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H. B. Harrison

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COMPUTER ANALYSIS

OF

PLANE FRAMES

by

H. B. Harrison

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SYNOPSIS

A detailed description is given of a computer program to carry out a general first order elastic analysis of any type of plane frame. It is capable of analyzing pinned or rigidly jointed frames, or mixtures of both systems. Shear and axial deformations may be taken into account in rigidly jointed frames. In addition, strains caused by temperature changes, shrinkage or lack-of-fit can be allowed for with little extra effort in data preparation. The source program has been written in the Fortran language so that it can be used on most currently available computers. Analysis is carried out by the displacement method so that considerations of frame redundancy do not arise. For a given machine there will be a maximum size of structure that can be accommodated, depending primarily upon the number and type of its members, the degrees of freedom of the frame, and the number of alternative load sets for which an analysis is desired. The program was developed to check the elastic behavior of a series of braced and unbraced multi-story steel frames being tested in the Fritz Engineering Laboratory of Lehigh University's Department of Civil Engineering as part of a program of research into the plastic behavior and design of multi-story steel frames.

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I INTRODUCTION

Since the widespread introduction of electronic computers considerable attention has been directed to automating the analysis of structural frames. Many computer programs which achieve the linear-elastic analysis of plane frames are in existence[(1),(2)] but the published accounts are principally descriptive and some do not include any more detailed information about the solution processes than can be contained in a general flow diagram. Many computers in the mid-fifties had highly individualized coding systems which took a long time to master and which were not compatible with machines from other makers. An awareness of this communication problem has resulted in the increasing use of algorithmic languages such as Fortran and Algol and most machines now have available compilers to handle one or both of these systems. In addition, these source languages are very compact in the sense that it is not necessary to state explicitly every arithmetic step in a program. This report includes the Fortran statements for the structural analysis program so that it will be of immediate use to engineers already familiar with Fortran and for those who are not so familiar, it will serve as a suitable introduction.

The program described in this report will handle any type of plane frame consisting of prismatic members. It will analyze for any number of load sets so that it is possible to obtain a complete matrix of influence coefficients for stress or deflection. For most structures, these are not required and the storage layout has been designed in such a way that for any given computer, larger frames can be accommodated if the information required concerning them is restricted. It is possible to either ignore, or take into account shear and axial strains in predominantly flexural structures. Pin jointed frames and mixtures of flexural and axially strained members, as in braced multi-story frames, may also be handled. In addition, the effects of temperature, shrinkage or lack of fit can be included. The basic method of analysis used is the displacement method which is generally to be preferred to the alternative force method[3] for structural frames on the grounds that it requires less data preparation at a cost of somewhat more arithmetic operations within the computer. This is certainly the case for frames of few redundancies but many degrees of freedom. This method is also simpler to understand, being a more general form of the traditional method of slope-deflection analysis.

The basic data to be presented to the computer consists of information about the degree of freedom, the number of flexural and axially strained members and their relevant stiffnesses. The frame topology is conveniently described by a statics matrix which can be prepared almost by inspection for most plane frames so that it is not necessary to construct it within the program from more basic data. The load-sets also are required, together with information about the temperature or shrinkage deformations of each frame element considered in isolation. The output has been arranged so that moments at each end of each flexural component are tabulated separately from the axial tensions for cases when both are required so that bending moment diagrams can be simply constructed. The frame deformations are also listed for each load set and a check computation of the load sets is finally made to give some idea of any accuracy loss due to rounding-off or machine errors during the computation.

II DISPLACEMENT ANALYSIS

The basic objective of any method of structural analysis is the determination of deformations and stresses and in order to compute the latter, the stress-resultants, that is, moments, shears, thrusts at any section are required. If the stress-resultants are known at any one section in a frame member, the complete stress condition for that member will be determined by statical considerations alone.

In Fig. 1(a) is shown part of a rigidly jointed plane frame and attention is confined to a typical member AB. It is evident that the complete stress condition for this member will be available if the values of the stress-resultants T, V and M_{AB} in Fig. 1(b) are known. An alternative arrangement of stress-resultants is shown in Fig. 1(c) where again the stress condition for the member can be determined from T, M_{AB}, and M_{BA}.

2.1 The member stiffness matrix

The deformations associated with the first system of stress resultants(Fig.1(b) have been shown in Fig. 2(a) where the deformations u_T, u_V and θ correspond to the forces T, V and the moment M_{AB}. For this system and neglecting at this stage deflections due to shear, the load-deformation relationships may be expressed conveniently in the matrix equation,

$$\begin{bmatrix} T \\ V \\ M_{AB} \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \cdot \begin{bmatrix} u_T \\ u_V \\ \theta_{AB} \end{bmatrix} \quad (1)$$

For the alternative system of stress-resultants (Fig. 1(c)) shown in Fig. 2(b), the load-deformation relationships take the form of the traditional slope-deflection equations for a prismatic member.

$$\begin{bmatrix} T \\ M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 4EI/L & 2EI/L \\ 0 & 2EI/L & 4EI/L \end{bmatrix} \cdot \begin{bmatrix} u_T \\ \phi_{AB} \\ \phi_{BA} \end{bmatrix} \tag{2}$$

Representing either of the above load-deformation equation sets by the matrix equation

$$(SR_{AB}) = (S_{AB}) \cdot (x_{AB}) \tag{3}$$

it is evident that the similar relationships for all members of a frame can be assembled in the one matrix equation,

$$\begin{bmatrix} (SR_{12}) \\ (SR_{34}) \\ \cdot \\ (SR_{MN}) \end{bmatrix} = \begin{bmatrix} (S_{12}) & \cdot & \cdot & \cdot \\ \cdot & (S_{34}) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & (S_{MN}) \end{bmatrix} \cdot \begin{bmatrix} (x_{12}) \\ (x_{34}) \\ \cdot \\ (x_{MN}) \end{bmatrix} \tag{4}$$

and the equation is represented conveniently as

$$(SR) = (S) \cdot (x) \tag{5}$$

It should be noted that while it is often convenient to group together in the above expressions the three load-deformation equations for each member, this procedure is not a necessary one and in the computer program described later, the axial load-deformation equations have been separated from the other pairs of equations and have all been placed together at the bottom of the lists. If axial strains are neglected, as in many flexural problems, these equations do not appear and hence for each member there are only two load-deformation relationships.

2.2 The Statics Matrix

The equations of statics for any frame represent mathematically the fact that the internal stress resultants must be in equilibrium with the applied loads. For a stable structure that is statically determinate, a unique set of stress resultants will equilibrate the external loads but an infinite set will do so for a redundant frame. The purpose of the analysis then is to find the particular set which also satisfies continuity. Since deformations can be measured conveniently by the movement of loads, whether real or virtual, it is necessary for the analysis to be undertaken with as many loads acting as the degree of freedom.

The degree of freedom of a structure is the count of all possible displacements and rotations of the joints. For the program described in this report, a change in direction of a member must be considered to take place at a joint and the same applies should any movement be restrained within a straight length such as at a support of a continuous beam. Load application positions along a beam may be considered as joints and then the computer will produce the deformations at the loads. However transverse loading on beams whether caused by point or distributed loads can be replaced with no loss of accuracy by the equilibrants of the fixing moments and shears (and tensions) if computer storage capacity is limited.

For a plane rigid frame, the degree of freedom $(\overline{L})$ is readily determined. If there are j joints altogether in the structure, 3 j deformations will determine the deformed shape of the frame since each joint may translate in two directions and may rotate as well. However, fixed bases do not deform and hinged bases only rotate so that if there are f fixed bases and h hinged supports,

$$\bar{L} = 3j - 3f - 2h$$

At an internal hinge, an additional degree of freedom will be present since two displacements and two rotations will be needed to define movement at such a node and consequently the degree of freedom of the frame can be expressed,

$$\bar{L} = 3j - 3f - 2h + h_i$$

Finally, for a flexural frame in which axial strains are neglected, the length of any member will not change so that a little consideration will show that if such a frame consists of m members,

$$\bar{L} = 3j - 3f - 2h + h_i - m$$

An alternative method of determining the degree of freedom of a frame depends upon the relationship between the degrees of freedom and of redundancy.

For any plane structure,

$$\bar{L} + R = 2\ NFM + NAM$$

where NFM is the count of the flexural members, NAM is the count of the axially strained members and R is the degree of redundancy. If axial stiffnesses are regarded as being indefinitely large, the relationship is

$$\bar{L} + R = 2\ NFM$$

Hence, the degree of freedom can be determined simply if the degree of redundancy of a structure is known.

Accordingly, it can be seen that for the whole structure there will be $\bar{L}$ equations of statics and most of these can be written down by inspection. In matrix form, these $\bar{L}$ equations of statics may be expressed,

$$(W) = (A) \cdot (SR) \tag{6}$$

The matrix (A) is called the statics matrix and will be of order $\bar{L} \times \bar{M}$ where $\bar{M}$ is a count of the stress resultants necessary to define conditions within the frame. For pin jointed frames, $\bar{M}$ will equal the number of frame members; for general flexural frames, $\bar{M}$ will be three times the member count or twice if axial strains are neglected. It can be noted that only for a determinate structure is the matrix (A) square so that an inversion of it will provide complete information about stress resultants as functions of the applied loads.

2.3 The Kinematics Matrix

The combination of Eqs. (5) and (6) will produce a matrix equation relating the applied loads to the relative deformations within members. What is required fundamentally is the relationship between loads and absolute deformations of the joints of a frame. This can be achieved if an equation relating relative to absolute deformations is available. Such a relationship takes the form,

$$(x) = (C) \cdot (X) \tag{7}$$

The matrix (C) is of order $\bar{M} \times \bar{L}$ and can be established by considering the relative deformations resulting from unit absolute displacements given to each load application point sequentially. It is referred to as the kinamatics matrix.[4] In establishing this matrix for any frame, it is soon noted that it is in fact the transpose of the statics matrix (A), and it can be shown from the principle of virtual work that this is necessarily the case.[4]

2.4 The Frame Stiffness Matrix

The combination of Eqs. (5), (6) and (7) will result in a matrix equation relating the applied loads to their movements.

$$(W) = (A) \cdot (S) \cdot (A^T) \cdot (X) \tag{8a}$$

The triple matrix product $(A \cdot S \cdot A^T)$ effectively expresses the values of load to produce unit deformations so that it can be called the frame stiffness matrix, denoted by (K). It will be a square matrix of order $\bar{L} \times \bar{L}$ and is invariably well conditioned and non-singular in a first-order analysis so that its invert will be the flexability matrix (F) for the structure.

$$(W) = (K) \cdot (X) \tag{8b}$$

$$\begin{aligned} (X) &= (K^{-1}) \cdot (W) \\ &= (F) \cdot (W) \end{aligned} \tag{9}$$

Thereafter, the stress resultants may be computed from the equation,

$$(SR) = (S) \cdot (A^T) \cdot (F) \cdot (W) \tag{10}$$

and, having proceeded this far, it is worthwhile to pre-multiply (SR) by the statics matrix (A) to recompute the load vector (W).

$$\begin{aligned} (A) \cdot (SR) &= (A) \cdot (S \cdot A^T \cdot F \cdot W) \\ &= (W) \end{aligned} \tag{11}$$

This procedure, suggested by Clough[5], provides a useful check upon the build up of error during the computation and the consequent significance of the results.

