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Welded Continuous Frames and Their Components

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BEHAVIOR OF TAPERED HAUNCHED CONNECTIONS

by

Jerome E. Smith

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1. SYNOPSIS

Theories for predicting the stress-distribution in haunched connections are reviewed and compared as a basis for selecting a satisfactory method of designing haunched connections in structures proportioned by the plastic method.

The results of a test carried out on a haunched steel corner connections joining a column and a sloping girder are next presented. The welded connection was of proportions that may be found in modern construction. The length of the girder (including the 12WF36 rolled section and a portion of the haunch) was approximately 12 feet and that of the column (16WF45 and haunch), approximately 9 feet.

The purpose of the test was to substantiate theoretical calculations made in the interest of the development of a design procedure for haunched corner connections.

The test showed that, in a connection of the proportions tested, an increase in haunch flange thickness of fifty per cent over rolled section flange thickness will force a plastic hinge to form outside the haunch. This assumes that adequate lateral support is supplied to prevent premature, inelastic buckling.

2. INTRODUCTION

2.1 PURPOSE

Since the use of haunched welded connections is often desirable in plastic design as well as in elastic design there is need for a simple yet accurate method of proportioning such haunches. The method should be such that it would fit into the philosophy of plastic design but could just as well be used by the elastic designer. Due to the poor rotation capacity that is generally observed in a haunched member, a design procedure should be developed which would assure elastic behavior of the haunched portion of a frame even when the structure has reached the ultimate load condition. This will mean for most structures that plastic hinges have developed in the prismatic beam sections adjacent to the haunch at ultimate load.

There are several reasons for the use of haunched connections in steel rigid frames. Not the least of these is the pleasing appearance they afford. Next is economy. A saving in weight of main frame members of 11 percent by the use of haunches has been demonstrated in Chapter 14 of Reference 1.

The use of haunches in plastic design may be necessary in order that rolled shapes may be used for the prismatic beam sections of the frame. This condition may be easily encountered for long span portal frames.

Frames designed on ultimate strength behavior having haunched connections which remain elastic will probably require less lateral bracing at the corners than the unhaunched frame.

The above discussion points out the need for a simple but sufficiently accurate procedure for the design of haunched connections in the elastic state. The primary objective of the present investigation was to determine the ratio between the flange thicknesses of the haunch and rolled sections joined required to ensure the formation of a plastic hinge in the rolled section while the haunch remained elastic. Since, in the interest of ease in fabrication, the depth of section, flange width, and web thickness in rolled shape and haunch are approximately equal at the section common to both, the ratio between flange thicknesses appears to be a logical criterion to consider.

Inseparable from this determination was an experimental evaluation of a method of calculating stresses in members with non-parallel flanges introduced by Harvey C. Olander[(2)*].

Information regarding the amount of lateral support required to adequately brace such a connection was also ascertained.

2.2 HISTORICAL REVIEW

Rigid frames for use as primary members were first introduced in this country in the 1920's when they were used in New York State parkways[(3)]. The marked increase in their popularity has been due to a number of factors. They make possible an economy

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* Numbers in parenthesis indicate the reference numbers in References.

in girder size due to the end restraint provided by the columns. The improvements in welding techniques have made it more practical to use built-up steel members. Rigid frames may be designed with proportions of pleasing appearance.

Connections for use in rigid frames may be divided into three classes: square; haunched; and, curved. These are shown in Figure 1. The connection tested in preparation of this report was similar to type 2B. It differed from 2B in that it incorporated a sloping girder and joined members of different sizes.

There are several methods of analysis available to determine the moments and forces present in rigid frames[3],[4],[5],[6],[7]. When moments and forces are known, unit stresses in the prismatic members of a rigid frame may be determined by the theories of flexure, direct stress, and shear. The use of these principles when applied to members with non-parallel flanges may lead to considerable error[8],[9]. Prior to the publication of Reference 2, theories for the analysis of such members led to methods generally too unwieldy to be used in the design office. Olander's method presents a simple analysis, based on the theory of the wedge, using formulae of familiar appearance.

2.3 REQUIREMENTS

As discussed in detail in Chapter 10 of Reference 1, there are four requirements that connections should satisfy to be acceptable.

1. "Connection must be adequate to develop plastic moment, M_p, of members joined.

2. "It is desirable, but not essential, that average unit rotation of connection materials not exceed that of an equivalent length of beams joined.

3. "To assure that all necessary plastic hinges will form, all connections must be proportioned to develop adequate rotation capacity, R.

4. "Obviously extra connecting materials must be kept to a minimum. Wasteful joint details will result in loss of over-all economy."

Since haunched connections may exhibit poor rotation capacity(1), it is desirable to cause the plastic hinge to form outside the haunch, in the rolled section joined. This may be accomplished by maintaining the entire haunch in an elastic state. Determining a means by which this might be done was the primary objective of the present investigation.

2.4 TEST PROGRAM - GENERAL

The test carried out was on a full scale haunched connection joining a 12WF36 girder with a 16WF45 column by means of a haunch with straight, nonparallel flanges. During the test, measurements of strain were recorded in order to determine the state of stress at several points in the connection. Strain measurements were also used to determine the rotation of various components of the specimen.

The testing of a connection with a curved compression flange has been proposed as a further correlation of theory with test.

3. ANALYSIS OF HAUNCHED CONNECTIONS

3.1 FLEXURE AND AXIAL FORCES

The following discussion purports to be an analysis of haunched connections in general. The tested connection is used as an illustration.

The method proposed by Osgood(8) is rationally developed from the equations of equilibrium and compatability and from the theory of the wedge. While the method is completely straightforward it has not achieved general use. This is probably due to the apparent difference between the expressions presented by Osgood and the better known formulae of conventional beam theory. This difference may be seen with the aid of Figure 2. A plate girder which is triangular (or trapeyoidal) in elevation and loaded at the intersection of its flanges (or extensions of them), point 0, is to be analyzed. The member is symmetrical with respect to a line bisecting the angle between the flanges and the area of a flange is assumed concentrated at its centroid.

The maximum radial stress due to the load P_o (passing through the centroid of the section and point 0) occurs at the centroid and is

$$f_{r1} = \frac{P_o}{wr\ (\alpha + \sin\alpha\cos\alpha) + 2\,A_f\,\cos^2\alpha} \qquad (3.1)$$

The maximum radial stress due to load V_o (normal to P_o and passing through point 0 occurs at the extremities of the section and is

$$f_{r2} = \frac{V_o \sin \alpha}{wr\ (\alpha - \sin \alpha \cos \alpha) + 2\ A_f \sin^2 \alpha} \qquad (3.2)$$

The maximum radial stress due to moment M_o (about point 0) also occurs at the extremities of the section and is

$$f_{r3} = \frac{2\ M_o}{r\ [wr\ (1 - 2\alpha \cot 2\alpha) + 4\ A_f\ \alpha]} \qquad (3.3)$$

F. Bleich(9) developed a theory based on the relationships between stress and strain with special regard to the rapid change of section and the curvature of the centerline in the connection. The theory unfolds rationally but, like Osgood's, terminates in expressions apparently too complicated to be used generally. The curved knee with nonparallel flanges discussed by Bleich is assumed symmetrical with respect to the plane of curvature. The external forces act in the same plane. The curved centerline of the beam may be defined as a line connecting the centers of gravity of a system of circular cylindrical sections passed through the beam such that the extreme fibres and section are mutually perpendicular at their intersection. The radial stress at any point a distance v from the centerline is given by the expression

$$\sigma = \frac{1}{\cos \alpha} \left[\frac{N}{A} - \frac{M}{\rho A} - \frac{Mv}{Z_1} \frac{\rho}{\rho + v} \right] \qquad (3.4)$$

which may more easily be understood by reference to Figure 3.

A is again the total area of the cylindrical cross section. Z_1 is a property analogous to the moment of inertia defined by the integral.

$$Z_1 = \rho \int_A \frac{v^2}{\rho + v} \, dA \qquad (3.5)$$

If $\rho > 2d$, Z_1 may be replaced by the moment of inertia, I.

The recommendations for design as set forth by Griffiths[6] do not suggest a method of analysis for a haunch with nonparallel flanges. Critical design sections are assumed to be at certain geometrical positions in the connection and these sections are checked against rules developed from previous work. This includes large scale model tests conducted at the National Bureau of Standards[10], [11],[12], and at Lehigh University[13], together with the theory developed by F. Bleich[9].

The critical design sections are taken

"(a) At the inside face of column and bottom of girder for a straight knee,

(b) At the points of tangency for a circular haunched knee,

(c) At the extremities and common intersection point for haunches made up of tapered, or trapezoidal, segments."[6]

A means of analysis of sections within the haunch itself is desirable since it is here that the most highly stressed fibres of the connection may be found.

The method for determining the stresses at any section in a member with nonparallel flanges presented by Olander is discussed in detail in Reference 2. A circular section is passed through the member so that it cuts the extremities of the member at right angles as shown in Figure 4a. The section is developed as shown in Figure 4b and its area A and moment of inertia I are obtained. All forces to the right of the section are resolved into the forces P_o, V_o, and M_o about 0, the center of the wedge formed by the cylindrical section and tangents to the extreme fibres of the section. P_o passes through the center of the wedge and the center of gravity of the section. (In the case shown the two flanges are not equal.) V_o passes through the center of the wedge and is normal to P_o. M_o is the moment about 0 of the forces to the right of the section. It is now possible to compute the stresses normal to the section by the familiar expression,

$$\sigma_r = \frac{P_o}{A} \pm \frac{Mc}{I} \tag{3.6}$$

in which M is the algebraic summation of M_o and V_o r.

The cylindrical section may be taken wherever an evaluation of the stresses is desired. The center of the wedge, 0, will shift with various sections when a curved flange is involved and will remain fixed if both flanges are straight.

This method is an approximation to the wedge theory and its accuracy varies with the angle included in the wedge and the geometrical proportions of the section. The results of investigating these variables may be seen in Figures 5, 6, and 7. Figures 5a and 5b show how the ratio of maximum fibre stresses computed by the two methods varies with the relative area of flange to web. In the case of Figure 5a, a load V_o is applied at the end of the wedge. In Figure 5b a moment is applied. Both figures also show the effect of variation in the angle α. For small angles the Olander simplification introduces negligible error, in stresses due to V_O and M_O. Even for the largest α, the maximum deviation was less than 5%. Two plots of the investigation of the wedge loaded axially are shown (Figures 5c and 5d). Although an error of 15 percent is quite high, the axial stress at the neutral axis is seldom of importance. The stress at the edge, or extremity, of the section combines with the flexural stress to give the critical value. Lest an error of 8 percent be thought prohibitive, it should be mentioned that the axial stress is usually a small percentage of the flexural stress so that 8 percent of a small percentage is an acceptable deviation.

Figures 6 and 7 show the agreement between the Olander and Osgood method in what could be an actual connection. Two sections are cut through the connection in order to show the variation in agreement between a large and small value of α.

3.2 SHEAR FORCES

The remarks made on the three earlier methods of analysis regarding stresses resulting from flexural and axial forces may be repeated for stresses due to shear forces. Either the formulae developed are too complex and unfamiliar or no analysis may be made in the haunch itself.

Again referring to Figure 2, the expression for maximum shear stress as given by Osgood is

$$\tau_{r\theta} = - \frac{M_o}{wr^2} \frac{[wr\ (\csc 2\alpha - \cot 2\alpha) + 2\ A_f]}{[wr\ (1 - 2\alpha \cot 2\alpha) + 4\ A_f\ \alpha]} \tag{3.7}$$

According to Bleich, the formula for the shear stress in the web of an I-shaped beam having nonparallel, curved flanges is of a complex nature and, since shear stresses are always small by comparison to fibre stresses, it is sufficiently accurate to compute shear stresses by the following design formula which neglects the curvature of the flanges:

$$\tau_{r\theta} = \frac{VQ}{Iw} - \frac{1}{3} \frac{M\bar{A}}{Iw} \tan \alpha \tag{3.8}$$

In this expression, V is the total shear force on the section and Q is the statical moment of the area $\bar{A}$ (Figure 3) about the axis through the center of gravity.

Olander suggests as an approximation the following simple formula

$$\tau_{r\theta} = \frac{VQ}{Iw} \tag{3.9}$$

in which V, the total shear force on the section, is equal to $\frac{M_o}{r}$ and Q is the statical moment, about the axis through the center of gravity, of the area outside the point at which the magnitude of the shear stress is desired. A comparison between this method and the more exact procedure was not made because the shear stresses are usually not critical except in the corner of the haunch.

4. DESIGN DETAILS

4.1 FLANGE THICKNESS

An objective of the overall project of which this report is a part is the development of design guides which will indicate the thickness of haunch flange required to maintain the haunch in an elastic condition. Until these guides may be formulated, this thickness will best be determined by a trial and error method. This is also true of the associated problem of locating the most highly stressed section. Expressions for these two values may be obtained but their solutions yield most readily to implicit methods. By assuming flange and width thickness and shape of haunch, the most critically stressed section may be found by analyzing several sections and plotting maximum stress as a function of wedge radius. With the critical section located, the flange thickness required to maintain the maximum stress below a certain value may be determined by analyzing the section with several different flange thicknesses and plotting the maximum stress as a function of the flange thickness. This was the procedure followed in the design of the tested connection.

The flange width was maintained equal to that of the rolled section adjacent to it. In the test connection the flange width was changed by tapering at the miter line. (Figure 8) In actual design, however, the width would probably be held constant at the greater rolled section flange width.

4.2 WEB THICKNESS

As a preliminary choice, the web thickness may be selected as a convenient value near that of the rolled sections joined. This size must then be investigated to determine whether or not it is adequate to resist the shearing forces present. In the expression for shearing stress,

$$\tau_{r\theta} = \frac{M_o\, Q}{r\, I\, w} \tag{3.9}$$

since the product of rI increases with r at a more rapid rate than does Q, the maximum shear stress will occur in the cylindrical section of smallest radius. In the haunch girder of the tested connection,

$$\frac{\tau_{max}}{P} = \frac{M_o\, Q}{P r I w} = \frac{(11.26)(35.1)}{72.3(385)(0.3125)} = 0.0455$$

in which P is the load on the specimen. In the haunch column,

$$\frac{\tau_{max}}{P} = 0.0306$$

For an anticipated ultimate load of 70.8 kips, a web thickness of 0.3125 inches would be adequate since the maximum shear stress would be

$$(0.0455)(70.8) = 3.22 \text{ ksi}.$$

Since the depth of the web increases considerably between the rolled section and the miter line it should also be investigated for compliance with Section 26(b) and (e) of the AISC Specification(14). The ability of the web to withstand the localized shear stresses is directly associated with the diagonal stiffener at the corner and will be considered below.

4.3 DIAGONAL STIFFENER AT CORNER

The corner diagonal stiffener refers to the plate joining the point of intersection of the two outside flanges with that of the two inside flanges. Its thickness was determined by two approximate methods. The first is described in detail in Section 10.3 of Reference 1. The primary objective of this method is to ensure that the connection does not fail to develop its computed plastic moment because of shear yielding in the web. This is accomplished by maintaining the moment at which yield commences due to shear, $M_{h(\tau)}$, at not less than the plastic moment M_p. The assumptions made in the development of this method are:

"(a) Maximum shear stress yield condition

"(b) Shear stress is uniformly distributed in web of knee

"(c) Web of knee carries shear stress, flange carries flexural stress."

It is also assumed that the flange carries the direct stress.

Equating the haunch moment at shear yield to the flexural strength,

$$\frac{wd^2 \sigma_y}{2(1 - \frac{d}{L})} = \sigma_y Z \tag{4.1}$$

the required web thickness is

$$w = \frac{2fS}{d^2} (1 - \frac{d}{L}) \tag{4.2}$$

and since f is slightly larger than 1.0 and $1 - \frac{d}{L}$ is slightly smaller than 1.0,

$$w \cong \frac{2S}{d^2} \tag{4.3}$$

This results in a required web thickness, for the cylindrical section of maximum radius passed through the haunch girder, of 0.513 inches. Assuming that a diagonal stiffener is actually spread over the entire corner of the connection and serves to uniformly thicken the web, the required thickness of stiffener is

$$t_s = \frac{\sqrt{2}}{b} \left(\frac{S}{d} - \frac{wd}{2}\right). \tag{4.4}$$

For the tested connection this meant a stiffener 0.512 inches thick was required.

The second method neglects the web entirely and assumes that the moment and direct force at the corner of the connection are resisted by the flanges alone. The flanges and stiffener are thought to be parts of a truss and the stiffener must be of sufficient thickness to resist any unbalance of forces brought into the "joint" by the flanges. By this method the required thickness was 0.84 inches. As a matter of practicality, the stiffener in the connection was cut from the same material as the flanges (13/16 inch plate).

4.4 SPLICE STIFFENERS

The stiffeners near the junction of the haunch and rolled section were originally to be placed in the haunch. Again in the interest of practicality their thickness was made equal to that of the haunch flange. When it was learned that a more common practice is to place the stiffener in the rolled section, they were not redesigned. This led to a much heavier stiffener being used than would be required by assuming truss action at the splice.

4.5 ANGLE BETWEEN HAUNCH FLANGES

The angle between the inner and outer flanges was chosen so that, as nearly as possible, a compressive stress of 33 ksi would be present all along the inside flange of the haunch. It is considered that this is the most severe condition possible in the haunch, in keeping with the requirement that the haunch remain elastic. The value of 33 ksi was chosen as it is the minimum allowable yield stress for A-7 steel for ASTM Specifications.

4.6 WELDING

The welding between the flange and web of the haunch was designed to resist the shear stress present at their common surface. The joint between the haunch and rolled section developed from an original proposal to butt both the haunch and rolled section against the 13/16 - inch stiffener. This involved making cutouts in both webs. This design was revised when the consequences of a possibly laminated stiffener were pointed out. There was also some feeling that cutouts might initiate cracks. The final welding design may be seen in Detail A of Figure 8.

5. PREDICTING BEHAVIOR OF CONNECTION

The behavior of the connection was predicted as a basis for evaluating the results of the test. These predictions involved dividing the connection into two parts: first, that part, in both column and girder, between the point of application of load and the cylindrical section passing through the intersection of the inside flanges of the haunch; and second, the remainder of the connection, in the vicinity of the diagonal stiffener.

5.1 PORTION BETWEEN POINT OF APPLICATION OF LOAD AND CYLINDRICAL SECTION OF MAXIMUM RADIUS

The rotation and deflection of the connection were assumed to be due entirely to the moment produced by the load. The rotation per unit length was taken, as usual, as

$$\phi = \frac{M}{EI} \tag{5.1}$$

The total rotation between two sections, one of which was rigidly fixed, was

$$\theta = \int \phi \, dx. \tag{5.2}$$

The deflection between these two sections was, then,

$$\delta = \int \phi x dx. \tag{5.3}$$

The moment, M, and moment of inertia, I, were as defined by the Olander method. Deformations in these portions of the connection were predicted assuming the girder and column to be rigidly fixed at the cylindrical section of maximum radius (the section passing through the intersection of the inside flanges of the haunch).

5.2 PORTION IN THE VICINITY OF THE DIAGONAL STIFFENER

In predicting the behavior of the balance of the connection (in the corner, proper), the method presented in Reference 15 was followed insofar as it was applicable. The shape of the portion of the connection under consideration, as well as the forces acting thereon, is shown in Figure 9. Use of this method involves the following assumptions:

1. The flexural and direct forces are taken by the flanges and the shear force by the web.
2. The flange forces decrease linearly from their value at B or D to zero at C, this force being taken by the web in shear.*
3. The unbalance between the two forces at A is taken as a direct force by the diagonal stiffener. This direct force decreases linearly from its value at A to zero at C, also being taken by the web in shear.

The deformation of the corner is due to two forces: the rotation due to shear, and the rotation due to moment.

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* It is recognized that this is an arbitrary assumption. However, since it correctly predicted experimental behavior in previous tests[15] it seemed reasonable to attempt another comparison on the same basis.

In predicting the rotation due to shear, it is convenient to think of the load on the connection, P, as divided into two parts: P_Q, deforming the stiffener; and P_R, deforming the web. In evaluating P_Q, the force in the stiffener (the unbalance between F_{iC} and F_{iG}) is $K_1 P_Q$. The average stress throughout the length of the stiffener is, therefore,

$$\frac{K_1 P_Q}{2 A_s} \tag{5.4}$$

where A_s is the area of the stiffener. The change in length of the stiffener due to this stress is

$$\Delta L_1 = \frac{K_1 P_Q}{2 A_s E} L_1 \tag{5.5}$$

where E is the modulus of elasticity and L_1 is the original length of the stiffener, $\overline{AC}$.

Now, consider the corner with the stiffener removed. Under load, point C moves to C' (Figure 9). Let $\overline{C'C''} = \overline{BC}$. Then $\overline{BC''} = \overline{BC} \sin\gamma \backsimeq \overline{BC}\,\gamma$ and $\overline{AC''} = \overline{AB} - \overline{BC}\,\gamma$.

$$\Delta L_2 = \overline{AC} - \overline{AC''} \tag{5.6}$$

$$= \overline{AC} - \sqrt{\overline{BC}^2 + \overline{AC''}^2}$$

$$= \overline{AC} - \sqrt{\overline{BC}^2 + (\overline{AB} - \overline{BC}\,\gamma)^2}$$

$$= \overline{AC} - \sqrt{\overline{BC}^2 + (\overline{AB}^2 - \overline{2AB}\ \overline{BC}\,\gamma)} \qquad (\gamma \ll 1.00)$$

Substituting the known values in this expression leads to

$$\Delta L_2 = K_2 \gamma \ . \tag{5.7}$$

where K_2 is computed from the previous expression. Since ΔL_1 must equal ΔL_2,

$$\frac{K_1 P_Q L_1}{2 A_s E} = K_2 \gamma \tag{5.8}$$

and,

$$P_Q = \frac{2 K_2 A_s E \gamma}{K_1 L_1} = K_3 \gamma \tag{5.9}$$

The portion of the load deforming the web will now be determined. The force originally taken by the stiffener is now assumed to be taken in shear by sides AB and AD. The flanges are removed and replaced by the shear forces they introduce into the web. The web is then in a state of pure shear and the shear stress on each of the four surfaces is computed. If (as in the case of the tested connection) these stresses are not all numerically equal, it is an indication that at least one assumption is not correct. This is already known to be true since there are actually direct forces on surfaces AB and AD. The average of the four shear stresses was used in predicting the behavior of the tested connection. This average shear stress

$$\tau = K_4 P_R \tag{5.10}$$

$$G\gamma = K_4 P_R$$

and

$$P_R = K_5 \gamma \tag{5.11}$$

In which G is the shear modulus of elasticity. Since

$$P = P_Q + P_R$$

$$P = K_3\gamma + K_5\gamma = K_6\gamma \qquad (5.12)$$

and the deformation of the connection due to shear deformation of the corner is

$$\gamma = \frac{P}{K_6} \qquad (5.13)$$

The constant K_1, depends upon the angle between the two inside flanges and the force in them. K_2 depends upon the dimensions of the corner. K_3 is simply a combination of K_1, K_2, the dimensions of the corner, and the modulus of elasticity of the material used in the corner. K_4 P_R is the average of the four values of shear obtained by dividing the shear force on a surface of the corner by the cross-sectional area of the surface. K_5 is the shear modulus divided by K_4 and K_6 is the sum of K_3 and K_5.

The rotation of the corner due to bending, λ, will now be considered. The elongation of the flange CD due to the force in the flange is

$$\delta_G = \frac{\sigma_G \overbrace{r_G\,\beta}^{CD}}{2E} \qquad (5.14)$$

in which $\frac{\sigma_G}{2}$ is the average stress in the flange between points D and C and r_G β is the arc length, AD. The rotation of the girder due to bending is

$$\lambda_G = \frac{\delta_G}{r_G\ \beta} = \frac{\sigma_G\ r_G\ \beta}{2E\ r_G\ \beta} = \frac{\sigma_G}{2E}\ . \tag{5.15}$$

Neglecting direct stress,

$$\sigma_G = \left(\frac{Mr\ \beta}{2I_F}\right)_G \tag{5.16}$$

where I_F is the moment of inertia of the two flanges about the axis through the center of gravity of the section.

Then

$$\lambda_G = \left(\frac{Mr\ \beta}{4EI_F}\right)_G \tag{5.17}$$

Likewise, in the column,

$$\lambda_C = \left(\frac{Mr\ \beta}{4EI_F}\right)_C \tag{5.18}$$

The total rotation of the corner due to bending is

$$\lambda = \lambda_G + \lambda_C. \tag{5.19}$$

Calculations of rotations and deflections of the corner were made assuming it to be rigidly fixed in the plane of the diagonal stiffener.

6. EXPERIMENTAL INVESTIGATION

6.1 DESCRIPTION OF TEST

The test was carried out on a full scale connection as detailed in Figure 8. Whenever a single test is used to determine whether a theory is adequate or not it must be designed to be "critical" in every respect possible. The following were done to meet this objective:

1. The connection was proportioned such that axial force in the rolled section would be just greater than the 15% "limit" for this factor (P_a/P_y in 12WF36 = 16%).

2. The shear was made close to what might be considered "critical" by using an a/d ratio of 3.5 and 3.9 for girder and column, respectively.

