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Elastic-plastic plane frame analysis.

H. B. Harrison

LEHIGH UNIVERSITY

DEPARTMENT OF CIVIL ENGINEERING

ELASTIC - PLASTIC PLANE FRAME ANALYSIS

(1) A compiled Fortran programme is now available which will carry out a first-order elastic-plastic analysis of plane frames using the GE225 computer. The method was outlined in a paper by C. K. Wang of Wisconsin in the December, 1963 Journal of the Structural Division, ASCE. The analysis is by the Displacement method with a sequential determination of the location and load factor when plastic hinges are formed. At each stage, the deformations and bending moments are printed and the angular rotations of plastic hinges are output after the collapse mechanism has been found. The maximum size frame that can be analysed using the GE225 would consist of 10 members with 15 degrees of freedom. Point application of loads, moments only can be considered and load application positions must be treated as joints. For larger problems the source programme is available in Fortran so that other larger computers could be used. In its present form, the programme does not take into account directly the effects of axial load upon stiffness or plastic moment, and only flexural members can be accomodated, so that braced frames cannot be analysed.

(2) In using the compiled programme on the GE225 machine, all that is necessary is to prepare the relevant data in the appropriate form as below:

(a) Card 1. Frame number (for identification) in Fortran Format I5.

(b) Card 2. Degrees of freedom and TWICE the number of members in Format 2I5.

(c) Cards 3,---J The statics matrix (all elements) in Format 7F10.4.

(d)	Cards L,---N	The stiffness matrix in Format F10.4 in the following sequence: S11, S12, S22, S23, S33, S34,-------Snn.
(e)	Cards M----P	Elements of the plastic moment vector in Format 7F10.4.
(f)	Card Q	The unit load set number in Format I5 (must be positive).
(g)	Cards R----S	The unit load set in Format 7F10.4 For more load sets, continue with set number, <u>the stiffness matrix again</u>, and the load set
(h)	Card T	If no more load sets, a negative integer in Format I5.
(i)	Cards W----W	Further frames, repeating the sequence (a) to (h).
(j)	Card X	For no further frames, a negative integer in Format I5. Hence the run will end when two negative integers have been read sequentially.
(k)	Cards Y,Z	Blank cards.

<u>Note on Fortran Format</u> examples of I5: Card 1 bbbb2, Card 2 bbb35, Card 3 bbb-4

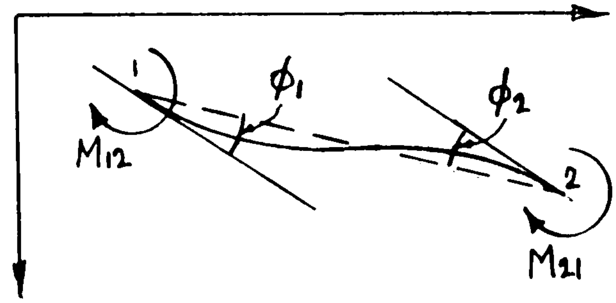
(b denotes space) examples of 2I5: Card 1 bbbb2bbb35, Card 2 bbb23bbbb3

examples of F10.4: Card 1 bbb-63.832, Card 2 bbb0.0bbbb

for 7F10.4, 7 such entries per data card are permitted, each within field of width 10.

(3) <u>The Stiffness Matrix</u> This matrix represents the assembled load-displacement relationship for all the frame elements and the form chosen follows the usual slope-deflection convention. For a member 1-2,

$$\begin{bmatrix} M_{12} \\ M_{21} \end{bmatrix} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \times \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$



For the whole frame, $M = S \cdot \phi$ where M and ϕ are column vectors and S is a triple-diagonal square matrix of order equal to twice the number of frame members. The ends of each member must be identified by numbers quite distinct from any joint numbering system.

The Statics Matrix This matrix represents the equilibrium equations for the frame.

$$W = A \cdot M$$

W is a column vector with as many elements as degrees of freedom and M is the column vector as above, listing the internal moments at member ends. Hence there will be twice as many elements in M as there are frame members.

The Plastic Moment Matrix is a column vector similar to M listing the moment capacities available at each end of all members.

The unit load set is the matrix W as above.

Note: The statics matrix A is the transpose of the more familiar displacement matrix A^T which relates kinematically the relative to absolute deformations.

$$\phi = A^T \cdot \Delta$$

Both matrices are readily assembled and if this is done, a useful check on mistakes is available.

Limitations:

In addition to the general limitation on frame size that can be accomodated by the Lehigh computer, there are two other important limitations on the efficiency of the programme.

(1) It is assumed that a plastic hinge once formed stays formed and if this is not the case for a frame, the results will not be of much use except that the load factor at collapse will err on the safe side. This follows from the fact that equilibrium and yield conditions will have been satisfied but not so the mechanism condition.

(2) It is also assumed that no strain reversal takes place in the frames of progressively detereorated stiffness that are analysed. However, the printed output is sufficient to indicate whether this phenomenon has occurred. It is not likely that this weakness will seriously limit the usefulness of the programme as the phenonemon has occurred only once in ten frames that have been analyzed by the author.

H. B. Harrison
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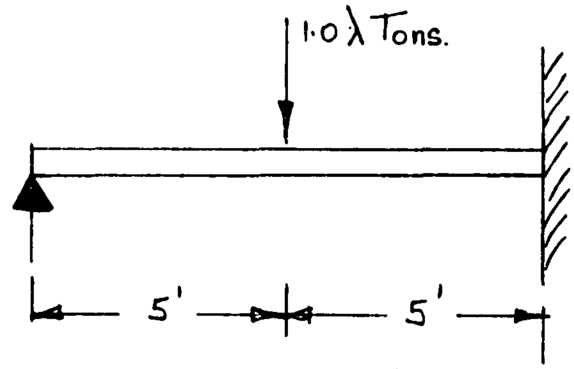
Example: Frame No. 2



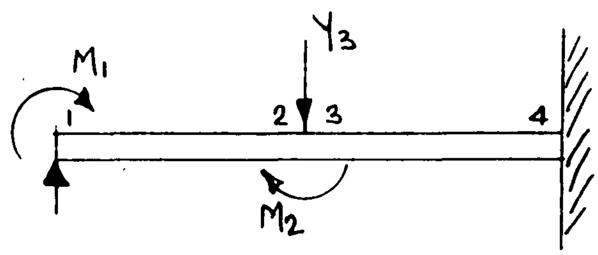
Degrees of freedom 3

Number of members 2

Assume $M_p = 40$ Ton ft.

Assume $EI = 5000$ Ton-ft.^{2}

Number member ends as shown



Statics Matrix (external loads as functions of stress resultants).

$$\begin{bmatrix} M_1 \\ M_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ -0.2 & -0.2 & 0.2 & 0.2 \end{bmatrix} X \cdot \begin{bmatrix} M_{12} \\ M_{21} \\ M_{34} \\ M_{43} \end{bmatrix}$$

Stiffness Matrix (stress resultants as functions of relative end slopes).

$$\begin{bmatrix} M_{12} \\ M_{21} \\ M_{34} \\ M_{43} \end{bmatrix} = \begin{bmatrix} 4000 & 2000 & 0 & 0 \\ 2000 & 4000 & 0 & 0 \\ 0 & 0 & 4000 & 2000 \\ 0 & 0 & 2000 & 4000 \end{bmatrix} X \begin{bmatrix} \phi_{12} \\ \phi_{21} \\ \phi_{34} \\ \phi_{43} \end{bmatrix}$$

The computer output for this example follows.

ELASTIC PLASTIC FIRST ORDER ANALYSIS OF FRAME NO. 2

THE STATICS MATRIX A

ROW 1	0.1000000E 01	0.	0.	0.
ROW 2	0.	0.1000000E 01	0.1000000E 01	0.
ROW 3	-0.2000000E-00	-0.2000000E-00	0.2000000E-00	0.2000000E-00

THE STIFFNESS MATRIX S

ROW 1	0.4000000E 04	0.2000000E 04	0.	0.
ROW 2	0.2000000E 04	0.4000000E 04	0.	0.
ROW 3	0.	0.	0.4000000E 04	0.2000000E 04
ROW 4	0.	0.	0.2000000E 04	0.4000000E 04

THE EXTERNAL LOAD VECTOR SET NO. 1

ROW 1	0.
ROW 2	0.
ROW 3	0.1000000E 01

DEFLECTIONS DUE TO UNIT LOADS

ROW 1	0.6250000E-03
ROW 2	-0.1562500E-03
ROW 3	0.1822917E-02

MOMENTS DUE TO UNIT LOADS

ROW 1	0.
ROW 2	-0.1562500E 01
ROW 3	0.1562500E 01
ROW 4	0.1875000E 01

PLASTIC HINGE NO. 1 FORMED AT POINT 4

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE(1)	0.2133333E 02	0.2133333E 02	
DEFLECTION	ADDITIONAL	CUMULATIVE	
X(1)	0.1333333E-01	0.1333333E-01	
X(2)	-0.3333333E-02	-0.3333333E-02	
X(3)	0.3888889E-01	0.3888889E-01	
MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M(1)	0.	0.	40.0000
M(2)	-33.3333	-33.3333	40.0000
M(3)	33.3333	33.3333	40.0000
M(4)	40.0000	40.0000	40.0000

DEFLECTIONS DUE TO UNIT LOADS

ROW 1 0.1250000E-02

ROW 2 0.

ROW 3 0.4166667E-02

MOMENTS DUE TO UNIT LOADS

ROW 1 0.

ROW 2 -0.2500000E 01

ROW 3 0.2500000E 01

ROW 4 0.

PLASTIC HINGE NO. 2 FORMED AT POINT 2

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE(2)	0.2666667E 01	0.2400000E 02	
DEFLECTION	ADDITIONAL	CUMULATIVE	
X(1)	0.3333333E-02	0.1666667E-01	
X(2)	0.	-0.3333333E-02	
X(3)	0.1111111E-01	0.5000000E-01	
MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M(1)	0.	0.	40.0000
M(2)	-6.6667	-40.0000	40.0000
M(3)	6.6667	40.0000	40.0000
M(4)	0.	40.0000	40.0000

DIVISION BY ZERO IN INVERSION

COLLAPSE MECHANISM HAS BEEN REACHED

	HINGE ROTATIONS
AT POINT(1)	-0.2182787E-10
AT POINT(2)	-0.2910383E-10
AT POINT(3)	-0.4365575E-10
AT POINT(4)	-0.3333333E-02

Plastic Design in High Strength Steel

THE ELASTIC-PLASTIC ANALYSIS

OF PLANE FLEXURAL FRAMES

by

H. B. Harrison

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SYNOPSIS

An account is given of a Fortran program for the elastic-plastic analysis of plane flexural frames. The program has been developed from one first written by Professor C. K. Wang and has proved to be useful in the study of steel structures.

With a minimum of input data, the program will enable a computer to carry out a series of elastic analyses of a steel structure. The position of each plastic hinge will be determined as it is formed and the load factor and deformed state of the structure will be output as each such event occurs. When the collapse mechanism is found, the rotations at each plastic hinge are computed as well as the deformations and load factor at the outset of failure.

Frames of moderate size can be analyzed by currently operating machines but an upper limit will exist for the frame size that can be handled by any given machine.

The limitations of the program are discussed in detail and several examples are given of its application.

CONTENTS

I INTRODUCTION

Professor C. K. Wang of the University of Wisconsin first described the basic principles of a computer program to analyze elastic-plastic structures in 1963[1] and made available to the author the Fortran coding of his scheme which was in a form suited to the CDC 1604 machine of Wisconsin. In modifying the program to suit the GE 225 machine at Lehigh University it soon became apparent that with the reduced storage capacity available, some attention should be paid to the reduction of the dimensioned arrays used by the program so that frames of reasonable size could be accommodated. The efficiency of the program has been improved in various ways which will be outlined in this report but the basic operating principles and solution techniques used originally by Wang are retained and due acknowledgement is paid for the ingenious way in which he has achieved his goal.

Basically, the program will carry out a series of first-order elastic analyses of a frame in which free hinges are successively introduced at those sections at which localized plastic hinges are assumed to develop at the load system is increased uniformly. Accordingly, it can be appreciated that the program must incorporate two distinct capabilities. The first is a system of "bookkeeping" in which a record is kept of the moments existing at all possible plastic hinge positions in a frame. The moments are compared with the available plastic moment capacity to detect whether or not the next plastic hinge is to form at any given position. The second capability is the utilization of a form of first order elastic analysis which can be applied simply and successively to frames of deteriorated stiffness as hinges are inserted. The type of displacement analysis described by Clough[2] and used by Wang is well suited for this purpose. Brief explanations will be given of both sections of the program since the functioning and limitations of the scheme can only be understood in their light.