At this stage, it should be pointed out that the procedure implied in Eq. (9) of first inverting the stiffness matrix (K) and then post-multiplying the result (F) by the load vector (W) will be wasteful of computer time unless as many load sets were to be considered as the degree of freedom of the frame. If only one load set is to be considered, it will be more efficient merely to solve the set of $\bar{L}$ simultaneous Eqs. (8b) and even if several sets are involved, equation solution will involve less machine time especially if a suitable solving routine is employed. The one used in the program described in this report is the system called Gauss-Jordan elimation as outlined by Salvadori[6].

2.5 Temperature Change Effects

A change of temperature will alter some deformations in all structures and in the case of redundant frames can affect the stress condition as well. Shrinkage strains and strains due to lack of fit of members in a frame will have results similar to those caused by temperature change. All can be accounted for in the deformation method of analysis by computing beforehand the effective "lack-of-fit" of each member of a structure, whether caused by temperature or shrinkage or by a genuine lack-of-fit, and listing them in a column matrix or vector which will be denoted by (x^H). These deformations will be relative within each member so there will be as many terms in (x^H) as in the relative deformation vector (x), that is, as many as there are stress resultants to be computed. The overall effect on the frame is calculable since the only change in the procedure outlined above will be to modify Eq. (5),

$$(SR) = (S) \cdot ((x) - (x^H)) \tag{12}$$

Eq. (12) represents in matrix form the physical situation that within each member the deformation is caused partly by stress and partly by initial lack-of-fit. The equations of statics and kinematics are unaffected by these considerations so that the basic frame equation to be solved, Eq.(8a), becomes,

$$(W) = (A) \cdot (S) \cdot (A^T) \cdot (X) - (A) \cdot (S) \cdot (x^H) \tag{13}$$

If the temperature terms are moved to the left hand side of Eq. (13), the effect is either to add more terms to the existing load vector or, if temperature stresses alone are of interest, the vector $(A) \cdot (S) \cdot (x^H)$

becomes the load vector. In the program developed in this report, it was decided to accommodate any number of alternative load vectors but the computed temperature load vector would be added only to the first real load vector, not to all of them. It was thought that a more useful program would result from such a technique.

2.6 Shear Deformations

Shear deformations will be taken into account in a general deformation analysis program by including appropriate terms in the elements of the member stiffness matrices shown in equations (1) or (2). It is readily shown from elementary analysis that the more correct form of Eq. (1), allowing for axial, shear and flexural deformations is,

$$\begin{bmatrix} T \\ V \\ M_{AB} \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{k_3L^3} & \frac{-6EI}{k_3L^2} \\ 0 & \frac{-6EI}{k_3L^2} & \frac{k_1}{k_3} \end{bmatrix} \begin{bmatrix} u_T \\ u_V \\ \theta_{AB} \end{bmatrix}$$

where $k_3 = 1 + \frac{12EI}{L^2\bar{A}G}$,

$$k_1 = \frac{4EI}{L} + 12(\frac{EI}{L})^2/ L\bar{A}G,$$

G = shear modulus

A = cross-sectional area, and

$\bar{A}$ = section area effective in resisting shear.

The alternative form of the member stiffness equations would appear as in Eq. (2) when allowance is made for shear deformations, with the substitutions of k_1/k_3 for the term 4EI/L and k_2/k_3 for the term 2EI/L. For this case, $k_2 = \frac{2EI}{L} - 12(\frac{EI}{L})^2/L\bar{A}G$

III THE COMPUTER PROGRAM

A principal objective in writing the program was the provision of an ability to analyze all types of plane frames without profligate use of computer storage capacity.

The limitation was accepted that members had to be prismatic but the program was devised to deal with triangulated or rigid frames. In the case of the latter, it could either take into account or ignore axial and shear strains. For any frame, the effects of temperature or shrinkage or lack-of-fit could be allowed for, if desired.

Accordingly, the program takes the form shown in the flow diagram of Fig. 3. The principal steps begin with reading of an integer which is regarded as the frame number if positive and if negative it is treated as a signal to terminate the program. Next, the degree of freedom, the count of flexural members, the count of axially strained members and the number of alternative load sets to be considered are input. The statics matrix is then input in an unabridged form and basically it will determine the form of output of deformations and stress resultants. This will be made clear in the examples given in Sect. V. Next, the data to construct the member stiffness matrix and the temperature vector are input. The first item for each member will be the flexural stiffness parameter (EI/L) followed by the shear stiffness factor ($1/L\bar{A}G$), the inverse form being chosen so that it may be set to zero if infinite shear stiffness is assumed, as is often the case in steel frames. The third term will be an angle representing the slope change along the member considered in isolation due to temperature strains if these are to be considered. Data for each flexural member is first input and is followed by data for each axially strained member. For these, there will be two items per member, the axial stiffness parameter (EA/L) and the initial oversize due to temperature or lack-of-fit. Even though the same member may be both flexurally and axially strained, the form of input set out above should be preserved. The member stiffness matrix is constructed with all flexural members delt with at first and then the axially strained members as is shown in Fig. 4. Finally, the load sets are read and these may be chosen in such a manner to utilize the program's ability to add temperature effects only to the first load set but not to the subsequent sets. Hence, if the effects of temperature or lack-of-fit alone are of interest, a null vector would be prepared for the first load set with the actual load set or sets following afterwards. If it is desired to obtain a complete flexability matrix by the inversion of the frame stiffness matrix, it is only necessary to specify at the beginning that as many load sets are to be analyzed as the degree of freedom and then to finally arrange for the input of load vectors which, if viewed side by side, would resemble a unit matrix. The form of equation solving routine used is that of Gauss-Jordan elimation with the largest pivot chosen at each stage and is the same used by C. K. Wang[4] but modified to deal with a succession of load sets.

A series of different frames can be analyzed by reading in more data beginning with the frame number as before and when this integer is negative the run will terminate.

The computer output will commence with the statics matrix (for verification) and is followed with the member stiffness matrix so that one may feel confident that it has been constructed correctly. The temperature vector is also output for the same reason. Next, the frame stiffness matrix will be printed and it should take a symmetrical form. The input load sets are printed out for verification and then a table entitled "deformation matrix for the frame" is output and it will be a flexibility matrix in the strict sense if the load sets took the form of a unit matrix as mentioned above. Otherwise as many columns will appear as the number of load sets and the elements will be the frame deformations caused by the loads in each set (with temperature deformations added to the first column if non-zero temperature terms had been previously input.)

If the frame undergoing analysis were rigid, the next table would consist of columns of member end moments, one column for each load set. The order of terms in each column will correspond to the order decided upon in the construction of the statics matrix. If axial strains were also considered, the next table will list the tension forces existing in each member. Finally, a check on computational accuracy is made by recalculating the load vectors and these are output and an inspection will provide some estimate of error build-up during the computation.

It should be noted that the type of input must necessarily correspond with the Fortran format statements in the source program.[7]

IV PROGRAM LIMITATIONS

The dimension statements at the head of the source program (Appendix A) will provide an upper limit on the size of structure that can be analyzed by the resulting object program. For any computer, it will be necessary to ensure that the total number of storage locations implied in the dimension statements is within the machine's capacity after allowing for the storage of the object program. It has been found that approximately 1900 locations are available for the arrays in the GE225 computer at Lehigh University. Within this upper limit, it is possible to vary the maximum values for degrees of freedom and number of load sets so that as many frame members as possible can be accommodated. The available store capacity C must not be exceeded by the total number of matrix elements which can be expressed,

$$C \geq (2\bar{L} + 1)\ (2NFM + NAM) + (2NFM + NAM)^2 + \bar{L}(\bar{L} + N) \tag{14}$$

where $\bar{L}$ = degree of freedom

NFM = count of flexural members

NAM = count of axially strained members

N = number of load sets

For a determinate pin-jointed frame, there will be as many members as the degree of freedom and a store capacity of approximately 1900 locations will be filled by a frame of 21 members if only one load set is applied. On the other hand, a flexural frame in which axial strains are neglected will be determinate if the degree of freedom is twice the number of members and 1900 locations would be filled by a frame of 11 members. The efficiency of the program increases with the degree of redundancy since the insertion of extra members in a determinate pin-jointed frame will not increase the degree of freedom. In a flexural frame the sum of the degrees of freedom $\bar{L}$ and of redundancy R will equal twice the count of the members so that for a given frame, the more redundant it is made through joint rigidity, the smaller will be the degree of freedom and hence the larger the frame to fill any available capacity.

$$\bar{L} + R = 2\ NFM \tag{15}$$

The ratio of L to R for multi-story rectangular frames with fixed bases is independant of the number of stories and is given by

$$\bar{L}/R = 1/3 + 2/3b \tag{16}$$

where b is the count of the bays. It can be seen that the ratio decreases from 1 for a single bay frame of any number of stories and approaches a value of 1/3 for a large number of bays. For single bay frames, 5 stories with 15 members would require a capacity of 2070 storage locations and would just exceed the capacity of the GE225 machine. On the other hand, a 7 bay, single story frame with 15 members would require 1560 locations and would be well within the available capacity. It can be seen that it is a relatively simple matter to decide whether a given frame is within the capacity of any given computer.

It has already been mentioned that a member in a flexural frame can sustain any type of lateral loading which in the analysis can be replaced by the equilibrants of the end fixing moments and shears so that load application positions do not necessarily have to be treated as joints. If this procedure is adopted, the computer will in effect produce a "complementary solution"[8] for the frame which should be added to the "particular solution" of zero deformation associated with the fixing moments and shears.

