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THE EFFECTIVE WIDTH OF CIRCULAR

CYLINDRICAL SHELLS REINFORCED BY RIBS

(A Theoretical Study)

by

Bruno Thürlimann

An Abridgment
of a Dissertation

presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Doctor of Philosophy

Lehigh University

1950

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Abstract

Formulas are derived for the effective width of circular cylindrical shells reinforced by ribs in the circumferential direction. In cases where the shell can be considered to extend to infinity the effective width depends on two parameters, $\sqrt{ah}$ and $\lambda = n \sqrt{\dfrac{h}{a}}$. The first parameter is a function of the radius "a" and the thickness h of the shell, the second contains in addition the factor n representing the influence of the stress distribution in circumferential direction.

Certain simplifications, the influence of which was investigated and found to be insignificant, are introduced in order to present a diagram for the effective width in different cases.

For the limiting case where the radius "a" of the shell increases to infinity the correspondence to the effective width of a T-Beam with a straight axis is established.

Introduction

The problem of the effective width of T-Beams with a straight axis (Fig. 1) was extensively investigated during the past 30 years (see Ref. (1) to (5))*. The

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*For list of references see p. 28

actual stress distribution in the flange is replaced by an imaginary constant stress distribution over the effective width. Taking instead of the actual flange a flange of width equal to the effective width, the ordinary beam theory (cross sections remain plane) can be used to calculate the fiber stresses and the deflection of the rib. The advantages of this procedure are quite obvious.

The case of a curved T-Beam was taken up by U. Finsterwalder (6), H. Bleich (7) and Th. v. Karman (8). In Ref. (6) the general unsymmetrical case is treated with certain simplifications and the solution is not developed for practical applications. H. Bleich investigates the bending of curved knees of T- and H-sections. In Ref. (8) a formula* for the effective width is given which does not coincide with the results of this abridgment.

The application of cylindrical shells stiffened by ribs in circumferential direction (Fig. 2) has entered many different fields, including shell arch roofs, airplane fuselages, pressure vessels, submarines, hot metal ladles, etc. The analysis of such structures

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*v. Karman gives the formula $\frac{b}{2} = 0.54 \cdot \sqrt{ah}$ without any derivation. In the present abridgment, it is shown that the numerical coefficient is not a constant.

is very involved, and there seems to be a specific need for establishing the effective width of cylindrical shells stiffened by ribs in order to simplify their analysis.

This paper is an abridgment of a doctoral dissertation[*] in which a comprehensive study of this problem was made. Herein the most important results, including some derivations, are presented.

I. Definition of the Effective Width:

Consider a circular cylinder of radius "a" and thickness h, stiffened by a rib and subjected to arbitrary radial loads in the plane of the rib (Fig. 2). The distribution of the direct forces $N_\varphi = \sigma_\varphi h$[**] of the shell in circumferential direction may be as shown in Fig. 2, $(N_\varphi)_{x=0}$ being the direct force along the rib. The width b of a circular ring of equal radius "a" and thickness h under the same loads shall be determined, under the assumption of a constant stress distribution $(N_\varphi)_{x=0}$ over the width of the ring, so as to make the rib stresses of both structures identical.

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* Bruno Thürlimann: "The Effective Width of Circular Cylindrical Shells Reinforced by Ribs", PhD-Dissertation, Lehigh University, 1950, Ref. (12).
** For list of notations see p. 25 , also Fig. 3.

The action of the rib on the shell may be found by taking equilibrium for a cut φ = constant:

$$S = \int N_\varphi \cdot dx \tag{1}$$

where S is the total force of the rib acting on the shell. The integral is taken over the entire length of the shell. The ring must resist to the same action of the rib (Fig. 2):

$$S = b(N_\varphi)_{x=0}$$

Hence the width b of the ring is:

$$b = \frac{S}{(N_\varphi)_{x=0}} \tag{2}$$

Physically, S may be thought of as being the force in a string stretched around the cylinder. If S is constant, the action of the string consists in a constant radial line load around the cylinder. In case S varies as a function of φ, tangential shear forces are acting on the shell in addition. From Eq. (1) it follows that S is positive as a compressive force in the string, N_φ being positive as a tensile force.

In summary, the effective width b of a cylinder is found by stretching around the cylinder a string under a string force S, calculating the direct force $(N_\varphi)_{x=0}$ directly under the string and applying Eq. (2). The imaginary T-section, composed of the rib as web and the effective width as flange, gives rib stresses equal to the one of the actual structure.

II. Calculation of the Effective Width:

In this chapter a solution of the differential
equations of cylindrical shells, acted upon by boundary
forces is presented. Then the effective width is
calculated by the above described procedure.

1. Circular Cylindrical Shell Under Boundary Forces:

In general, 10 forces and moments are acting on an
infinitesimal shell element $dx \cdot ad\varphi$ of a circular
cylindrical shell (Fig. 3). The displacements u, v
and w in axial, circumferential and radial direction
respectively, are shown in the same Fig. The general
solution of the differential equations for arbitrary
conditions at the boundaries x = constant is very
complicated.* Miesel derived an approximate solution
sufficiently close for any practical application.**
Assume a variation of the stresses in circumferential
direction in form of the function cos $n\varphi$ (n being
the number of complete waves) and furthermore, the
second boundary $x = l$ sufficiently far removed, to
be of no influence on the boundary x=o. Then, any
unknown quantity H, where H stands for a force, moment
or displacement, has the form:

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* See Ref. (9), p. 123 and Ref. (10), p. 31.
** Ref. (10), p. 48. Miesel's notations were changed
to conform with the ones adopted in this paper.

$$H = \bar{H}Ce^{-\mu_2\xi}\left[k_1\sin(\mu_1\xi + \psi)+k_2\cos(\mu_1\xi + \psi)\right]\cos n\varphi \quad (3)$$

In the following Table A the most important forces, moments and displacements are given in this form. The 2 constants of integration are C and ψ. μ_1 and μ_2 are coefficients depending on the shell dimensions, Poisson's ratio ν and the number n of the harmonic under consideration. $\bar{H}$, k_1 and k_2 are constants depending on the quantity H. The string force S as defined by Eq. (1) is:

$$S_n(x) = \int_x^\infty N_\varphi \, dx \quad (4)$$

By replacing N_φ by its value from the Table A, $S_n(x)$ becomes:

$$S_n(x) = EC\frac{2h}{k}\int_x^\infty e^{-\mu_2\xi}\left[\mu_2\left(\mu_1^2 - \frac{1}{4}\lambda^4(1-\frac{1}{n^2})\ (2+\nu)\right)\sin(\mu_1\xi+\psi)\right.$$
$$\left.+\ \mu_1\left(\mu_2^2 + \frac{1}{4}\lambda^4(1-\frac{1}{n^2})\ (2+\nu)\right)\ \cos(\mu_1\xi+\psi)\right]\cos n\varphi \, dx$$

and performing the integration:

$$S_n(x) = E\ \frac{2ha}{k}\ Ce^{-\mu_2\xi}\left[-\frac{1}{4}\lambda^4(1-\frac{1}{n^2})(2+\nu)\ \sin\ (\mu_1\xi+\psi)\right.$$
$$\left.+\ \mu_1\mu_2\ \cos(\mu_1\xi +\psi)\right]\ \cos n\varphi \quad (5)$$