3. The connection was proportioned so that the stress on the compression flange would be as uniform as practicable (see Figure 10). This places it in the most critical condition with regard to lateral buckling.

4. Residual stresses were neglected although a previous haunched connection had shown them to be a factor that influenced connection strength.

5. No allowance was made for the fact that the yield stress of the haunch might be less than that of the members joined (and this turned out to be the case!).

Loads were applied to the connection through end fixtures welded to the ends of the rolled sections. The connection was placed in the universal testing machine so that the end fixture pin on the girder was directly above that on the column (Figure 11).

Four lateral support rods were attached to each of the three stiffeners as may also be seen in Figure 11. A typical rod is shown in detail in Figure 12. SR-4 electrical strain gages were placed on the rods in order to measure the force required to prevent the connection from buckling laterally. In order that the lateral support rods themselves would not buckle, they were tensioned by means of a turnbuckle to a load of 3 kips each (approximately half the load that would cause yielding in the dynamometer) prior to the application of load to the connection. Whenever the total load in a rod approached either zero or 6 kips, the rod was tightened or relaxed, respectively.

Throughout the portion of the test during which the connection remained elastic, loads were applied in definite load increments. This was possible since the connection would support the load placed upon it. As parts of the connection began to yield it became necessary to load the specimen on a "deflection" criterion. Additional load was applied until a specified additional deflection had occurred. At this point no further increases in load were made and readings of load, deflection, and time were recorded. As soon as deflection and load settled to a reasonably constant value all instrument and gage readings were taken and the process repeated.

Rotation measurements were taken in order to determine the rotation of five portions of the connection. These included the two joints between rolled section and haunch, the girder and column of the haunch, and the corner of the connection. The locations of the dials used to measure these rotations may be seen in Figure 13. Figure 13 also shows the location of the dial gage used to measure the overall deflection of the connection.

SR-4 gages were also used to measure the strains at many locations in the connection. These strain readings were taken in order to experimentally verify the method used to calculate the stresses in the connection. The locations of these gages are shown in Figures 14, 15, and 16.

An ordinary surveyor's transit was used to read lateral deflection of the compression flange at each of the three stiffeners (Figure 13).

A plumb bob and a horizontal scale were used to measure the increase in the distance between the load line and intersection of outside flanges (Figure 13). This gives an indication of the deformed shape of the connection and provides a means for correcting the moment at the haunch due to increase in moment arm.

The properties of the various pieces of steel used in the test were determined from coupons cut from the material. The results of the tests carried out on these coupons may be seen in Table 1 in the Appendix. The tests were carried out in a mechanical, screw-type testing machine at a slow, laboratory rate. The static yield stress was used in determining first yield and the plastic moment. The coupon was strained into the plastic range and the testing machine was stopped. The load on the coupon would slowly decrease until (after eight or ten minutes) it reached a constant value. The static yield stress was determined by dividing this constant value by the cross sectional area of the coupon.

6.2 RESULTS

The results of the experimental investigation and their correlation with theory are now presented. They are discussed in the next section.

Figure 17 is a curve showing the relationship between the load sustained by the connection and the unit rotation experienced by the portion of the connection in the vicinity of the junction of the 12WF36 and haunch. This rotation was measured by a rotation indicator using dial gages. The predicted curve, assuming an idealized stress-strain relationship, is shown together with predicted values of first yield (P_{yc}), ultimate load (P_u), and ultimate load as modified by the influence of direct stress (P_{uc}).

The theoretical P-ϕ curve in Figure 17 for the elastic portion is based on the values of moment and curvature at several sections within the gage length. The correlation between theory and test is quite satisfactory, and the hinge rotated the desired amount.

Figures 18, 19, 20, and 21 are photographs showing the connection in the vicinity of the hinge in the 12WF36 girder after the completion of the test. Figure 18 shows the location of the hinge in the connection while Figures 19, 20, and 21 show the yielding (indicated by the dark lines in the whitewash) in the compression flange, web, and tension flange, respectively.

Another curve of load average unit rotation relationship is shown in Figure 22. This rotation was obtained from SR-4 electrical strain gages located in the web of the 12WF36 member as shown (gage numbers 31 and 32). These were also in the portion of the connection in which the plastic hinge formed.

The rotation of the 16WF45, 12WF36, and haunch portion of the connection, as measured by mechanical strain gages, is shown in Figure 23. A comparison of the experimental and theoretical curves for the haunch clearly shows the large amount of yielding which took place in this part of the knee.

The total rotation of the connection, including the haunch and a part of each rolled section is shown in Figure 25.

The relationship between the load and the deflection of the connection, as indicated by mechanical dial gage number 11, is shown in Figure 24. The deflection was predicted using the measured moment of inertia of the sections but neglecting deformation due to direct force.

The agreement between the predicted and experimental behavior of the corner of the connection in the region of the diagonal stiffener may be seen in Figure 26. Although shear yielding in the web began at a load of less than 25 kips, this had a small effect on the behavior of the corner.

The results of applying the theory presented under "Predicting Behavior of Connection" (Equations 5.3, 5.13, and 5.19), to a previously tested connection may be seen in Figure 27. This connection joined two lengths of 8BL13 rolled sections. In a building, the column would have been vertical and the girder horizontal. The haunched portion of the connection was made of material of dimensions similar to those of the rolled sections. No attempt was made to strengthen the haunch beyond increasing the depth. The theory satisfactorily predicts the elastic slope.

Information regarding lateral forces and displacements are shown in Figure 28. In order to prevent buckling of the lateral support rods they were pretensioned and maintained under tension during as much of the test as possible. The net force required to support the connection against lateral motion at two points, A and B, is shown in Figure 28 (a) and (q). The lateral buckle in the connection occurred between these two points (Figure 38). The deflection at the two points as well as the deflection at the center of the buckle may also be seen in Figure 28 (b), (d), and (e).

The variable relationship between the load and moment at two sections in the connection may be seen in Figure 29. This data is obtained from the mirror gage. The relationship used in predicting deformations is shown as a dashed line. The motion of the section common to the 16WF45 and the haunch was interpolated from that of the corner of the connection assuming a linear deformation between the load point and the corner. This curve shows that the haunch actually sustained a moment greater than the value implied in the load values previously presented. While they could be corrected, it would only serve to substantiate the conclusions. The results would appear somewhat better.

The agreement between experimental strains and those computed by the Olander method may be seen in Figures 10 and 30. Figure 30 shows the strains in the cylindrical section of minimum radius in the haunch girder for four different loads. Figure 10 is a graphical presentation of the strains along the tension and compression flange of the haunch girder for the same four loads. In the cases where more than one SR-4 electrical strain gage was placed laterally across a flange, the average of the readings was plotted.

The lateral distribution of strain across the flanges of a typical section may be seen in Figure 31. The strains are plotted for both tension and compression flanges at four different loads and compared to the predicted values.

Figure 32 is a comparison between the predicted and experimental variation of strain on the tension flange between the cylindrical section of maximum radius and the outside corner of the connection. The strains are again plotted for various loads. As indicated, the theoretical curve was based on the assumption of zero stress at the outer corner and a linear variation to the point of (assumed) maximum stress.

7. DISCUSSION OF RESULTS

7.1 FORMATION OF HINGE

One of the primary objectives of the test was to cause a plastic hinge to form in the rolled section adjacent to the haunch. For this purpose, the haunched portion of the connection was made stronger than the rolled shapes adjacent to it. The lateral support system was designed to prevent lateral buckling of the connection prior to the formation of the hinge. That a hinge actually formed may be seen in several figures. The load average unit rotation relationship in the vicinity of the hinge may be seen in Figure 17. The connection sustained a load higher than its computed ultimate load through a hinge rotation ten times larger than the elastic rotation of the 12WF36 in which the hinge formed. Physical evidence of hinge formation (characterized by flaking of whitewashed mill scale) is seen in the compression flange (Figure 19) and the tension flange and web (Figures 20 and 21).

The plot of load average unit rotation made with the aid of two SR-4 electrical strain gages in the vicinity of the plastic hinge (Figure 22) gives another picture of the behavior of the connection at this point. This is because the gages were in a position less apt to be affected by welding residual stresses and the stiffener.

7.2 ELASTICITY OF HAUNCH

The haunch was intended to remain elastic during the formation of the hinge. That this objective was not attained may be seen in the load-rotation relationship for the haunch in Figure 23. Photographic evidence is shown in Figure 33. Although this photograph was taken at a load of 68.6 kips, a considerable portion of the yielding had taken place prior to reaching a load of 40 kips. The haunch was designed so that the maximum stress in the compression flange would not exceed 33 ksi (the minimum allowable yield stress for A-7 steel). As may be seen in Table 1, the actual static yield stress of the haunch flange material was 27.7 ksi. Thus the flange yielded prior to attaining the ultimate load of the connection. The residual stresses introduced into the haunch by the welding also contributed to its early inelastic behavior. In spite of this inelastic action in the haunch, the connection was still able to meet the desired objective.

While the web of the haunch yielded locally at low loads (Figure 34 was taken at a load of 35 kips) it proved adequate to resist web buckling. The yield lines may be seen to emanate from points where high welding residual stresses would be expected.

7.3 DEFORMATION OF THE HAUNCH

The deformation of the haunch, Figures 23 and 26, follows in general the predicted behavior. Variations at low load are due to the low yield strength of the haunch flange material. The agreement between predicted and experimental load-vs-average unit rotation behavior for the corner of the haunch (Figure 26) indicate that the rather arbitrary assumptions made in the prediction still result in reasonable correlation with test results.

7.4 STRESS DISTRIBUTION IN THE HAUNCH

The angle between the flanges of the haunch was chosen so that, as nearly as possible, the stress would be constant along the compression flanges. The degree to which this was accomplished in the girder may be seen in Figure 10. The large variations at comparatively low loads again show the effect of low yield strength and residual stresses.

7.5 DIAGONAL STIFFENER AT THE CORNER

The diagonal stiffener was thicker than required (see Eq. 4.4). But in spite of this it yielded as is seen in Figure 35 taken subsequent to the test. It was made of the same material as the haunch flange and thus had a lower yield stress than assumed. Further, the welding introduced compressive stresses in the plates. Therefore yielding was to be expected at lower than predicted loads. Figure 36 shows the relationsip between measured and predicted strains. This, along with Figure 32, is evidence that the forces in the outside flanges of the haunch do not decrease to zero at the outside corner of the haunch.

7.6 SIMULTANEOUS FORMATION OF TWO HINGES

The distance between the load point and the beginning of the haunch, in both the column and the girder, had been designed so that, if the yield strength of the 12WF36 and the 16WF45 had been equal, hinges would have formed in them simultaneously. This did not occur, as may be seen in Figures 23 and 37. The reason for this, as may be seen in Table 1, is that the 12WF36 girder material had a yield strength considerably lower than that of the 16WF45 column. But the fact remains that the haunch adequately supported the moment corresponding to the predicted hinge moments at each end and is therefore adequate insofar as design objectives are concerned.

7.7 LATERAL SUPPORT

Because the plastic hinge was able to rotate through a relatively large angle while the connection sustained a load above or near its ultimate load, it may be said that the lateral support was adequate. Figure 28 shows the relationship between lateral force and lateral deflection at the extremities of the flange in which the buckle occurred as well as the deflection in the center of the buckle. The relationship between lateral force and deflection at a certain point is that the tensile force required to prevent large lateral displacement was largest on the side of the connection opposite to the side to which the point tended to move. This is logical and the small displacements allowed by the lateral force is another reason for saying it was adequate.

7.8 ULTIMATE FAILURE

The failure of the connection was due to lateral buckling of the compression flange of the haunch girder. This buckle may be seen in Figure 38, taken subsequent to removal of load from the specimen.

8. DESIGN SUGGESTIONS

The results of the investigation are summarized in the design suggestions that follow.

8.1 GEOMETRY OF HAUNCH

The geometry of the haunch will usually be dictated by architectural requirements. If not, the proportions may be selected so that a desirable economic compromise is achieved between cost of rolled section and expense of haunch fabrication.

For proportions similar to the connection tested, the angle between the flanges (β) should be not less than about 11 degrees (or 0.2 radians). This will cause the small end of the haunch to be the most highly stressed section. This location is, of course, a function of the distance between the point of zero moment in the member and the beginning of the haunch. Therefore the angle between flanges will vary for different proportions.

8.2 THICKNESS OF WEB

As discussed in section 4.2, the web thickness may be selected as a convenient value near that of the sections joined and then investigated for adequacy in resisting shear stresses (Eq. 3.9) and compliance with AISC Specifications (Section 26 (b) and (e)). It will also be a factor in selecting thicknesses of diagonal stiffeners.

8.3 THICKNESS OF FLANGE

The overall program of which this report is a part will result in guides to design to cover all possible shapes of corner connections. As an example of these guides, Figure 39 is submitted. Theoretical investigations were carried out on symmetrical connections joining (at right angles) three widely varying sizes of rolled shapes. The investigations involve finding the most highly stressed section in a haunch and then determining how thick the haunch flange must be in order to maintain the maximum stress below a certain limit.

Until more work has been done along these lines, both of the above steps must be done by trial and error methods, as detailed in the Appendix.

8.4 DIAGONAL STIFFENER

The diagonal stiffener will be adequate if fabricated from the haunch flange material.

8.5 END STIFFENERS

For small angles (less than 20 degrees) between the haunch flanges, the end stiffeners should be made of material no thinner than the flange of the rolled section in which it is placed. For larger angles, the stiffener should be investigated in a similar manner to the diagonal stiffener (see section 4.3).

8.6 LATERAL SUPPORT

Lateral support should be provided for both the tension and compression flanges of the haunch at their junction with the rolled section and at the extremities of the diagonal stiffeners. Each pair of rods used in the test had an area of 1.57 square inches which was 8.3% of the maximum cross-sectional area supported.

9. SUMMARY

This report includes the following:

1. Four methods of designing haunched corner connections were discussed. The reasons for selection of the Olander method as the preferred procedure were given in Section 3. Section 3 also contains the results of a comparison between Olander's and a more rigorous method of analysis (Osgood's) which show the former to give satisfactory results.

2. A description of a test on a full scale, welded, haunched corner connection is given in Section 6.

3. The agreement between predicted behavior of the connection (as determined by the methods of Section 5) and the experimental results are discussed in Section 7.

4. Design suggestions relevant to geometry of haunch, thickness of web and flange, diagonal and end stiffeners, and lateral support are made in Section 8.

5. A suggested sample design guide is presented in Figure 39.

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11. NOMENCLATURE

a = Distance from point of inflection to plastic hinge

A = Total area

$\bar{A}$ = Portion of the area away from the center of gravity on a surface on which the magnitude of unit shearing stress is desired

A_f = Area of one flange

A_s = Area of stiffener

A_w = Area of web

b = Width of flange

c = Distance from neutral axis to extreme fiber of section

c_1 = Ratio of haunch to rolled section flange thickness required for certain allowable stress

d = Depth of section

dA = Differential element of area

dx = Differential element of length

E = Young's modulus of elasticity

E_{st} = Modulus of elasticity in strain hardening range.

f = Shape factor; ratio of $\frac{M_p}{M_y}$

f_r = Radial unit stress as computed by Osgood method.

F_i = Total force in inside flange

F_o = Total force in outside flange

G = Shearing modulus of elasticity

I = Moment of inertia

I_F = Moment of inertia of flanges about center of gravity

$K_1 \ldots K_6$ = Constants, depending on the shape and material of a connection

L = Distance from point of inflection in a member to haunch point

L_1 = Length of diagonal stiffener in corner of connection

M = Total moment on a section

M_h = Moment about the haunch point

$M_{h(\tau)}$ = Moment about the haunch point at which yield occurs due to shear force

M_o = Moment about vertex of wedge

M_p = Ultimate moment that can be reached according to simple plastic theory; plastic moment

M_{PR} = Plastic moment in a rolled shape

M_y = Moment causing first yield in section

N = Force on section parallel to centerline and passing through center of gravity

0 = Vertex of wedge

P = Total load on connection passing through points of inflection

P_a = Axial load on rolled section

P_o = Force on vertex of wedge passing through center of gravity of cylindrical section

P_Q = That portion of P which deforms diagonal stiffener

P_R = That portion of P which deforms web in corner of connection

P_u = Theoretical ultimate load on connection

P_{uc} = Theoretical ultimate load on connection modified by effect of axial force

P_y = Axial load on rolled section sufficient to cause yielding

P_{yc} = Theoretical load on connection causing first yield due to moment and axial force

Q = Statical moment of $\bar{A}$ about center of gravity

r = Radius of wedge

S = Section modulus; $\frac{I}{c}$

t = Thickness of flange

t_s = Thickness of diagonal stiffener

v = Distance from neutral axis to some fiber in section

V = Total shear force on section

V_o = Force on vertex of wedge normal to P_o

w = Thickness of web

WW = Whitewash

X = A variable length

Z = Plastic modulus

Z_1 = Property analogous to I; $Z_1 = \rho \int_A \frac{v^2}{\rho + v} dA$

α = Wedge angle between extreme fiber and fiber at center of gravity

β = Wedge angle between extreme fibers

γ = Total rotation of corner due to shearing stresses

Δ = The change in ....

δ = Deflection

ϵ = Unit strain

ϵ_{st} = Unit strain at beginning of strain hardening

ϵ_y = Unit strain at yield

$\dot{\epsilon}$ = Unit strain per second of time

θ = Total angle change

λ = Total rotation of corner due to flexural stresses

ρ = Radius of curvature of centerline of section

σ = Radial fiber stress due to flexure and direct stress as computed by F. Bleich method

σ_r = Radial fiber stress due to flexure, direct stress, or both as computed by Olander method

σ_u = Maximum load carried by tensile coupon divided by original cross-sectional area

σ_y = Yield stress

σ_{yst} = Static yield stress

τ = Average unit shear stress on four surfaces of corner of connection

$\tau_{r\theta}$ = Unit shear stress on cylindrical section

$\emptyset$ = Average unit rotation

C and G, when used as subscripts, refer to column and girder, respectively.

The haunch point is the intersection of the centerlines of the rolled sections joined by a corner connection.

12. A P P E N D I X

TABLE 1

Coupon Data

Material: Structural Steel

Coupon No.	Shape	Condition	Location	Dimensions L - b - t	Type of Test	Instrument	σ_{yst} ksi	σ_y ksi Mill Report	σ_u ksi	E_{st} ksi	ϵ_y x10-3 in/in	ϵ_{st} x10-3 in/in	% elongation (gage length)	Speed	Remarks
A1	5/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.31	T	(2)	34.0	39.8	60.8	730.	1.41	15.4			$\dot{\epsilon}$ unknown,(3),20.5^K
A3	5/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.31	T	(2)	34.3	↑	60.5	957.	1.27	18.0	$\frac{29.7}{(8)}$	$\frac{7.5}{54.2}$ (4)	(3) 20^K
A5	5/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.31	T	(2)	34.8	↓	61.4	667.	1.22	17.5	$\frac{28.1}{(8)}$	$\frac{7.3}{32.1}$ (4)	(3) 20^K
A6	5/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.31	T	(2)	35.1	39.8	62.0	712.	1.31	16.9			$\dot{\epsilon}$ unknown
B3	13/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.80	T	(2)	28.1	↑ No mill report available ↓	57.6	671.	1.01	11.4	$\frac{34.4}{(8)}$	$\frac{10.4}{62.5}$ (4)	(3) 46.^{K}
B1	13/16"P_L	(1)		22-1$\frac{1}{2}$ - 0.81	T	(2)	27.3		56.8	550.	0.96	10.2	$\frac{35.9}{(8)}$	$\frac{5.4}{66.7}$ (4)	(3) 44.^{K}
C1	12WF36	(1)	Top Web	22-1$\frac{1}{2}$ - 0.32	T	(2)	34.2		60.3	605.	1.18	18.7	$\frac{29.7}{(8)}$	$\frac{6.9}{54.1}$ (4)	(3) 20.5^K
C3	12WF36	(1)	Ctr. Web	22-1$\frac{1}{2}$ - 0.32	T	(2)	36.0		61.9	476.	1.20	21.0	$\frac{31.2}{(8)}$	$\frac{12.1}{50.0}$ (4)	(3) 22.^{K}
C5	12WF36	(1)	Flge.	22-1$\frac{1}{2}$ - 0.46	T	(2)	33.1		59.3	578.	1.25	18.0	$\frac{30.5}{(8)}$	$\frac{5.8}{54.1}$ (4)	(3) 29.^{K}
C7	12WF36	(1)	Flge.	22-1$\frac{1}{2}$ - 0.46	T	(2)	33.2		59.3	550.	1.50	24.9	$\frac{28.1}{(8)}$	$\frac{--}{45.4}$ (4)	(3) 30.7^K
D1	16WF45	(1)	Top Web	22-1$\frac{1}{2}$ - 0.37	T	(2)	35.7		64.8	668.	1.55	24.6	$\frac{28.1}{(8)}$	$\frac{8.8}{50.0}$ (4)	(3) 26.5^K
D3	16WF45	(1)	Ctr. Web	22-1$\frac{1}{2}$ - 0.36	T	(2)	39.6		66.1	742.	1.25	18.8	$\frac{28.9}{(8)}$	$\frac{6.7}{58.3}$ (4)	(3) 27.5^K
D5	16WF45	(1)	Flge.	22-1$\frac{1}{2}$ - 0.47	T	(2)	36.2		65.0	652.	1.22	18.4	$\frac{28.1}{(8)}$	$\frac{7.1}{58.4}$ (4)	(3) 34.^{K}
D7	16WF45	(1)	Flge.	22-1$\frac{1}{2}$ - 0.48	T	(2)	35.9		65.2	695.	1.41	14.6	$\frac{28.1}{(8)}$	$\frac{8.3}{50.0}$ (4)	(3) 35.^{K}

(1)-As delivered (2)-8" G.L. Microformer (3)-Cross heads separated at 0.025"/min. to load appearing in "Remarks" column and at 0.3"/min. from there to breaking (4)- μ"/"/sec to yielding/μ"/"/sec thru yielding.

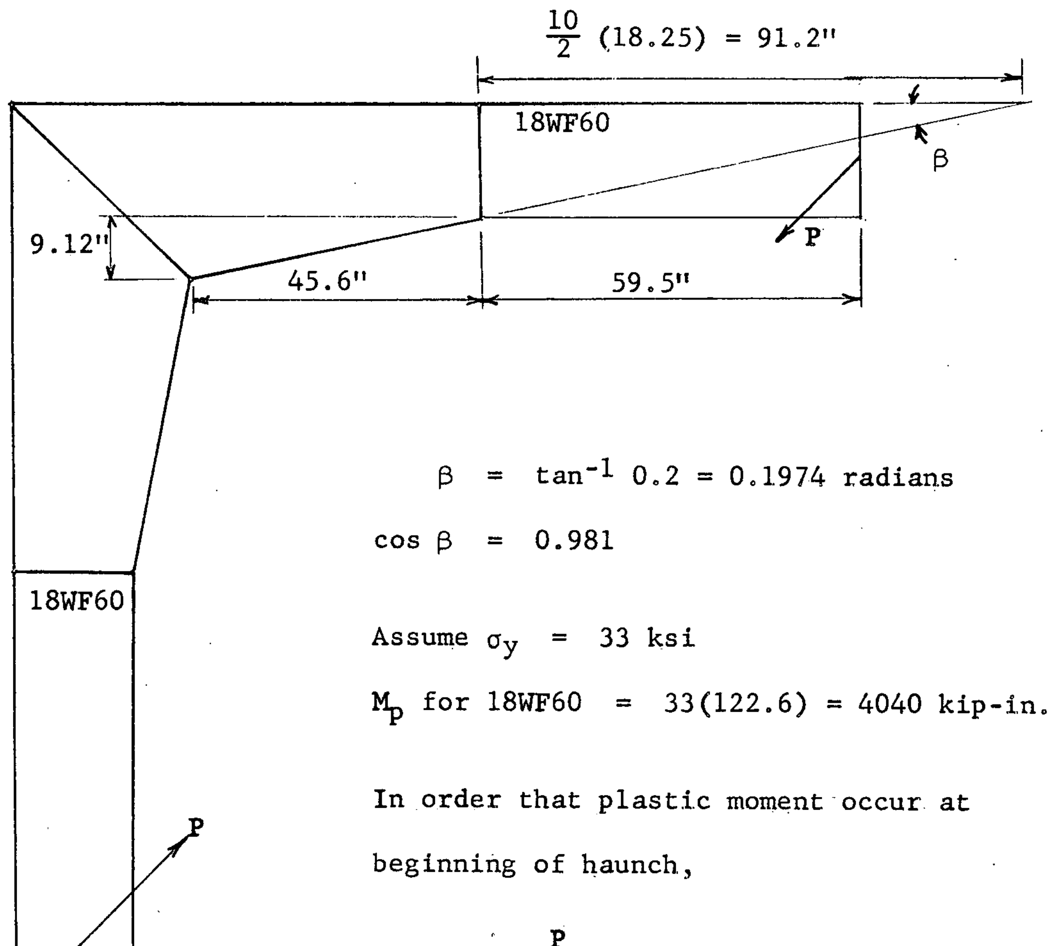
TABLE 2

Summary of Results of Cross Section Measurements

		A	d	b	W	I	S	Z	f
		$''^2$	$''$	$''$	$''$	$''^4$	$''^3$	$''^3$	
12WF36	Handbook	10.59	12.24	6.56	0.305	280.8	45.9	51.42	1.12
	Measured	10.40	12.22	6.58	0.323	270.1	44.2	49.80	1.13
	% Difference	-1.8	-0.2	+0.3	+5.9	-3.8	-3.7	-3.1	+0.9
16WF45	Handbook	13.24	16.12	7.04	0.346	583.3	72.4	82.0	1.13
	Measured	13.09	16.16	7.08	0.359	570.3	70.7	80.3	1.14
	% Difference	-1.1	+0.2	+0.6	+3.8	-2.2	-2.4	-2.1	+0.9

SAMPLE CALCULATIONS TO DETERMINE HAUNCH FLANGE THICKNESS

Given: A connection joining two 18WF60 members with the proportions shown below:



$$\beta = \tan^{-1} 0.2 = 0.1974 \text{ radians}$$

$$\cos \beta = 0.981$$

Assume $\sigma_y = 33$ ksi

M_p for 18WF60 $= 33(122.6) = 4040$ kip-in.

