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Multi-dimensional regression analysis, June 1969

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MULTI-DIMENSIONAL REGRESSION ANALYSIS

(USERS' MANUAL FOR PROGRAMS CURVE AND MULTI)

by

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1. THEORY

A problem, which is encountered rather frequently in experimental research, is to find a functional expression for a relationship between a number of variables from a set of data obtained by experiments. The most widely used method for solving this problem is the "Least Squares Fit".

The basis of this method is to approximate the data $D_L(x,y,z)$, (which are assumed to be a function of three independent variables) by a regression function $\overline{D}(x,y,z)$ in such a way that the sum of squares of the residuals $r_L(x,y,z) = D_L(x,y,z) - \overline{D}(x,y,z)$ becomes a minimum.

$$S = \sum_{L=1}^{n} [D_L(x,y,z) - \overline{D}(x,y,z)]^2 = \min$$

n = total number of data

For a four-dimensional regression analysis (three independent variables) the regression function has the following form:

$$\overline{D}(x,y,z) = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} a_{ijk} \cdot f_i(x) \cdot g_j(y) \cdot h_k(z)$$

where a_{ijk} are unknown coefficients of the regression subfunctions $f_i(x)$, $g_j(y)$, and $h_k(z)$

These coefficients can be determined by minimizing the sum of squares of residuals:

$$\frac{\partial S}{\partial a_{ijk}} = \frac{\partial}{\partial a_{ijk}} \left[\sum_{L=1}^{n} (D_L - \bar{D})^2 \right] = 2 \sum_{L=1}^{n} \left[(\bar{D}_L - D) \cdot f_i g_j h_k \right] = 0$$

The minimization process leads to a system of m x m linear simultaneous equations where m is the product of the total number of subfunctions $f_i(x)$, $g_j(y)$, and $h_k(z)$:

$$m = n_i \times n_j \times n_k$$

Solving these equations gives the regression coefficients a_{ijk}, from which the values of the regression function $\bar{D}(x,y,z)$ can be determined for arbitrary values of x, y, and z.

As an illustration of the basic procedure the following equations for a two-dimensional regression analysis (one independent variable x) are obtained:

$$\bar{D}(x) = \sum_{i=1}^{n_i} a_i f_i(x) = a_1 f_1(x) + a_2 f_2(x) + \ldots + a_{ni} f_{ni}(x)$$

$$S = \sum_{L=1}^{n} [D_L(x) - \bar{D}(x)]^2$$

$$\frac{\partial S}{\partial a_i} = 2 \sum_{L=1}^{n} [D_L(x) - \bar{D}(x)] \frac{\partial \bar{D}(x)}{\partial a_i} = 0$$

$$\sum_{L=1}^{n} [D_L(x) - \bar{D}(x)] f_i(x) = 0$$

$$\sum_{L=1}^{n} D_L f_i(x) = \sum_{L=1}^{n} \bar{D}(x) f_i(x)$$

$$\begin{bmatrix} \sum_{L=1}^{n} f_1 f_1 & \sum_{L=1}^{n} f_1 f_2 & \cdots & \sum_{L=1}^{n} f_1 f_{ni} \\ \sum_{L=1}^{n} f_2 f_1 & \sum_{L=1}^{n} f_2 f_2 & \cdots & \sum_{L=1}^{n} f_2 f_{ni} \\ \cdots\cdots & \cdots\cdots & & \cdots\cdots \\ \sum_{L=1}^{n} f_{ni} f_1 & \sum_{L=1}^{n} f_{ni} f_2 & \cdots & \sum_{L=1}^{n} f_{ni} f_{ni} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot\cdot \\ a_{ni} \end{bmatrix} = \begin{bmatrix} \sum_{L=1}^{n} D_L f_1 \\ \sum_{L=1}^{n} D_L f_2 \\ \cdots\cdots \\ \sum_{L=1}^{n} D_L f_{ni} \end{bmatrix}$$

The governing equations for the three and four-dimensional regression analysis are found in a way similar to those for a two-dimensional problem.

2. PROGRAM CURVE

2.1 Purpose

The purpose of the program CURVE is a regression analysis of data with one or two independent variables.

This analysis is based on the following regression functions:

one independent variable:

$$\bar{D}(x) = \sum_{i=1}^{n_i} a_i f_i(x)$$

two independent variables:

$$\bar{D}(x,y) = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} a_{ij} f_i(x) g_j(y)$$

The data have to be arranged in the following form:

D (I,J) I = 1,...,NR subscript related to variable x

J = 1,...,NP subscript related to variable y

where x = constant for all values D(I,J=1..NP)

y = constant for all values D(I=1...NR,J)

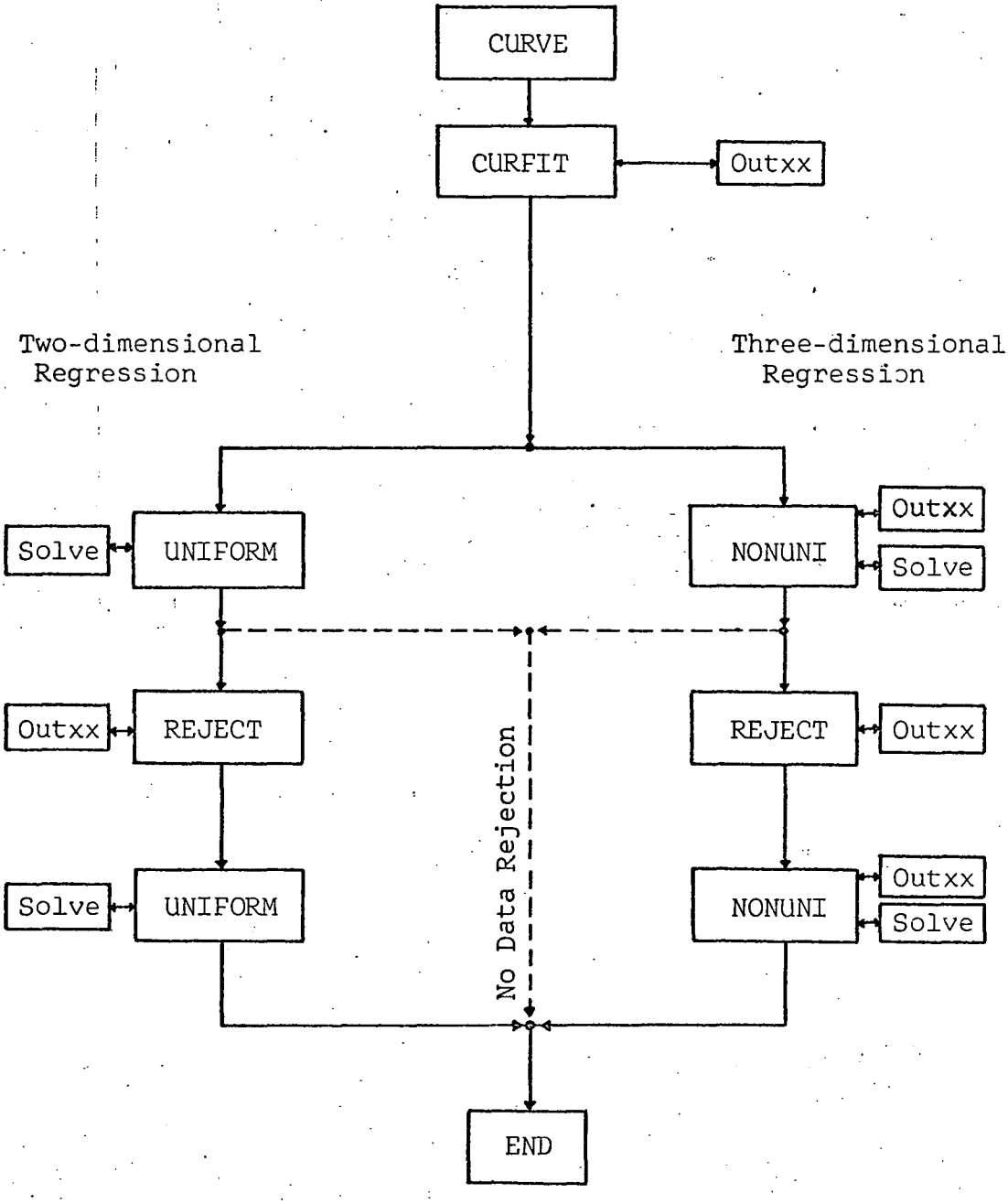
For two-dimensional problems (one independent variable) y is assumed to have a constant value of 1.

By selecting the proper code numbers the following types of regression subfunctions can be utilized:

Code Number	$f_i(u)$, $g_j(u)$
1	1
2	u
3	$\sqrt{u}$
4	1/u
5	u^2
6	u^3
7	$\log_{10} u$
8	$\sqrt[4]{u}$

Unless special provisions are made the program will in a subsequent step reject data whose residuals after the first least squares fit lie beyond a defined constant band-width. This band-width is twice the standard error of estimate (standard deviation of residuals) multiplied by a factor which has to be specified by the user. In a final step the program will repeat the regression analysis for those data which were not rejected.

2.2 Logical Flow Chart



2.3 Description of Subprograms

(1) Main program CURVE

Purpose: Input of data

1. control variables
2. data
3. arguments of variable x (and y)
4. code numbers of regression subfunctions $f_i(x)$ (and $g_j(y)$)

The source program is written in such a form, that after executing the regression analysis with a certain set of functions $f_i(x)$ (or $f_i(x)$ and $g_j(y)$) other sets of functions will be read in and the regression analysis will be repeated.