V DATA PREPARATION

The first card in the data set must contain a positive integer in Format I5 and it is regarded as the identifying number for the frame. The last card can be any negative integer in the same format and its function is to terminate the run. The second card, in Format 4I5, must list the degree of freedom, the count of the flexural members, the count of the axially strained members and the number of different load sets. The construction of the matrices shown in Fig. 4 will be governed by these four integers. The major effort in data preparation concerns the statics matrix which is next input. Several examples will be shown later of the construction of this matrix. After the statics matrix, the elements of the member stiffness matrix are input, the flexural members being first with one card per member listing in Format 3F10.5 the flexural stiffness parameter (EI/L) the shear parameter (1/LAG) and the slope change angle caused by temperature. If one or both of the latter quantities are ignored in the analysis, zero should be punched in their place. However, if there are no flexural members, only the cards containing the axial stiffness terms (EA/L) and the initial oversize due to temperature for each member are read. These cards should be punched in Format 2F10.5. Finally the load sets are input, each in Format 7F10.5, and the order of terms must correspond with that already decided upon when the statics matrix was established.

The preparation of the statics matrix is best explained by examples. Referring to Fig. 5, the pin-jointed frame numbered 1, will be studied. Such a frame clearly has only 2 degrees of freedom associated with the 2 possible displacements of the joint O. The equations of statics relate the applied loads X and Y to the stress resultants which are called T_{AO}, T_{BO}, T_{CO}, T_{DO} and T_{EO}. These will be as follows,

$$X = 1/2\ T_{AO} - 1/2\ T_{CO} - \frac{\sqrt{2}}{2} T_{DO} - T_{EO}$$

$$Y = \frac{\sqrt{3}}{2} T_{AO} + T_{BO} + \frac{\sqrt{3}}{2} T_{CO} + \frac{\sqrt{2}}{2} T_{DO}$$

Expressing these equations in matrix form,

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0.5 & 0.0 & -0.5 & -0.707 & -1.0 \\ 0.866 & 1.0 & 0.866 & 0.707 & 0.0 \end{bmatrix} \cdot \begin{bmatrix} T_{AO} \\ T_{BO} \\ T_{CO} \\ T_{DO} \\ T_{EO} \end{bmatrix}$$

or (W) = (A) · (SR)

where A is the statics matrix. The solution for this problem is shown in Appendix B.

The frame number 2 in Fig. 5 has 7 degrees of freedom as indicated and the equations of statics can be expressed in matrix form by referring to the joint force diagrams in the figure.

$$\begin{bmatrix} M_A \\ M_B \\ X_B \\ Y_B \\ M_C \\ X_C \\ Y_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 1/360 & 1/360 & -1/360 & -1/360 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/360 & 1/360 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ T_{AB} \\ T_{BC} \\ T_{BD} \\ T_{CE} \end{bmatrix}$$

Frame 2 will be studied in detail to demonstrate how flexural, shear, axial and temperature strains, as well as lack-of-fit, may be taken into account. Suppose that at construction, the temperature was 70^{o}F and the spring support CE was 1.5 in too short. At working conditions, the temperature above the beam is 120^{o}F and below the beam, 80^{o}F.

For the 18WF50 beam, EI/L = 66,716.6 kip. in. and EA/L = 1225.1 kip/in. For the springs BD, CE, EA/L is effectively 10.0 and 4.0 as shown on figure. The shear parameter ($1/L\bar{A}G$) for the beam is computed to be 0.000 000 036 kip^{-1} $in.^{-1}$ Considered in isolation, the spring BD would increase in length 0.001608 in. due to a 10°F rise in temperature. The spring CE would increase by 0.000804 in. in length, but it is initially too short by 1.5 in. so both effects are accommodated by specifying its initial oversize as -1.499196 in.

With regard to the effect of temperature on the beam, it can be assumed that a uniform temperature gradient exists between top and bottom flanges. For a mean temperature rise of 30°F, each beam would increase in length by 0.072360 in. The higher temperature at the top of the beam will produce a negative curvature in each isolated beam with a slope change angle readily computed as 0.00536 radians.

The load vector can be formed from the equilibrants of the fixing moments and shears associated with the live load intensity of 0.1 kip/ft. together with the dead load of the beam. The fixing moments and shears will be,

$$M_{FAB} = -135.0 \text{ kip in.} \qquad V_{FAB} = 2.25 \text{ kip}$$

$$M_{FBA} = 135.0 \text{ kip in.} \qquad V_{FBA} = 2.25 \text{ kip}$$

$$M_{FBC} = -45.0 \text{ kip in.} \qquad V_{FBC} = 0.75 \text{ kip}$$

$$M_{FCB} = 45.0 \text{ kip in.} \qquad V_{FCB} = 0.75 \text{ kip}$$

The equilibrating joint loads and moments are simply deduced from the above values so that the applied load vector becomes,

M_A	M_B	X_B	Y_B	M_C	X_C	Y_C
135.0	-90.0	0.0	-3.0	-45.0	0.0	-0.75

The data cards for the analysis of frame 2 would be ordered as follows:

Data			Format	Card No.
2			I5	1
7 2 4 2			4I5	2
(statics matrix)			7F10.5	3-10
66716.6	.000 000 036	-.00536	3F10.5	11
"	"	"	"	12
1225.1	.07236		2F10.5	13
"	"		"	14
10.0	.001608		"	15
4.0	-1.499196		"	16
(load vector)			7F10.5	17,18
-2			I5	19

The computer solution for this problem is given in Appendix C

It has been explained that the preparation of the statics matrix constitutes most of the preparatory work necessary in using the program. For rectangular frameworks of a rigid kind, this can be done by inspection but more work is required for frames with sloping members such as pitched-roof portals. The statics matrix for these frames has been found to be somewhat easier to prepare when axial strains are taken into account rather than when they are neglected which is the usual approach to this type of problem. A simple example is shown in Fig. 6 and the forces acting on the joints are shown in Fig. 7 for the case when axial strains are neglected. The rafter thrust (Th) is evaluated in terms of member end moments from the condition of vertical equilibrium at the joint C. There is no independent freedom for vertical movement at C since the horizontal movements at both column-rafter connections have been treated as independent degrees of freedom. It it had been desired to obtain directly the vertical movement at the apex, a degree of freedom could have considered at that position rather than at the right hand column. Alternatively, a joint with 2 degrees of freedom similar to joint B could have been considered as existing slightly to one side of the apex. This problem does not exist when axial strains are accounted for as in Fig. 6(c). For this case, the joint forces (moments are omitted for clarity) are shown in Fig. 8. The statics matrices for both cases are shown in the Appendix D.

The sign convention adopted in the examples is that in which clockwise moments are regarded as positive and the positive directions of displacements are those coinciding with the force directions on the various figures. However, it should be noted that the program itself is not dependent upon any sign convention and it will work for any consistent convention adopted in the preparation of the input data.

VI CONCLUSION

The Fortran program described in this report has been thoroughly tested and run on both the GE 225 computer at Lehigh University and the IBM 7074 machine at the Bethlehem Steel Company. The only changes necessary for the latter computer concerned the input and output statements. The word READ was replaced by READ INPUT TAPE 1, and the word PRINT was replaced by WRITE OUTPUT TAPE 2, with the rest of the program remaining identical. The appropriate call cards were required for both machines. Compile time on the GE225 was approximately 8 minutes and running time for a problem of maximum size for this machine was approximately 4 minutes. The IBM 7074 would compile and run in the one operation and the same frame would be analyzed in a total time of approximately 2 minutes.

The storage capacity available for data with the GE 225 was approximately 1900 locations but it was found that 6900 locations were available with the IBM 7074. Since the required capacity for a frame is proportional to the square of the number of members, as in Eq. (14), the IBM machine was capable of analyzing frames of approximately twice the size that could be accommodated by the GE 225. It is evident that a structural engineer will always have structures which can exceed the available capacity of a high speed store in a computer and methods have been proposed[(9)] for dealing with specialized problems.

VII ACKNOWLEDGEMENTS

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VITA

The author is on leave from the University of Sydney.

VIII NOMENCLATURE

(A)	the statics matrix
(A^T)	the transposed statics matrix
A	cross-sectional area
$\bar{A}$	section area effective in redisting shear
b	count of the bays in a rectangular frame
(C)	the kinematics matrix (= (A^T))
C	computer storage capacity available for data
E	Young's modulus
(F)	the frame flexibility matrix
f	count of the fixed bases in a frame
G	shear modulus
h	count of the hinged supports of a frame
h_i	count of the internal hinges in a rigid frame
H_N	an applied horizontal load, N^{th} in a list of loads
I	second moment of area
j	count of the joints in a frame
(K)	the frame stiffness matrix (= (F^{-1}))
k_1,k_2,k_3	parameters defined in the text
L	length of a prismatic member
$\bar{L}$	degree of freedom of a structure
M	an applied bending moment
$\bar{M}$	count of the stress resultants in a frame
M_A	an external moment applied to joint A
M_{MN}	moment applied at end M to member MN

M_N	an applied external moment, N^{th} in a list
M_{FAB}	moment to fix the end A of member AB
m	count of the members in a rigid frame
N	count of the alternative load sets
NAM	count of axially strained members of a frame
NFM	count of flexural members of a frame
R	degree of redundancy
(S)	the member stiffness matrix
(SR)	matrix vector of stress resultants
(S_{MN})	the stiffness matrix for member MN
T	an applied tension force
T_{AB}	the tension force in member AB
u_T, u_V	displacements in directions of forces T,V
V	an applied normal shear force
V_N	an applied vertical load, N^{th} in a list
V_{FAB}	shear to fix the end A of member AB
(W)	matrix vector of the applied forces, moments
X,Y	applied joint forces
(X)	matrix vector of frame displacements
(x)	matrix vector of relative member displacements
(x_{AB})	matrix vector of relative displacements for member AB
(x^H)	matrix vector of relative displacements caused by temperature or similar effects
θ_{AB}	slope at end A relative to the tangent at B
ϕ_{AB}	slope at end A relative to the line AB

Appendix A Fortran Statements

The statements which follow are in a form used as a source program for the IBM 7074 machine. The names chosen for the variables and the arrays conform in general to the terminology used in the text. There are some differences due to Fortran variable requirements and some of the dimensioned arrays are used several times in the program to conserve storage spaces.

