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WELDED CONTINUOUS FRAMES AND THEIR COMPONENTS

PROGRESS REPORT Q

INELASTIC LOCAL BUCKLING OF WF-SECTIONS

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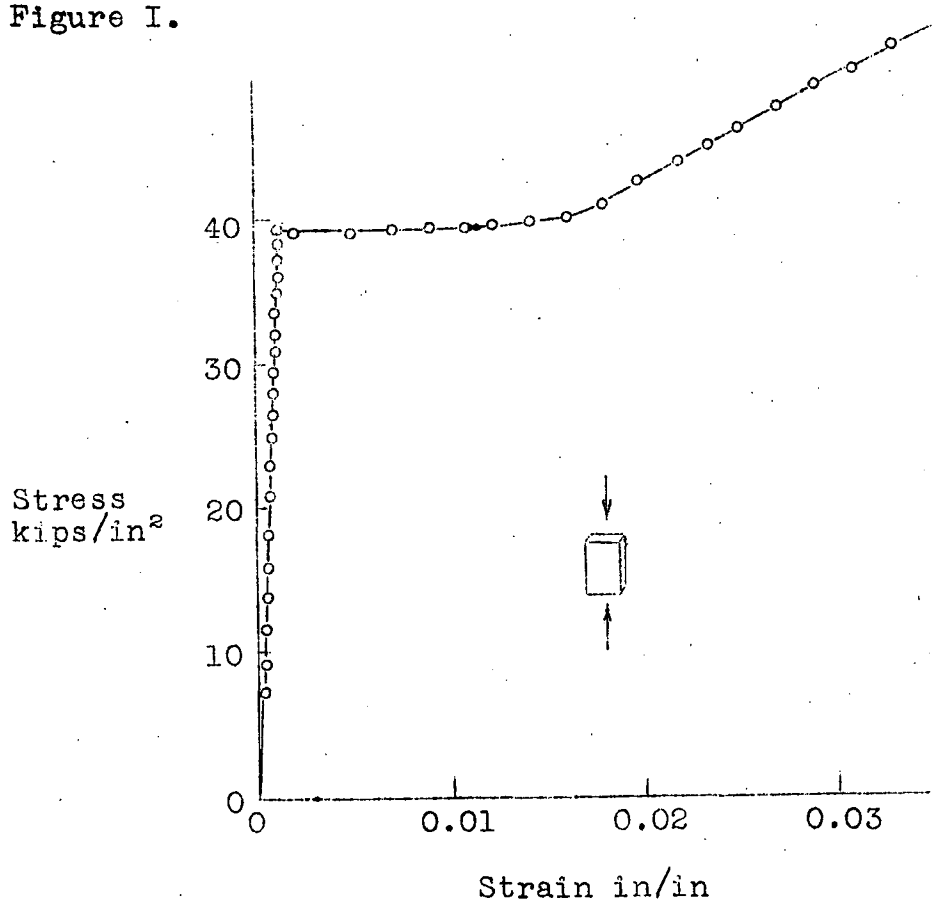
I. INTRODUCTION

During recent years there has been concentrated research into the inelastic behavior of steel structures. It is now apparent that some of the new theories formulated may be applied to the design of certain type structures, thus bringing an economy of material. However, the analysis of the ultimate strength of continuous structures, based on the simple plastic theory, is realistic only if plastic hinges are properly developed. The seriousness of premature inelastic buckling of structural steel members has been revealed by the results of a research program on "Welded Continuous Frames and Their Components" being carried out in Fritz Laboratory at Lehigh University. Some beams, columns and connections developed their calculated plastic hinge strength and some buckled before reaching the calculated value. Of those which did develop the theoretical plastic hinge moment, some collapsed immediately, while others continued to carry the hinge moment as straining was increased.

Structural members of perfectly plastic material theoretically have no resistance to buckling when the average compressive stress has reached the yield point, no matter whether the buckling concept is based on the tangent modulus theory or the double modulus theory. Thus, the compressed elements of structural members should buckle when the compressive yield stress is reached. However, as evidenced by a large number of tests, such buckling does not necessarily occur in the compression flanges of some of the rolled shapes.

The usual coupon compression tests of structural steel specimens have always shown considerable resistance to buckling in the plastic range. This is evident, for example, in the typical compression stress-strain diagram obtained from a small coupon shown in Figure I.