In order that plastic moment occur at beginning of haunch,

$$59.5 \frac{P}{\sqrt{2}} = 4040$$

$$P = 96.3 \text{ kips}$$

$$M_o = \frac{96.3}{\sqrt{2}} \left[\frac{18.25}{2} - (91.2 - 59.5)\right]$$

$$= 68.0 \left[-22.6\right] = -1530 \text{ kip-in.}$$

$$\alpha = \frac{\beta}{2} = 0.0987 \text{ radians}$$

$$\cos\alpha = 0.995$$

$$\sin\alpha = 0.0985$$

$$P_o = 68.0 \quad (0.995 + 0.098) = 74.3 \text{ kips}$$

$$V_o = 68.0 \quad (0.995 - 0.098) = 60.9 \text{ kips}$$

$$r_{min} = \frac{91.2}{0.981} = 93.0 \text{ in.}$$

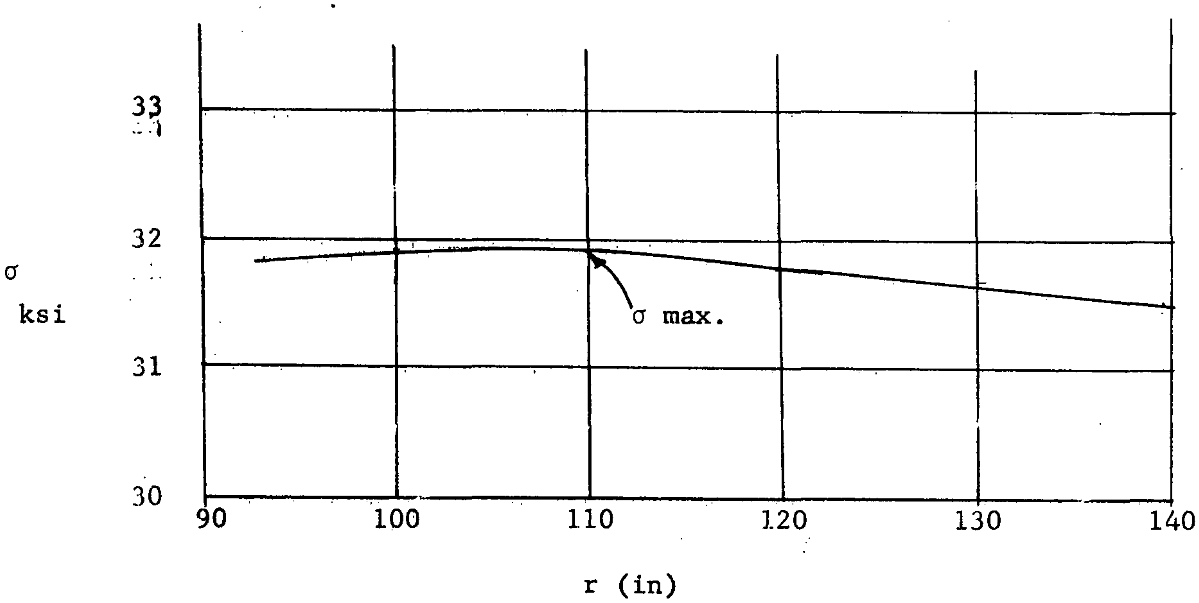
$$r_{max} = \frac{136.8}{0.981} = 139.5 \text{ in.}$$

As a first approximation, let $C_1 = 1.5$

(see table on next page)

$C_1 = 1.5$

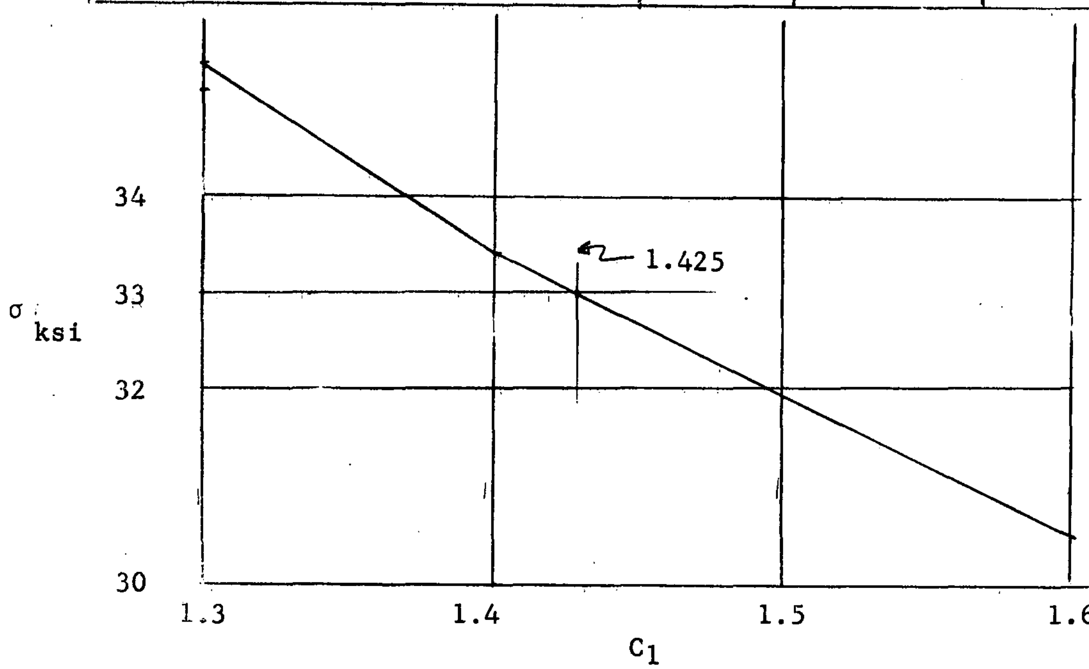
$r =$	93	100	110	120	130	139.5
$r\beta =$	18.35	19.75	21.73	23.66	25.62	27.55
$C_1 t = 1.5\ (0.695) =$	1.042	1.042	1.042	1.042	1.042	1.042
$2\ C_1 t =$	2.084	2.084	2.084	2.084	2.084	2.084
$r\beta - C_1 t =$	17.31	18.70	20.69	22.62	24.58	26.51
$r\beta - 2\ C_1 t =$	16.27	17.66	19.65	21.58	23.54	25.47
$2\ A_f = 2\ (7.56)(C_1 t) =$	15.78	15.78	15.78	15.78	15.78	15.78
$A_w = 0.416\ (r\beta - 2\ C_1 t) =$	6.75	7.34	8.16	8.95	9.79	10.60
$A =$	22.53	23.12	23.94	24.73	25.57	26.38
$\frac{2\ A_f}{4}\ (r\beta - C_1 t)^2 =$	1181	1380	1689	2020	2383	2772
$\frac{A_w}{12}\ (r\beta - 2\ C_1 t)^2 =$	149	191	263	347	452	574
$I =$	1330	1571	1952	2367	2835	3346
$V_o\ r =$	5660	6090	6700	7300	7910	8500
$M =$	4130	4560	5170	5770	6380	6970
$\frac{Mr\beta}{2I} =$	28.5	28.7	28.8	28.8	28.8	28.7
$\frac{Po}{A} =$	3.3	3.2	3.1	3.0	2.9	2.82
$\sigma =$	31.8	31.9	31.90	31.8	31.7	31.52



$r = 110$

$r\beta = 21.73$

C_1 =	1.3	1.4	1.5	1.6
$C_1 t$ =	0.904	0.973	1.042	1.112
$2\ C_1 t$ =	1.808	1.956	2.084	2.224
$r\beta - C_1 t$ =	20.826	20.757	20.69	20.618
$r\beta - 2\ C_1 t$ =	19.922	19.774	19.65	19.506
$2\ A_f = 2(7.56)\ C_1 t$ =	13.67	14.79	15.78	16.81
$A_w = 0.416\ (r\beta - 2\ C1t)$ =	8.29	8.23	8.16	8.11
A =	21.96	23.02	23.94	24.92
$\frac{2\ A_f}{4}\ (r\beta - C_1 t)^2$ =	1482	1593	1689	1786
$\frac{A_w}{12}\ (r\beta - 2\ C_1 t)^2$ =	274	268	263	257
I =	1756	1861	1952	2043
$M = V_o r - M_o = (60.9)(110) - 1530$ =	5170	5170	5170	5170
$\frac{Mr\beta}{2I}$ =	31.99	30.18	28.8	27.5
$\frac{Po}{A}$ =	3.39	3.23	3.1	2.98
σ =	35.38	33.41	31.9	30.48



Therefore, for this connection, the thickness of the haunch flange must be at least 1.425 (0.695) = 0.990 in. This is based upon an allowable stress of 33 ksi.

13. F I G U R E S

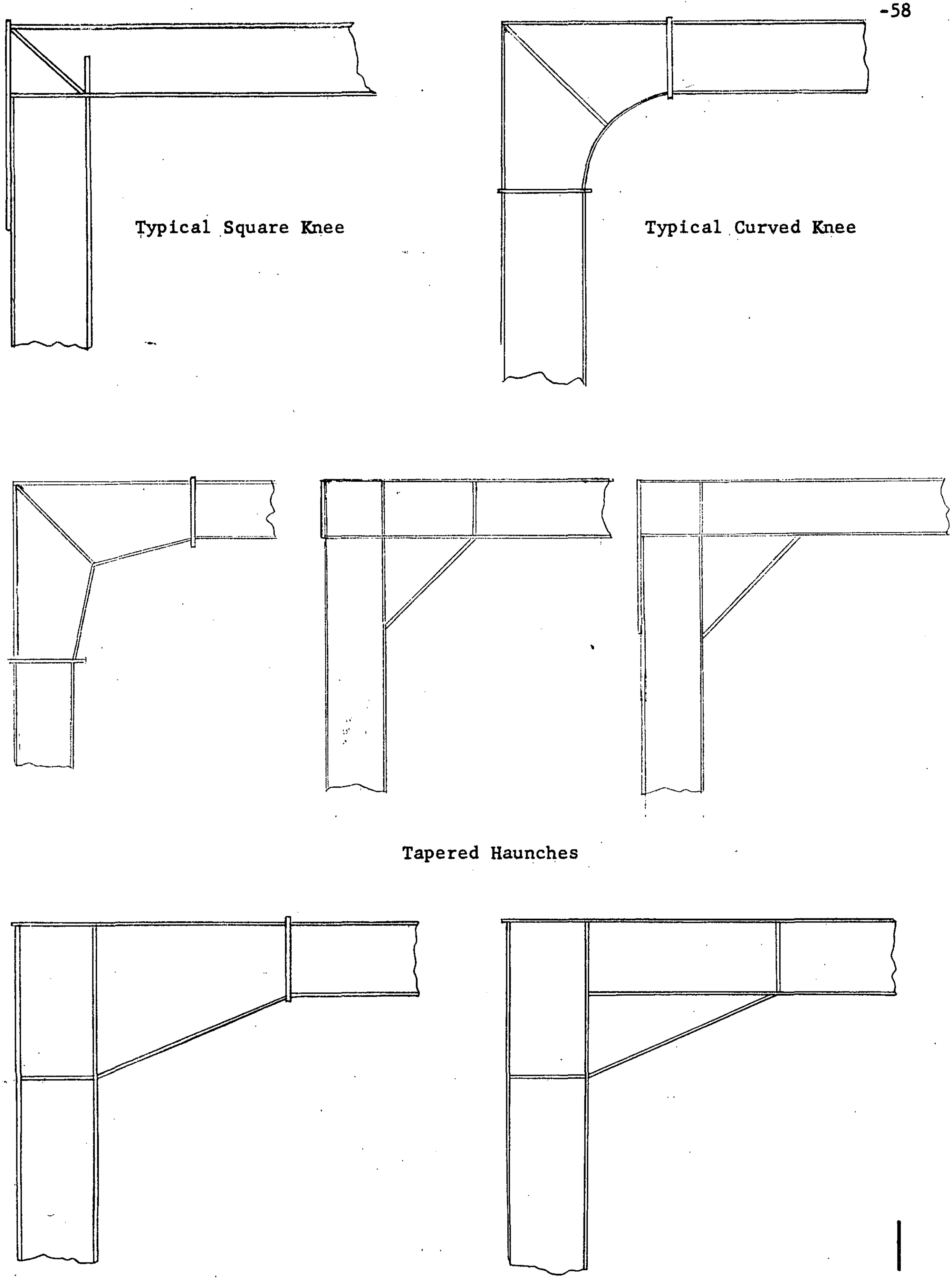


Fig. 1 PORTAL FRAME KNEES

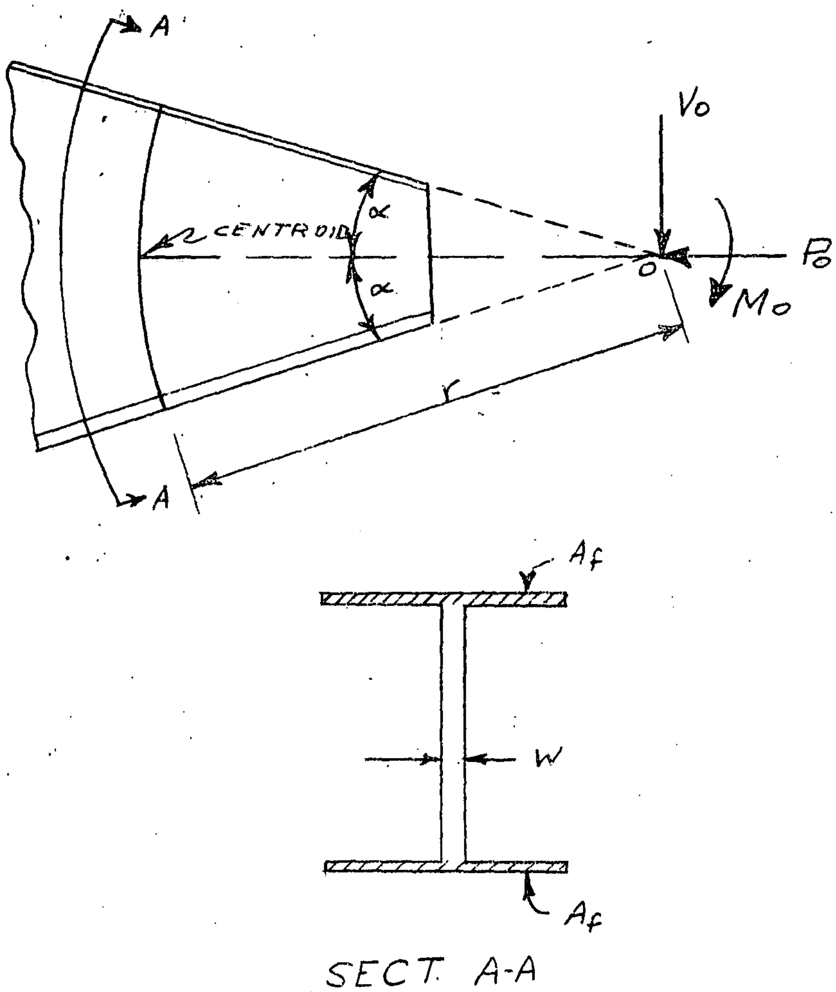


Fig. 2 TERMINOLOGY USED IN DISCUSSION OF OSGOOD'S METHOD

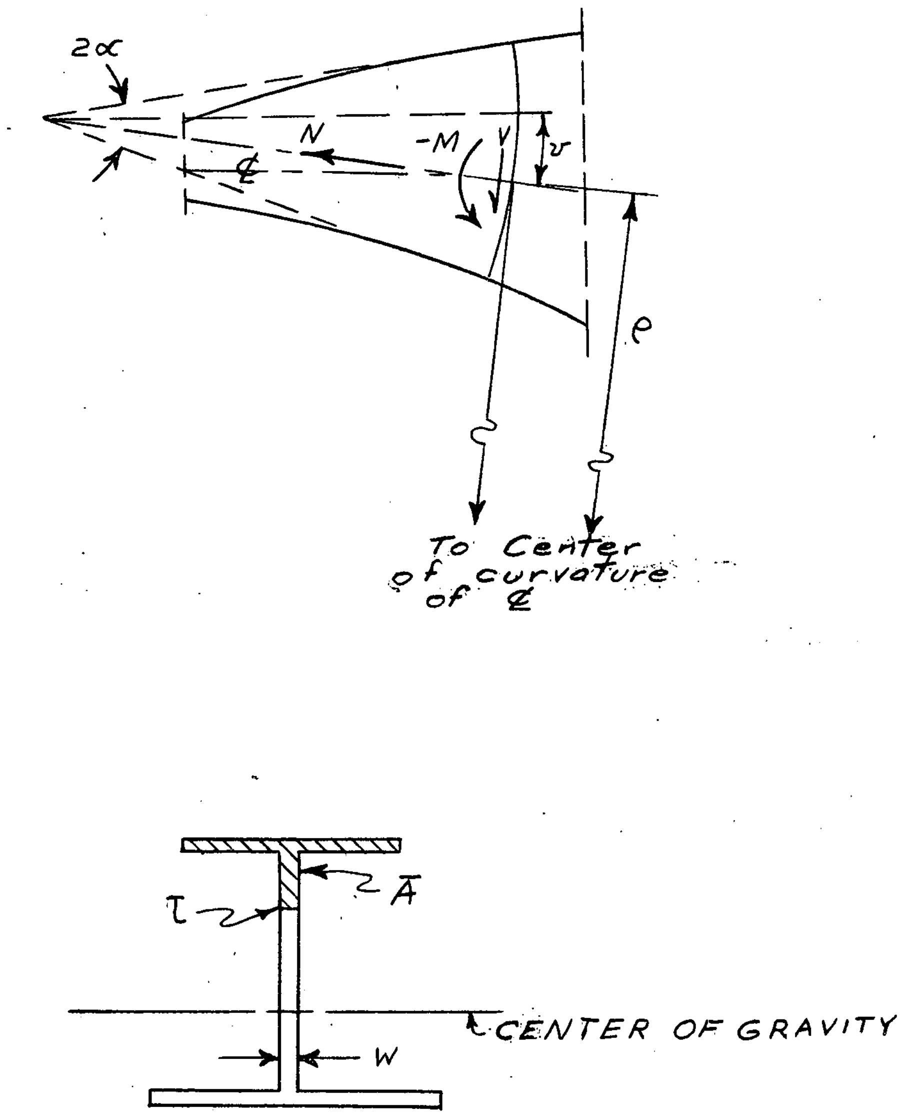


Fig. 3 TERMINOLOGY USED IN DISCUSSION OF F. BLEICH'S METHOD

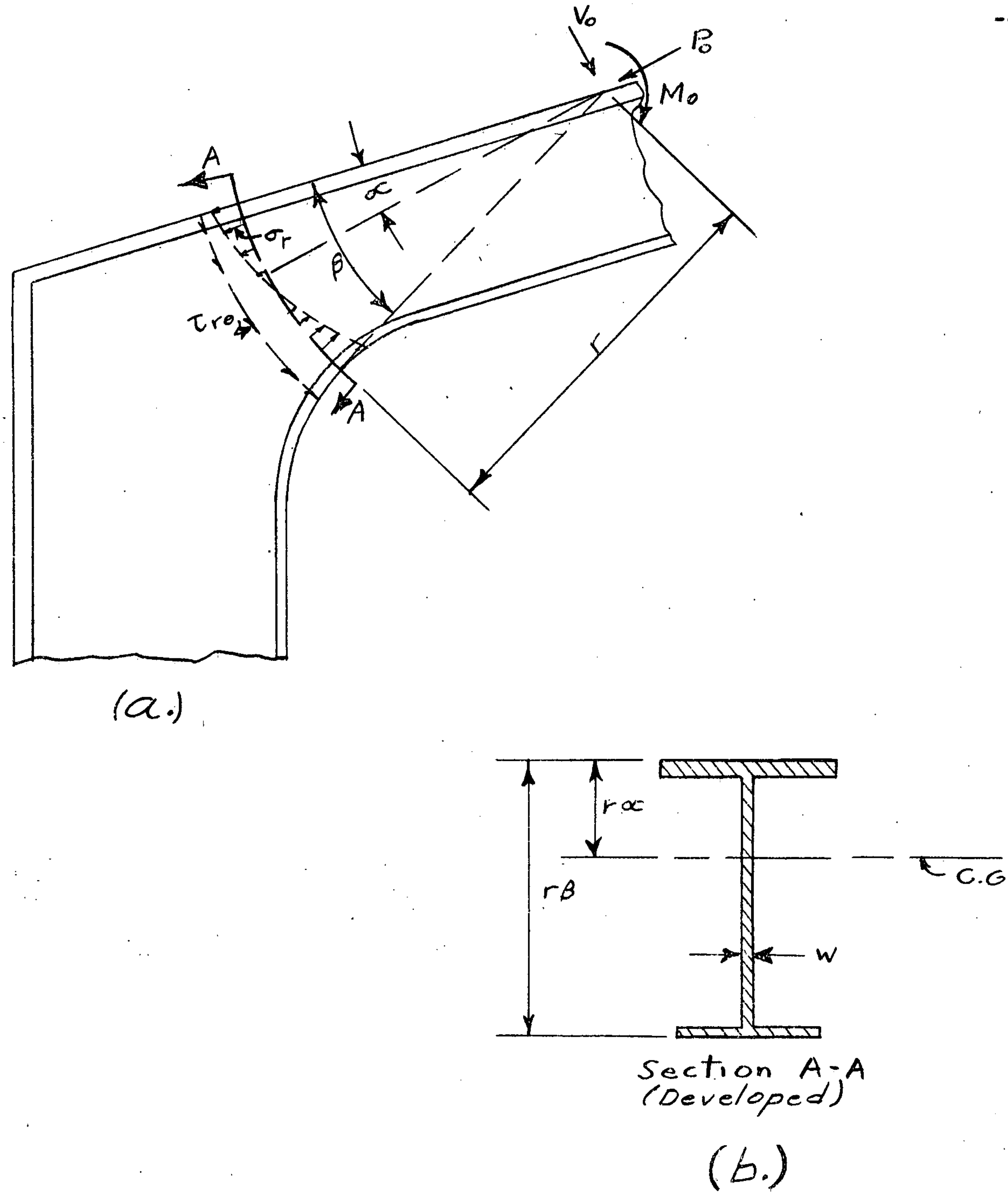
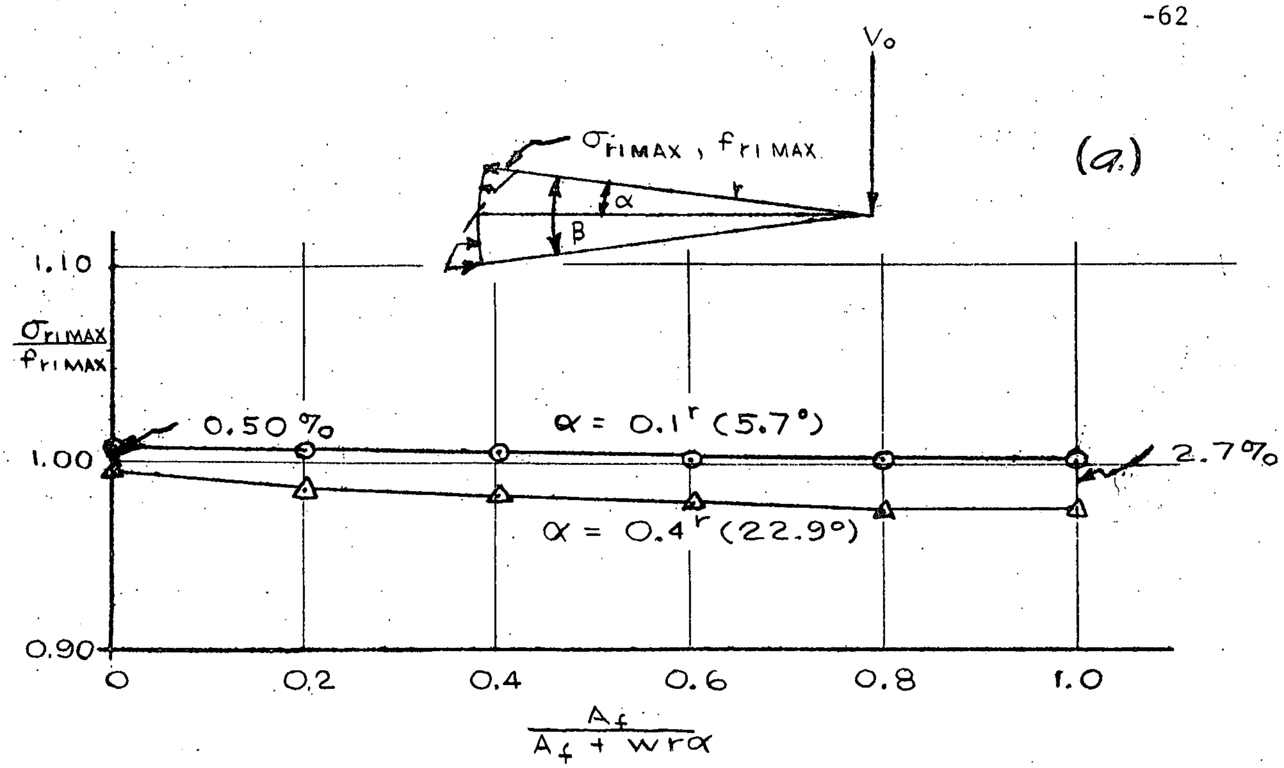


