1949

Relaxation method applied to torsion, July 18, 1949

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Relaxation Method Applied To Torsion

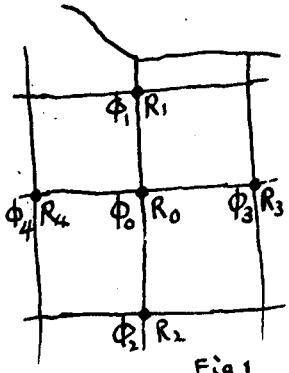


Fig 1

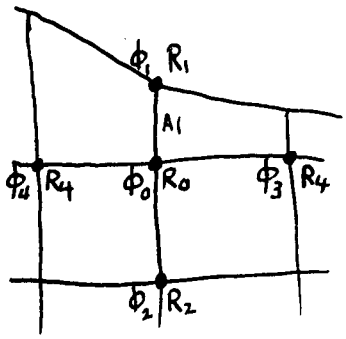


Fig 2

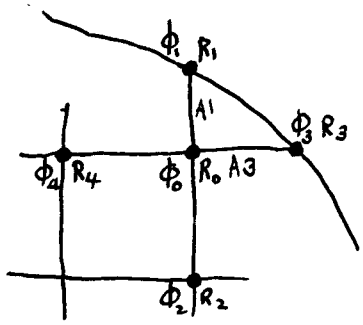


Fig 3

Formulas for Residuals:

(A) Regular star: (Fig 1.) $R_0 = \phi_1 + \phi_2 + \phi_3 + \phi_4 + 250 - 4\phi_0$

(B) One unequal leg at ϕ_1: (Fig 2). $R_0 = (II)_{S=A1}\,\phi_2^{\text{opp. to } \phi_1} + \phi_3 + \phi_4 + 250 - [(I)_{S=A1} + 2]\,\phi_0$

(C) Two unequal legs at ϕ_1 & ϕ_3: (Fig 3). $R_0 = (II)_{S=A1}\,\phi_2^{\text{opp. to } \phi_1} + (II)_{S=A3}\,\phi_4^{\text{opp. to } \phi_3} + 250 - [(I)_{S=A1} + (I)_{S=A3}]\,\phi_0$

Relaxation of Residuals:

(A) Regular star (Fig 1.) If change ϕ_0 by an amount m
Then R_0 should be changed by $-4m$
R_1, R_2, R_3 and R_4 changed by $+m$.

(B) Irregular star, One unequal leg. (Fig 2.)

All the same <u>except</u> that changes in R_0 due to the changes in ϕ_0 or ϕ_2 are different.

(i) If change ϕ_0 by m, R_0 should be changed by $-[(I)_{S=A1} + 2]\,m$

(ii) If change ϕ_2 by m, R_0 " " " " $+ (II)_{S=A1}\,m$

(C) Irregular star, Two unequal leg (Fig 3.)

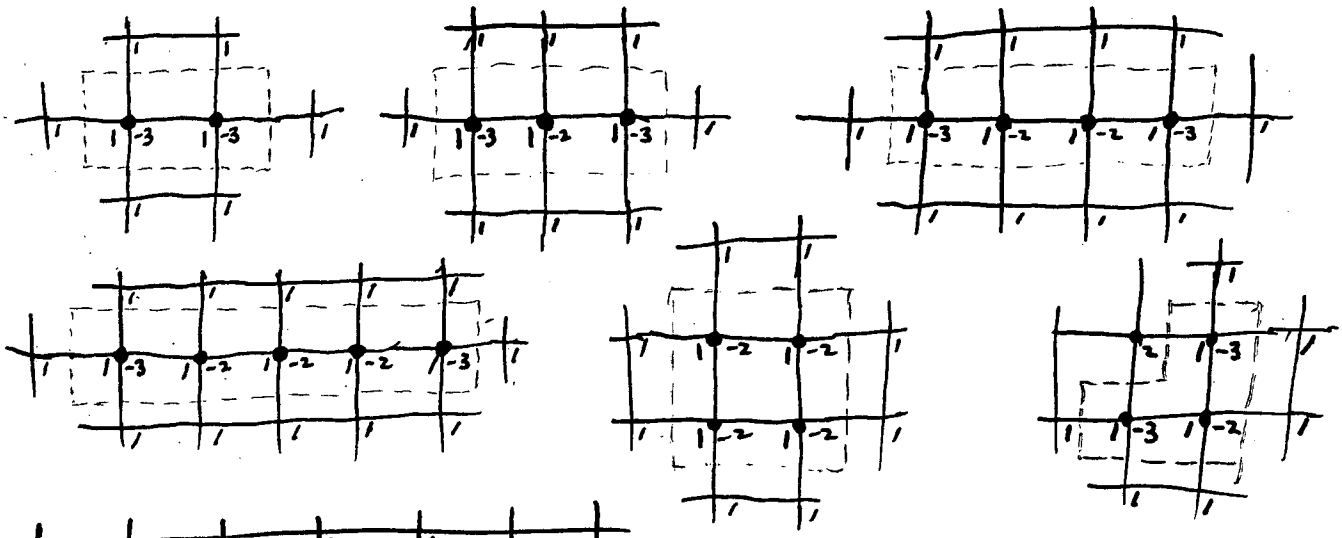
All the same <u>except</u> that changes in R_0 due to the changes in ϕ_0, ϕ_2 or ϕ_4 are different.

