1947

# Investigation in torsion, February 1947

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INVESTIGATION IN TORSION BY F. K. CHANG

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--This report is submitted to Prof. B. G. Johnston for the course C. E. 213, Structural Research. --

This investigation was made under the direction of Prof. B. G. Johnston and part of the material is taken from his notes on the course C. E. 221, Structural Members and Frames. Special acknowledgment is made here for his conduction throughout the work .

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## 1. Comparison of Torsion Properties Between Parallel Flange and Sloped Flange Sections

The torsion properties of parallel flange and sloped flange sections as rolled by different mills are certainly different. It is very interesting to find out how much is the difference and whether is it possible to have a compromised value for both cases. As the Bethlehem Manual of Steel Construction ,1934 edition, has the K- values prepared for sloped flange sections , it is also interesting to know whether is it on safe side to use these values for parallel flange sections.

The calculation of the constant,K, is based on Lyse-Johnston formula as given in their paper " Structural Beams in Torsion ".

All WF shapes with nominal depth from 36 to 16 inclusive and 14 WF 38 to 30; 12WF 36 to 27 ; lo WF 29 to 21; 8WF 20 and 17 rolled in the Behtlehem Steel Company have sloped flanges. The same sizes mentioned above rolled by Carnegie-Illinois Steel Corporation have parallel flanges.

The following 15 sizes are selected in such a way to cover all ranges to represent the general cases.

| Sections | | Sloped flange Sections 5% slope | Parallel flanges Sections | % Difference |
|---|---|---|---|---|
| B36a 36&16½ | 300# | 68.799 | 66.047 | 4.167% |
| | 260 | 44.777 | 42.915 | 4.339 |
| | 230 | 30.943 | 29.507 | 4,867 |
| B24a 24&12 | 120 | 8.845 | 8.402 | 5.272 |
| | 110 | 6.919 | 6.547 | 5.681 |
| | 100 | 5.245 | 4.955 | 5.852 |
| B16 16&8½ | 50 | 1.621 | 1.552 | 4.445 |
| | 40 | 0.853 | 0.810 | 5.300 |
| | 36 | 0.590 | 0.559 | 5.540 |
| B14 14&6¾ | 38 | 0.860 | 0.819 | 5.000 |
| | 34 | 0.614 | 0.583 | 5.310 |
| | 30 | 0.410 | 0.387 | 5.940 |
| B10 10&5¾ | 29 | 0.625 | 0.594 | 5.210 |
| | 21 | 0.229 | 0.214 | 7.000 |
| B8 8&5¼ | 17 | 0.160 | 0.151 | 5.96 |
| | | | Avg. | 5.330 |

It is evident that (1) parallel flange sections always have a smaller K values; (2) the max. difference is 7% and the average is 5% and (3) it is on dangerous side in most cases to use the K-value of sloped flange sections for parallel flange sections.

Since the difference is not very large, one compromised value for both cases is possible in publishing in the handbook.

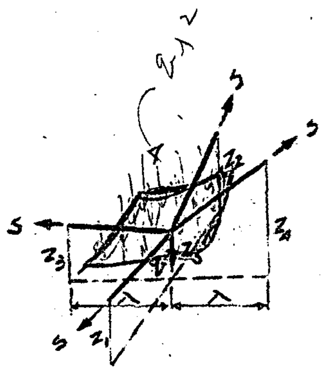
2. Discrepancy Between Lyse-Johnston Formula and Numerical Method

The computation in the above article is based on the Lyse-Johnston formula from soap film experiment. The numerical method as discussed later is very useful in torsion problems. It is worthwhile to see how these two methods check each other. For a 10 by 5 by 30 lbs joist in R. A. Skelton Handbook, the K-value by Lyse-Johnston formula is 0.942 and Mr. F. S. Shaw used the same section in Dorman and Long Handbook and obtained a K-value of 0.896 by numerical method which is about 5% lower.

This discrepancy , if computation is right , is due to two factors, (1) the accuracy of numerical method and (2) the values of different coefficients in Lyse-Johnston formula.

The first factor has been fully discussed in Mr. Shaw's paper "The Torsion of a Rolled Steel Joist Section by Relaxation Methods." and the second factor will be discussed later.

