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ELASTIC-PLASTIC ANALYSIS OF FRAMES - INCLUDING AXIAL FORCE EFFECT ON MOMENT CAPACITY

by

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ABSTRACT

A method is presented for analyzing the elastic-plastic behavior of frames which account for the reduction of plastic moment capacity due to the presence of axial force. A computer program, based on the one originally developed by Wang and modified by Harrison, is presented. While the original program could produce the elastic-plastic load-deflection curve for a frame, it could not account for the gradually decreasing bending capacity of the frame members which resulted from the increasing axial force present in them. The reported program has been modified to include this effect.

An explanation of the basic method used by the original program is presented. Following a discussion on the interaction of axial force with moment capacity, the new method is presented and the required modifications to the original computer program are described.

Explanation of input data and sample runs have been included.

Some possibilities for design use of the program are presented and discussed in detail.

Problems and limitations encountered in the use of the program are discussed and several suggested solutions to these difficulties are outlined.

1. INTRODUCTION

When a structural steel section is subjected to some axial force P, its full plastic moment, M_p, is reduced to some lesser value which is called the reduced plastic moment and is denoted by M_{pc}. The relation between P and M_{pc} can be determined analytically for most cross-sections.[1,2] Consequently, if P has some known value and if the relationship between the loads applied to the member and the moments induced in it is known, then the load required to form a plastic hinge at some point in the member can also be determined. (When the moment in a member reaches M_p (of no axial force is present) or M_{pc}, a plastic hinge is said to have formed. Rotation can occur with no change in moment.)

However, if this member is a part of some frame its axial force will usually be some complex function of the loads on the structure. The problem of finding the frame load required to form a plastic hinge at a particular location therefore becomes much more involved. A direct solution to this problem is difficult and a trial-and-error procedure becomes the logical approach in the analysis.

A method is presented in this report for determining the load-deflection history of a frame, accounting for this axial force effect.

For the purpose of explanation and illustration the simple one-bay, one-story frame shown in Fig. 1a will be considered. This frame is used to approximate the behavior of the bottom story of the six-story frame in Fig. 1b.

Each column top load of 5W corresponds to one-half of the gravity loading on the upper stories of the larger frame. The lateral load used will be some percentage of the total gravity load. Designation of a particular percentage fully defines the relative proportions of the lateral and gravity loads.

The analysis reported is based upon a method for performing a first-order, elastic-plastic analysis of a general plane frame which ignores the effect of axial load. The method was originally developed by C. K. Wang[3] of the University of Wisconsin and later modified by H. B. Harrison[4] at Lehigh University. The approaches to the elastic-plastic analysis developed by Wang and Harrison relied heavily on the use of a computer as does the approach presented here. However, an understanding of the mechanics of the computer program used here is not necessarily a pre-requisite to an understanding of the method. The report is organized with this purpose in mind. In the second chapter the theory of the simple-plastic computer analysis as Wang and Harrison developed it is presented. The chapter also explains the interaction between axial force on a member and its moment capacity. Chapter 3 deals with the effect that axial force has on the analysis of a frame.

The computer program used in the analysis is presented in chapter four. Examples of input data and output results are discussed and some of the major difficulties encountered are mentioned. The fifth chapter contains an analysis of the example frame mentioned above. Much of the data obtained from the computer program is explained and discussed.

It is hoped that this explanation along with the accompanying examples will enable one who is unfamiliar with computer methods to make intelligent use of the program. Also, for the benefit of those who may wish to attempt further modifications and for those who have some programming background and desire more detailed information, a flow diagram for the main program is contained in Appendix A. A listing of each subroutine used in the Fortran program is contained in Appendix B.

2. BASIS OF ANALYSIS

2.1 THE ELASTIC, SIMPLE-PLASTIC ANALYSIS

The elastic simple-plastic analysis, as developed by Wang and Harrison, provides the foundation for the method of analysis reported here. A schematic load-deflection curve, typical of Harrison's solution is shown in Fig. 2, where H is the lateral load and Δ_H is the sway deflection of the column tops.

The method begins with an elastic analysis of the frame which determines the slope of line $\overline{OA}$. Point A represents the formation of the first plastic hinge in the frame and it is located as follows. At each of i possible plastic hinge locations in the frame, the moment corresponding to a given load can be expressed as:

$$M_i = m_i H \tag{1}$$

in which m_i is the moment resulting from the application of a unit load to the frame and H is the applied load. If axial load is assumed to have no effect, a plastic hinge will form at a point when $M_i = M_{pi}$. The location and load corresponding to the formation of the first plastic hinge can be determined by substituting M_{pi} for M_i in Eq. 1, solving for H, and then selecting the lowest value.

The analysis continues by inserting a real hinge in the frame at the location of the plastic hinge (shown as (1) in Fig. 2) and performing another elastic analysis. In this way, new values of m_i are found, and the slope of the second segment is determined. A new version of Eq. 1 can be written in order to locate point B which corresponds to the formation of the second plastic hinge.

$$m_{i1} H_1 + m_{i2} H_2 = M_i \tag{2}$$

Double subscripts have been introduced to the m's to indicate which point or location on the frame they refer to (i) and which elastic analysis they result from (1 or 2). For values of load between H_1 and H_2 (see Fig. 2), Eq. 2 computes the moment at each point. (Note that $m_{i2} = 0$ for the first plastic hinge location due to the presence of a real hinge there). As before, substituting M_{pi} for M_i, solving for H_2, and selecting the lowest value will locate point B on the plot.

The procedure of analyzing a successive series of elastic frames with real hinges at the plastic hinge locations is continued until a failure mechanism forms, as indicated by a horizontal line for some appropriate load-deflection plot for the frame.

In general, the equations which govern the loads corresponding to the various stages of hinge formation and the locations of these hinges can be written in the following form.

$$\begin{aligned}
m_{1,1} H_2 + m_{1,2} H_2 + \ldots\ldots m_{1,h-1} H_{h-1} + m_{1,h} H_h &= M_{p1} \\
m_{2,1} H_1 + m_{2,2} H_2 + \ldots\ldots m_{2,h-1} H_{h-1} + m_{2,h} H_h &= M_{p2} \\
& \qquad\qquad (3) \\
m_{a,1} H_1 + m_{a,2} H_2 + \ldots\ldots m_{a,h-1} H_{h-1} + m_{a,h} H_h &= M_{pa}
\end{aligned}$$

in which h is the number of the plastic hinge under investigation and a is the number of possible hinge locations for the frame.

The values of H are known between H_1 and H_{h-1} inclusive by previous calculations. Also, once H_1, H_2, . . . H_{h-1} are found, their values do not change as additional load is applied. Therefore, Eqs. 3 are a series of equations in one unknown with each equation resulting in some value for H_h. Comparison of these values determines the location of the new hinge. A similar series must be examined for each new hinge. Because the values of H_1, H_2 . . . H_{h-1} do not change after they are first determined, one pass up the load-deflection curve is sufficient to evaluate the complete load-deflection history of the frame. In other words, once a plastic hinge forms at some point, it is not influenced by any additional load that may be placed on the frame at a later time.

It should be noted that regardless of the magnitude of the column top loads in Fig. 2 they have no effect on the capacity of the frame when it is analyzed in the above manner, since it is assumed that the presence of axial load in a member has no effect on its moment capacity. The validity of this assumption is investigated next.

2.2 THE EFFECT OF AXIAL FORCE ON MOMENT CAPACITY

If a member is subjected to some axial force P, its available moment capacity M_p is reduced. This reduced value of M_p, defined as M_{pc}, can be determined analytically, and for most sections, curves and equations have been developed which relate P to M_{pc} once M_p and the yield load P_y are known.[1,2] For most wide-flange shapes bent about their major axis, such curves fall in a relatively narrow band, and simple expressions can be used to approximate them.

Figure 3 shows a non-dimensional plot of the moment versus axial force relationship for strong-axis bending of wide-flange shapes. The solid curves represent the upper and lower limits of the exact interaction relations for wide-flange shapes.[2] The dotted straight lines are the usual approximations for these curves as suggested in Ref. 1. The approximate curves are expressed analytically by the following equations:

$$M_{pc} = M_p \qquad (0 \leq P \leq .15\ P_y) \qquad (4)$$

$$M_{pc} = 1.18\ M_p\ (1 - P/P_y) \qquad (.15\ P_y < P \leq P_y) \qquad (5)$$

Note that Eq. 4 indicates there is no reduction of moment capacity due to axial force if P is less than 0.15 P_y. The limits imposed on the application of Eq. 4 therefore provide an upper bound to the axial loads which may be present in a structure if an elastic simple-plastic analysis of it is to be valid. If the axial loads reach some value greater than 0.15 P_y Eq. 5 must be used and various modifications in the analysis are required. These modifications are the subject of the next chapter.

3. INCLUSION OF THE AXIAL FORCE EFFECT IN A FRAME ANALYSIS

The effect of axial force can be included in the analysis by modifying the equations which govern the formation of plastic hinges in the frame. Previously, all plastic hinges had to satisfy the requirements imposed by Eq. 3. If the effect of axial force is considered, these requirements are applicable only to those plastic hinges which form under an axial force less than 0.15 P_y, as is shown by Eq. 4. All other plastic hinges must satisfy Eq. 5.

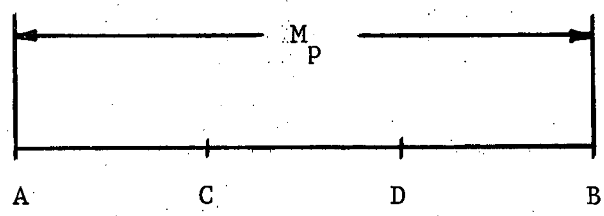
The generalized forms of Eqs. 4 and 5 are

$$\sum_{i=1}^{h} m_i H_i = M_{pc} = M_p \qquad \text{for } 0 \le P \le .15\ P_y \tag{6}$$

$$\sum_{i=1}^{h} m_i H_i = 1.18\ M_p \left[1 - \frac{\sum_{i=1}^{b} n_i H_i}{P_y} \right] \qquad \text{for } .15\ P_y < P \le P_y \tag{7}$$

where n is the unit axial force in the member, b is the number of the point on the load-deflection curve which is currently being computed, and h is the number of the plastic hinge the equation is being applied to. Note that b is always greater than or equal to h. The reason for this will be seen below. Equation 6 was presented and discussed in Chapter 2 (as Eq. 3), but some additional explanation will be of value in understanding its function in the modified analysis.

Let the line AB below represent the plastic moment capacity M_p at some point in a structural member whose axial force never exceeds $0.15\ P_y$.



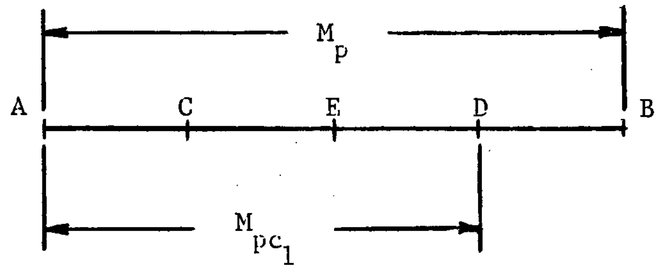
As load is placed on the structure, a bending moment equal to the unit moment m times the applied load will be induced at the point. The line segment AC is the moment present at the point (m_1 times H_1) when the first plastic hinge forms in the frame. (The first plastic hinge is assumed to form somewhere other than at the point in question.) Additional increments of load on the frame will result in similar additions to the moment at the point, such as CD, which is equal to m_2 times H_2. If the point in question is involved in the failure mechanism for the frame, the bending moment at the point will finally reach M_p and a plastic hinge will form. At this stage in the loading, Eq. 6 is satisfied as shown below:

$$m_1\ H_1 + m_2\ H_2 + m_3\ H_3 = M_p \qquad (8)$$

$$(AC + CD + DB = AB)$$

Axial force had no effect in the above case because it was less than the critical value of $0.15\ P_y$.

If the axial force at some point exceeds $0.15\ P_y$, the behavior of the point through various stages of loading can be traced with the aid of the diagram below.



As before, let the line AB represent M_p for the point. When the first increment of load is applied to the frame, some moment and some axial load will be induced at this location. Because the axial force P is assumed to exceed 0.15 P_y, some reduction in M_p will take place. When the first plastic hinge forms in the frame, conditions at this point are represented by a reduction in capacity DB due to axial force and a bending moment at the point equal to AC. Of the remaining bending capacity that portion which is usable (M_{pc1}), is now CD rather than CB as in the previous case. If the second plastic hinge forms at this location, the next increment of load will cause the added moment CE and reduction in capacity due to axial force ED to meet at point E thereby satisfying Eq. 7 for this location in the frame. As more load is applied to the frame, no additional moment will occur at this location because a real hinge is inserted at the location of each plastic hinge for the purpose of each subsequent elastic analysis. Hence all further unit moments for this point will be zero. This is not the case for the unit axial force at this location. In most cases, the unit axial force will continue to have some value other than zero until the mechanism load is reached. Unless the formation of a hinge at the point under discussion resulted in the formation of a failure mechanism, additional frame loads will be added to those already present. In order to reach this mechanism load Eq. 7 shows that at the location in question, the additional axial force resulting from this increase in frame loads causes M_{pc} to decrease. This in turn requires adjustment of segments AC and CE in order to maintain equilibrium at the point.

In summary the analysis of a frame can be thought of as the determination of a series of points on its load-deflection curve. Each of these points represents the formation of one new plastic hinge. The curve becomes horizontal and the analysis terminates when enough hinges have formed to produce a failure mechanism. At each of the points on the load-deflection curve, all the plastic hinges then formed must satisfy either Eq. 6 or Eq. 7 depending on the value of axial force at each point. Those for which Eq. 7 is applicable experience a weakening or an inability to carry their previous moment due to an increase in axial force as more load is applied to the frame. Hence, as each of the plastic hinges form, a readjustment or a redistribution of moment must take place in the frame to account for the weakening of the previous hinges. This redistribution must be such that each hinge will satisfy its pertinent equation. This redistribution can be accomplished in many ways.

One method is to apply some internal moment to each plastic hinge that is effected by axial force. This internal moment would be equal in magnitude to the reduction in moment capacity at each point due to the increase in axial force. Because the value of these internal moments cannot be determined until the increment of load to form the next plastic hinge is determined and because the increment of load cannot be determined until the internal moments are known, the problem is non-linear, and a trial-and-error procedure becomes the most logical approach for computer analysis. This method has the disadvantage of introducing to the frame an additional degree of freedom for each internal moment applied to the frame. If a computer-matrix solution based on the displacement method is used, the introduction of additional degrees of freedom to the frame will use up a noticeable percentage of the available computer storage space and may severely limit the size of frame which can be handled by a particular machine.

A second approach, and the one which is used by the present analysis, is to make successive adjustments on the various increments of load between the formation of plastic hinges until either Eq. 6 or Eq. 7 is satisfied at each plastic hinge location. This procedure can be exemplified by reference to Fig. 4. The upper curve in Fig. 4 is a schematic of the elastic, simple-plastic load-deflection curve for the frame shown where the effect of axial force on bending capacity is neglected. Points A_o, B_o, C_o, and D_o represent the formation of plastic hinges on the frame at points 1, 2, 3, and 4 respectively. For the purpose of explaining the method of solution, it will be assumed that the axial force does not alter the order of formation of the plastic hinges from that of the elastic, simple-plastic analysis. For the most part this is a correct assumption for simple frames. However, in complex frames involving a large number of hinges in the failure mechanism, it may not always be true. The solution begins with an elastic analysis of the original unaltered frame which produces unit moments and unit axial loads for each of the possible hinge locations on the frame. Equation 7 is applied to each of these points and a value of load required to form a plastic hinge at each is determined. If axial force is found to be less than 0.15 P_y, Eq. 6 is substituted for Eq. 7. In an actual analysis, the magnitude of P is checked against 0.15 P_y for the location involved and the proper equation is applied at each hinge. For this explanation, P will be assumed greater than 0.15 P_y for all members. The smallest of these loads determines that the first plastic hinge forms at point 1 on the frame and at a load corresponding to A_1 on the plot. The axial force effect lowers A from A_0 to A_1. A real hinge is then inserted at location 1 and a second elastic analysis is performed to determine the slope of the next segment. On the basis of Eq. 7, point B_1 is found and the second plastic hinge forms at location 2 on the frame. The increase in load in moving from A_1 to B_1 increases the axial force at location 1 and thus reduces the moment capacity at that point. In order to satisfy Eq. 7 at location 1, P_1 must decrease to the value represented by point A_2. Point B is computed again and the cycle is repeated until Eq. 7 is satisfied at points A and B (Locations 1 and 2) simultaneously. Once agreement is obtained for both points, point B is recorded as a point on the actual load-deflection plot. Next, a third elastic analysis on a frame with two real hinges is performed and the process repeats. Successive hinges are introduced until a mechanism is formed, ending the analysis.

Figure 5 is a schematic plot comparing the curves produced by the elastic, simple-plastic analysis of Wang and Harrison (dashed) and the analysis including the effect of axial force (solid). The general trend is for axial force to decrease the slope of each segment on the curve. Straight lines are not strictly correct for the lower curve beyond the first segment. Each segment is a curve which starts at the lower point, tangent to the elastic slope for the frame (the slope of the corresponding segment of the upper curve) and gradually curves to reach the next higher point. The curves have been drawn straight because the information which is necessary to produce the exact line is not produced by the analysis presented in this report.

4. THE COMPUTER PROGRAM

4.1 INDEXING SYSTEMS

To write a successful computer program for even the simplest of problems it is necessary to institute some convenient and logical system of identifying the quantities involved. Some of the systems used by the program are shown in Fig. 6.

Figure 6a shows the method of identifying members in a frame. Each point of load application is considered to be a joint in the frame and as a result, a beam subjected to quarter point loading is broken down into three members. (Intermediate supports in a continuous beam would be treated in the same manner.) Each member is assigned a number (circles on Fig. 6a) as is each member end (squares on Fig. 6a). The even end number is equal to twice the member number and the odd end number is taken as one less than the even number.

These member end numbers are used to identify the internal forces acting on the member ends. The sign convention for these internal forces (Fig. 6b) is moment positive counter-clockwise on the member end and axial force positive in tension. The choice of a sign convention is purely arbitrary and any consistent convention can be used. The numbering of these end forces is exemplified in Fig. 6b. The member numbering system is also used to identify which location the unit moments and axial loads produced by each elastic analysis pertain to. These unit quantities are indexed in exactly the same manner as the member end forces and the same sign convention is followed.

The second indexing system, shown in Fig. 6c, arises from the need to identify deflections and loads. Each joint, unless it is restrained, has three degrees of freedom; a horizontal and vertical deflection and a rotation. For each degree of freedom there is a corresponding load whose point of application and direction coincides with the possible movement.

Although loads will not usually be acting at all possible locations on the frame at one time, the matrix methods used require space to be reserved for all possibilities. Hence, the numbering system for loads and deflections is the same. Figure 6c shows the deflections and loading possibilities for the example frame. There are four vertical, four horizontal, and four rotational deflections possible as well as their corresponding loads, and they are indicated in their positive directions. The sequence of numbering is completely arbitrary although listing all vertical, all horizontal, and all rotational quantities in sequence lends order to the output.

The step-wise nature of the analysis provides the need for a third indexing system. Each of the initial elastic slopes on the load-deflection plot is produced by an elastic analysis which also produces the unit moments and axial forces mentioned previously as well as the unit deflections. Each of these pieces of data must be associated with a location on the frame and with the analysis which produced it. For example, a unit moment is of no value unless it is known that it applied to location number 2 and that it was produced by an analysis of the frame with plastic hinges formed at locations 4 and 7. Hence, two subscripts will be necessary to completely identify unit axial loads, moments, and deflections. The first subscript will be the number of the location (2 for the above example) and the second would be a 3 if the analysis of a frame with plastic hinges at locations 4 and 7 is the third one to be performed.

A fourth system of subscripts evolved partly because of the trial-and-error system employed in the analysis and the resulting necessity for the program to be able to refer to previous results, and secondly because of a need for some mechanism to terminate the run if convergence could not be obtained in a reasonable number of tries. The procedure adopted in the program could be improved on to increase its efficiency. However, the method used enjoys the advantage of simplicity over other possibilities.

This fourth system introduces a second subscript to the increment of applied load H indicating which trial number it is a result of. (The first subscript refers to the segment of the load-deflection curve it pertains to.) For example, on the first try at satisfying Eq. 6 and/or Eq. 7 at each of three plastic hinge locations, $H_{(1,1)}$, $H_{(2,1)}$, and $H_{(3,1)}$ would be produced, while the second try would result in $H_{(1,2)}$, $H_{(2,2)}$, and $H_{(3,2)}$, and so on.

4.2 INPUT DATA

After labeling the various quantities in the manner described in Section 4.1, the data required for an analysis can be assembled.

The program is designed to accommodate more than one analysis with each run so at the start of each analysis an identifying frame number JJ is read in. This number may be any fixed point Integer from 0 to 99999. It is used as an identifying number which appears on the first page of the output for each analysis. In addition, after the data for the last frame to be analyzed, a negative frame number should be included. This will serve as a trigger to terminate the run.

Next, in order that the program might construct internally several of the arrays required by the analysis, two more fixed point integers are input. They are the degrees of freedom L and the number of members NM. (For the example frame L and NM would be 12 and 5 respectively.) From NM two other quantities are found which are also used internally for matrix operations:

$$M = 2 \times NM \tag{9}$$

$$N = 3 \times NM \tag{10}$$

where M is the number of member ends in the frame and N is the number of internal forces in the frame.

The statics matrix A which relates the applied loads on the frame to the internal member end forces follows L and NM as input data. The matrix is determined by knowing that for each degree of freedom in the frame an equation of equilibrium can be written.