II ELASTIC-PLASTIC ANALYSIS

It is often the case that the form of an analysis carried out by hand would not be a desirable one to program for a computer. Neal and Symonds[3] have proposed a method for estimating the deformations near collapse of rigid frames and it has been used by Heyman[4] and Vickery[5] in a study of the effects of deformation and strain hardening on the collapse load. Heyman[6] has subsequently used a different approach based on Virtual Work to achieve the same end. In all methods, the mechanism of failure is found previously and the deformations at failure are determined by first finding the position where the last-to-form plastic hinge would occur. These methods have had the common aim of avoiding the onerous computation of load factors and deformations as each plastic hinge is formed when the load intensity is progressively increased. This latter approach is probably the best to use with a computer as intuitive judgements are eliminated. In such a method of computation, once it has been decided that a plastic hinge exists at some position, the next stage in the analysis concerns the same initial frame with a free hinge at the position nominated, but subjected to a new loading system. The new system would consist of the original set of unit loads together with a moment of the full plastic value acting as an external action on the ends of the members meeting at the "hinge". The method is demonstrated in Fig. 1 for a propped cantilever where the results of the first elastic analysis shown in Fig. 1 (b) indicate that the first plastic hinge will form at position C. Inserting a free hinge at C, it can be seen that the second and final analysis shown in Fig. 1 (c) is that of a simply supported beam with an extra external action, namely the moment M_p, acting at C. This approach presents no problems for a hand solution, but it would be inefficient for a machine solution because of the necessity of providing for the extra degree of freedom and the corresponding new loading term in the dimensioning of the various matrices affected by the degree of freedom. If provision had to be made for an extra degree of freedom at every position where a hinge was likely to form, a small frame would rapidly fill the available data storage capacity of a computer.

The alternative system used by Wang does not involve the same difficulties and is illustrated for the propped cantilever shown again in Fig. 2. The results of the first elastic analysis are shown in Fig. 2(b) and in row 4 of Table I. The load factors in row 5, obtained by dividing the available moment capacity at each position by the unit moment at the same position, determine where the first plastic hinge will form. This will be the case at that position where the load factor is smallest as shown in row 6. The moments at all positions when the first hinge has formed are shown in row 8 and the residual moment capacity is shown in row 9 of the table and also in Fig. 2(c). With a free hinge inserted at position 4 in Fig. 2, the frame is again analyzed for the original loading system as in row 10 with the load factors determined by dividing the residual moment capacities by the unit load moments. It is in this respect, illustrated in Fig. 2(c) that the machine solution devised by Wang differs from the hand solution technique.

It can also be seen from Table I that the procedure is essentially cyclical. It is feasible to calculate the deformation at each stage but these results have not been included in the tabulation. A collapse mechanism will have been reached in the analysis when the structure has been converted into a mechanism. The numerical indication of such a phenomenon can be in several forms. It may be that the coefficients in the stiffness equations would form a singular matrix so that zero division would be encountered in an attempted solution and would end the analysis. If this does not occur, the computed deformations would be very large which would indicate that the load-deflection diagram has become horizontal. Wang has explained the computer indications of frame failure in reference (1) though some of his collapse criteria have been eliminated in the present program for reasons which will be explained later.

The method chosen for systematizing an elastic-plastic analysis has been explained and its success as part of a computer program will obviously depend upon the provision of a method of elastic analysis which will deal in a simple fashion with the insertion of hinges in rigid frames.

Table I

Numerical Analysis of Propped Cantilever Problem

Position in Beam (Fig. 2)	1.	2.	3.	4.
1. Initial moment capacity	40	40	40	40
2. Cumulative moments to date	0	0	0	0
3. Available moment capacity	40	40	40	40
4. Elastic Analysis 1				
Moments due to unit loading	0	-1.56	1.56	1.83
5. Load factors (row 3 ÷ row 4)*	—	25.64	25.64	21.33
6. Smallest load factor (SLG)				21.33
7. Unit moments x SLG	0	-33.33	33.33	40
8. Cumulative moments to date	0	-33.33	33.33	40
9. Avail. capacity (row 1 - row 8)	40	6.67	6.67	0
10. Elastic Analysis 2				
Moments due to unit loading with hinge at position 4	0	-2.5	2.5	0
11. Load factors (row 9 ÷ row 10)*	—	2.66	2.66	*
12. Smallest load factor (SLG)		2.66		
13. Unit moments x SLG	0	-6.67	6.67	0
14. Cumulative moments to date	0	-40	40	40
15. Avail. capacity (row 1 - row 14)	40	0	0	0
16. Elastic Analysis 3 with hinges at positions 4 and 2	(Either zero division or very large deformations will result)			

* Positions where moment is near zero are not included in search for smallest load factor to avoid premature zero division stop.

(The computer output for this problem is shown in Appendix B)

III ELASTIC FLEXURAL ANALYSIS

The displacement method of frame analysis can be formulated in many forms, all with the common characteristic that the load-displacement behavior of a frame as a whole is built up from a knowledge of the load-displacement relationship for its component members. In the case of a flexural frame, the elementary component will be a straight prismatic member as shown at (a) in Fig. 3 and if axial and shear stiffnesses are assumed infinite, the load displacement relationships take the form of the simple slope-deflection equations,

$$M_{AB} = \frac{2EI}{L}(2\ \phi\ AB + \phi\ BA)$$

$$M_{BA} = \frac{2EI}{L}(\phi\ AB + 2\ \phi\ BA)$$

which can be expressed in the matrix form,

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \cdot \begin{bmatrix} \phi AB \\ \phi BA \end{bmatrix}$$

or

$$(SR_{AB}) = (S_{AB}) \cdot (\phi_{AB}) \qquad (1)$$

For all the members of a frame, the similar equations for each member may be assembled in the matrix equation,

$$\begin{bmatrix} (SR_{AB}) \\ (SR_{BC}) \\ (SR_{CD}) \end{bmatrix} = \begin{bmatrix} (S_{AB}) & \cdot & \cdot \\ \cdot & (S_{BC}) & \cdot \\ \cdot & \cdot & (S_{CD}) \end{bmatrix} \cdot \begin{bmatrix} (\phi AB) \\ (\phi BC) \\ (\phi CD) \end{bmatrix}$$

or

$$(SR) = (S) \cdot (\phi) \qquad (2)$$

where (S) is called the member stiffness matrix and (SR) will be a column matrix or vector listing the moments acting at the ends of all frame members. It is usually a simple matter to write down the equations of statics which relate these moments (called stress resultants) to the applied loads.

$$(W) = (A) \cdot (SR) \tag{3}$$

The load vector (W) must have as many terms as the degree of freedom of the structure since deformations are measured by the movement of loads (whether real or virtual) in a displacement analysis. If the degree of freedom is $\bar{L}$ and the number of members is NM, then the statics matrix (A) will be of order $\bar{L}$ x 2NM. Only for a statically determinate structure will $\bar{L}$ = 2NM so that inversion of (A) is then possible and the stress resultants will be known in terms of loads without any further analysis. Finally, the relative deformations within each member (Ø) can be expressed in terms of movements of the loads (X) by a kinematics matrix C,

$$(\emptyset) = (C) \cdot (X) \tag{4}$$

and it can be shown[1] that the matrix (C) is the transpose of the statics matrix (A). Hence, the load-displacement equations for the whole structure can be expressed,

$$(W) = (A) \cdot (S) \cdot (A^T) \cdot (X) \tag{5}$$

where the triple matrix product $(A \cdot S \cdot A^T)$ is the stiffness matrix (K) of the frame. For a given set of loads (W), the displacements can be determined by standard equation solution programs. Thereafter, the moments at the ends of each frame member can be computed from Eqs. (2) and (4),

$$(SR) = (S) \cdot (A^T) \cdot (X) \tag{6}$$

This form of first order frame analysis can accommodate the modification associated with the insertion of a hinge within a structure. There are two ways in which the modification can be made. The obvious way is to consider the extra degree of freedom involved and to add a row to the matrix (A) (and a corresponding column to A^T) leaving the member stiffness matrix (S) unchanged. It has been explained earlier that this approach would be impracticable in a computer program as all possible changes in the degree of freedom would have to be accounted for in the initial dimensioning and establishment of the statics matrix (A). The alternative approach adopted by Wang was to keep (A) and effectively $\bar{L}$ unchanged and modify the member stiffness matrix (S). The procedure can be understood by referring to Fig. 3(b). If a hinge is present at the end A of member AB, the slope-deflection equations become,

$$M_{AB} = 0 = \frac{2EI}{L}(2\phi_{AB} + \phi_{BA}) \ ; \ M_{BA} = \frac{2EI}{L}(\phi_{AB} + \phi_{BA}) \text{ and hence,}$$

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{3EI}{L} \end{bmatrix} \cdot \begin{bmatrix} \phi AB \\ \phi BA \end{bmatrix} \qquad (7)$$

Similarly, if a hinge were to exist at the end B as in Fig. 3(c),

$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi AB \\ \phi BA \end{bmatrix} \qquad (8)$$

By adopting a numerical system rather than an alphabetic system for identifying the ends of each member, with the odd number always smaller than the even number, Wang was able to achieve the necessary changes to the matrix (S) in accordance with Eqs. (7) and (8) using the computed location of any hinge. For example, the substitution of a hinge at a position 16 in any frame would necessitate the following alterations to the (S) matrix.

$$S^1\ (15,15) = \frac{3}{4}\ S(15,15)$$

$$S^1\ (15,16) = S^1\ (16,16) = S^1(16,15) = 0$$

where the primes denote the new values. If a hinge occurred at position 15,

$$S^1\ (15,15) = S^1\ (15,16) = S^1(16,15) = 0$$

$$S^1\ (16,16) = \frac{3}{4}\ S(16,16)$$

Simple tests exist in computer languages for detecting whether a number is odd or even and then the appropriate changes to the member stiffness matrix (S) can be made.

IV DESCRIPTION OF PROGRAM

The Fortran program is included in the Appendix A and the principal stages in its operation are shown in the flow diagram in Fig. 4. The first step is to dimension the arrays and it should be understood that the program will analyze frames whose arrays cannot exceed the initially set sizes but which can be of any size smaller than the initially set values. A discussion of the limitations on frames sizes that can be accommodated by a given machine will be given in the next section. The first item of data must be the identifying number of the frame which if negative, is regarded as the exit signal. Next, the degree of freedom $\overline{L}$ and the member count NM are read and these two numbers will control the sizes of all the subsequent arrays built within the store for the frame being studied. All the elements of the statics matrix are then read in row by row. This is followed by the member stiffness and plastic moment data with one card per member containing the EI/L and M_p values. From this information, the member stiffness matrix (S) and the plastic moment vector will be constructed in the store. The (S) matrix is output for checking, together with headings and the full statics matrix (A). Wang's original program has been modified considerably in this region by incorporating the ability to analyze the same frame for a series of different loading conditions. Accordingly, the next item input has to be the identifying number of the load set which is to follow. For load sets other than the first, the completely deteriorated member stiffness matrix is reconstructed before the analysis proceeds. It the load set number is negative, the program will look for data for a new frame and if no further frames are to be studied, the final card has to contain a negative integer in the place of a frame identifying number. Hence, the final two cards in any run will contain negative integers. With the load set input and printed for checking purposes, the program proceeds with the analysis by building the frame stiffness matrix (K) from the member stiffness matrix (S) and the statics matrix (A) according to equation (5). The equations are solved for deformations and if these are too large, an indication is given that the frame has reached the collapse condition. Deformations exceeding the value of 10^4 are regarded as being too large. It this is not the case, the moments are computed using Eqs. (2) and (4) and the smallest load factor sought so that the position of a plastic hinge can be found.

This part of the program follows Wang's original scheme except for one alteration. It was found that erroneous results were produced for some frames by the original program because the load factors were computed by dividing the residual moment capacity by the absolute value of the moments caused by unit loading. Such a procedure is satisfactory provided the unit load moments at the critical positions are of the same sign in the successive analysis of the frames of deteriorated stiffness. It may well be the case that the moment at the position with the least reserve of strength may be decreasing under increasing load. A test has been incorporated in the section of the program concerned with the finding of the smallest load factor to determine whether such is the case and if so, the position in question is not included in the search for the smallest load factor.

