimum weight design as might be the case in an aircraft structure so that the more realistic criterion is that of minimum cost. When only one grade of steel was readily available, the minimum weight and minimum cost designs would be identical if the other factors of fabrication, erection as suggested by Neal were discounted. This situation needs some clarification when a variety of steel grades can be obtained readily for the same sectional shape. Many of the studies made of minimum weight design have been concerned with the minimization of a term called the weight function x;

$$x = \sum_{\text{frame}} M_p L \tag{1}$$

where M_p is the plastic moment and L the length of a prismatic member. It follows that the minimum cost design will be the minimum weight design which utilizes the grade of steel for which the yield point - cost ratio is a maximum. Since the strength of the new steels is known(6) to rise at a faster rate than the cost, it would follow that a minimum cost design would need to incorporate the steel of highest strength. The very light frameworks that would appear if this theory was followed would be quite flexible at working loads but the second order effects of deformation would become of such significance that the real strength of a frame could be well below the figure arrived at by simple plastic theory. It would appear that the great success of the simple plastic theory in predicting accurately the failure loads for flexural frames of one or two stories has been occasioned by the influence of strain-hardening in mild steel.(7) As stated by Horne(8), strain-hardening "is a property of mild steel which is not taken into account in the simple plastic theory, and which has a pronounced beneficial effect on failure loads". It has also been claimed[9] that strain-hardening is essential for the formation of a plastic mechanism in most frames. It is not yet clearly established whether the strain-hardening stiffness for high-strength steels is comparable with that for mild steel and further research may be needed to clarify the position.[10]

In recommendations for the plastic design of single story pitched frames, Baker[9] has not accounted for strain-hardening but has shown that it is possible to design plastically in high-strength steel to the B.S.968 Specification by computing simply the reduction in simple plastic collapse load caused by finite deformations. The assumption is made that the peak load-carrying capacity is reached when the last plastic hinge is formed corresponding to the rigid-plastic mechanism of failure. When axial forces are considerable, it could be the case that the peak load would be reached before the formation of all the plastic hinges appropriate to a rigid-plastic failure mechanism. The extent of the beneficial influence of strain-hardening upon the behavior of complex structures in high-strength steel is uncertain because deformations could be very considerable and the strain-hardening modulus may well be less than the figure appropriate to mild steel.

It can be seen that the accurate calculation of the maximum load-carrying capacity of a flexible frame in high-strength steel will be difficult and yet no real decisions can be made about the economics of competitive designs in steels of various grades without some reasonable estimate of maximum frame strength.

III. FRAME STRENGTH ESTIMATION

It has been mentioned that the maximum load-carrying capacity of a rigid frame must be a function of the yield strength of the material and the overall frame stiffness. The accurate prediction of the maximum load becomes more difficult when high-rise frames are considered in which the material is high-strength steel.

It has been proposed[12] that the strength prediction of a frame might be estimated using the harmonic mean of the plastic failure load and the elastic stability load. Even though this approach may be overly conservative when compared with model tests[13], it does at least involve the principal factors which influence frame behavior and so it seems reasonable to use it to compare the strengths of alternative fictitious designs in a study of frame economics.

For the frame examples studied in this work, the plastic failure load has been computed using a computer program[14] which will trace the formation of plastic hinges in a systematic fashion by first-order elastic analysis until the frame has been converted into a mechanism. The limitations of this program have been discussed elsewhere[15]. The load factor computed in this way is no different from that which could be arrived at by simple plastic theory based on the concept of a rigid-plastic material. In addition, no direct allowance can be made for the reduction in plastic moment due to axial stresses. The elastic stability loads for the frame examples have been computed from the results of a second-order elastic analysis[16] again carried out by a computer program. The effects of primary bending moments have not been taken into account but the errors involved in this approximation are likely to be negligible[17] for the type of frame considered. The development of these programs was considered to be a prerequisite in any study of frame economics which had the aim of proceeding farther into the topic than merely a general discussion of the various factors involved.

In a study of frame economics, a distinction is apparent between the dual approaches of design and analysis. In a design study, a frame of fixed geometry and strength is required so that the substitution of high-strength steel for mild steel will be accompanied by a decrease in the size and weight of the member concerned. Consequently, there would be a decrease in frame stiffness which would be reflected in a lower elastic critical load. This behavior can be represented diagramatically as in Fig. 1. The diagram is qualitative only and the difference between the plastic and the elastic critical load factors is much less than would be the case in a practical frame. The full lines represent the behavior of a frame in mild steel and the broken lines refer to a frame of equal strength in high-strength steel. There will be a decrease in the elastic critical load in the latter case so that there must be an increase in the load factor from simple plastic theory so that both frames will have equal strength. Evidently the design in high-strength steel will be more flexible than the other so that there would be larger deformations both at the working load level and at the maximum load. In addition, there may be fewer plastic hinges formed at the maximum load in the case of the design in high-strength steel but larger plastic rotations may have occurred at positions where the early hinges were developed.

On the other hand, the approach in an analytical study could be different. The object would be the estimation of frame strength as high-strength material is progressively substituted for mild steel in various members of a frame of constant weight and section sizes. It follows that the elastic critical load would be constant but the load factors deduced from simple plastic theory would increase. However, there would not be the same proportional increase in the real load factors at failure. The behavior is illustrated in Fig. 2. The deformations at working load values would increase in approximate proportion to the working load levels but the deformations at maximum load would probably be comparable.

Both approaches to the problem have been followed in the studies described in this report.

