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WELDED CONTINUOUS FRAMES AND THEIR COMPONENTS

Progress Report No. ~~24~~ 25

PLASTIC ANALYSIS AND DESIGN OF SQUARE RIGID FRAME KNEES

by

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A B S T R A C T

This paper contains sample calculations and a description of the plastic design of a square knee for a single-span rigid frame. Welds are designed using the concepts of plastic design as well as those of elastic design in order that the parallelism can be seen. The methods of design presented here can be used as a guide in designing comparable connections for rectangular portal frames. A theoretical analysis of a straight knee with diagonal stiffeners is also presented and leads to expressions for the reinforcement required within the knee to prevent undue deformation. A further analysis is made of the rotation and deflection of the connection.

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1. INTRODUCTION

Several corner connections were designed to be used as test specimens for a program on Welded Continuous Frames and Their Components carried out at Lehigh University.⁽¹⁾ The object of this series of tests was to determine whether connections made from a range of sizes of rolled shapes would have similar behavior, verifying that the fundamental principles of plastic design apply regardless of the size of member joined in a connection. The corner connections were all of a square type which would be typical in a flat-roofed rigid frame building. The results of the tests were satisfactory, and verified the design principles.

This paper presents in detail a sample design of one of the large connections in the series. The analysis of the loads and deformations in both the elastic and plastic ranges is given as well as the design of welds and details by elastic and plastic design methods.

The analysis of straight knees containing diagonal stiffeners is developed in the latter portion of this report. It contains an analysis leading to the required thickness of the knee web. When the web is found to be deficient, it is necessary to prevent the occurrence of excessive deformation due to the high shear stresses that are present. Therefore, the development of an analysis leading to the required diagonal stiffener thickness is given. Finally, derivations of the deformations and rotations that occur in straight knees with diagonal stiffeners are shown.

2. SAMPLE DESIGN OF A WELDED CORNER CONNECTION

Sample calculations from the design of a straight corner connection to determine member size, web reinforcement and welds are given in this section. An analysis is also made to determine the stresses and theoretical deformations. These calculations use the equations and concepts given in Reference 1 and handbook properties from the AISC Handbook.

2.1 SELECTION OF MEMBER

Assume from the geometry and statics of the structure being designed, that the forces shown in Fig 1 are known to be acting on the knee.

The required plastic modulus is found from

$$\begin{aligned} Z &= \frac{M_p}{\sigma_y} = \frac{9000}{33} \\ &= 273 \text{ in}^3 \end{aligned} \quad \dots (1)$$

Where M_p is the plastic moment and σ_y the yield stress

use 24WF100	($Z = 278.3 \text{ in}^3$)
	$S = 248.9 \text{ in}^3$
	$d = 24.0 \text{ in}$
	$w = 0.468 \text{ in}$
	$t = 0.584 \text{ in}$

2.2 REQUIRED REINFORCEMENT

The required web thickness is obtained from

$$\begin{aligned} w_r &= \frac{\sqrt{3}S}{d^2} \\ &= \frac{\sqrt{3} \times 248.9}{24^2} \end{aligned} \quad \dots (2)$$

Giving: $= 0.749 \text{ in} > 0.468$

Hence, a diagonal stiffener is required. The thickness

of this stiffener is obtained from

$$t_s = \frac{\sqrt{2}}{b} \left(\frac{S}{d} - \frac{wd}{\sqrt{3}} \right) \quad \dots(3)$$

$$= \frac{\sqrt{2}}{12} \left(\frac{248.9}{24} - \frac{(0.468)(24)}{\sqrt{3}} \right)$$

$$= 0.457 \text{ in.}$$

Used 3/4 in Plate

Equations 2 and 3 are developed in the section titled: Analysis of straight knees with diagonal stiffeners.

2.3 YIELD MOMENT AND AXIAL FORCE:

For purposes of analyzing the knee tested, it was necessary to obtain additional stresses and forces acting on the entire structure.

Fig. 2 shows the proportions of the test connection. The ^{dimension} distance "a" is _{the distance} to an assumed inflection point.

The yield moment is found from

$$M_y = \sigma_y S \quad \dots(4)$$

$$= 33 \times 248.9$$

$$= 8,210 \text{ in-kips}$$

The plastic moment of the section is found from

$$M_p = \sigma_y Z \quad \dots(5)$$

$$= 33 \times 278.3$$

$$9,170 \text{ in-kips}$$

To determine the allowable axial force at first yield, the combined bending and axial force formula is used

$$\sigma = \frac{P}{A} + \frac{M}{S} \quad \dots(6)$$

Therefore:

$$\sigma_y = \frac{P_y}{\sqrt{2}} \left(\frac{1}{A} + \frac{a}{S} \right)$$

$$P_y = \frac{33\sqrt{2}}{\frac{1}{29.43} + \frac{96}{248.9}}$$

$$= 111.4 \text{ kips}$$

The moment at the haunch can then be determined by statics as:

$$\begin{aligned} M_h(y) &= \frac{Py}{\sqrt{2}} (a + d/2) \quad \dots(7) \\ &= \frac{111.4}{\sqrt{2}} \times 108 \\ &= 8,500 \text{ in-kips} \end{aligned}$$

2.4 MODIFICATION OF PLASTIC MOMENT DUE TO AXIAL FORCE

The modified plastic moment is found by determining the interaction between the plastic moment, M_p , of the section and the reduction due to axial load. The axial load is assumed to be carried by a small area of web near the centroid of the section. An ultimate value of P is assumed from which the depth of web, y_a , (Fig. 3) required to support the axial load is found.

As an example, assume $P = 135$ kips; then:

$$\begin{aligned} y_a &= \frac{P}{\sqrt{2}w\sigma_y} \quad \dots(8) \\ &= \frac{135}{\sqrt{2} \times 33 \times 0.468} \\ &= 6.18 \text{ inches} \end{aligned}$$

The plastic modulus of this area is the first moment of the area.

Hence:

$$Z_a = \frac{w y_a^2}{4} \quad \dots(9)$$

Therefore, the bending moment of the area carrying the axial load is

$$\begin{aligned} M_a &= \sigma_y Z_a \quad \dots(10) \\ &= \frac{33 \times 0.468 \times 6.18^2}{4} \\ &= 144.3 \text{ in-kips} \end{aligned}$$

Hence, the reduced plastic moment is

$$\begin{aligned} M_{pc} &= M_p - M_a \quad \dots(11) \\ &= 9,170 - 144 \\ &= 9,026 \text{ in-kips} \end{aligned}$$

From statics

$$\begin{aligned}
 P_u &= \frac{\sqrt{2} M_{pc}}{a} && \dots(12) \\
 &= \frac{\sqrt{2} \times 9,026}{96} \\
 &= 133 \text{ kips}
 \end{aligned}$$

Next assuming $P_u = 133$ kips and repeating the process gives from statics:

$$P_u = 133 \text{ k}$$

2.5 SHEAR STRESS WITHOUT WEB REINFORCEMENT

The shear stress on a knee without reinforcement can be obtained by noting that the shear stress is $\tau = \frac{F_o}{wd}$, where F_o is the flange force as obtained in the chapter, "Analysis of Straight Knees with Diagonal Stiffeners"

Hence:

$$\begin{aligned}
 \tau_y &= \frac{M_{hy}}{wd^2} (1 - d/L) && \dots(13) \\
 &= \frac{8500}{0.468 \times 24^2} (1 - 24/108) \\
 &= 24.25 \text{ ksi}
 \end{aligned}$$

This exceeds $0.578 \sigma_y$ therefore additional reinforcement is required.