## 3. The Numerical Method



Instead of considering a membrane as in the membrane analogy method, we consider a fine net with load "q" at unit area and tension "s" per unit length. As shown in the left figure, We have:

$$s\lambda\left(\frac{z_3-z_5}{\lambda}\right)+s\lambda\left(\frac{z_2-z_5}{\lambda}\right)+s\lambda\left(\frac{z_1-z_5}{\lambda}\right)+s\lambda\left(\frac{z_4-z_5}{\lambda}\right)=-q\lambda^2$$

$$\therefore z_1+z_2+z_3+z_4-4z_5=-\frac{q}{s}\lambda^2$$

$$\text{Or}\quad \Phi_6=\frac{1}{4}\left[\Phi_1+\Phi_2+\Phi_3+\Phi_4+2G\theta\lambda^2\right]$$

For simplicity in calculation, we use a constant "a" in place of the term $2G\theta\lambda^2$. The values of $\phi$'s obtained then are the coefficients of $\frac{2G\theta\lambda^2}{a}$. In most cases, we take "a" as 1. When the net becomes very fine, it approaches a membrane and the result is very accurate.

Based on above formula, we can find the $\phi$ values either by solving simultaneous equations or by numerical method.

When the number of simutaneous equations is 10 or above it is recommended to use numerical procedure or the combination of these two method. The numerical method is (1) assigning arbitrary $\phi$ values or prefered solving simultaneous equations of very coarse net as a guide; (2) using the improvement formula $\Phi_5=\frac{1}{4}(\Phi_1+\Phi_2+\Phi_3+\Phi_4+a)$ to improve the $\Phi$ values and using the improved values as soon as available.

The convergence by using improvement formula is very slow and sometimes more than 30 cycles is needed. Labor of improvement can be largely reduced by using difference function and extrapolation.

Assume the true value of $\phi$ is w , and e the error, then

$\phi = \omega + e$

When this is improved, we obtain

$\phi' = \omega + e'$

The difference between $\phi'$ and $\phi$ , we call the difference function $\delta$.

$\delta = \phi' - \phi = e' - e$

If we apply improvement process to the difference functions , we get successively

$\delta' = \phi'' - \phi' = e'' - e'$

$\delta'' = \phi''' - \phi'' = e''' - e''$

$\delta^{(n-1)} = \phi^{n} - \phi^{(n-1)} = e^{n} - e^{(n-1)}$

Then, $\phi' + \delta' + \delta'' + \delta''' + \cdots\cdots + \delta^{(n-1)} = \omega + e^{n} = \phi^{n}$.

This shows that if we improve the difference function "n" times and add the sum of these functions to $\phi'$, we can get a result as accurate as improving the value $\phi$ itself "n" times. As $\delta$ is difference between $\phi'$ and $\phi$ , the constant term" a" is cancelled out, the improvement formula for $\delta$ becomes :

$\delta_5 = \frac{1}{4}(\delta_1 + \delta_2 + \delta_3 + \delta_4)$

If more accurate result is desired , we can extrapolate the $\delta$ values. As $\delta$- value improves many Cycles, it becomes a smooth function decreasing uniformly by a factor $r$, as

$\delta^{k} = \delta^{k}$; $\delta^{(k+1)} = r\delta^{k}$; $\delta^{(k+2)} = r^2\delta^{k}$

so, $\delta^{k} + \delta^{(k+1)} + \delta^{(k+2)} + \cdots\cdots = \delta^{k}(1 + r + r^2 + \cdots\cdots) = \delta^{k}/(1-r)$

Based on above principle , the procedu-re is outlined as follows:

(a) Chose trial function $\phi$ ,or if prefered, solve simultaneous equations of very coarse net as a guide.

(b) Improve $\phi$ once, using improvement formula $\phi_5 = \frac{1}{4}(\phi_1 + \phi_2 + \phi_3 + \phi_4 + a)$ and obtain $\phi'$ .

(c) Calculate difference functions, $\delta = \phi' - \phi$.

(d) Improving the $\delta$ values in the same order as improving $\phi$. The improvement formula is $\delta_5 = \frac{1}{4}(\delta_1 + \delta_2 + \delta_3 + \delta_4)$ , until a smooth function decreasing uniformly is obtained.

(e) Find mean value of $\delta^k / \delta^{k+1} = r$

(f) New function $\bar{\phi} = \phi' + \delta' + \delta'' + \delta''' + \dots + \frac{\delta^k}{1-r}$

(g) Improve $\bar{\phi}$ once, get $\bar{\phi}'$. If unsatisfactory, repeat this process again.

