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Plastic Design in High Strength Steel

THE SECOND-ORDER ELASTIC ANALYSIS OF PLANE RIGID FRAMES

by

H. B. Harrison

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SYNOPSIS

A computer program is described which will carry out both a first-order and a second-order elastic analysis of plane rigid frames. The program will accept data describing a frame in terms of its joint co-ordinates, member properties and connections together with the loads for which an analysis is required. A sequence of load sets can be analyzed so that it is possible to use the results to compute the elastic stability load for a frame. In the second order analysis, account is taken of the change in flexural stiffness of a member caused by axial load and in addition, the equilibrium equations are formulated for the deformed shape of the frame. The analysis is based on the displacement method using matrix techniques with the second order solution obtained by an iterative process.

The frame size that can be handled by the program is a function of the store capacity available in any computer and the program limitations in this regard are discussed in detail. The program is in the Fortran language and was developed to aid in the computation of frame strengths in a study of the economics of using high-tensile steel in rigid structures.

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I. INTRODUCTION

When high strength steels are used in the design of rigid frames, the weight and cost reductions are quite attractive when the frame strength can be accurately estimated by simple plastic theory.(1) However, this theory will overestimate frame strength whenever axial stresses are large as in multi-story frames even of mild steel and also whenever the deformations under load are sufficiently large to invalidate the formulation of equilibrium in terms of the unloaded frame configuration. The latter effect becomes serious when lighter sections of high strength steel are substituted in a frame design for stiffer sections of equal plastic strength in mild steel. It has been shown(2,3) that the effects of strain-hardening in mild steel can compensate for the deformation effects in some frames, but such an influence is likely to be less significant when more flexible frames are proportioned in high-strength steel.

In any economic study of the value in using high-strength steel in building frames, it is necessary to have available a method for estimating the maximum frame strength so that design comparisons may be made. But the problem of computing the real maximum strength of a frame, allowing for plasticity and deformation is still quite a formidable one. It has been suggested(4) that the maximum strength of a steel frame might be estimated using one half of the harmonic mean of the plastic failure load and the elastic stability load. Many formulae for pinned steel columns are based upon a similar principle. Computer programs are available[5,6] which can lessen considerably the effort required in determining the plastic failure load for a frame but the accurate calculation of the elastic stability load is a formidable problem. Analytical solutions for some classes of frames are available[7] and computational methods using the Southwell Plot method have also been proposed.[8] It is possible to predict the elastic stability load with close accuracy using results obtained from a second-order elastic analysis. A second-order analysis differs from a first-order or linear analysis because of the necessity to take into account the changes in flexural stiffness of each member caused by axial loading and as well, the equations of equilibrium must be formulated for the deformed shape of the frame rather than for its initial unloaded shape. A second-order deformation curve will become close to horizontal as the loads approach the elastic stability values. This behavior is illustrated in Fig. 1 for the case of a centrally-loaded beam subjected to axial compression. If the numerical values for load and transverse deflection are known at 4 or 5 points on such a load-deflection curve, the elastic stability or Euler load can be predicted accurately by plotting the ratio of transverse load to transverse deflection against the value of axial load. This is shown in Fig. 2. In effect, the former ratio is a measure of the resistance to flexure or stiffness of the member and the stiffness becomes zero at the Euler load. The prediction of the critical load can be made from the results of second-order analysis carried out at load intensities well below the elastic values. The same approach can be used to estimate the stability loads of frames if a convenient method is available to solve for deformations at various intensities of the applied loads. The program described in this report has been constructed with this in view.

For a given set of frame loads, the program will first perform a linear analysis, computing and printing both deformation and stress resultants and including in the latter the axial forces in all members. With the axial forces known, new values for the flexural stiffness of the components are computed, using the well known stability functions.(9) At the same time, the co-ordinates of all joints are adjusted to correspond to the deformed state computed in the first analysis and the equations of statics are then automatically adjusted. The whole cycle is repeated until satisfactory convergence is detected. As changes in the statics equations are incorporated in the program, it can be appreciated that the stability loads predicted from the results will be those corresponding to cases where sway is not prevented in rigid frames. If changes in the equilibrium formulation were not allowed for, the higher non-sway critical loads would be computed.

II. BASIC PROGRAM REQUIREMENTS

It is evident that a second-order elastic analysis program must include three distinct capacities. Firstly, it must have incorporated in it a set of instructions that will construct the equations of equilibrium of a frame using as data the co-ordinates of all the joints together with a list of the members and data concerning their individual properties and the joints to which they are connected. In addition the types of elastic deformation possible at each joint, or the degree of freedom of the joint must be specified. If such a routine were available, it can be appreciated that in the iterative second-order analysis, the equations of statics could be reconstructed in each cycle using the joint co-ordinates altered slightly by the displacements computed in the previous cycle.

The second requirement in the program is the ability to set up and solve the linear displacement equations which relate the frame loads and deformations. From a knowledge of displacements, the stress resultants in each member can be calculated and these will include the axial forces in each member. Evidently an efficient equation solving routine would be an important part of the program. Finally, it is necessary to compute for each cycle in the second-order process the stiffness coefficients of each member accounting for the axial forces in each. In the case of tension members, the flexural stiffnesses will be larger than the values appropriate for zero axial load and the opposite is the case for compression members.

The three requirements outlined above are quite closely interconnected in the program. The coefficients of the equations of equilibrium are stored in what is called the statics matrix and the coefficients of the member stiffness equations are stored in a member stiffness matrix. The deformation equations for the complete frame are constructed in each cycle using these two arrays.

III. DISPLACEMENT ANALYSIS

The linear elastic analysis of a frame is achieved in the displacement method by establishing the load-deformation equations for the complete structure using the load-deformation equations for its component members, together with the equilibrium equations for the frame. The method has been explained in detail elsewhere[10] and will be outlined briefly here. The loads applied to a frame can be listed in a column matrix (W) and there will be as many terms in (W) as the degree of freedom of the structure since deformations are conveniently measured by the movement of loads, whether real or virtual. The equations of statics relate the applied loads to the internal stress resultants of which there will be three for each member in a frame. There could be a moment, a shear and an axial tension at one section in a member, or more conveniently, the moments at each end together with the axial tension force. If the stress resultants (SR) for all the frame members are assembled in a list, then the equations of statics can be expressed,

$$(W) = (A) \cdot (SR) \tag{1}$$

where (A) is called the statics matrix. For one member the stress resultants are related to the relative deformations within the member by the member stiffness equations which can take the form of the usual slope-deflection equations of conventional analysis. For all the frame members, these equations can be assembled in the matrix equation,

$$(SR) = (S) \cdot (x)$$

Finally, the relative deformations (x) are related to the joint displacements (X) by a kinematics matrix which can be shown[5] to be the transpose of the statics matrix. Hence the load deformation equations for a frame can be expressed,

$$(W) = (A) \cdot (S) \cdot (A^T) \cdot (X) \tag{3}$$

and with the displacements (X) obtained by any suitable solution technique, the stress resultants (SR) may be obtained from Equation (2),

$$(SR) = (S) \cdot (A^T) \cdot (X) \tag{4}$$

In a first order analysis, the statics matrix (A) is formulated for the undeformed shape of a frame and the member stiffness matrix (S) involves the flexural stiffness of each member in the absence of axial load. The second-order analysis can be obtained using the first-order solution as a starting point and altering in each iterative cycle both of these matrices, accounting for the computed deformations and axial loads. The way in which these alterations can be achieved will be explained in more detail.

IV. THE STATICS MATRIX

The establishment of a statics matrix for a plane frame is a simple matter in a hand computation when the intuition and structural sense of a designer can be used most effectively. The computer does not possess these qualities and so must be made to follow out a strictly determined course. The routine developed for this purpose can be understood with reference to the diagram in Fig. 3. The two joints P and Q in Fig. 3a are joined by the member N. The end of the member at joint P is numbered R and at joint Q is numbered S. The stress condition within the member will be fully defined by the two end moments M_R and M_S and the tension force T_N as shown in Fig. 3b. The equilibrium equations for each joint are the equations relating the joint forces in frame co-ordinates to the stress resultants M_R, M_S and T_N. All of the forces acting on the joints are shown in Fig. 3c. The computer has to be supplied with information sufficient to define the frame geometry and the member properties and the details of how this is done will be described in Section VII. The construction of the equations of statics can be understood with reference to the flow diagram in Fig. 4. Each joint in a frame is studied in turn and the members framing into a given joint are detected. If the joint has a degree of freedom in the x direction of the frame co-ordinate system, the coefficients of the statics matrix will be computed and stored in the appropriate array location. The joint is then tested to see whether it may move elastically in the y direction and then whether it may rotate. For each joint, the computer must study all the frame members so that it can deal with those meeting at the particular joint. The indexing problem is quite formidable and the detailed steps used to successfully construct the statics matrix for any frame can be understood by studying that section of the Fortran program shown in Appendix A. Only the basic structural and logical principles are shown in Figs. 3 and 4.

V. THE MEMBER STIFFNESS MATRIX

The member stiffness matrix is denoted by (S) in Eq. (2) and it represents the collected action - displacement relationships that exist for each member of a framework expressed in member co-ordinates. It will be a square matrix of order equal to three times the count of the members in a frame since for each member, as in Fig. 5 the slope-deflection equations may be expressed in the 3 x 3 matrix equation,

$$\begin{bmatrix} T \\ M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 4EI/L & 2EI/L \\ 0 & 2EI/L & 4EI/L \end{bmatrix} \begin{bmatrix} u_T \\ \phi_{AB} \\ \phi_{BA} \end{bmatrix} \tag{5}$$

The coefficients in Eq. (5) are shown in the form used in a linear-elastic analysis where no account is taken of the change in flexural stiffness of a member caused by axial load. The flexural stiffness of a member will be decreased in the presence of axial compression and conversely will be increased by an axial tension so that Eq. 5 may be expressed in a more general form as in Eq. 6.

$$\begin{bmatrix} T \\ M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & S\cdot EI/L & CS\cdot EI/L \\ 0 & CS\cdot EI/L & S\cdot EI/L \end{bmatrix} \begin{bmatrix} u_T \\ \phi_{AB} \\ \phi_{BA} \end{bmatrix} \tag{6}$$

Tables for the stability functions S and C have been prepared in various forms by different workers[11,12] but it is simpler for a computer to calculate these coefficients at each stage in an iteration cycle for each member. It can be shown from the elementary analysis of a single member as in Fig. 5, that, in the case of a compression member,

$$S = \frac{B\ (B + \text{Cot } B - B\ \text{Cot}^2 B)}{1 - B\ \text{Cot } B} \tag{7}$$

and
$$CS = \frac{B\ (B - \text{Cot } B + B\ \text{Cot}^2 B)}{1 - B\ \text{Cot } B} \tag{8}$$

where $B = \frac{\pi}{2}\sqrt{T/P_E}$ and

T is the axial force in the member whereas

P_E is the Euler critical load for the member

In the case of a tension member similar expressions are applicable:

$$S = \frac{B\ (B - \text{Coth } B + B\ \text{Coth}^2 B)}{B\ \text{Coth } B - 1} \tag{9}$$

and
$$CS = \frac{B\ (B + \text{Coth } B - B\ \text{Coth}^2 B)}{B\ \text{Coth } B - 1} \tag{10}$$

VI. PROGRAM OPERATION

The program operation is explained in the flow diagram shown in Fig. 6. At the beginning, the maximum limits to the array dimensions are specified but the program will analyze structures where the arrays are less than the maximum values stated. The frame identification number is read and should it be negative, it will be regarded as the signal to terminate the run. The frame description is contained in the next three blocks on the flow diagram. The count of the joints including supports and the count of the members are numbers which will largely control the construction within the store of all the arrays needed in the solution. The arrays containing the input data have been set out diagrammatically in Fig. 7, whereas those shown in Fig. 8 are the matrices developed within the store which are needed to achieve a solution. Referring again to Fig. 6, the statics and member stiffness matrices are constructed and the Euler loads of all members are found. At this stage, the load set identifying number is read and if this is negative, it serves as an indicator that no further load sets for a frame are to be studied and control is returned to begin the analysis of another frame. It can be seen that the last two items of data in any run will be negative integers.

