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Reference values for test girders.

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Submitted to the

Welded Plate Girder Project Committee

REFERENCE VALUES
FOR TEST GIRDERS

including

Cross Sectional Constants,
Yield, Plastic, and Critical Loads,
Deflections

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Abstract

In an investigation of the static carrying capacity of welded plate girders conducted at Fritz Engineering Laboratory, Lehigh University, both theoretical and experimental studies have been undertaken. In all the reports of this Welded Plate Girder Project, comparisons will be made with such standard reference values as "yield loads", "plastic loads", "critical loads", and computed deflections. It is the purpose of this report to establish these loads and deformations while stating the assumptions and definitions used in their evaluation.

1. Introduction

An investigation of the behavior of thin-web, welded plate girders has been under way at Lehigh University and is now essentially complete. The experimental phase of this program was begun in the summer of 1958 and ended one year later. At its completion, a total of thirty-three ultimate load tests had been conducted on thirteen full-size girders. The description of these girder tests and the reporting of the results are the objectives of additional papers presently in preparation. Here, the girders and tests are described only insofar as necessary for explaining numerical computations of specific reference values.

A girder section can be subjected to bending, shear, or a combination of both these loadings. In this research project all three conditions were investigated. Consequently, three different test setups were used as shown in Figs. 1, 2, and 3. This classifies the thirteen plate girders into the following three groups:

Group	shown in	subjected to	Girders
1	Fig. 1	bending	G1, G2, G3, G4, G5
2	Fig. 2	shear	G6, G7
3	Fig. 3	combined	E1, E2, E4, E5, G8, G9

Throughout the entire project the girders are termed as given in the last column of this table. For example, G1 is the notation used for girder No. 1.

The cross section of some girders changed within their lengths. The reason for this design was to confine failure to a certain region whose loading conditions were well defined. This region was the test section proper as indicated in Figs. 1 and 2. The end sections, flanking the test section, only differed in web thickness for the first group, whereas in the second group cover plates were also added over a portion of their length. In the third group the cross section did not change and the entire girder was the test section proper. The four girders termed E1, E2, E4, and E5 were fabricated by splicing the undamaged end sections of the correspondingly numbered girders G1, G2, G4 and G5, and reinforcing them with cover plates.

Each girder was subjected to at least two ultimate load tests. After causing failure in a particular panel, the load was removed and the panel reinforced. All bending girder failures occurred in the compression flange and thus reinforcement consisted of welding small steel plates to this flange. Diagonal and transverse stiffeners were used to strengthen the girders of the other two groups. Since no major deformations were caused in panels adjacent to the one which failed in the first test, referred to as T1, a second test, T2, could be conducted. In some cases this process was repeated and additional tests, such as T3 and T4, were carried out.

Of all the possible parameters influencing the carrying capacity of plate girders, the investigation was restricted to the following four:

1. Loading condition: $\zeta = \tau/\sigma = \frac{\text{shear stress}}{\text{normal stress}}$

2. Type of cross section: various shapes of compr. flanges

3. Web slenderness: $\beta = b/t = \frac{\text{web depth}}{\text{web thickness}}$

4. Stiffener spacing: $\alpha = a/b = \frac{\text{panel length}}{\text{panel depth}}$

The first parameter was taken care of by choosing the three test setups previously mentioned. The second parameter was of special importance to the bending girders, wherefore three different shapes of the compression flanges were chosen as illustrated with cross sections I, II, and III shown in Fig. 4. Denoting with "a" the stiffener spacing, "b" the web depth, and "t" the web thickness, the third and fourth parameters completely defined the shape of a web panel. In the test program, the third parameter was varied by building pairs of girders which only differed in the web slenderness, such as G2 and G4, or G3 and G5. Finally, the fourth parameter was accounted for by subdividing the test section into panels with different lengths. After failure occurred in a longer panel, it was reinforced and thus failure could be forced to occur in a shorter panel.

The parametric values of all the girders' test sections are listed in Table 1. Furthermore, the added sketches indicate where each girder failed, how the obtained test was termed, and where reinforcement plates were welded to the flanges or webs. Taking, as an example, girder E4, the sketches give the following information: The first test of this girder, T1, caused failure in the left-hand end panel whose stiffener spacing was 1.5 times the web depth. The girder was then reinforced by subdividing each of the two larger panels with new transverse stiffeners. Thus failure was forced to occur in the right-half of the girder where the spacing of the stiffeners was 0.75 of the depth. This happened in the panel adjacent to the loading stiffener and furnished the second test, T2. Welding a reinforcing stiffener across this damaged panel allowed for a third test T3 in a panel whose aspect ratio was $\alpha = 0.5$.

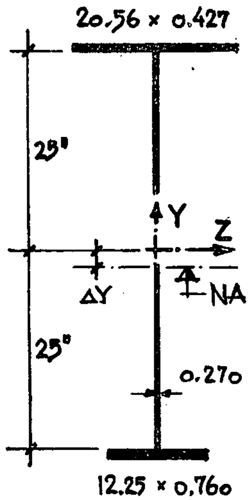
In the following sections detailed computations will be presented, beginning with the moment of inertia and section moduli (Sec. 2), then the values of some reference moments (Sec. 3), the web buckling stresses (Sec. 4), and finally, the deflections (Sec. 5).

2. Cross Sectional Constants

In this section will be presented the moments of inertia for all girders with their corresponding section moduli. To compute these values, it is necessary to know the cross sectional shapes and dimensions. The former can be found in Fig. 4, while the latter are summarized in Table 2. Here, the sizes of the flanges and webs are listed, together with the four different cover plates which were used.

A typical computation of the cross sectional constants is carried out below. The procedure was first to find the moment of inertia I_z about the Z-axis which was located at the mid-depth of the web. Then, after determining the actual centroid of the section, the moment of inertia about the neutral axis was found by means of the parallel axis theorem. Finally, dividing this value by the distance to the extreme fibers e_a and e_b, the section moduli S_a and S_b were obtained. As seen, the indexes "a" and "b" distinguish between quantities above and below the neutral axis, respectively.

Computation of Section Moduli of G1-T1, Test Section



PL	Dimensions in	Area in^2	Y in	$Q_z = Y.A$ in^3	I_z in^4
T.Fl.	20.56x0.427	8.78	+25.21	+221.3	5580
Web	50x0.270	13.50			2810
B.Fl.	12.25x0.760	9.31	-25.38	-236.3	6000
		31.59		-15.0	14390

$\Delta Y = Q_z/A = -15.0/31.59 = -0.47$ in

$\underline{I_m} = I_z - (\Delta Y)^2 A = 14,390 - 0.47^2 x 31.59 = \underline{14,380\ in^4}$

$e_a = 25 + 0.47 + 0.43 = 25.90$ in

$e_b = 25 - 0.47 + 0.76 = 25.29$ in

$\underline{S_a} = I_n/e_a = 14,380/25.90 = \underline{555\ in^3}$

$\underline{S_b} = I_n/e_b = 14,380/25.29 = \underline{568\ in^3}$

Following the above procedure, all necessary cross sectional constants were computed for the girders and are presented in Table 3. In the first three columns of this table are given the properties of the test section, namely, the moment of inertia I_m and the corresponding section moduli S_a and S_b. Next, the moments of inertia of the bending and shear girders' end sections, I_e, are added. Finally, some special moments of inertia are given in the last column which will now be explained for each girder:

- G1. After the first test on this girder was completed, its top flange width was reduced by flame cutting to 13.56 inches. Thus, for computations involving G1-T2 (second test of girder 1) this new width must be used, resulting in I = 12,210 in^4 and a neutral axis at Y = -3.158 in.

