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DEVELOPMENT OF DESIGN INFORMATION FOR USE WITH USS "T-1" STEEL

EFFECT OF COLD-BENDING ON COLUMN STRENGTH

by

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ABSTRACT

This report is a summary of the experimental work of the second part of the research investigation on the strength of USS "T-1" Steel round columns. The purpose of this study is to find out the effect of cold-straightening on the ultimate load carrying capacity of round column members. Controlled cold-bending tests on curved round bars were first carried out as an idealized procedure simulating the actual cold-straightening operation in mill practice. Residual stresses caused by the cold-bending were then measured by both the modified boring-out method and by the beam dissection method. Stub column test results indicated the compressive property of the steels as influenced by the presence of the residual stress, and proved the effectiveness of heat treatment for stress-relieving. The results of the full scale column tests verified the theoretical analyses on the ultimate strength of round columns, which take into account both the effects of the residual stress and out-of-straightness of the members.

I. INTRODUCTION

A research project on the "Development of Design Information for Use with USS 'T-1' Steel" has been in progress at Lehigh University since 1957. The purpose of the investigation was the determination of the true ultimate load carrying capacity of round columns. The first part of the research program was concerned primarily with the investigation of the influence of residual stresses caused by the heat treatment of the material, as well as with the effect of initial deflections on the column strength in the inelastic range. An extensive study by both theoretical and experimental means has been carried out.[1,2]*

In addition to these investigations, it was considered of practical importance to determine quantitatively the effect of the cold-straightening of the column members on their ultimate strength. Rolled steel sections are usually straightened by such operations as cold-bending, gagging, rotarizing, etc. Since these operations involve plastic deformations of the material, residual stresses will be introduced into the column members due to the straightening process. For wide flange shape columns Huber[3] investigated this problem in 1956, showing that the secant formula is too safe for short columns with a large eccentricity, whereas for medium length columns the secant formula results in unconservative designs. However, the reported test data which concern themselves only with low magnitudes of cold-bending residual stresses seem not sufficient enough to draw conclusions for more general cases. The ultimate strength of columns failing in the inelastic range depends entirely on the magnitude and distribution of the cold-straightening residual stresses,

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*The numbers in parentheses refer to the list of references. (Chapter X) which in turn depend on the shape of the cross section and on the amount of initial curvature. It must be emphasized that in practical cases there might exist many possibilities of large initial curvatures (e.g. kinks, knuckles, etc.) in rolled column members.

For this reason a series of controlled tests, including coupon tests, cold-bending tests, residual stress measurements, stub column tests and full scale column tests, are required for obtaining comprehensive information on the effect of cold-straightening on "T-1" Steel round column strength. These tests were conducted on the various items of material as classified in Ref. 1. Theoretical investigations on such problems as deflection analysis of cold-bending, development of the modified boring-out method for determination of non-symmetric residual stresses, and analysis of the ultimate strength of cold-straightened columns, has been conducted and reported in Ref. 2.

In this report the experimental work on the second phase of the program will be discussed and the results obtained will be compared with the theoretical predictions.

II. TEST PROGRAM AND DESCRIPTION OF MATERIAL

II.1 Test Program

In this phase of the investigation, as noted in the introduction of this report, the following types of experiments were carried out: tension coupon tests, cold-bending tests, residual stress measurements by the modified boring-out method and by the beam-dissection method, stub column tests, and column tests. Table 1 summarizes the test specimens and their test designations. The classification of the materials used in this series of tests and their heat treatments are given below:(1)

USS "T-1" Steel

Item No. 1-A	Quenched, bent, tempered and air-cooled (as delivered).
Item No. 1-B	Quenched, bent, tempered, air-cooled and cold-straightened.
Item No. 2	Quenched, bent, tempered, air-cooled, cold-straightened and stress relieved.

The bars of Item No. 1-A had been originally subjected to hot-bending and then heat-treated by tempering and air-cooling in order to prepare the initially curved specimens which are required for the current study. Since it is the main objective of this investigation to find out the quantitative effect of cold-straightening on ultimate strength of column members, a controlled cold-straightening test on the curved specimens was performed by applying a known magnitude of bending moment. After having been straightened, each specimen was cut into two pieces: one piece was supplied for the test specimen of

Item No. 1-B; another piece was heat-treated for stress-relieving by the United States Steel Corporation, being designated as Item No. 2.

A supplementary series of tests was also carried out for AISI 1020 Carbon Steel bars. Following is the material classification, where a designation similar to USS"T-1" Steel specimens was used.

AISI 1020 Carbon Steel

Item No. 6-A	Bent and stress-relieved (as delivered).
Item No. 6-B	Bent, stress-relieved and cold-straightened.
Item No. 7	Bent, stress-relieved, cold-straightened and stress-relieved.

Figure 1 gives the location of the various specimens in the original bar lengths before and after cold-straightening or stress-relieving.

II.2 Tension Coupon Tests

Standard tension coupon tests were carried out in a 120,000 lb. screw-type universal testing machine, following the same testing procedure as the one reported in Ref. 1. In this series of investigations, however, special attention was paid to the existance of the Bauschinger Effect in cold-bent specimens which had been partly subjected to yielding in compression. For this particular purpose three coupons were prepared from three different parts of the cross section in each specimen: (a) one from the part where yielding in compression had taken place during cold-bending, (b) one from the part yielded in tension, and (c) another specimen from the center portion of the cross section where yielding had never occurred.

The mechanical properties obtained from the tension coupon test results are summarized in Table 2. Although the number of the test data is not quite sufficient enough to draw a definite conclusion, it can be seen from these results that the yield stress of cold-bent steels (Item No. 1-B and 2) is slightly higher than that of the material which is not subjected to pre-yielding (Item No. 1-A).

The Bauschinger Effect was hardly observed from the tests. This is perhaps due to the fact that yielding of the material in compression due to cold-bending was not pronounced enough (the maximum strain was about twice as large as the elastic limit strain.) to reduce the proportional limit in the subsequent tension test to a measurable extent. If, however, specimens are straightened by an application of a higher value of the cold-bending moment, then the proportional limit might be lowered considerably and thus to effect the column strength accordingly.[4]

It was also confirmed from this series of tests that the idealized elastic-fully plastic stress-strain relation is a good approximation for the stress-strain curve of USS "T-1" Steel. This was pointed out already in the previous report.[1]

III. COLD-BENDING OF BEAMS

An initially curved column member can be straightened by an application of load which causes the same amount of permanent deformations in the reverse direction as the original deflection. Since the initial deflected shape of actual column members would be of arbitrary shape, the final shape could not be perfectly straight unless multi-point loadings were applied at necessary points and in the proper directions through cold-bending. As a matter of fact, a conventional method called "gagging" is frequently used in the engineering practice to make steel members straight within some allowable tolerance for the out-of-straightness. This operation is essentially a "three-point loading", (one concentrated load between two supports) piecewise at initially curved portions of the specimen. However, it would be more convenient for the current investigation to perform a simply supported beam test with two-point loading (see sketch in Fig. 3) so that one can obtain a column specimen containing uniformly distributed residual stresses along the length of the member, thereby facilitating the column strength analysis.

A theoretical analysis of the load-deflection relationship of a beam subjected to cold-bending should be carried out prior to the test in order to be able to predict the necessary and sufficient amount of the maximum load which must be applied on the specimen. It should be pointed out that such a preparation of the load-deflection curve would be required especially when the maximum load is close to the full plastic load of the member, because an application of excessive load might produce a plastic hinge in the beam, and consequently no suitable column test specimen would be available.

Since this analysis will be dealing with the general "two-point loading system", which includes the "three-point loading system" as a special case, the results of the solution will provide a basic information on the cold-straightening of column members.

III.1 Load-Deflection Relationship and the Residual Stresses

Using the simple plastic theory applied to the problem of the flexure of beams with a circular cross section, a load-deflection analysis has been carried out for the case of a simply supported beam (length L) which is subjected to a symmetric two-point loading, each load p being at a distance ℓ from the end of the beam.(2) This analysis has shown that the deflection at the center of the beam, $u(\frac{L}{2})$, can be expressed by the following formula in terms of $\beta = \frac{M_o}{M_p}$ ($M_o = p \cdot \ell$), M_p being the full plastic moment of the beam. (The formula has been derived through the double integration procedure of beams, without taking into account the effect of thermal residual stresses.)

$$\frac{u(\frac{L}{2})}{u^*} = \frac{9\pi}{8} \frac{\gamma^2}{\beta^2(3-\gamma^2)} \left[\frac{3\pi^2}{256} + \frac{1}{2}\beta^2\left(\frac{1}{\gamma^2} - 1\right) F(\beta) + \beta \cdot F_1(\beta) - F_2(\beta) \right] \qquad \text{(1)}$$

where

$$\gamma \equiv \frac{\ell}{L/2}$$

$$u^* = \frac{M_p L^2}{24 E I}(3 - \gamma^2)$$,Elastic deflection at the center when the beam is subjected to a moment equal to M_p.

$F(\beta)$, $F_1(\beta)$ and $F_2(\beta)$ are the given functions depending only upon the ratio $\beta = \frac{M_o}{M_p}$. (See Nomenclature of this report.)

Through numerical integration procedures these functions have been computed for various values of β, and are illustrated by the curves in Fig. 2.

When the beam is completely unloaded after applying the load p, the residual deflection $\overline{u}$ will be obtained by subtracting an elastic deflection u_e (caused by load p) from the one given by Eq. 1.

$$\overline{u} = u - u_e \qquad \ldots\ldots\ldots (2)$$

A uniformly curved beam, of which the initial deformation at the center is equal to δ, can be straightened by applying a bending moment β which produces the same amount of residual deflection (given by Eq. 2) as δ. Since

$$\frac{u_e\left(\frac{L}{2}\right)}{u^*} = \frac{M_o}{M_p} = \beta \qquad \ldots\ldots\ldots (3)$$

it follows that

$$\delta = u\left(\frac{L}{2}\right) - \beta\, u^* \qquad \ldots\ldots\ldots (4)$$

Figure 3 illustrates the relationship between initial deformations δ and the required amount of the cold-bending moment $\beta = \frac{M_o}{M_p}$ for several values of parameter $\gamma = \frac{\ell}{L/2}$. It can be seen from this figure that the two-point loading system of a small value of γ (e.g. $\gamma < 0.5$) can make a curved specimen straight more efficiently by applying a lower load than the case of a concentrated loading at the center ($\gamma = 1.0$), without causing a danger of producing a plastic hinge in the beam.

Since the beam is subjected to plastic deformation by cold-bending, there will be residual stress remaining in the specimen. If pre-existing residual stresses caused by heat treatment are neglected, then the distribution of the residual stress due to the cold-straightening can be determined simply as the sum of the stress produced by the loading at the maximum moment $M_o (= p\ell)$ and the stress which corresponds to the unloading process, assuming elastic behavior at unloading. Hence, the axial residual stress σ_z (at a distance x from the bending axis) is given by the following formula:

$$\left.\begin{aligned} \frac{\sigma_z(\xi)}{\sigma_y} &= 1 + \frac{16}{3\pi}\beta\cdot\xi && \text{for} \quad -1 \leq \xi \leq -\frac{1}{F(\beta)} \\ &= -F(\beta)\cdot\xi + \frac{16}{3\pi}\beta\cdot\xi && \text{for} \quad -\frac{1}{F(\beta)} \leq \xi \leq \frac{1}{F(\beta)} \\ &= -1 + \frac{16}{3\pi}\beta\cdot\xi && \text{for} \quad \frac{1}{F(\beta)} \leq \xi \leq 1 \end{aligned}\right\} \quad (5)$$

where $\xi \equiv \frac{x}{R}$.

The maximum stress will take place at the extreme fiber of the cross section ($\xi = \pm 1$) or at $\xi = \pm \frac{1}{F(\beta)}$, depending upon the value of the applied bending moment $\frac{M_o}{M_p} = \beta$. Figure 4 shows the relationship between the maximum residual stress due to cold-straightening and the applied bending moment β. By using this result together with the δ vs. β curves in Fig. 3, one can directly estimate the maximum probable value of residual stress in a column specimen straightened by a cold-bending operation with the given information being the magnitude of its initial out-of-straightness.

III.2 Cold-Bending Tests and Their Results

Cold-bending tests with two-point loading were conducted in a 300,000 lb. hydraulic type testing machine on specimens of Items No. 1-A and 6-A (See Art. 2.1).

The specimens were stress-relieved USS "T-1" Steel and AISI 1020 carbon steel round beams of 2-3/4 inch diameter, with approximately five inches of maximum initial deflection within a fifteen foot length. The shape of the initial deflection was that of single curvature in one plane without significant twisting in other planes.

The general set-up of the specimen in the testing machine is illustrated by Fig. 5. Since it was anticipated that, according to the preliminary analysis made in the foregoing article, extremely large deflections (approximately two feet at the center) and slopes would occur at the maximum load, particular attention had to be paid to the arrangement of the end supports and loading fixtures. (See Photograph 1) The end supports should be able to rotate freely around an axis of bending (simply supported condition) and at the same time move lengthwise smoothly in order to prevent the development of axial thrust. These conditions were realized by using a pair of sliding support blocks as shown in Fig. 6. When a load was applied, the loading blocks, containing two rollers in each set (See Fig. 6), were able to follow the lengthwise movements of the specimen, so that an unfavorable effect of extra frictional forces which would introduce unknown bending moments into the beam, could be eliminated. To prevent the loading blocks from slipping down along the specimen, a pair of steel bands were attached on the specimen by tack welding. Two lateral supports were provided on one base beam so as to prevent the lateral movement of the test specimen which was allowed to deflect only in a vertical plane.

The actual distance ℓ between the loading point and the support was measured at every step of load reading, and the corresponding bending moment $M_o = p.\ell$ in the middle part of the beam was recorded. This value was checked by strain readings according to the ordinary beam theory in the elastic range by means of SR-4 gages affixed on both top and bottom surfaces at two locations in the middle of the specimen.

The deflection of the beam was measured at five points along the member by an $\frac{1}{100}$ inch scale. (See Fig. 5)

Typical results of a cold-bending test are shown in Fig. 7, indicating the relationship between deflection $u(\frac{L}{2})$ at the center of the beam against the bending moment M_o as compared with a theoretical curve for the case of $\gamma = 0.2$ obtained by the analysis in Art. III.1. It should be pointed out that the beams must be straightened within an acceptable tolerance of straightness for column test specimens and with a known bending moment M_o, so that one can directly estimate the locked-in residual stresses. For this reason it was assured that as the beams were straightened, satisfactory correlation with the theoretical predictions existed. Figure 8 shows the deflected shape of the beam at several loads.

After cold-straightening (See Photograph 2) the beams were cut into proper lengths for each test specimen of the two groups; Items

No. 1-B and No. 2 for "T-1" Steel specimens, and Items No. 6-B and No. 7 for the carbon steel specimens. The test specimens of Items No. 2 and 7 were stress-relieved by heat treatment. (See Art. II.1)

IV. MEASUREMENTS OF RESIDUAL STRESSES CAUSED BY COLD-BENDING

IV.1 Modified Boring-Out Method

a) Introduction

When steel column members are subjected to heat treatment and then straightened by cold-bending operations, the final magnitude and distribution of the residual stresses locked in the specimen will be so complicated that an analytical solution taking into account the pre-existing thermal residual stresses becomes too involved for numerical or graphical methods.[3] Since the residual stresses caused only by cold-bending has an anti-symmetric distribution pattern with respect to the bending axis, as shown in Eq. 5, the ordinary "boring-out method" which is useful for measurements of polar-symmetric residual stresses in circular cylinders is not applicable.