For example (see Fig. 7) the upper left column top joint on the frame of Fig. 6 produces 3 equations.

$$\Sigma F_x = 0$$

$$W_5 = \frac{M_2 + M_1}{\ell_1} - P_2 \qquad (11)$$

$$\Sigma F_y = 0$$

$$W_1 = P_1 + \frac{M_3 + M_4}{\ell_2} \qquad (12)$$

$$\Sigma M = 0$$

$$W_9 = M_2 + M_3 \qquad (13)$$

Writing equilibrium equations similar to Eqs. 11, 12, or 13 for each degree of freedom in the frame and presenting the results in matrix form results in the statics matrix of Table 1. The size of this matrix is (L x N).

The development of the statics matrix for a non-rectangular frame such as a gable frame is more involved, but the same techniques apply.

A list of section properties for each member in the frame follows the statics matrix. The four variables required are indicated and defined below:

$$\begin{aligned} SDAT &= EI/\ell \\ PM &= M_p \\ EAOL &= EA/\ell \\ PY &= P_y \end{aligned} \qquad (14)$$

where E = modulus of elasticity, I = moment of inertia, A = cross-sectional area, and ℓ = member length.

A load set number, KK, is next in the input data. It is treated in much the same manner as was the frame number JJ above. KK is used to identify a particular load set on the output and if it is negative, the program assumes there are no additional load sets to be applied to the current frame and goes to the beginning to see if there is another frame to be analyzed. As mentioned previously, should it encounter a negative frame number there, it will exit from the program. Following KK is a column matrix (L x 1) of the applied loads. The load matrix (PL) for the example frame subjected to gravity loads at the quarter points of the beam and a horizontal force to the right (equal in the magnitude to the gravity loads) applied at the left column top appears in Table 2. Note that all possible applied forces must appear in the array even if they have no value for the given loading case.

For a particular frame, any number of load sets may be applied during a single run with a negative KK as the final card in the deck of load sets. A sample deck setup for the analysis of two frames with two sets of applied loads each is shown in Table 3.

Experience with the Lehigh GE225 computer has indicated that a card with "END" appearing in the first 3 columns should be the last card in the data deck. With the large number of data cards required for each analysis, it is quite conceivable that one may be lost causing the machine to read into the next analysis or even the next program in its attempt to obtain the required amount of data. Encountering any card with alphaneumeric characters on it will halt the analysis.

In the preparation of the data for an analysis, care should be taken to use consistent units throughout. All of the numerical examples in this report are based on kips and inches.

4.3 PROGRAM OUTPUT

Copies of the program and some example outputs are contained in their entirety in B and C. Most questions concerning specific difficulties can be resolved by referring to this area. However, some general explanation is in order and this can be found in Table 4.

4.4 PROBLEMS

4.4.1 Storage Space

The program storage requirements severely limit the size of frame which can be accommodated on all but the largest machines. Several alternative solutions to this limitation are available.

The simplest and perhaps most obvious move would be to use a larger capacity machine. However, this is not always possible or desirable. It seems that regardless of the available machine capacity, there will always be frames of interest which exceed this capacity. Hence, several things have been done to the program to economize on the available capacity within a particular machine.

One way to eliminate wasted storage is to utilize common storage. Ordinarily if each of two subroutines use a variable of the same name, it is considered by the machine to be two separate and discrete quantities in each of the subroutines. If it is desired to transfer the value of this variable from one subroutine to the other, it must be listed in the subroutine call statement as an argument. As a result, although both subroutines are using the same variable, two storage locations are required in the machine memory. If this variable is an array of 100 points, 200 locations would then be required. By listing the variable in identical common statements at the beginning of each subroutine, each subroutine will store the value or values of the variable in the same or common storage locations so they will be freely available to each.

Hence, one location is all that is necessary to store one variable. The program contained in this report uses nine subroutines and a main program. The use of common statements reduces its data storage requirements to one-tenth of that required previously.

Additional economy may be obtained through the use of magnetic tapes. With most installations, it is possible to store segments of a program on tape until they are called for by other segments of the program. It is also possible to store data on tapes for future use. Although, conceptually it is possible to use both of these mechanisms simultaneously to reduce storage requirements, most machines lack the necessary hardware to accommodate the large number of tapes required and only one can usually be used. The reported program uses tapes for data storage. On the GE225 computer, three tapes are needed to accomplish this, which, combined with the systems tape which controls the program execution, equals the total tape capacity of the machine.

Of the methods used to fit a particular size frame into the storage available within a given machine, the above mentioned ones are concerned primarily with making more efficient use of a machine through modifications to the program and its use of machine hardware. However, it is possible in some cases to decrease the size of the data arrays which must be stored. If some beforehand knowledge can be obtained, either through hand calculations, another program, intuition, etc., of the location of the plastic hinges in the failure mechanism, the loads on members which are not directly involved in the mechanism can be replaced with fixed end moments and shears. This, by eliminating some degrees of freedom, will reduce the size of all the arrays involved and may mean the difference between running a problem and not running it on a particular machine.

4.4.2 Change in the Order of Formation of Plastic Hinges

As was mentioned in Chapter 3, during the trial-and-error procedure to satisfy either Eq. 6 or 7 at each plastic hinge location on the frame, successive adjustments are made in each of the legs of the load-deflection curve. If, for example, three plastic hinges are present in the frame, the first to form and the lowest on the load-deflection curve is in the beam (a member insensitive to axial load), the second is in the column (a member sensitive to axial load) and if the point which is being investigated for being the third and next hinge to form is a location with a large amount of remaining moment capacity, the following is likely to occur. With each trial, the location of the first point on the plot will not vary noticeably because of its comparative insensitivity to axial load. The second point on the plot will however be lowered due to the increasing axial load at its location resulting from the third increment of load. It is conceivable that the length of the second segment could be reduced to zero or a negative value. Zero indicates that plastic hinge Nos. 1 and 2 now form at identical loads. A negative value for the second segment indicates that now the plastic hinge that previously formed second will form first. In other words, there has been a change in the order of formation of the hinges. This, however, does not imply that if the frame were to be tested the plastic hinges would form in this new order but rather that the equation governing the second hinge now is satisfied at a lower load than that which satisfies the equation governing the first hinge.

The solution to this difficulty can be shown with the aid of Fig. 8. At the point where this change in hinge order occurred, frames A, B, and C had been analyzed. Traveling up the load-deflection curve entailed utilizing the unit moments and axial loads, etc. from these elastic analyses of the frames to satisfy the governing equations at each plastic hinge. (Eqs. 6 and 7) Now, with the negative second segment and with the resulting new order of formation for the hinges, a new frame must be analyzed (Frame E in Fig. 8). After this frame is analyzed the regular analysis can proceed using the unit values of frames A, E, and C in the usual fashion. If by chance the second segment had become exactly zero, analyses of frames A and C would be sufficient and no new frame analysis would be required.

One of the major difficulties involved in this procedure is how to identify a given frame with hinges at various locations and how to record whether or not it has been previously analyzed. This may seem to be a trivial problem until one considers that for the example frame, with an assumption that four hinges would be required in most failure mechanisms, the computer could be called on to analyze any of more than 50 different frames. In view of the storage problems mentioned above, numbering all 50 would use an excessive number of machine storage locations. The solution to this problem would be based on the creation of a column matrix whose number of columns equals the number of hinges in the failure mechanism (more if the exact number is unknown). Each row would record the analysis of one frame. If hinges formed at locations 4, 7, and 11 (in order), the first four rows would be as shown below:

0	0	0	0
4	0	0	0
4	7	0	0
4	7	11	0

This array indicates that the first frame analyzed contained no plastic hinges, the second contained one at location 4, and so on.

An element by element search of this matrix would determine whether or not a given frame had been analyzed and, if it had been, which one it was (first, second, third, or fourth analysis). Knowing the number of the frame in which certain plastic hinges were present would allow the program to locate the unit axial forces, unit moments, and unit deflections pertaining to that frame. In summary, once a change in hinge formation occurs, reference to the above matrix would determine if the new required frame (Frame E in Fig. 8) had been solved. If not, it would be analyzed and that fact would be recorded in the matrix. If it had been previously analyzed and recorded the analysis could continue immediately using the previously computed information. From the standpoint of storage, note that this matrix would require approximately twenty-four locations to record all pertinent data as compared to fifty or more locations if all possible frames which could be analyzed were simply numered. The occurrence of the hinge reversal phenominon increases rapidly as larger frames are encountered, and it is in this area that the ease of identifying frames with this matrix becomes most apparent. For frames only slightly larger than the example frame, the matrix can be expected to require less than one-quarter as many storage locations as there are possible frames to be analyzed.

5. RESULTS AND DESIGN CHARTS FOR THE EXAMPLE FRAME

Much of a computer program's value lies in its ability to perform a large number of computations in a short period of time. The program contained in this report produces a complete load-deflection history of the example frame under some prescribed loading in approximately five minutes on the GE225 machine. Therefore, with only a minimum of effort on the part of the operator, curves can be developed which, for a given frame, will completely predict the behavior of the frame throughout its range of loading possibilities. The development of such curves for the example frame is the subject of this chapter.

5.1 BOUNDS ON POSSIBLE FRAMES

If the only limitation imposed on the construction of a frame is that its geometry must coincide with that of the example frame, an almost unlimited number of such frames could be constructed from the various rolled steel wide-flange shapes available.[5] However, many of these possible frames would behave in a similar fashion when subjected to the same loading. Therefore, if some bounding cases could be analyzed, nearly all behavior possibilities could be predicted with a minimum of effort.

In order to pick such bounding cases, it is helpful to use some parameters which indicate some of the properties of a given frame.

The first of these is the ratio of the column stiffness to beam stiffness, G, as shown below[6]

$$G = \frac{I_c/\ell_c}{I_b/\ell_b} \tag{15}$$

where I_c = the moment of inertia of the column, ℓ_c = length of the column, I_b = moment of inertia of the beam, and ℓ_b = length of the beam. The second parameter is the slenderness ratio, ℓ/r_x where r_x = the radius of gyration of a section about its major axis.

For most building frames, Ref. 6 places the upper and lower limiting values of G at 3.0 and 0.5 respectively. For each of these values of G, several combinations of ℓ/r_x for the beam and column are possible. Using some common values of ℓ/r_x, Fig. 9 represents some theoretical possibilities for each extreme of G.

In general, for most frames and in particular for the example frame, many of the possible combinations of G, ℓ/r_x for the column, and ℓ/r_x for the beam cannot be constructed with the rolled shapes currently available. For instance, for the example frame with ℓ = 180", a G of 0.5 would require that

$$I_b = 4\ I_c \tag{16}$$

and if,

$$\ell_c/r_{x_c} = 30 \tag{17}$$

and

$$\ell_b/r_{x_b} = 25 \tag{18}$$

Then

$$r_{x_c} = 6.0 \tag{19}$$

and

$$r_{x_b} = 14.4 \tag{20}$$

The "Plastid Section Modulus Table" of Ref. 5 shows that there are no rolled sections with $r_x \approx 14.4$ whose moment of inertia equals approximately four times the moment ofinertia of any section with $r_x \approx 6.0$. (The sections listed in the "Plastic Section Modulus Table" have been rearranged and listed for convenience in Table 5 in order of their radius of gyration.) Similarly, several other combinations of ℓ/r_x and G are impossible to construct for the example frame and all such cases are indicated by crosses in the appropriate boxes in Fig. 9.

Frames satisfying the conditions corresponding to each of the remaining boxes in Fig. 9 can be constructed and two typical cases are indicated by circles. The analysis of these two cases will provide the bounds mentioned above on the behavior of the example frame.

5.2 RESULTS FOR EACH CASE

Figure 10 shows the two frames represented by the circles of Fig. 9. The case of G = 0.5 will be referred to as the strong beam-weak column case while for G = 3.0 the term strong column-weak beam will be used. The sections indicated on the frames of Fig. 10 were chosen by the procedure explained above (Eqs. 15 to 20).

As shown in Fig. 1 the example frame is assumed to be the bottom story of a six-story frame which results in a 5:1 ratio between column top loads and beam gravity loads. The ratio of a point load on the beam to the side load is given by β. For the six-story frame the side load is given as a percentage of the total vertical load by $100/12\ \beta$.

Figures 11 and 12 are plots of horizontal load versus horizontal deflection of the column tops for several loading conditions on each frame. Each curve represents a different ratio of horizontal to vertical loading as indicated by the values of β. The horizontal load, as a percentage of the total vertical load, is also shown.

Each point shown on each curve corresponds to the formation of a plastic hinge in the frame. The location and order of formation for these hinges is shown on the small diagram of the frame at the end of each curve. The numbers located next to each plastic hinge indicate the sequence of formation.

Note the preponderence of sway mechanisms for the strong beam-weak column case and the corresponding presence of beam mechanisms for the strong column-weak beam case. The relative strengths of the beam and column obviously play a key role in the determination of the type of failure mechanism.

In Fig. 11 the case of H = 1% of the gravity load was not produced by the program. This is because, by coincidence, the hinge reversal phenomenon of Section 4.4 occurs for this particular ratio of loads. This is evidenced by comparing the order of hinge formation for $\beta = 4.16$ and $\beta = 16.7$. By introducing slightly different values of β than that corresponding to the 1% case, close approximation to the 1% line could be obtained. However, this was not attempted for this figure.

In Fig. 11 the analysis of the frame with $\beta = 16.7$ produced some unusual results. The third and fourth segments of the load-deflection plot for this case indicate an increase in frame stiffness over that shown by the second segment. This would not normally be expected because the introduction of each successive plastic hinge to the frame brings it closer to failure by removing one degree of indeterminancy. As a result the stiffness of the frame will usually be reduced with the formation of each new plastic hinge. However, in this case, the results produced by the program are correct and they can be explained with the aid of Fig. 13.

The four frames of Fig. 13 are the frames corresponding to each of the four segments of the load-deflection plot of Fig. 11 for $\beta = 16.7$. Because a symmetrical frame subjected to symmetrical gravity loading (W) will experience no sidesway, the unit deflection (δ_H) at the column tops of such a frame is a function of horizontal load (H) only as is shown by frames (A) and (C) of Fig. 13. For an unsymmetrical frame, such as (B) or (D) of Fig. 13 symmetrical gravity load will cause some sway. As a result, the unit column top deflection (δ_H) for such a frame will be a function of both lateral (H) and gravity (W) loads.

The frames analyzed to produce the lower curve of Fig. 11 ($\beta = 16.7$) alternate between the symmetrical and unsymmetrical cases shown by Fig. 13. Hence, only the second and fourth segments of the load-deflection curve are functions of both the gravity and lateral loads.

If the relative contribution of W to δ_H is much higher than that of H as would be the case for the frame of Fig. 11 with H = 1/2%(W), the value of the unit deflection δ_H for the second analysis (frame B of Fig. 13) would be greater than that for the first analysis (frame A of Fig. 13). A flat slope would appear on the H - Δ_H plot. Following a similar line of reasoning, it can be seen that the slope of the third segment should increase from that of the second segment, which it does.

Note that for the cases of β = 2.08, 2.78, and 4.16 in Fig. 11, none of the frames analyzed are symmetric after the first. Hence, the above effect does not appear.

This effect can also be seen in the lower three curves of Fig. 12. Here, however, the effect of symmetry explained above is combined with another problem. The failure mechanism for the lower three analyses is a beam mechanism and as a result, the plotting of H versus Δ_H is not too indicative of the behavior of the frames. For these three cases, a plot of H versus vertical beam deflection would be more appropriate.

In summary, the axial force effect lowers the value of the load required to form a particular mechanism in a frame and quite often will cause the plastic hinges involved in that mechanism to form in a different order. If the influence of the axial force is great enough, a failure mechanism which is at least partly different from that predicted by simple plastic theory (neglecting the effect of axial force on moment capacity) will result.

Usually the stiffness of the structure decreases as the effect of axial force increases. This can be seen by comparing the slopes of corresponding segments on each of two load-deflection plots such as those in Figs. 11 or 12. Again, note that straight lines are not strictly correct as was explained at the end of Chapter 2. However, the fact that the exact curves for each segment are curves which are concave downward in itself indicates that axial force decreases frame stiffness.

5.3 DESIGN CHARTS

For any particular frame geometry, non-dimensional interaction curves can be constructed which will allow rapid and easy design of the frame under combined vertical and horizontal loading while at the same time accounting for the effect of axial load. Figure 14 is an example of such curves as developed for the example frame. The non-dimensional plots were produced by analyzing the two bounding frames of Fig. 10 under various combinations of loading throughout the range of possible ratios. The limiting cases for these loading combinations or ratios are simply the case of no horizontal force (H = 0% of W) which produces the simple beam mechanism load and secondly, the case of no vertical force ($\beta = 0$) which produces the simple sway mechanism load. All other failure mechanisms will be produced by some combination of loads between these two extremes.

For each of the analyses within the above range, curves similar to those of Figs. 11 and 12 are produced. Picking the peak value of load for each curve and plotting non-dimensionally one-half the vertical load over P_y of one column against the horizontal load H over the simple sway mechanism load results in the curves of Fig. 14. The curves of Fig. 14 indicate the interaction between three quantities: the lateral load, the gravity load, and the section properties of the sections used. With this plot a knowledge of any two of these will allow the determination of the third. If one of the two known quantities is the frame section properties, the procedure is a direct one. Knowing the section properties, G can be computed and consequently the proper curve in Fig. 14 can be picked. Either H/H_{sway} or $6\ \beta\ H/P_y$ can be computed from the value of the known load, depending on which load is known. Hence, the point on the plot which indicates the maximum load the frame can sustain is defined by the fact that one of its coordinates is known and it must lie on a given curve.

To design a frame if only the values of the loads are known, it is necessary to pick a set of trial sections first. Knowing the loads and picking some trial section allows the proper curve (value of G) to be chosen and the coordinates of the point defined by the loads to be determined. The relative locations of the point and the curve indicate whether the trial frame has any reserve strength and if so, how much. With a small number of trials, it is possible to pick sections which will cause the point defined by the loads to fall very near the limiting curve. The closer the point falls to the curve the more efficient the frame will be under the given loading.

The curves of Fig. 14 can also be used to predict the type of failure mechanism and the order of formation of the plastic hinges in each case. To do this it is only necessary to refer back to the load-deflection plots which were used to produce the curves of Fig. 14.

6. SUMMARY

A method for analyzing frames to determine their elastic-plastic load-deflection behavior has been presented. The method accounts for the weakening effect of axial force on the plastic moment capacity of the members. Both the method of analysis and the computer program which was developed to perform it are based on a method and a computer program developed by Wang and Harrison which neglects this axial force effect. The basic difference between the two methods lies in the fact that an analysis which includes the effect of axial force is non-linear while one which does not include it is linear. As a result of the non-linearity involved, the method presented in this report uses a trial-and-error procedure.

In order to use the reported program to analyze a frame, it is necessary to systematize the various groups of input information that are needed in the course of the analysis. The methods and systems used are explained in Chapter 3 and exemplified in Fig. 6 and appendix C. Typical program output is also given in Chapter 3 and an example run is contained in appendix C.

The two basic problems encountered in the use of the program have been discussed. They are: (1) Storage capacity, that is, how to make efficient use of available machine capacity and how to modify a given problem so that it requires less space in the computer. (2) Change in the order of formation of plastic hinges during analysis.

The solution to the first difficulty consists of program and input data modifications discussed in Chapter 4. The second problem, that of hinge order reversal, arises from the trial-and-error procedure employed by the program. A detailed explanation and solution have been presented in Chapter 4.

The results of several analyses and some possible uses for the program in design have been presented. In particular, a method for producing a series of curves for a given frame which show the interaction of frame gravity loads, lateral loads, and some parameters indicating frame properties have been shown.

7. NOMENCLATURE AND PROGRAM SYMBOLS

Text	Program	Definition
	A	Statics matrix.
A		Cross-sectional area.
	AA	The increased capacity of the point in question. (The x-axis intersection for the straight line approximation to the interaction formula)
	ABC	Dummy variable used to read past unwanted sections on tape.
m_{ij}	AM	Array of unit moments for each elastic frame analyzed.
n_{ij}	AN	Array of unit axial loads for each elastic frame analyzed.
	ASAT1	Temporary storage array for one column of the matrix ASAT.
	ASAT	Matrix A times matrix S times matrix A transposed.
	ASM	Subroutine which adjusts the stiffness matrix to account for the formation of the last plastic hinge.
	ATX	Array used in the computation of the plastic hinge rotations.
P	AX	Total axial load at some point due to all increments of load on the frame.
	B	Intermediate variable (no general definition).
	BB	The axial load at the point in question which results from previous increments of load on the structure.
	C	Intermediate variable (no general definition).
	CAX	Cumulative axial load at a point.
	CC	The axial load at the point in question which results from increments of load being applied to the structure after a plastic hinge has formed at the point.

Text	Program	Definition
	CM	Moment at a point in the frame.
M_{pc}	CPM	Reduced plastic moment capacity.
	CX	An array of the deflections in the frame at the formation of a plastic hinge.
	CXX	Identical to CX.
	D	Intermediate variable (no general definition).
	DATA	Subroutine which inputs new frame data.
M_i	DD	The moment induced at the point in question due to previous increments of load being applied to the structure.
	DELTAP	Subroutine which computes the increment of load required to form the next plastic hinge.
	DM	Array used in the computation of the plastic hinge rotations.
E		Modulus of elasticity.
EAOL	EAOL	Matrix which stores the value of EA/L for each frame member.
m_{ij}	EE	The unit moment for the point in question for this increment of load.
G		Ratio of column stiffness to beam stiffness.
	GG	A function of the unit axial load for this increment at the point in question.
H		Horizontal load.
H_{sway}		Sway mechanism load.
	I	Counter.
I		Moment of inertia.
	IA	Point being investigated for being the next hinge to form.
	IBZ	Number of the location on the frame where the next plastic hinge is known to have formed or is currently assumed to be forming.
	ICYC	Cycle number in the trial and error procedure used to find the next increment of load.

