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Welded Built-up Columns

EFFECT OF STRAIN RATE ON THE YIELD STRESS OF STRUCTURAL STEELS

by

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ABSTRACT

This report summarizes the results of an investigation into the influence of strain rate on the yield stress of three structural steels - A36, A441 and T-1. Tensile coupons were tested to obtain the experimental results. A "static yield stress level" is used to eliminate the influence of strain rate. A relationship between the ratio of dynamic yield stress level to static yield stress level and strain rate has been established; a relationship expressing the difference between these two stress levels and strain rate is also developed as an approximation to the first relationship. Confidence limits have been found using variance analysis. A method to estimate the static yield stress level from mill test data is proposed.

CONTENTS

1. INTRODUCTION

1.1 Purpose and Scope

The yield stress of steel is affected directly by the rate of straining. This may be regarded as a property of steel; this phenomenon has been studied and observed on numerous occasions. Generally speaking the greater the speed of straining, the higher the yield stress tends to become until the limit when the ultimate load is reached without yielding.[1]

Since a particular type of steel could have an infinite number of such values depending on the definition, the speed of testing of a coupon is of the utmost importance when defining yield stress. Actually there are many definitions for the yield stress and justification exists, to a greater or lesser degree, for using any particular value in design. Specifications do not take account of the size effect in the coupons, and the differences in testing machines. Although the ASTM has tentative specifications limiting the maximum testing speed, it would appear that some investigators use lower speeds than others with the result that discrepancies as high as 20% exist in the measured value for yield stress. In addition the testing speed and strain rate are themselves two different quantities without a defined relationship. Hence the use of the term "yield stress" has limited value, unless it is qualified by a strain rate.

However, it should be noted that strain rate does not account for all the variation between tests; it cannot account for material differences or manufacturing methods. The difference due to chemistry and manufacturing procedures can be evaluated more clearly if these superimposed artificial discrepancies of strain rate are removed.

In this report the influence of strain rate on the yield stress is studied. Tensile coupons of ASTM A36 and A441 designation steels and of one quenched and tempered (Q-T) steel* were used. These coupons were tested in connection with various research projects on columns, beams, beam-columns and frames. The coupons were of the plate type and were cut from lengths of structural shapes in the as-rolled condition, which were not subjected to straightening operations in the mill. In all the tests, the strain rate was measured in the plastic range.

1.2 Definitions

Some of the terms used to define the strength of steel are "yield point", "yield strength" and "yield stress level". ASTM definitions for yield point and yield strength are given in Section 6. However, in this report, the following terms are used - "(upper) yield point", "lower yield point","(dynamic) yield stress level" and "static yield stress level". These terms are defined in Section 6 and are also shown in Fig. 1.

Both the upper and lower yield points are used as the basis for designating the yield stress. Indeed it is common practice in the testing of coupons to record the "yield" as the highest reading indicated by the free "follower" pointer on the load indicator dial, the actual

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*ASTM has no designation for this type of steel. Some of the trade names are T-1, NAX-90, etc. T-1 steel was used in this study. The abbreviation Q-T will be used hereafter.

load having dropped somewhat. In this report emphasis is given to the static yield stress level and dynamic yield stress level. By its definition, the static yield stress level is independent of the speed of testing. It is also not affected by size of specimen, testing machine, etc. (Most loads on structures such as buildings are static.) Therefore it is a more uniform standard for comparison. The dynamic yield stress level may be defined for materials that show no increase in stress, with increase in strain when testing at a constant strain rate. This depends on the speed of testing.

It is impossible to test a coupon at zero strain rate and obtain a stress-strain diagram. However, a method for obtaining the stress corresponding to zero strain rate will be described in Section 2. Assuming that the strain rate is the only factor that influences the dynamic yield stress level, a relationship between the strain rate and the ratio of dynamic yield stress level to static yield stress level can be established; for convenience this ratio will be termed "dynamic yield stress ratio". This ratio will always be greater than unity.

1.3 Influencing Factors

As mentioned in Section 1.2, the dynamic yield stress level is influenced by the speed of testing, size of specimen, testing machine, etc. In this report the emphasis is essentially on the speed of testing. The speed of testing can be defined in two different ways - "rate of separation of crosshead" and "strain rate"; and these have entirely different meanings.

Elastic Range

The schematic diagram of a testing machine is shown in Fig. 2. Assuming that the crosshead separation speed, V is constant, the following equation for the strain rate in the elastic range can be obtained:[2]

$$\dot{\epsilon} = \frac{V}{L} \cdot \frac{1}{\left(1 + \frac{EAL_1^3}{24E_1I_1L} + \frac{EAL_2}{2E_2A_2L} + \frac{EAL_3}{2E_3A_3L}\right)} \qquad (1)$$

where $\dot{\epsilon}$ is the elastic strain rate; other symbols are defined in Fig. 2. In deriving this equation it is assumed that all the deformations are elastic and that there is no slip in the grips, etc. Thus, the strain rate is a function of the crosshead speed, (V), the testing machine, (E_1, I_1, A_1, L_1) and the specimen size (A,L). For a given value of V, the strain rate will have different values depending on the testing machine and specimen size. This relationship is shown in Fig. 3 for a 60 kip and a 300 kip testing machine, for a V of 1/16 in. per minute per inch of gage length.[2] The curves show that for specimens of thickness between 1/8 in. and 2 in. the actual strain rate may vary from about 100 microinch per in. per sec to 600 microinch per in. per sec.

The ASTM standards introduce another problem that concerns the speed of testing. ASTM specifies the rate of separation of crossheads, since the specification of strain rate is not a practicable method of controlling machines currently used in production testing. In section 10d (1) of ASTM A370-61T, the rate of separation of crossheads is limited to 1/16 in. per min per inch of gage length for coupons with reduced cross sections. It is also prescribed that the rate of separation of the crossheads under load shall not exceed 1/2 in. per min per inch of gage length. (This actually defines the upper limit of the speed of testing.) In section 10d (3) of ASTM A370-61T the speed of testing is limited by the rate of stressing of 100,000 psi per min.

Suppose that the rate of separation of crossheads of 1/16 in. per min. per inch of gage length is used. For a 0.505 in. diam. coupon of 2 in. gage length, the crosshead separation speed is 1/8 in. per min, and for a plate type coupon of 8 in. gage length, the crosshead separation speed is 1/2 in. per min. Here it is assumed that the two coupons will have the same strain rate of 1/16 in. per inch per min or 1042 microinch per inch per sec. If the rate of stressing of 100,000 psi per min. is used as the criterion, the corresponding calculated strain rate would be 55 microinch per in. per sec. In the foregoing calculation it is assumed that all of the deformation goes into the specimen in the indicated gage length. Thus the two criterions are too far apart.

Plastic Range

The strain induced in a specimen in the elastic range was shown to be influenced by the response of the testing machine. In the plastic range all the extension is absorbed by the specimen and the testing machine does not undergo further deformation.[2] This would indicate that the crosshead speed may be linearly proportional to the strain rate.

Thus the same crosshead speed will be producing a small strain rate in the elastic range and a relatively higher strain rate in the plastic range. Results of tests conducted to study this aspect are reported.

1.4 Previous Research

The influence of strain rate on the dynamic yield stress ratio has been investigated by many researchers.[3-6] In Fig. 4a are shown the curves presented in these quoted references, relating to the ratio of yield stress and the average elastic strain rate. Also, results of tensile coupon tests conducted at a rate simulating the mill rate are shown.[2] The simulated mill tests show a large variation in stress ratios. The reference stress σ_y, was taken as the yield stress of a specimen located in the immediate neighborhood of the simulated mill specimen. From the nature of the relationship in Fig. 4a it can be seen that for low strain rates, a small change in strain rate has resulted in relatively large changes of the dynamic yield stress ratio. Also, in the elastic range, at a given valve opening and very low crosshead speeds, a considerable variation of the strain rate has been observed.

The relationship between dynamic yield stress ratio and strain rate in the plastic range also has been observed to be similar to that in the elastic range.[2] Figure 4b shows some recorded values. It has been observed in this investigation that a straining rate in the elastic range of 1 microinch per in. per sec. corresponds to a plastic strain rate which varies from 5 to 20 microinch per in. per sec. depending on the area of the specimen. The 5- to 20- fold increase in the plastic range is said to be due to the fact that the testing machine heads and screws are undergoing no deformation, all of the extension being concentrated in the yielding zone.

The results of a number of tensile coupon tests of A7 steel conducted at different strain rates in the plastic range have been reported.[7] The dynamic yield stress ratio is plotted against strain rate. Limits within which the test points may lie have been suggested.

Fisher and Viest have conducted similar tests and analyzed their results from a statistical point of view.[8] But all of their dynamic yield stress ratios have been measured at one specified strain rate, so that the limits can not be projected over a wide range of strain rates.

In the investigation reported in this paper, the tests were conducted at various strain rates and the results have been analyzed statistically. The coupons were not obtained specially for this investigation and hence may not represent a wide range of thicknesses.

2. DESCRIPTION OF TESTS

2.1 General Test Program

The tension coupons used in this investigation conformed to ASTM standards. They were of the plate type, 20 in. long and had an 8 in. gage length; the width was 2 in. at the grips and reduced to 1 1/2 in. over the gage length of 8 in. as shown in Fig. 5.

The coupons of A36 steel were cut from hot rolled square hollow tube shapes, while those of A441 steel were cut from webs and flanges of W shapes. The Q-T steel coupons were obtained from plates and bar stock. The locations of these coupons are shown in Fig. 6. Table 1 gives the thicknesses and sources of all coupons tested.

The coupons were tested in a 120,000 lb. capacity universal, mechanical testing machine of the screw-power-type with a positive control over the speed of the crosshead. Automatic electronic recording equipment was used to plot the load-strain curve, which usually covered the plastic range entirely on the recording paper of size 8½ in. x 11 in. An 8 in. extensometer with solenoid was used to relay the strain to the plotting device. A timing device attached to the autographic recorder made a record of time in seconds along the strain axis of the load-strain plot. (Such a trace is shown at the top of Fig. 8.) This trace made possible the calculation of the strain rate. This set up is shown in Fig. 7. In this figure can be seen a solenoid connected to a timer.

The crosshead speed of the testing machine could be controlled whereas strain rate could not be controlled. A few pilot tests indicated how this speed should be set in order to obtain a certain strain rate approximately; the actual strain is obtained as described in Section 2.2. In Table 2 are shown the range of setting used in this test program. In the plastic range each coupon could be tested at two or three different crosshead speeds and thus it was possible to obtain two or three dynamic yield stress ratios at corresponding strain rates.