According to some studies made, it has been attributed to the yielding process of steel. The same reason might have kept some of the beam, column and connection specimens from buckling in the early plastic region. This is discussed in further detail in the next section.

To establish an effective value equivalent to the tangent modulus for steel for calculating the buckling strength of small steel coupons in the plastic range, there is proposed a series of compression tests of precisely aligned steel specimens which would be strained through the plastic range into the strain hardening range. Results will be applied to the analysis of local buckling strength of WF flanges. A further program is recommended for the investigation of the local buckling strength of various WF sections tested as short columns and beams. This series would be followed by tests of WF shapes under bending moment.

The ultimate aim of this investigation is to establish a specification for the required geometric projection of I and WF shapes so that plastic hinges can be developed and maintained through a considerable range of rotation under various loading conditions without reduction due to local buckling. Emphasis will be given to designating the geometric properties of those available sections which meet the requirements of strength and of rotation capacity.

Discussions of lateral buckling of bending members in the plastic range are also included.

II. INELASTIC BUCKLING CRITERION OF STRUCTURAL STEEL

Shanley has proved that an ideal column will start to bend at a load equal to its tangent modulus load.* According to the average stress and strain diagram of structural steel, the tangent modulus in the plastic range prior to strain hardening is zero. Thus, the criterion of the tangent modulus load is without real meaning in the limiting case. Therefore, in columns loaded with a slight eccentricity (provided the columns are made of a material which exhibits

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*The tangent modulus load P_t is computed from the expression

$$P_t = \frac{\pi^2 E_t I}{(KL)^2}$$

where E_t = tangent modulus determined from compressive stress-strain diagram

I = moment of inertia

L = actual length of column

K = constant depending on end condition

($K = 1$ for pin-ended column; $K = \frac{1}{2}$ for fixed-ended column) the above-stated zero slope through the plastic range) the average compression stress on the column cross-section can never reach the yield point. Ideally loaded columns made of perfectly plastic material** are actually unstable when the average stress reaches the yield point. Such a column may collapse at any instant when the average stress reaches the yield point regardless of the length of the column.

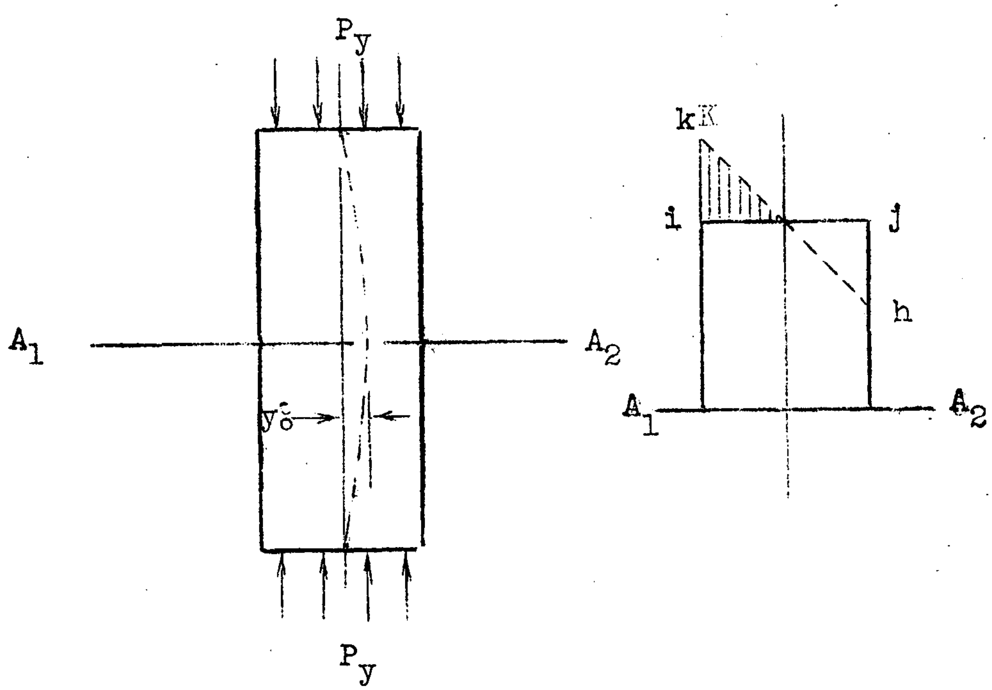


Fig. 2

**The stress-strain diagram is assumed to consist of two straight lines: one with a slope of E up to the lower yield point stress; the other with zero slope.