A method called the "modified boring-out method" was suggested by Lambert[5] to measure residual stresses of an arbitrary distribution pattern in a solid cylinder by separating them into a polar-symmetric and an anti-symmetric part. The principle of this method is well described in Ref. 5, and the following formula to determine the anti-symmetric part of the residual stresses from measured strains, $\bar{\varepsilon}_z$ (anti-symmetric part of the strain readings in the axial direction), has been developed in Ref. 2.

$$\sigma_z(\bar{\xi}_i) = \frac{\frac{\pi}{8}\cdot E \cdot S(\bar{\rho}_i)\cdot \frac{N}{i} - \sum_{j=1}^{i-1} \alpha_{ij}\cdot\sqrt{\frac{i-j+\frac{1}{2}}{N}}\cdot \sigma_z(\bar{\xi}_j)}{\alpha_{ii}\sqrt{\frac{1}{2N}}} \qquad (i = 1, 2, 3, \ldots\ldots N)$$

where $\bar{\xi}_i \equiv \left(\frac{x_i}{R}\right)^2 = \frac{i-\frac{1}{2}}{N}$ (x_i : coordinate - see Fig. 11.a)

$$\left.\begin{array}{ll} \bar{\rho}_i \equiv \left(\frac{r_i}{R}\right)^2 = \frac{i}{N} \quad , & (r_i : \text{the drilling radius}) \\ \\ S(\bar{\rho}_i) = (1-\bar{\rho}_i^{\,2}) \cdot \bar{\varepsilon}_z(\bar{\rho}_i) \quad , & \text{Strain function.} \\ \\ \alpha_{ij} = \text{MacLaurin's coefficient given in Appendix C of} & \\ \quad\quad \text{Ref. 2.} & \end{array}\right\} \quad \ldots\ldots\ldots (6)$$

While the test procedure of this method is essentially the same as that of the ordinary boring-out method discussed in the previous report (see Ref. 1, Sect. 3.1, a) it should be pointed out that the above formula has been derived for a case where the specimen is bored out only from the inside up to a possible maximum diameter for the drilling. As was emphasized in Ref. 1, the procedure of using two successive steps, boring-out from the inside and turning-down from the outside (which was called the "combined method") would be the most suitable way of determining the polar-symmetric residual stresses in the entire range of the cross section. However, when the anti-symmetric residual stresses are computed by the strain function $S(\bar{\rho}_i)$ according to Eq. 6 it can be seen directly from the definition of the strain function that the effect of a possible error in the measurement of strain $\bar{\varepsilon}_z(\bar{\rho}_i)$ due to drillings at the vicinity of strain gages would be small when the drilling radius becomes close to the radius of the specimen, i.e. $\bar{\rho}_i \rightarrow 1$.

b) Test Procedure

The 2-3/4 inch diameter solid cylindrical specimens were cut from both the cold-straightened bars and from the stress-relieved bars into 8 inch lengths. Four pairs of AX-5 type SR-4 strain gages were applied in intervals of 90° around the outside surface at the mid-length of the specimen. Neoprene brushing compound protected the strain gages from damage due to the large amount of oil coolant used. For the purpose of checking whether the gages were properly installed on the specimen or not, compression tests were performed on each specimen by applying low loads (the average stress not exceeding 1/8 of yield stress of the material) in a testing machine; a satisfactory linearity between stress and strain was thus assured.

At first, the test specimen was drilled out by a boring machine from 1/2 inches to 2-1/4 inches in diameter using 1/8 inch increments, each corresponding strain change being recorded after the temperature of the specimen became stabilized. Then the test specimen was replaced into a lathe where it was again bored-out for strain measurements up to approximately 2-11/16 inches in diameter by using a boring bar supported at both ends. This boring bar was able to drill the specimen by 1/64 inch thick layers without causing any significant eccentricity in boring. Photograph 3 shows the general view of the test set-up in the lathe and a typical specimen tested by this procedure.

c) Test Results

The change of strain in both axial and tangential direction which was released by the successive boring-out sequences was measured at the four prescribed positions of the outside surface of the specimen, and the readings were recorded as ε_z^i and ε_θ^i (i = 1, 2, 3 and 4), respectively. Generally speaking, these strains can always be split into two sets, a polar-symmetric set (ε_z^o, ε_θ^o) and an anti-symmetric set ($\overline{\varepsilon}_z$, $\overline{\varepsilon}_\theta$) in the following way:

$$\varepsilon_z^i = \varepsilon_z^o + \overline{\varepsilon}_z^i, \quad \varepsilon_\theta^i = \varepsilon_\theta^o + \overline{\varepsilon}_\theta^i$$

The polar-symmetric set of strains, which were computed as the average value of four readings such that $\varepsilon_z^o = \frac{1}{4} \sum_{i}^{i\sim4} \varepsilon_z^i$ and $\varepsilon_\theta^o = \frac{1}{4} \sum_{i}^{i\sim4} \varepsilon_\theta^i$, were converted into a polar-symmetric part of residual stresses by the ordinary Sach's formula given by Eq. 3.1 of Ref. 1. In Fig. 9.a and b the measured polar-symmetric strains (T. Nos. 13-13-1, 13-13-2 and 15-13 of Item No. I-B) are plotted against the boring diameter. Using these experimental data, a typical example of the corresponding residual stress distribution was obtained as indicated by three lines in Fig. 10, where each line represents axial, tangential and radial components of the residual stresses in a cold-straightened test specimen (T. No. 15-13). In the same figure are also shown the residual stress distributions in test specimens, T. No. 13-3 and T. No. 13-4 of Item No. 1-A for which the measurements were performed before the specimens were subjected to the cold-straightening operation. It can be seen from a comparison between these results that the cold-straightening introduces little perturbation on the distribution of the polar-symmetric residual stresses caused mainly by heat treatment of the material.

If tangential stress due to cold-bending is negligible, as assumed in the ordinary beam theory, then the anti-symmetric residual stress remaining in the cold-straightened specimen can be considered as uniaxial (in the axial direction only) and will be obtained from Eq. 6 by taking $\bar{\varepsilon}_z$ such that

$$\bar{\varepsilon}_z = \frac{1}{2} (\bar{\varepsilon}_z^{\,3} - \bar{\varepsilon}_z^{\,1}),$$

where $\bar{\varepsilon}_z^{\,3}$ and $\bar{\varepsilon}_z^{\,1}$ are the anti-symmetric strains in the plane of bending. (See Fig. 11a)

In Fig. 11.b are shown experimental data of the anti-symmetric strains $\bar{\varepsilon}_z$ measured from the same test specimens for which the polar-symmetric strain readings have been demonstrated in Fig. 9. A typical example of the strain function $S(\bar{\rho}_i)$ as plotted against drilling radius $\bar{\rho}_i$ is illustrated in Fig. 11.c. Furthermore the final result of the anti-symmetric residual stress distribution in a cold-straightened round specimen is shown in Fig. 11.d. It is compared with theoretical results which have been obtained from Eq. 5 and which are given by a curve in the same figure. Good correlation can be seen indicating that this modified boring-out procedure is a satisfactory method for measurement of residual stress of arbitrary distribution in solid cylinders.

IV.2 Beam Dissection Method

The modified boring-out method described in the foregoing article, which takes into account the triaxiality of the thermal residual stresses in a round specimen, has been found to be quite general, complete and useful for measuring residual stresses of any type of distribution pattern. This procedure, however, involves a considerable amount of work for the completion of the whole process of drilling. Therefore it seems worthwhile to develop an approximate but simpler method to furnish enough information for the column strength problem. If the influence of both tangential and radial components of residual stresses are negligible, then the "beam dissection method" would be the most suitable way of finding the magnitude and the distribution of the axial residual stresses.

The principle and the procedure of the beam dissection method are essentially the same as for the case of flat plates or wide flange shapes containing uniaxial residual stress, except for a slight modification in the process of dissectioning. The method consists of two steps: (1) cutting out a flat piece of plate from the middle part of the round bar in the principal plane of cold-bending, (2) slicing this plate into narrow strips. Figure 12 shows a schematic sketch of the test specimen taken from a cold-straightened bar, indicating the whole process of this measurement. From the 8 inch long, 2-3/4 inch diameter round bar specimen, a 1/2 inch thick plate was sawed and machined flat; 5 inch gage marks were punched on both sides of the plate. This plate was sliced into 1/4 inch wide strips by a metal sawing machine. The length within the gage marks was measured before and after the sectioning by a modified Whittemore strain gage (made in Fritz Laboratory) which contains a $\frac{1}{10,000}$ inch dial gage. (See Photograph 4)

Since the resultant of the internal bending moment due to the originally locked-in residual stresses within a material from which the flat plate is to be cut out is not necessarily equal to zero, it should be anticipated that a change in the length of the plate will take place after the primary cut. This change of length, measured at gage points

A-A' and B-B' (shown in Fig. 12), must be taken into account as a correction factor for the primary cut applied to the readings corresponding to the second sectioning in order to obtain an actual distribution of the original residual stresses.

The test results of the residual stress distribution in cold-straightened specimens (T.Nos. 12-14, 13-14 and 16-14) as measured by the beam dissection method are shown in Fig. 13 where a theoretical curve calculated by Eq. 5 is also illustrated for purposes of comparison. It may be concluded from the satisfactory correlation between the theoretical prediction, the tests results, and the results obtained by the modified boring-out method as demonstrated in the previous article, that the beam dissection method can be used for the measurement of the residual stresses in cold-straightened round bars if thermal residual stresses due to heat treatments are comparatively low.

V. COLUMN STRENGTH

V.1 Introduction

A theoretical analysis of the behavior of high strength steel round columns containing anti-symmetric residual stress caused by cold-straightening has been presented in Art. V.3 of Ref. 2, and curves for obtaining their ultimate strength has been prepared for the case of $\beta = 0.883$. (See Fig. 14) Also, it has been pointed out that a perfectly straight member of such columns subjected to a monotonically increasing axial thrust starts to deflect laterally as soon as the load reaches a certain value at which first yield takes place in some part of the cross section. With a further application of an infinitesimal incremental load internal stress distribution corresponding to the additional loading will not be symmetric any more due to the effect of the localized yield. Therefore such a column member of "undeflected shape" is not in general in a balanced condition between internal and external moments, and the "bent configuration" is the only possible state which satisfies the equilibrium condition. In other words, there exists no bifurcation point on a load-deflection curve and thus the behavior of column members containing anti-symmetric residual stresses cannot be considered as a buckling phenomenon. Consequently the "tangent modulus formula" for the prediction of the critical load of cold-straightened columns is not applicable.

As has been shown in Ref. 2, the load carrying capacity of cold-straightened round columns should be determined only by finding their true ultimate strength which depends primarily upon both the magnitude of residual stress caused by the cold-bending and the amount of unavoidable out-of-straightness of the member.

In order to verify the analytical results of the ultimate strength solution, experimental investigations on 2-3/4 inch diameter round specimens of both "T-1" Steel and Carbon Steel, including stub column tests and full scale column tests, were carried out. The stub column tests were included in the test program for the following purposes: (1) to visualize the overall effect of the cold-straightening residual stress on compressive members by comparing the test results with the theoretical prediction on average stress-strain relationship for short columns, and (2) to check the effectiveness of stress-relieving heat treatment for the materials of Item Nos. 2 and 7.

V.2 Stub Column Tests

Eighteen stub column tests (on ten "T-1" Steel and eight Carbon Steel specimens) were conducted in an 800,000 lb. screw type machine by following the same test procedure as the one reported in Ref. 1. In addition to the measurement of the shortening of the specimen over a five inch gage length, the lateral deflections of the stub columns containing anti-symmetric residual stress are also measured by dial gages. (Photograph 5) The lateral deflection at the mid-length of the stub column was obtained after eliminating an effect of movement of the specimen (considered as rigid body displacement) due to the horizontal movement of cross head of the machine when a load increment is applied on the specimen. It was checked from the measurements for specimens of Item No. 1-B that the maximum lateral deflection was in all cases less than 0.01 inch.

Figure 15 shows stub column test results (T.No. 16-15) as compared with a theoretical curve for the average stress-strain relationship of an extremely short column which was subjected to a cold bending moment of $\frac{M_\alpha}{M_p} = \beta = 0.883$.

To illustrate both the influence of the residual stress caused by cold-straightening and the effectiveness of stress-relieving on compression members, comparisons are made in terms of tangent modulus E_t versus average-stress relationship for three different kinds of materials, that is non-cold-straightened, cold-straightened and stress-relieved steels. (See Fig. 16) It can be seen that, for both "T-1" Steel and Carbon Steel specimens, the heat treatment of stress-relieving was remarkably effective for restoring the tangent modulus level as high as that of the original material prior to the cold-straightening.

Although for columns containing anti-symmetric residual stress the tangent modulus itself can not be utilized directly as a criterion of column strength,* it is true that the larger value of tangent modulus assures the stronger resistance against bending of the column resistance against bending of the column member and thus its higher ultimate strength.

V.3 Column Tests

Full scale column tests with concentric loading were carried out in the 800,000 lb. testing machine under flat-ended condition. The

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*For the case of round columns containing polar-symmetric residual stresses, the buckling load has been given in terms of the tangent modulus, E_t. (See Ref. 1) testing procedure was the same as that described in the previous report.[1]

Seven 2-3/4 inch diameter column test specimens were prepared from the cold-straightened and from the stress-relieved bars. Particular attention was paid to the selection of test specimens from the straightest portions of the cold-straightened bars (Nos. 1-A and 6-A). Before testing, initial out-of-straightness of the specimens was measured by $\frac{1}{1000}$ inch dial gages at every six inch interval along the member.

After the bars were cold-straightened, a reference line was marked on each specimen, identifying a principal plane of the bending in which the maximum values of residual stresses (both in tension and in compression) were locked-in. In general cases of cold-straightened column members the principal plane of the initial deflection remained after cold-straightening would not necessarily coincide with that of the cold-bending. However, the ultimate strength of such column members can be regarded as intermediate between two limiting values for the following extreme cases which have been solved analytically in Ref. 2:

(a) the convex side of the member in the initial deflection contains maximum compressive residual stress,

(b) the convex side of the member contains maximum tensile residual stress.

Since it was an essential objective of these experiments to check the analytical solutions directly with the test results, the column test specimens were selected from cold-straightened bars for which the two principal planes (i.e. of the initial deflection and of the cold-bending) are as close as possible to each other in order to assimilate the conditions used in the analysis. For those specimens tested in this series the angle between the two principal planes was limited to less than about twenty degrees.

From strain gage readings taken at seven locations along the column member, principal curvatures were measured in the same way as has been described in Ref. 1, and a typical example of the curvature variation is shown in Fig. 17. It can be seen also from these results that the inflection points of the deflected column shape remained at an almost constant location as the load increased. The distance between the two inflection points determined the effective column length, kL. Their values are listed in Table 3.

The following assumption has been made in the theoretical treatment concerning the relationship between deflection d and curvature ϕ at mid-height of the columns.[2]

$$d = \left(\frac{kL}{\pi} \right)^2 \phi$$

Test data of column deflections against curvatures are plotted in Fig. 18, being compared with the straight lines corresponding to the formula. It can be seen from their correlation that this assumption is satisfactory until the failure load reaches.

Figure 19 shows typical examples of load-deflection curves which have been prepared by a numerical computation for cold-straightened column members of $\frac{M_o}{M_p} = \beta = 0.883$ and of generalized slenderness ratio of $\eta = 0.664$. (See Ref. 2) In the same figure are also plotted column test results of T. No. 16-16 for a comparison with the theoretical curve for $\frac{d_o}{R} = 0.036$ which corresponds to the value of the initial deflection of the specimen.

Table 3 summarizes the column test results as compared with the theoretical predictions of their ultimate strength. It can be noted that their correlation is fairly good for both cold-straightened and stress-relieved materials.