2.2 Method of Testing

Each coupon was tested at a crosshead speed of 1/16 in. per min per in. of gage length up to the yield point, or until the plot had shown that the coupon was straining in the plastic range. Once the load-strain diagram showed that the strain was in the plastic range, the machine was stopped to record the static yield stress level. At this stage the strain would be a little less than twice the yield strain. The plot of the load-strain curve dropped from its original course indicating a decrease in the load; however slight increase in strain was noticed. The load indicator came to a stable position in about ~~5~~ 3 minutes*

Then the timer and the machine, set at a selected speed were started to record the speed and the corresponding dynamic yield stress level, as shown in Fig. 8. After the load-strain plot had been recorded for about 2 in. on the recording paper, the timer and the machine were

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*For longer periods of time, the decrease in load will be negligibly more than that for 5 minutes.

stopped to obtain a second recording of static yield stress level, and the operation was repeated. On the available length of paper each coupon could be tested at two or three different speeds in the plastic range, depending on the onset of strain-hardening.

The strain rate is obtained by dividing the strain by the corresponding time interval in seconds. The dynamic yield stress level is usually a horizontal line on the plot in the same interval of strain. The measurement of static yield stress level may present some difficulty. If a line is drawn joining the points 1, 2, and 3 in Fig. 8, this line determines the static yield stress level. If this line is horizontal there is no problem. If this line is slightly inclined to the horizontal, then the ordinate at 0.5 per cent strain is to be taken as the static yield stress level. This value is used to compute the dynamic yield stress ratio from the dynamic yield stress level.

2.3 Special Tests

A few special tests were conducted with the object of clarifying or testing a particular fact.

a. Comparison of 0.505 in. diam coupon and plate type coupon

Usually 0.505 in. diam coupon has a gage length of 2 in., while a plate type coupon has a gage length of 8 in. This test was intended primarily to check if crosshead speed and stress rate are consistent. Two coupons, one each of the round and flat type were tested at the ASTM speed of 1/16 in. per min per inch of gage length and the stress rate and strain rate were computed in the elastic range and plastic range.

b. Plate type coupon tested at ASTM speed

One plate type coupon of 8 in. gage length was tested at the ASTM speed of 1/16 in. per min per in. of gage length. The object of this test was to show that the strain rate has to be measured directly and not computed on the basis of the gage length and the crosshead speed. For this purpose, the strain rate was measured in the plastic range by the method described previously. It will be shown that computed strain rate and measured strain rate are different.

c. Test to show that the static yield stress level is not influenced by the testing machine.

The method of measuring static yield stress level was described earlier. This test was intended to show that the drop in the load is wholly a property of strain rate rather than of the momentum or elasticity of the testing machine.

3. TEST RESULTS AND DISCUSSION

The results of the special tests are presented and discussed first to make the doubtful facts clear before discussing the results of the general test program.

3.1 Special Tests

a. Comparison of 0.505 in. diam coupon and plate type coupon

The results of the coupon tests are given in Table 3. In the elastic range the strain rates are 107 microinch per in. per sec for the 0.505 in. coupon and 94 microinch per in. per sec for the plate type coupon, although both of them were tested at the ASTM crosshead speed of 1/16 in. per min per inch of gage length. The strain rate corresponding to ASTM speed is theoretically 1042 microinch per in. per sec. Therefore these two tests show that in the elastic range the crosshead speed and strain rate have no definite relationship and are influenced by the gage length. Also the stress rates were found to be 192,500 psi per min and 169,500 psi per min for the 0.505 in. and plate type coupons respectively. These values indicate that the machine response is also different for different gage lengths.

In the plastic range the strain rates were 780 microinch per in. per sec for the 0.505 in. coupon and 870 microinch per in. per sec for the plate type coupon. Neither of these values are anywhere near to the expected value of 1042 microinch per in. per sec. This shows that the strain rate must be measured directly from the strain recorded and should not be computed from the crosshead speed. The main reason for this is that the coupon elongates over a length greater than the gage length and consequently the specified crosshead speed of 1/16 in. per in. per min produces a considerably lower strain rate than 1042 microinch per in. per sec. Again, the measured strain rate is not likely to be consistent since the length of the coupon progressively increases in the plastic range, depending on the gage length.

b. Plate type coupon tested at ASTM speed

The result of this test is shown in Fig. 8. This coupon was tested at a crosshead speed of 0.2 in. per min. Assuming that the actual length of coupon elongating is 9 in., the crosshead speed corresponds to a strain rate of 370 microinch per in. per sec theoretically. On the basis of 8 in. gage length this strain rate would have been 417 microinch per in. per sec. In the plastic range, the measured strain rates were 280, 340 and 360 microinch per in. per sec. Even if this adjusted gage length of 9 in. was accepted as satisfactory the error would be 25 per cent, 8 per cent, and 3 per cent respectively. This again shows that the lengthening of the coupon in the plastic range alters the strain rate and is neither proportional to the crosshead speed nor consistent with an adjusted gage length.

c. Test to show that the static yield stress level is not influenced by the testing machine

In Section 2, the method of measuring static yield stress level has been described. In Fig. 9 a typical stress-strain curve is shown.

The position of the static yield stress level is checked by "jogging" the load slightly, that is, a small increase in strain rate with an immediate reduction back to zero. If the static position were a function of the elasticity of the machine, (that is, a function of the momentum of the elastic recovery of the testing machine), then the static position would take up some other level of equilibrium due to the smaller momentum of the "jogging", as shown in the inset of Fig. 9. In the case of hydraulic machines strain reversal and lower equilibrium load may be recorded due to leakage of oil in the system.

3.2 General Tests

The results of the tensile coupon tests conducted in this series are given in Table 1; the static yield stress level, the strain rate and the corresponding dynamic yield stress level in the plastic range are listed. For some of the coupons dynamic yield stress level could be measured at one or two strain rates only, since strain-hardening was noticed prematurely. From this data the dynamic yield stress ratio was computed. Test data on strain rate and corresponding dynamic yield stress ratios are shown in Figs. 10, 11 and 12 for A36 steel, A441 steel and Q-T steel respectively.

The dynamic yield stress ratio and strain rate are related by an equation,

$$\frac{\sigma_{yd}}{\sigma_{ys}} = 1 + k\,\dot{\varepsilon}^{n} \qquad (2)$$

where σ_{yd} = dynamic yield stress level
σ_{ys} = static yield stress level
σ_{yd}/σ_{ys} = dynamic yield stress ratio
$\dot{\epsilon}$ = strain rate
k, n = constants.

Similarly, the difference between dynamic yield stress level and static yield stress level and strain rate are related by an equation,

$$\sigma_{yd} - \sigma_{ys} = c\,\dot{\epsilon}^{m} \tag{3}$$

where c and m are constants. Equation (3) is an approximation to equation (2) obtained by assuming that the static yield stress level is the same for all the coupons.

The estimated curves of best fit were obtained by regression analyses. For the steels studied in this investigation, the equations take the following form:

Steel	No. of Samples	Equation (2)	Equation (3)
A36	189	$\frac{\sigma_{yd}}{\sigma_{ys}} = 1+0.021\dot{\epsilon}^{0.26}$	$\sigma_{yd}-\sigma_{ys} = 0.87\dot{\epsilon}^{0.24}$
A441	39	$= 1+0.020\dot{\epsilon}^{0.18}$	$= 1.06\dot{\epsilon}^{0.18}$
Q-T	29	$= 1+0.023\dot{\epsilon}^{0.08}$	$= 2.57\dot{\epsilon}^{0.08}$

These equations satisfy the condition that at zero strain rate the dynamic yield stress level is the same as the static yield stress level. It was assumed that the measured strain rates were free from error.

In Figs. 10, 11 and 12, the two boundary curves on each side of the estimated curves represent the 95 per cent confidence limits of

σ_{yd}/σ_{ys} and $(\sigma_{yd}-\sigma_{ys})$. That is, corresponding to any given value of strain rate, two limits were found for σ_{yd}/σ_{ys} and $(\sigma_{yd}-\sigma_{ys})$ such that the true value would lie between these limits in 95 per cent of the cases. It can be noticed that at least 95 per cent of the test data lies within these limits. The limiting curves are a function of the number of samples, in addition to the other factors involved. In these figures are also shown the histograms of static yield stress level. The mean static yield stress levels of each steel shown in these diagrams are 38.1 ksi for A36 steel, 54.2 ksi for A441 steel and 110.9 ksi for Q-T steel.

It can be noticed from the confidence limits for the three steels that the range of variation decreases with increase in the mean static yield stress level. For example at a strain rate of 1040 micro-inch per in. per sec, the range of variations are as given below:

Steel	Mean σ_{ys} (ksi)	Range of Variation	
		$\frac{\sigma_{yd}}{\sigma_{ys}}$ (%)	$(\sigma_{yd}-\sigma_{ys})$ ksi
A36	38.1	9.0	3.7
A441	54.2	7.0	3.4
Q-T	110.9	2.5	2.7

This shows that even though the variation in σ_{yd}/σ_{ys} is very much different for different steels, the actual variation in $(\sigma_{yd}-\sigma_{ys})$ is more or less the same. Nevertheless, it matters much more in the case of A36 steel which has a lower static yield stress level than in the case of Q-T steel which has a higher static yield stress level.

The estimated curves for all the steels are shown in Fig. 13 a. This figure indicates that at extremely low strain rates, there is very rapid increase in the dynamic yield stress ratio. At higher strain rates the curves are asymptotic to horizontal lines. This is significant in the case of Q-T steel. This shows that at higher strain rates the influence of strain rate on the dynamic yield stress ratio of Q-T steel is very small and almost constant.

It can be inferred from Fig. 13 a that for a given strain rate, the dynamic yield stress ratio decreases as the mean static yield stress level increases. For example, at 1040 microinch per in. per sec (which is the desired effect of the ASTM speed of 1/16 in. per min per inch of gage length) the average dynamic yield stress ratios are as follows:

Steel	Mean σ_{ys}	σ_{yd}/σ_{ys}
A36	38.1	1.127
A441	54.2	1.071
Q-T	110.9	1.040

Therefore when coupons of these steels are tested at a crosshead speed that could produce a strain rate of 1040 microinch per in. per sec the dynamic yield stress level can be expected to exceed the static yield stress level by approximately 13 per cent, 7 per cent and 4 per cent for A36, A441 and Q-T steels respectively. In other words, if no static yield stress level has been recorded as in the case of a mill test, the static yield stress level can be obtained by reducing the recorded dynamic yield stress level by 11 per cent, 7 per cent and 4 per cent for A36, A441 and Q-T steels respectively.