IV. STUDIES OF FRAMES OF CONSTANT STIFFNESS

The plane frame which was chosen as the basis of the studies in this report is shown in Fig. 3. It is a frame that has been used by Heyman[18] as an example to demonstrate a method of deflection calculation at plastic failure. It was chosen for the purposes of this study because it is sufficiently complex to demonstrate the consequences of using steels of different strengths and yet it is of a size where the computer programs could accommodate the analyses within the core store of the available computers. The sections used in the analyses were the U. S. wide-flange equivalents to the British universal sections used by Heyman and the details are shown in Table 1. In order to limit the scope of the study, and to correspond more closely with practical requirements, it was decided not to vary the material between pairs of members. Hence, the top two beams in any study will be identical and likewise the lower two beams. Each column was considered as being of the one grade of steel in either 24 ft. or 48 ft. lengths and the matching columns in either the lower two panels, or the upper two panels were considered identical. Hence, instead of having to consider variations in the twelve members of the frame, attention could be confined to a study of the effects of variation in material among four member groups.

Altogether, eight frames were examined initially in which various combinations of members pairs were assumed to consist of either 36 ksi or 45 ksi steel. The details are shown in Fig. 4. All of the frames were identical in the corresponding member section sizes so that the same elastic critical load was relevant for all. The loads shown in Fig. 3 have been regarded as unit values so that all of the results are given in terms of load factors rather than absolute values. The principal results from computer analyses of eight frame variations are given in Table II. It can be seen that a complete set of 13 plastic hinges (one more than the degree of redundancy) developed at failure in only one frame and this was the case when high-strength material was used in the beams with low strength columns. The load-sway diagrams for each frame are shown in Fig. 5 and the sequence of plastic hinge formation is also shown in the figure. As all the frames are of the same weight, the strength/weight ratio is a maximum for frame No. 2 which is to be expected as it consists wholly of the high-strength steel. But the real criterion for efficiency is the strength/cost ratio and to evaluate this, some figures have to be assumed for the unit costs of the two grades of steel. For this purpose, the figures of $111 and $127 per ton have been used, these being quoted rates for ASTM A36 and V45 steel from the Bethlehem Steel Corporation in February, 1962. The cost data for the eight frames are set out in Table III where the maximum load factor has been computed assuming it to be half the harmonic mean of the plastic and the elastic critical load factors (Merchant Formula).

Two matters of interest can be seen from Table III. Firstly, the most efficient frame from the standpoint of strength/cost ratio is No. 3 which consists of high-strength steel in all beams with mild steel in all the columns. But the strength/cost ratio is not significantly larger than those for frames 2 and 8 and is probably associated with the fact that more plastic hinges are formed at the plastic failure of frame 3 than for any of the others. From the view point of purely plastic design, it is a more efficient design. Secondly, it can be seen that if steel of the one yield strength is to be used throughout the frame, the use of the high-strength steel as in frame No. 2 is to be preferred but the percentage gain in strength/cost ratio (8%) is considerably less than either the increase in unit cost of the material (14%), or the increase in yield strength itself (25%). Accordingly, it is informative to compare data for frames of uniform material of higher yield stress than 45 ksi and these results are shown in Table IV. The strength/cost ratio is still highest for the 55 ksi steel but the rate of increase in the ratio is decreasing and it seems likely that an optimum yield strength would occur for the frame example in the region of 60-70 ksi if the material costs were to increase in the same manner as the increase over the 36-55 ksi range. It is evident that the 4-story frame used as an example is too small a structure to demonstrate any appreciable reduction in load factor from the plastic value since the elastic critical load is approximately 20 times the plastic failure load. Nevertheless, the principle is demonstrated that for any frame of constant strength material, there will exist an optimum value for yield stress to maximize the strength cost ratio and it seems likely that the stress will decrease with increasing size of frame. The advantages in using a mixture of steel grades are also likely to become more pronounced in the case of larger structures.

These analyses illustrate the consequences of utilizing a variety of steel grades but the designer's problem is somewhat different as has been explained earlier. The design problem is to achieve a specified frame strength at a minimum cost so that this problem called for some further consideration.

V. STUDIES OF FRAMES OF CONSTANT STRENGTH

For the purposes of this study it was decided to regard Frame No. 10 of 55 ksi steel throughout as the basic design so that the problem could be investigated if section shapes could be located with the same plastic strength as those used in Frame 10 but using lower strength steel. The sections chosen are shown in Table V. Three frames were studied which had approximately the same strength as Frame No. 10 and the construction details are shown in Fig. 6. The load-deflection diagrams obtained from plastic analysis are plotted in Fig. 7 and the sequence of hinge formation is also shown. The principal results are set out in Tables VI and VII. It can be seen from the latter table that the substitution of large sections in mild steel for the small sections in high-strength steels raises the elastic critical load considerably as a consequence of the increased frame stiffness so that the frame strengths were not identical when computed from the harmonic mean of the elastic critical and plastic load factors. Hence the more significant figures to compare are the strength/cost ratios set out in the last column of Table VII. Here again it can be seen that the most efficient design from the strength/cost ratio standpoint was that of Frame No. 10 which utilized high-strength steel for all members, but the results for Frame No. 12 are quite significant. In this example, the beams were all of low strength steel but the columns were the same as those in Frame No. 10, all being of 55 ksi steel. It can be noted that whereas Frame No. 12 was 19% heavier than Frame No. 10, it was only 4 ½% more in cost. Further, the working load deformations of Frame No. 12 are considerably less than those for Frame No. 10 as can be seen in Fig. 7. On the other hand, the extra 3 inches depth in all the beams of Frame No. 12 could constitute a significant disadvantage. The maximum plastic hinge rotations are also shown in Table VI and in all cases, these rotations occurred at the first-formed plastic hinge which was in the beam at the lowest level. Significantly more rotation capacity can be seen to be required for the high-strength design (Frame No. 10) than for the others.

