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PRESTRESSED CONCRETE BRIDGE MEMBERS

PROGRESS REPORT 20

THE CALCULATION OF FLEXURAL STRESSES IN A PRESTRESSED CONCRETE MEMBER

by

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E R R A T A:

In Pages 3 and 12

e_{cs} = the tensile strain which occurs in the concrete at the steel level during the application of the moment M, minus the value of e_{ce}.

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S Y N O P S I S

The necessity of knowing the flexural stresses induced in a prestressed member by an applied moment arises in a number of instances, notably when the flexural fatigue properties are to be determined.

Equations are here set up for the calculation of the stress-moment relations for the steel reinforcement and the extreme fibers of the concrete of a rectangular, prestressed concrete section. The equations indicate the state of stress in the section as the applied moment is increased from zero to the failure point.

CONTENTS

INTRODUCTION

1. Introduction

Before the flexural fatigue properties of a prestressed concrete member can be calculated it is necessary to know, for a number of cross sections in the member, the relation between applied moment and the resulting stresses in the steel reinforcement and in the extreme fibers of the concrete. In this report equations are set up for the calculation of the stress-moment relations for a rectangular, prestressed concrete section.

Depending upon the magnitude of the applied moment, there may or may not be flexural cracks present in the concrete. In either case the steel and concrete stresses are calculated using the following data:

(I) Equations of static equilibrium

(II) The assumption of a linear strain distribution in the cross sections of the concrete.

(III) The given stress-strain relation for the steel

(IV) An assumed stress-strain relation for the concrete

Prior to cracking the concrete and steel stresses are reasonably small and a linear relation between stress and strain may be assumed for both materials. Thus the stresses in an uncracked section are calculated in a reasonably straight forward manner.

At higher stages of loading the concrete stress-strain relation is noticeably non-elastic and a more complicated relation must be assumed. The complexity of the problem is further increased by the possibility of yielding in the steel and by the fact that, after cracking, the position of the neutral axis becomes an unknown quantity to be determined. The solution for the cracked section is therefore more lengthy than for the uncracked case. However a direct solution has been made possible by the construction of an intercept chart which removes the necessity of solving simultaneous equations.

2. Notation and Sign Convention

The following notation, which follows that suggested by the A. C. I. - A.S.C.E. Joint Committee 323*, is used.

α = $\frac{E_c}{f'_c} \cdot e_{cu}$

a = internal lever arm

A_c = b.h = area of entire concrete section

b = width of beam

C = total compressive force in the concrete

d = depth to C.G. of steel

e = eccentricity of C.G. of steel with respect to the C.G. of the concrete area

e_c = concrete strain

e_{cs} = total concrete strain at the steel level

e_{cu} = ultimate concrete strain

e_{c1} = concrete strain at the top fibre

e_s = steel strain

E = $\frac{e_c}{e_{cu}}$

E_1 = $\frac{e_{c1}}{e_{cu}}$

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* "Proposed Definitions and Notations for Prestressed Concrete"

A.C.I. Journal Oct. 1952 V 49 P 85

E_c = initial modulus of elasticity of the concrete

f_c = concrete stress

f_c' = concrete ultimate stress

f_s = steel stress

f_{se} = steel stress due to effective prestress

$F = \frac{f_c}{f_c'}$

F = effective prestress force after deduction of all losses

f_L^t = concrete stress in top fiber due to applied moment M_L

f_L^b = concrete stress in bottom fiber due to applied moment M_L

f_F^t = concrete stress in top fiber due to the effective prestressing force F

f_F^b = concrete stress in the bottom fiber due to the effective prestressing force F

h = total depth of beam

I = total moment of inertia of the uncracked section

I_c = moment of inertia of the uncracked concrete

kd = depth to the neutral axis of the cracked section

k_2kd = depth to the resultant compressive force in the cracked section

$k_2' = (1-k_2)$

$n = \frac{E_s}{E_c}$

$$p = \frac{A_s}{b.d}$$

$$p' = \frac{A_s}{b.h}$$

T = total tensile force at a cracked section

$\bar{x}$ = distance from C.G. of the steel to the N.A. of the composite uncracked section.

Sign Convention: In the following, compressive stresses are taken as positive and tensile stresses as negative.

U N C R A C K E D   S E C T I O N

3. Uncracked Section, Approximate Solution

In the loading stage prior to cracking, a first approximation to the behavior of a prestressed member may be obtained by neglecting the increase in steel stress, which occurs with increased moment as a result of the small elastic deformation in the beam. Then we may write:

$$M = F.a$$

where M = applied moment

F = $f_{se}As$ = effective prestressing force in the steel

a = lever arm, i.e., the distance between the center of gravity of the compressive force and the center of gravity of the tension force.

It follows that "a" increases linearly with M as shown in Figure I.

The concrete stresses are obtained as the sum of the stresses at initial prestress plus the stresses due to the applied load. Thus, with compressive stresses positive,

$$f_c^t = f_F^t + f_L^t \qquad f_c^b = f_F^b + f_L^b \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

but

$$f_F^t = + \left(\frac{F}{A_c} - \frac{F.e}{I_c} \cdot \frac{h}{2}\right) \ldots\ldots\ldots\ldots 3$$

i.e.

$$f_F^t = + \frac{F}{A_c}\left(1 - \frac{6e}{h}\right)$$

and

$$f_L^t = + \frac{M}{I_c} \cdot y \quad \ldots\ldots\ldots\ldots \quad 4$$

and so

$$f_c^t = \frac{F}{A_c}\left(1 - \frac{6e}{h}\right) + \frac{6M}{bh^2}$$

$$\ldots\ldots\ldots\ldots \quad 5$$

similarly

$$f_c^b = \frac{F}{A_c}\left(1 + \frac{6e}{h}\right) - \frac{6M}{bh^2}$$

The steel stresses have of course been assumed to remain constant at f_{se} , the steel stress due to effective prestress.

The error involved in the above method may be expected to be less than 10%.

4. Stresses at an Uncracked Section

To take into account the variation in steel stress, the analysis is made by treating the section as a composite member, calculating the stresses resulting from the applied moment, M, and adding them, as before, to the initial stresses due to the effective prestress.

Referring to Figure 2, the position of the neutral axis of the composite section is found as:

$$\bar{x} = \frac{A_c \cdot e}{A_c + (n-1)A_s} \quad \ldots\ldots\ldots\ldots \quad 6$$

where e is the distance from the center of gravity of the concrete area A_c to the center of gravity of the steel area A_s, and $\bar{x}$ is the distance from the center of gravity of the steel area to the neutral axis.