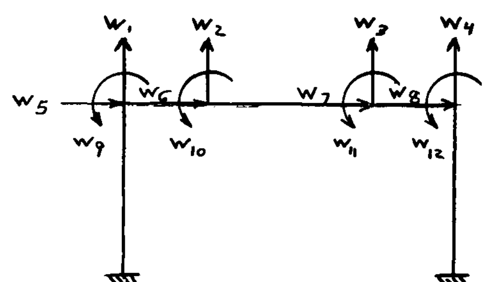
Text	Program	Definition
h	IH	Number of the plastic hinge being investigated.
	II	Switching parameter used in subroutine deltap.
	INDEX	Switching parameter which controls where the main program goes after control leaves a subroutine.
	IORD	Matrix which records the order of the plastic hinge formation.
	IP1	Counter.
b	IPT	Number of the point currently under investigation.
	IT1	Tape number.
	IT2	Tape number.
	IXZ	Index which controls the printing of intermediate results in subroutine DELTAP.
	J	Counter.
JJ	JJ	Frame number.
	K	Counter.
KK	KK	Load set number.
	KZ	One less than the number of the hinge currently under investigation.
L	L	Degrees of freedom in the frame.
ℓ		Length.
	LDSET	Subroutine which inputs the loads.
	LOWEST	Subroutine which picks the next plastic hinge.
	LP1	Frame degrees of freedom plus one.
M,a	M	Twice the number of members in a frame.
	MATRIX	Subroutine which performs an elastic analysis of a frame.
	MM	One less than twice the number of members in a frame.
N	N	Three times the number of members in a frame.
NM	NM	Number of frame members.
	NPASS	Switching parameter which causes the stiffness and statics matrices to be printed at the beginning of each new frame analysis.
	NPH	The number of the location where the next plastic hinge occurs.

Text	Program	Definition
	OUTPUT	Subroutine which prints out a table of information summarizing the conditions in the frame after the formation of each plastic hinge.
	P	Increment of load.
	PHR	Subroutine which computes and prints the internal rotations undergone by each plastic hinge at collapse load.
	PL	Matrix of the applied loads.
M_p	PM	Matrix of the plastic moment capacities for all the frame members.
	POPY	Ratio of P to P_y at a point.
	PP	Final computed load required to form a plastic hinge at some location.
	PT	Matrix which stores the value of the load to form a plastic hinge at some location for each of several tries.
P_y	PY	Matrix of the yield loads for all frame members.
	RAX	Axial load at a point due to loads applied to the structure after a plastic hinge forms at the point.
	RM	Induced moment at a point.
	RR	Axial load at a point due to one increment of load applied after a plastic hinge appears at the point.
r_x		Radius of gyration.
	RZ	Moment induced at a point due to one increment of load.
	S	Stiffness matrix.
	SAT	Matrix S times matrix A transposed.
	SAT1	Temporary storage array for one column of the matrix SAT.
	SAX	Axial load at a point due to loads applied to the structure before a plastic hinge forms at the point.

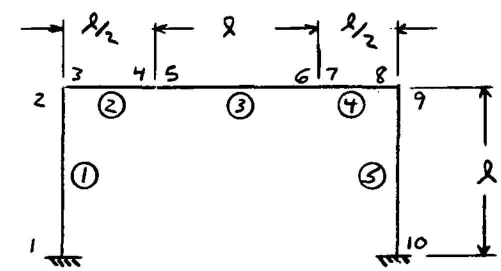
Text	Program	Definition
	SB	Intermediate variable (no general definition).
	SC	Intermediate variable (no general definition).
SDAT	SDAT	Matrix which stores the value of EI/L for each frame member.
	SS	Axial load at a point due to one increment of load applied before a plastic hinge appears at the point.
	TEMP	Temporary variable name.
	UNDEFL	Subroutine which computes the unit deflection, moments, and axial loads.
	UX	Array of unit deflections for each elastic frame analyzed.
W		Vertical load.
β		The ratio of a beam point load to the side load. (For the example frame).
Δ_H		Horizontal deflection.
δ_H		Unit horizontal deflection.

TABLE 1 STATICS MATRIX (A) FOR EXAMPLE FRAME

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	P_1	P_2	P_3	P_4	P_5
W_1	0	0	$2/\ell$	$2/\ell$	0	0	0	0	0	0	1	0	0	0	0
W_2	0	0	$-2/\ell$	$-2/\ell$	$1/\ell$	$1/\ell$	0	0	0	0	0	0	0	0	0
W_3	0	0	0	0	$-1/\ell$	$-1/\ell$	$2/\ell$	$2/\ell$	0	0	0	0	0	0	0
W_4	0	0	0	0	0	0	$-2/\ell$	$-2/\ell$	0	0	0	0	0	0	1
W_5	$1/\ell$	$1/\ell$	0	0	0	0	0	0	0	0	0	-1	0	0	0
W_6	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0
W_7	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0
W_8	0	0	0	0	0	0	0	0	$1/\ell$	$1/\ell$	0	0	0	1	0
W_9	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
W_{10}	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
W_{11}	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
W_{12}	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0



Load Numbering and
Sign Convention



Member Numbering

TABLE 2 MATRIX OF APPLIED LOADS (PL)

$$
\begin{array}{c}
W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \\ W_9 \\ W_{10} \\ W_{11} \\ W_{12}
\end{array}
\begin{bmatrix}
0 \\ -1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

TABLE 3 SEQUENCE OF INPUT DATA

		INPUT ITEM(S)	FORTRAN SYMBOL	FORTRAN II FORMAT
Analysis of First Frame		Frame No.	KK	I 5
		Degree of Freedom and number of members	L, NM	2 I 5
		Statics Matrix	A	7 F 10.4
		Section Properties	SDAT PM EAOL PY	4 F 10.4
	Load Applied to First Frame	Load Set Number	KK	I 5
		Load Matrix	PL	7 F 10.4
		Load Set Number	KK	I 5
		Load Matrix	PL	7 F 10.4
		Load Set Number (negative)	KK	I 5
Analysis of Second Frame		Frame No.	KK	I 5
		Degree of Freedom and number of members	L, NM	2 I 5
		Statics Matrix	A	7 F 10.4
		Section Properties	SDAT PM EAOL PY	4 F 10.4
	Load Applied to Second Frame	Load Set Number	KK	I 5
		Load Matrix	PL	7 F 10.4
		Load Set Number	KK	I 5
		Load Matrix	PL	7 F 10.4
		Load Set Number (negative)	KK	I 5
		Frame No. (negative)	JJ	I 5
		END CARD		END

TABLE 4 PROGRAM OUTPUT SEQUENCE

ITEM	COMMENT
1. Frame Number	Printed on the top of the first page for identification purposes.
2. Statics Matrix	Listed for reference and checking (to insure correct punching of input data.)
3. Member Stiffness Matrix	Created internally from section properties. Listed for reference and checking.
4. Load Set Number	Listed for identification.
5. Load Set	Listed for reference and checking.
The following four items are repeated in sequence until the failure mechanism is formed.	
6. Unit Deflections Unit Moments Unit Axial Loads	Produced by each elastic analysis of the frame.
7. Number and location of plastic hinge	Values of each load increment for each trial, as well as the totals for each trial are also output for examination here.
8. Table	The total moment, M_{pc}, M_p, axial load, P_y, and the ratio P/P_y are listed for each member end in the frame.
9. Deflections	The value of the deflection for each of the "L" degrees of freedom in the frame is computed and listed.
After the failure load is reached the following item is produced.	
10. Hinge Rotations	The internal rotations undergone by each plastic hinge are listed.
If additional load sets are to be run, the output begins again with item 4. If no load sets remain, but additional frames do, item 1 will appear next. If nothing remains to be run, the machine will exit from the program.	

TABLE 5 SECTION PROPERTIES LISTED BY RADIUS OF GYRATION

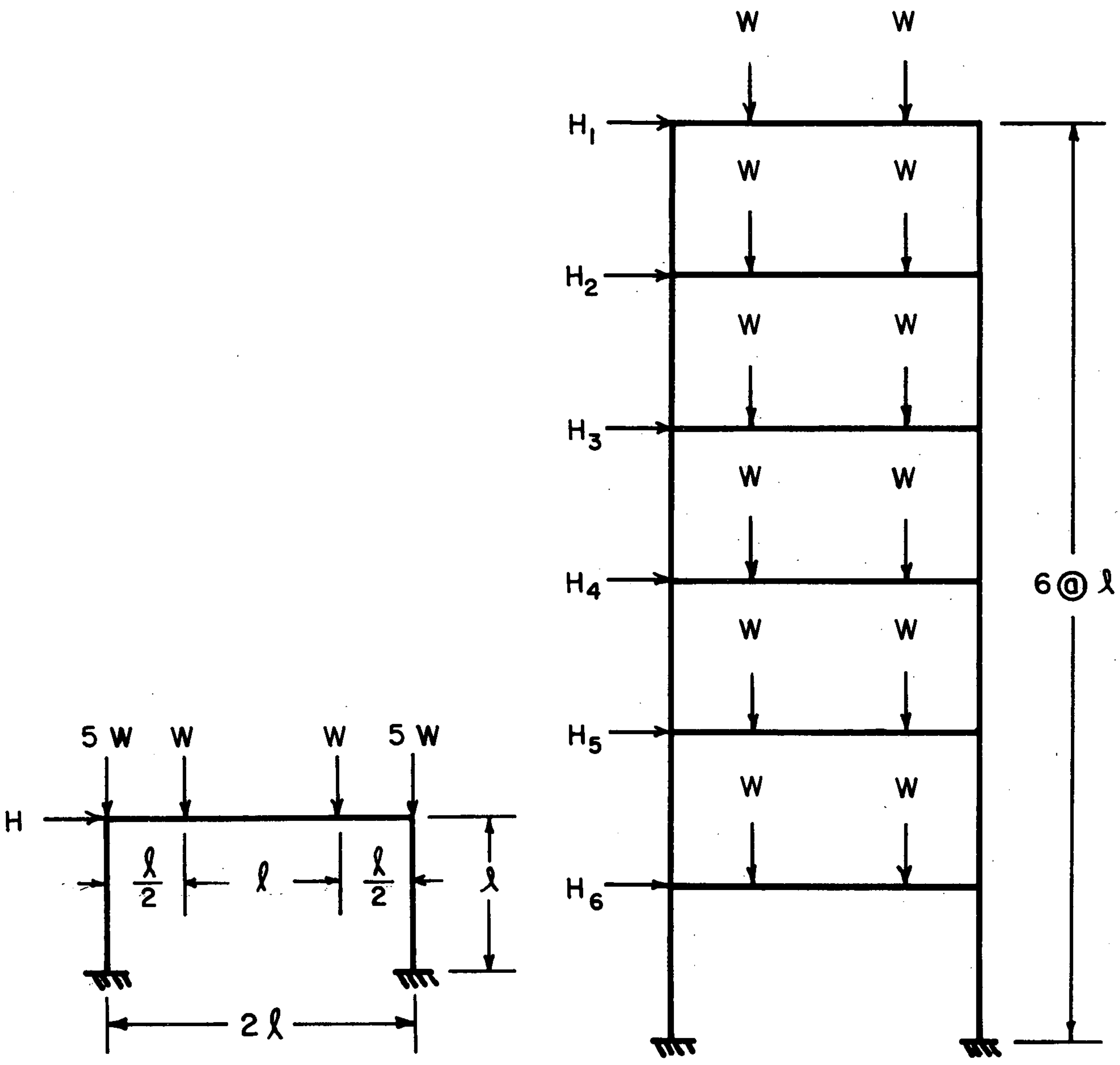
SECTION	RADIUS OF GYRATION RX	MOMENT OF INERTIA IX	PLASTIC MODULUS Z	AREA A	DEPTH D
3I 7	1.15	2.9	2.3	2.17	3.00
3I 6	1.23	2.5	1.9	1.64	3.00
4I 9	1.56	6.7	4.0	2.76	4.00
4I 8	1.64	6.0	3.5	2.21	4.00
4M 13	1.65	10.4	6.1	3.82	4.00
4WF 13	1.72	11.3	6.3	3.82	4.16
5I 15	1.87	15.0	7.4	4.29	5.00
5I 10	2.05	12.1	5.6	2.87	5.00
5M 19	2.08	23.8	11.1	5.56	5.00
5WF 16	2.13	21.3	9.6	4.70	5.00
5WF 18	2.16	25.4	11.4	5.45	5.12
6I 17	2.28	26.0	10.5	5.02	6.00
6JR 4	2.37	7.3	2.8	1.30	6.00
6I 12	2.46	21.8	8.4	3.61	6.00
6B 12	2.48	21.7	8.3	3.53	6.00
6M 22	2.49	41.0	15.6	6.62	6.00
6M 25	2.53	47.0	17.9	7.35	6.00
6M 20	2.57	38.8	14.6	5.88	6.00
6B 16	2.59	31.7	11.6	4.72	6.25
6WF 20	2.66	41.7	15.0	5.90	6.20
7I 20	2.66	41.9	14.4	5.83	7.00
6WF 25	2.69	53.5	19.0	7.37	6.37
7JR 5	2.74	12.1	4.0	1.61	7.00
7I 15	2.80	36.2	11.9	4.43	7.00
8I 23	3.09	64.2	19.2	6.71	8.00
8JR 6	3.12	18.7	5.4	1.92	8.00
8B 13	3.21	39.5	11.4	3.83	8.00
8M 20	3.22	60.7	17.4	5.88	8.00
8M 22	3.23	68.3	19.7	6.61	8.00
8I 18	3.26	56.9	16.3	5.34	8.00
8B 15	3.29	48.0	13.6	4.43	8.12
8M 28	3.31	90.1	25.6	8.23	8.00
8M 17	3.35	56.4	15.7	5.00	8.00
8WF 17	3.36	56.4	15.8	5.00	8.00
8M 18	3.38	62.1	17.4	5.44	8.00
8M 34	3.40	115.5	32.7	10.09	8.00
8WF 24	3.42	82.5	23.1	7.06	7.93
8WF 20	3.45	69.2	19.1	5.88	8.14
8M 24	3.45	83.8	23.3	7.06	8.00
8WF 28	3.45	97.8	27.1	8.23	8.06
8M 33	3.45	112.8	31.6	9.59	8.00
8WF 35	3.50	126.5	34.7	10.30	8.12
8WF 40	3.53	146.3	39.9	11.76	8.25
8WF 48	3.61	183.7	49.0	14.11	8.50
8WF 58	3.65	227.3	59.9	17.06	8.75
8WF 67	3.71	271.8	70.1	19.70	9.00
10I 35	3.76	145.8	35.2	10.22	10.00
10JR 9 $	3.85	39.0	9.2	2.64	10.00
10M 29	3.92	131.5	30.9	8.55	9.88
10B 15 $	3.95	68.8	16.0	4.40	10.00
10B 17 $	4.05	81.8	18.6	4.98	10.12
10I 25	4.07	122.1	28.0	7.38	10.00
10M 21	4.14	104.4	23.7	6.10	9.90
10WF 21	4.14	106.3	24.1	6.19	9.90
10M 23	4.16	116.6	26.5	6.73	9.88

SECTION	RADIUS OF GYRATION RX	MOMENT OF INERTIA IX	PLASTIC MODULUS Z	AREA A	DEPTH D
10WF 25	4.26	133.2	29.5	7.35	10.08
10WF 39	4.27	209.7	47.0	11.48	9.94
10WF 29	4.29	157.3	34.7	8.53	10.22
10WF 45	4.33	248.6	55.0	13.24	10.12
10WF 54	4.39	305.7	67.0	15.86	10.12
10B 19	4.41	96.2	21.6	5.61	10.25
10WF 60	4.41	343.7	75.1	17.66	10.25
10WF 66	4.44	382.5	82.8	19.41	10.38
10WF 72	4.46	420.7	90.7	21.18	10.50
10WF 77	4.49	457.2	97.7	22.67	10.62
12I 50	4.55	301.6	60.7	14.57	12.00
10WF 89	4.55	542.4	114.4	26.19	10.88
12JR 12	4.57	72.2	14.3	3.45	12.00
10WF100	4.61	625.0	130.1	29.43	11.12
12B 16 $	4.65	105.3	20.6	4.86	12.00
10WF112	4.67	718.7	147.5	32.92	11.36
12I 35	4.72	227.0	44.4	10.20	12.00
12I 41	4.77	268.9	52.5	11.94	12.00
12B 19 $	4.81	130.1	24.8	5.62	12.16
12I 32	4.83	215.8	41.6	9.26	12.00
12B 22	4.91	155.7	29.4	6.47	12.31
12WF 27 $	5.06	204.1	38.0	7.97	11.96
12WF 31 $	5.11	238.4	44.0	9.12	12.09
12WF 40	5.13	310.1	57.6	11.77	11.94
12WF 36	5.15	280.8	51.4	10.59	12.24
12WF 45	5.15	350.8	64.9	13.24	12.06
12WF 50	5.18	394.5	72.6	14.71	12.19
12WF 53	5.23	426.2	78.2	15.59	12.06
12WF 58	5.28	476.1	86.5	17.06	12.19
12WF 79	5.34	663.0	119.3	23.22	12.38
12WF 85	5.38	723.3	129.1	24.98	12.50
14B 17 $	5.40	147.3	24.7	5.05	14.00
12WF 92	5.40	788.9	140.2	27.06	12.62
12WF 99	5.43	858.5	151.8	29.09	12.75
12WF106	5.46	930.7	163.4	31.19	12.88
12WF120	5.51	1071.7	186.4	35.31	13.12
14B 22 $	5.52	197.4	33.0	6.47	13.72
12WF133	5.59	1221.2	209.7	39.11	13.38
14B 26 $	5.63	242.6	39.9	7.65	13.89
12WF161	5.70	1541.8	259.2	47.38	13.88
14WF 30 $	5.73	289.6	47.1	8.81	13.86
15I 50	5.74	481.1	76.5	14.59	15.00
14WF 43 $	5.82	429.0	69.7	12.65	13.68
12WF190	5.82	1892.5	311.5	55.86	14.38
14WF 34 $	5.83	339.2	54.5	10.00	14.00
14WF 48	5.86	484.9	78.5	14.11	13.81
14WF 38 $	5.87	385.3	61.5	11.17	14.12
14WF 53	5.90	542.1	87.1	15.59	13.94
15I 43	5.95	441.8	68.6	12.49	15.00
14WF 61	5.98	641.5	102.4	17.94	13.91
14WF 68	6.02	724.1	114.8	20.00	14.06
14WF 74	6.05	796.8	125.6	21.76	14.19
14WF 78	6.09	851.2	134.0	22.94	14.06
14WF 84	6.13	928.4	145.4	24.71	14.18
14WF111	6.23	1266.5	196.0	32.65	14.37

$ CHECK SHAPES SO MARKED FOR COMPLIANCE WITH FORMULA (25), SECTION 2.6 OF THE AISC SPECIFICATION, WHEN SUBJECTED TO COMBINED AXIAL FORCE AND PLASTIC BENDING MOMENT AT ULTIMATE LOADING.

TABLE 5 SECTION PROPERTIES LISTED BY RADIUS OF GYRATION (continued)