VI. CONCLUSIONS

It is evident that the availability of a range of structural steels of different strengths and unit costs has given rise to a considerable problem where economic design of building frames is concerned. Much of the research work that has been done in the past on minimum weight design was based on the concepts of simple theory and is of no great assistance to a designer who requires a frame of minimum cost which will no longer be closely associated with a design of minimum weight. There will always be occasions when functional requirements necessitate columns of small overall dimensions as well as shallow beams but it is difficult to allow for such advantages in any quantitative study. The disadvantages of having larger deformations at working load levels would need to be assessed as well, but all of these factors have been largely ignored in the present study and maximum frame strengths were considered as being of primary importance. The distinction between the approaches of design and analysis was emphasized in this study and separate chapters were devoted to each topic. A further problem in assessing the efficiencies of alternative designs for high-rise frames is the lag that has often occurred between the development of design procedures and methods of analysis. The design of a high-rise frame can be achieved by considering isolated groups of members[19] which are proportioned with live load intensitites which are not constant but depend upon the area supported. The overall strength of such structures, besides being difficult to compute at the present time can also be a largely irrelevant quantity. The sway deformations at working load levels would need to be assessed after the preliminary design has been completed and may well reduce the advantage of the extensive use of high-strength steels except in the case of braced frames.

In all of the examples described in this report, the most efficient structures on the basis of strength/cost ratio were seen to be those in high-strength steel but the trends indicate that this would not always be the case if larger frames were studied. It was significant that frames in mild steel which were very much heavier than alternative designs in high-strength steel were only a few percent higher in cost. The extraneous factors of supply, ease of fabrication and transportation, and the reduction in foundation costs could well govern the economic advantage of a design in high-strength steel.