The above statement can be demonstrated as follows:

Let P_y be the load at which the average compression stress on the column reaches the yield point

$$P_y = A\sigma_y$$

where "A" equals the total cross-section area of the column, σ_y the lower yield point stress.

Suppose the ideal column is loaded with axial load P_y and a small displacement is made along the column such as the dotted lines show in Fig. 2. Considering a cross-section $A_1 - A_2$, let the corresponding displacement at that section be y_0. Then the section is under a total axial load P_y and a moment $M = P_y y_0$. In order to balance both the moment and the axial load, the stress distribution must be changed from ij to kh. But the stress cannot be any higher than σ_y if the material is perfectly plastic, and the added stress indicated by the shaded area in Figure 2 is therefore impossible. The external moment will not be balanced. It is obvious that the column will collapse in bending as soon as the small displacement is made.

This reasoning would lead to a conclusion that a member will never sustain yield point stress in compression if it is made of perfectly plastic material.

In the simple plastic theory of structural analysis, sections at which strains exceed the yield point strain are assumed to develop "plastic hinges". In the case of WF sections there is, from the above discussion, a possibility of buckling of the compression flange. This would naturally reduce the value of the plastic hinge moment. Therefore, the problem of instability of structural steel members becomes very important when consideration is given to plastic design.

Actually none of the metals used in engineering structures are perfectly plastic. In the case of structural steel, for example, strain hardening commences at a strain of about 15 times the yield point strain. A typical example was shown in Fig. 1. But the stress-strain curve from ordinary tension and compression tests usually do not give enough information for the analysis of stability problems in the plastic range; i.e., whereas, according to the coupon test, the column should buckle...it does not. The mechanism of yielding of steel has been observed to affect the buckling strength of compression members.

In a simple tension or compression test of a structural steel bar, one will find that Lüder's lines do not all appear at once when the yield point is reached. Yield lines usually are initiated in some places and then gradually spread over the whole specimen. While the yield lines are progressing, the region where the yield lines were initiated might have developed all its plastic strain and reached the strain hardening range locally. The specimen cannot be considered as perfectly plastic even though a portion of the stress and strain diagram is observed to be parallel to the abscissa. The compression member can therefore be expected to have, theoretically, a buckling strength of tangent modulus load with the modulus chosen from the ordinary stress-strain curve at the starting point of the strain hardening range, ϵ_s, Fig. 3,

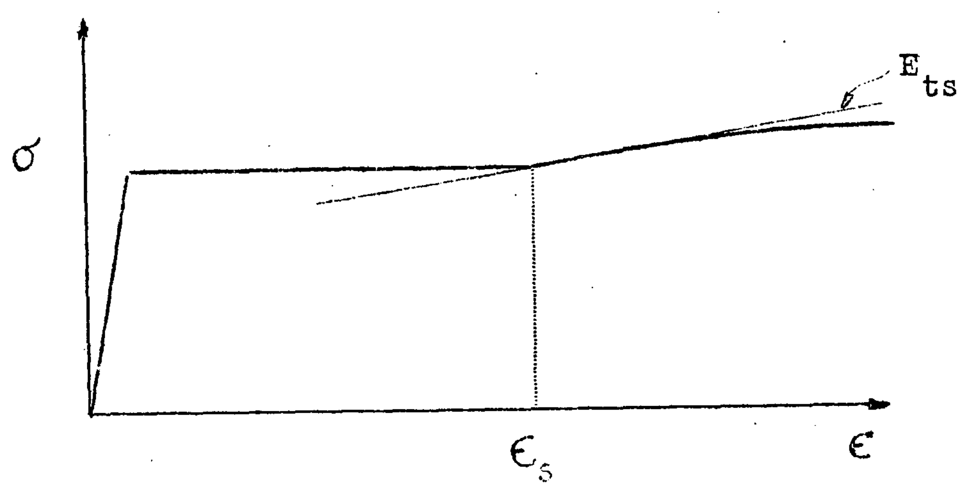


Fig. 3

How does this yielding process affect the buckling strength of a compression member? It can be demonstrated by the following analogical example:

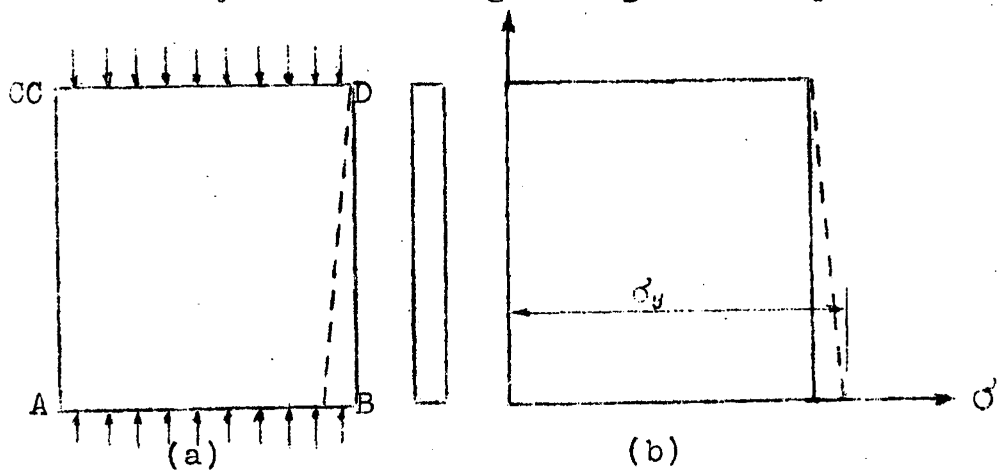


Fig. 4

Suppose the compression member is slightly tapered on one side, as shown in dotted lines in Fig. 4 (a). Instead of having a uniform stress throughout its length, the stress distribution along the member will be as shown by the dotted lines, Fig. 4 (b). The root section AB will reach the yield point first, and as stress increases the yield zones will progress to reach the top section CD. If the mechanical properties of the material are homogeneous, every section will be strain hardened as soon as the stress exceeds σ_y. During this progression of yielding, the strain hardening zone and elastic zone are separated by only an infinitesimally thin plane which has perfect plasticity.

In this case the short compression member will, however, have a buckling strength at least equal to the tangent modulus load, using as the tangent of the stress and strain diagram the value at the starting point of the strain hardening region as before.

To summarize the above discussion there are two extreme cases:

First, if the material is perfectly plastic and stressed to the yield point, the compression member will be unstable and will bend no matter what the L/r ratio of the compression member is. The column curve will appear as in Fig. 5.

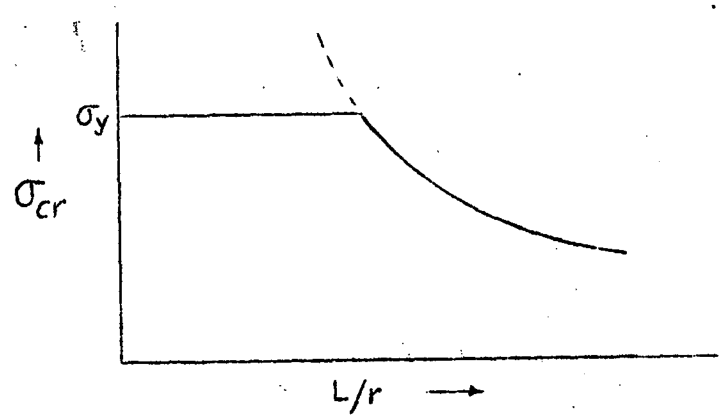


Fig. 5

Secondly, if the plastic flow in a compression member is established plane by plane, and all the planes are strain hardened as plastic flow progresses, the compression member will then have a buckling strength of tangent modulus load as defined above. The column curve will be as shown in Fig. 6.