(2) Subprogram CURFIT

Purpose: Calling of regression subroutines in prescribed order (see the logical flow chart)

Output: original data

arguments of variable x (and y)

(3) Subprogram OUTXX

Purpose: print out of arrays (original data, residuals, fitted data), output is printed in columns of 8 (Format E14.4), headings are provided by the calling program

(4) Subprogram SOLVE

Purpose: Solution of systems of simultaneous equations (modified version of the matrix pack subroutine)

(5) Subprogram REJECT

Purpose:
1. calculation of total number of data, sum of squares of residuals, standard error of estimate (standard deviation of residuals) before rejection
2. rejection of data, whose residuals lie beyond a certain (constant) band-width
3. calculation of total number of data, sum of squares and standard error of estimate after rejection

Output:
1. residuals before rejection
2. total number of data, sum of squares and standard error before rejection
3. rejected data
4. residuals after rejection
5. total number of data, sum of squares, and standard error after rejection

(6) Subroutine UNIFORM

Purpose: Two dimensional regression analysis (one independent variable x)

1. scaling of variable x (logarithmic or linear scale)
2. calculation of values of regression subfunctions $f_i(x)$
3. generating and solving the system of simultaneous equations for the regression coefficients
4. calculation of fitted data
5. calculation of residuals

Output:
1. list of regression subfunctions $f_i(x)$ and code numbers of selected functions
2. coefficient matrix and right-hand side of system of simultaneous equations
3. regression coefficients
4. fitted data

(7) Subroutine NONUNI

Purpose: Three dimensional regression analysis (two independent variables x and y)

1. scaling of variable x (logarithmic or linear scale)
2. calculation of values of regression subfunctions $f_i(x)$ and $g_j(y)$
3. generating and solving the system of simultaneous equations for the regression coefficients

4. calculation of fitted data
5. calculation of residuals

Output:
1. list of regression subfunctions $f_i(x)$, $g_j(y)$ and code number of selected functions
2. coefficient matrix and right hand side of system of simultaneous equations
3. regression coefficients
4. fitted data

2.4 Input

Card A Control Variables FORMAT (4I5, F10.0)

Cols.

1-5 NP = maximum number of data per value of variable x
(NP $\leq$ 40)

6-10 NR = maximum number of values for variable x
(NR $\leq$ 165)

15 scaling of variable x
1 $x' = x$ (linear scale)
2 $x' = \log_{10} x$ (logarithmic scale)

20 number of independent variables
1 one independent variable x
2 two independent variables x and y

21-30 PRO = band-width factor for rejection
(PRO = 1.50....2.0)

Card B Data FORMAT (10F8.0)

Data D(I,J) have to be arranged row by row

D(1,J = 1,2,...,NP)

D(2,J = 1,2,...,NP)

.....

Card C Arguments of variable x FORMAT (10F8.0)

Two-dimensional regression analysis:

Card D1: Number of regression subfunctions FORMAT (I1)

NF = total number of regression subfunctions $f_i(x)$

$NF \leq 8$

Card E1: Code numbers for regression subfunctions:

FORMAT (8(I1, 1x))

code number i for regression

subfunctions $f_i(x)$ see page 5

In case the regression analysis should be repeated with different types of subfunctions an unrestricted number of cards of type D1 and E1 can be added.

Card F1: Blank card

Terminal for repetition of regression analysis

Three-dimensional regression analysis:

Card D2: Arguments of variable y FORMAT (10F8.0)

Card E2: Number of regression subfunctions FORMAT (2(I1,1x))

Col. 1 NF = total number of subfunctions $f_i(x)$

$$NF \leq 8$$

Col. 3 NG = total number of subfunctions $g_j(y)$

$$NG \leq 8 \qquad NF * NG \leq 30$$

Card F2: Code number of regression subfunctions $f_i(x)$

FORMAT (8(I1,1x))

see page 5

Card G2: Code number of regression subfunctions $g_j(\ )$

FORMAT (8(I1,1x))

see page 5

In case the regression analysis should be repeated with different types of subfunctions an unrestricted number of cards of type E2, F2, and G2 can be added.

Card H2: Blank Card

Terminal for repetition of regression analysis.

2.5 Limitations, Remarks

(1) Maximum values for control variables

NR $\leq$ 165 maximum number of arguments for variable x

NP $\leq$ 40 maximum number of data per argument of variable x or maximum number of arguments for variable y respectively

165 x 40 = 6600 maximum number of data

NF $\leq$ 8 maximum number of subfunctions $f_i(x)$

NG $\leq$ 8 maximum number of subfunctions $g_j(y)$

NF * NG $\leq$ 30

(2) Sequence of code numbers

The code numbers i (and j) for the selection of regression subfunctions $f_i(x)$ (and $g_j(y)$) have to be arranged in increasing order, for instance 1, 4, 5 not 1,5,4

(3) Arrangement of data

x(I) = constant for all values D(I,J = 1...NP)

y(J) = constant for all values D(I=1...NR, J)

(4) Zero and negative arguments of x and y

In case the independent variables $x' = \frac{x}{\log_{10}x}$ and y have arguments less than or equal to zero, a value of zero is assigned to the following subfunctions:

Code Number	$f_i(u)$, $g_j(u)$
3	$\sqrt[2]{u}$
4	$1/u$
7	$\log_{10}u$
8	$\sqrt[4]{u}$

(5) Zero values of data

Zero values of data D(I,J) = 0.0 are considered as missing data. In case it is their true value, a very small quantity should be assigned to them.

(6) Rejection of data

In case the rejection of data and a subsequent second least squares fit should be suppressed, card CURVE 56 in subprogram CURFIT (NOREJ = 2) has to be replaced by NOREJ = 1.

(7) Required field length

CM = $100\ 000_8$

2.6 Nomenclature

Arrays

STR (165,40)	Data D(I,J)
AGE (165)	Variable X(I)
X (40)	Variable Y(J)
RES (165,40)	Residuals
ST (165)	Fitted data $\bar{D}(I)$ one independent variable
DAT (165,40)	Fitted data $\bar{D}(I,J)$ two independent variables
AG (165)	Scaled variable x'(I)
F (8,165)	Regression subfunctions $f_i(x)$
G (8,40)	Regression subfunctions $g_j(y)$
KT (8)	code numbers for selected sub-functions $f_i(x)$

KX (8)	code numbers for selected sub-functions $g_j(y)$
A (30,30)	coefficient matrix for system of simultaneous equations

Variables

NR	=	total number of arguments for variable x
NP	=	total number of arguments for variable y or total number of data per argument of variable x
NF	=	n_i = total number of selected subfunctions $f_i(x)$
NG	=	n_j = total number of selected subfunctions $g_j(y)$
SNOM	=	total number of data
PRO	=	band-width factor

Branching Indices

NSE	Scaling of variable x NSE = 1 $x' = x$ (linear scale) NSE = 2 $x' = \log_{10} x$ (logarithmic scale)
NCODE	number of independent variables NCODE = 1 one independent variable x NCODE = 2 two independent variables x and y

NOREJ	rejection of data NOREJ = 1 no rejection of data NOREJ = 2 rejection of data with subsequent second least squares fit

