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Welded Continuous Frames and Their Components

THE SPACING OF LATERAL BRACING IN PLASTIC DESIGN

by

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S Y N O P S I S

A design procedure for the determination of the spacing of lateral supports in plastically designed steel frames is proposed. The procedure is applicable to structures which are subjected primarily to bending forces and which are fabricated from rolled, mild steel sections. The report is a continuation of a previous paper⁽²⁾ in which a theoretical basis is presented for the spacing of lateral braces. This procedure is simplified for practical use and modified as a result of experiments. These experiments are described, and finally design recommendations are presented.

1. I N T R O D U C T I O N

A plastically designed steel structure fails when a sufficient number of plastic hinges have developed to form a mechanism. At those hinges which do not form last the member is required to undergo a certain amount of inelastic rotation without a decrease of the moment capacity. If the hinge is one of the first ones to develop, this rotation may be quite considerable.⁽¹⁾

The member may not deliver the required rotation at the full plastic moment if its rotation capacity is reduced by instability. One such instability effect is lateral-torsional buckling. Its occurrence may be postponed until the member has delivered its required rotation if lateral bracing is provided at sufficient intervals along the compression flange of the member.

The purpose of this report is to derive formulas by which the necessary distance between lateral braces may be determined. These rules represent simplifications to a method presented recently⁽²⁾ and they apply to structural steel (ASTM-A7) wide-flange sections which are subjected primarily to bending forces. The paper also contains a report on tests which were conducted to substantiate the theoretical derivations.

2. A B R I E F R E V I E W O F P R E V I O U S W O R K

In a paper entitled "The Lateral-Torsional Buckling of Yielded Structural Steel Members" M. W. White⁽²⁾ developed a method for determining the spacing of lateral supports for beams so that lateral buckling is delayed until after the required hinge rotations have taken place. This method is an extension of the classical elastic lateral buckling theory for finding the critical lengths of elastically restrained beams which are subjected to end moments about their strong axis. It is assumed that in the case of inelastic lateral buckling, portions of the beam are strain-hardened and portions are elastic. By applying the appropriate strain-hardening and elastic moduli of the respective sections to the differential equations of lateral-torsional buckling and then matching boundary conditions at the common juncture of the two regions, the critical length of the beam was obtained as the eigenvalue of the differential equations.

The critical length between lateral braces is expressed by the following formula:

$$\left(\frac{L}{r_y}\right) = 18 \nu_f \nu_s \nu_f \nu_\alpha \sqrt{\frac{M_p}{M_o}} \quad (1)$$

where

L_{cr} = Critical unsupported length.

r_y = Weak axis radius of gyration.

M_p = Plastic moment of the section.

M_o = The larger of the two end moments.

ν_ρ = Correction factor for moment ratio.

ν_s = Correction factor for St. Venant's torsion.

ν_γ = Correction factor for the degree of end fixity.

ν_α = Correction factor for the effect of partial strain-hardening.

If all four correction factors in Eq. 1 are equal to 1.0, L_{cr} represents the critical length of a beam which is subjected to uniform moment. It also means that the whole beam is strain-hardened (strain-hardening modulus $E_{st}=900$ ksi and shear modulus $G_{st} = 2,500$ ksi⁽³⁾), that the contribution of St. Venant's torsion* to the lateral buckling strength is neglected, and that adjacent portions of the beam do not restrain it from lateral buckling. The four correction factors modify this "base length" to include the effects of non-uniform moment, (ν_ρ), St. Venant's torsion, (ν_s),

*It has been shown(2) that for the relatively short segments under consideration here, lateral buckling is resisted mainly by the warping stiffness.

degree of end fixity, (ν_y), and partial strain-hardening, (ν_α). Each of these effects are beneficial and thus the correction factors increase the base length.

Equation 1 represents an initial motion (eigenvalue) solution, and it does not give the ultimate lateral buckling strength of the member.* The concept is analogous to the tangent modulus theory of axially loaded columns, and as such the solution gives a lower bound to the true strength of the member.

The results of this work are applicable to structural steel wide-flange beams when it can be assumed that the effects of the axial force and the shear force are small. The loads are assumed to be applied to the beams at the points of lateral support.

Whereas the method represents a rational solution to a very complicated problem, there are still some objections against using it in design. These are the following:

- (1) For the case where the member is subjected to a nearly uniform moment, the method gives results which are overly conservative when compared with tests.
- (2)

*A study of the "reduced modulus" strength and "ultimate" strength of beams failing by lateral buckling is now underway at Lehigh University. Preliminary considerations reveal that the results of the "tangent modulus" approach are quite conservative.

- (2) The design is an iterative procedure, involving the determination of the critical length of each unbraced segment by trial and error.
- (3) Numerous charts are required for finding the correction factors.
- (4) One of the parameters necessary for finding ν_α is the inelastic rotation at each plastic hinge. This quantity varies according to geometry and loading and its determination involves considerable effort.⁽¹⁾

In the following section simplifications will be developed which will enable the determination of the correction factors in a more straightforward manner. Ultimately one simple equation, dependent only on the ratio of the end moments, is suggested.

3. SIMPLIFICATIONS OF THE THEORY

3.1 SIMPLIFICATION FOR ν_ϕ

The correction factor ν_ϕ accounts for the increase of the critical length when the beam is not under uniform moment. The ratio of the smaller to the larger end moment

is ρ , and it can vary from +1.0 to -1.0. The increase of the critical length is shown to be dependent on ρ and on the degree of end fixity. (2) The relationship between ν_ρ and ρ is developed in Ref. 2 and it is shown in Fig. 1 for the cases of simply supported and fully fixed ends. The true value of ν_ρ is in the region bounded by the two curves.

For reasons of simplification the two curves in Fig. 1 can be replaced by a straight line without introducing an appreciable error. The simplified expression for ν_ρ is

$$\nu_\rho = 1.34 - 0.34\rho \quad (2)$$

This straight line is shown as a heavy solid line in Fig. 1. (The maximum possible error is about 5%)

3.2 SIMPLIFICATION FOR ν_s .

The basic critical length was derived on the premise that St. Venant's torsional stiffness will not resist lateral buckling. This is a reasonable assumption when $\rho = 1.0$. However, it is shown in Ref. 2 that ν_s increases as ρ becomes smaller. The correction factor ν_s is also shown to increase as the cross sectional parameter $\frac{dZ}{K}$ (where d is the depth of the section, Z is the plastic modulus, and

K is St. Venant's torsion constant) decreases.

The relationship between ρ , $\frac{dz}{K}$ and ν_s is shown in Fig. 2. The degree of end fixity is seen to have little effect on ν_s , except where ρ approaches -1.0. However, $\frac{dz}{K}$ influences ν_s considerably, especially if $\frac{dz}{K}$ is less than 600. A tabulation of $\frac{dz}{K}$ for most of the rolled wide-flange sections (Appendix C, Ref. 2) shows that for sections which are generally used as beams the value of $\frac{dz}{K}$ ranges from about 500 to 1000. From Fig. 2 it can be seen that for this range ν_s may be represented as a straight line (heavy solid line) without introducing an error on the unsafe side of more than 2%. Thus, the simplified equation for ν_s is

$$\nu_s = 1.08 - 0.04\rho \quad (3)$$

3.3 SIMPLIFICATION FOR ν_α .

The base critical length of $18 r_y$ is obtained by assuming that the whole member is strain-hardened. The whole beam is assumed to be uniformly yielded because of the equal end moments. If the end moments are unequal, this assumption is not true since only parts of the beam are yielded. The assumption is now made that some parts

of the length are fully elastic, while others are fully strain-hardened. The elastic portion of the length is governed by the elastic moduli E and G , and the strain-hardened portion is governed by E_{st} and G_{st} . By matching boundary conditions at the junction of the two regions, the differential equations are solved numerically with the aid of an IBM-650 electronic computer.

The results are plotted in a series of charts (Figs. 12 to 16 in Ref. 2) which show the relationship between the correction factor ν_{α} and the coefficient α (where αL is the length of the strain-hardened region, as illustrated in the inset of Fig. 3) for various end conditions and moment ratios. It can be shown (Fig. 16, Ref. 2) that the end conditions have little effect on this relationship. Average curves for various values of ρ are given in Fig. 3. From this figure it can be seen that ρ also has little influence on the relations between α and ν_{α} . In fact, an average of all the curves may be used without impairing the accuracy of ν_{α} significantly. This average curve is shown as a heavy curve in Fig. 3. In the subsequent calculations this average curve will be used.

The value of the correction factor for partial strain-

hardening may be obtained thus from Fig. 3 if the extent of strain hardening αL along the length of the beam segment is known. In Ref. 2 it was shown that αL is dependent on the inelastic rotation at the hinge (determined by the methods of Ref. 1), on the moment ratio ρ , and on the ratio of the curvature at the start of strain-hardening to that at the elastic limit, ϕ_{st}/ϕ_y . The following equation represents this relationship:

$$\frac{\theta_R}{L\phi_p} = \left[\frac{\alpha(1-\rho) - 0.085}{0.915(1-\rho)} \right] \left[\left(\frac{\phi_{st}}{\phi_p} - 1 \right) + \frac{1}{2} \left\{ \frac{\alpha(1-\rho) - 0.085}{1 - \alpha(-\rho)} \right\} \right] + \frac{\alpha}{2} \quad (4)$$

where

θ_R = Inelastic angle change of the segment under investigation.

ϕ_p = Elastic curvature when $M = M_p$. This curvature is equal to:

$$\phi_p = \frac{M_p}{EI_x} \quad (5)$$

ϕ_{st} = Curvature at the start of strain-hardening.

The derivation of Eq. 4 (pp. 72-77, Ref. 2) is based on the following reasoning: For an assumed idealised moment-curvature relationship (which includes the influence of residual stress and strain-hardening) the length of the yielded zone and the magnitude of the moment at the hinge

can be determined if the inelastic rotation θ_R and the moment ratio ρ are known. If the length of the yielded zone is taken as the length of the strain-hardened zone αL , a safe solution results since not all of the yielded zone is strain-hardened.

It is furthermore assumed that the total inelastic rotation of the hinge is distributed between the two segments joining at the hinge in accordance with the ratio of the moment gradients. That is

$$\theta_R g_R = \theta_L g_L \quad (6)$$

where

θ_R, θ_L = Inelastic angle change to the right and to the left of the hinge, respectively.

g_R, g_L = Moment gradient of the right and the left segment, respectively. (The relationships defining θ , g_R and g_L are shown with the aid of an illustrative example in Fig. 4.)

Since the total inelastic rotation θ is equal to

$$\theta = \theta_R + \theta_L \quad (7)$$

the inelastic rotation of the hinge end of the right segment is

$$\theta_R = \frac{\theta}{1 + g_R/g_L} \quad (8)$$

For given values of $\frac{\theta_R}{L\phi_p}$ and β and for an assumed average value of $\phi_{st}/\phi_y = 12$, α may be computed from Eq. 4. The corresponding value of ν_α is then obtained from Fig. 3. Combining the curve from Fig. 3 and α from Eq. 5, a set of curves showing the relationship between β and ν_α for various constant values of $\frac{\theta_R}{L\phi_p}$ may be constructed. These curves are shown in Fig. 5.

3.4 THE CRITICAL LENGTH WITHOUT THE INFLUENCE OF END RESTRAINT

It will be convenient in future derivations to have a solution for the critical buckling length of the beam when the effect of end restraint is not included. For this reason Eq. 1 can be written as follows:

$$\left(\frac{L}{r_y}\right)_{cr} \cdot \frac{1}{\nu_\beta} = 18 \nu_\beta \nu_s \nu_\alpha \sqrt{\frac{M_o}{M_p}} \quad (9)$$

Since the critical length is derived for $M_o = M_p$, the radical in Eq. 9 is equal to 1.0. For given values of β and

$\frac{\theta_R}{L\phi_p}$, the critical length may now be computed with the aid

of Eqs. 2 and 3 and Fig. 5. A plot of ρ versus $(L/r_y)_{cr} \cdot \frac{1}{\psi_y}$ for constant values of $\frac{\theta_R}{L\theta_p}$ is shown in Fig. 6. The curves are nearly straight lines, and they could be represented as such (Fig. 7).