SECTION	RADIUS OF GYRATION RX	MOMENT OF INERTIA IX	PLASTIC MODULUS Z	AREA A	DEPTH D
16B 26 $	6.24	298.1	43.9	7.65	15.65
14WF119	6.26	1373.1	210.9	34.99	14.50
14WF127	6.29	1476.7	225.9	37.33	14.62
14WF136	6.31	1593.0	242.7	39.98	14.75
14WF142	6.32	1672.2	254.8	41.85	14.75
14WF150	6.37	1786.9	270.2	44.08	14.88
16B 31 $	6.39	372.5	53.8	9.12	15.84
14WF158	6.40	1900.6	286.3	46.47	15.00
14WF167	6.42	2020.8	302.9	49.09	15.12
14WF176	6.45	2149.6	321.3	51.73	15.25
16WF 36 $	6.49	446.3	63.9	10.59	15.85
14WF184	6.49	2274.8	337.5	54.07	15.38
14WF193	6.51	2402.4	355.1	56.73	15.50
14WF202	6.54	2538.8	373.6	59.39	15.63
14WF211	6.56	2671.4	391.7	62.07	15.75
14WF219	6.59	2798.2	408.0	64.36	15.87
16WF 40 $	6.62	515.5	72.7	11.77	16.00
16WF 58	6.62	746.4	106.2	17.04	15.86
14WF228	6.62	2942.4	427.2	67.06	16.00
14WF320	6.63	4141.7	592.2	94.12	16.81
16WF 45 $	6.64	583.3	82.0	13.24	16.12
14WF237	6.65	3080.9	445.4	69.69	16.12
16WF 64	6.66	833.8	117.9	18.80	16.00
16WF 50 $	6.68	655.4	92.7	14.70	16.25
14WF246	6.68	3228.9	464.5	72.33	16.25
18I 70	6.70	917.5	123.8	20.46	18.00
16WF 71	6.70	936.9	131.6	20.86	16.16
16WF 78	6.74	1042.6	145.5	22.92	16.32
14WF264	6.74	3526.0	502.4	77.63	16.50
14WF287	6.81	3912.1	551.6	84.37	16.81
16WF 88	6.87	1222.6	169.0	25.87	16.16
14WF314	6.90	4399.4	611.5	92.30	17.19
16WF 96	6.93	1355.1	186.0	28.22	16.32
14WF342	6.99	4911.5	673.0	00.59	17.56
18I 55	7.07	795.5	103.5	15.94	18.00
14WF370	7.08	5454.2	737.3	08.78	17.94
14WF398	7.17	6013.7	803.0	16.98	18.31
14WF426	7.26	6610.3	869.3	25.25	18.69
18WF 45 $	7.30	704.5	89.6	13.24	17.86
18WF 50 $	7.38	800.6	100.8	14.71	18.00
18WF 55 $	7.41	889.9	111.6	16.19	18.12
18WF 64 $	7.46	1045.8	131.8	18.80	17.87
18WF 60 $	7.47	984.0	122.6	17.64	18.25
18WF 70	7.49	1153.9	144.7	20.56	18.00
18WF 77	7.54	1286.8	160.5	22.63	18.16
18WF 85	7.57	1429.9	177.6	24.97	18.32
20I 95	7.59	1599.7	192.0	27.74	20.00
20I 75	7.60	1263.5	151.5	21.90	20.00
18WF 96	7.70	1674.7	206.0	28.22	18.16
18WF105	7.75	1852.5	226.5	30.86	18.32
20I 85	7.78	1501.7	177.3	24.80	20.00
18WF114	7.79	2033.8	247.9	33.51	18.48
20I 65	7.83	1169.5	137.3	19.08	20.00
21WF 55 $	8.40	1140.7	125.4	16.18	20.80
21WF 62 $	8.53	1326.8	144.1	18.23	20.99
21WF 82	8.53	1752.4	191.6	24.10	20.86
21WF 68 $	8.59	1478.3	159.8	20.02	21.13
21WF 96	8.60	2088.9	226.3	28.21	21.14
21WF 73 $	8.64	1600.3	172.1	21.46	21.24
21WF112	8.92	2620.6	278.0	32.93	21.00
21WF127	8.99	3017.2	317.8	37.34	21.24
21WF142	9.03	3403.1	357.0	41.76	21.46
24I 100	9.05	2371.8	238.8	29.25	24.00
24I 90	9.21	2230.1	220.5	26.30	24.00
24I 120	9.26	3010.8	298.0	35.13	24.00
24I 80 $	9.46	2087.2	203.0	23.33	24.00
24WF 68 $	9.53	1814.5	175.5	20.00	23.71
24I 106	9.53	2811.5	273.0	30.96	24.00
24WF 76 $	9.68	2096.4	200.1	22.37	23.91
24WF 84 $	9.78	2364.3	224.0	24.71	24.09
24WF 94 $	9.85	2683.0	253.0	27.63	24.29
24WF100 $	10.08	2987.3	278.3	29.43	24.00
24WF110 $	10.12	3315.0	307.7	32.36	24.16
24WF120 $	10.15	3635.3	336.6	35.29	24.31
24WF130 $	10.24	4009.5	369.2	38.21	24.25
24WF145	10.34	4561.0	416.0	42.62	24.49
24WF160	10.42	5110.3	463.7	47.04	24.72
27WF 84 $	10.69	2824.8	243.2	24.71	26.69
27WF 94 $	10.87	3266.7	277.7	27.65	26.91
27WF102 $	10.96	3604.1	304.4	30.01	27.07
27WF114 $	11.03	4080.5	342.8	33.53	27.28
27WF145 $	11.26	5414.3	452.0	42.68	26.88
27WF160	11.31	6018.6	504.3	47.04	27.06
27WF177	11.36	6728.6	556.9	52.10	27.31
30WF 99 $	11.70	3988.6	312.0	29.11	29.64
30WF108 $	11.85	4461.0	345.5	31.77	29.82
30WF116 $	12.00	4919.1	377.6	34.13	30.00
30WF124 $	12.11	5347.1	407.4	36.45	30.16
30WF132 $	12.17	5753.1	436.7	38.83	30.30
30WF172 $	12.48	7891.5	593.0	50.65	29.88
30WF190 $	12.57	8825.9	659.6	55.90	30.12
30WF210	12.64	9872.4	733.9	61.78	30.38
33WF118 $	13.02	5886.9	414.3	34.71	32.86
33WF130 $	13.23	6699.0	466.0	38.26	33.10
33WF141 $	13.39	7442.2	513.2	41.51	33.31
33WF152 $	13.50	8147.6	558.3	44.71	33.50
33WF200 $	13.71	11048.2	754.4	58.79	33.00
33WF220 $	13.79	12312.1	836.2	64.73	33.25
33WF240	13.88	13585.1	918.2	70.52	33.50
36WF135 $	14.01	7796.1	509.1	39.70	35.55
36WF150 $	14.29	9012.1	579.8	44.16	35.84
36WF160	14.38	9738.6	623.3	47.09	36.00
36WF170 $	14.47	10470.0	666.7	49.98	36.16
36WF182 $	14.52	11281.5	716.9	53.54	36.32
36WF194 $	14.56	12103.4	767.2	57.11	36.48
36WF230 $	14.88	14988.4	942.7	67.73	35.88
36WF245 $	14.95	16092.2	1008.0	72.03	36.06
36WF260 $	15.00	17233.8	1076.0	76.56	36.24
36WF280	15.12	18819.3	1167.0	82.32	36.50
36WF300	15.17	20290.2	1255.0	88.17	36.72



(a) (b)

Fig. 1 Example Frame

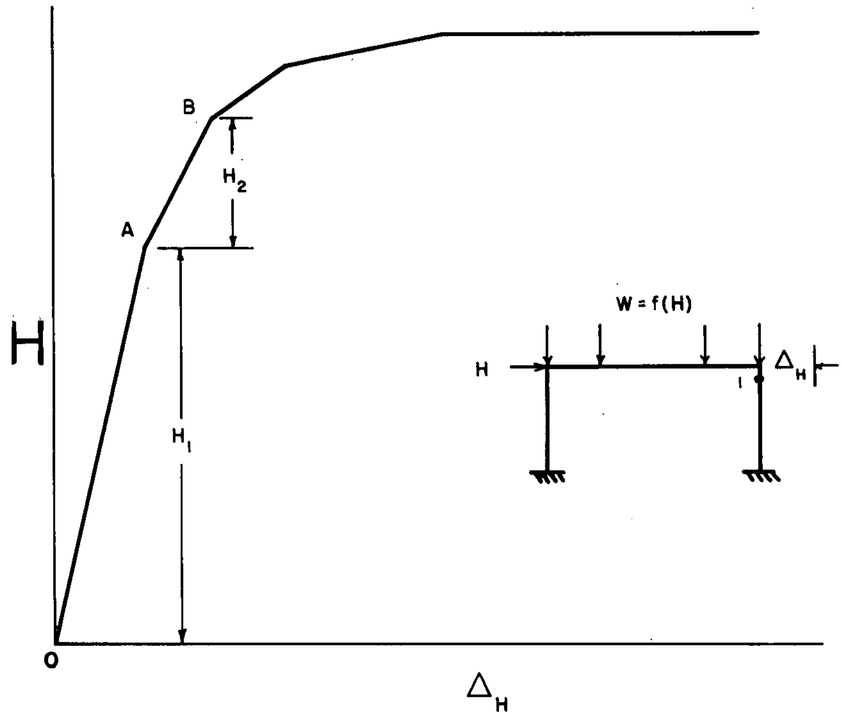


Fig. 2 Schematic Elastic Simple-Plastic Plot Neglecting Axial Force Effects

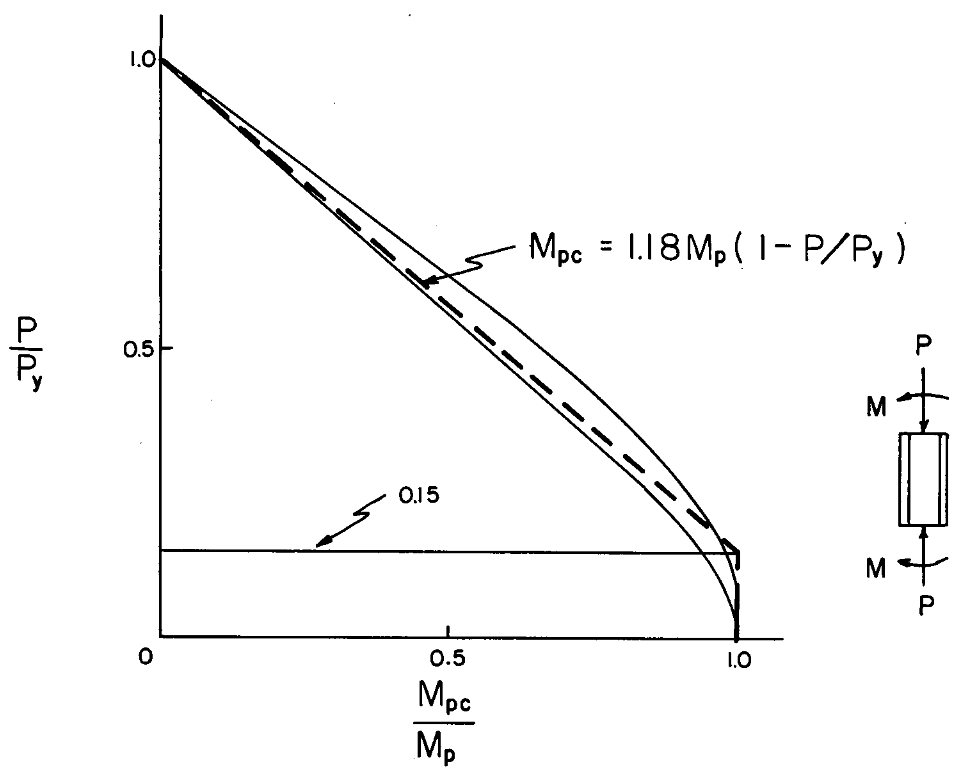


Fig. 3 Interaction Curves

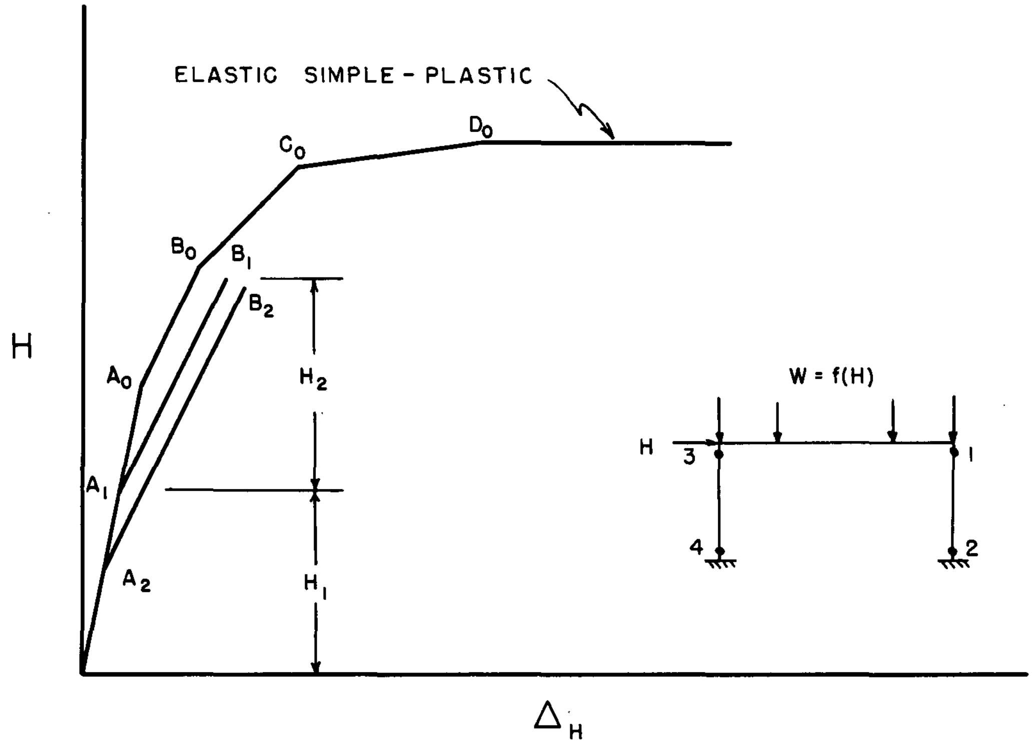


Fig. 4 Schematic Plot Indicating Successive Trials for the Determination of a Second Plastic Hinge

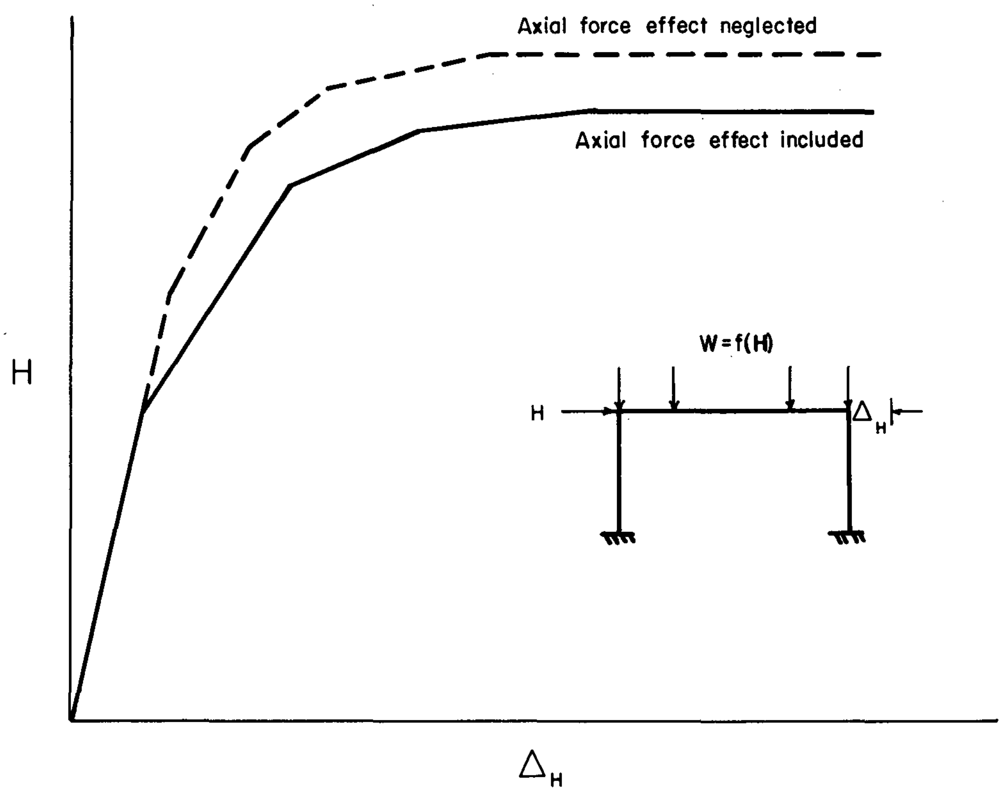


Fig. 5 Comparison of Load-Deflection Plots with and without the Effect of Axial Force

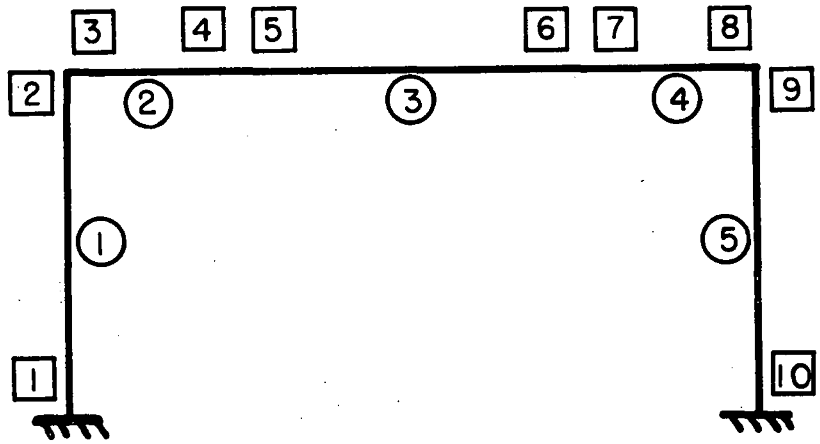


Fig. 6a Member Numbering Systems

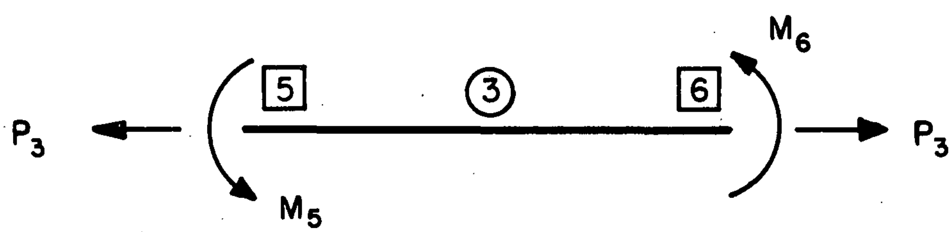


Fig. 6b Internal Force Numbering and Sign Convention

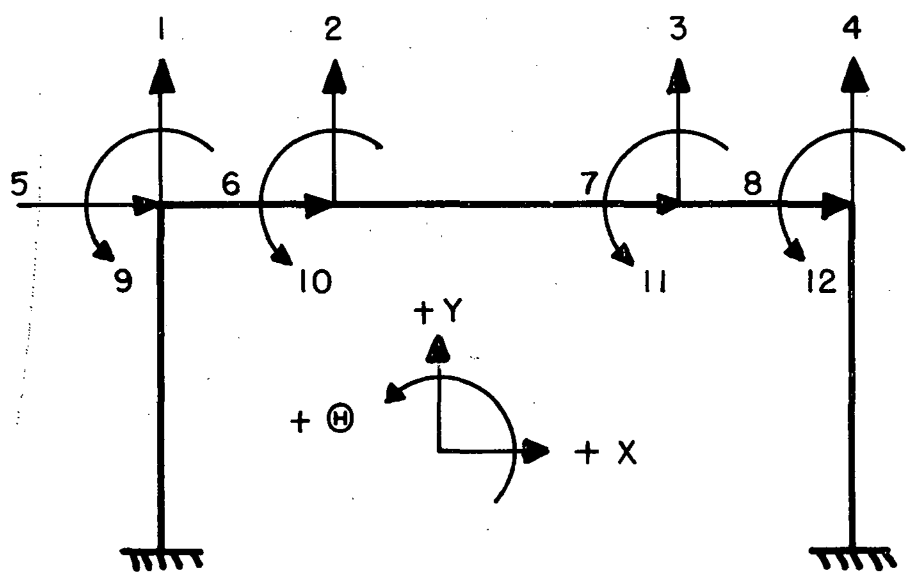


Fig. 6c Deflection and Applied Load Numbering and Sign Convention

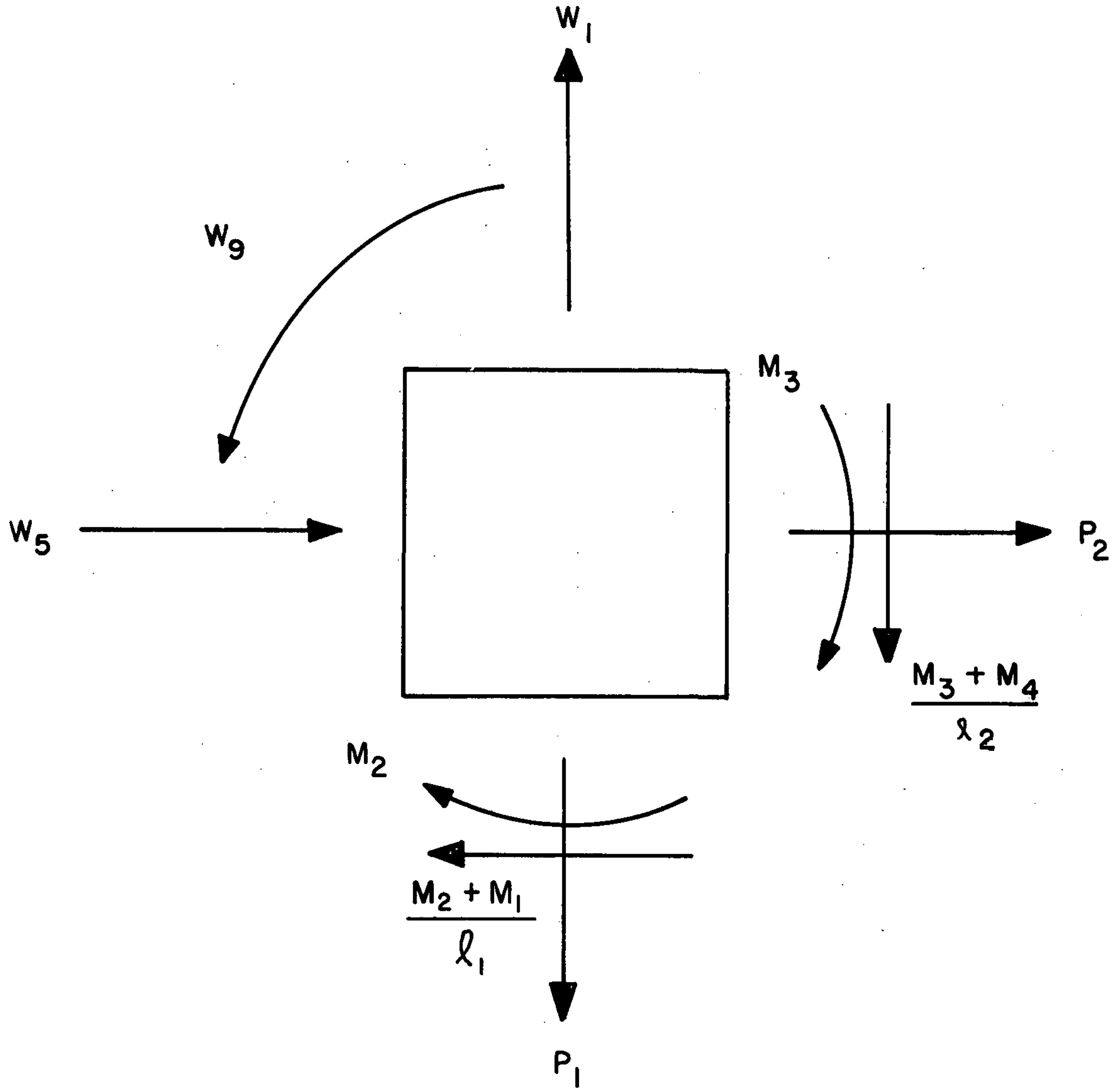
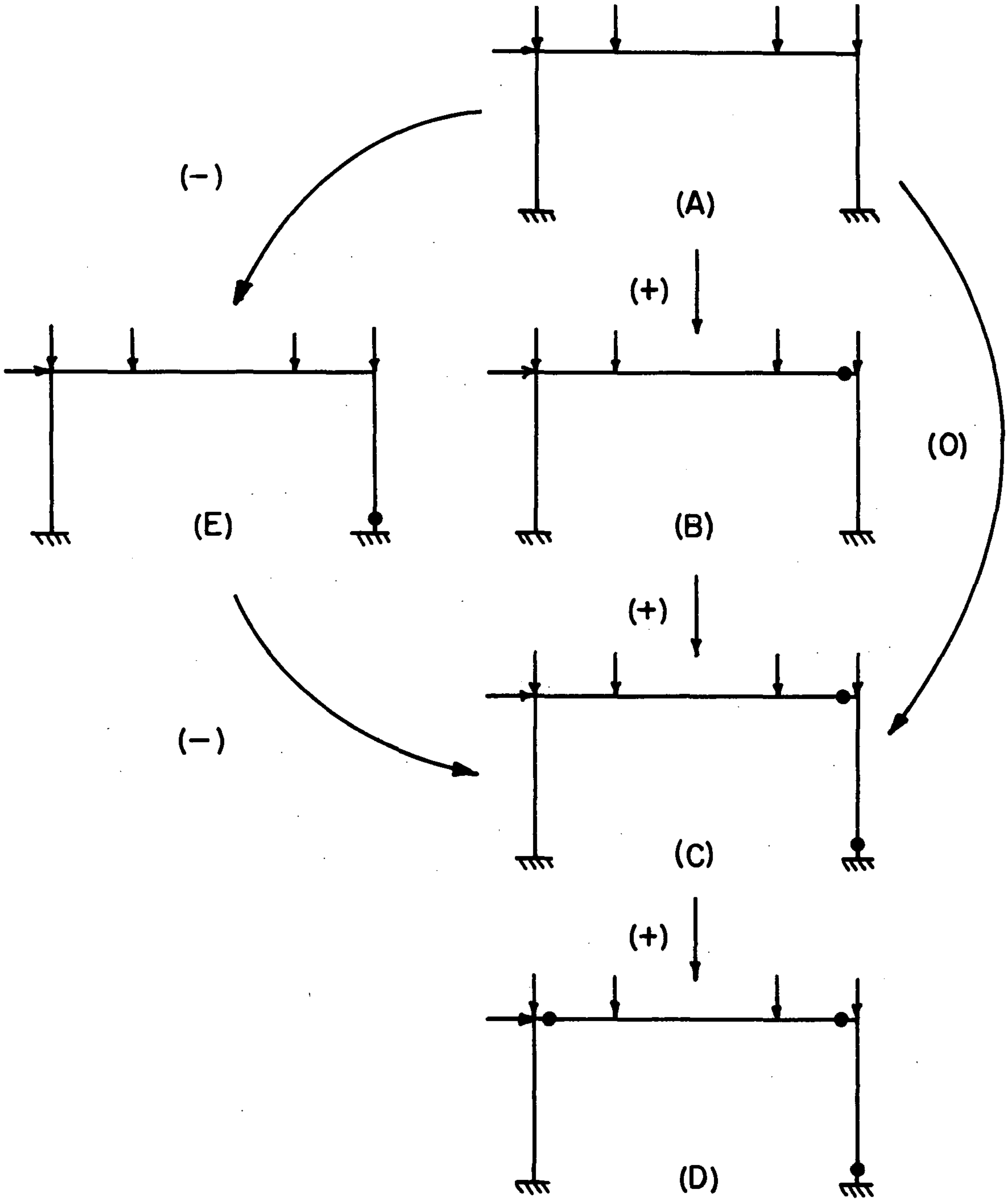


Fig. 7 Freebody of the Upper Left Column Top on the Example Frame



The quantity in parenthesis indicates the length of the second segment on the load-deflection plot for the frame.