VI. SUMMARY OF RESULTS

The following is a summary of the experimental results presented in this report:

(1) From the standard tension coupon test results it was observed that the yield stress of cold-bent "T-1" Steels (Item No. 1-B and 2) was several percent higher than that of the non-cold worked material (Item No. 1-A). (Table 2) However, it may be still premature to draw a conclusive statement on this characteristic of steels because of the limited number of the test results.

(2) Controlled cold-bending tests (two-point loading) were performed on curved round bars in order to prepare acceptably straight specimens for subsequent column tests. The bars were cold-straightened by uniform bending moment of about 85~88 percent of the full plastic moment. (See Fig. 7)

(3) Non-symmetric residual stress in round column specimens which is caused by both heat-treatments and cold-straightening, was measured by the so-called "modified boring-out method", separating the residual stress into a polar-symmetric set and an anti-symmetric part. It was found that the magnitude and distribution of residual stress in the polar-symmetric set were very similar to that of thermal residual stresses measured from the non-cold worked material. The anti-symmetric part was almost equal to the theoretical values computed by Eq. 5. (See Fig. 11.d) These stresses have been derived for an estimation of cold-bending residual stress.

(4) An approximate but simpler method called "beam dissection method" also was used for the residual stress measurement to compare the test results with those obtained by the modified boring-out method. Their close correlation suggests the appropriateness of such a simple sectioning method for the determination of the residual stress in solid cylinders. (See Fig. 13)

(5) In order to confirm the analytical work on the compressive properties of round columns as influenced by the presence of residual stress due to cold-bending, stub column tests were carried out on those materials, i.e. non-cold-bent steel, cold-bent steel and stress-relieved steel. Test results showed a satisfactory correlation with the theoretical prediction in the average stress-strain relationship of the short columns. (Fig. 15) Furthermore, they indicated that heat-treatment for stress-relieving was quite effective to improve the compressive property. (Fig. 16)

(6) There exists no buckling phenomenon in the behavior of cold-straightened columns which contain anti-symmetric residual stress; therefore the tangent modulus formula can not be ulitized for a prediction of the column strength. The load carrying capacity of such column members can be determined by their true ultimate load which depends upon both the magnitude of the cold-straightening residual stress and the out-of-straightness remaining after the cold-bending operation.

The column test results showed fairly good agreement with the results of theoretical analysis which have been reported in Ref. 2. (Table 3)

VII. ACKNOWLEDGEMENTS

This report presents the results of the second part of the research investigation on the strength of circular columns of USS "T-1" Steel, currently being sponsored by the United States Steel Corporation. The work is being carried out under the direction of Dr. Theodore V. Galambos at the Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, of which Professor William J. Eney is Director.

The authors wish to express their sincere appreciation to Dr. Robert L. Ketter who directed the earlier phase of this research program.

All tests were performed at Fritz Engineering Laboratory. Acknowledgement is also due to Mr. Kenneth R. Harpel, Foreman of the Laboratory and to the Laboratory staff for their whole-hearted cooperation.

Mr. Dian P. Jen assisted in the testing. His assistance is gratefully acknowledged.

VIII. NOMENCLATURE

Notations:

E	Young's modulus
E_t	Tangent modulus
I	Moment of inertia
L	Length of beam or column
M_o	Uniform bending moment ($= p \cdot \ell$)
M_p	Full plastic moment
N	Number of division , See Eq. 6
P	Axial thrust
P_{max}	Maximum load
P_y	Full plastic load
R	Radius of cross section
d	Deflection of column at mid-length
d_o	Initial deflection of column for effective length kL at mid-height, (after cold-straightening)
k	Effective column length factor
ℓ	Distance between loading point and end of beam
p	Lateral load for cold-bending
r_i	Drilling radius, See Eq. 6
r_o	Radius of gyration ($= R/2$)
u	Lateral deflection of beam
u^*	Elastic deflection at center when beam is subjected to a moment equal to M_p
$\bar{u}$	Residual deflection
u_e	Elastic deflection when beam is subjected to M_o

x Coordinate in the direction of lateral load p

α_{ij} MacLaurin's coefficients

β Parameter for cold-straightening, $= M_o/M_p$

γ Parameter for loading point $= \dfrac{\ell}{L/2}$

δ Initial deflection of beam at center (before cold-straightening)

$\varepsilon_z{}^i$, $\varepsilon_\theta{}^i$ Measured strains, axial and tangential, respectively (i = 1,2,3 and 4)

$\varepsilon_z{}^o$, $\varepsilon_\theta{}^o$ Symmetric strains, axial and tangential, respectively

$\bar{\varepsilon}_z{}^i$, $\bar{\varepsilon}_\theta{}^i$ Anti-symmetric strains, axial and tangential, respectively

ξ Non-dimensionalized coordinate $= \dfrac{X}{R}$

$\bar{\xi}_i = \left(\dfrac{X_i}{R} \right)^2$ (i = 1, 2,.... N)

η Generalized slenderness ratio of columns

$$= \frac{\frac{kL}{r_o}}{\pi \sqrt{\frac{E}{\sigma_y}}}$$

$\bar{\rho}_i = \left(\dfrac{r_i}{R} \right)^2$ (i = 1,2,.... N)

σ_r, σ_θ, σ_z Residual stress in radial, tangential and axial direction respectively.

σ_y Yield stress

ϕ Curvature

Functions:

$$F(\beta) = \frac{1}{\cos\left[f^{-1}(\beta)\right]}$$, where $f^{-1}(\beta)$ is the inverse function of

$$f(\beta) = \frac{3}{8\cos\beta}\left(\frac{\pi}{2} - \beta + \frac{1}{4}\sin 4\beta\right) + \sin^3\beta$$

$F_1(\beta)$ = First integral of $F(\beta)$

$$= \int_{\frac{3\pi}{16}}^{\beta} F(\beta)\,d\beta$$

$F_2(\beta)$ = Second integral of $F(\beta)$

$$= \int_{\frac{3\pi}{16}}^{\beta} F_1(\beta)\,d\beta$$

$S(\bar{\rho})$ = Strain function

$$= (1-\bar{\rho}^2)\cdot\bar{\varepsilon}_z(\bar{\rho})$$

IX. TABLES, FIGURES AND PHOTOGRAPHS

TABLE 1

SUMMARY OF TEST SPECIMENS

	Item	Bar No.	Tension Coupon Test	Residual Stress Measurement: Boring-Out or Combined Method	Residual Stress Measurement: Modified Boring-Out Method	Residual Stress Measurement: Beam Dissect-ion Method	Stub Column Test	Column Test	Cold-Bending Test
USS "T-1" STEEL (Heat No. 22C887)	1-A	11							11-10
		12							12-10
		13	13-1 13-2	13-3 13-4			13-5		13-10
		15					15-5		15-10
		16	16-1-a 16-1-b 16-1-c				16-5		16-10
	1-B	12				12-14	12-15	12-17	
		13	13-11-a 13-11-b 13-11-c		13-13-1 13-13-2	13-14	13-15	13-16	
		15			15-13		15-15		
		16	16-11-a 16-11-b 16-11-c			16-14	16-15	16-16	
	2	12					12-25	12-26	
		13	13-21-a 13-21-b 13-21-c		13-23		13-25		
		16	16-21-a 16-21-b 16-21-c				16-25	16-26	

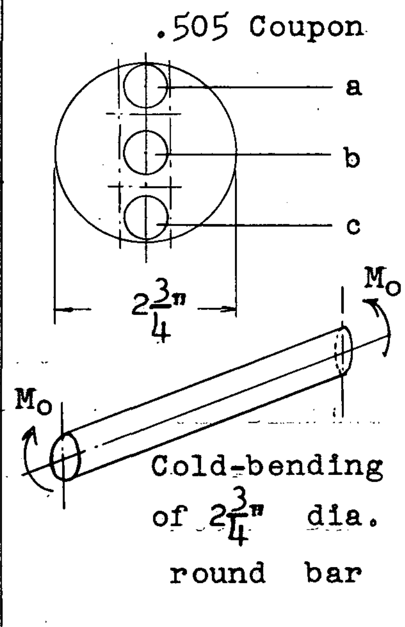
Table 1 (Concluded)

	Item	Bar	Tension Coupon Test	Residual Stress Measurement: Boring-out or Combined Method	Residual Stress Measurement: Modified Boring-Out Method	Residual Stress Measurement: Beam Dissect-ion Method	Stub Column Test	Column Test	Cold-Bending Test
AISI 1020 CARBON STEEL	6-A	C	C-1 C-2	C-3 C-4			C-5		C-10
		D	D-1 D-2				D-5		D-10
		E	E-1 E-2				E-5		
		F	F-1 F-2				F-5		F-10
	6-B	C			C-13		C-15		
		D	D-11-a D-11-b D-11-c			D-14			
		F	F-11-a F-11-b F-11-c			F-14	F-15	F-16	
	7	C					C-25		
		D						D-26	
		F	F-21-a F-21-b F-21-c				F-25		

Heat No. 25C168 (Bar No. C & D)
Heat No. 28C943 (Bar No. E & F)

TABLE 2

TENSION COUPON TEST RESULTS

Item No.	Test No.	Young's Modulus (x 10^3 ksi)	Yield Stress (ksi) Static Level	Yield Stress (ksi) Strain Rate = 80 $\frac{\text{micro in.}}{\text{in/sec.}}$	Yield Stress (ksi) 0.2% Offset*	Ultimate Tensile Strength (ksi)	Coupons (a, b and c) Taken from Cold-bent Bars
1-A	13-1	28.8	119	123	117.0	127.5	
	13-2	30.9	121.5	124.5	117.0	130	
	16-1-a	30.0	117.5	118	114.3	126	
	16-1-b	30.0	117.5	119	114.3	126	
	16-1-c	30.0	120	121.5	114.3	129.5	
1-B	13-11-a	30.0	127	129		135.5	
	13-11-b	30.7	126.5	128.5		134	
	13-11-c	30.6	120.5	122		128.5	
	16-11-a	30.9	126	129		135	
	16-11-b	30.0	127	127.5		134.5	
	16-11-c	29.9	127	128.5		135	
2	13-21-a	30.6	129	129.5		136	
	13-21-b	30.9	129	131		136	
	13-21-c	30.2	123.5	124		131.5	
	16-21-a	30.2	123	125		132	
	16-21-b	30.2	124.5	126		133.5	
	16-21-c	30.0	124.5	126		134	

* This data was supplied by the United States Steel Corporation

Table 2 (Concluded)

Item No.	Test No.	Young's Modulus (x 10^3 ksi)	Yield Stress (ksi)			Ultimate Tensile Strength (ksi)	Coupons (a, b and c) Taken from Cold-bent Bars
			Static Level	Strain Rate = 80 $\frac{\text{micro in.}}{\text{in/sec.}}$	0.2 % Offset*		
6-A	C-1	29.1	36.2	37.0	37.1	72.3	
	C-2				37.1	72.3	
	D-1	29.8	30.7	33.6	37.1	59.6	
	D-2	29.8	31.0	32.5	37.1	59.7	
	E-1	29.1	34.4	35.4	39.1	63.0	
	E-2	29.4	34.3	35.6	39.1	62.6	
	F-1	29.4	34.0	35.0	39.1	62.0	
	F-2	28.8	34.0	35.0	39.1	62.3	
6-B	D-11-a	29.1	29.3	31.3		58.5	
	D-11-b	29.2	34.3	36.5		59.6	
	D-11-c	28.5	31.5	31.2		59.3	
	F-11-a	30.0	35.1	36.2		63.4	
	F-11-b	29.7	35.0	35.9		63.3	
	F-11-c	30.8	35.0	35.7		64.1	
7	F-21-a	29.0	33.7	34.1		62.9	
	F-21-b	29.7	34.0	35.0		62.7	
	F-21-c	30.4	32.3	33.0		62.7	

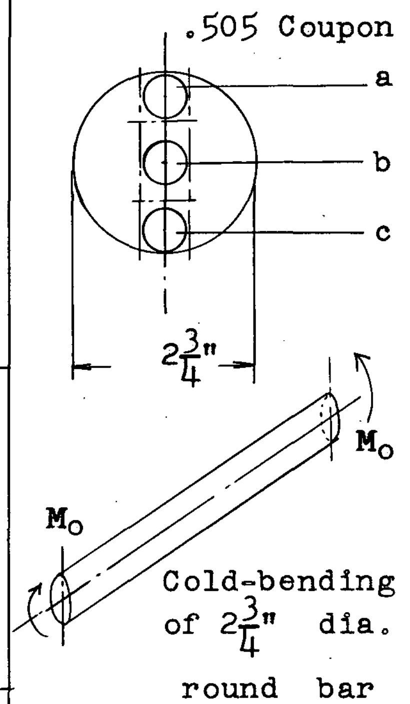


TABLE 3

COLUMN TEST RESULTS

Item No.	Test No.	Actual Length	Effective Length	Generalized Slenderness Ratio	Initial Deflection	Maximum Load (Test Results)		Ultimate Load (Theory)
		L (in)	kL (in)	$\left(\frac{kL}{r_o}\right)/\pi \cdot \sqrt{\frac{E}{\sigma_y}}$	$\frac{d_o}{R}$	P_{max} (kips)	$\frac{P_{max}}{P_y}$	$\frac{P_{max}}{P_y}$
1-B	12-17	62	36.2	1.100	0.025	464.5	0.600	0.57
	13-16	40	21.0	.618	.011	694.1	.918	.91
	16-16	40	22.4	.664	.036	589.9	.773	.76
2	12-26	40	23.7	.675	.010	650.6	.941	.95
	16-26	50.5	25.6	.729	.065	548.2	.726	.74
6-B	F-16	65	42.6	.665	.007	200.0	.922	.92
7	D-26	35.5	*			174.7	.910	

* Local yielding occurred only at the vicinity of the end of the column so that no inflection point of deflection was observed.