- G2, G3, G4, G5. After completion of the first tests of these bending girders, a steel plate was welded on either side of the top flange. Each of these two plates had an area of one square inch, the same distance from the neutral axis as the centroid of the unreinforced flange, and extended over the longest panel, i.e., were 75 inches long, wherefore this new I-value is computed.

- G6, G7. These values are the moments of inertia of the sections under the reactions where cover plates were added, i.e., ℄ D shown in Fig. 2.

- E1. The outside cover plates of girder E1 were terminated 75 inches from its ends and, therefore, two moments of inertia are needed to compute deflections. The value shown in the last column applies to the end portions of the girder. Since the third test produced failure within this region, the values without cover plates must be used for calculations concerning E1-T3.

3. Reference Moments and Loads

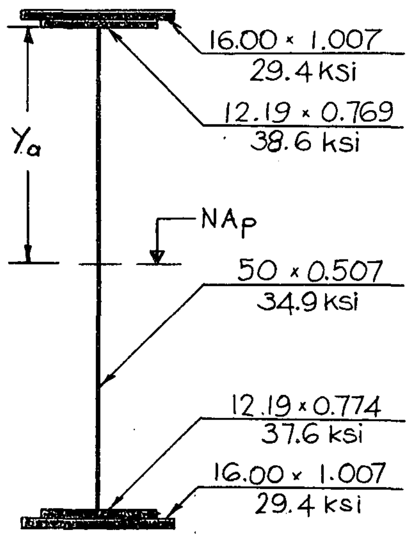
This section is devoted to the computation of the flange moment, yield moment, and plastic moment with the corresponding loads of the latter two. These moments are defined in Ref. 1, p. 34. Their definitions are repeated below with the modifications needed to take into account the different yield stresses of the component plates.

The flange moment, M_f, is defined as the moment carried by the flanges alone when the stresses over the flanges are equal to the yield stress. For a symmetrical girder whose yield stresses are the same for both flanges, this would simply be computed as $M_f = A_f \sigma_{yf} h$, where A_f and σ_{yf} are the area and yield stress of one flange and h is the distance between the centroids of the flanges. The actual girders tested exhibited a certain degree of dissymmetry in shape and yield stress. Therefore, the area and yield stress of the compression flange are arbitrarily selected to be used for the computation. Incidentally, the alternate use of the tension flange properties would not lead to any great differences. Check computations show that their use would, at most, underestimate the value calculated with the compression flange properties by 2.5%. When more than one plate comprises a flange, a weighted yield stress of the compression flange was used. This weighted stress, $\bar{\sigma}_y$, will be defined as $\bar{\sigma}_y = \Sigma A \sigma_y / \Sigma A$ where A and σ_y are the areas and yield stresses of the component plates and Σ indicates their summation.

The yield moment, M_y, is the moment which initiates nominal yielding in the most extreme fiber. In the case where the yield stresses of the flanges would be the same, it would be computed as $M_y = \sigma_y S$, where the smaller value of the section modulus, S, would be used. Since the yield stresses of the flanges differed, the definition that $M_y = \sigma_{ya} S_a$ is adopted, where σ_{ya} and S_a are the yield stress and section modulus of the compression flange. As in the case of the flange moment, the defined value of M_y is, at most, underestimated by 2.5% when bottom flange properties are substituted. Using the procedure adopted previously, a weighted yield stress was used when the flange is composed of a number of plates.

The plastic moment, M_p, is the limiting value of the moment which would be reached upon applying an infinite curvature to a section, neglecting the effect of strain-hardening. Usually it is calculated as the product of the girder's yield stress and plastic modulus, Z. This method assumes a section whose yield stress is constant for all its elements. As such, it can not be used in computations involving the test girders, most of whose component parts yielded at different stress levels. Therefore, this moment will be evaluated from the relation that $M_p = \Sigma A \sigma_y y_p$, where the A and σ_y are the area and yield stress of a section's elements and y_p is the distance from the plastic neutral axis, NA_p, to the centroid of each element.

COMPUTATION OF PLASTIC MOMENT OF E2



$$[\Sigma A\sigma_y]_a - [\Sigma A\sigma_y]_b = 0$$

$$16.11 \times 29.4 + 9.37 \times 38.6 + (0.507y_a)34.9 - 0.507(50-y_a)34.9 - 9.44 \times 37.6 - 16.11 \times 29.4 = 0$$

$$474 + 362 + 17.69y_a - (885 + 17.69y_a) - 355 - 474 = 0$$

$$y_a = 24.8 \text{ in.}$$

$$M_p = \Sigma(A\sigma_y)y_p$$

$$= 474 \times 26.07 + 362 \times 25.19 + 439 \times 12.4 + 446 \times 12.6 + 355 \times 25.59 + 474 \times 26.48$$

$$\underline{M_p = 54{,}100 \text{ k-in}}$$

This plastic neutral axis is found from the equilibrium condition that the sum of the normal forces over the entire cross section must vanish. Using the subscripts a and b mentioned before, this condition is expressed as $[\Sigma A\sigma_y]_a - [\Sigma A\sigma_y]_b = 0$. As a sample computation, the plastic moment of E2 has been calculated above. All necessary static yield stresses are listed in Table 4.

To calculate the yield and plastic loads, the spans of the girders enter. Again, due to the different test setups, three groups are distinguished: bending, shear, and combined loading.

The bending group has a constant moment over the test section, $M = 150P$. Thus the yield and plastic loads are simply computed as $P_y = M_y/150$ and $P_p = M_p/150$, where the moments are expressed in kip-inches and the loads in kips.

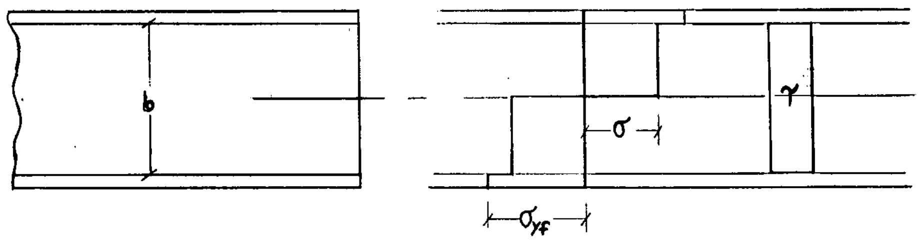
The shear group, although subjected to a relatively small variable moment, was considered to be under pure shear. Therefore, the yield load is the load that initiates nominal yielding at the neutral axis in the web and is computed from $V_y = \tau_{yw}It/Q$ where V_y is the shear force at first nominal yielding, τ_{yw} the shear yield stress of the web, Q and I are the static moment and moment of inertia about the neutral axis, and t is the thickness of the web. From Fig. 2 it can be seen that V = P. Substituting this value in the preceding equation and using the Mises yield condition that $\sigma_{yw} = \sqrt{3}\,\tau_{yw}$, the yield load will be evaluated as $P_y = \dfrac{\sigma_{yw}It}{\sqrt{3}\,Q}$ where σ_{yw} is the yield stress of the web. The plastic load is defined as the load which causes the web to completely yield due to shear, $P_p = \sigma_{yw}A_w/\sqrt{3}$, A_w being the area of the web.

