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G. R. Spalding

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FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

# THE EFFECT OF PRESTRESS ON ELASTIC COLUMNS

by
George R. Spalding

Supervisors: Professor W. J. Eney
Professor H. J. McCrodden

Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

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ABSTRACT

The extremely long and slender member, the elastic column, fails due to buckling. The carrying capacity of such a member may be increased if the natural tendency of the column to buckle is opposed.

The buckling of an elastic column can be opposed by the use of prestressing wires placed triangularly in the member. The degree of increase in the load carrying capacity is dependent upon the initial prestress introduced, within limits.

A prestressed member appears to be a more flexible element than a similar non-prestressed member.

A third characteristic of such columns is their ability to recover their original position. In the tests, the prestressed member, upon removal of the failure load, recovered well and was still of use as a load-carrying column. The non-prestressed column, however, recovered very little and was useless as a load carrying element.

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# THE EFFECT OF PRESTRESS ON ELASTIC COLUMNS

## INTRODUCTION

For the past decade prestressed concrete has risen to great heights in the field of structural design. There has been much written on its use in beams and girders. Prestressed concrete columns, however, have been ignored to date.

It is the purpose of this report to investigate the effects of pre-tensioning on concrete columns in the elastic range. Particular emphasis has been placed on the load carrying capacity of the columns.

The report herein presented includes a mathematical analysis of the effects of prestress on a long slender concrete column whose failure is governed by Euler loading, and the procedure and results of the testing of two columns, one prestressed and one non-prestressed.

The author acknowledges the guidance and inspiration of the director of the project, Howard J. McCrodden, past professor of structural engineering at Lehigh University. Without Professor McCrodden's help and criticism this report never would have been written.

Thanks must also be given to Professor William J. Eney for his interest and guidance as director of the project after the departure of Professor McCrodden.

## ANALYTICAL DEVELOPMENT

It has been proved that pretensioning alone will never cause buckling in a concrete column, but that an increasing prestressing force will ultimately crush the concrete when the compressive stress for that concrete is reached. It has also been calculated that a centrally prestressed column will support no load in excess of the Euler load. It is necessary to know these two points before continuing.

Suppose, however, that rather than running the wire through a concrete column vertically, we place them in a triangular fashion as shown in Figure 1a and apply an increasing axial load P. At the critical or ultimate value of P the member will assume the deflected position shown in Figure 1b. In Figure 1a F is the stress in the wires causing a prestressing force R in the concrete and two equal and opposite lateral forces $Q_2$. When the column is

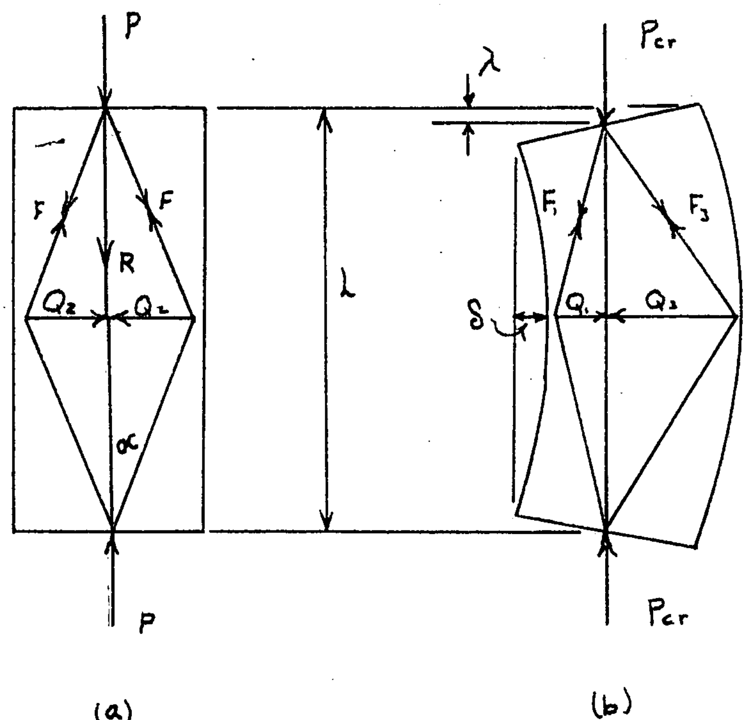



FIG 1

deflected due to loading (Figure 1b) one wire has shortened, reducing its stress, and consequently, its lateral component from $Q_2$ to $Q_1$, while the other wire has increased in length increasing its lateral component from $Q_2$ to $Q_3$.

The load $\underline{P}$ will be critical when the strain energy of bending within the column, $\Delta V$, is equal to the external work done on the column.

We may then say for the critical condition:

$$W = W_V + W_L$$

$W_V$ = work done on column in vertical direction

$W_L$ = work done on column in lateral direction

$$\Delta V = \int_0^{\ell} \frac{M^2 dx}{2\,EI} \qquad \text{where} \qquad M = -EI\,\frac{d^2y}{dx^2}$$

Assuming a sine curve of deflection for approximate solution:

$$y = a \sin \frac{\pi x}{\ell} \qquad a = \delta \quad \text{when} \quad x = \frac{\ell}{2}$$

$$y = \delta \sin \frac{\pi x}{\ell} \qquad \frac{dy}{dx} = \delta\,\frac{\pi}{\ell} \cos \frac{\pi x}{\ell}$$

$$\frac{d^2y}{dx^2} = -\delta\,\frac{\pi^2}{\ell^2} \sin \frac{\pi x}{\ell}$$

$$\Delta V = \int_0^{\ell} \frac{EI}{2} \left(\frac{d^2y}{dx^2}\right)^2 dx$$

$$\Delta \qquad \Delta V = \frac{EI}{2}\,\delta^2\,\frac{\pi^4}{\ell^4}\,\frac{\ell}{\pi} \int_0^{\ell} \frac{\pi}{\ell} \sin \frac{\pi x}{\ell}\,dx$$

$$\Delta V = \frac{EI}{2}\,\delta^2\,\frac{\pi^3}{\ell^3} \left[\frac{\pi x}{2\ell} - \frac{1}{4} \sin \frac{2\pi x}{\ell}\right]_0^{\ell}$$

$$\Delta V = \frac{EI\,\delta^2\,\pi^4}{4\,\ell^3}$$

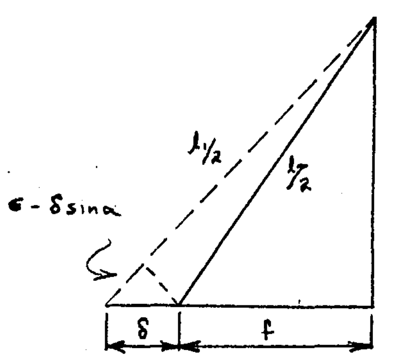
Calculation of vertical work on column ($W_V$)

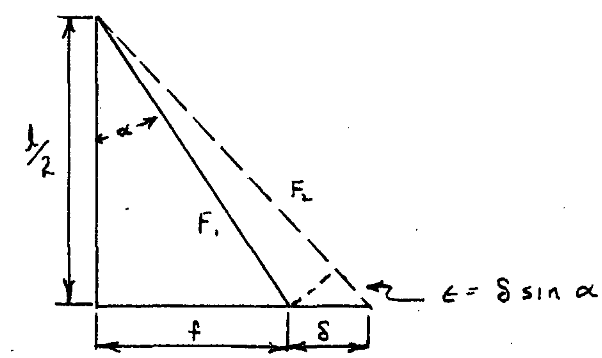
$$W_V = \frac{P_{cr}}{2} \int_0^{\ell} \left(\frac{dy}{dx}\right)^2 dx \qquad \text{where} \qquad y = \delta \sin \frac{\pi x}{\ell}$$

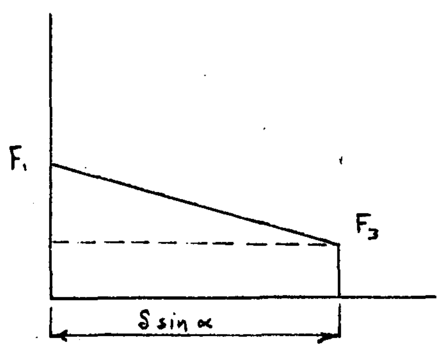
$$W_V = \frac{P_{cr}}{2}\,\delta^2\,\frac{\pi}{\ell}\int_0^{\ell}\cos^2\frac{\pi x}{\ell}\;\frac{\pi}{\ell}\,dx$$

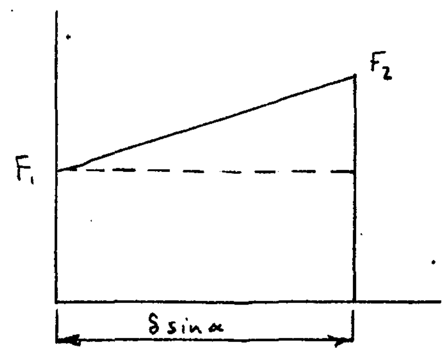
$$W_V = \frac{P_{cr}\;\delta^2\;\pi^2}{4\ell}$$

Calculation of lateral work ($W_L$)




$$W_L = \left(\frac{F_2 - F_3}{2}\right)\delta \sin\alpha$$

$$F_3 = F_1 - \frac{\ell_1/2 - \ell_3/2}{\ell_1/2}\,A_s\,E\;; \qquad F_2 = F_1 + \frac{\ell_2/2 - \ell_1/2}{\ell_1/2}\,A_s\,E$$

$$F_2 - F_3 = \left[\frac{\ell_2 - \ell_1}{\ell_1} + \frac{\ell_1 - \ell_3}{\ell_1}\right] A_s\,E$$

$$F_2 - F_3 = \frac{\ell_2 - \ell_3}{\ell_1} A_s E \qquad \ell_2 - \ell_3 = \delta \sin\alpha$$

$$\ell_1 = \ell \sec\alpha$$

$$F_2 - F_3 = \frac{(2)\ \delta \sin\alpha}{\ell \sec\alpha} A_s E = (2) \frac{\delta}{\ell} \sin\alpha \cos\alpha A_s E$$

$$W_L = -\frac{(2)}{2} \frac{\delta^2}{\ell} \sin^2\alpha \cos\alpha A_s E \qquad \sin\alpha = \frac{2f}{\ell}$$

$$= -\frac{(2)}{2} \frac{\delta^2}{\ell^3} f^2\ 4 A_s E \qquad \cos\alpha = 1$$

Multiply by 2 for 2 ends of member.

$$W_L = -\frac{8\ \delta^2 f^2}{\ell^3} A_s E$$

$$\Delta V = W_V + W_L$$

$$\frac{E\ I\ \delta^2 \pi^4}{4\ \ell^3} = \frac{P_{cr}\ \delta^2 \pi^2}{4\ell} - \frac{8\ \delta^2 f^2}{\pi^2 \ell^2} A_s E$$

$$P_{cr} = \frac{E\ I\ \pi^2}{\ell^2} + \frac{32\ f^2}{\pi^2 \ell^2} A_s E$$

Expression for buckling load:

Let

$$\frac{F^2}{\ell^2} = \sin^2\alpha$$

For small angles $< 10°$ $\quad \sin^2\alpha \approx \alpha^2$

At 10° assuming $\sin^2\alpha = \alpha$ gives error of about 3%.

We then have:

$$\frac{P_{cr}}{P_e} = 1 + \frac{32}{\pi^2} \frac{\alpha^2}{P_e} A_s E$$

$$\frac{P_{cr}}{P_e} = 1 + 3.3 \frac{\alpha^2}{P_e} A_s E$$

Discussion of Mathematical Analysis

It must be kept in mind that the above is an approximate solution, for only first order variables were considered, the loss of prestress due to loading was neglected and only one point of wire contact (at the center) was assumed.

It is of interest to note that the critical load for a pretensioned column is independent of the prestress introduced. This seems only reasonable, for the increase in load carrying capacity depends only on a lengthening and shortening of the wires and not upon their initial stress (assuming, of course, sufficient initial strain so that the wire that shortens remains slightly stressed). A more rigorous analysis taking into account second order variation would naturally show $\underline{P_{cr}}$ as a function of $\underline{R}$ (initial prestress).

My analysis shows $\underline{P_{cr}}$ to be a function of the variable $\underline{\alpha}$ (initial angle between the wires and the center line of the member).

## EXPERIMENTATION

### Column Size

Determination of the column size was the first problem to be considered. The member had to be well into the elastic range in order to give significant results. Whether or not a member is elastic depends upon the $\ell/r$ ratio and the modulus of elasticity of the material used. The modulus of elasticity of concrete is generally expressed as a function of the compressive strength of the concrete ($f_c$). Thus, in order to determine the $\ell/r$ ratio sufficient for a column to be considered elastic, the compressive strength of the concrete (to be obtained from the design mix) must be plugged into an empirical formula based on test results. That E value is then, in turn, placed in Euler's formula for elastic columns.

There are as many empirical formulas for the modulus of elasticity of concrete as there are types of aggregate or research assistants who wish to run tests. For example, the A.C.I. suggests the expression $E = 1000\, f_c$ as a fair guess, while August Komendant in his recent book "Prestressed Concrete Structures" suggests $E = [f_c/(2300 + f_c)][8.15 \times 10^6]$. These two formulas could not be more divergent when applied to high strength concrete. As a consequence, it seemed that a good guess as to the modulus of elasticity would do as well as anything. However, in order to be on the safe side in calculating the required $\ell/r$ ratio necessary for elastic columns, Mr. Komendant's formula was used since it gave considerably more conservative values. This was done, as tabulated below, by finding E for various $f_c$ values and by using these values of E in Euler's formula for the critical load of an elastic column to determine the minimum $\ell/r$ value.

$$E = \frac{f_c}{2300 + f_c}\, 8.15 \times 10^6 \qquad \text{(Komendant)}$$

$$f_c = \frac{\pi^2 E}{(\ell/r)^2} \qquad \text{(Euler)}$$

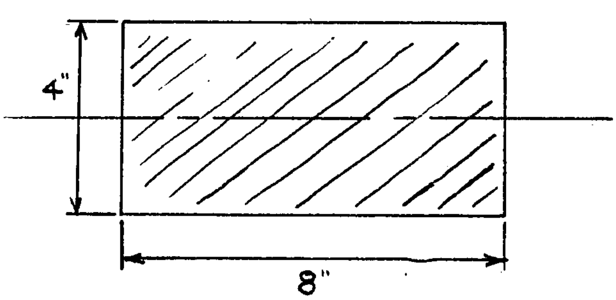
| $f_c$ | E | $(\ell/r)^2$ | $\ell/r$ |
|---|---|---|---|
| 2000 | $3.8 \times 10^6$ | 18,450 | 136 |
| 3000 | $4.6 \times 10^6$ | 14,460 | 120 |
| 4000 | $5.2 \times 10^6$ | 12,620 | 113 |
| 5000 | $5.6 \times 10^6$ | 10,860 | 104 |
| 6000 | $5.9 \times 10^6$ | 9,540 | 98 |

It was assumed from the onset that a minimum of 5000 psi compressive strength could be attained and, consequently, an $\ell/r$ ratio of 104 would give an elastic column. However, the effects of prestressing would be more pronounced the greater the difference between the Euler load and the crushing load. For this reason the author decided that, if possible, he would realize an $\ell/r$ ratio of 125.

Another controlling factor was the testing machines available. The small 300,000 lb machine was more desirable in all respects except that it would limit the length of column to about 5 ft. This was really a smaller member than could be properly analyzed and so there only remained the large 800,000 lb machine.