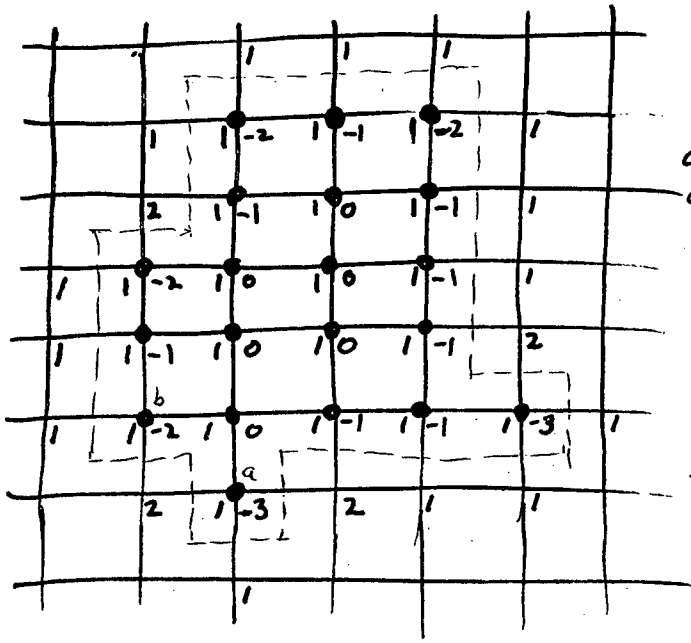
(i) If change ϕ_0 by m, R_0 should be changed by $-[(I)_{S=A1} + (I)_{S=A3}]\,m$

(ii) If " ϕ_2 " m, R_0 " " " " $+ (II)_{S=A1}\,m$

(iii) If " ϕ_4 " m, R_0 " " " " $+ (II)_{S=A3}\,m$

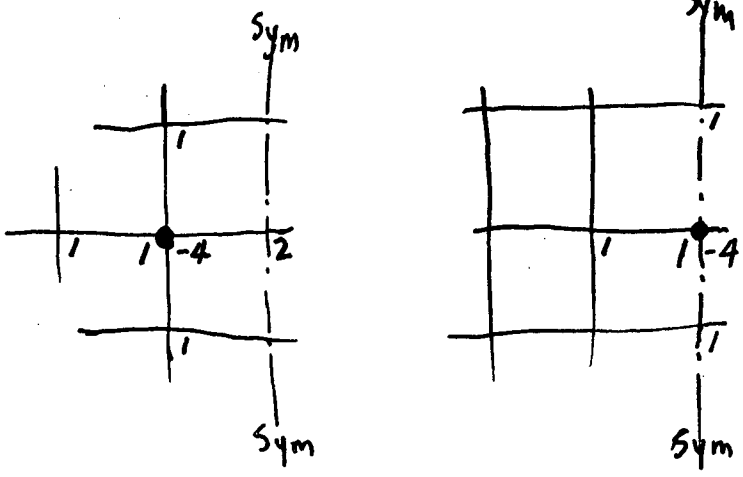
Block relaxation operators :





Rule: *All points like a, having an alteration to the residual of -3, are connected directly to 3 points that remain undisplaced, whilst all pts like b, having a residual alteration of -2, are connected directly to 2 points that remain undisplaced. Of the points that do not move, at any one such pt there is an alteration to the residual of +1 for each direct connection from that point to a point that is being displaced.*

Techniques with Symmetry :



Notes :

(1) The constant "250" in the first 3 formulas may be of different values for different cases

(2) All ϕ-values along the boundary equals zero

(3) $(II)_{S=A1}$, for example, means the value in column (II) of the table on p. 34 "NUMERICAL SOLUTION OF LAPLACES & POISSONS EQUATIONS" corresponding S value equal to A1. A1 is the ratio of unequal leg to full leg.

