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SYNOPSIS

A method for estimating the probable fatigue life of prestressed concrete flexural members is presented. Extensive use is made of the results presented in the first three parts of this paper. Reasonable agreement is obtained between computed mean fatigue life and observed fatigue life for the beam tests, but it is emphasized that considerable variability is inherently associated with the fatigue phenomenon and that use of statistical methods is essential to an adequate treatment of the problem.

INTRODUCTION

In preceding parts of this paper, the results of beam fatigue tests were reported[1], the fatigue properties of 7/16-in. prestressing strand were determined[2], and an analysis was developed for the determination of the steel stresses in prestressed concrete members subjected to fatigue loading[3]. In this final part of the paper the information of Parts II and III is utilized to predict the probable fatigue life of the beams discussed in Part I. A comparison is made between observed and predicted mean fatigue lines and limitations of the results are presented. Finally the results are discussed as to their possible effect on specifications.

NOTATION

a_i	a fractional portion of the total number of cycles corresponding to $N_i(P)$
A_c	area of concrete section
A_s	cross sectional area of longitudinal tension steel
b	width of rectangular beam
d	effective depth of beam
D	standard deviation of log N
e	distance from center of gravity of A_s to center of gravity of A_c
E	non-dimensionalized concrete strain; $E = \frac{\epsilon_c}{\epsilon_u}$
E_1	value of E at extreme concrete compression fiber
f_{s1}	total steel stress at moment $M_1 > M_{on}$
F_n	prestressing force in beam during the n-th load cycle
F_{se}	prestressing force in test beam just prior to first load cycle
f'_c	ultimate cylinder strength of the concrete
f'_r	modulus of rupture of the concrete
f_{sn}	steel stress during the n-th load cycle
f^b_{cF}	concrete stress at the bottom fiber due to the prestress force
h	full depth of concrete section
I	moment of inertia of steel-concrete transformed section about centroidal axis
I_c	moment of inertia of concrete area about its centroidal axis
k	dimensionless factor defining depth to neutral axis at a cracked section
k_2	dimensionless factor defining location of compressive force in concrete compressive stress block

k_3	dimensionless factor relating concrete strength in beam and cylinder
m	$\frac{E_s}{E_c}$
M	applied moment
M_{on}	moment in n-th cycle at which cracks begin to open
N	number of cycles
N(P)	number of cycles at probability of failure P
p	reinforcement ratio; $\frac{A_s}{bd}$
P	probability of strand failure at or before N cycles
P_{01}, P_{02}	overloads
Q	probability of beam failure at or before N cycles
R	stress interval; $R = S_{max} - S_L$
S_L	fatigue limit corresponding to S_{min}
S_{max}	maximum stress level in a repeated load cycle
S_{min}	minimum stress level in a repeated load cycle
u	number of strands in the beam at depth d
$\overline{X}$	distance from the center of gravity of the steel area to the centroidal axis
α	dimensionless parameter defining the shape of the concrete stress-strain relation: $\alpha = E_{cn} \frac{\epsilon_u}{f'_c}$
ϵ_{s1}	total steel strain at the cracked section at moment $M_1 > M_{on}$
ϵ_{sF}	steel strain due to prestressing force F_n
ϵ_{cF}	elastic strain in concrete at the steel level due to prestressing force F_n
ϵ_u	concrete strain in cylinder at f'_c
ψ	compatibility factor

FATIGUE LIFE OF PRESTRESSED BEAMS -- STEEL FAILURE

In order to predict the fatigue life of a given beam, it is necessary first to determine, from the known or assumed load history, the corresponding stress history for the reinforcing steel. To make the transformation from load history to stress history, use is made of stress-moment curves which may be computed for any particular beam cross section using the equations presented in Part III of this paper. If the response of the beam to load remains constant throughout the major portion of its fatigue life, only one stress-moment relation has to be obtained. If, however, the response of the beam varies as a result of the fatigue loading, the load history must be broken into a number of intervals depending upon the rate of change of beam response, and a stress-moment relation must be computed for each interval.

It was observed in the beam fatigue tests[(1)] that, after an initial sequence of repeated loadings during which considerable changes took place in the deflections, deformations, and cracking patterns, the beams settled down to a fairly consistent response to the repeated loadings. Values of the compatibility factor, ψ, computed from deformation measurements on the beams under test also remained fairly constant after the initial sequence of loadings[(3)]. The results of these tests thus indicate that a single stress-moment relation would normally be sufficient - at least for beams similar to those tested - for determining the stress history, and in the following discussion it will be assumed that the response of the beam to load remains constant.