To the tangential shear force in circumferential direction $N_{x\varphi}$ (see Fig. 3), the twisting moment $M_{x\varphi}$ contributes a component $\frac{1}{a}M_{x\varphi}$. Expressed as a function of the string force $S_n(x)$, the total tangential shear force T becomes:

TABLE A

a   radius of cylinder
h   thickness of cylinder
ν Poisson's ratio for the material
n   the harmonic under consideration (number of complete
        cosine-waves of the stresses in circumferential
        direction)
C, ψ constants of integration

$$\xi = \frac{x}{a}$$

$$\mu_1 = \sqrt[4]{3}\sqrt{\frac{a}{h}}\sqrt{\sqrt{1 + \tfrac{1}{2}\lambda^4\left(1 - \tfrac{1}{n^2}\right)} - \frac{1}{\sqrt{3}}\lambda^2}$$

$$\lambda = n\sqrt{\frac{h}{a}}$$

$$k = \frac{12a^2}{h^2}$$

$$\mu_2 = \sqrt[4]{3}\sqrt{\frac{a}{h}}\sqrt{\sqrt{1 + \tfrac{1}{2}\lambda^4\left(1 - \tfrac{1}{n^2}\right)} + \frac{1}{\sqrt{3}}\lambda^2}$$

$$k' = k\left[1 + \tfrac{1}{2}\lambda^4\left(1 - \tfrac{1}{n^2}\right)\right]$$

H	H	k_1	k_2
$E\,\dfrac{\partial w}{\partial x}$	$-E$	1	0
N_x	$Eh\left(\dfrac{1}{k} + \dfrac{n^2-1}{k'}\right)$	μ_2	$-\mu_1$
N_φ	$E\,\dfrac{2h}{k}$	$\mu_2\left[\mu_1^2 - \tfrac{1}{4}\lambda^4\left(1 - \tfrac{1}{n^2}\right)(2+\nu)\right]$	$\mu_1\left[\mu_2^2 + \tfrac{1}{4}\lambda^4\left(1 - \tfrac{1}{n^2}\right)(2+\nu)\right]$
M_x	$E\,\dfrac{ah}{k}$	$\mu_2\left(1 - \dfrac{n^2\nu}{\sqrt{k'}}\right)$	$-\mu_1\left(1 + \dfrac{n^2\nu}{\sqrt{k'}}\right)$
M_φ	$E\,\dfrac{ah}{k}$	$\mu_2\left(\nu - \dfrac{n^2}{\sqrt{k'}}\right)$	$-\mu_1\left(\nu + \dfrac{n^2}{\sqrt{k'}}\right)$
$S_n(x)$	$E\,\dfrac{2ah}{k}$	$-\tfrac{1}{4}\lambda^4\left(1 - \tfrac{1}{n^2}\right)(2+\nu)$	$\mu_1 \cdot \mu_2$

General Case:

$$H = ECe^{-\mu_2\xi}\left[k_1 \cdot \sin(\mu_1\xi + \psi) + k_2 \cdot \cos(\mu_1\xi + \psi)\right]\cos n\varphi$$

Special Case:

$$M_x = E\,\frac{ah}{k}\,Ce^{-\mu_2\xi}\left[\mu_2\left(1 - \frac{n^2\nu}{\sqrt{k'}}\right)\sin(\mu_1\xi + \psi) - \mu_1 \cdot \left(1 + \frac{n^2\nu}{\sqrt{k'}}\right)\cos(\mu_1\xi + \psi)\right]\cos n\varphi$$

$$T = N_{x\varphi} + \frac{1}{a} M_{x\varphi} = \frac{1}{a} \frac{S_n(x)}{\partial \varphi} \tag{6}$$

This shear force T must be used for designing the diagonal steel in reinforced concrete shells adjacent to stiffeners.

2. Effective Width of an Infinitely Long Cylinder
(Poisson's ratio $\nu = o$)

The following derivations are greatly simplified if Poisson's ratio ν is taken equal to zero. The influence of this simplification is insignificant for concrete. In the case of steel ($\nu = 0.3$) the error involved amounts to about 2.5%.

In the middle part of an infinitely long cylinder a string force S is applied (Fig. 4). S can have any variation. It is always possible to present it in form of a Fourier series. The effective width will be derived for the nth term of this series. Consider the unit string force:

$$S = S_n \cos n\varphi \qquad = 1 \cos n\varphi \tag{7}$$

Each of the two parts on both sides of the string will carry half of this string force. The continuity for the 2 parts requires that the slope in x-direction at x = 0 is zero. By using Table A the 2 conditions take the form:

$$x = 0: \quad S_n(0) = E \frac{2ah}{k} C\mu_1\mu_2 \cos\psi \cos n\varphi = \frac{1}{2} \cos n\varphi$$

$$E \frac{\partial w}{\partial x} = - EC \sin\psi \cos n\varphi = 0$$

The second of these equations requires $\psi = 0$ and the other constant of integration becomes:

$$\left. \begin{aligned} C &= \frac{1}{4} \frac{k}{Eah\mu_1\mu_2} \\ \psi &= 0 \end{aligned} \right\} \quad (8)$$

N_φ is calculated by replacing in Table A the two constants C and ψ by the expressions (8):

$$(N_\varphi)_{x=0} = \frac{1}{2a\mu_2}\left[\mu_2^2 + \frac{1}{2}\lambda^4 \cdot (1-\frac{1}{n^2})\right] \cos n\varphi \quad (9)$$

The effective width is the ratio of the applied string force S to the direct Force N_φ at x = 0 (Eq. (2)):

$$b = \frac{S}{(N_\varphi)_{x=0}} = \frac{2a}{\mu_2\left[1 + \dfrac{\lambda^4(1-\frac{1}{n^2})}{2\mu_2^2}\right]}$$