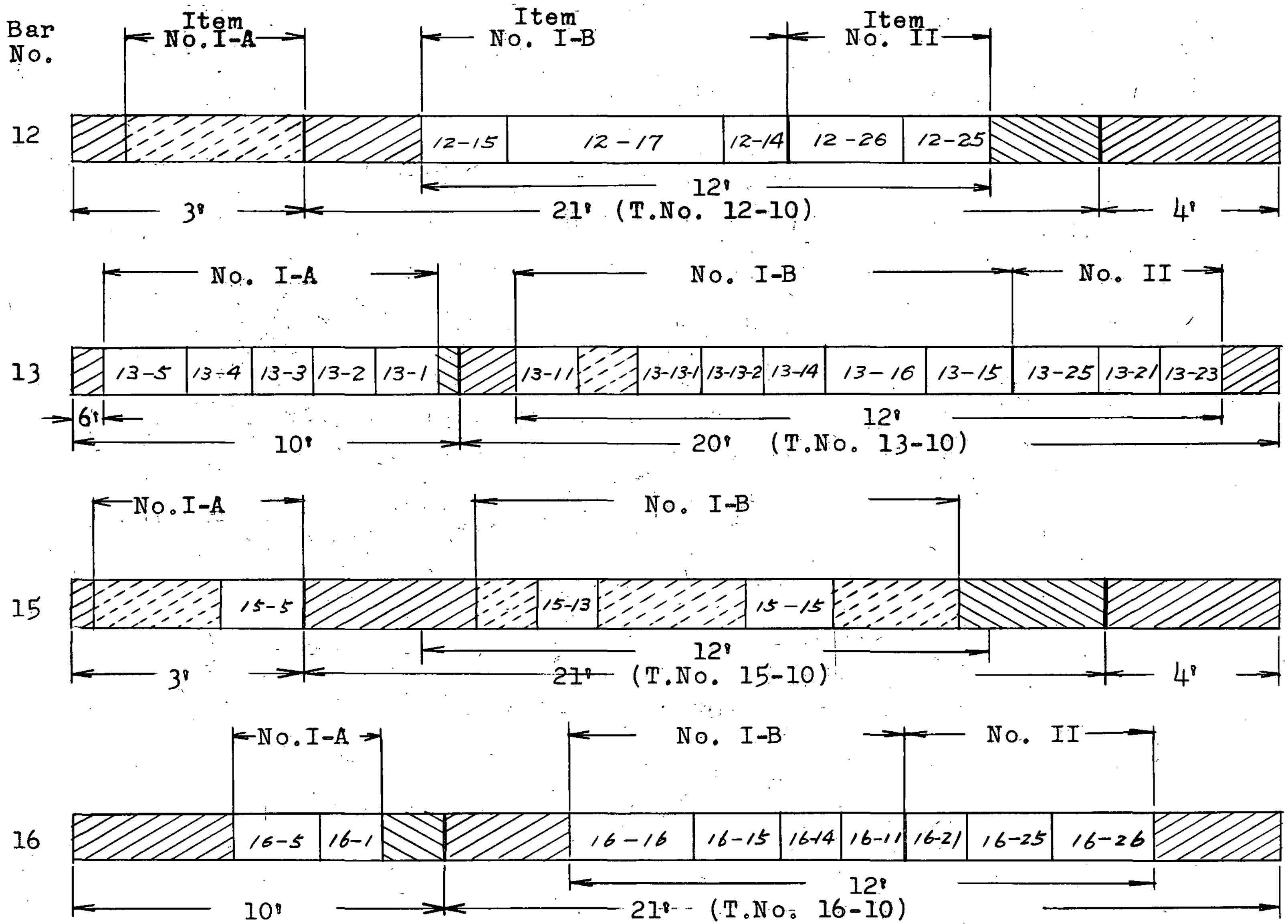


Fig. 1 - POSITION OF INDIVIDUAL SPECIMENS

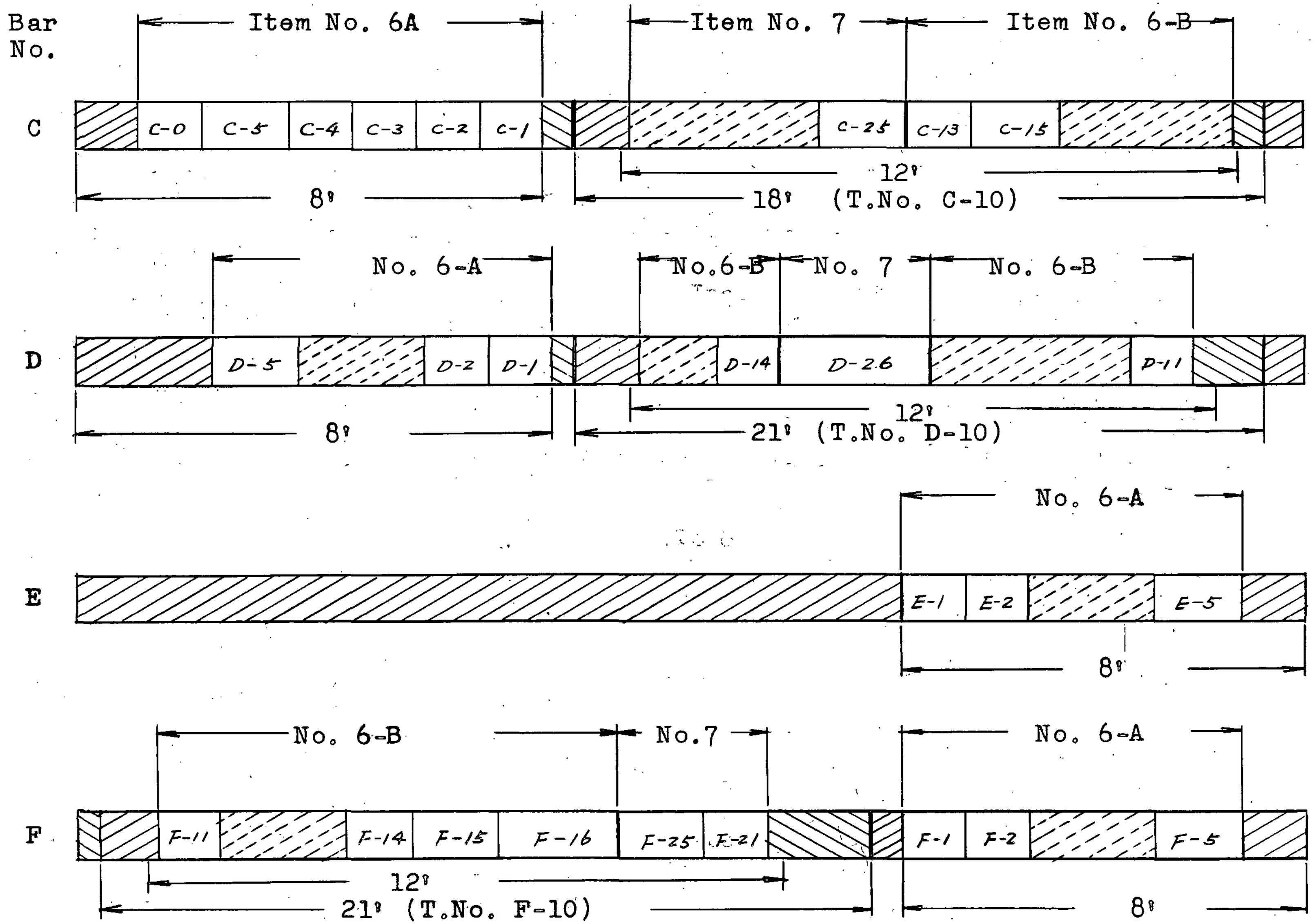


Fig. 1 - POSITION OF INDIVIDUAL SPECIMENS (Concluded)

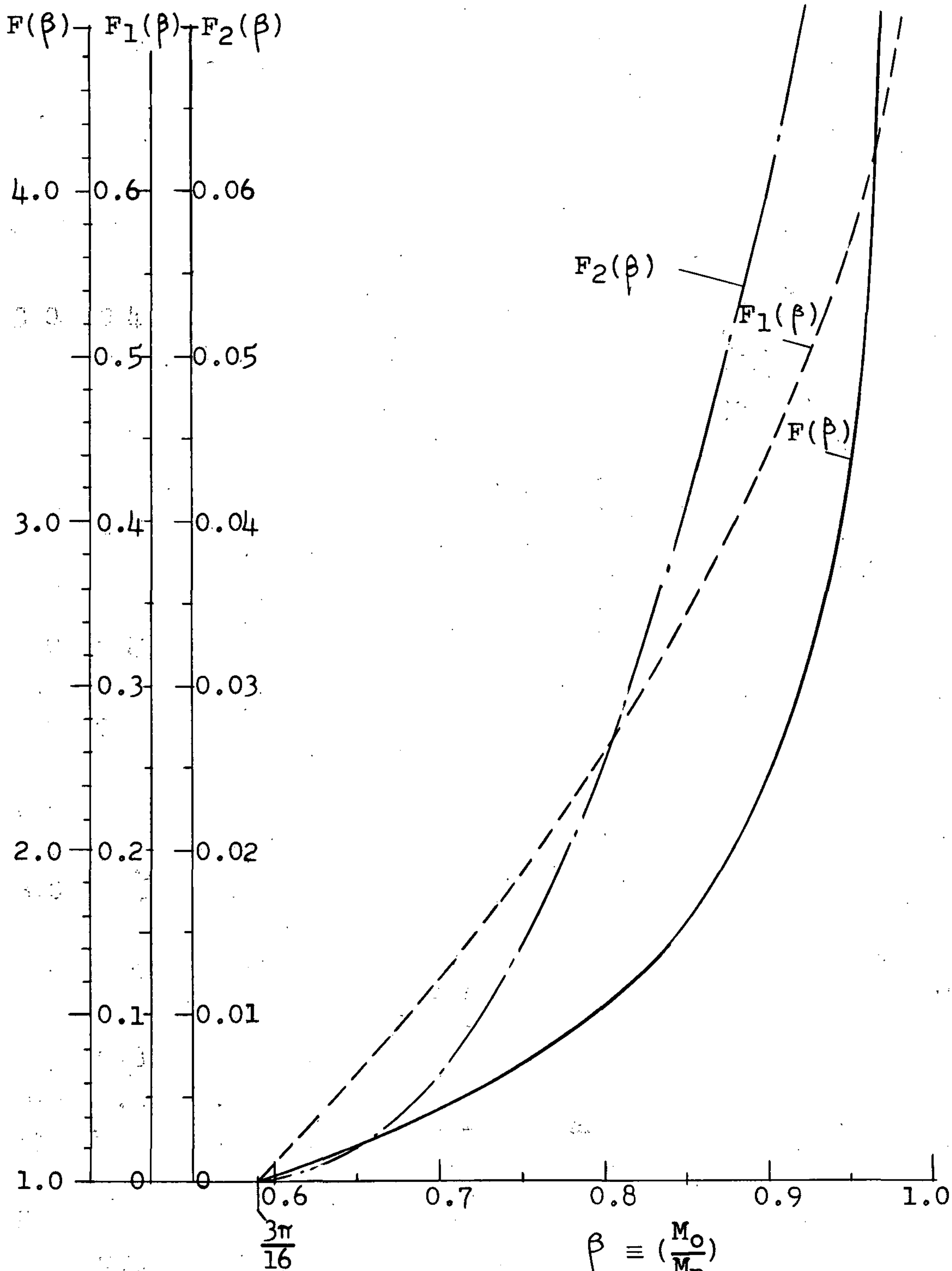


Fig. 2 - FUNCTIONS $F(\beta)$, $F_1(\beta)$, and $F_2(\beta)$, AGAINST β

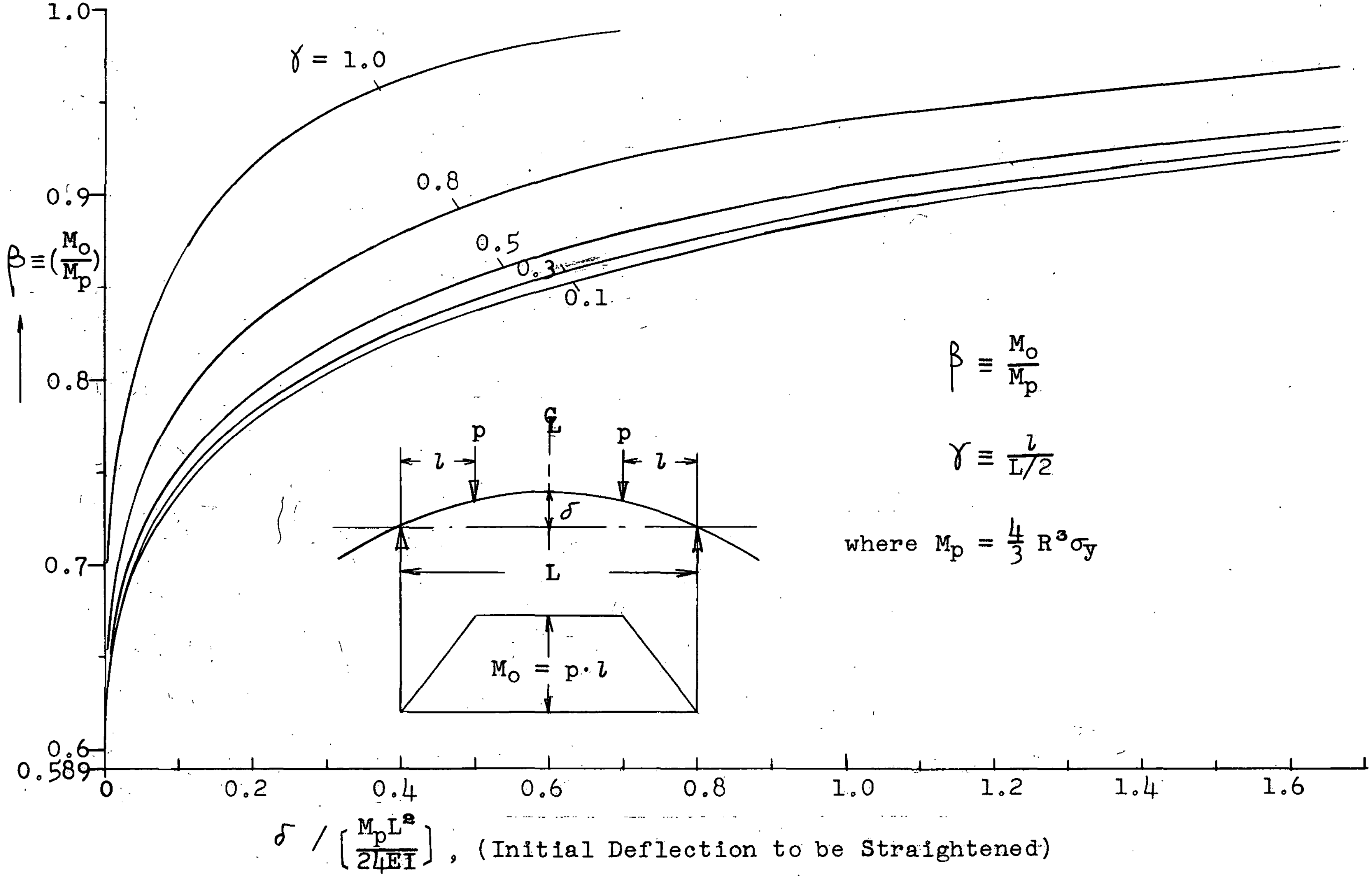


Fig. 3 - RELATIONSHIP BETWEEN INITIAL DEFLECTION δ AND REQUIRED BENDING MOMENT FOR COLD-STRAIGHTENING $\beta \equiv \frac{M_o}{M_p}$

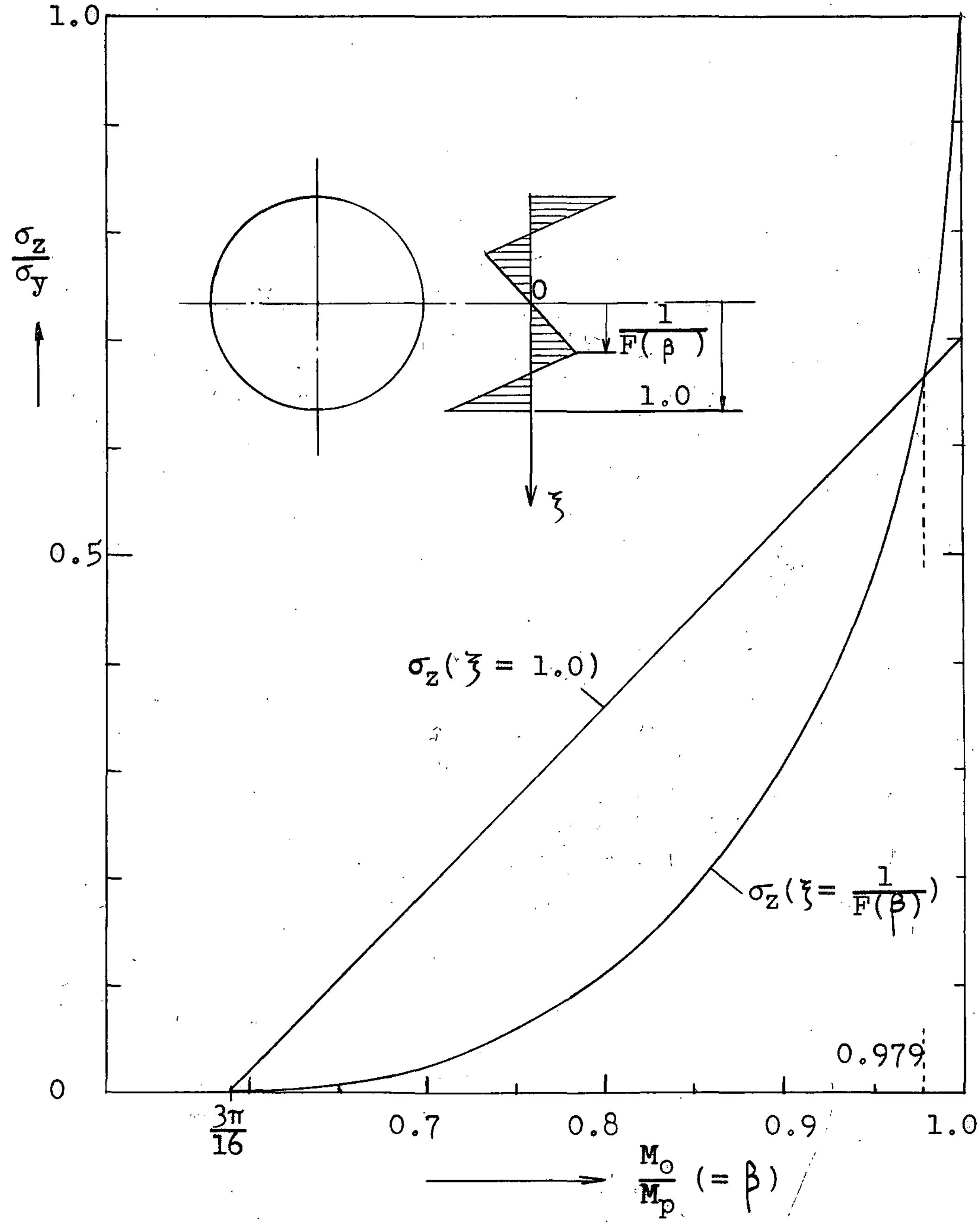


Fig. 4 - MAXIMUM RESIDUAL STRESSES DUE TO COLD-BENDING

1. Cross Head of Testing Machine
2. Loading Beam
3. Loading Fixtures
4. Supporting Blocks
5. Base Beam
6. Lower Bed of Testing Machine
7. Lateral Supports
8. Test Specimen
9. Level String for Deflection Gage
10. Deflection Gage
11. SR-4 Strain Gages