2.7 PROGRAM LISTING

```
      PROGRAM CURVE (INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)                CURVE   1
*                                                                          CURVE   2
*************************************************************************CURVE   3
*                                                                        *CURVE   4
*   REGRESSION ANALYSIS OF DATA WITH ONE OR TWO INDEPENDENT VARIABLES    *CURVE   5
*                                                                        *CURVE   6
*************************************************************************CURVE   7
*                                                                          CURVE   8
      COMMON/UNIA/STR(165,40),RES(165,40),AGE(165),AG(165),F(8,165),       CURVE   9
     1ST(165),KT(8),NSE,NCODE,PRO,SNOM                                     CURVE  10
      COMMON/NONA/DAT(165,40),G(8,40),X(41),KX(8)                          CURVE  11
      DIMENSION A(30,30)                                                   CURVE  12
      IO=2                                                                 CURVE  13
C                                                                          CURVE  14
C     INPUT DATA                                                           CURVE  15
C                                                                          CURVE  16
      READ(IN,400) NP,NR,NSE,NCODE,PRO                                     CURVE  17
      DO 100 I=1,NR                                                        CURVE  18
      READ(IN,401) (STR(I,N),N=1,NP)                                       CURVE  19
  100 CONTINUE                                                             CURVE  20
      READ(IN,401) (AGE(I),I=1,NR)                                         CURVE  21
      GO TO (101,102)NCODE                                                 CURVE  22
C                                                                          CURVE  23
C     ONE INDEPENDENT VARIABLE                                             CURVE  24
C                                                                          CURVE  25
  101 READ(IN,402) NF                                                      CURVE  26
      IF(NF.EQ.0) GO TO 150                                                CURVE  27
      READ(IN,403) (KT(J),J=1,NF)                                          CURVE  28
      NG=1                                                                 CURVE  29
      NFG=NF                                                               CURVE  30
      CALL CURFIT(NP,NR,NF,NG,NFG,A)                                       CURVE  31
      GO TO 101                                                            CURVE  32
C                                                                          CURVE  33
C     TWO INDEPENDENT VARIABLES                                            CURVE  34
C                                                                          CURVE  35
  102 READ(IN,401) (X(I),I=1,NP)                                           CURVE  36
  103 READ(IN,402) NF,NG                                                   CURVE  37
      IF(NF.EQ.0) GO TO 150                                                CURVE  38
      READ(IN,403) (KT(J),J=1,NF)                                          CURVE  39
      READ(IN,403) (KX(I),I=1,NG)                                          CURVE  40
      NFG=NF*NG                                                            CURVE  41
      CALL CURFIT(NP,NR,NF,NG,NFG,A)                                       CURVE  42
      GO TO 103                                                            CURVE  43
  400 FORMAT(4I5,F10.0)                                                    CURVE  44
  401 FORMAT(10F8.0)                                                       CURVE  45
  402 FORMAT(I1,3X,I1)                                                     CURVE  46
  403 FORMAT(8(I1,1X))                                                     CURVE  47
  150 CALL EXIT                                                            CURVE  48
      END                                                                  CURVE  49
```

```
      SUBROUTINE CURFIT(NP,NR,NF,NG,NFG,A)                              CURVE 50
      COMMON/UNIA/STR(165,40),RES(165,40),AGE(165),AG(165),F(8,165),    CURVE 51
     1ST(165),KT(8),NSE,NCODE,PRO,SNOM                                  CURVE 52
      COMMON/NONA/DAT(165,40),G(8,40),X(41),KX(8)                       CURVE 53
      DIMENSION A(NFG,NFG)                                              CURVE 54
      IO=2                                                              CURVE 55
      NOREJ=2                                                           CURVE 56
      GO TO (100,103)NCODE                                              CURVE 57
C                                                                       CURVE 58
C     ONE INDEPENDENT VARIABLE                                          CURVE 59
C                                                                       CURVE 60
  100 DO 101 I=1,NR                                                     CURVE 61
      DO 101 N=1,NP                                                     CURVE 62
      RES(I,N)=0.0                                                      CURVE 63
  101 CONTINUE                                                          CURVE 64
      WRITE(IO,300)                                                     CURVE 65
      WRITE(IO,301)                                                     CURVE 66
      CALL OUTXX(STR,AGE,NP,NR)                                         CURVE 67
      WRITE(IO,302)                                                     CURVE 68
      CALL UNIFORM(NP,NR,NF,A)                                          CURVE 69
      GO TO (150,102)NOREJ                                              CURVE 70
  102 WRITE(IO,303)                                                     CURVE 71
      CALL REJECT(NP,NR)                                                CURVE 72
      WRITE(IO,304)                                                     CURVE 73
      CALL UNIFORM(NP,NR,NF,A)                                          CURVE 74
      GO TO 150                                                         CURVE 75
C                                                                       CURVE 76
C     TWO INDEPENDENT VARIABLES                                         CURVE 77
C                                                                       CURVE 78
  103 DO 104 I=1,NR                                                     CURVE 79
      DO 104 N=1,NP                                                     CURVE 80
      RES(I,N)=0.0                                                      CURVE 81
  104 CONTINUE                                                          CURVE 82
      WRITE(IO,305)                                                     CURVE 83
      WRITE(IO,301)                                                     CURVE 84
      CALL OUTXX(STR,AGE,NP,NR)                                         CURVE 85
      WRITE(IO,306)                                                     CURVE 86
      WRITE(IO,307)                                                     CURVE 87
      P=NP                                                              CURVE 88
      NPH=P/2.0+0.6                                                     CURVE 89
      X(NP+1)=0.0                                                       CURVE 90
      DO 105 I=1,NPH                                                    CURVE 91
      J=I+NPH                                                           CURVE 92
      WRITE(IO,308) I,X(I),J,X(J)                                       CURVE 93
  105 CONTINUE                                                          CURVE 94
      WRITE(IO,302)                                                     CURVE 95
      CALL NONUNI(NP,NR,NF,NG,NFG,A)                                    CURVE 96
      GO TO (150,106)NOREJ                                              CURVE 97
  106 WRITE(IO,303)                                                     CURVE 98
      CALL REJECT(NP,NR)                                                CURVE 99
      WRITE(IO,304)                                                     CURVE100
      CALL NONUNI(NP,NR,NF,NG,NFG,A)                                    CURVE101
  300 FORMAT(1H1,//,51X,*ONE INDEPENDENT VARIABLE*,/)                   CURVE102
  301 FORMAT(//,10X,15HORIGINAL DATA :,/)                               CURVE103
  302 FORMAT(1H1,//,10X,25HFIRST LEAST SQUARES FIT :,/)                 CURVE104
  303 FORMAT(1H1,//,10X,16HDATA REJECTION :,/)                          CURVE105
  304 FORMAT(1H1,//,10X,26HSECOND LEAST SQUARES FIT :,/)                CURVE106
```

```
  305  FORMAT(1H1,//,51X,*TWO INDEPENDENT VARIABLES*,/)                CURVE107
  306  FORMAT(/////,10X,*Y-COORDINATES*,/)                             CURVE108
  307  FORMAT(10X,2(*POINT*,11X,*Y*,20X),/)                            CURVE109
  308  FORMAT(10X,2(I3,8X,E11.4,15X))                                  CURVE110
  150  RETURN                                                          CURVE111
       END                                                             CURVE112
```

```
      SUBROUTINE OUTXX(S,AGE,NP,NR)                                    CURVE113
      DIMENSION S(165,40),AGE(165)                                     CURVE114
      IO=2                                                             CURVE115
      DO 100 L=1,NP,8                                                  CURVE116
      NA=L                                                             CURVE117
      NE=L+7                                                           CURVE118
      IF(NE.GT.NP) NE=NP                                               CURVE119
      WRITE(IO,300) (N,N=NA,NE)                                        CURVE120
      WRITE(IO,301)                                                    CURVE121
      DO 100 I=1,NR                                                    CURVE122
      WRITE(IO,302) AGE(I),(S(I,N),N=NA,NE)                            CURVE123
  100 CONTINUE                                                         CURVE124
  300 FORMAT(///,11X,*X*,6X,8(11X,I3))                                 CURVE125
  301 FORMAT(/)                                                        CURVE126
  302 FORMAT(5X,E12.4,5X,8E14.4)                                       CURVE127
      RETURN                                                           CURVE128
      END                                                              CURVE129
```

```
      SUBROUTINE SOLVE(C,B,X,N)                                        CURVE130
      DIMENSION C(N,N),B(N),X(N),A(30,31),IROW(30)                     CURVE131
      IO=2                                                             CURVE132
      M=N+1                                                            CURVE133
      DO 100 I=1,N                                                     CURVE134
      IROW(I)=I                                                        CURVE135
      A(I,M)=B(I)                                                      CURVE136
      DO 100 J=1,N                                                     CURVE137
      A(I,J)=C(I,J)                                                    CURVE138
  100 CONTINUE                                                         CURVE139
      DET=1.                                                           CURVE140
      DO 106 I=1,N                                                     CURVE141
      BIG=0.0                                                          CURVE142
      DO 101 II=I,N                                                    CURVE143
      IF(BIG.GE.ABS(A(II,I))) GO TO 101                                CURVE144
      BIG=ABS(A(II,I))                                                 CURVE145
      K=II                                                             CURVE146
  101 CONTINUE                                                         CURVE147
      IF(BIG.GT.0.0) GO TO 102                                         CURVE148
      WRITE(IO,300)                                                    CURVE149
      CALL EXIT                                                        CURVE150
  102 IF(K.EQ.I) GO TO 104                                             CURVE151
      L=IROW(I)                                                        CURVE152
      IROW(I)=IROW(K)                                                  CURVE153
      IROW(K)=L                                                        CURVE154
      DO 103 J=1,M                                                     CURVE155
      Z=A(I,J)                                                         CURVE156
      A(I,J)=A(K,J)                                                    CURVE157
      A(K,J)=Z                                                         CURVE158
  103 CONTINUE                                                         CURVE159
  104 Z=A(I,I)                                                         CURVE160
      DET=Z*DET                                                        CURVE161
      DO 105 J=I,M                                                     CURVE162
      A(I,J)=A(I,J)/Z                                                  CURVE163
  105 CONTINUE                                                         CURVE164
      IF(I.EQ.N) GO TO 107                                             CURVE165
      II=I+1                                                           CURVE166
      DO 106 K=II,N                                                    CURVE167
      Z=A(K,I)                                                         CURVE168
      DO 106 J=I,M                                                     CURVE169
      A(K,J)=A(K,J)-Z*A(I,J)                                           CURVE170
  106 CONTINUE                                                         CURVE171
  107 X(N)=A(N,M)                                                      CURVE172
      I=N                                                              CURVE173
  108 Z=0.0                                                            CURVE174
      DO 109 J=I,N                                                     CURVE175
      Z=Z+A(I-1,J)*X(J)                                                CURVE176
  109 CONTINUE                                                         CURVE177
      I=I-1                                                            CURVE178
      X(I)=A(I,M)-Z                                                    CURVE179
      IF(I.GT.1) GO TO 108                                             CURVE180
  300 FORMAT(////,10X,*SINGULAR COEFFICIENT MATRIX*)                   CURVE181
      RETURN                                                           CURVE182
      END                                                              CURVE183
```

```
      SUBROUTINE REJECT(NP,NR)                                          CURVE184
      COMMON/UNIA/STR(165,40),RES(165,40),AGE(165),AG(165),F(8,165),    CURVE185
     1ST(165),KT(8),NSE,NCODE,PRO,SNOM                                  CURVE186
      IO=2                                                              CURVE187
      WRITE(IO,300)                                                     CURVE188
      CALL OUTXX(RES,AGE,NP,NR)                                         CURVE189
C                                                                       CURVE190
C     COMPUTE SUM OF SQUARES AND STANDARD ERROR                         CURVE191
C                                                                       CURVE192
      SUM=0.0                                                           CURVE193
      DO 100 I=1,NR                                                     CURVE194
      DO 100 L=1,NP                                                     CURVE195
      SUM=RES(I,L)*RES(I,L)+SUM                                         CURVE196
  100 CONTINUE                                                          CURVE197
      WRITE(IO,301) SNOM                                                CURVE198
      WRITE(IO,302) SUM                                                 CURVE199
      STD=SQRT(SUM/SNOM)                                                CURVE200
      WRITE(IO,303) STD,PRO                                             CURVE201
C                                                                       CURVE202
C     REJECT DATA,  COMPUTE NEW SUM OF SQUARES AND STANDARD ERROR       CURVE203
C                                                                       CURVE204
      STD=PRO*STD                                                       CURVE205
      WRITE(IO,304)                                                     CURVE206
      DO 101 I=1,NR                                                     CURVE207
      DO 101 L=1,NP                                                     CURVE208
      IF(ABS(RES(I,L)).LE.STD) GO TO 101                                CURVE209
      SNOM=SNOM-1.0                                                     CURVE210
      SUM=SUM-RES(I,L)*RES(I,L)                                         CURVE211
      RES(I,L)=2.0E+300                                                 CURVE212
      WRITE(IO,305) AGE(I),L                                            CURVE213
  101 CONTINUE                                                          CURVE214
      WRITE(IO,306)                                                     CURVE215
      CALL OUTXX(RES,AGE,NP,NR)                                         CURVE216
      WRITE(IO,307) SNOM                                                CURVE217
      WRITE(IO,308) SUM                                                 CURVE218
      STD=SQRT(SUM/SNOM)                                                CURVE219
      WRITE(IO,309) STD,PRO                                             CURVE220
  300 FORMAT(//,10X,*RESIDUALS BEFORE REJECTION*)                       CURVE221
  301 FORMAT(////,10X,*TOTAL NO. OF DATA BEFORE REJECTION*,14X,F7.0)    CURVE222
  302 FORMAT(/,10X,*SUM OF SQUARES BEFORE REJECTION*,13X,E11.4)         CURVE223
  303 FORMAT(/,10X,*STANDARD ERROR BEFORE REJECTION*,13X,E11.4,10X,*BANDCURVE224
     1-WIDTH FACTOR*,5X,F6.3)                                           CURVE225
  304 FORMAT(///,10X,*REJECTED DATA :*,11X,*X*,12X,*POINT*,/)           CURVE226
  305 FORMAT(30X,E11.4,9X,I2)                                           CURVE227
  306 FORMAT(//////,10X,*RESIDUALS AFTER REJECTION*)                    CURVE228
  307 FORMAT(////,10X,*TOTAL NO. OF DATA AFTER REJECTION*,14X,F7.0)     CURVE229
  308 FORMAT(/,10X,*SUM OF SQUARES AFTER REJECTION*,13X,E11.4)          CURVE230
  309 FORMAT(/,10X,*STANDARD ERROR AFTER REJECTION*,13X,E11.4,10X,*BAND-CURVE231
     1WIDTH FACTOR*,5X,F6.3)                                            CURVE232
      RETURN                                                            CURVE233
      END                                                               CURVE234
```

```
      SUBROUTINE UNIFORM(NP,NR,NF,A)                                    CURVE235
      COMMON/UNIA/STR(165,40),RES(165,40),AGE(165),AG(165),F(8,165),    CURVE236
     1ST(165),KT(8),NSE,NCODE,PRO,SNOM                                  CURVE237