If μ_2 is replaced by its value given in Table A,

$$b = 1.52 \sqrt{ah} \frac{1}{\sqrt{\sqrt{1+\frac{1}{2}\lambda^4(1-\frac{1}{n^2})}+\frac{1}{\sqrt{3}}\lambda^2}} \cdot \frac{1}{1+\dfrac{\lambda^4(1-\frac{1}{n^2})}{2\mu_2^2}}$$

$$(10)$$

In case of axial symmetry the number n of the waves of the string force S around the cylinder is zero (string force S is constant). $\lambda = n\sqrt{\frac{h}{a}}$ being proportional to n

will be zero too, and Eq. (10) reduces to:

$$\lambda = 0: \qquad b = 1.52 \sqrt{ah} \qquad (11)$$

Eq. (10) is essentially a function of the two para-
meters $\sqrt{ah}$ and λ . It can be shown that in practical
applications ($\frac{h}{a} < \frac{1}{5}$; $\frac{nh}{a} < \frac{1}{2}$) the terms $\frac{1}{n^2}$ and $\lambda^4 (1 - \frac{1}{n^2})$
may be safely neglected.* Then the effective width
becomes:

$$b = 1.52 \sqrt{ah} \quad \frac{1}{\sqrt{\sqrt{1 + \frac{1}{2}\lambda^4} + \frac{1}{\sqrt{3}}\lambda^2}} \qquad (12)$$

which is an expression in terms of the two parameters
$\sqrt{ah}$ and λ only. For the purpose of tabulating Eq. (12)
the following form is chosen:

$$b = K \cdot \sqrt{ah} \qquad (12a)$$

$$K = \frac{1.52}{\sqrt{\sqrt{1 + \frac{1}{2}\lambda^4} + \frac{1}{\sqrt{3}}\lambda^2}} \qquad (12b)$$

Fig. 5 is a graph representing the coefficient K
for the present $(\beta l = \infty)$ and several other cases as a
function of λ . Note the rapid decrease of the
effective width by increasing λ .

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* See Ref. (12), p. 37 for an investigation of this
simplification.

The cross bending stress σ_x under the rib will be calculated. The bending moment M_x at $x = 0$ is: (Table A and Eq. (8))

$$(M_x)_{x=0} = -\frac{1}{4\mu_2} \cos n\varphi \qquad (13)$$

Computing the bending stress σ_x and the direct stress σ_φ (Eq. (9)) the ratio of the two stresses becomes:

$$\left|\frac{\sigma_x}{\sigma_\varphi}\right|_{x=0} = \frac{6M_x}{hN_\varphi} = \frac{3a}{h\left[\mu_2^2 + \frac{1}{2}\lambda^4(1-\frac{1}{n^2})\right]}$$

$$\left|\frac{\sigma_x}{\sigma_\varphi}\right|_{x=0} = \frac{1.7321}{\left(\sqrt{1+\frac{1}{2}\lambda^4(1-\frac{1}{n^2})} + \frac{1}{\sqrt{3}}\lambda^2\right)} \cdot \frac{1}{1 + \frac{\lambda^4(1-\frac{1}{n^2})}{2\mu_2^2}}$$

Using the same simplifications as for Eq. (10)

$$\left|\frac{\sigma_x}{\sigma_\varphi}\right|_{x=0} = \frac{1.7321}{\sqrt{1+\frac{1}{2}\lambda^4} + \frac{1}{\sqrt{3}}\lambda^2} \qquad (14)$$

The use of Eq. (14) is quite obvious. It gives with a minimum of calculation the maximum cross bending stress σ_x if the the direct stress σ_φ is known. Note that

for axially symmetrical loads ($n = \lambda = 0$), the maximum cross bending stress is 1.73 times the stress σ_φ .

Expressions for the direct force N_φ and the bending moment M_x at a distance x from an applied string force $S = S_n \cos n\varphi$ can be calculated. In Ref. (12) tables of these values as function of the paramenters βx and λ are computed. By means of these tables N_φ and M_x in the shell are readily determined, once the string force S is known.

3. Effective Width of a Semi-infinite Cylinder:

A unit string force $S=1 \cos n\varphi$, making n complete cosine-waves around the circumference, is applied to the end of a semi-infinite cylinder. The boundary conditions for the free end x=o are

$$x=o: \qquad S_n(0) = 1 \cos n\varphi$$
$$M_x = 0 \qquad\qquad \Big\} \text{(15)}$$

The derivations of the effective width may be found in Ref. (12), p. 46. Eventually the following expression is arrived at:

$$b = 0.38 \; \sqrt{ah} \; \frac{1}{\sqrt{\sqrt{1+\tfrac{1}{2}\lambda^4} + \tfrac{1}{\sqrt{3}}\lambda^2}} \left[1 + \frac{\lambda^2}{3\sqrt{1+\tfrac{1}{2}\lambda^4}}\right] \text{(16)}$$

Using the form:

$$b = K \sqrt{ah} \qquad\qquad (16a)$$

where:
$$K = \frac{0.38}{\sqrt{\sqrt{1+\frac{1}{2}\lambda^4}+\frac{1}{\sqrt{3}}\lambda^2}}\left[1+\frac{\lambda^2}{3\sqrt{1+\frac{1}{2}\lambda^4}}\right] \qquad (16b)$$

values of K for different values of λ are given in the graph of Fig. 5 (case $\beta l = 0$). The effective width is readily determined by use of this diagram.

For axial symmetry (n = λ = o), Eq. (16) reduces to:

$$b = 0.38 \sqrt{ah} \qquad\qquad (17)$$

The ratio of the maximum cross bending stress σ_x (occurs at a certain distance from the free end x=o) to the direct stress σ_φ at x=o is for the case of axial symmetry:

$$\lambda = 0: \qquad \left|\frac{\sigma_x}{\sigma_\varphi}\right|_{max} = 0.56$$

With increasing λ this ratio decreases rapidly ($\lambda = 1$; $\left|\frac{\sigma_x}{\sigma_\varphi}\right| = 0.30$)

Fig. 5 allows the calculation of the effective width for a cylinder extending on one side of the rib to infinity* and having a finite overhang l on the other

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* A boundary at a finite distance d may be considered to be at infinity if $\beta d > 2.4$ (See Ref. (12), p. 25).

side $(0 \leq l \leq \infty)$.

In Ref. (12) the derivation of new solutions by superposition of the two cases given herein, (infinitely long and semi-infinite cylinder) is shown. In addition the effective width for some special cases, under condition of axial symmetry, is calculated (effective width of the curved flange of a knee of H-section, etc.)