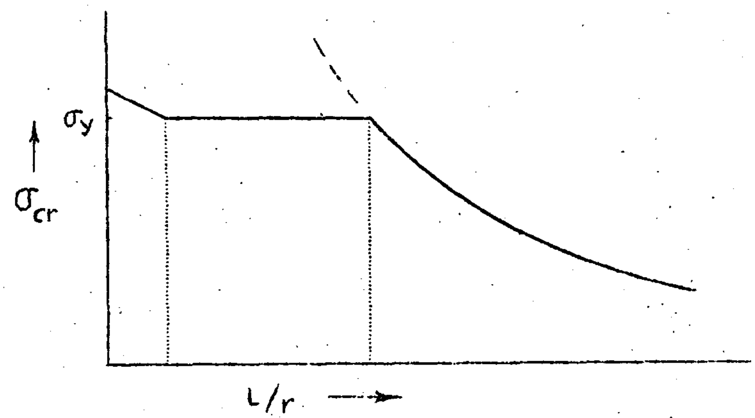


Fig. 6

The practical case may lie between the above two extremes. Apparently the tangent modulus load defined as above will become the upper limit of the buckling strength for the short compression member. ? See Prog Report 5

For rectangular sections the L/t ratio which corresponds to the point at which the compression specimen begins to carry greater loads than the yield point stress may be calculated as follows: Referring to Figs. 3 and 7

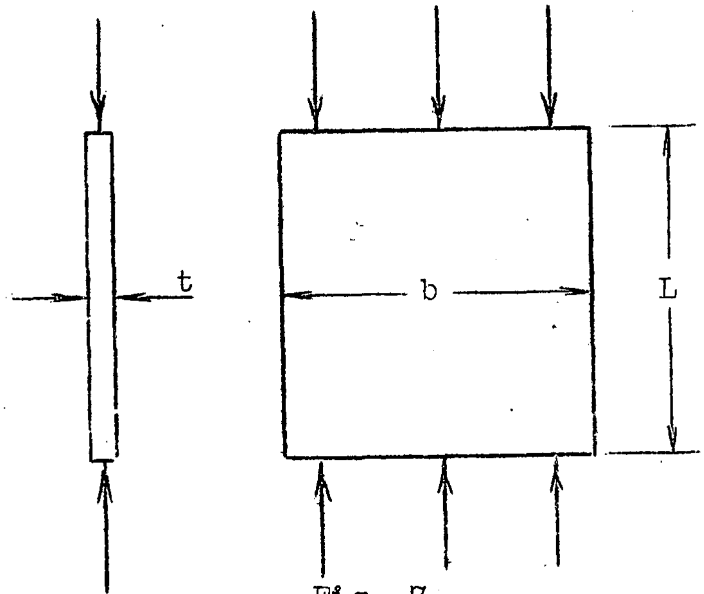


Fig. 7

$$\frac{d\sigma}{d\epsilon} = E_{ts} \equiv 700 \text{kips/inch}^2$$

$$I = \frac{bt^3}{12}$$

$$\sigma_y = 40.36 \text{ kips/inch}^2$$

$$P = a\sigma_y = bt\sigma_y$$

$$P = \frac{E_{ts} I \pi^2}{L^2}$$

$$\frac{L}{t} = 3.75$$

$$\frac{L}{t} = 7.5$$ for the case of fixed ends.

Compression coupons have usually been tested at an L/t ratio of 4. Test conditions simulate fixed ends.

No bending in the plastic range is observed. In previous work by Johnston and Opila[9] their experimentally determined critical L/t value was similar to that computed. Tests of very carefully aligned short compression members of various $\frac{L}{t}$ ratios are planned to evaluate an effective value of the tangent modulus at the starting point of the strain hardening region to predict the buckling strength of steel structural members.

III. INELASTIC LOCAL BUCKLING STRENGTH OF COMPRESSION FLANGES OF WIDE FLANGE SECTIONS

A certain length of the flanges of a wide flange section will become plastic when a plastic hinge is formed at the section. The yielded area is generally a function of the moment gradient and the shape factor of the member. In the case shown in Fig. 8, the length of the yielded flange will be