The moment of inertia of the composite section is

$$I = \frac{b.h^3}{12} + b.h(e-\bar{x})^2 + (n-1)A_s\,\bar{x}^2$$

i.e.

$$I = A_c\left[\frac{h^2}{12} + (e-\bar{x})^2 + (n-1).p'.\bar{x}^2\right] \quad \ldots\ldots 7$$

where $p' = \frac{A_s}{b.h}$ = Proportion of steel reinforcement to total concrete area.

The stresses in the concrete due to the applied moment M_L are:

$$f_L^t = + \frac{M}{I}\left(\frac{h}{2} + e - \bar{x}\right)$$

$$f_L^b = - \frac{M}{I}\left(\frac{h}{2} - e + \bar{x}\right)$$

and the total concrete stresses are given by

$$f_c = f_F + f_L$$

i.e.

$$f_c^t = + \frac{F}{A_c} \left(1 - \frac{6e}{h}\right) + \frac{M}{I} \left(\frac{h}{2} + e - \bar{x}\right)$$

$$f_c^b = + \frac{F}{A_c} \left(1 + \frac{6e}{h}\right) - \frac{M}{I} \left(\frac{h}{2} - e + \bar{x}\right) \qquad \ldots\ldots\ldots\ldots 8$$

The additional steel stress due to M is

$$f_{sL} = n \frac{M}{I} \bar{x}$$

and the total steel stress is therefore

$$f_s = f_{se} + n . \frac{M}{I} x$$

or

$$f_s = \frac{F}{A_s} + n . \frac{M}{I} . x \qquad \ldots\ldots\ldots\ldots 9$$

It should be noted that in the above equations "I" refers to the total moment of inertia of the cross-section and is given by Equation 7.

C R A C K E D   S E C T I O N

5. Stress-Strain Relation for Concrete

A cubic parabola is here used to represent the stress-strain relation for the concrete and is plotted, in dimensionless form, in Figure 3. The general equation for such a curve,

$$F = A_1 E^3 + A_2 E^2 + A_3 E + A_4 \quad \ldots\ldots\ldots\ldots \quad 10$$

where $F = \dfrac{f_c}{f_c'}$

and $E = \dfrac{e_c}{e_{cu}}$,

is convenient to work with, but at the same time contains a sufficient number of coefficients, A_1, A_2, A_3, A_4, to give a reasonable approximation to the actual stress-strain relation.

The coefficients are evaluated such that

(a) when $E = 0$, $\dfrac{dF}{dE} = \alpha = \dfrac{E_c}{f_c'} . e_{cu}$ (i.e. the slope of the stress strain curve at zero stress is equal to the initial modulus of elasticity of the concrete)

(b) when $E = 1.0$, $\dfrac{dF}{dE} = 0$ (the slope of the stress strain curve is zero at ultimate stress)

(c) when $E = 0$, $F = 0$, and

(d) $E = 1.0$, $F = 1.0$

The resulting expression, in dimensionless form, is

$$F = \alpha.E - (2\alpha - 3)E^2 + (\alpha - 2)E^3 \quad \ldots\ldots\ldots \quad 11$$

where $\alpha = \frac{E_c}{f'_c} \cdot e_{cu}$ 12

or, in the more usual notation,

$$\frac{f_c}{f'_c} = \frac{E_c}{f'_c} .e . \left(\frac{e_c}{e_{cu}}\right) - \left[2\frac{E_c}{f'_c} .e - 3\right]\left(\frac{e_c}{e_{cu}}\right)^2 + \left[\frac{E_c}{f'_c} . e - 2\right]\left(\frac{e_c}{e_{cu}}\right)^3$$

. . . . 11 a

For equation 11 to represent a monotonically increasing curve between E = 0 and E = 1, as shown in Figure 3, a limitation must be placed on the value of α, the initial slope of the curve. If the initial slope is too steep, the curve reaches its maximum value at a smaller value of E and then becomes a minimum at E = 1. This is illustrated in Figure 3.

Thus it is stipulated that

$$\frac{d^2F}{dE^2} \leqslant 0 \quad \text{at} \quad E = 1.0$$

i.e. $\quad -2(2\alpha - 3) + 3\times 2\times(\alpha - 2)\,E \leqslant 0 \quad \text{at } E = 1.0$

$\therefore \quad \alpha \leqslant 3$ 13

The desirable values of α to give a good approximation to the actual stress-strain relation are discussed later.

6. Strain Distribution

A linear strain distribution is assumed at the section under consideration, as in Figure 4. The total strain in the steel, when moment M is applied, is

$$e_s = e_{se} + e_{ce} + e_{cs} \quad \ldots \ldots \ldots \ldots \quad 14$$

where e_{se} = strain in the steel due to effective prestress

e_{ce} = strain in the concrete at the steel level due to effective prestress.

e_{cs} = total strain in concrete at steel level after the moment M is applied.

also e_{c1} = total strain in concrete at the top fiber after the moment M is applied

The usual assumption of "perfect" bonding is implied in the above equation.

Thus the total steel strain, e_s, can be thought of as the sum of two components, e_{cs} and $(e_{se} + e_{ce})$. The latter quantity may be considered to have a constant value for a given beam. For a pretensioned beam, it is equal to the total initial steel pre-stress e_{si} minus inelastic losses. If the losses can be ignored

$$(e_{se} + e_{ce}) = e_{si}$$

Rewriting equation 14

$$e_s - (e_{se} + e_{ce}) = e_{cs} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 15$$

From Figure ~~5e~~ 4c it can be seen that

$$\frac{e_{c1}}{kd} = \frac{e_{c1} + e_{cs}}{d}$$

i.e.

$$e_{cs} = \frac{1-k}{k} e_{c1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 16a$$

dividing throughout by e_{cu} we obtain

$$\frac{e_{cs}}{e_{cu}} = \frac{1-k}{k} \frac{e_{c1}}{e_{cu}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 16$$

Now the strain at a distance "x" above the level of the neutral axis is

$$e_c = e_{c1} \cdot \frac{x}{kd} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 17a$$

and again dividing by e_{cu}

$$\frac{e_c}{e_{cu}} = \frac{e_{c1}}{e_{cu}} \cdot \frac{x}{kd} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 17b$$

or

$$E = \frac{e_{c_1}}{e_{cu}} \cdot \frac{x}{kd} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 17$$

hence

$$\frac{dE}{dx} = \frac{e_{c_1}}{e_{cu}} \frac{1}{kd} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 18$$

Equations 16 and 18 will be used later.