3.3 Prediction of Static Yield Stress Level

The relationship between $(\sigma_{yd}-\sigma_{ys})$ and strain rate for the three steels are shown in Fig. 13 b. The difference $(\sigma_{yd}-\sigma_{ys})$ is a narrow band for all the steels. As mentioned above, the variation is more or less the same. An estimated average curve relating $(\sigma_{yd}-\sigma_{ys})$ and $\dot{\varepsilon}$ for all the three steels is shown in Fig. 13 b. This can be expressed as an equation:

$$\sigma_{yd} - \sigma_{ys} = 3.2 + 0.001\dot{\varepsilon} \qquad (4)$$

where $\sigma_{yd} - \sigma_{ys}$ is in ksi and $\dot{\varepsilon}$ is strain rate in microinch per in. per sec. This equation can be used for any of the three steels and if a reliable strain rate is computed the error involved in predicting the static yield stress level will be quite small. For example, consider the stress-strain curve shown in Fig. 14. This coupon has been tested at a crosshead speed of 0.30 in. per min in the region where the dynamic yield stress level is 41.0 ksi. Assuming the gage length is 9 in. the strain rate would be 560 microinch per in. per sec. From equation (4) for this strain rate $\sigma_{yd} - \sigma_{ys} = 3.8$ ksi so that $\sigma_{ys} = 41.0 - 3.8 = 37.2$ ksi. The measured σ_{ys} for this coupon is 37.0 ksi; the predicted value is in error by 0.5 per cent.

In equation (4) the strain rate has a very small influence on $(\sigma_{yd}-\sigma_{ys})$. Therefore even if there is error in estimating the gage length and calculating the strain rate from crosshead speed, there will not be appreciable error in the predicted static yield stress level.

The problem of strain rate and of the determination of its effect on the yield stress can be solved by a substantial number of tests on a wide variety of testing machines. Steel from different manufacturers must also be subjected to exhaustive tests. It is expected that the outcome of such tests would show a similarity on the relationship of dynamic yield stress ratio to strain rate for different types of testing machines and different steels. This trend has been indicated from the reasonable correlation between previous tests, and the series of tests described in this report.

4. SUMMARY

This section summarizes the results of a limited number of tests on the influence of strain rate on the yield stress of A36 and A441 steels of ASTM designation and of a quenched and tempered steel (T-1 steel). The thickness of the coupons varied from 1/4 in. to 3/4 in.

In this report the dynamic yield stress level (σ_{yd}) is defined as the average stress during actual yielding in the plastic range, which remains fairly constant provided the strain rate remains constant. The static yield stress level (σ_{ys}) is defined as the average stress during actual yielding in the plastic range at zero strain rate; this also remains fairly constant. A method of determining these values are described in Section 2. The ratio of dynamic yield stress level to static yield stress level is termed "dynamic yield stress ratio" (σ_{yd}/σ_{ys}).

Relationships between σ_{yd}/σ_{ys} and the strain rate, and ($\sigma_{yd}-\sigma_{ys}$) and the strain rate are obtained from the results of tests conducted at various strain rates on a number of tensile coupons. The strain rate was measured directly over the gage length, with the help of special apparatus. A method for predicting static yield stress level from a standard coupon test is proposed.

The following are the important conclusions of this study:

1. There is no simple relationship between crosshead speed and strain rate; the strain rate and stress rate are influenced by the machine re-

sponse in the elastic range. Gage length of the coupon affects the strain rate in both the elastic range and plastic range. Strain rate must be measured directly from the coupon elongation.

2. Strain rate depends on the strain itself, since at every instant the length over which a coupon elongates progressively increases, causing the same crosshead speed to produce different strain rates.

3. Static yield stress level is a property of steel and is not a function of the momentum of the elastic recovery of the testing machine.

4. The relationship between dynamic yield stress ratio and strain rate can be expressed by

$$\frac{\sigma_{yd}}{\sigma_{ys}} = 1 + k\,\dot{\varepsilon}^{n} \qquad (2)$$

An approximate form of this equation is

$$\sigma_{yd} - \sigma_{ys} = c\,\dot{\varepsilon}^{m} \qquad (3)$$

The constants k, n, c and m are evaluated by a regression analyses of the test data (Figs. 10 through 13).

5. Confidence limits have been found for the above equations such that the true value would lie within these limits in 95 per cent of the cases. These limits also define the range of variation at any strain rate. The range of variation decreases with increase in the mean static yield stress level.

6. The dynamic yield stress ratio increases rapidly at low strain rates and very slowly at higher strain rates; also it decreases with increase in static yield stress level.

7. The difference $(\sigma_{yd}-\sigma_{ys})$ is essentially the same for A36, A441 and Q-T steels. They lie in a narrow band over a wide range of strain rates.

8. An average curve relating $(\sigma_{yd}-\sigma_{ys})$ and strain rate is proposed:

$$\sigma_{yd} - \sigma_{ys} = 3.2 + 0.001\dot{\varepsilon} \tag{4}$$

With the help of this equation, it is possible to predict the static yield stress level of a specimen from a standard tensile coupon test; the strain rate is computed approximately from the crosshead speed.

9. In order to define the influence of strain rate on yield stress exactly, a substantial number of tests must be conducted on a large number of specimens of various thicknesses, made by different manufacturers, in a wide variety of testing machines.

5. ACKNOWLEDGEMENTS

This report presents the results of an experimental study and analysis into the effect of strain rate on the yield stress of some structural steels. This is a part of the research program on the influence of residual stress on column strength carried out at the Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania. Professor William J. Eney is head of the Department and of the Laboratory.

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6. NOMENCLATURE

Symbols

σ = average stress

σ_p = proportional limit in terms of stress

σ_{uy} = upper yield point

σ_y = lower yield point

σ_{yd} = dynamic yield stress level

σ_{ys} = static yield stress level

ϵ = strain

$\dot{\epsilon}$ = strain rate = $d\epsilon/dt$

Glossary

Yield Point - The first stress in a material, less than the maximum attainable stress, at which an increase in strain occurs without an increase in stress. Note: It should be noted that only materials that exhibit the unique phenomenon of yielding have a yield point.

Yield Strength - The stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain. The deviation is expressed in terms of strain.

The above two definitions are as given in ASTM designation E6-61T. ASTM also describes various methods to determine the above quantities.

In this report the following terms have been used.

The (upper) yield point - σ_{uy}, is "the first stress in a material less than the maximum attainable stress at which an increase in strain occurs without an increase in stress." (This is the same as ASTM definition for yield point.)

The lower yield point - σ_{ly}, is the lowest level of yield stress immediately following the upper yield point maintaining a constant strain rate.

The (dynamic) yield stress level - σ_{yd}, is the average stress during actual yielding in the plastic range, which remains fairly constant, provided the strain rate remains constant.

The static yield stress level - σ_{ys}, is the average stress during actual yielding in the plastic range, at zero strain rate; this also remains fairly constant.

7. APPENDIX

A short note on the statistical theory used in this report.[9]

1. Estimated Curves

Suppose that x and y are two variables, and it is desired to predict y as a function of x. It will be assumed that x and y have a linear relationship of the form

$$Y_i = a + bx_i \qquad \text{(A1)}$$

where Y_i is the predicted value of y corresponding to any $x = x_i$. Equation (A1) is the "linear regression of y on x". The constants a and b are estimated by the "method of least squares"; in this process the quantity

$$L = \sum_1^N (y_i - Y_i)^2 = \sum_1^N (y_i - a - bx_i)^2 \qquad \text{(A2)}$$

is minimized with respect to a and b.

From this,

$$a = \bar{y} - b\bar{x}, \qquad b = \frac{\sum_1^N (x_i-\bar{x})(y_i-\bar{y})}{\sum_1^N (x_i-\bar{x})^2} \qquad \text{(A3)}$$

where

$$\bar{x} = \frac{1}{N}\sum_1^N x_i, \qquad \bar{y} = \frac{1}{N}\sum_1^N y_i .$$

If the relationship between two variables u and v, is non-linear, as for example,

$$V_i = 1 + \alpha u_i^{\beta} \qquad \text{(A4)}$$

where V_i is the predicted value of V corresponding to $u = u_i$, and α β are two constants, this equation can be written as

$$\log(V_i - 1) = \log\alpha + \beta \log u_i$$

That is

$$x_i = \log u_i, \quad y_i = \log(v_i - 1), \quad a = \log\alpha, \; b = \beta$$

In this report, the assumed relationship between σ_{yd}/σ_{ys} and $\acute{\epsilon}$ is of the form

$$\frac{\sigma_{yd}}{\sigma_{ys}} = 1 + k\acute{\epsilon}^{n} \qquad (2)$$

which is similar to equation (A4).

2. Confidence Limits

Consider that in an analysis for which equation (A1) is valid, the constants a and b have been calculated using N pairs of observations on x and y. Suppose in a new test for a given $x = x_c$, the value of y_c is obtained. The predicted value of y is

$$Y_c = a + bx_c \qquad \text{(A6)}$$

Since a and b have been computed from equation (A3) for a particular sample of N pairs of observations, it is clear that their values and consequently Y_c will fluctuate from sample to sample. In other words, if a series of (a,b) values is calculated from several samples, each containing N pairs of observations then equation (A6) will give different predictions for y for the same $x = x_c$. Nevertheless, for any given sample, the prediction in equation (A6) would be "reliable" if the difference $(y_c - Y_c)$ is small. In fact the degree of reliability of this prediction is measured by "Average $(y_c - Y_c)^2$", which is the average of $(y_c - Y_c)^2$ that would be obtained from a and b for different samples. It can be shown that

$$\text{Average} \quad (y_c - Y_c)^2 = \sigma^2 \left[1 + \frac{1}{N} + \frac{(x_c - \bar{x})^2}{(x_i - \bar{x})^2} \right] \qquad \text{(A7)}$$

where σ = standard deviation of y.

A good estimate of σ^2 is given by

$$\sigma^2 = \frac{1}{N-2} \sum_{1}^{N} (y_i - a - bx_i)^2 \qquad \text{(A8)}$$

The quantity Average $(y_c - Y_c)^2$ is called the "variance of y about the estimated regression line", and is represented by V(y). If this variance is small, the prediction is good.

If the probabilistic behavior of y is assumed (namely, y is normally distributed) then it can be shown that for $x = x_c$

$$\text{Probability} \quad (Y_c - t_\alpha S < y < Y_c + t_\alpha S) = 1 - \alpha \qquad \text{(A9)}$$

where $S = \sqrt{V(y)}$

and t_α is a factor that can be obtained from probability tables for a given α and N. α is usually a small preassigned quantity. The interval,

$Y_c - t_\alpha S$ to $Y_c + t_\alpha S$ is called the 100(1-α) per cent "confidence interval" for y when x = x_c, and y = $Y_c \mp t_\alpha S$ are called the lower and upper confidence limits.