Variable	In Text	In Program
Statics matrix	(A)	A(I,J)
Member stiffness	(S)	S(I,J)
Frame stiffness matrix	(K)	ASAT(I,J)
Temperature vector	(x^H)	FIH(I)
Degree of freedom	$\overline{L}$	L
Stiffness parameter	EI/L	EK
Shear parameter	$1/\overline{L}AG$	SK
Temperature slope change	-	TFY
Applied load vector	(W)	ASAT(I,L+1)
Deformation vector	(X)	ASAT(I,L+1)
Stress resultant vector	(SR)	S(I,J)(reused)

```
C
C          FORTRAN PROGRAM  6.  FOR  IBM 7074
C          FIRST  ORDER  ANALYSIS  OF  PLANE  STRUCTURES.
C          ACCOUNTING FOR FLEXURAL AND/OR AXIAL STRAINS AND/OR
C          TEMPERATURE OR SHRINKAGE OR LACK OF FIT OF MEMBERS.
C
C          AUTHOR, H.R.HARRISON     APRIL, 1965.
C
C
C          SPECIFICATION  OF  MATRIX  DIMENSIONS.
C
C
      DIMENSION A[27,51],S[51,51]
      DIMENSION SAT[51,27],ASAT[27,40]
      DIMENSION EIH[51]
C
C          READ  FRAME  NUMBER  AND  EXIT  IF  NEGATIVE.
C
    1 READ INPUT TAPE 1, 10, JJ
   10 FORMAT [I5]
      IF  [JJ]  11,20,20
   11 CALL EXIT
C
C
C          READ  DEGREES  OF FREEDOM,  NUMBER  OF  FLEXURAL  MEMBERS,
C          NUMBER  OF  AXIALLY  STRAINED  MEMBERS, NUMBER OF LOAD SETS.
C
   20 READ INPUT TAPE 1, 30, L, NFM, NAM, N
   30 FORMAT [4I5]
      M = 2 * NFM
      JK = M + 1
      JL = M + NAM
C
C
C          INPUT  THE  STATICS  MATRIX  A.
C
      READ INPUT TAPE 1, 40, [[A[I,J], J = 1,JL], I = 1,L]
   40 FORMAT [7F10.5]
C
C
C          INPUT  THE  MAIN  ELEMENTS,  BUILD  THE  STIFFNESS  MATRIX.
C
      IF [NFM]  50,100,50
   50 DO 60   I = 1,M
      DO 60   J = 1,M
      S[I,J] = 0.
   60 CONTINUE
      DO 90   I = 1,M
      ITEST = I/2 * 2 - I
      IF [ITEST]  70,90,90
   70 READ INPUT TAPE 1, 81, EK, SK, TFY
   80 FORMAT[2F10.5]
   81 FORMAT[3F10.5]
      R = 12.0 * EK * SK
      S[I,I] = [4.0 * EK + R * EK]/[1.0 + R]
```

Appendix A   Fortran Statements

```
      S(I+1,I+1) = S(I,I)
      S(I+1,I) = (2.0 + EK - R * EK)/(1.0 + R)
      S(I,I+1) = S(I+1,I)
      FIH(I) = TFY / 2.0
      FIH (I + 1) = -FIH(I)
   90 CONTINUE
C
      IF(NAM)  93,140,93
   93 IF (M)  94,100,94
   94 DO 95   I = 1,M
      DO 95   J = JK,JL
   95 S(I,J) = 0.
      DO 96   I = JK,JL
      DO 96   J = 1,M
   96 S(I,J) = 0.
  100 DO 130   I = JK,JL
      DO 130   J = JK,JL
      IF (I-J)  110,120,110
  110 S(I,J) = 0.
      GO TO 130
  120 READ INPUT TAPE 1, 80, S(I,J), FIH(I)
  130 CONTINUE
C
C
C           OUTPUT IDENTIFICATION   AND   THE   INPUT   MATRICES.
C
  140 WRITE OUTPUT TAPE 2, 150, JJ
  150 FORMAT(43H1 LINEAR   ELASTIC   ANALYSIS   OF   FRAME   NO.,   I3///)
      WRITE OUTPUT TAPE 2, 160
  160 FORMAT (22H0 THE   STATICS   MATRIX//)
      DO 170   I = 1,L
  170 WRITE OUTPUT TAPE 2, 180, I, (A(I,J), J = 1,JL)
  180 FORMAT(4H0ROW, I3, 1X, 7E16.7/(8X, 7E16.7))
      WRITE OUTPUT TAPE 2,   190
  190 FORMAT(32H1 THE  MEMBER   STIFFNESS   MATRIX//)
      DO 200   I = 1,JL
  200 WRITE OUTPUT TAPE 2, 180, I, (S(I,J), J = 1,JL)
      WRITE OUTPUT TAPE 2, 201
  201 FORMAT(48H0 THE TEMPERATURE, SHRINKAGE, LACK OF FIT VECTOR//)
      WRITE OUTPUT TAPE 2, 203, (FIH(I), I = 1,JL)
  203 FORMAT(14H0COLUMN VECTOR,1X,7E15.7/(15X, 7E15.7))
C
C
C           FORM   THE   MATRIX   PRODUCT   S . AT
C
      DO 220   I = 1,JL
      DO 220   J = 1,L
      SAT (I,J) = 0.
      DO 210   K = 1,JL
  210 SAT(I,J) = SAT(I,J) + S(I,K) * A(J,K)
  220 CONTINUE
C
C
C           FORM   THE   MATRIX   PRODUCT   A . SAT
C
```

```
      DO 240  I = 1,L
      DO 240  J = 1,L
      ASAT[I,J] = 0.
      DO 230  K = 1,JL
  230 ASAT[I,J] = ASAT[I,J] + A[I,K] * SAT[K,J]
  240 CONTINUE
C
C
C             OUTPUT  THE  FRAME  STIFFNESS  MATRIX
C
      WRITE OUTPUT TAPE 2, 250
  250 FORMAT[31H1 THE  FRAME  STIFFNESS  MATRIX//]
      DO 260  I = 1,L
  260 WRITE OUTPUT TAPE 2, 180, I, [ASAT[I,J], J = 1,L]
C
C
C             INPUT  THE  LOAD  SETS,  STORE  BESIDE  ASAT  MATRIX.
C
      DO 270  K = 1,N
      J = L + K
  270 READ INPUT TAPE 1, 40, [ASAT[I,J], I = 1,L]
C
C
C             ADD TEMPERATURE ETC. EFFECTS, IF ANY, TO FIRST LOAD SET.
C
      K = L + 1
      DO 275  I = 1,L
      DO 275  J = 1,JL
      ASAT[I,K] = ASAT[I,K] + SAT[J,I] * FIH[J]
  275 CONTINUE
C
C
C             OUTPUT  THE  LOAD  SETS  FOR  IDENTIFICATION.
C
      WRITE OUTPUT TAPE 2, 280
  280 FORMAT[26H1 THE  APPLIED  LOAD  SETS//]
      KK = L + 1
      KJ = L + N
      DO 290  I = 1,L
  290 WRITE OUTPUT TAPE 2, 180, I, [ASAT[I,J], J = KK,KJ]
C
C
C             SOLVE  EQUATIONS  BY  GAUSS  JORDAN  ELIMINATION.
C
      DO 410  I = 1,L
      IP1 = I + 1
      TEMP = ABSF[ASAT[I,I]]
      K = I
      DO 310  J = I,L
      IF[ABSF[ASAT[J,I]] - TEMP]  310,310,300
  300 K = J
      TEMP = ABSF[ASAT[J,I]]
  310 CONTINUE
      IF [K-I] 320,340,320
  320 DO 330  J = I,KJ
```

```
      TEMP = ASAT[I,J]
      ASAT[I,J] = ASAT[K,J]
  330 ASAT[K,J] = TEMP
  340 IF [ASAT[I,I]]  370,350,370
  350 WRITE OUTPUT TAPE 2,  360
  360 FORMAT[30H0DIVISION BY ZERO IN INVERSION]
      GO TO 600
  370 TEMP = 1./ASAT[I,I]
      DO 380  J = I,KJ
  380 ASAT[I,J] = ASAT[I,J] * TEMP
      DO 410  J = 1,L
      IF [I-J]  390,410,390
  390 TEMP = ASAT[J,I]
      DO 400  K = IP1,KJ
  400 ASAT[J,K] = ASAT[J,K] - TEMP * ASAT[I,K]
  410 CONTINUE
C
C
C           OUTPUT  THE  FRAME  DEFORMATION  MATRIX.
C
      WRITE OUTPUT TAPE 2, 420
  420 FORMAT[43H1 THE  DEFORMATION  MATRIX  FOR  THE  FRAME///]
      DO 430  I = 1,L
  430 WRITE OUTPUT TAPE 2, 180, I, [ASAT[I,J], J = KK,KJ]
C
C
C           COMPUTE S.FIH, STORE IN S AND TRANSFER TO FIH LATER.
C
      DO 436  I = 1,JL
      TEMP = 0.0
      DO 435  J = 1,JL
  435 TEMP = TEMP + S[I,J] * FIH[J]
      S[I,1] = TEMP
  436 CONTINUE
      DO 437  I = 1,JL
  437 FIH[I] = S[I,1]
C
C
C           COMPUTE  THE  STRESS  RESULTANT  MATRIX   SAT . DELTA
C
      DO 450  I = 1,JL
      DO 450  J = 1,N
      S[I,J] = 0.
      JN = L + J
      DO 440  K = 1,L
  440 S[I,J] = S[I,J] + SAT[I,K] * ASAT[K,JN]
  450 CONTINUE
C
C
C           SUBTRACT S.FIH FROM FIRST COLUMN OF STRESS RESULTANTS.
C
      DO 455  I = 1,JL
  455 S[I,1] = S[I,1] - FIH[I]
C
C
```

```
C         OUTPUT THE STRESS RESULTANT INFORMATION.
C
      IF(NAM) 490,460,490
  460 WRITE OUTPUT TAPE 2, 470
  470 FORMAT(47H1 THE   MOMENTS   AT   THE   ENDS   OF   ALL   MEMBERS///)
      DO 480  I = 1,M
  480 WRITE OUTPUT TAPE 2, 180, I, (S(I,J), J = 1,N)
      GO TO 555
C
  490 IF(NFM)  530,500,530
  500 WRITE OUTPUT TAPE 2, 510
  510 FORMAT(40H1 THE   AXIAL   TENSIONS   IN   ALL   MEMBERS///)
      DO 520  I = 1,JL
  520 WRITE OUTPUT TAPE 2, 180, I, (S(I,J), J = 1,N)
      GO TO 555
C
  530 WRITE OUTPUT TAPE 2, 470
      DO 540  I = 1,M
  540 WRITE OUTPUT TAPE 2, 180, I, (S(I,J), J = 1,N)
      WRITE OUTPUT TAPE 2, 510
      DO 550  I = JK,JJ
  550 WRITE OUTPUT TAPE 2, 180, I, (S(I,J), J = 1,N)
C
C
C         INTERNAL  CHECK  ON  COMPUTATIONAL  ACCURACY
C         RECALCULATE  THE  LOAD  SET  MATRIX.
C
  555 DO 570  I = 1,L
      DO 570  J = 1,N
      ASAT(I,J) = 0.
      DO 560  K = 1,JL
  560 ASAT(I,J) = ASAT(I,J) + A(I,K) * S(K,J)
  570 CONTINUE
      WRITE OUTPUT TAPE 2, 580
  580 FORMAT(45H1 ACCURACY  CHECK.   THE   APPLIED   LOAD   SETS//)
      DO 590  I = 1,L
  590 WRITE OUTPUT TAPE 2, 180, I, (ASAT(I,J), J = 1,N)
  600 WRITE OUTPUT TAPE 2, 610, JJ
  610 FORMAT(38H0 ANALYSIS  COMPLETED  FOR  FRAME  NO., I3)
      GO TO 1
      END
```

```
LINEAR ELASTIC ANALYSIS OF FRAME NO.1

THE STATICS MATRIX

ROW 1   0.5000000E 00   0.0000000E 00  -0.5000000E 00  -0.7071067E 00  -0.1000000E 01
ROW 2   0.8660254E 00   0.1000000E 01   0.8660254E 00   0.7071067E 00   0.0000000E 00

THE MEMBER STIFFNESS MATRIX

ROW 1   0.8660254E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00
ROW 2   0.0000000E 00   0.1000000E 01   0.0000000E 00   0.0000000E 00   0.0000000E 00
ROW 3   0.0000000E 00   0.0000000E 00   0.8660254E 00   0.0000000E 00   0.0000000E 00
ROW 4   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.7071067E 00   0.0000000E 00
ROW 5   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.0000000E 00   0.1000000E 01
THE TEMPERATURE, SHRINKAGE, LACK OF FIT VECTOR

COLUMN VECTOR  0.0000000E 00  0.0000000E 00  0.0000000E 00  0.0000000E 00  0.0000000E 00

THE FRAME STIFFNESS MATRIX

ROW 1   0.1786566E 01  -0.3535533E 00
ROW 2  -0.3535533E 00   0.2652591E 01

THE APPLIED LOAD SETS

ROW 1   0.1000000E 01   0.0000000E 00   0.3000000E 01
ROW 2   0.0000000E 00   0.1000000E 01   0.5000000E 01

THE DEFORMATION MATRIX FOR THE FRAME

ROW 1   0.5748969E 00   0.7662571E-01   0.2107819E 01
ROW 2   0.7662571E-01   0.3872030E 00   0.2165892E 01

THE AXIAL TENSIONS IN ALL MEMBERS

ROW 1   0.3064070E 00   0.3235821E 00   0.2537131E 01
ROW 2   0.7662571E-01   0.3872030E 00   0.2165892E 01
ROW 3  -0.1914684E 00   0.2572223E 00   0.7117065E 00
ROW 4  -0.2491355E 00   0.1552886E 00   0.2903640E-01
ROW 5  -0.5748969E 00  -0.7662571E-01  -0.2107819E 01

ACCURACY CHECK.  THE APPLIED LOAD SETS

ROW 1   0.1000000E 01   0.1800000E-07   0.3000000E 01
ROW 2   0.2000000E-07   0.1000000E 01   0.5000000E 01
ANALYSIS COMPLETED FOR FRAME NO.1
```