$$l = \frac{f - 1}{f} L$$

where f is the shape factor $\frac{M_p - M_y}{M_y}$

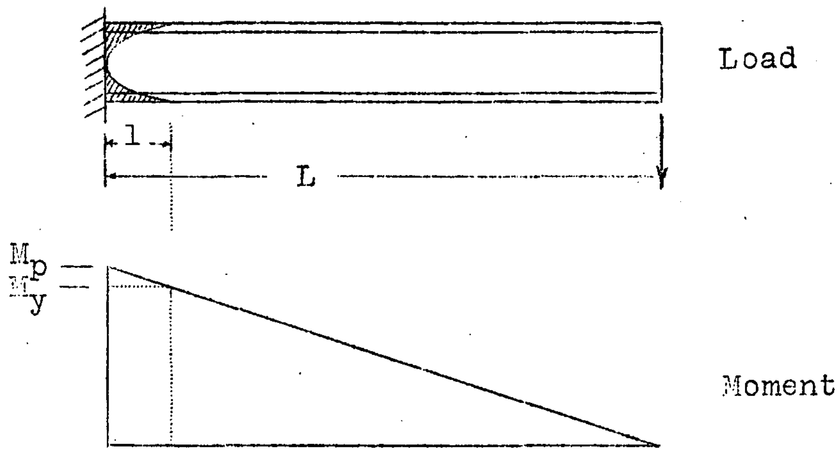


Fig. 8

The buckling problem of the compression flange of the WF shape in the yielded area can be simulated by the buckling problem of a plate with the following boundary conditions:

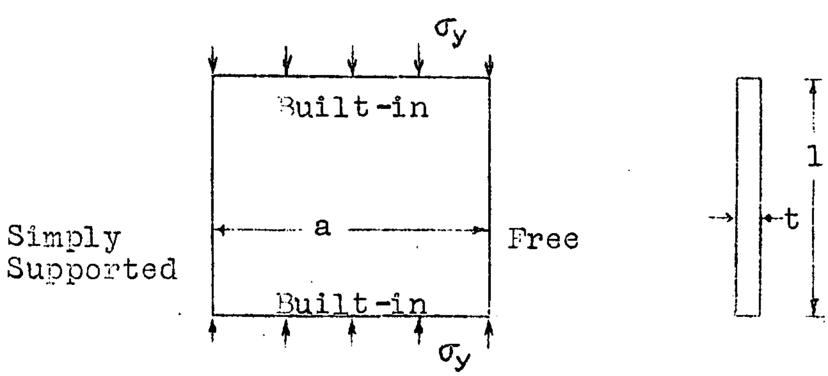


Fig. 9

From the discussions in the previous section it is obvious that the strain hardening part of the stress and strain curve is important in the analysis of the buckling strength of this plate under yield stress.

Analytic solutions to the problem of plastic buckling of a plate with non-linear stress-strain relations have been studied by a number of authors: Timoshenko (7), Ilyusion (4), Stowell and Lundquist (6), Handleman and Prager (3), Bijlaard (1), and is summarized by Bleich (2).

In both Handleman and Ilyusion's solutions, von Kármán's concept of double modulus column strength were used. Stowell and Lundquist treated the problem from Shanley's concept of tangent modulus column strength. However, all these solutions cannot be applied to structural steel in the plastic range without modification. On the one hand, the solutions are for a continuously strain hardening material and on the other, it is seen that the yielding process is of pronounced influence.

It is not the purpose of this report to give detail analysis of the formulation of the solutions to this problem. It will be reported separately after a part of the proposed tests are accomplished.

IV. INELASTIC BUCKLING OF THE WEB OF WF SECTIONS

The web of a deep wide flange section will buckle locally as the plastic zone penetrates close to the neutral axis. The problem becomes more involved than that of the flanges alone. The plastic zone no longer contains only a plate. It contains the flange and a part of the web with very irregular boundaries. It is hoped that time will permit analytic studies and tests. In the latter will be varied the depth of the section, thickness of the web, and width of the flanges.

V. INELASTIC LATERAL BUCKLING OF STRUCTURAL MEMBERS

In elastic theory it is obvious that the lateral buckling strength of the "a" beam with two ends built into the wall (Fig. 10 a) is higher than a simply supported beam as shown in Fig. 10 b .