VII. ACKNOWLEDGEMENTS

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FIGURES

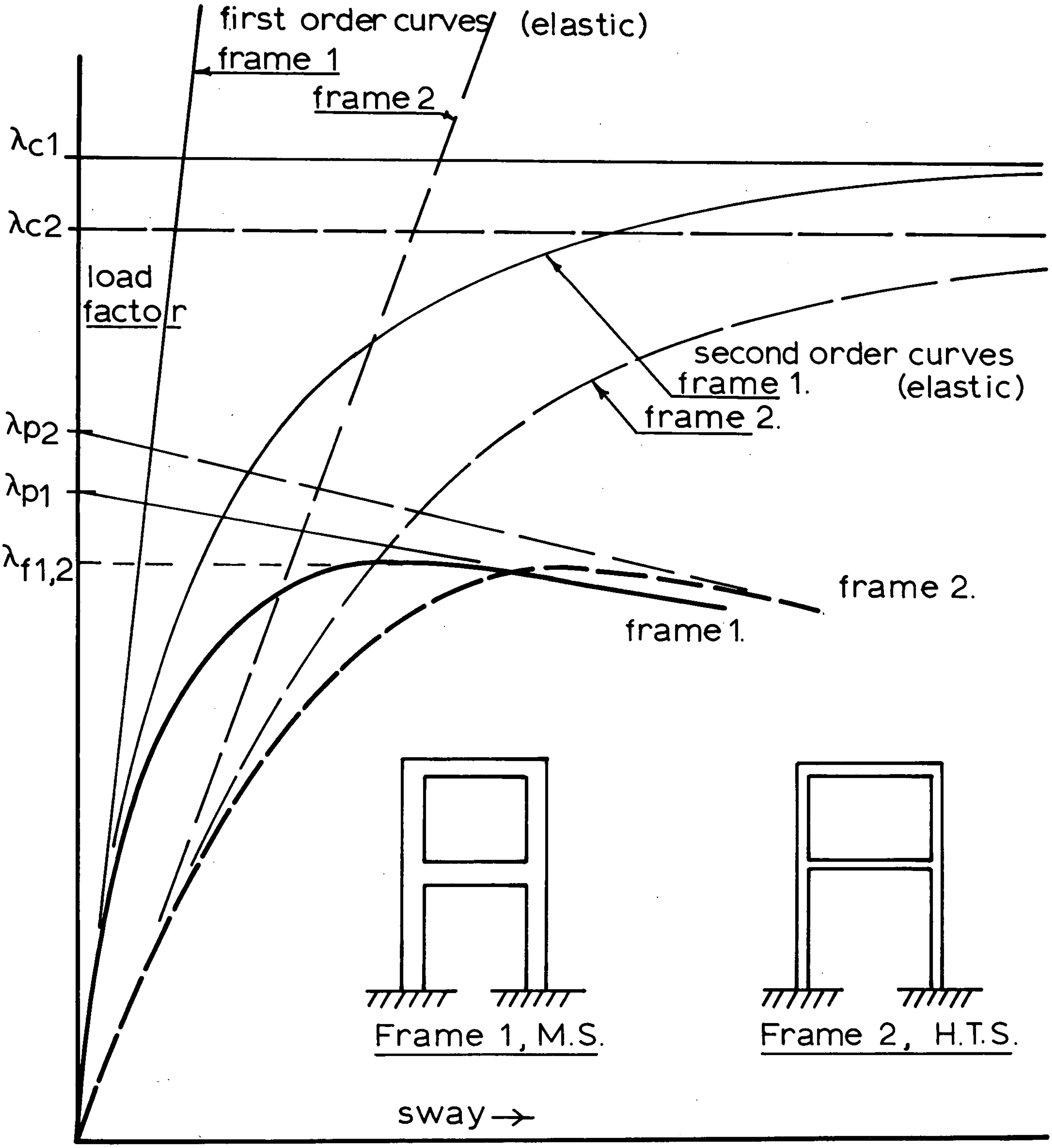