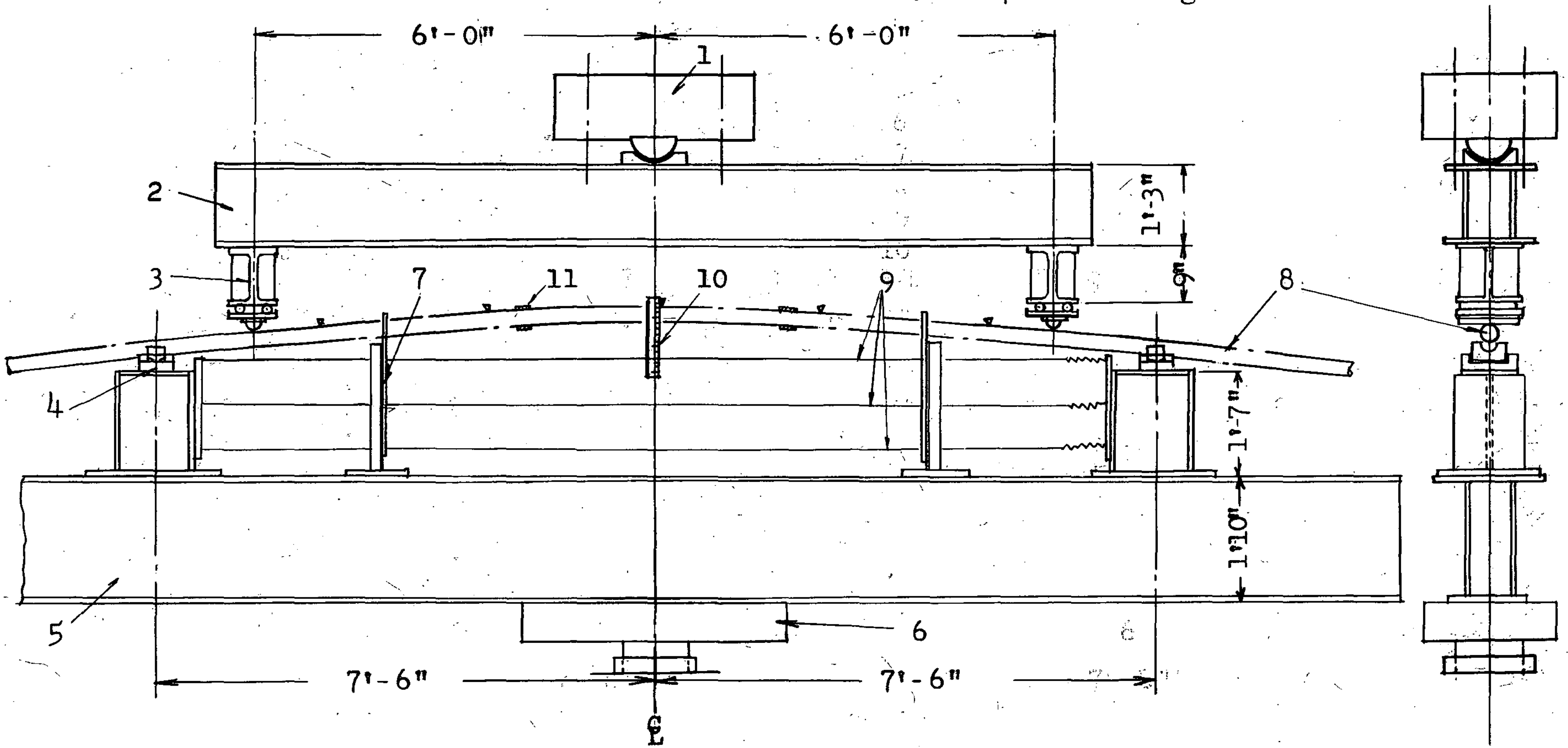


Fig. 5 - COLD-BENDING TEST SETUP

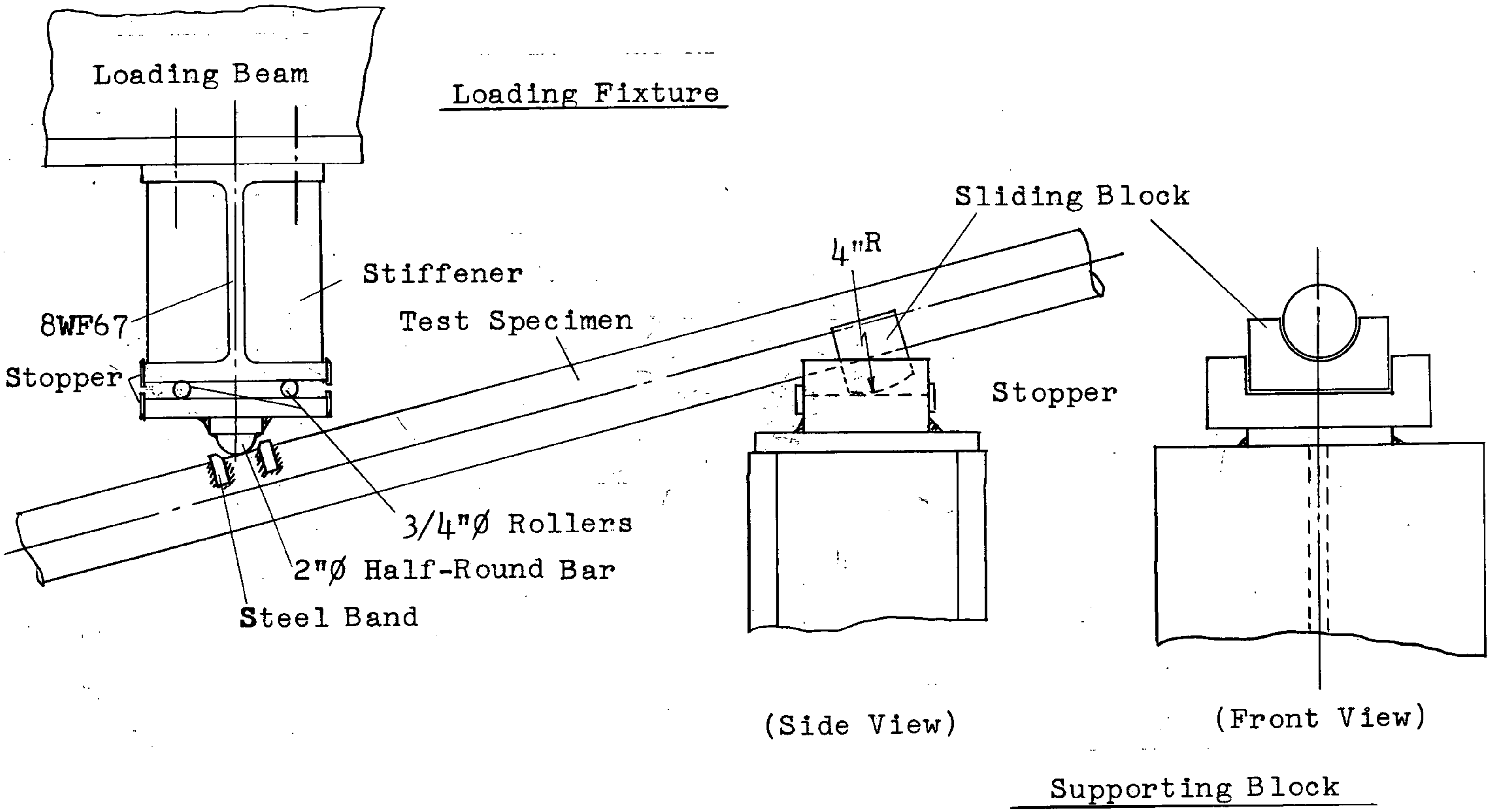


Fig. 6 - DETAIL OF TEST FIXTURES

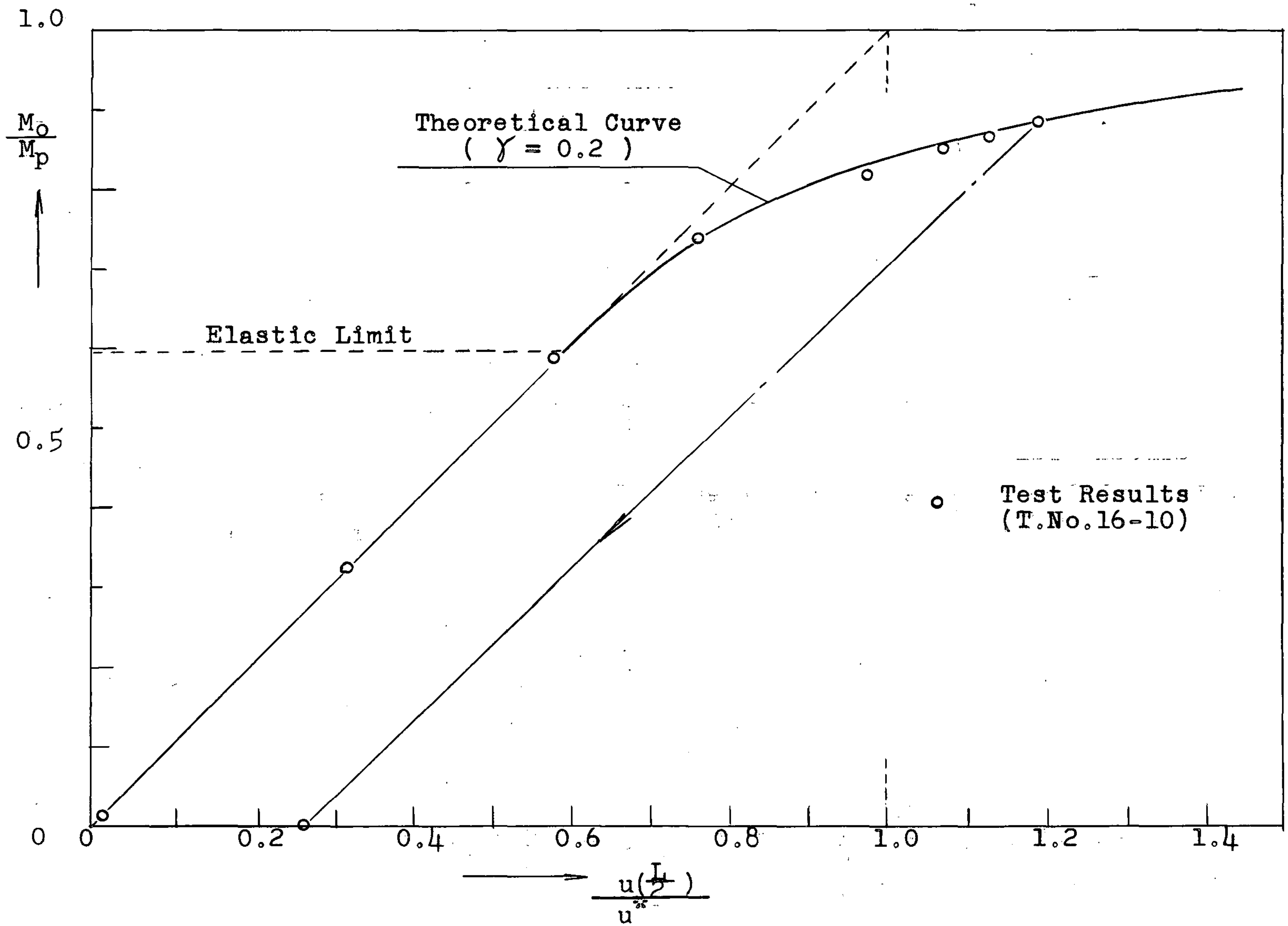


Fig. 7 - MOMENT-DEFLECTION RELATIONSHIP

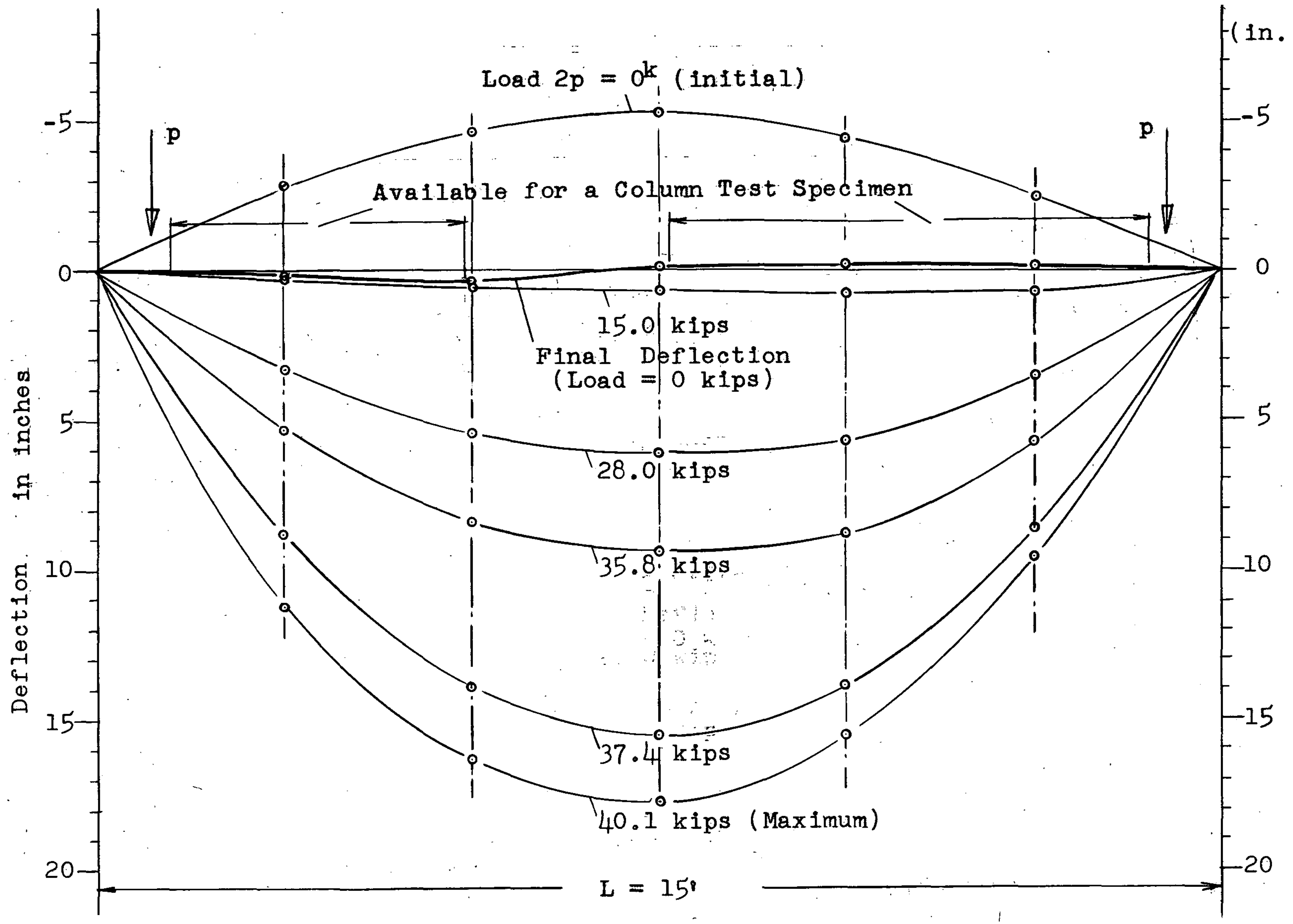


Fig. 8 - DEFLECTION OF BEAM (COLD-STRAIGHTENING TEST)