      DIMENSION A(NF,NF),B(8),R(8)                                      CURVE238
      IO=2                                                              CURVE239
      WRITE(IO,300)                                                     CURVE240
      WRITE(IO,301)                                                     CURVE241
      WRITE(IO,302) (KT(L),L=1,NF)                                      CURVE242
C                                                                       CURVE243
C     LOG. OR NON-LOG. SCALE FOR VARIABLE X                             CURVE244
C                                                                       CURVE245
      GO TO (100,102) NSE                                               CURVE246
  100 DO 101 I=1,NR                                                     CURVE247
      AG(I)=AGE(I)                                                      CURVE248
  101 CONTINUE                                                          CURVE249
      WRITE(IO,303)                                                     CURVE250
      GO TO 104                                                         CURVE251
  102 DO 103 I=1,NR                                                     CURVE252
      AG(I)=0.0                                                         CURVE253
      IF(AGE(I).GT.0.0) AG(I)=ALOG10(AGE(I))                            CURVE254
  103 CONTINUE                                                          CURVE255
      WRITE(IO,304)                                                     CURVE256
C                                                                       CURVE257
C     COMPUTE VALUES OF REGRESSION SUBFUNCTIONS F                       CURVE258
C                                                                       CURVE259
  104 DO 107 I=1,NR                                                     CURVE260
      F(1,I)=1.0                                                        CURVE261
      F(2,I)=AG(I)                                                      CURVE262
      F(5,I)=AG(I)*AG(I)                                                CURVE263
      F(6,I)=F(5,I)*AG(I)                                               CURVE264
      IF(AG(I).LE.0.0) 105,106                                          CURVE265
  105 F(3,I)=F(4,I)=F(7,I)=F(8,I)=0.0                                   CURVE266
      GO TO 107                                                         CURVE267
  106 F(3,I)=SQRT(AG(I))                                                CURVE268
      F(4,I)=1.0/AG(I)                                                  CURVE269
      F(7,I)=ALOG10(AG(I))                                              CURVE270
      F(8,I)=SQRT(F(3,I))                                               CURVE271
  107 CONTINUE                                                          CURVE272
      DO 108 J=1,NF                                                     CURVE273
      KK=KT(J)                                                          CURVE274
      DO 108 I=1,NR                                                     CURVE275
      F(J,I)=F(KK,I)                                                    CURVE276
  108 CONTINUE                                                          CURVE277
C                                                                       CURVE278
C     GENERATE SYSTEM OF SIMULTANEOUS EQUATIONS                         CURVE279
C                                                                       CURVE280
      DO 109 J=1,NF                                                     CURVE281
      B(J)=0.0                                                          CURVE282
      DO 109 K=1,NF                                                     CURVE283
      A(J,K)=0.0                                                        CURVE284
  109 CONTINUE                                                          CURVE285
      SNOM=0.0                                                          CURVE286
      DO 111 I=1,NR                                                     CURVE287
      DO 111 L=1,NP                                                     CURVE288
      IF(STR(I,L).EQ.0.0) GO TO 111                                     CURVE289
      IF(RES(I,L).GT.1.0E+300) GO TO 111                                CURVE290
      SNOM=SNOM+1.0                                                     CURVE291
```

```
      DO 110 J=1,NF                                                    CURVE292
      B(J)=STR(I,L)*F(J,I)+B(J)                                        CURVE293
      DO 110 K=1,NF                                                    CURVE294
      A(J,K)=F(K,I)*F(J,I)+A(J,K)                                      CURVE295
  110 CONTINUE                                                         CURVE296
  111 CONTINUE                                                         CURVE297
C                                                                      CURVE298
C     SOLVE SIMULTANEOUS EQUATIONS                                     CURVE299
C                                                                      CURVE300
      CALL SOLVE(A,B,R,NF)                                             CURVE301
      WRITE(IO,305)                                                    CURVE302
      DO 112 J=1,NF                                                    CURVE303
      WRITE(IO,306) B(J),(A(J,K),K=1,NF)                               CURVE304
  112 CONTINUE                                                         CURVE305
      WRITE(IO,307)                                                    CURVE306
      WRITE(IO,308) (R(J),J=1,NF)                                      CURVE307
C                                                                      CURVE308
C     COMPUTE FITTED DATA                                              CURVE309
C                                                                      CURVE310
      DO 114 I=1,NR                                                    CURVE311
      ST(I)=0.0                                                        CURVE312
      DO 113 J=1,NF                                                    CURVE313
      ST(I)=F(J,I)*R(J)+ST(I)                                          CURVE314
  113 CONTINUE                                                         CURVE315
  114 CONTINUE                                                         CURVE316
      WRITE(IO,309)                                                    CURVE317
      DO 115 I=1,NR                                                    CURVE318
      WRITE(IO,310) AGE(I),ST(I)                                       CURVE319
  115 CONTINUE                                                         CURVE320
C                                                                      CURVE321
C     COMPUTE RESIDUALS                                                CURVE322
C                                                                      CURVE323
      DO 116 I=1,NR                                                    CURVE324
      DO 116 L=1,NP                                                    CURVE325
      RES(I,L)=0.0                                                     CURVE326
      IF(STR(I,L).NE.0.0) RES(I,L)=STR(I,L)-ST(I)                      CURVE327
  116 CONTINUE                                                         CURVE328
  300 FORMAT(//,10X,*LIST OF FUNCTIONS OF X :   1- CONST.   2- X       3CURVE329
     1- SQRT(X)    4- 1/X*)                                            CURVE330
  301 FORMAT(37X,46H5- X**2     6- X**3    7- LOG10(X)   8- X**.25,//) CURVE331
  302 FORMAT(10X,21HUSED FUNCTIONS OF X :,6X,8(I1,2X))                 CURVE332
  303 FORMAT(//,10X,*VARIABLE X = X  (NON-LOG. SCALE)*,/)              CURVE333
  304 FORMAT(//,10X,*VARIABLE X = LOG10(X)*,/)                         CURVE334
  305 FORMAT(//,10X,*SIMULTANEOUS EQUATIONS*,/)                        CURVE335
  306 FORMAT(10X,E12.5,10X,8(F12.5,1X))                                CURVE336
  307 FORMAT(////,10X,34HCOEFFICIENTS A(I) :      A(I)*F(I),/)         CURVE337
  308 FORMAT(6X,8E15.4)                                                CURVE338
  309 FORMAT(///,16X,*X*,13X,*FITTED DATA*,/)                          CURVE339
  310 FORMAT(1X,2(9X,E11.4))                                           CURVE340
      RETURN                                                           CURVE341
      END                                                              CURVE342
```

```
      SUBROUTINE NONUNI(NP,NR,NF,NG,NFG,A)                              CURVE343
      COMMON/UNIA/STR(165,40),RES(165,40),AGE(165),AG(165),F(8,165),    CURVE344
     1ST(165),KT(8),NSE,NCODE,PRO,SNOM                                  CURVE345
      COMMON/NONA/DAT(165,40),G(8,40),X(41),KX(8)                       CURVE346
      DIMENSION A(NFG,NFG),R(30),B(30)                                  CURVE347
      IO=2                                                              CURVE348
      WRITE(IO,300)                                                     CURVE349
      WRITE(IO,301)                                                     CURVE350
      WRITE(IO,302) (KT(L),L=1,NF)                                      CURVE351
      WRITE(IO,303)                                                     CURVE352
      WRITE(IO,304)                                                     CURVE353
      WRITE(IO,305) (KX(L),L=1,NG)                                      CURVE354
C                                                                       CURVE355
C     LOG. OR NON-LOG. SCALE FOR VARIABLE X                             CURVE356
C                                                                       CURVE357
      GO TO (100,102) NSE                                               CURVE358
  100 DO 101 I=1,NR                                                     CURVE359
      AG(I)=AGE(I)                                                      CURVE360
  101 CONTINUE                                                          CURVE361
      WRITE(IO,306)                                                     CURVE362
      GO TO 104                                                         CURVE363
  102 DO 103 I=1,NR                                                     CURVE364
      AG(I)=0.0                                                         CURVE365
      IF(AGE(I).GT.0.0) AG(I)=ALOG10(AGE(I))                            CURVE366
  103 CONTINUE                                                          CURVE367
      WRITE(IO,307)                                                     CURVE368
C                                                                       CURVE369
C     COMPUTE VALUES OF REGRESSION SUBFUNCTIONS F AND G                 CURVE370
C                                                                       CURVE371
  104 DO 107 I=1,NR                                                     CURVE372
      F(1,I)=1.0                                                        CURVE373
      F(2,I)=AG(I)                                                      CURVE374
      F(5,I)=AG(I)*AG(I)                                                CURVE375
      F(6,I)=AG(I)*F(5,I)                                               CURVE376
      IF(AG(I).LE.0.0) 105,106                                          CURVE377
  105 F(3,I)=F(4,I)=F(7,I)=F(8,I)=0.0                                   CURVE378
      GO TO 107                                                         CURVE379
  106 F(3,I)=SQRT(AG(I))                                                CURVE380
      F(4,I)=1.0/AG(I)                                                  CURVE381
      F(7,I)=ALOG10(AG(I))                                              CURVE382
      F(8,I)=SQRT(F(3,I))                                               CURVE383
  107 CONTINUE                                                          CURVE384
      DO 110 L=1,NP                                                     CURVE385
      G(1,L)=1.0                                                        CURVE386
      G(2,L)=X(L)                                                       CURVE387
      G(5,L)=X(L)*X(L)                                                  CURVE388
      G(6,L)=X(L)*G(5,L)                                                CURVE389
      IF(X(L).LE.0.0) 108,109                                           CURVE390
  108 G(3,L)=G(4,L)=G(7,L)=G(8,L)=0.0                                   CURVE391
      GO TO 110                                                         CURVE392
  109 G(3,L)=SQRT(X(L))                                                 CURVE393
      G(4,L)=1.0/X(L)                                                   CURVE394
      G(7,L)=ALOG10(X(L))                                               CURVE395
      G(8,L)=SQRT(G(3,L))                                               CURVE396
  110 CONTINUE                                                          CURVE397
      DO 111 J=1,NF                                                     CURVE398
      KK=KT(J)                                                          CURVE399
```

```
      DO 111 I=1,NR                                                    CURVE400
      F(J,I)=F(KK,I)                                                   CURVE401
  111 CONTINUE                                                         CURVE402
      DO 112 J=1,NG                                                    CURVE403
      KK=KX(J)                                                         CURVE404
      DO 112 L=1,NP                                                    CURVE405
      G(J,L)=G(KK,L)                                                   CURVE406
  112 CONTINUE                                                         CURVE407
C                                                                      CURVE408
C     GENERATE SYSTEM OF SIMULTANEOUS EQUATIONS                        CURVE409
C                                                                      CURVE410
      DO 113 J=1,NFG                                                   CURVE411
      B(J)=0.0                                                         CURVE412
      DO 113 K=1,NFG                                                   CURVE413
      A(J,K)=0.0                                                       CURVE414
  113 CONTINUE                                                         CURVE415
      SNOM=0.0                                                         CURVE416
      DO 115 I=1,NR                                                    CURVE417
      DO 115 N=1,NP                                                    CURVE418
      IF(STR(I,N).EQ.0.0) GO TO 115                                    CURVE419
      IF(RES(I,N).GT.1.0E+300) GO TO 115                               CURVE420
      SNOM=SNOM+1.0                                                    CURVE421
      DO 114 J=1,NF                                                    CURVE422
      DO 114 L=1,NG                                                    CURVE423
      NV=(J-1)*NG+L                                                    CURVE424
      B(NV)=STR(I,N)*F(J,I)*G(L,N)+B(NV)                               CURVE425
      DO 114 JH=1,NF                                                   CURVE426
      DO 114 LH=1,NG                                                   CURVE427
      NH=(JH-1)*NG+LH                                                  CURVE428
      A(NV,NH)=F(JH,I)*G(LH,N)*F(J,I)*G(L,N)+A(NV,NH)                  CURVE429
  114 CONTINUE                                                         CURVE430
  115 CONTINUE                                                         CURVE431
C                                                                      CURVE432
C     SOLVE SIMULTANEOUS EQUATIONS                                     CURVE433
C                                                                      CURVE434
      CALL SOLVE(A,B,R,NFG)                                            CURVE435
      WRITE(IO,308)                                                    CURVE436
      DO 116 I=1,NF                                                    CURVE437
      JE=(I-1)*NG+1                                                    CURVE438
      JA=JE+NG-1                                                       CURVE439
      WRITE(IO,309) (R(J),J=JE,JA)                                     CURVE440
  116 CONTINUE                                                         CURVE441
C                                                                      CURVE442
C     COMPUTE FITTED DATA                                              CURVE443
C                                                                      CURVE444
      DO 117 I=1,NR                                                    CURVE445
      DO 117 L=1,NP                                                    CURVE446
      DAT(I,L)=0.0                                                     CURVE447
      DO 117 J=1,NF                                                    CURVE448
      DO 117 K=1,NG                                                    CURVE449
      NV=(J-1)*NG+K                                                    CURVE450
      DAT(I,L)=F(J,I)*G(K,L)*R(NV)+DAT(I,L)                            CURVE451
  117 CONTINUE                                                         CURVE452
      WRITE(IO,310)                                                    CURVE453
      CALL OUTXX(DAT,AGE,NP,NR)                                        CURVE454
C                                                                      CURVE455
C     COMPUTE RESIDUALS                                                CURVE456
C                                                                      CURVE457
```

```
      DO 118 I=1,NR                                                    CURVE458
      DO 118 L=1,NP                                                    CURVE459
      RES(I,L)=0.0                                                     CURVE460
      IF(STR(I,L).NE.0.0) RES(I,L)=STR(I,L)-DAT(I,L)                   CURVE461
  118 CONTINUE                                                         CURVE462
  300 FORMAT(//,10X,*LIST OF FUNCTIONS OF X :   1- CONST.   2- X      3CURVE463
     1- SQRT(X)    4- 1/X*)                                           CURVE464
  301 FORMAT(37X,46H5- X**2     6- X**3    7- LOG10(X)   8- X**.25,//) CURVE465
  302 FORMAT(10X,21HUSED FUNCTIONS OF X :,6X,8(I1,2X))                 CURVE466
  303 FORMAT(//,10X,*LIST OF FUNCTIONS OF Y :   1- CONST.   2- Y      3CURVE467
     1- SQRT(Y)    4- 1/Y*)                                           CURVE468
  304 FORMAT(37X,46H5- Y**2     6- Y**3    7- LOG10(Y)   8- Y**.25,//) CURVE469
  305 FORMAT(10X,21HUSED FUNCTIONS OF Y :,6X,8(I1,2X))                 CURVE470
  306 FORMAT(//,10X,*VARIABLE X = X  (NON-LOG. SCALE)*,/)              CURVE471
  307 FORMAT(//,10X,*VARIABLE X = LOG10(X)*,/)                         CURVE472
  308 FORMAT(//,10X,42HCOEFFICIENTS A(I,J) :     A(I,J)*F(I)*G(J),/)   CURVE473
  309 FORMAT(6X,8E15.4)                                                CURVE474
  310 FORMAT(1H1,//,10X,13HFITTED DATA :,//)                           CURVE475
      RETURN                                                           CURVE476
      END                                                              CURVE477
```