The shear stress at the ultimate load is

$$\tau_u = \frac{M_{hp}}{wd^2} (1-d/L) \quad \dots(14)$$

from statics we obtain

$$M_{h(p)} = \frac{P_u}{2} (a + d/2)$$

Hence:

$$\tau_u = \frac{10,150}{0.468 \times 24^2} (1 - 24/108)$$

Giving:

$$\tau_u = 29.2 \text{ ksi}$$

2.6 SHEARING STRESS AT ULTIMATE LOAD WITH DIAGONAL STIFFENER

The shearing stress on a knee with a diagonal stiffener is developed ^{later in this report} ~~in the following chapter.~~

$$\text{Hence: } \tau = K_3 \frac{M_h(p)}{d} (1 - d/L) G \quad \dots(15)$$

$$\begin{aligned} \text{Where } K_3 &= \frac{1}{wGd + \frac{t_s b_s E}{2\sqrt{2}}} \\ &= \frac{1}{0.468 \times 11.5 \times 10^3 \times 24 + \frac{0.75 \times 11.53 \times 30 \times 10^3}{2\sqrt{2}}} \\ &= 0.453 \times 10^{-5} / \text{kip} \end{aligned}$$

~~Hence:~~

$$\begin{aligned} \text{Therefore, } \tau &= \frac{0.453}{24} \times 10^{-5} \times 10,150 (1 - 24/108) 11.5 \times 10^3 \\ &= 17.15 \text{ ksi} \end{aligned}$$

2.7 ROTATION OF THE KNEE AT YIELD

The rotation at yield is given by

$$\theta_y = \frac{M_r(y)}{d} \frac{(L-d)}{(L-d/2)} \left\{ K_3 + \frac{(1 + K_2)}{E A_f} \right\} \quad \dots(16)$$

where $M_r(y)$ is the moment at the junction of rolled beam and connection

$$\begin{aligned} \text{Hence: } M_r(y) &= P_y \frac{a}{\sqrt{2}} \\ &= \frac{111.4 \times 96}{\sqrt{2}} \\ &= 7,580 \text{ in-kips} \end{aligned}$$

and

$$\begin{aligned} K_2 &= \frac{1}{1 + \frac{2\sqrt{2} wdG}{t_s b_s E}} \\ &= \frac{1}{1 + \frac{2\sqrt{2} \times 0.468 \times 24 \times 11.5 \times 10^3}{0.75 \times 11.53 \times 30 \times 10^3}} \end{aligned}$$

Giving:

$$= 0.468$$

Therefore:

$$\theta_y = \frac{7.580}{24} \frac{(108-24)}{(108-12)} \left\{ 0.453 \times 10^{-5} + \frac{(1 + 0.458)}{12 \times 0.775 \times 30 \times 10^3} \right\}$$

Giving:

$$\theta_y = 0.00269 \text{ radians}$$

The rotation analysis of straight knees with diagonal stiffeners is developed in the following chapter.

2.8 DEFLECTION BETWEEN END PINS AT THE YIELD LOAD

Since this corner connection was to be tested by applying loads as shown in Fig 2, it was desirable for comparison with experimental results, to predict the deflection of the legs in the direction of the loads. Even though this calculation would not be needed in the design of a portal frame, it is shown here.

The deflection yield is

$$\delta_y = \sqrt{2} \left\{ \frac{M_x a^2}{3EI} + \frac{\theta L}{2} \right\} \quad \dots(17)$$

Hence:

$$\delta_y = \sqrt{2} \left\{ \frac{7,580 \times 96^2}{3 \times 30 \times 10^3 \times 2987.3} + \frac{0.00269 \times 108}{2} \right\}$$

Giving:

$$\delta_y = 0.584 \text{ inches}$$

2.9 ELASTIC DESIGN OF WELDS

These welds are proportioned to follow Section 15 of the AISC Specification at a load at which the maximum combined stress in the rolled section at the edge of the connection is 20 ksi. From the combined bending and axial force formula the load at this stress is found to be 67.5 kips and the bending moment at the rolled section would be 4590 in-kips.

- 1) Fillet Welds ~~for a possible lap joint of end plate to flange of column~~ ^{FOR A POSSIBLE LAP JOINT OF END PLATE TO FLANGE OF COLUMN (Fig 4)}

Since the flange force must be transferred to the end plate,

(Fig. 4) the area of weld required is

$$\begin{aligned}
 A &= \frac{\text{Flange Force}}{13.6} = \frac{\text{Allowable Elastic Stress} \times \text{Plate area}}{\text{Allowable Weld Stress}} \\
 &= \frac{20 \times 12 \times 0.775}{13.6} \quad \dots(18) \\
 &= 13.68 \text{ in}^2
 \end{aligned}$$

Hence the total length of weld required is

$$L = \frac{\text{Area Weld}}{0.707 D} \quad \dots(19)$$

Giving:

$$D = 3/8 \text{ in; } L = 51.6 \text{ in.}$$

$$D = 1/2 \text{ in; } L = 38.7 \text{ in.}$$

It is evident that too great a length of lap joint is required using fillet welds. There ^{fore,} as a matter of economy a butt weld would be more suitable to use.

2) Fillet Welds Between Column Web and Beam Flange (Fig 5)

These welds must develop the combined stresses due to bending, the axial load component and the shear force from the transverse component of load.

The tensile or compressive direct stress on the weld is

$$\begin{aligned}
 \sigma_D &= P/A \sqrt{2} \\
 \sigma_D &= \frac{67.5}{\sqrt{2} \times 24.98} \\
 &= 1.622 \text{ ksi}
 \end{aligned}$$

The direct force per inch of weld is then:

$$\begin{aligned}
 f_D &= \sigma_D \frac{W}{2} \\
 &= \frac{1.622 \times 0.468}{2} \\
 &= 0.33 \text{ kips/in}
 \end{aligned}$$

The maximum force per inch due to bending in the web (Fig. 6) is:

$$f_m = \frac{3M_w}{L^2}$$

where

$$M_w = M - M_f$$

and

$$M_f = F(d-t)$$

$$M_w = 4.590 - 186 (24-0.775)$$

Giving:

$$M_w = 280 \text{ in-kips}$$

Hence:

$$\begin{aligned} f_m &= \frac{3M_w}{L^2} \\ &= \frac{3 \times 280}{22.45^2} \\ &= 1.664 \text{ kips/in} \end{aligned}$$

The force per inch per line of welding due to shear

$$f_v = \frac{P}{2\sqrt{2} \cdot (d-2t)}$$

Hence:

$$f_v = \frac{67.5}{2\sqrt{2} \times 22.45}$$

Giving:

$$= 1.062 \text{ kips/in}$$

The resultant force vector is then

$$\begin{aligned} R &= \sqrt{(f_D + f_m)^2 + f_v^2} \\ R &= \sqrt{(1.664 + 0.38)^2 + 1.062^2} \\ &= 2.30 \text{ kips/in} \end{aligned}$$

Hence the size fillet weld required is

$$\begin{aligned} D &= \frac{R}{0.707 \times 13.6} \\ &= \frac{2.30}{0.707 \times 13.6} \\ &= 0.24 \text{ inches} \end{aligned} \quad \dots(20)$$

Nominal Size = 1/4 in.

2)9. Forty-Five Degree Fillet Welds at Ends of Diagonal Stiffener (Fig. 7)

The force in the diagonal stiffener is given by

$$F_s = \sqrt{2} K_2 \times \text{Flange Force} \quad \dots(21)$$

Hence:

$$\begin{aligned} F_s &= \sqrt{2} \times 0.458 \times 20 \times 0.775 \\ &= 106.6 \text{ kips} \end{aligned}$$

This force is obtained by noting that $F_1 = K_2 F_o$ (eq 52) as developed in the following chapter.

The force per inch of weld is then

$$\begin{aligned} &= \frac{F_s}{2b - w} \\ &= \frac{106.6}{24 - 0.468} \end{aligned}$$

Giving:

$$= 4.61 \text{ kips/in}$$

Therefore, the size fillet weld required is

$$\begin{aligned} D &= \frac{\text{Force per inch}}{13.6 \cos 22.5^\circ} \quad \dots(22) \\ &= \frac{4.61}{13.6 \times 0.9239} \end{aligned}$$

Giving:

$$= 0.367 \text{ inches}$$

Nominal Size = 3/8 in.

The minimum fillet weld provisions of the AWS Building Code were followed in selecting the welds between the diagonal stiffener and beam web.

2)9. Fillet Weld for Web of Beam to End Plate (Fig. 8)

These welds are designed to transmit the elastic flange force of the column into the web of the beam by shear.

Hence, size of fillet weld required is

$$D = \frac{\text{Flange Force}}{2(d-2t) 13.6 \times 0.707} \quad \dots(23)$$

Therefore:

$$D = \frac{20 \times 12 \times 0.775}{2 \times 22.45 \times 13.6 \times 0.707}$$

Giving:

$$= 0.431 \text{ inches}$$

Nominal Size = 7/16 in.

5) Fillet Weld Required Between Column Flange and Beam Flange (Fig. 9)

These welds were first designed as fillet weld which would develop the elastic flange force when the corner was subjected to an opening corner load condition.

The size fillet weld required is

$$D = \frac{\text{Flange Force}}{(2b-w) 13.6 \times 0.707} \quad \dots(24)$$

$$= \frac{20 \times 12 \times 0.775}{(24-0.468) 13.6 \times 0.707}$$

Giving:

$$0.823 \text{ inches}$$

Nominal Size = 7/8 in.

Hence, a butt weld would be more satisfactory at this point.

6.9 Fillet Welds Between Vertical Side Stiffener and Web of Beam (Fig 10)

These welds are obtained by assuming that the maximum possible flange force must be borne by the beam web and stiffener and that the concentrated load is distributed to the web on planes bounded by 45° angles from the point of application of load.