With the reading in of the elements of the load set, the computer will then develop the frame stiffness matrix as in Eq. (3). As only one load set is examined at a time, this stiffness matrix is not inverted but rather an efficient equation solving routine is used (Gauss - Jordan elemination)(13) to find the frame deformations and then the stress-resultants are available using Eq. (4). The complete solution is printed after completing the first cycle so that the linear-elastic solution is available. To obtain the second-order solution, the program will now execute an iteration procedure.

(1) Firstly, the coefficients in the member stiffness matrix are modified using the values of axial load computed in the first cycle and the stability functions expressed in Eqs. (7) to (10).

(2) Secondly, the joint cc-ordinates are adjusted to allow for the displacements computed in the first cycle.

(3) Then, the statics matrix is reconstructed on the basid of the new set of joint co-ordinates.

(4) The whole cycle is repeated until satisfactory convergence is obtained.

Various tests are incorporated in the program to guard against endless cycling which might occur if the applied loads be near the critical values. After the second-order deformations and stress-resultants are printed, the original member stiffness and statics matrices are reconstructed and a further load set can be examined.

VII. DEMONSTRATION EXAMPLES

The use of the program in predicting the elastic-stability loads for plane frames will be demonstrated for the two structures shown in Fig. 9. The portal frame shown in Fig. 9a is a case for which the analytical solution is available.[7] The four story frame shown in Fig. 9b has been used previously as a plastic analysis problem by Heyman.[14] The systems of joint and member identification chosen for each frame are also shown in Fig. 9. The origin of co-ordinates in each case has been placed at the lower left-hand support. The co-ordinate, joint-type, and connection matrices for example 1 would be read from cards punched in the appropriate Fortran format. The complete data input for this problem is set out in Table I. It can be seen that the joint details are completely specified in the sequence of cards from 3 - 8. The first two entries on each card are the co-ordinates of a joint. The units or zeros in the remaining three columns provide the computer with the details of the degree of freedom of a joint. For instance the zeros on card 3 show that joint 1 is not free to displace in the x or y direction while the one in the final column indicates freedom of rotation. The sequence adopted corresponds with that outlined in the flow diagram in Fig. 4. The total degree of freedom of the frame is the sum of the units for all joints. Cards 9 - 13 contain data relevant to each member. The first two figures shown for card 12 show that member number 4 is connected to joints 4 and 5, with member end numbers of 7 and 8 respectively shown by the next two numbers. The elastic modulus for the material is 30,000 ksi, the second moment of area is 100 in.^{4}, the cross-sectional area is 10 in.^{2} and the length of the member is 100 in. (This last item is not strictly required since the joint co-ordinates could be utilized to compute the member length). Card 14 indicates that load set number 1 is to follow and the elements for the load set are contained in the next two cards. With 14 degrees of freedom for frame example 1, there are 14 elements in each load set and the order has to follow strictly the order of the units (ones) in the joint-type matrix (cards 3 - 8). The first unit is shown for rotation of joint number 1 in card 3 and hence the first statics equation constructed by the computer will be that concerned with the moment equilibrium of joint 1. As no external moment is applied to the frame at joint 1, the first entry on card 15 is zero. The next unit is on card 4 referring to displacement of joint number 2 and so the statics equation for equilibrium in the x direction at that joint will be the next constructed in the store. For a load of 0.1 kips applied at joint 2 in the x-direction, this figure is entered as the second element on card 15. The vertical loads of 10 kips each acting on the beam of frame 1 are shown with negative signs on cards 15 and 16 since the positive y direction is upwards. The final two negative integers shown on cards 17, 18 serve to terminate the run of the program.

The computer output for frame 1 is shown in the Appendix B and the load-deflection curves have been plotted in Fig. 10. The ratio of vertical to horizontal loading was maintained at 100 for this example and the analysis was made for five load sets. The convergence test adopted to determine the stage at which the interaction could cease in the second order analysis was based upon a comparison of deformations. Agreement to 0.5% between the results for the final two cycles was considered satisfactory. However, provision had to be made to exempt from testing any deformations of less than 10^{-6} in absolute value. The combined effect of rounding-off errors in a machine which worked to 8 places and the fact that rotations as well as displacements were included in the test of deformations would have otherwise caused endless cycling. A measure of the frame sway stiffness was obtained from the ratio of the horizontal force to the second-order horizontal displacement and the decrease in stiffness with increasing load can be seen in Fig. 11. By extending the curve to the horizontal axis, a quite accurate figure can be obtained for the elastic-stability load of the frame as can be seen in the figure.

The data for the 4-story frame used as example 2 was prepared in a similar way. The convergence limits for this case had to be much wider (5%) than in the previous example because of an excessive build-up of round-off errors in a machine which worked to 8 figures. For the same reason, larger values of horizontal load had to be applied to obtain convergence within the limit of

20 cycles which was imposed to ensure that estimates of running time on the machine were not exceeded. It can be understood that ideally, the frame stiffness should be estimated for an infinitesimal disturbing force. Four points were obtained on the load-deflection curve when the ratio of vertical to horizontal load was kept at 6 and three points were obtained for a ratio of 12 (Fig. 12). The curves of deteriorated frame stiffness for both cases are shown in Fig. 13 and while a considerable extrapolation is needed to estimate the critical load, the errors involved will be less than those associated with alternative methods such as those based upon estimates of the effective length of individual column lengths.[15]

It is also possible to estimate the elastic critical load of a frame from the first order and second order solutions for only one load set. This can be done by reversing the procedure devised by Horne[2] for finding the second order load-deformation curve from the first-order curve and the elastic critical load. This method is based upon the equation,

$$v_2 = \frac{v_1}{1 - V/V_c} \tag{12}$$

where v_1, v_2 are the first order and second order deformations respectively, associated with a loading parameter V and V_c is the corresponding critical load parameter. As the program will produce values of v_1 and v_2 for a given load set V, it is a simple matter to compute V_c from Equation (12). These calculations have been set out in Tables II and III for the frame examples 1 and 2. In the case of frame 1, there is a progressive increase in the computed critical load as the load system increases in intensity. On the other hand there is a scatter in the results for frame 2 which can be attributed to the 5% convergence limits set for deformations in the solutions in this problem. In example 1, the corresponding limit was 0.5%. In effect, the use of Equation (12) will produce values for critical load which correspond to a linear extrapolation in Figures 11 and 13 of the lines joining each plotted point to the first order solution for stiffness which is plotted in each figure on the vertical axis. Since it is evident that there can be only one realistic critical load for a frame under a given pattern of loading, the question then arises as to which of the figures given for critical load in Tables II and III are the most reliable. The results in Table III for frame example 2 are not sufficiently accurate for any significant conclusions to be reached in this regard although any of them would serve as a good estimate of the critical load for practical purposes. Closer limits were applied to the convergence of successive deformations in the case of frame example 1 so that the five values for the critical load which increase smoothly from 44 to 59 kips need some explanation. In the first place, the right-hand side of Equation (12) represents only the first term in an infinite series where all the remaining terms have been dismissed[2] as being unimportant. Secondly, the computer has allowed for the change in stiffness of the beam as well as the columns and the analysis was carried out for finite, though small, values of horizontal load.

Since the axial shortening of members has been accounted for in the analysis, a horizontal deformation would exist at the beam level in the absence of a horizontal disturbing force. If this small deformation were subtracted from the total computed deformation for any load set to obtain the extra movement associated with the horizontal load, the curve shown in Fig. 11 would be straightened and the scatter in the computed values for critical load shown in Table II would be diminished.

However, the result is encouraging because the estimates of critical load made from analyses for loads well below the critical values came out to be low and consequently, on the safe side. The computational advantage when low load values are studied lies in the more rapid convergence to the second-order results as can be seen from Table II. The cycle count has not been shown for frame example 2 because the results were obtained from two computer runs with coarse convergence limits and no conclusion can be drawn from such information.

VIII. PROGRAM LIMITATIONS

In the present form, the program does not make use of a tape backing store so that the size of frame which can be accommodated by any machine will be limited by the capacity of the high speed store. The store required (C) for the arrays can be readily estimated from Figures 7 and 8.

$$C \geq 3NM\ (3NM + 2\bar{L} + 4) + \bar{L}\ (\bar{L} + 3) + 7\ JCT \qquad (11)$$

where NM is the count of the members,

JCT is the count of the joints and

$\bar{L}$ is the degree of freedom of the structure.

A frame of 15 members with 10 joints and 30 degrees of freedom would require a store of 5965 locations and when the program was run on an IBM 7074 machine, the total store available after compilation was 6315 locations. With a knowledge of the store available for any machine, it is a simple matter (using Equation 11) to check whether the program would work for a given frame.

Frames with pinned or fixed bases may be analyzed by the program but the restriction applies that all other joints within the frame be rigid. A pinned internal joint could be simulated by a fictitious short member of negligible inertia but such a member would have to be included in the overall count of members. The size of frame to be handled would be reduced considerably by the presence of a few internal hinges. Hence, the program in its present form is not ideally suited to the problem of the determination of deteriorated stability loads in a frame where hinges are inserted sequentially. However, this problem could be handled without great complication in a modified form of the present program, for the presence of a hinge at the end of a member could be indicated by using a negative integer for the member and identifier in the MCON matrix as in Fig. 7. The effect of one hinge would be allowed for by adding one further degree of freedom to the frame and an extra row would then be required in the statics matrix without any other changes being necessary.

IX. CONCLUSION

The two examples which have been analyzed demonstrate that the program can deal just as readily with frames subjected to primary bending moments (Example 1) as with others where these are negligible (Example 2). It allows also for the increase in flexural stiffness of tension members as well as the decrease in stiffness for compression members. The length changes due to axial load are also accounted for in the analysis. Further, the changes in stiffness of all the members of a frame are considered whereas it is usual in hand computations to consider only the most heavily loaded compression members. The equations of statics are formulated for the deformed state of a frame using the amended joint co-ordinates so that in this regard the treatment may be classified as following large deflection theory.[16] However, there are some inconsistences in this regard as the Euler load for each member is computed always for the initial length and the assumption is made that curvatures may be represented by the usual second derivative of y with respect to x.

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XI. NOMENCLATURE

A	Cross-sectional area
(A)	Statics matrix
(A^T)	Transposed statics matrix
B	Axial load parameter $= \frac{\pi}{2} \left(\frac{T}{P_E}\right)^{\frac{1}{2}}$
C	Computer capacity for data
E	Elastic modulus
I	Second moment of area
JCT	Count of frame joints
L	Length of a member
$\bar{L}$	Degree of freedom of a frame
M_{AB}	Clockwise moment at end A of member AB
M_R	Clockwise moment at end R of a member
N	Count of members in a frame
NM	Count of members in a frame
P, Q	Identifying numbers of two frame joints
P_E	The Euler load for a member
R, S	Identifying numbers of the ends of a member
(S)	Member stiffness matrix
(SR)	Stress resultant vector
S, CS	Stability functions
T	Axial force in a member
T_N	Axial force in member N

u_T	Axial deformation
v_1, v_2	First and second order deformations
(W)	Applied load vector
W	An applied load
(x)	Relative deformations vector
(X)	Absolute deformations vector
$\emptyset_{AB}$	Slope change at end A of member AB relative to line AB

XII. APPENDICES

Appendix A - Program Listing

The statements which follow are in form compatible with a CDC 3200 computer but only minor changes would be required for other machines. The input and output statements together with those for the entry and exit may require correction. The listing does not conform exactly with the flow chart of Fig. 6 in that it will cause the deformations and stress resultants to be printed at the end of each cycle in the second-order solution. A great amount of output is not entailed when a limit of 20 cycles is specified for any problem, and the results at the end of each cycle are some consolation if convergence has not occurred.