Fig. 8 Change in the Order of Formation of Plastic Hinge

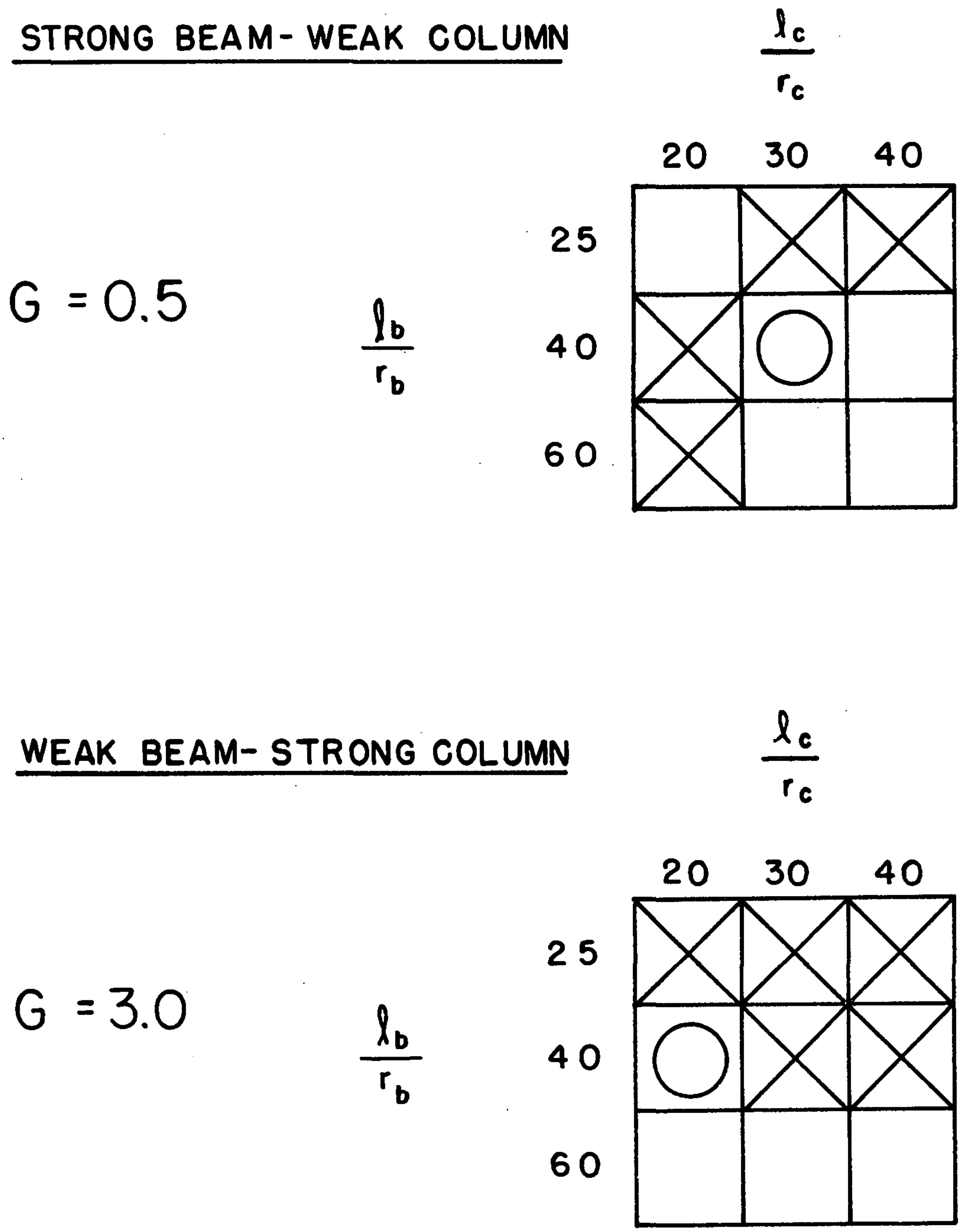
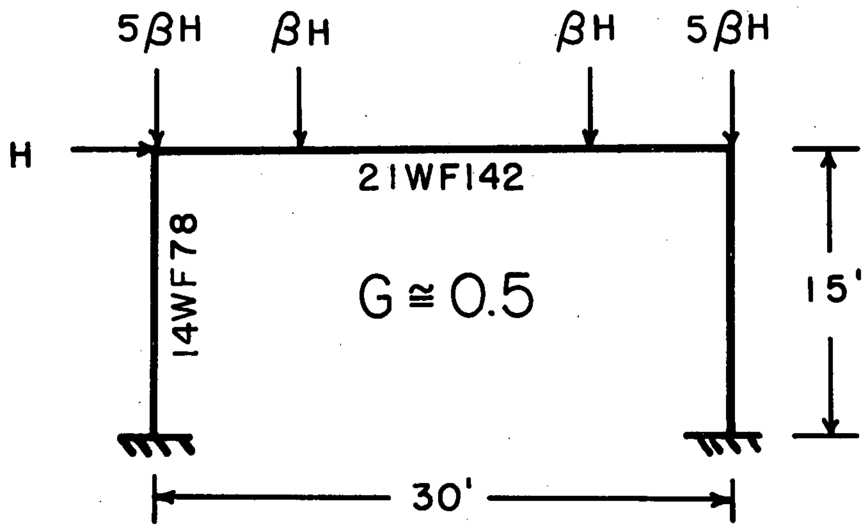