7. Equilibrium Equations

Considering the concrete compressive stress block above the neutral axis of the cracked section, as shown in Figure 5, we have:

$$C = b.\int_{o}^{kd} f_c \, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 19$$

which may be written as

$$C = b.f_c' \cdot \int_{o}^{e_{c_1}} \frac{f_c}{f_c'} \cdot \frac{dx}{de_c} de_c$$

better to write

$$C = b.f_c' \int_0^{E_1} \frac{f_c}{f_c'} \cdot \frac{dx}{dE} . dE$$

with $\frac{f_c}{f_c'} = F$

etc

or, with

$$\frac{f_c}{f_c'} = F,$$

$$\frac{e_c}{e_{cu}} = E$$

and

$$\frac{e_{c_1}}{e_{cu}} = E_1,$$

$$C = b.f_c' \int_0^{E_1} F \frac{dx}{dE} \, dE \quad \ldots\ldots\ldots\ldots\ldots 20$$

but from equation 18

$$\frac{dx}{dE} = \frac{kd}{E_1} \quad \ldots\ldots\ldots\ldots\ldots 21$$

hence we can rewrite (20) in dimensionless form as

$$\frac{C}{b.d.f_c'} = \frac{1}{E_1} \cdot k \int_0^{E_1} F.dE$$

i.e.

$$\frac{C}{b.d.f_c'} = \frac{k}{E_1}\left[\frac{\alpha}{2} E_1{}^2 - \frac{2\alpha-3}{3} E_1{}^3 + \frac{\alpha-2}{4} E_1{}^4\right]$$

or

$$\frac{C}{b.d.f_c'} = k\left[\frac{\alpha}{2} E_1 - \frac{2\alpha-3}{3} E_1{}^2 + \frac{\alpha-2}{4} E_1{}^3\right] \ldots\ldots 22$$

In the usual notation this is

$$\frac{C}{b.d.f'} = k\left\{\frac{1}{2}\cdot\frac{E_c}{f'_c}\cdot e_{cu}\left(\frac{e_{c1}}{e_{cu}}\right) - \left[\frac{2}{3}\cdot\frac{E_c}{f'_c}\cdot e_{cu} - 1\right]\left(\frac{e_{c1}}{e_{cu}}\right)^2\right.$$

$$\left. + \left[\frac{1}{4}\cdot\frac{E_c}{f'_c}\cdot e_{cu} - \frac{1}{2}\right]\left(\frac{e_{c1}}{e_{cu}}\right)^3\right\} \quad \ldots\ldots\ldots\ldots \quad 22a$$

8. Value of α

It is generally accepted that e_{cu}, the ultimate strain in the top fiber of the beam, is a constant, the values quoted by different investigators lie between 0.003 and 0.004. It is also usual for the ratio $\frac{E_c}{f'_c}$ to be assumed, for practical purposes, a constant. Choosing e_{cu} as 0.003 and a value of 1000 for $\frac{E_c}{f'_c}$ the parameter α becomes 3 which, as can be seen from equation 13, is the maximum value it may take.

The value

$$\alpha = 3. \quad \ldots\ldots\ldots\ldots \quad 23$$

will be used here. Equation 22 then becomes

$$\frac{C}{b.d.f'_c} = k\left[\frac{1}{4}E_1^3 - E_1^2 + \frac{12}{8}E_1\right] \quad \ldots\ldots\ldots\ldots \quad 24$$

9. Position of the Center of Gravity of the Compressive Stress Block

To determine the value of the applied moment, M, in terms of the internal stresses, it is necessary to know the length of the internal lever arm and hence of the position of the resultant compressive force C. M is given by

$$M = C\, d(1-k_2k) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 25$$

The parameter k_2 is chosen to represent the position of the force C and may be found by a consideration of the geometry of the cubic parabola. Referring to Figure 6 it can be seen that the value of k_2 will depend upon the value of E_1, but although E_1 may vary between zero and unity only a small variation can occur in k_2. Limits to the value of k_2 can easily be found by replacing the parabola OPA, on the one hand by the straight line OBA, and, on the other, by the rectangle OF_1A; thus it is shown that the value of k_2 must always be between 0.5 and 0.33.

In evaluating k_2 it will be convenient to work with the quantity $k_2' = (1-k_2)$, then, from Figure 7,

$$k_2' . E_1 \int_0^{E_1} F \, dE = \int_0^{E_1} E.F. \, dE$$

considering the left hand side of the equation,

$$k_2' \, E_1 \int_0^{E_1} F \, dE = k_2' \, E_1 \int_0^{E_1} \left[\alpha E - (2\alpha-3)E^2 + (\alpha-2)E^3 \right] dE$$

$$= k_2' . \left[\alpha \frac{E_1{}^3}{2} - (2\alpha-3) \frac{E_1{}^4}{3} + (\alpha-2) \frac{E_1{}^5}{4} \right]$$

and now considering the right hand side,

$$\int_0^{E_1} E.F.dE = \int_0^{E_1} \left[\alpha E^2 - (2\alpha-3)E^3 + (\alpha-2)E^4 \right] dE$$

$$= \frac{\alpha}{3} E_1^3 - \frac{2\alpha-3}{4} E_1{}^4 + \frac{\alpha-2}{5} E_1{}^5$$

Hence, with $\alpha = 3$,

$$k_2' = \frac{1 - \frac{3}{4} E_1 + \frac{1}{5} E_1{}^2}{\frac{3}{2} - E_1 + \frac{1}{4} E_1{}^2}$$

and

$$k_2 = 1 - \frac{1 - \frac{3}{4} E_1 + \frac{1}{5} E_1^2}{\frac{3}{2} - E_1 + \frac{1}{4} E_1^2} \quad \ldots\ldots\ldots\ldots \quad 25$$

Equation 25 is plotted in Figure 7 which shows the variation to be from 0.33 for $E_1 = 0$, to 0.4 for $E_1 = 1.0$.

10. Calculation of Stresses

Summarizing the equations applying to the cracked section;

$$F = \alpha E - (2\alpha - 3)E^2 + (\alpha - 2)E^3 \quad \ldots\ldots\ldots\ldots \quad 11$$

$$e_{cs} = e_s - (e_{se} + e_{ce}) \quad \ldots\ldots\ldots\ldots \quad 15$$

$$\frac{e_{cs}}{e_{cu}} = \frac{1-k}{k} E_1 \quad \ldots\ldots\ldots\ldots \quad 16$$

$$\frac{C}{b.d.f_c'} = k \left[\frac{1}{4} E_1^3 - E_1^2 + \frac{12}{8} E_1 \right] \quad \ldots\ldots\ldots\ldots \quad 24$$

$$M = C.d\ (1 - k_2 k) = f_s.A_s\ d\ (1 - k_2 k) \quad \ldots\ldots\ldots\ldots \quad 25$$

$$k_2 = 1 - \frac{1 - \frac{3}{4} E_1 + \frac{1}{5} E_1^2}{\frac{3}{2} - E_1 + \frac{1}{4} E_1^2} \quad \ldots\ldots\ldots\ldots \quad 26$$

Equations (16) and (24), which contain as unknowns $\frac{e_{cs}}{e_{cu}}$, k, E_1 and $\frac{C}{b.d.f_c'}$, have been used to construct the intercept chart in Figure 7.