The charts in Fig. 7 give a solution to the problem of determining critical bracing spacing if the end restraints due to the adjacent members are neglected. Even though the solution has been simplified considerably, it is still necessary to compute the inelastic angle change θ_R . One further step toward simplicity would be to assume a value for the inelastic rotation which is an average upper limit for most structures. Although no studies on the most probable hinge angle are available, it is believed that a value of $\frac{\theta_R}{L\theta_p} = 3.0$ would represent a reasonable value. The critical length then would be expressed by the following linear equation (shown as a heavy dashed line in Fig. 7):

$$\left(\frac{L}{r_y}\right)_{cr} \cdot \frac{1}{\psi_y} = 48 - 30\rho \quad (10)$$

Experiments have shown (as will be discussed later in this report) that if ρ is near +1.0, Eq. 10 gives conservative results. The tests indicate that the critical length need not be less than $30 r_y$. Thus the simplified

expressions for the bracing spacing with the exclusion of the effect of end restraint are as follows:

$$\begin{aligned} \left(\frac{L}{r_y}\right)_{cr} \cdot \frac{1}{\nu_f} &= 30; \quad \text{for } 1.0 \gg \rho \gg 0.6 \\ \left(\frac{L}{r_y}\right)_{cr} \cdot \frac{1}{\nu_f} &= 48 - 30\rho; \quad \text{for } 0.6 \gg \rho \gg -1.0 \end{aligned} \quad (11)$$

3.5 SIMPLIFICATION FOR ν_f .

In order to complete the solution to the problem it is still necessary to determine the effect on the critical length of the end restraint due to the adjacent spans. One could assume, of course, that the ends of the beam segment under consideration are simply supported ($\nu_f = 1.0$). However, it has been shown⁽⁴⁾ that under uniform moment the critical length is doubled if the ends of the beams are fixed. Since the restraint offered by the adjacent beams will be between these two extremes, L_{cr} obtained for simple supports could be quite conservative.

The following two assumptions are made in the subsequent derivations:

(1) Since the added refinement of including the effect of more than one adjacent span is not justified, the

ends of the two neighboring spans are assumed laterally simply supported.

(2) It is assumed that the effective lateral buckling length is the same as the effective length of axially loaded columns. This use of the well-known effective length concept of centrally loaded columns⁽⁵⁾ for the lateral buckling of beams is not quite correct if the moment ratio is not equal to +1.0, but it is sufficiently accurate for the practical cases under consideration.*

The two assumptions above reduce the problem to finding the effective length of the center span BC of the three-span column ABCD shown in Fig. 8a. The column is subjected to an axial force P, and the outer ends of the spans AB and CD are simply supported.

The restraint of the outside spans may be replaced by springs acting at the ends of the critical span BC (Fig. 8b). The spring coefficient k of each spring can be expressed approximately as

$$k = \frac{3EI_y}{L} (1 - P/P_{cr}) \quad (12)$$

where $\frac{EI_y}{L}$ is the stiffness of the restraining beam,

*For further discussion of this see Ref. 4 or p. 40-42 of Ref. 2. It is shown in Ref. 2 that $\nu_y = 1.8$ when $f = -1.0$, thus giving a maximum error of 10%.

and $1 - P/P_{cr}$ represents the reduction in stiffness due to axial force. Since $P/P_{cr} = \left(\frac{L}{L_{cr}}\right)^2$, Eq. 12 can be written as

$$k = \frac{3EI_y}{L} \left[1 - \left(\frac{L}{L_{cr}}\right)^2 \right] \quad (13)$$

Equation 13 is compared with an exact solution for an axially loaded column in Fig. 9, and it is seen to give conservative results. The approximation of Eq. 13 is also conservative for the case of uniform moment (compressive forces uniformly distributed along the flange) or for zero moment ratio (Fig. 9).

Thus the spring constants of the two springs at the ends of beam segment BC are

$$k_\ell \cong \frac{3EI_\ell}{L_\ell} \left[1 - \left(\frac{L_\ell}{L_{\ell cr}}\right)^2 \right] \quad (14)$$

$$k_s = \frac{3EI_s}{L_s} \left[1 - \left(\frac{L_s}{L_{s cr}}\right)^2 \right]$$

The subscripts ℓ and s refer to the larger and the smaller spring constant, respectively.

The correction factor ν_y is defined as the ratio of the simply supported critical length of the center span in Fig. 8a to the restrained critical length. Knowing the spring constants k_ℓ and k_s , L_{cr} of the column shown in Fig. 8b can be determined by the slope deflection method, modified to include the effect of the axial force.⁽⁵⁾ The variables of the solution can be grouped into the ratios $\frac{k_s}{k_\ell}$, $\frac{k_\ell L}{3EI_y}$ and ν_y . (The derivations are not reproduced in this report.) The relationships between these three ratios are shown in Fig. 10. This figure shows ν_y versus $\frac{k_s}{k_\ell}$ for constant values $\frac{k_\ell L}{3EI_y}$.

For a continuous beam of constant section and equally braced spans the value of $\frac{k_\ell L}{3EI_y}$ is less than 1.0. That is, $I_s = I_\ell = I$, $L_s = L_\ell = L$, and thus $\frac{k_\ell L}{3EI} = \left[1 - \left(\frac{L}{L_{cr}}\right)^2\right]$ and $\frac{k_s}{k_\ell} = \left[\frac{1 - \left(\frac{L}{L_{scr}}\right)^2}{1 - \left(\frac{L}{L_{\ell cr}}\right)^2} \right]$. It may be seen in Fig. 10

that in this case the relationship between ν_y and $\frac{k_s}{k_\ell}$ can be approximated by a family of straight lines. The equation of these lines is

$$\nu_y = 1.0 + 0.18 \left[1 - \left(\frac{L}{L_{\ell cr}}\right)^2 \right] + 0.20 \left[1 - \left(\frac{L}{L_{scr}}\right)^2 \right] \quad (15)$$

The next step is to represent $(L_\ell)_{cr}$ and $(L_s)_{cr}$ in terms of the moment ratios of the two adjacent spans, ρ_s and ρ_ℓ , and the center span, ρ_{cr} . Under the moment diagram shown in the inset of Fig. 11 for example, the segment AB remains elastic and thus it gives a larger restraint to the critical segment BC than the segment CD which is next to the plastic hinge at C and is therefore partially plastified. Hence $\rho_\ell = \rho_{AB}$, $\rho_s = \rho_{CD}$ and $\rho_{cr} = \rho_{BC}$.

The following equations have been derived in the Appendix for the critical lengths of the adjacent segments:

$$\left(\frac{L_\ell}{r_y}\right)_{cr} = \frac{107}{\sqrt{|\rho_{cr}|}} \left[1.34 - 0.34 \rho_\ell \right] \quad (16)$$

$$\text{for } 0 \leq |\rho_{cr}| \leq 0.9$$

$$\left(\frac{L_\ell}{r_y}\right)_{cr} = 151 - 38 \rho_\ell - 1100 (|\rho_{cr}| - 0.9) \quad (17)$$

$$\text{for } 0.9 < |\rho_{cr}| < 1.0$$

$$\left(\frac{L_\ell}{r_y}\right)_{cr} = 48 - 30 \rho_\ell \quad (18)$$

for $|\rho_{cr}| = 1.0$ or if one end of the adjacent span contains a plastic hinge. (Span CD, Fig. 11)

If Eqs. 17 or 18 yield critical lengths smaller than $30 r_y$, $L_{\ell} = 30 r_y$ should be used, as was suggested for Eq. 11. For the adjacent segment furnishing the smaller restraint, $(L_s)_{cr}$ values may be obtained by using f_s instead of f_{ℓ} in Eqs. 16, 17 and 18. Equations 16, 17 and 18 are represented in Fig. 11 for various values of f_{cr} .

In the preceding discussion it was demonstrated that the correction factor ν_y can be determined if the simple plastic analysis moment diagram is available. In every day design practice it is not necessary to compute ν_y as precisely as outlined above. A value of $\nu_y = 1.25$ can be assumed as an appropriate value for typical restraint situations. Using this correction factor, the following simplified equation is obtained:

$$(L/r_y)_{cr} = 60 - 40 f \quad (19)$$

but not less than 30.

4. EXPERIMENTAL INVESTIGATION

Two series of tests were conducted to substantiate the assumptions of the lateral buckling theory. Each of

these test series consisted of four beam tests. In each test two symmetrically spaced loads were applied transversely to a simply supported beam (see inset in Fig. 14). The beam segment between the loads (segment BC in Fig. 14) was the critical segment. Lateral braces were supplied at the load points (B and C), preventing lateral movement of the compression flange.

The first four beam tests were experiments where the critical segment was subjected to a uniform moment and these have already been reported.⁽²⁾ Only the final results are given here (Table 2 and Fig. 18) for purposes of comparison. The side segments of these beams were boxed in to provide full lateral fixity, thereby making the correction factor $\psi_y = 2.0$ under the applied uniform moment gradient.

The arrangement of the test set-up is shown in Fig. 13 for four further tests (LB-5, LB-6, LB-7 and LB-8). Load was applied to the test beam at two equally spaced locations through a loading beam. The test beam was supported on two 12 in. diameter rollers placed on the base beam, which in turn rested on the weighing platform of the testing machine. The whole test set-up could be

shifted along the longitudinal axis of the beam, such that any combination of the ratio between the two loads, and thus the moment ratio ρ of the critical span, could be achieved.

The critical span of the test beam was the segment between the load points. Lateral bracing at the load points was accomplished by angle studs clamped to the base beam and extending upward on each side of the test beam (see section A-A in Fig. 12). Lateral support was given to the top flange only.

The instrumentation consisted of level bars placed at the loading points and the support points, measuring the rotation in the plane of bending; three additional level-bars, placed at the load points and at the mid-section of the critical segment, were used to measure the rotation of the mid-section with respect to the laterally supported load-points; deflection dials, measuring over a nine inch gage length, were placed on the top and the bottom flange to measure the average strain from which the curvature could be computed. A schematic view of the instrumentation is shown in Fig. 13.

Readings were taken at equal load increments as long as the beam remained elastic. During plastic deformations, increments of transverse deflection were gaged by a control dial at mid-span (see Fig. 13), measuring the relative distance between the base beam and the specimen. An initial load reading was taken after the desired deflection had been reached and maintained, and a final load reading was taken after all the gages had been read. The average of these two readings was taken as the recorded load.

Test curves for each of the four tests are shown in Figs. 14 through 17. In each figure an inset is included which shows the location and the relative magnitudes of the loads. Also shown is the moment diagram, indicating the location of the plastic moment M_p and the moment ratio ρ . Two curves, giving the relationship between the moment (non-dimensionalized by M_p) and the rotation at each reading point, are shown in the figures: One curve (circles) indicates the total rotation of the critical segment (obtained by adding the rotation of the level bar at the load points, see inset in Fig. 13) in the plane of bending. The other curve shows the lateral rotation of the center of the test beam. This curve is shown because it determines the load at which lateral buckling started.

The four test specimens were cut from the same piece of a 10WF21 beam. The physical properties of the material were obtained by tensile coupons cut from the web and the flanges of the beams. Table 1 shows the average static yield stress of the flanges, and the values of A, Z and I_x as determined from the measured dimensions. The plastic moment M_p , and the elastic curvature ϕ_p were computed from the above quantities, and they are also shown in Table 1.

The test results for all eight tests are shown in Table 2. In this table, the value of ν_f represents the lateral restraint of the neighboring segments, and it is computed by the methods outlined in Section 3 of this report. The rotation θ_R (non-dimensionalized as $\frac{\theta_R}{L\phi_p}$) is the plastic rotation in the critical segment at the point of maximum moment. The value of θ_R is calculated by subtracting the elastic rotation at $M = M_p$ from the rotation corresponding to the maximum rotation in the plane of the applied loads.

The test results are plotted on the ρ versus $\frac{L}{\nu_f r_y}$ curves obtained previously (Fig. 7) in Fig. 18. The inelastic rotation of each test is shown in parentheses

after each test point. From Fig. 18 it may be observed that if the average inelastic rotation $\Theta_R/L\theta_p$ is assumed to equal 3.0, the lower limit for the critical slenderness ratio need not be less than 30. The test points lying below this limit all exhibited rotation capacities well over $3L\theta_p$. Therefore the rule which stipulated that the minimum critical length be $30 r_y$ (for $\nu_y = 1.0$) (Eq. 11) is justified by tests.

5. RECOMMENDATIONS FOR DESIGN

The design simplifications for the spacing of lateral bracing of members subjected primarily to bending moments can be summarized in the following suggested procedure:

- (1) Obtain the ultimate bending moment diagram of the structure by the plastic theory.
- (2) Determine the member sizes and the inelastic rotations at each hinge.⁽¹⁾
- (3) Assume the spacing of the lateral braces, and determine the moment ratios of each critical span and its adjacent spans from the moment diagram in accordance with the assumed spacing.

- (4) Determine the critical length exclusive of the influence of the restraint of the side spans,

$$(L/r_y)_{cr} \frac{1}{\psi_f} \quad \text{from Fig. 6. The parameter } \frac{\Theta_R}{L\phi_p}$$

is computed by the following formulas:

$$\Theta_R = \frac{\Theta}{1 + g_R/g_L} \quad (6)$$

$$\phi_p = \frac{M_p}{EI_x} \quad (5)$$

In case the inelastic hinge rotation Θ is not computed, the critical length is obtained by $(L/r_y)_{cr} \left(\frac{1}{\psi_f} \right) = 48-30 \rho$ (Eq. 10) but not smaller than 30. If the assumed spacing of the lateral supports is smaller than the critical length obtained in the preceding operation, the spacing is conservative.