Fig. 9 Possible Bounding Cases for the Construction of the Example Frame

STRONG BEAM-WEAK COLUMN



STRONG COLUMN-WEAK BEAM

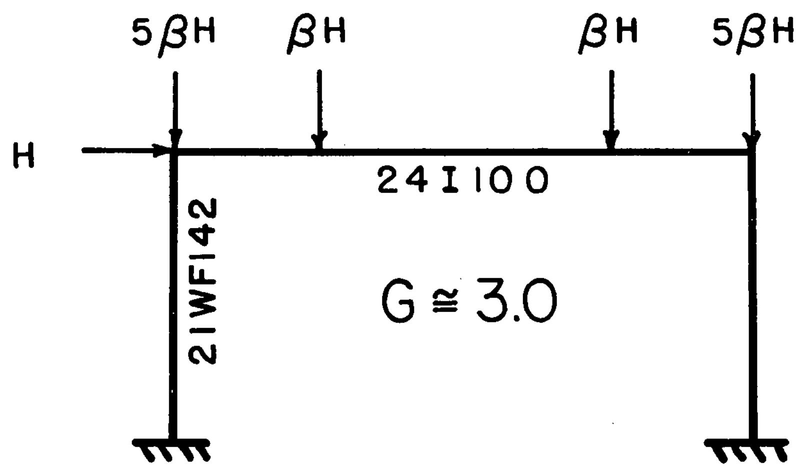


Fig. 10 Bounding Frames

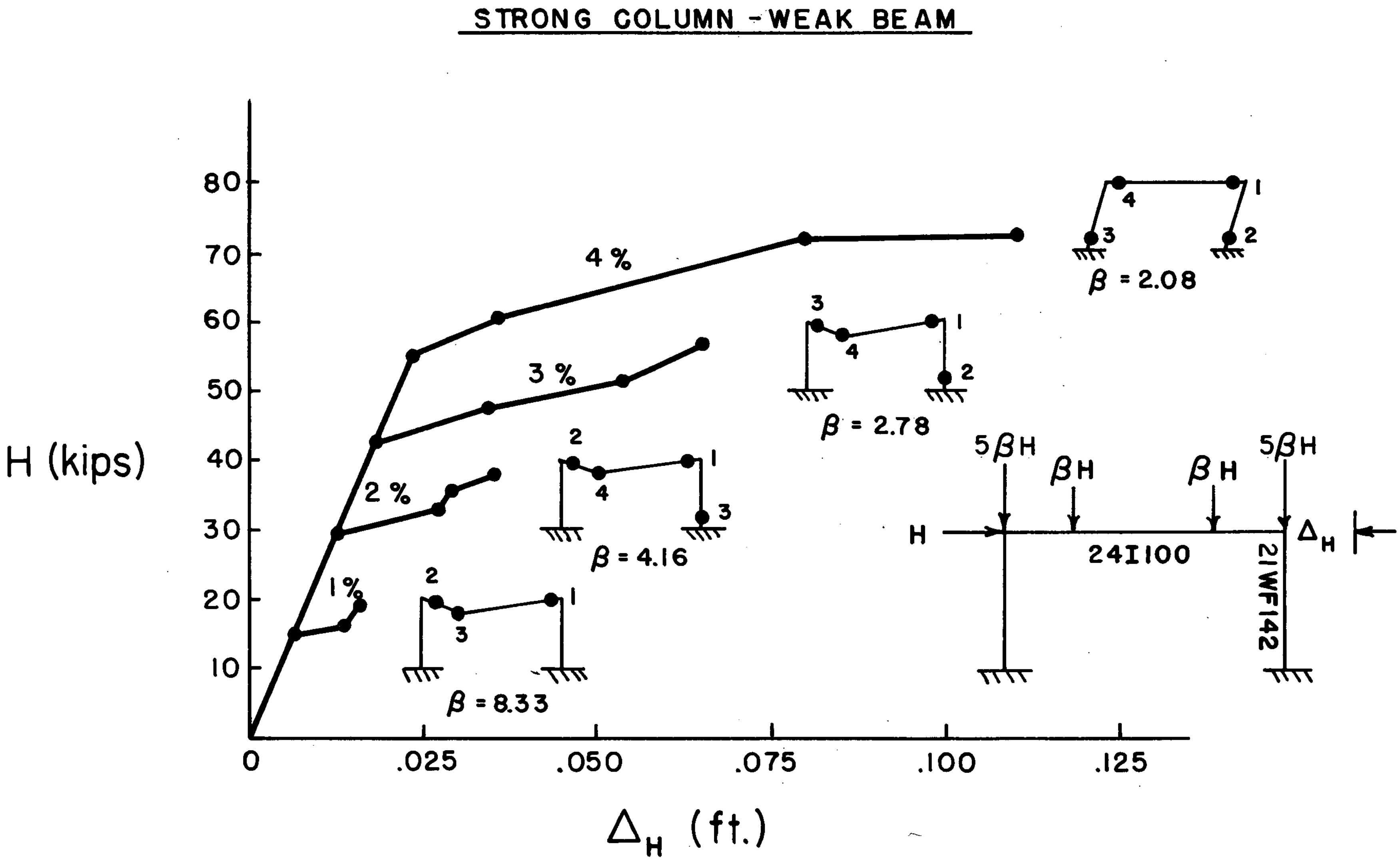


Fig. 11 Results for the Case G = 3.0

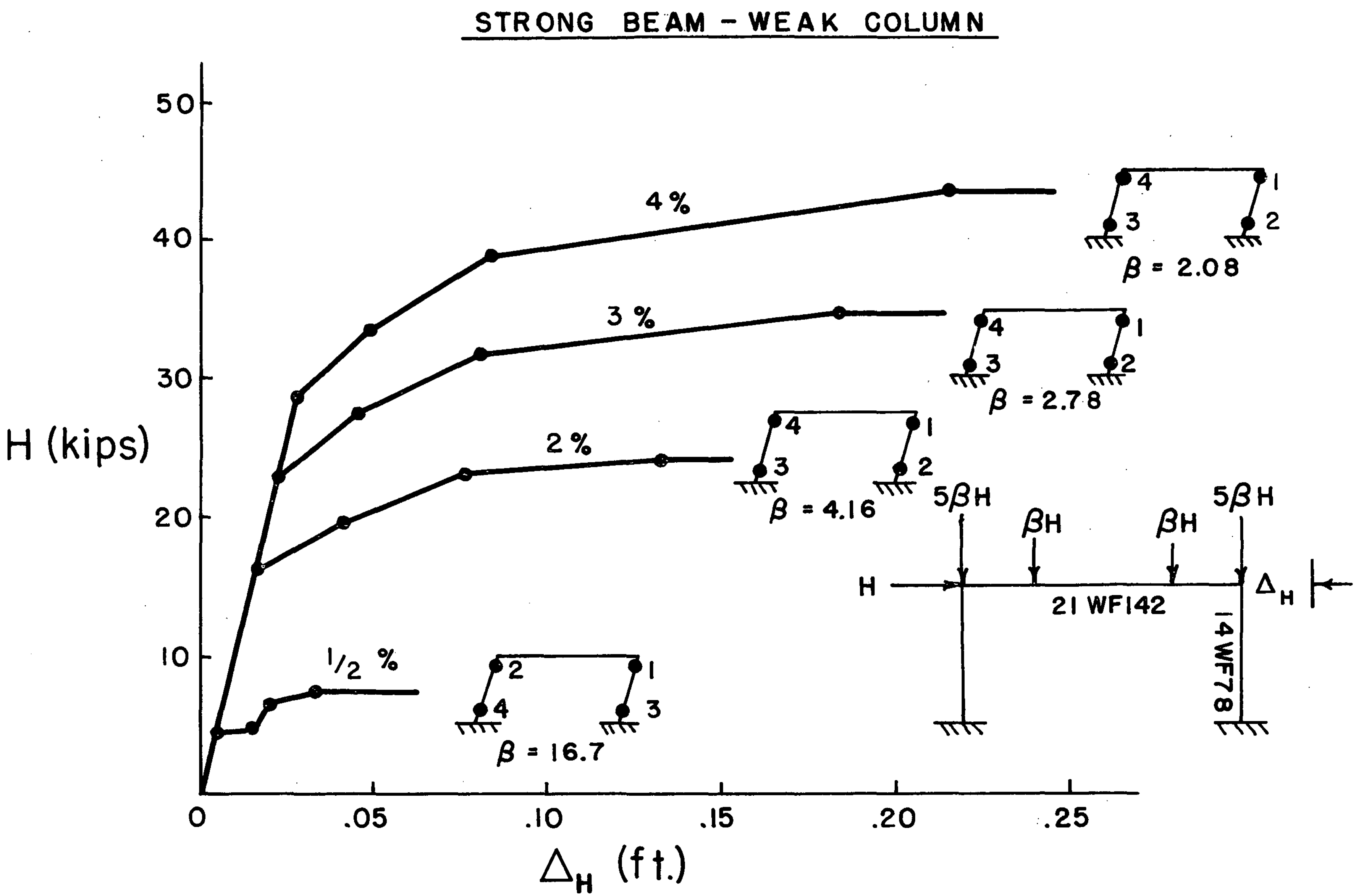


Fig. 12 Results for the Case G = 0.5

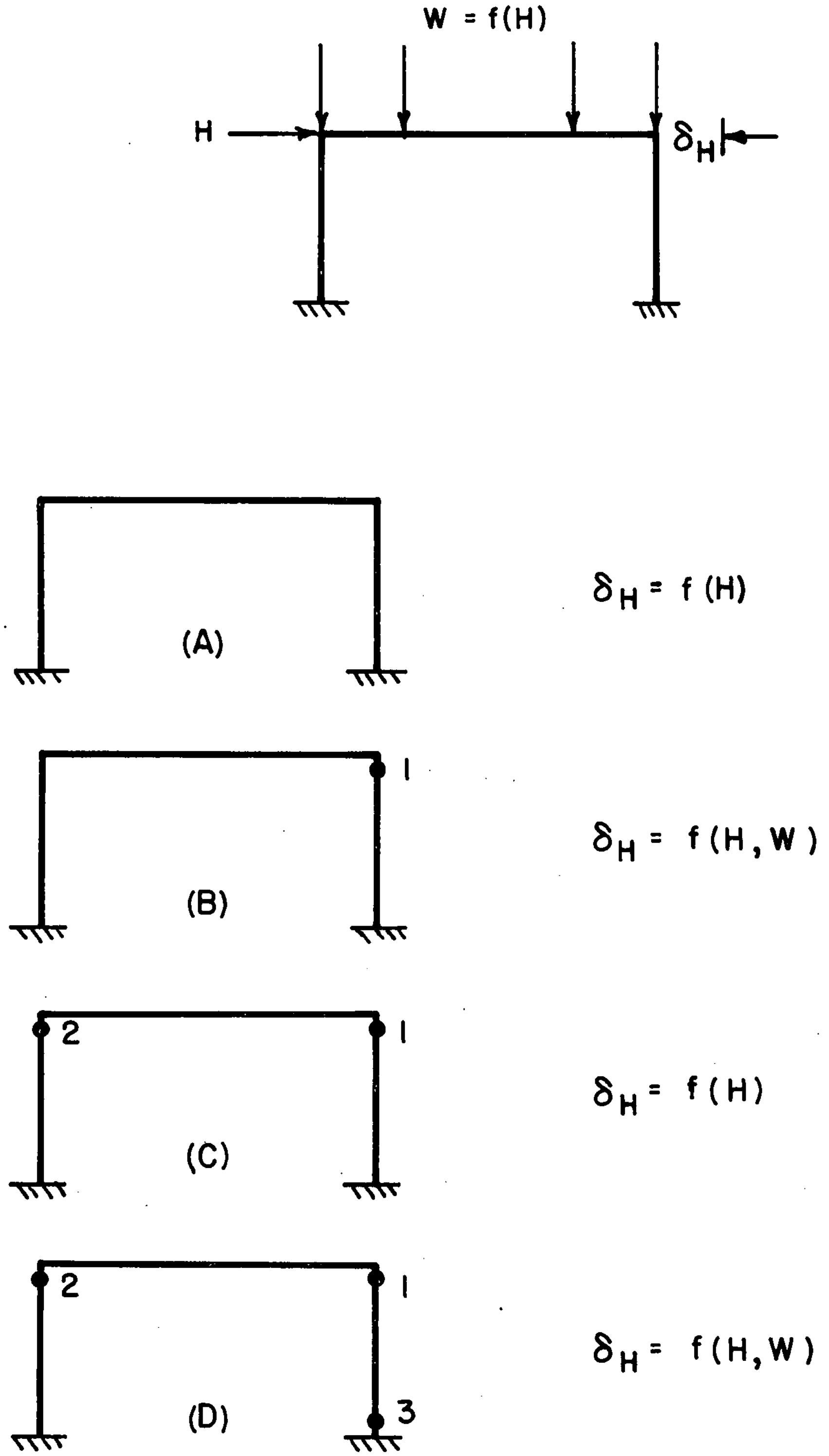


Fig. 13 Effect of Frame Symmetry on Unit Horizontal Deflection

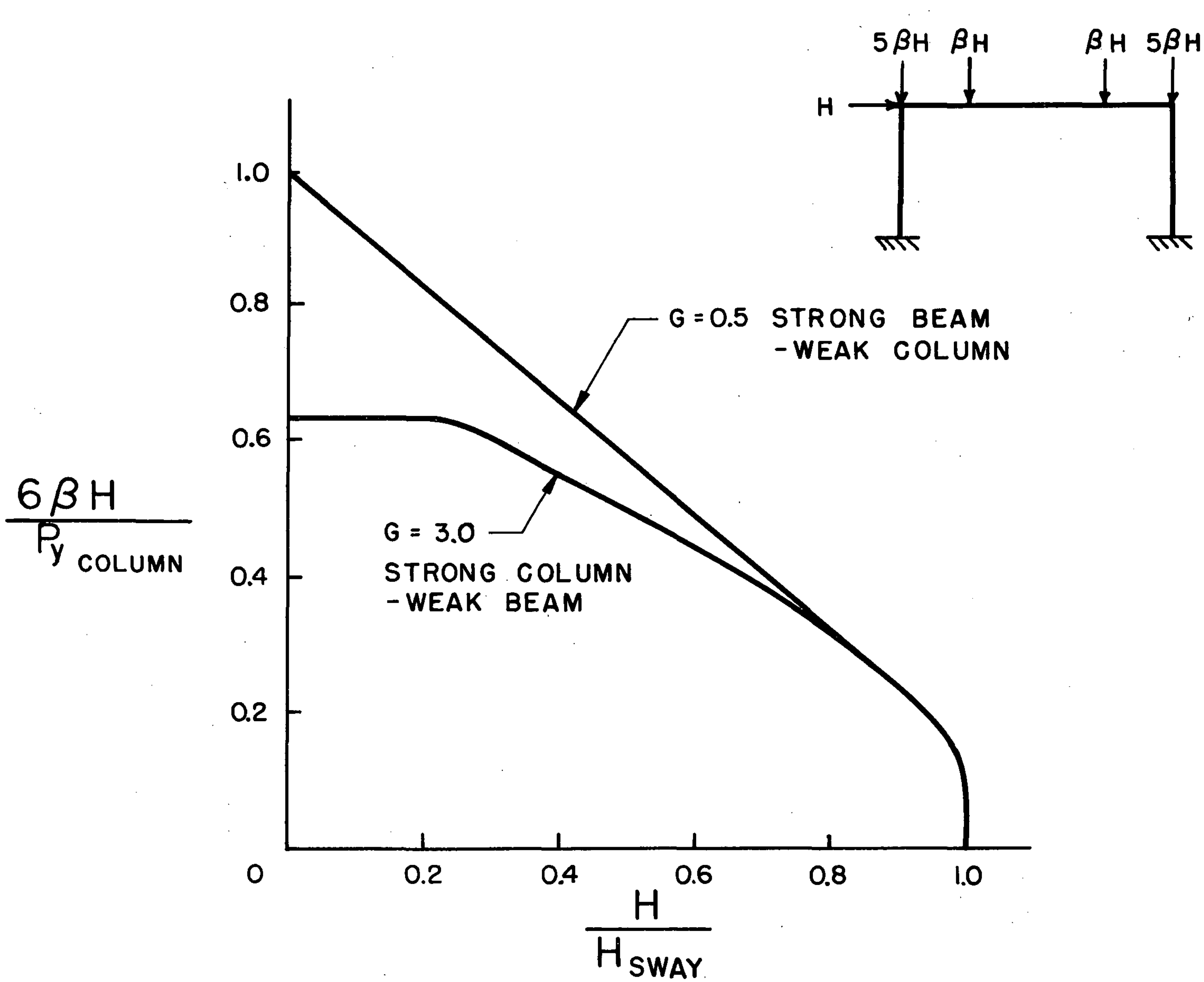
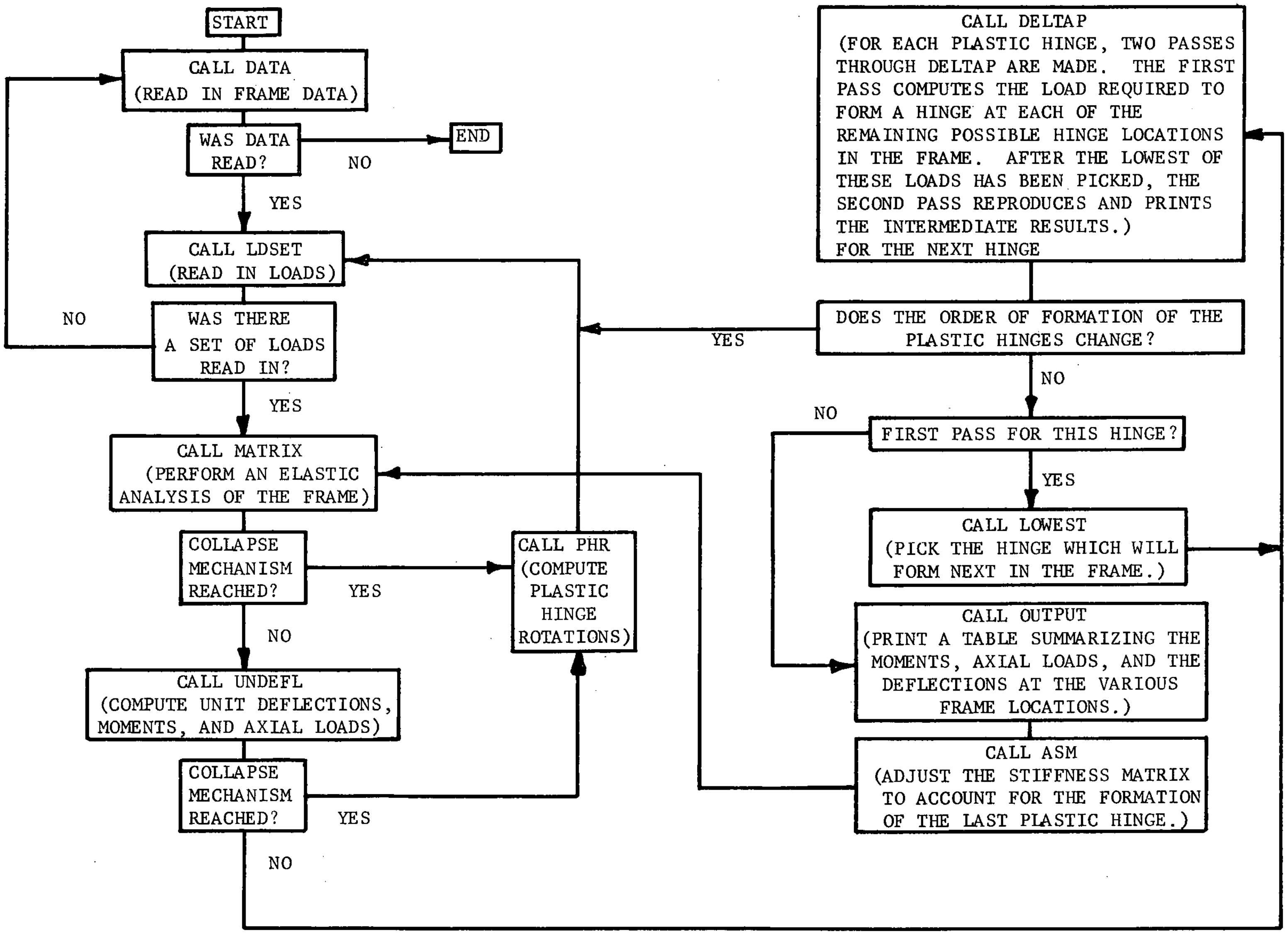