Appendix A   Fortran Program

```
C
C
C         LEHIGH UNIVERSITY        FRITZ LABORATORY.
C
C         SECOND ORDER ELASTIC PLANE FRAME ANALYSIS.
C
C         BEAM SHEAR STIFFNESS ASSUMED INFINITE.
C
C         AUTHOR, H.B.HARRISON,        JULY, 1965.
C
C
C
C         SPECIFICATION OF ARRAY DIMENSIONS.
C
      PROGRAM 3202
      DIMENSION A(24,36), S(36,36), SAT(36,24)
      DIMENSION ASAI(24,25), SDAT(48), DELTA(24)
      DIMENSION EULER(15), SLOAD(30), SR(45)
      DIMENSION CORD(10,2), MCON(15,4), JTYPE(10,3)
      DIMENSION SCOR(10,2)
C
C
C         INPUT FRAME NO., EXIT IF NEGATIVE.
C
   10 READ(60,20)   JJ
   20 FORMAT(I5)
      IF(JJ) 30,40,40
   30 STOP
C
C
C         READ COUNT OF JOINTS, COUNT OF FRAME MEMBERS.
C
   40 READ(60,50)   JCT,NM
   50 FORMAT(2I5)
      M2 = 2 * NM
      M3 = 3 * NM
C
C
C         READ JOINT CO ORDINATES AND TYPE OF DEFORMATION.
C
      DO 55  I = 1,JCT
   55 READ(60,56) (CORD(I,J), J = 1,2),(JTYPE(I,J),  J = 1,3)
   56 FORMAT(2F10.5,3I5)
C
C
C         READ MEMBER CONNECTION DATA AND PROPERTIES E,I,A,L.
C
      DO 57 I = 1,NM
      JI = 4*I - 3
      K = JI + 3
   57 READ(60,58) (MCON(I,J), J = 1,4), (SDAT(N), N = JI,K)
   58 FORMAT(4I5,4F10.5)
      L = 0
      DO 61  I = 1,JCT
      DO 61  J = 1,3
      L = L + JTYPE(I,J)
   60 FORMAT(7F10.5)
   61 CONTINUE
      NCYCL = 0
      DO 59  I = 1,JCT
```

```
      DO 59  J = 1,2
   59 SCOR(I,J) = CORD(I,J)
C
C
C            BUILD THE STATICS MATRIX  A
C
  611 NJ = 0
      NK = 0
      DO 613 IA=1,L
      DO 613 IJ=1,M3
  613 A(IA,IJ)=0.0
      DO 69  J = 1,JCT
      DO 68  M = 1,NM
      NA = NJ
      IF(J-MCON(M,1))  63,62,63
   62 JF = MCON(M,2)
      MJ = MCON(M,3)
      MF = MCON(M,4)
      GO TO 65
   63 IF(J-MCON(M,2))  68,64,68
   64 JF = MCON(M,1)
      MJ = MCON(M,4)
      MF = MCON(M,3)
   65 X = CORD(JF,1) - CORD(J,1)
      Y = CORD(JF,2) - CORD(J,2)
      DIST = SQRTF(X*X + Y*Y)
      SYN = Y/DIST
      CSN = X/DIST
      NN = 2*NM+M
      IF(JTYPE(J,1))  67,67,66
   66 NA = NA + 1
      A(NA,MJ) = SYN/DIST
      A(NA,MF) = SYN/DIST
      A(NA,NN) = -CSN
C
   67 IF(JTYPE(J,2))  691,691,681
  681 NA = NA + 1
      A(NA,MJ) = -CSN/DIST
      A(NA,MF) = -CSN/DIST
      A(NA,NN) = -SYN
  691 IF(JTYPE(J,3))  682,682,692
  692 NA = NA + 1
      A(NA,MJ) = 1.0
  682 IF(NA - NK)  68,68,683
  683 NK = NA
   68 CONTINUE
      NJ = NK
   69 CONTINUE
      IF(NCYCL)  240,695,280
  695 CONTINUE
C
C
C            PRINT TITLES AND THE STATICS MATRIX (A).
C
      WRITE(61,70)  JJ
   70 FORMAT(47H1THE ELASTIC SECOND ORDER ANALYSIS OF FRAME NO., I3///)
      WRITE(61,80)
   80 FORMAT( 21H0THE  STATICS  MATRIX//)
      DO 90  I = 1,L
   90 WRITE(61,100) I, (A(I,J), J = 1,M3)
  100 FORMAT(4H0ROW,I3,1X, 7E15.7/(8X, 7E16.7))
C
```

```
C
C          BUILD THE MEMBER STIFFNESS MATRIX, CALC. THE EULER LOAD.
C
      DO 130  I = 1,M3
      DO 130 J = 1,M3
  130 S(I,J) = 0.0
      DO 150  I = 1,M2
      ITEST = I/2 * 2 - I
      IF(ITEST) 140,150,150
  140 K = 2*I - 1
      S(I,I) = 4.0*SDAT(K) * SDAT(K+1)/ (SDAT(K+3))
      S(I+1,I+1) = S(I,I)
      S(I+1,I) = 0.5 * S(I,I)
      S(I,I+1) = S(I+1,I)
  150 CONTINUE
      DO 160  I = 1,NM
      J = 4 * I - 3
      K = M2 + I
      S(K,K) = SDAT(J) * SDAT(J+2)/ SDAT(J+3)
  160 CONTINUE
      WRITE(61,170)
  170 FORMAT(32H1 THE  MEMBER  STIFFNESS  MATRIX//)
      DO 180  I = 1,M3
  180 WRITE(61,100) I, (S(I,J), J = 1,M3)
C
      PI = 3.14159265
      DO 190 I = 1,NM
      J = 4*I - 3
  190  EULER(I) = PI*PI*SDAT(J)*SDAT(J+1)/(SDAT(J+3)*SDAT(J+3))
      WRITE(61,200)
  200 FORMAT(37H0 THE  EULER  LOAD  FOR  EACH  MEMBER//)
      WRITE(61,210) (EULER(I), I = 1,NM)
  210 FORMAT(8X, 7E15.7/(8X,7E16.7))
C
C
C          READ THE LOAD SET NO., TEST IF NEGATIVE
C          FOLLOW WITH THE LOAD SET.
C
  240 READ(50,20)  KK
      IF(KK) 10,250,25.
  250 READ(60,60) (SLOAD(I) , I = 1,L)
      DO 275 I=1,L
  275 DELTA(I) = 0.0
      NCYCL = 0
C
  280 CONTINUE
C
C
C          SET UP AND SOLVE THE DEFORMATION EQUATIONS.
C
      DO 300  I = 1,M3
      DO 300  J = 1,L
      SAT(I,J) = 0.0
      DO 290  K = 1,M3
  290 SAT(I,J) = SAT(I,J) + S(I,K) * A(J,K)
  300 CONTINUE
      DO 320  I = 1,L
      DO 320  J = 1,L
      ASAT(I,J) = 0.0
      DO 310  K = 1,M3
  310 ASAT(I,J) = ASAT(I,J) + A(I,K) * SAT(K,J)
  320 CONTINUE
```

```
      KJ = L+1
      DO 330  I = 1,L
  330 ASAT(I,KJ) = SLOAD(I)
C
      DO 460  I = 1,L
      IP1 = I + 1
      TEMP = ABSF(ASAT(I,I))
      K = I
      DO 350  J = I,L
      IF(ABSF(ASAT(J,I)) - TEMP) 350,350,340
  340 K = J
      TEMP = ABSF(ASAT(J,I))
  350 CONTINUE
      IF(K-I) 360,380,360
  360 DO 370  J = I,KJ
      TEMP = ASAT(I,J)
      ASAT(I,J) = ASAT(K,J)
  370 ASAT(K,J) = TEMP
  380 IF(ASAT(I,I)) 420,390,420
  390 GO TO  670
C
C
  420 TEMP = 1.0/ASAT(I,I)
      DO 430  J = I,KJ
  430 ASAT(I,J) = ASAT(I,J) * TEMP
      DO 460  J = 1,L
      IF(I-J)  440,460,440
  440 TEMP = ASAT(J,I)
      DO 450  K = IP1, KJ
  450 ASAT(J,K) = ASAT(J,K) - TEMP * ASAT(I,K)
  460 CONTINUE
C
C
C             TEST FOR EXCESSIVE DEFORMATIONS.
C
      XLMT = 10000.0
      DO 465  I = 1,L
      IF(ABSF(ASAT(I,KJ)) - XLMT)  465,465,710
  465 CONTINUE
C
C
C             CALCULATE THE STRESS RESULTANTS.
C
      DO 480  I = 1,45
      SR(I) = 0.0
      DO 470  J = 1,L
  470 SR(I) = SR(I) + SAT(I,J) * ASAT(J,KJ)
  480 CONTINUE
      NITS = 20
      NCYCL = NCYCL + 1
      IF(NCYCL - NITS) 485,690,690
C
C
C             OUTPUT FIRST ORDER DEFORMATIONS AND STRESS RESULTANTS.
C
  485 IF(NCYCL - 1) 490,490,5501
  490 WRITE(61,500)  KK
  500 FORMAT(45H1 FIRST  ORDER  ANALYSIS  FOR  LOAD  SET  NO., I3///)
      WRITE(61,260)  KK
  260 FORMAT(21H0 THE  LOAD  SET  NO., I3)
      WRITE(61,270) (SLOAD(I), I = 1,L)
  270 FORMAT(8X, 7E15.7/(8X,7E16.7))
```

```
5501 WRITE(61,5502)   NCYCL
5502 FORMAT(29H0 RESULTS AT END OF CYCLE NO.,  I3///)
     WRITE(61,520)
 520 FORMAT(22H0   FRAME   DEFORMATIONS///)
     WRITE(61,270) (ASAT(I,KJ), I = 1,L)
     WRITE(61,530)
 530 FORMAT(28H0   MOMENTS   AT   MEMBER   ENDS///)
     WRITE(61,270) (SR(I),I = 1,M2)
     WRITE(61,540)
 540 FORMAT(31H0   AXIAL   TENSIONS   IN   MEMBERS///)
     J = M2 + 1
     WRITE(61,270) (SR(I), I = J,M3)
C
C
C             COMPARISON OF ITERATED DEFORMATIONS.
C
     DO 560   I = 1,L
     TEST = ABSF(DELTA(I)/100.0)
     IF(ABSF(ASAT(I,KJ)) - 0.0001)  560,555,555
 555 CONTINUE
     IF(ABSF(DELTA(I)-ASAT(I,KJ)) - TEST)  560,560,570
 560 CONTINUE
     GO TO 610
C
C
C             MODIFY THE MEMBER STIFFNESS MATRIX BECAUSE OF AXIAL LOADS.
C
 570 DO  605  I = 1,NM
      M2I = M2+I
      RLOAD = SR(M2I)/EULER(I)
     IF(RLOAD)  590,580,580
 580 B2 = PI * SQRTF(RLOAD)
     EZP = EXPF(B2)
     COTH = (EZP + 1.0)/(EZP - 1.0)
     B = B2/2.0
     DEN = B * COTH - 1.0
     SK = B*(B-COTH + B*COTH*COTH)/DEN
     CS = B*(B + COTH - B*COTH*COTH)/DEN
     GO TO 600
C
 590 RLOAD = -1.0 * RLOAD
     B = (PI/2.0) * SQRTF(RLOAD)
     COT = COSF(B)/SINF(B)
     DEN = 1.0 - B*COT
     SK = B*(B + COT - B*COT*COT)/DEN
     CS = B*(B - COT + B*COT*COT)/DEN
C
 600 EIL = SDAT(4*I-3) * SDAT(4*I-2)/SDAT(4*I)
     S(2*I,2*I) = SK * EIL
     S(2*I-1,2*I-1) = SK*EIL
     S(2*I,2*I-1) = CS * EIL
     S(2*I-1,2*I) = S(2*I,2*I-1)
 605 CONTINUE
     DO 606 I = 1,L
 606 DELTA(I) = ASAT(I,KJ)
C
C
C             CHANGE THE JOINT COORDINATES.
C
     LCT = 0
     DO 6099 J = 1,JCT
     DO 6098 I = 1,3
```

```
      IF(JTYPE(J,I))  6098,6098,6097
 6097 LCT = LCT + 1
      IF(I - 3)  6095,6098,6095
 6095 CORD(J,I) = SCOR(J,I) + DELTA(LCT)
 6098 CONTINUE
 6099 CONTINUE
      GO TO 611
C
C
C          IF CONVERGENCE, PRINT SECOND ORDER RESULTS.
C
  610 WRITE(61,620)  KK
  620 FORMAT(46H0 SECOND  ORDER  ANALYSIS  FOR  LOAD  SET  NO., I3///)
      WRITE(61,260)  KK
      WRITE(61,270) (SLOAD(I), I = 1,L)
      WRITE(61,520)
      WRITE(61,270) (ASAT(I,KJ), I = 1,L)
      WRITE(61,530)
      WRITE(61,270) (SR(I), I = 1,M2)
      WRITE(61,540)
      J = M2 + 1
      WRITE(61,270) (SR(I), I = J,M3)
      WRITE(61,630)  NCYCL
  630 FORMAT(32H0  NUMBER OF  ITERATION  CYCLES , I3///)
      WRITE(61,640)  JJ,KK
  640 FORMAT(29H0ANALYSIS COMPLETED FRAME NO.,I3,13H LOAD SET NO.,I3)
C
C
C          RE BUILD THE MEMBER STIFFNESS MATRIX (S).
C
  645 DO 660  I = 1,M2
      ITEST = I/2*2-I
      IF (ITEST)  650,660,660
  650 K = 2*I - 1
      S(I,I) = 4.0*SDAT(K) * SDAT(K+1) / SDAT(K+3)
      S(I+1,I+1) = S(I,I)
      S(I+1,I) = 0.5 * S(I,I)
      S(I,I+1) = S(I+1,I)
  660 CONTINUE
C
C
C          REVERT TO INITIAL CO ORDINATES, REBUILD STATICS MATRIX.
C
      DO 661  I = 1,JCT
      DO 661  J = 1,2
  661 CORD(I,J) = SCOR(I,J)
      NCYCL = -1
      GO TO 611
C
C
C          ERROR DESCRIPTIONS.
C
  670 WRITE(61,680)  NCYCL
  680 FORMAT(48H0  ZERO DIVISION IN EQUATION SOLUTION. CYCLE NO.,I3//)
      WRITE(61,640)  JJ,KK
      GO TO 645
  690 WRITE(61,700)  NITS
  700 FORMAT(22H0  NO  CONVERGENCE  IN, I3, 13H  ITERATIONS.//)
      WRITE(61,170)
      DO 1800  I = 1,M3
 1800 WRITE(61,100) I, (S(I,J), J = 1,M3)
      WRITE(61,640)  JJ,KK

      GO TO 645
  710 WRITE(61,720)  KK
  720 FORMAT(50H0 DEFORMATION  LIMIT  EXCEEDED  BY  LOAD  SET  NO.,I3//)
      WRITE(61,640)  JJ,KK
      GO TO 645
C
      END
```