4. Discussion of the Equations for the Effective Width:

A few remarks may be made about the physical meaning of the derived equations. In case of axial symmetry no direct shear forces $N_{x\varphi}$, $N_{\varphi x}$ and twisting moments $M_{x\varphi}$, $M_{\varphi x}$ are present in the shell. The effective width b, given by Eq. (11) or (17) is solely a function of $\sqrt{ah}$. b depends on the ability of the shell to escape radially (displacement w). The factor $\sqrt{ah}$ describes this property. In the general case, the parameter $\lambda = n\sqrt{\dfrac{h}{a}}$ enters the expressions for the effective width (Eq. (12) and (16)). λ is proportional to the tangential shear forces T transmitted by the string (or in the actual case by the rib) to the shell (compare Eq. (6) and (7)). It takes into account the lag of the direct shear forces $N_{x\varphi}$ and $N_{\varphi x}$ on the effective width.

In summary, two effects govern the effective width of a cylindrical shell, the escaping in radial direction and

the lag of the direct shear forces. In the T-Beam
problem (flange plate is plane) the first factor
does not enter.

III. The Effective Width of Cylindrical Shells in Case the Radius of the Shell Increases to Infinity

1. **The Problem:**

The question arises as to what the effective width
of a cylindrical shell becomes if the radius "a" of the
shell approaches infinity. Obviously the axis of the
rib and the middle plane of the shell become straight and
the effective width should be identical with the
effective width of a flat plate reinforced by a rib
(T-Beam). No difficulty exists in proving that the
differential equations of the shell reduce to the
differential equation of a flat plate if the radius "a"
is increased to infinity.

Nevertheless this does not prove that the equations
for the effective width of cylindrical shells given in
the previous two chapters will check with those of the
corresponding T-Beams. For Miesel's approximate solution
(Eq. (3)) was used for the calculations and further
simplifications were introduced (p. 10) in order to
get expressions depending on two parameters λ and $\sqrt{ah}$ only.

Hence the limiting process "a"→∞ will be applied to the derived formulas for the effective width directly and these results will be compared to the corresponding equations of a T-Beam with a straight axis.

2. <u>Effective Width of a T-Beam With a Straight Axis:</u>

Solutions of this problem may be found in Ref. (1) to (5). The following two cases are presented without derivations, taken from Ref. (11), p. 112.

a) T-Beam with an infinitely wide flange:

A continuous T-Beam with a flange sufficiently large to be considered as extending to infinity is supported by equidistant supports with spans L (Fig. 6). y is the coordinate in the direction of the rib, x is taken perpendicular to it. The load acting on the rib is:

$$p = p_0 \cos \frac{\pi}{L} y$$

If Poisson's ratio ν is assumed to be zero, the effective width of the beam is

$$b = \frac{4}{3} \frac{L}{\pi} = 0.424 \ L \qquad\qquad (18)$$

A simple beam of equal span L has the same effective width if cross-beams at the supports are provided and are adequate to carry shear forces τ_{yx}.

b) Beam with an infinitely wide flange on one side:

Differing from the previous case, the beam has an infinitely wide flange on one side only (Fig. 6b). All other conditions are equal. By neglecting the torsional stiffness of the rib, the effective width is:

$$b = \frac{1}{2} \frac{L}{\pi} = 0.159 \, L \tag{19}$$

The only variable in the two equations (18) and (19) is the span L. It is quite obvious that the effective width increases to infinity if the span $L \rightarrow \infty$. This fact will be of importance in the following discussion.

3. **Effective Width of Cylindrical Shells for the Limiting Case "a" $\rightarrow \infty$:**

The string force $S = S_n \cos n\varphi$ and hence the direct stress σ_φ varies in n complete cosine-waves around the cylinder (Fig. 7). The length of one half-wave is L and the angle corresponding to this arc length L is:

$$\alpha = \frac{L}{a}$$

The number n of complete waves around the cylinder is hence:

$$n = \frac{\pi}{\alpha} = \frac{\pi a}{L}$$

And the parameter λ (Table A) as a function of the half-wave length L becomes:

$$\lambda = n\sqrt{\frac{h}{a}} = \frac{\pi a}{L}\sqrt{\frac{h}{a}} = \frac{\pi}{L}\sqrt{ah} \qquad (20)$$

Consider the radius "a" increasing to infinity. During this process the half-wave length L or the number n of the waves can be kept constant. If the latter is done the length L becomes infinitely long. This would then correspond to a T-Beam of infinite span, a case which certainly does not have any practical meaning. Therefore the length L will be kept constant.

a)   Infinitely long cylinders:

The following equation for the effective width was derived ((Eq. (12)):

$$b = 1.52\sqrt{ah}\;\frac{1}{\sqrt{\sqrt{1+\frac{1}{2}\lambda^4}+\frac{1}{\sqrt{3}}\lambda^2}}$$

Substituting $\lambda = \frac{\pi}{L}\sqrt{ah}$ and rearranging

$$b = \frac{1.52\,L}{\pi}\;\frac{1}{\sqrt{\sqrt{\frac{1}{\lambda^4}+\frac{1}{2}}+\frac{1}{\sqrt{3}}}}$$

Now, in the limit as $\lambda \to \infty$

$$(b)_{a \to \infty} = 0.427\,L \qquad (21)$$

This checks within 0.5% the value of the effective width of the corresponding T-Beam (Eq.(18)).

b)   Semi-infinite cylinder:

The effective width is given by Eq. (16):

$$b = 0.38 \sqrt{ah} \; \frac{1}{\sqrt{1 + \frac{1}{2}\lambda^4} + \frac{1}{\sqrt{3}}\lambda^2} \left[1 + \frac{\lambda^2}{3\sqrt{1 + \frac{1}{2}\lambda^4}} \right]$$

Substituting $\lambda = \frac{\pi}{L} \cdot \sqrt{ah}$ and rearranging

$$b = \frac{0.38 \, L}{\pi} \; \frac{1}{\sqrt{\sqrt{\frac{1}{\lambda^4} + \frac{1}{2}} + \frac{1}{\sqrt{3}}}} \left[1 + \frac{1}{3\sqrt{\frac{1}{\lambda^4} + \frac{1}{2}}} \right]$$

the limit "a" $\rightarrow \infty$ (and in consequence $\lambda \rightarrow \infty$) gives:

$$\underline{\underline{(b)_{a \rightarrow \infty} = 0.157 \; L}} \tag{22}$$

The difference between this value and the one of
the corresponding T-Beam (Eq. (19) is 1.4%.

c)   The case of axial symmetry:

The effective width for $\lambda = 0$ (axial symmetry)
is given by Eq. (11) and (17) respectively. Obviously,
if the radius "a" is increased to infinity, b becomes
infinitely large. But the corresponding T-Beam has a
span L = ∞ whose effective width is b = ∞ (Eq. (18)
and (19). The correspondence between the two problems is
established for this special case too. Actually, the
stability will set an upper limit for the effective
width. A short treatment of this problem may be found
in Ref. (12), p. 66.