When the beam is subjected only to repeated load cycles of constant magnitude, the stress history will consist of repeated stress cycles of constant magnitude. After the magnitude of the stress cycle has been determined from the stress-moment relation, Eqs. 2.4 and 2.6* may be used to determine the mean fatigue life, $\overline{N}$, and the standard deviation, D, for a single strand element subjected to this stress cycle. These two values may be used in Eq. 2.2 to determine the number of cycles, N, corresponding to <u>any</u> probability level P.

If there are u similar strands present in the beam section at the same level, then the probability of beam failure at or before N cycles is

$$Q = 1 - (L - P)^u \tag{4.1}$$

Thus, the mean fatigue life of a beam may be obtained from Eq. 2.2 as the number of cycles N which correspond to a probability of failure in a single strand

$$P = 1 - (0.5)^{1/u}$$

Should the strands be placed at z different levels, with u_1, u_2, $\ldots u_i$, $\ldots u_z$ strands in the first, second, ...i-th, ...z-th levels, then the probability of beam failure at or before N cycles is

$$Q = 1 - (1 - P_1)^{u_1} \; (1 - P_2)^{u_2} \ldots (1 - P_i)^{u_i} \ldots (1 - P_z)^{u_z} \tag{4.2}$$

where P_i is the probability of failure at or before N cycles for an individual strand subjected to the repeated stress cycles which occur in the steel at the i-th level.

* 2.6 indicates that this equation is given in Part II as Eq. 2.6

In the case of a beam subjected to cumulative damage loading, the load history may be given as a curve relating load magnitude and relative frequency of occurrence (load-frequency distribution), a load-frequency histogram, or a block of load cycles as used in Part I[1]. In each case the load history can be expressed, either exactly or approximately, as a block of load cycles, and the stress-moment relation may then be used to make the transformation into a corresponding block of stress cycles. Equation 2.13 will then indicate, for a strand element subjected to this repeated load block, the probability of failure, P, corresponding to any number of cycles, N. Equation 4.1 or 4.2 may be used to determine from P the probability of fatigue failure of the beam at or before N cycles.

Comparison with Test Results

Strand fatigue failure took place in all of the beam fatigue tests described in Part I[1], and a comparison may now be made between observed and predicted mean fatigue lives.

Stress-moment relations were computed for the six beams and were used to determine the magnitude of the stresses in the reinforcement in the beams under the test loadings. In making the computations, a ψ value of 1.0 and a k_3 value of 0.85 were adopted. Creep-relaxation losses in the steel were not measured but were assumed to be 4 percent[4]. Values of applied moments and corresponding steel stresses for the six beams are shown in Table 1.

Since the beams contained three strands and a predicted mean fatigue life was desired, values of u = 3 and Q = 0.5 are substituted in Eq. 4.1

to give a value of 0.206 for P. Thus, the mean fatigue life of a beam is equal to the fatigue life at the 0.206 probability level of a single strand subjected to the stress history of the steel in the beam. The relation between the stress interval R and log N has been determined for the 0.206 probability level from Eqs. 2.2, 2.4, and 2.6, and is plotted in Fig. 1.

Use of the 0.206 probability line in Fig. 1, together with values computed for R, yields the predicted mean fatigue life for beams F1, F2, and F4, which were subjected to constant cycle loading. In the case of beams F5, F7, and F8, which were all subjected to cumulative damage loading, the 0.206 probability line provides values of N(0.206) which may be substituted in Eq. 2.13 to give the predicted mean fatigue life. The complete calculations for fatigue life of beam F7 are shown in the Appendix. Although a comparison of the computed and predicted fatigue lives in Table 1 shows a slight tendency for the method to over-estimate fatigue life, agreement is generally quite good, especially considering the variability of the phenomenon being studied.

FATIGUE LIFE OF PRESTRESSED BEAMS -- CONCRETE FAILURE

Since the state of stress in a strand in a beam is essentially simple tension, fatigue data obtained from strand tension tests may be used directly in the calculation of beam fatigue life. Also, since the strands are present as discrete elements, the "size effect" involved in the prediction of the fatigue life of u strands from the fatigue data for one strand is taken into account quite simply, using Eq. 4.1. A study of fatigue failure in the concrete compression zone of the beam, however, is complicated by both size effect and the presence of the stress gradient.