- (5) Obtain the correction factor for the restraint of the adjacent segments ψ_f from Fig. 10. The spring constants of the adjacent segments k_ℓ and k_s are determined by the following formulas:

$$k_l = \frac{3EI_l}{L_l} \left[1 - \left(\frac{L_l}{L_{lcr}} \right)^2 \right] \quad (14)$$

$$k_s = \frac{3EI_s}{L_s} \left[1 - \left(\frac{L_s}{L_{scr}} \right)^2 \right]$$

(6) The critical lengths L_{lcr} and L_{scr} of the adjacent segments are

$$\left(\frac{L_l}{r_y} \right)_{cr} = \frac{107}{\sqrt{|\rho_{cr}|}} \left[1.34 - 0.34 \rho_l \right] \quad (16)$$

$$\text{for } 0 \leq |\rho_{cr}| \leq 0.9$$

$$\left(\frac{L_l}{r_y} \right)_{cr} = 151 - 38 \rho_l - 1100 (|\rho_{cr}| - 0.9) \quad (17)$$

$$\text{for } 0.9 < |\rho_{cr}| < 1.0$$

$$\left(\frac{L_l}{r_y} \right)_{cr} = 48 - 30 \rho_l \quad (18)$$

$$\text{for } |\rho_{cr}| = 1.0$$

(7) In case the spacing of the braces is uniform

($L_l = L_{cr} = L_s$), and a beam of constant cross

section is used, the value of ν_y can be obtained

from the following formula:

$$\nu_y = 1.0 + 0.18 \left[1 - \left(\frac{L}{L_{l \text{ cr}}} \right)^2 \right] + 0.20 \left[1 - \left(\frac{L}{L_{s \text{ cr}}} \right)^2 \right] \quad (15)$$

where $L_{s \text{ cr}}$ and $L_{l \text{ cr}}$ are computed by Eqs. 15, 16 or 18.

(8) For the most rapid and simplified operation (ν_y assumed to be 1.25), the assumed bracing spacing may be checked by the application of the following simple formula:

$$\left(\frac{L}{r_y} \right)_{\text{cr}} = 60 - 40 \rho \quad (19)$$

but not less than 35.

Example problems for the determination of the bracing spacing are given in Refs. 2, 6 and 7.*

*The equations for the critical length of the adjacent spans in Ref. 7 are different from the equations developed in this report. This is due to a difference in derivation and due to an arithmetical error in the preliminary report. Therefore, the equation

$$\left(\frac{L_l}{r_y} \right)_{\text{cr}} = \frac{134}{\sqrt{|\rho_{\text{cr}}|}} + 60 (1 - \rho_l)$$

in Ref. 7 (Eq. 6.21) is incorrect.

6. S U M M A R Y

In the preceding sections of this paper recommendations are made for the determination of the spacing between lateral braces in plastically designed beams. The rules are applicable for plastically designed steel frames if the members are primarily subjected to bending moments and if they are fabricated of parallel-flanged members.

The recommendations are based on simplifications made on a theory which was proposed by White.⁽²⁾ For routine calculations a simple equation is given (Eq. 19); if the problem warrants a more precise analysis, formulas and charts are provided.

7. ACKNOWLEDGEMENTS

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8. NOMENCLATURE

- d = Depth of section
 E = Modulus of elasticity
 g = Moment gradient
 G = Shear modulus
 I_x = Moment of inertia about x-axis of section
 I_y = Moment of inertia about y-axis of section
 I_w = Warping constant
 K = St. Venant's torsion constant
 k = Spring constant of restraining segment
 L = Length of unbraced span
 M_o = Larger of two end moments
 M_p = Plastic moment
 r_y = Weak axis radius of gyration
 Z = Plastic modulus
 α = Extent of yielding in the beam
 θ = Inelastic rotation
 v_α = Correction factor for the effect of practice strain-hardening
 v_δ = Correction factor for the degree of end fixity
 v_ρ = Correction factor for moment ratio
 v_s = Correction factor for St. Venant's torsion

ρ = Moment ratio

σ_y = Yield Stress

ϕ = Curvature

ϕ_p = Elastic curvature at $M = M_p$

Subscripts:

st = Strain-hardening

l = Larger

s = Smaller

cr = Critical

R = Right

L = Left

9. APPENDIX

DETERMINATION OF L_{cr} FOR THE ADJACENT SPANS.

For the three-span beam shown in the inset of Fig. 11, the absolute value of the moment at the support B is equal to

$$M_B = |\rho_{BC}| M_p = |\rho_{BC}| Z \sigma_y \quad (A-1)$$

The critical length for the span AB is ⁽²⁾

$$(L_{AB})_{cr} = \pi \nu_{\rho_{AB}} \sqrt{\frac{E \sqrt{I_w I_y}}{M_B}} \quad (A-2)$$

The coefficient $\nu_{\rho_{AB}}$ represents the correction factor for the moment ratio of beam AB; the rest of the equation is the critical length due to uniform moment if St.

Venant's torsion is neglected (Eq. 4.11, Ref. 2). For simplicity it is assumed that the beam AB is not restrained at its ends. Due to this assumption and due to the neglect of St. Venant's torsion (which has considerable influence in elastic buckling), the critical length obtained from Eq. A-2 is smaller than the actual L_{cr} of the beam. The influence of this on the value of ν_{ρ} , is however, quite small, since the variations of $(L_{cr})_{cr}$ cause little

change in the term $1 - \left(\frac{L}{L_{cr}}\right)^2$ if $L \ll L_{cr}$

(which is the usual case).

Substituting A-1 in A-2, noting that the warping constant $I_{\omega} = \frac{I_y d^2}{4}$ for wide-flange shapes, and rearranging the equation, the critical length of the adjacent span is equal to

$$\left(\frac{L_{AB}}{r_y}\right)_{cr} = \pi \nu_{\phi} \sqrt{\frac{E}{|\rho_{BC}| 2\sigma_y} \left(\frac{I_y d}{Z r_y^2}\right)} \quad (A-3)$$

The ratio $\sqrt{\frac{I_y d}{Z r_y^2}} \approx 1.6$ for most rolled wide-flange sections. (2) The correction factor ν_{ϕ} is given by Eq. 2; substitution of ν_{ϕ} and the constants $E = 30 \times 10^6$ psi and $\sigma_y = 33$ ksi into Eq. A-3 gives the following equation for L_{cr} :

$$\left(\frac{L_{AB}}{r_y}\right)_{cr} = \frac{107}{\sqrt{|\rho_{BC}|}} \left[1.34 - 0.34 \rho_{AB} \right] \quad (A-4)$$

Equation A-4 is only valid if the span AB remains elastic (that is $\rho_{BC} \leq 0.9$ - see Ref. 2). If the adjacent span is next to a plastic hinge (span CD in Fig. 11), or if $\rho_{BC} = 1.0$, the critical length is

determined by Eq. 10. A graphical representation of Eq. A-4 for various constant ratios of ρ_{BC} is given in Fig. 11. Also shown in this figure is the curve for $\rho_{BC} = 1.0$ (Eq. 10). If a straight line variation between the limit of elastic buckling and buckling when one end of the adjacent span contains a plastic hinge is assumed, the following equation can be derived for the range $0.9 < |\rho_{BC}| < 1.0$:

$$\left(\frac{L_{AB}}{r_y}\right)_{cr} = 151 - 38 \rho_{AB} - \left[\rho_{BC} - 0.9\right] \left[1030 - 80 \rho_{AB}\right] \quad (A-5)$$

In this equation, the term $80 \rho_{AB}$ is small when compared with 1030, and therefore the equation may be abbreviated to

$$\left(\frac{L_{AB}}{r_y}\right)_{cr} = 151 - 38 \rho_{AB} - 1100 (\rho_{BC} - 0.9) \quad (A-6)$$

A line representing Eq. A-6 for $\rho_{BC} = 0.95$ is shown in Fig. 11.

TABLE 1

Section and Material Properties (measured)

Cross Section	10WF21
Yield stress, σ_y	38.3 ksi (static yield level of flange material)
Area	6.39 in ² (6.19 in ² Handbook value)
Plastic Modulus, Z	24.87 in ³
Moment of Inertia, I _x	109.90 in ⁴ (106.3 in ⁴ Handbook value)
Plastic Moment	953 kip-in
Elastic Curvature, $\frac{M_p}{EI_x} = \phi_p$	0.000289 rad/in

TABLE 2

Test No. Results

Test	Section	ν_y	$\frac{L}{\nu_y r_y}$	f_{cr}	$\frac{\theta}{L\phi_p}$ at M_{max}
LB-1	10WF29	2.00	11.4	1.000	23.5
LB-2	10WF29	2.00	20.5	1.000	5.9
LB-3	16WF36	2.00	11.2	1.000	6.2
LB-4	10WF29	2.00	35.8	1.000	0.5
LB-5	10WF21	1.27	37.8	0.388	4.8
LB-6	10WF21	1.30	29.6	0.706	4.6
LB-7	10WF21	1.40	20.6	0.912	8.9
LB-8	10WF21	1.45	19.9	0.980	10.4