In the final analysis, the matter was settled when two 11-in. channels exactly 160 in. (13 ft - 4 in.) long were left at the author's disposal. These made ideal side forms.

The cross section of the member was chosen to be 8 in. × 4 in. and the length to be 160 in. The 8 in. × 4 in. section was used, as the rectangular shape would insure bending about the desired axis, allow for 1 in. cover and 2 in. between the wires, and insure a satisfactorily high $\ell/r$ ratio. Calculation of $\ell/r$ value is given below.




axis of bending

$r = \sqrt{I/A}$

$$r^2 = \frac{bh^3}{12\ bh} = \frac{h^2}{12}$$

$$r = \frac{h}{\sqrt{12}} = \frac{4}{\sqrt{12}} = 1.15 \text{ in.}$$

$$\ell/r = \frac{160}{1.15} = 138$$

## Forms

As has been previously mentioned two columns were made. It was desired to have one form contain both members, if possible, in order to economize on materials and facilitate pouring.

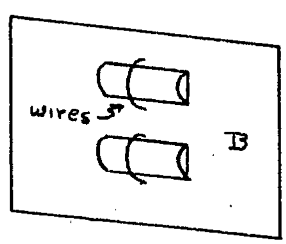
Two 11 in. channels 13 ft - 4 in. in length served as side forms while a board 9 in. deep, 1 in. thick and 13 ft - 4 in. long was placed between them to separate the two columns as shown in Figure 5. The jacking frames bearing against each end of the channels were the end forms. These frames will be gone into in more detail in the next section. A plywood base was placed under the channels.

Clamps were used to hold the channels against buckling under the load due to the stressed wires. The board was covered with asbestos paper to keep the concrete from adhering to its surface and was held in place by 4-in. sticks of wood. A picture of the entire setup is shown on page 10.

## Prestressing

Originally it was planned to use 20 1/10-in. round wired each carrying a load of 1,500 lb or 191 psi. Twenty wires each carrying a load of 1500 lb is, or course, only 30,000 lb prestress. But 20 wires was the absolute maximum that could be used due to reasonable cover requirements. It would have been much more expedient to use the now popular 3/8-in. strand for then the proper amount of stress could have been induced, cover requirements adhered to, and the trouble of stressing large numbers of wires avoided. Twenty wires were still found exceedingly difficult to stress because the wires were so close together as will be seen more clearly later. As a consequence the number of wires was reduced to 14 each carrying 1,430 lb (185,000 psi) and totaling 20,000 lb of prestress. It can be seen from the Appendix that 185,000 psi is about as close to the yield point of the steel wire as could be considered safe.

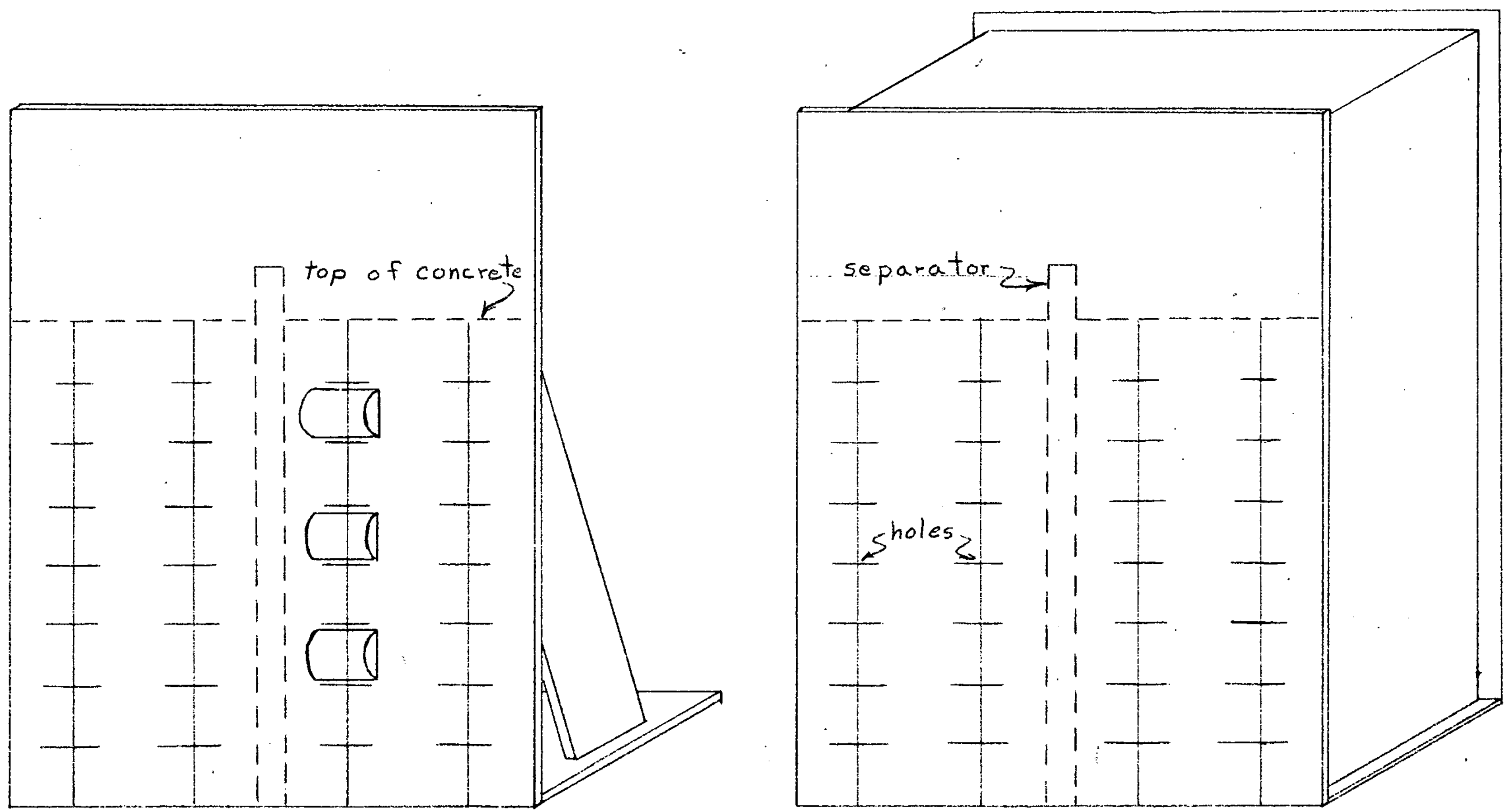
The jacking frames used were reinforced metal plates 1/4-in. thick containing 1/8-in. diameter holes through which the wires pass. A detail drawing of the frame is shown on page 12. As can be seen from Figure 2b on page 12, a 1-in. length of 7/8-in. half round rod is welded between every two holes, with the exception of the bottom hole (due to the odd number). From sketch below, this allows one wire to be run from frame A, through a hole in frame B, through the hole below, and back to frame A.




The advantage of the above setup is that it allows the wires to be gripped at one end only, a great saving regardless of method of gripping.

Before going into the method of stressing and gripping, a word or two should be said about wiring pattern. From the mathematical development it is known that a triangular effect of the wiring is desired. The sketch

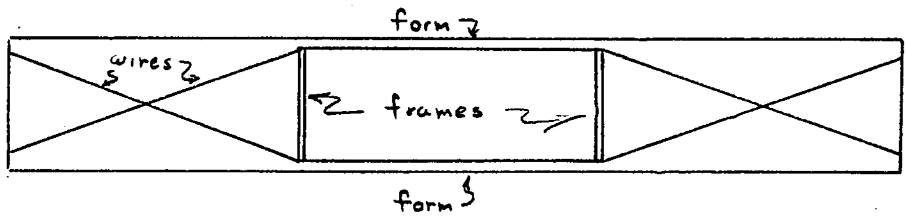
# JACKING FRAMES




(a) (b)

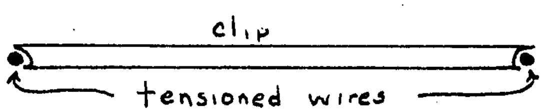
FIG. 2

below is a plan view of the wires for each column.

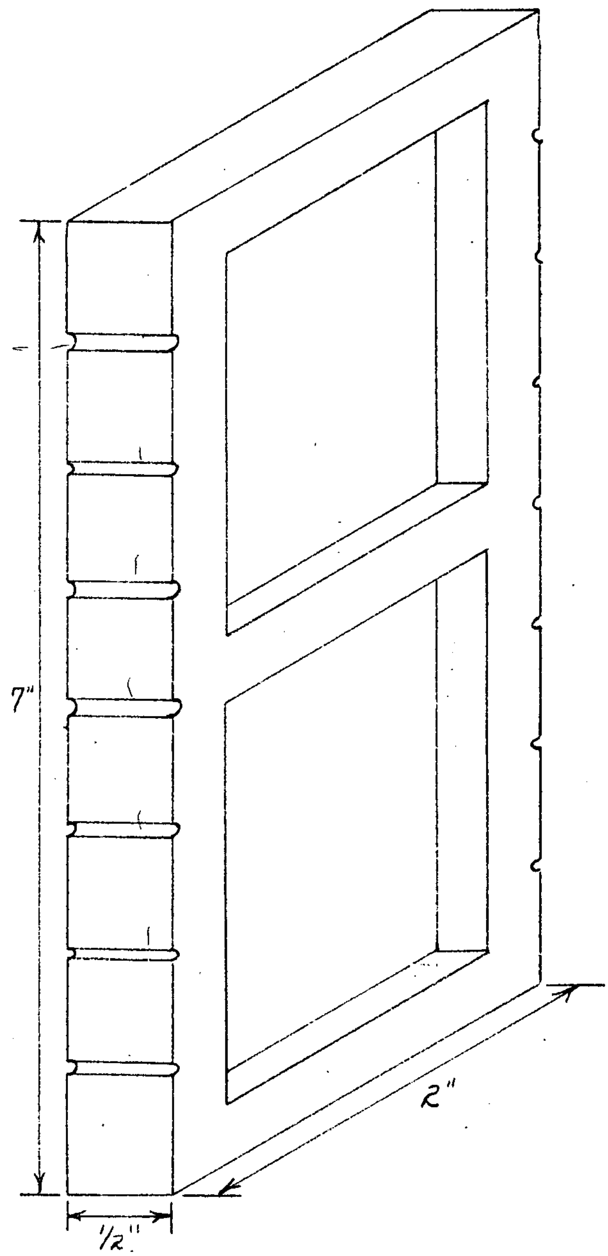



It must be realized that there are six more wires below the ones shown above spaced 7/8 of an inch apart. A detailed scetch of the frames separating the wires is shown on page 14. Two frames placed as above seemed more advisable than one frame in the center of the member as it was feared stress concentrations would be built up around the metal frame. These concentrations, it seemed, would have much less effect on the ultimate load of the column if they were placed 2-1/2 ft each side of the center than if they were at the critical section. This matter will be discussed in greater detail in the section on results.

I am sure that my method of separating the strands can be greatly improved upon. For example small clips, as sketched below, of high strength steel could be used under some circumstances. These clips would separate




two wires, but the wires would have to be on exactly the same level. Note that my wires cross between separating frames and jacking frames. This means that one wire is slightly below its partner. If such a condition existed




SEPARATING FRAMES

FIG. 3

when the two wire clips were used the clips would tend to rotate.

I am sure that a little thought would bring forth numerous ideas for separators much better than the ones used.

Wooden copies of the separators were used in the non-prestressed column as they carried little load.

The method used in gripping the 1/10-in. diameter wires could not have been more economical of time and money, nor more reliable. I used what is called a Nico-Press sleeve made by The National Telephone Supply Company of Cleveland, Ohio*. The Nico-Press sleeves used in my work were approximately 3 in. in length and 1/8-in. inside diameter. The outside diameter was approximately 1/2 in. The gripping is accomplished by making a series of presses in the sleeve, the number depending on the strength required, with a Nico-Press tool supplied by the manufacturer.

I used two sleeves on each wire and put about five presses in each sleeve. This I found adequate when the wires were stressed 185,000 psi**. A sketch of the Nico-Press sleeve is found on page 16 along with a diagrammatic sketch of the tool used to make the necessary presses.

Inside the sleeves bearing against the jacking frames were the jacks. Below is a detail of one of the jacks. Essentially the jacks are 1/4-in. bolts, 1-1/2 in. in length (thread length) screwed inside a 1-1/2-in. long threaded sleeve. The wire passes through the center of the bolt and is held

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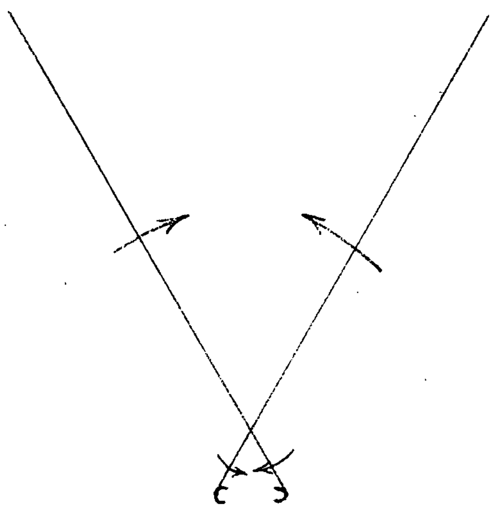
* The Nico-Press sleeve was first used in prestressed concrete, to grip small diameter wires, by Cesar A. Buenaventura, Research Assistant at Lehigh University. Mr. Buenaventura suggested this method to me after completing tests that proved beyond a doubt their value. See Bond of Wires in Prestressed Concrete by C. A. Buenaventura.

**Mr. Buenaventura's report gives exactly the required number of presses for a given stress.

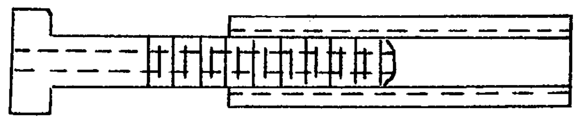
NICO - PRESS SLEEVE



NICO - PRESS TOOL

FIG. 4

by the Nico-Press sleeve as shown. The wire is then stressed by unscrewing the bolt which elongates the wire, thus inducing stress. Due to the large amount of elongation necessary to stress the wires to 185,000 psi it was, in my case, necessary to use two jacks on each wire.



The wires in the non-pretensioned column were pulled hand tight and the bolts unscrewed so as to just bear firmly.

In the prestressed member, fourteen wires had to be tensioned. In order to accomplish this, A-2 (SR-4) strain gages were placed on the two top and one bottom wires. These three wires were then strained until the desirable stress (185,000 lb) was realized. The remaining ten wires were then strained until they vibrated with the same natural frequency as the four wires of known stress. This was done by plucking the questionable strand with the thumb and then plucking one of the known wires. When the sounds were the same from both strands the two stresses were equal. This was a very tedious operation as the tightening of one wire invariably changed the stresses of the others. This can be understood at once when it is realized that the frames separating the strands made each wire stress dependent on the others. The data taken while stressing the three wires with the strain gages is shown on page A-16 of the appendix.

Some question was raised as to whether or not the gages on the wires would tend to slightly reduce the frequency of those strands and give erroneous frequencies when plucked. It is my opinion that one A-2 SR-4 strain gage on a 1/10-in.-diameter wire over a span of approximately 5 feet will have a negligible deadening effect.