Appendix B Solution to Frame Example 1

```
FIRST ORDER ANALYSIS FOR LOAD SET NO. 1


THE LOAD SET NO. 1
        0.0000000E 00   0.1000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00  -0.1000000E 02   0.0000000E 00
        0.0000000E 00  -0.1000000E 02   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00
 FRAME DEFORMATIONS


       -0.5663644E-02   0.2257991E 00  -0.9900191E-02   0.1358528E-01   0.2253381E 00  -0.1460057E 01   0.9917830E-02
        0.2248769E 00  -0.1449017E 01  -0.1008310E-01   0.2244159E 00  -0.1009996E-01  -0.1308416E-01   0.7664157E-02
 MOMENTS AT MEMBER ENDS


        0.2000000E-04   0.3849785E 03  -0.3849780E 03  -0.6050251E 03   0.6050258E 03  -0.5950297E 03   0.5950291E 03
        0.4149655E 03  -0.4149662E 03   0.3000000E-04
 AXIAL TENSIONS IN MEMBERS


       -0.9900191E 01  -0.1383000E 01  -0.1383600E 01  -0.1383000E 01  -0.1009996E 02
SECOND ORDER ANALYSIS FOR LOAD SET NO. 1


THE LOAD SET NO. 1
        0.0000000E 00   0.1000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00  -0.1000000E 02   0.0000000E 00
        0.0000000E 00  -0.1000000E 02   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00
 FRAME DEFORMATIONS


       -0.5567435E-02   0.2921230E 00  -0.1016704E-01   0.1386222E-01   0.2701764E 00  -0.1477572E 01   0.9992229E-02
        0.2697171E 00  -0.1464517E 01  -0.1018772E-01   0.2481566E 00  -0.1032126E-01  -0.1327104E-01   0.7985080E-02
 MOMENTS AT MEMBER ENDS


        0.1700000E-03   0.3789434E 03  -0.3789437E 03  -0.6110624E 03   0.6110620E 03  -0.5992546E 03   0.5992536E 03
        0.4143151E 03  -0.4143156E 03   0.2000000E-04
 AXIAL TENSIONS IN MEMBERS


       -0.9883486E 01  -0.1227377E 01  -0.1372804E 01  -0.1225177E 01  -0.1011678E 02
 NUMBER OF ITERATION CYCLES 4


ANALYSIS COMPLETED FRAME NO. 1 LOAD SET NO. 1
```

Appendix C Solution to Frame Example 2

FIRST ORDER ANALYSIS FOR LOAD SET NO. 5

THE LOAD SET NO. 5

0.8000000E 02	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00
-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03
0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00
0.0000000E 00	-0.4800000E 03	0.0000000E 00				

FRAME DEFORMATIONS

0.9443196E 01	-0.1235885E 01	0.7713683E-02	0.9413706E 01	-0.1373401E 01	0.7543234E-02	0.7018565E 01
-0.1110289E 01	0.1248262E-01	0.7018329E 01	-0.1238068E 01	0.1239062E-01	0.4302519E 01	-0.8623471E 00
0.1186773E-01	0.4302603E 01	-0.9641530E 00	0.1188531E-01	0.1723736E 01	-0.4925535E 00	0.1053464E-01
0.1723719E 01	-0.5511608E 00	0.1053157E-01				

MOMENTS AT MEMBER ENDS

0.3236957E 04	0.3211677E 04	0.5382565E 04	0.5368921E 04	0.5702671E 04	0.5705555E 04	0.5102443E 04
0.5101938E 04	-0.3236960E 04	-0.2554009E 04	-0.3211674E 04	-0.2517488E 04	-0.2828556E 04	-0.2916613E 04
-0.2851429E 04	-0.2923794E 04	-0.2786053E 04	-0.2976961E 04	-0.2781758E 04	-0.2975625E 04	-0.2125480E 04
-0.3634128E 04	-0.2126312E 04	-0.3634520E 04				

AXIAL TENSIONS IN MEMBERS

-0.3978700E 02	-0.3185000E 00	0.1247000E 00	-0.2390000E-01	-0.4620871E 03	-0.4979122E 03	-0.9122206E 03
-0.1007777E 04	-0.1360532E 04	-0.1519467E 04	-0.1812186E 04	-0.2027812E 04		

SECOND ORDER ANALYSIS FOR LOAD SET NO. 5

THE LOAD SET NO. 5

0.8000000E 02	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00
-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03
0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00	0.0000000E 00	-0.4800000E 03	0.0000000E 00
0.0000000E 00	-0.4800000E 03	0.0000000E 00				

FRAME DEFORMATIONS

0.1920651E 02	-0.1839013E 01	0.1055252E-01	0.1917722E 02	-0.2082450E 01	0.1035727E-01	0.1573629E 02
-0.1632065E 01	0.2019417E-01	0.1573737E 02	-0.1863825E 01	0.2007802E-01	0.1064285E 02	-0.1212831E 01
0.2554713E-01	0.1064537E 02	-0.1405299E 01	0.2546491E-01	0.4372163E 01	-0.5929833E 00	0.2661573E-01
0.4372563E 01	-0.7085702E 00	0.2666189E-01				

MOMENTS AT MEMBER ENDS

0.4351374E 04	0.4322877E 04	0.8677881E 04	0.8660642E 04	0.1229898E 05	0.1228548E 05	0.1288892E 05
0.1289649E 05	-0.4351367E 04	-0.3077498E 04	-0.4322870E 04	-0.3050146E 04	-0.5600374E 04	-0.4951172E 04
-0.5610487E 04	-0.4977026E 04	-0.7347795E 04	-0.7229605E 04	-0.7308445E 04	-0.7184891E 04	-0.5659301E 04
-0.8310807E 04	-0.5711592E 04	-0.8060782E 04				

AXIAL TENSIONS IN MEMBERS

-0.3929671E 02	0.1648074E 01	0.3856172E 01	0.6412107E 00	-0.4547658E 03	-0.5030457E 03	-0.8856843E 03
-0.1030335E 04	-0.1296277E 04	-0.1577700E 04	-0.1705668E 04	-0.2130289E 04		

NUMBER OF ITERATION CYCLES 12

ANALYSIS COMPLETED FRAME NO. 2 LOAD SET NO. 5

XIII. TABLES AND FIGURES

TABLE I. DATA INPUT FOR FRAME EXAMPLE 1

Data								Format	Card No.	Remarks
1								I5	1	Frame identification number
6	5							2I5	2	Count of joints, members
0·0	0·0	0	0	1				2F10·5, 3I5	3	
0·0	300·0	1	1	1				" "	4	
100·0	300·0	1	1	1				" "	5	Joint details
200·0	300·0	1	1	1				" "	6	
300·0	300·0	1	1	1				" "	7	
300·0	0·0	0	0	1				" "	8	
1	2	1	2	30000·0	100·0	10·0	300·0	4I5, 4F10·5	9	
2	3	3	4	"	"	"	100·0	" "	10	
3	4	5	6	"	"	"	100·0	" "	11	Member details
4	5	7	8	"	"	"	100·0	" "	12	
5	6	9	10	"	"	"	300·0	" "	13	
1								I5	14	Load set identification number
0·0	0·1	0·0	0·0	0·0	-10·0	0·0		7F10·5	15	
0·0	-10·0	0·0	0·0	0·0	0·0	0·0		"	16	Load set
-1								I5	17	Load sets complete indicator
-1								"	18	Frames complete indicator

TABLE II. ANALYSIS OF FRAME EXAMPLE 1

Load Set	Unit Vertical Load (kip)	Horizontal Load (kip)	1st Order Sway (in.)	2nd Order Sway (in.)	Cycles for Convergence	Critical Load (kip)
1	10	0·1	0·2258	0·2921	4	44·04
2	20	0·2	0·4516	0·7629	6	49·01
3	30	0·3	0·6774	1·5470	8	53·37
4	40	0·4	0·9032	3·0404	12	56·90
5	50	0·5	1·1290	7·2009	26	59·30

TABLE III. ANALYSIS OF FRAME EXAMPLE 2

Load Set	Unit Vertical Load (kip)	Horizontal Load (kip)	1st Order Sway (in.)	2nd Order Sway (in.)	Critical Load (kip)
1	150	25	2·9510	3·4970	961
2	240	40	4·7216	6·3122	952
3	360	60	7·0824	11·7227	909
4	480	80	9·4432	19·2065	944
5	150	12·5	1·4755	1·6898	1183
6	240	20	2·3608	3·0820	1026
7	360	30	3·5412	5·5832	984