In his program, Wang incorporated four separate tests to determine whether the collapse load for a frame had been reached. One of those tests involved the minimum load factor which, if too small, would indicate that the load-deflection curve for a frame was close to horizontal. However, it was found that this test would frequently terminate prematurely the analysis for any frame where two plastic hinges might form simultaneously. This test has been omitted from the present program since it is considered that a deformation limitation will determine effectively whether or not the load-displacement curve might be horizontal. Only one of the other two tests for collapse which were provided by Wang has been retained. This is the test which outputs the message "division by zero in inversion" and it effectively determines the stage at which a row and column in the frame stiffness matrix (K) contains only zero terms. In theory, this is the only necessary test but the other is required because rounding-off errors in the floating point arithmetic could delay the program termination and invalidate the final calculation of plastic hinge rotations.

After the mechanism of failure has been found and the load factor and the cumulative moments and deformations at the maximum load are printed, the final computation concerns the amount of plastic rotation that would have occurred at the positions of all plastic hinges except the last formed. Referring to equation (2), the relative end slopes ($\emptyset$) could be calculated by pre-multiplying the list of cumulative moments (SR) by the inverted form of the member stiffness matrix.

$$(\emptyset) = (S)^{-1} \cdot (SR) \tag{9}$$

It can be noted that a simple inversion of (S) is not possible because at the final stages of an analysis, this matrix does not exist in its original form. Wang inverted (S) and stored the data at the beginning of his program but was aware of the fact that considerable economy of storage capacity would result if the elements of (S) were stored as a list and the matrix reconstructed in its actual or inverted form when required. This has been done in the present program.

The slopes computed from equation (9) will be the same as those which can be calculated from equation (4) only at those positions where no plastic deformation has occurred. Accordingly, the amount of plastic hinge rotation can be expressed,

$$(\phi_P) = (S^{-1}) \cdot (SR) - (A^T) \cdot (X) \tag{10}$$

where the lists (SR) and (X) are the moments and deformations in the frame at the stage when the last plastic hinge has just been formed.

Finally, control is returned to see if any further load sets are to be studied for the frame in question. If so, the member stiffness matrix would need to be completely reconstructed as it would have been altered considerably in the course of the analysis for the first load set. If no further load sets are available, the program will commence the analysis of another problem. If there are no further frames to be studied, the run will terminate.

V PROGRAM LIMITATIONS

The program will perform a first-order elastic-plastic analysis of rigid planes of prismatic members and in its present form is strictly limited to this form of analysis. Since axial stiffness of members is assumed to be infinite, the axial forces present in the members are not calculated explicitly so that it is not possible to arrange for a progressive decrease in plastic moment capacity caused by the presence of axial load. However, it is always possible to account approximately for this effect by reading initial values for plastic moments, already reduced by the estimated axial loads at failure. To account explicitly for axial strains, the member stiffness matrix would consist of (3 x 3) units for each member instead of the (2 x 2) units currently specified so that for a limited computer store capacity, the size of frame to be handled would be curtailed drastically. To account for second-order effects in the displacement analysis, the axial forces in members would be needed with the capacity disadvantage mentioned above, but then the reduction in stiffness of each member could be readily computed and the member stiffness matrix modified progressively in essentially iterative solution procedure. Running time would increase greatly as a result.

The statics matrix also would require progressive modification to account for sway deformations and whereas programs can always be written to do this for any specific frame, it is difficult to visualize a general program that could account for the phenomenon for any type of rigid frame. The great advantage of Wang's scheme is that it can be used for any type of plane frame as a standard program.

The main limiting factor in the use of a general program for frame analysis is storage capacity since the use of matrix methods has the disadvantage that quite extensive arrays can be generated by only moderately sized structures. It is evident that methods can always be developed to utilize tapes as a backing store for a specific machine but the generality of a program is then lost. It is anticipated that core store capacities of computers of the next generation will be greatly in excess of those currently available, so that it will be possible to analyze with an elastic-plastic program the range of sizes of steel frames for which such an analysis is currently relevant.

In its original form, Wang's program required a storage capacity which can be expressed,

$$C \geqslant (\bar{L} + 2MN)^2 + 4NM^2 + 3\bar{L} + 14NM \tag{11}$$

where C = capacity,

$\bar{L}$ = degree of freedom, and

NM = the number of members in a frame

As has been explained, there is no need to store the inverted form of the member stiffness matrix if this can be generated when required from a one dimensional list of member stiffness parameters. The capacity required for the modified program can be expressed,

$$C \geqslant (\bar{L} + 2NM)^2 + 3\bar{L} + 15NM \tag{12}$$

As an example, a three-story, two bay rigid frame subjected to two-point loading on each beam would have 36 degrees of freedom and 27 members so that the original program would require a capacity of 11502 locations. The modified program would require the reduced capacity of 8613 locations. (The data capacity of the Lehigh GE 225 computer when using the elastic-plastic program was found to be 1860 whereas with an IBM 7074 machine, the capacity was 6850 locations.) It is apparent that load application positions have to be treated as joints so that a beam under two-point loading constitutes three members. Consequently, the available capacity of a medium sized machine such as the GE 225 will be fully utilized by frames of only moderate size.

One further limiting factor should be mentioned. It can sometimes occur in steel frames that a plastic hinge which is formed early in the loading history may not be required in the collapse condition. The moment at such a section would decrease in magnitude and a plastic hinge would not then exist. This phenomenon cannot be accounted for in the present program as the process of free hinge insertion is irreversible. The calculated load factor for such a problem would err on the safe side since the equilibrium and yield conditions would be satisfied but not the mechanism condition. This phenomenon has been mentioned by Finzi[7]. The example of a two-span beam, which has been used by Neal[8] to demonstrate this phenomenon, is shown in Fig. 5. For the loads shown at (a), an elastic analysis will produce a maximum moment at the point(4) as can be seen in (b). However, a simple plastic analysis will predict a failure mechanism with plastic hinges at (3) and (6) but not at (4). This can be deduced from the moment diagram shown at (d) in the figure. The results obtained from a computer analysis of this problem are in the Appendix C. It can be seen that the computer correctly detects the formation of the first hinge at position (4) and the second at (6) as shown at (c) in Fig. 5 but cannot account for the closing of the first formed hinge thereafter. Accordingly, it arrives at an invalid collapse mechanism with a load factor smaller than the correct one. Consequently, it is desirable for any frame to check the collapse mechanism arrived at by the computer to see whether or not it is valid.

A related problem is that of the formation of a plastic hinge under a distributed load. In such a case, the loading must be replaced by equivalent point loads, as many being chosen as the computer capacity will accommodate.

VI CONCLUSION

The Wang program is a very powerful tool in the analysis and design of steel structures and has been used to study the economics of steel frame design using the various grades of high tensile steel currently available. Any such study would evidently involve the analysis of many trial designs and, for frames other than simple one story portals, the computational problem would be insuperable without the use of a computer program such as the one described. As a final example, the frame analyzed by Heyman[6] is shown in Fig. 6 and a selection of pages from the computer output is shown in the Appendix D. The computed load-sway curve is shown in Fig. 7. The complete print out for this frame consisted of over 40 pages and the total time for both compilation and execution on an IBM 7074 was less than 3 minutes. The preparation of the statics matrix which is the collection of all the equations of equilibrium for the structure was a simple matter taking less than half an hour. The print out of this matrix is also shown in the Appendix D. A more detailed explanation of statics matrices has been given elsewhere.[9]

The sign convention adopted in the solution of Heyman's problem is that in which clockwise moments acting on the ends of members are regarded as positive together with downward vertical loads. The program itself is not dependent upon any particular sign convention and will operate successfully as long as a self-consistent convention is adopted in the statics matrix and in the vector of applied loads.

VII ACKNOWLEDGEMENTS

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At the time of writing, the author was on study leave from the University of Sydney, Australia.

NOMENCLATURE

(A)	the statics matrix
(A^T)	the statics matrix transposed (= (C))
(C)	the kinematics matrix
C	computer capacity for data
E	Young's modulus
I	second moment of area
I,J	identifying integers
(K)	the frame stiffness matrix
L	length of a prismatic member
$\bar{L}$	degree of freedom
NM	count of the members in a frame
M_P	full plastic moment of resistance
M_{AB}	moment applied at end A of member AB
(S)	the member stiffness matrix for a frame
(S_{AB})	member stiffness matrix for member AB
(SR)	stress resultant vector for a frame
(SR_{AB})	stress resultant vector for member AB
SLG	smallest load factor
S(I,J)	an element in the member stiffness matrix
S'(I,J)	a new value for S(I,J)
$(S)^{-1}$	the inverted member stiffness matrix
(W)	the applied load vector
W	an applied point load
(X)	the frame deformation vector
$(\emptyset_{AB})$	relative deformation vector for member AB

$(\emptyset)$	relative deformation vector for a frame
$(\emptyset_P)$	plastic hinge rotation vector

IX APPENDIXES

Appendix A — The Fortran Program

The statements of the program are contained in the following pages. (pp. 25 - 31) They are shown in the form used by the IBM 7074 computer but the only changes necessary for the GE 225 machine are the following substitutions:

READ for READ INPUT TAPE 1,

and

PRINT for WRITE OUTPUT TAPE 2,

Format requirements in Fortran impose some limitations on the choice of names for variables but as far as possible, the names used correspond with those used in the text. The identification of the principal variables used in the program is shown in Table II.

Table II Identification of Variables

Variable	In Text	In Program
the statics matrix	(A)	A(I,J)
load factors	-	ALG(I)
frame stiffness matrix	(K)	ASAT(I,J)
frame deformation vector	(X)	ASAT(I,L+1)
relative deformation vector	$(A^T)\cdot(X)=(\emptyset)$	ATX(I)
cumulative load factor	-	CLG
cumulative moment vector	-	CM(I)
relative deformation vector	$(S^{-1})\ (SR)=(\emptyset)$	DM(I)
plastic hinge rotation vector	$(\emptyset_P)$	H(I)
frame identification number	-	JJ
load set identification number	-	KK
location of plastic hinge	-	NPH
analysis stage number	-	NCYCL
the applied unit load vector	(W)	P(I)
initial plastic moment vector	-	PM(I)
the member stiffness matrix	(S)	S(I,J)
an intermediate matrix product	$(S)\cdot(A^T)$	SAT(I,J)
smallest load factor	-	SALG
moments caused by unit loads	(SR)	SATX(I)
member stiffness data vector	-	SDAT(I)

```
C
C
C         LEHIGH UNIVERSITY    FRITZ LABORATORY    APRIL, 1965.
C
C         FIRST ORDER ELASTIC PLASTIC PLANE FRAME ANALYSIS.
C
C         BASIC PROGRAM BY C.K.WANG, UNIV. OF WISCONSIN, 1963.
C
C         MODIFIED FOR IBM 7074 BY H.B.HARRISON.
C
C
C
C         SPECIFY THE MAXIMUM SIZES OF ALL THE MATRICES.
C
      DIMENSION A(30,48), S(48,48), SAT(48,30), SATX(48)
      DIMENSION P(30), ASAT(30,31), PM(48), ALG(48)
      DIMENSION CX(30), CM(48), SDAT(24), DM(48)
      DIMENSION ATX(48), H(48)
C
C
C         INPUT FRAME NUMBER, EXIT IF NEGATIVE.
C
    9 READ INPUT TAPE 1,  13, JJ
   13 FORMAT (1I5)
      IF (JJ)  99,5,5
C
C
C         INPUT THE DEGREES OF FREEDOM L, MEMBER COUNT NM.
C
    5 READ INPUT TAPE 1, 23, L, NM
   23 FORMAT (2I5)
      M = 2 * NM
C
C
C         INPUT ALL ELEMENTS OF THE STATICS MATRIX  A.
C
      READ INPUT TAPE 1, 35,((A(I,J), J=1,M), I=1,L)
   35 FORMAT(7F10.4)
C
C
C         INPUT THE MEMBER PROPERTIES, EI/L AND Mp.
C
      DO 166  I = 1,NM
      READ INPUT TAPE 1, 167, SDAT(I), PMI
      K = 2*I
      PM(K) = PMI
      PM(K-1) = PM(K)
  166 CONTINUE
  167 FORMAT(2F10.4)
C
C
C         BUILD THE MEMBER STIFFNESS MATRIX  S.
C
      DO 160  I = 1,M
      DO 160  J = 1,M
```