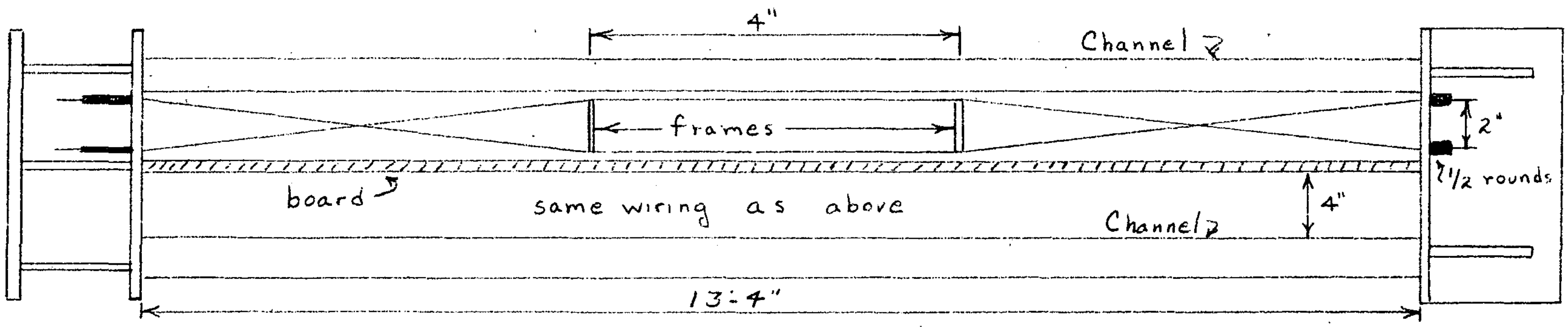
After all wires were stressed to the desired amount they were left overnight to allow creep and strain hardening to take effect. During this time of course the stress dropped off. The stress was again raised to 185,000 lb. It remained constant.

## TEST PROGRAM

The testing was done in the large 800,000-lb machine. The first problem encountered in testing was the setup for end connections. The mathematical analysis was done on the assumption that the ends were pin connected, consequently this condition had to be as nearly duplicated as possible. This was done by making the two metal caps as shown on page 21. The cap was essentially a 6 in. × 4 in. metal plate with a 6-in.-long 1-in. round rod welded to the plate parallel to the axis of bending. These caps were held with neat cement paste to give a smooth bearing surface and allowed the ends of the member to rotate about the desired axis.

The columns were lifted into the testing machine by the overhead crane and aligned with a level rod and plumb-bob. When the desired alignment was obtained, the member was held against sidesway in both directions by a wooden frame. It was then lowered to bear against the cap containing neat cement paste. The top cap was then cemented in place. After the capping compound had hardened, a slight load (1000 lb) was applied and the wooden frames removed.

Deflection readings were made possible by placing a one-foot rule at the centerline of the column perpendicular to the axis of the member along the



4"
Channel
frames
2"
1/2 rounds
board
same wiring as above
4"
Channel
13'-4"


FORMS

FIG. 5

line of deflection. A plumb-bob was hung from the center of the top of the column reaching about 1/2 inch from the base. The bob string remained stationary and consequently as the column deflected the deflection could easily be read from the ruler. This deflection setup is shown on page 21.

In order to determine initial eccentricity of the two columns, measurements of the cross section were taken each foot of length. Also a plumb-bob was dropped along the face of each member and measurements taken from this string to the face to determine eccentricity.

In the case of the non-prestressed member the load was applied in 10,000-lb increments up to 30,000 lb and 5000-lb at a time from there up. The prestressed column received loading in 10,000-lb increments up to 50,000 lb and 5000 lb at a time from there up. At each increase in load the deflection was read and cracks were noted and sketches made as they occurred.

All data regarding initial eccentricity and dimensions of the cross section are given in the appendix.

## Testing of Non-Prestressed Member

As has been stated before, the testing of both members was done in the 800,000-lb testing machine at Fritz Laboratory.

An initial load of 1000 lb was applied to the non-prestressed column and an initial centerline deflection observed. The centerline reading was 56.0. This scale could be and was read to the nearest 1/100 of an inch. The load was applied in 10,000-lb increments up to 30,000 lb with no evident deflection. At 35,000 lb a deflection of 0.05 in. was recorded.

At a load of 50,000 lb and a deflection of 0.18 in. very small cracks appeared on the tension face of the column about 3.5 feet each side of the

TEST SET-UP

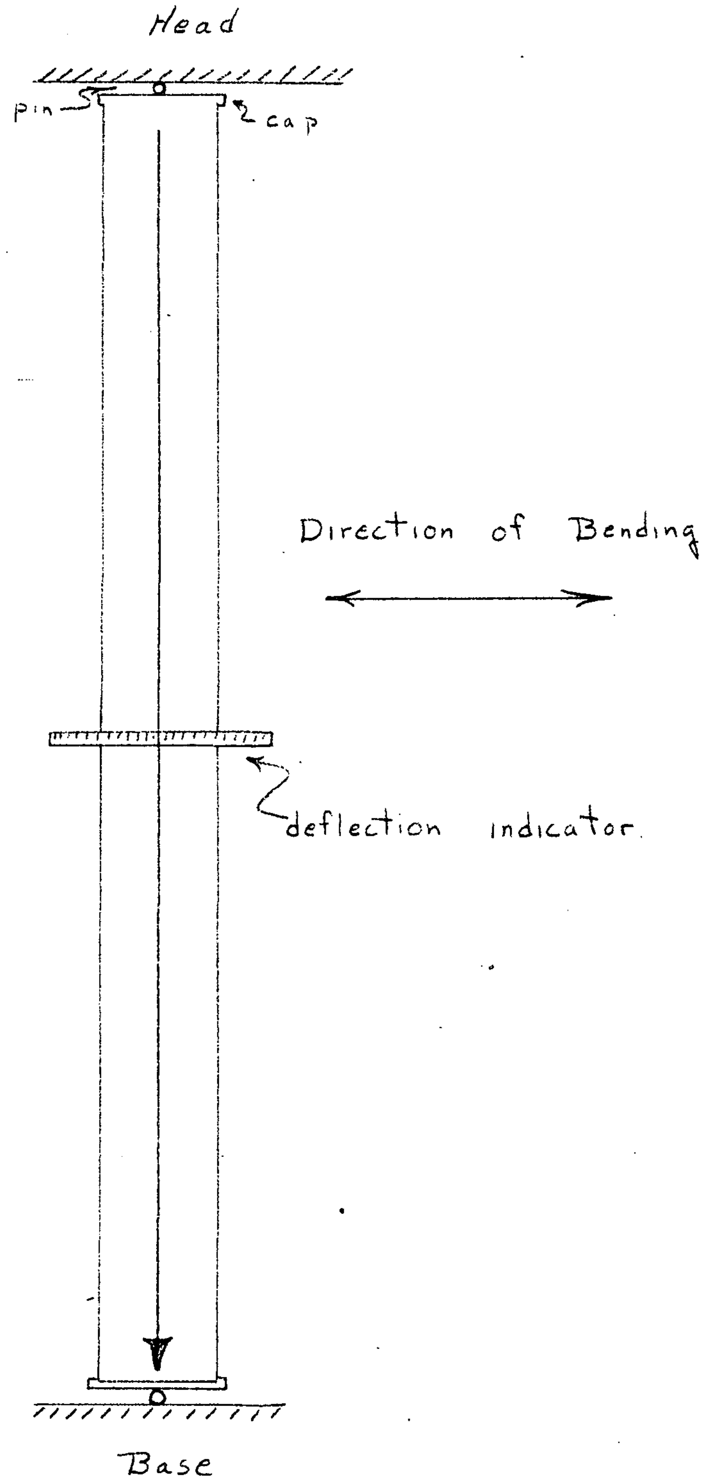



FIG. 6

centerline. This was the section of the member containing the wooden frames used to separate the wires. It is extremely probable that stress concentrations were set up in this region.

The first cracks to appear at the center of the column were first noted at a load of 65,000 lb. The deflection was 0.36 in. Also the cracks previously mentioned, about 3.5 ft each side of the column center, were more distinct but did not open.

A bit of trouble was encountered in obtaining a load of 75,000 lb and when this value was reached the deflection went to 0.74 in.

Upon trying to increase the load above 75,000 lb the member deflected to 5.60 in., cracked badly on the tension face and dropped the load off to 30,000 lb. The tension cracks in the neighborhood of the wooden frames were opened about 1/4 of an inch. There were two large and several small cracks at the center of the column. The two large cracks were perhaps 1/8 of an inch or more.

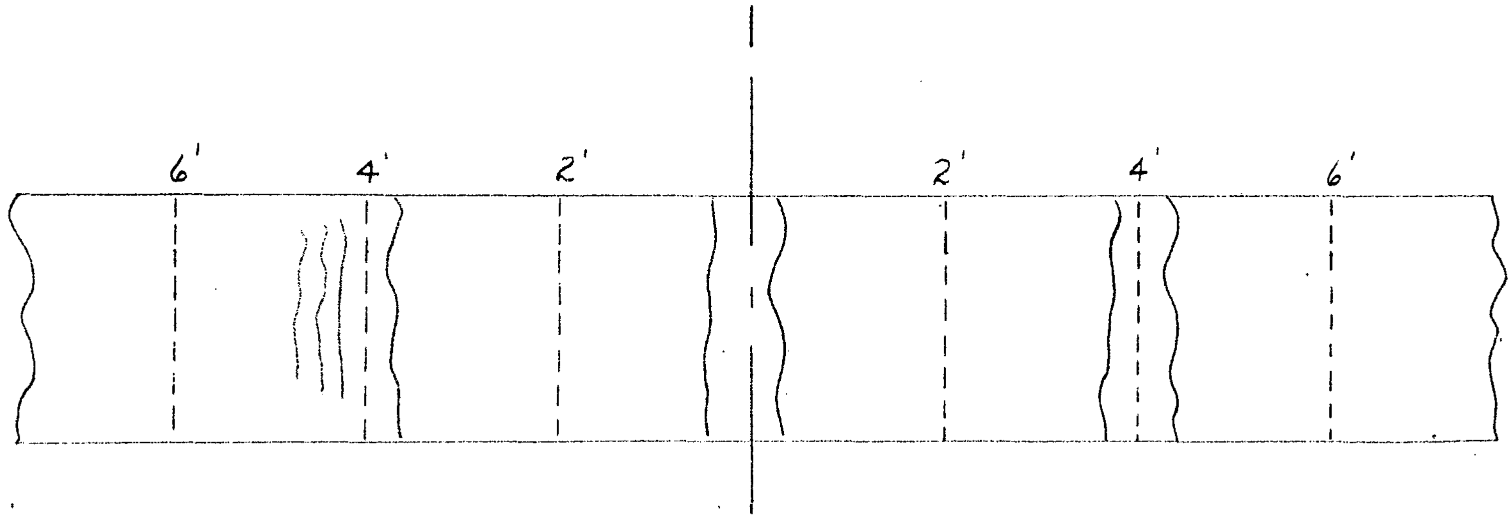
The load was dropped to 2000 lb and the deflection measured at 2.50 in. The centerline cracks closed but were still visible. The cracks 3.5 ft from the center, however, remained open about 1/8 of an inch.

## Testing of Prestressed Member

In the case of the prestressed member the load was applied in 10,000 lb increments up to 50,000 lb. A deflection of 0.03 in. was recorded at 10,000 lb. This deflection increased regularly and a deflection at centerline of 0.26 in. was read at 50,000 lb. There were no visible cracks. At a load of 70,000 lb the deflection was 1.20 in., but there were no visible cracks and everything seemed to be in order.

SKETCH OF CRACKS

NON-PRESTRESSED MEMBER




TENSION FACE

NO VISIBLE CRACKS ON COMPRESSION FACE

FIG. 7

Separators placed 4' each side of center line

The test had to be halted for 12 to 15 minutes and during this time the 70,000-lb loading was maintained. The member held the load for about 10 minutes and then the load dropped off slightly. When the testing was resumed the column deflected from 1.20 in. to 3.40 in. but the load would not go back to 70,000 lb. Cracks a and b appeared as indicated in sketch on page 25. Crack a was noticeable but did not open while crack b opened about 1/8 of an inch. There were also several extremely small cracks that were barely perceptable.

The load was then dropped off to 2000 lb and the centerline deflection read 0.67 in. All cracks disappeared completely except crack b (the large one) which closed but remained visible.

The loading was once more increased to 30,000 lb. The deflection was read at 2.35 in. All of the cracks remained invisible except the major one (b) which opened about 3/8 of an inch, indicating a yielding of the steel at this point. The load then dropped off to 8000 lb while the deflection increased to 6 in. and cracks appeared on the compression face opposite the major crack on the tension side. The concrete on the compression face spalled indicating failure of the concrete in compression.

The load was then dropped to 1000 lb and the deflection dropped to 2 in., then increased to 5000 lb and deflection reading jumped to 7 in. The load then dropped to 2000 lb and the centerline deflection was 10 in.

The column was then taken from the testing machine by the crane and placed on the floor. The major crack (b) was the only one visible on the tension face and it was open 1/4 in. The concrete in the vicinity of the compression crack was well spalled. There was a permanent set of 3.7 in. in the column at centerline.

SKETCH OF CRACKS

PRESTRESSED MEMBER

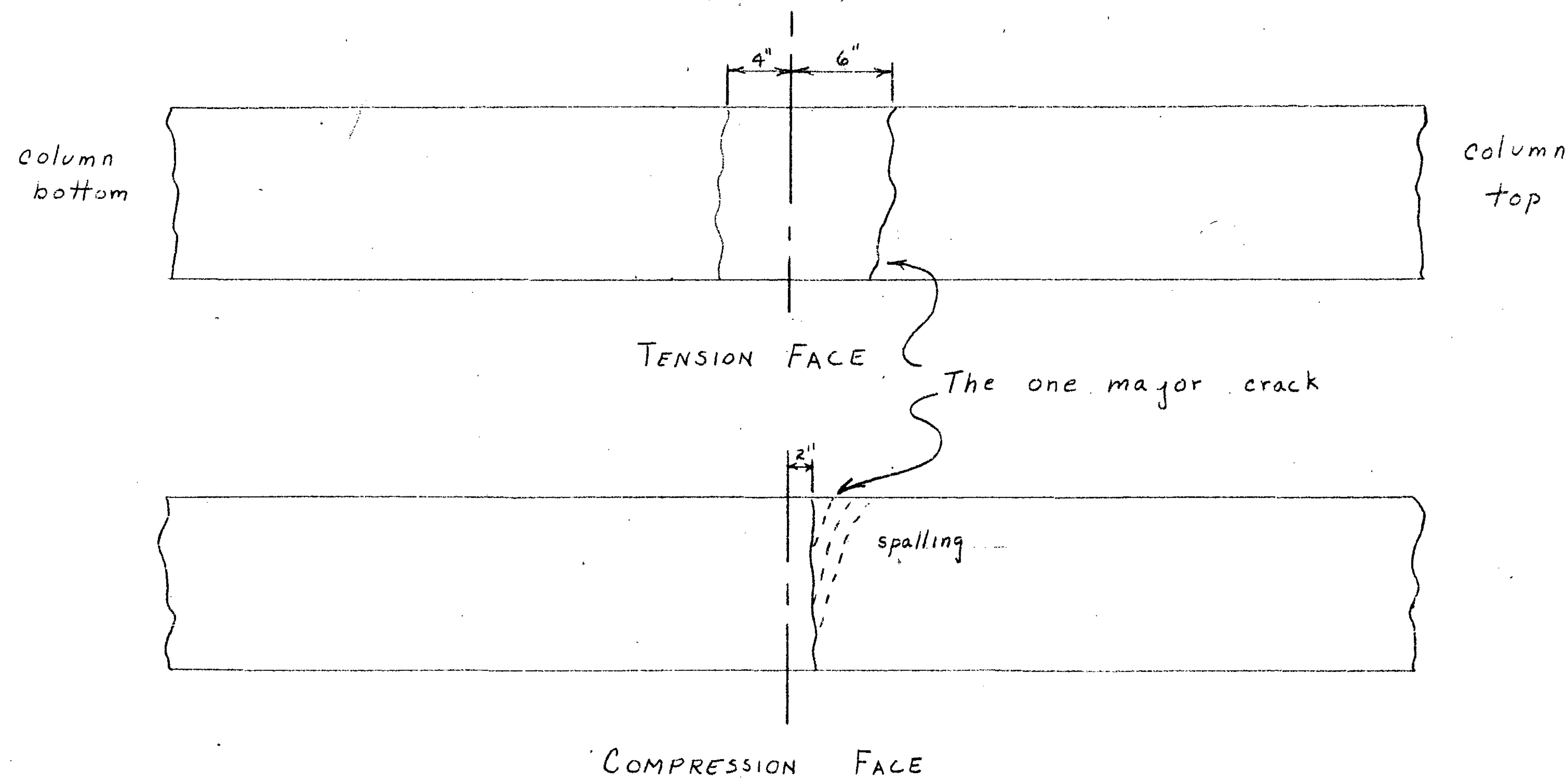



FIG. 8

DISCUSSION OF RESULTS

In order for experimental results to validate any theory many tests must be conducted. Consequently it was not my desire to draw any definite conclusions concerning the action of elastic prestressed columns under axial loading, but rather to see if pretensioning had any effects and, if so, what they might be.