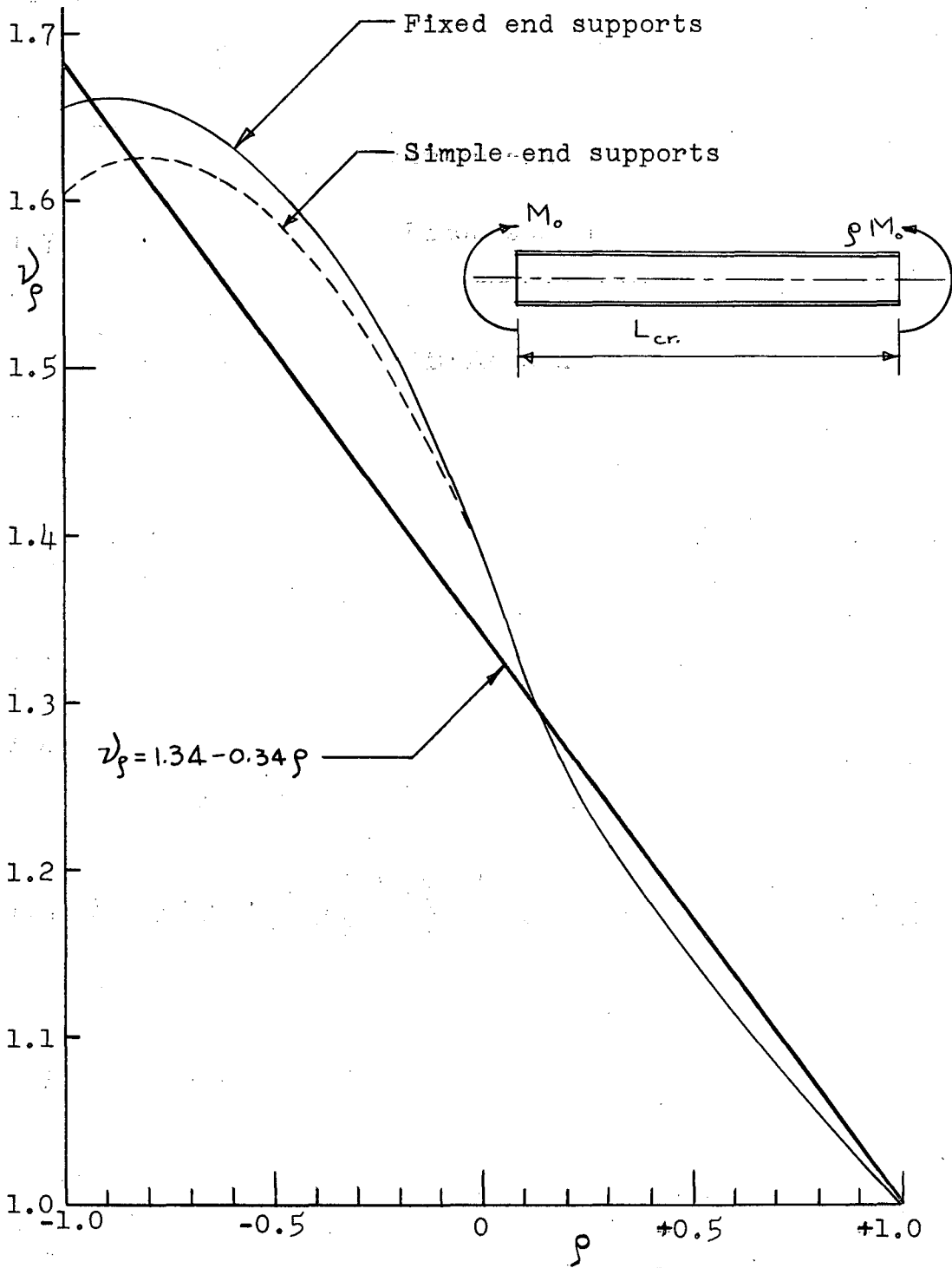


Fig. 1 - CORRECTION FACTOR FOR MOMENT RATIO

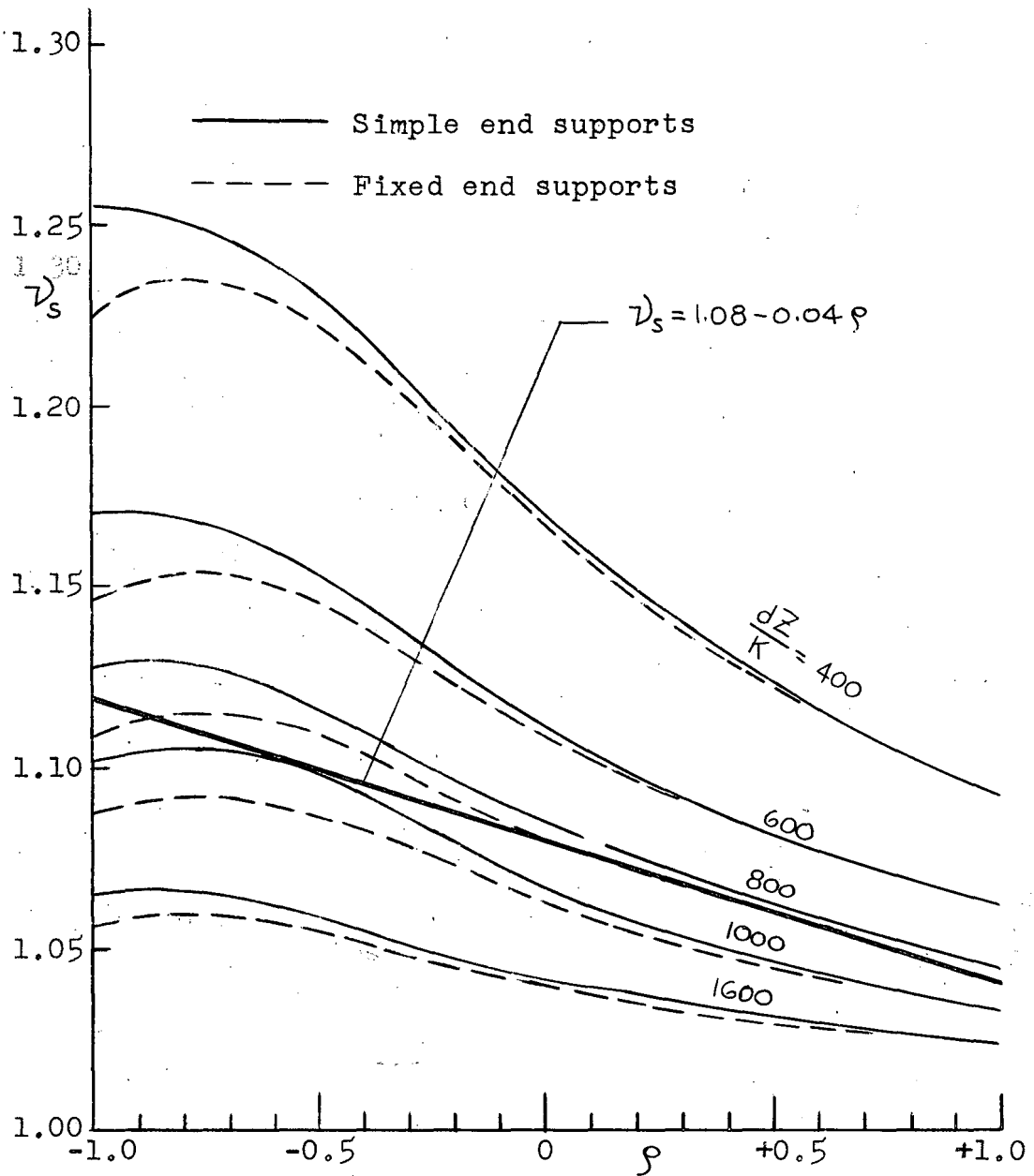


Fig. 2 - CORRECTION FACTOR FOR ST. VENANT'S TORSION

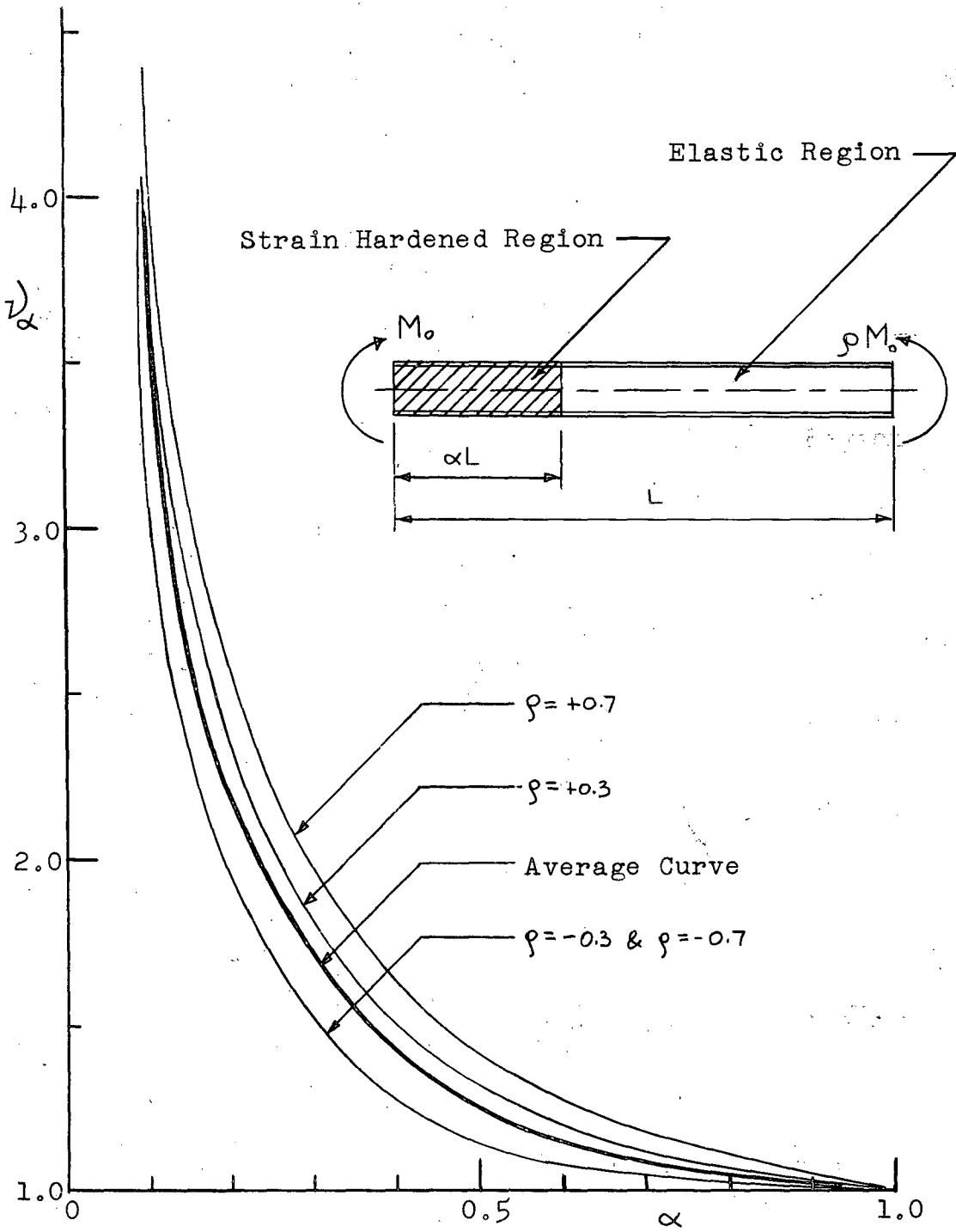


Fig. 3 - AVERAGE α VS. ν_α CURVE FOR ANY BOUNDARY CONDITION AND MOMENT RATIO

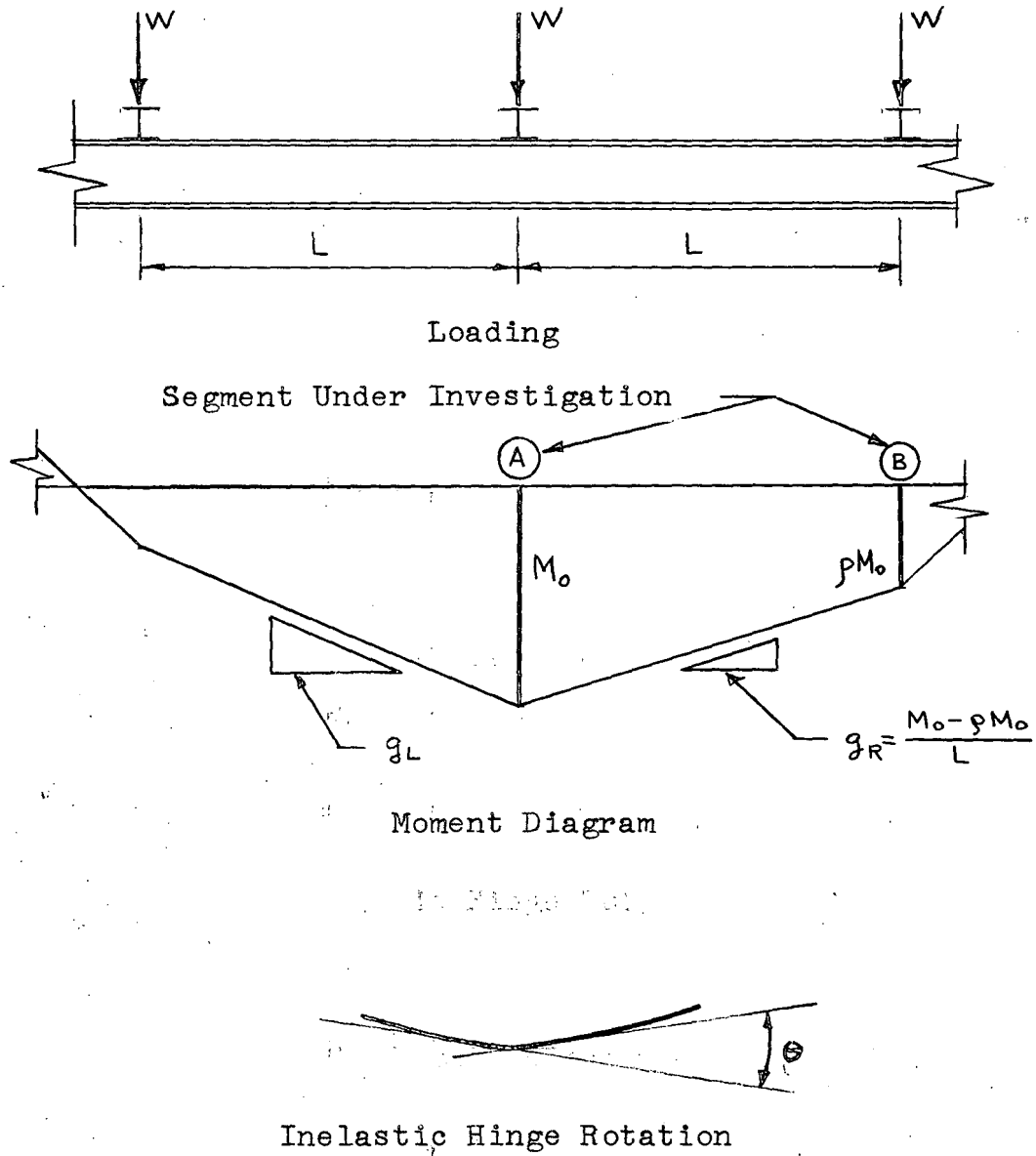


Fig. 4 - DEFINITION OF MOMENT GRADIENT AND HINGE ANGLE