$T = K G_1 \theta = 2V$

$\therefore K = \frac{2V}{G\theta}$

$= \frac{2 \times 5331.6213 \, G\theta\lambda^4}{G\theta}$

$= 10663.2426\lambda^4$

K connected = 10663.2426
K disconnected = 7690.6667
Difference 2972.5759

Difference $= \Delta D^4$ $D = 12.25$

$D = \frac{(n+\ell)^2 + w(\ell + \frac{w}{4})}{2\ell + n} = \frac{(8+6.4)^2 + 6(6.4+\frac{6}{4})}{2\times 6.4 + 8} = 12.248$

$\alpha = \frac{2972.5759}{(12.25)^4} = \frac{2972.5759}{22,504.0514} = 0.132$

ψ_1	ψ_2	ψ_m	
1.0510	1.0104	1/2	
0.8606	1.0154	1/2	
1.1298	0.9673	1/2	
0.9965	0.9955	1/2	
1.1298	0.9673	1/2	

eqn 23

eqn 23

$V = 2616.3732\left(\frac{1}{9}\right)\left(\frac{2G\theta}{a}\right)\lambda^4 \times 2 + 42.5182\left(\frac{2G\theta}{a}\right)\lambda^4 \times 2$

$= 5331.6213 \, G\theta\lambda^4$

4651.3301
680.2912
5331.6213

$K = \frac{1}{3}n^3 b = \frac{1}{3}\times 6^3 \times 12\lambda^4 = 864\lambda^4$

$K = \frac{1}{3}\times 8^3 \times 40\lambda^4 = 6826.6667\lambda^4$

7690.6667λ^4

2.9851
14.2112
9.0908
23.9276
16.3664
45.2016
15.7718
127.5545
42.5182

$\ell = 6.4$

$\frac{\ell}{n} = 0.8$

$\frac{w}{n} = 0.75$

$\frac{w}{n} = 0.4$

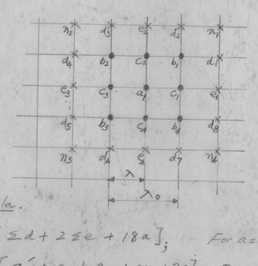
2483.5858

n=8

w=6

point D-2

CROSS SECTION 10 X 10.
TRADE ☆ MARK



Φ: ① $a_1' = \frac{1}{16}[\Sigma d + 2\Sigma e + 18a]$; For $a=0.25$, $a_1' = \frac{1}{16}[\Sigma d + 2\Sigma e + 4.5]$

② $b_1' = \frac{1}{4}[a_1' + e_1 + e_2 + n_1 + 2a]$; For $a=0.25$, $b_1' = \frac{1}{4}[a_1' + e_1 + e_2 + n_1 + 0.50]$

③ $c_1' = \frac{1}{4}[a_1' + b_1' + b_2' + e_1 + a]$; For $a=0.25$ $c_1' = \frac{1}{4}[a_1' + b_1' + b_2' + e_1 + 0.25]$

All values of Φ's are coefficient of $\frac{2 q_{pp}^2}{a}$ or $2\Phi 4\lambda^2$ or $2\Phi 0\lambda_0^2$ while $\lambda_0 = \frac{1}{2}\lambda$

Ψ. δ: ① $a_1' = \frac{1}{16}[\Sigma d + 2\Sigma e]$

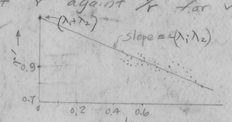
② $b_1' = \frac{1}{4}[a_1' + e_1 + e_2 + n_1]$

③ $c_1' = \frac{1}{4}[a_1' + b_1' + b_2' + a]$

Extrapolation.

① Compute r & r' at various points where $r = \frac{\delta^{[K-1]}}{\delta^{[K-2]}}$; $r' = \frac{\delta^{[K]}}{\delta^{[K-1]}}$

② plot r' against $\frac{1}{K}$ for various pts.



Solve for π_1 & λ_2.

③ Compute the coefficient

$\frac{(1-\lambda_1-\lambda_2)}{(1-\lambda_1)(1-\lambda_2)}$ & $\frac{1}{(1-\lambda_1)(1-\lambda_2)}$.

④ $\Psi = \psi' + \delta' + \delta'' + \cdots + \delta^{[K-2]} + \frac{1-\lambda_1-\lambda_2}{(1-\lambda_1)(1-\lambda_2)}\delta^{[K-1]} + \frac{1}{(1-\lambda_1)(1-\lambda_2)}\delta^K$

J. of Appl. physic

$q_{pp} = \frac{1}{16}[\ \ -\ \ -\ \ ]$

$b_{pp} = \frac{1}{4}[$ 4 corner pts $]$

$c_{pp} = \frac{1}{4}[$ 4 mid. pts $]$

- 4.24

+ 11.45

Starting Block

POINT C3

POINT C1

(1) Improvement formula for ϕ:

$$\phi_0 = \tfrac{1}{4}(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \tfrac{1}{4}) \; ; \quad a = \tfrac{1}{4}.$$

(2) Improvement formula for δ:

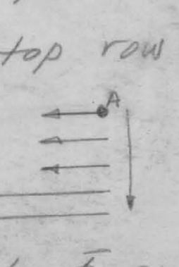
$$\delta_0 = \tfrac{1}{4}(\delta_1 + \delta_2 + \delta_3 + \delta_4)$$

(3) Order of improvement:

Starting from pt A.