Fig. 4 MANNER IN WHICH SECTION IS CUT AND FORCES ARE RESOLVED IN OLANDER METHOD



A_f = AREA OF ONE FLANGE
$wr\alpha$ = AREA OF ONE-HALF WEB
σ = STRESS BY SIMPLIFIED METHOD (OLANDER)
f = STRESS BY MORE RATIONAL METHOD (OSGOOD)

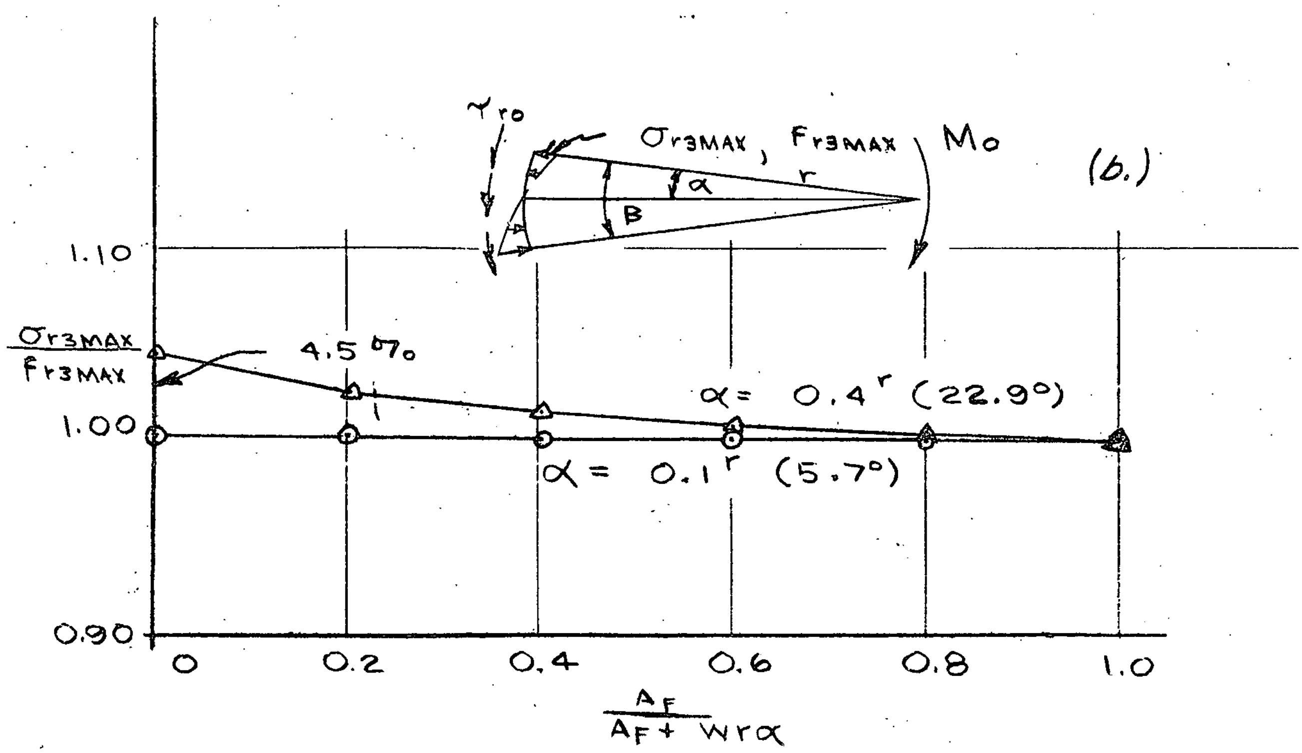
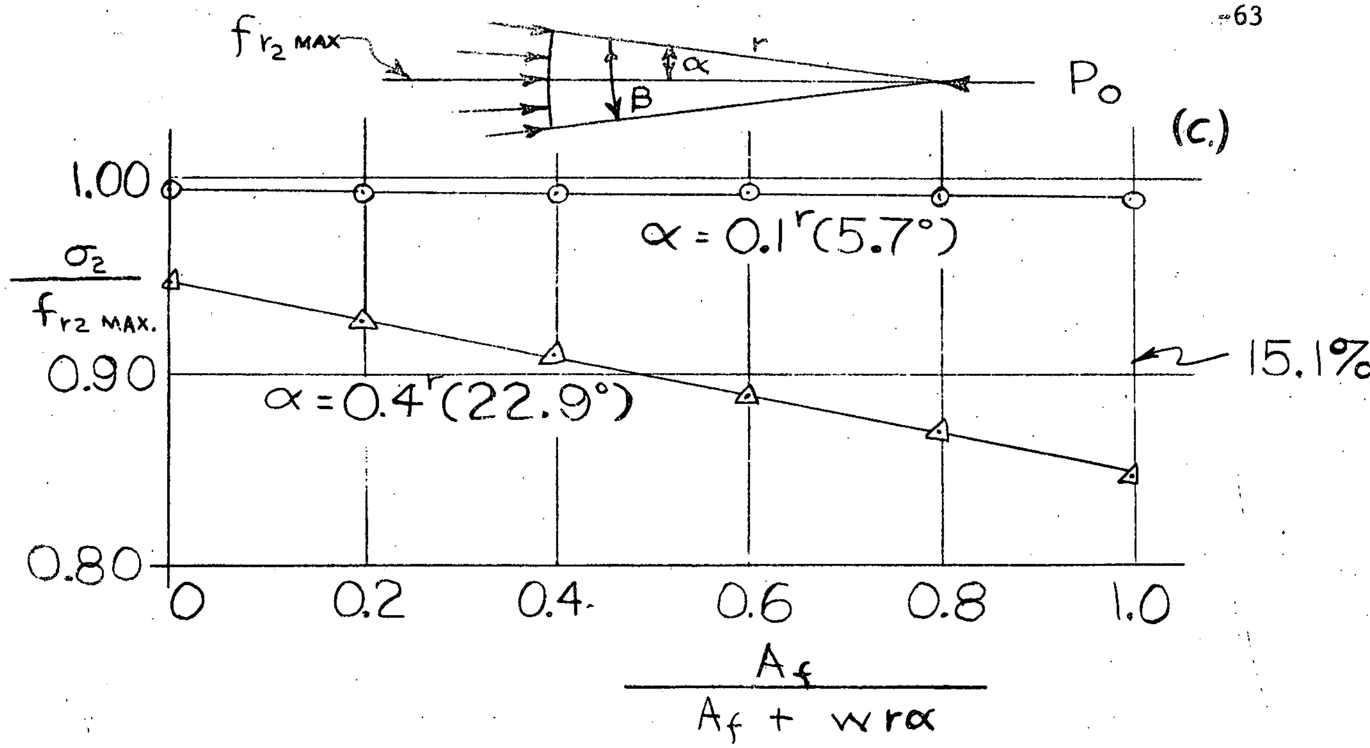


Fig. 5 (a, b) COMPARISON BETWEEN OLANDER AND OSGOOD METHODS FOR SHEAR AND MOMENT LOADINGS

5

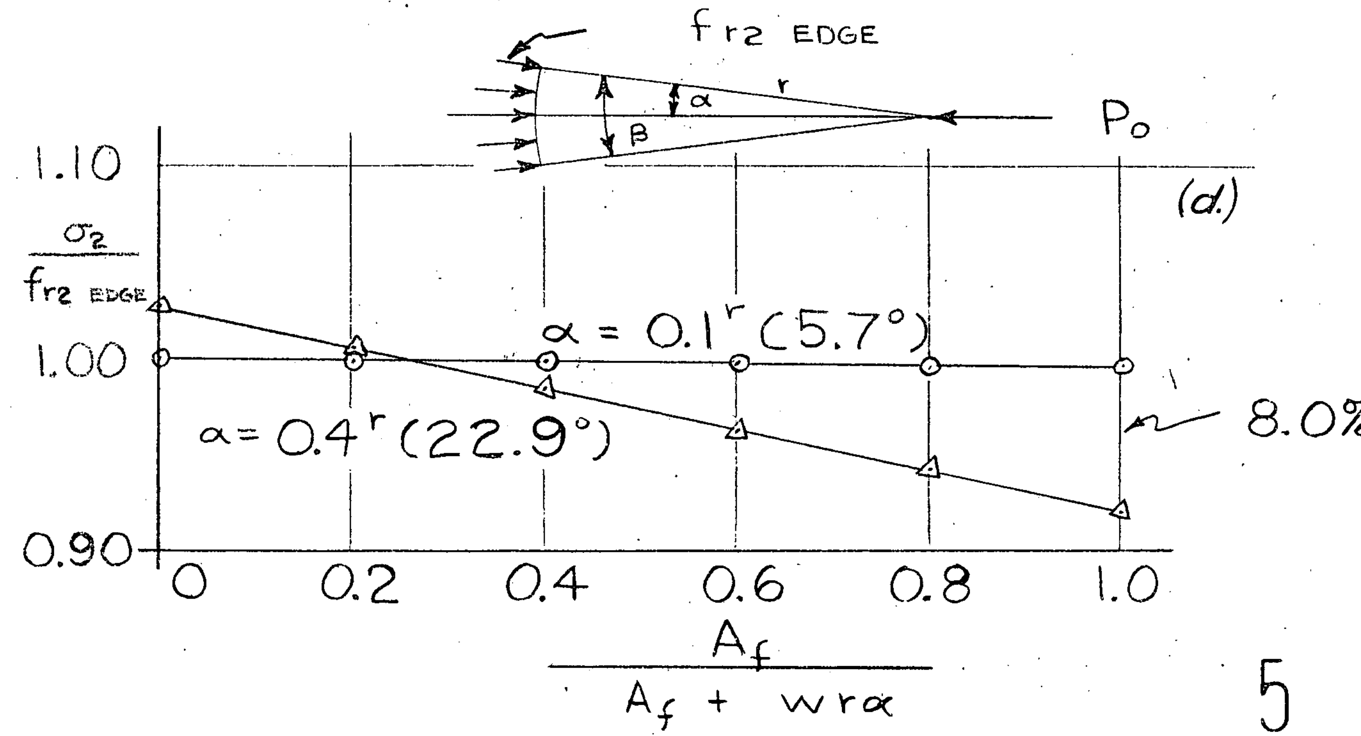


A_f = AREA OF ONE FLANGE

$wr\alpha$ = AREA OF ONE-HALF WEB

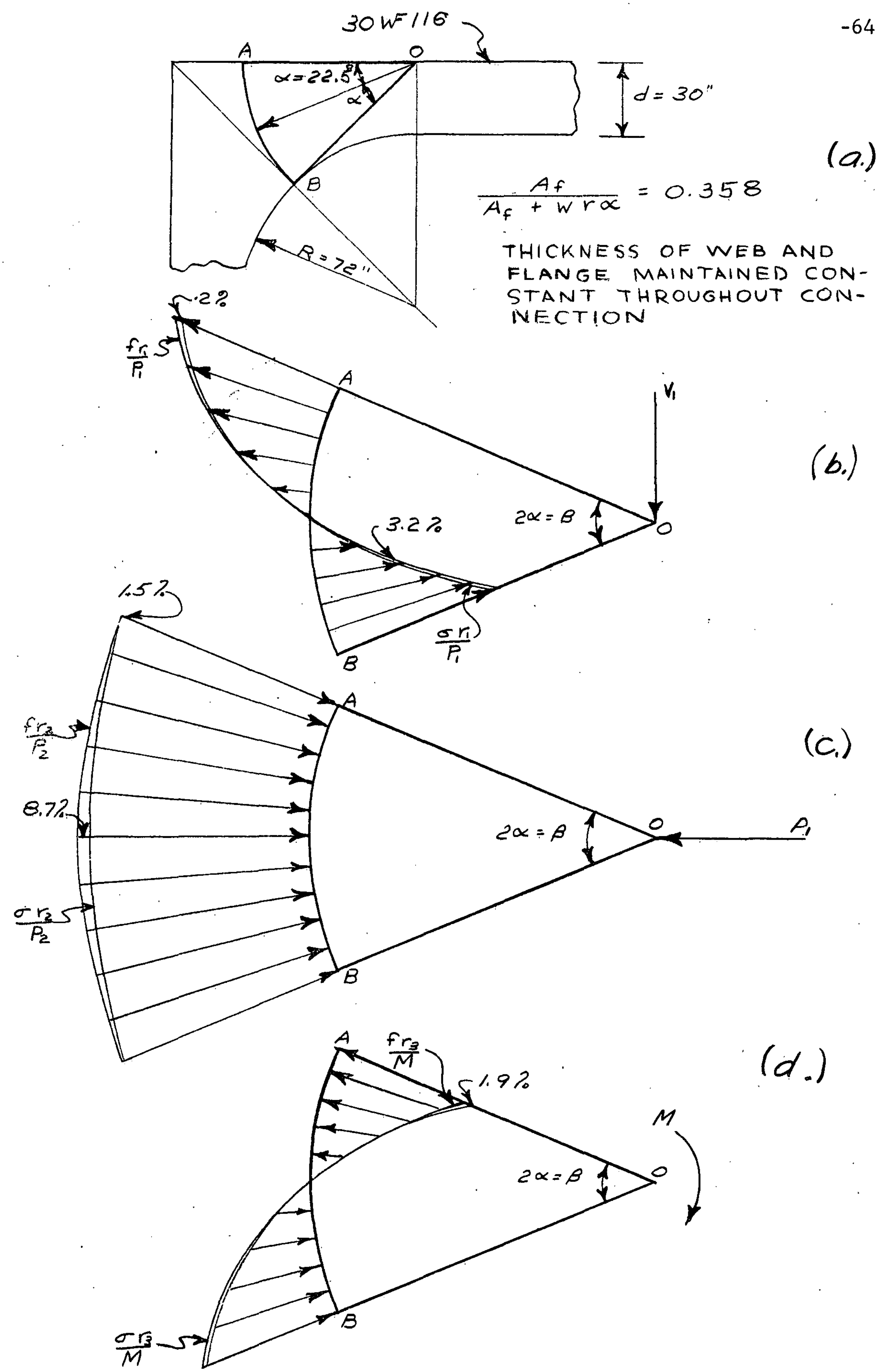
σ = STRESS BY SIMPLIFIED METHOD (OLANDER)

f = STRESS BY MORE RATIONAL METHOD (OSGOOD)