The allowable compressive force to be carried by the web is

$$P_w = 24w (t + 2k) \quad \dots(25)$$

$$= 24(0.468) (0.775 \times 3.125)$$

$$= 43.8 \text{ kips}$$

Hence, the force to be carried by the vertical stiffener is

$$F_s = P_F - P_w$$

where P_F is the flange force

P_F = allowable plate stress x plate area

$$P_F = 20 (12) (0.775)$$

$$= 186 \text{ kips}$$

Therefore:

$$F_s = 186 - 43.8$$

$$= 142.2 \text{ kips}$$

Therefore, the size fillet weld required is

$$D = \frac{F_s}{4 \times 0.707 \times 13.6 (d/2 - k)} \quad \dots(26)$$

$$= \frac{142.2}{4 \times 0.707 \times 13.6 (12 - 1.56)}$$

$$= 0.332 \text{ inches}$$

Nominal Size = 3/8 in.

2.10 PLASTIC DESIGN OF WELDS

In this section, the sizes of welds necessary to meet the requirements of section 12 of Reference 1 at the maximum load of the connection are calculated.

2.10.1 Possible Fillet Weld for Lap Joint of End Plate to Flange of Column (Fig 4)

The length of lap joint required to transfer the plastic flange force is obtained as follows:

Area of weld required is:

$$A = \frac{\text{Flange Force}}{\text{Allowable Weld Stress}} = \frac{\text{Yield stress x Plate Area}}{22.4}$$

$$= \frac{307}{22.4}$$

...(27)

Gives:

$$= 13.7 \text{ in}^2$$

Hence the length of the weld required is:

$$L = \frac{\text{Area}}{0.707D} \quad \dots(19)$$

Therefore:

$$D = 5/16 \text{ in}; \quad L = 62.0 \text{ in}$$

$$D = 7/16 \text{ in}; \quad L = 44.3 \text{ in}$$

A butt weld is found to be more suitable in this case.

2.10 Column Web Fillet Weld to Beam Flange (Fig 5)

These welds are designed to develop the combined tensile forces of bending, the most severe axial load component in the web and the shear force from the transverse component of load. The direct tensile or compression force per inch of weld due to axial load and bending is:

$$\begin{aligned} f_D &= 33 w/2 \\ &= 33 \times \frac{0.468}{2} \end{aligned}$$

Gives:

$$= 7.72 \text{ kips/in}$$

Force per inch due to shear

$$\begin{aligned} f_v &= \frac{\sqrt{2} Pu}{4(d-2t)} \\ &= \frac{133}{2\sqrt{2} \times 22.45} \end{aligned}$$

Gives:

$$= 2.1 \text{ kips/in}$$

Therefore resultant force per inch

$$\begin{aligned} R &= \sqrt{f_D^2 + f_v^2} \\ R &= \sqrt{2.1^2 + 7.72^2} \end{aligned}$$

Gives:

$$= 8.0 \text{ k/in}$$

However:

$$D = \frac{R}{0.707 \times 22.4} \quad \dots(28)$$

$$= \frac{8.0}{0.808 \times 22.4}$$

Gives:

$$= 0.508 \text{ inches}$$

$$\text{Nominal Size} = 9/16 \text{ in.}$$

2) Forty-Five Degree Fillet Welds to Ends of Diagonal Stiffener (Fig 7)

The force in the diagonal stiffener is obtained as

$$F_s = \sqrt{2} K_2 \sigma_y b t \quad \dots(21)$$

$$= \sqrt{2} \times 0.458 \times 33 \times 12 \times 0.775$$

Gives:

$$= 176.4 \text{ kips}$$

Hence from

$$D = \frac{F_s}{22.4 \times 2 (b-w) \cos 22.5^\circ} \quad \dots(29)$$

$$= \frac{176.4}{22.4 \times 2 (12-0.468) 0.9239}$$

We obtain:

$$= 0.370 \text{ inches}$$

$$\text{Nominal Size} = 3/8 \text{ inches}$$

2) Fillet Welds for End Plate to Beam Web (Fig 8)

The welds must transmit the plastic flange force from the column into the beam web by shear.

The plastic flange force is

$$P_F = \sigma_y b t$$

$$= 33 \times 12 \times 0.775$$

Gives:

$$P_F = 307 \text{ kips}$$

Therefore, size fillet weld required is

$$D = \frac{P_F}{0.707 \times 22.4 \times 2 (d-2t)} \dots(30)$$

$$= \frac{307}{0.707 \times 22.4 \times 2 \times 22.45}$$

Gives: = 0.434 inches

Nominal Size = 7/16 inches

3) 10 Possible Fillet Welds to Connect Column Flange to Beam Flange (Fig 9)

These welds were first designed as fillet welds to develop the plastic flange force of the column when the knee is subjected to an opening load condition.

Required size of fillet weld is

$$D = \frac{\text{Flange Force}}{0.707 \times 22.4 (2b-w)} \dots(31)$$

$$= \frac{307}{0.707 \times 22.4 \times 23.53}$$

Gives: = 0.829 inches

Nominal Size = 7/8 inches

Hence, a butt weld is found to be more suitable in this case.

3) 10 Fillet Welds Between Vertical Stiffener and Web of Beam (Fig 10)

Flange force to be carried by the web is

$$P_w = \sigma_y w (t + 2k) \dots(32)$$

$$= 33 \times 0.468 (0.775 + 3/125)$$

Gives: = 60.2 kips

Hence, the force in the vertical stiffener is

$$F_s = P_F - P_w$$

$$= 307 - 60.2$$

Gives:

$$= 246.8 \text{ kips}$$

Therefore, size fillet weld required is

$$D = \frac{F_s}{4 \times 0.707 \times 22.4 (d/2-k)} \quad \dots(33)$$

Therefore:

$$D = \frac{246.8}{4 \times 0.707 \times 22.4 (12-1.56)}$$

Gives:

$$= 0.350 \text{ inches}$$

Nominal Size = 3/8 in.

3. ANALYSIS OF STRAIGHT KNEES WITH DIAGONAL STIFFENERS

Two analyses are considered in this section:

- (1) an analysis leading to a required thickness of diagonal stiffener to prevent undue deformation of the knee web due to shear force, and,
- (2) an analysis of the rotation and deflection of a straight knee with diagonal stiffeners.

3.1 DIAGONAL STIFFENERS FOR STRAIGHT CONNECTIONS

From Reference 2 and referring to Fig. 11

$$F_o = \frac{M_h}{d} (1 - d/L) \quad \dots(34)$$

where M_h = moment at point H

d = depth of section

L = length of connections leg

The desired moment at point H is M_p . There^{fore,} equation 34 becomes

$$F_o = \frac{\sigma_y Z}{d} (1 - d/L) \quad \dots(35)$$

Since the force F_o must be resisted by the web in shear, the magnitude of the shear stress is:

$$\tau = \frac{\sigma_y Z}{wd^2} (1-d/L) \quad \dots(36)$$

noting that

$$Z = fS$$

We obtain

$$\tau = \frac{\sigma_y fS(1-d/L)}{wd^2} \quad \dots(37)$$

However, since f is slightly greater than unity and the quantity $(1-d/L)$ is slightly less, their product is approximately equal to unity. Therefore:

$$= \frac{\sigma_y S}{wd^2} \quad \dots(38)$$

However, τ , cannot exceed $\frac{\sigma_y}{\sqrt{3}}$

Hence:
$$\frac{\sigma_y}{\sqrt{3}} = \frac{\sigma_y S}{wd^2}$$

$$w_r = \frac{\sqrt{3}S}{d^2} \quad \dots(2)$$

The required web thickness can also be expressed as a function of M_p by noting that $S = \frac{Z}{f} = \frac{M_p}{f\sigma_y}$. Then,

$$w_r = \frac{\sqrt{3} M_p}{f \sigma_y d^2} = \frac{\sqrt{3} M_p}{(1.12) (33) d^2} = 0.565 \frac{M_p}{d^2}$$

Hence:

$$w_r \cong 0.6 \frac{M_p}{d^2} \quad \dots(39)$$

If the required web thickness, w_r , is greater than that of the rolled section, it is necessary to provide a diagonal stiffener. The force F_o is then resisted by the web acting in shear and the diagonal stiffener acting in compression. The shear resistance of the web, F_w , is given by

$$F_w = \frac{\sigma_y}{\sqrt{3}} wd \quad \dots(40)$$

(where w = the thickness of the web) and the resistance

of the diagonal stiffener is given by

$$F_s = \sigma_y \frac{bt_s}{\sqrt{2}} \quad \dots(41)$$

Since $F_o = F_w + F_s$, then from equations 35, 40 and 41

$$\frac{\sigma_y Z}{d} (1 - d/L) = \frac{\sigma_y}{\sqrt{3}} wd + \frac{\sigma_y}{\sqrt{2}} bts \quad \dots(42)$$

Solving this equation for t_s ,

$$t_s = \frac{\sqrt{2}}{b} \left(2 \frac{(1-d/L)}{d} \frac{wd}{\sqrt{3}} \right) \quad \dots(43)$$

Since $Z = fS$, and since the quantity $f (1 - d/L)$ is very nearly equal to 1.0, equation (43) reduces, finally to

$$t_s = \frac{\sqrt{2}}{b} \left(\frac{S}{d} - \frac{wd}{\sqrt{3}} \right) \quad \dots(3)$$

which gives the required thickness of diagonal stiffener in order that the connection be capable of resisting the plastic moment, M_p , applied at the intersection of neutral lines of the beam and girder.