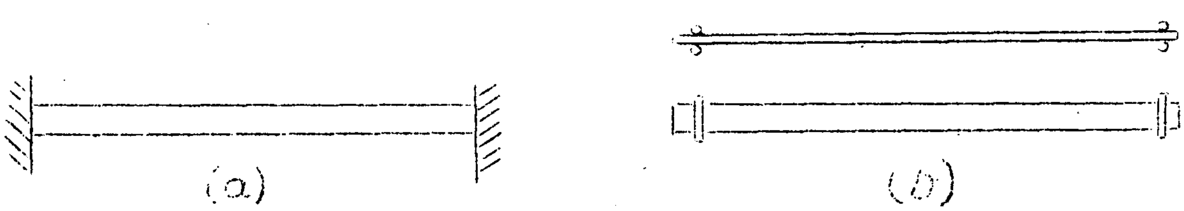


Fig. 10

The formation of plastic hinges at the two supports of a built-in beam under uniform loading naturally changes the boundary conditions of the beam at the two ends. Hence, the lateral buckling strength of the beam is reduced. When a third plastic hinge is introduced at the center of the beam the lateral buckling strength will be further decreased. Tests have shown that beams buckled laterally in presence of the third plastic hinge at the center, although good lateral support was provided.

The worst possible case is that of a bending member under constant moment.

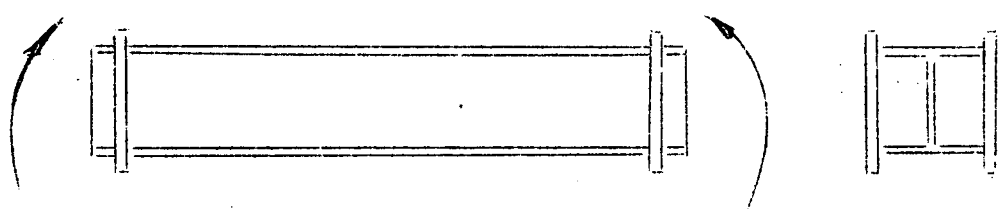


Fig. 11

According to the assumption that the yielded part of the beam is perfectly plastic, it is then evident that the lateral buckling strength of the beam will be equivalent to a beam considering only the elastic part under the same moment. According to the assumption that the yielded part of the beam will have a tangent modulus strength of strain hardening region of steel, the lateral buckling strength of this beam can be approximated by regarding the beam as having different moduli in the elastic part and plastic part. The actual lateral buckling strength of such a beam may be in between the above two limits.

No test program is specially planned for this lateral buckling problem. However, in the continuous beam test proposed below, lateral deflection data will be collected as a preliminary study.

VI. PROPOSED TEST PROGRAM

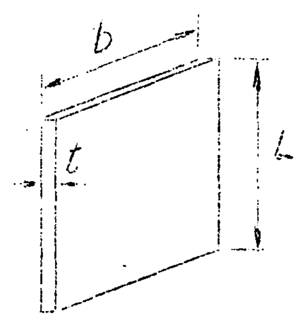
Three series of tests are proposed. One or two continuous beam tests are also suggested after the three series have been completed.

(a) Test of Short Compression Coupons

This series of tests is the same as the ordinary compression test of coupons except the L/t ratios are varied. The basic compressive stress and strain diagram is to be determined by the data collected from part of this series. The effective value of the tangent modulus at the beginning of the strain hardening range is studied by varying the L/r ratios of the specimens.

Comparisons will be made between annealed and as-delivered material, rectangular cross-sections and round sections. The primary purpose for this is to see if the yielding process is influenced by the shape of cross-section and by the condition of the material (annealed or as-delivered). In the latter case, is there any difference in the yielding process in small annealed and as-delivered specimens? There may be an influence of the locked-up stresses not relieved by the usual sectioning process. If the yielding process is changed by these factors there would be a resultant influence on the effective tangent modulus.

1. Annealed Rectangular Coupons



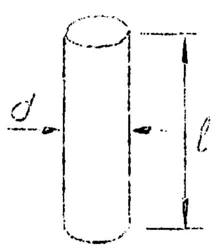
No.	t	L	L/t
1	3/8"	$1\frac{1}{2}$"	4
2	"	1-7/8"	5
3	"	2-$\frac{1}{4}$"	6
4	"	2-5/8"	7
5	"	3	8
6	"	3-3/8"	9

SR4 gages for plastic range or clip gage will be used on both sides to detect bending. Copper plates will be used as shims at both ends to secure uniform pressure.

2. <u>As-Delivered Rectangular Coupons</u>

Testing procedure and dimensions will be the same as for the annealed coupons. Possibly this group should be tested in duplicate.