Consider first the non-prestressed member. The area of reinforcement (14 - 1/10-in. wires) in this column was about 3/100 of one percent, 0.03%, and consequently may be considered negligible. The Euler load for both members based on an average modulus of elasticity value of $5.04 \times 10^6$ psi (see appendixx pages A-18 to A-25) is:

$$P_e = \frac{\pi^2 EA}{(\frac{\ell}{r})^2} = \frac{\pi^2 \times 5.04 \times 10^6 \times 32}{(138)^2}$$

$$P_e = 84{,}000 \text{ lb}$$

The non-prestressed colum n carried actually 75,000 lb and, at this value the centerline deflection was 0.74 in. The entire load-deflection curve may be found on page 32.

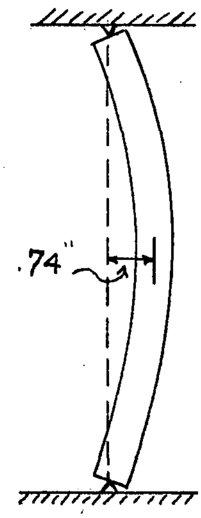
From the plot of $\delta/P$ vs $\delta$ on page A-15 the $P_{cr}$ value has been calculated as 85,000 lb and the initial deflection is seen to be 0.11 in. This value may be verified by considering the deflected shape to be a sine curve. Then

$$\delta_i = a_1 \sin \frac{\pi x}{\ell} + a_2 \sin \frac{2\pi x}{\ell} + a_3 \sin \frac{3\pi x}{\ell} + \ldots$$

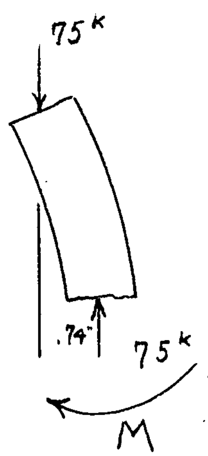
represents the deflection at any point and each term represents one mode of vibration. Page A-2 of the appendix shows this analysis carried out based on initial eccentricity reading taken before testing. The results give an initial eccentricity of 0.108 in. which closely verifies Southwell's Method (above).

It should also be noted that $P_{cr}$ and $P_e$ are almost identical values.

Regarding the method of failure, consider the member just at failure in the position shown.




It is seen that, taking the upper half of the member, the section is held in equilibrium by a couple of 75 k × .74 in. and an opposing moment M. This moment M may be represented as shown below




$M = 75 \times .74 = 55.5$ in.-kips

$T = C$

$T = \frac{55.5}{2.67} = 20.8$ k

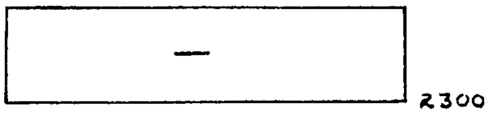
$\frac{20.8}{8} = 2.6$ k/in. ; $\frac{0 + S_m}{2} = S_a = 2.6$

$S_m = 2.6$ k/sq in. (tension)

max tension stress due to bending = 2600 psi.

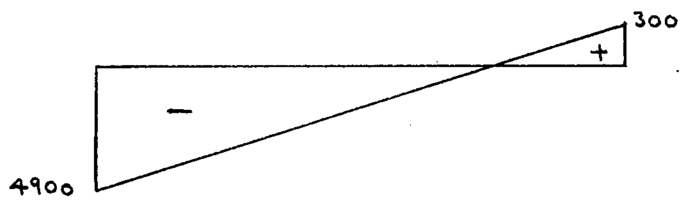
This value is conservative due to shift in axis.

Stress due to axial load:




$$\frac{75 \text{ k}}{32} = 2300 \text{ psi (compression)}$$

Superimposing the two stress diagrams we have:




There was no evidence of the crushing of the concrete as indeed there shouldn't have been due to high strength. But the stresses do indicate a possible failure in tension, which is what might have happened.

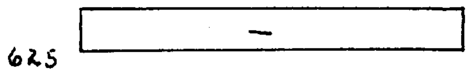
In analyzing the cracks, it should be noted that although there was definite cracking at the centerline, equally large if not larger cracks appeared about 3.5 feet from the midsection where the wooden separators were placed. Evidently concentrated stresses were set up at these points and this could possibly have been the reason for failure.

Although this column was more rigid than the prestressed member, as can be seen from the load vs deflection curve of page 32, its recovery after the load was taken off was far inferior. When the load was dropped from 75,000 lb to 2000 lb the centerline deflection was 2.58 in.

The prestressed member, which had 14 1/10-in. wires causing 20,000 lb of prestress (625 psi - compression), also had an Euler load of 84,000 lb. This column carried a maximum of 70,000 lb and reached a maximum centerline deflection of 0.85 in.

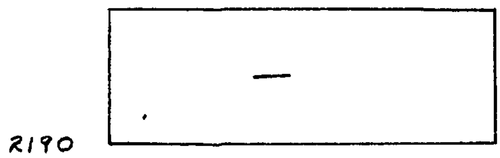
From the plot of $\delta/P$ vs $\delta$ on page A-13 of the Appendix the critical load for this member is shown to be 89,000 lb.

Consider now a section taken from the center of the member. First there is a uniform load of 20,000 lb or 625 psi acting in compression due to prestress.

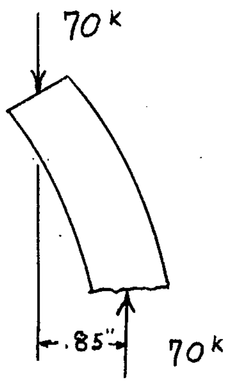



There is also an axial compression of 70,000 lb of

$$\frac{70{,}000}{32} = 2190 \text{ psi.}$$




Considering half of the member in equilibrium we have



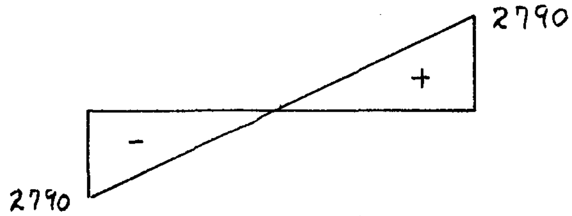

$$M = 70{,}000 \times .85 = 59.5 \text{ in.-kips}$$

$$T = C = \frac{59{,}500 \text{ in.-lb}}{2.67 \text{ in.}} = 22{,}300 \text{ lb}$$

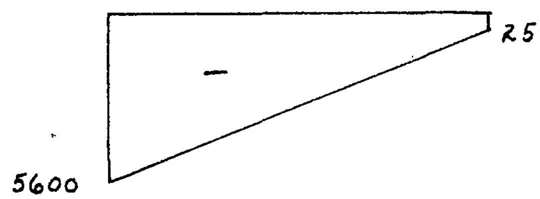
$$\frac{22{,}300}{8} = 2790 \text{ lb/in.}$$

$$\text{average stress} = \frac{2790}{2} = 1395 \text{ psi}$$

maximum stress = 2790 psi




Totaling the three stress diagrams:

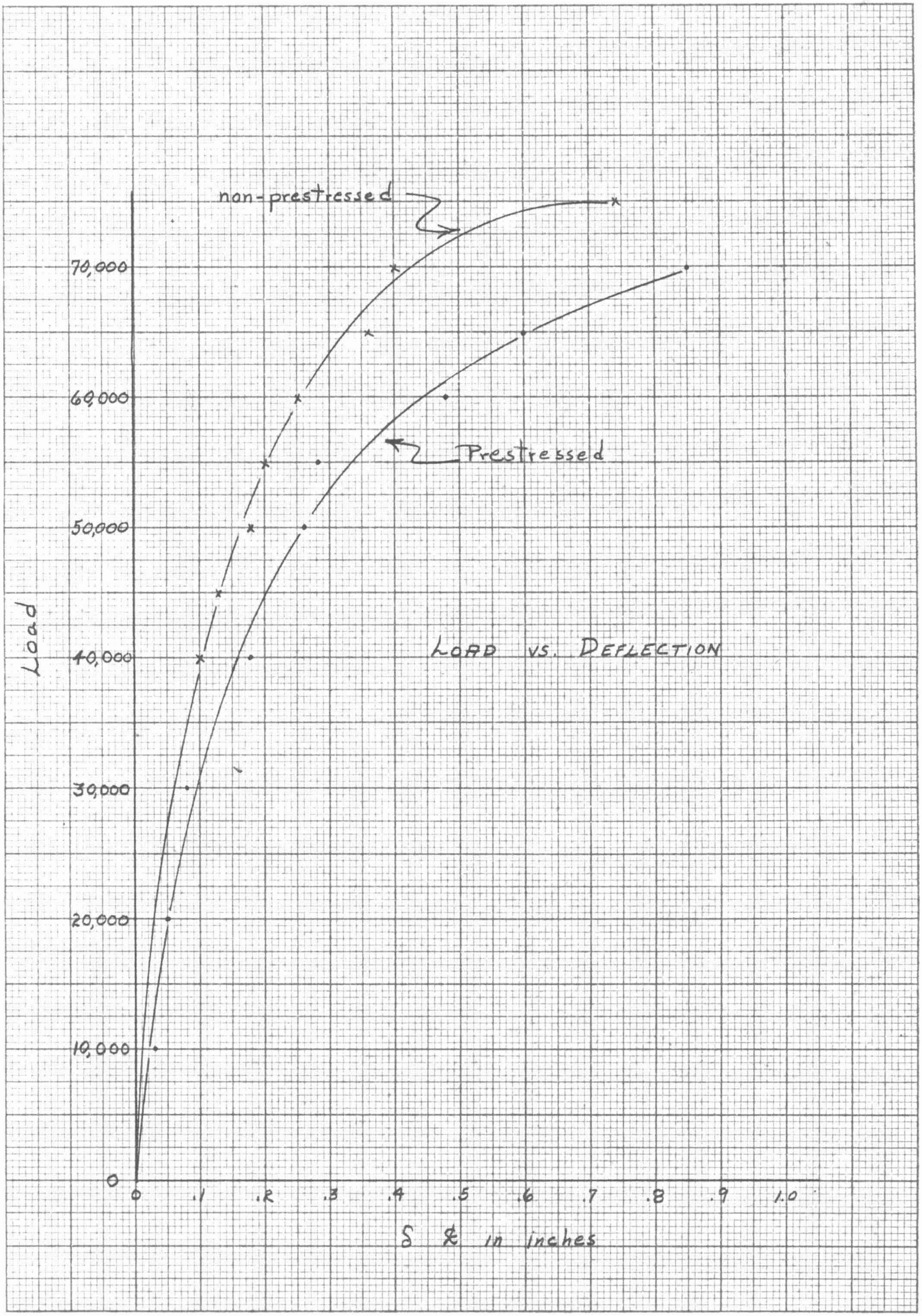



The 5600 psi maximum conpression is not as high as the cylinder strength's, which were between 6600 and 7600 psi.

There was also definite evidence of a yielding of the stressing wires after failure which would release the compression due to prestress and allow a cracking on the tension face of the column. This cracking would then shift the neutral axis toward the compression face and very likely cause compression failure. A yielding of the steel was a very likely occurrance for the wires were originally stressed to 185,000 psi. The yield point of this wire is not much over 200,000 psi (220,000 is definitely above yield stress).

Another factor which I am sure had a definite bearing on the load carrying capacity of this member was the fact that when the load reached 70,000 lb the test was halted for about 15 to 20 minutes. The 70,000 lb load was maintained for several minutes but then slowly dropped off, and could not be regained. The high stresses to which both the concrete and the steel were exposed at this loading undoubtedly caused creep in the steel and plastic flow in the concrete. Had it not been for this delay I feel sure the column would have carried more than the 70,000 lb, perhaps not a great deal more, but more.

It is also my personal feeling that a basic mistake in judgement was made when I prestressed rather than post-tensioned the test columns. The analytical analysis was made without considering the effect of bonding in prestressing. Post-tensioning would have most ideally duplicated the theoretical column analyzed mathematically. The wires could have been placed in the true triangular position and bond would not have been present.



non-prestressed
Prestressed
LOAD VS. DEFLECTION
Load
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
δ ℄ in inches

# APPENDIX

## DESIGN MIX

Coarse Aggregate:

Bulk specific gravity = 2.70

Apparent specific gravity = 2.83 3/4-in. stone

Absorption = 1.76%

Fine Aggregate:

Bulk Specific gravity = 2.58

Apparent specific gravity = 2.62

Absorption = 1.01%

High Early Strength Portland Cement

4800 psi in 7 days

5 gal/sack - $H_2O$

46% sand

170 lb/sack - sand

200 lb/sack - stone

1 yard

335 lb - $H_2O$

8 sack - cement

1360 lb - sand

1600 lb - stone

I need:

$$2 \times \frac{32}{144} \times 13.3 = 5.9$$

$$4 \times .2 = .8$$

$$\overline{6.7}$$

mix 4 at 1.8 = 7.2 cu ft

1 batch

$H_2O$ $- \frac{1.8}{27} \times 335 = 21.6$ lb

cement $- \frac{1.8}{27} \times 8 \times 94 = 50$ lb

sand $- \frac{1.8}{27} \times 1360 = 90.6$ lb

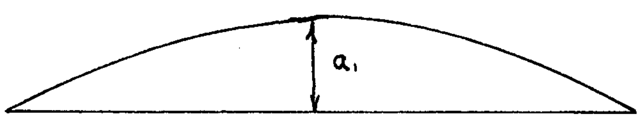
stone $- \frac{1.8}{27} \times 1600 = 100.7$ lb

CALCULATION OF CENTERLINE DEFLECTION FOR NON-PRESTRESSED MEMBER

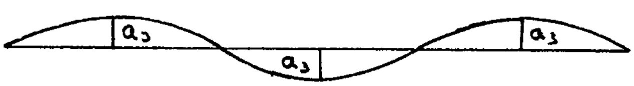
Let:

$$y_o = a_1 \sin \frac{\pi x}{\ell} + a_2 \sin \frac{2\pi x}{\ell} + a_3 \sin \frac{3\pi x}{\ell} + \ldots$$

represent the deflected shape.