A statistical approach to the size effect and stress gradient problems has been made by Fowler(5), but his work, being concerned with materials such as steel which exhibit similar stress-strain properties in tension and compression, is not directly applicable to concrete. A considerable amount of work, both analytic and experimental, will be required before concrete fatigue life under stress gradients can be predicted using fatigue test data obtained from axially loaded specimens.

A simple lower bound estimate of the fatigue life of over-reinforced concrete beams can however be obtained by determining the fatigue life of a piece of plain concrete similar to the concrete in the beam, with cross sectional area equal to the area of the concrete stress block, and subjected to a pattern of repeated stresses which are uniform over the cross section and equal in value to the stresses in the extreme fiber of the beam.

Stress-moment relations for the concrete top fiber may be determined using the equations derived in Part III(3). The non-dimensional concrete top-fiber strain, E_1, is evaluated during the steel stress computations; the corresponding value of concrete stress is given by Eq. 3.7. The stress history for the concrete top fiber may then be obtained from the stress-moment relation and the known or assumed load history. Fatigue test data obtained from axially loaded test specimens may then be used to estimate a lower bound value for beam fatigue life.

DISCUSSION

Before a summary is made of the results of this investigation, several important aspects of the study of beam failure by steel fatigue will be discussed, in order to emphasize limitations involved in the present approach.

Limited Applicability of Strand Fatigue Data

The essentially empirical nature of the strand fatigue data has already been mentioned. It is therefore emphasized that the strand tests were extremely limited in scope, being restricted to a specimen of one particular size and length, in unrusted condition, and obtained from one manufacturer. Compared with the extensive and systematic data required by the nature of the problem, the experimental work described can be regarded as little more than a series of pilot tests. The further need for large, statistically designed experimental programs is obvious. An important aim of such future work would be to separate the "inherent" from the "experimental" variability associated with fatigue test data.

Size Effect

Equations 4.1 and 4.2 take into account the influence of the number of strands in the beam, and so represent an allowance for size effect in the amount of steel in the cross section. There is another size effect to be considered in the longitudinal direction. If the steel stress were uniform along the length of the beam the entire size effect for the steel would be represented by the following equation,

$$Q = 1 - (1 - P)^{u.v}$$

where v is the ratio of the length of the beam to the length of the strand test specimen. However, an examination of the deformations measured in the test beams indicates that steel stress varies greatly along the length of the beam, even in regions of constant moment, and in fact will attain a maximum value only at the widest crack. This can be seen in Figs. 14, 15, and 16, in Part I[1].

An accurate analysis of the longitudinal size effect would require the determination of the steel stress at each section along the beam, the calculation of the probability of failure in each increment of beam length, and finally, the combined probability of failure for the entire length. Such a procedure, even if it were possible to evaluate accurately the variation in steel stress along the beam, is clearly not feasible. In this investigation it has been assumed that failure will always occur in the region of maximum steel stress, which exists at the widest crack. Experimental values of ψ shown in Fig. 10 Part III[3], were accordingly obtained only from deformation measurements in the gage length in which failure eventually occurred. In all six beam tests the wire failures took place in the gage length which gage the largest tensile deformation readings.

Considering Eq. 4.1, it is seen that the likelihood of fatigue failure increases greatly with the number of strands in the cross section. It should, however, be remembered that beam fatigue failure has here been associated with a failure of one of the wires of a strand. When there is a very large number of discrete steel elements present in the cross section, the consequences of failure of one or even several of them is far less serious, and it may be necessary in such a situation arbitrarily to define beam fatigue failure when beam stiffness or static ultimate strength has been reduced below some specified limit.

Variability in Response of Beam to Load

An examination of the results obtained in Part II[2] indicates extreme sensitivity of strand fatigue life to small changes in the maximum and minimum stress levels. At the 60 percent minimum stress level, for example, a change in maximum stress level of only 14 percent, from 71 to 85, is sufficient to change the mean fatigue life from infinity to approximately 70,000. This sensitivity becomes more pronounced, of course, in the range of large N and small S values, where the mean curve is approaching its asymptotic value. Computations for beam fatigue life show a like sensitivity of beam fatigue life to small variations in the loading, particularly in the maximum load level, and also to small errors in the computed steel stresses.