In obtaining the separate points on the stress moment curve, it will be convenient to first assume a steel stress, calculate the corresponding values of k, E, f_c and then the applied moment.

The procedure is as follows:

(a) choose values of e_s and f_s from a point on the stress-strain curve of the steel and find e_{cs} as

$$e_{cs} = e_s - (e_{se} + e_{ce})$$

and thence $\frac{e_{cs}}{e_{cu}}$, remembering that $(e_{se} + e_{ce})$ is a known quantity for a given beam which depends only on the initial prestressing force and the losses. $e_{cu} = 0.003$.

(b) Calculate $C = T = f_s A_s$ and hence $\frac{C}{b.d.f_c'}$

(c) Enter the intercept chart shown in Figure 7 using the values of $\frac{e_{cs}}{e_{cu}}$ and $\frac{C}{b.d.f_c'}$ given in steps (a) and (b), hence find k and E_1 .

(d) Using the concrete stress strain relation Equation 11 and the value of E_1 from step (c), find F_1 and f_{c1}.

(e) Calculate the moment M as

$$M = f_s A_s d(1-k_2 k)$$

where k_2 can be read off Figure 7 for the corresponding value of E_1.

Thus the steel and concrete stresses f_s and f_{c1}, given in steps (a) and (d), correspond to the moment M given in step (e).

11. Ultimate Strength

The ultimate flexural moment for the section is obtained from the equations in section 10 by placing F = 1.0 and E = 1.0.

$$e_s = e_{se} + e_{ce} + e_{cu} \cdot \frac{1-k}{k} \quad \ldots\ldots\ldots\ldots \quad 27$$

$$k = \frac{f_s A_s}{0.75\ b.d.f_c'} \quad \ldots \quad 28$$

$$M = f_s.A_s.d(1-k_2k) \quad \ldots \quad 29$$

$$k_2 = 0.4 \quad \ldots \quad 30$$

The calculation is carried out by choosing a trial value of f_s, calculating k with equation 28 and hence e_s with equation 27. The correct value of f_s has been chosen when the value of e_s, obgained from equation 27, agrees with the value obtained from the steel stress strain curve.

SAMPLE CALCULATION

12. Calculation of the Stress-Moment Relations for a Prestressed Beam

To illustrate the use of the equations, the stress-moment relations are now obtained for a prestressed beam. The details of the section, which are given below, refer to test beam A8 in the series of tests described in Progress Report 18. A full description of the manufacture and testing of the beam is given in that report.

The relevent data are,

$b = 8$ in

$h = 18$ in

$d = 13$ in

$A_s = 0.653$ in

$f'_c = 6260$ lb/sq. in

$F_i = 96.33$ kips, $f_{si} = \dfrac{96.33}{0.653} = 147.5$ kips/in.^{2}

$F_o = 92.47$ kips, $f_{so} = \dfrac{92.47}{0.653} = 141.3$ kips/in.^{2}

$F = 85.73$ kips, $f_{se} = \dfrac{85.73}{0.653} = 131.2$ kips/in.^{2}

$e_{se} + e_{ce} = 0.0060$

Ultimate Flexural Moment

Try f_s = 250 kips/sq. in. $e_s = 0.0127$ (Figure 8)

$k = 0.335$ (Equation 28)

$e_s = 0.0126$ (Equation 27)

This is sufficiently close to 0.0127

$$M = 250{,}000 \times 0.653 \times 13 \times (1 - 0.4 \times 0.335) \quad \text{(Equation 29)}$$

$$= 1.835 \times 10^6 \text{ in. lb.}$$

(Observed ultimate moment = 1.810×10^6 in. lb.)

Uncracked Section

$$\bar{x} = \frac{144 \times 4}{144 + (5-1)0.653} = 3.93 \text{ in} \quad \text{(Equation 6)}$$

$$I = \frac{8 \times 18^3}{12} + 8 \times 18(4-3.93)^2 + (5-1)0.653 \times 3.93^2$$

(Equation 7)

$$I = 3940 \text{ in}^4$$

(a) Zero Moment

F = 85.73 kips, f_{se} = 131.2 kips/sq in.

$$f_c^t = + \frac{85.73}{144}\left(1 - \frac{6 \times 4}{18}\right) \quad \text{(Equation 8)}$$

$= -199$ lb/sq in (negative sign means tension)

$$f_c^b = + \frac{F}{A_c} (1 + \frac{6e}{h}) \qquad \text{(Equation 8)}$$

$$= + 1390 \text{ lb/sq in}$$

(b) Cracking Moment

Taking $f_t' = - \frac{1}{10} f_c'$

$$= - 626 \text{ lb/sq in}$$

and substituting values in equation 8

$$- 626 = \frac{85.730}{144} (1 + \frac{24}{18}) - \frac{M}{3938} (9 - 4 + 3.93)$$

$$M = 879{,}000 \text{ in lb}$$

(Observed cracking moment = 863,000 in lb)

$$f_s = \frac{F}{A_s} + n. \frac{M}{I} x \qquad \text{(Equation 9)}$$

$$= 137{,}000 \text{ lb./sq in.}$$

$$f_c^t = - 199 + \frac{879{,}000}{3938} (9 + 4 - 3.93) \qquad \text{(Equation 8)}$$