Fig. 14 Design Chart for the Example Frame

APPENDIX A - MAIN PROGRAM FLOW CHART



APPENDIX B PROGRAM LISTINGS


```
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
   90 CALL DATA
      GO TO (99, 87),INDEX
   87 CALL LDSET
      GO TO ( 90, 91),INDEX
   91 CALL MATRIX
      GO TO ( 98, 106,   1),INDEX
    1 CALL UNDEFL
      GO TO (100, 106, 98), INDEX
  100 CALL DELTAP
      GO TO ( 98, 99, 108, 112, 87), INDEX
  108 CALL LOWEST
      GO TO ( 106, 100), INDEX
  112 CALL OUTPUT
      CALL ASM
      GO TO 91
  106 CALL PHR
      GO TO 87
   98 PRINT 96
   96 FORMAT (48HNONSENSE TEST RESULT----- INDEX VALUE IN ERROR,    )
   99 CALL EXIT
      END
```

```
      SUBROUTINE DATA
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     READ THE FRAME NUMBER, EXIT IF NEGATIVE
C
    9 READ 13, JJ
   13 FORMAT (I5)
      IF (JJ)  88,5,5
   88 INDEX = 1
      RETURN
    5 REWIND 2
      REWIND 3
      REWIND 4
C
C     READ DEGREES OF FREEDOM AND NUMBER OF MEMBERS
C
      READ 23, L, NM
   23 FORMAT (2I5)
      M = 2 * NM
      N = 3 * NM
      MM = M + 1
C
C     READ IN THE STATICS MATRIX
C
      READ 35, ((A(I,J), J=1,N), I=1,L)
   35 FORMAT(7F10.4)
C
C     READ EI/L, PLASTIC MOMENT, EA/L, AND PY FOR EACH MEMBER
C
      DO 166  I = 1,NM
      K = 2*I
      READ 167, SDAT(I), PM(K), EAOL(I), PY(K)
      PY(K-1) = PY(K)
  166 PM(K-1) = PM(K)
  167 FORMAT(4F10.4)
      WRITE TAPE 2,((A(I,J), I= 1,L),J = 1, N)
      WRITE TAPE 2,( PM(I), I = 1, M)
      WRITE TAPE 2,( PY(I), I = 1, M)
      WRITE TAPE 2,( SDAT(I), I = 1, NM)
      WRITE TAPE 2,( EAOL(I), I = 1, NM)
      NPASS = 1
      INDEX = 2
      RETURN
      END
```

```
      SUBROUTINE LDSET
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     READ THE LOAD SET NUMBER.  IF IT IS NEGATIVE, READ A NEW FRAME
C     NO..
C
  708 READ 13, KK
   13 FORMAT (I5)
      IF (KK-1)89, 81, 81
   89 INDEX = 1
      RETURN
C
C     SET ALL THE ELEMENTS OF THE STIFFNESS MATRIX TO ZERO
C
   81 DO 160  I = 1,N
      DO 160  J = 1,N
  160 S(I,J) = 0.
      REWIND 2
      REWIND 3
      REWIND 4
      READ TAPE 2,((A(I,J), I = 1, L), J = 1, N)
      READ TAPE 2, ABC
      READ TAPE 2, ABC
      READ TAPE 2, (SDAT(I), I = 1, NM)
      READ TAPE 2, ( EAOL(I), I = 1, NM)
C
C     CONSTRUCT THE STIFFNESS MATRIX FROM KNOWN DATA
C
      DO 161  I = 1,M, 2
      K = I/2 + 1
      S(I,I) = 4.0 * SDAT(K)
      S(I+1,I+1) = S(I,I)
      S(I+1,I) = 0.5 * S(I,I)
  161 S(I,I+1) = S(I+1,I)
      DO 11 I = MM, N
      J = I - M
   11 S(I,I) = EAOL(J)
C
C     IF NPASS IS POSITIVE,
C     PRINT OUT TITLES, THE STATICS MATRIX, AND THE STIFFNESS MATRIX
C
      IF (NPASS) 707, 82, 82
   82 PRINT 97, JJ
```

```
   97 FORMAT (18H1ANALYSIS OF FRAME, I3, //)
      PRINT 7
    7 FORMAT (19H0THE STATICS MATRIX//)
      DO 1 I=1,L
    1 PRINT 21,I, (A(I,J), J=1,N)
   21 FORMAT(4H0ROW,I3,1X, 7E16.7/(8X,7E16.7))
      PRINT 17
   17 FORMAT (21H1THE STIFFNESS MATRIX//)
      DO 2 I=1,N
    2 PRINT 21,I,(S(I,J),J=1,N)
      NPASS = -1
  707 PRINT 27, KK
   27 FORMAT (13H1LOAD SET NO., I3)
C
C     READ IN AND PRINT OUT THE LOAD SET.
C
      READ 35, (PL(I), I=1,L)
   35 FORMAT (7F10.4)
      DO 3 I=1,L
    3 PRINT 21, I, PL(I)
      IH = 0
      WRITE TAPE 2,( PL(I), I = 1, L)
      WRITE TAPE 2,( (S(I,J), I = 1, N),J = 1, N)
      INDEX = 2
      RETURN
      END
```

```
          SUBROUTINE MATRIX
          COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
         1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
         2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
         3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
          EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
         1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
         2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
         3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
         41))
          DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
         1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
         2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
         3ATX(14), CXX(15), SAT1(15), ASAT1(15)
       15 IH = IH + 1
          IF (IH/2 * 2 - IH) 67, 68, 92
       92 INDEX = 1
          RETURN
       67 IT1 = 3
          IT2 = 4
          GO TO 69
       68 IT1 = 4
          IT2 = 3
C
C         CONSTRUCT THE MATRIX SAT ( S TIMES A TRANSPOSED)
C        (ALL THE FOLLOWING MATRIX OPERATIONS STORE THEIR INTERMEDIATE
C        RESULTS ON TAPE.)
C
       69 DO 57 I = 1, N
          DO 20 J=1,L
          SAT1(J) = 0.
          DO 20 K=1,N
       20 SAT1(J) = SAT1(J) + S(I,K) *A(J,K)
       57 WRITE TAPE IT1,(SAT1(II), II = 1, L)
          REWIND IT1
          DO 58 I = 1, N
       58 READ TAPE IT1,(SAT(I,J), J = 1, L)
          REWIND IT1
          WRITE TAPE IT1,((SAT(I,J), I = 1, N),J= 1, L)
C
C         CONSTRUCT THE MATRIX ASAT (A TIMES S TIMES A TRANSPOSED)
C
          DO 63 I = 1, L
          DO 40 J=1,L
          DO 40 K=1,N
          ASAT1(J) = 0.
       40 ASAT1(J) = ASAT1(J) +A(I,K) * SAT(K,J)
       63 WRITE TAPE IT1,(ASAT1(II), II = 1, L)
          REWIND IT1
          READ TAPE IT1, ABC
          DO 64 I = 1, L
       64 READ TAPE IT1,(ASAT (I,J), J= 1, L)
          REWIND IT1
          READ TAPE IT1, ABC
          LP1=L+1
```

```
      DO 50 I=1,L
   50 ASAT(I,LP1) = PL(I)
C
C     SOLVE THE EQUATIONS FOR THE UNIT DEFLECTIONS
C
      DO 60 I=1,L
      IP1=I+1
      TEMP=ABSF(ASAT(I,I))
      K=I
      DO 61 J=I,L
      IF (ABSF(ASAT(J,I))-TEMP)  61,61,62
   62 K=J
      TEMP = ABSF(ASAT(J,I))
   61 CONTINUE
      IF (K-I)  72,71,72
   72 DO 45 J=I,LP1
      TEMP = ASAT(I,J)
      ASAT(I,J) = ASAT(K,J)
   45 ASAT(K,J) = TEMP
   71 IF (ASAT(I,I))  16,147,16
  147 PRINT 347
  347 FORMAT (30H0DIVISION BY ZERO IN INVERSION)
      INDEX = 2
      RETURN
   16 TEMP = 1./ASAT(I,I)
      DO 70 J=I,LP1
   70 ASAT(I,J)=ASAT(I,J)*TEMP
      DO 60 J=1,L
      IF (I-J) 59,60,59
   59 TEMP=ASAT(J,I)
      DO 80 K=IP1,LP1
   80 ASAT(J,K)=ASAT(J,K)-TEMP*ASAT(I,K)
   60 CONTINUE
      INDEX = 3
      RETURN
      END
```

```
      SUBROUTINE UNDEFL
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     PRINT OUT THE UNIT DEFLECTIONS
C     ADD A COLUMN TO THE UNIT DEFLECTION MATRIX
C
      PRINT 511
  511 FORMAT ( 17H1UNIT DEFLECTIONS)
      KZ = IH - 1
      IF (IH - 1) 92, 74, 73
   73 READ TAPE IT2, ABC
      READ TAPE IT2,((UX(I,J), I = 1, L),J = 1, KZ)
   74 DO 51 I=1,L
      UX(I,IH) = ASAT(I,LP1)
   51 PRINT 21, I, ASAT(I,LP1)
      WRITE TAPE IT1,((UX(I,J), I = 1, L),J = 1, IH)
      REWIND IT1
      READ TAPE IT1,((SAT(I,J), I = 1, N),J = 1, L)
      READ TAPE IT1, ABC
C
C     CHECK TO SEE IF THE DEFLECTIONS EXCEED AN ARBITRARY MAXIMUM LIMIT
C     IF THEY DO, SAY SO AND GO ON TO COMPUTE THE HINGE ROTATIONS
C
      DO 311 I=1,L
      TEMP = ABSF(ASAT(I,LP1)) - 1.E+02
      IF (TEMP) 311,647,647
  311 CONTINUE
      GO TO 303
  647 PRINT 847
  847 FORMAT (21H0DEFLECTION TOO LARGE)
      INDEX = 2
      RETURN
C
C     COMPUTE AND PRINT UNIT MOMENTS AND AXIAL LOADS
C     ALSO ADD COLUMNS TO THE UNIT MOMENT AND UNIT AXIAL LOAD MATRICES
C     IF THE VALUE OF THE MOMENT COEF IS BELOW A GIVEN VALUE, SET IT TO
C     ZERO.
C
  303 DO 120 I=1,N
      SATX(I) = 0.
      DO 120 K=1,L
  120 SATX(I)=SATX(I)+SAT(I,K)*ASAT(K,LP1)
      PRINT 522
```

```
522 FORMAT (13H1UNIT MOMENTS)
    IF (IH - 1) 92, 76, 75
 75 READ TAPE IT2,((AM(I,J), I = 1, M),J = 1,KZ)
    READ TAPE IT2,((AN(I,J), I = 1, M),J = 1,KZ)
 76 DO 94 I = 1, M
    PRINT 21, I, SATX(I)
 21 FORMAT(4H0ROW,I3,1X, 7E16.7/(8X,7E16.7))
    IF ( SATX(I)) 56, 94, 56
 56 IF (ABSF(SATX(I)) - .001) 54, 54, 94
 54 SATX(I) = 0.
    PRINT 55, I
 55 FORMAT (24X, 14HUNIT MOMENT AT, I3, 1X, 11HSET TO ZERO)
 94 AM(I,IH) = SATX(I)
    PRINT 6
  6 FORMAT (17H0UNIT AXIAL LOADS)
    DO 93 I = MM, N
    K = I - M
    J = 2 * K
    PRINT 10, K, SATX(I)
 10 FORMAT(7H0MEMBER, I3, 1X, E16.7)
    AN(J,IH) = SATX(I)
 93 AN(J-1,IH) = SATX(I)
    WRITE TAPE IT1,((AM(I,J), I = 1, M),J = 1, IH)
    WRITE TAPE IT1,((AN(I,J), I = 1, M),J = 1, IH)
    REWIND 2
    II = 1
    INDEX = 1
    RETURN
 92 INDEX = 3
    RETURN
    END
```

```
      SUBROUTINE DELTAP
C
C     PLACE THE FOLLOWING QUANTITIES IN COMMON STORAGE SO THEY WILL BE
C     AVAILABLE TO ALL SUBROUTINES,
C
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
C
C     THE FOLLOWING EQUIVALENCE STATEMENTS RESULT FROM THE USE OF
C     TAPES.  NO TWO VARIABLES OR ARRAYS WHICH ARE EQUIVALENCED TO ONE
C     ANOTHER ARE EVER NEEDED AT THE SAME TIME.
C
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     11)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
C
C     DIMENSION THE FOLLOWING ARRAYS,
C
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     IF CONTROL IS COMING FROM THE SUBROUTINE WHICH PICKS THE LOWEST
C     VALUE OF LOAD TO FORM THE NEXT HINGE, GO DIRECTLY TO STATEMENT
C     31 AND REPRODUCE THE PREVIOUS RESULTS.  IF CONTROL IS COMING FROM
C     UNDEF (IF UNIT DEFLECTIONS AND FORCES HAVE JUST BEEN COMPUTED,)
C     AND THEREFORE IF THE NEXT PLASTIC HINGE HAS NOT YET BEEN LOCATED,
C     READ IN THE ARRAYS REQUIRED,
C
      GO TO ( 110, 31),II
C
C     THE FOLLOWING 6 STATEMENTS RETURN CONTROL TO THE MAIN PROGRAM
C     TO DETERMINE WHAT MEANING THE THREE VALUES OF INDEX WILL CONVEY
C     TO THE MAIN PROGRAM, REFER TO THAT PROGRAM,
C
  107 INDEX = 1
      RETURN
  109 INDEX = 2
      RETURN
  111 INDEX = 4
      RETURN
C
C     FROM TAPE, READ THE DATA REQUIRED FOR THE COMING SERIES OF
C     COMPUTATIONS.
C
  110 READ TAPE 2, ABC
      READ TAPE 2,( PM(I), I = 1, M)
      READ TAPE 2,( PY(I), I = 1, M)
      IXZ = 1
C
```

```
C       IF IH IS ONE, THE ARRAYS IORD AND P HAVE NOT BEEN USED YET AND
C       HENCE CANNOT BE READ IN.
C
           IF (IH - 1)107, 77, 78
        78 READ TAPE IT2,(IORD(I) , I = 1, KZ)
           READ TAPE IT2, (P(IPT, 1), IPT = 1, KZ)
C
C        BEGIN LOOP TO DETERMINE THE NEXT INCREMENT OF LOAD
C
        77 DO 28 IA = 1, M
C
C        IF THE MOMENT COEFICIENT IS ZERO AT A GIVEN POINT, SET THE VALUE
C        OF P AT A HIGH VALUE AND BEGIN THE LOOP AGAIN.
C       (ZERO MOMENT COEFFICIENT INDICATES THAT EITHER A PLASTIC HINGE HAS
C       ALREADY FORMED AT THE POINT OR ELSE BY COINCIDENCE, THE COEFF.
C       IS ZERO AND THEREFORE A PLASTIC HINGE SIMPLY WILL NOT FORM AT THIS
C       POINT DURING THIS INCREMENT OF LOAD.
C
           IF (AM(IA,IH)) 31, 32, 31
        32 PP(IA) = 1000.
           GO TO 28
C
C       INITIALIZE THE CYCLE NUMBER.
C
        31 ICYC = 0
C
C       INCREMENT THE CYCLE NUMBER.
C
        24 ICYC = ICYC + 1
C
C       SET THE TOTAL LOAD FOR THE CYCLE TO ZERO AT THE START.
C
           PPT = 0.
C
C       IF THIS IS THE FIRST CYCLE, USE THE LAST TRY FOR THE PREVIOUS
C       HINGE AS A FIRST GUESS.
C       THE VARIABLE KZ IS ONE LESS THAN IH.
C       OTHERWISE, START AT THE BEGINNING (AT THE BOTTOM OF THE LOAD
C       DEFLECTION CURVE)
C
           IF (ICYC - 1)107, 83, 84
        83 IPT = KZ
           GO TO 22
        84 IPT = 0
C
C        IF 40 CYCLES HAVE BEEN COMPLETED WITHOUT CONVERGENCE, SAY SO AND
C        REDO THAT PARTICULAR LOOP, PRINTING AS EACH VALUE IS FOUND.
C       CALL EXIT WHEN THIS HAS BEEN DONE.
C
           IF (ICYC - 40) 22, 22, 4
         4 PRINT 8, IA
         8 FORMAT (4HIPT., I3, 1X, 17HWILL NOT CONVERGE //)
           IF (IXZ)   5, 107, 42
        42 IXZ = -1
           GO TO 31
```

```
C
C     INCREMENT THE POINT NUMBER.
C
   22 IPT = IPT + 1
C
C     REFERING TO STATEMENT LABELED 12 BELOW, IT CAN BE SEEN THAT THE
C     QUANTITIES AA, BB, CC, DD, EE, AND GG ARE THE MAJOR VARIABLES
C     INVOLVED IN THE DETERMINATION OF THE LOAD INCREMENT P[IPT,ICYC],
C     [ THE VARIABLES O AND Q ARE USED ONLY TO INDICATE SIGN.]
C     WHERE-
C     AA = THE INCREASED CAPACITY OF THE POINT IN QUESTION, [THE X-AXIS
C     INTERSECTION FOR THE STRAIGHT LINE APPROXIMATION TO THE INTER-
C     ACTION FORMULA.]
C     BB = THE AXIAL LOAD AT THE POINT IN QUESTION WHICH RESULTS FROM
C     PREVIOUS INCREMENTS OF LOAD ON THE STRUCTURE.
C     CC = THE AXIAL LOAD AT THE POINT IN QUESTION WHICH RESULTS FROM
C     INCREMENTS OF LOAD BEING APPLIED TO THE STRUCTURE AFTER A PLASTIC
C     HINGE HAS FORMED AT THE POINT.
C     DD = THE MOMENT INDUCED AT THE POINT IN QUESTION DUE TO PREVIOUS
C     INCREMENTS OF LOAD BEING APPLIED TO THE STRUCTURE.
C     EE = THE UNIT MOMENT FOR THE POINT IN QUESTION FOR THIS INCREMENT
C     OF LOAD.
C     GG = A FUNCTION OF THE UNIT AXIAL LOAD FOR THIS INCREMENT AT THE
C     POINT IN QUESTION.
C
C     THE FOLLOWING TESTS DETERMINE WHERE ON THE LOAD DEFLECTION CURVE
C     THE PRESENT COMPUTATIONS LIE AND CONSEQUENTLY WHETHER OR NOT EACH
C     OF THE VARIABLES ABOVE HAS SOME VALUE OTHER THAN ZERO.
C     ALSO, THE TESTS SERVE TO DETERMINE WHETHER THE POINT IN QUESTION
C     IS A KNOWN PLASTIC HINGE OR WHETHER THE PROGRAM IS JUST TESTING
C     THE POINT TO SEE IF IT IS THE NEXT PLASTIC HINGE.
C
      IF [IPT - 1]107, 116, 117
  116 IF [IH-1]107, 118, 119
  118 CC = 0.
      BB = 0.
      DD = 0.
      IBZ = IA
      GO TO 121
  119 BB = 0.
      DD = 0.
      GO TO 122
  117 SB = 0.
      KK = IPT-1
      IF [ IH - IPT] 107, 101, 102
  101 IBZ = IA
      GO TO 103
  102 IBZ= IORD[IPT]
  103 DO 123 J = 1, KK
      B = P[J,ICYC]*AN[IBZ,J]
  123 SB = SB + B
      BB = SB * 1.18 * PM[IBZ] / PY[IBZ]
      DD = 0.
      DO 124 J = 1, KK
      D = P[J,ICYC] * AM[IBZ,J]
```

```
      124 DD = DD + D
      122 IF(ICYC-1)109, 126, 125
      125 IF(IH-IPT)107, 126, 127
      126 CC = 0.
          IF (IH - IPT)107, 104, 105
      104 IBZ = IA
          GO TO 121
      105 IBZ = IORD(IPT)
          GO TO 121
      127 SC = 0.
          KK = IPT + 1
          IBZ = IORD(IPT)
          DO 128 J = KK, IH
          C = P(J,ICYC-1)*AN(IBZ,J)
      128 SC = SC + C
          CC = SC + 1.18 * PM(IBZ) / PY(IBZ)
      121 AA = 1.18 * PM(IBZ)
          EE = AM(IBZ,IPT)
          GG = 1.18 * PM(IBZ) * AN(IBZ,IPT) / PY(IBZ)
C
C     IF THE UNIT MOMENT AT THE POINT HAS A DIFFERENT SIGN THAN THE
C     MOMENT THAT HAS BEEN INDUCED PREVIOUSLY AT THIS POINT, THE
C     VARIABLE Q ALTERS THE EQUATION FOR P(IPT,ICYC) SO THAT THE
C     QUANTITY DD WILL PROVIDE ADDITIONAL MOMENT CAPACITY AT THE POINT
C     RATHER THAN LESS AS IS THE USUAL CASE.
C
          IF (DD/EE) 6,   7, 7
        6 Q = -1.
          GO TO 11
        7 Q = 1.
C
C     IF THE UNIT AXIAL LOAD AT THE POINT HAS A DIFFERENT SIGN THAN THE
C     PREVIOUS AXIAL LOAD AT THE POINT, THE QUANTITY O  ALTERS THE EQUAT
C     *ION FOR P(IPT,ICYC) SO THAT THE MOMENT CAPACITY AT THE POINT WILL
C     BE INCREASED.
C
       11 IF (( BB + CC )/ GG ) 9, 10 , 10
        9 O = -1.
          GO TO 12
       10 O = 1.
C
C     COMPUTE THE VALUE OF THE INCREMENT OF LOAD.
C
       12 P(IPT, ICYC) = (AA*ABSF(BB + CC)*O - ABSF(DD)*Q)/(ABSF(EE)+ABSF(GG
         1))
C
C      BEGIN CHECK TO SEE IF AXIAL LOAD REDUCTION IS WARRENTED.
C     FIRST, COMPUTE THE AXIAL LOAD AT THE POINT UNDER CONSIDERATION.
C     TWO CONTRIBUTIONS ARE CONSIDERED.  THE AXIAL LOAD SAX DUE TO
C     PREVIOUS INCREMENTS OF LOAD AND RAX DUE TO LATER INCREMENTS OF
C     LOAD.
C
          SAX = 0.
          DO 129 J = 1, IPT
          SS = AN(IBZ,J) * P(J,ICYC)
```

```
  129 SAX = SAX + SS
      RAX = 0.
      IF[IH-1]107,  36, 131
  131 IF[ICYC-1]107,  36, 37
   37 IF[IH-IPT]107,  36, 38
   38 KK = IPT + 1
      DO 39 J = KK, IH
      RR = AN[IBZ,J] * P[J,ICYC-1]
   39 RAX = RAX + RR
   36 AX = SAX + RAX
C
C      NEXT, SEE IF P/PY IS GREATER THAN .15.  IF IT IS, SKIP THE NEXT
C      BLOCK OF INSTRUCTIONS.  IF NOT, COMPUTE THE INCREMENT OF LOAD P
C      TO FORM THE NEXT HINGE NEGLECTING THE EFFECT OF AXIAL LOAD.
C
      IF[ABSF[AX]/PY[IBZ]-.15] 41, 230, 230
  230 IF [IXZ] 132, 232, 232
   41 RM = 0.
      IF[IPT-1]107, 44, 43
   43 KK = IPT - 1
      DO 133 J = 1, KK
      RZ = AM[IBZ,J] * P[J,ICYC]
  133 RM = RM + RZ
C
C     THIS TEST DETERMINES WHETHER THE SIGN OF THE MOMENT HAS CHANGED
C     AND IF IT HAS, THE INCREMENT OF LOAD IS COMPUTED AS IF THERE HAD
C     BEEN AN INCREASE IN THE AVAILABLE MOMENT CAPACITY RATHER THAN A DE
C     -CREASE AS USUAL.
C
      IF [RM/AM[IBZ,IPT]] 48,107, 44
   48 P[IPT,ICYC] = [PM[IBZ] + ABSF[RM]] / ABSF[AM[IBZ,IPT]]
      GO TO 49
   44 P[IPT,ICYC] = [PM[IBZ] - ABSF[RM]]/ ABSF[AM[IBZ,IPT]]
   49 IF [IXZ] 231, 232, 232
C
C     IF THE AXIAL LOAD AT THE POINT IS LESS THAN .15 PY PRINT THIS ON
C     THE OUTPUT IF IXZ IS NEGATIVE. [ THE VALUE OF IXZ DETERMINES
C     WHETHER OR NOT INTERMEDIATE RESULTS WILL BE PRINTED.  IF IXZ IS
C     POSITIVE, PRINTING WILL BE SUPPRESSED. IF IT IS NEGATIVE PRINTING
C     WILL OCCUR.]
C
  231 PRINT 46, IPT
   46 FORMAT [23HNO REDUCTION FOR HINGE , I2]
C
C     PRINT THE VALUE OF THE NEXT INCREMENT OF LOAD.
C
  132 PRINT 135,IPT, ICYC, P[IPT,ICYC]
  135 FORMAT [2HP[,I2,1H,,I2,4H] = , F10.4, //]
C
C     IF THE VALUE OF THE INCREMENT OF LOAD HAS TURNED NEGATIVE, THIS
C     INDICATES THAT THE AXIAL LOAD AT THE POINT HAS BUILT UP TO THE
C     POINT WHERE THE HINGE WILL NOW FORM AT A LOAD LESS THAN THE
C     PREVIOUS PLASTIC HINGE FORMED AT, IN OTHER WORDS, THERE HAS BEEN
C     A CHANGE IN THE ORDER OF FORMATION OF THE PLASTIC HINGES IN THE
C     FRAME. [ THIS IS NOT TO INDICATE THAT IF THE FRAME WERE TESTED THE
```

```
C     HINGES WOULD FORM IN A DIFFERENT ORDER, IT IS JUST THAT FOR THE
C     PURPOSES OF THE ANALYSIS, THE ORDER HAS CHANGED.)
C
  232 IF (P(IPT,ICYC))  1, 79, 79
    1 IF (IPT - 1) 107, 32, 2
    2 PRINT 3
    3 FORMAT (27H1ABORT, HINGE ORDER CHANGES )
C
C     TERMINATE THIS RUN.
C
    5 INDEX = 5
      RETURN
C
C     IF THIS IS THE FIRST PLASTIC HINGE TO FORM IN THE FRAME, ONE TRY
C     WILL PRODUCE AN EXACT VALUE FOR THE LOAD INCREMENT.  THEREFORE
C     CHECK THE NEXT LOCATION ON THE FRAME.
C     OTHERWISE, GO ON TO THE NEXT LEG ON THE LOAD DEFLECTION CURVE.
C
   79 IF (IH - 1)107, 14, 18
   14 PP(IA) = P(IPT, ICYC)
      GO TO 233
   18 PPT = PPT + P(IPT, ICYC)
      IF (IH - IPT)107, 19, 22
   19 PT(ICYC) = PPT
C
C     CHECK THE QUANTITY IXZ TO SEE IF THE RESULTS OBTAINED SHOULD BR
C     PRINTED,  THEY WILL BE PRINTED IF 1) THE HINGE HAS BEEN PICKED
C     AND THE COMPUTATIONS ARE BEING REDONE SO THEY MAY BE LISTED, OR
C     2) IF CONVERGENCE COULD NOT BE OBTAINED.
C
      IF (IXZ) 34, 52, 52
   34 PRINT 33, PPT
   33 FORMAT ( 27X, 20HTOTAL FOR THIS CYCLE, F10.4, ///)
   52 IF (ICYC - 1)107, 24, 25
C
C     IF THE DIFFERENCE BETWEEN TWO SUCESSIVE VALUES OF THE TOTAL
C     LOAD REQUIRED TO FORM A PLASTIC HINGE IS LESS THAN SOME ARBITRARY
C     QUANTITY, CONSIDER CONVERGENCE OBTAINED.  OTHERWISE DO THE
C     COMPUTATION AGAIN.
C
   25 IF (ABSF(PT(ICYC - 1)-PT(ICYC)) - .001) 234, 234, 24
  234 IF (IXZ) 235, 26, 26
  235 PRINT 238, IH
  238 FORMAT (30H0CONVERSION OBTAINED FOR HINGE, I3)
   26 PP(IA) = PT(ICYC)
  233 IF (IXZ) 111, 28, 28
C
C     GO BACK AND CHECK THE NEXT POSSIBLE PLASTIC HINGE LOCATION.
C
   28 CONTINUE
      INDEX = 3
      RETURN
      END
```

```
      SUBROUTINE LOWEST
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     PICK THE SMALLEST VALUE OF THE LOAD P.
C     DETERMINE THE LOCATION OF THE NEXT PLASTIC HINGE.
C
      AA = 1000.
      IF ( IH - 1) 1, 1, 2
    2 ITEMP = NPH
    1 DO 236 I = 1, M
  237 IF (AA - PP(I)) 236, 239, 239
  239 AA = PP(I)
      NPH = I
  236 CONTINUE
      IF (ITEMP - NPH ) 3, 4, 3
    4 PRINT 5
    5 FORMAT ( 24H1SAME HINGE PICKED TWICE  )
      INDEX = 1
      RETURN
    3 IXZ = -1
      IA = NPH
C
C     PRINT OUT THE LOCATION DECEIDED ON FOR THE NEXT HINGE AND GO BACK
C     AND RECALCULATE THE QUANTITIES ASSOCIATED WITH THIS POINT
C     PRINTING AS EACH RESULT IS OBTAINED.
C
      PRINT 242, IH, NPH
  242 FORMAT (18H1PLASTIC HINGE NO., I3, 4X, 13HFORMED AT PT.,  I3,//)
      II = 2
      INDEX = 2
      RETURN
      END
```

```
      SUBROUTINE OUTPUT
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     ADD ANOTHER ELEMENT TO THE IORD MATRIX ( THIS MATRIX KEEPS TRACK
C     OF THE ORDER IN WHICH HINGES HAVE FORMED.)
C
  240 IORD(IH) = NPH
      WRITE TAPE IT1,(IORD(I), I = 1, IH)
      WRITE TAPE IT1, (P(IPT, ICYC), IPT = 1, IH)
C
C     PRINT OUT A TABLE GIVING THE VALUES OF TOTAL MOMENT, MPC,
C     PLASTIC MOMENT, AXIAL LOAD, PY, AND THE RATIO OF AXIAL LOAD TO
C     PY FOR EACH POINT ON THE FRAME.
C
      PRINT 241
  241 FORMAT (4H PT,,3X,9HTOT. MOM.,6X,3HMPC,10X,2HMP,7X,8HAX. LOAD,6X,
     22HPY,10X, 4HP/PY,//)
      DO 247 I = 1, M
      CM(I) = 0.
      CAX = 0.
      DO 243 J = 1, IH
      CM(I) = CM(I) + AM(I,J) * P(J,ICYC)
  243 CAX = CAX + AN(I,J) * P(J, ICYC)
      POPY = ABSF(CAX)/PY(I)
      IF (POPY - .15 ) 245, 246, 246
  245 CPM = PM(I)
      GO TO 247
  246 CPM = 1.18 * PM(I) * (1.- POPY)
  247 PRINT 244, I, CM(I), CPM, PM(I), CAX, PY(I), POPY
  244 FORMAT ( 1H0, I2, 6F12.3 )
      WRITE TAPE IT1,(CM(I), I = 1, M)
      REWIND IT1
      READ TAPE IT1, ABC
      READ TAPE IT1,((UX(I,J), I = 1,L),J = 1, IH)
      DO 85 I = 1, 5
   85 READ TAPE IT1, ABC
C
C    PRINT OUT THE TOTAL DEFLECTIONS UP TO THIS POINT FOR EACH DEGREE
C    OF FREEDOM IN THE FRAME.
C
      PRINT 248
  248 FORMAT(33H0TOTAL DEFLECTIONS AT THIS STAGE.  )
      PRINT 249
```

```
249 FORMAT ( 4HOPT., 4X, 10HDEFLECTION,//)
    DO 251 I = 1, L
    CX(I) = 0.
    DO 250 J = 1, IH
250 CX(I) = CX(I) + P(J,ICYC) * UX(I,J)
251 PRINT 252, I, CX(I)
252 FORMAT ( I3, E16.7,//)
    WRITE TAPE IT1,(CX(I) , I = 1, L)
    READ TAPE 2, ABC
    READ TAPE 2, ABC
    READ TAPE 2,(PL(I) , I = 1,L)
    READ TAPE 2,((S(I,J), I = 1,N),J = 1, N)
    BACKSPACE 2
    REWIND 3
    REWIND 4
    RETURN
    END
```

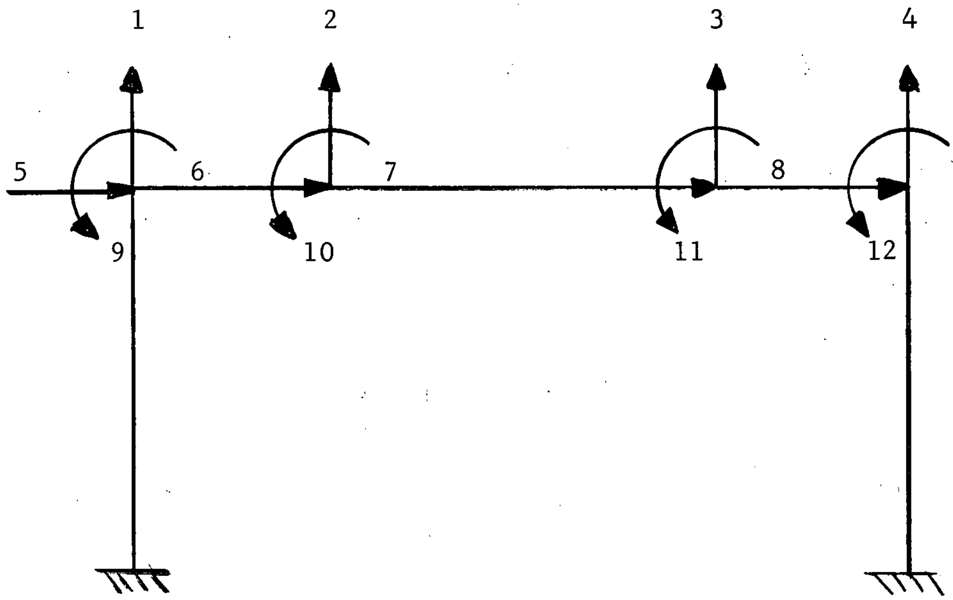
```
      SUBROUTINE ASM
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     ALTER THE STIFFNESS MATRIX TO ACCOUNT FOR THE FORMATION OF THE
C     LAST PLASTIC HINGE.
C
      IF((NPH/2 * 2)- NPH) 211, 210, 210
  210 S(NPH-1,NPH-1)=0.75*S(NPH-1,NPH-1)
      S(NPH,NPH) = 0.
      S(NPH-1,NPH) = 0.
      S(NPH,NPH-1) = 0.
      GO TO 212
  211 S(NPH+1,NPH+1) = 0.75*S(NPH+1,NPH+1)
      S(NPH,NPH) = 0.
      S(NPH,NPH+1) = 0.
      S(NPH+1,NPH) = 0.
  212 WRITE TAPE 2,((S(I,J), I = 1, N),J = 1, N)
      REWIND 2
      READ TAPE 2,((A(I,J), I = 1, L),J = 1, N)
      RETURN
      END
```

```
      SUBROUTINE PHR
      COMMON A, S, SDAT, EAOL, PL, L, NM, M, N, MM, I, J, K, NPASS, KK,
     1JJ, IH, IT1, IT2, ABC, LP1, IP1, TEMP, KZ, IXZ, IPT, IA, ICYC,
     2IBZ, AA, BB, CC, DD, EE, B, SB, D, SC, C, GG, SAX, SS, RAX, AX,
     3RR, RM, NPH, CAX, POPY, CPM, INDEX, II
      EQUIVALENCE (A(1), ASAT(1), AM(1)), (A(166), AN(1)), (S(295), DM(1
     1)), (S(309), ATX(1)), (S(1), SAT(1), UX(1), PY(1)), (S(15), PM(1))
     2, (S(391),PT(1)), (S(49), IORD(1)), (S(57), PP(1)), (S(71), P(1)),
     3(S(323), CXX(1)), (PL(1), CM(1)), (SDAT(1), CX(1), SAT1(1), ASAT1(
     41))
      DIMENSION A(15,21), S(21,21), SDAT(7), EAOL(7), PL(15), PM(14),
     1PY(14), CX(15), CM(14), SAT(21,15), ASAT(15,16), PP(14), UX(15,8),
     2SATX(21), AM(15,11), AN(15,11), IORD(8), P(8,40), PT(40), DM(14),
     3ATX(14), CXX(15), SAT1(15), ASAT1(15)
C
C     RE-ENTRY POINT FOR THE COMPUTATION OF PLASTIC HINGE ROTATIONS.
C
   47 PRINT 408
  408 FORMAT (36H0COLLAPSE MECHANISM HAS BEEN REACHED)
      REWIND 2
      REWIND IT2
      READ TAPE 2, ((A(I,J), I = 1, L), J = 1, N)
      READ TAPE 2, ABC
      READ TAPE 2, ABC
      READ TAPE 2,(SDAT (I), I= 1, NM)
      DO 86 I = 1, 6
   86 READ TAPE IT2, ABC
      READ TAPE IT2,(CM(I), I = 1,M)
      READ TAPE IT2,(CXX(I), I = 1, L)
C
C     CONSTRUCT THE INVERTED STIFFNESS MATRIX (FLEXIBILITY  MATRIX.)
C
      DO 163  I = 1,M
      DO 163  J = 1,M
  163 S(I,J) = 0.
      DO 164  I = 1,M, 2
      K = I/2 + 1
      S(I,I) = 1.0/(3.0 * SDAT(K))
      S(I+1,I+1) = S(I,I)
      S(I,I+1) = -0.5*S(I,I)
  164 S(I+1,I) = S(I,I+1)
      DO 134 I = 1, M
      DM(I) = 0.
      DO 134 K=1,M
  134 DM(I) = DM(I) + S(I,K) * CM(K)
      DO 136 I = 1, M
      ATX(I) = 0.
      DO 136 K=1,L
  136 ATX(I) = ATX(I) + A(K,I)*CXX(K)
C
C     PRINT OUT THE ROTATIONS AT EACH POINT.
C
      PRINT 138
  138 FORMAT (16H0HINGE ROTATIONS)
      PRINT 93
```

```
   53 FORMAT (4HOPT.,4X,8HROTATION,//)
      DO 139 I= 1, M
      H = DM(I) * ATX(I)
  139 PRINT 252, I, H
  252 FORMAT ( I3, E16.7, //)
C
C     RETURN TO THE READ IN POINT FOR KK TO SEE IF THERE ARE ANY MORE
C     LOAD SETS.
C
      RETURN
      END
```