The combined group was subjected to both shear and moment. Since the moment varied throughout the girder's length, a cross section in the failed panel was selected at which the reference loads for each test were to be evaluated. This section was chosen to be at a distance of one-half the web depth away from the maximum moment in the panel or at the middle of the panel when its length is less than its depth. It is realized that this method of evaluating the yield and plastic loads differs from the usual procedures used for a beam. Thus, the yield load is defined as the load which initiates yielding in the critical cross section of a girder. In general, yielding first occurs at the intersection of the web and flange where the yield condition, $\sigma_{yw} = \sqrt{\sigma^2 + 3\tau^2}$, is used to evaluate the yield load. Substituting the values of $\sigma = \frac{MY}{I} = \frac{Px}{2} \cdot \frac{25}{I}$ and $\tau = \frac{VQ}{It} = \frac{PQ}{2It}$ into the equation above, where M = Px/2 and V = P/2 from Fig. 3, the yield load for the interaction girders will be

$$P_y = \frac{\sigma_{yw}}{\sqrt{(25x/2I)^2 + 3(Q/2It)^2}},$$

x being the distance from the end of the girder span to the critical cross section. If yielding does not begin at the aforementioned point, the bending or shear case discussed before applies.

The plastic load of any single test on a girder is defined as the load producing plastification at the critical cross section of the failed panel. The presence of shear in the combined group of girders reduced their full plastic moments M_p. For these girders, an expression for a modified plastic moment M_{ps} was developed from considerations which follow. The stress condition sketched on the following page is the basis for evaluating M_{ps}, (see Ref. 2). It will be assumed that the flanges have fully yielded,



thereby providing the flange moment, M_f, and that a constant normal stress σ is present over the web accompanied by the constant shearing stress τ. From the sketch, the modified moment M_{ps} is $M_{ps} = M_f + \frac{\sigma t b}{2} \cdot \frac{b}{2}$. An expression for σ is obtained from the yield criterion $\sigma_{yw} = \sqrt{\sigma^2 + 3\tau^2}$, where $\tau = \frac{V}{bt} = \frac{M_{ps}}{btx}$. Substituting the value of σ in the first equation yields $M_{ps} = M_f + \frac{1}{4} tb^2 \sqrt{\sigma_{yw}^2 - 3(M_{ps}/btx)^2}$. After solving for M_{ps} and observing that $M_{ps} = P_p x/2$,

$$P_p = \frac{2}{xa}\left[M_f + \sqrt{aM_w^2 - (a-1)M_f^2}\right]$$

where a is a constant, $a = 1 + \frac{3}{16}\left(\frac{b}{x}\right)^2$, and M_w is the portion of the full plastic moment M_p contributed by the web, $M_w = \sigma_{yw} tb^2/4$. When a negative number results under the radical sign, the shear case explained before must be used to obtain P_p. Physically this result implies that the web yields due to shear before the yield stress is reached in the flanges.

In Table 5 are summarized the reference moments and loads for all test girders. Unlike the bending and shear groups, which had constant moments and shears over their test sections, the combined group had a variable moment over its test sections which results in two or more yield and plastic loads for each girder.

4. Web Buckling Stresses

An additional reference value with which the obtained ultimate load can be compared is the conventionally computed web buckling stress or load. It is the objective of this section to establish these stresses and loads for all the girders.

The general equation for the ideal critical stress of an isolated web panel is

$$\left.\begin{matrix}\sigma_{cri}\\ \tau_{cri}\end{matrix}\right\} = k\,\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 = k\,\frac{\pi^2 E}{12(1-\nu^2)}\cdot\frac{1}{\beta^2}$$

where σ_{cri} and τ_{cri} are the ideal critical normal and shearing stresses, respectively. The factor $\frac{\pi^2 E}{12(1-\nu^2)}$ is a constant dependent only on the material properties, i.e., the modulus of elasticity E and Poisson's ratio ν, while $\beta = b/t$ is the slenderness ratio of the web. Finally, the buckling coefficient k is a variable depending on the loading and boundary conditions and, in general, also on the panel's aspect ratio $\alpha = a/b$. Values for this coefficient can be found in such literature as Ref. 3, 4, and 5.

Before presenting computations, some detail information must be specified in order that the web buckling stresses of the actual girders can be computed. In general, the procedures of Ref. 6 and 7 have been adopted for the details which follow:

The constant, $\frac{\pi^2 E}{12(1-\nu^2)}$, for steel plates is equal to 26,750 ksi.

Web panels are considered pin ended on all sides.

The proportional limit of the web material is taken as $\sigma_p = 0.8\sigma_y$. If the ideal critical stress σ_{cri} is less than σ_p, it is equal to the critical stress σ_{cr}, $\sigma_{cri} = \sigma_{cr}$. Whenever it exceeds this value, the critical stress σ_{cr} is found from a reduction procedure, $\sigma_{cr} = \sigma_y(1 - \frac{0.16\sigma_y}{\sigma_{cri}})$, a relation derived from Eq. (64), Ref. 6. Similarly $\tau_{cr} = \tau_y(1 - \frac{0.16\tau_y}{\tau_{cri}})$, where τ is the shear stress.

When a moment gradient exists in a panel, the critical section is considered to be at a distance of one-half the web depth from the maximum moment in the panel. In the case where the panel's length is smaller than its depth, this section is at the middle of the panel.

The critical shear force of a panel subjected to pure shear is computed as the product of the critical shear stress and the area of the web, $V_{cr} = \tau_{cr} A_w$.

In all cases, the unsupported web depth "b" is taken as the clear web depth, which is 50 inches for all girders.

Finally, when the neutral axis is equal to or less than 1/2 inch away from the web's geometric center, it is assumed to coincide with the centerline.

The general cases of bending, shear, and combined loading are presented next.

For bending, the general formula for the normal critical stress σ_{cri} applies. The k-value is the only remaining unknown. Since all the girders except G1-T2 had their neutral axes less than 1/2 inch away from the web's geometric center, the k-value is k = 23.9. In G1-T2 the neutral axis shifted 3.62 inches down from the web's centerline and thus the compressive stresses are higher than the tensile stresses. In accordance with Ref. 7, this leads to a k-value of k = 18.6. The critical load P_{cr} is determined from the relation that $\sigma_{cr} = M_{cr}Y/I$, where $M_{cr} = P_{cr}x/2$, see Fig. 1.