Appendix A   Fortran Statements

```
  160 S[I,J] = 0.
      DO 161  I = 1,M
      ITEST = I/2*2-I
      IF[ITEST]  162,161,161
  162 K = I/2 + 1
      S[I,I] = 4.0 * SDAT[K]
      S[I+1,I+1] = S[I,I]
      S[I+1,I] = 0.5 * S[I,I]
      S[I,I+1] = S[I+1,I]
  161 CONTINUE
C
C
C           OUTPUT TITLES, THE STATICS MATRIX A, STIFFNESS MATRIX 'S.
C
      WRITE OUTPUT TAPE 2, 97, JJ
   97 FORMAT [50H1ELASTIC PLASTIC FIRST ORDER ANALYSIS OF FRAME NO.,I3/]
      WRITE OUTPUT TAPE 2, 7
    7 FORMAT[21H0THE  STATICS  MATRIX//]
      DO 1 I=1,L
    1 WRITE OUTPUT TAPE 2, 21,I, [A[I,J], J = 1,M]
   21 FORMAT[4H0ROW,I3,1X, 7E16.7/[8X,7E16.7]]
      WRITE OUTPUT TAPE 2, 17
   17 FORMAT[23H1THE  STIFFNESS  MATRIX//]
      DO 2 I=1,M
    2 WRITE OUTPUT TAPE 2, 21, I, [S[I,J], J = 1,M]
C
C          INPUT THE LOAD SET NO. KK,IF NEG. COMMENCE THE NEXT FRAME.
C
  708 READ INPUT TAPE 1, 13, KK
      IF[KK-1] 9,707,800
C
C          IF KK IS UNITY, BYPASS THE NEXT BLOCK OF INSTRUCTIONS.
C          IF KK IS GREATER THAN UNITY, REBUILD THE S MATRIX.
C
  800 DO 901  I = 1,M
      DO 901 J = 1,M
      S[I,J] = 0.
  901 CONTINUE
      DO 931 I = 1,M
      ITEST = [I/2*2] - I
      IF [ITEST] 932,931,931
  932 K = I/2 + 1
      S[I,I] = 4.0 * SDAT[K]
      S[I+1,I+1] = S[I,I]
      S[I,I+1] = 0.5 * S[I,I]
      S[I+1,I] = S[I,I+1]
  931 CONTINUE
C
  707 WRITE OUTPUT TAPE 2,  27,  KK
   27 FORMAT[33H1THE EXTERNAL LOAD VECTOR SET NO., I3]
C
C
C          INPUT THE LOAD SET VECTOR P.
C
      READ INPUT TAPE 1, 35,[P[I], I = 1,L]
```

```
      DO 3 I=1,L
    3 WRITE OUTPUT TAPE 2, 21, I, P[I]
C
C
C         SET TO ZERO THE VARIABLES NCYCL,CLG AND THE ARRAYS CX,CM,SAT.
C
      NCYCL = 0
      CLG = 0.
      DO 24 I=1,L
   24 CX[I] = 0.
      DO 26 I=1,M
   26 CM[I] = 0.
C
C
C         RE ENTRY POINT FOR SUCCESSIVE ANALYSES OF DETERIORATED FRAMES,
C
   15 DO 10 I=1,M
      DO 10 J=1,L
      SAT[I,J]=0.
C
C
C         POST MULTIPLY S BY TRANSPOSED A TO GET MATRIX SAT.
C
      DO 20 K=1,M
   20 SAT[I,J]=SAT[I,J]+S[I,K]*A[J,K]
   10 CONTINUE
C
C
C         PREMULTIPLY SAT BY A TO GET MATRIX ASAT.
C
      DO 30 I=1,L
      DO 30 J=1,L
      ASAT[I,J]=0.
      DO 40 K=1,M
   40 ASAT[I,J]=ASAT[I,J]+A[I,K]*SAT[K,J]
   30 CONTINUE
C
C
C         SOLVE THE SLOPE DEFLECTION EQUATIONS FOR THE GIVEN LOAD SET,
C         STORE THE SOLUTION IN THE LAST COLUMN OF THE MATRIX ASAT.
C
      LP1=L+1
      DO 50 I=1,L
   50 ASAT[I,LP1]=P[I]
      DO 60 I=1,L
      IP1=I+1
      TEMP=ABSF[ASAT[I,I]]
      K=I
      DO 61 J=I,L
      IF [ABSF[ASAT[J,I]]-TEMP]  61,61,62
   62 K=J
      TEMP = ABSF[ASAT[J,I]]
   61 CONTINUE
      IF [K-I]  72,71,72
   72 DO 45 J=I,LP1
```

```
      TEMP = ASAT(I,J)
      ASAT(I,J) = ASAT(K,J)
   45 ASAT(K,J) = TEMP
   71 IF (ASAT(I,I))  16,147,16
  147 WRITE OUTPUT TAPE 2, 347
  347 FORMAT (30H0DIVISION BY ZERO IN INVERSION)
      GO TO 47
   16 TEMP = 1./ASAT(I,I)
      DO 70 J=I,LP1
   70 ASAT(I,J)=ASAT(I,J)*TEMP
      DO 60 J=1,L
      IF (I-J) 59,60,59
   59 TEMP=ASAT(J,I)
      DO 80 K=IP1,LP1
   80 ASAT(J,K)=ASAT(J,K)-TEMP*ASAT(I,K)
   60 CONTINUE
C
C
C          PRINT THE DEFORMATIONS DUE TO UNIT LOADS,TEST IF TOO LARGE.
C
      WRITE OUTPUT TAPE 2, 511
  511 FORMAT (30H0DEFLECTIONS DUE TO UNIT LOADS)
      DO 51 I=1,L
   51 WRITE OUTPUT TAPE 2, 21, I, ASAT(I,LP1)
      DO 311 I=1,L
      TEMP = ABSF(ASAT(I,LP1)) - 1.E+04
      IF (TEMP) 311,647,647
  311 CONTINUE
      GO TO 303
  647 WRITE OUTPUT TAPE 2, 847
  847 FORMAT (21H0DEFLECTION TOO LARGE)
      GO TO 47
C
C
C          COMPUTE AND OUTPUT MOMENTS DUE TO THE UNIT LOADS.
C
  303 DO 110 I=1,M
      SATX(I) = 0.
      DO 120 K=1,L
  120 SATX(I)=SATX(I)+SAT(I,K)*ASAT(K,LP1)
  110 CONTINUE
      WRITE OUTPUT TAPE 2, 522
  522 FORMAT (26H0MOMENTS DUE TO UNIT LOADS)
      DO 52 I=1,M
   52 WRITE OUTPUT TAPE 2, 21, I, SATX(I)
C
C
C          CALCULATE THE LOAD FACTOR ALG AT EACH END OF EACH MEMBER.
C
      DO 201 I=1,M
      IF(ABSF(SATX(I)) - 1.0) 202,202,203
  202 ALG(I) = 1.E20
      GO TO 201
  203 ALG(I) = (PM(I)-ABSF(CM(I)))/ABSF(SATX(I))
  201 CONTINUE
```

```
C
C
C        FIND POSITION AND VALUE OF SMALLEST LOAD FACTOR SALG.
C
      SALG = 1.E20
      DO 204  I = 1,M
      TEST = CM(I) * SATX(I)
      IF (TEST)  204,205,205
  205 IF(ALG(I) - SALG) 1206,204,204
 1206 SALG = ALG(I)
      NPH = I
  204 CONTINUE
C
C
C        FACTOR UNIT MOMENTS BY SALG AND GET CUMULATIVE MOMENTS.
C
  302 DO 207 I=1,M
      SATX(I) = SALG*SATX(I)
  207 CM(I) = CM(I) + SATX(I)
C
C
C        CALCULATE THE CUMULATIVE LOAD FACTOR. CLG.
C
  304 CLG = CLG + SALG
C
C
C        MULTIPLY UNIT DEFLECTIONS BY SMALLEST LOAD FACTOR SALG.
C
      DO 206 I=1,L
      ASAT(I,LP1) = SALG*ASAT(I,LP1)
C
C
C        CALCULATE CUMULATIVE DEFLECTIONS.
C
  206 CX(I) = CX(I) + ASAT(I,LP1)
C
C        UPDATE THE CYCLE NUMBER NCYCL.
C
      NCYCL = NCYCL + 1
C
C
C        OUTPUT CYCLE NO. AND LOCATION OF PLASTIC HINGE.LOAD FACTOR ETC
C
      WRITE OUTPUT TAPE 2, 401, NCYCL, NPH
  401 FORMAT(18H1PLASTIC HINGE NO., I3, 2X, 15HFORMED AT POINT,I3/)
      WRITE OUTPUT TAPE 2, 402
  402 FORMAT(12H0LOAD FACTOR,3X,10HADDITIONAL,9X,10HCUMULATIVE)
      WRITE OUTPUT TAPE 2, 403, NCYCL, SALG, CLG
  403 FORMAT (6H0STAGE, I4, E18.7, E18.7)
C
C
C        OUTPUT ADDITIONAL AND CUMULATIVE DEFLECTIONS.
C
      WRITE OUTPUT TAPE 2, 404
  404 FORMAT(12H0DEFORMATION, 3X, 10HADDITIONAL,9X,10HCUMULATIVE/)
```

```
      DO 208 I=1,L
  208 WRITE OUTPUT TAPE 2, 405, I, ASAT(I,LP1), CX(I)
  405 FORMAT (3H AT, I4, E22.7, E19.7)
C
C
C         OUTPUT ADDITIONAL AND CUMULATIVE MOMENTS WITH PL.MOMENT CAP.
C
      WRITE OUTPUT TAPE 2, 406
  406 FORMAT(7H0MOMENT8X,10HADDITIONAL,9X10HCUMULATIVE10X,8HPLAS MOM/)
      DO 209 I=1,M
  209 WRITE OUTPUT TAPE 2, 407, I, SATX(I), CM(I), PM(I)
  407 FORMAT (3H AT, I4, F18.4,2F19.4)
C
C
C         CHANGE THE STIFFNESS MATRIX ACCORDING TO  WHERE THE LAST
C         PLASTIC HINGE WAS FOUND.
C
      ITEST = (NPH/2 * 2) - NPH
      IF (ITEST) 211,210,210
  210 S(NPH-1,NPH-1)=0.75*S(NPH-1,NPH-1)
      S(NPH,NPH) = 0.
      S(NPH-1,NPH) = 0.
      S(NPH,NPH-1) = 0.
      GO TO 212
  211 S(NPH+1,NPH+1) = 0.75*S(NPH+1,NPH+1)
      S(NPH,NPH) = 0.
      S(NPH,NPH+1) = 0.
      S(NPH+1,NPH) = 0.
C
C
C         RETURN CONTROL TO ANALYSE THE DETERIURATED FRAME.
C
  212 GO TO 15
C
C
C         COMPUTE THE HINGE ROTATIONS ONCE THE COLLAPSE MECHANISM HAS
C         BEEN FOUND. FIRST, INVERT THE  S  MATRIX.
C
   47 WRITE OUTPUT TAPE 2, 408
  408 FORMAT (36H0COLLAPSE MECHANISM HAS BEEN REACHED)
      DO 163  I = 1,M
      DO 163  J = 1,M
  163 S(I,J) = 0.
      DO 164  I = 1,M
      ITEST = I/2*2-I
      IF(ITEST)  165,164,164
  165 K = I/2 + 1
      S(I,I) = 1.0/(3.0 * SDAT(K))
      S(I+1,I+1) = S(I,I)
      S(I,I+1) = -0.5*S(I,I)
      S(I+1,I) = S(I,I+1)
  164 CONTINUE
      DO 133 I=1,M
      DM(I) = 0.
      DO 134 K=1,M
```

```
  134 DM[I] = DM[I] + S[I,K] * CM[K]
  133 CONTINUE
      DO 135 I=1,M
      ATX[I] = 0.
      DO 136 K=1,L
  136 ATX[I] = ATX[I] + A[K,I]*CX[K]
  135 CONTINUE
      DO 137 I=1,M
  137 H[I] = DM[I] - ATX[I]
      WRITE OUTPUT TAPE 2, 138
  138 FORMAT [1H0,14X,15HHINGE ROTATIONS/]
      DO 139 I=1,M
  139 WRITE OUTPUT TAPE 2, 140, I, H[I]
  140 FORMAT [9H AT POINT, I4,E15.7]
C
C
C          RETURN CONTROL TO SEE IF ANY MORE LOAD SETS.
C
      GO TO 708
C
   99 CALL EXIT
      END
```

ELASTIC PLASTIC FIRST ORDER ANALYSIS OF FRAME NO. 2

PROPPED CANTILEVER PROBLEM

THE STATICS MATRIX

ROW 1	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 2	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00
ROW 3	-0.2000000E 00	-0.2000000E 00	0.2000000E 00	0.2000000E 00