In the stress computations, a number of factors are involved which cannot be evaluated precisely in most practical situations, and it is important to observe the effect of variations in these quantities on beam fatigue life. The quantities k_3, ψ, and prestress losses are particularly important in this respect. Although losses due to concrete creep and shrinkage can be measured accurately in laboratory test beams, accurate prediction of these quantities, especially under field conditions, is almost impossible because of inherent variability in concrete properties. In addition to the concrete losses, a certain amount of loss occurs due to creep and relaxation in the steel. While losses in the prestressing force do not materially affect the maximum steel stress level in the loaded beam, they directly affect the minimum steel stress level.

To observe the variation in values of predicted fatigue life, stress calculations were made for beam F7 using k_3 values of 0.85 and 1.0, ψ values of 0.7, 1.0, and 1.3, and prestress losses of 2 and 4 percent. Stress-moment relations were plotted for each calculation, values were thus obtained for steel stresses in the beam due to the applied loadings, and values of beam fatigue life were then determined from Eq. 2.13. Results of five different sets of calculations are contained in Table 2, which shows the effect on fatigue life of variations of the parameters from the previously assumed values of $k_3 = 0.85$, $\psi = 1.0$, and 4 percent steel losses. Variations in the factors of the order considered are seen to vary the mean fatigue life by 20 to 30 percent. It should be noted that beam F7 was subjected to particularly heavy overloading which caused a large proportion of the fatigue damage in the beam. The value of the stress interval R for this overload is large, in the order of 9; in cases where R is small, the corresponding variation in beam fatigue life, due to variations in k_3, ψ, and steel loss, will be larger, and may well exceed 100 percent.

Since it will not be possible in a practical situation to predict any of these factors with exactitude, variability in predicted beam fatigue life is likely to be much greater even than that indicates by the variability in the strand fatigue data. In such a situation, it would seem advisable to treat not only the fatigue properties of the materials as random variables but also the response of the beam to load. Thus, quantities such as f'_c, k_3, α, F_n, and ψ, which have been introduced in Part III(3), would be considered not as single valued parameters but as statistics with associated frequency distributions. Such a procedure, however, is clearly not feasible until very extensive experimental work is conducted to determine the frequency distributions for each random variable.

The reasonable agreement obtained between predicted mean and observed fatigue life for the test beam indicates the appropriateness of the methods developed in this investigation. By adopting suitably conservative values for parameters which are not known exactly, the equations may be used to check the safety against fatigue failure of partially prestressed members which are cracked under load.

Effect on Beam Fatigue Life of Repeated Overloadings

In the cumulative damage tests on beams F5, F7, and F8, the predominant load level produced approximately zero stress in the concrete at the bottom fiber; the first overload, P_{01}, was large enough to cause the tension cracks to open, and produced a stress in the steel approximately equal to the fatigue limit, the second overload, P_{02}, opened the crack further and caused an overstress of considerable magnitude in the strand. In beam F5 the steel stress level corresponding to load P_{01} was just below the fatigue limit and hence, according to the findings of Part I[2], did not cause fatigue damage. Failure was brought about in this beam by the repeated application of load P_{02}. Load P_{01} produced stress levels in beam F7 and F8 above the fatigue limit and contributed significantly to fatigue damage.

The reasonable correlation of theory with experiment for these three beams, together with the conclusion of Part II that stress levels smaller than the fatigue limit do not contribute to strand fatigue, indicates that loadings which cause opening and closing of the tension cracks will only begin to affect beam fatigue life when they produce overstress in the steel reinforcement.

SUMMARY AND CONCLUSIONS

An investigation was conducted into the fatigue life of prestressed concrete beams subjected to both constant cycle and cumulative damage loadings. Attention was given primarily to beams which are under-reinforced with respect to fatigue failure, i.e., to beams in which fatigue of the tension steel would precede fatigue in the concrete compression zone.

An experimental study was made of the fatigue properties of 7/16 inch diameter high strength prestressing strand. An empirical relation between maximum and minimum stress level and probable fatigue life was developed from the constant cycle test data. The results of cumulative damage tests showed good correlation with mean fatigue life predicted by Miner's theory, and a generalized form was developed to apply at all probability levels.

A theoretical analysis was made of the behavior of prestressed concrete beams under repeated loadings. Equations were derived for the stresses in the steel and in the extreme concrete compressive fibers in members of rectangular and I-shaped sections subjected to repeated loadings.