FIG 1 FRAMES OF CONSTANT STRENGTH

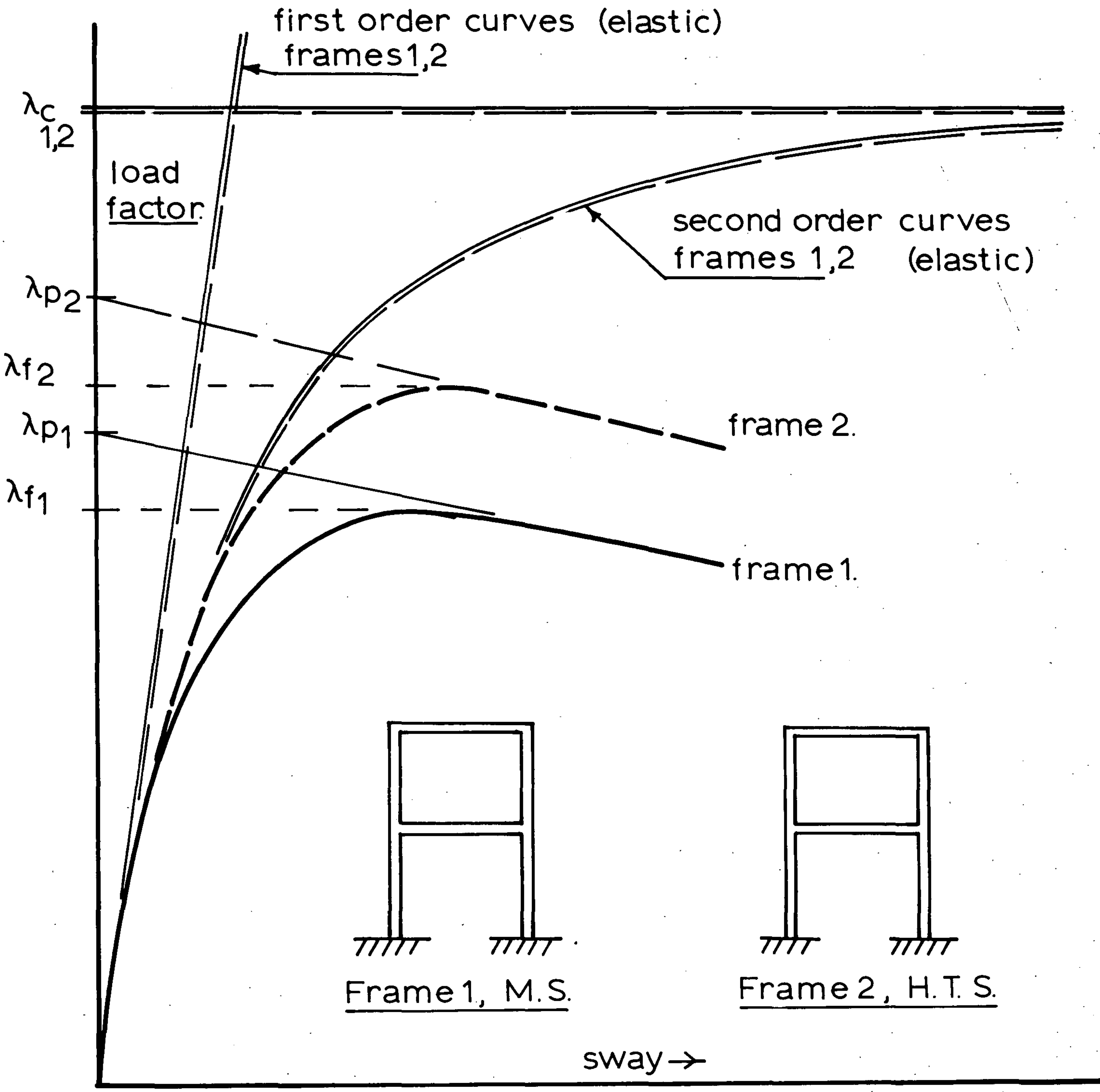
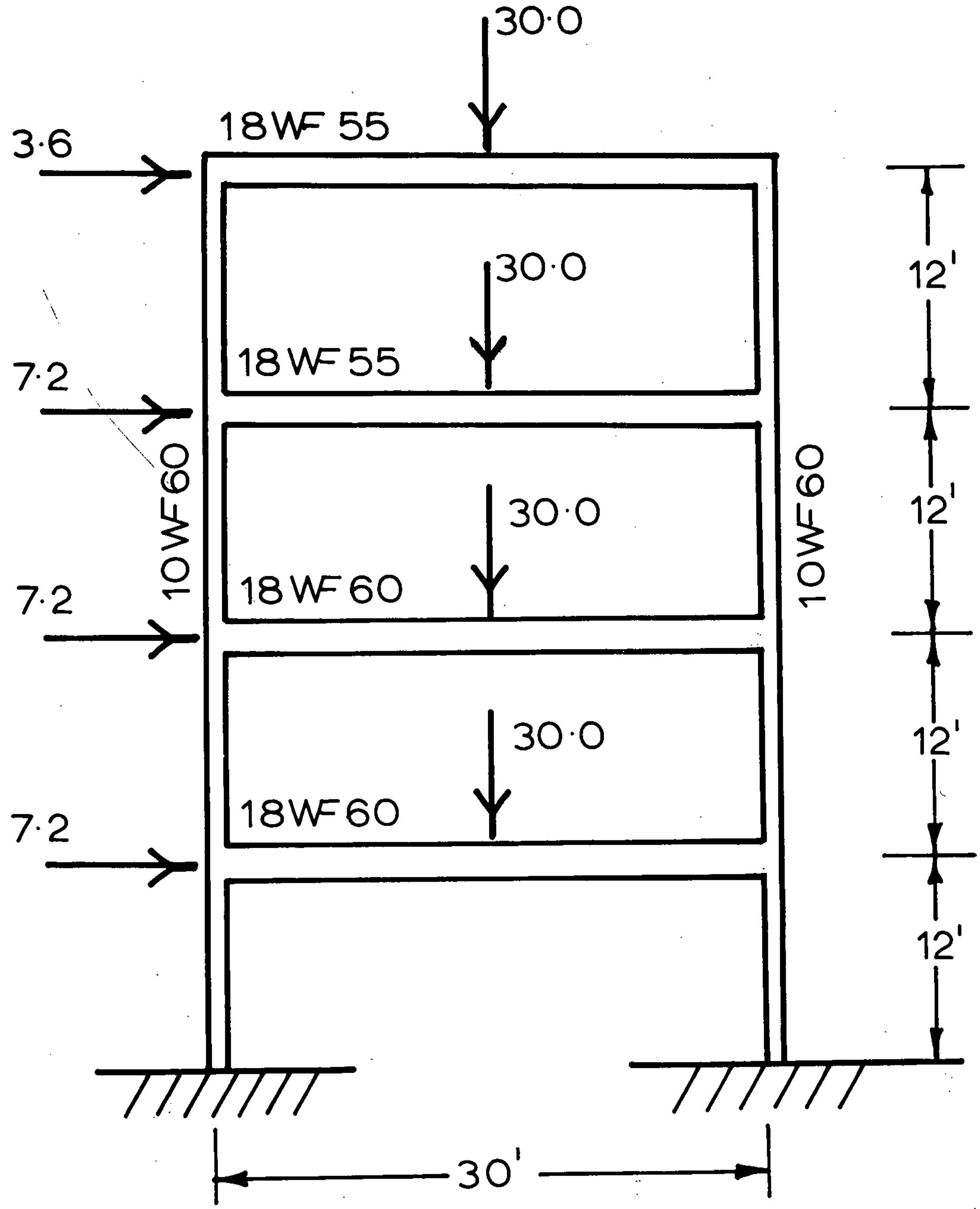