In this report $\alpha = 0.05$. The regression line in equation (A6) gives the relationship between σ_{yd}/σ_{ys} and $\acute{\epsilon}$(See Figs. 10 through 13). The two curves bounding this regression curve are confidence limits. The observed data are plotted to show that 95 percent of y values for any given x are lying inside the confidence interval.

3. Relationship between σ_{yd}, σ_{ys} and $\acute{\epsilon}$

In Section 4 the relationship between σ_{yd}/σ_{ys} and $\acute{\epsilon}$ is given by

$$\sigma_{yd}/\sigma_{ys} = 1 + k\acute{\epsilon}^n \qquad (2)$$

This can be written as

$$\sigma_{yd} - \sigma_{ys} = k \cdot \sigma_{ys} \ \acute{\epsilon}^n$$

By taking logarithms of both sides of this equation

$$\log(\sigma_{yd} - \sigma_{ys}) = \log k + \log \sigma_{ys} + n \log \acute{\epsilon} \qquad (A10)$$

If it is assumed that σ_{ys} is constant, that is, all the test specimens have the same static yield stress level, then the term log σ_{ys} can be combined with log k and

$$\log(\sigma_{yd} - \sigma_{ys}) = \log c + m \log \acute{\epsilon}$$

or

$$\sigma_{yd} - \sigma_{ys} = c\dot{\epsilon}^{m} \tag{3}$$

This is an approximation to equation (2).

Regression lines of the form (3) and corresponding confidence intervals are also shown in Figs. 10 through 13.

Table 1. Summary of Coupon Tests

Project 296. A36 Steel. (Coupons from hollow tube square box shapes)

Coupon No.	Thickness	σ_{ys}	1		2		3	
			$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}
AA-E1	5/16	37.3	-	-	-	-	-	-
AA-E2	5/16	35.3	80	36.5	200	37.0	-	-
AA-E3	5/16	35.6	110	37.8	230	38.4	-	-
AA-E4	5/16	37.1	160	39.6	280	40.6	-	-
AA-F1	5/16	35.0	110	37.4	200	38.3	-	-
AA-F2	5/16	34.9	80	37.2	420	38.3	-	-
AA-F3	5/16	35.8	110	39.5	-	-	-	-
AA-F4	5/16	37.0	140	39.5	640	40.8	-	-
AA-G1	5/16	34.6	45	36.5	240	37.7	-	-
AA-G2	5/16	34.8	75	37.4	700	38.6	-	-
AA-G3	5/16	35.0	110	37.6	470	38.8	-	-
AA-G4	5/16	36.8	140	39.7	510	41.3	-	-
AA-H1	5/16	34.8	90	36.9	410	38.0	-	-
AA-H2	5/16	34.8	140	37.7	500	38.5	-	-
AA-H3	5/16	34.5	140	37.5	600	38.6	-	-
AA-H4	5/16	36.4	75	39.5	150	40.4	200	40.9
GG-A1	1/4	35.8	40	37.1	115	38.2	300	38.9
GG-A2	1/4	37.3	80	39.4	175	40.5	-	-
GG-A3	1/4	35.6	120	40.9	240	-	-	-
GG-A4	1/4	37.3	150	39.9	370	40.1	-	-
GG-G1	1/4	39.4	45	41.7	130	42.5	220	41.7
GG-G2	1/4	41.4	70	44.1	380	45.5	-	-
GG-G3	1/4	35.8	105	38.1	530	39.8	-	-
GG-G4	1/4	39.0	140	42.0	600	44.0	-	-

Table 1 - Continued

Coupon No.	Thickness	σ_{ys}	1		2		3	
			$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}
GG-H1	1/4	36.9	-	-	-	-	-	-
GG-H2	1/4	37.7	55	40.0	170	42.0	630	43.8
GG-H3	1/4	37.3	120	40.1	220	41.0	-	-
GG-H4	1/4	38.6	150	41.0	310	43.0	800	44.2
GG-K1	1/4	38.0	80	40.3	-	-	-	-
GG-K2	1/4	38.8	110	41.7	490	43.8	1200	-
GG-K3	1/4	40.4	120	43.1	200	44.5	-	-
GG-K4	1/4	39.6	90	42.2	140	42.2	200	43.2
BB-C1	5/32	36.2	-	-	-	-	-	-
BB-C2	5/32	35.0	50	37.4	170	39.0	250	40.6
BB-C3	5/32	36.4	65	38.5	220	39.1	490	40.4
BB-C4	5/32	38.6	110	41.1	600	42.4	-	-
BB-D1	5/32	35.9	130	39.2	200	39.6	-	-
BB-D2	5/32	35.0	160	37.2	310	37.5	-	-
BB-D3	5/32	37.1	140	39.7	320	40.1	-	-
BB-D4	5/32	34.6	120	36.0	430	38.1	-	-
DD-I1	1/4	38.4	25	40.1	53	40.5	103	41.9
DD-I2	1/4	39.7	57	42.5	-	-	608	47.3
DD-I3	1/4	39.2	102	42.0	255	42.5	837	44.1
DD-I4	1/4	39.2	142	43.5	337	43.2	975	43.8
DD-J1	1/4	38.3	27	39.8	57	40.2	125	40.6
DD-J2	1/4	38.7	75	41.3	362	44.3	988	46.3
DD-J3	1/4	38.4	105	40.8	525	41.8	1188	43.0
DD-J4	1/4	38.1	140	41.9	625	42.9	1250	43.2
DD-K1	1/4	37.5	23	39.8	56	39.9	115	40.4
DD-K2	1/4	39.1	60	42.1	750	45.0	200	42.2
DD-K3	1/4	38.4	112	40.9	912	42.6	283	42.8
DD-K4	1/4	38.4	-	-	925	45.0	312	41.6

Table 1 - Continued

Coupon No.	Thickness	σ_{ys}	1		2		3	
			$\dot{\varepsilon}$	σ_{yd}	$\dot{\varepsilon}$	σ_{yd}	$\dot{\varepsilon}$	σ_{yd}
DD-M1	1/4	37.2	75	39.7	1000	41.2	-	-
DD-M2	1/4	38.6	-	-	1125	42.7	108	41.8
DD-M3	1/4	37.9	137	41.0	-	-	650	43.0
DD-M4	1/4	37.9	82	40.2	162	40.9	263	42.4
CC-G1	3/16	44.6	33	47.0	90	47.4	117	47.7
CC-G2	3/16	43.5	37	45.7	82	46.5	128	46.9
CC-G3	3/16	42.8	-	-	74	45.7	144	46.6
CC-G4	3/16	44.2	35	46.3	73	47.0	117	47.4
CC-H1	3/16	46.1	49	48.3	86	49.0	135	48.3
CC-H2	3/16	43.0	-	-	44	44.8	113	45.9
CC-H3	3/16	42.3	28	45.2	53	45.6	114	46.0
CC-H4	3/16	45.0	25	47.0	51	47.4	113	48.4
EEE-E1	1/2	36.8	20	38.5	56	39.2	150	40.0
EEE-E2	1/2	38.5	44	40.1	107	41.0	205	42.0
EEE-E3	1/2	37.0	18	38.6	54	39.2	231	40.5
EEE-E4	1/2	38.9	78	41.1	368	42.7	675	42.3
EEE-G1	1/2	37.0	59	39.1	563	41.0	885	41.3
EEE-G2	1/2	39.1	31	41.4	107	41.6	420	43.1
EEE-G3	1/2	37.6	102	40.2	920	42.6	30	39.8
EEE-G4	1/2	39.2	965	43.8	24	41.5	63	41.9
EE-E1	1/2	36.7	11	37.9	319	39.6	-	-
EE-E2	1/2	37.7	15	39.3	400	40.8	910	41.6
EE-E3	1/2	38.4	45	40.1	470	41.4	1060	42.0
EE-E4	1/2	38.2	47	40.0	725	41.5	1165	41.9
EE-F1	1/2	38.0	300	41.6	390	42.1	-	-
EE-F2	1/2	39.1	440	42.8	6.5	40.8	-	-
EE-F3	1/2	39.1	750	42.6	915	43.1	11	40.8
EE-F4	1/2	39.0	10.4	40.5	1150	43.2	1210	43.5

Project 297. A441 Steel. (Coupons from WF shapes)

Coupon No.	Thickness	σ_{ys}	1		2		3	
			$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}
HT-3	7/16	53.3	95	55.6	208	55.9	560	56.6
HT-4	7/16	53.4	42	55.1	110	55.3	425	55.7
HT-7	7/16	53.6	105	55.6	130	56.1	900	57.5
HT-5	5/16	51.4	71	54.1	210	54.9	375	54.9
HT-6	5/16	52.5	36	53.7	122	55.4	595	56.0
HT-12	1/4	66.5	65	68.3	33	68.2	900	69.6
HT-15	7/16	54.4	34	56.8	58	57.0	1070	58.5
HT-17	7/16	56.1	115	59.3	36	59.6	1230	61.2
HT-16	1/4	59.8	125	62.0	1040	63.3	1500	63.1
HT-22	11/16	51.0	48	53.3	298	54.2	646	54.4
HT-23	11/16	49.3	50	53.2	412	54.4	583	54.6
HT-24	7/16	51.2	42	53.3	638	54.7	1020	54.6
HT-25	7/16	52.0	32	54.0	850	55.0	1235	55.5

Project 290. T-1 Steel. (Coupons from plates and bar stock)

Coupon No.	Thickness	σ_{ys}	1		2		3	
			$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}	$\dot{\epsilon}$	σ_{yd}
T2-1AA	1/2	109.3	-	112.5	175	113.4	682	114.1
T2-1AB	1/2	111.2	95	114.2	231	115.0	868	115.3
T2-1AC	1/2	110.6	119	114.0	283	114.2	1025	114.9
T2-3A	1/2	111.2	-	114.0	375	114.6	1012	115.0
T2-5A	1/2	108.7	94	112.2	481	113.0	1280	113.9
T2-7A	1/2	111.0	131	113.8	647	114.0	1562	114.6
T5-1	1/2	110.6	54	115.1	775	116.1	172	-
T5-3	1/2	110.9	72	115.0	1000	116.0	234	115.8
T5-5	1/2	111.0	119	115.0	1100	116.1	320	115.9
T5-7	1/2	112.2	50	115.5	1165	117.1	450	117.0
T5-9	1/2	112.8	83	116.7	1390	118.0	520	-

Table 2: Range of Crosshead Speed for 8 in. Gage Length Coupons (in./min)

0 - 0.015
0.015 - 0.03
0.03 - 0.05
0.05 - 0.075
0.10 - 0.15
0.15 - 0.20
0.20 - 0.25
0.25 - 0.30
0.30 - 0.35
0.35 - 0.45
0.45 - 0.55
0.55 - 0.65
0.65 - 0.75

0.50 - ASTM Speed

The crosshead speed is set in any of the 13 ranges. With the available markings on the dial, an exact speed can not be set.