FOR EQS. 4 THROUGH 8

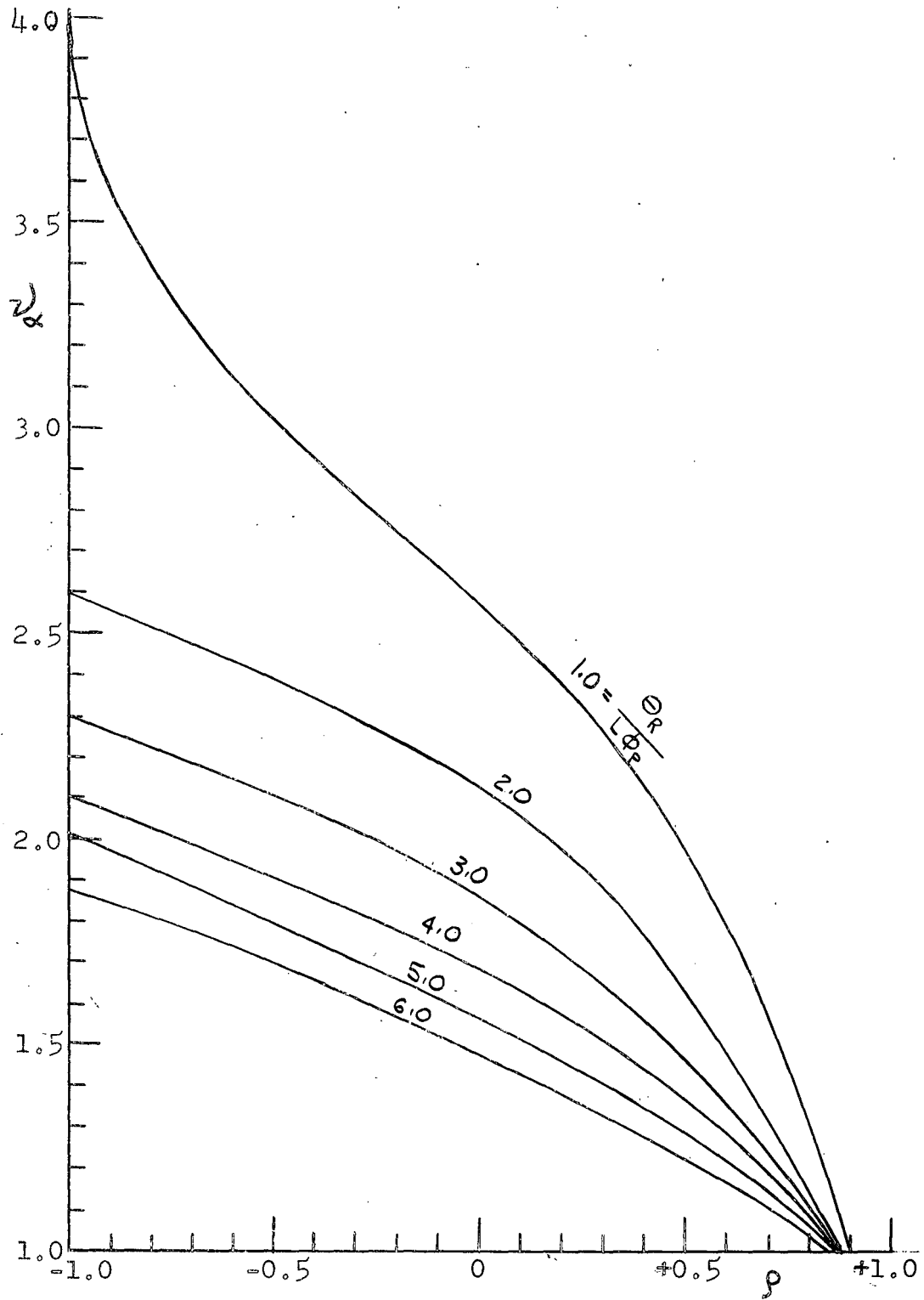


Fig. 5 - CORRECTION FACTOR FOR PARTIAL STRAIN HARDENING

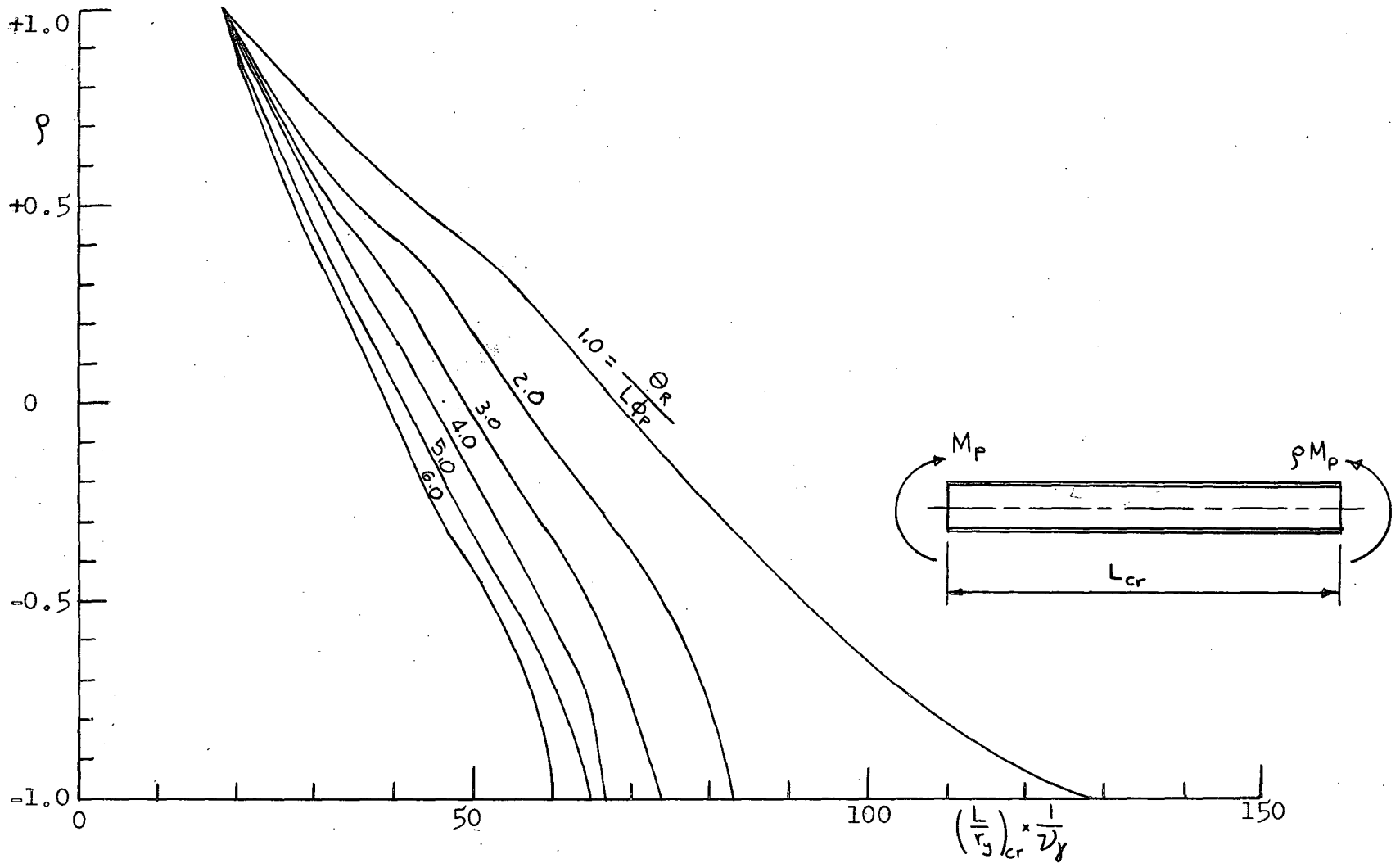


Fig. 6 - Laterally Unsupported Critical Length of Beams

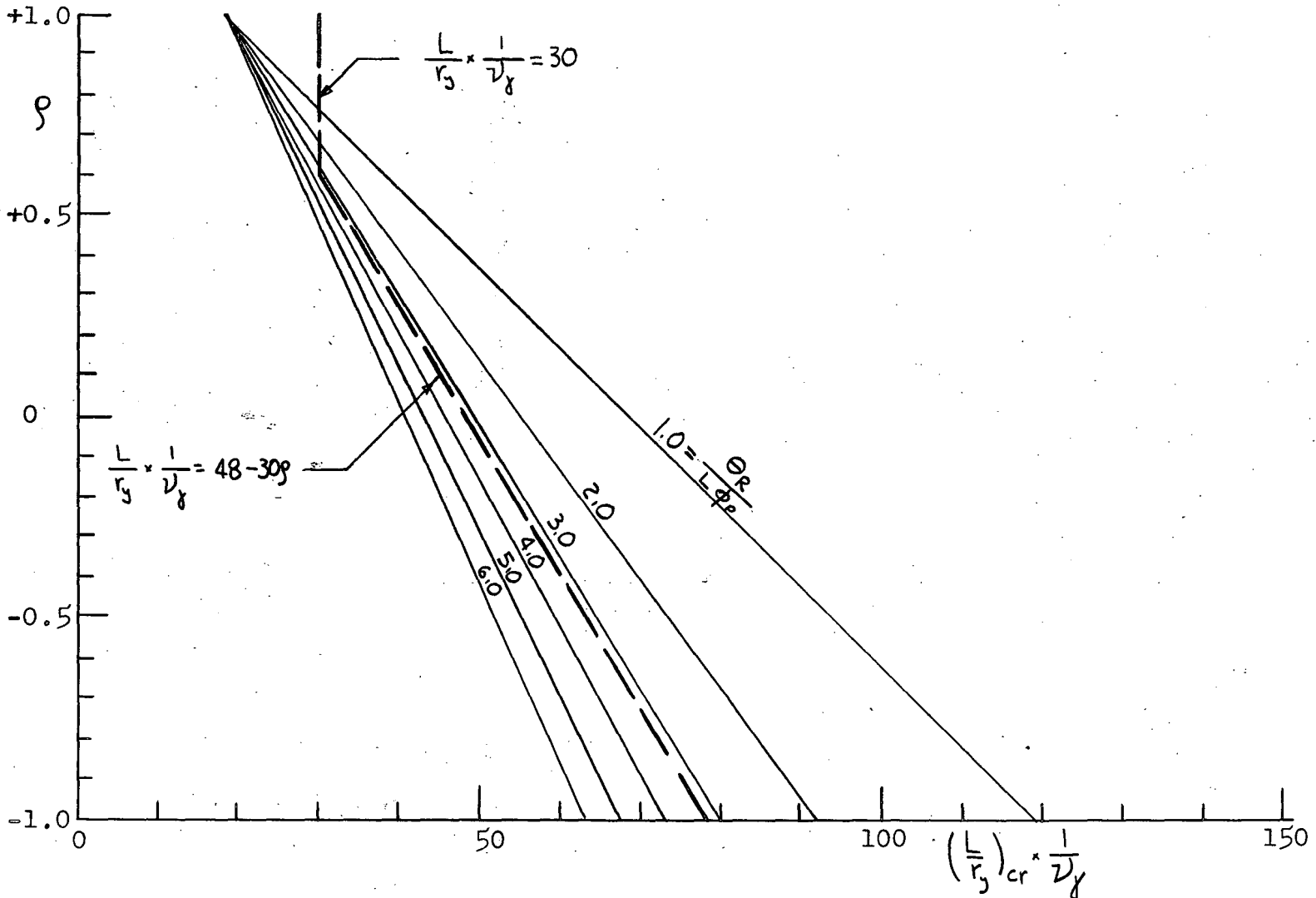


Fig. 7 - LINEAR APPROXIMATIONS FOR THE CRITICAL LENGTH

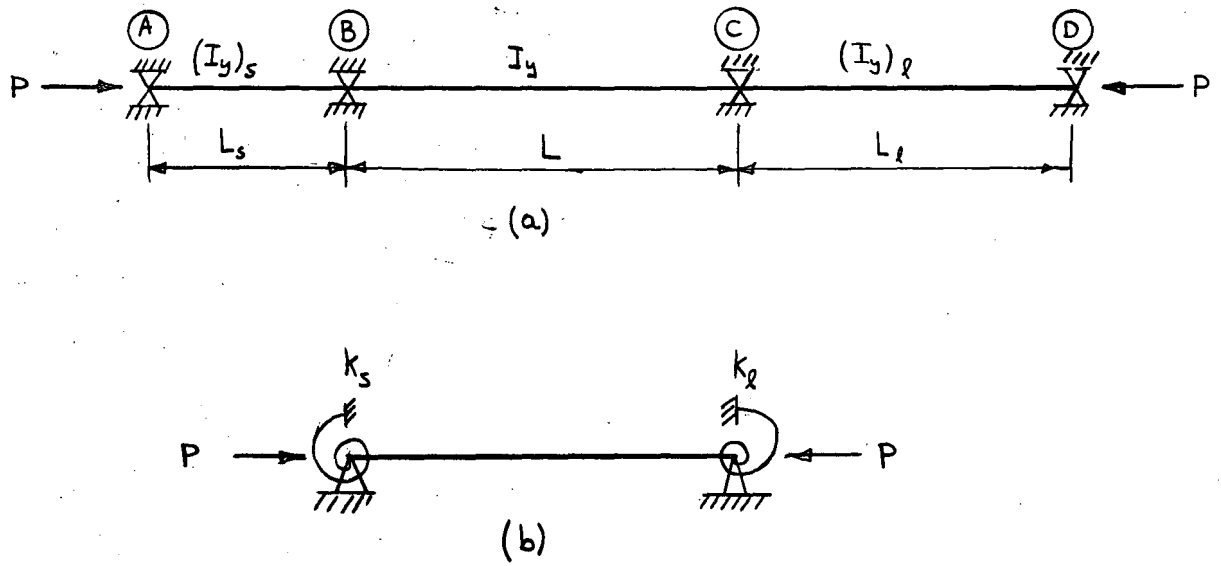