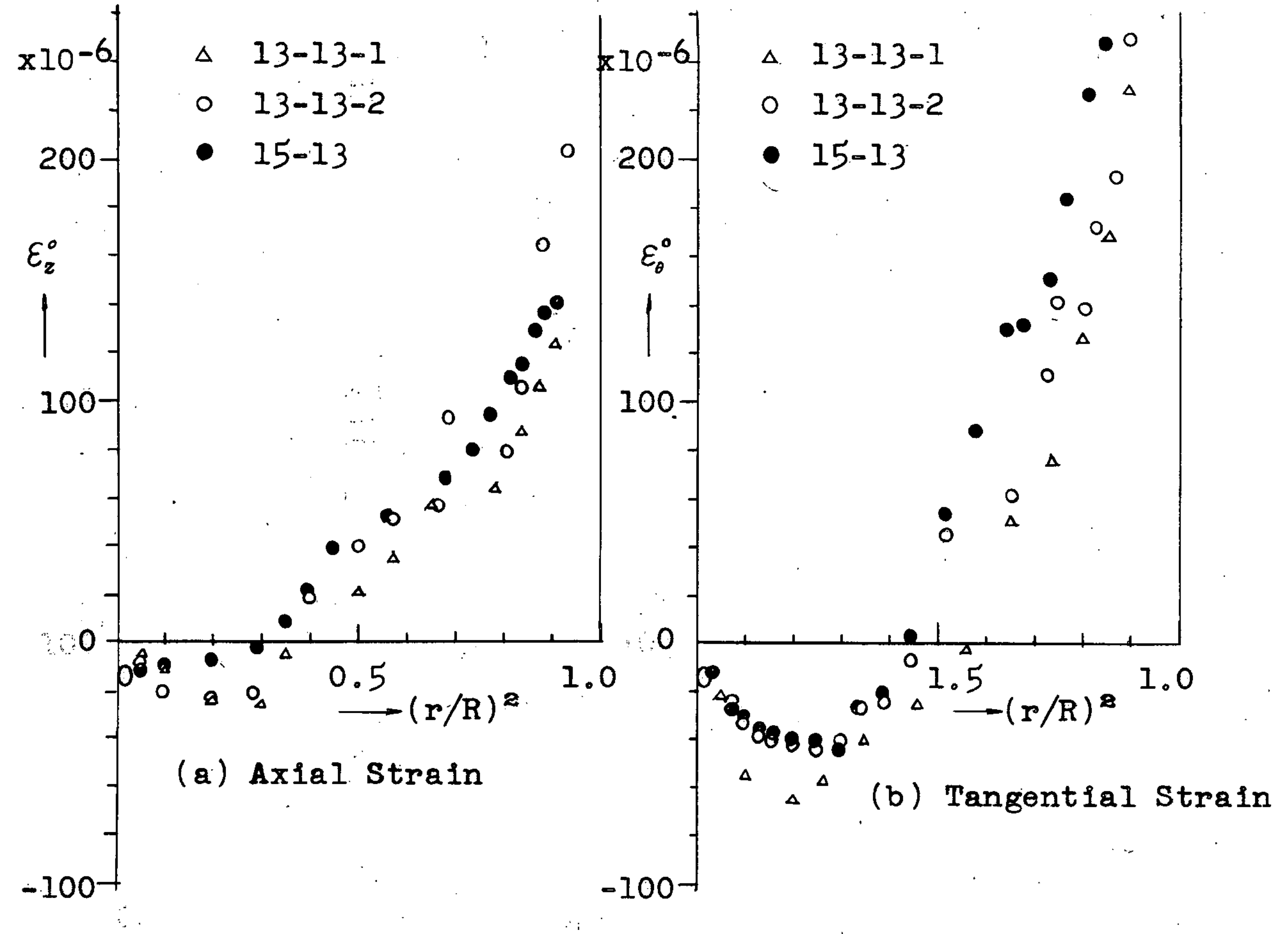
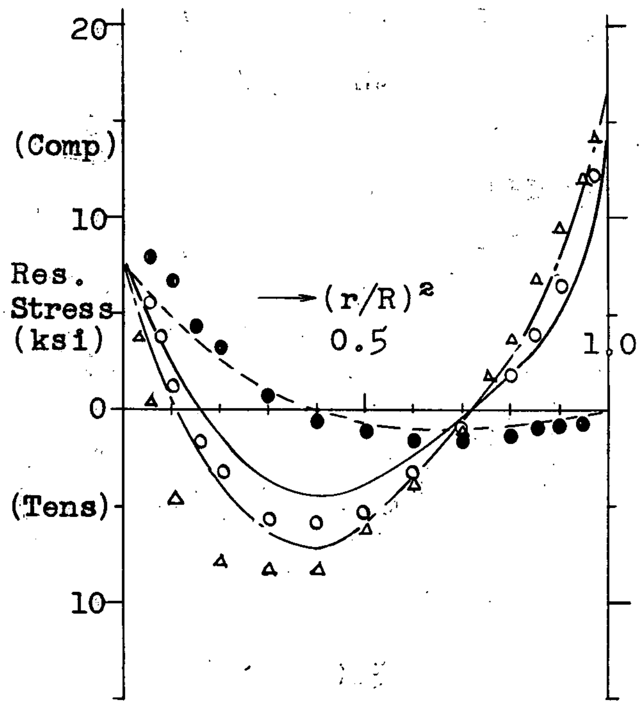


Fig. 9 - MEASURED STRAINS (POLAR-SYMMETRIC)



	Before Cold-Bending T.No. 13-3,4	After Cold-Bending T. No. 15-13
σ_r	●	– – – – – – –
σ_θ	△	—— · —— · —
σ_z	○	————————

Fig. 10 - MEASURED RESIDUAL STRESSES (POLAR-SYMMETRIC)
BY MODIFIED BORING-OUT METHOD

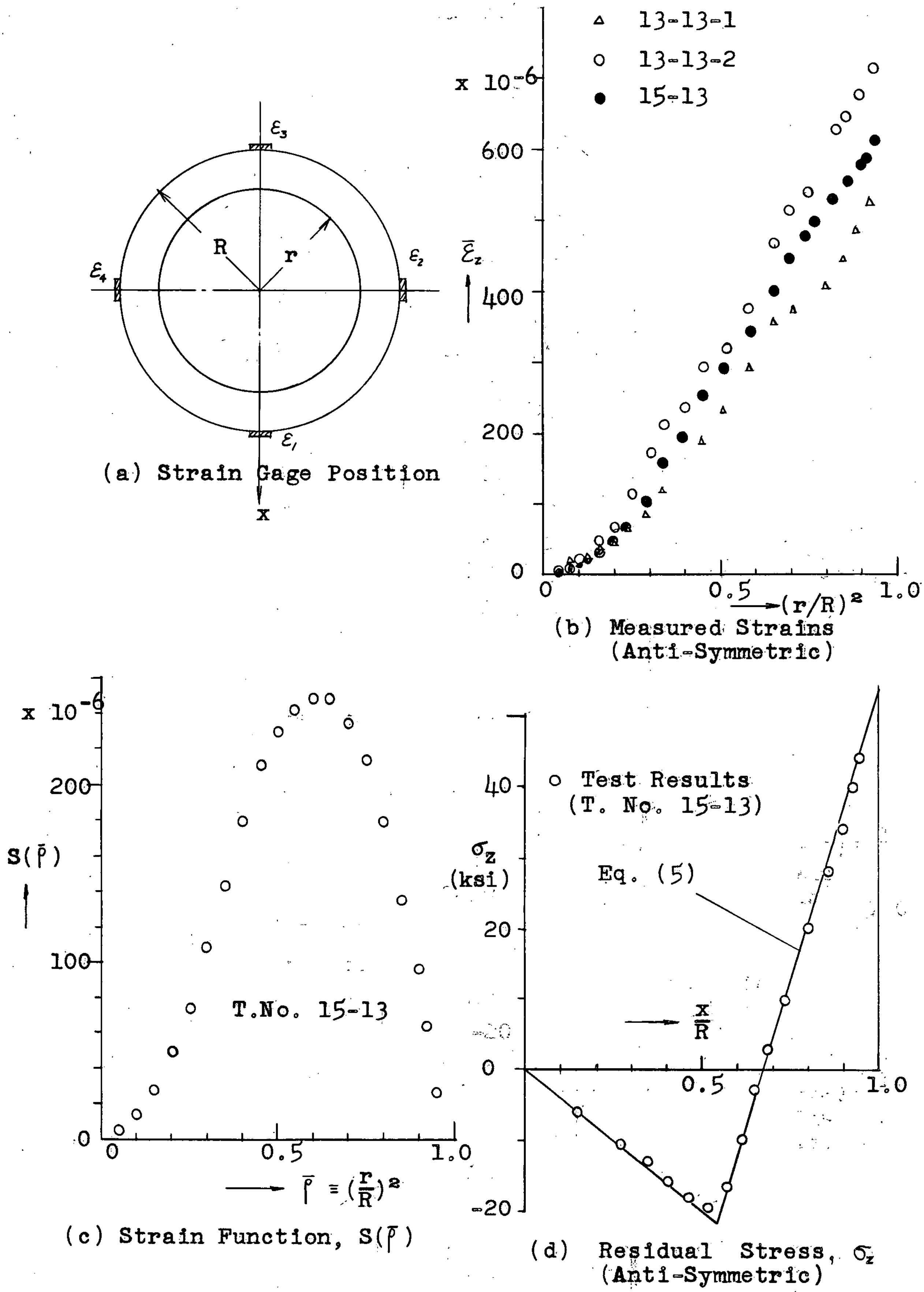


Fig. 11 - RESIDUAL STRESS MEASUREMENT
BY MODIFIED BORING-OUT METHOD

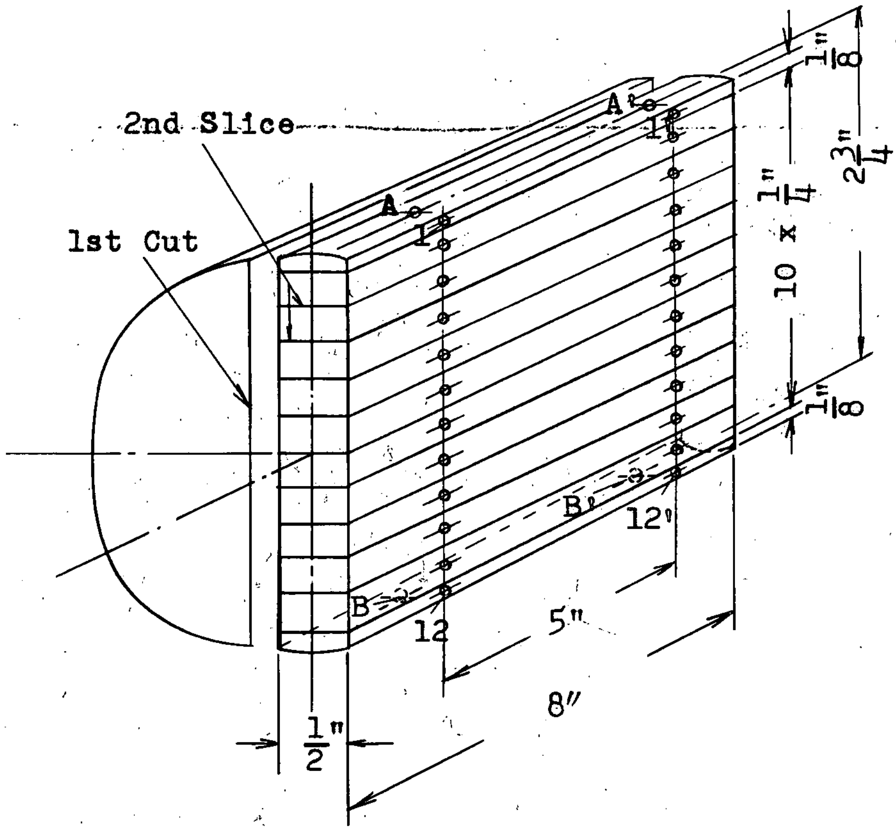


Fig. 12 - TEST SPECIMEN FOR RESIDUAL STRESS MEASUREMENT BY BEAM DISSECTION METHOD

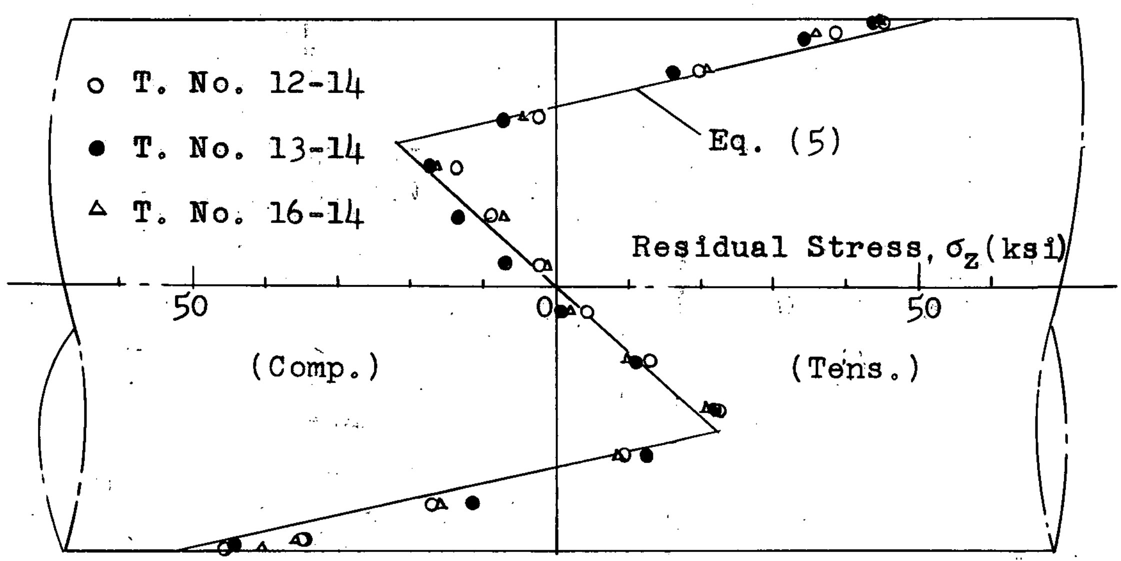


Fig. 13 - RESIDUAL STRESS IN COLD-STRAIGHTENED ROUND BARS (MEASURED BY BEAM DISSECTION METHOD)