4. Conclusions

1. It was shown that the effective width of cylindrical shells reduces to the effective width of the corresponding T-Beams with a straight axis if the radius of the shell is increased to infinity. The check is complete from the practical point of view (differences of 0.5% and 1.4%), but it is not a mathematically exact one. This is to be expected as an approximate solution (Eq. (3)) was used for the derivation of the effective width of cylindrical shells.

2. The close correspondence established between the two problems for the limiting case "a" = ∞ may be considered as a justification for the use of the approximate solution Eq. (3).

3. The derived formulas for the effective width are not limited by a certain value of the radius "a". They are based on the general principles of the theory of elasticity. The stability of the shell and of the combination of the shell and the rib gives an upper limit, a problem which exists equally for the effective width of the T-Beam.

Imaginary stress distribution

Actual stress distribution

$\frac{1}{2}b$ $\frac{1}{2}b$

Flange (flat plate)

Rib

T - Beam

Fig. 1

Cylinder

Ring

a

h

$-X$ O X

$(N_\varphi)_{x=0}$

$(N_\varphi)_{x=0}$

$\frac{1}{2}b$ $\frac{1}{2}b$
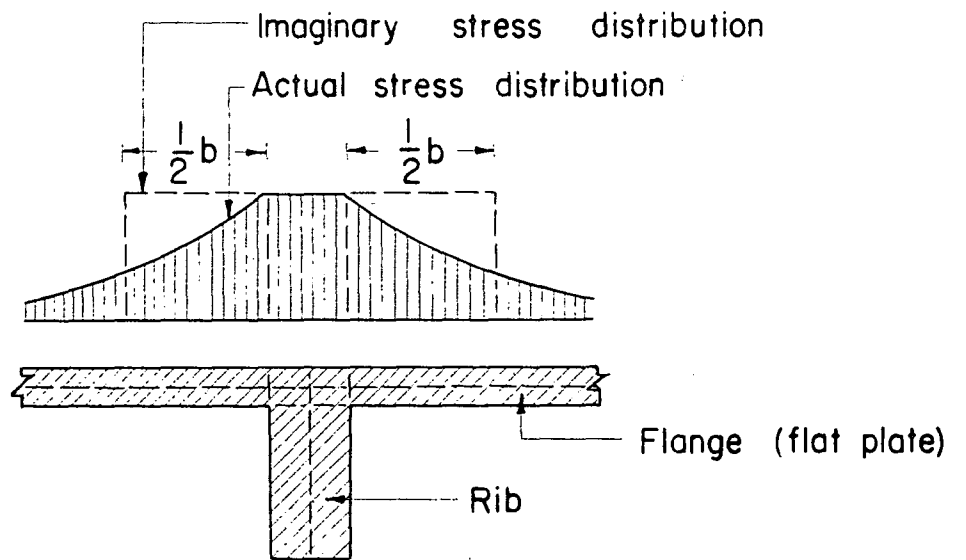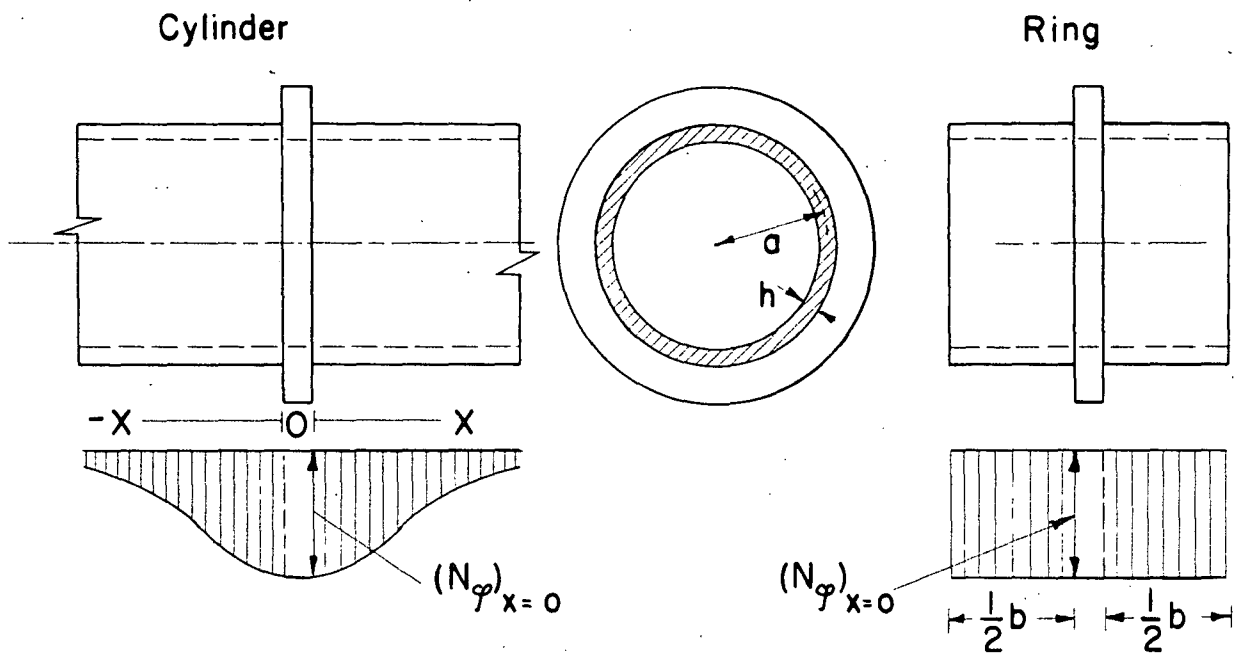
Fig. 2