3.2 ROTATION ANALYSIS OF STRAIGHT KNEES WITH DIAGONAL STIFFENERS*

The rotation of the knee is made up of two parts:

- 1) Rotation due to shear, designated as δ , and
- 2) Rotation due to bending, designated as β .

Since comparisons are made with experimentally determined rotation values, there is a third component to be considered if the measurement is made at a point other than at the precise end of the connection:

- 3) Rotation due to bending of the rolled section over the length, r , between the end of the knee and point of rotation measurement, designated as θ_r .

Hence the total knee rotation is $\theta = \delta + \beta + \theta_r \quad \dots(44)$

*Based in part on Reference 3, Appendix A.

Two different approaches were used in Reference 2 to predict the moment-rotation characteristics of straight knees with diagonal stiffeners. It is the purpose of this section to refine the solution of this problem.

Rotation due to shear in the square knee A B C D (Fig. 12) reinforced with diagonal stiffeners will be found by making the same assumption that was implied in section (3.1): ~~above~~ the flange force F_0 is resisted in part by the knee web and in part by the component of thrust from the diagonal stiffener AC. Thus in Fig. 13a although the decrease in stress is linear from D to C, the flange at point C retains a stress of magnitude F_1/A_F . The resultant force is transmitted to the diagonal stiffener.

The problem to be solved, then, is the relation between the force F_2 transmitted by the exterior flange to the web due to shearing action (represented by the triangular distribution in Fig. 13a) and the force F_1 transmitted to the diagonal stiffener. This may be done by connecting the continuity condition at point C; then the moment-shear deformation relationship may be developed.

Consider the plate A B C D with diagonal stiffener and loaded with end compressive forces, P, as shown in Fig. 13b. This simulates the loading applied to the stiffener by the flange force F_1 . Fig. 13c shows the shear stresses acting on the web, the stresses introduced by the flange force F_2 . The variation in normal stress along the stiffener due to the loading of Fig. 13b would resemble Fig. 14a; the stress will decrease toward the center of the plate as the stiffener transmits load to the plate by shear. On the other hand, the shear loading of Fig. 13c will cause stresses along the stiffener somewhat like those of Fig. 14b. Normal stresses will gradually increase towards the center of the stiffener. When the two loadings of Figs. 13b and 13c are added together

to give the loading due to the flange force, F_0 , it will be assumed that the resultant stresses in the stiffener are uniform as shown in Fig. 14c. It will be assumed that the web plate remains in a state of pure shear and the contraction along line AC of Fig. 13c will thus remain uniform.

Since the total shortening of the stiffener must equal the contraction due to the shear stresses in order that the continuity condition be satisfied, then in the general case (referring to Fig. 15)

$$\int_0^a \epsilon_w(x) dx = \int_0^a \epsilon_s(x) dx \quad \dots(45)$$

where the subscripts w and s refer to web and stiffener, respectively.

According to the assumptions made above, equation 45 reduces in this problem to ⁽⁴⁾

$$\delta/2 = \epsilon_x \quad \dots(46)$$

where the subscripts w and s have been dropped, being uniform and equal along line AC. Now

$$\delta = \frac{\tau}{G}; \quad \epsilon_x = \frac{\sigma_x}{E} \quad \dots(47)$$

and

$$\tau = \frac{F_2}{A_w}; \quad \sigma_x = \frac{P}{A_s} = \frac{F_1 \sqrt{2}}{A_s} \quad \dots(48)$$

Therefore, from equations 46, 47 and 48

$$\frac{F_2}{2A_w G} = \frac{F_1 \sqrt{2}}{A_s E}$$

or

$$F_2 = \frac{2 A_w G \sqrt{2}}{A_s E} F_1 \quad \dots(49)$$

But

$$F_1 + F_2 = F_0 \quad \dots(50)$$

and therefore, if we let

$$K_1 = 2A_w G \sqrt{2}/A_s E \quad \dots(51)$$

then

$$F_1 + K_1 F_1 = F_0$$

or

$$F_1 = \frac{F_0}{1+K_1} = K_2 F_0 \quad \dots(52)$$

where

$$K_2 = \frac{1}{1 + \frac{2A_w G \sqrt{2}}{A_s E}} \quad \dots(53)$$

Similarly,

$$F_2 = (1-K_2) F_0 \quad \dots(54)$$

The rotation of the knee due to these forces is equal to γ .

Since

$$F_0 = \frac{M_h}{L} \left(\frac{L}{d} - 1 \right) \quad \dots(55)$$

Then equations 54 and 55 and the first of equations 47 and 48 may be used to determine the moment-deformation (shear) relationship.

Making the substitutions,

$$\gamma = K_3 \frac{M_h}{d} (1 - d/L) \quad \dots(56)$$

where

$$K_3 = \frac{1-K_2}{G A_w} = \frac{1}{Gwd + \frac{t_s b_s E}{2 \sqrt{2}}} \quad \dots(57)$$

According to assumptions made above, the extension of the flanges, δ , will be given by

$$\delta = \left(\frac{\sigma_D + \sigma_C}{2} \right) \frac{d}{E} \quad \dots(58)$$

Using Fig. 16, the total "bending" rotation at the knee is

$$\beta = 2 \theta_a = a \frac{\delta}{d} = \frac{\sigma_D + \sigma_C}{E}$$

Now from Fig. 13a and equation 52

$$\sigma_C = \frac{F_1}{F_0} \sigma_D = K_2 \sigma_D$$

and from Fig. 13a and Equation 55

$$\sigma_D = \frac{M_h}{LA_F} \left(\frac{L}{d} - 1 \right)$$

Then the total bending rotation is given by

$$\beta = (1+K_2) \frac{M_h}{A_f d E} (1 - d/L) \quad \dots(59)$$

The rotation θ_r , due to flexure of the rolled section over length r is given by

$$\theta_r = \frac{M_r}{EI} (2r - r^2/a) \quad \dots(60)$$

therefore

$$\theta_r = \frac{M_h}{EI} (1 - 1/2L) (2r - r^2/a) \quad \dots(61)$$

When r is small, the term r^2/a may be neglected. Then the total rotation is given by a summation of the values determined from equations (56), (59) and (61) or

$$\theta = \delta + \beta + \theta_r \quad \dots(44)$$

$$\theta = K_3 \frac{M_h}{d} (1-d/L) + (1+K_2) \frac{M_h}{A_f d E} (1 - d/L) + \frac{M_h}{EI} (1 - d/2L) (2r - r^2/a) \quad \dots(62)$$

$$\theta = M_r \frac{(1-d/L)}{(1-d/2L)} \left\{ \frac{K_3}{d} + \frac{(1+K_2)}{A_f d E} + \frac{(1 - d/2L)}{(1-d/L)} \frac{(2r - r^2/a)}{EI} \right\} \quad \dots(63)$$

The deflection between the end pins of the knee can be found by considering the deflection due to the rotation of the knee from the shear and bending deformations, as well as the deflection due to flexure of the rolled section over a length "a".

Hence:

$$\delta = \sqrt{2} \left\{ \frac{M_r a^2}{3EI} + \frac{\theta L}{2} \right\} \quad \dots(17)$$

The results of this analysis of shear and bending deformations are compared with experimental results for two tests using WF shapes of widely differing geometry in Figs. 17^A and 18. The initial portion of the moment rotation curve of Connection L from Reference 2 is shown in Fig. 17^A, equation (63) being used to plot the theoretical curve shown by the dotted line. In Fig. 18 is a similar comparison using the results of a frame test with an 8WF 40 shape. In the second case, load is plotted against the total rotation of the knee.

In view of the agreement between theory and test of two markedly different cross-sections, this analysis affords a satisfactory explanation of experimental behavior.