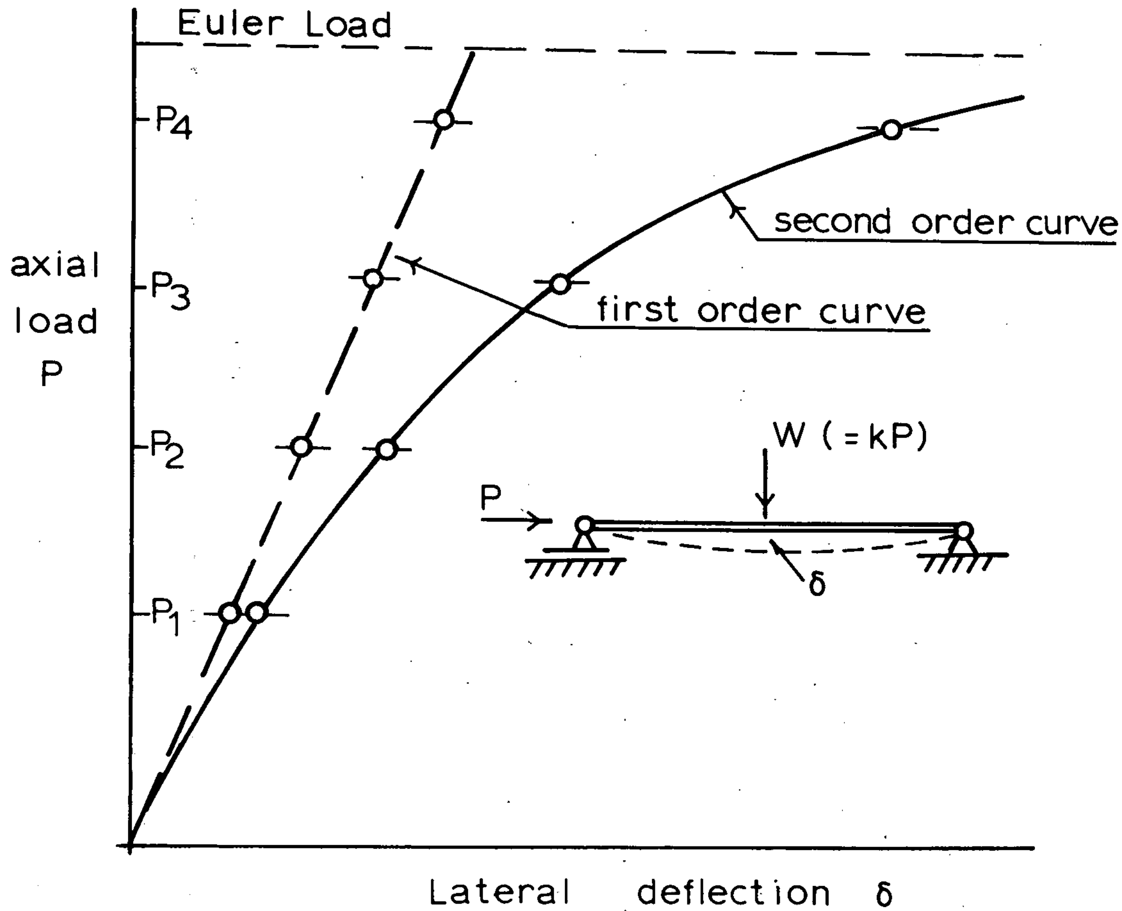

FIG 1 TRANSVERSELY LOADED STRUT

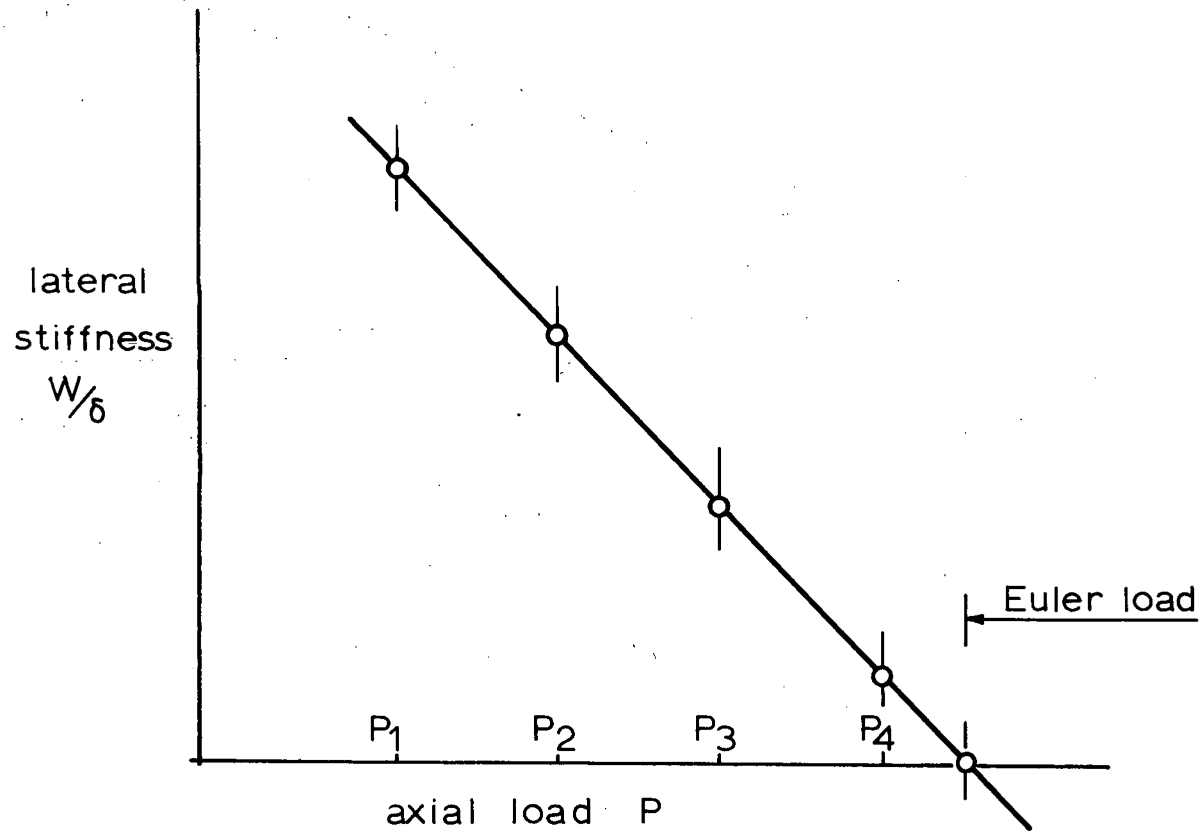

FIG 2 STIFFNESS - AXIAL LOAD VARIATION

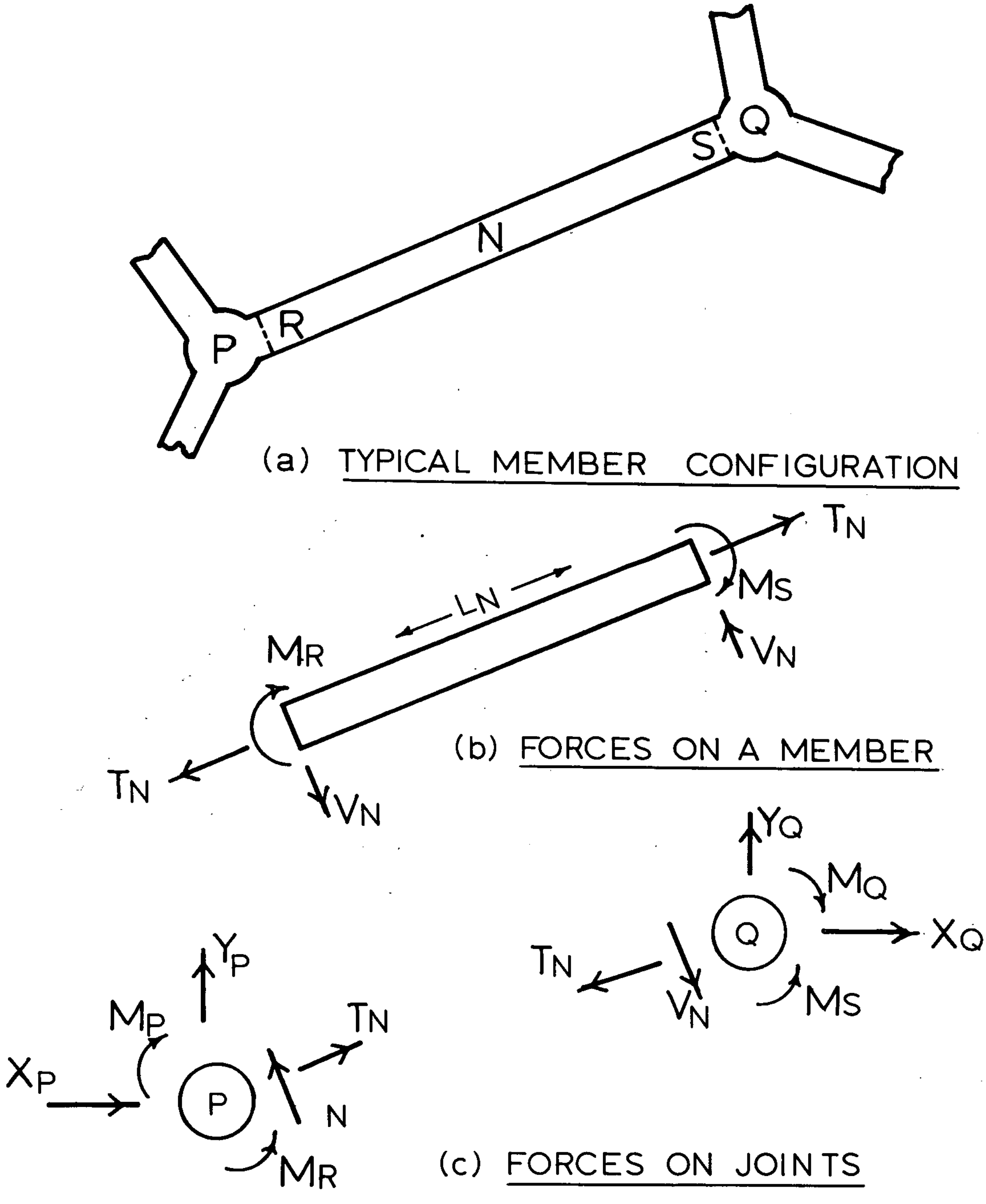

FIG 3 MEMBER AND JOINT EQUILIBRIUM

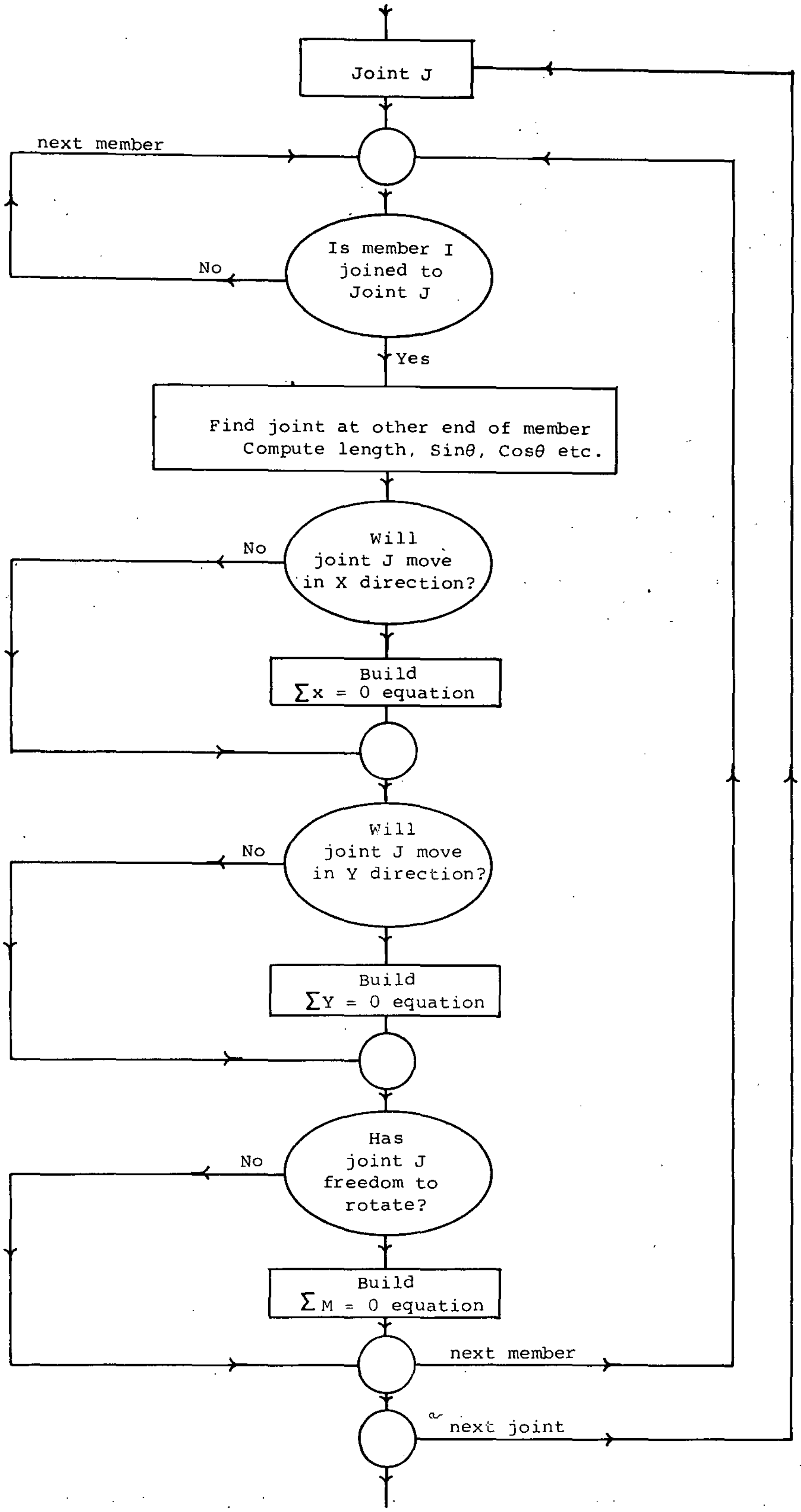