From right to left in each row

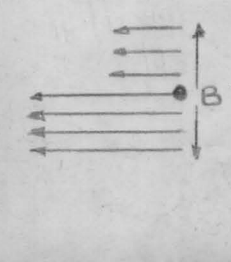
From top row to bottom row



Except Φ and Φ' – From pt B.

From right to left in each row and

move downward and upward separately.



(4) r

0.769
0.812
0.700
0.700
0.722
0.714
0.736
0.727
0.782
0.770
0.837
0.809
0.823
0.880
0.920
0.923

avg r = 0.790

$\frac{1}{1-r} = 4.76$

Vertical strip (from pt A down to pt B), summed values:

Row	Left column	Right column
A	0.3750	0.5000
1	1.5000	0.5000
2	6.0016	2.0004
3	3.0008	1.0004
4	6.0048	2.0020
5	3.0064	1.0024
6	6.0288	2.0104
7	3.0304	1.0112
8	6.1376	2.0488
9	3.1536	1.0538
10	6.6528	2.2292
11	3.6936	1.2396
12	8.9776	2.9928
13 (B)	6.2752	2.0006

At B: $+243 = \delta$, $+150 = \delta'$, $\Phi = 0.9265$, $\Phi_2 = 0.9528$, $\Phi = 1.0001$, $\Phi' = 1.0003$; $+16 \times 4.76 = 76$. Total 84.5548

Horizontal grid, summed values:

Edge value	Edge sum															Total
0.8750	3.500	14.0016	7.0024	14.0080	7.0096	14.0368	7.0336	14.1136	7.0968	14.3424	7.3184	15.2704	8.4128	21.2208	5.7384	244.6604
1.5000	3.0000	12.0016	6.0020	12.0080	6.0088	12.0336	6.0296	12.1060	6.0104	12.2776	6.2360	12.8176	6.7088	14.3128	3.6724	375.8796
1.8750	7.5000	30.0032	15.0048	30.0224	15.0232	30.0864	15.0736	30.2432	15.1952	30.6224	15.4928	31.5296	16.1440	33.1152	9.3656	699.3012
2.0000	4.0000	16.0016	8.0028	16.0120	8.0124	16.0456	8.0380	16.1232	8.0956	16.2904	8.2172	16.6272	8.4280	17.0664	4.2874	870.5890
1.8750	7.5000	30.0032	15.0040	30.0224	15.0216	30.0816	15.0680	30.2144	15.1624	30.4152	15.3400	30.9328	15.6040	31.4336	7.8804	1190.2926
1.5000	3.0000	12.0016	6.0020	12.0088	6.0080	12.0288	6.0252	12.0792	6.0592	12.1696	6.1152	12.3072	6.1920	12.4440	3.1166	1317.8500
0.8750	3.5000	14.0016	7.0024	14.0096	7.0088	14.0304	7.0264	14.0848	7.0624	14.1728	7.1160	14.3024	7.1864	14.4208	3.6108	1466.3856

Edge ℄ Bo ℄ Bo ℄ Bo ℄ Bo ℄ Bo ℄

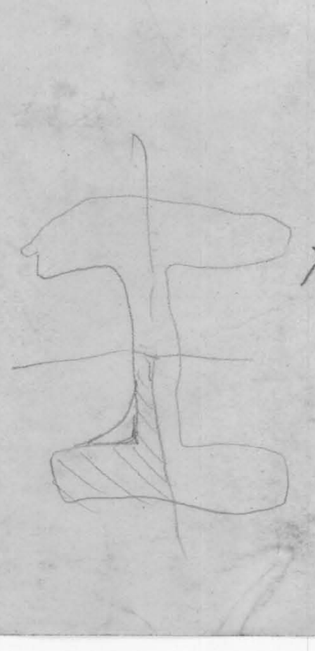
$V = 1466.3856\,[\times\tfrac{1}{36}](240)\lambda_0^4 \times 2$ ← We consider only one-half

$K = \frac{2V}{G\theta} = 325.8634\,\lambda_0^4, \quad \lambda_0 = \tfrac{1}{2}\omega.$

Investigation in Torsion

By F. K. Chong

POINT C2



$\lambda = \frac{\lambda_0}{2}$ $\frac{4}{16}$ $\frac{1}{3}$ · $\frac{1}{3}$

	I	II
0.150	13.3	1.74
0.450	4.4	1.4
0.740	2.9	1.15
0.950	2.1	1.02

POINT D3 Scale 80