A method of determining the probable fatigue life of under-reinforced members was presented, which uses the data obtained from the strand fatigue tests, together with the equations derived in the analysis of beam behavior.

Static and fatigue tests were conducted on eight prestressed concrete beams of rectangular section. Although considerable changes in deformations and deflections took place in the early load cycles, the beams settled down quickly to a consistent response to load which was maintained over the major portion of the load history. Steel fatigue failures occurred in all beams which were fatigue tested. Satisfactory agreement was obtained between computed mean fatigue life and observed fatigue life.

Finally, a method was indicated for obtaining a lower bound estimate for the fatigue life of over-reinforced members by using the equations derived in the theoretical analysis to determine the stress history of the concrete in the extreme compression fiber, and applying data on concrete fatigue life obtained from fatigue tests on axially loaded specimens.

The following conclusions are indicated by the experimental and theoretical work comprising this investigation:

(1) The response of a prestressed concrete beam may be expected to vary considerably as a result of the application of fatigue loading. This variation is probably due to creep effects, changes in the concrete stress-strain relation, and progressive bond failure between the tension steel and surrounding concrete in the vicinity of the tension cracks. However, after an initial sequence of repeated loadings, representing perhaps ten percent of the fatigue life, the beam normally settles down to a fairly regular and consistent response to load.

When the fatigue loading is particularly severe, a continuous change in beam response may occur up to failure. Such severe fatigue loading would rarely be encountered under field conditions; in most cases the fatigue properties of a member may be studied by assuming a constant response of beam to load.

(2) A steel fatigue failure of a prestressed beam occurs by successive fracture of the elements of steel reinforcement in the beam. A considerable number of load cycles may separate the first and second steel failures, but the interval separating successive failures will tend to decrease as the number of failed elements increases. Failure of each steel element is accompanied by a corresponding decrease in beam rigidity.

When the total area of steel reinforcement is contained in a small number of elements, it is advisable to define beam fatigue failure as failure of the first steel element. When there are a large number of steel elements present in the section, beam fatigue failure may better be defined arbitrarily as the failure of some proportion of the elements. The proportion would be chosen from a consideration of allowable decreases in beam rigidity and factor of safety against static loading.

(3) The fatigue life of a beam which fails by steel fatigue and is subjected to a known load history may be estimated using the fatigue properties of the reinforcing steel, together with an analysis of the response of the beam to load.

(4) Quantitative information on material fatigue properties must at present come from experimental studies, and such information is there-

fore restricted in application. Because of the variability inherent in material fatigue properties, simple S-N curves and fatigue envelopes are inadequate representations of constant cycle fatigue properties. Statistical interpretation of strand fatigue test data is necessary in any satisfactory treatment.

(5) The results of the investigation of strand fatigue properties indicated that stress cycles in the loading history which are smaller than the fatigue limit will not contribute to fatigue failure in the strand. Thus, beam loadings which cause flexural cracks to open should <u>not</u> shorten beam fatigue life provided the stresses induced in the strand reinforcement are smaller than the fatigue limit. The use of partial prestressing techniques should not therefore lead to problems of premature fatigue failure, provided a conservative estimate of the stresses in the reinforcement, together with steel fatigue data, indicates adequate fatigue life for the beam. In other words, it can be concluded that the specification permitting no tension in the concrete in the maximum moment region of the beam can be safely revised to permit some percentage of the modulus of rupture as an allowable tensile stress.

The application of the results of this investigation must be limited to members of normal weight concrete pretensioned with strand. Also an adequate distance from first cracks to end of strand must be mentioned to develop the force in the strand.

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APPENDIX

EXAMPLE SOLUTION OF FATIGUE LIFE OF AN UNDER-REINFORCED BEAM

The procedure proposed in this paper is illustrated by a numerical calculation of the probable fatigue life of test beam F7.

BEAM AND MATERIAL PROPERTIES

$b = 6.31''$

$d = 8.00''$

$h = 12.06''$

$\bar{x} = 1.92''$

$e = 1.97''$

$A_s = 0.3267\ in^2$

$p = 0.00648$

$I_c = 920\ in^4$

$I = 929.4\ in^4$

$m = 6.4$ (first load cycle)

$f'_c = 6.22$ ksi

$\epsilon_u = 0.0023^{**}$

$F_1 = F_n = 36.30^k$

$\epsilon_{sF} = 0.00397^{*}$

$\epsilon_{cF} = 0.00014^{**}$

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* Including measured concrete creep and shrinkage losses and 4 percent steel creep loss.