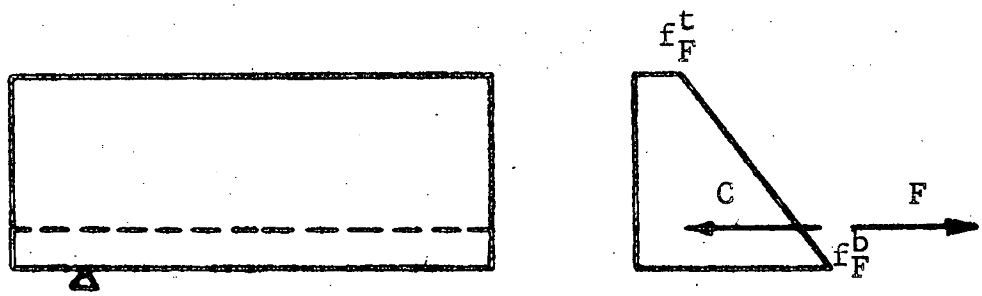
$$= 1840 \text{ lb/sq. in.}$$

Cracked Section

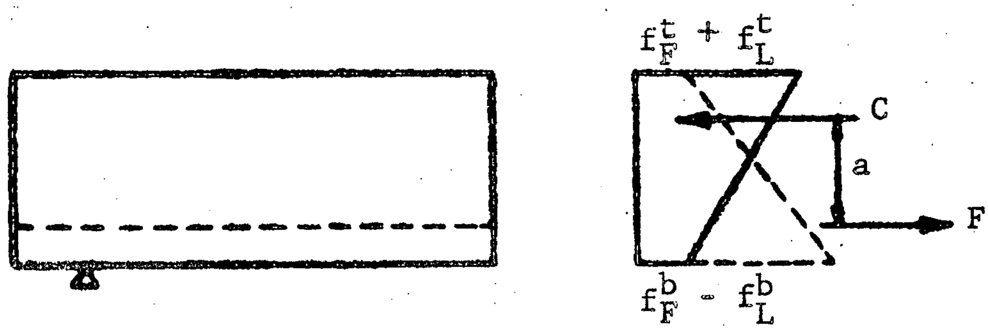
The calculations for the cracked section have been carried out in Tabular form on the following page. The procedure used is described on pages 20 and 21.

CALCULATIONS FOR CRACKED SECTION

f_s kips/in^2	$\frac{f_s}{f_u}$	f_sA_s	e_s	e_{cs}	$\frac{e_{cs}}{e_{cu}}$	$\frac{C}{b.d.f'_c}$	E_1	k	k_2	$\frac{f_c}{f'_c}$	M kip-in.	$\frac{M}{M_u}$
160	0.604	104.5	0.0064	0.0004	0.133	0.161	0.18	0.70	0.34	0.45	1 035	0.563
180	.680	117.5	.0069	.0009	.300	.181	.30	.50	.35	.66	1 260	.685
200	.756	130.6	.0076	.0016	.516	.201	.41	.44	.36	.80	1 430	.777
210	.794	137.1	.0080	.0020	.666	.211	.48	.42	.36	.86	1 512	.822
220	.832	143.7	.0085	.0025	.833	.221	.54	.39	.37	.90	1 598	.868
230	.869	150.2	.0093	.0033	1.083	.231	.64	.37	.37	.95	1 682	.914
240	.905	156.7	.0101	.0041	1.377	.241	.74	.36	.38	.98	1 750	.950



(a) Stresses due to effective prestressing force, F.



(b) stresses when moment M_L, less than the cracking moment, is applied.

Figure 1: UNCRACKED SECTION

(Approximate Solution for Concrete Stresses)