Fig. 8 - DEFINITION FOR EQS. 11 THROUGH 16

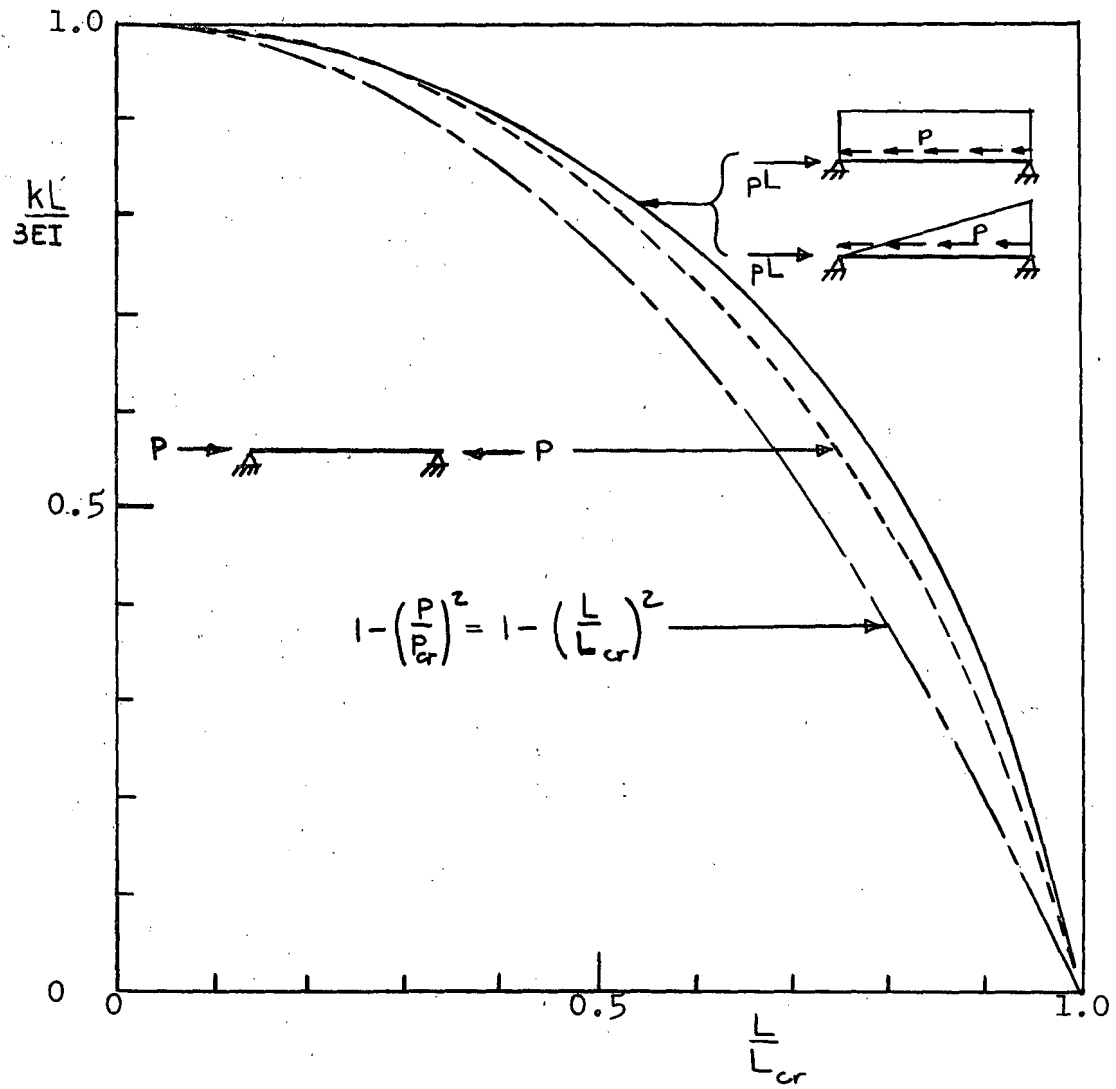


Fig. 9 - STIFFNESS FACTORS UNDER VARIOUS LOADINGS

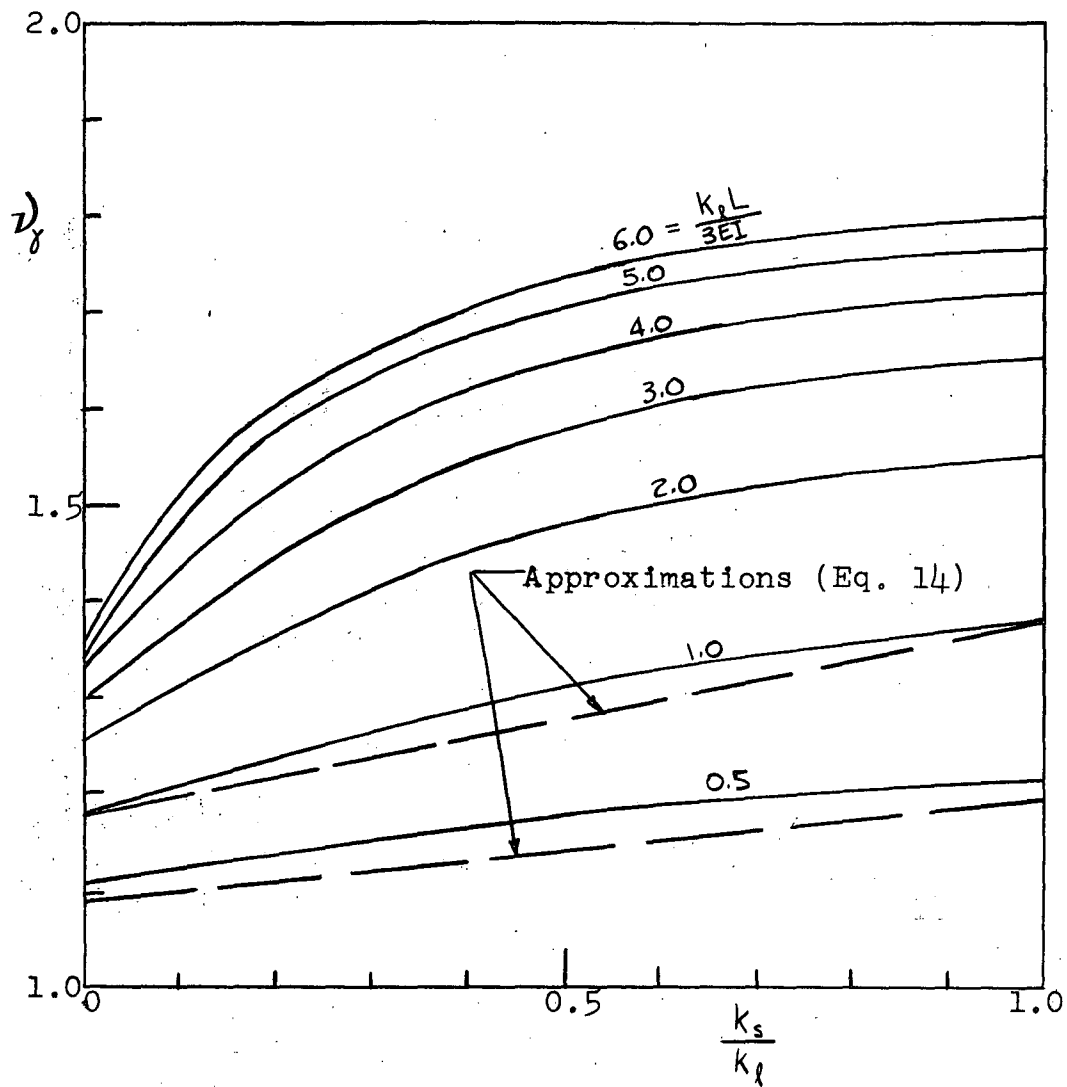


Fig. 10 - EFFECT OF END RESTRAINT ON v_y

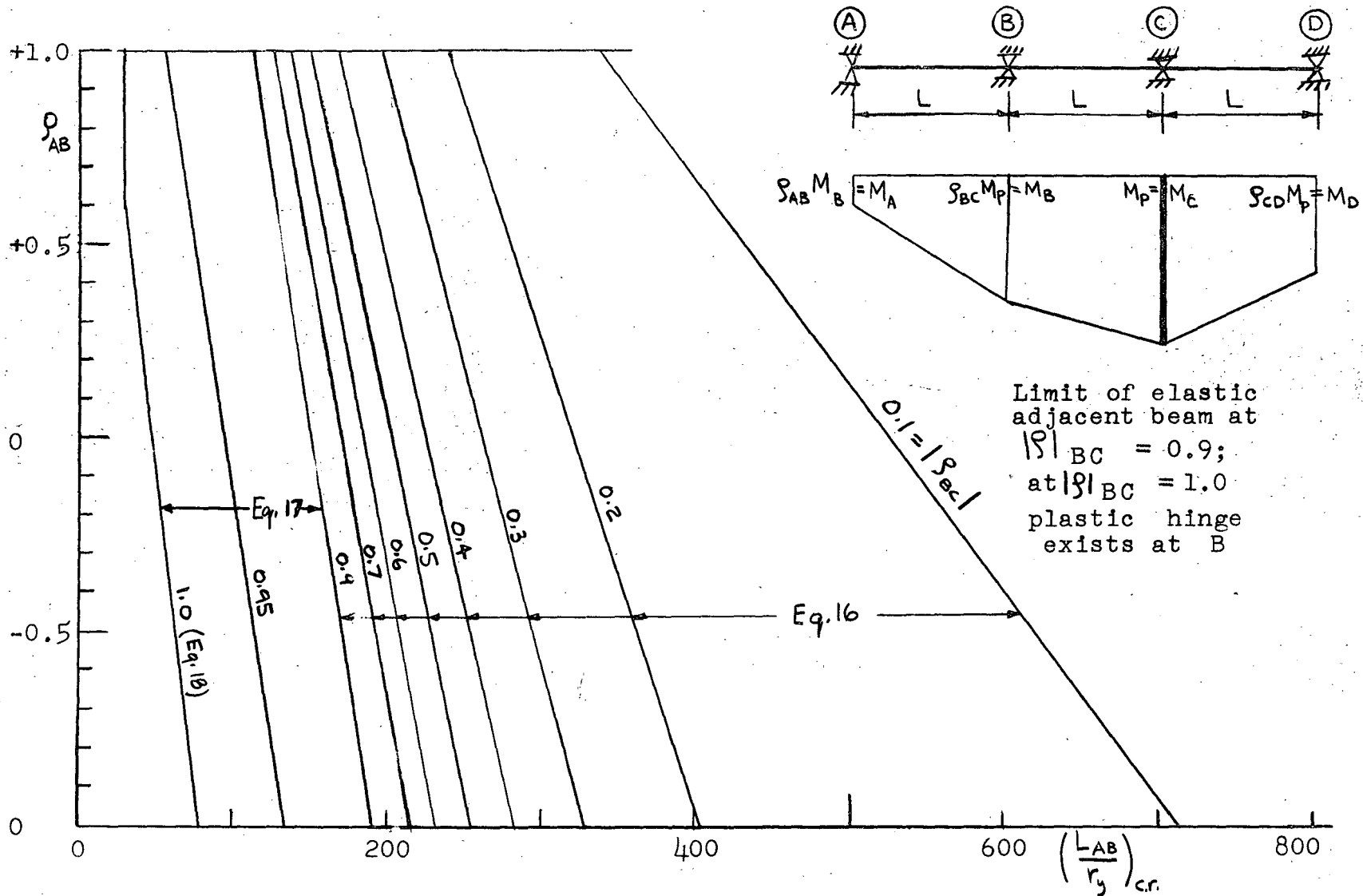
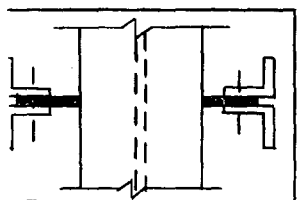
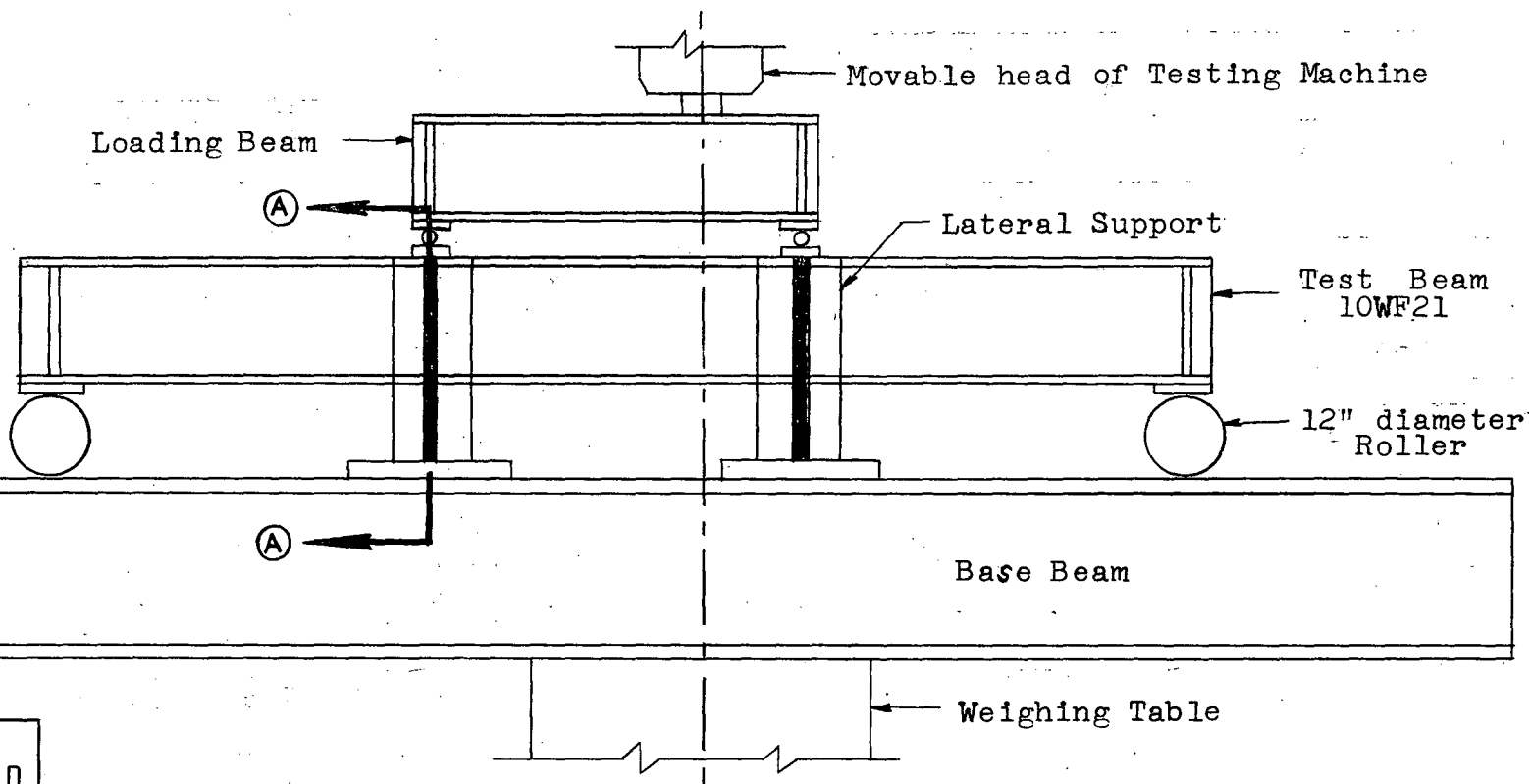
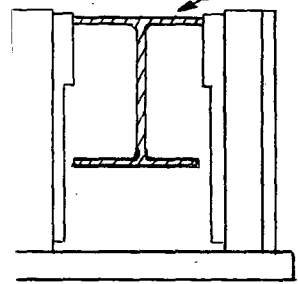