FIG 2 FRAMES OF CONSTANT STIFFNESS



(load units-kip)

FIG 3 FRAME EXAMPLE WITH UNIT LOADING

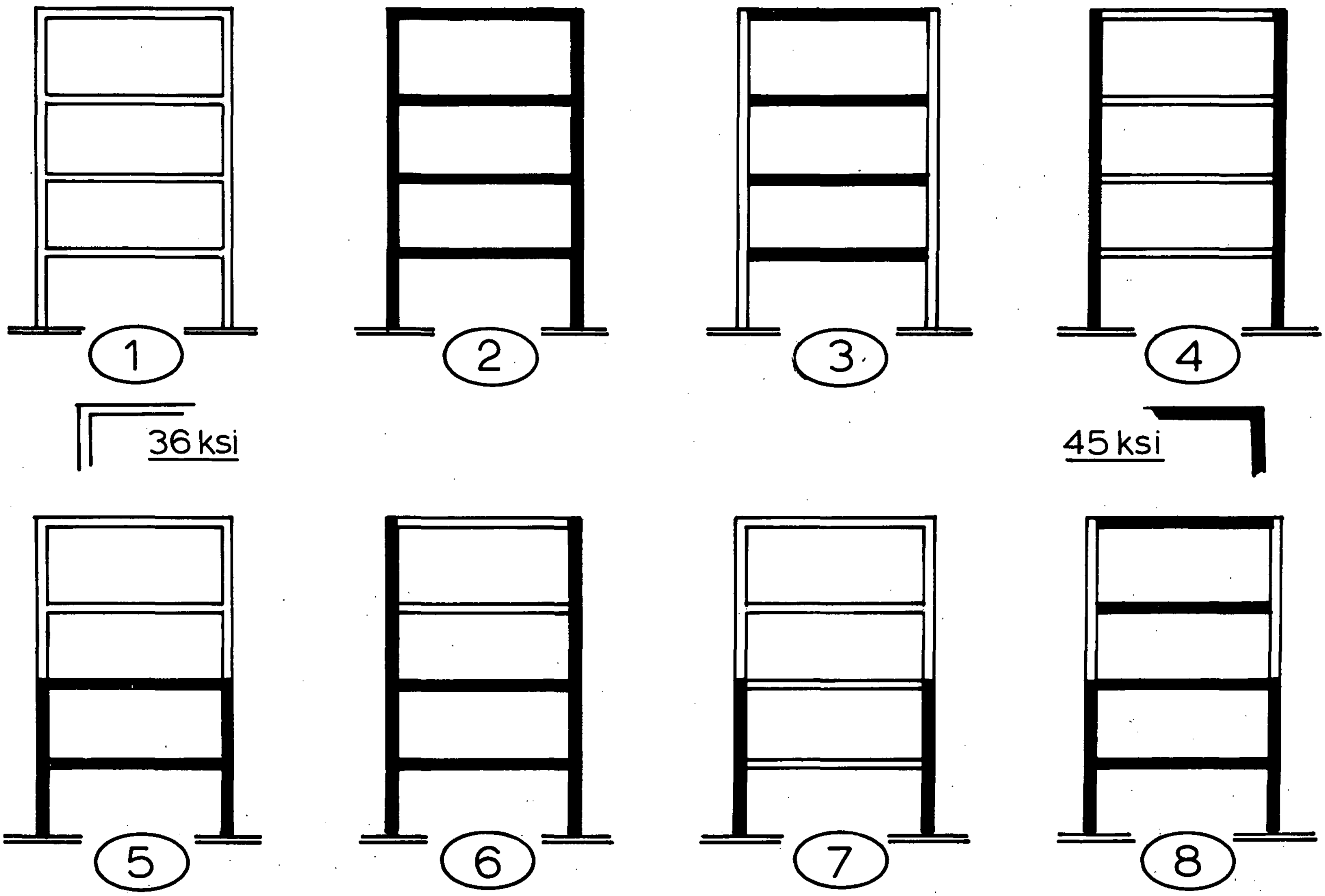


FIG 4 FRAMES OF CONSTANT STIFFNESS

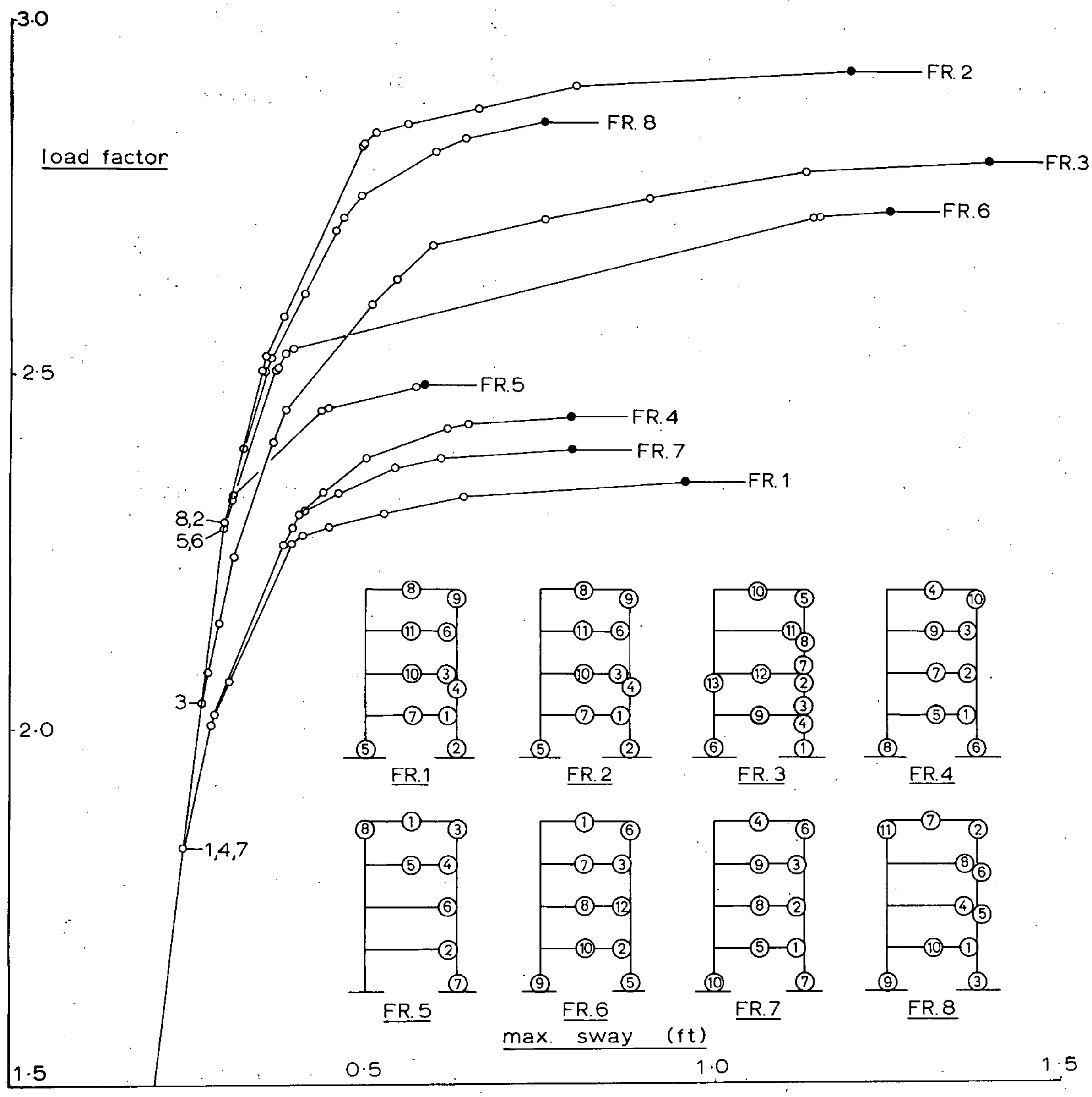


FIG 5 LOAD SWAY CURVES FRAMES 1 – 8

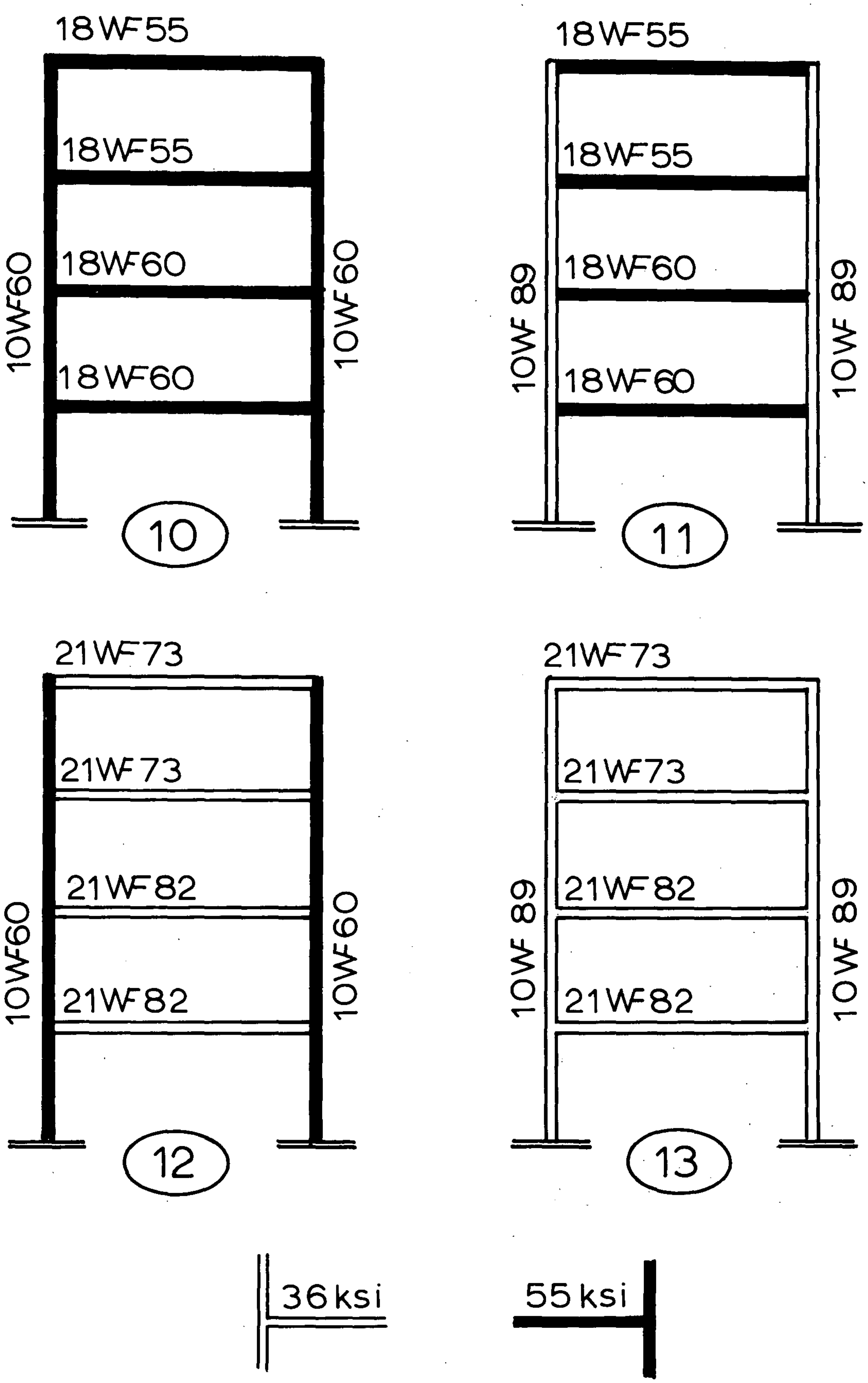