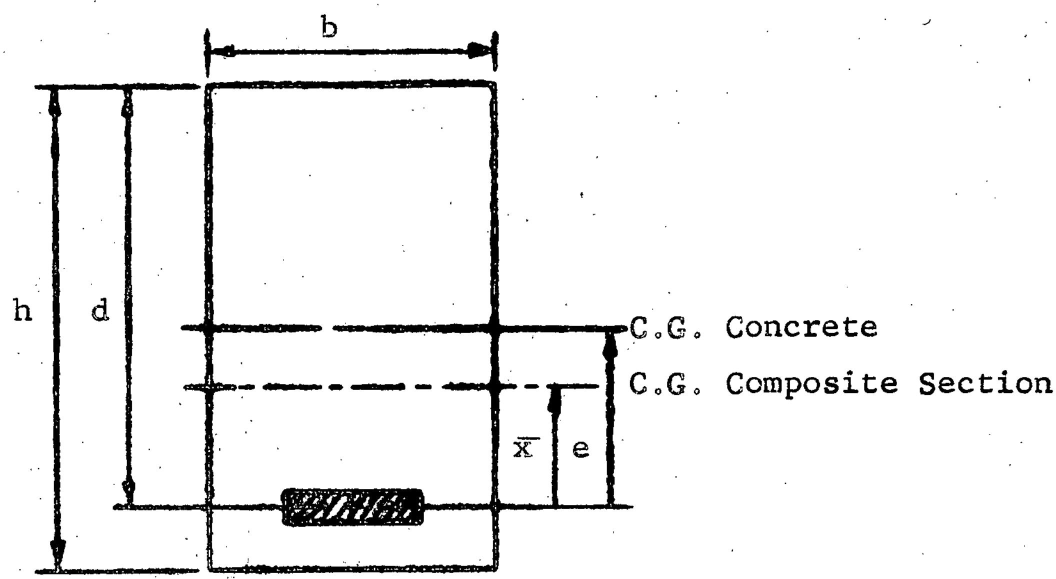


Figure 2: UNCRACKED SECTION

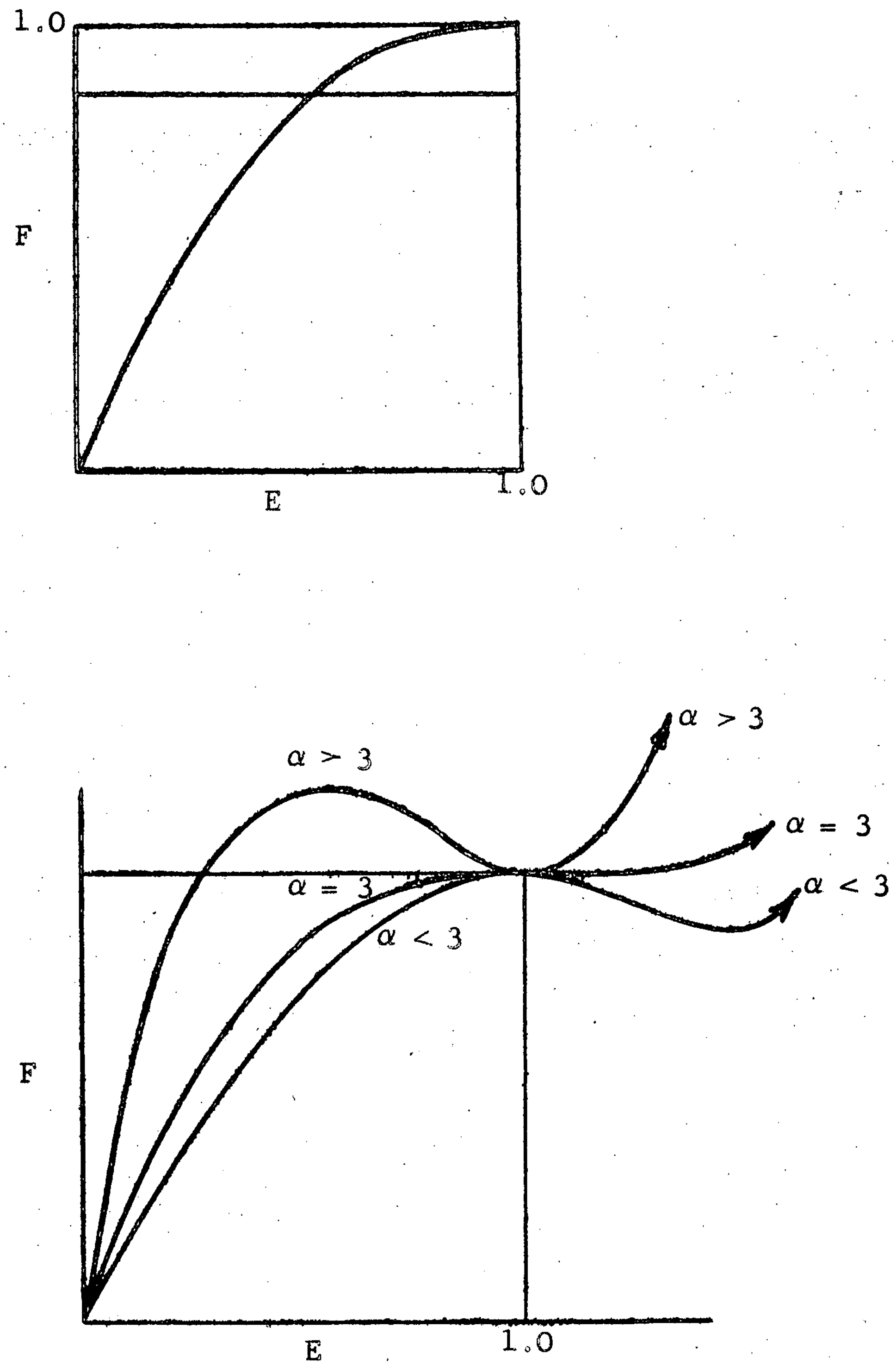
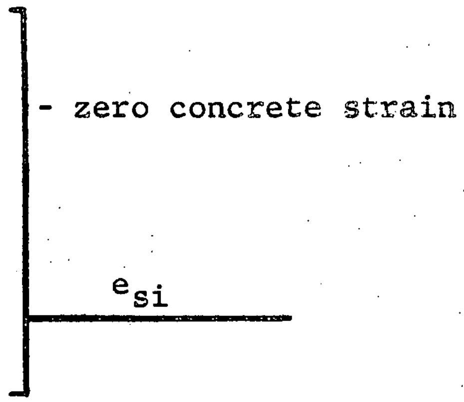
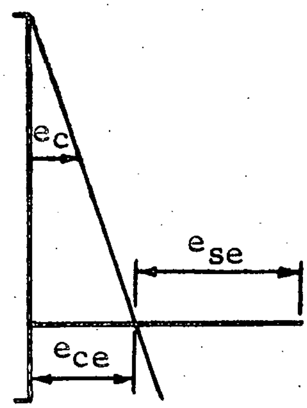


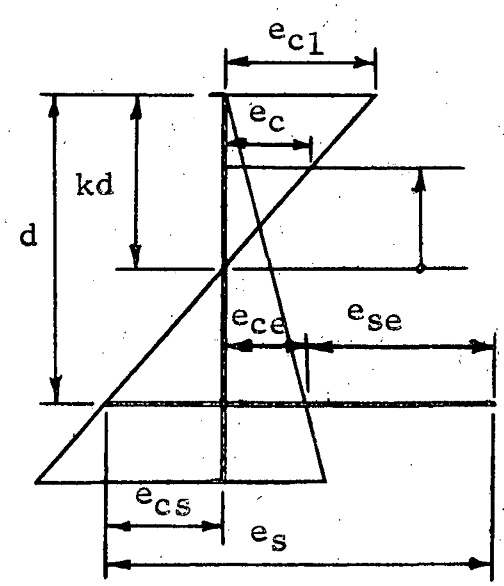
Figure 3: CUBIC PARABOLA



(a) Strain prior to stress transfer



(b) Strain due to effective prestress



(c) Total strain at moment, M.