3. <u>Round Sections</u>



	d	L	L/d
1	1/2"	$\sqrt{3}$	$2\sqrt{3}$
2	"	$5\sqrt{3}/4$	$5\sqrt{3}/2$
3	"	$3\sqrt{3}/2$	$3\sqrt{3}$
4	"	$7\sqrt{3}/4$	$7\sqrt{3}/2$
5	"	$2\sqrt{3}$	$4\sqrt{3}$
6	"	$9\sqrt{3}/4$	$9\sqrt{3}/2$

Round sections are to be tested in as-delivered condition.

(b) <u>Tests of Short Columns</u>

Short columns of both rectangular and WF sections are to be tested to give information for selecting the best specimens for the bending tests to follow. Results are also expected to give indication as to whether the previous analysis is in the right direction.

Some of these test proportions may require modification as a result of the first series of tests.

1. Annealed Rectangular Specimens

These tests differ from series (I).1 in that they are larger. The objective is to examine the size-effect and to see if this changes the effective tangent modulus.

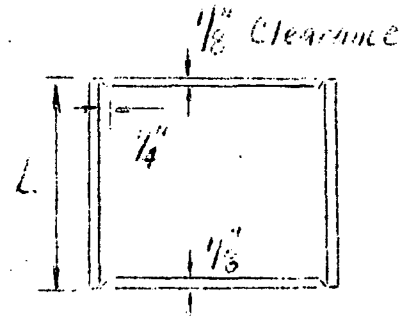
No.	t	L	b	L/t
1	1"	5.5	2	5.5
2	1"	7	2	7
3	1"	8.5	2	8.5
4*	1"	7	2*	7

*No. 4 is tapered 1:10 to give a stress gradient as shown in Fig. 4.

2. WF Sections (Annealed)

This series is to see how much influence the restraining action of the web will have on the results and to see how good a correlation can be obtained with the results of the tests in (I.) and (II.)1. The first step will be a pilot test to see if a sufficiently uniform stress distribution can be obtained in the flanges if the web is trimmed as shown.

The lengths shown in the table following are only tentative, since the tests, (possibly two of each shape), will be carried out in the vicinity of the critical L/t determined in previous tests of rectangular specimens.



Series No.	No. Tests	Section	L	L/t
1	2	8WF67	9"	-
2	2	8WF40	6"	-
3	2	8WF31	6"	-
4	2	8WF24	6"	-
5	2	8WF17	4"	-
6	2	8WF13	4"	-

Both in this series and in the one to follow, further consideration will have to be given to the problem of residual stress, a variable which has been observed to aggravate the local buckling tendency.

(c) Test of WF Shapes Under Bending Moments (Annealed)

This series of tests will contain two groups. First will be a series of simply supported beams, each simulating two cantilever beams.

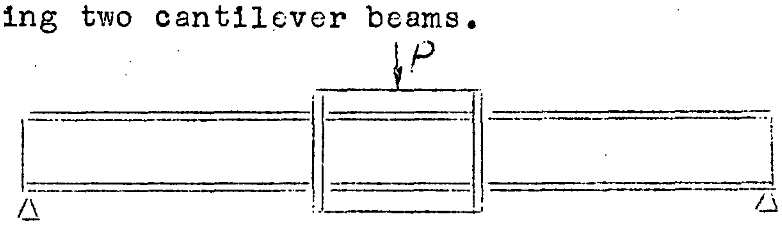


Fig. 12

It will contain around ten specimens with different sections and moment arms. Specific dimensions of the specimens will be determined after the first two sets of tests are completed.

The second group will contain fewer specimens and will be loaded under constant moment as shown in Fig. 13.

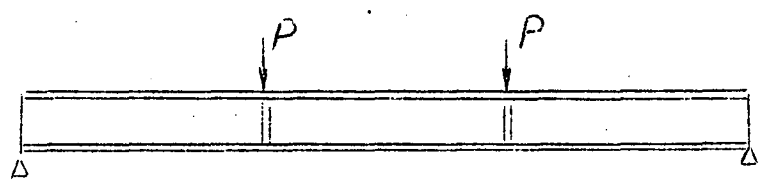


Fig. 13

All the specimens will be annealed and lateral supports will be provided in the test set-up.

One or two continuous beams are proposed to verify the theory in the indeterminate structures.

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