For shear, the general formula for the critical shearing stress τ_{cri} applies. The buckling coefficient of $k = 4.00 + \frac{5.34}{\alpha^2}$ is used when the panel's length/depth ratio α is equal to or less than unity, and $k = 5.34 + \frac{4.00}{\alpha^2}$ is used when $\alpha \geq 1$. In this case, P_{cr} is evaluated from $\tau_{cr} = V_{cr}/bt$, where $V_{cr} = P_{cr}$, see Fig. 2.

For combined bending and shear,

$$\sigma_{vcri} = \frac{\sqrt{\sigma^2+3\tau^2}}{\sqrt{(\sigma/\sigma_{cri})^2+(\tau/\tau_{cri})^2}} \quad ,$$

where σ_{vcri} is the equivalent ideal critical stress for combined loading (Vergleichsspannung), σ the extreme bending stress of the web $\sigma = MY/I$, and τ the average shearing stress $\tau = V/bt$. M and V are the applied moment and shear in the considered cross section respectively. The formula above is only applicable to girders whose neutral axes coincide with their web's geometric centers and as such applies to all the girders of the combined group whose axes were all less than 1/2" away from the centerline of the web. After reducing the ideal stress for inelastic action, if necessary, the critical load P_{cr} is obtained from the equation $\sigma_{vcr}=\sqrt{\sigma^2+3\tau^2}$, where σ and τ are both functions of P, the applied load.

As an example, the critical load of the first test of E2, a girder under combined loading, shall be computed. From the previous tables and figures the properties of E2-T1 are as follows: $\alpha = 3.0$, $\beta = 99$, $A_w = 25.3$ in^2, $I = 39{,}620$ in^4 and $\sigma_{yw} = 34.9$ ksi. From Table 1 it is seen that the long panel failed in this first test. Since a moment gradient exists in this panel, the critical cross section is at a distance of 25" to the left of the center bearing stiffener or x = 125" from the end of the girder. Knowing this data and using $k_\sigma = 23.9$ and $k_\tau = 5.79$ $(\alpha>1)$, the critical stresses due to moment and shear are evaluated as $\sigma_{cri} = 65.2$ ksi and $\tau_{cri} = 15.8$ ksi. The stress at the compressive edge of the web is $\sigma = \frac{MY}{I} = \frac{P.125}{2.39{,}620}\,25 = 0.0394P$ [ksi], and the average shearing stress over the section is $\tau = \frac{V}{A_w} = \frac{P}{2\cdot 25.3} = 0.0198P$ [ksi]. Substituting these values into the equation for the combined critical stress,

$$\sigma_{vcri} = \frac{\sqrt{(0.0394P)^2+3(0.0198P)^2}}{\sqrt{\left(\frac{0.0394P}{65.2}\right)^2+\left(\frac{0.0198P}{15.8}\right)^2}} = \frac{0.0522P}{0.00139P} = 37.5 \text{ ksi}$$

The proportional limit is $\sigma_p = 0.8\sigma_y = 0.8\text{x}34.9 = 27.9$ ksi.

Since $\sigma_{vcri} > \sigma_p$, the reduction for the inelastic range applies as follows:

$$\sigma_{vcr} = 34.9\left(1 - 0.16\,\frac{34.9}{37.5}\right) = 29.8 \text{ ksi}$$

Since $\sigma_{vcr} = \sqrt{\sigma^2+3\tau^2} = 0.0522P_{cr}$, $P_{cr} = 29.8/0.0522$, or

$$\underline{P_{cr} = 570 \text{ kips}}$$

In Table 6 and 7 are summarized the critical stresses and loads for all girders. The bending and shear groups never exceed the elastic limit and thus $\sigma_{cri} = \sigma_{cr}$ and $\tau_{cri} = \tau_{cr}$. The center panel of E5-T1 was subjected to pure moment and therefore no entries are made under the columns for shear.

5. Deflections

In order to check on the elastic behavior of the girders, their predicted deflections are evaluated in this section. Due to the different loading conditions, the girders can be divided into the groupings decided on in Sec. 1, i.e., bending, shear, and combined loading girders. The centerline deflections are obtained for the bending and combined groups while the end deflections are given for the shear girders.

The method of Virtual Work is used to obtain all deflections. In this method a unit load is applied to the girder at the point where the deflection is desired and its resulting moment, m, and shear, v, diagrams drawn. Then the deflection under this "dummy" load is computed as the sum of the bending and shear contributions:

$$v = \int \frac{Mm}{EI}\, dx + \int \frac{Vv}{GA_w}\, dx$$

In this expression M and V are the moment and shear due to the actual loading, E = 30,000 ksi is the modulus of elasticity, and G = 11,530 ksi is the shearing modulus. All integrals extend over the entire girder length where the origin of the x-axis is taken at the end of each girder span. BJ: high?

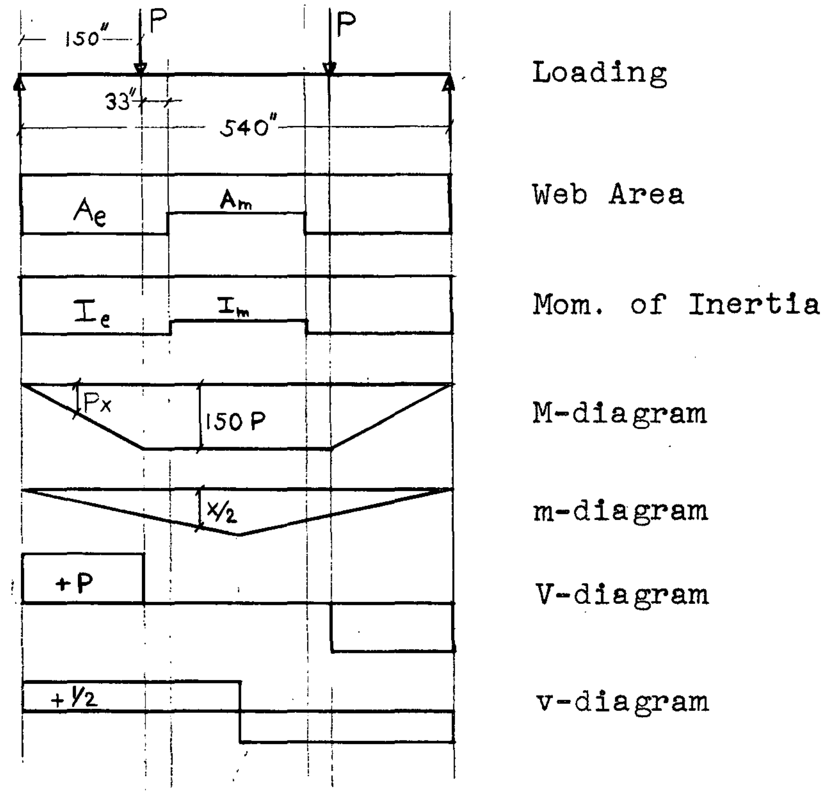
The units for all quantities and dimensions are kips and inches, exclusively.