Fig. 11 - THE CRITICAL LENGTH OF ADJACENT BEAMS



Support to Top Flange Only



Section A-A

Fig. 12 - SCHEMATIC VIEW OF TEST SETUP

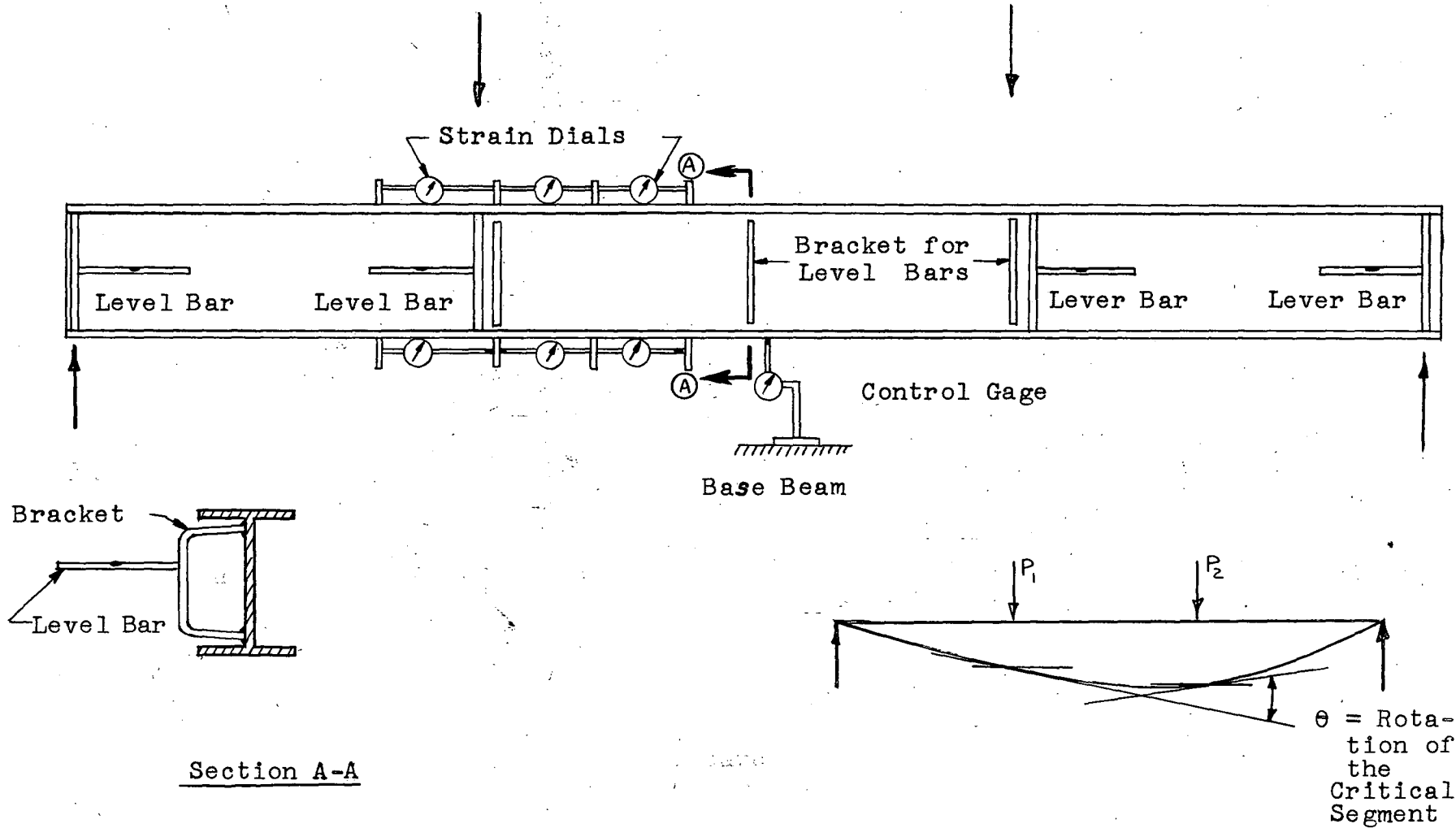


Fig. 13 - SCHEMATIC SKETCH OF THE INSTRUMENTATION

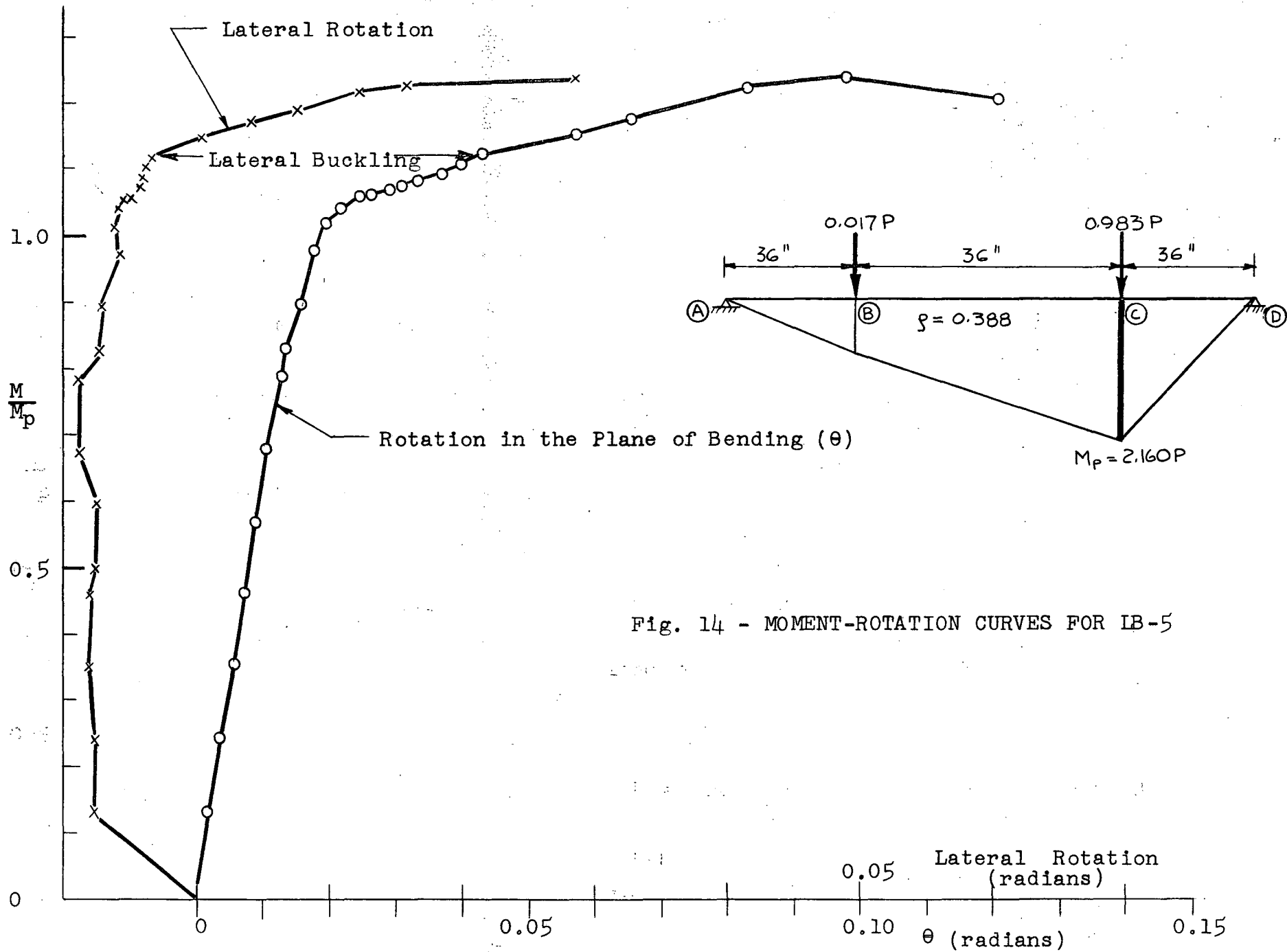


Fig. 14 - MOMENT-ROTATION CURVES FOR LB-5

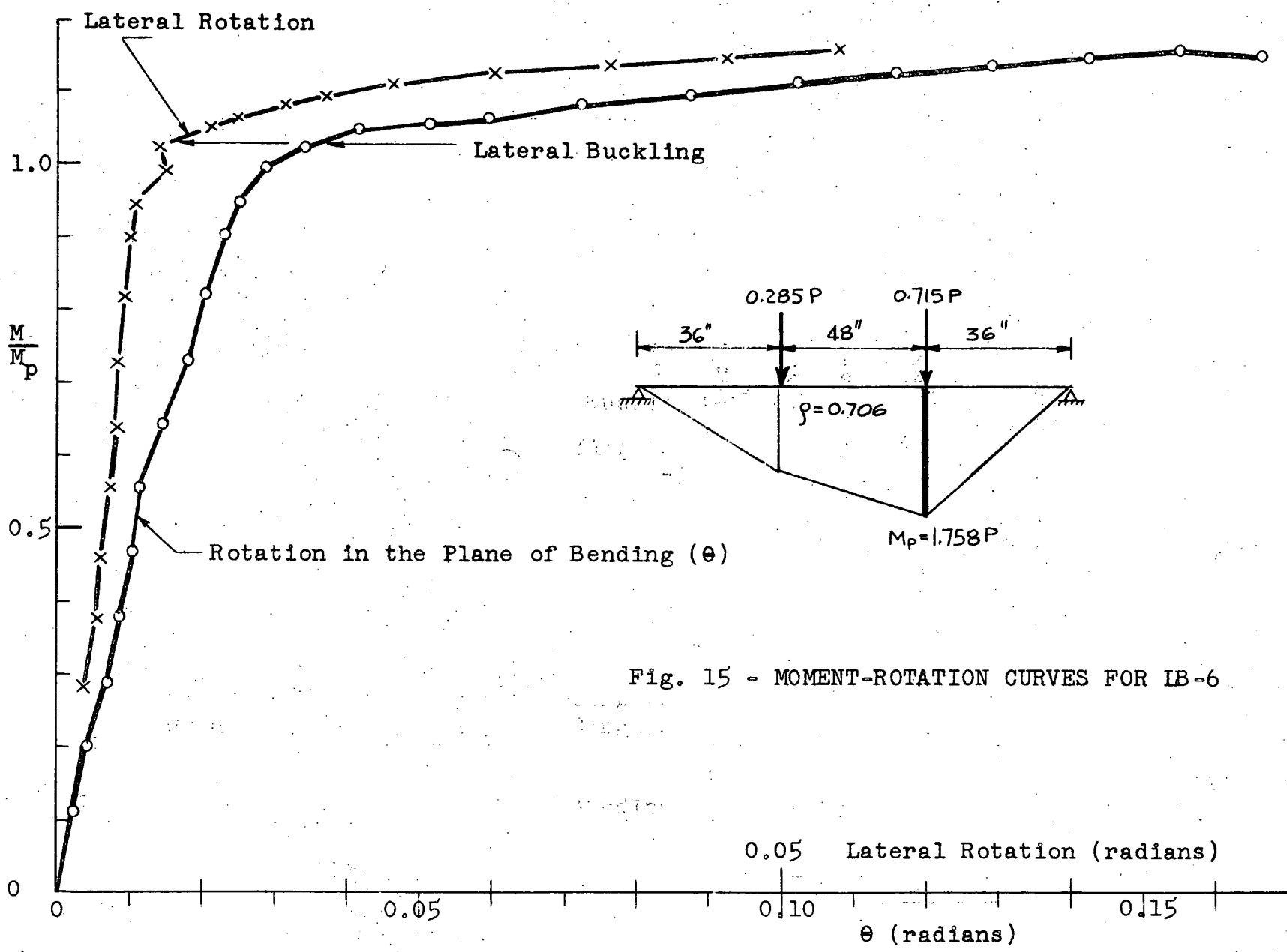