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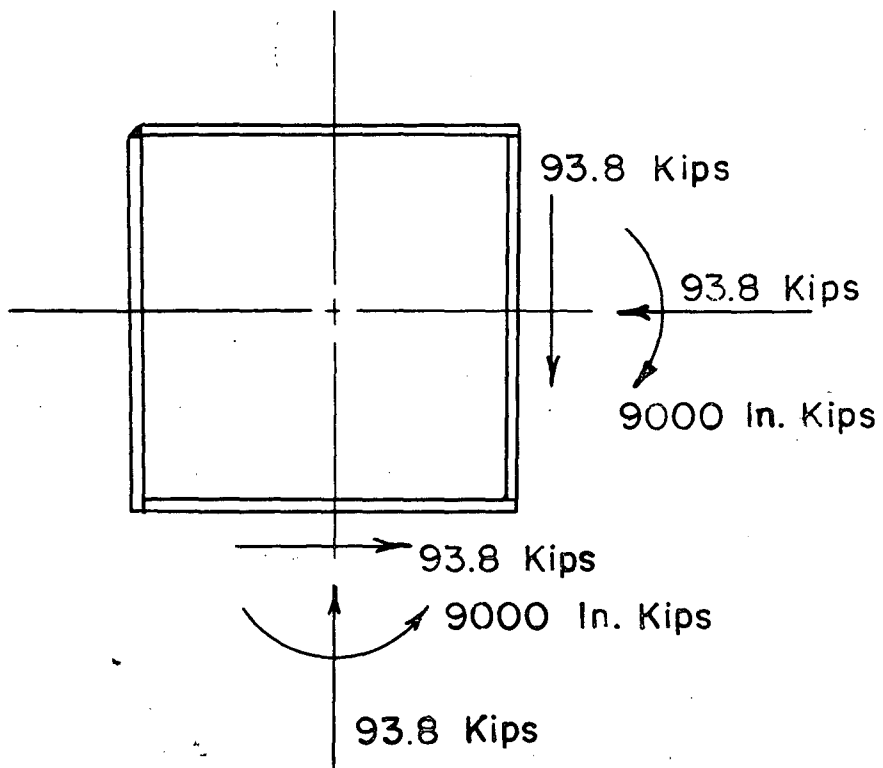