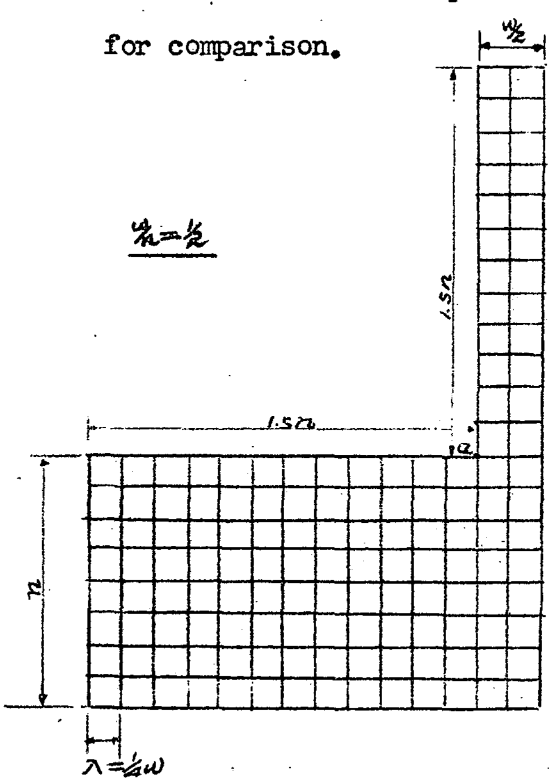
After we obtain the correct $\phi$ values , it is necessary to compute the volume under the $\phi$ surface. The method of double application of Simpson's Rule seems to be very satisfactory. For a part of a strip of indefinite lenth, the K-values by exact formula and by using the Simpson's Rule to find the volume are identically the same.

A square of side length "b" has been solved by both numerical method and simultaneous equation method. In the former method , the net size is $\lambda = b/8$ and in the later method ten simultaneous equations are solved. The K- value obtained in both method is $0.1386b^4$ while the exact solution by Timoshenko is o.1406,only 1.42% off.

4. Comparison of the coefficient "$\alpha$" in Lyse-Johnston Formula Obtained By Soap Film Experiment And Numerical Method

Concerning the added torsional rigidity due to the connection of the flange and web and also due to the fillet at this point of a I-section, Trayer and March assume the additional K equal to $\alpha D^4$ while D is the diameter of the largest circle that can be inscribed at the juncture of the web and flange. Professor Lyse and Johnston in their paper " Structural Beams In Torsion" have these coefficients prepared by soap film experiment. In order to find out the source of the discrepancy between the Lyse-Johnston formula and the numerical method it is suggested to compare the coefficient "$\alpha$" first.

Two values of the coefficients "$\alpha$" are computed by numerical method for radius r equals zero and w/n equals $\frac{1}{2}$ and 1 respectively for comparison.



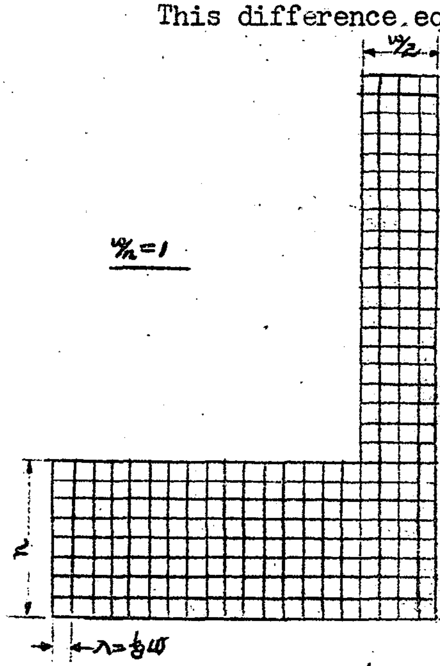

For w/n= $\frac{1}{2}$ , we use a net $\lambda = \frac{1}{4}w$ In the calculation, assume that both the flanges and web are continuous at farther ends and assume that the "hump" due to connection of flange and web does not affect the $\phi$ values at 1.5n away from the point a. This assumption is found reasonable after the calculation is made. Theoretically, the "hump" does affect every point in the net, but the effect is very small for points farther away from the junction and can be neglected without error. The volume is computed by using Simpson's Rule. Instead of finding the volume of each square separately, it is much convenient to find out a coefficient for each point once and multiply the individual $\phi$ by its coefficient and add them together.

The value of K when the web and flange are connected : 325.6834
The value of K when the web and flange disconnected : 314.6666
The difference 11.1968

This difference equals $\alpha D^4$ while $D = 4.25$

Therefore $\alpha = 0.034$

By soap film experiment $\alpha = 0.081$




For w/n =1, we use a net as shown $\lambda = \frac{1}{8}w$

K connected 706.5124

K disconnected 640.0000

Difference 66.5124

while $D = 5.00$

Therefore $\alpha = 0.1064$

By soap film experiment $\alpha = 0.0940$

The point for w/n=1 is very close by both methods and that for w/n=$\frac{1}{2}$ is far off. The tendency of the curve by numerical method can be found only after one or more points with w/n ratio between 1 and $\frac{1}{2}$ is computed. It is early now to draw a conclusion.