5

Fig. 5 (c, d) COMPARISON BETWEEN OLANDER AND OSGOOD METHODS FOR AXIAL LOAD



6

Fig. 6 COMPARISON BETWEEN OLANDER AND OSGOOD METHODS FOR AN ACTUAL CONNECTION

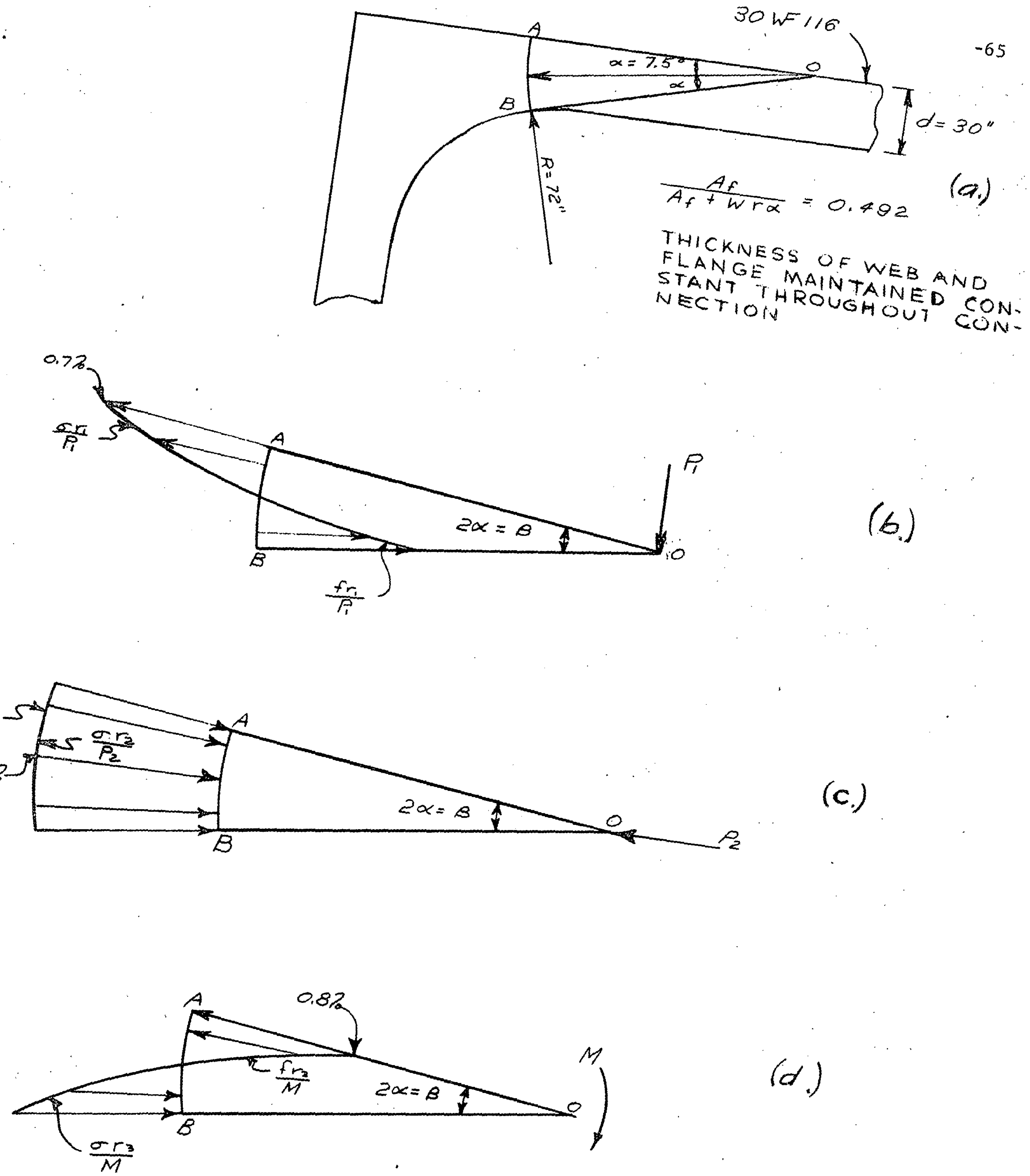


Fig. 7 COMPARISON BETWEEN OLANDER AND OSGOOD METHODS FOR AN ACTUAL CONNECTION

7

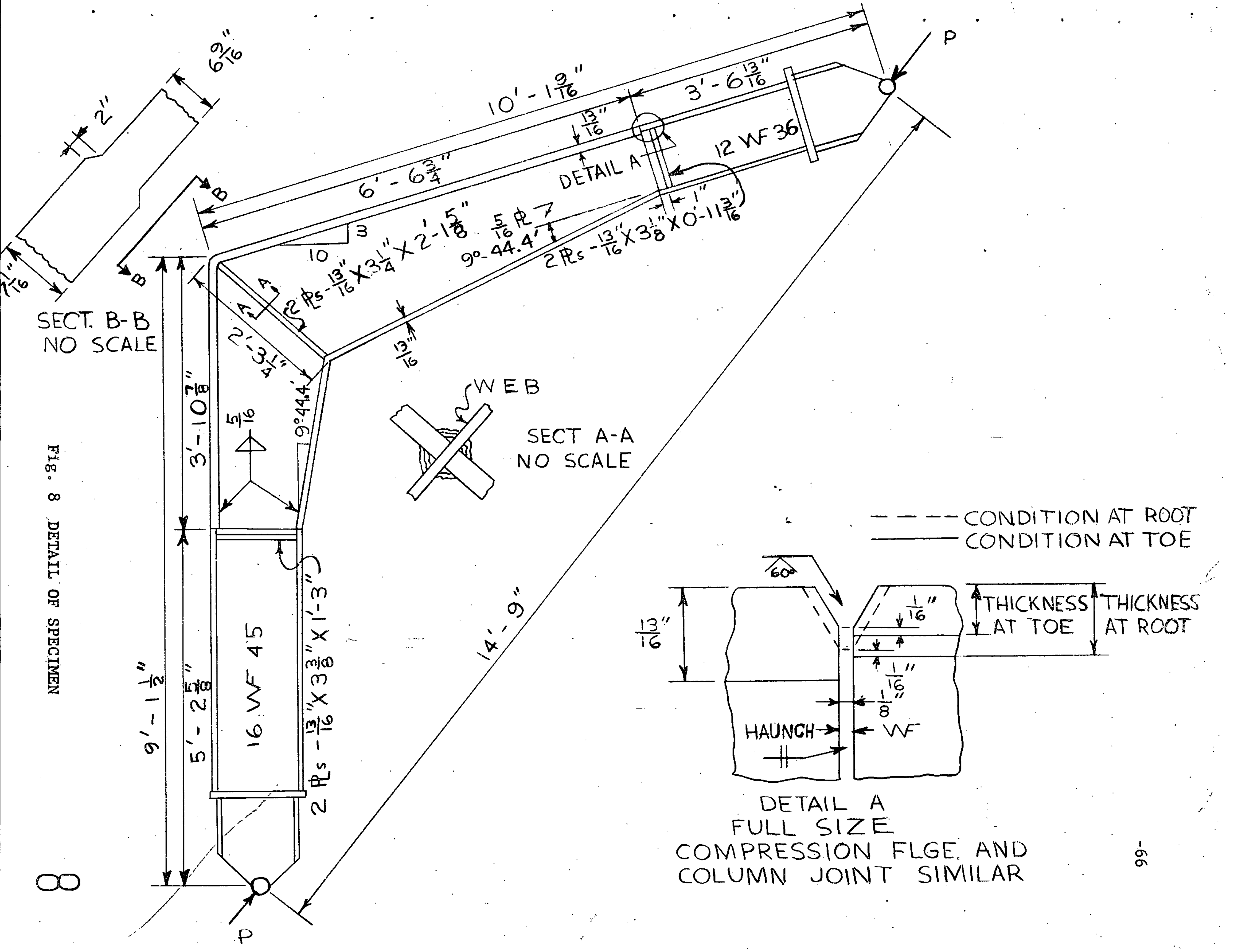


Fig. 8 DETAIL OF SPECIMEN

8

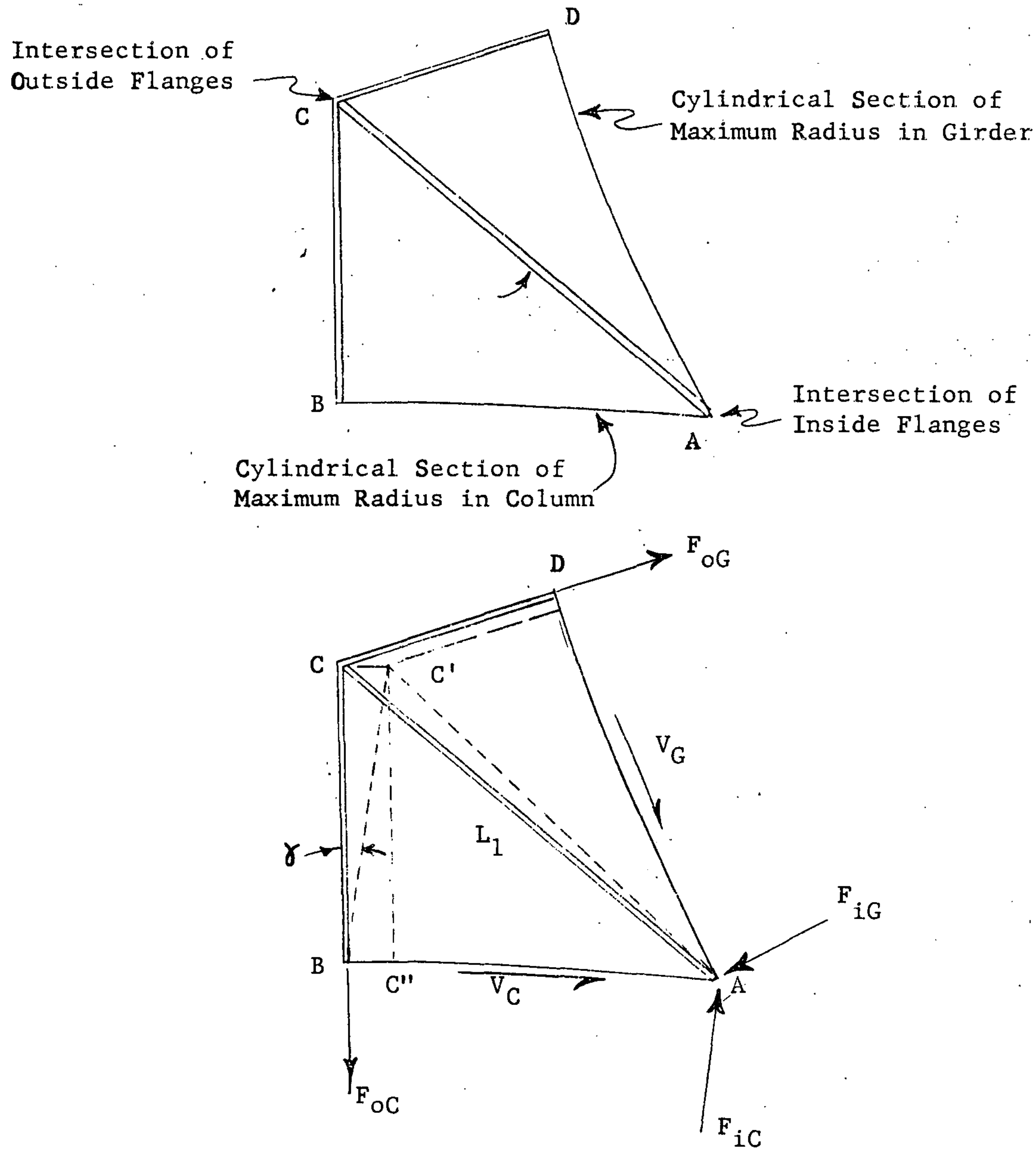


Fig. 9 ASSUMED CONDITIONS IN CORNER OF CONNECTION

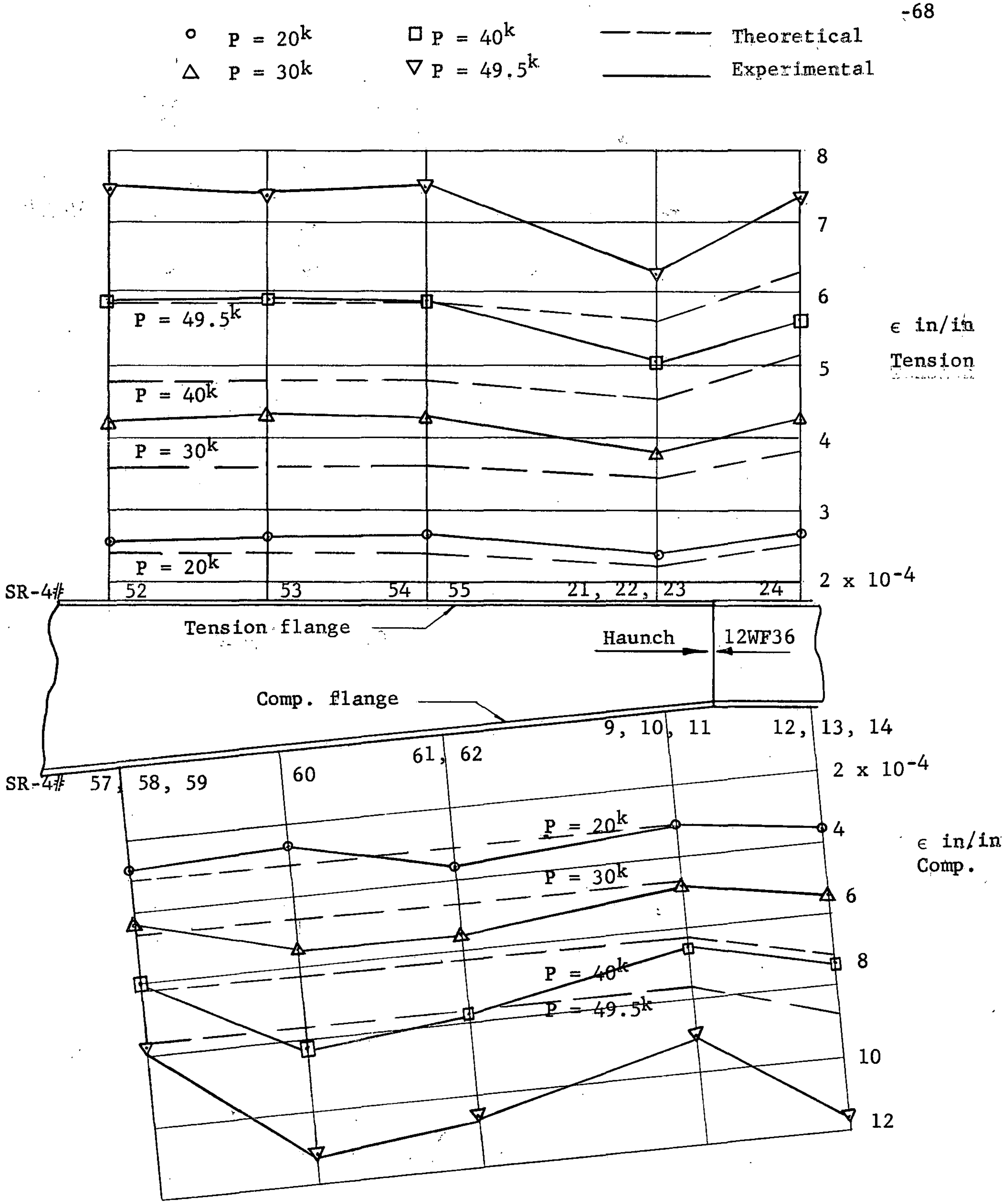


Fig. 10 THEORETICAL AND EXPERIMENTAL STRAINS IN HAUNCH GIRDER FLANGES

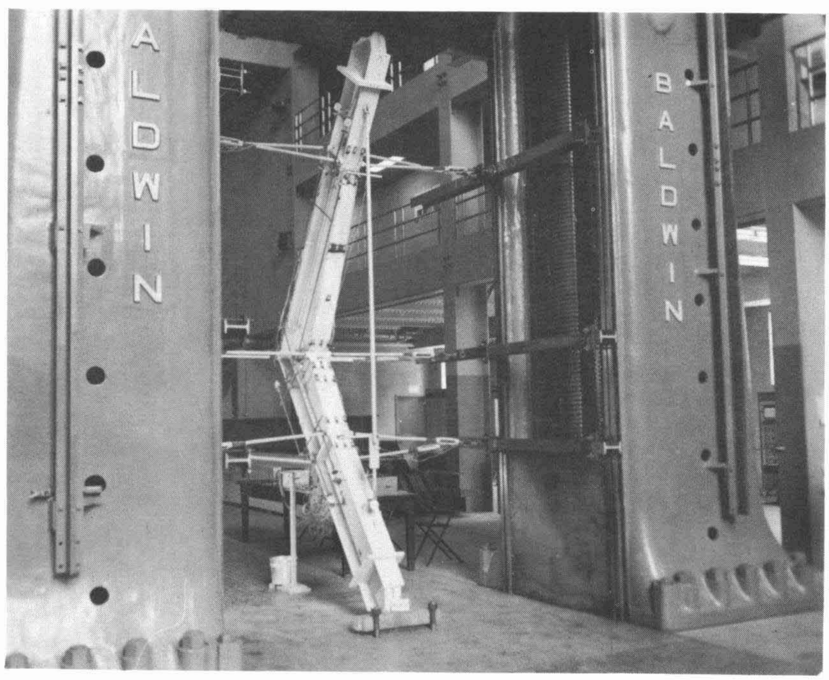


Fig. 11 OVERALL VIEW OF SPECIMEN IN TESTING MACHINE
PRIOR TO APPLICATION OF LOAD

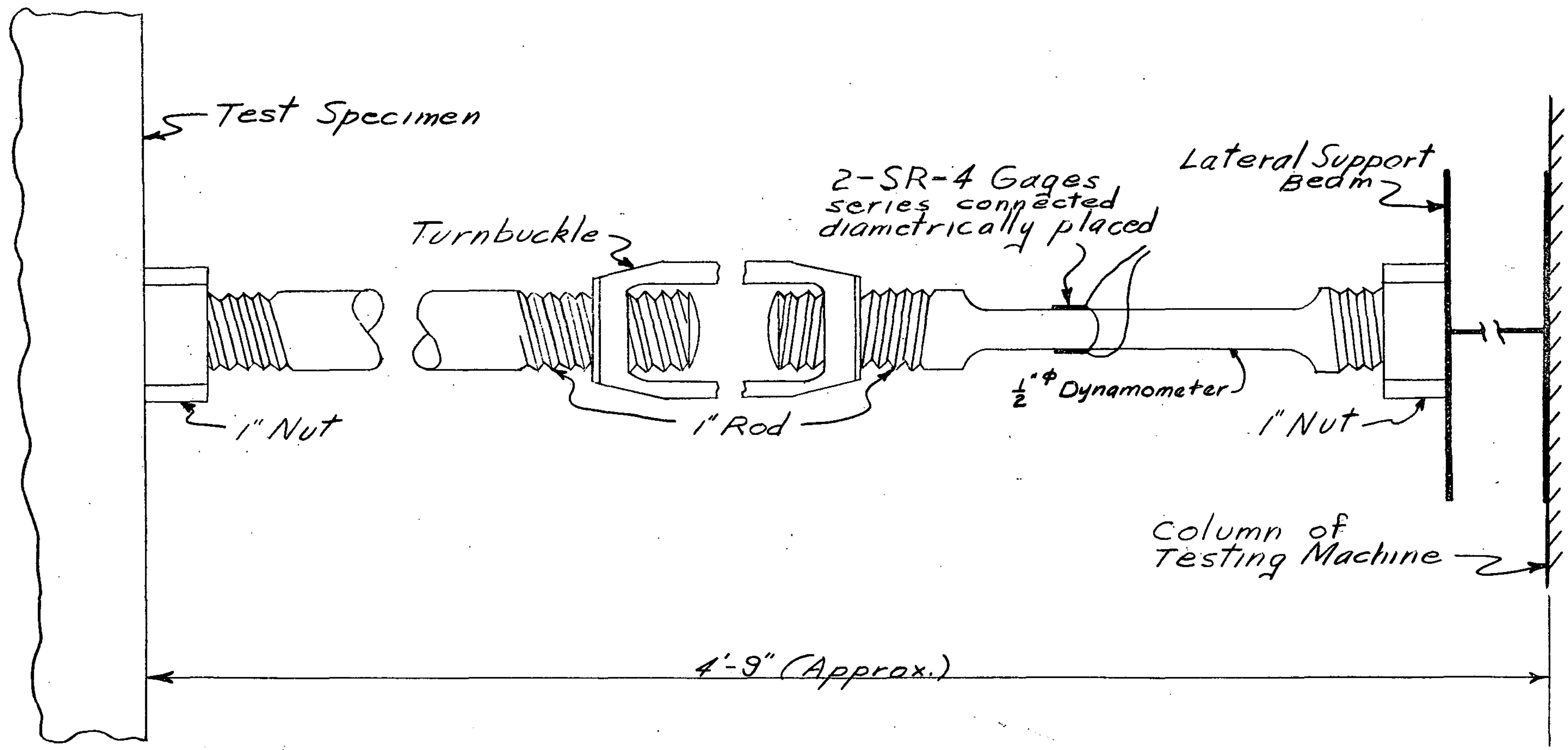


Fig. 12
Detail of Lateral Support Rod

12

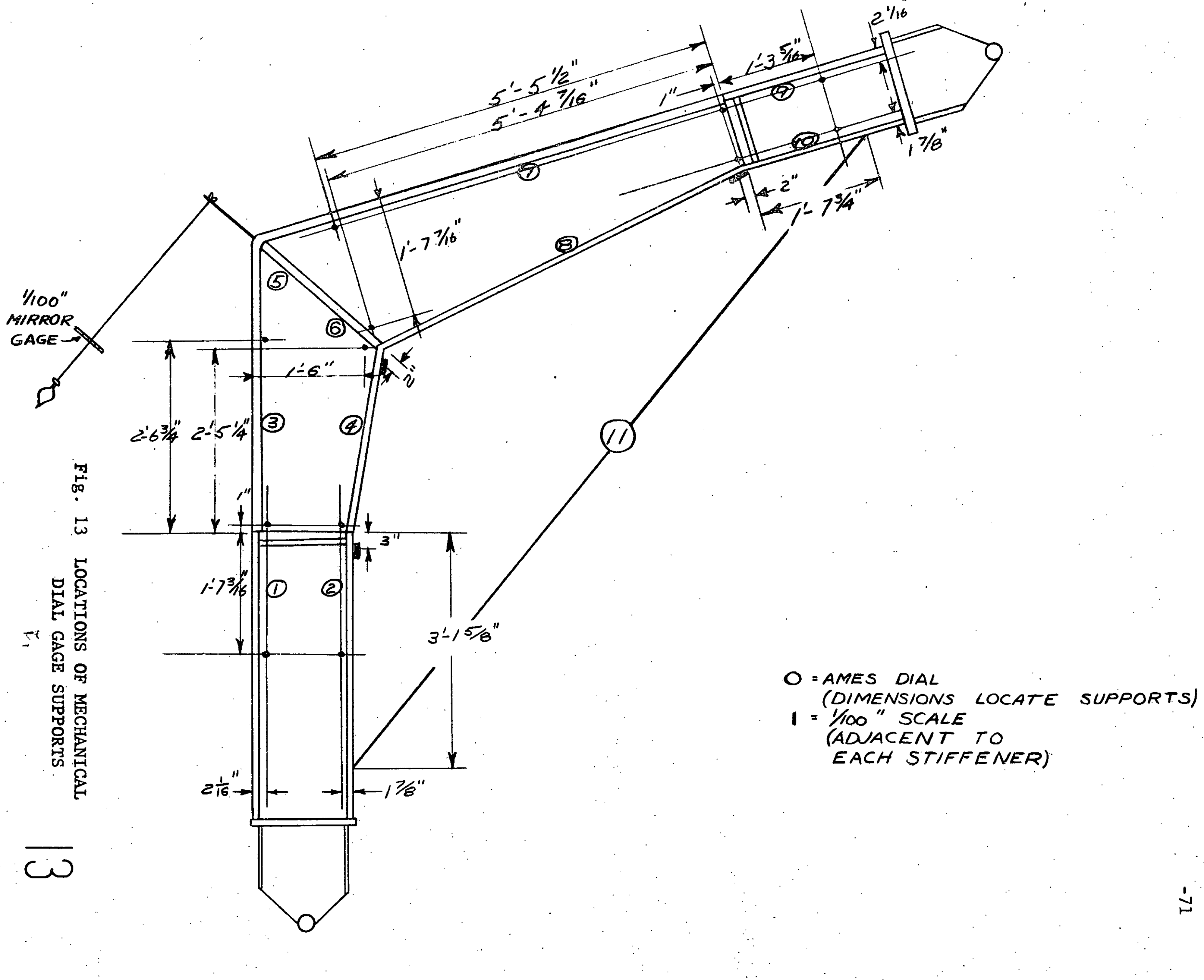


Fig. 13 LOCATIONS OF MECHANICAL DIAL GAGE SUPPORTS

13

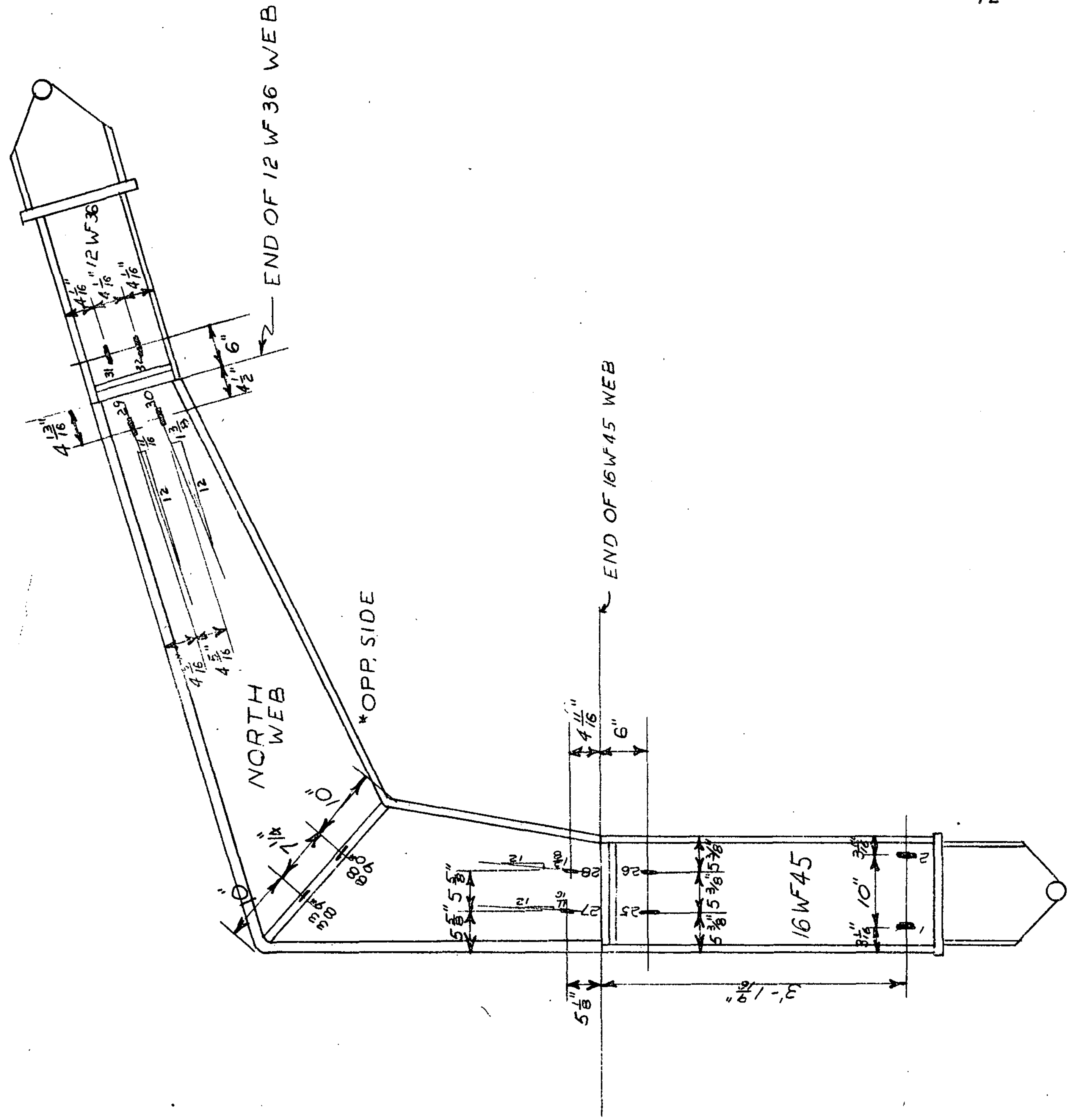


Fig. 14 SR-4 GAGE LOCATIONS IN WEBS AND STIFFENERS. ALL GAGES TYPE A1. GAGES ON ONE SIDE ONLY EXCEPT AS NOTED

14

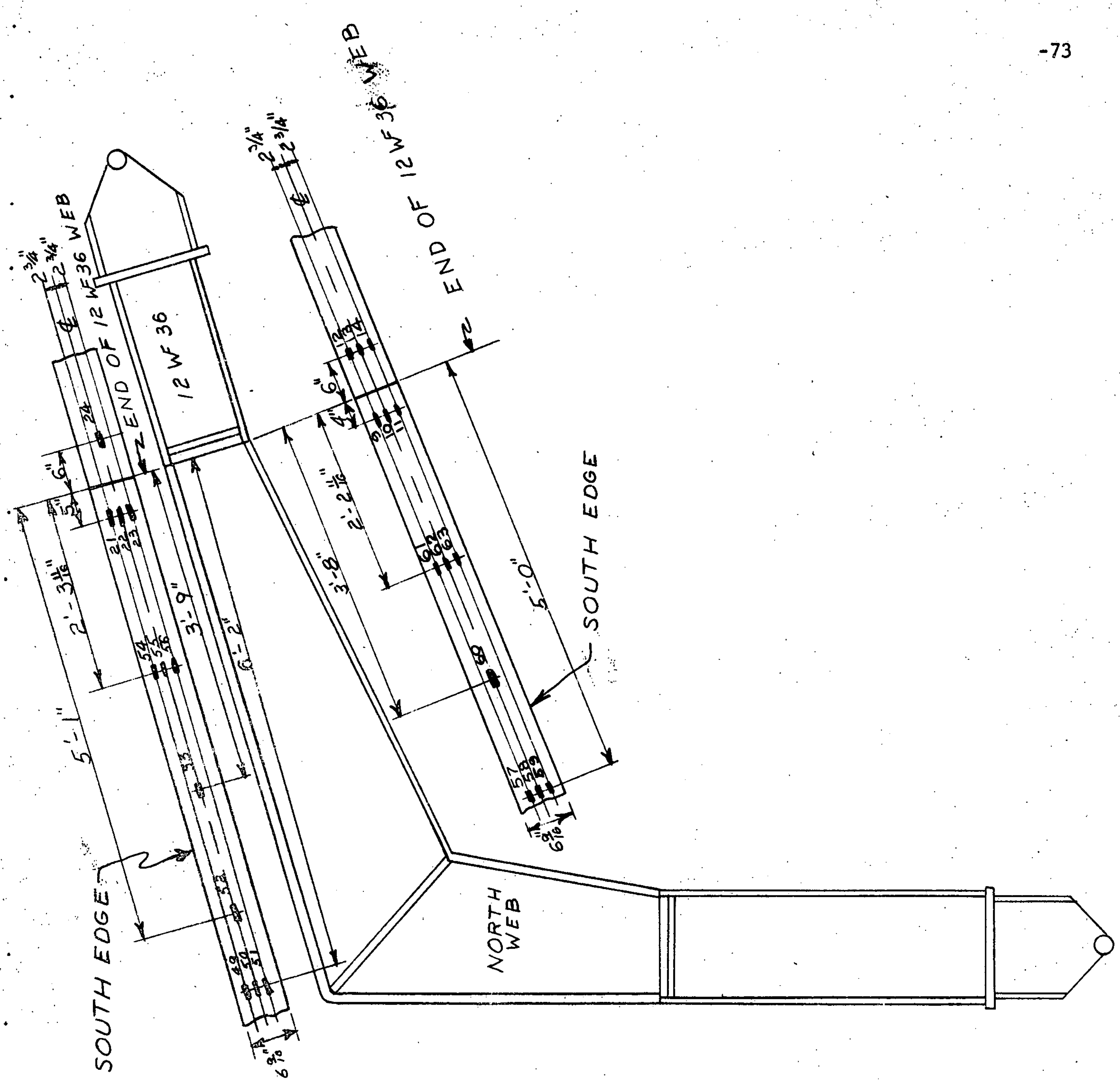
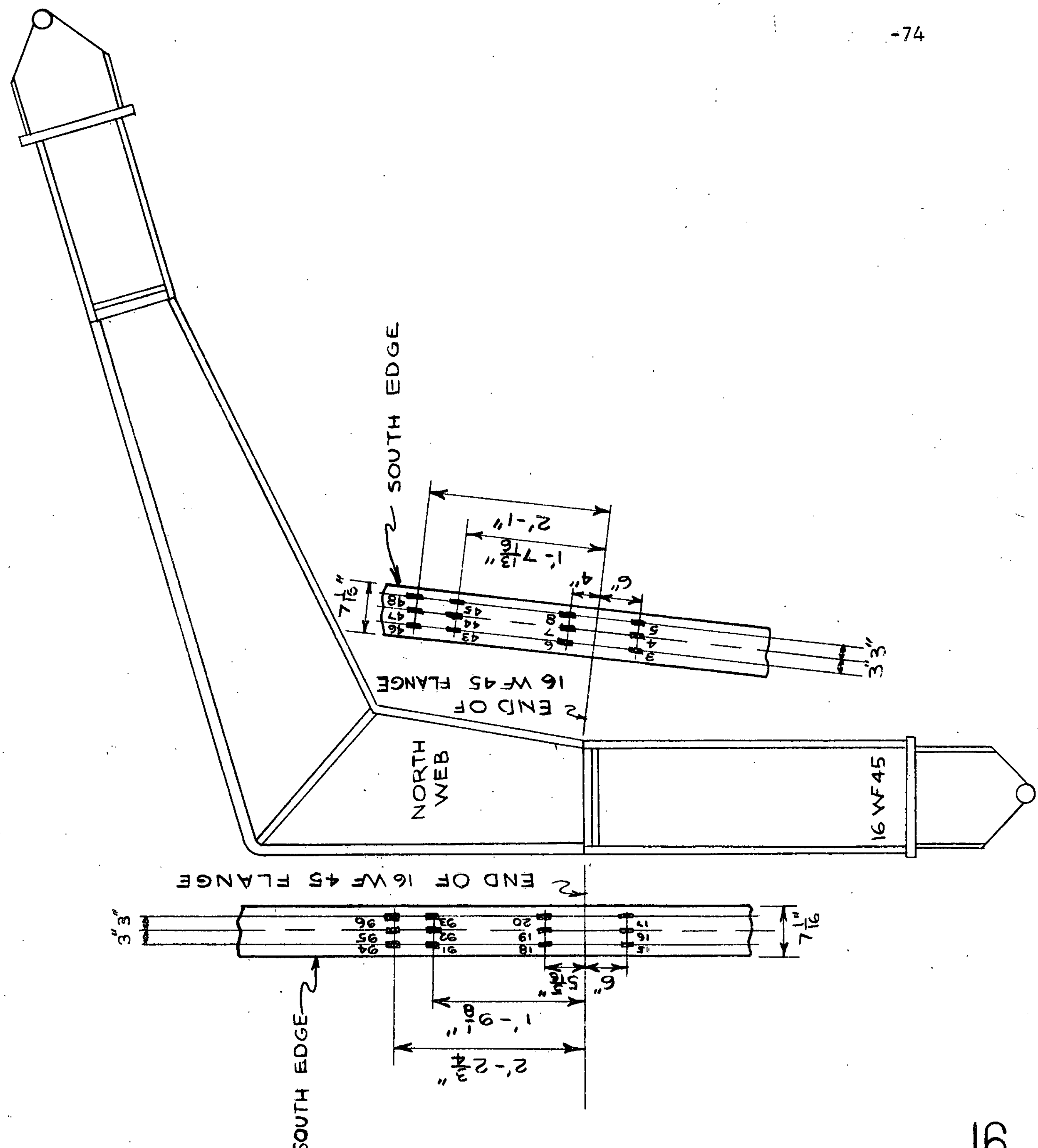