THE STIFFNESS MATRIX

ROW 1	0.4000000E 04	0.2000000E 04	0.0000000E 00	0.0000000E 00
ROW 2	0.2000000E 04	0.4000000E 04	0.0000000E 00	0.0000000E 00
ROW 3	0.0000000E 00	0.0000000E 00	0.4000000E 04	0.2000000E 04
ROW 4	0.0000000E 00	0.0000000E 00	0.2000000E 04	0.4000000E 04

THE EXTERNAL LOAD VECTOR SET NO. 1

ROW 1	0.0000000E 00
ROW 2	0.0000000E 00
ROW 3	0.1000000E 01

DEFLECTIONS DUE TO UNIT LOADS

ROW 1	0.6250000E-03
ROW 2	-0.1562500E-03
ROW 3	0.1822917E-02

MOMENTS DUE TO UNIT LOADS

ROW 1	-0.1000000E-06
ROW 2	-0.1562500E 01
ROW 3	0.1562500E 01
ROW 4	0.1875000E 01

Appendix B Solution to Propped Cantilever

PLASTIC HINGE NO. 1 FORMED AT POINT 4

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE 1	0.2133333E 02	0.2133333E 02	
DEFORMATION	ADDITIONAL	CUMULATIVE	
AT 1	0.1333333E-01	0.1333333E-01	
AT 2	-0.3333333E-02	-0.3333333E-02	
AT 3	0.3888889E-01	0.3888889E-01	
MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	-0.0000	-0.0000	40.0000
AT 2	-33.3333	-33.3333	40.0000
AT 3	33.3333	33.3333	40.0000
AT 4	40.0000	40.0000	40.0000

DEFLECTIONS DUE TO UNIT LOADS

ROW 1 0.1250000E-02

ROW 2 0.0000000E 00

ROW 3 0.4166667E-02

MOMENTS DUE TO UNIT LOADS

ROW 1 -0.3000000E-06

ROW 2 -0.2500000E 01

ROW 3 0.2500000E 01

ROW 4 0.0000000E 00

PLASTIC HINGE NO. 2 FORMED AT POINT 2

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE 2	0.2666667E 01	0.2400000E 02	
DEFORMATION	ADDITIONAL	CUMULATIVE	
AT 1	0.3333333E-02	0.1666667E-01	
AT 2	0.0000000E 00	-0.3333333E-02	
AT 3	0.1111111E-01	0.5000000E-01	
MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	-0.0000	-0.0000	40.0000
AT 2	-6.6667	-40.0000	40.0000
AT 3	6.6667	40.0000	40.0000
AT 4	0.0000	40.0000	40.0000

DEFLECTIONS DUE TO UNIT LOADS

ROW 1 0.1000000E 05

ROW 2 -0.1000000E 05

ROW 3 0.5000000E 05

DEFLECTION TOO LARGE

COLLAPSE MECHANISM HAS BEEN REACHED

HINGE ROTATIONS

AT POINT 1 -0.8000000E-09
AT POINT 2 0.2000000E-08
AT POINT 3 -0.1000000E-08
AT POINT 4 -0.3333334E-02

Appendix B (cont.)

ELASTIC PLASTIC FIRST ORDER ANALYSIS OF FRAME NO. 13

NEAL - FINZI PROBLEM

THE STATICS MATRIX

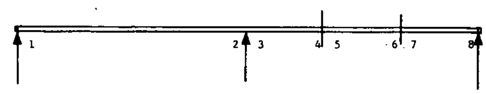
ROW 1	0.0000000E 00	0.0000000E 00	-0.1000000E 01	-0.1000000E 01	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00
ROW 2	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.1000000E 01	-0.1000000E 01	0.1000000E 01	0.1000000E 01
ROW 3	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 4	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01

THE STIFFNESS MATRIX

ROW 1	0.1333333E 01	0.6666667E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 2	0.6666667E 00	0.1333333E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 3	0.0000000E 00	0.0000000E 00	0.4000000E 01	0.2000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 4	0.0000000E 00	0.0000000E 00	0.2000000E 01	0.4000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.4000000E 01	0.2000000E 01	0.0000000E 00	0.0000000E 00
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.2000000E 01	0.4000000E 01	0.0000000E 00	0.0000000E 00
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.4000000E 01	0.2000000E 01
ROW 8	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.2000000E 01	0.4000000E 01

THE EXTERNAL LOAD VECTOR SET NO. 1

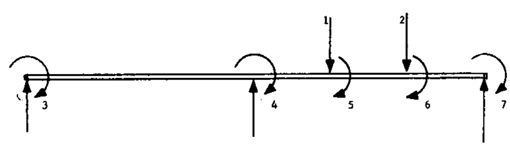
ROW 1	0.1300000E 01
ROW 2	0.7000000E 00
ROW 3	0.0000000E 00
ROW 4	0.0000000E 00
ROW 5	0.0000000E 00
ROW 6	0.0000000E 00
ROW 7	0.0000000E 00



MEMBER IDENTIFICATION SEQUENCE

DEFLECTIONS DUE TO UNIT LOADS

ROW 1	0.5629626E 00
ROW 2	0.5870367E 00
ROW 3	-0.2583332E 00
ROW 4	0.5166664E 00
ROW 5	0.3972220E 00
ROW 6	-0.3444443E 00
ROW 7	-0.7083330E 00



DEFORMATION IDENTIFICATION SEQUENCE

NEAL - FINZI PROBLEM

MOMENTS DUE TO UNIT LOADS

ROW 1	0.2000000E-07
ROW 2	0.5166664E 00
ROW 3	-0.5166663E 00
ROW 4	-0.7555552E 00
ROW 5	0.7555549E 00
ROW 6	-0.7277775E 00
ROW 7	0.7277773E 00
ROW 8	-0.1000000E-06

Appendix C Solution to Neal-Finzi Problem

PLASTIC HINGE NO. 1 FORMED AT POINT 4

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE 1	0.1323530E 01	0.1323530E 01

DEFORMATION	ADDITIONAL	CUMULATIVE
AT 1	0.7450979E 00	0.7450979E 00
AT 2	0.7769607E 00	0.7769607E 00
AT 3	-0.3419117E 00	-0.3419117E 00
AT 4	0.6838235E 00	0.6838235E 00
AT 5	0.5257352E 00	0.5257352E 00
AT 6	-0.4558823E 00	-0.4558823E 00
AT 7	-0.9374999E 00	-0.9374999E 00

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	0.0000	0.0000	1.0000
AT 2	0.6838	0.6838	1.0000
AT 3	-0.6838	-0.6838	1.0000
AT 4	-1.0000	-1.0000	1.0000
AT 5	1.0000	1.0000	1.0000
AT 6	-0.9632	-0.9632	1.0000
AT 7	0.9632	0.9632	1.0000
AT 8	-0.0000	-0.0000	1.0000

DEFLECTIONS DUE TO UNIT LOADS

ROW 1	0.2199998E 01
ROW 2	0.1216666E 01
ROW 3	-0.8249992E 00
ROW 4	0.1649998E 01
ROW 5	-0.9249992E 00
ROW 6	-0.1099999E 01
ROW 7	-0.1274999E 01

MOMENTS DUE TO UNIT LOADS

ROW 1	0.1000000E-06
ROW 2	0.1649998E 01
ROW 3	-0.1649998E 01
ROW 4	0.0000000E 00
ROW 5	-0.1400000E-05
ROW 6	-0.3500009E 00
ROW 7	0.3499996E 00
ROW 8	-0.4000000E-06

PLASTIC HINGE NO. 2 FORMED AT POINT 6

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE 2	0.1050416E 00	0.1428572E 01

DEFORMATION	ADDITIONAL	CUMULATIVE
AT 1	0.2310912E 00	0.9761892E 00
AT 2	0.1278005E 00	0.9047611E 00
AT 3	-0.8665922E-01	-0.4285709E 00
AT 4	0.1733184E 00	0.8571419E 00
AT 5	-0.9716337E-01	0.4285718E 00
AT 6	-0.1155456E 00	-0.5714279E 00
AT 7	-0.1339279E 00	-0.1071428E 01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	0.0000	0.0000	1.0000
AT 2	0.1733	0.8571	1.0000
AT 3	-0.1733	-0.8571	1.0000
AT 4	0.0000	-1.0000	1.0000
AT 5	-0.0000	1.0000	1.0000
AT 6	-0.0368	-1.0000	1.0000
AT 7	0.0368	1.0000	1.0000
AT 8	-0.0000	-0.0000	1.0000

DEFLECTIONS DUE TO UNIT LOADS

ROW 1	0.1820832E 01
ROW 2	0.4375000E 07
ROW 3	-0.7374996E 00
ROW 4	0.1474999E 01
ROW 5	0.4374999E 07
ROW 6	-0.4375000E 07
ROW 7	-0.4375001E 07

DEFLECTION TOO LARGE

COLLAPSE MECHANISM HAS BEEN REACHED

HINGE ROTATIONS

AT POINT 1	0.2000000E-07
AT POINT 2	-0.4000000E-07
AT POINT 3	0.0000000E 00
AT POINT 4	0.3571410E 00
AT POINT 5	-0.4000000E-07
AT POINT 6	-0.1000000E-07
AT POINT 7	-0.2000000E-07
AT POINT 8	-0.8000000E-07

ELASTIC PLASTIC FIRST ORDER ANALYSIS OF FRAME NO. 9

HEYMAN'S FRAME

THE STATICS MATRIX

ROW 1	-0.6666670E-01	-0.6666670E-01	0.6666670E-01	0.6666670E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 2	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.6666670E-01	-0.6666670E-01	0.6666670E-01
	0.6666670E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 3	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	-0.6666670E-01	-0.6666670E-01	0.6666670E-01	0.6666670E-01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 4	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.6666670E-01	-0.6666670E-01
	0.6666670E-01	0.6666670E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	-0.8333330E-01	-0.8333330E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	-0.8333330E-01	-0.8333330E-01			
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.8333330E-01
	-0.8333330E-01	0.8333330E-01	0.8333330E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	-0.8333330E-01	-0.8333330E-01	0.8333330E-01	0.8333330E-01			
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.8333330E-01	-0.8333330E-01	0.8333330E-01
	0.8333330E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.8333330E-01	-0.8333330E-01
	0.8333330E-01	0.8333330E-01	0.0000000E 00	0.0000000E 00			
ROW 8	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	-0.8333330E-01	-0.8333330E-01	0.8333330E-01	0.8333330E-01	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.8333330E-01	-0.8333330E-01	0.8333330E-01	0.8333333E-01
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 9	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 10	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 11	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01			
ROW 12	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 13	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 14	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00			
ROW 15	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 16	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 17	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01
	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 18	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 19	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01
	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			
ROW 20	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00			

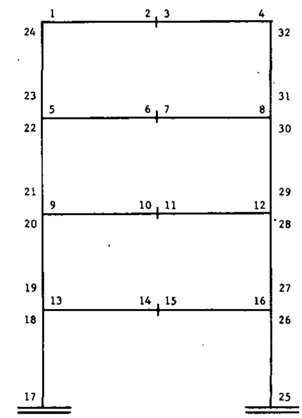
Appendix D Computer Solution for Heyman Frame

PLASTIC HINGE NO. 1 FORMED AT POINT 16

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE 1	0.1742112E 01	0.1742112E 01

DEFORMATION	ADDITIONAL	CUMULATIVE
AT 1	0.7561517E-01	0.7561517E-01
AT 2	0.5351284E-01	0.5351284E-01
AT 3	0.5307721E-01	0.5307721E-01
AT 4	0.5642525E-01	0.5642525E-01
AT 5	0.2291726E 00	0.2291726E 00
AT 6	0.2061622E 00	0.2061622E 00
AT 7	0.1532738E 00	0.1532738E 00
AT 8	0.7213038E-01	0.7213038E-01
AT 9	0.5519973E-02	0.5519973E-02
AT 10	-0.3615000E-03	-0.3615000E-03
AT 11	-0.4073974E-02	-0.4073974E-02
AT 12	0.3911923E-02	0.3911923E-02
AT 13	-0.1030965E-02	-0.1030965E-02
AT 14	0.2119359E-03	0.2119359E-03
AT 15	0.5899208E-02	0.5899208E-02
AT 16	-0.1801213E-02	-0.1801213E-02
AT 17	0.1305644E-02	0.1305644E-02
AT 18	0.7413990E-02	0.7413990E-02
AT 19	-0.2335401E-02	-0.2335401E-02
AT 20	0.1927612E-02	0.1927612E-02