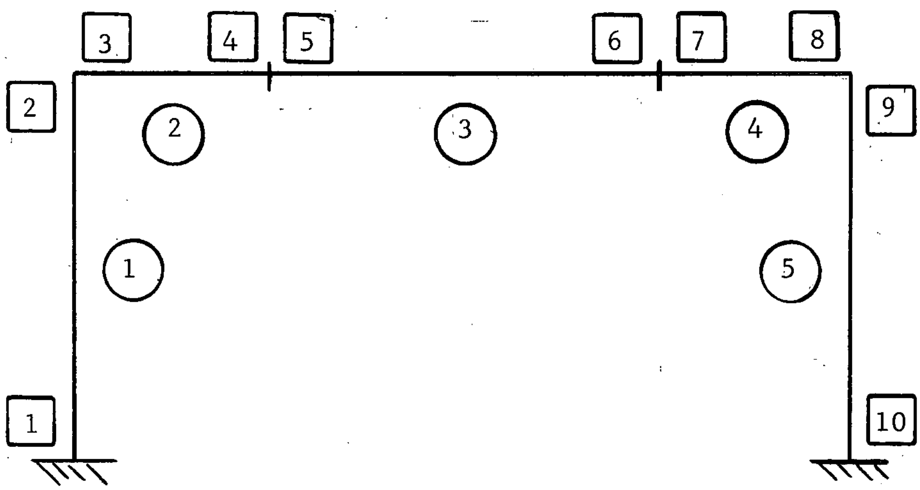
APPENDIX C ANALYSIS OF THE EXAMPLE FRAME

INDEXING SYSTEMS FOR THIS EXAMPLE



DEFLECTION AND LOAD NUMBERING

Columns - 14 W 78 Beams - 21 W 142



MEMBER IDENTIFICATION

- Member Numbers

- Member End Numbers

ANALYSIS OF FRAME 7

THE STATICS MATRIX

ROW 1	0.	0.	0.1333333E-00	0.1333333E-00	0.	0.	0.
	0.	0.	0.	0.1000000E 01	0.	0.	0.
	0.						
ROW 2	0.	0.	-0.1333333E-00	-0.1333333E-00	0.6666667E-01	0.6666667E-01	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 3	0.	0.	0.	0.	-0.6666667E-01	-0.6666667E-01	0.1333333E-00
	0.1333333E-00	0.	0.	0.	0.	0.	0.
	0.						
ROW 4	0.	0.	0.	0.	0.	0.	-0.1333333E-00
	-0.1333333E-00	0.	0.	0.	0.	0.	0.
	0.1000000E 01						
ROW 5	0.6666667E-01	0.6666667E-01	0.	0.	0.	0.	0.
	0.	0.	0.	0.	-0.1000000E 01	0.	0.
	0.						
ROW 6	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.1000000E 01	-0.1000000E 01	0.
	0.						
ROW 7	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.1000000E 01	-0.1000000E 01
	0.						
ROW 8	0.	0.	0.	0.	0.	0.	0.
	0.	0.6666667E-01	0.6666667E-01	0.	0.	0.	0.1000000E 01
	0.						
ROW 9	0.	0.1000000E 01	0.1000000E 01	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 10	0.	0.	0.	0.1000000E 01	0.1000000E 01	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 11	0.	0.	0.	0.	0.	0.1000000E 01	0.1000000E 01
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 12	0.	0.	0.	0.	0.	0.	0.
	0.1000000E 01	0.1000000E 01	0.	0.	0.	0.	0.
	0.						

THE STIFFNESS MATRIX

ROW 1	0.4728880E 05	0.2364440E 05	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 2	0.2364440E 05	0.4728880E 05	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 3	0.	0.	0.3781220E 06	0.1890610E 06	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 4	0.	0.	0.1890610E 06	0.3781220E 06	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 5	0.	0.	0.	0.	0.1890612E 06	0.9453060E 05	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 6	0.	0.	0.	0.	0.9453060E 05	0.1890612E 06	0.
	0.	0.	0.	0.	0.	0.	0.
	0.						
ROW 7	0.	0.	0.	0.	0.	0.	0.3781220E 06
	0.1890610E 06	0.	0.	0.	0.	0.	0.
	0.						
ROW 8	0.	0.	0.	0.	0.	0.	0.1890610E 06
	0.3781220E 06	0.	0.	0.	0.	0.	0.
	0.						
ROW 9	0.	0.	0.	0.	0.	0.	0.
	0.	0.4728880E 05	0.2364440E 05	0.	0.	0.	0.
	0.						
ROW 10	0.	0.	0.	0.	0.	0.	0.
	0.	0.2364440E 05	0.4728880E 05	0.	0.	0.	0.
	0.						
ROW 11	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.4588000E 05	0.	0.	0.
	0.						
ROW 12	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.1670400E 06	0.	0.
	0.						
ROW 13	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.6352000E 05	0.
	0.						
ROW 14	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.1670400E 06
	0.						
ROW 15	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.
	0.4588000E 05						

LOAD SET NO. 3

ROW 1	-0.2080000E 02
ROW 2	-0.4160000E 01
ROW 3	-0.4160000E 01
ROW 4	-0.2080000E 02
ROW 5	0.1000000E 01
ROW 6	0.
ROW 7	0.
ROW 8	0.
ROW 9	0.
ROW 10	0.
ROW 11	0.
ROW 12	0.

UNIT DEFLECTIONS

ROW 1	-0.5390036E-03
ROW 2	-0.2523466E-02
ROW 3	-0.2391322E-02
ROW 4	-0.5490522E-03
ROW 5	0.9983079E-03
ROW 6	0.9883555E-03
ROW 7	0.9684507E-03
ROW 8	0.9584983E-03
ROW 9	-0.2731933E-03
ROW 10	-0.2040362E-03
ROW 11	0.2094626E-03
ROW 12	0.2237525E-03

UNIT MOMENTS

ROW 1	-0.1738613E 01
ROW 2	-0.8198105E 01
ROW 3	0.8198104E 01
ROW 4	0.2127302E 02
ROW 5	-0.2127302E 02
ROW 6	0.1781527E 02
ROW 7	-0.1781527E 02
ROW 8	-0.1511361E 02
ROW 9	0.1511361E 02
ROW 10	0.9823117E 01

UNIT AXIAL LOADS

MEMBER 1	-0.2472948E 02
MEMBER 2	-0.1662450E 01
MEMBER 3	-0.1662449E 01
MEMBER 4	-0.1662447E 01
MEMBER 5	-0.2519052E 02

PLASTIC HINGE NO. 1 FORMED AT PT. 8

P(1, 1) = 16.0349

PT.	TOT. MOM.	MPC	MP	AX. LOAD	PY	P/PY
1	-27.878	246.592	402.000	-396.535	825.840	0.480
2	-131.456	246.592	402.000	-396.535	825.840	0.480
3	131.456	1071.000	1071.000	-26.657	1503.360	0.018
4	341.111	1071.000	1071.000	-26.657	1503.360	0.018
5	-341.111	1071.000	1071.000	-26.657	1503.360	0.018
6	285.666	1071.000	1071.000	-26.657	1503.360	0.018
7	-285.666	1071.000	1071.000	-26.657	1503.360	0.018
8	-242.345	1071.000	1071.000	-26.657	1503.360	0.018
9	242.345	242.345	402.000	-403.927	825.840	0.489
10	157.513	242.345	402.000	-403.927	825.840	0.489

TOTAL DEFLECTIONS AT THIS STAGE.

PT.	DEFLECTION
1	-0.8642870E-02
2	-0.4046353E-01
3	-0.3834462E-01
4	-0.8803999E-02
5	0.1600777E-01
6	0.1584818E-01
7	0.1552901E-01
8	0.1536943E-01
9	-0.4380628E-02
10	-0.3271700E-02
11	0.3358713E-02
12	0.3587849E-02

UNIT DEFLECTIONS

ROW 1	-0.5459480E-03
ROW 2	-0.3662038E-02
ROW 3	-0.3712537E-02
ROW 4	-0.5421078E-03
ROW 5	0.4017866E-02
ROW 6	0.4014089E-02
ROW 7	0.4006534E-02
ROW 8	0.4002757E-02
ROW 9	-0.4576735E-03
ROW 10	-0.3171101E-03
ROW 11	0.3150363E-03
ROW 12	0.4765678E-03

UNIT MOMENTS

ROW 1	0.8178591E 01
ROW 2	-0.2642825E 01
ROW 3	0.2642824E 01
ROW 4	0.2921788E 02
ROW 5	-0.2921788E 02
ROW 6	0.3053930E 02
ROW 7	-0.3053930E 02
ROW 8	-0.4768372E-06

UNIT MOMENT AT 8 SET TO ZERO

ROW 9	0.
ROW 10	0.9464278E 01

UNIT AXIAL LOADS

MEMBER 1	-0.2504809E 02
MEMBER 2	-0.6309566E 00
MEMBER 3	-0.6309533E 00
MEMBER 4	-0.6309786E 00
MEMBER 5	-0.2487191E 02

PLASTIC HINGE NO. 2 FORMED AT PT. 10

P[2, 1] = 3.5718

TOTAL FOR THIS CYCLE 3.5718

P[1, 2] = 14.3100
P[2, 2] = 5.3361

TOTAL FOR THIS CYCLE 19.6461

P[1, 3] = 13.4580
P[2, 3] = 6.2075

TOTAL FOR THIS CYCLE 19.6655

P[1, 4] = 13.0371
P[2, 4] = 6.6380

TOTAL FOR THIS CYCLE 19.6751

P[1, 5] = 12.8293
P[2, 5] = 6.8506

TOTAL FOR THIS CYCLE 19.6798

P[1, 6] = 12.7266
P[2, 6] = 6.9556

TOTAL FOR THIS CYCLE 19.6822

P[1, 7] = 12.6759
P[2, 7] = 7.0075

TOTAL FOR THIS CYCLE 19.6833

P[1, 8] = 12.6508
P[2, 8] = 7.0331

TOTAL FOR THIS CYCLE 19.6839

CONVERSION OBTAINED FOR HINGE 2

PT.	TOT. MOM.	MPC	MP	AX. LOAD	PY	P/PY
1	35.526	193.472	402.000	-489.014	825.840	0.592
2	-122.300	193.472	402.000	-489.014	825.840	0.592
3	122.300	1071.000	1071.000	-25.469	1503.360	0.017
4	474.613	1071.000	1071.000	-25.469	1503.360	0.017
5	-474.613	1071.000	1071.000	-25.469	1503.360	0.017
6	440.163	1071.000	1071.000	-25.469	1503.360	0.017
7	-440.163	1071.000	1071.000	-25.469	1503.360	0.017
8	-191.200	1071.000	1071.000	-25.469	1503.360	0.017
9	191.200	190.834	402.000	-493.607	825.840	0.598
10	190.834	190.834	402.000	-493.607	825.840	0.598

TOTAL DEFLECTIONS AT THIS STAGE.

PT.	DEFLECTION
1	-0.1065854E-01
2	-0.5767935E-01
3	-0.5636279E-01
4	-0.1075865E-01
5	0.4088741E-01
6	0.4073493E-01
7	0.4042999E-01
8	0.4027752E-01
9	-0.6674977E-02
10	-0.4811488E-02
11	0.4865552E-02
12	0.6182395E-02

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UNIT DEFLECTIONS

ROW  1   -0.5430028E-03
ROW  2   -0.3941991E-02
ROW  3   -0.3915028E-02
ROW  4   -0.5450530E-03
ROW  5    0.5448868E-02
ROW  6    0.5448868E-02
ROW  7    0.5448868E-02
ROW  8    0.5448867E-02
ROW  9   -0.5150484E-03
ROW 10   -0.3369616E-03
ROW 11    0.3380688E-03
ROW 12    0.5049607E-03

UNIT MOMENTS

ROW  1    0.1358903E 02
ROW  2    0.1411018E 01
ROW  3   -0.1411019E 01
ROW  4    0.3225827E 02
ROW  5   -0.3225827E 02
ROW  6    0.3155276E 02
ROW  7   -0.3155276E 02
ROW  8   -0.4768372E-06
                    UNIT MOMENT AT  8 SET TO ZERO
ROW  9    0.
ROW 10    0.

UNIT AXIAL LOADS

MEMBER  1   -0.2491297E 02
MEMBER  2    0.3814697E-05
MEMBER  3    0.5245209E-05
MEMBER  4   -0.4673004E-04
MEMBER  5   -0.2500703E 02
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PLASTIC HINGE NO.  3    FORMED AT PT.  1

P( 3, 1) =     5.6614
                              TOTAL FOR THIS CYCLE    5.6614

P( 1, 2) =     9.8896
P( 2, 2) =     6.4334
P( 3, 2) =     7.3802
                              TOTAL FOR THIS CYCLE   23.7032

P( 1, 3) =     9.3446
P( 2, 3) =     5.9513
P( 3, 3) =     8.0136
                              TOTAL FOR THIS CYCLE   23.3096

P( 1, 4) =     9.2698
P( 2, 4) =     5.6447
P( 3, 4) =     8.2951
                              TOTAL FOR THIS CYCLE   23.2096

P( 1, 9) =     9.3271
P( 2, 9) =     5.2433
P( 3, 9) =     8.5941
                              TOTAL FOR THIS CYCLE   23.1645

P( 1,10) =     9.3299
P( 2,10) =     5.2322
P( 3,10) =     8.6019
                              TOTAL FOR THIS CYCLE   23.1640

CONVERSION OBTAINED FOR HINGE  3
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PT.	TOT. MOM.	MPC	MP	AX. LOAD	PY	P/PY
1	143.462	143.462	402.000	-576.079	825.840	0.698
2	-78.178	143.462	402.000	-576.079	825.840	0.698
3	78.178	1071.000	1071.000	-18.812	1503.360	0.013
4	628.831	1071.000	1071.000	-18.812	1503.360	0.013
5	-628.831	1071.000	1071.000	-18.812	1503.360	0.013
6	597.415	1071.000	1071.000	-18.812	1503.360	0.013
7	-597.415	1071.000	1071.000	-18.812	1503.360	0.013
8	-141.008	1071.000	1071.000	-18.812	1503.360	0.013
9	141.008	141.056	402.000	-580.267	825.840	0.703
10	141.168	141.056	402.000	-580.267	825.840	0.703

TOTAL DEFLECTIONS AT THIS STAGE.

PT.	DEFLECTION
1	-0.1255620E-01
2	-0.7661273E-01
3	-0.7541214E-01
4	-0.1264750E-01
5	0.7720692E-01
6	0.7709430E-01
7	0.7686907E-01
8	0.7675645E-01
9	-0.9373891E-02
10	-0.6461324E-02
11	0.6510625E-02
12	0.8924699E-02

UNIT DEFLECTIONS

ROW 1	-0.5331299E-03
ROW 2	-0.4880432E-02
ROW 3	-0.4593806E-02
ROW 4	-0.5549259E-03
ROW 5	0.1695467E-01
ROW 6	0.1695467E-01
ROW 7	0.1695467E-01
ROW 8	0.1695467E-01
ROW 9	-0.7073771E-03
ROW 10	-0.4035064E-03
ROW 11	0.4152766E-03
ROW 12	0.6001377E-03

UNIT MOMENTS

ROW 1	0.
ROW 2	0.1500005E 02
ROW 3	-0.1500005E 02
ROW 4	0.4245004E 02
ROW 5	-0.4245004E 02
ROW 6	0.3495001E 02
ROW 7	-0.3495001E 02
ROW 8	-0.7152557E-06

UNIT MOMENT AT 8 SET TO ZERO

ROW 9	0.
ROW 10	0.

UNIT AXIAL LOADS

MEMBER 1	-0.2446000E 02
MEMBER 2	0.7629395E-04
MEMBER 3	-0.6484985E-04
MEMBER 4	0.1525879E-04
MEMBER 5	-0.2546000E 02

PLASTIC HINGE NO. 4 FORMED AT PT. 2

P[4, 1] = 7.6297

TOTAL FOR THIS CYCLE 7.6297

P[1, 2] = 5.5598
P[2, 2] = 4.3857
P[3, 2] = 7.1289
P[4, 2] = 9.5486

TOTAL FOR THIS CYCLE 26.6230

P[1, 3] = 5.7353
P[2, 3] = 3.9156
P[3, 3] = 6.4644
P[4, 3] = 10.0619

TOTAL FOR THIS CYCLE 26.1772

P[1, 8] = 6.2519
P[2, 8] = 3.5142
P[3, 8] = 6.2000
P[4, 8] = 10.2605

TOTAL FOR THIS CYCLE 26.2266

P[1, 9] = 6.2552
P[2, 9] = 3.5072
P[3, 9] = 6.2039
P[4, 9] = 10.2605

TOTAL FOR THIS CYCLE 26.2268

CONVERSION OBTAINED FOR HINGE 4

PT.	TOT. MOM.	MPC	MP	AX. LOAD	PY	P/PY
1	102.113	102.113	402.000	-648.066	825.840	0.785
2	102.113	102.113	402.000	-648.066	825.840	0.785
3	-102.113	1071.000	1071.000	-12.611	1503.360	0.008
4	671.227	1071.000	1071.000	-12.611	1503.360	0.008
5	-671.227	1071.000	1071.000	-12.612	1503.360	0.008
6	772.901	1071.000	1071.000	-12.612	1503.360	0.008
7	-772.901	1071.000	1071.000	-12.612	1503.360	0.008
8	-94.539	1071.000	1071.000	-12.612	1503.360	0.008
9	94.539	94.582	402.000	-661.176	825.840	0.801
10	94.639	94.582	402.000	-661.176	825.840	0.801

TOTAL DEFLECTIONS AT THIS STAGE.

PT.	DEFLECTION
1	-0.1412525E-01
2	-0.1031598E-00
3	-0.9940210E-01
4	-0.1441100E-01
5	0.2281042E-00
6	0.2280287E-00
7	0.2278777E-00
8	0.2278022E-00
9	-0.1376740E-01
10	-0.8619118E-02
11	0.8773426E-02
12	0.1236149E-01

UNIT DEFLECTIONS

ROW 1	-0.5440279E-03
ROW 2	-0.3844548E-02
ROW 3	-0.3844547E-02
ROW 4	-0.5440279E-03
ROW 5	0.1365333E 04
ROW 6	0.1365333E 04
ROW 7	0.1365333E 04
ROW 8	0.1365333E 04
ROW 9	-0.4950780E-03
ROW 10	-0.3300519E-03
ROW 11	0.3300519E-03
ROW 12	0.4950780E-03

DEFLECTION TOO LARGE

COLLAPSE MECHANISM HAS BEEN REACHED

HINGE ROTATIONS

PT.	ROTATION
1	-0.1376736E-01
2	-0.9094947E-11
3	-0.1818989E-11
4	0.3637979E-11
5	0.
6	0.1455192E-10
7	0.3637979E-10
8	0.9640644E-10
9	-0.2621692E-01
10	-0.1385121E-01

EXIT CALLED.

REFERENCES

1. WRC-ASCE Joint Committee
COMMENTARY ON PLASTIC DESIGN IN STEEL,
ASCE Manual No. 41, 1961

2. Beedle, L. S.
PLASTIC DESIGN OF STEEL FRAMES,
Wiley, New York, 1958

3. Wang, C. K.
GENERAL COMPUTER PROGRAM FOR LIMIT ANALYSIS,
Journal of the Structural Division, ASCE,
ST6, December 1963

4. Harrison, H. B.
THE ELASTIC-PLASTIC ANALYSIS OF PLANE FLEXURAL FRAMES,
Fritz Engineering Laboratory Report No. 297.16,
Lehigh University, July 1965

5. AISC
MANUAL OF STEEL CONSTRUCTION,
AISC, New York, 1964

6. Column Research Council
GUIDE TO DESIGN CRITERIA FOR METAL COMPRESSION MEMBERS,
CRC, 1960

A C K N O W L E D G M E N T S

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Dr. Lynn S. Beedle is Director of the Laboratory and Acting Head of the Department. The study was part of a general investigation "Plastic Design in High Strength Steel" sponsored jointly by the Welding Research Council and the Department of the Navy, with funds supplied by the American Institute of Steel Construction, American Iron and Steel Institute, Bureau of Ships, and Bureau of Yards and Docks.

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