CONCISTENT RIGHT-HAND
SIGN CONVENTION, in $u-v-w$
$$x-\phi-z$$
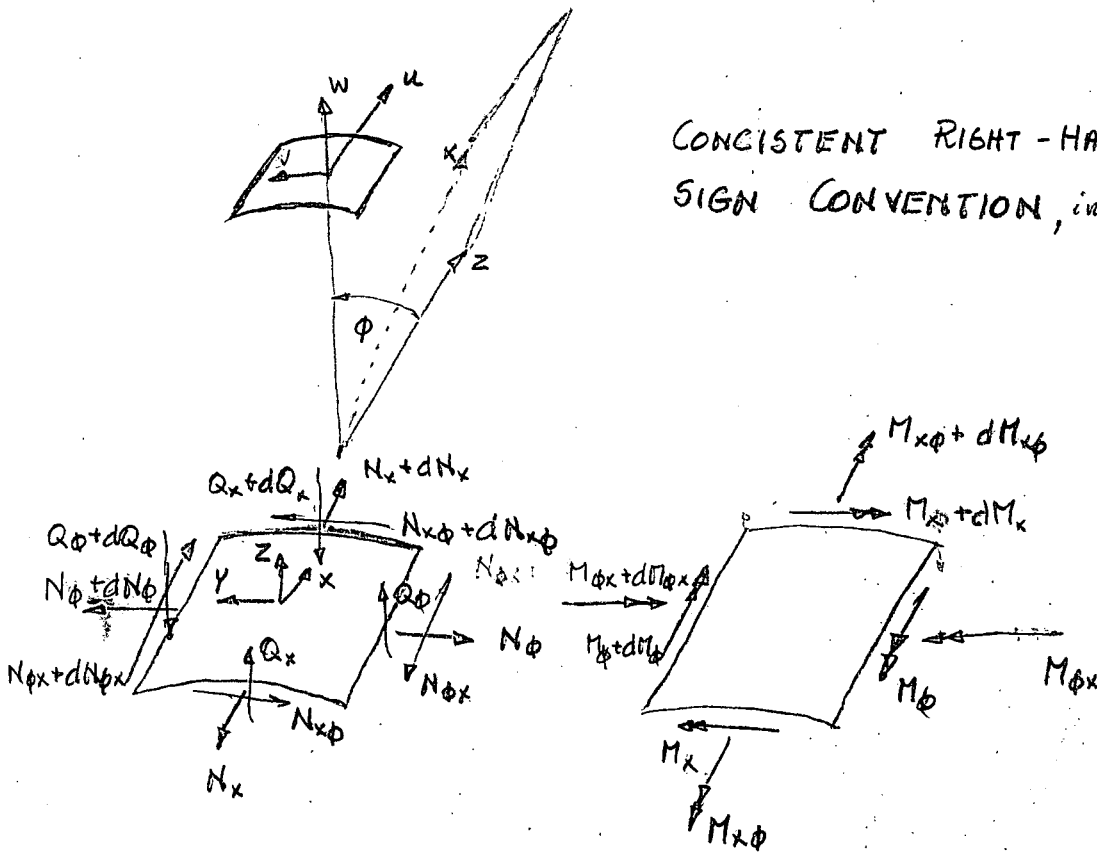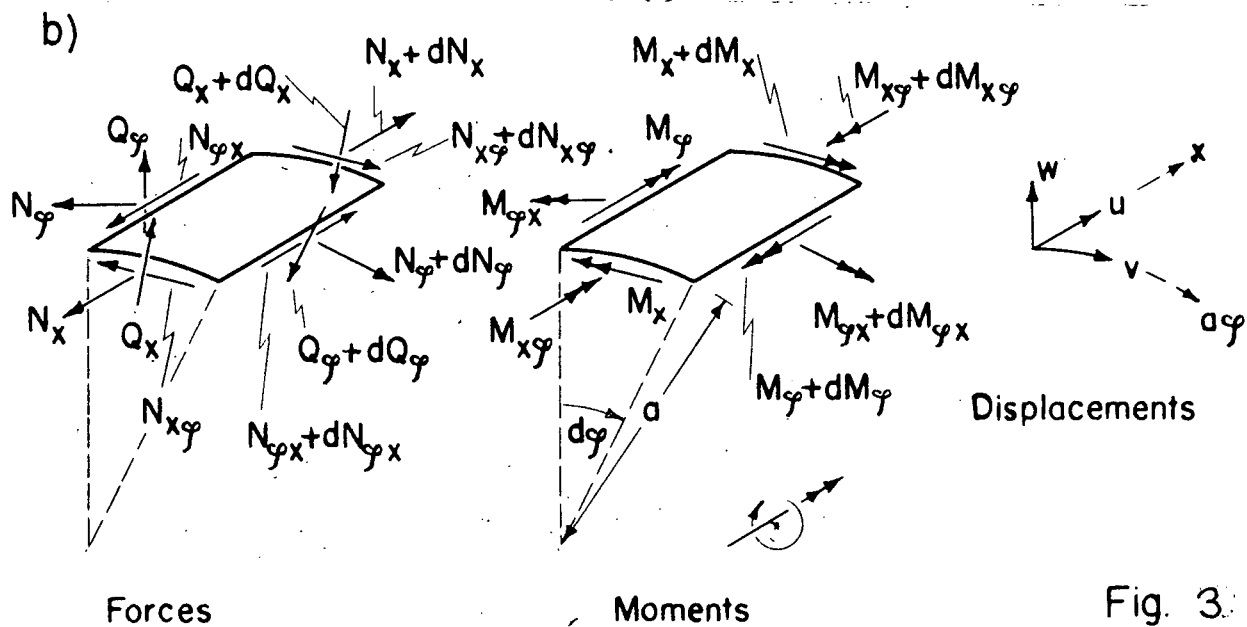
b)

$Q_x + dQ_x$ $N_x + dN_x$

Q_φ $N_{\varphi x}$

N_φ $N_{x\varphi} + dN_{x\varphi}$

N_x Q_x $N_\varphi + dN_\varphi$

$N_{x\varphi}$ $Q_\varphi + dQ_\varphi$

$N_{\varphi x} + dN_{\varphi x}$

Forces

$M_x + dM_x$ $M_{x\varphi} + dM_{x\varphi}$

M_φ

$M_{\varphi x}$

M_x

$M_{x\varphi}$ $M_{\varphi x} + dM_{\varphi x}$

$d\varphi$ a $M_\varphi + dM_\varphi$

Moments

w x
u
v
aφ

Displacements

Fig. 3

a)

Variation of String Force S

String

φ

S_2

$S = S_2 \cos 2\varphi$

φ

S_3

$S = S_3 \cos 3\varphi$

Fig. 4

$\beta l = \infty$

$\beta l = 10$

$\beta l = 0.8$

$\beta l = 0.6$

$\beta l = 0.4$

$\beta l = 0.2$

$\beta l = 0$

Infinitely long cylinder
-Eq (12b)-

Semi-infinite cylinder
-Eq (16b)-

Effective width $b = K\sqrt{ah}$

$\lambda = n\sqrt{\dfrac{h}{a}}$

$\beta = \dfrac{1.3161}{\sqrt{ah}}$

$S = S_n \cos n\varphi$

Fig. 5

p_o
Load $p = p_o \cos \dfrac{\pi}{L} y$

O
$-y$ y

A

$\frac{1}{2} L$ $\frac{1}{2} L$ L

A

Section A - A :