Figure 4: CRACKED SECTION

Strain Distribution at Various Stages of Loading

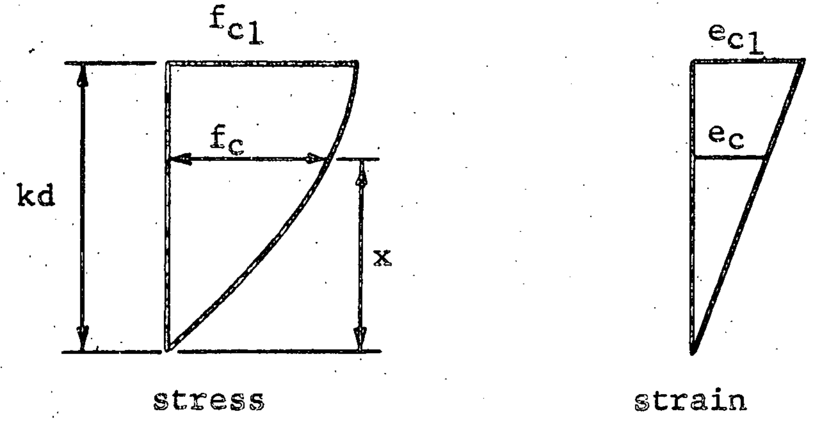


Figure 5: CRACKED SECTION

Distribution of stress and strain above the Neutral Axis

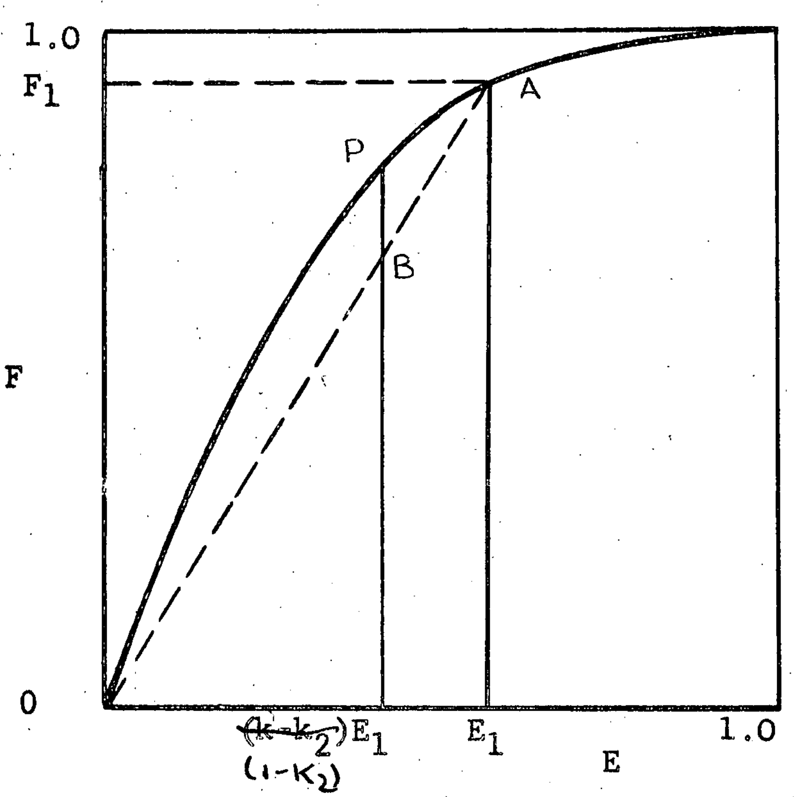


Figure 6: CENTER OF GRAVITY OF STRESS BLOCK

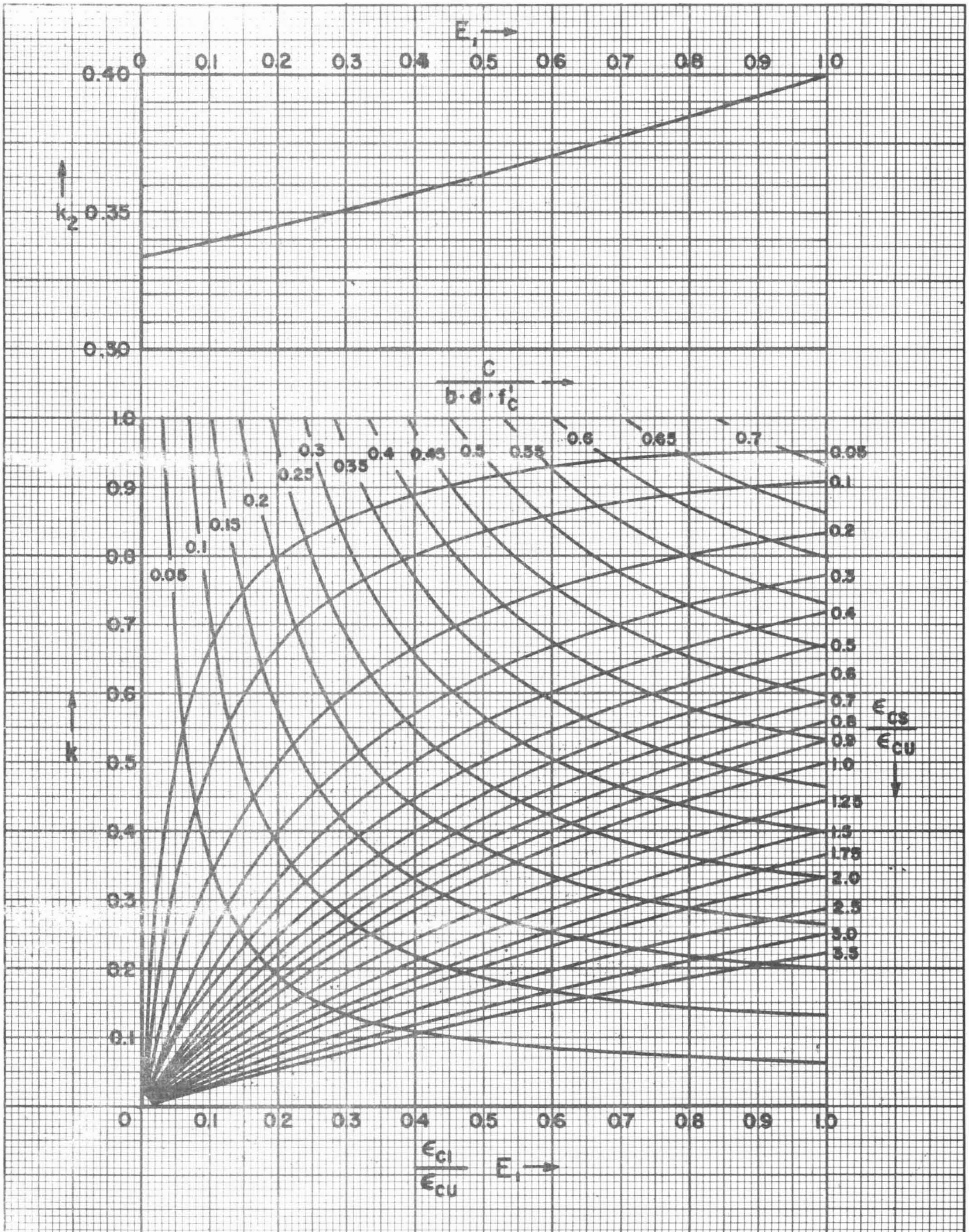