Fig. 15 SR-4 GAGE LOCATIONS IN FLANGES OF GIRDER.
ALL GAGES TYPE A1

15



16

Fig. 16 SR-4 GAGE LOCATIONS IN FLANGES OF COLUMN.
ALL GAGES TYPE A1

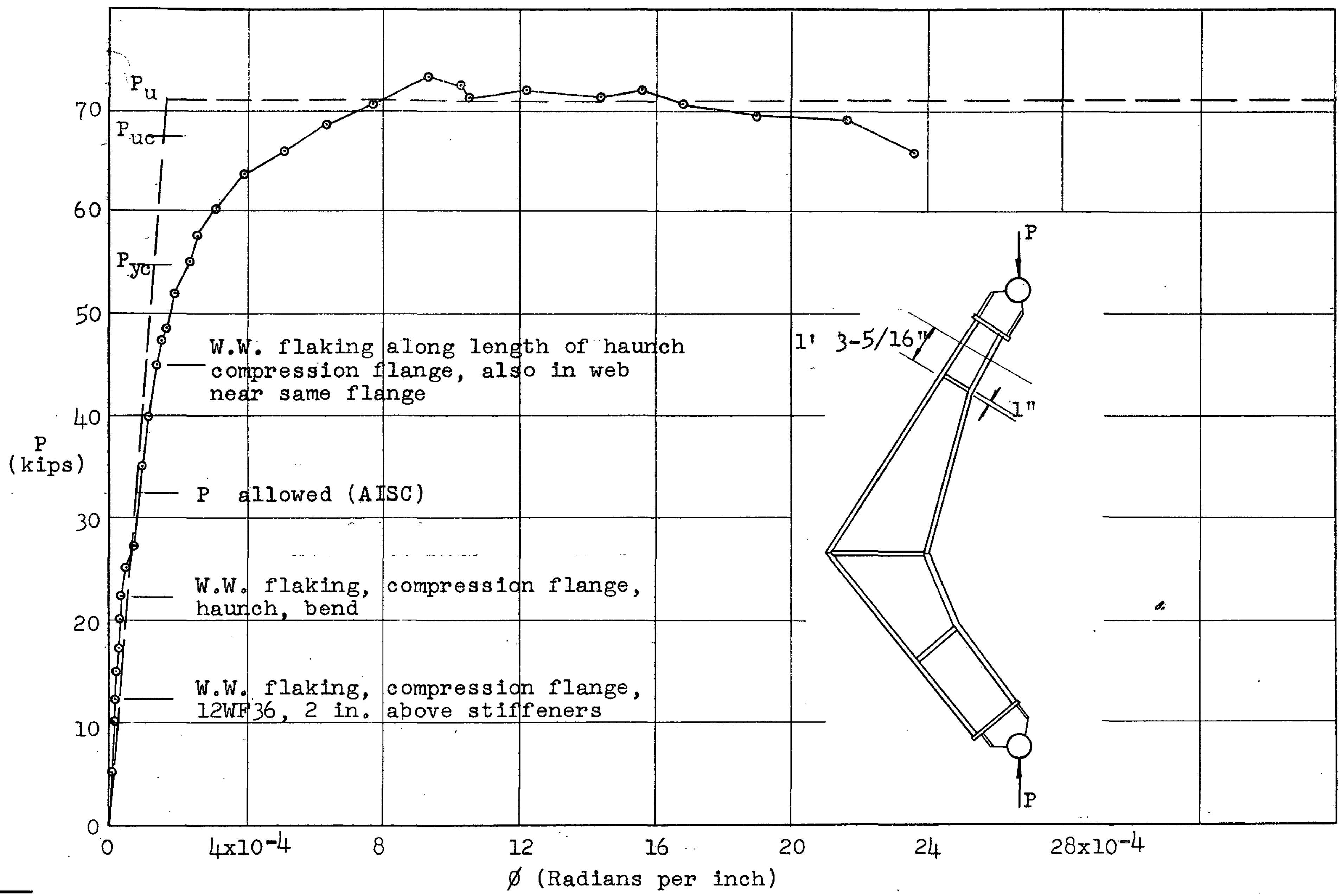


FIG. 17 - LOAD VS. AVERAGE UNIT ROTATION IN LENGTH INDICATED

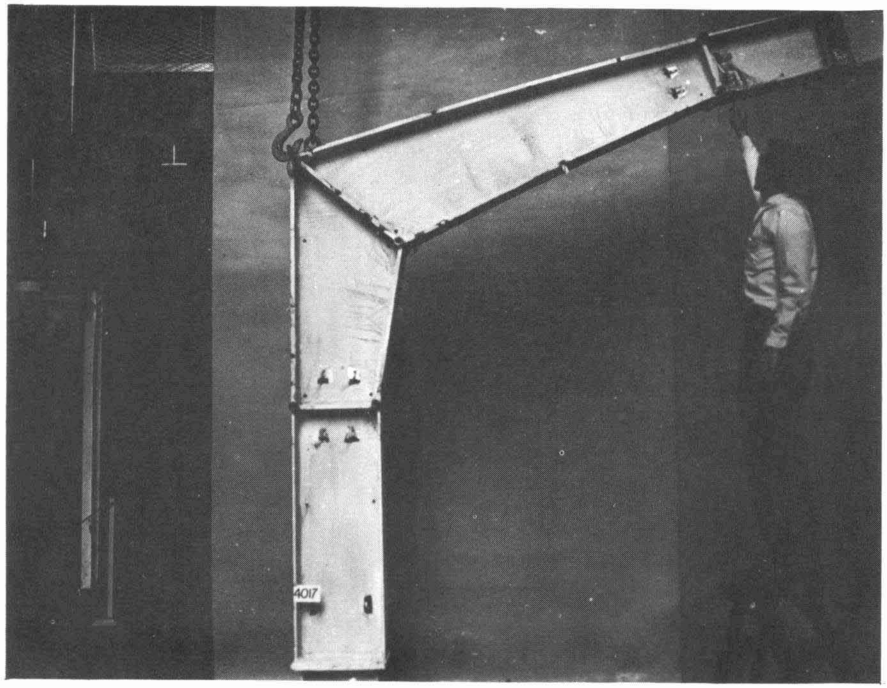


Fig. 18 LOCATION OF PLASTIC HINGE INDICATED. CONNECTION AS IT WOULD APPEAR IN BUILDING

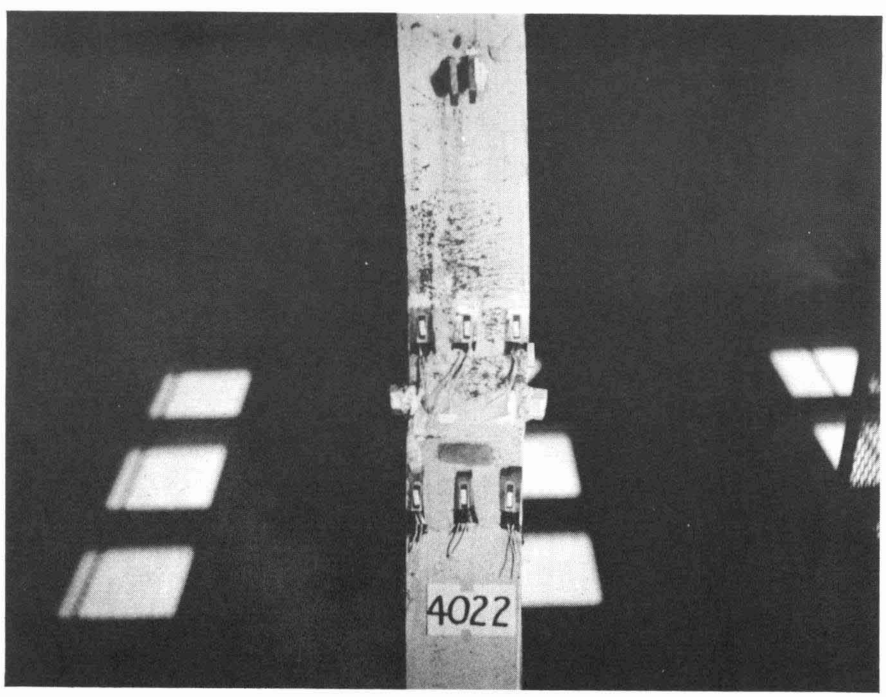


Fig. 19 YIELDING IN COMPRESSION FLANGE IN VICINITY OF PLASTIC HINGE

18,19

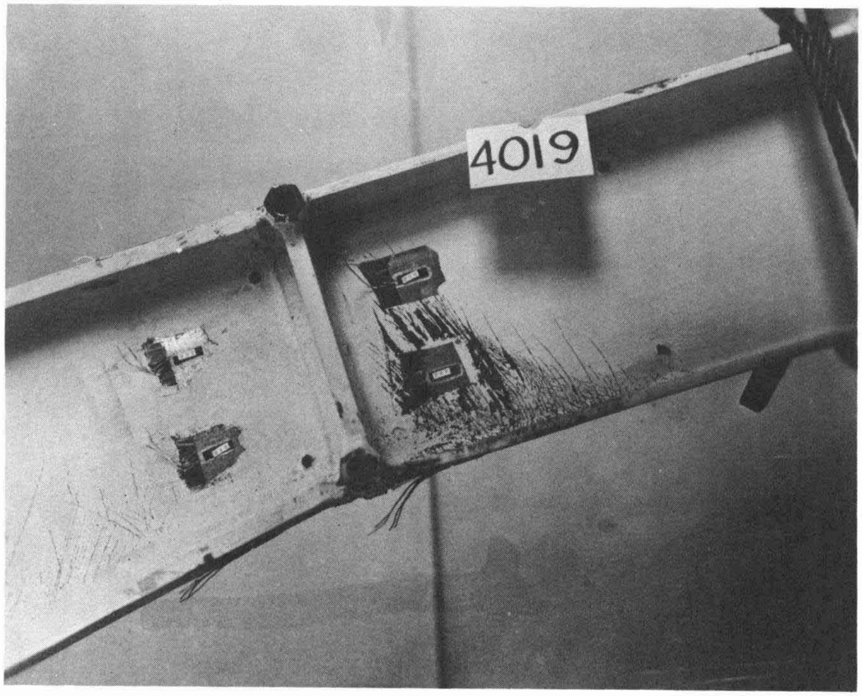


Fig. 20 YIELDING IN WEB IN VICINITY OF PLASTIC HINGE

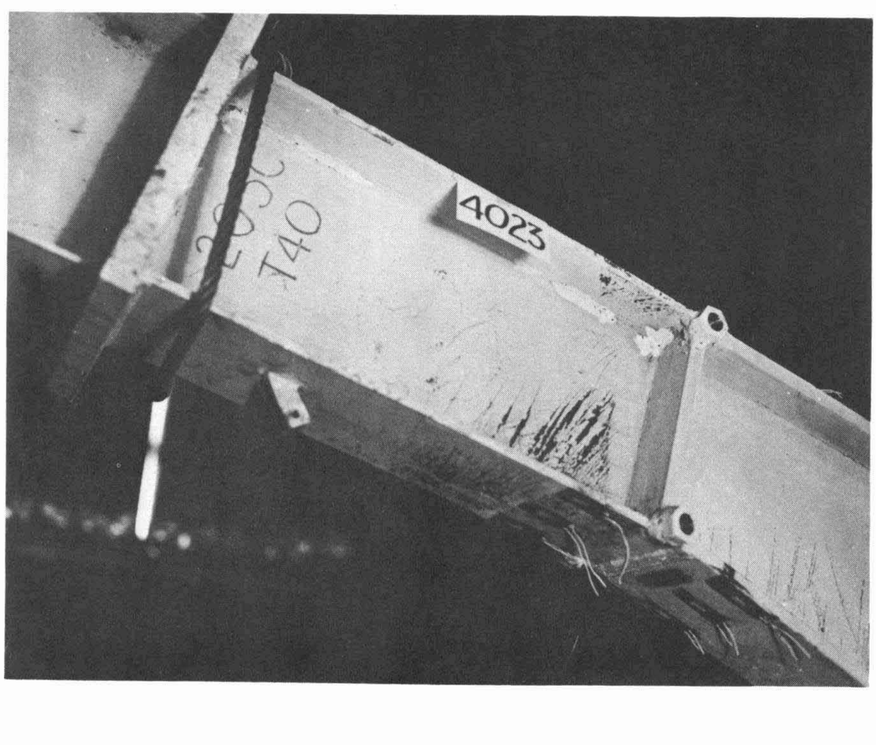


Fig. 21 YIELDING THROUGHOUT DEPTH OF SECTION IN VICINITY OF PLASTIC HINGE

20,21

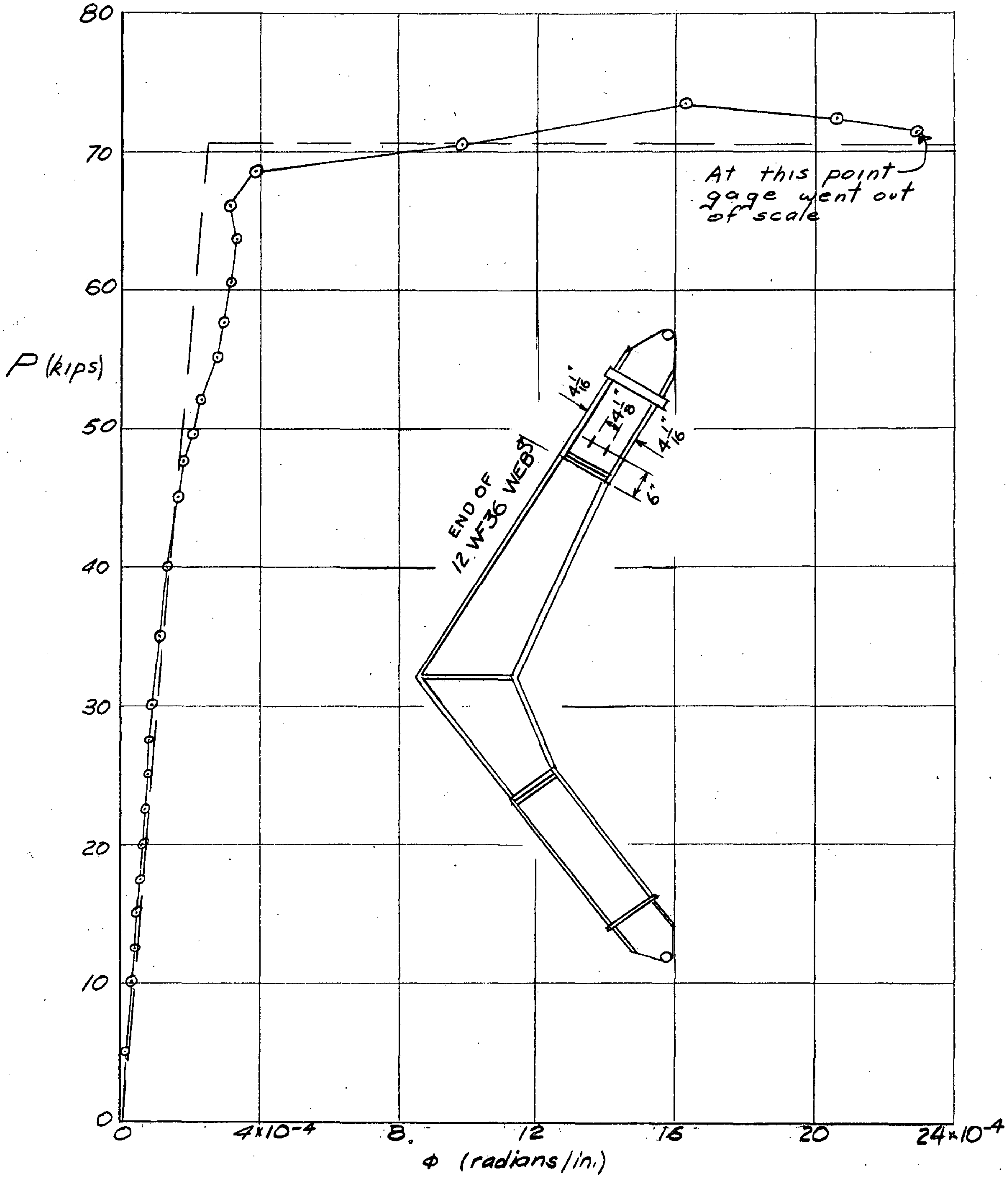


Fig. 22 LOAD VS. UNIT ROTATION AS MEASURED BY SR-4 GAGES IN VICINITY OF PLASTIC HINGE

22

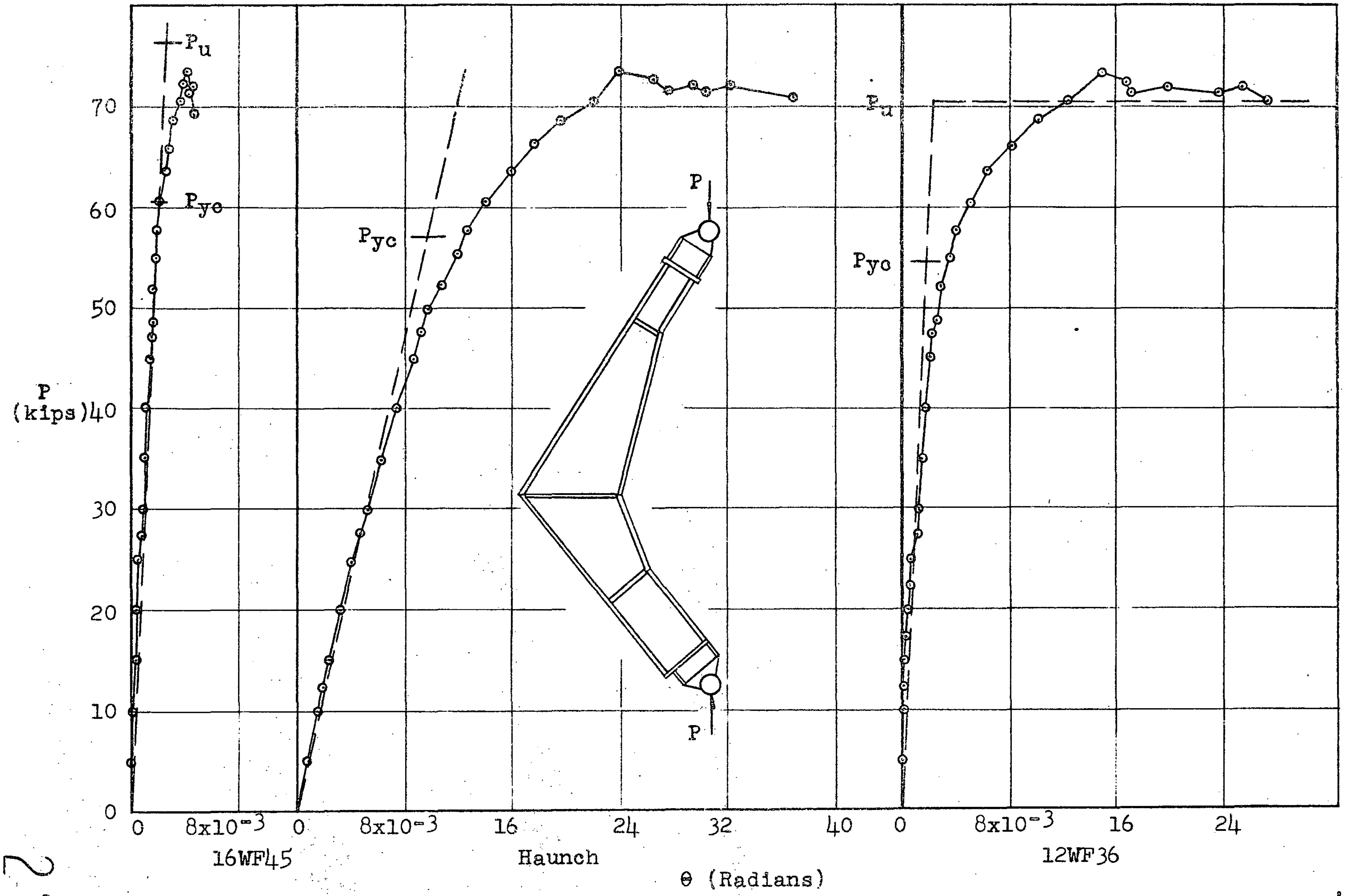


FIG. 23 - ROTATION OF CONNECTION COMPONENTS