FIG 1 FORCES AND MOMENTS ACTING ON THE KNEE

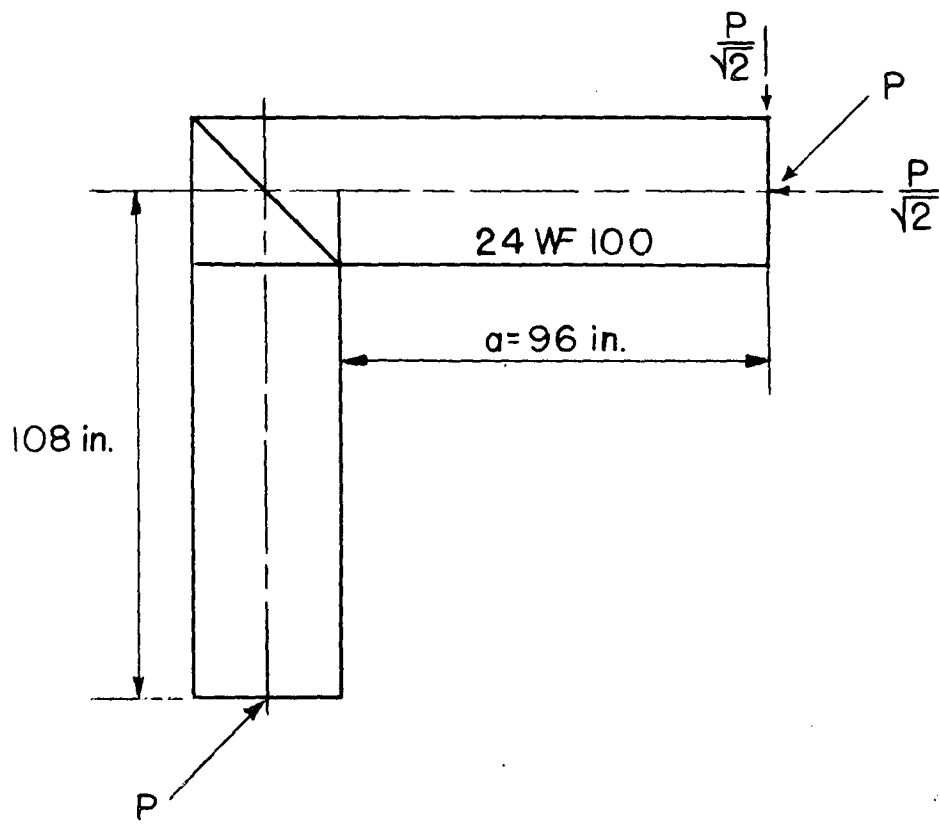


FIG 2 GEOMETRY OF THE TEST CONNECTIONS

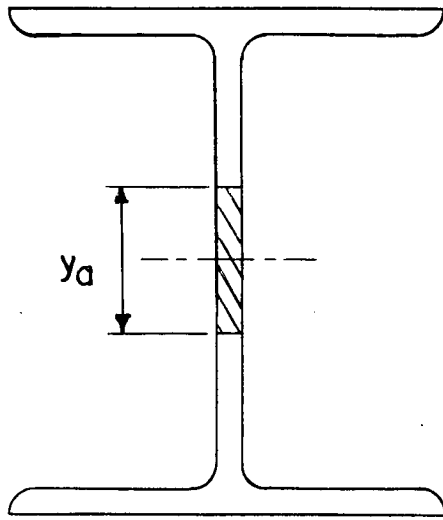


FIG 3 CROSS-SECTION SHOWING AREA CARRYING AXIAL FORCE

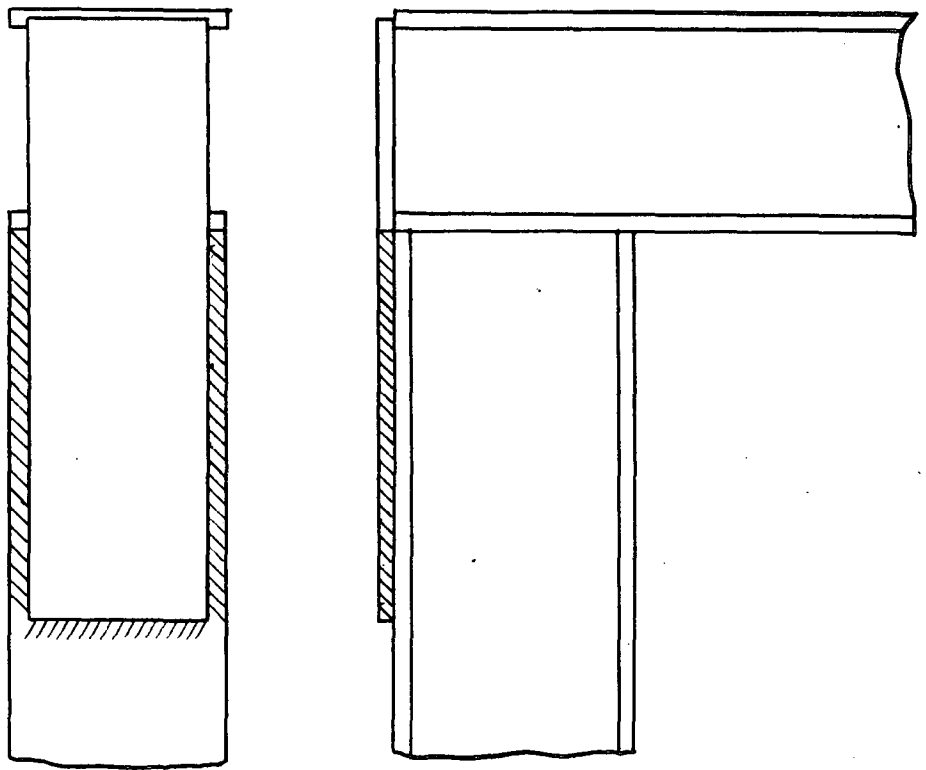


FIG 4 FILLET WELDS FOR END PLATE TO COLUMN FLANGE LAP JOINT

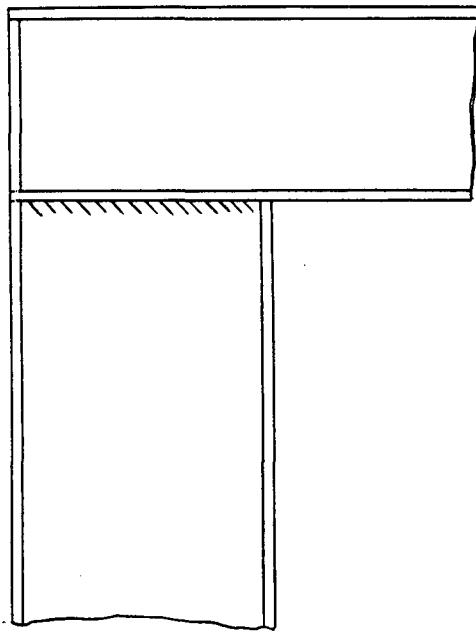


FIG 5 COLUMN WEB TO BEAM FLANGE WELD

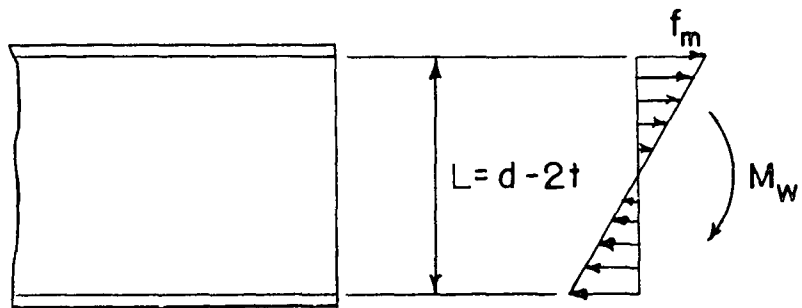


FIG 6 RESISTING MOMENT OF WEB

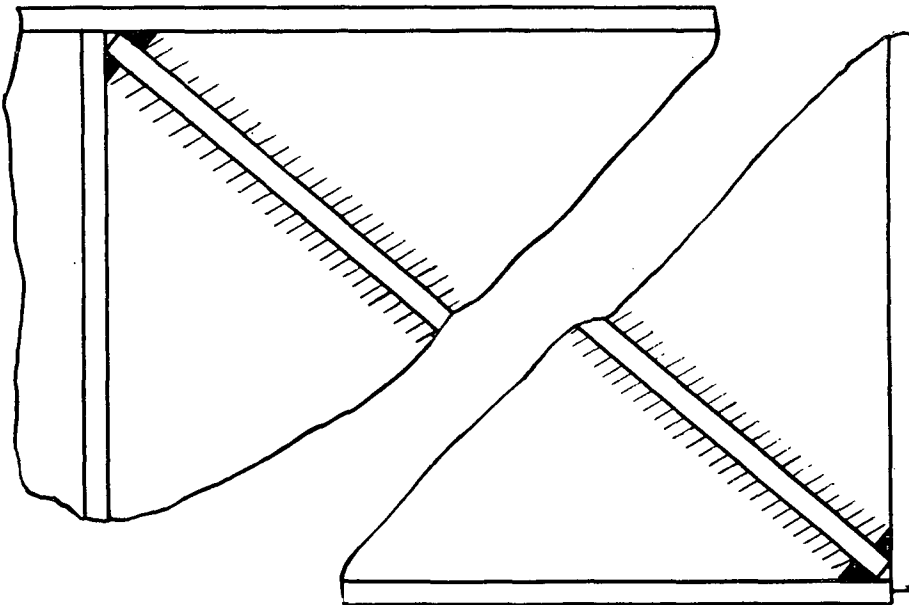


FIG 7 WELDS AT ENDS OF DIAGONAL STIFFENER AND BETWEEN BEAM WEB AND STIFFENER