Appendix B Machine Solution to Problem 1

LINEAR ELASTIC ANALYSIS OF FRAME NO. 2

THE STATICS MATRIX

ROW 1	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 2	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 3	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	-0.1000000E 01	0.0000000E 00	0.0000000E 00
ROW 4	0.2777770E-02	0.2777770E-02	-0.2777770E-02	-0.2777770E-02	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.0000000E 00	0.0000000E 00
ROW 7	0.0000000E 00	0.0000000E 00	0.2777770E-02	0.2777770E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01

THE MEMBER STIFFNESS MATRIX

ROW 1	0.2612596E 06	0.1278263E 06	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 2	0.1278263E 06	0.2612596E 06	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 3	0.0000000E 00	0.0000000E 00	0.2612596E 06	0.1278263E 06	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 4	0.0000000E 00	0.0000000E 00	0.1278263E 06	0.2612596E 06	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1225083E 04	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1225083E 04	0.0000000E 00	0.0000000E 00
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 02	0.0000000E 00
ROW 8	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.4000000E 01

THE TEMPERATURE, SHRINKAGE, LACK OF FIT VECTOR

COLUMN VECTOR	-0.2680000E-02	0.2680000E-02	-0.2680000E-02	0.2680000E-02	0.7236000E-01	0.7236000E-01	0.1608000E-02	-0.1499196E 01

THE FRAME STIFFNESS MATRIX

ROW 1	0.2612596E 06	0.1278263E 06	0.0000000E 00	0.1080791E 04	0.0000000E 00	0.0000000E 00	0.0000000E 00
ROW 2	0.1278263E 06	0.5225192E 06	0.0000000E 00	0.0000000E 00	0.1278263E 06	0.0000000E 00	0.1080791E 04
ROW 3	0.0000000E 00	0.0000000E 00	0.2450167E 04	0.0000000E 00	0.0000000E 00	-0.1225083E 04	0.0000000E 00
ROW 4	0.1080791E 04	0.0000000E 00	0.0000000E 00	0.2200876E 02	-0.1080791E 04	0.0000000E 00	-0.6004378E 01
ROW 5	0.0000000E 00	0.1278263E 06	0.0000000E 00	-0.1080791E 04	0.2612596E 06	0.0000000E 00	0.1080791E 04
ROW 6	0.0000000E 00	0.0000000E 00	-0.1225083E 04	0.0000000E 00	0.0000000E 00	0.1225083E 04	0.0000000E 00
ROW 7	0.0000000E 00	0.1080791E 04	0.0000000E 00	-0.6004378E 01	0.1080791E 04	0.0000000E 00	0.1000438E 02

Appendix C Machine Solution to Problem 2

THE APPLIED LOAD SETS

ROW 1	-0.2226013E 03	0.1350000E 03
ROW 2	-0.9000000E 02	-0.9000000E 02
ROW 3	0.0000000E 00	0.0000000E 00
ROW 4	-0.2983920E 01	-0.3000000E 01
ROW 5	0.3126013E 03	-0.4500000E 02
ROW 6	0.8864703E 02	0.0000000E 00
ROW 7	-0.6746784E 01	-0.7500000E 00

THE DEFORMATION MATRIX FOR THE FRAME

ROW 1	-0.7794632E-03	0.1832692E-02
ROW 2	0.2068282E-02	-0.8982499E-04
ROW 3	0.7235999E-01	0.0000000E 00
ROW 4	-0.2621597E 00	-0.3074845E 00
ROW 5	0.6265017E-02	-0.6633544E-03
ROW 6	0.1447200E 00	0.0000000E 00
ROW 7	-0.1731986E 01	-0.1781444E 00

THE MOMENTS AT THE ENDS OF ALL MEMBERS

ROW 1	0.1350001E 03	0.1349999E 03
ROW 2	-0.2002184E 03	-0.1215279E 03
ROW 3	0.1102183E 03	0.3152787E 02
ROW 4	-0.4500012E 02	-0.4500006E 02

THE AXIAL TENSIONS IN ALL MEMBERS

ROW 5	-0.1500000E-04	0.0000000E 00
ROW 6	-0.9000000E-05	0.0000000E 00
ROW 7	-0.2637677E 01	-0.3074845E 01
ROW 8	-0.9311612E 00	-0.7125774E 00

ACCURACY CHECK. THE APPLIED LOAD SETS

ROW 1	0.1350001E 03	0.1349999E 03
ROW 2	-0.9000011E 02	-0.9000005E 02
ROW 3	-0.6000000E-05	0.0000000E 00
ROW 4	-0.3000000E 01	-0.3000000E 01
ROW 5	-0.4500012E 02	-0.4500006E 02
ROW 6	-0.9000000E-05	0.0000000E 00
ROW 7	-0.7500000E 00	-0.7500001E 00

ANALYSIS COMPLETED FOR FRAME NO. 2

LINEAR ELASTIC ANALYSIS OF FRAME NO. 31

THE STATICS MATRIX

ROW 1	-0.1000000E 00	-0.1000000E 00	0.0000000E 00	0.0000000E 00	0.1250000E 00	0.1250000E 00	-0.6250000E-01
	-0.6250000E-01	0.0000000E 00	0.0000000E 00				
ROW 2	0.0000000E 00	0.0000000E 00	-0.1000000E 00	-0.1000000E 00	0.1000000E 00	0.1000000E 00	-0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00				
ROW 3	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.1250000E 00	-0.1250000E 00	0.6250000E-01
	0.6250000E-01	-0.1000000E 00	-0.1000000E 00				
ROW 4	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00				
ROW 5	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00				
ROW 6	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01
	0.0000000E 00	0.0000000E 00	0.0000000E 00				
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.1000000E 01	0.1000000E 01	0.0000000E 00				

Appendix D Statics Matrices for Problem 3

Case 1 Axial Strains Neglected

LINEAR ELASTIC ANALYSIS OF FRAME NO. 32

THE STATICS MATRIX

ROW 1	-0.1000000E 00	-0.1000000E 00	0.3713900E-01	0.3713900E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.9284750E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						
ROW 2	0.0000000E 00	0.0000000E 00	-0.3713900E-01	-0.3713900E-01	0.3713900E-01	0.3713900E-01	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.9284750E 00	-0.9284750E 00	0.0000000E 00
	0.0000000E 00						
ROW 3	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.3713900E-01	-0.3713900E-01	-0.1856950E-01
	-0.1856950E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.9284750E 00	-0.9284750E 00
	0.0000000E 00						
ROW 4	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1856950E-01
	0.1856950E-01	-0.1000000E 00	-0.1000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.9284750E 00
	0.0000000E 00						
ROW 5	0.0000000E 00	0.0000000E 00	0.9284750E-01	0.9284750E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.1000000E 01	0.3713900E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						
ROW 6	0.0000000E 00	0.0000000E 00	-0.9284750E-01	-0.9284750E-01	0.9284750E-01	0.9284750E-01	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.3713900E 00	0.3713900E 00	0.0000000E 00
	0.0000000E 00						
ROW 7	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.9284750E-01	-0.9284750E-01	0.4642370E-01
	0.4642370E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.3713900E 00	-0.3713900E 00
	0.0000000E 00						
ROW 8	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.4642370E-01
	-0.4642370E-01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.3713900E 00
	-0.1000000E 01						
ROW 9	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						
ROW 10	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						
ROW 11	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.1000000E 01	0.1000000E 01
	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						
ROW 12	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.1000000E 01	0.1000000E 01	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
	0.0000000E 00						