To illustrate the procedure followed in calculating deflections, the expression for the centerline deflections of the bending girders is developed next. In the example shown on the next page, the loading is first pictured together with sketches of the areas and moments of inertia of the girders. Then the moment and shear diagrams, both for the real and dummy loadings, are drawn. The integrals are next written down with the first three integrals representing the moment component and the last including the shear contribution. Observing the symmetry of the loading and cross sectional properties, the integrals need only be evaluated over half the length of the girder and then doubled to obtain the final value.

Substituting the properties of G1-T1, into the resulting equation (a), where I_e, I_m, and A_e equal 15,550 in^4, 14,380 in^4, and 19.10 in^2, respectively, the centerline deflection for the applied load of P = 100 kips would be 1.172 inches. In this case the shear component is 5.8% of the total deflection.

Using the same procedure as above, the equations needed to evaluate all required deflections are obtained. These expressions are listed on page 22 together with the cases to which they apply.

Centerline Deflection of Bending Girders



$$v_{\mathfrak{C}} = \int \frac{Mm}{EI}\,dx + \int \frac{Vv}{GA_w}\,dx$$

$$= 2\left[\int_0^{150} \frac{(Px)(x/2)}{EI_e}\,dx + \int_{150}^{183} \frac{(150P)(x/2)}{EI_e}\,dx + \int_{183}^{270} \frac{(150P)(x/2)}{EI_m}\,dx + \int_0^{150} \frac{(P)(1/2)}{A_wG}\right]$$

$$= 2x10^3\left[\frac{562.5P}{EI_e} + \frac{412.1P}{EI_e} + \frac{1478P}{EI_m} + \frac{0.0750P}{A_wG}\right]$$

$$v_{\mathfrak{C}} = P\left[64.97/I_e + 98.53/I_m + 0.01301/A_e\right] \qquad \text{(a)}$$

All bending girders, (except G1), were reinforced with steel plates after their first test. With a new cross section present, whose moment of inertia I is listed in the "special sections" of Table 3, the expression for the centerline deflection in the second test is:

$$v_{\text{¢}} = P(64.97/I_e + 54.93/I_m + 43.59/I + 0.01301/A_e) \quad \text{(b)}$$

The shear girders have a maximum deflection at their ends. Observing that these girders have cover plates at their reaction points, thus having special moments of inertia I, the equation for end deflections is:

$$v_e = P(5.689/I_e + 7.316/I_m + 25.39/I + 0.0132/A_e + 0.0075/A_m) \quad \text{(c)}$$

Girder E1, the first of the girders under combined loading, had its second cover plates terminated 75 inches from its ends. Letting the moments of inertia of the section with and without the second cover plates be I_m and I, the relation for centerline deflection is:

$$v_{\text{¢}} = P(19.88/I_m + 2.344/I + 0.00689/A_w) \quad \text{(d)}$$

All other girders under combined loading had constant cross sections throughout their lengths. As such, the centerline deflections can be found from Eq. (d) by substituting $I = I_m$. The resulting equation is:

$$v_{\text{¢}} = P(22.23/I_m + 0.00689/A_w) \quad \text{(e)}$$

A summary of girder deflections, computed from the above equations, is given in Table 8. Here, the bending and shear components of the total deflection are listed, together with the equation that is applied to determine them. The centerline deflection is given for the bending and combined groups while the end deflection is listed for the shear girders. As a matter of interest, the percentage of the shear contribution to the total deflection is included. All deflections are evaluated for P = 100 kips.

6. Acknowledgements

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Part 1 contains the specifications, Part 2 gives additional recommendations.

8. List of Nomenclature

1. Capital Letters - preferably used for quantities which do not have linear dimensions

A : Area of cross section

E : Modulus of elasticity, 30,000 ksi

G : Girder, used with a number, e.g., G2 refers to girder No. 2; also shear modulus, 11,530 ksi

I : Moment of inertia

M : Bending moment

NA : Neutral axis

P : Applied load

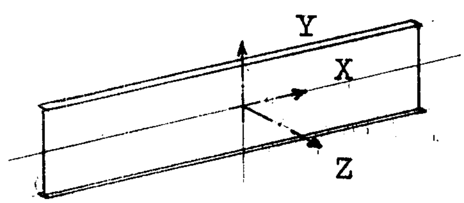
Q : Statical moment of area

S : Section modulus

T : Test, used with a number, e.g., T1 refers to the first test on a girder

V : Shear force

X,Y,Z : Cartesian coordinates having their origin in the middle of the girder



2. Small Letters - preferably used for linear dimensions

a : Panel length

b : Panel depth, b = 50" for all girders

c : One-half the flange width

d : Flange thickness

e : Distance from NA to the extreme fiber of the flange

h : Distance between the centroids of the flanges

k : Buckling constant

t : Web thickness

u,v,w : Displacements in the X,Y,Z directions

x : Longitudinal coordinate with origins at either end of a girder's span

BJ: ? b, d, t, w

GW seconded

Bill likes this, but could shift in design

3. Greek Letters - used for nondimensional parameters and stresses

$\alpha = a/b$: Panel length to panel depth

$\beta = b/t$: Web depth to web thickness

ν : Poisson's ratio (= 0.3)

σ : Normal stress

τ : Shear stress

4. Subscripts

?

a : Above, e.g.	S_a : Section modulus above NA
b : Below, e.g.	e_b : Distance from NA to extreme fiber of bottom flange
cr : Critical, e.g.	σ_{cr} : Critical normal stress
℄ : Centerline, e.g.	$v_℄$: Centerline deflection
e : End, e.g.	I_e : Moment of inertia of end sections

f : Flange, e.g.	M_f	:	Moment contributed by flanges
i : Ideal, e.g.	τ_{cri}	:	Ideal critical shearing stress before inelastic reductions
m : Middle or test, e.g.	I_m	:	Moment of inertia of middle or test section
n : Neutral axis, e.g.	I_n	:	Moment of inertia about NA
p : Plastic, e.g.	P_p	:	Load causing the plastic moment
v : Combined, e.g.	σ_{vcr}	:	Critical stress under combined loading
w : Web, e.g.	A_w	:	Area of the web
y : Yield, e.g.	σ_{yw}	:	Yield stress of web

Table 1 - Summary of Girder Parameters

Girder	Loading	Cross Section	Web Slenderness: β	Stiffener Spacing: α T1 / T3	Stiffener Spacing: α T2 / T4
G1	Pure Bending	I	185	1.50	0.75
G2	Pure Bending	II	185	1.50	0.75
G3	Pure Bending	III	185	1.50	0.75
G4	Pure Bending	II	388	1.50	0.75
G5	Pure Bending	III	388	1.50	0.75
G6	Shear	II	259	1.50 / 0.50	0.75
G7	Shear	II	255	1.00	1.00
E1	Combined Bending and Shear	IV	131	3.00 / 1.50	1.50 / 1.00
E2	Combined Bending and Shear	V	99	3.00	1.50
E4	Combined Bending and Shear	VI	128	1.50 / 0.50	0.75
E5	Combined Bending and Shear	III	128	0.36	0.75
G8	Combined Bending and Shear	II	254	3.00 / 1.50	1.50 / 1.00
G9	Combined Bending and Shear	II	382	3.00 / 1.50	1.50