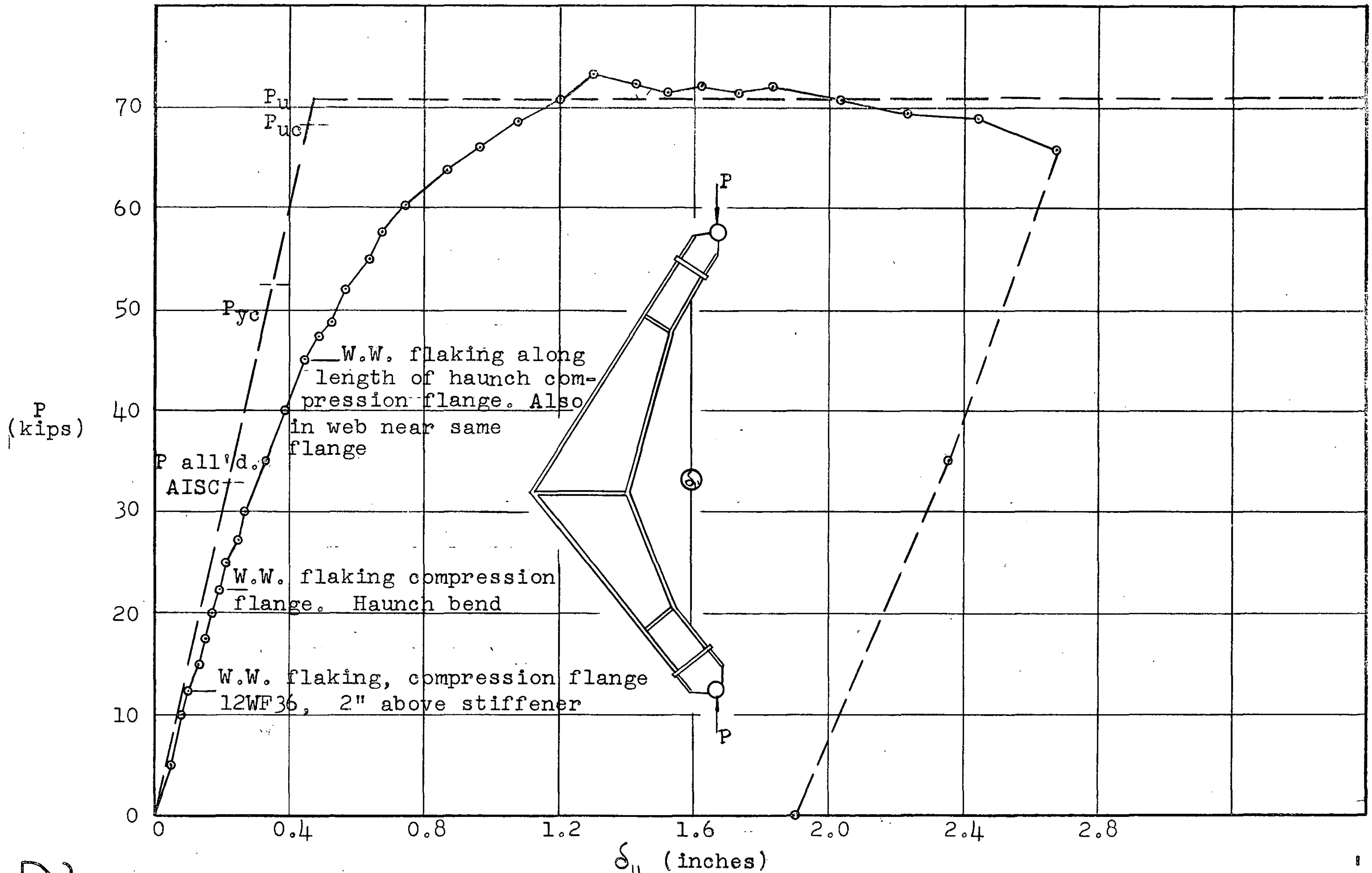


FIG. 24 - LOAD VS DEFLECTION OF CONNECTION

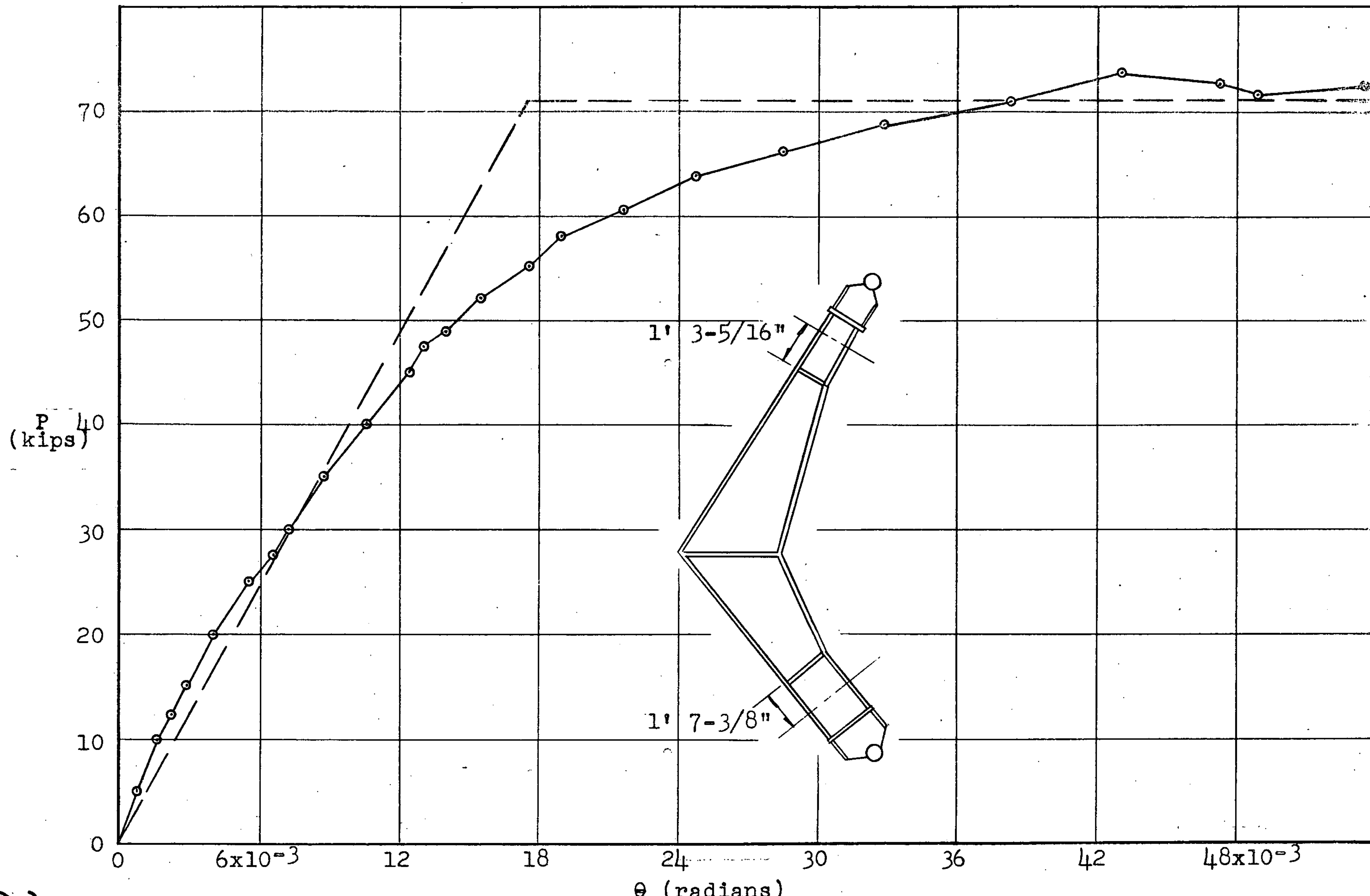


FIG. 25 - LOAD VS ROTATION OF CONNECTION

25

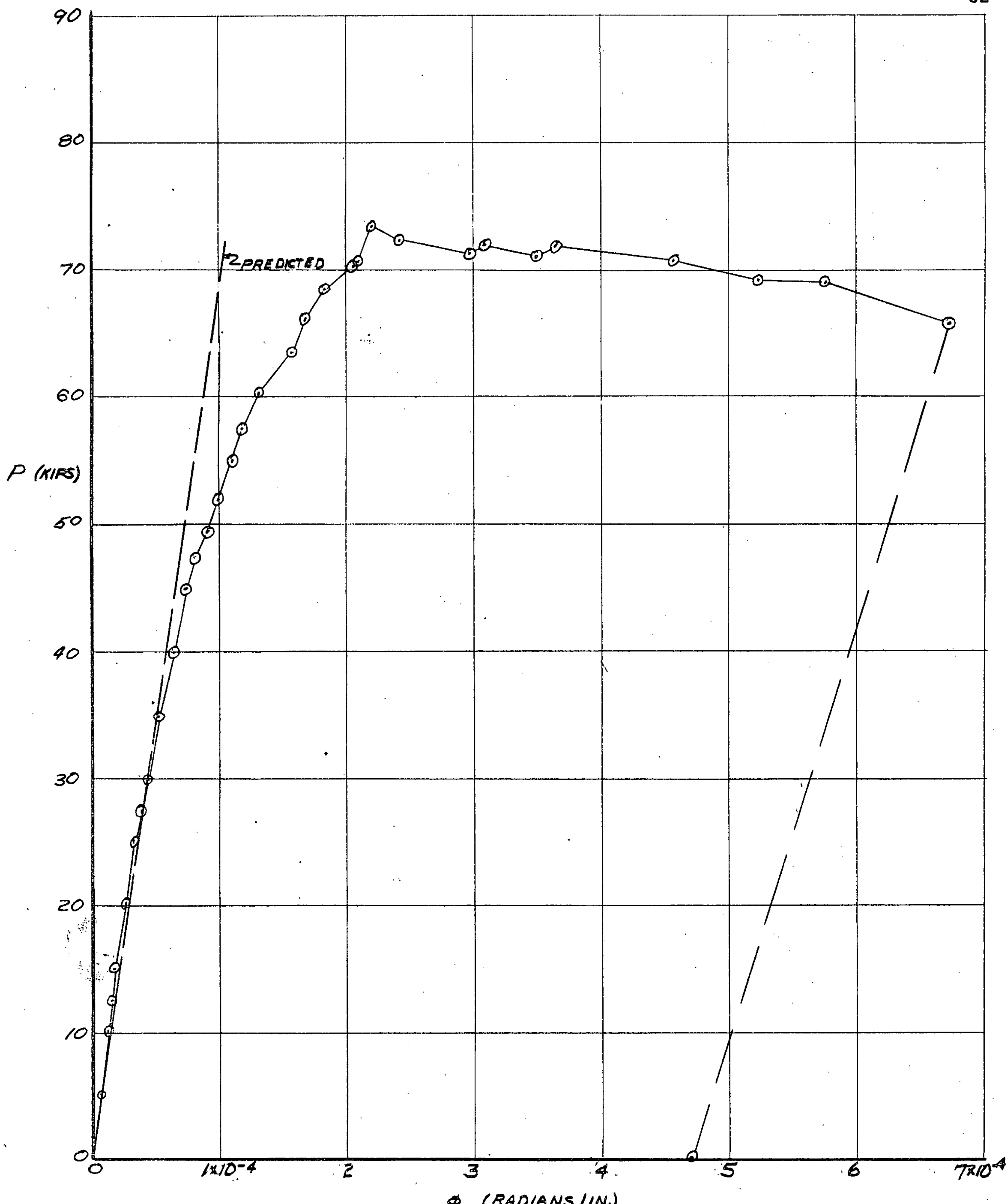


Fig. 26 LOAD VS. AVERAGE UNIT ROTATION IN CORNER OF CONNECTION

26

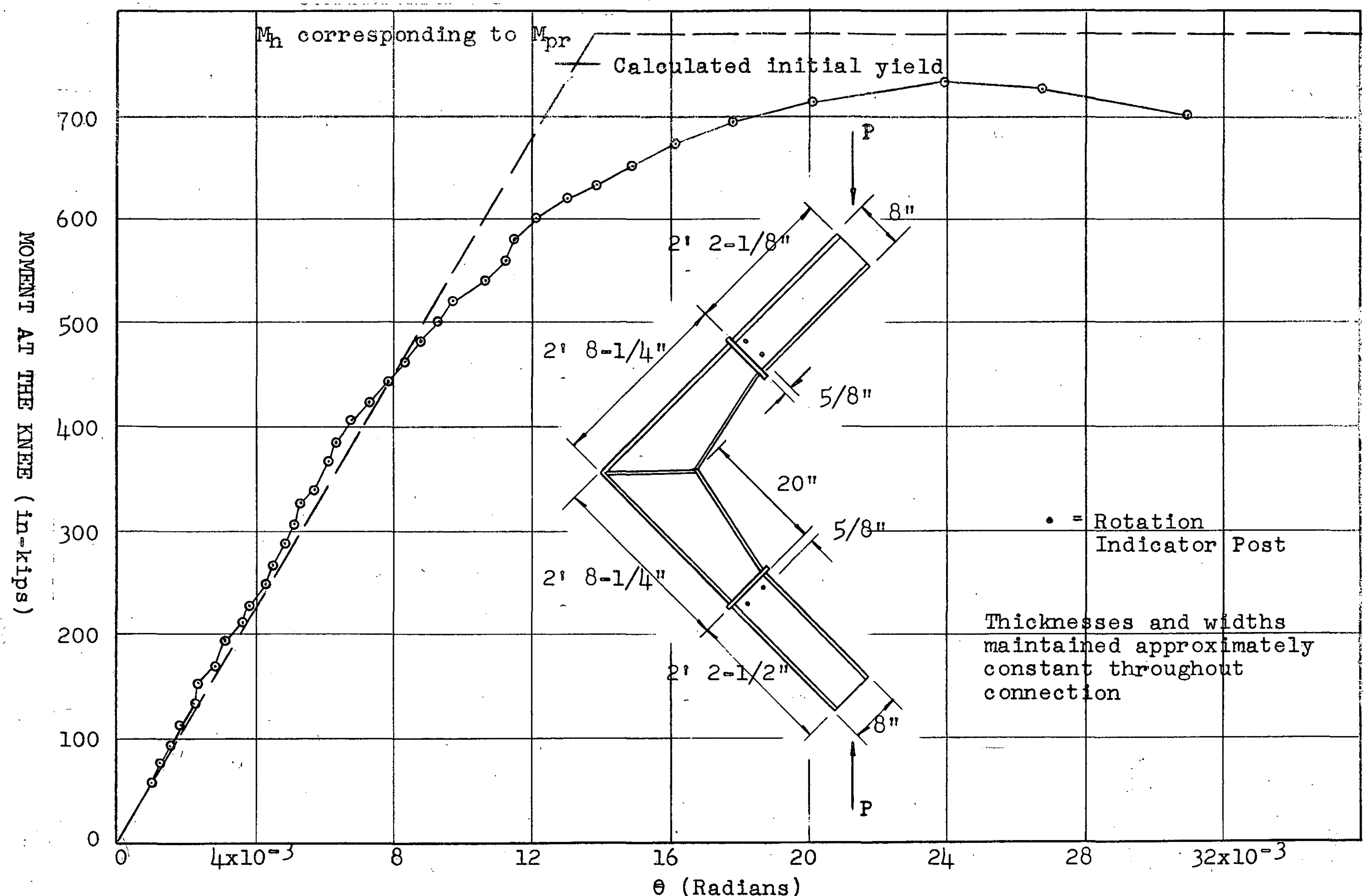


FIG. 27 - MOMENT VS. CURVATURE FOR PREVIOUSLY TESTED CONNECTION

27

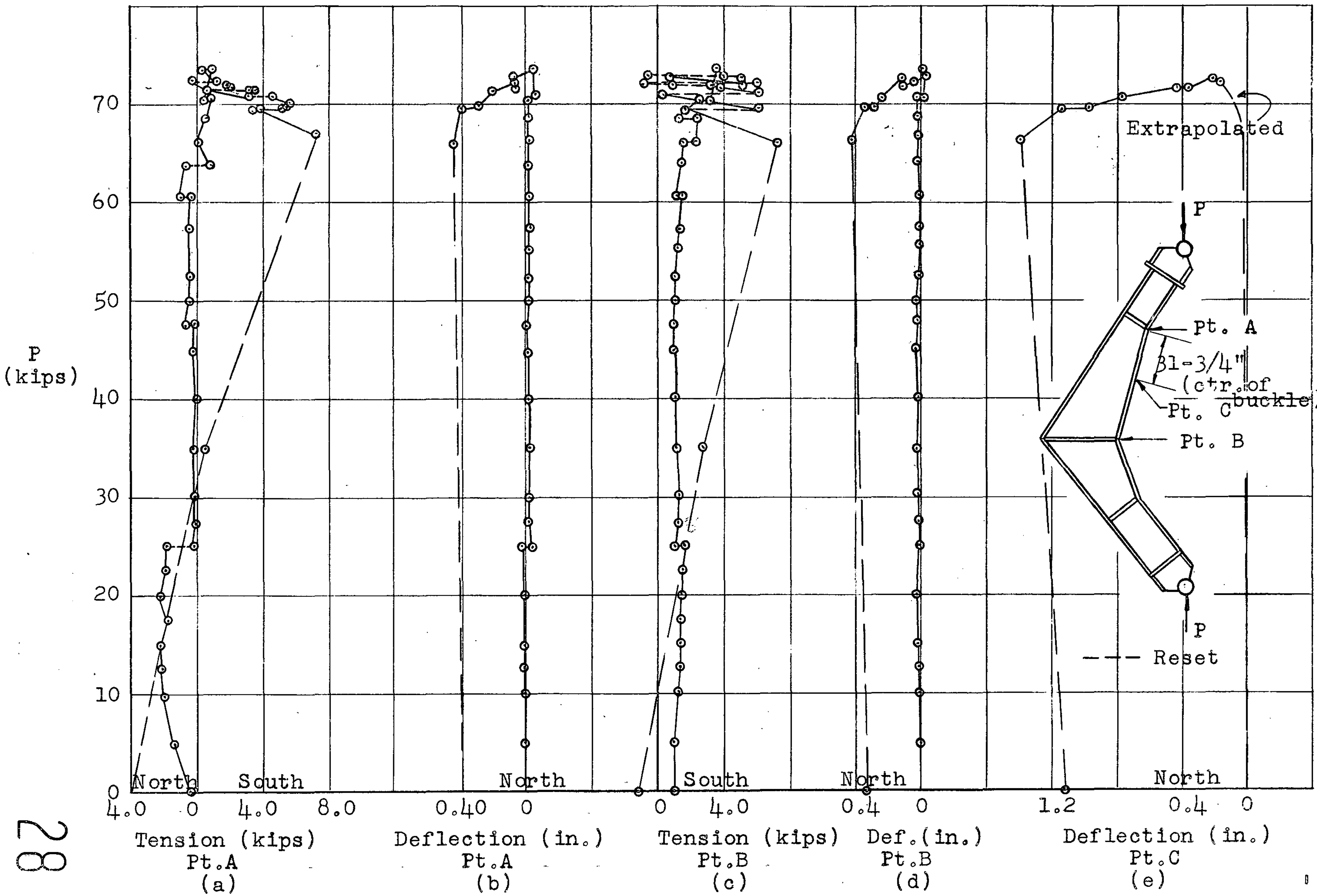


FIG. 28 - LATERAL SUPPORT DATA

28

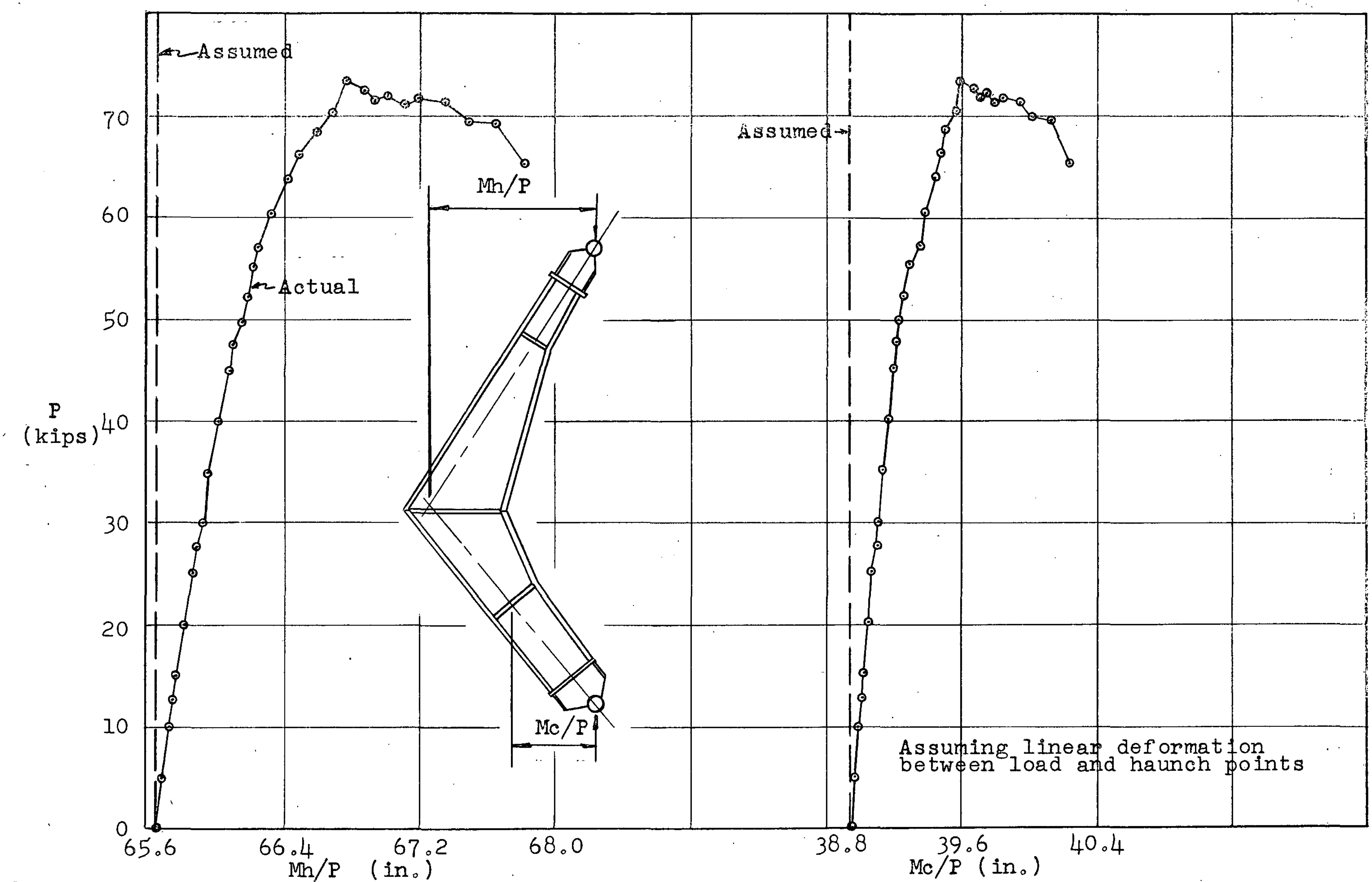


FIG. 29 - ASSUMED AND ACTUAL MOMENT ARMS

29

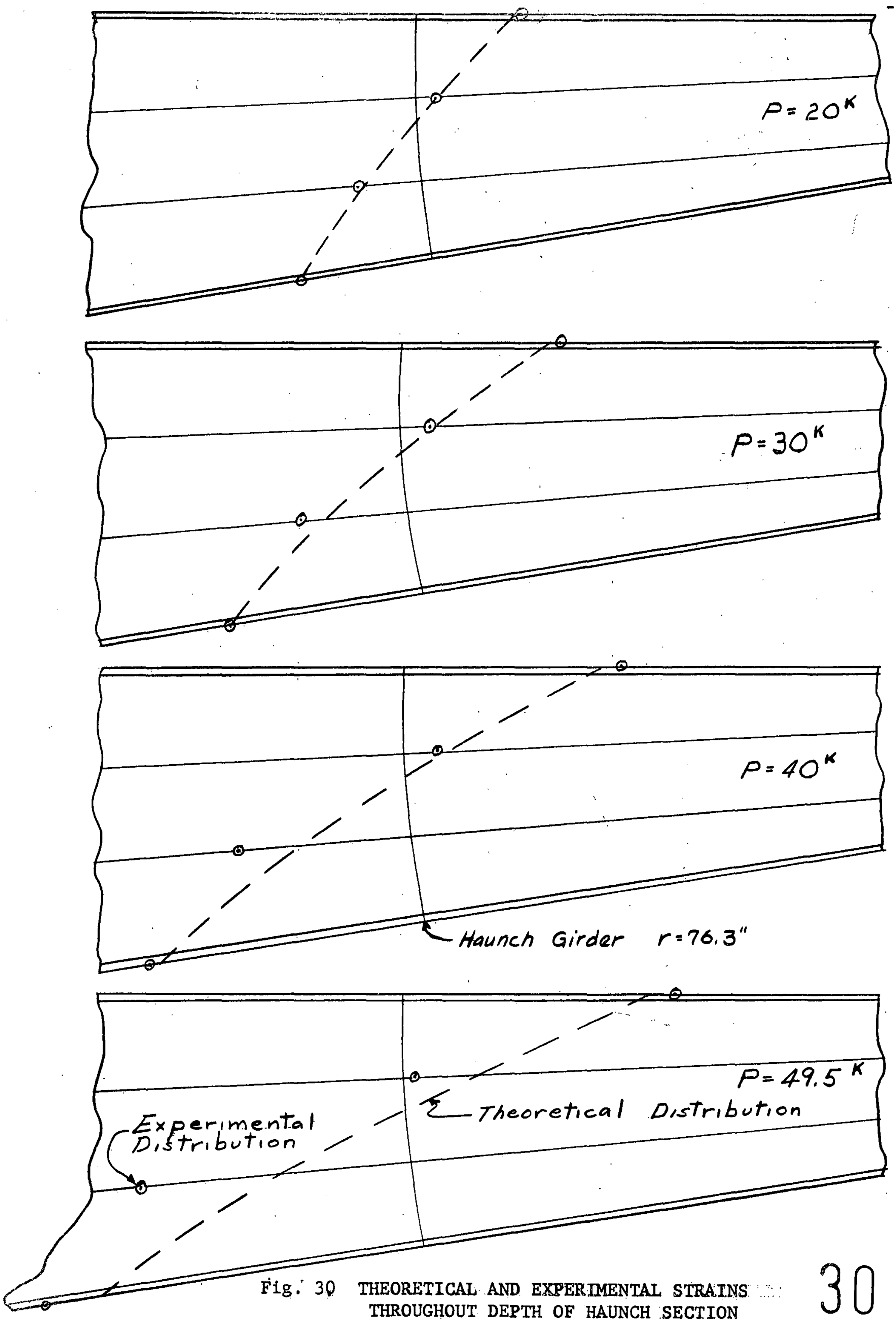


Fig. 30 THEORETICAL AND EXPERIMENTAL STRAINS THROUGHOUT DEPTH OF HAUNCH SECTION

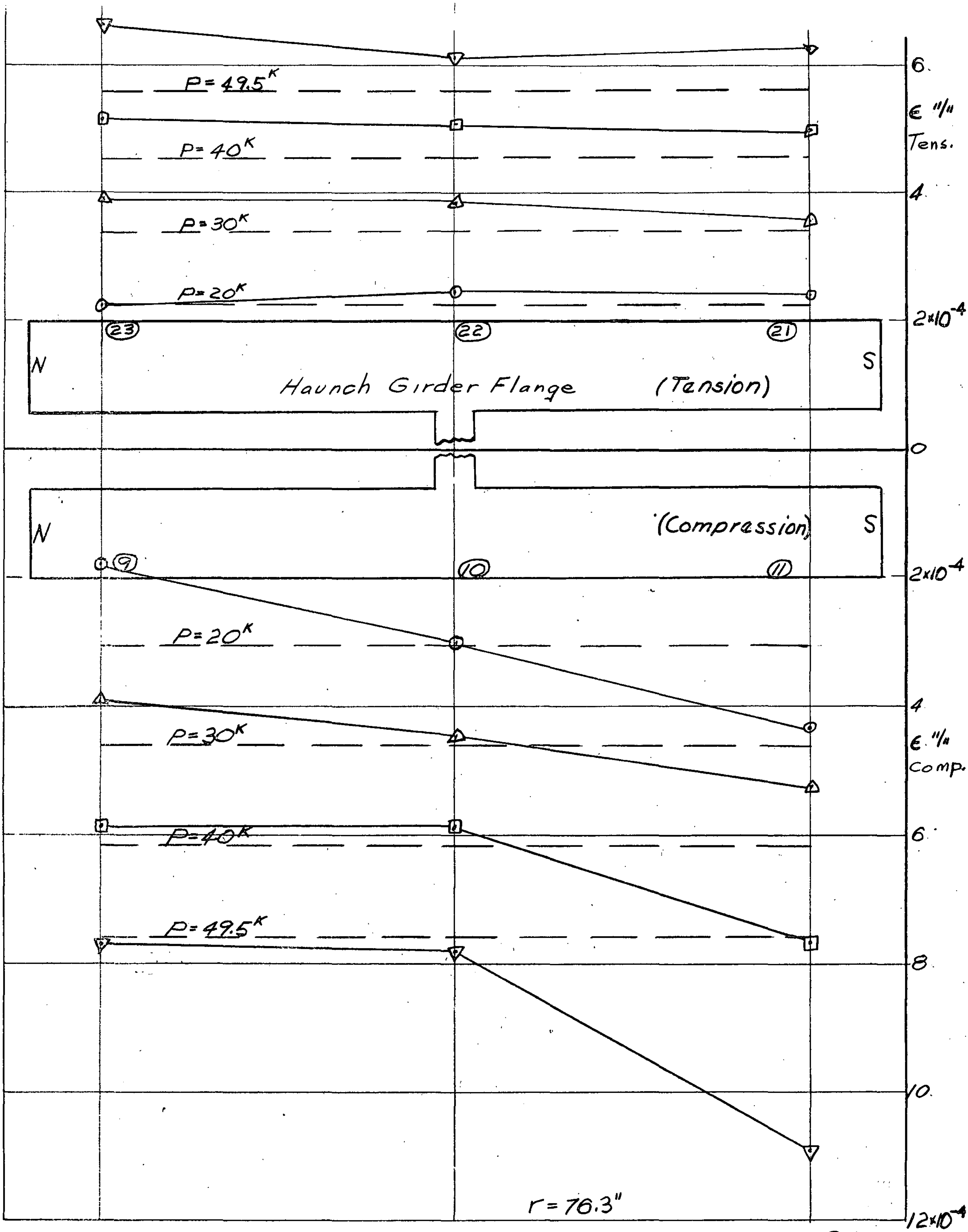


Fig. 31 LATERAL DISTRIBUTION OF STRAIN ACROSS HAUNCH GIRDER FLANGE

31

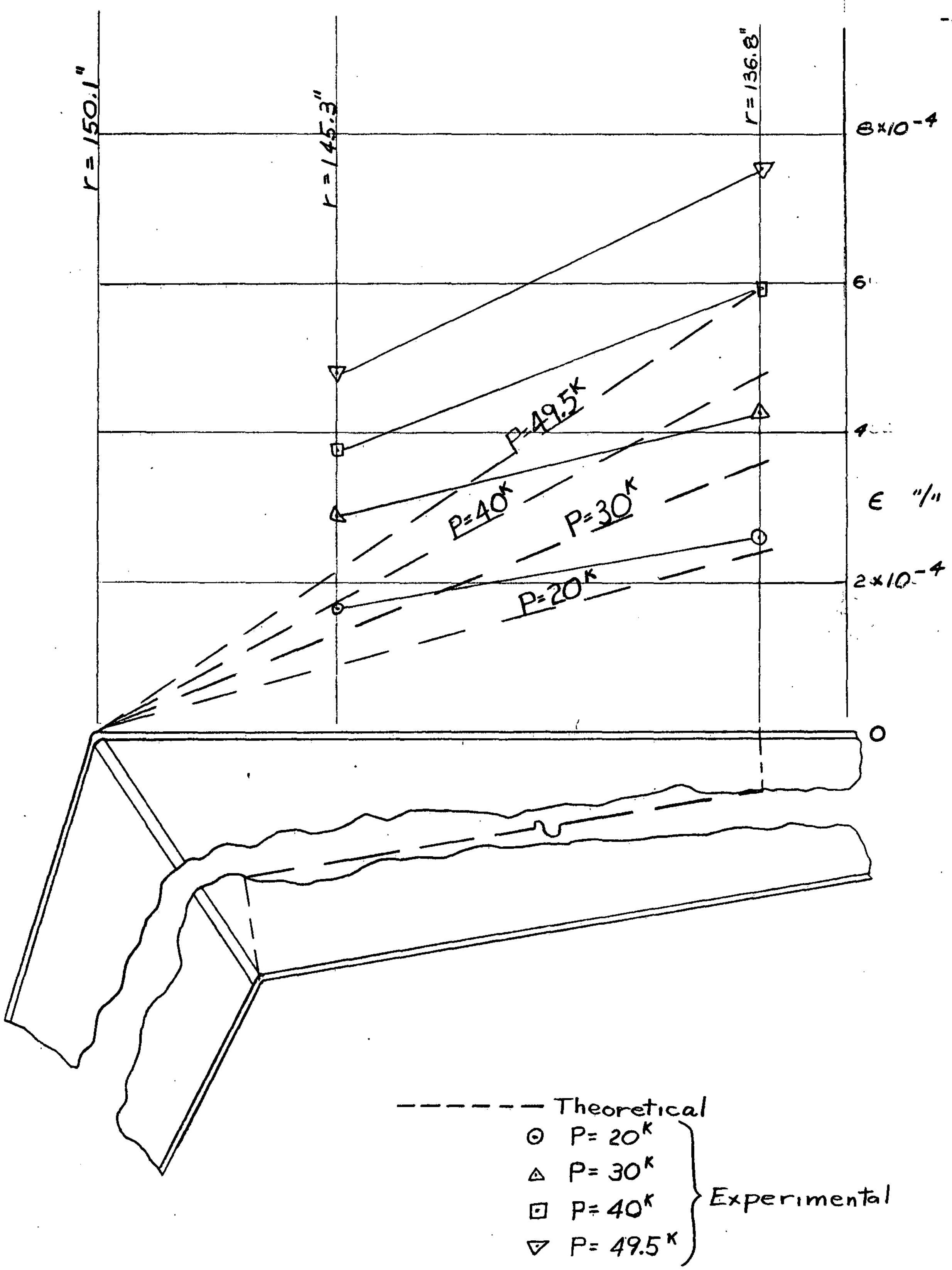


Fig. 32 THEORETICAL AND EXPERIMENTAL STRAINS IN HAUNCH GIRDER FLANGE IN CORNER