$a_1 \sin \frac{\pi x}{\ell}$




$a_3 \sin \frac{3\pi x}{\ell}$

From page A-8 of Appendix, 1/8 inch equals initial centerline deflection and 1/16 inch equals initial quarter-point deflection.

$x = \ell/2 \qquad 1/8 = a_1 \sin \frac{\pi}{2} - a_3 \sin \frac{3\pi}{2}$

$x = \ell/4 \qquad 1/16 = a_1 \sin \frac{\pi}{4} + a_3 \sin \frac{3\pi}{4}$

$$1/8 = a_1 - a_3$$

$$1/16 = .707\ a_1 + .707\ a_3$$

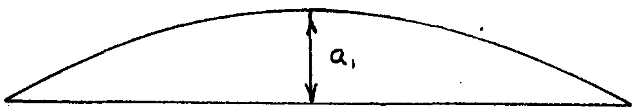
$$1.414\ a_1 = \frac{2.414}{16}$$

$$\underline{a_1 = 0.108 \text{ inches}}$$

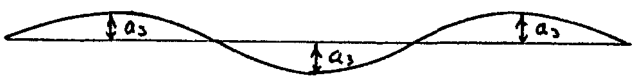
CALCULATION OF CENTERLINE DEFLECTION FOR PRESTRESSED MEMBER

Deflected shape may be represented by:

$$y_o = a_1 \sin \frac{\pi x}{\ell} + a_2 \sin \frac{2\pi x}{\ell} + a_3 \sin \frac{3\pi x}{\ell} + \ldots$$




$a_1 \sin \frac{\pi x}{\ell}$




$a_3 \sin \frac{3\pi x}{\ell}$

From page A-9 of Appendix, 1/4 inch equals initial centerline deflection and 1/8 inch equals initial quarter point deflection.

$x = \ell/2 \qquad 1/4 = a_1 \sin \frac{\pi}{2} - a_3 \sin \frac{3\pi}{2}$

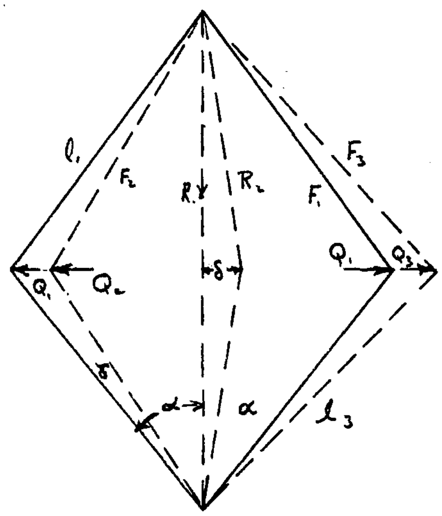
$x = \ell/4 \qquad 1/8 = a_1 \sin \frac{\pi}{4} + a_3 \sin \frac{3\pi}{4}$

$$1/4 = a_1 - a_3$$

$$1/8 = .707\ a_1 + .707\ a_3$$

$$a_1 = .214 \text{ inches}$$

## CALCULATION OF INFLUENCE OF THE ANGLE $\alpha$ CONSIDERING SECOND ORDER VARIABLES



1 indicates initial position

2 & 3 indicate final position

$$R = 2\,F_2 \cos \alpha$$

$$\ell_1 = \frac{L}{2} \sec \alpha \qquad \ell_3 = \left(\frac{L}{2} \cos \gamma\right) \sec (\alpha + \gamma)$$

$$\ell_2 = \left(\frac{L}{2} \cos \gamma\right) \sec (\alpha - \gamma)$$

$$R_{(2-3)} = F_2 \cos (\alpha - \gamma) + F_3 \cos (\alpha + \gamma)$$

$$Q_2 = 2\,F_2 \sin (\alpha - \gamma) \qquad Q_3 = 2\,F_3 \sin (\alpha + \gamma)$$

$$F_2 = F_1 - \frac{\ell_1 - \ell_2}{\ell_1} A_s E \qquad F_3 = F_1 + \frac{\ell_3 - \ell_1}{\ell_1} A_s E$$

$$F_2 = F_1 - \frac{\frac{L}{2} \sec \alpha - \frac{L}{2} \cos \gamma \sec (\alpha - \gamma)}{\frac{L}{2} \sec \alpha} A_s E$$

$$F_3 = F_1 + \frac{\frac{L}{2} \cos \gamma \sec (\alpha + \gamma)}{\frac{L}{2} \sec \alpha} - \frac{L}{2} \sec \alpha\, A_s E$$

$$R_{2-3} = F_2 \cos (\alpha - \gamma) + F_3 \cos (\alpha + \gamma)$$

$$= \cos (\alpha + \gamma) \left[F_1 - \frac{\sin \alpha \sin \gamma}{\cos (\alpha - \gamma)} A_s E\right] + \cos (\alpha + \gamma) \left[F_1 + \frac{\sin \alpha \sin \gamma}{\cos (\alpha + \gamma)} A_s E\right]$$

$$= F_1 \cos (\alpha - \gamma) - \sin \alpha \sin \gamma\, A_s E + F_1 \cos (\alpha + \gamma) + \sin \alpha \sin \gamma\, A_s E$$

$$R_{2-3} = F_1 \cos(\alpha - \gamma) + F_1 \cos(\alpha + \gamma)$$

$$\tan \gamma = \frac{\delta}{L/2} = \frac{2\delta}{L}$$

$$R_{2-3} = F_1 (\cos \alpha \cos \gamma + \sin \alpha \sin \gamma) + F_1 (\cos \alpha \cos \gamma - \sin \alpha \sin \gamma)$$

$$= F_1 (\cos \frac{L}{m} + \sin \frac{2\delta}{m}) + F_1 (\cos \alpha \frac{L}{m} - \sin \alpha \frac{2\delta}{m})$$

$$= 2 F_1 \cos \alpha (\frac{L}{m})$$

$$R_1 = 2 F_1 \cos \alpha$$

$$R_{2-3} = 2 F_1 \frac{L}{n} \frac{L}{m} = R_1 \cos \gamma$$

when $\delta = e$ $\quad R_{2-3} = 2 F_1 \cos^2 \alpha = 2 F_1 (\frac{1 + \cos 2\alpha}{2})$

$$F_2 = F_1 - \frac{\sec \alpha - \cos \gamma \sec (\alpha - \gamma)}{\sec \alpha} A_s E$$

$$= F_1 - \frac{\frac{1}{\cos \alpha} - \frac{\cos \gamma}{\cos (\alpha - \gamma)} A_s E}{1/\cos \alpha}$$

$$= F_1 - \frac{\frac{\cos \alpha}{\cos \alpha} - \frac{\cos \alpha \cos \gamma}{\cos (\alpha - \gamma)}}{1} A_s E$$

$$= F_1 - (1 - \frac{\cos \alpha \cos \gamma}{\cos \alpha \cos \gamma + \sin \alpha \sin \gamma}) A_s E$$

$$F_3 = F_1 + \frac{\cos \gamma \sec (\alpha + \gamma) - \sec \alpha}{\sec \alpha} A_s E$$

$$= F_1 + \frac{\frac{\cos \gamma}{\cos (\alpha + \gamma)} - \frac{1}{\cos \alpha}}{1/\cos \alpha} A_s E$$

$$= F_1 + \frac{\frac{\cos \alpha \cos \gamma}{\cos (\alpha + \gamma)} - \frac{\cos \alpha}{\cos \alpha}}{1} A_s E$$

$$= F_1 + (\frac{\cos \alpha \cos \gamma}{\cos \alpha \cos \gamma - \sin \alpha \sin \gamma} - 1) A_s E$$

$$F_2 = F_1 - \left(\frac{\cos\alpha\cos\gamma - \cos\alpha\cos\gamma + \sin\alpha\sin\gamma}{\cos\alpha\cos\gamma + \sin\alpha\sin\gamma}\right) A_s E$$

$$F_3 = F_1 + \left(\frac{\cos\alpha\cos\gamma - \cos\alpha\cos\gamma + \sin\alpha\sin\gamma}{\cos\alpha\cos\gamma - \sin\alpha\sin\gamma}\right) A_s E$$

$$F_2 = F_1 - \frac{\sin\alpha\sin\gamma}{\cos(\alpha-\gamma)} A_s E \qquad F_3 = F_1 + \frac{\sin\alpha\sin\gamma}{\cos(\alpha+\gamma)} A_s E$$

---

$$Q_2 = 2\sin(\alpha-\gamma)\left[F_1 - \frac{\sin\alpha\sin\gamma}{\cos(\alpha-\gamma)} A_s E\right]$$

$$Q_3 = 2\sin(\alpha+\gamma)\left[F_1 + \frac{\sin\alpha\sin\gamma}{\cos(\alpha+\gamma)} A_s E\right]$$

$$Q_2 = 2 F_1 \sin(\alpha+\gamma) - 2\sin\alpha\sin\gamma\tan(\alpha-\gamma) A_s E$$

$$Q_3 = 2 F_1 \sin(\alpha+\gamma) + 2\sin\alpha\sin\gamma\tan(\alpha+\gamma) A_s E$$

$$Q_2 = 2 F_1 (\sin\alpha\cos\gamma - \cos\alpha\sin\gamma) - 2\sin\alpha\sin\gamma\tan(\alpha-\gamma) A_s E$$

$$Q_3 = 2 F_1 (\sin\alpha\cos\gamma + \cos\alpha\sin\gamma) + 2\sin\alpha\sin\gamma\tan(\alpha+\gamma) A_s E$$

$$Q_2 = 2F_1 (\sin\alpha\cos\gamma - \cos\alpha\sin\gamma) - 2 A_s E \sin\alpha\sin\gamma \left(\frac{\sin\alpha\cos\gamma - \cos\alpha\sin\gamma}{\cos\alpha\cos\gamma + \sin\alpha\sin\gamma}\right)$$

$$Q_2 = 2F_1 (\sin\alpha\cos\gamma - \cos\alpha\sin\gamma) - 2 A_s E \left(\frac{\sin^2\alpha\sin\gamma\cos\gamma - \sin\alpha\cos\alpha\sin^2\gamma}{\cos\alpha\cos\gamma + \sin\alpha\sin\gamma}\right)$$

$$Q_3 = 2F_1 (\sin\alpha\cos\gamma + \cos\alpha\sin\gamma) + 2A_s E \sin\alpha\sin\gamma \left(\frac{\sin\alpha\cos\gamma + \cos\alpha\sin\gamma}{\cos\alpha\cos\gamma - \sin\alpha\sin\gamma}\right)$$

$$Q_3 = 2F_1 (\sin\alpha\cos\gamma + \cos\alpha\sin\gamma) + 2A_s E \left(\frac{\sin^2\alpha\sin\gamma\cos\gamma + \sin\alpha\cos\alpha\sin^2\gamma}{\cos\alpha\cos\gamma - \sin\alpha\sin\gamma}\right)$$

$$Q_2 = 2 F_1 [\sin(\alpha-\gamma)] - \frac{2\sin(\alpha-\gamma)\sin\alpha\sin\gamma}{\cos(\alpha-\gamma)} A_s E$$

$$Q_3 = 2 F_1 \sin(\alpha+\gamma) + \frac{2\sin(\alpha+\gamma)\sin\alpha\sin\gamma}{\cos(\alpha+\gamma)} A_s E$$

$$Q_3 - Q_2 = 2F_1\Big[\sin(\alpha+\gamma) - \sin(\alpha-\gamma)\Big] + 2\sin\alpha\sin\gamma\, A_sE\Big[\tan(\alpha+\gamma) + \tan(\alpha-\gamma)\Big]$$

$$= 2F_1\Big[2\cos(\tfrac{1}{2}\,2\alpha)\sin(\tfrac{1}{2}\,2\gamma)\Big] + 2\sin\alpha\sin\gamma\, A_sE\left(\frac{2\sin 2\alpha}{\cos 2\gamma + \cos 2\alpha}\right)$$

$$= 4\,F_1\,(\cos\alpha\sin\gamma) + 4\sin\alpha\sin\gamma\, A_sE\left(\frac{2\sin\alpha}{\cos 2\gamma + \cos 2\alpha}\right)$$

$$= 2\,R_1\sin\gamma + A_sE\sin\gamma\,\frac{8\sin^2\alpha}{\cos 2\gamma + \cos 2\alpha}$$

$$= 2\,R_1\sin\gamma + A_sE\sin\gamma\,\frac{8\sin^2\alpha}{2\cos\gamma + 2\cos\alpha}$$

$$Q_3 - Q_2 = \underline{\underline{2\,R_1\sin\gamma}} + A_sE\sin\gamma\,\frac{4\sin^2\alpha}{\cos\gamma + \cos\alpha}$$

my value

# INITIAL ECCENTRICITY OF NON-PRESTRESSED MEMBER

| Offset | | eccentricity |
|---|---|---|
| 1 1/2" | 1 | 0 |
| 1 1/2" | 2 | 0 |
| 1 1/2" | 3 | 0 |
| 1 3/8" | 4 | 1/8" |
| 1 3/8" | 5 | 1/8" |
| 1 3/8" | 6 | 1/8" |
| 1 3/8" | 7 | 1/8" |
| 1 3/8" | 8 | 1/8" |
| 1 3/8" | 9 | 1/8" |
| 1 1/2" | 10 | 0 |
| 1 1/2" | 11 | 0 |
| 1 1/2" | 12 | 0 |
| 1 1/2" | 13 | 0 |

# INITIAL ECCENTRICITIES OF PRESTRESSED MEMBER

| Offset | | eccentricity |
|---|---|---|
| 1 1/2" | 1 | 0 |
| 1 1/2" | 2 | 0 |
| 1 1/2" | 3 | 0 |
| 1 3/8" | 4 | 1/8" |
| 1 3/8" | 5 | 1/8" |
| 1 1/4" | 6 | 1/4" |
| 1 1/4" | 7 | 1/4" |
| 1 1/4" | 8 | 1/4" |
| 1 1/4" | 9 | 1/4" |
| 1 3/8" | 10 | 1/8" |
| 1 3/8" | 11 | 1/8" |
| 1 1/2" | 12 | 0 |
| 1 1/2" | 13 | 0 |

## Load-Deflection Readings

## Prestressed

| Load | Reading | Deflection (in.) | Remarks |
|---|---|---|---|
| 0 | 60.0 | 0 | |
| 10 | 60.3 | .03 | |
| 20 | 60.5 | .05 | |
| 30 | 60.7 | .07 | |
| 40 | 61.8 | .18 | |
| 50 | 62.6 | .26 | |
| 55 | 62.8 | .28 | |
| 60 | 64.8 | .48 | |
| 65 | 66.0 | .60 | |
| 70 | 72.0 | 1.20 | This load held for about 12-15 minutes |
| 2 | 66.7 | .67 | |
| 30 | 83.5 | 2.35 | |
| 8 | 120.0 | 6 | |
| 1 | 80 | 2 | |
| 5 | 120 + 1" | 7 | |
| 2 | 120 + 4" | 10 | |