**Values of ϵ_u and ϵ_{cF} from test measurements. In design calculations, values of ϵ_u and ϵ_{cF} may be estimated.

FIRST LOADING STAGE, $M \leq M_{on}$

a) Cracking Moment in First Load Cycle, M_{o1}

$$\text{Take } f'_r = 0.10\, f'_c = 0.622 \text{ ksi}$$

$$f^b_{cF} = -F_{se}\left[\frac{1}{A_c} + \frac{e}{I_c}\ \frac{h}{2}\right]$$

$$f^b_{cF} = -36.30\left[\frac{1}{76.09} + \frac{1.97\ (6.03)}{920}\right]$$

$$= -0.950 \text{ ksi}$$

$$M_{o1} = I\,\frac{f'_r - f^b_{cF}}{\frac{h}{2} - e + \bar{x}}$$

$$= \frac{929.4\ (1.572)}{5.98}$$

$$= 244 \text{ in-k}$$

$$f_s = \frac{F_{se}}{A_s} + m\ \frac{M_{o1}}{I}\ \bar{x}$$

$$= 110 + \frac{6.4\ (244)\ 1.92}{929.4}$$

$$= 113.2 \text{ ksi}$$

b) Cracking Moment, $n > 1$

$$M_{on} = I\,\frac{-f^b_{cF}}{\frac{h}{2} - e + \bar{x}}$$

$$= \frac{929.4\ (0.950)}{5.98}$$

$$= 147.6 \text{ in-k}$$

$$f_s = 110 + \frac{6.4\ (147.6)\ 1.92}{929.4}$$

$$= 112.2 \text{ ksi}$$

SECOND LOADING STAGE, $M > M_{on}$

The stress-moment calculations for $M > M_{on}$ are presented in Table 3. The numerical values in columns 1 to 12 are calculated using the following step-by-step procedure:

Column

1. Choose steel stresses, f_{s1}, at suitable intervals.
2. Obtain corresponding steel strains, ϵ_{s1}, from stress-strain curve.
3. Compute $\epsilon_{s1} - \epsilon_{sF} - \epsilon_{cF}$; for Bm F7, $(\epsilon_{sF} + \epsilon_{cF}) = 0.00411$.
4. Compute $\dfrac{(\epsilon_{s1} - \epsilon_{sF} - \epsilon_{cF})}{\psi}$
5. Compute $\dfrac{A_s \quad f_{s1}}{bd\ k_3\ f'_c}$
6. Make trial values of k until equations 3.15 and 3.13a are satisfied simultaneously.

$$\frac{f_{s1}\quad A_s}{bd\ k_3\ f'_c} = k\left[\frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3\right] \tag{3.15}$$

$$\frac{(\epsilon_{s1} - \epsilon_{sF} - \epsilon_{cF})}{\epsilon_u\ \psi} = E_1\left[\frac{1-k}{k}\right] \tag{3.13a}$$

To evaluate quickly the right side of Eq. 3.15 it is convenient to plot k against E_1 for various values of the quantity $\frac{f_{s1} \; A_s}{bd \; k_3 \; f'_c}$.

The value of α in Eq. 3.15 is determined from Eq. 3.8, $\alpha = \frac{E_{cn} \epsilon_u}{f'_c}$. Note that the average value of E_{cn} will usually be less than the initial tangent modulus at the first load cycle, E_{co}. In this example, $E_{cn} = 3.61 \times 10^6$ psi is the average of the results of the concrete cylinder stress-strain tests with pre-loadings[1].

$$\alpha = \frac{3.61 \times 10^6 \; (0.0023)}{6220} = 1.38 \qquad \text{Take } \alpha = 1.40.$$

7. Determine k_2 from Eq. 3.11 again it is convenient to plot k_2 against E_1.
8. Value of E_1.
9. Value of ϵ_{c1}.
10. Obtain k k_2, i.e. Column 6 x column 7.
11. Hence $(1 - k \, k_2)$.
12. Compute M_1 from

$$M_1 = f_s \, A_s \, d \, (1 - k_2 \, k) \tag{3.18}$$

MEAN FATIGUE LIFE OF BEAM

Values of f_{s1} and M_1 may now be used to plot a stress-moment curve and hence obtain the following stresses corresponding to the applied load:

	Load Kips	Moment in.k	Stresses %
P_{min}	= 3.80	136.8	44.4
P_{pred}	= 7.05	253.7	51.1
P_{01}	= 9.09	327.3	60.3
P_{02}	= 10.37	373.8	67.2

$$S_L = 0.8\,(S_{min}) + 23$$
$$= 0.8\,(44.4) + 23$$
$$= 58.6$$

$S_{pred} - S_L$ = negative, therefore an understress

$S_{01} - S_L = 60.3 - 58.6 = 1.7$

$S_{02} - S_L = 67.2 - 58.6 = 8.6$

From the 0.206 probability line in Fig. 1, the following values of $\overline{\log N}$ and hence $\overline{N}$ are obtained.

R	$\overline{\log N}$	N
8.6	5.144	139,400
1.7	6.225	1,678,000

Substituting values in Eq. 2.13

$$N(P) = \frac{1}{\sum \frac{a_i}{N_i(P)}}$$

$$N(0.206) = \frac{1}{\frac{0.1}{139,400} + \frac{0.3}{1,678,000}}$$

$$= \frac{10^6}{0.718 + 0.179}$$

$$N(0.206) = 1.12 \times 10^6 \text{ cycles}$$

The mean fatigue life of the beam is equal to the number of cycles for which the probability of fatigue failure in one strand is 0.206. Thus the predicted mean fatigue life of the beam is 1.12×10^6 cycles.

TABLE 1 - COMPARISON OF PREDICTED AND OBSERVED BEAM FATIGUE LIVES

Beam	Moments, in-k.				Stresses, % Static Ult.				$\bar{N}_p$ x 10^6	N_e x 10^6
	M_{min}	M_{pred}	M_{01}	M_{02}	S_{min}	S_{pred}	S_{01}	S_{02}		
F1	162	436	-	-	62.5	79.8	-	-	.179	.225
F2	162	436	-	-	62.0	79.4	-	-	.191	.164
F4	162	436	-	-	62.0	79.8	-	-	.170	.139
F5	136.8	254.6	329	366.4	46.9	52.0*	60.6	66.0	2.300	1.947
F7	136.8	253.7	327.3	373.8	44.4	51.1*	60.3	67.2	1.120	1.167
F8	136.8	256.3	327.0	375.2	45.4	50.0*	60.3	67.6	1.310	1.136

Notes: * Understress

$\bar{N}_p$ = Predicted mean fatigue life

N_e = Observed fatigue life

TABLE 2 - EFFECT OF PARAMETERS k_3, ψ, % STEEL LOSSES ON MEAN FATIGUE LIFE*

k_3	ψ	% Steel Losses	S_{min} %	S_{o1} %	S_{o2} %	S_L %	$\bar{N}$ x 10^6
0.85	0.7	4	44.4	58.8	65.0	58.6	1.96
0.85	1.0	4	44.4	60.3	67.2	58.6	1.12
0.85	1.3	4	44.4	61.1	68.7	58.6	0.78
0.85	1.0	2	45.4	60.3	67.2	59.3	1.47
1.00	1.0	4	44.4	59.9	66.8	58.6	1.32

*Computation made for Beam F7

TABLE 3 STRESS MOMENT CALCULATIONS FOR $M > M_{on}$, Beam F7

Cracked Section:

$\psi = 1.0$ $k_1 = 0.85$ $\alpha = 1.4$

(1) f_{s1}	(2) ϵ_{s1}	(3) $\epsilon_{s1}-(A)^*$	(4) $\frac{\epsilon_{s1}-(A)}{\psi}$	(5) $\frac{A_s f_{s1}}{bd\ k_1\ f'_c}^{**}$	(6) k	(7) k_2	(8) E_1	(9) ϵ_{c1}	(10) $k\ k_2$	(11) $1-kk_2$	(12) M^{***}
120	.00434	.00023	.00023	0.148	0.736	0.334	0.28	.00064	0.245	0.755	238
140	.00498	.00087	.00087	0.172	0.541	0.336	0.45	.00104	0.182	0.818	300
160	.00570	.00159	.00159	0.197	0.469	0.339	0.615	.00141	0.159	0.841	353
180	.00650	.00239	.00239	0.222	0.429	0.346	0.78	.00179	0.148	0.852	403
200	.00742	.00331	.00331	0.246	0.404	0.356	0.975	.00224	0.144	0.856	449