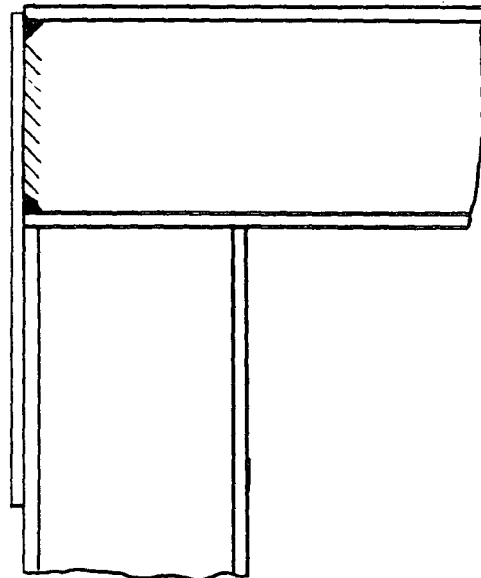


FIG 8 X BEAM WEB TO END PLATE FILLET WELDS

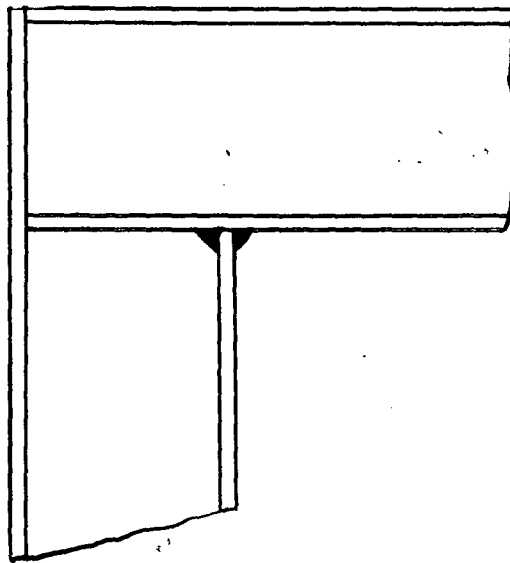


FIG 9 WELD BETWEEN COLUMN AND BEAM FLANGES

X

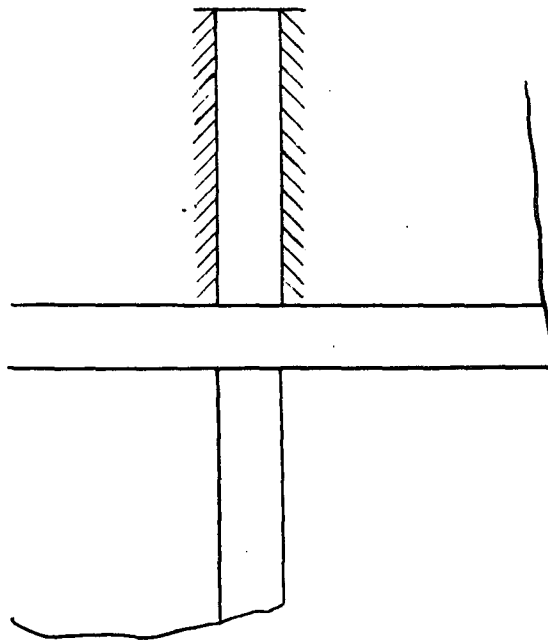


FIG 10 WELD BETWEEN BEAM WEB AND HALF-DEPTH STIFFENER

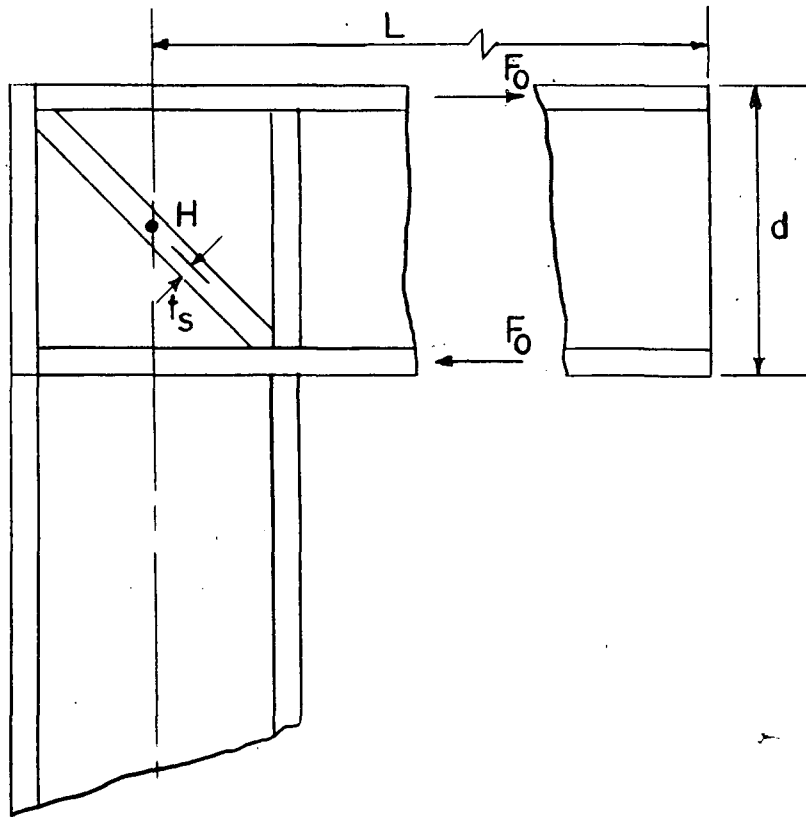


FIG 11 DIMENSIONS AND ASSUMED FORCES ON TYPICAL STRAIGHT KNEE

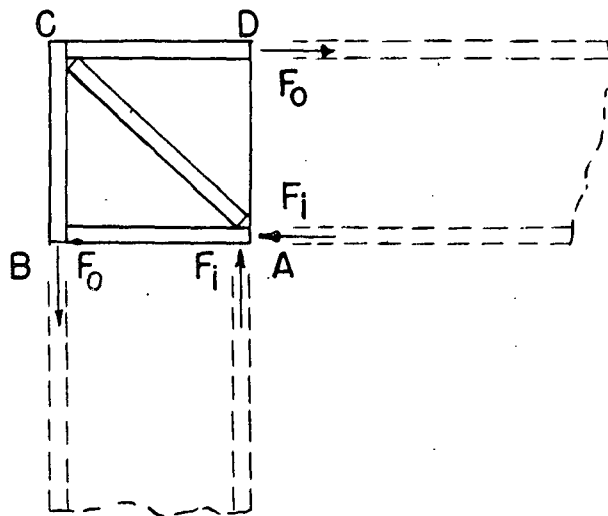


FIG 12 LOADING ON SQUARE KNEE

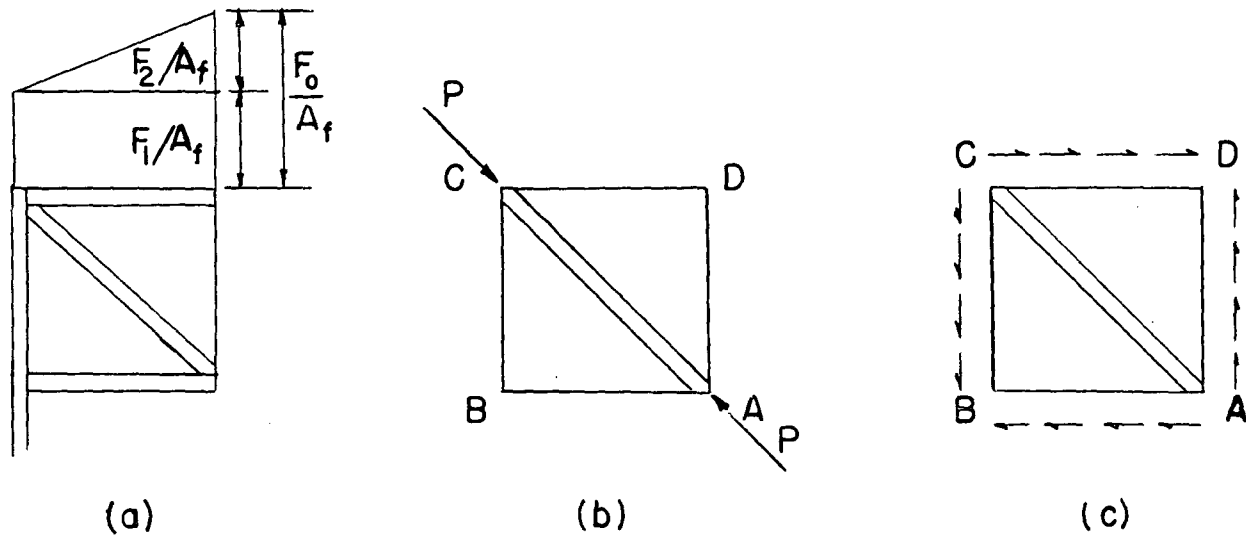


FIG 13 ASSUMED STRESS DISTRIBUTION IN FLANGES AND LOADING ON DIAGONAL STIFFENER AND WEB