Case 2 Axial Strains Considered

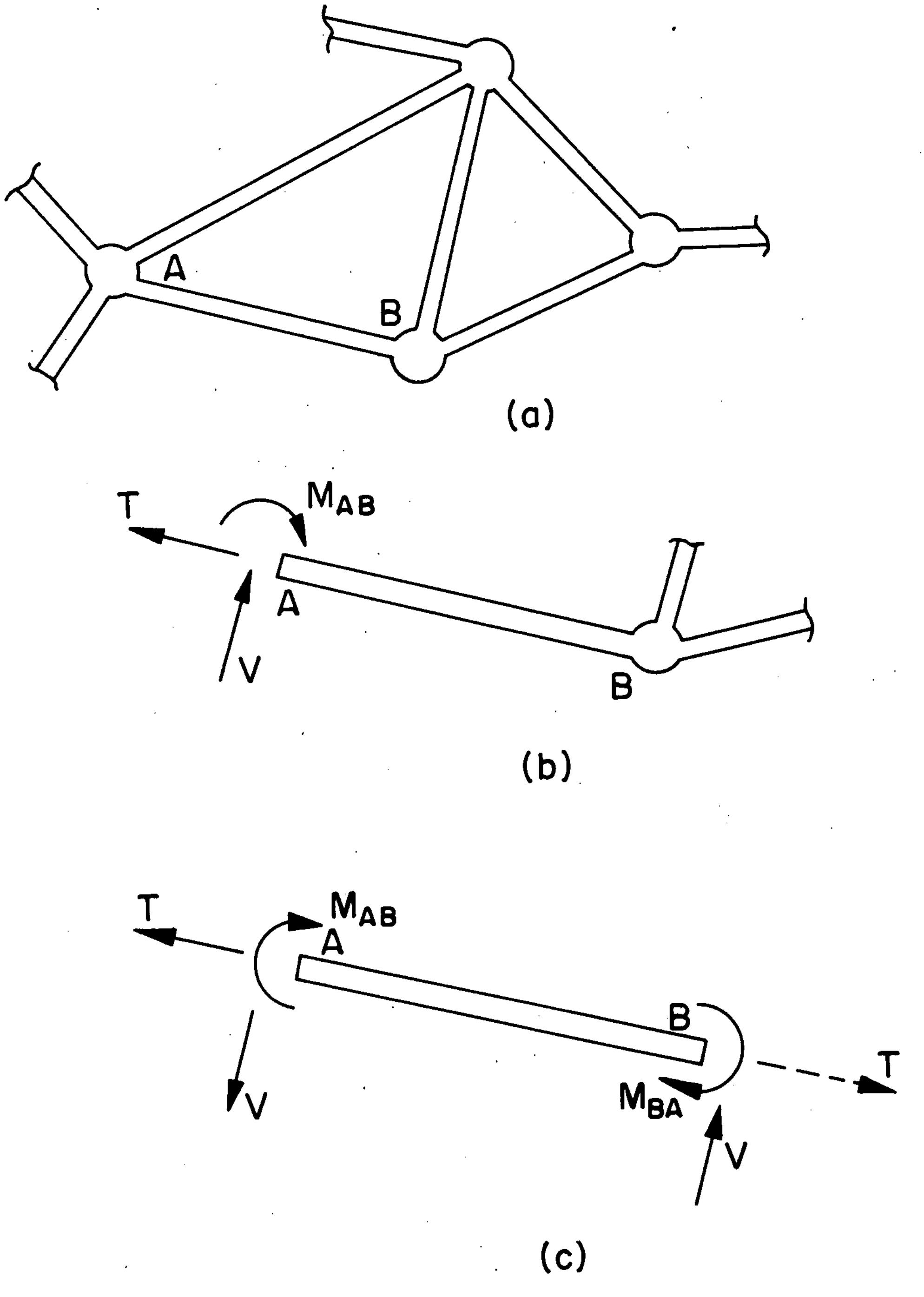


Fig. 1 Stress Resultants for a Member of a Plane Frame

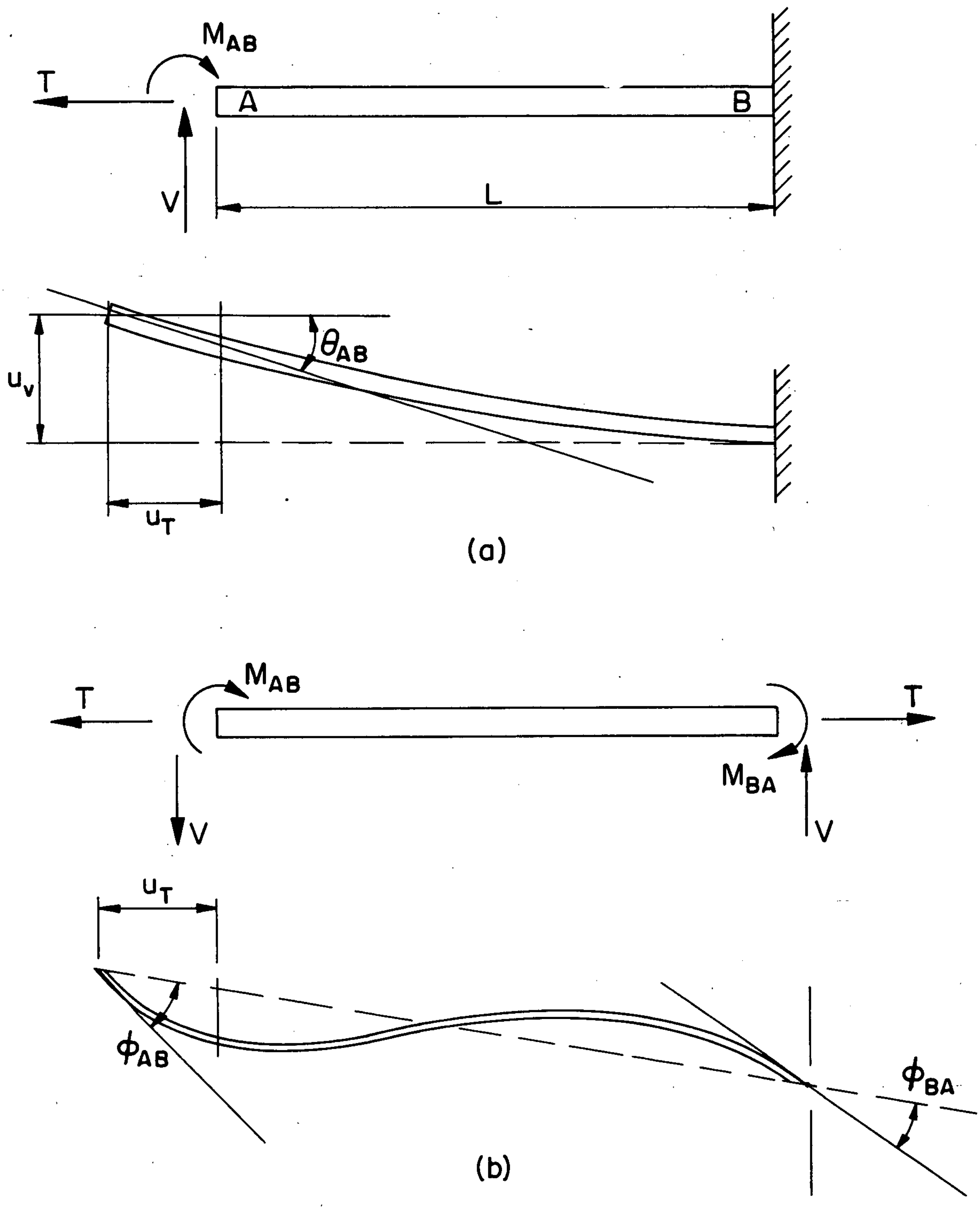


Fig. 2 Alternative Load-Deformation Systems