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	-109.8803	-109.8803	318.3330
AT 2	-255.2833	-255.2833	318.3330
AT 3	255.2833	255.2833	318.3330
AT 4	163.5027	163.5027	318.3330
AT 5	-96.6562	-96.6562	318.3330
AT 6	-218.8554	-218.8554	318.3330
AT 7	218.8554	218.8554	318.3330
AT 8	249.5827	249.5827	318.3330
AT 9	-16.8987	-16.8987	350.0000
AT 10	-227.3768	-227.3768	350.0000
AT 11	227.3768	227.3768	350.0000
AT 12	312.2976	312.2976	350.0000
AT 13	33.0057	33.0057	350.0000
AT 14	-233.4777	-233.4777	350.0000
AT 15	233.4777	233.4777	350.0000
AT 16	350.0000	350.0000	350.0000
AT 17	-126.8332	-126.8332	213.3330
AT 18	-38.2772	-38.2772	213.3330
AT 19	5.2717	5.2717	213.3330
AT 20	-12.8215	-12.8215	213.3330
AT 21	29.7204	29.7204	213.3330
AT 22	5.9834	5.9834	213.3330
AT 23	90.6731	90.6731	213.3330
AT 24	109.8804	109.8804	213.3330
AT 25	-192.3649	-192.3649	213.3330
AT 26	-169.3407	-169.3407	213.3330
AT 27	-180.6593	-180.6593	213.3330
AT 28	-188.0883	-188.0883	213.3330
AT 29	-124.2090	-124.2090	213.3330
AT 30	-137.2727	-137.2727	213.3330
AT 31	-112.3098	-112.3098	213.3330
AT 32	-163.5026	-163.5026	213.3330



MEMBER IDENTIFICATION SEQUENCE

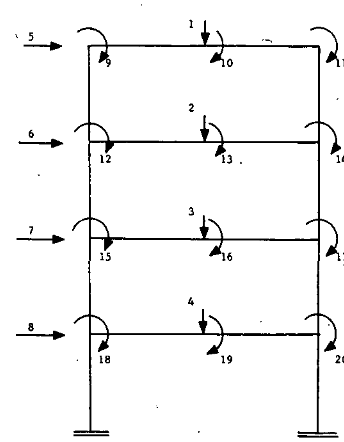
HEYMAN'S FRAME

PLASTIC HINGE NO. 2 FORMED AT POINT 25

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE 2	0.1608411E 00	0.1902953E 01

DEFORMATION	ADDITIONAL	CUMULATIVE
AT 1	0.7022406E-02	0.8263758E-01
AT 2	0.4815532E-02	0.5832837E-01
AT 3	0.5488834E-02	0.5856605E-01
AT 4	0.1484072E-01	0.7126597E-01
AT 5	0.3892903E-01	0.2681016E 00
AT 6	0.3667761E-01	0.2428399E 00
AT 7	0.3078326E-01	0.1840570E 00
AT 8	0.1452562E-01	0.8665599E-01
AT 9	0.5177058E-03	0.6037679E-02
AT 10	-0.3466457E-04	-0.3961646E-03
AT 11	-0.3790476E-03	-0.4453022E-02
AT 12	0.3630792E-03	0.4275002E-02
AT 13	-0.1044763E-03	-0.1135441E-02
AT 14	0.5482599E-04	0.2667619E-03
AT 15	0.7730907E-03	0.6672299E-02
AT 16	-0.2412891E-03	-0.2042502E-02
AT 17	0.1920655E-03	0.1497709E-02
AT 18	0.1608577E-02	0.9022567E-02
AT 19	-0.3557223E-04	-0.2370973E-02
AT 20	0.1875937E-02	0.3803549E-02

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	-9.9812	-119.8615	318.3330
AT 2	-23.6370	-278.9204	318.3330
AT 3	23.6370	278.9204	318.3330
AT 4	15.1231	178.6258	318.3330
AT 5	-8.4408	-105.0970	318.3330
AT 6	-19.9998	-238.8552	318.3330
AT 7	19.9998	238.8552	318.3330
AT 8	23.9381	273.5207	318.3330
AT 9	5.6614	-11.2373	350.0000
AT 10	-22.0650	-249.4418	350.0000
AT 11	22.0650	249.4418	350.0000
AT 12	33.9100	346.2076	350.0000
AT 13	5.8339	38.8396	350.0000
AT 14	-39.1062	-272.5839	350.0000
AT 15	39.1062	272.5838	350.0000
AT 16	0.0000	350.0000	350.0000
AT 17	-24.1615	-150.9947	213.3330
AT 18	-4.9480	-43.2252	213.3330
AT 19	-0.8859	4.3859	213.3330
AT 20	-10.8653	-23.6868	213.3330
AT 21	5.2039	34.9243	213.3330
AT 22	0.3065	6.2899	213.3330
AT 23	8.1343	98.8074	213.3330
AT 24	9.9812	119.8616	213.3330
AT 25	-20.9681	-213.3330	213.3330
AT 26	1.4390	-167.9017	213.3330
AT 27	-1.4389	-182.0982	213.3330
AT 28	-21.5518	-209.6402	213.3330
AT 29	-12.3581	-136.5671	213.3330
AT 30	-13.9973	-151.2701	213.3330
AT 31	-9.9408	-122.2506	213.3330
AT 32	-15.1231	-178.6258	213.3330



DEFORMATION IDENTIFICATION SEQUENCE

HEYMAN'S FRAME

Appendix D (cont.)

PLASTIC HINGE NO. 11 FORMED AT POINT 7

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE 11	0.2017249E-01	0.2233189E 01

DEFORMATION	ADDITIONAL	CUMULATIVE
AT 1	0.7338172E-01	0.2126757E 00
AT 2	0.1432835E-01	0.9152327E-01
AT 3	0.1047941E 00	0.2057828E 00
AT 4	0.1386438E 00	0.3778240E 00
AT 5	0.3018041E 00	0.9125587E 00
AT 6	0.2580480E 00	0.8139163E 00
AT 7	0.2038937E 00	0.6510125E 00
AT 8	0.1071458E 00	0.3350324E 00
AT 9	0.4647286E-02	0.1429244E-01
AT 10	-0.4892073E-02	-0.8470476E-02
AT 11	-0.4892077E-02	-0.1271767E-01
AT 12	0.2404476E-02	0.9047118E-02
AT 13	-0.2776098E-03	-0.1809422E-02
AT 14	0.4079600E-02	0.6897758E-02
AT 15	0.6764885E-02	0.1908631E-01
AT 16	-0.6986274E-02	-0.9450560E-02
AT 17	0.4729477E-02	0.1119317E-01
AT 18	0.9021518E-02	0.3055573E-01
AT 19	0.9353626E-02	0.1610212E-01
AT 20	0.8495571E-02	0.2331155E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
AT 1	-9.0792	-154.9372	318.3330
AT 2	0.0000	-318.3330	318.3330
AT 3	0.0032	318.3365	318.3330
AT 4	0.0031	213.3363	318.3330
AT 5	41.1791	-49.9351	318.3330
AT 6	-25.1280	-318.3329	318.3330
AT 7	25.1282	318.3330	318.3330
AT 8	0.0000	318.3330	318.3330
AT 9	-9.0772	45.0656	350.0000
AT 10	0.0000	-350.0000	350.0000
AT 11	0.0003	350.0002	350.0000
AT 12	0.0000	350.0000	350.0000
AT 13	-9.0776	45.0655	350.0000
AT 14	0.0001	-349.9998	350.0000
AT 15	0.0000	350.0000	350.0000
AT 16	0.0000	350.0000	350.0000
AT 17	0.0000	-213.3330	213.3330
AT 18	1.6609	-59.4314	213.3330
AT 19	7.4169	14.3666	213.3330
AT 20	-19.5373	-122.6293	213.3330
AT 21	28.6153	77.5649	213.3330
AT 22	-23.4674	-42.3477	213.3330
AT 23	-17.7110	92.2839	213.3330
AT 24	9.0782	154.9363	213.3330
AT 25	0.0000	-213.3330	213.3330
AT 26	-7.7623	-189.2230	213.3330
AT 27	7.7624	-160.7768	213.3330
AT 28	0.0000	-213.3330	213.3330
AT 29	0.0001	-136.6664	213.3330
AT 30	-7.7624	-187.9727	213.3330
AT 31	7.7626	-130.3601	213.3330
AT 32	0.0000	-213.3330	213.3330

DEFLECTION TOO LARGE

COLLAPSE MECHANISM HAS BEEN REACHED

HINGE ROTATIONS

AT POINT	1	-0.3000000E-09
AT POINT	2	0.1615364E-01
AT POINT	3	0.9000000E-09
AT POINT	4	-0.3000000E-09
AT POINT	5	0.0000000E 00
AT POINT	6	0.1400000E-08
AT POINT	7	-0.1700000E-08
AT POINT	8	-0.8707182E-02
AT POINT	9	-0.1900000E-08
AT POINT	10	0.1408325E-01
AT POINT	11	0.4000000E-09
AT POINT	12	-0.2064373E-01
AT POINT	13	-0.1500000E-08
AT POINT	14	0.0000000E 00
AT POINT	15	-0.3702211E-01
AT POINT	16	-0.4423154E-01
AT POINT	17	0.1767095E-01
AT POINT	18	-0.1100000E-08
AT POINT	19	0.2600000E-08
AT POINT	20	0.2800000E-08
AT POINT	21	0.1600000E-08
AT POINT	22	0.1000000E-08
AT POINT	23	0.2700000E-08
AT POINT	24	0.1300000E-08
AT POINT	25	0.2129304E-01
AT POINT	26	0.1800000E-08
AT POINT	27	0.4900000E-08
AT POINT	28	0.7718326E-02
AT POINT	29	0.2700000E-08
AT POINT	30	0.2400000E-08
AT POINT	31	0.4500000E-08
AT POINT	32	0.1266886E-01

Appendix D (cont.)

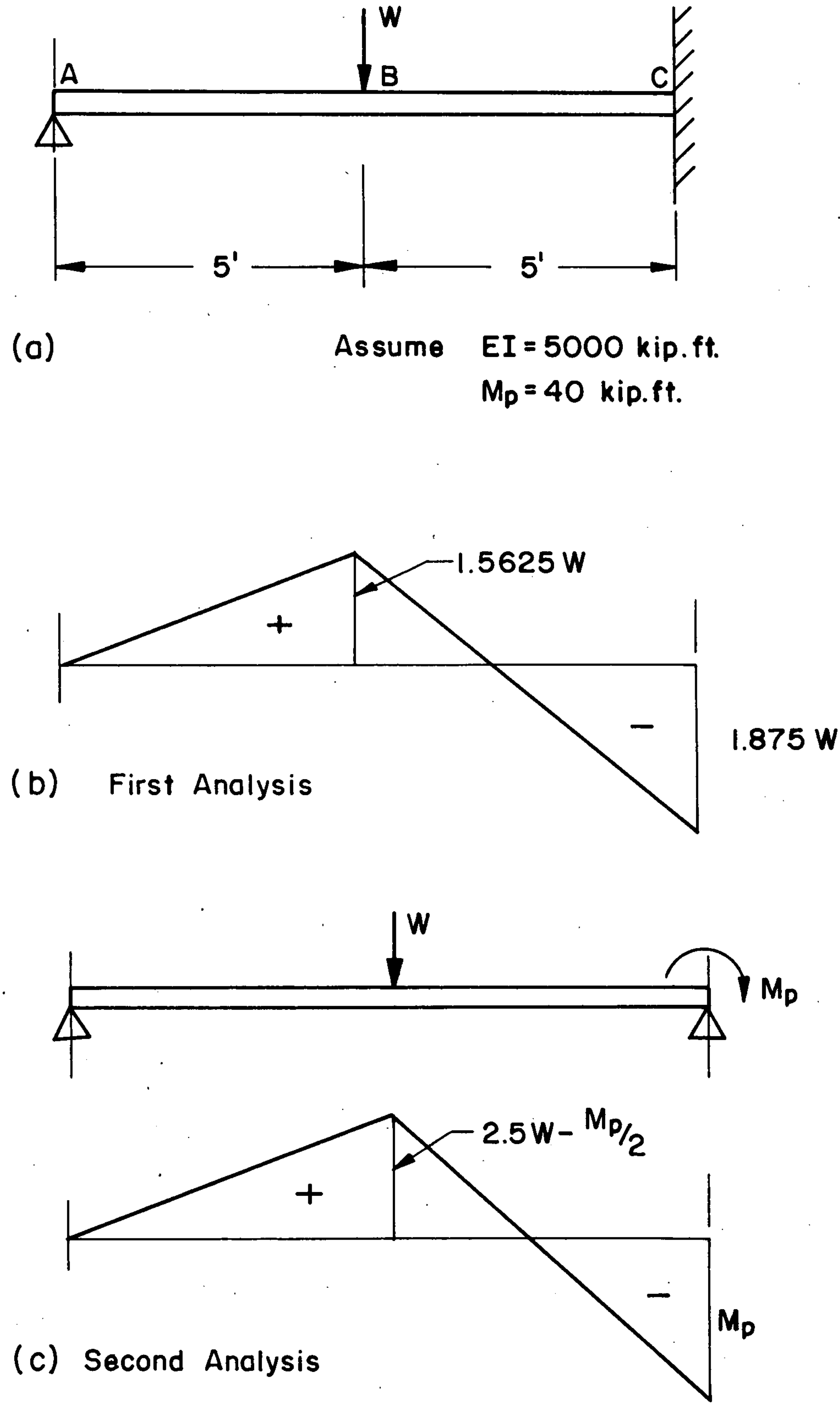


Fig. 1 Elastic-Plastic Analysis (manual computation)