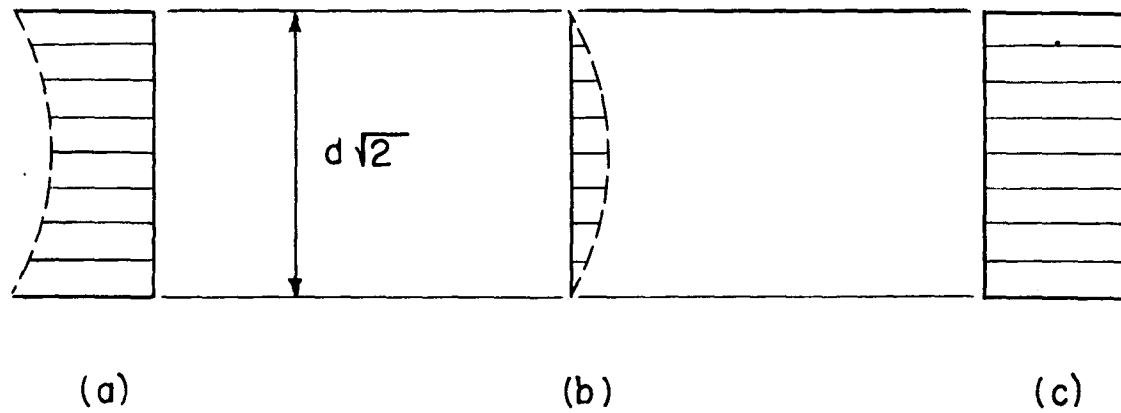


FIG 14 VARIATION IN STRESS ALONG DIAGONAL STIFFENER

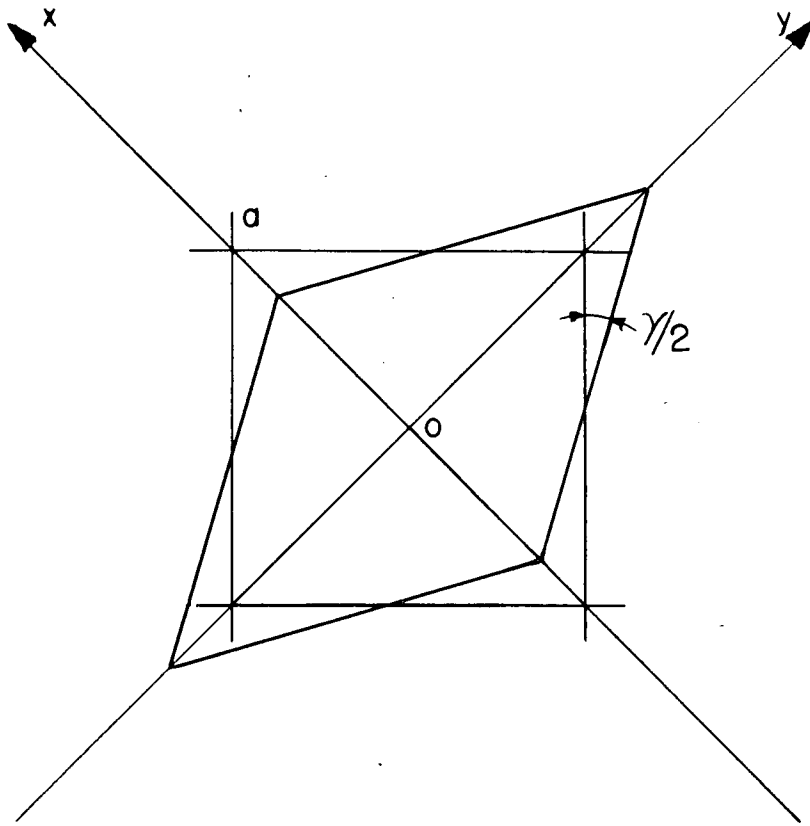


FIG 15 DEFORMATION OF KNEE WEB PLATE

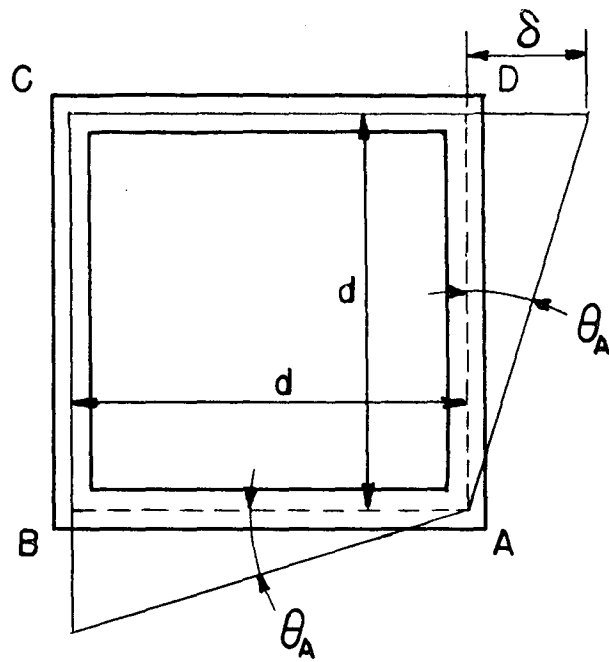


FIG 16 ASSUMED KNEE DEFORMATION FOR MEMBERS OF EQUAL DEPTH

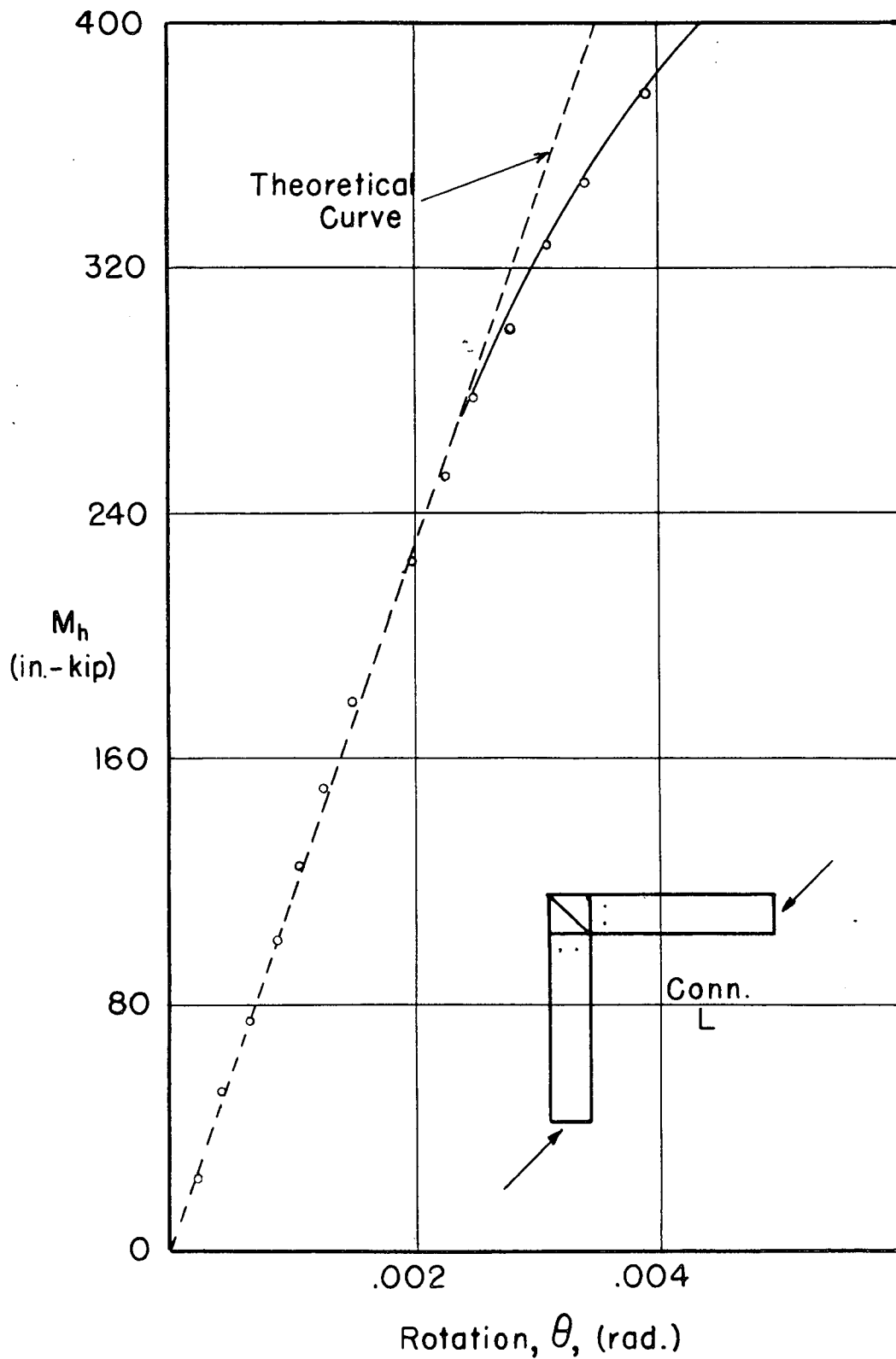


FIG 17 MOMENT ANGLE CHANGE RELATIONSHIP OF CONNECTION AT THE KNEE. 8B13 ROLLED SHAPE

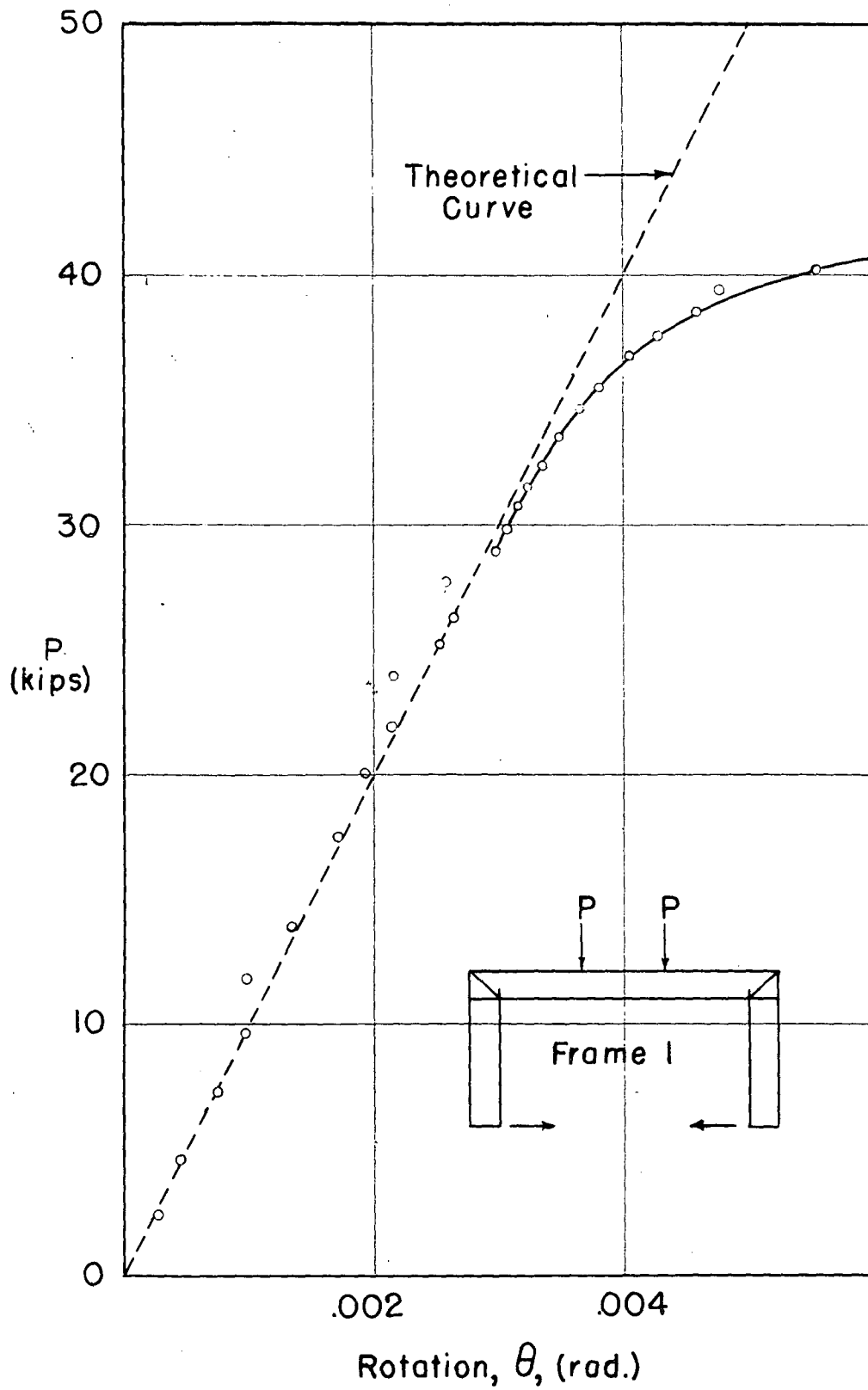


FIG 18 ROTATION AT KNEE WITH INCREASE OF LOAD FOR PORTAL FRAME. 8WF40 ROLLED SHAPE