FIG 6 FRAMES OF CONSTANT STRENGTH

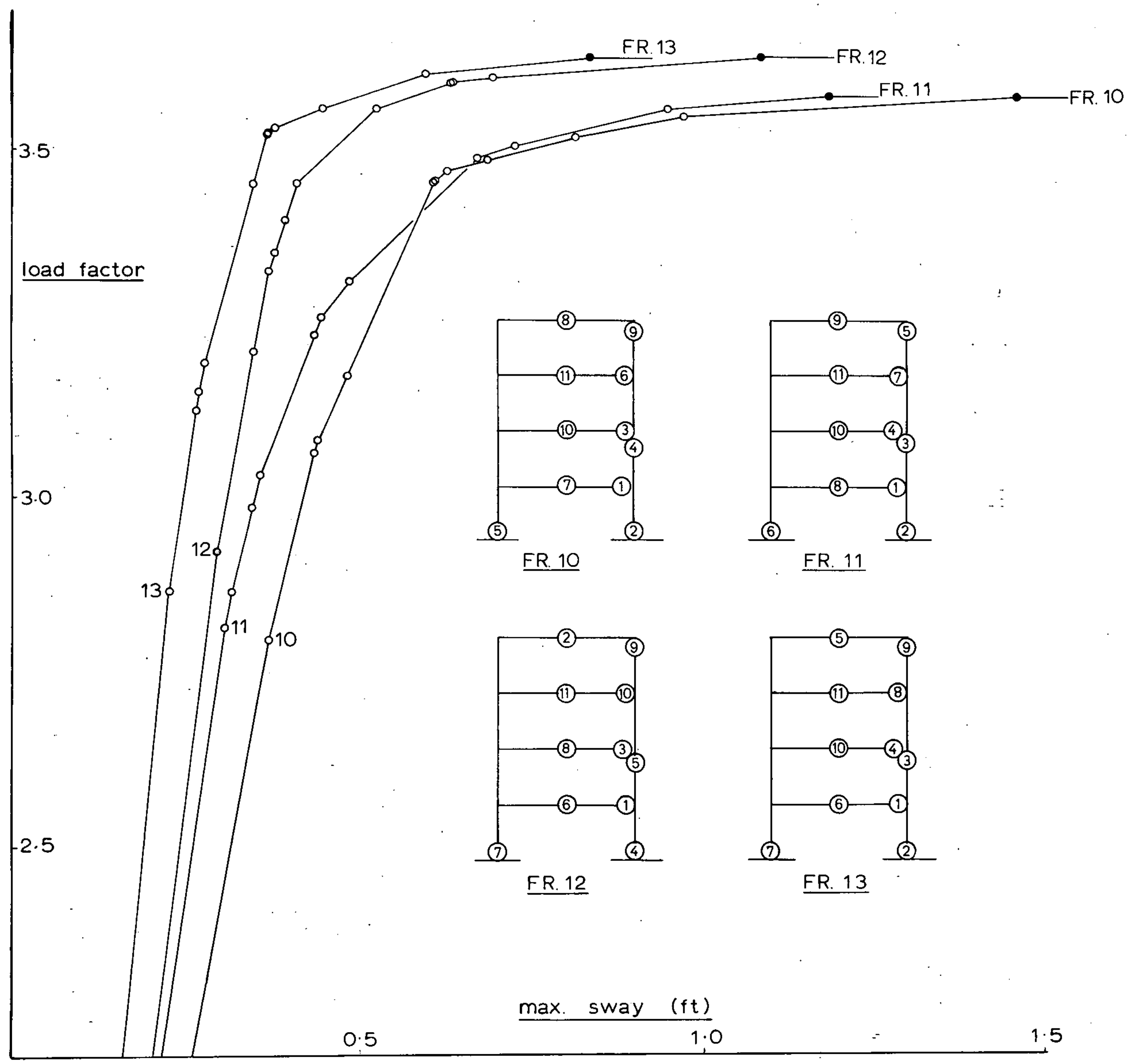


FIG 7 LOAD SWAY CURVES FRAMES 10 — 13

TABLES

TABLE I

SECTION DATA FOR CONSTANT STIFFNESS FRAMES

Section	Inertia ($in.^4$)	Area ($in.^2$)	Plastic Modulus ($in.^3$)
18WF55	889.9	16.19	111.6
18WF60	984.0	17.64	122.6
10WF60	343.7	17.66	75.1