3. PROGRAM MULTI

3.1 Purpose

The purpose of the program MULTI is a regression analysis of data with three independent variables x, y, and z. This analysis is based on the following regression functions:

$$\bar{D}(x,y,z) = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} a_{ijk}\, f_i(x)\, g_j(y)\, h_k(z)$$

The data have to be arranged in the following form

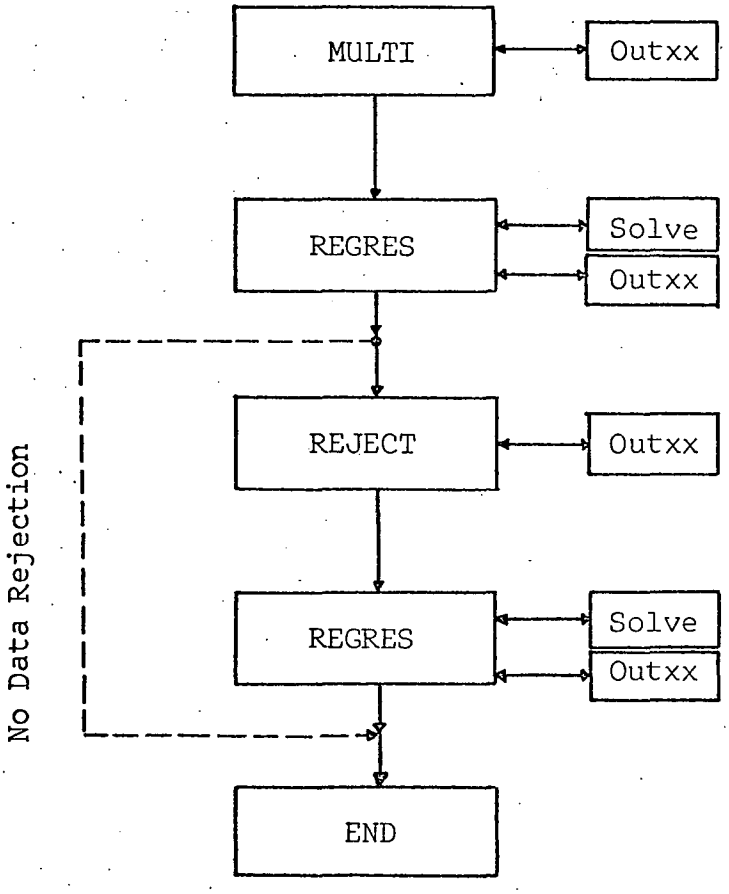
D(I,J)	I = 1,...,NR	subscript related to variable x
	J = 1,...,NP	subscript related to variables y and z

where x = constant for all values D(I,J=1...NP)

y = constant, z = constant } for all values D(I=1...NR,J)

The types of subfunctions, which can be selected for $f_i(x)$, $g_j(y)$, and $h_k(z)$ as well as the information about data rejection are identical to those given on page 5 for program CURVE.