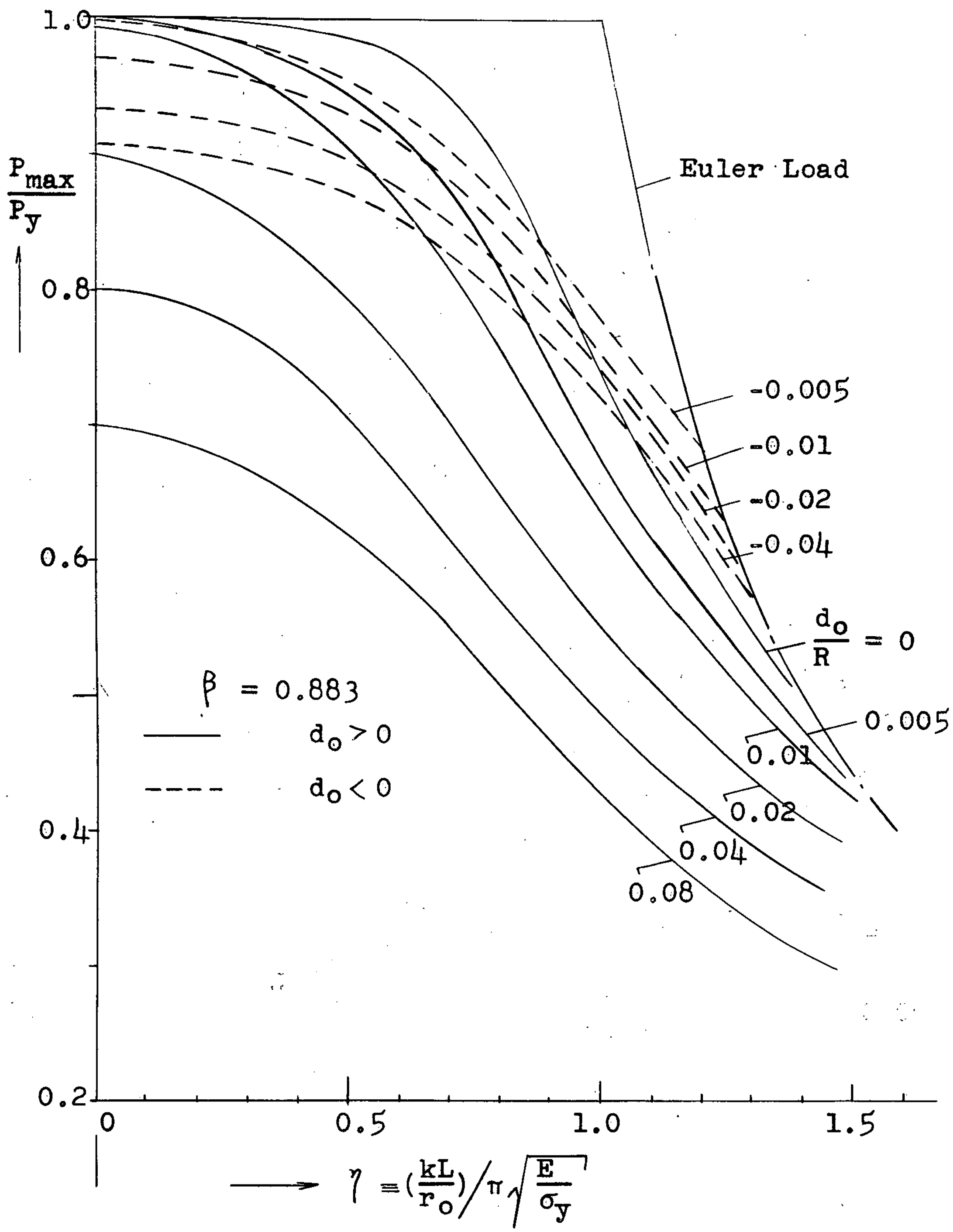


Fig.14-ULTIMATE STRENGTH OF COLD-STRAIGHTENED COLUMNS

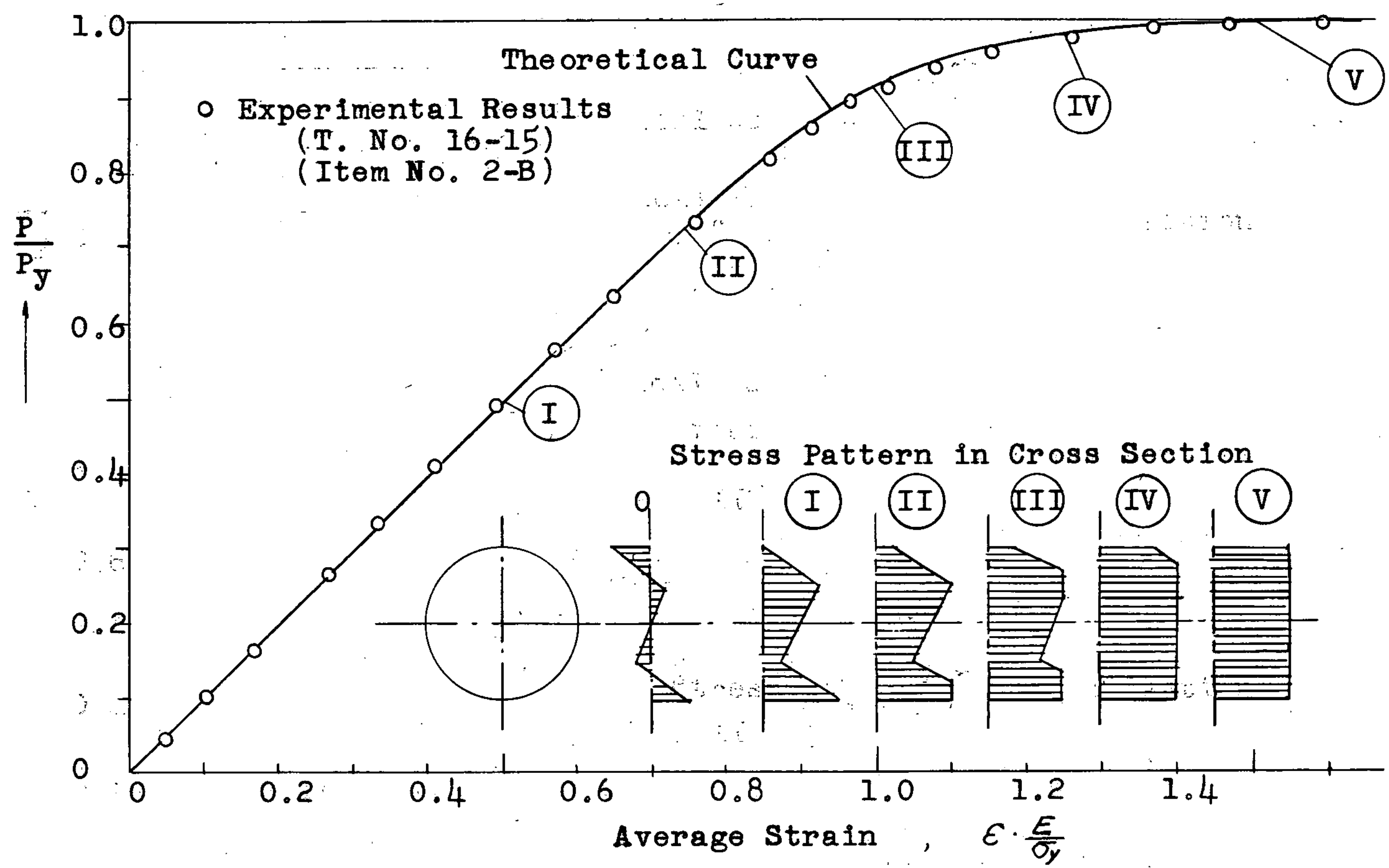


Fig. 15 - AVERAGE STRESS-STRAIN CURVE OF STUB COLUMN

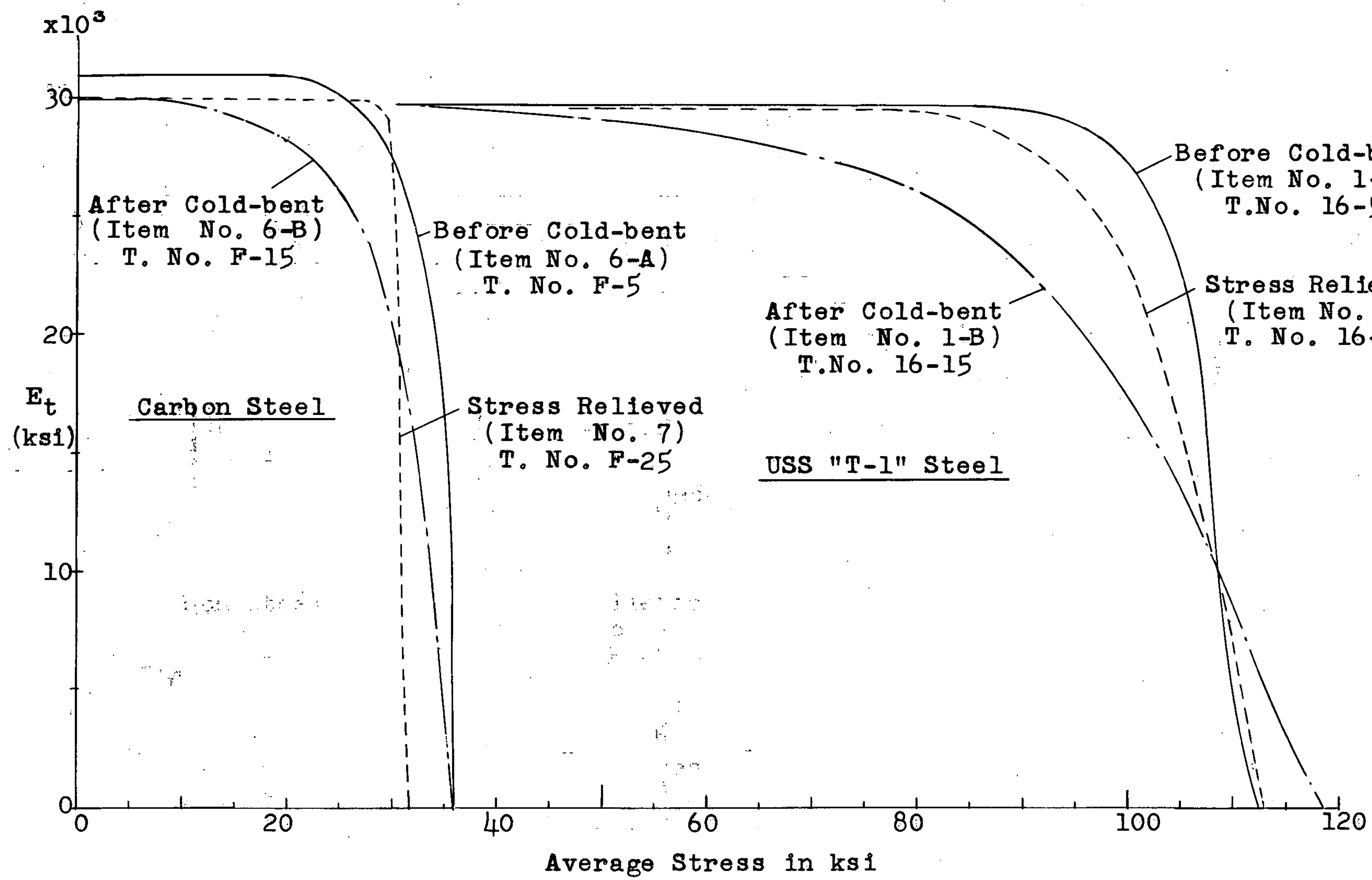


Fig. 16 - EFFECT OF BOTH COLD-BENDING AND STRESS-RELIEVING ON TANGENT MODULUS OF STUB COLUMNS

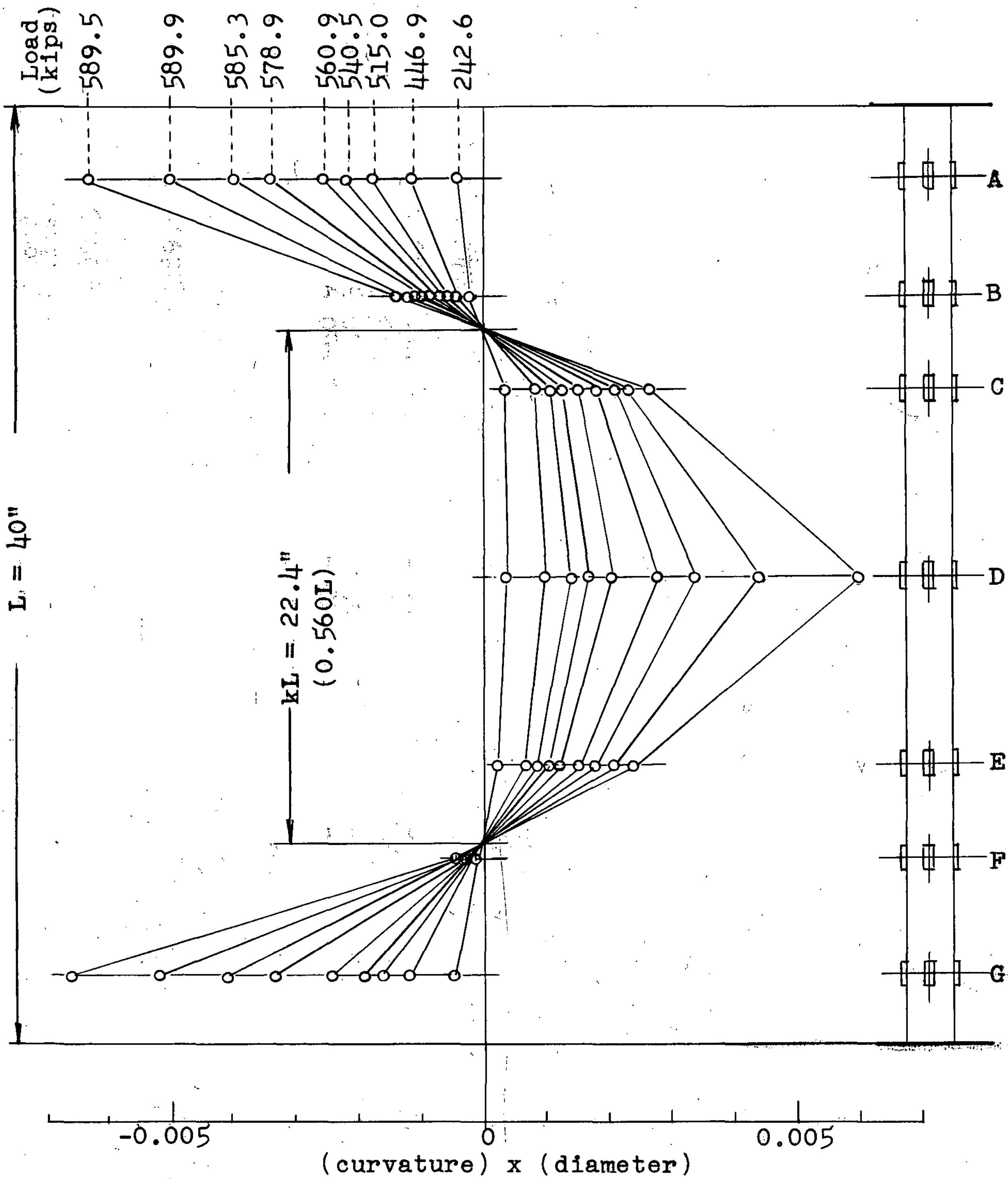


Fig. 17 - CURVATURE VARIATION ALONG COLUMN LENGTH

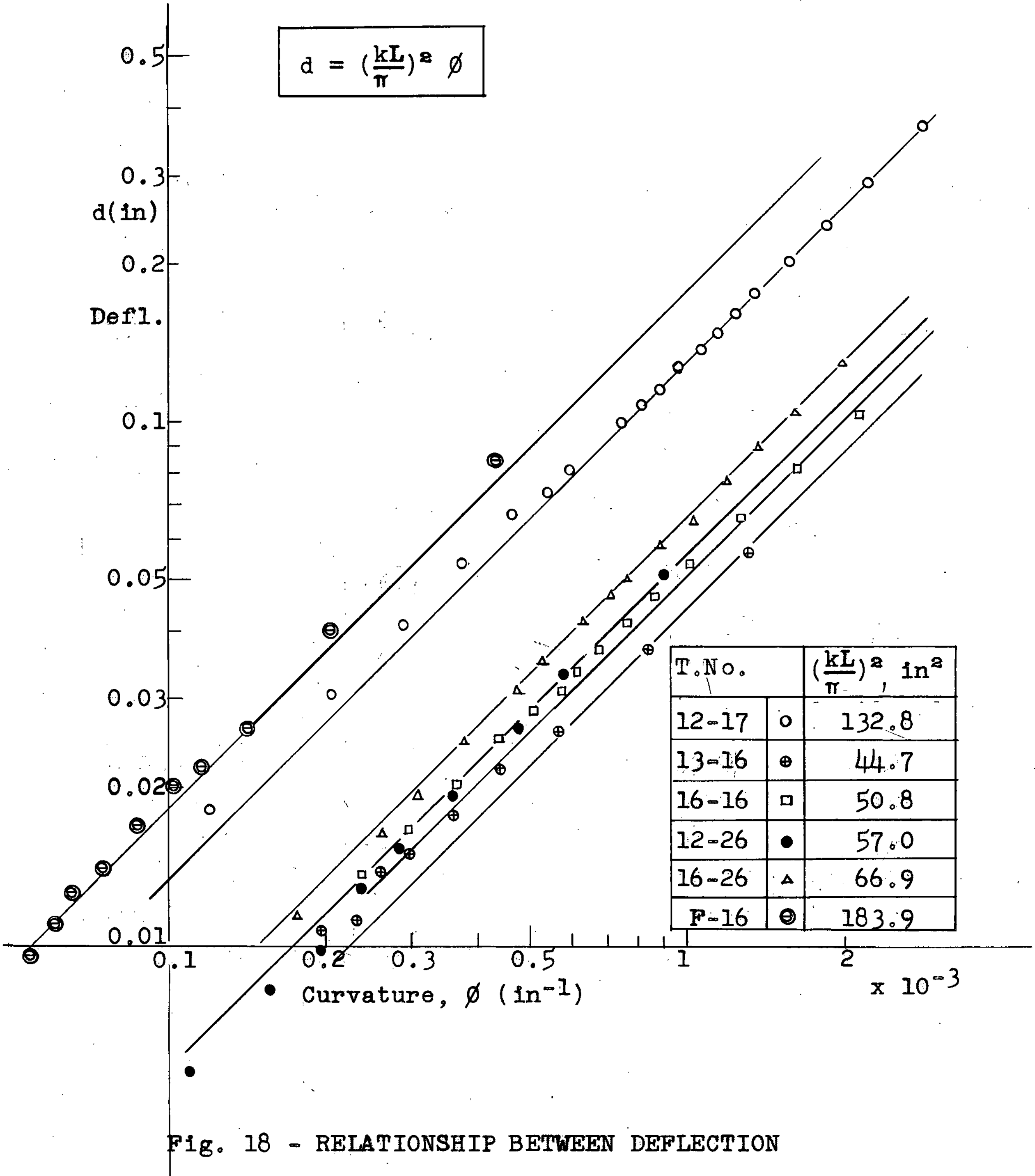


Fig. 18 - RELATIONSHIP BETWEEN DEFLECTION AND CURVATURE OF COLUMNS

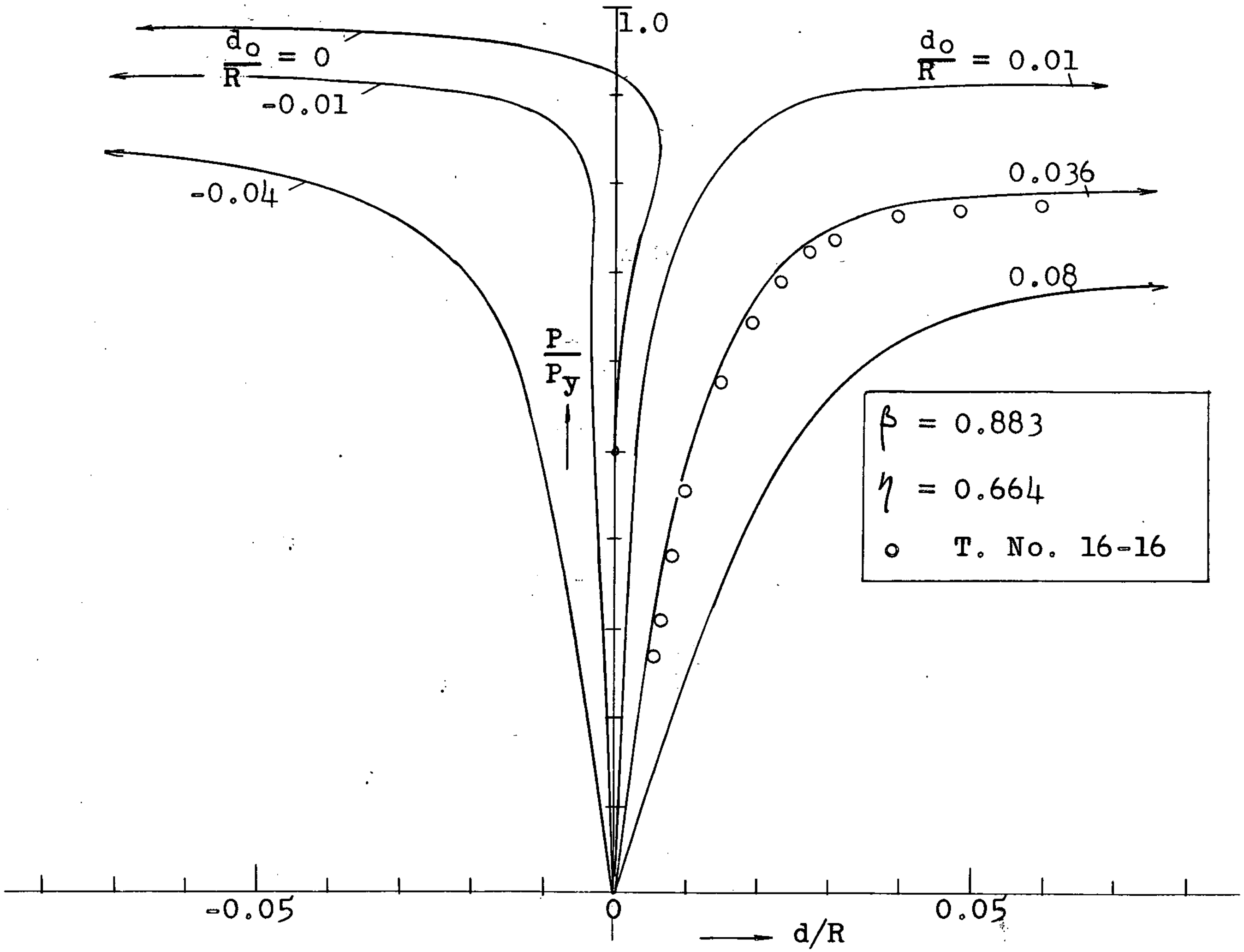


Fig. 19 - LOAD-DEFLECTION CURVES OF COLD-STRAIGHTENED COLUMNS