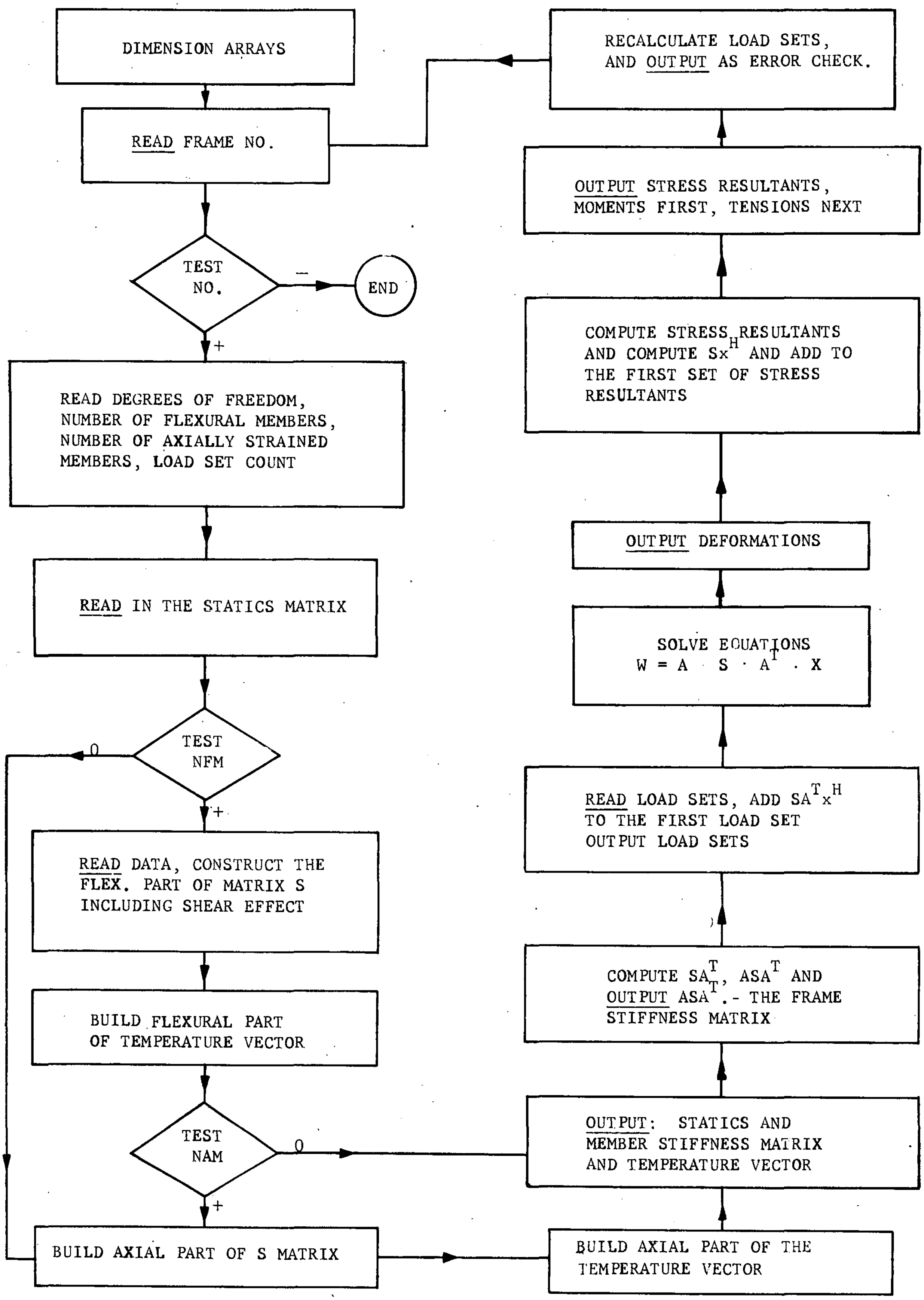


Fig. 3 Flow Diagram for the Displacement Analysis Program

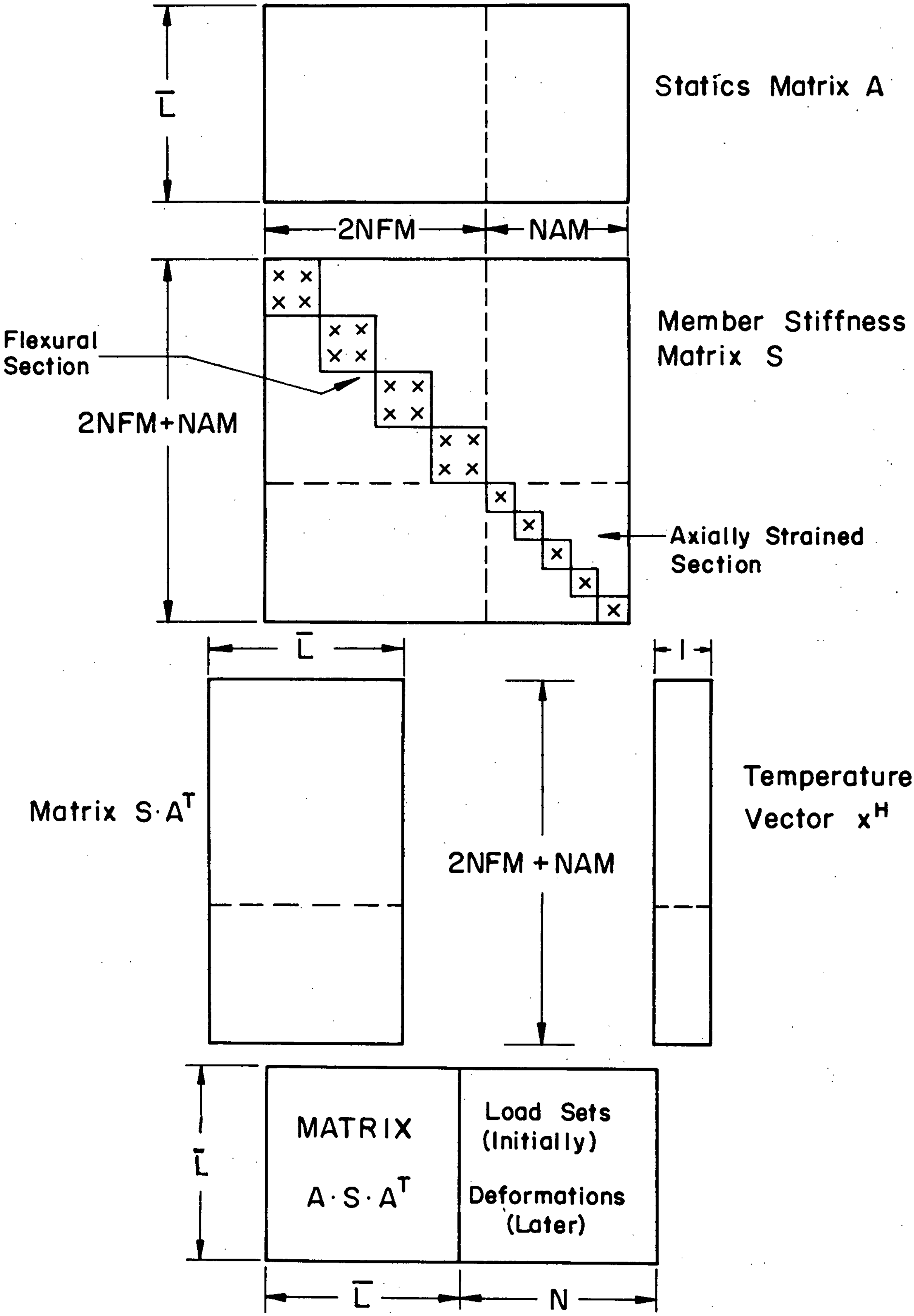
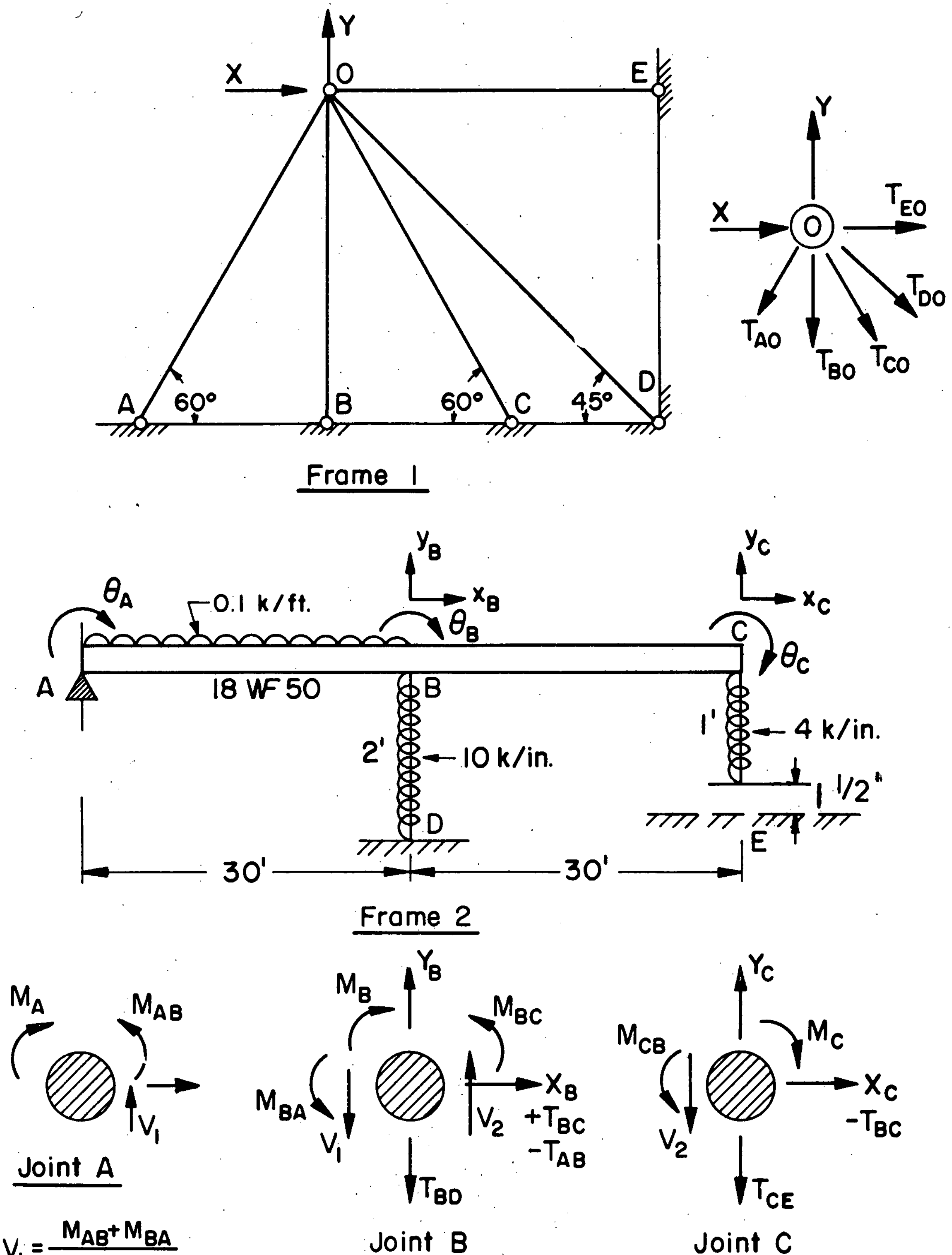