3.2 Logical Flow Chart



3.3 Description of Subprograms

(1) Main Program MULTI:

Purpose: Input of data

1. control variables
2. data
3. arguments of variable x, y, and z
4. code numbers for regression subfunctions $f_i(x)$, $g_j(y)$, and $h_k(z)$

Calling of regression subroutines in prescribed order

Output: Original data

arguments of variables x, y and z

headings

The source program is written in such a form, that after executing the regression analysis with a certain set of functions $f_i(x)$, $g_j(y)$, $h_k(z)$ other sets of functions will be read in and the regression analysis will be repeated.

(2) Subprogram REGRES

Purpose: Four-dimensional regression analysis (three independent variables x, y, z)

1. scaling of variable x (logarithmic or linear scale)
2. calculation of values of regression subfunctions $f_i(x)$, $g_j(y)$, $h_k(z)$

3. generating and solving the system of simultaneous equations for the regression coefficients
4. calculation of fitted data
5. calculation of residuals

Output: 1. list of regression subfunctions $f_i(x)$, $g_j(y)$, $h_k(z)$ and code numbers of selected functions
2. regression coefficients
3. fitted data

(3) Other subprograms

The remaining subprograms OUTXX, SOLVE, and REJECT, used in this program, are identical to those described on page 7 for program CURVE.

3.4 Input

Card A Control Variables FORMAT (3I5, 5X, F10.0)

Cols.

1-5 NP = maximum number of values for variable y (or z) ($NP \leq 40$)

6-10 NR = maximum number of values for variable x ($NR \leq 140$)

15 scaling of variable x

1 $x' = x$ (linear scale)

2 $x' = \log_{10} x$ (logarithmic scale)

21-30 PRO = band-with factor for rejection

(PRO =1.5...2.0)

Card B Data FORMAT (10F8.0)

Data D(I,J) have to be arranged row by row

D(1, J=1,2,...,NP)

D(2, J=1,2,...,NP)

Card C Arguments of variable x

Card D Arguments of variable y FORMAT (10F8.0)

Card E Arguments of variable z

Card F Number of regression subfunctions FORMAT (3(I1,1x)

Col. 1 NF = total number of subfunctions $f_i(x)$ $NF \leq 8$

Col. 3 NG = total number of subfunctions $g_j(y)$ $NG \leq 8$

Col. 5 NH = total number of subfunctions $h_k(z)$ $NH \leq 7$

Card G, Card H, Card I } code numbers of regression subfunctions { $f_i(x)$, $g_j(y)$, $h_k(z)$

(see page 5)

FORMAT (8(I1, 1x)

In case the regression analysis should be repeated with different types of subfunctions, an unrestricted number of cards of type G, H, and I can be added.

Card J: Blank Card

Terminal for repetition of regression analysis

3.5 Limitations, Remarks

(1) Maximum values for control variables

$NR \leq 140$ maximum number of arguments for variable x

$NP \leq 40$ maximum number of arguments for variables y and z

$140 * 40 = 5600$ maximum number of data

$NF \leq 8$ maximum number of subfunctions $f_i(x)$

$NG \leq 8$ maximum number of subfunctions $g_j(y)$

$NH \leq 7$ maximum number of subfunctions $h_k(z)$

$NF * NG * NH \leq 50$

(2) Arrangement of data

X(I) = constant for all values D(I,J=1,...,NP)

Y(J), Z(J) = constant for all values D(I=1,...,NR,J)

For further limitations and remarks concerning sequence of code numbers, zero and negative arguments of x, y, z, zero values of data, rejection of data, and required field length see page 13.

3.6 Nomenclature

Arrays

STR (140,40)	data D(I,J)
AGE (140)	variable X(I)
X (40)	variable Y(J)
Y (40)	variable Z(J)
RES (140,40)	residuals
DAT (140,40)	fitted data $\bar{D}$(I,J)
AG (140)	scaled variable x'(I)
F(8,140)	regression subfunctions $f_i(x)$
G(8,40)	regression subfunctions $g_j(y)$
H(8,40)	regression subfunctions $h_k(z)$
KT (8)	code numbers for selected subfunctions $f_i(x)$
KX (8)	code numbers for selected subfunctions $g_j(y)$
KY (8)	code numbers for selected subfunctions $h_k(z)$
A (50,50)	coefficient matrix for system of simultaneous equations

Variables

NR = total number of arguments of variable x

NP = total number of arguments of variable y (or z)

NF = n_i = total number of selected subfunctions $f_i(x)$

NG = n_j = total number of selected subfunctions $g_j(y)$

NH = n_k = total number of selected subfunctions $h_k(z)$

SNOM = total number of data

PRO = band-width factor

Branching Indices

NSE Scaling of variable x

NSE = 1 $x' = x$ (linear scale)

NSE = 2 $x' = \log_{10} x$ (logarithmic scale)

NOREJ rejection of data

NOREJ = 1 no rejection of data .