Location of Failure ~~ and Reinforcements --- (T1, T3 | T2, T4)

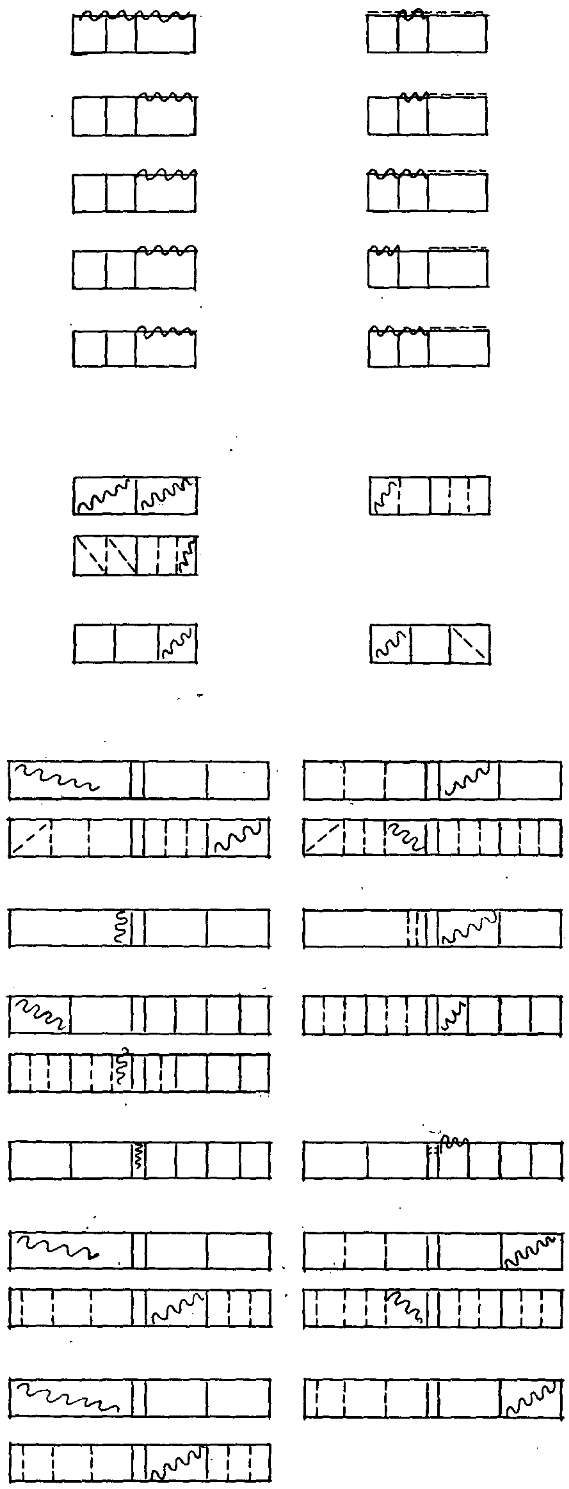


Table 2

Summary of Cross Sectional Dimensions in inches

Girder	Cross Sect.	Top Flange Width 2c	Top Flange Thickness d	Bottom Flange Width 2c	Bottom Flange Thickness d	Web Thickness Test t	Web Thickness End t
G1	I	20.56	0.427	12.25	0.760	0.270	0.382
G2	II	12.19	0.769	12.19	0.774	0.270	0.507
G3	III	8.62	0.328	12.19	0.770	0.270	0.492
G4	II	12.16	0.774	12.19	0.765	0.129	0.392
G5	III	8.62	0.328	12.25	0.767	0.129	0.392
G6	II	12.13	0.778	12.13	0.778	0.193	0.369
G7	II	12.19	0.769	12.19	0.766	0.196	0.381
E1	IV	20.56	0.427	12.25	0.760	0.382	---
E2	V	12.19	0.769	12.19	0.774	0.507	---
E4	VI	12.16	0.774	12.19	0.765	0.392	---
E5	III	8.62	0.328	12.25	0.767	0.392	---
G8	II	12.00	0.752	12.00	0.747	0.197	---
G9	II	12.00	0.755	12.00	0.745	0.131	---

Cover Plates:
- PL A: 15.04 x 0.882, PL C: 16.00 x 1.007
- PL B: 18.00 x 0.750, PL D: 11.19 x 0.510

Table 3

Summary of Moments of Inertia and Section Moduli

see p 7

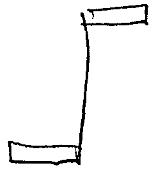
Girder	Test Section I_m in^4	Test Section S_a in^3	Test Section S_b in^3	End Sect. I_e in^4	Spec. Sect. I in^4
G1	14,380	555	568	15,550	12,210
G2	14,940	578	581	17,400	16,170
G3	16,220	488	620	18,530	17,790
G4	13,420	522	519	16,160	14,640
G5	14,710	443	561	17,450	16,950
G6	14,180	550	550	16,010	23,750
G7	14,100	548	547	16,030	23,760
E1	52,920	1,922	1,968	----	33,670
E2	39,620	1,480	1,485	----	----
E4	34,390	1,292	1,287	----	----
E5	17,480	524	672	----	----
G8	13,640	531	528	----	----
G9	12,960	505	501	----	----

Table 4

Summary of Static Yield Stresses
(kips per square inch, ksi)

Girder	Cross Sect.	Flanges Top	Flanges Bottom	Webs Test	Webs End
G1	I	35.4	35.8	33.0	41.7
G2	II	38.6	37.6	35.3	34.9
G3	III	35.5	38.1	33.7	37.3
G4	II	37.6	37.0	43.4	40.0
G5	III	35.5	37.0	45.7	40.0
G6	II	37.9	37.9	36.7	--
G7	II	37.6	37.6	36.7	--
E1	IV	35.4	35.8	41.7	--
E2	V	38.6	37.6	34.9	--
E4	VI	37.6	37.0	40.0	--
E5	III	35.5	37.0	40.0	--
G8	II	41.3	41.3	38.2	--
G9	II	41.8	41.8	44.5	--