## Load-Deflection Readings

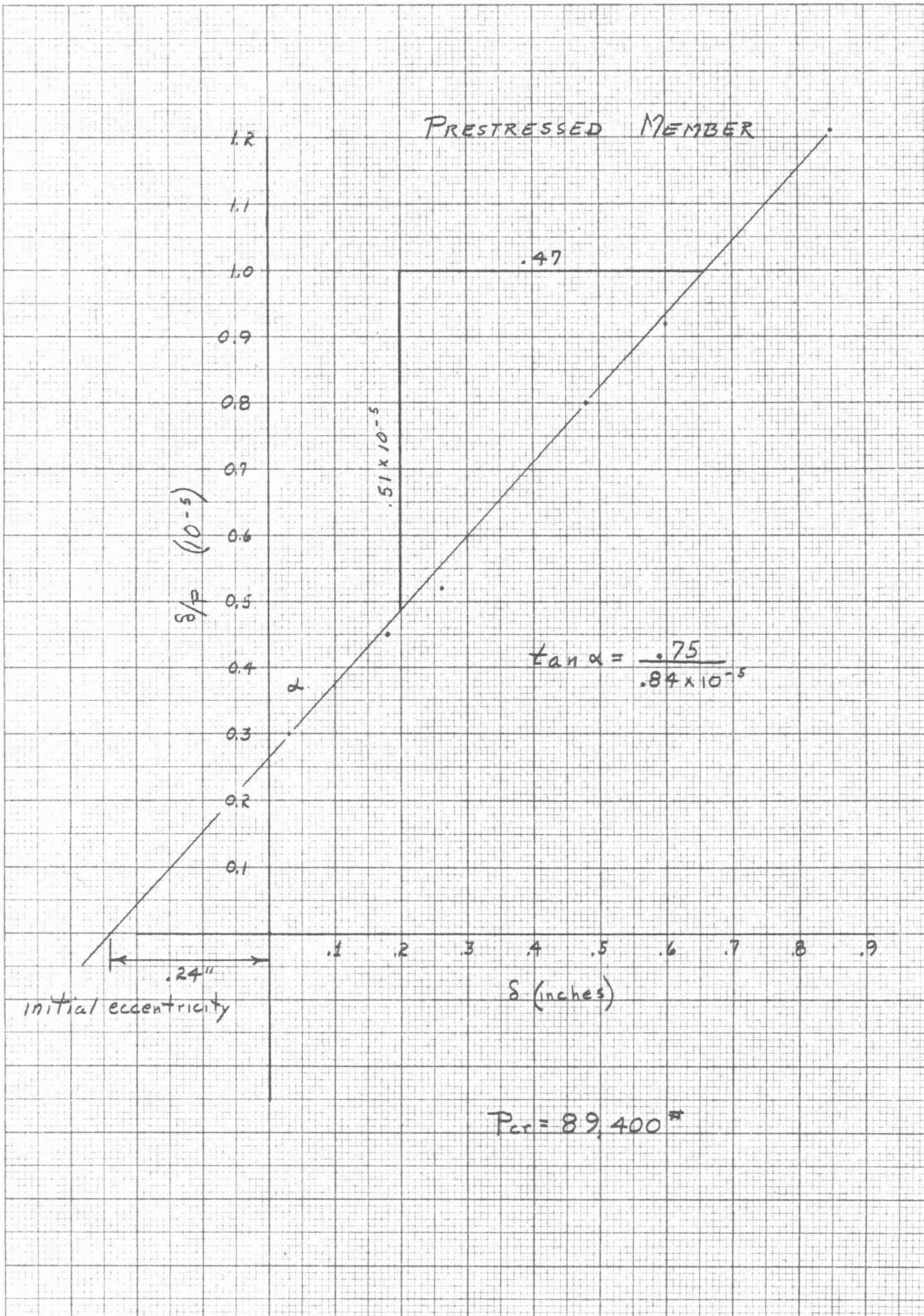
## Non-Prestressed

| Load | Reading | Deflection (in.) | Remarks |
|---|---|---|---|
| 1 | 56.0 | 0 | |
| 10 | 56.0 | 0 | |
| 20 | 56.0 | 0 | |
| 30 | 56.0 | 0 | |
| 35 | 55.5 | .05 | |
| 40 | 55.0 | .1 | |
| 45 | 54.7 | .13 | |
| 50 | 54.2 | .18 | |
| 55 | 54.0 | .20 | |
| 60 | 53.5 | .25 | |
| 65 | 52.4 | .36 | |
| 70 | 52.0 | .40 | |
| 75 | 48.6 | .74 | |
| 2 | 0 | 6.0 | |

## Data for $\delta$ vs $\delta/P$ Plot

## Prestressed Data

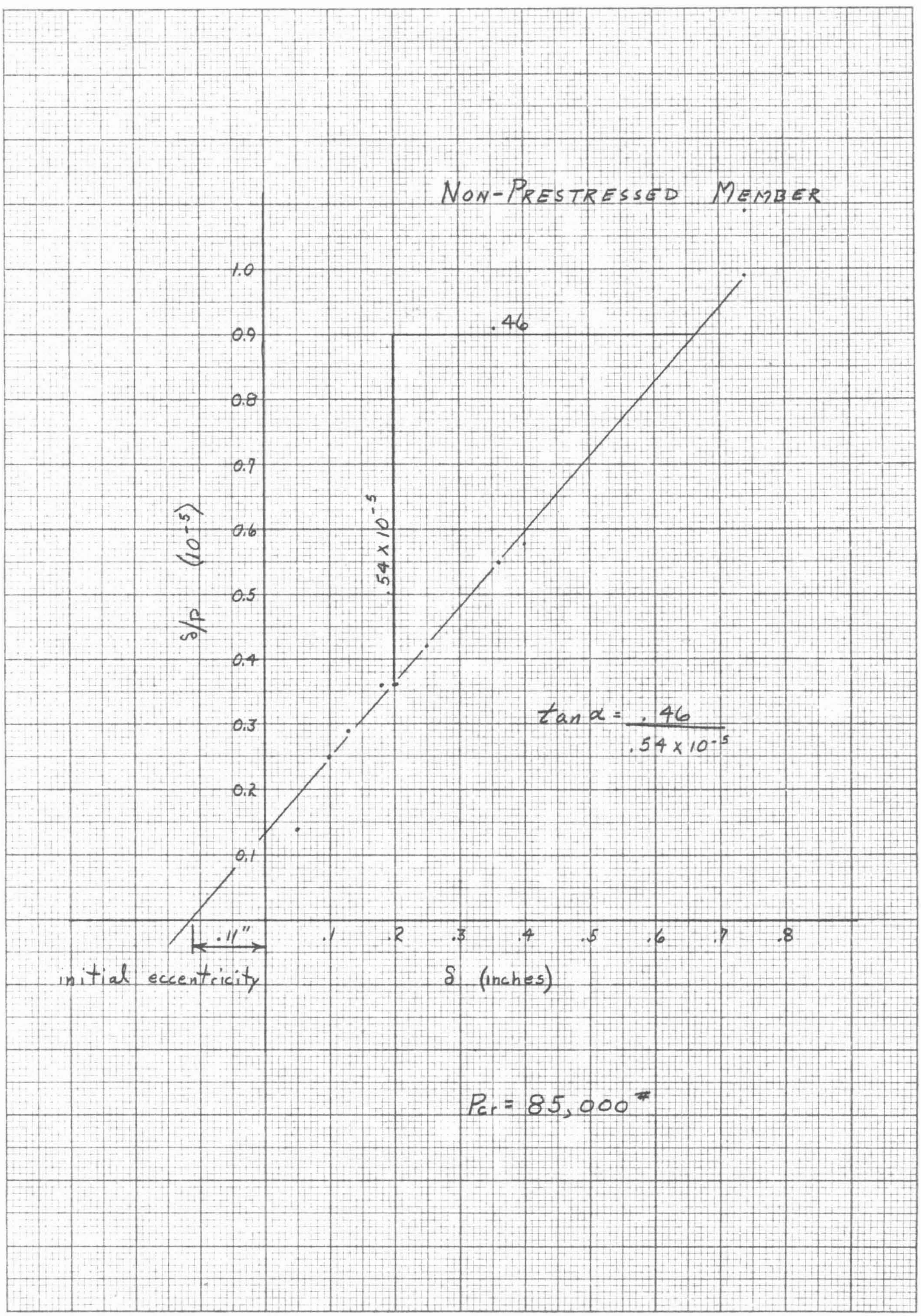
| $\delta$ | P | $\delta/P$ | | |
|---|---|---|---|---|
| .03 | 10,000 | .00 00 0300 | = | .3 $\times 10^{-5}$ |
| .05 | 20,000 | .00 00 0250 | = | .25 $\times 10^{-5}$ |
| .07 | 30,000 | .00 00 0234 | = | .234 $\times 10^{-5}$ |
| .18 | 40,000 | .00 00 045 | = | .45 $\times 10^{-5}$ |
| .26 | 50,000 | .00 00 052 | = | .52 $\times 10^{-5}$ |
| .28 | 55,000 | .00 00 051 | = | .51 $\times 10^{-5}$ |
| .48 | 60,000 | .00 00 080 | = | .80 $\times 10^{-5}$ |
| .60 | 65,000 | .00 00 092 | = | .92 $\times 10^{-5}$ |
| .85 | 70,000 | .00 00 121 | = | 1.21 $\times 10^{-5}$ |


PRESTRESSED MEMBER
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
δ/P (10⁻⁵)
.47
.51 x 10⁻⁵
α
tan α = .75 / .84 x 10⁻⁵
.1
.2
.3
.4
.5
.6
.7
.8
.9
.24"
initial eccentricity
δ (inches)
Pcr = 89,400 #

## Data for $\delta$ vs $\delta/P$ Plot

## Non-Prestressed Data

| $\delta$ | P | $\delta/P$ | | |
|---|---|---|---|---|
| .05 | 35,000 | .00 00 014 | = | $.14 \times 10^{-5}$ |
| .10 | 40,000 | .00 00 025 | = | $.25 \times 10^{-5}$ |
| .13 | 45,000 | .00 00 029 | = | $.29 \times 10^{-5}$ |
| .18 | 50,000 | .00 00 036 | = | $.36 \times 10^{-5}$ |
| .20 | 55,000 | .00 00 036 | = | $.36 \times 10^{-5}$ |
| .25 | 60,000 | .00 00 042 | = | $.42 \times 10^{-5}$ |
| .36 | 65,000 | .00 00 055 | = | $.55 \times 10^{-5}$ |
| .40 | 70,000 | .00 00 058 | = | $.58 \times 10^{-5}$ |
| .74 | 75,000 | .00 00 099 | = | $.99 \times 10^{-5}$ |




10 X 10 TO THE 1/2 INCH 359-11 KEUFFEL & ESSER CO. MADE IN U.S.A.

Stress-Strain Readings for Stressing Wires

| Stress | Gage 1 | Gage 2 | Gage 3 |
|---|---|---|---|
| 6,000 | 6-880 | 9-310 | 9-210 |
| 24,500 | 7-520 | 9-950 | 9-850 |
| 43,000 | 8-160 | 10-590 | 10-490 |
| 61,500 | 8-800 | A0-230 | A0-130 |
| 80,000 | 9-440 | A0-870 | A0-770 |
| 98,500 | 10-080 | A1-510 | A1-410 |
| 117,000 | 10-720 | A2-150 | A2-050 |
| 135,000 | A0-360 | A2-790 | A2-690 |
| 154,000 | A1-000 | A3-430 | A3-330 |
| 172,500 | A1-640 | A4-070 | A3-970 |
| 185,000 | A2-070 | A4-500 | A4-400 |

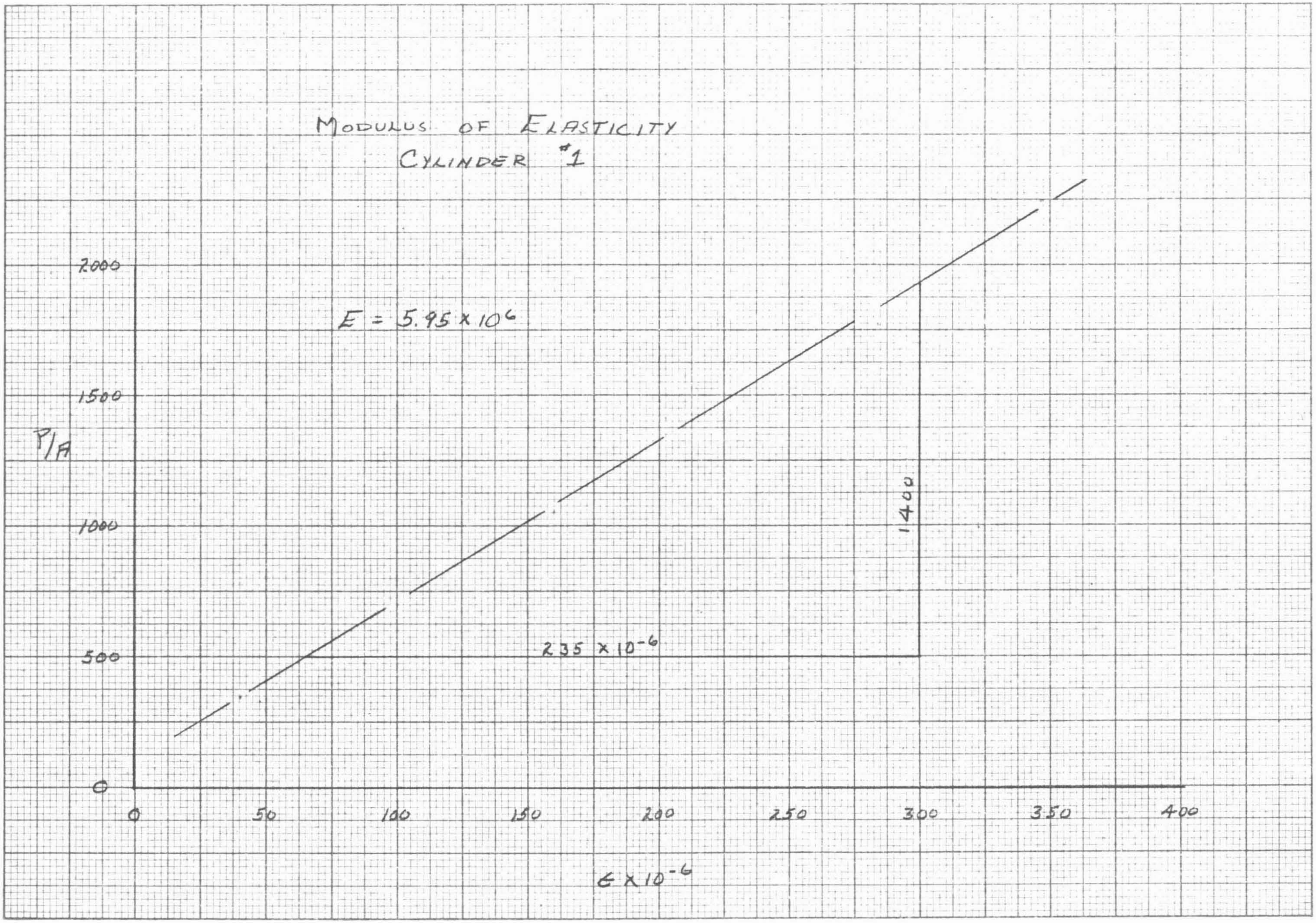
## Modulus of Elasticity Test

### Cylinder No 1

| Load | Reading | $\Delta R$ (in $\times 10^{-6}$) | Stress (psi) |
|---|---|---|---|
| 0 | 6-700 | 0 | 0 |
| 2,000 | 6-700 | 0 | 70.8 |
| 10,000 | 644 | 36 | 354 |
| 20,000 | 609 | 81 | 708 |
| 30,000 | 549 | 151 | 1060 |
| | | | |
| 2,000 | 693 | 0 | 70.8 |
| 10,000 | 652 | 41 | 354 |
| 20,000 | 593 | 100 | 708 |
| 30,000 | 535 | 158 | 1060 |
| 40,000 | 472 | 221 | 1420 |
| 50,000 | 410 | 283 | 1770 |
| 60,000 | 346 | 347 | 2125 |
| | | | |
| 2,000 | 682 | 0 | 70.8 |
| 60,000 | 340 | 342 | 2125 |