Table 3. Results of 0.505 in. and Plate Type Coupon Tests

No.	Type	Area (in^2)	Gage Length (in)	ELASTIC RANGE			PLASTIC RANGE	
				Strain Rate $d\epsilon/dt$ (micro-in/in/sec)	Stress Rate $d\sigma/dt$ (psi/min)	Time Measured (sec)	Strain Rate $d\epsilon/dt$ micro-in/in/sec	Time Measured (sec)
1	0.505 in Round	0.20	2	107	192,500	7	780	12
2	1.5x0.5 Flat	0.76	8	94	169,500	8	870	10

Notes:

1. Both the coupons are of the same material.
2. Both the coupons were tested at a crosshead speed of 1/16 in/min/in of gage length.
3. Crosshead speed of 1/16 in/min/in of gage length = 1042 microin/in/sec
   Stress Rate of 100,000 psi/min = 55 microin/in/sec

$$E = \frac{d\sigma/dt}{d\epsilon/dt}$$

4. E assumed equal to 30,000 ksi.

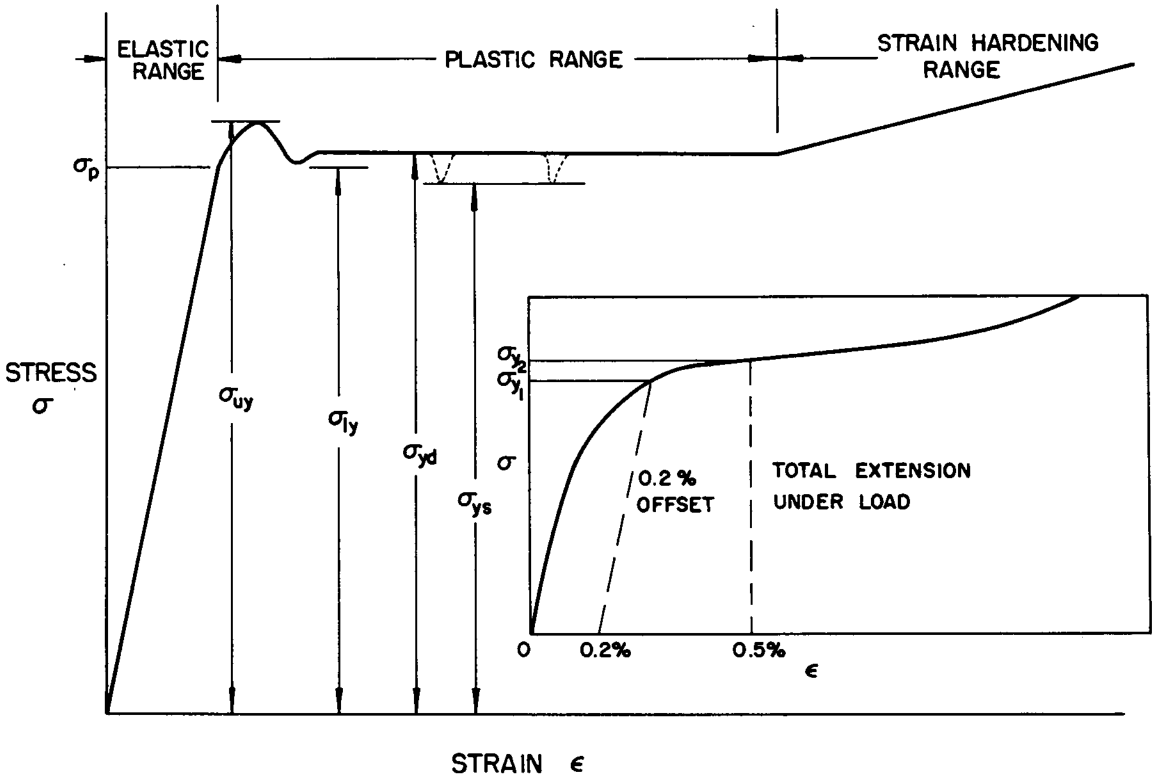


Fig. 1 Stress-Strain Curve-Definition of Terms

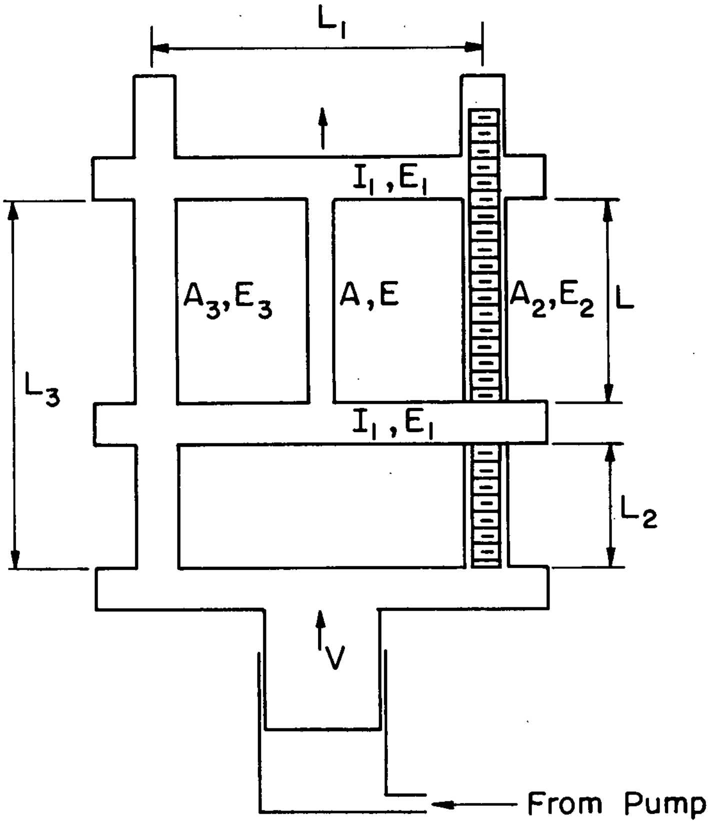


Fig. 2 Schematic of a Testing Machine

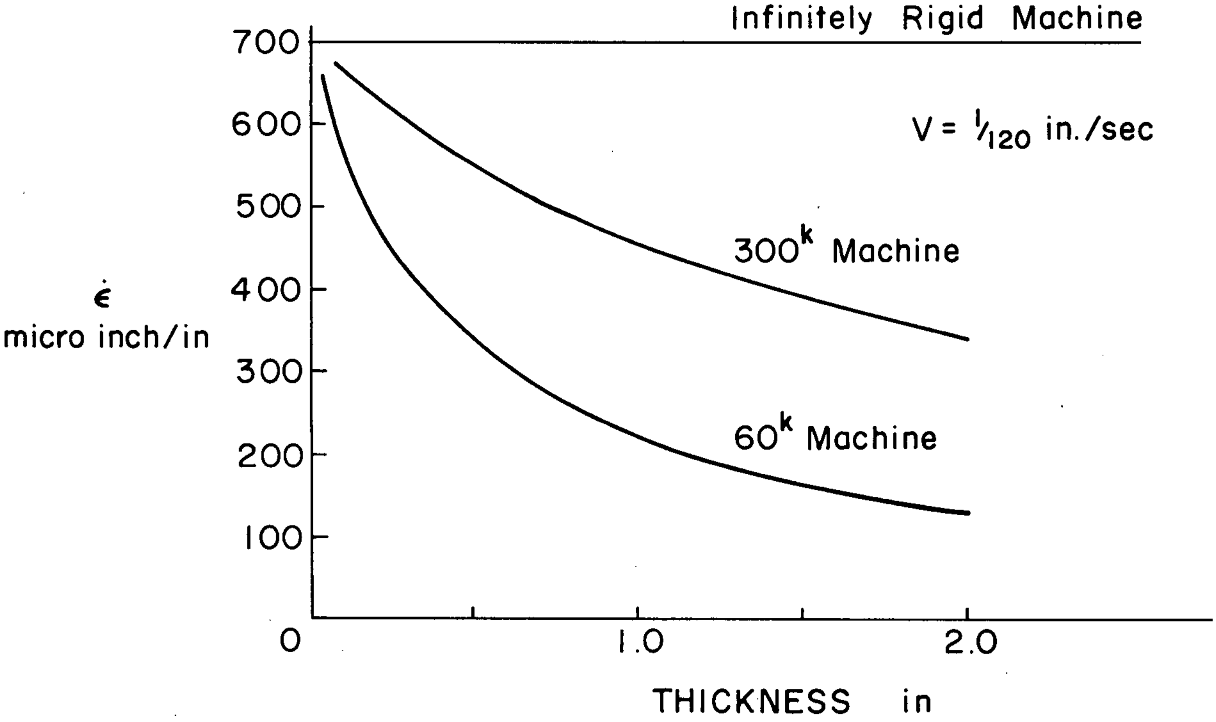


Fig. 3 Relationship between Strain Rate, Coupon Thickness and Crosshead Speed

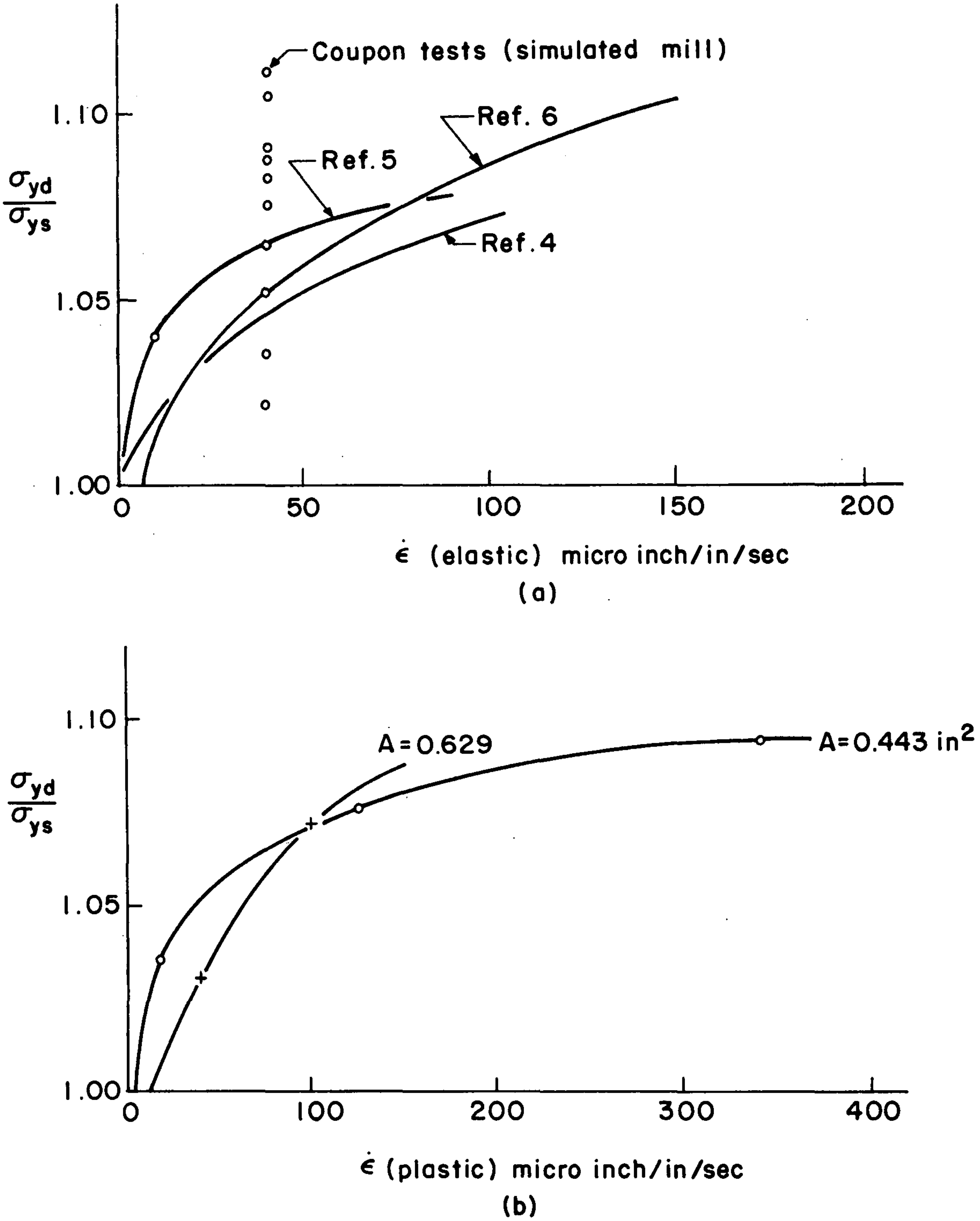


Fig. 4 Relationship between Dynamic Yield Stress Ratio and Strain Rate

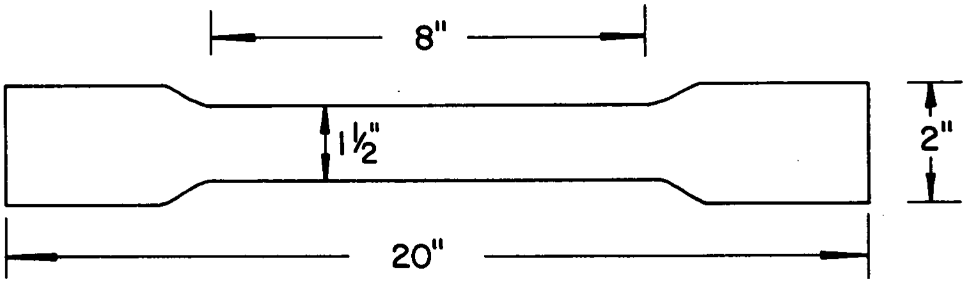


Fig. 5 Typical Tension Coupon