Cover Plates { PL A: 30.1, PL C: 29.4
PL B: 29.8, PL D: 33.5 }



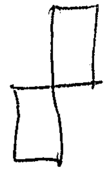


Table 5

Summary of Reference Moments and Loads

Girder	Test	M_f k-in	M_y k-in	M_p k-in	P_y kips	P_p kips
G1	T1	15,700	19,600	21,900	131	148
	T2	10,400	15,100	18,700	101	118
G2	T1,T2	18,400	22,300	24,200	149	167
G3	T1,T2	16,600	17,300	23,600	116	156
G4	T1,T2	18,000	19,600	21,200	130	139
G5	T1,T2	16,500	15,700	21,300	105	134
G6	T1,T2,T3	18,200	20,800	22,600	193	205
G7	T1,T2	17,900	20,600	22,300	196	208
E1	T1,T2,T4	58,000	60,000	68,600	826	920
	T3	36,600	40,700	47,100	905	920
E2	T1,T2	43,400	48,500	54,100	716	855
	T1				880	905
E4	T2	39,100	43,000	48,600	658	691
	T3				639	666
E5	T1	16,500	18,600	27,500	248	367
	T2				358	386
G8	T1,T3,T4	18,900	21,900	23,600	280	368
	T2				410	434
G9	T1,T3	19,200	21,100	22,700	264	354
	T2				324	336

Table 6

Summary of Critical Stresses and Loads

Bending Girders

Girder	Test	α	β	k	σ_{cr} ksi	P_{cr} kips
G1	T1	1.50	185	23.9	18.7	70.1
	T2	0.75		18.6	14.5	41.9
G2	T1	1.50	185	23.9	18.7	74.1
	T2	0.75				
G3	T1	1.50	185	23.9	18.7	82.1
	T2	0.75				
G4	T1	1.50	388	23.9	4.25	15.3
	T2	0.75				
G5	T1	1.50	388	23.9	4.25	17.0
	T2	0.75				

Shear Girders

Girder	Test	α	β	k	τ_{cr}	P_{cr}
G6	T1	1.50	259	7.12	2.84	27.4
	T2	0.75		13.5	5.38	51.9
	T3	0.50		25.4	10.1	97.6
G7	T1	1.00	255	9.34	3.84	37.6
	T2	1.00				

Table 7

Summary of Critical Stresses and Loads

Girders under Combined Loading

Girder	Test	α	β	Bending k	Bending σ_{cri} ksi	Shear k	Shear σ_{cri} ksi	Combined σ_{vcri} ksi	Combined σ_{vcr} ksi	P_{cr} kips
E1	T1	3.00	131	23.9	37.2	5.79	9.01	18.0	18.0	332
	T2	1.50				7.12	11.1	21.8	21.8	402
	T3	1.50				7.12	11.1	20.3	20.3	415
	T4	1.00				9.34	14.6	27.4	27.4	506
E2	T1	3.00	99	23.9	65.2	5.79	15.8	37.5	29.8	570
	T2	1.50				7.12	19.4	44.1	30.4	584
E4	T1	1.50	128	23.9	39.0	7.12	11.6	21.3	21.3	445
	T2	0.75				13.5	22.0	38.5	33.4	513
	T3	0.50				25.4	41.4	46.9	34.5	517
E5	T1	0.36	128	23.9	39.0	--	--	39.0	33.4	314
	T2	0.75				13.5	22.0	38.8	33.4	322
G8	T1	3.00	254	23.9	9.91	5.79	2.40	5.99	5.99	41.5
	T2	1.50				7.12	2.96	5.59	5.59	56.4
	T3	1.50				7.12	2.96	6.96	6.96	48.3
	T4	1.00				9.34	3.88	8.26	8.26	57.3
G9	T1	3.00	382	23.9	4.38	5.79	1.06	2.32	2.32	12.9
	T2	1.50				7.12	1.31	2.36	2.36	16.8
	T3	1.50				7.12	1.31	2.78	2.78	15.5

Table 8

Summary of Girder Deflections
(in inches and under nominal load P=100 kips)

Girder	Test	Eq. Used	Deflections due to Bending	Shear	Total	% Shear/Total
G1	T1	(a)	1.104	0.068	1.172	5.8
	T2	(a)	1.225		1.293	5.3
G2	T1	(a)	1.033	0.051	1.084	4.7
	T2	(b)	1.008		1.059	4.8
G3	T1	(a)	0.958	0.053	1.011	5.2
	T2	(b)	0.931		0.984	5.4
G4	T1	(a)	1.136	0.066	1.202	5.5
	T2	(b)	1.102		1.168	5.6
G5	T1	(a)	1.042	0.066	1.108	6.0
	T2	(b)	1.000		1.066	6.2
G6	All	(c)	0.194	0.150	0.344	43.6
G7	All	(c)	0.194	0.147	0.341	43.1
E1	All Tests	(d)	0.045	0.034	0.079	43.1
E2	All Tests	(e)	0.056	0.026	0.082	31.7
E4	All Tests	(e)	0.064	0.033	0.097	34.0
E5	All Tests	(e)	0.127	0.033	0.160	20.6
G8	All Tests	(e)	0.163	0.066	0.229	28.8
G9	All Tests	(e)	0.172	0.099	0.271	36.5