Ultimate load = 216,250 lb

Ultimate stress = 7660 psi


MODULUS OF ELASTICITY
CYLINDER #1
E = 5.95 x 10^6
2000
1500
1000
500
0
P/A
1400
235 x 10^-6
0
50
100
150
200
250
300
350
400
ε x 10^-6

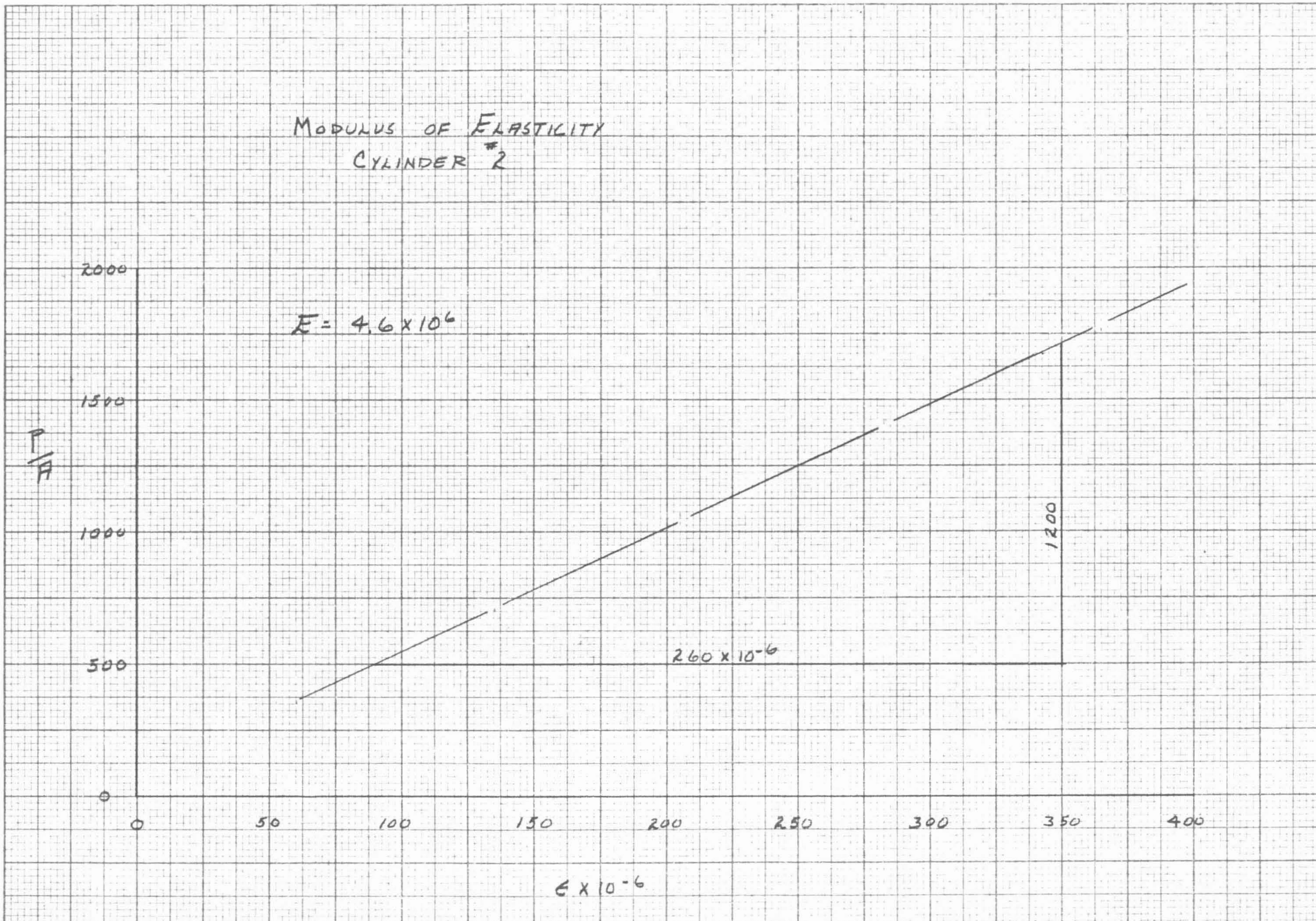
Modulus of Elasticity Test

Cylinder No 2

| Load | Reading | $\Delta R$ (in $\times 10^{-6}$) | Stress (psi) |
|---|---|---|---|
| 0 | 5-492 | | 0 |
| 2,000 | 5-470 | 0 | 70.8 |
| 10,000 | 400 | 70 | 354 |
| 20,000 | 328 | 142 | 708 |
| 30,000 | 252 | 218 | 1060 |
| 2,000 | 464 | 0 | 70.8 |
| 10,000 | 402 | 62 | 354 |
| 20,000 | 330 | 134 | 708 |
| 30,000 | 260 | 204 | 1060 |
| 40,000 | 181 | 283 | 1420 |
| 50,000 | 101 | 363 | 1770 |
| 60,000 | 021 | 443 | 2125 |
| 2,000 | 458 | 0 | 70.8 |
| 60,000 | 026 | 432 | 2125 |

Ultimate load = 224,500 lb

Ultimate stress = 7950 psi


MODULUS OF ELASTICITY
CYLINDER #2
E = 4.6 x 10^6
2000
1500
1000
500
0
P/A
1200
260 x 10^-6
0
50
100
150
200
250
300
350
400
ε x 10^-6

A-20

## Modulus of Elasticity Test

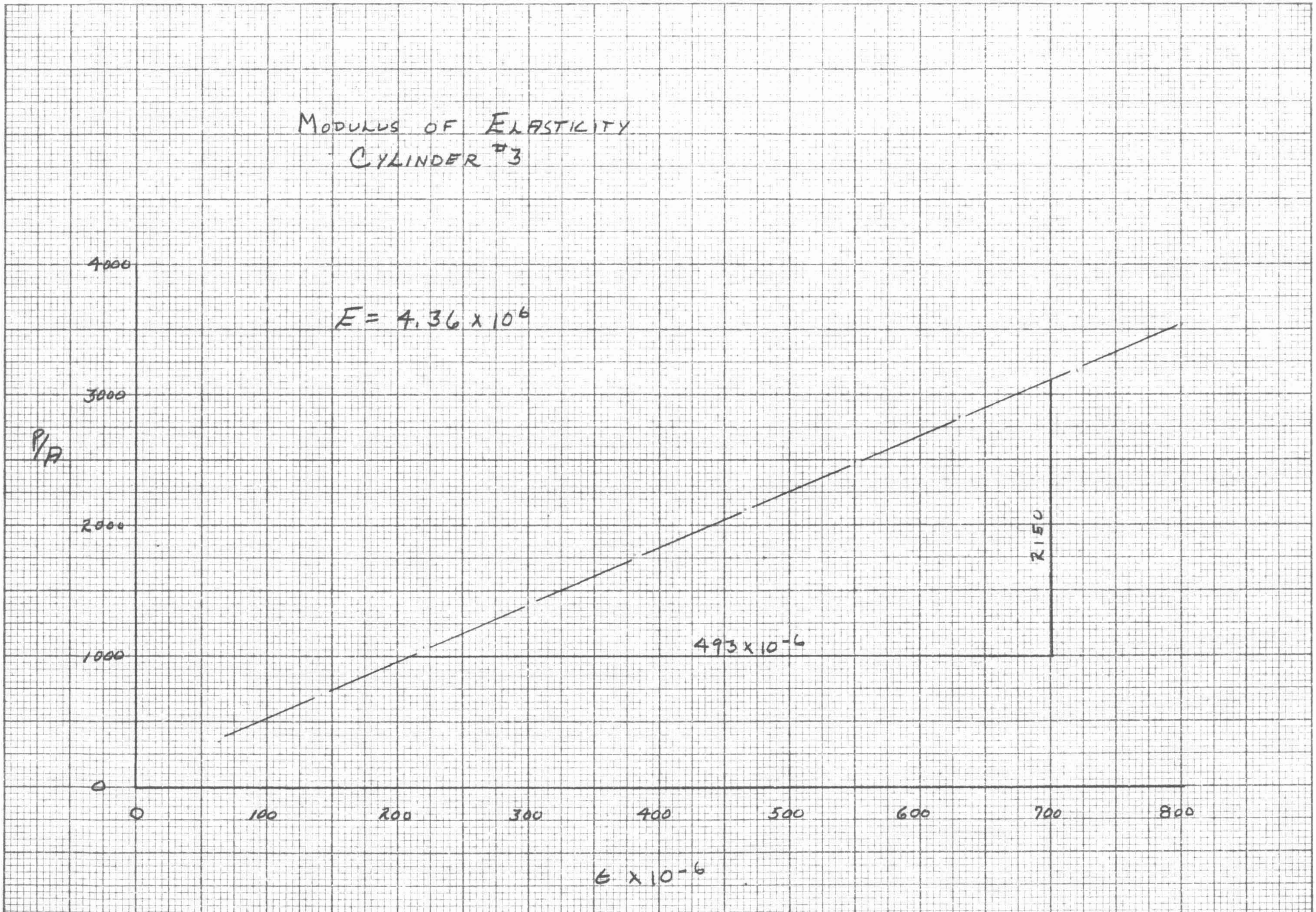
## Cylinder No 3

| Load | Reading | $\Delta R$ (in $\times 10^{-6}$) | Stress (psi) |
|---|---|---|---|
| 0 | 4-1580 | | 0 |
| 2,000 | 4-1561 | | 70.8 |
| 10,000 | 1500 | 61 | 354 |
| 20,000 | 1422 | 139 | 708 |
| 30,000 | 1340 | 221 | 1060 |
| | | | |
| 2,000 | 1551 | | 70.8 |
| 10,000 | 1488 | 63 | 354 |
| 20,000 | 1410 | 141 | 708 |
| 30,000 | 1332 | 219 | 1060 |
| 40,000 | 1249 | 302 | 1420 |
| 50,000 | 1169 | 382 | 1770 |
| 60,000 | 1085 | 466 | 2125 |
| 70,000 | 1002 | 549 | 2480 |
| 80,000 | 921 | 630 | 2840 |
| 90,000 | 832 | 719 | 3190 |
| 100,000 | 752 | 799 | 3540 |
| | | | |
| 2,000 | 1520 | 0 | 70.8 |
| 60,000 | 1047 | 473 | 2125 |
| 100,000 | 724 | 796 | 3540 |

Ultimate load = 203,500 lb

Ultimate stress = 7200 psi

K+E 10 X 10 TO THE 1/2 INCH 359-11
KEUFFEL & ESSER CO. MADE IN U.S.A.




A-22

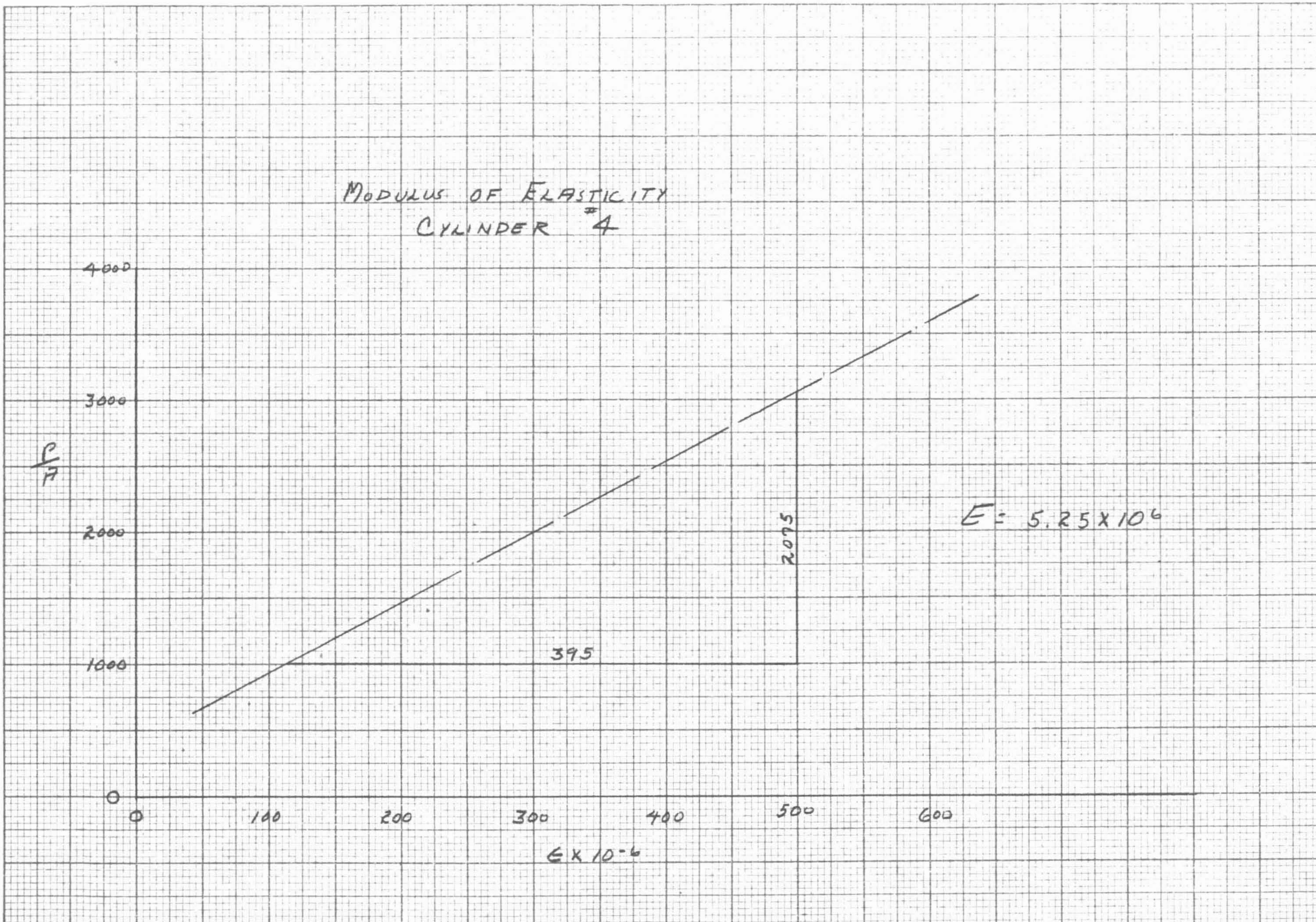
## Modulus of Elasticity Test

### Cylinder No 4

| Load | Reading | ΔR (in x $10^{-6}$) | Stress (psi) |
|---|---|---|---|
| 0 | 5-1238 | | 0 |
| 2,000 | 5-1223 | 0 | 70.8 |
| 10,000 | 1190 | 33 | 354 |
| 20,000 | 1139 | 84 | 708 |
| 30,000 | 1088 | 135 | 1060 |
| | | | |
| 2,000 | 1223 | 0 | 70.8 |
| 10,000 | 1188 | 35 | 354 |
| 20,000 | 1139 | 84 | 708 |
| 30,000 | 1086 | 137 | 1060 |
| 40,000 | 1003 | 220 | 1420 |
| 50,000 | 969 | 254 | 1770 |
| 60,000 | 905 | 318 | 2125 |
| 70,000 | 840 | 383 | 2480 |
| 80,000 | 772 | 451 | 2840 |
| 90,000 | 702 | 521 | 3190 |
| 100,000 | 631 | 592 | 3540 |
| | | | |
| 2,000 | 1205 | 0 | 70.8 |
| 60,000 | 883 | 322 | 2125 |
| 100,000 | 630 | 575 | 3540 |