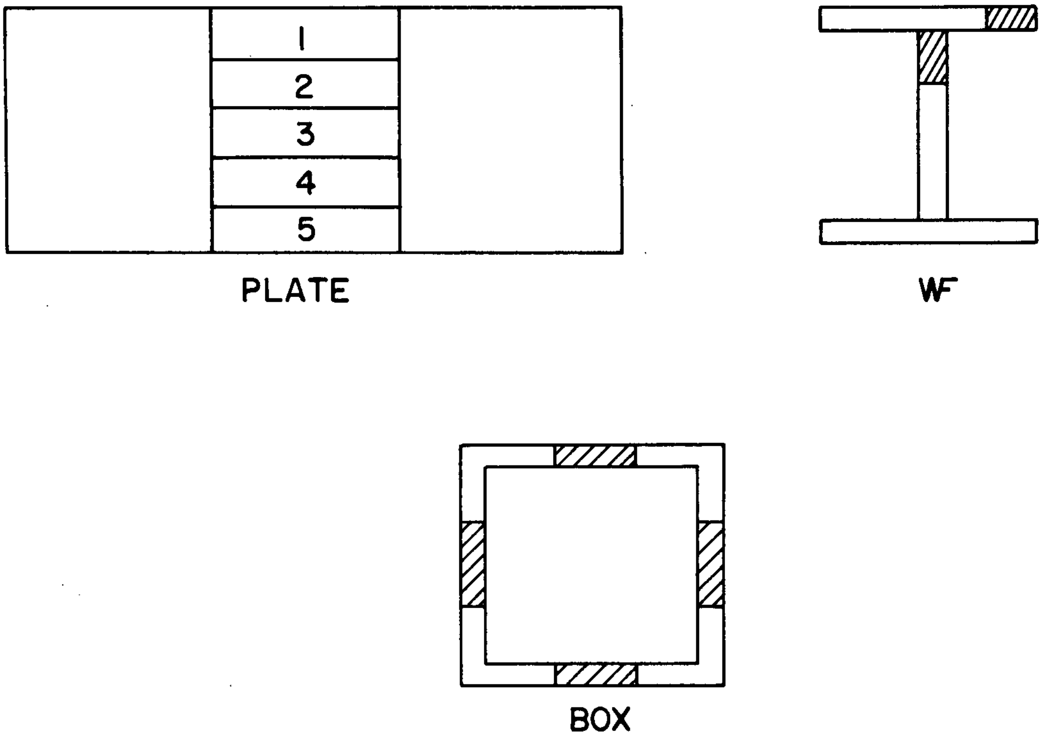


Fig. 6 Location of Coupons

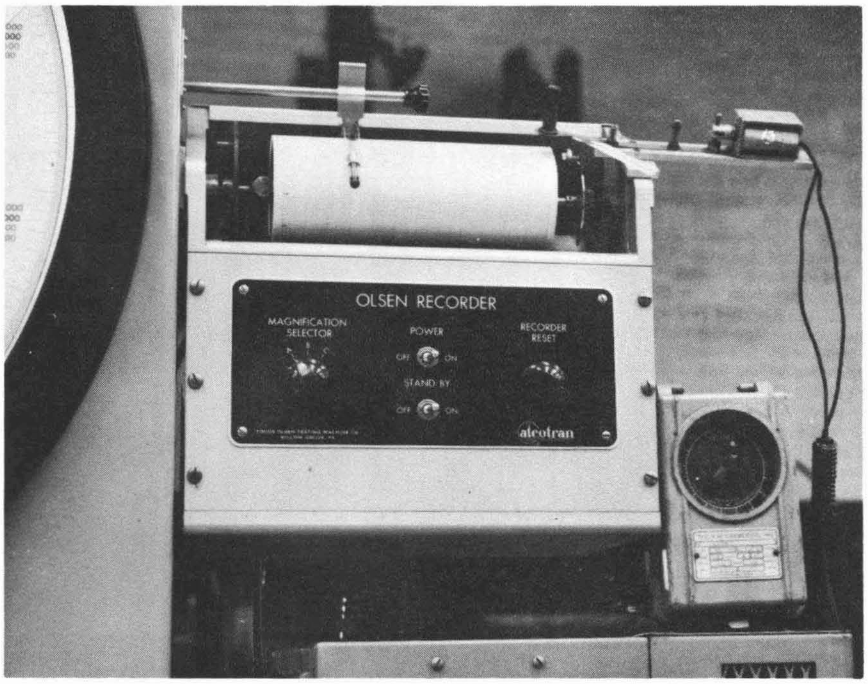


Fig. 7 Time Recording Device

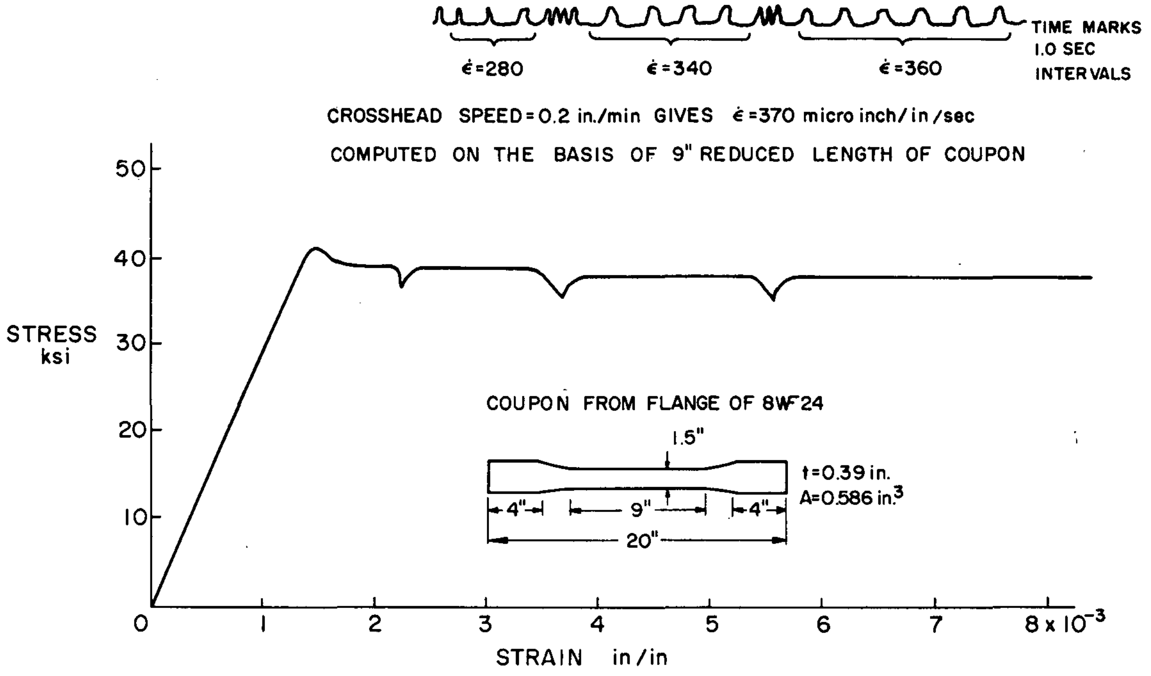


Fig. 8 Typical Stress-Strain-Time Diagram

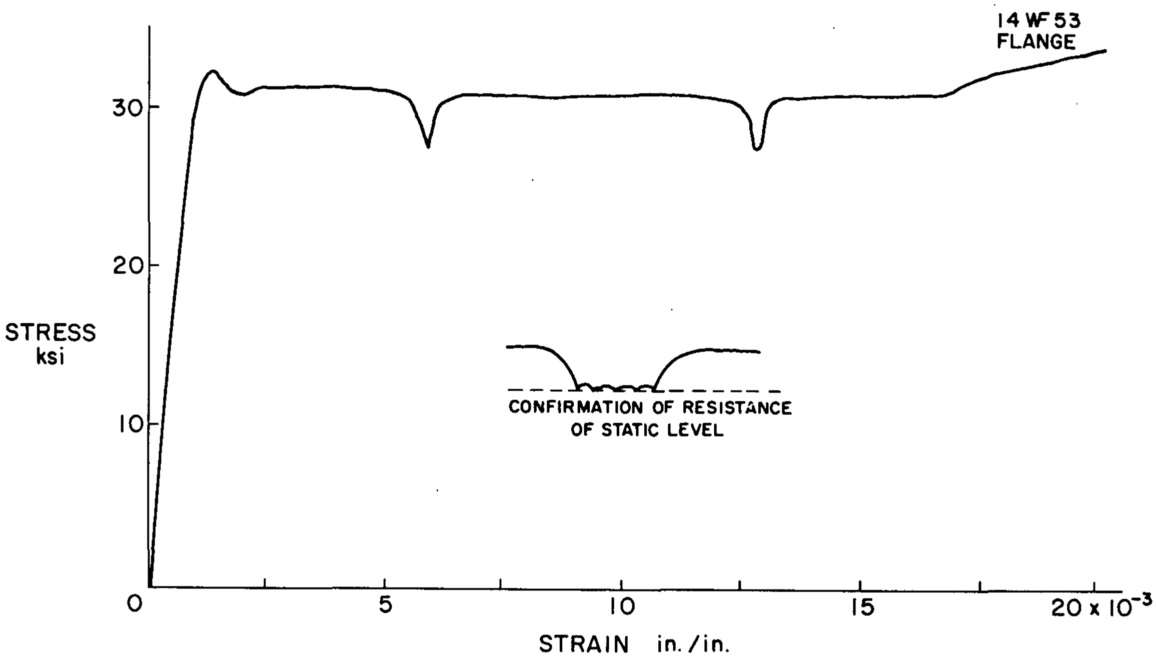


Fig. 9 Stress-Strain Curve

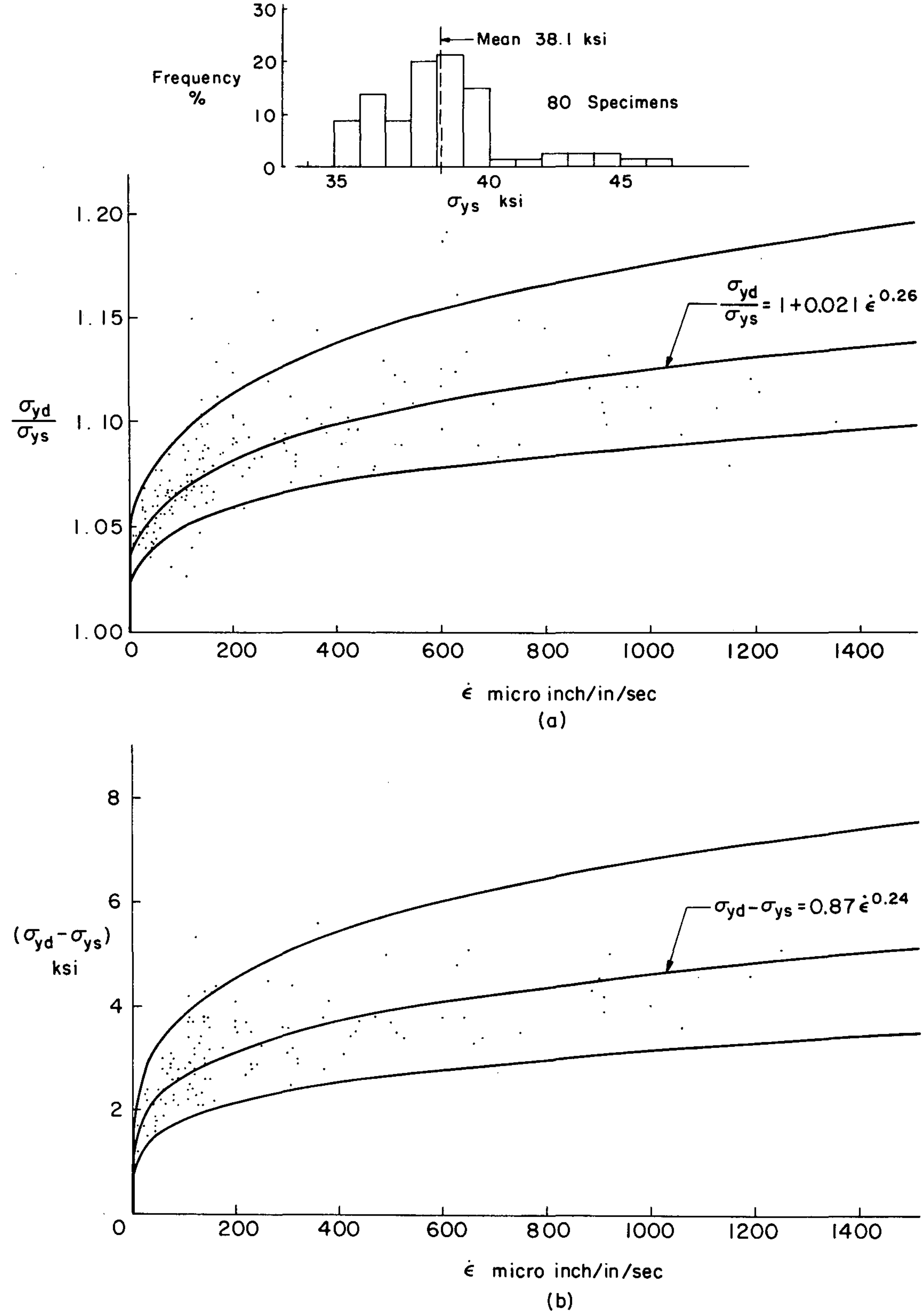


Fig. 10 Relationship between Dynamic Yield Stress Level, Static Yield Stress Level and Strain Rate - A36 Steel

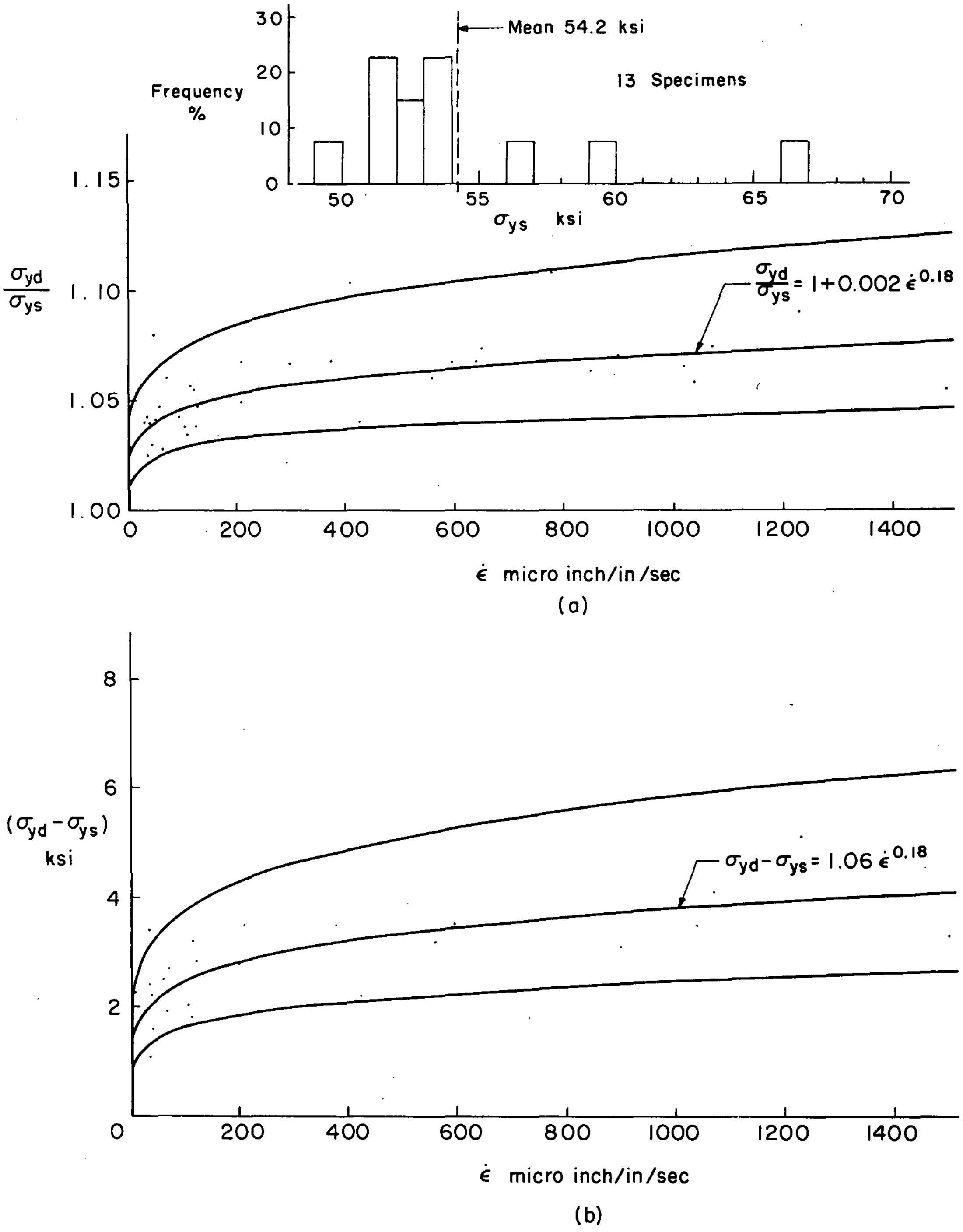


Fig. 11 Relationship between Dynamic Yield Stress Level, Static Yield Stress Level and Strain Rate - A441 Steel