a) $-X$ O O X

b)   O X

Fig. 6

S_n

Middle surface
of the shell

String force $S = S_n \cos n\varphi$

L

α

a
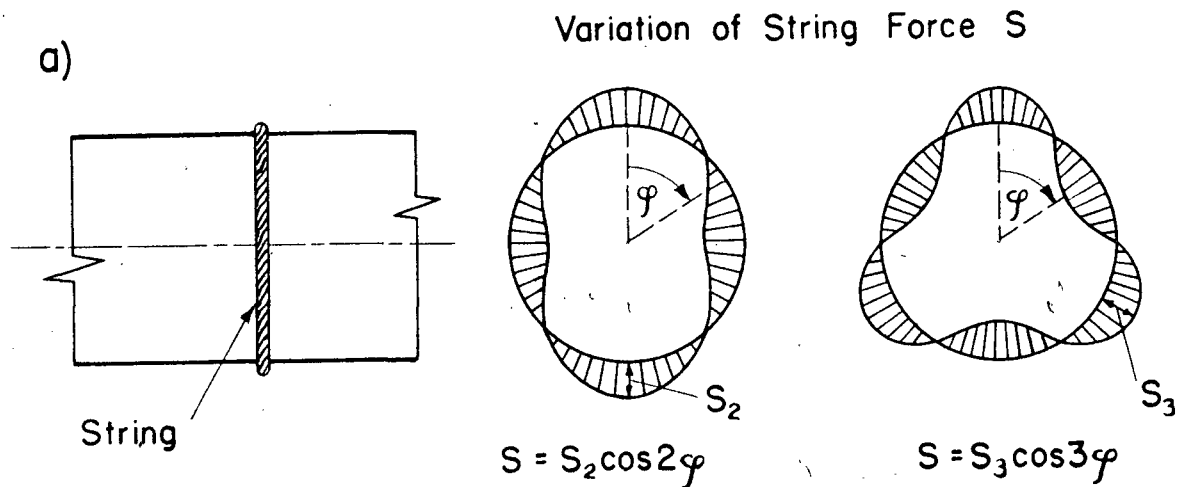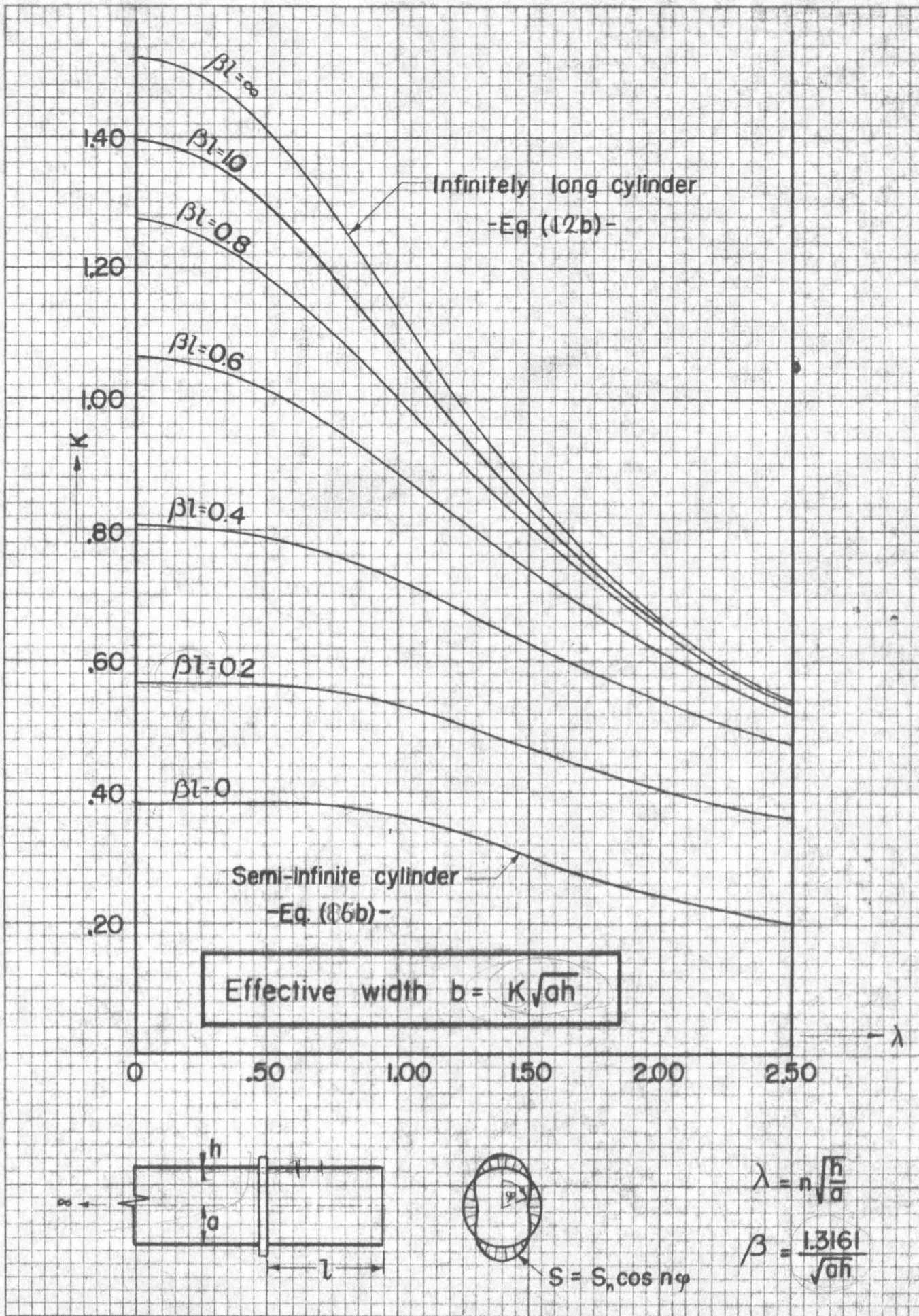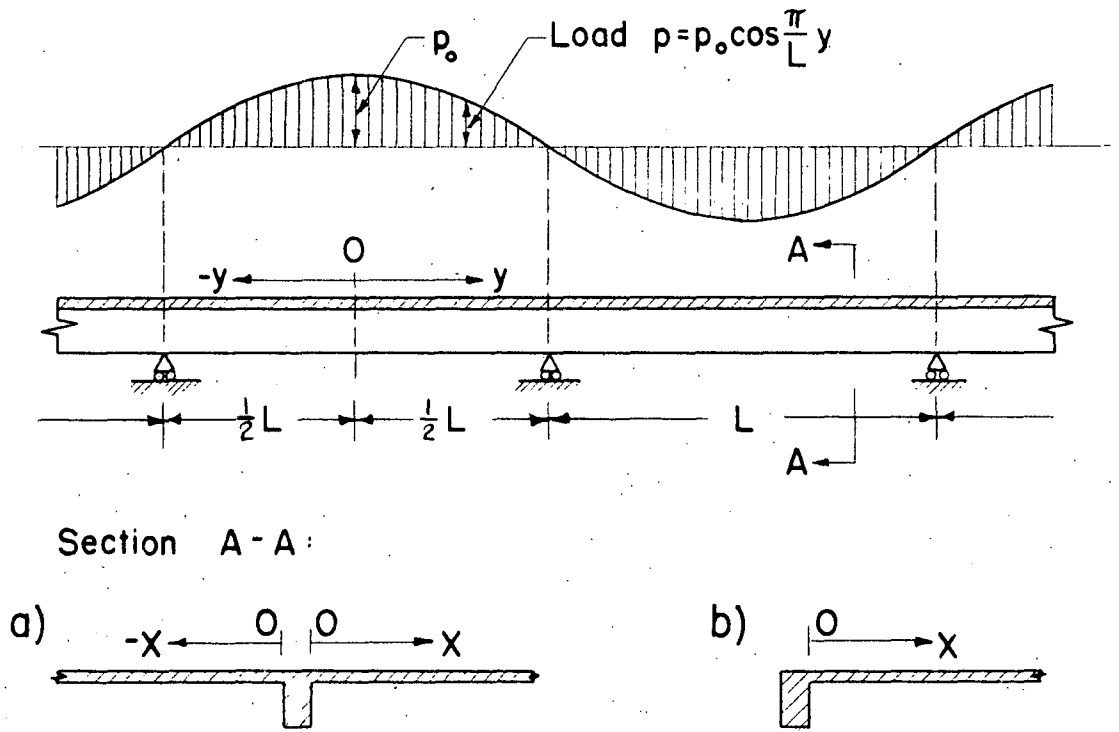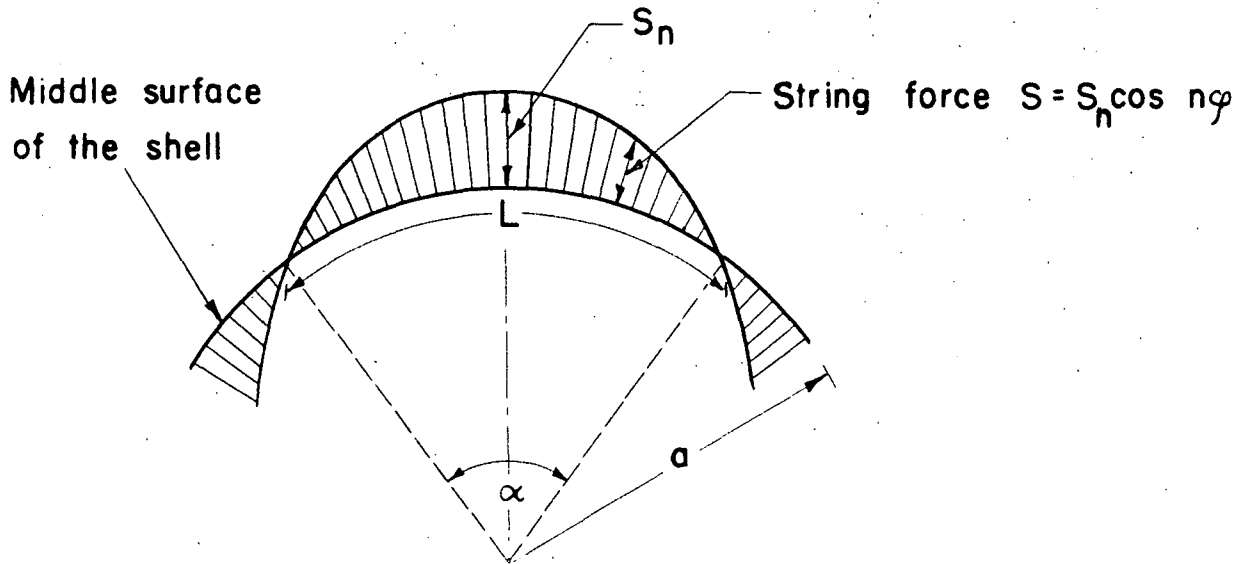
Fig. 7