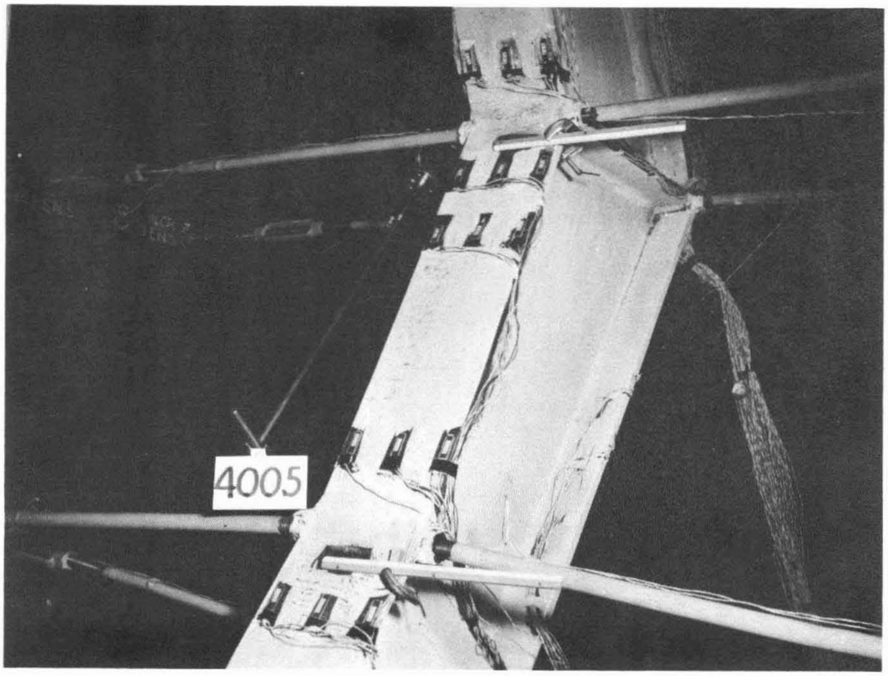


Fig. 33 EXTENT OF YIELDING IN HAUNCH COLUMN FLANGE AND WEB
LOAD = 68.6 KIPS

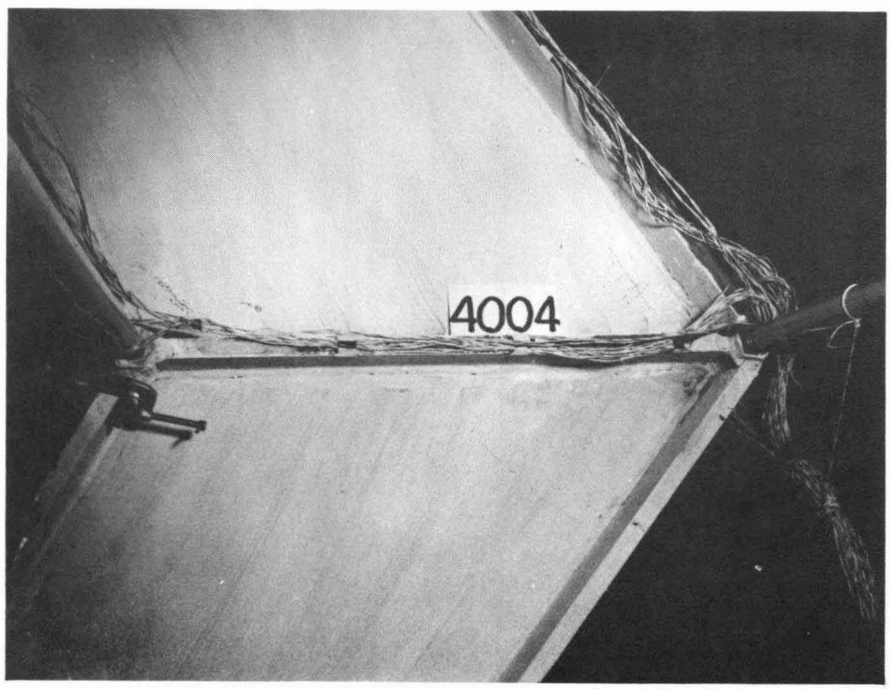


Fig. 34 EXTENT OF YIELDING IN WEB IN CORNER
LOAD = 35.0 KIPS

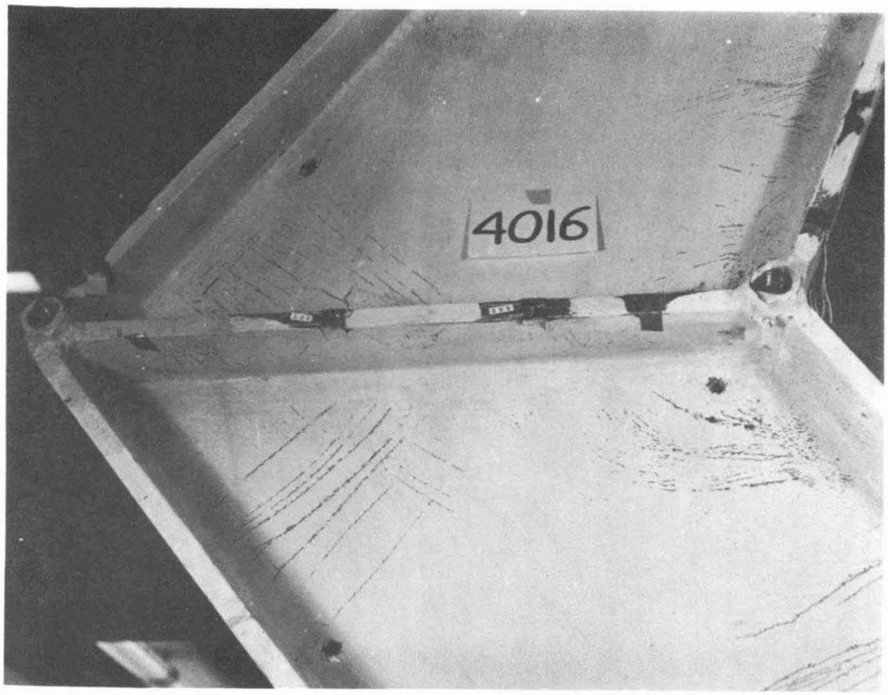


Fig. 35 EXTENT OF YIELDING IN WEB AND STIFFENER IN CORNER AT END OF TEST

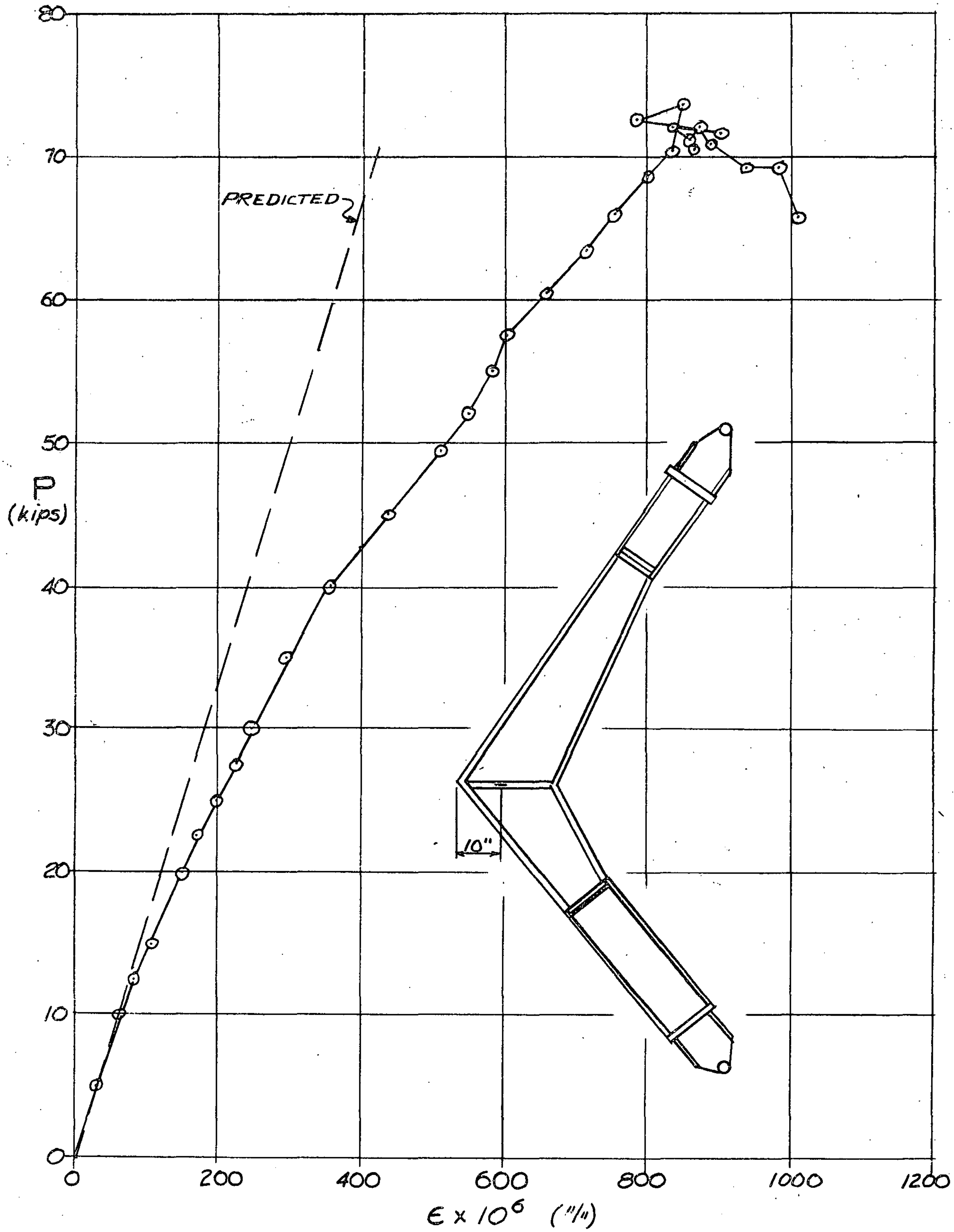


Fig. 36 PREDICTED AND AVERAGE EXPERIMENTAL STRAINS FOR 2 SR-4 GAGES ON EDGE OF DIAGONAL STIFFENER. ONE GAGE EACH ON NEAR AND FAR SIDE OF CONNECTION

36

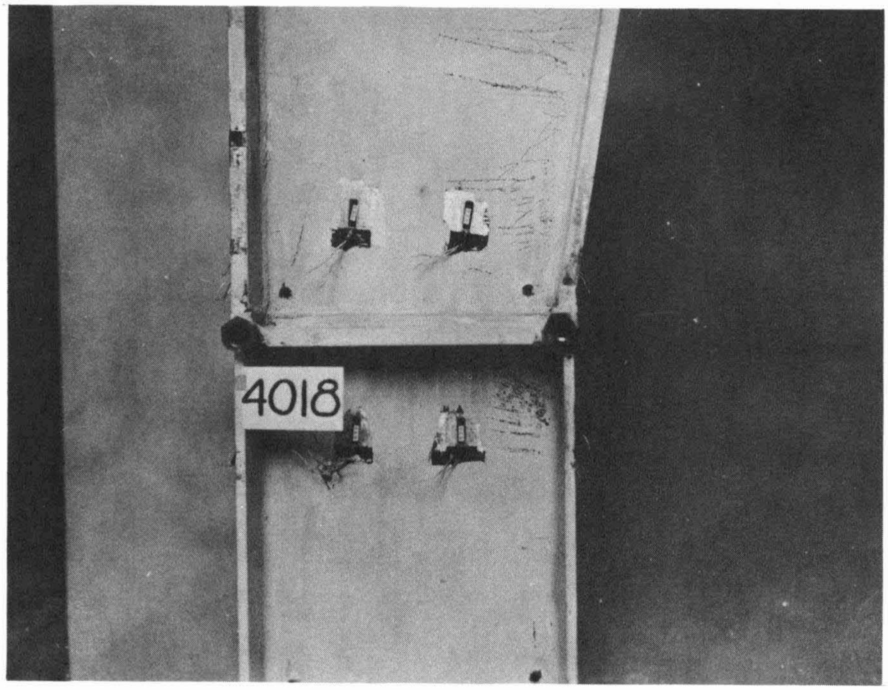
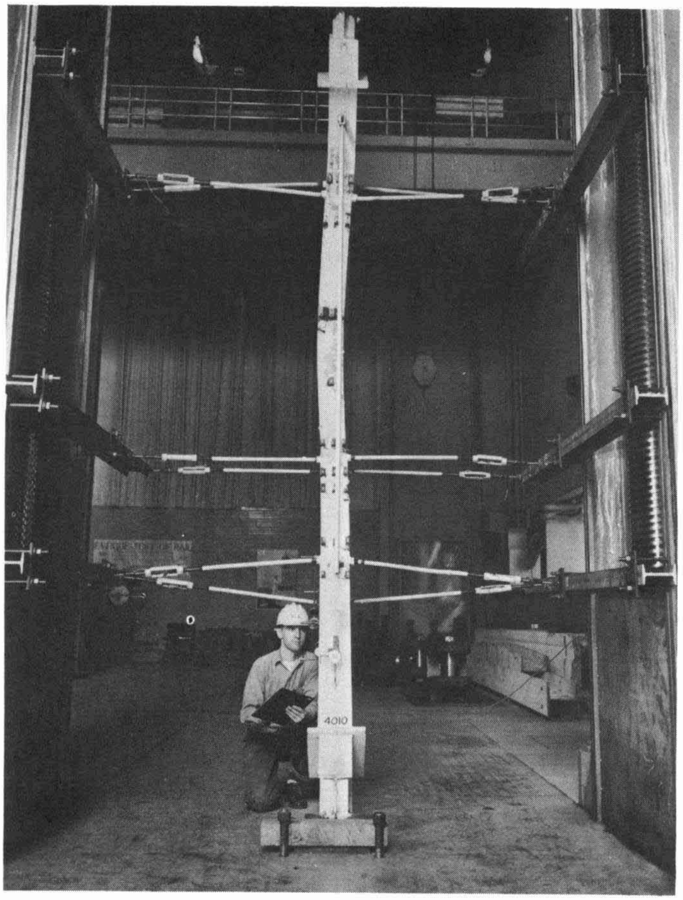


Fig. 37 EXTENT OF YIELDING AT JUNCTION OF HAUNCH COLUMN AND 16WF45



37,38

Fig. 38 VIEW OF COMPRESSION FLANGES SUBSEQUENT TO REMOVAL OF LOAD
NOTE LATERAL SUPPORT RODS AND LATERAL BUCKLE IN HAUNCH GIRDER

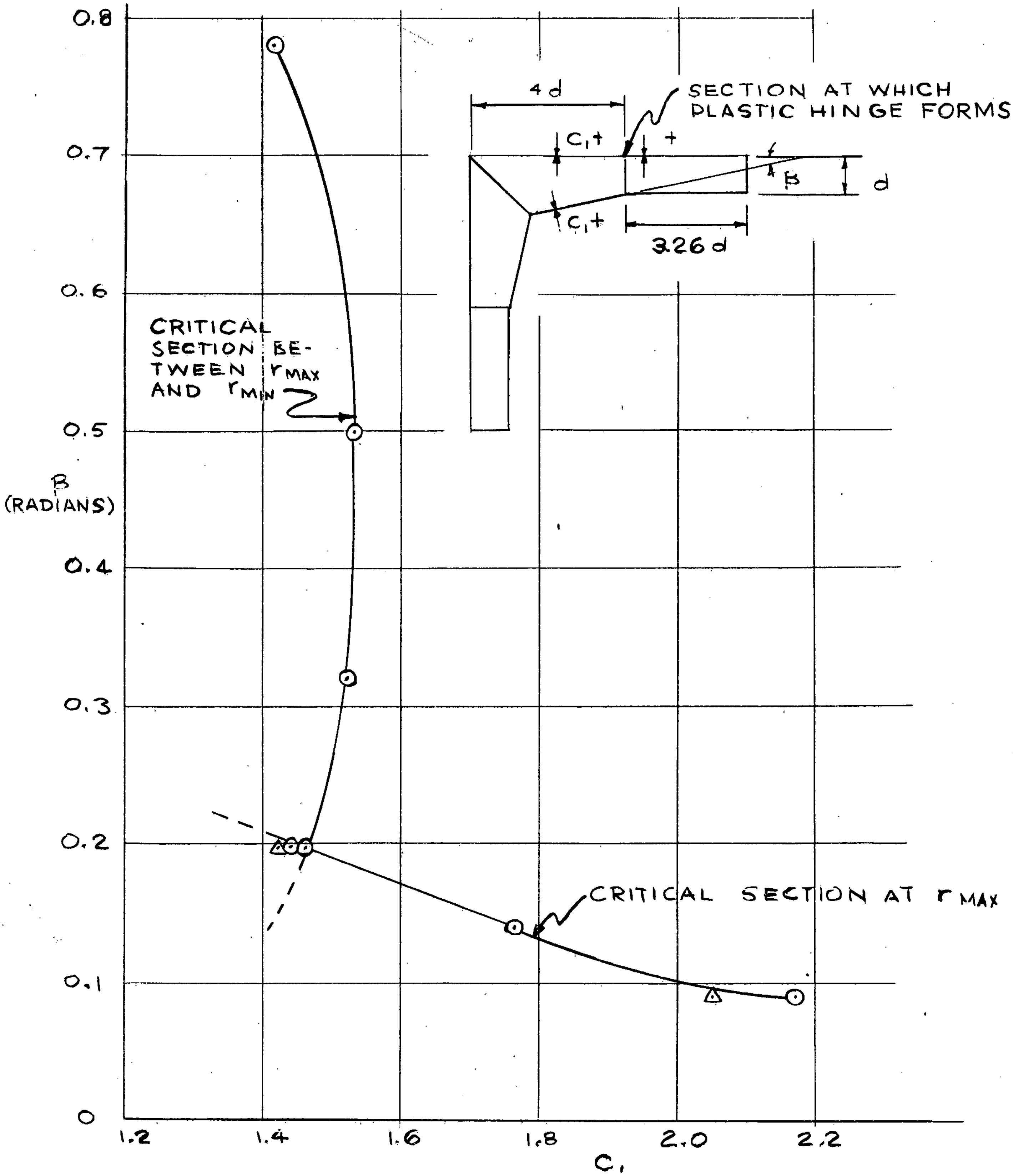


Fig. 39 SUGGESTED DESIGN GUIDE