Ultimate load = 188,000 lb

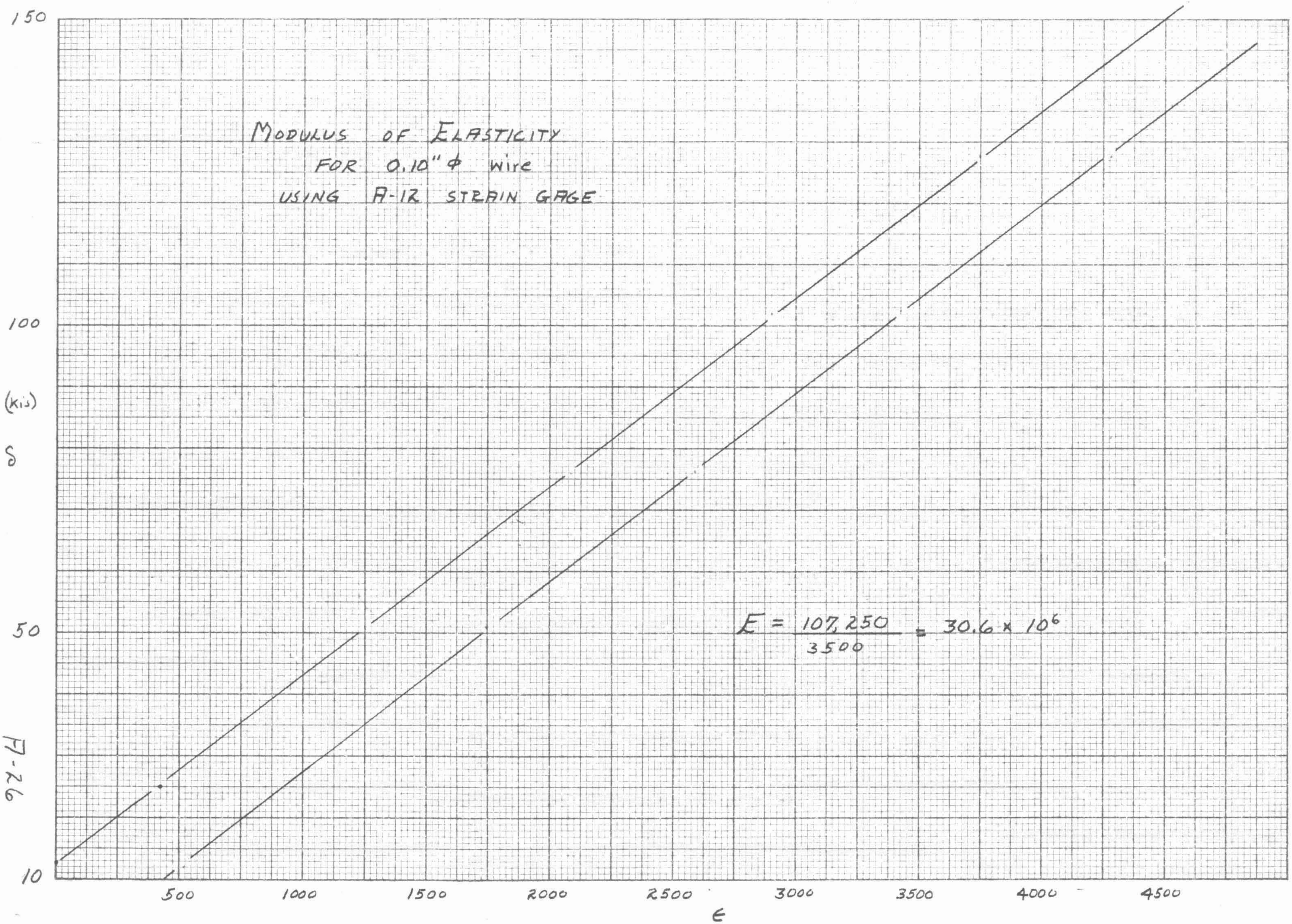
Ultimate stress = 6660 psi


MODULUS OF ELASTICITY
CYLINDER #4
4000
3000
2000
1000
0
P/A
0
100
200
300
400
500
600
ε x 10^-6
395
2075
E = 5.25 x 10^6

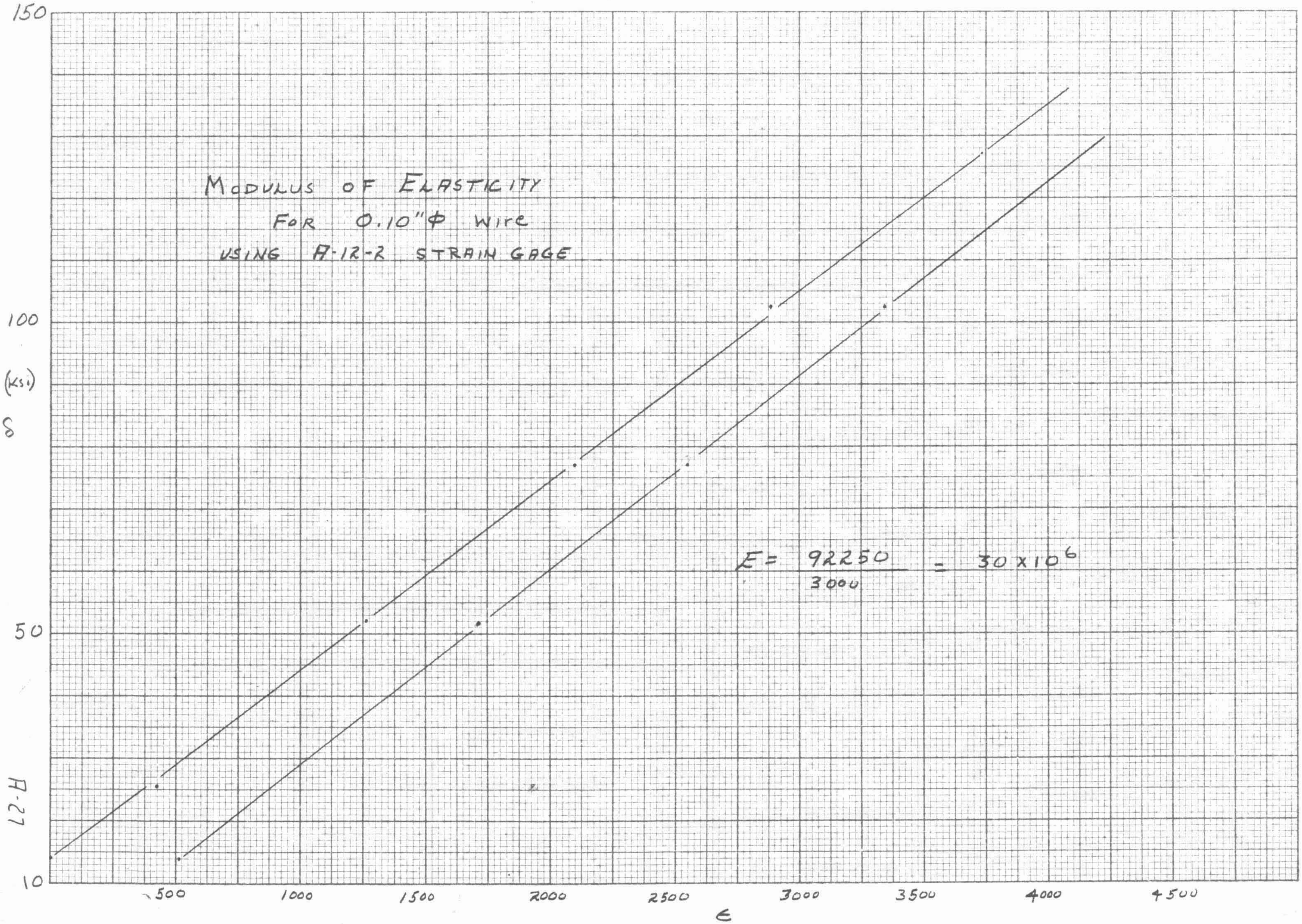
## Modulus of Elasticity Test

May 1, 1953
0.10" $\phi$ wire
Project 235
GRS & CAB

| Load | G.F. 2.06 A-12 Readings | $\Delta R$ | 2.10 A-12-2 Readings | $\Delta R$ | Stress |
|---|---|---|---|---|---|
| 100 | 5-1620 | 0 | 5-1960 | 0 | 12,750 |
| 200 | 7-030 | +410 | 6-1400 | 440 | 25,500 |
| 400 | 7-869 | 1249 | 7-1229 | 1269 | 38,250 |
| 600 | 7-1700 | 2080 | 8-1058 | 2098 | 76,500 |
| 800 | 8-1510 | 2890 | 9-862 | 2902 | 102,500 |
| 1000 | 9-1369 | 3749 | 10-702 | 3742 | 127,500 |
| 1200 | 10-1209 | 4589 | A2-088 | 5228 | 153,000 |
| 100 | 6-1131 | +511 | 7-461 | 501 | 12,750 |
| 400 | 7-1365 | 1745 | 8-660 | 1700 | 38,250 |
| 600 | 8-1190 | 2570 | 9-479 | 2591 | 76,500 |
| 800 | 9-1040 | 3420 | 10-321 | 3361 | 102,500 |
| 1000 | 10-888 | 4268 | A1-730 | 4770 | 127,500 |
| 100 | 6-1120 | +500 | 7-400 | 440 | 12,750 |
| 400 | 7-1361 | +1741 | 8-600 | 1700 | 38,250 |
| 800 | 9-1041 | +3421 | 10-321 | 3361 | 102,500 |
| 1000 | 10-890 | +4270 | A1-730 | 4770 | 127,500 |
| 100 | 6-1129 | +509 | 7-408 | 448 | 12,750 |


MODULUS OF ELASTICITY
FOR 0.10" φ wire
USING A-12 STRAIN GAGE
$E = \frac{107,250}{3500} = 30.6 \times 10^6$
150
100
50
10
(kis)
S
500
1000
1500
2000
2500
3000
3500
4000
4500
ϵ

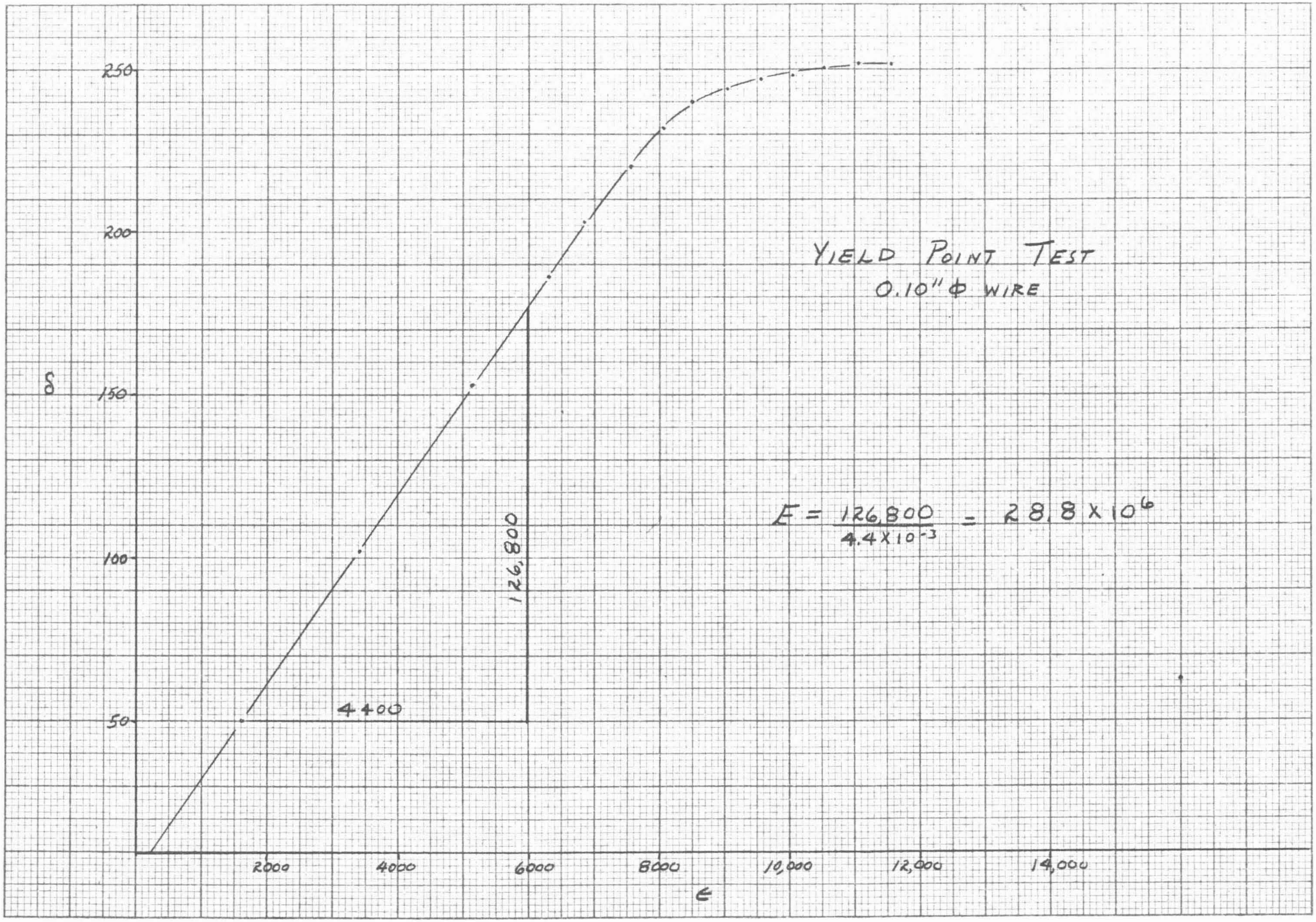
F7-26


MODULUS OF ELASTICITY
FOR 0.10"Φ WIRE
USING A-12-2 STRAIN GAGE
$E = \frac{92250}{3000} = 30 \times 10^6$
150
100
(ksi)
δ
50
10
500
1000
1500
2000
2500
3000
3500
4000
4500
ϵ

Yield Point Test

May 29, 1953

| Load | Reading | E | δ |
|---|---|---|---|
| 100 | 7-970 | 0 | 12,730 |
| 400 | 9-240 | 1270 | 51,000 |
| 800 | 10-910 | 2940 | 102,000 |
| 1200 | A3-150 | 5180 | 153,000 |
| 1600 | A4-890 | 6920 | 204,000 |
| 1730 | A5-500 | 7530 | 220,000 |
| 1820 | A6-000 | 8030 | 232,000 |
| 1880 | A6-500 | 8530 | 240,000 |
| 1915 | A7-000 | 9030 | 244,000 |
| 1942 | A7-500 | 9530 | 247,500 |
| 1957 | A8-000 | 10030 | 249,000 |
| 1970 | A8-500 | 10530 | 251,000 |
| 1975 | A9-000 | 11030 | 252,000 |
| 1981 | A9-500 | 11530 | 252,500 |
| 1985 | A10-000 | 12030 | 252,500 |
| 1987 | A10-500 | 12530 | 253,000 |
| 1992 | A11-000 | 13030 | 254,000 |
| 1994 | A11-500 | 13530 | 254,000 |


YIELD POINT TEST
0.10" Φ WIRE
E = 126,800 / 4.4 X 10^-3 = 28.8 X 10^6
250
200
150
100
50
δ
4400
126,800
2000
4000
6000
8000
10,000
12,000
14,000
ε