Fig. 4 Arrays Used in the Displacement Program



$$V_1 = \frac{M_{AB} + M_{BA}}{AB}$$

$$V_2 = \frac{M_{BC} + M_{CB}}{BC}$$

Fig. 5 Frame Examples 1,2

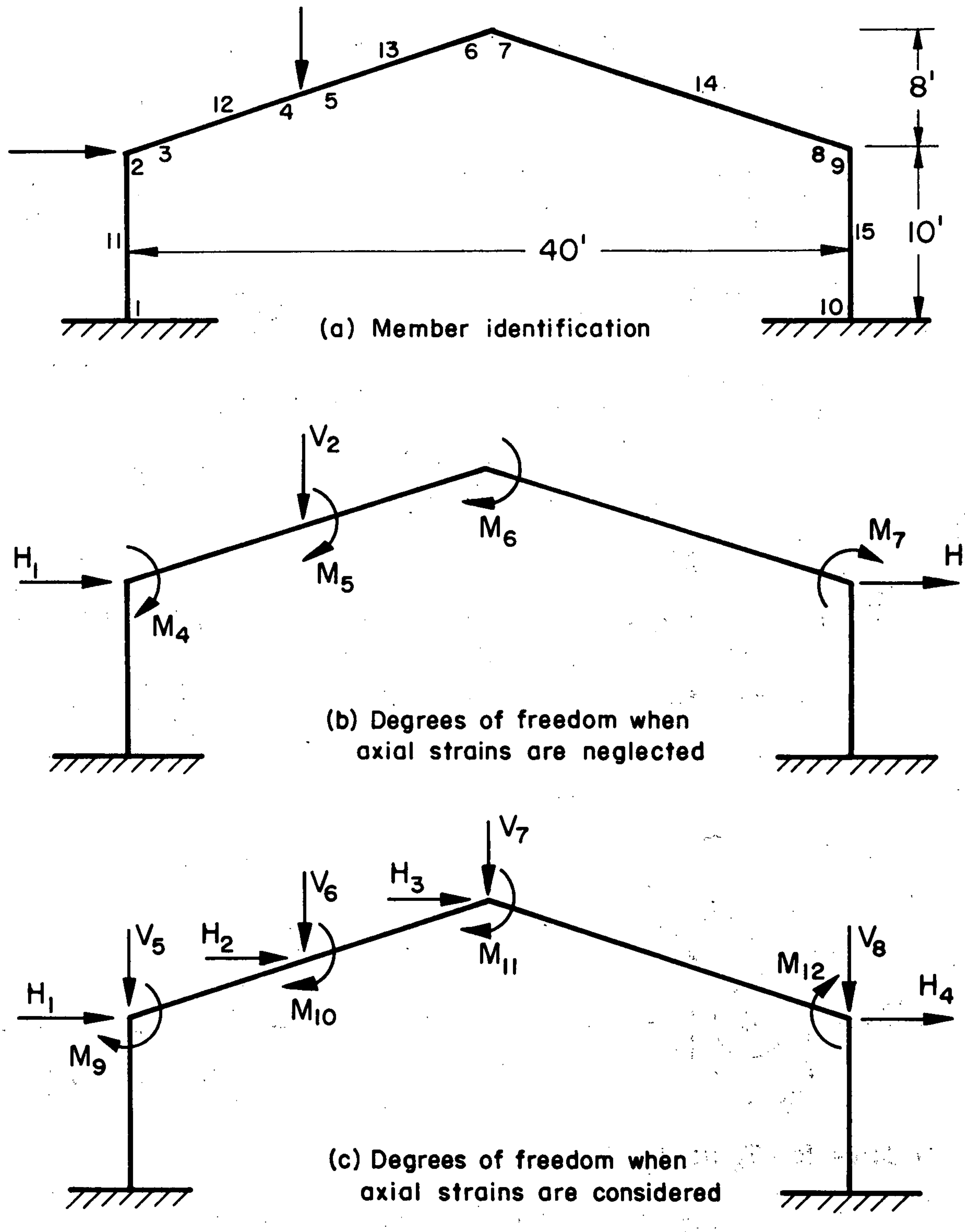
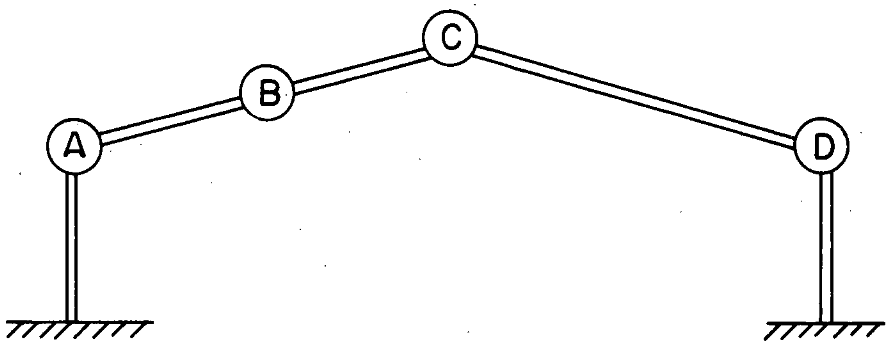


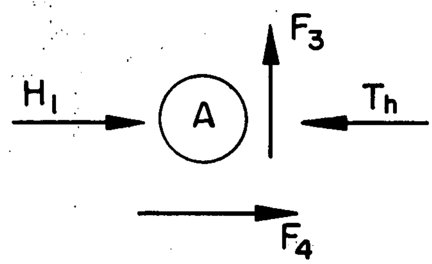
Fig. 6 Frame Example 3

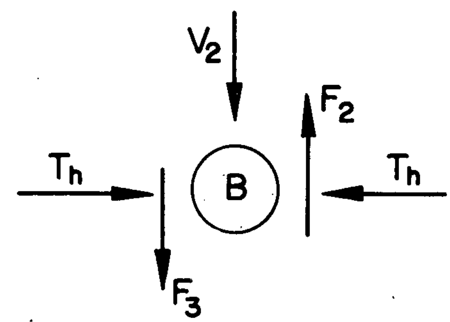


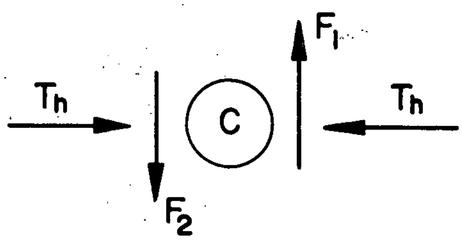
$$F_1 = \frac{M_{78} + M_{87}}{20} + \frac{2}{5} T_h \quad ; \quad F_2 = \frac{M_{56} + M_{65}}{10} - \frac{2}{5} T_h$$

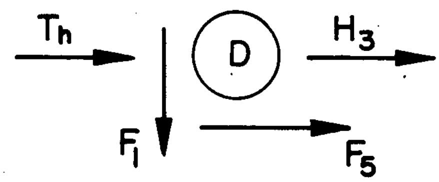
$$F_3 = \frac{M_{34} + M_{43}}{10} - \frac{4}{5} T_h \quad ; \quad F_4 = \frac{M_{12} + M_{21}}{10}$$

$$F_5 = \frac{M_{910} + M_{109}}{10}$$



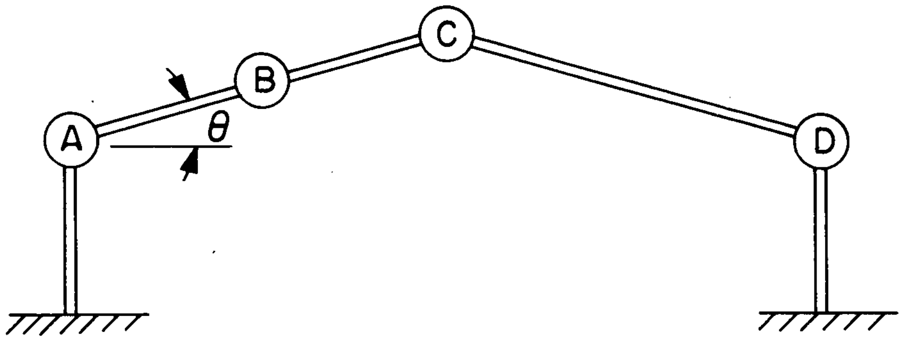






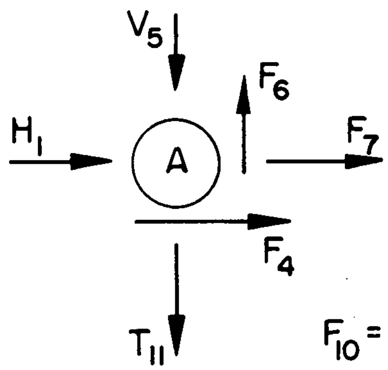
(Solve for T_h as $F_1 = F_2$)

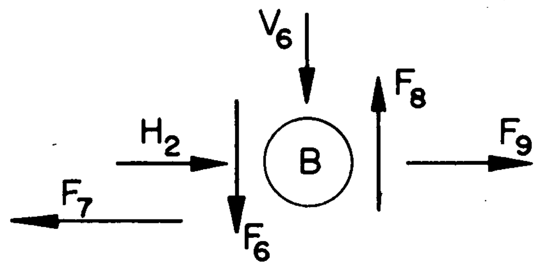
Fig. 7 Joint Forces for Frame 3 When Axial Strains are Neglected



$$F_6 = \frac{M_{34}+M_{43}}{10}\cos\theta + T_{12}\sin\theta \; ; \quad F_7 = T_{12}\cos\theta - \frac{M_{34}+M_{43}}{10}\sin\theta$$

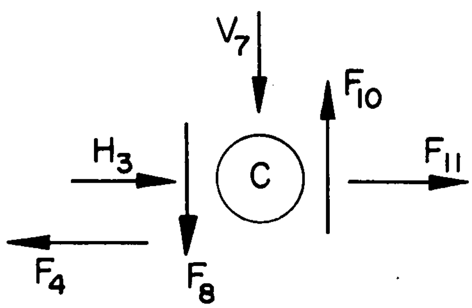
$$F_8 = \frac{M_{56}+M_{65}}{10}\cos\theta + T_{13}\sin\theta \; ; \quad F_9 = T_{13}\cos\theta - \frac{M_{56}+M_{65}}{10}\sin\theta$$





$$F_{10} = \frac{M_{78}+M_{87}}{20}\cos\theta - T_{14}\sin\theta$$

$$F_{11} = \frac{M_{78}+M_{87}}{20}\sin\theta + T_{14}\cos\theta$$



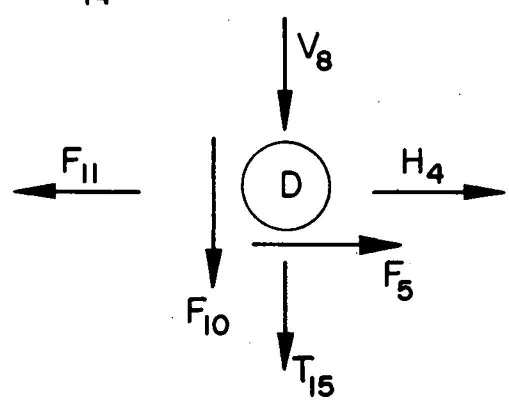


Fig. 8 Joint Forces for Frame 3 When Axial Strains are Considered

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