Figure 7: INTERCEPT CHART

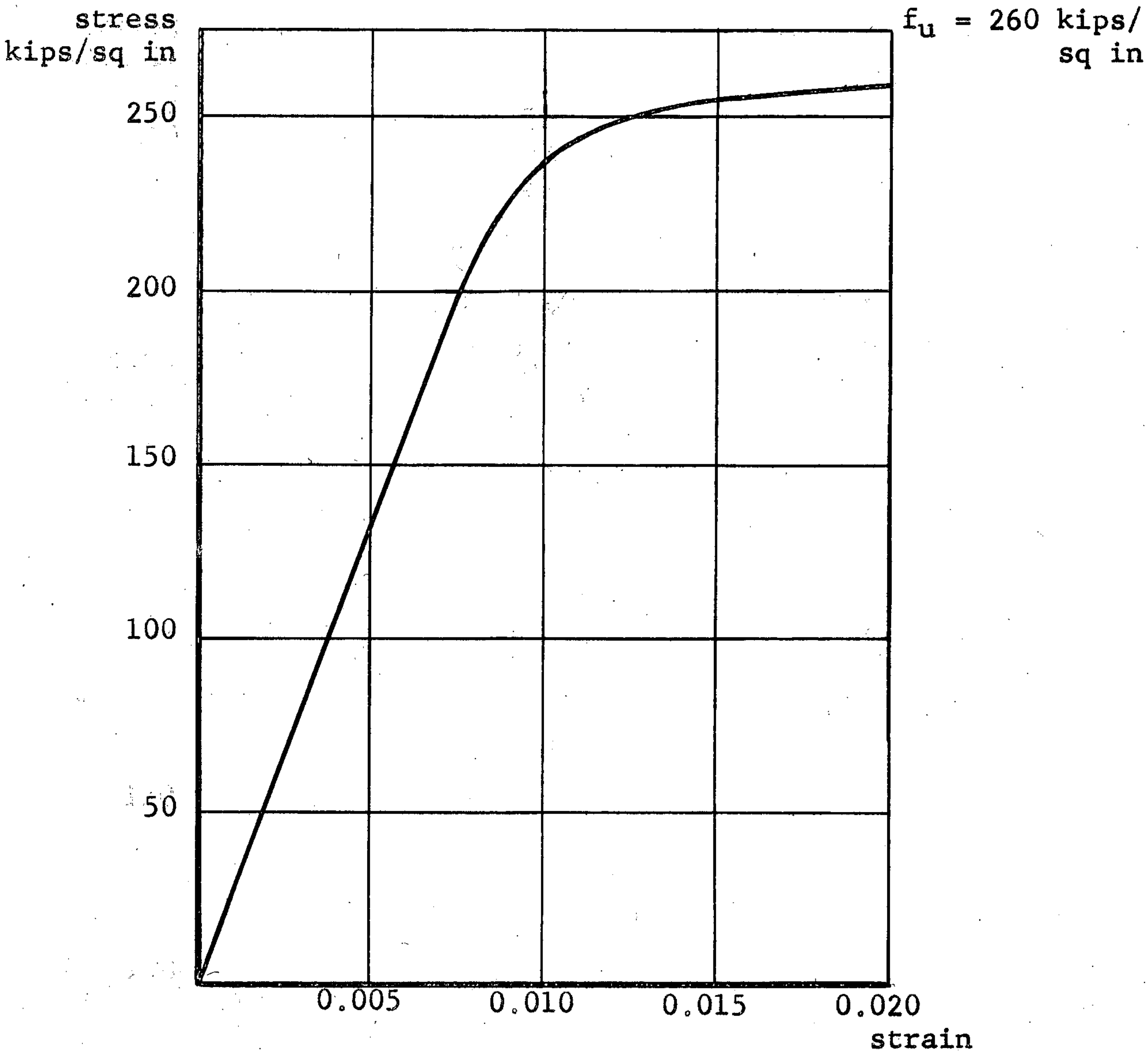
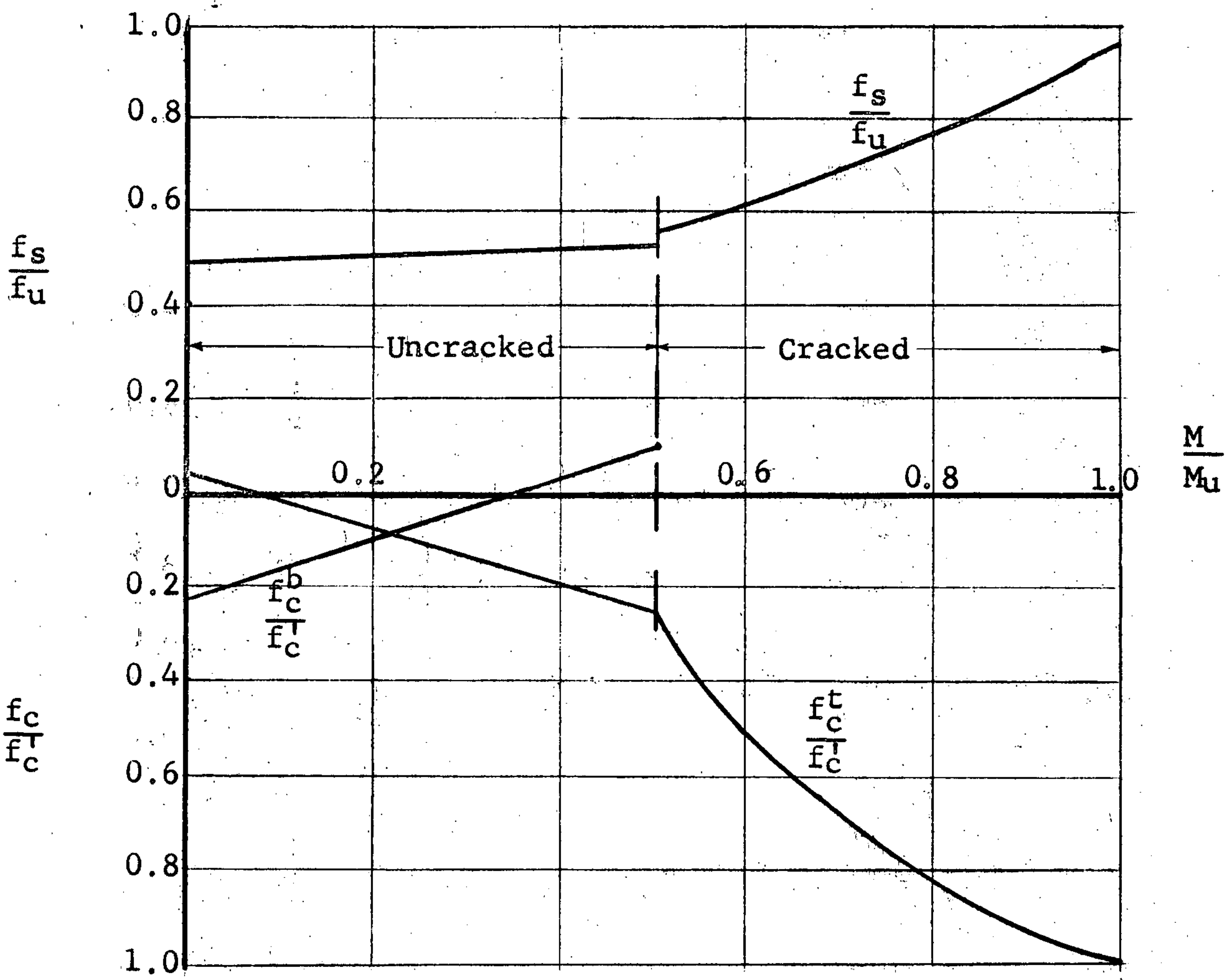


Figure 8: STRESS-STRAIN CURVE FOR 7/16" DIAMETER HIGH TENSILE STEEL STRAND



f_s = steel stress

f_c^b = stress in concrete bottom fiber

f_c^t = stress in concrete top fiber

Figure 9: STRESS MOMENT RELATIONS