$*(A) = \epsilon_{sF} + \epsilon_{cF} = 0.00411$

$** \quad \frac{A_s}{bd\ k_1\ f'_c} = .001227 \quad \frac{in^2}{kips}$

$*** \quad M = f_s\ A_s\ d\ (1 - k\ k_2)$

$A_s\ d = 0.3267\ (8) = 2.62$

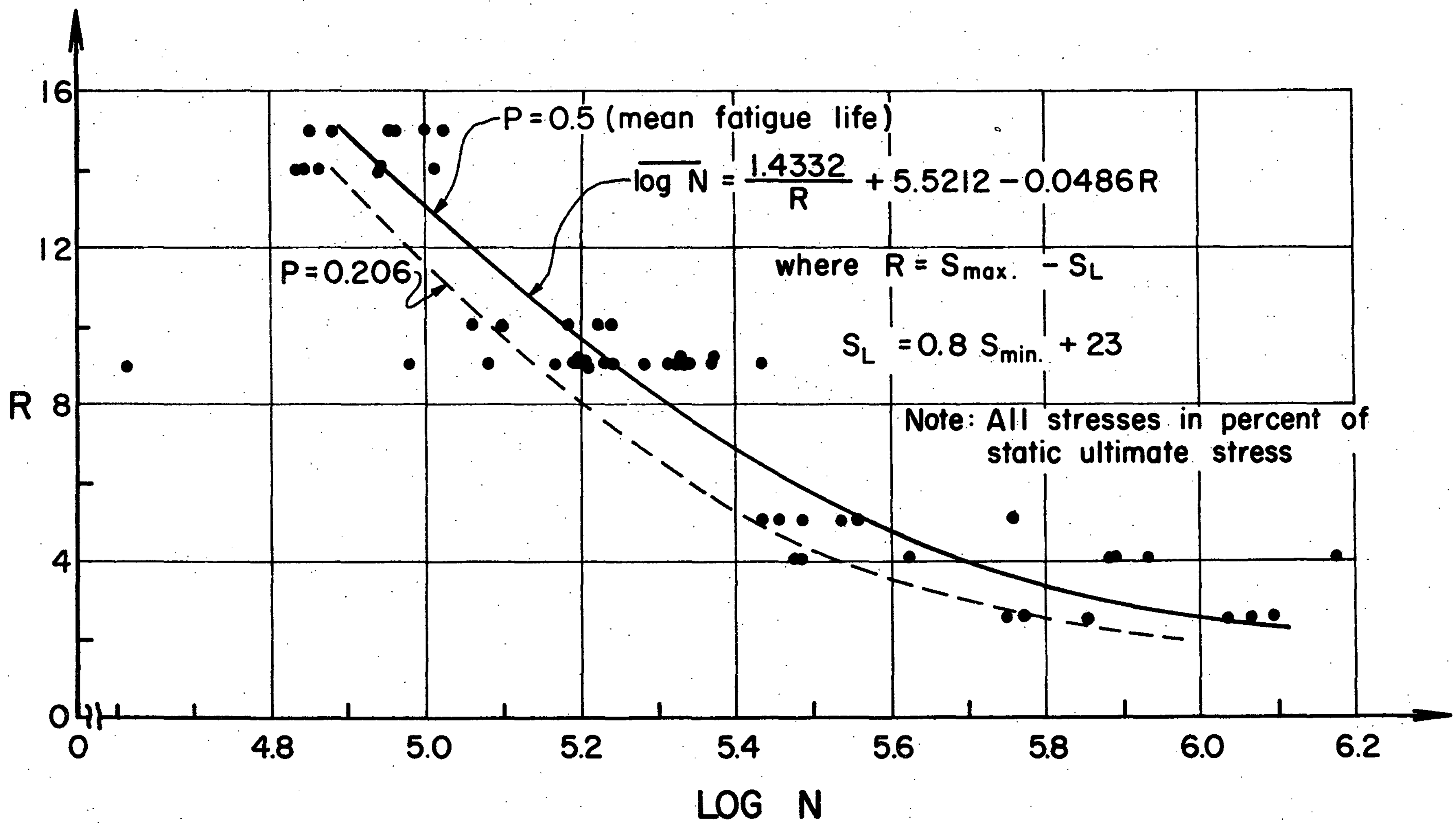

FIG. I R VERSUS LOG N

ASEE

WSE CLH instructions

CLH
DVH
JHB
Bad