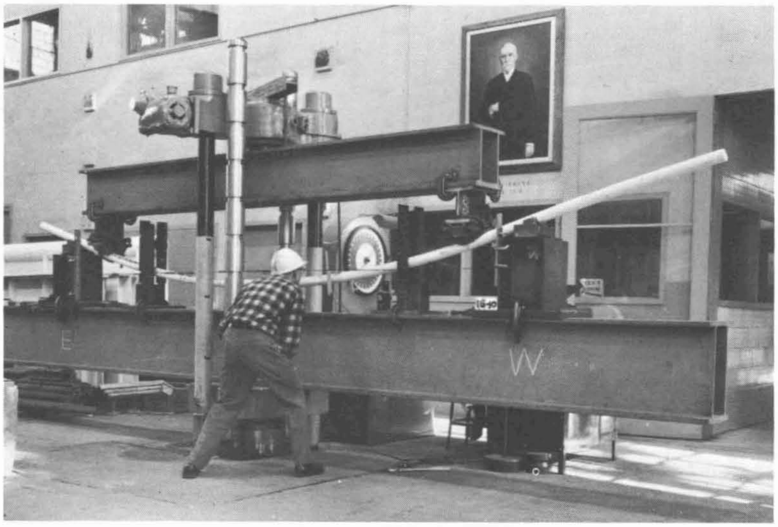


Photo. 1 Cold Bending Test (T. No. 16-10)
(At the maximum load)

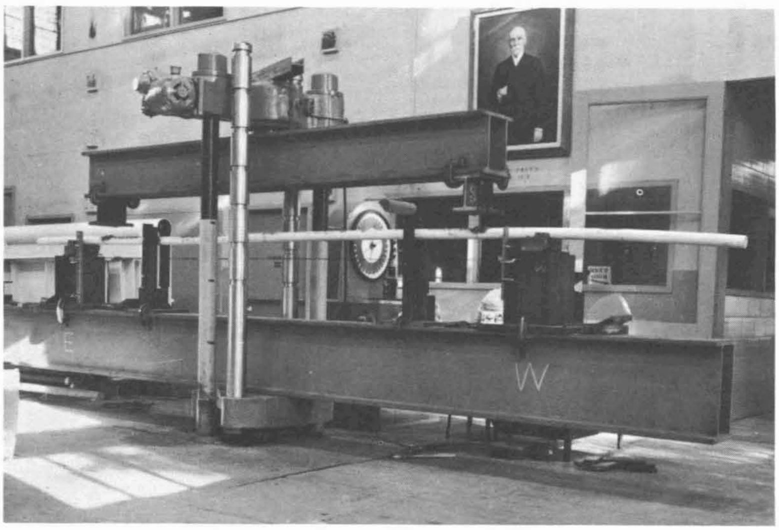


Photo. 2 Cold-Straightened Bar
(After releasing load)

Tested Specimen
(T. No. 13-13-2)

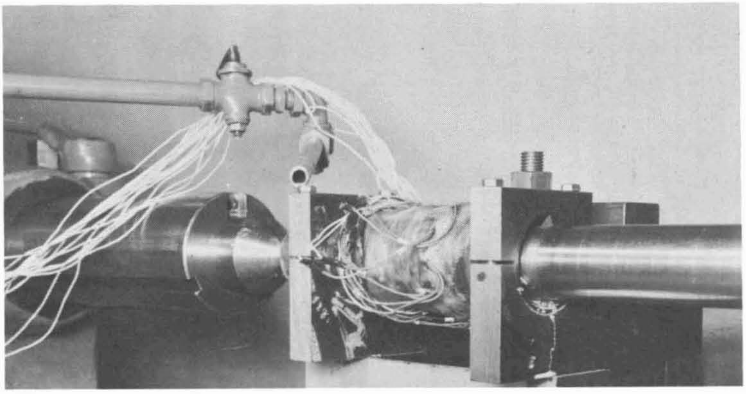


Photo. 3 Residual Stress Measurement
By Modified Boring-out Method

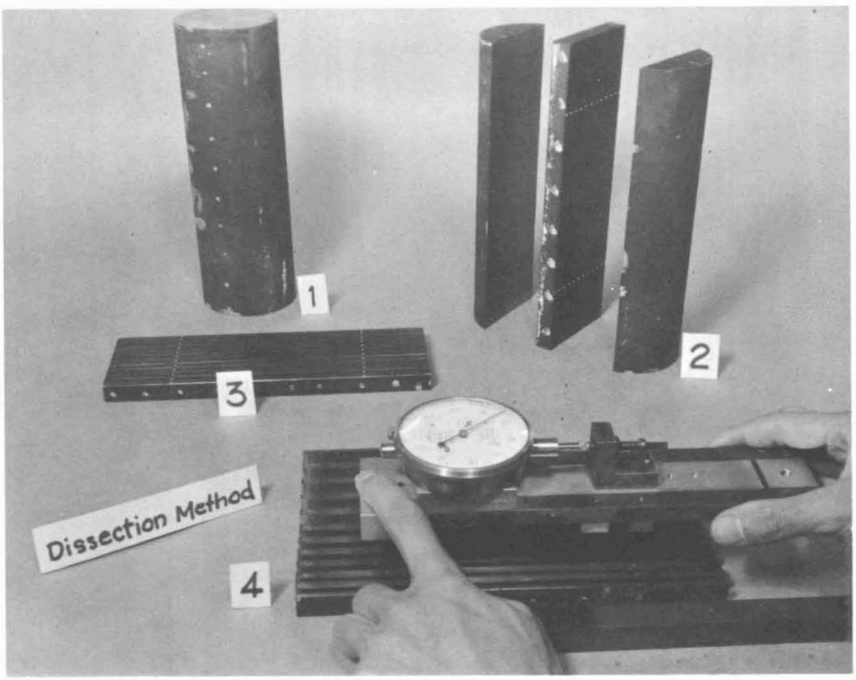


Photo. 4 Residual Stress Measurement
By Beam Dissection Method
(T. No. 13-14)

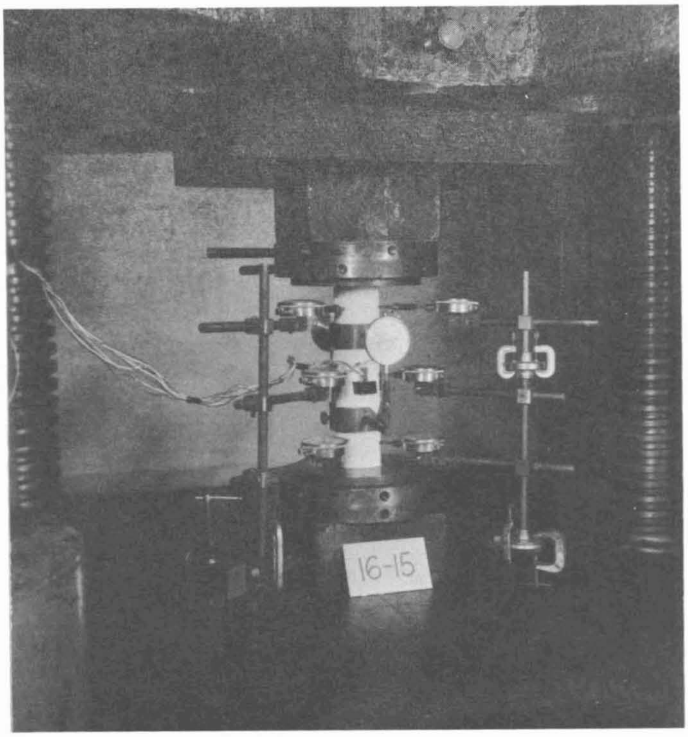


Photo. 5 Stub Column Test
(T. No. 16-15)

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