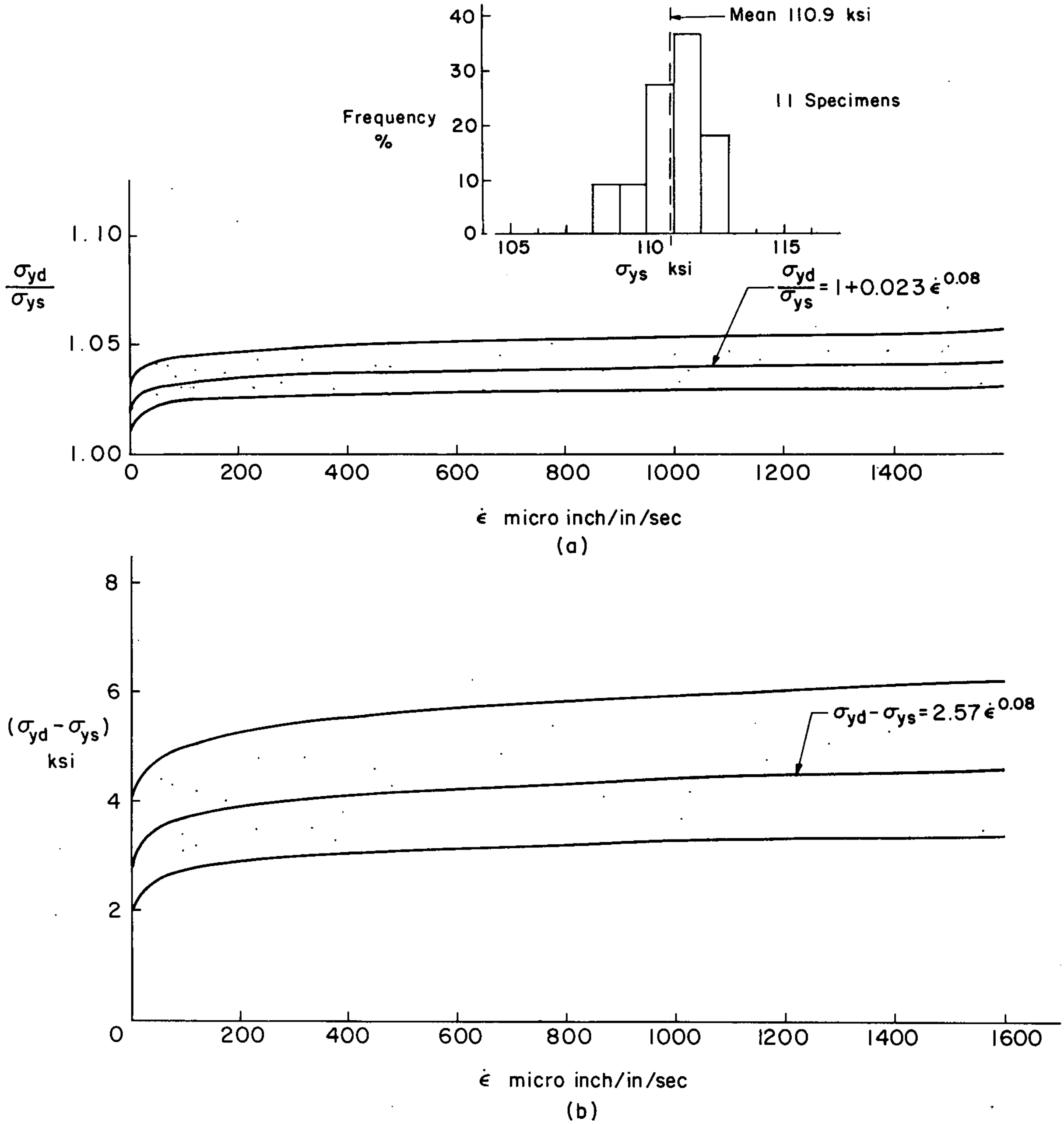


Fig. 12 Relationship between Dynamic Yield Stress Level, Static Yield Stress Level and Strain Rate - T-1 Steel

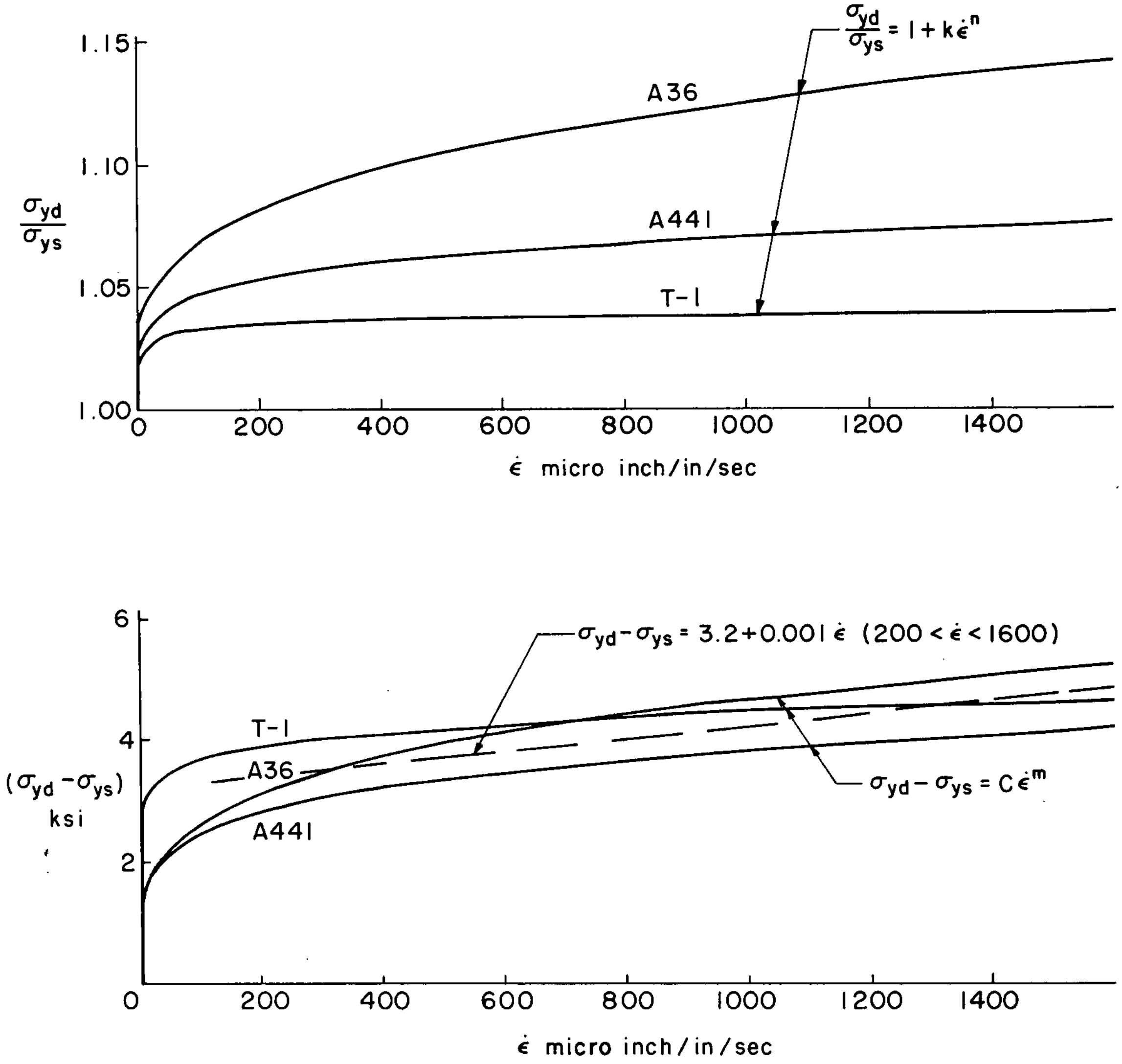
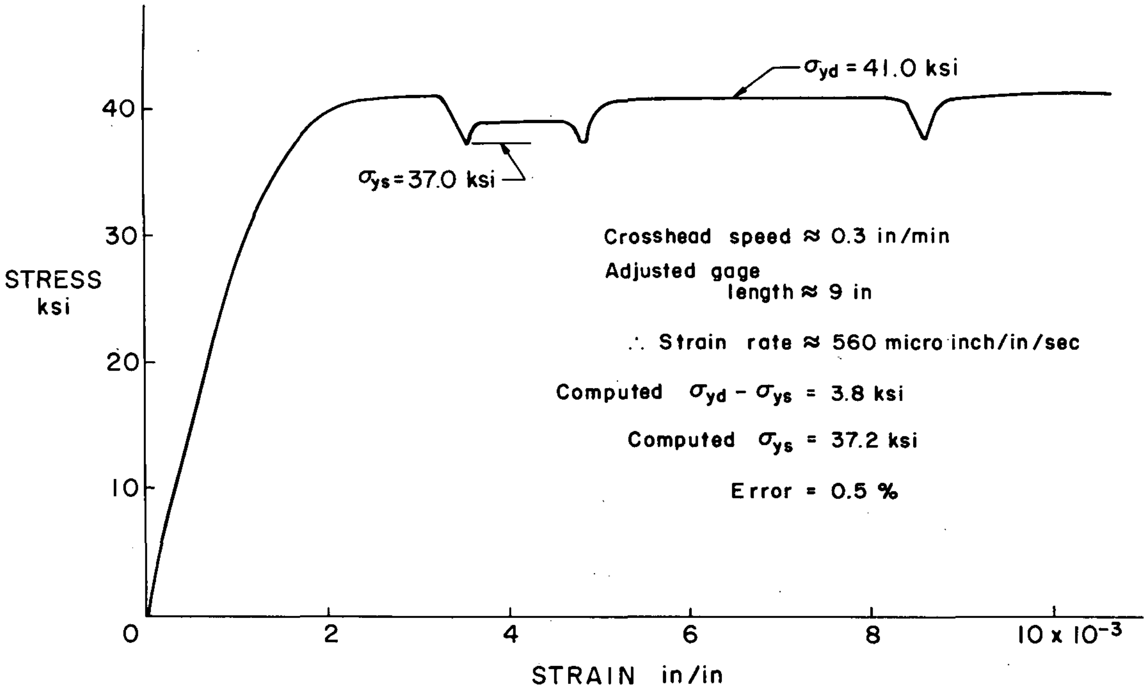


Fig. 13 Estimated Curves relating Dynamic Yield Stress Level, Static Yield Stress Level, and Strain Rate - A36, A441 and T-1 Steel



Crosshead speed ≈ 0.3 in/min

Adjusted gage length ≈ 9 in

∴ Strain rate ≈ 560 micro inch/in/sec

Computed $\sigma_{yd} - \sigma_{ys}$ = 3.8 ksi

Computed σ_{ys} = 37.2 ksi

Error = 0.5 %

Fig. 14 Stress-Strain Curve of Test Coupon~~s~~ EEE-G1

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