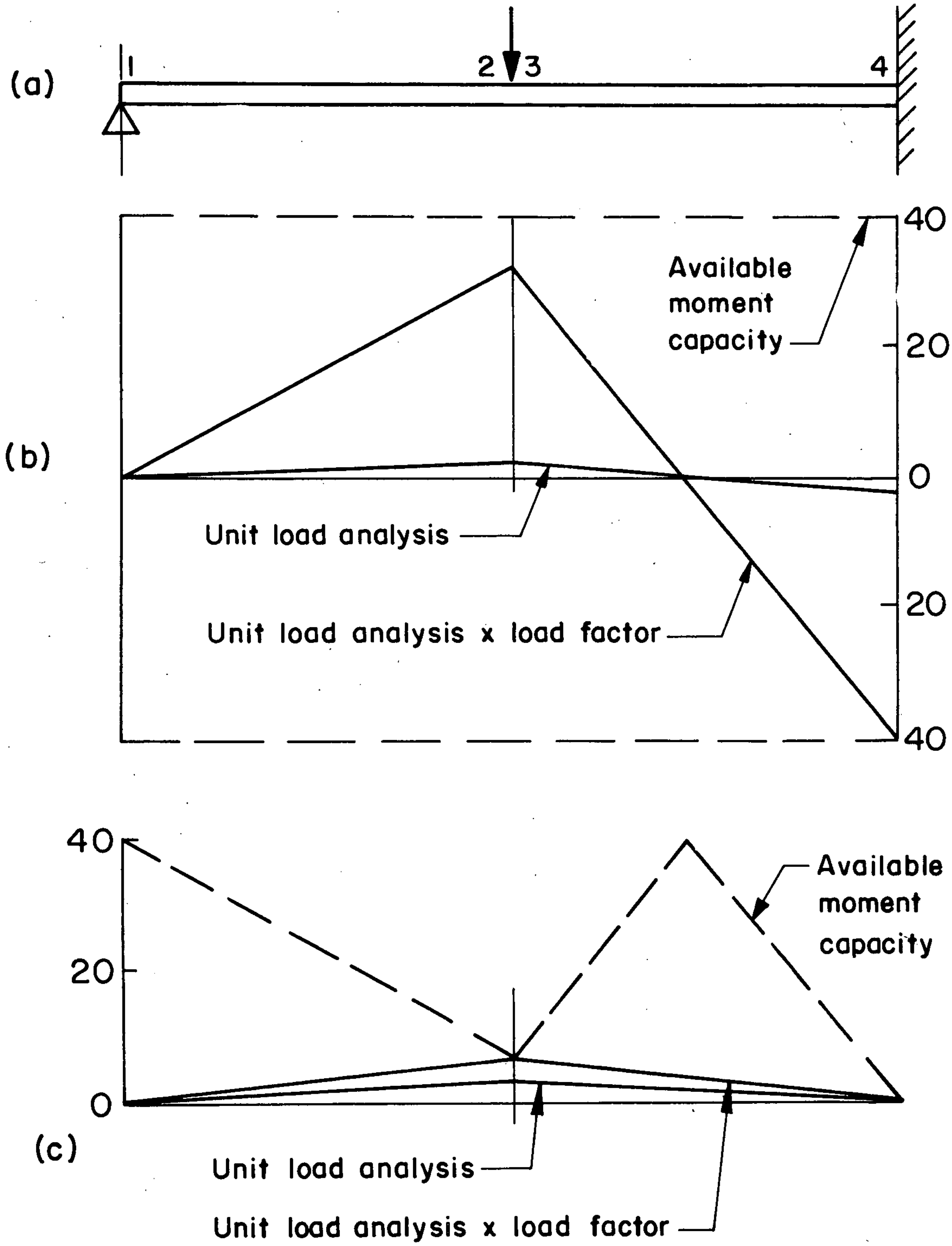


Fig. 2 Elastic-Plastic Analysis (machine computation)