FIG.4 CONSTRUCTION OF STATICS MATRIX

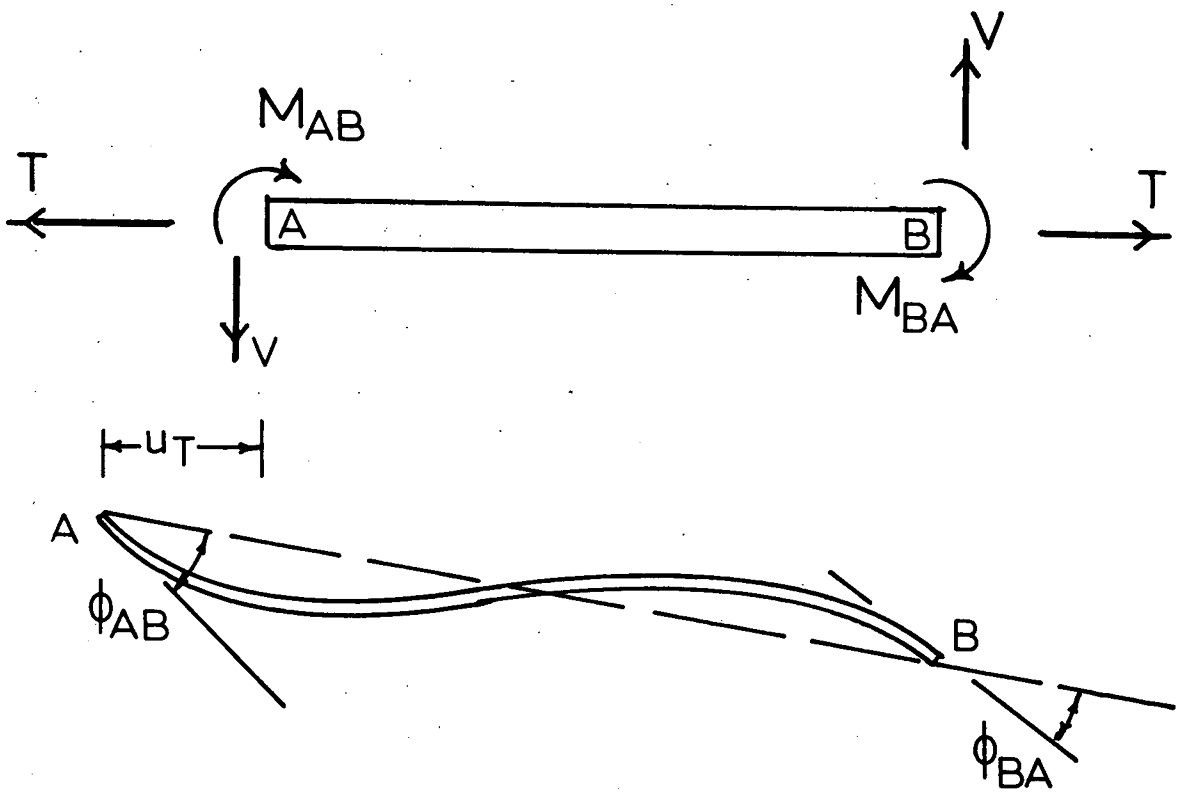

FIG 5 MEMBER FORCES, DEFORMATIONS

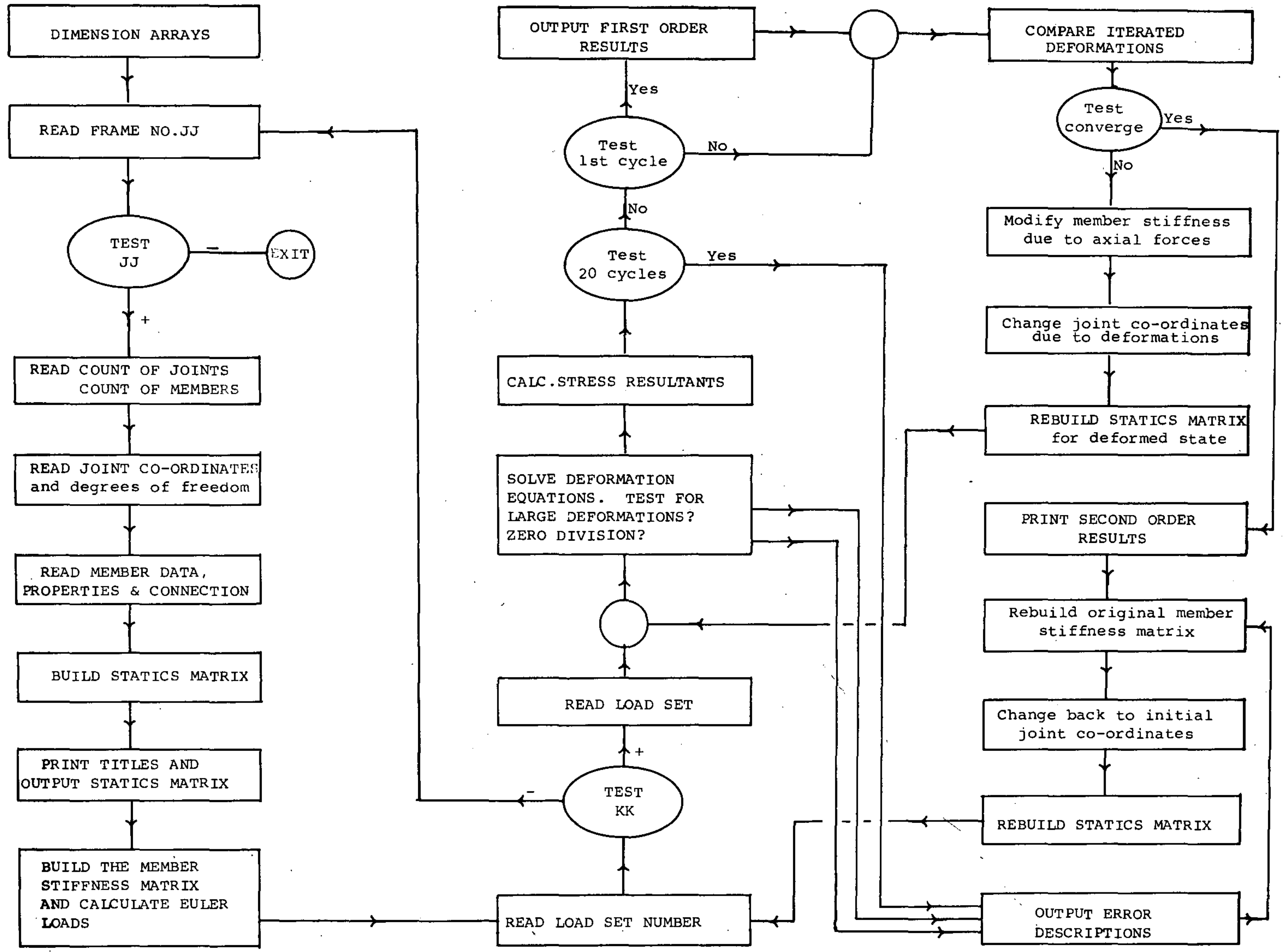

FIG.6 FLOW CHART FOR SECOND ORDER ELASTIC ANALYSIS

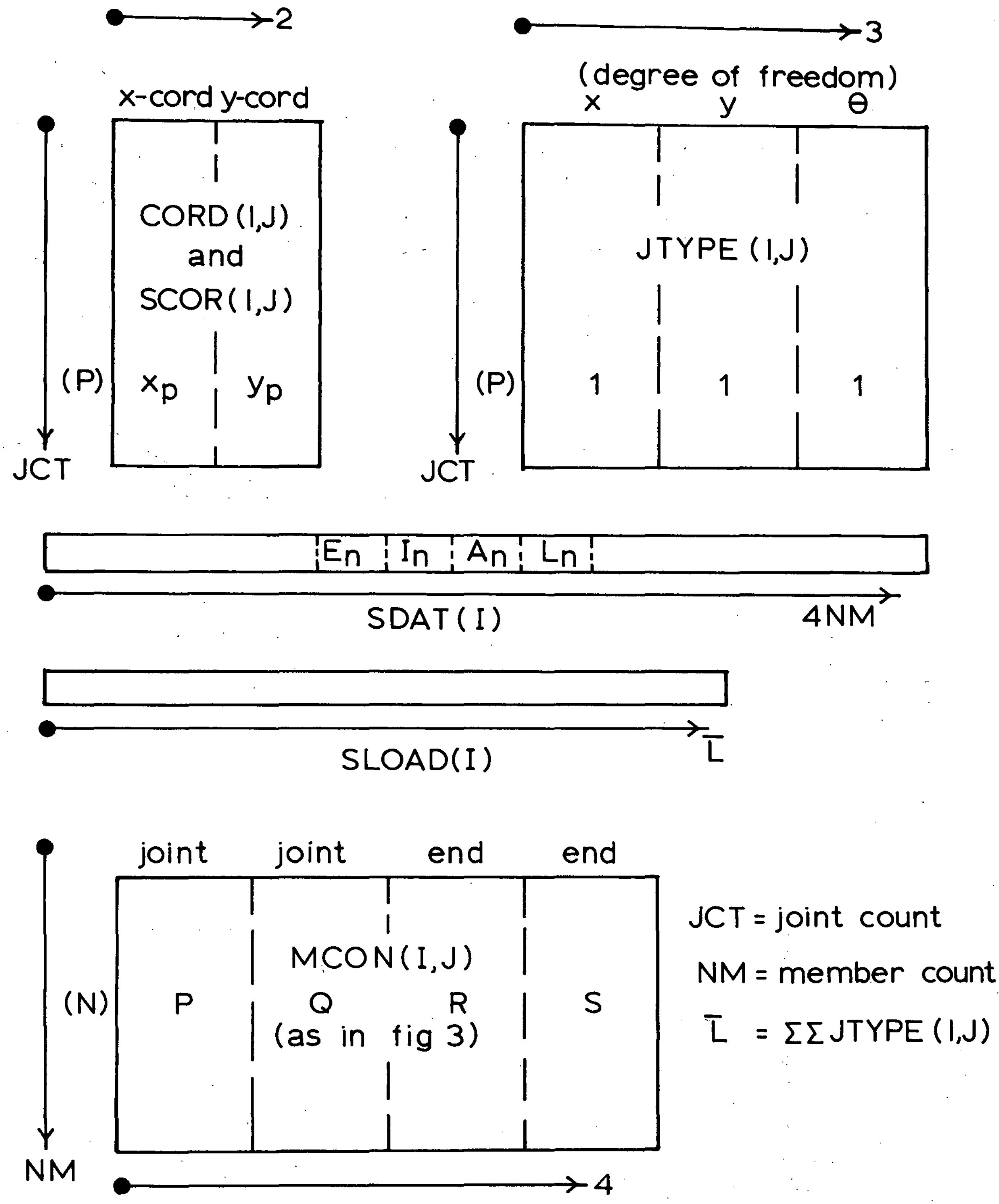