TABLE II

RESULTS OF PLASTIC ANALYSIS FOR FRAMES OF CONSTANT STIFFNESS

Frame No.	Construction Details Yield Strengths in -				Conditions at Development of first plastic hinge		
	Top Beams	Lower Beams	Top Columns	Lower Columns	Load Factor	Max.Vert. Defl. (ft.)	Max.Sway Defl. (ft.)
1	36	36	36	36	1.8337	0.0796	0.2412
2	45	45	45	45	2.2921	0.0995	0.3015
3	45	45	36	36	2.0377	0.0884	0.2681
4	36	36	45	45	1.8337	0.0796	0.2412
5	36	45	36	45	2.2868	0.0993	0.3008
6	36	45	45	45	2.2868	0.0993	0.3008
7	36	36	36	45	1.8337	0.0796	0.2412
8	45	45	36	45	2.2921	0.0995	0.3015

Frame No.	Conditions at Development of last plastic hinge			Total Number of plastic hinges	Maximum plastic hinge Rotation at failure (r)
	Load Factor	Max.Vert. Defl. (ft.)	Max.Sway Defl. (ft.)		
1	2.3514	0.3971	0.9586	11	0.0465
2	2.9393	0.4964	1.1984	11	0.0581
3	2.8098	0.5973	1.3991	13	0.0797
4	2.4417	0.3357	0.7949	10	0.0376
5	2.4893	0.3953	0.5831	8	0.0407
6	2.7375	0.8001	1.2554	10	0.0962
7	2.3966	0.3219	0.7973	10	0.0303
8	2.8617	0.3684	0.7567	11	0.0330

TABLE III

COST-STRENGTH DATA FOR CONSTANT STIFFNESS FRAMES

Frame No.	Plastic Load Factor	Critical Elastic Load	Maximum Load Factor	Frame Cost $ US	Strength/Cost Ratio ($\times 10^{-5}$)
1	2.3514	61.80	2.265	702	323
2	2.9393	61.80	2.806	804	349
3	2.8098	61.80	2.688	758	355
4	2.4417	61.80	2.349	749	314
5	2.4893	61.80	2.393	755	317
6	2.7375	61.80	2.621	778	337
7	2.3966	61.80	2.307	726	318
8	2.8617	61.80	2.735	782	350

TABLE IV

COST-STRENGTH DATA FOR CONSTANT STIFFNESS FRAMES OF UNIFORM MATERIAL

Frame No.	1	2	9	10
Material Yield (ksi)	36	45	50	55
Material Cost ($/ton)	111	127	131	138
Frame Cost ($)	702	804	830	873
Plastic Failure Load Factor	2.3514	2.9393	3.2658	3.5890
Elastic Critical Load Factor	61.80	61.80	61.80	61.80
Maximum Load Factor	2.265	2.806	3.102	3.392
Strength/Cost Ratio	323	349	374	389

TABLE V

SECTIONS OF APPROXIMATELY EQUAL STRENGTH

Section	Plastic Moment in 55 ksi steel (kip-ft.)	Plastic Moment in 36 ksi steel (kip-ft.)
18 WF 55	511.5	
18 WF 60	561.9	
10 WF 60	344.2	
21 WF 73		516.3
21 WF 82		574.8
10 WF 89		343.2

TABLE VI

RESULTS OF PLASTIC ANALYSES FOR FRAMES OF APPROXIMATELY CONSTANT STRENGTH

Frame No.	Construction Details Sections - Material in - (ksi)				Conditions at Development of First Plastic Hinge		
	Top Beams	Lower Beams	Top Columns	Lower Columns	Load Factor	Max. Vert. Defln.(ft.)	Max. Sway Defln.(ft.)
10	18WF55 55 ksi	18WF60 55 ksi	10WF60 55 ksi	10WF60 55 ksi	2.7968	0.1214	0.3679
11	18WF55 55 ksi	18WF60 55 ksi	10WF89 36 ksi	10WF89 36 ksi	2.8140	0.1062	0.3038
12	21WF73 36 ksi	21WF82 36 ksi	10WF60 55 ksi	10WF60 55 ksi	2.9233	0.0846	0.2933
13	21WF73 36 ksi	21WF82 36 ksi	10WF89 36 ksi	10WF89 36 ksi	2.8663	0.0721	0.2244

Frame No.	Conditions at Development of Last Plastic Hinge			Total Number of Plastic Hinges	Maximum Plastic Hinge Rotation at Failure (r)
	Load Factor	Max. Vert. Defln.(ft.)	Max. Sway Defln.(ft.)		
10	3.5890	0.3378	1.4647	11	0.0712
11	3.5874	0.4198	1.1881	11	0.0503
12	3.6420	0.5196	1.0865	11	0.0589
13	3.6404	0.3468	0.8330	11	0.0401

TABLE VII

COST-WEIGHT-STRENGTH DATA FOR FRAMES OF APPROXIMATELY EQUAL STRENGTH

Frame No.	Plastic Load Factor	Critical Elastic Load Factor	Maximum Load Factor
10	3.5890	61.80	3.392
11	3.5874	79.13	3.432
12	3.6420	80.13	3.485
13	3.6404	107.46	3.521

Frame No.	Frame Weight (kip)	Frame Cost ($)	Strength/Weight Ratio ($\times 10^{-2}$)	Strength/Cost Ratio ($\times 10^{-5}$)
10	12.66	873	268	389
11	15.44	950	222	361
12	15.06	913	231	382
13	17.84	990	197	356

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