Fig. 15 - MOMENT-ROTATION CURVES FOR LB-6

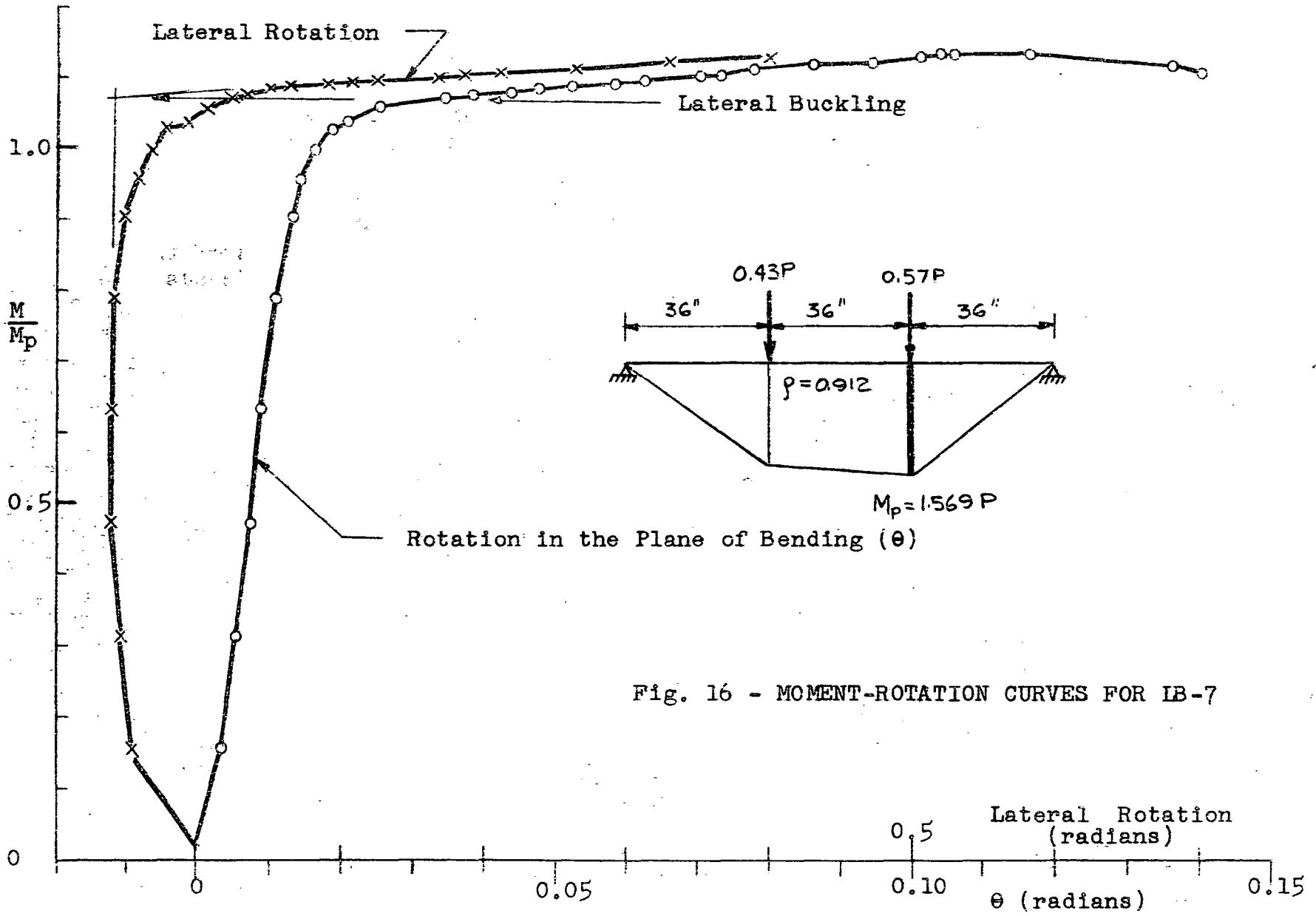


Fig. 16 - MOMENT-ROTATION CURVES FOR LB-7

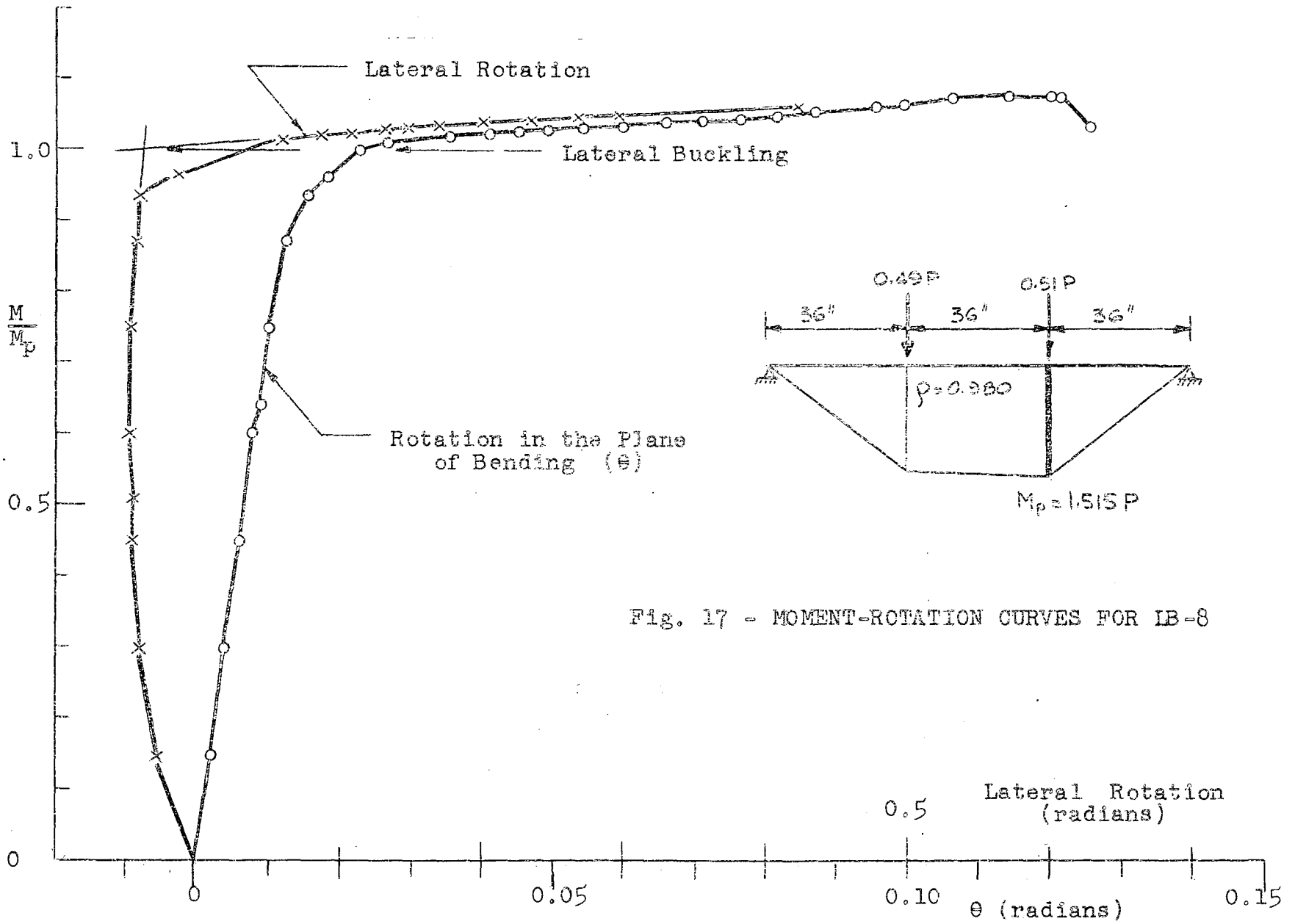


Fig. 17 - MOMENT-ROTATION CURVES FOR LB-8

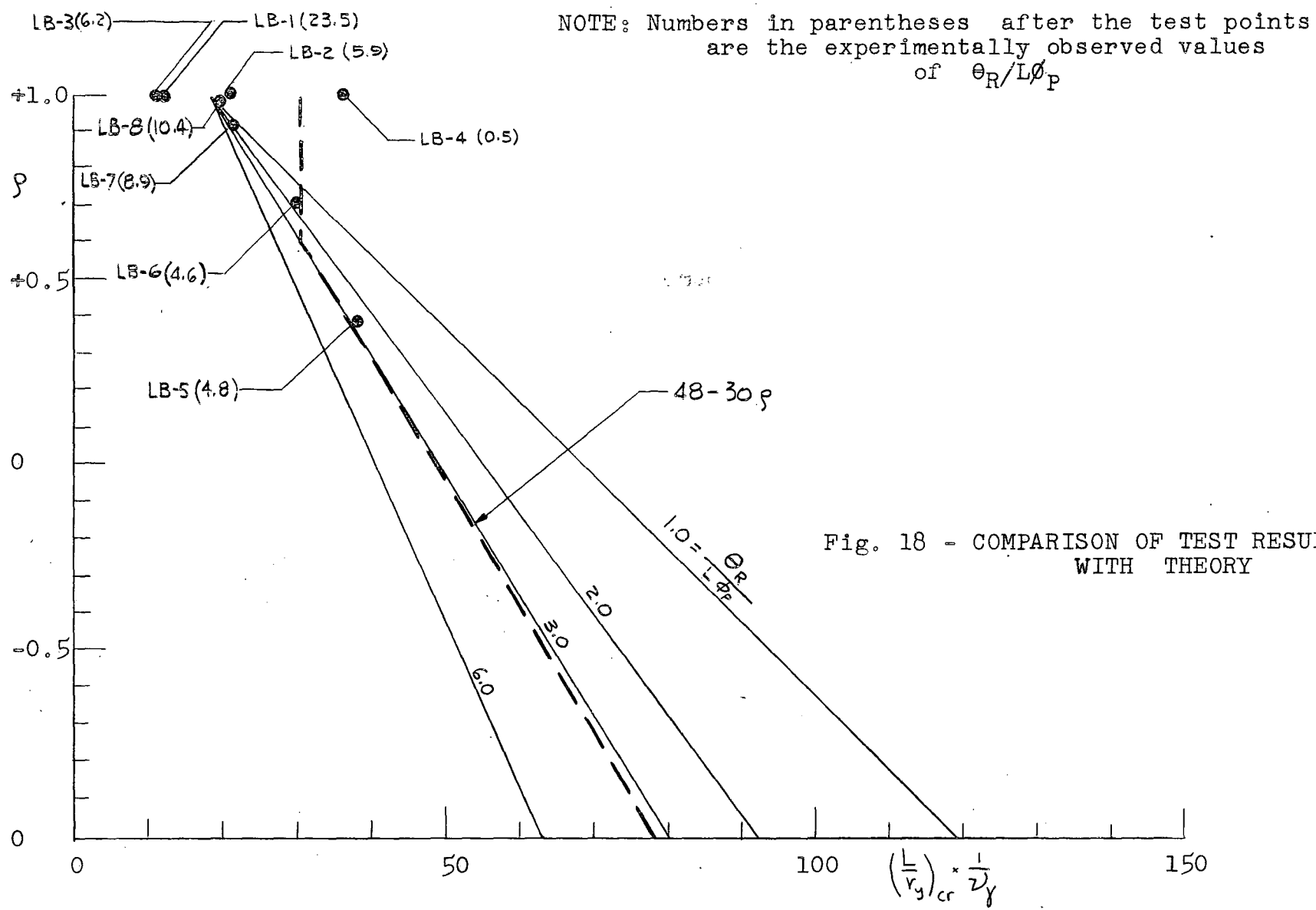


Fig. 18 - COMPARISON OF TEST RESULTS WITH THEORY

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