FIG 7 DATA INPUT MATRICES

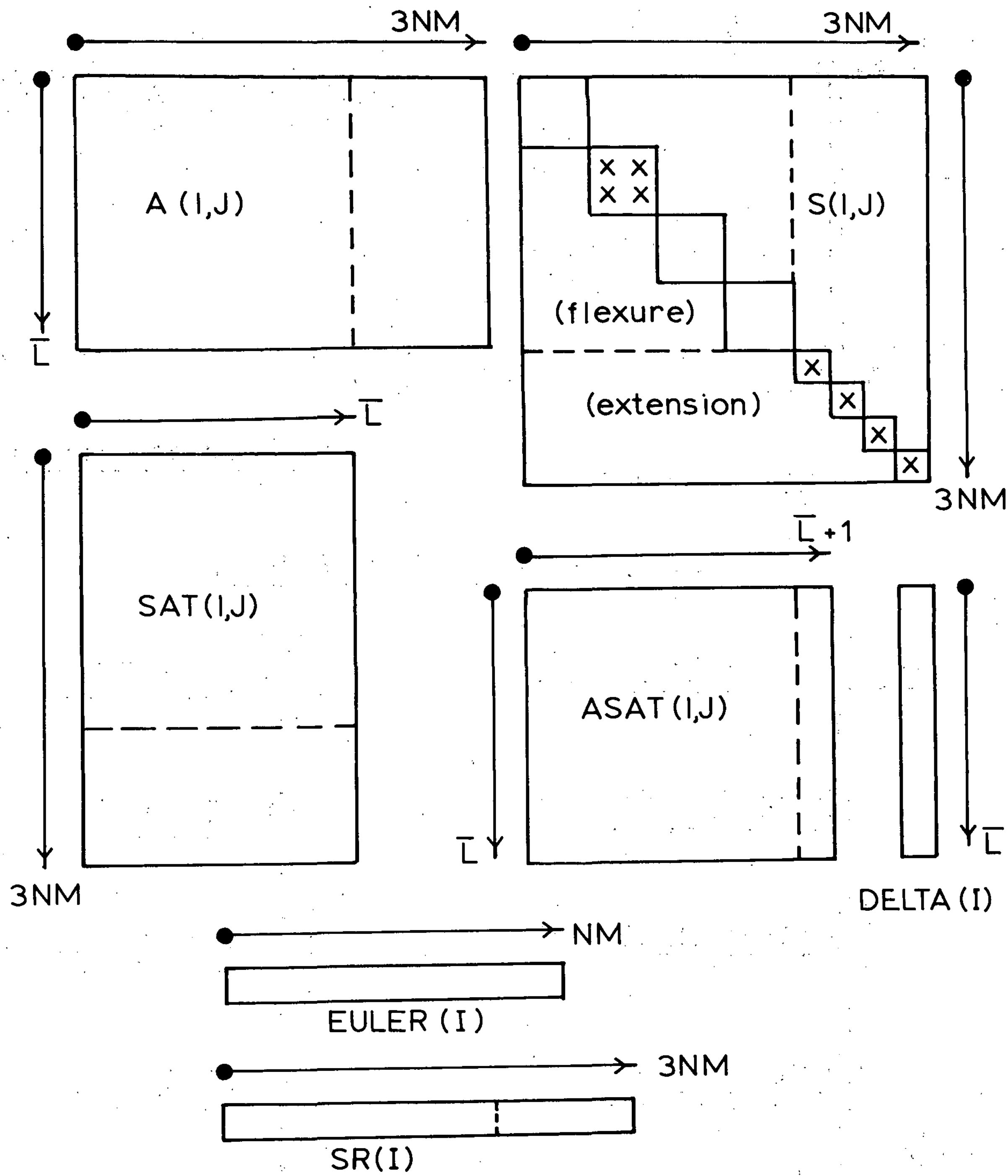

FIG 8 GENERATED MATRICES

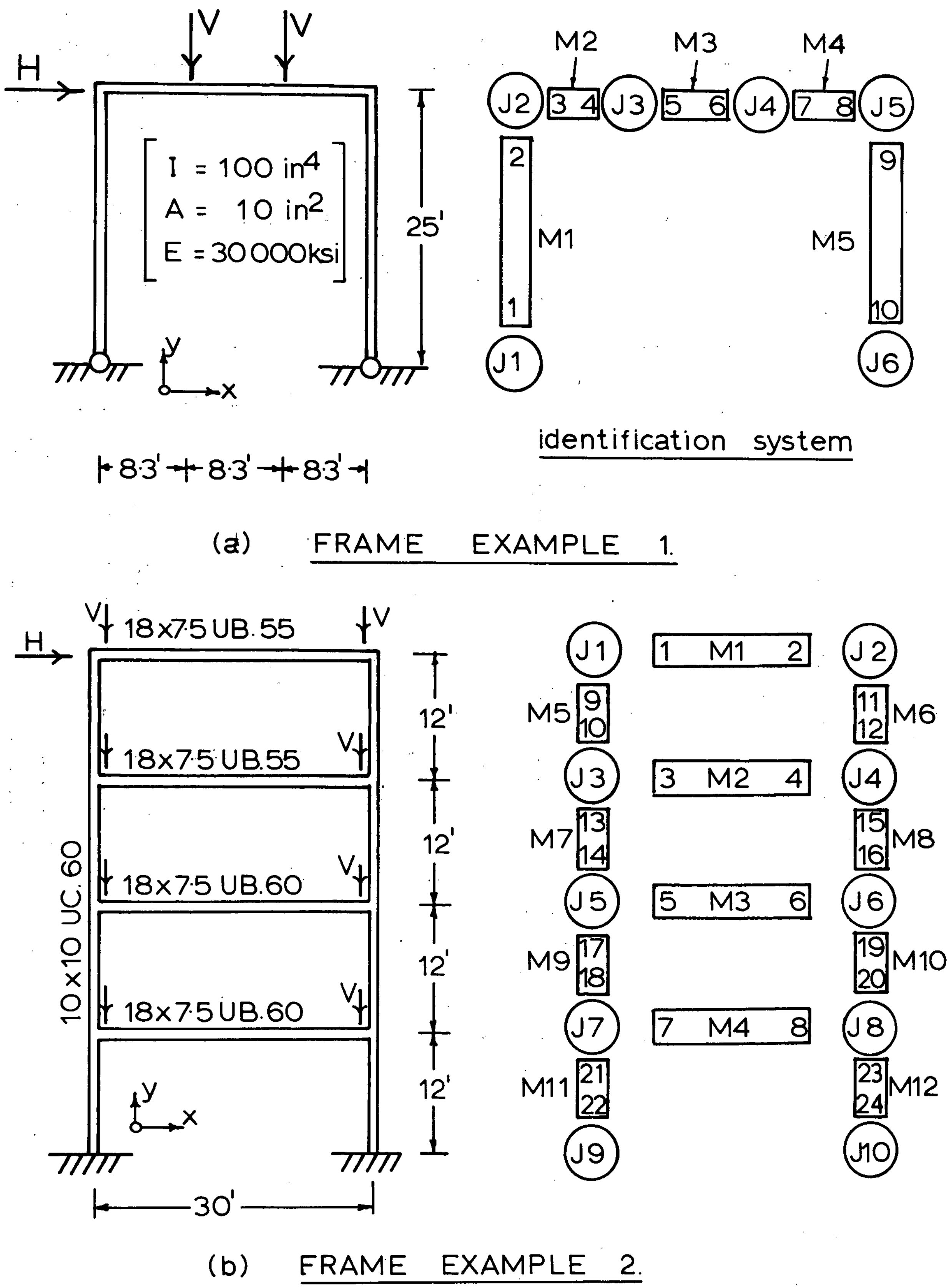

FIG 9 DEMONSTRATION EXAMPLES

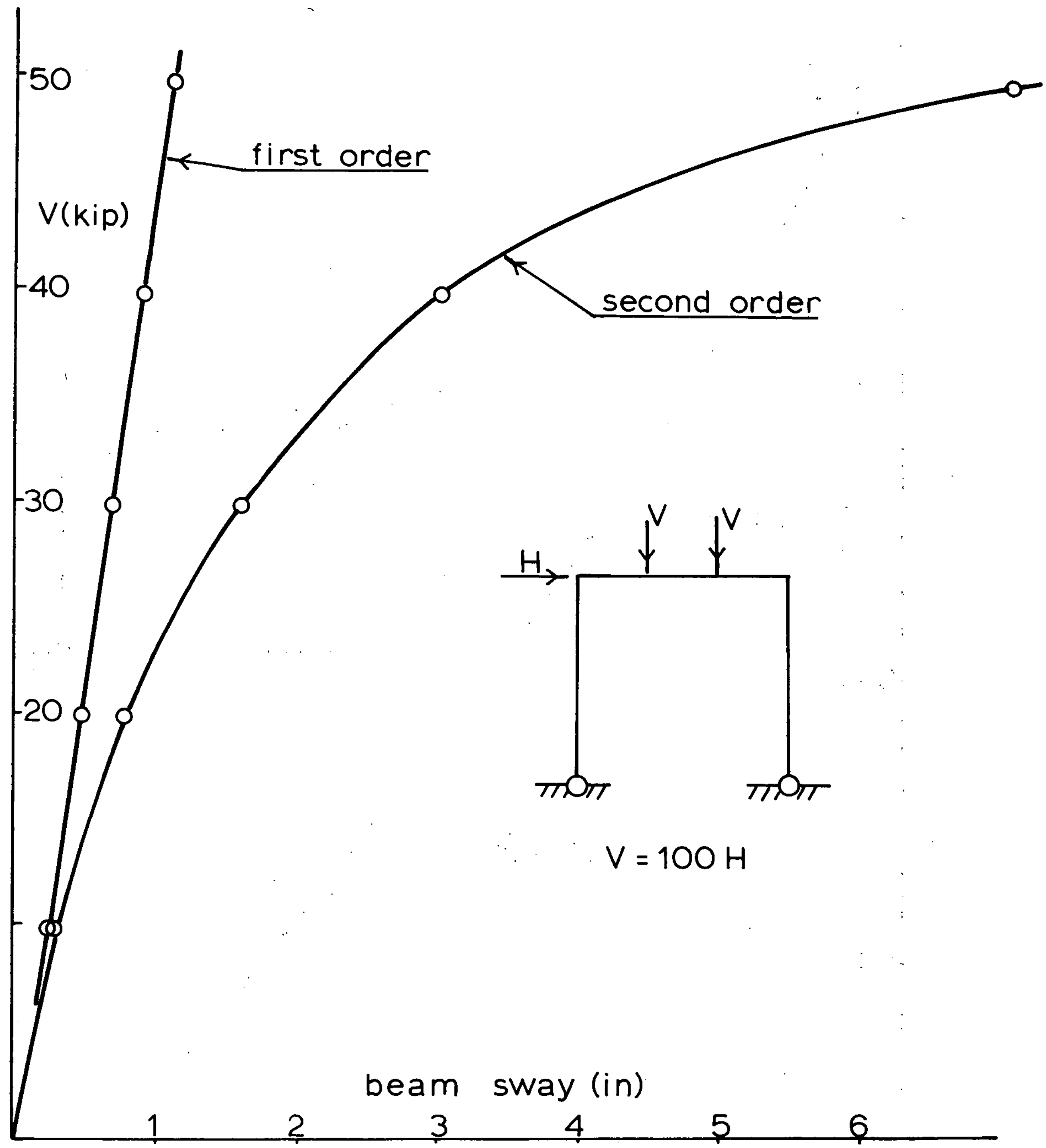

FIG 10 LOAD-SWAY CURVES FOR FRAME 1.