NOREJ = 2 rejection of data with subsequent second least squares fit

3.7 PROGRAM LISTING

```
      PROGRAM MULTI (INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)                MULTI  1
C                                                                          MULTI  2
C*****************************************************************************MULTI  3
C                                                                          *MULTI  4
C     REGRESSION ANALYSIS OF DATA WITH THREE INDEPENDENT VARIABLES         *MULTI  5
C                                                                          *MULTI  6
C*****************************************************************************MULTI  7
C                                                                          MULTI  8
      COMMON/TRID/STR(140,40),DAT(140,40),RES(140,40),AGE(140),AG(140),    MULTI  9
     1X(41),Y(41),F(8,140),G(8,40),H(8,40),KT(8),KX(8),KY(8),SNOM,NSE,     MULTI 10
     2PRO                                                                  MULTI 11
      DIMENSION A(50,50)                                                   MULTI 12
      IN=1                                                                 MULTI 13
      IO=2                                                                 MULTI 14
      NOREJ=2                                                              MULTI 15
C                                                                          MULTI 16
C     INPUT DATA                                                           MULTI 17
C                                                                          MULTI 18
      READ(IN,400) NP,NR,NSE,PRO                                           MULTI 19
      DO 100 I=1,NR                                                        MULTI 20
      READ(IN,401) (STR(I,N),N=1,NP)                                       MULTI 21
  100 CONTINUE                                                             MULTI 22
      READ(IN,401) (AGE(I),I=1,NR)                                         MULTI 23
      READ(IN,401) (X(I),I=1,NP)                                           MULTI 24
      READ(IN,401) (Y(I),I=1,NP)                                           MULTI 25
  101 READ(IN,402) NF,NG,NH                                                MULTI 26
      IF(NF.EQ.0) GO TO 150                                                MULTI 27
      READ(IN,402) (KT(I),I=1,NF)                                          MULTI 28
      READ(IN,402) (KX(I),I=1,NG)                                          MULTI 29
      READ(IN,402) (KY(I),I=1,NH)                                          MULTI 30
C                                                                          MULTI 31
C     OUTPUT DATA                                                          MULTI 32
C                                                                          MULTI 33
      WRITE(IO,300)                                                        MULTI 34
      CALL OUTXX(STR,AGE,NP,NR)                                            MULTI 35
      WRITE(IO,301)                                                        MULTI 36
      WRITE(IO,302)                                                        MULTI 37
      P=NP                                                                 MULTI 38
      NPH=P/2.0+0.6                                                        MULTI 39
      NFH=NF*NG*NH                                                         MULTI 40
      X(NP+1)=Y(NP+1)=0.0                                                  MULTI 41
      DO 102 I=1,NPH                                                       MULTI 42
      J=I+NPH                                                              MULTI 43
      WRITE(IO,303) I,X(I),Y(I),J,X(J),Y(J)                                MULTI 44
  102 CONTINUE                                                             MULTI 45
C                                                                          MULTI 46
C     CALL REGRESSION ROUTINES                                             MULTI 47
C                                                                          MULTI 48
      DO 103 I=1,NR                                                        MULTI 49
      DO 103 N=1,NP                                                        MULTI 50
      RES(I,N)=0.0                                                         MULTI 51
  103 CONTINUE                                                             MULTI 52
      WRITE(IO,304)                                                        MULTI 53
      CALL REGRES(NP,NR,NF,NG,NH,NFH,A)                                    MULTI 54
      GO TO (150,104) NOREJ                                                MULTI 55
  104 WRITE(IO,305)                                                        MULTI 56
      CALL REJECT(NP,NR)                                                   MULTI 57
```

```
      WRITE(IO,306)                                                    MULTI 58
      CALL REGRES(NP,NR,NF,NG,NH,NFH,A)                                MULTI 59
  300 FORMAT(1H1,///,10X,*ORIGINAL DATA :*,//)                         MULTI 60
  301 FORMAT(/////,10X,*COORDINATES*,/)                                MULTI 61
  302 FORMAT(10X,2(*POINT*,11X,*Y*,18X,*Z*,20X),/)                     MULTI 62
  303 FORMAT(10X,2(I3,8X,E11.4,8X,E11.4,15X))                          MULTI 63
  304 FORMAT(1H1,//,10X,*FIRST LEAST SQUARES FIT*,/)                   MULTI 64
  305 FORMAT(1H1,//,10X,*DATA REJECTION*,/)                            MULTI 65
  306 FORMAT(1H1,//,10X,*SECOND LEAST SQUARES FIT*,/)                  MULTI 66
  400 FORMAT(3I5,5X,F10.0)                                             MULTI 67
  401 FORMAT(10F8.0)                                                   MULTI 68
  402 FORMAT(8(I1,1X))                                                 MULTI 69
  150 CALL EXIT                                                        MULTI 70
      END                                                              MULTI 71
```

```
      SUBROUTINE OUTXX(S,AGE,NP,NR)                                    MULTI 72
      DIMENSION S(140,40),AGE(140)                                     MULTI 73
      IO=2                                                             MULTI 74
      DO 100 L=1,NP,8                                                  MULTI 75
      NA=L                                                             MULTI 76
      NE=L+7                                                           MULTI 77
      IF(NE.GT.NP) NE=NP                                               MULTI 78
      WRITE(IO,300) (N,N=NA,NE)                                        MULTI 79
      WRITE(IO,301)                                                    MULTI 80
      DO 100 I=1,NR                                                    MULTI 81
      WRITE(IO,302) AGE(I),(S(I,N),N=NA,NE)                            MULTI 82
  100 CONTINUE                                                         MULTI 83
  300 FORMAT(///,11X,*X*,6X,8(11X,I3))                                 MULTI 84
  301 FORMAT(/)                                                        MULTI 85
  302 FORMAT(5X,E12.4,5X,8E14.4)                                       MULTI 86
      RETURN                                                           MULTI 87
      END                                                              MULTI 88
```

```
      SUBROUTINE SOLVE(C,B,X,N)                                    MULTI  89
      DIMENSION C(N,N),B(N),X(N),A(50,51),IROW(50)                 MULTI  90
      IO=2                                                         MULTI  91
      M=N+1                                                        MULTI  92
      DO 100 I=1,N                                                 MULTI  93
      IROW(I)=I                                                    MULTI  94
      A(I,M)=B(I)                                                  MULTI  95
      DO 100 J=1,N                                                 MULTI  96
      A(I,J)=C(I,J)                                                MULTI  97
  100 CONTINUE                                                     MULTI  98
      DET=1.                                                       MULTI  99
      DO 106 I=1,N                                                 MULTI100
      BIG=0.0                                                      MULTI101
      DO 101 II=I,N                                                MULTI102
      IF(BIG.GE.ABS(A(II,I))) GO TO 101                            MULTI103
      BIG=ABS(A(II,I))                                             MULTI104
      K=II                                                         MULTI105
  101 CONTINUE                                                     MULTI106
      IF(BIG.GT.0.0) GO TO 102                                     MULTI107
      WRITE(IO,300)                                                MULTI108
      CALL EXIT                                                    MULTI109
  102 IF(K.EQ.I) GO TO 104                                         MULTI110
      L=IROW(I)                                                    MULTI111
      IROW(I)=IROW(K)                                              MULTI112
      IROW(K)=L                                                    MULTI113
      DO 103 J=1,M                                                 MULTI114
      Z=A(I,J)                                                     MULTI115
      A(I,J)=A(K,J)                                                MULTI116
      A(K,J)=Z                                                     MULTI117
  103 CONTINUE                                                     MULTI118
  104 Z=A(I,I)                                                     MULTI119
      DET=Z*DET                                                    MULTI120
      DO 105 J=I,M                                                 MULTI121
      A(I,J)=A(I,J)/Z                                              MULTI122
  105 CONTINUE                                                     MULTI123
      IF(I.EQ.N) GO TO 107                                         MULTI124
      II=I+1                                                       MULTI125
      DO 106 K=II,N                                                MULTI126
      Z=A(K,I)                                                     MULTI127
      DO 106 J=I,M                                                 MULTI128
      A(K,J)=A(K,J)-Z*A(I,J)                                       MULTI129
  106 CONTINUE                                                     MULTI130
  107 X(N)=A(N,M)                                                  MULTI131
      I=N                                                          MULTI132
  108 Z=0.0                                                        MULTI133
      DO 109 J=I,N                                                 MULTI134
      Z=Z+A(I-1,J)*X(J)                                            MULTI135
  109 CONTINUE                                                     MULTI136
      I=I-1                                                        MULTI137
      X(I)=A(I,M)-Z                                                MULTI138
      IF(I.GT.1) GO TO 108                                         MULTI139
  300 FORMAT(////,10X,*SINGULAR COEFFICIENT MATRIX*)               MULTI140
      RETURN                                                       MULTI141
      END                                                          MULTI142
```

```
      SUBROUTINE REJECT(NP,NR)                                          MULTI143
      COMMON/TRID/STR(140,40),DAT(140,40),RES(140,40),AGE(140),AG(140), MULTI144
     1X(41),Y(41),F(8,140),G(8,40),H(8,40),KT(8),KX(8),KY(8),SNOM,NSE,  MULTI145
     2PRO                                                               MULTI146
      IO=2                                                              MULTI147
      WRITE(IO,300)                                                     MULTI148
      CALL OUTXX(RES,AGE,NP,NR)                                         MULTI149
C                                                                       MULTI150
C     COMPUTE SUM OF SQUARES AND STANDARD ERROR                         MULTI151
C                                                                       MULTI152
      SUM=0.0                                                           MULTI153
      DO 100 I=1,NR                                                     MULTI154
      DO 100 L=1,NP                                                     MULTI155
      SUM=RES(I,L)*RES(I,L)+SUM                                         MULTI156
  100 CONTINUE                                                          MULTI157
      WRITE(IO,301) SNOM                                                MULTI158
      WRITE(IO,302) SUM                                                 MULTI159
      STD=SQRT(SUM/SNOM)                                                MULTI160
      WRITE(IO,303) STD,PRO                                             MULTI161
C                                                                       MULTI162
C     REJECT DATA,  COMPUTE NEW SUM OF SQUARES AND STANDARD ERROR       MULTI163
C                                                                       MULTI164
      STD=PRO*STD                                                       MULTI165
      WRITE(IO,304)                                                     MULTI166
      DO 101 I=1,NR                                                     MULTI167
      DO 101 L=1,NP                                                     MULTI168
      IF(ABS(RES(I,L)).LE.STD) GO TO 101                                MULTI169
      SNOM=SNOM-1.0                                                     MULTI170
      SUM=SUM-RES(I,L)*RES(I,L)                                         MULTI171
      RES(I,L)=2.0E+300                                                 MULTI172
      WRITE(IO,305) AGE(I),L                                            MULTI173
  101 CONTINUE                                                          MULTI174
      WRITE(IO,306)                                                     MULTI175
      CALL OUTXX(RES,AGE,NP,NR)                                         MULTI176
      WRITE(IO,307) SNOM                                                MULTI177
      WRITE(IO,308) SUM                                                 MULTI178
      STD=SQRT(SUM/SNOM)                                                MULTI179
      WRITE(IO,309) STD,PRO                                             MULTI180
  300 FORMAT(//,10X,*RESIDUALS BEFORE REJECTION*)                       MULTI181
  301 FORMAT(///,10X,*TOTAL NO. OF DATA BEFORE REJECTION*,14X,F7.0)     MULTI182
  302 FORMAT(/,10X,*SUM OF SQUARES BEFORE REJECTION*,13X,E11.4)         MULTI183
  303 FORMAT(/,10X,*STANDARD ERROR BEFORE REJECTION*,13X,E11.4,10X,*BANDMULTI184
     1-WIDTH FACTOR*,5X,F6.3)                                           MULTI185
  304 FORMAT(///,10X,*REJECTED DATA :*,11X,*X*,12X,*POINT*,/)           MULTI186
  305 FORMAT(30X,E11.4,9X,I2)                                           MULTI187
  306 FORMAT(//////,10X,*RESIDUALS AFTER REJECTION*)                    MULTI188
  307 FORMAT(////,10X,*TOTAL NO. OF DATA AFTER REJECTION*,14X,F7.0)     MULTI189
  308 FORMAT(/,10X,*SUM OF SQUARES AFTER REJECTION*,13X,E11.4)          MULTI190
  309 FORMAT(/,10X,*STANDARD ERROR AFTER REJECTION*,13X,E11.4,10X,*BAND-MULTI191
     1WIDTH FACTOR*,5X,F6.3)                                            MULTI192
      RETURN                                                            MULTI193
      END                                                               MULTI194
```

```
      SUBROUTINE REGRES(NP,NR,NF,NG,NH,NFH,A)                          MULTI195
      COMMON/TRID/STR(140,40),DAT(140,40),RES(140,40),AGE(140),AG(140), MULTI196
     1X(41),Y(41),F(8,140),G(8,40),H(8,40),KT(8),KX(8),KY(8),SNOM,NSE, MULTI197
     2PRO                                                              MULTI198
      DIMENSION A(NFH,NFH),R(50),B(50)                                 MULTI199
      IO=2                                                             MULTI200
      WRITE(IO,300)                                                    MULTI201
      WRITE(IO,301)                                                    MULTI202
      WRITE(IO,302) (KT(L),L=1,NF)                                     MULTI203
      WRITE(IO,303)                                                    MULTI204
      WRITE(IO,304)                                                    MULTI205
      WRITE(IO,305) (KX(L),L=1,NG)                                     MULTI206
      WRITE(IO,306)                                                    MULTI207
      WRITE(IO,307)                                                    MULTI208
      WRITE(IO,308) (KY(L),L=1,NH)                                     MULTI209
C                                                                      MULTI210
C     LOG. OR NON-LOG. SCALE FOR VARIABLE X                            MULTI211
C                                                                      MULTI212
      GO TO (100,102) NSE                                              MULTI213
  100 DO 101 I=1,NR                                                    MULTI214
      AG(I)=AGE(I)                                                     MULTI215
  101 CONTINUE                                                         MULTI216
      WRITE(IO,309)                                                    MULTI217
      GO TO 104                                                        MULTI218
  102 DO 103 I=1,NR                                                    MULTI219
      AG(I)=0.0                                                        MULTI220
      IF(AGE(I).GT.0.0) AG(I)=ALOG10(AGE(I))                           MULTI221
  103 CONTINUE                                                         MULTI222
      WRITE(IO,310)                                                    MULTI223
C                                                                      MULTI224
C     COMPUTE VALUES OF REGRESSION SUBFUNCTIONS F, G, AND H            MULTI225
C                                                                      MULTI226
  104 DO 107 I=1,NR                                                    MULTI227
      F(1,I)=1.0                                                       MULTI228
      F(2,I)=AG(I)                                                     MULTI229
      F(5,I)=AG(I)*AG(I)                                               MULTI230
      F(6,I)=AG(I)*F(5,I)                                              MULTI231
      IF(AG(I).LE.0.0) 105,106                                         MULTI232
  105 F(3,I)=F(4,I)=F(7,I)=F(8,I)=0.0                                  MULTI233
      GO TO 107                                                        MULTI234
  106 F(3,I)=SQRT(AG(I))                                               MULTI235
      F(4,I)=1.0/AG(I)                                                 MULTI236
      F(7,I)=ALOG10(AG(I))                                             MULTI237
      F(8,I)=SQRT(F(3,I))                                              MULTI238
  107 CONTINUE                                                         MULTI239
      DO 110 L=1,NP                                                    MULTI240
      G(1,L)=1.0                                                       MULTI241
      G(2,L)=X(L)                                                      MULTI242
      G(5,L)=X(L)*X(L)                                                 MULTI243
      G(6,L)=X(L)*G(5,L)                                               MULTI244
      IF(X(L).LE.0.0) 108,109                                          MULTI245
  108 G(3,L)=G(4,L)=G(7,L)=G(8,L)=0.0                                  MULTI246
      GO TO 110                                                        MULTI247
  109 G(3,L)=SQRT(X(L))                                                MULTI248
      G(4,L)=1.0/X(L)                                                  MULTI249
      G(7,L)=ALOG10(X(L))                                              MULTI250
      G(8,L)=SQRT(G(3,L))                                              MULTI251
```