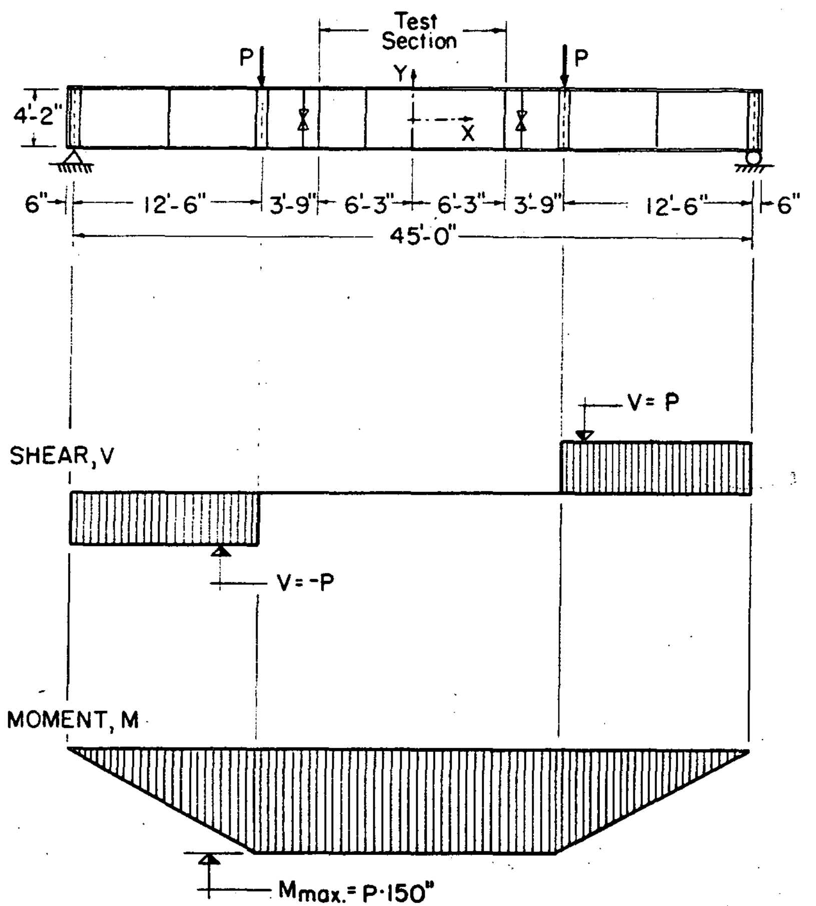


Fig. 1 - Test Setup of Bending Girders

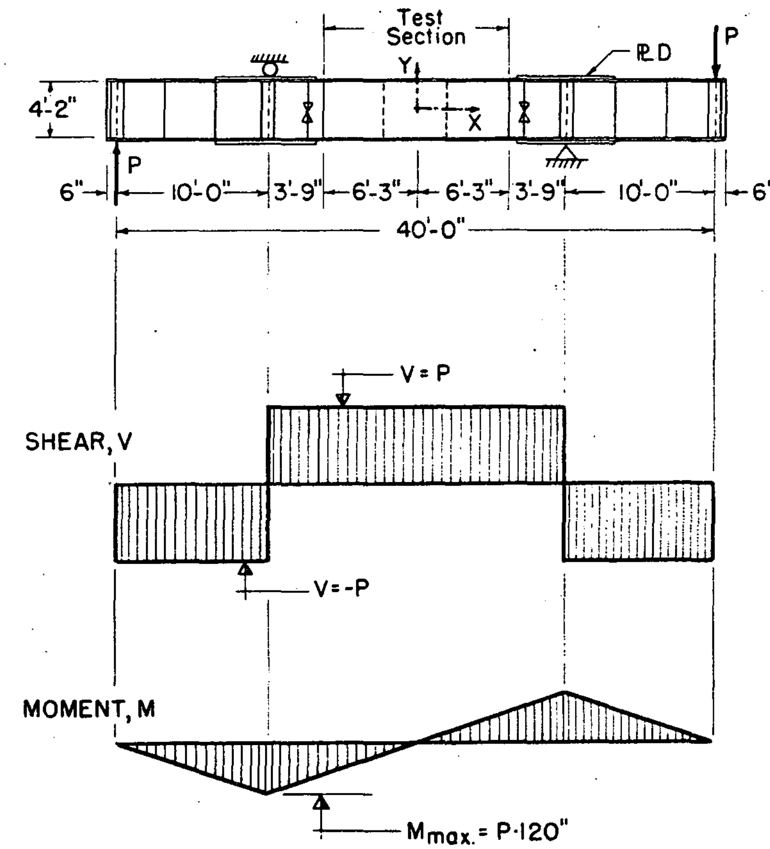


Fig. 2 - Test Setup of Shear Girders

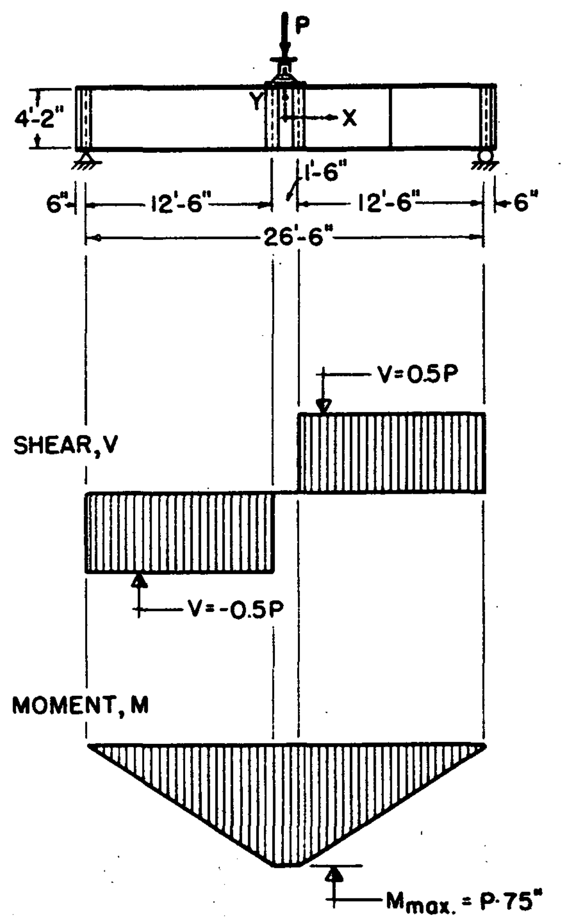


Fig. 3 - Test Setup of Interaction Girders

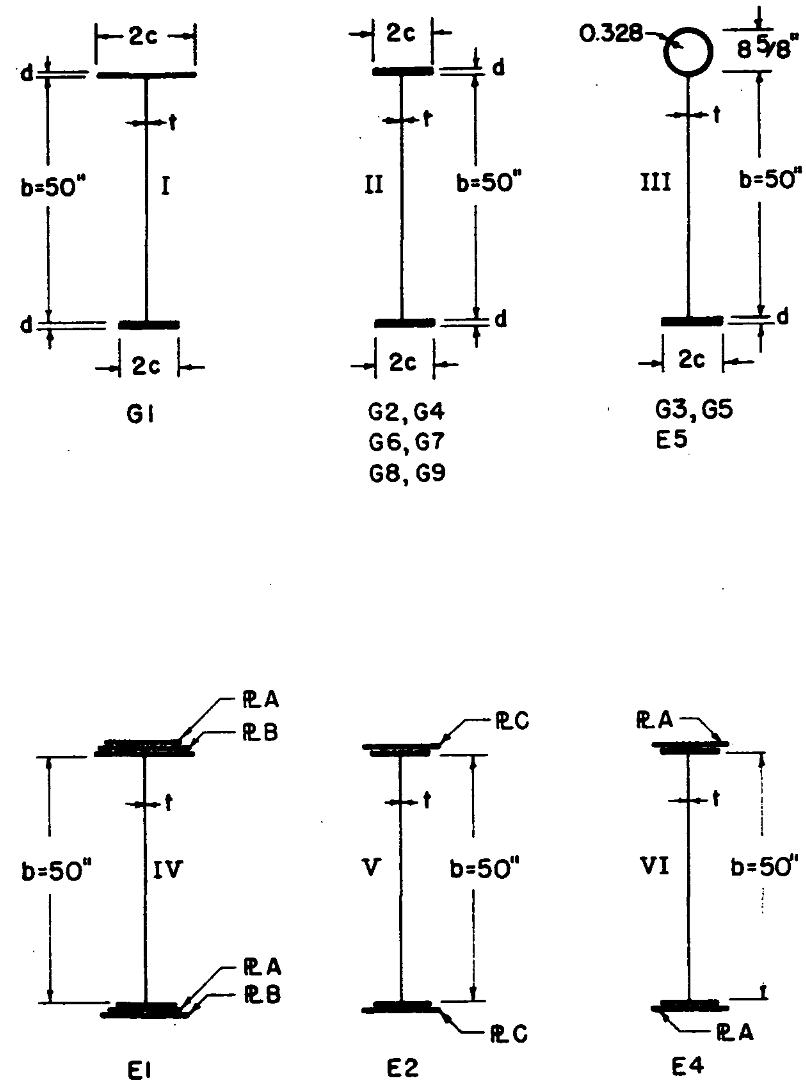


Fig. 4 - Girder Cross Sections