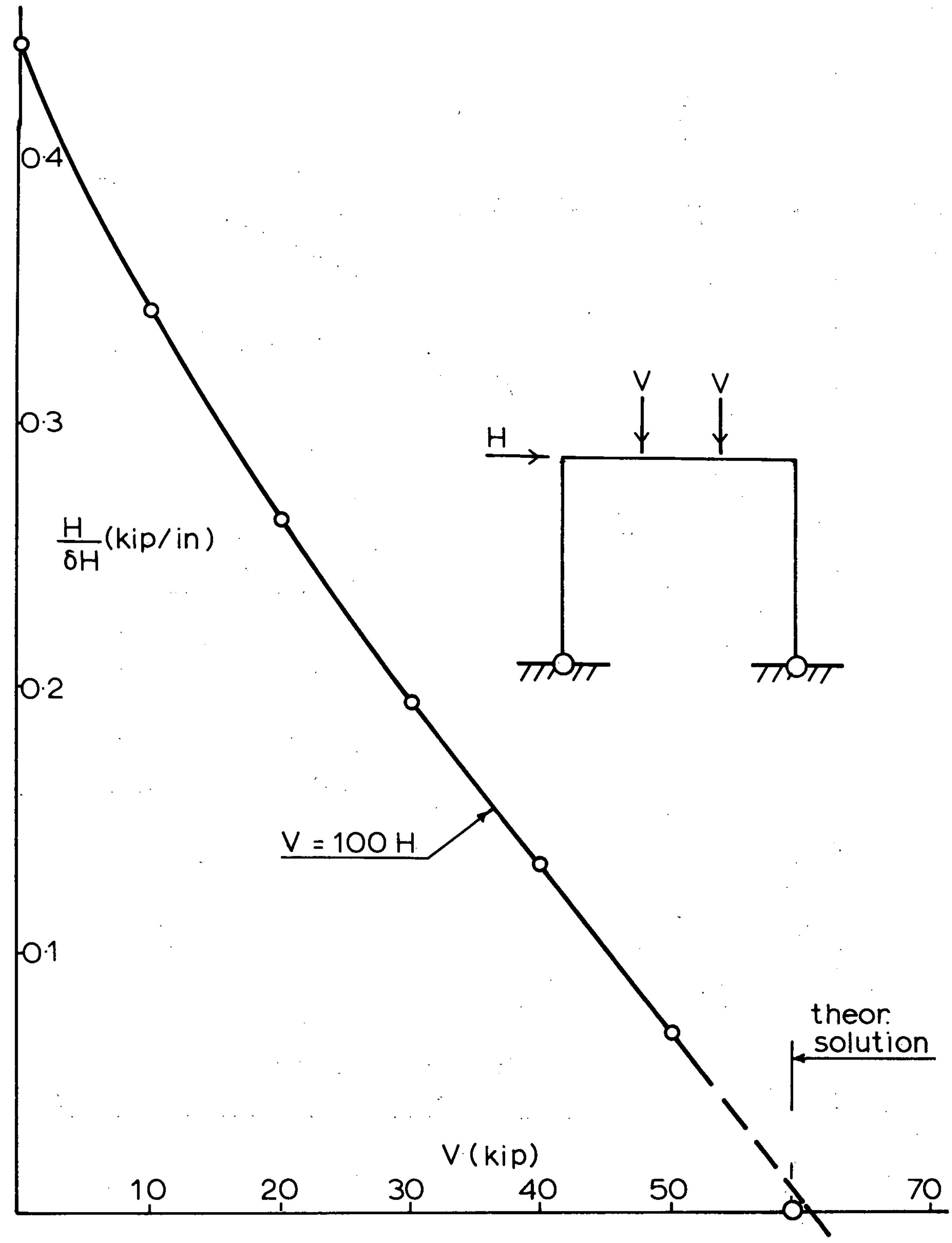

FIG 11 STIFFNESS-LOAD CURVE, FRAME 1

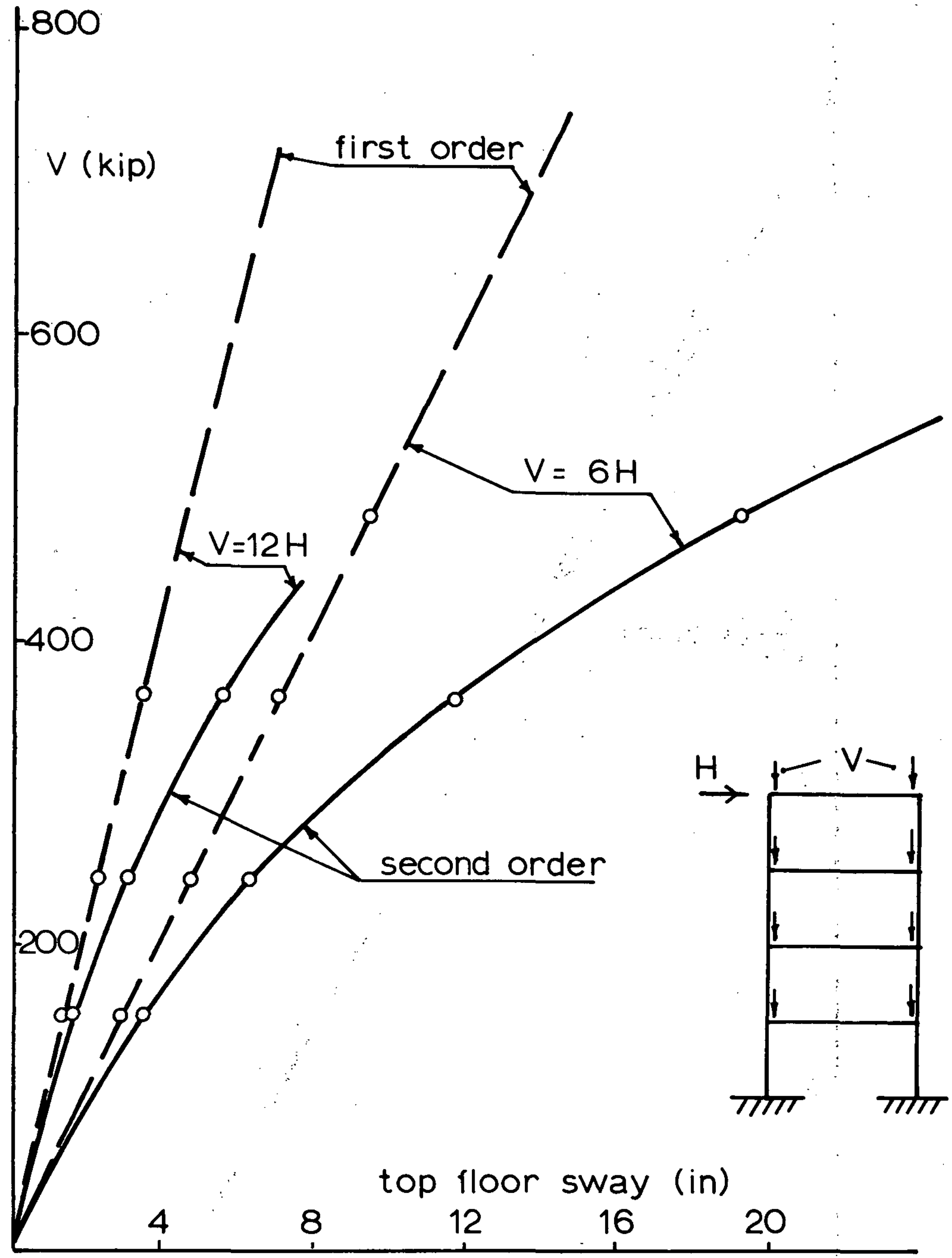

FIG 12 LOAD-SWAY CURVES FOR FRAME 2

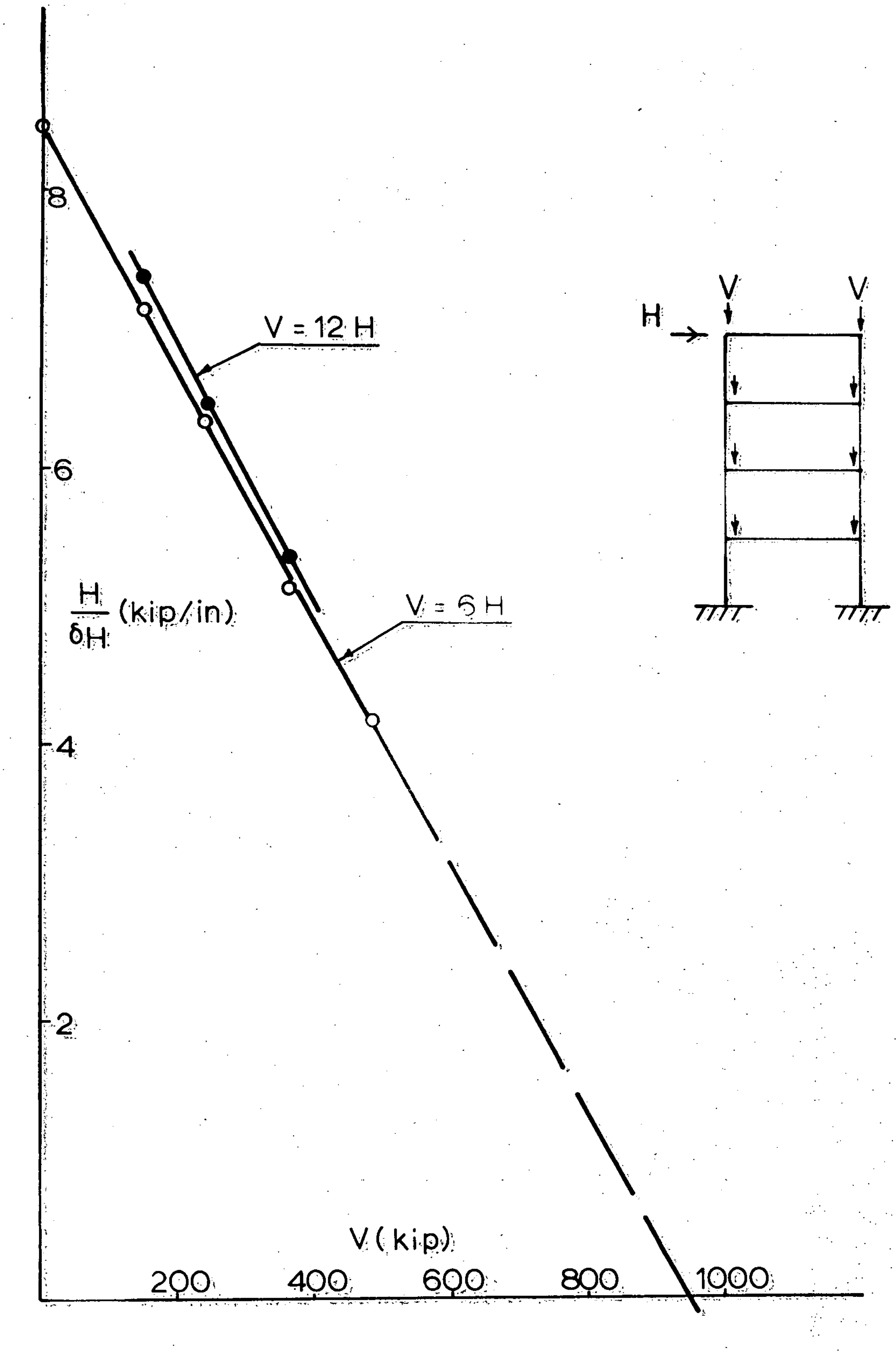

FIG 13 STIFFNESS-LOAD CURVE, FRAME 2

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