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  110 CONTINUE                                                    MULTI252
      DO 113 L=1,NP                                               MULTI253
      H(1,L)=1.0                                                  MULTI254
      H(2,L)=Y(L)                                                 MULTI255
      H(5,L)=Y(L)*Y(L)                                            MULTI256
      H(6,L)=Y(L)*H(5,L)                                          MULTI257
      IF(Y(L).LE.0.0) 111,112                                     MULTI258
  111 H(3,L)=H(4,L)=H(7,L)=H(8,L)=0.0                             MULTI259
      GO TO 113                                                   MULTI260
  112 H(3,L)=SQRT(Y(L))                                           MULTI261
      H(4,L)=1.0/Y(L)                                             MULTI262
      H(7,L)=ALOG10(Y(L))                                         MULTI263
      H(8,L)=SQRT(H(3,L))                                         MULTI264
  113 CONTINUE                                                    MULTI265
      DO 115 J=1,NF                                               MULTI266
      KK=KT(J)                                                    MULTI267
      IF(J.EQ.KK) GO TO 115                                       MULTI268
      DO 114 I=1,NR                                               MULTI269
      F(J,I)=F(KK,I)                                              MULTI270
  114 CONTINUE                                                    MULTI271
  115 CONTINUE                                                    MULTI272
      DO 117 J=1,NG                                               MULTI273
      KK=KX(J)                                                    MULTI274
      IF(J.EQ.KK) GO TO 117                                       MULTI275
      DO 116 L=1,NP                                               MULTI276
      G(J,L)=G(KK,L)                                              MULTI277
  116 CONTINUE                                                    MULTI278
  117 CONTINUE                                                    MULTI279
      DO 119 J=1,NH                                               MULTI280
      KK=KY(J)                                                    MULTI281
      IF(J.EQ.KK) GO TO 119                                       MULTI282
      DO 118 L=1,NP                                               MULTI283
      H(J,L)=H(KK,L)                                              MULTI284
  118 CONTINUE                                                    MULTI285
  119 CONTINUE                                                    MULTI286
C                                                                 MULTI287
C     GENERATE SYSTEM OF SIMULTANEOUS EQUATIONS                   MULTI288
C                                                                 MULTI289
      DO 120 J=1,NFH                                              MULTI290
      B(J)=0.0                                                    MULTI291
      DO 120 K=1,NFH                                              MULTI292
      A(J,K)=0.0                                                  MULTI293
  120 CONTINUE                                                    MULTI294
      SNOM=0.0                                                    MULTI295
      NGH=NG*NH                                                   MULTI296
      DO 122 I=1,NR                                               MULTI297
      DO 122 N=1,NP                                               MULTI298
      IF(STR(I,N).EQ.0.0) GO TO 122                               MULTI299
      IF(RES(I,N).GT.1.0E+300) GO TO 122                          MULTI300
      SNOM=SNOM+1.0                                               MULTI301
      DO 121 J=1,NF                                               MULTI302
      DO 121 K=1,NG                                               MULTI303
      DO 121 L=1,NH                                               MULTI304
      NV=(J-1)*NGH+(K-1)*NH+L                                     MULTI305
      B(NV)=STR(I,N)*F(J,I)*G(K,N)*H(L,N)+B(NV)                   MULTI306
      DO 121 JH=1,NF                                              MULTI307
      DO 121 KH=1,NG                                              MULTI308
      DO 121 LH=1,NH                                              MULTI309
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      NS=(JH-1)*NGH+(KH-1)*NH+LH                                        MULTI310
      A(NV,NS)=F(JH,I)*G(KH,N)*H(LH,N)*F(J,I)*G(K,N)*H(L,N)+A(NV,NS)    MULTI311
  121 CONTINUE                                                          MULTI312
  122 CONTINUE                                                          MULTI313
C                                                                       MULTI314
C     SOLVE SIMULTANEOUS EQUATIONS                                      MULTI315
C                                                                       MULTI316
      CALL SOLVE(A,B,R,NFH)                                             MULTI317
      WRITE(IO,311)                                                     MULTI318
      DO 123 J=1,NF                                                     MULTI319
      JI=(J-1)*NGH                                                      MULTI320
      WRITE(IO,312)                                                     MULTI321
      DO 123 K=1,NG                                                     MULTI322
      JA=JI+(K-1)*NH+1                                                  MULTI323
      JE=JA+NH-1                                                        MULTI324
      WRITE(IO,313) J,K,(R(M),M=JA,JE)                                  MULTI325
  123 CONTINUE                                                          MULTI326
C                                                                       MULTI327
C     COMPUTE FITTED DATA                                               MULTI328
C                                                                       MULTI329
      DO 124 I=1,NR                                                     MULTI330
      DO 124 N=1,NP                                                     MULTI331
      DAT(I,N)=0.0                                                      MULTI332
      DO 124 J=1,NF                                                     MULTI333
      DO 124 K=1,NG                                                     MULTI334
      DO 124 L=1,NH                                                     MULTI335
      NV=(J-1)*NGH+(K-1)*NH+L                                           MULTI336
      DAT(I,N)=F(J,I)*G(K,N)*H(L,N)*R(NV)+DAT(I,N)                      MULTI337
  124 CONTINUE                                                          MULTI338
      WRITE(IO,314)                                                     MULTI339
      CALL OUTXA(DAT,AGE,NP,NR)                                         MULTI340
C                                                                       MULTI341
C     COMPUTE RESIDUALS                                                 MULTI342
C                                                                       MULTI343
      DO 125 I=1,NR                                                     MULTI344
      DO 125 N=1,NP                                                     MULTI345
      RES(I,N)=0.0                                                      MULTI346
      IF(STR(I,N).NE.0.0) RES(I,N)=STR(I,N)-DAT(I,N)                    MULTI347
  125 CONTINUE                                                          MULTI348
  300 FORMAT(//,10X,*LIST OF FUNCTIONS OF X :   1- CONST.   2- X       3MULTI349
     1- SQRT(X)    4- 1/X*)                                             MULTI350
  301 FORMAT(37X,46H5- X**2     6- X**3     7- LOG10(X)   8- X**.25,//) MULTI351
  302 FORMAT(10X,21HUSED FUNCTIONS OF X :,6X,8(I1,2X))                  MULTI352
  303 FORMAT(//,10X,*LIST OF FUNCTIONS OF Y :   1- CONST.   2- Y       3MULTI353
     1- SQRT(Y)    4- 1/Y*)                                             MULTI354
  304 FORMAT(37X,46H5- Y**2     6- Y**3     7- LOG10(Y)   8- Y**.25,//) MULTI355
  305 FORMAT(10X,21HUSED FUNCTIONS OF Y :,6X,8(I1,2X))                  MULTI356
  306 FORMAT(//,10X,*LIST OF FUNCTIONS OF Z :   1- CONST.   2- Z       3MULTI357
     1- SQRT(Z)    4- 1/Z*)                                             MULTI358
  307 FORMAT(37X,46H5- Z**2     6- Z**3     7- LOG10(Z)   8- Z**.25,//) MULTI359
  308 FORMAT(10X,21HUSED FUNCTIONS OF Z :,6X,8(I1,2X))                  MULTI360
  309 FORMAT(//,10X,*VARIABLE X = X  (NON-LOG. SCALE)*,/)               MULTI361
  310 FORMAT(//,10X,*VARIABLE X = LOG10(X)*,/)                          MULTI362
  311 FORMAT(///,10X,51HCOEFFICIENTS A(I,J,K) :     A(I,J,K)*F(I)*G(J)*HMULTI363
     1(K),/)                                                            MULTI364
  312 FORMAT(/)                                                         MULTI365
  313 FORMAT(10X,*I =*,I2,5X,*J =*,I2,5X,7E15.4)                        MULTI366
  314 FORMAT(1H1,//,10X,13HFITTED DATA :,//)                            MULTI367
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      RETURN                                                    MULTI368
      END                                                       MULTI369
```

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