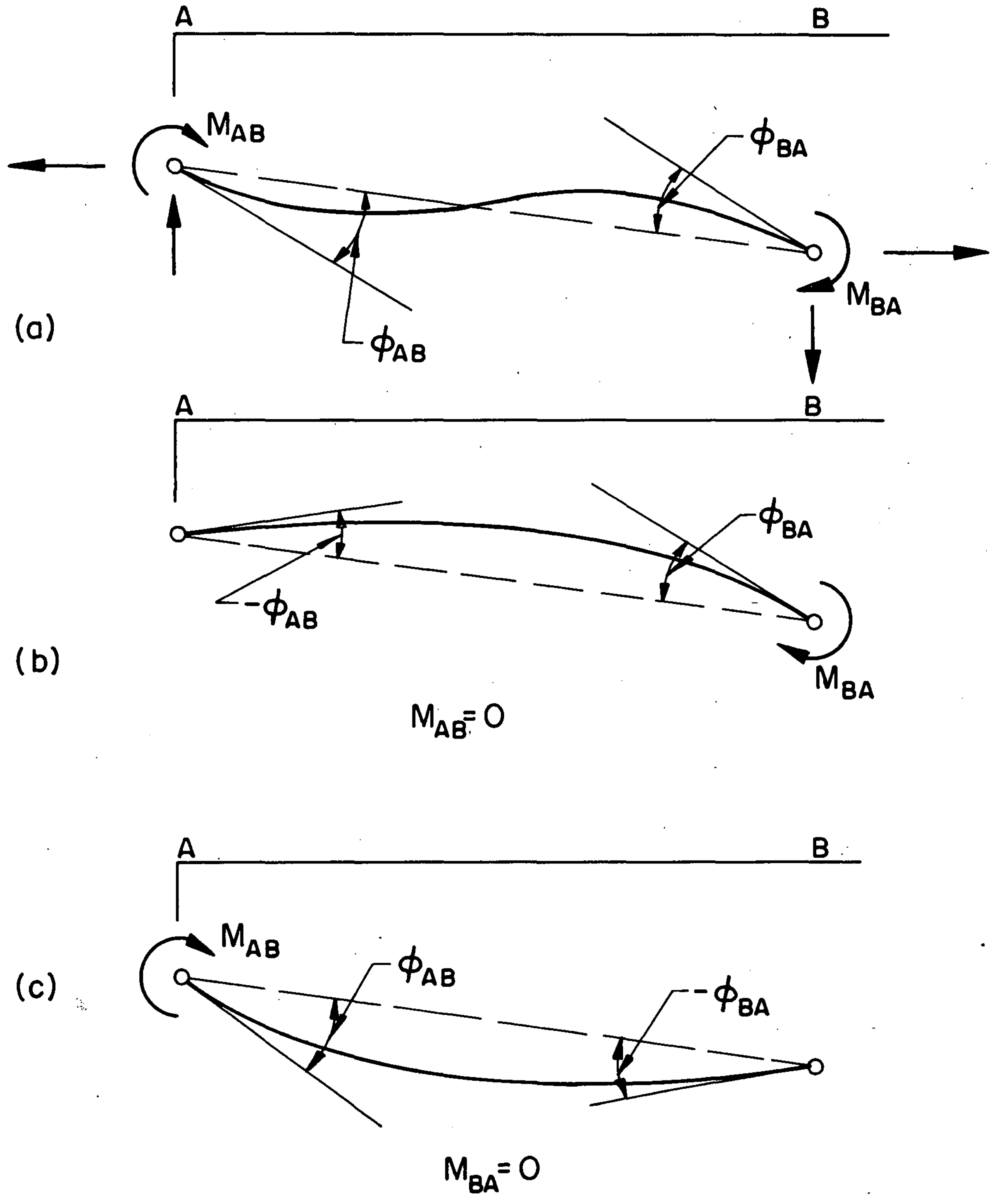


Fig. 3 Load Displacement Relations

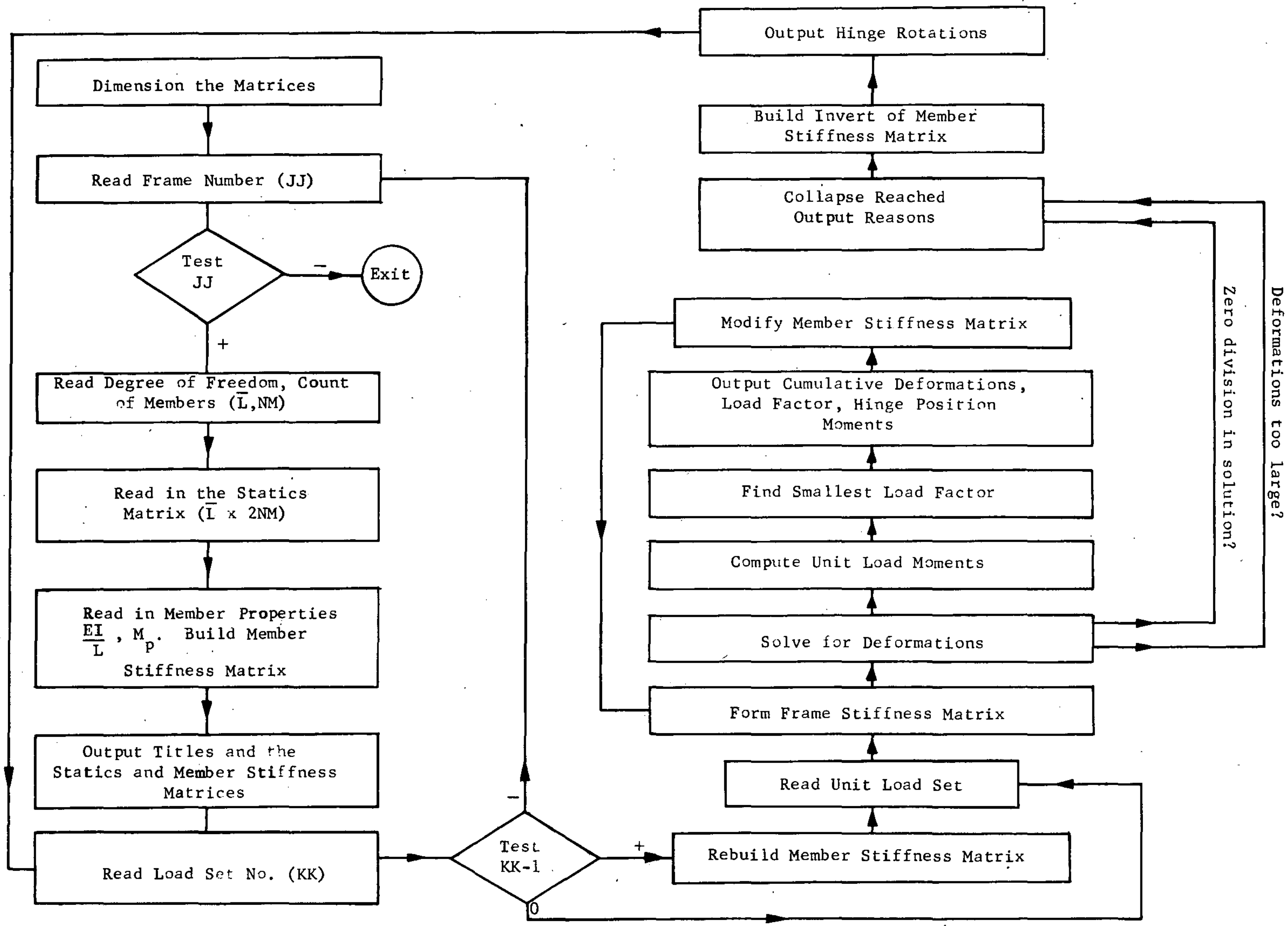


Fig. 4 Flow Diagram

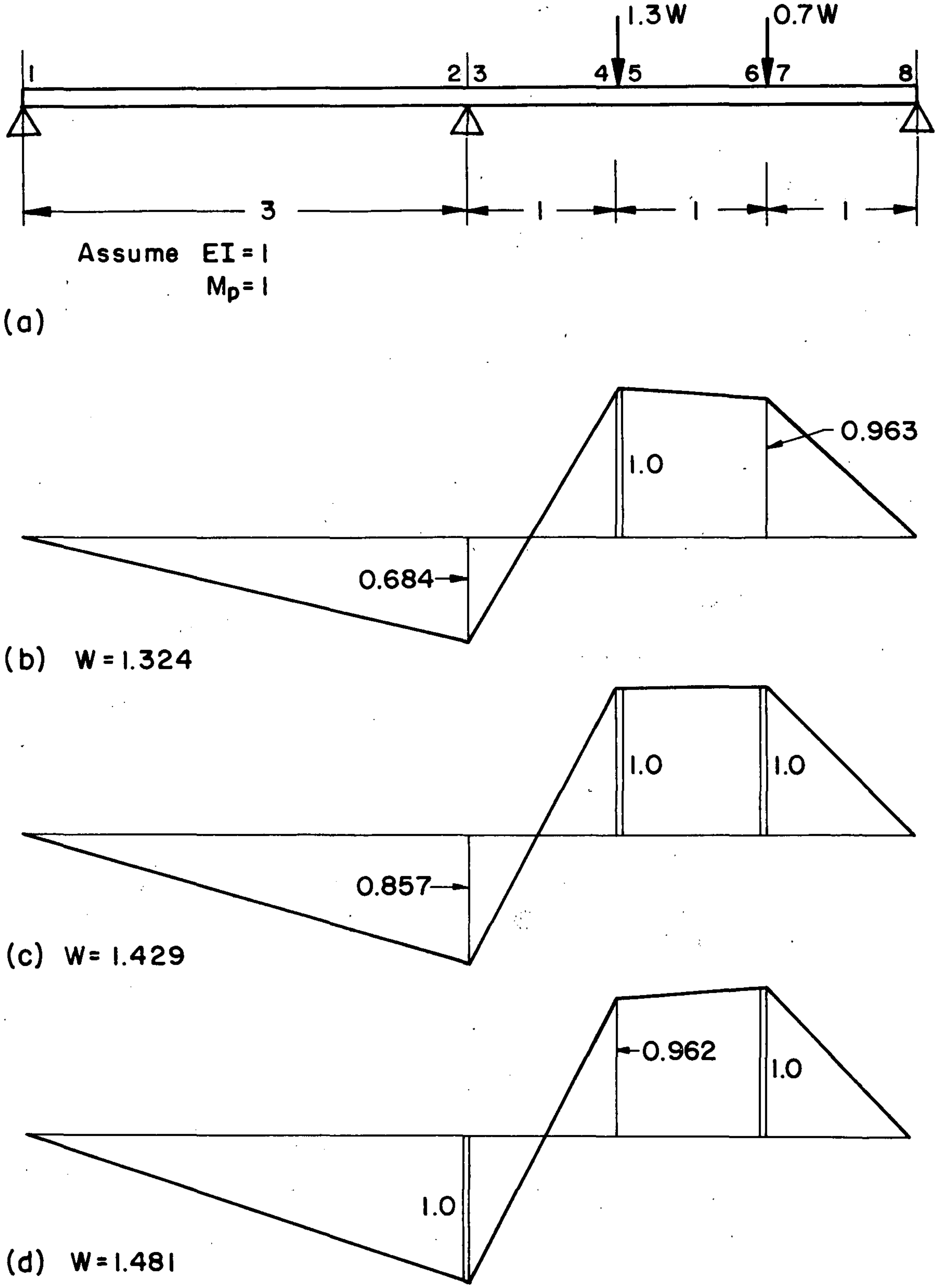
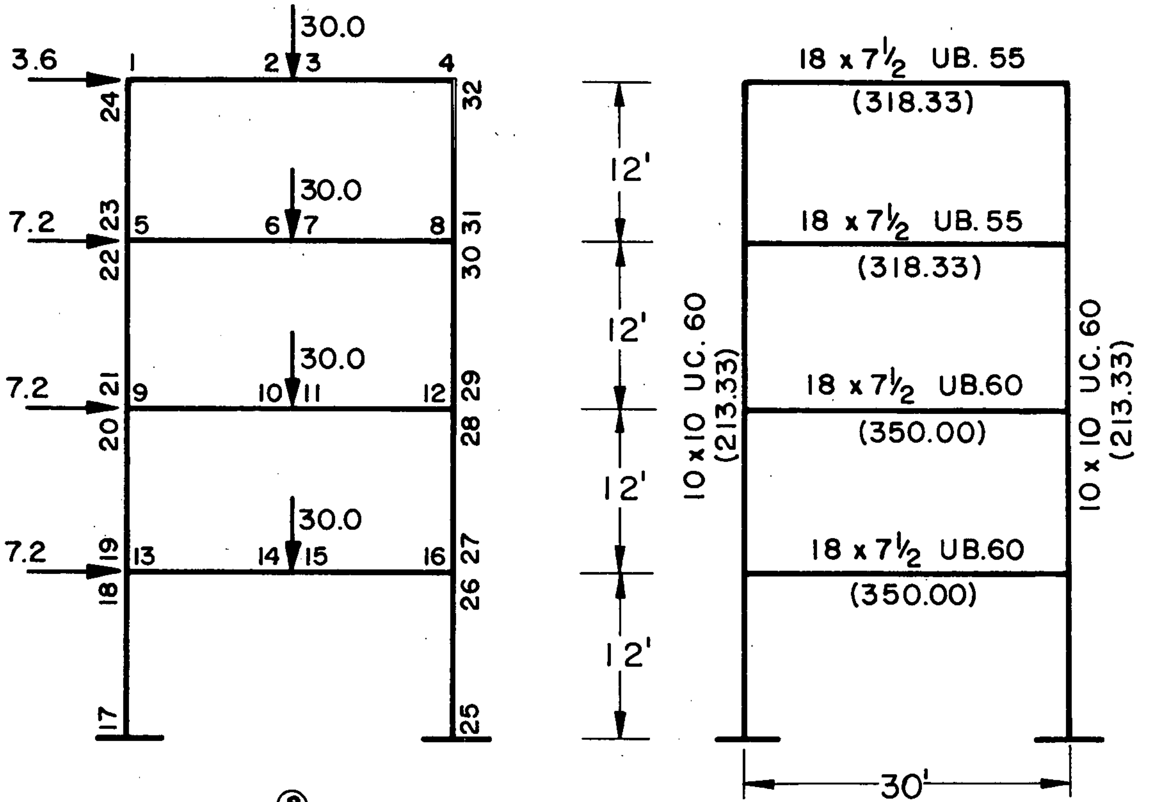
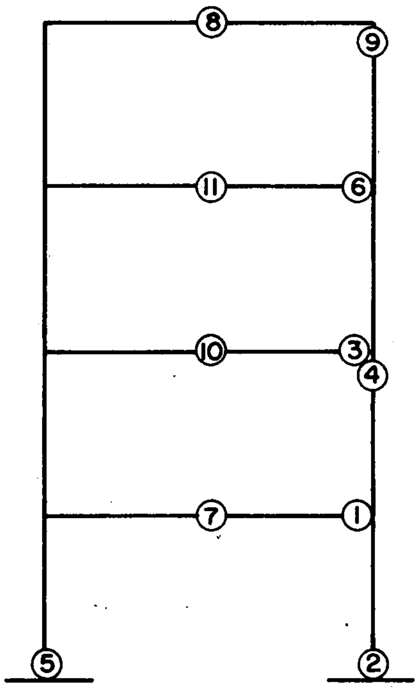


Fig. 5 Neal-Finzi Problem





UB - Universal Beam

UC - Universal Column

Units - Kip, Feet

HINGE No.	AT POINT	LOAD FACTOR	TOP SWAY
1	16	1.742	0.229
2	25	1.903	0.268
3	12	1.920	0.273
4	28	1.950	0.287
5	17	2.145	0.375
6	8	2.150	0.379
7	15	2.161	0.390
8	2	2.171	0.429
9	32	2.191	0.500
10	10	2.213	0.611
11	7	2.233	0.913

Fig. 6 Heyman's Frame

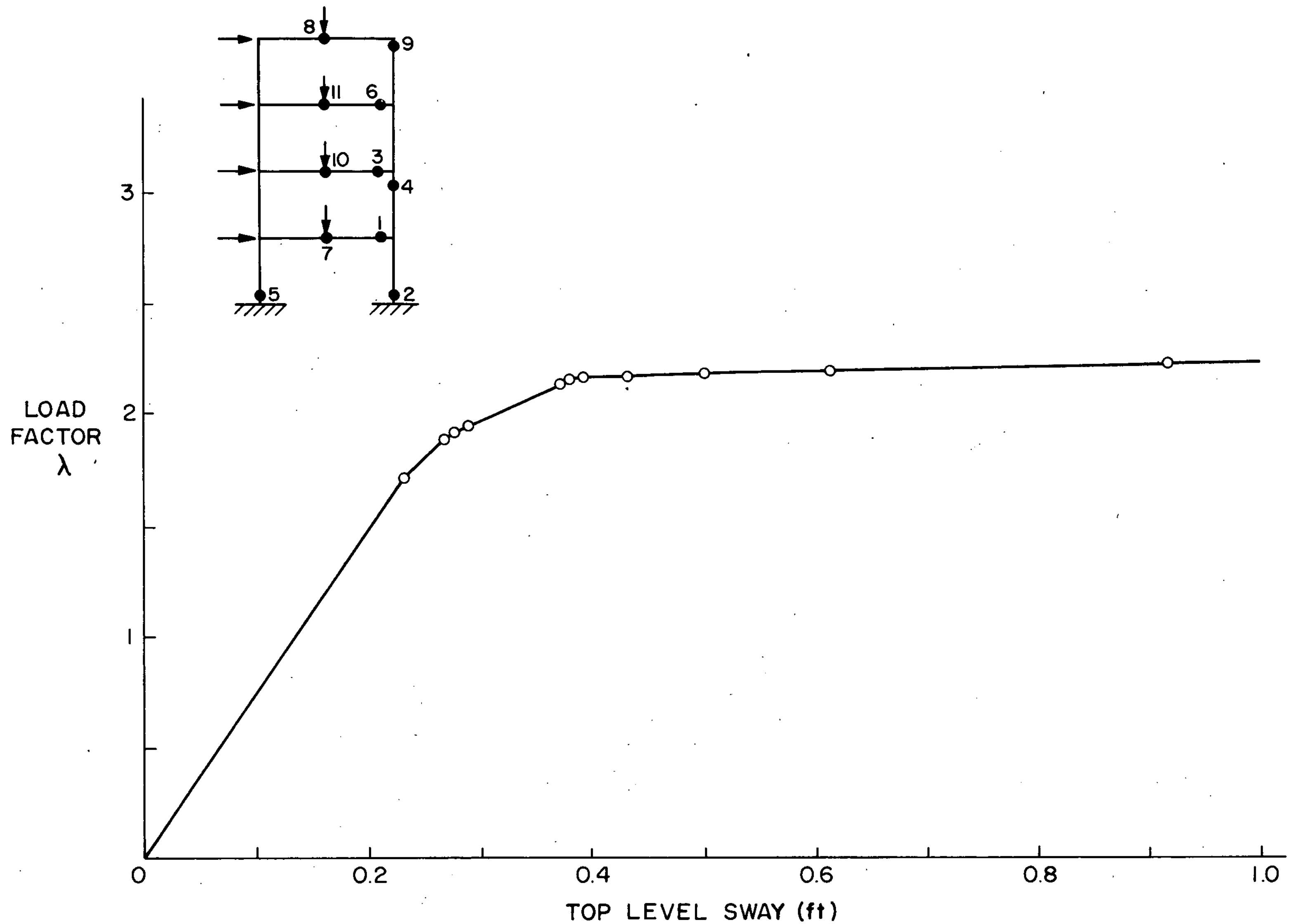


Fig. 7 Computed Load-Sway Curve

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