NOTATIONS

Roman Alphabet

a	radius of the shell
b	effective width
C	constant of integration, Eq. (3) or Table A
e	base of natural logarithm
E	modulus of elasticity
h	thickness of the shell
H	symbol for any force, moment etc., of the shell
$\overline{H}$	coefficient depending on the quantity H under consideration, Table A
k, k'	coefficients defined in Table A
k_1, k_2	coefficients defined in Table A
$K (\beta l, \lambda)$	function of the two parameters βl and λ, used for the effective width
l	length of the shell in axial direction
L	span of T-Beam or half-wave length of the string force S (Fig. 6 and 7)
M_x	bending moment per unit width of the shell in axial direction
M_φ	bending moment per unit width of the shell in circumferential direction
$M_{x\varphi}$, $M_{\varphi x}$	twisting moments per unit width of the

shell

n n^{th} term of a Fourier series (number of complete cosine-waves of the string force S around the cylinder)

N_x direct force per unit width of the shell in axial direction

N_φ direct force per unit width of the shell in circumferential direction

$N_{x\varphi}$, $N_{\varphi x}$ direct shear forces per unit width of the shell

p, p_o distributed line load, Fig. 6

Q_x normal shear force per unit width of the shell acting on a face x = constant

Q_φ normal shear force per unit width of the shell acting on a face φ = constant

S   string force S defined by Eq. (1)

$S_n(x)$ string force at a distance x in the case of the nth harmonic, defined by Eq. (5)

T   total shear force per unit width of shell in circumferential direction, Eq. (6)

u   displacement in axial direction, Fig. 3

v   displacement in circumferential direction, Fig. 3

w   displacement in radial direction, positive outward, Fig. 3

x	coordinate of the shell in axial direction
y	coordinate of the T-Beam in direction of its axis

Greek Alphabet

α	angle corresponding to a half-wave of the string force (Fig. 7)
β	shell coefficient, depending on the thickness h and the radius "a" of the shell, Fig. 5
λ	parameter of the effective width, see Table A
$\mu_1; \mu_2$	numerical coefficients, defined in Table A
ν	Poisson's ratio
σ	stress
σ_x	bending stress in axial direction
σ_φ	direct stress in circumferential direction
$\tau_{x\varphi}$	shear stress of the shell on a cut x = constant in circumferential direction
τ_{yx}	shear stress in the flange of a T-Beam
φ	angular coordinate of the shell in circumferential direction
ξ	dimensionless coordinate in x-direction ($\xi = \frac{x}{a}$)
ψ	angle, constant of integration, Eq. (3) or Table A

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