1964

Beam-column experiments, Proc. ASCE, Vol. 90 (ST2) April 1964 Reprint 242 (64-23)

T. V. Galambos

R. C. Van Kuren

Welded Continuous Frames and Their Components

BEAM-COLUMN EXPERIMENTS

by

R. Carlton Van Kuren

and

Theodore V. Galambos

This work has been carried out as part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Institute of Research, Lehigh University
Column Research Council (advisory)
Office of Naval Research (Contract Nonr 610(03))
Bureau of Ships
Bureau of Yards and Docks

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pa.

June 1961

Fritz Engineering Laboratory Report No. 205A.30

SYNOPSIS

This report contains a description of 42 beam-column experiments conducted to study the strength and deformation characteristics of as-rolled wide-flange columns subjected to axial force and bending moments about the strong axis. The effects of axial force, length, member size, lateral bracing, and loading conditions are studied. The scope of the tests, the test setup, and the experimental procedures are described. The effects of the variations of the various parameters are discussed. Finally, the results are compared with various theories and with empirical interaction curves.

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I. INTRODUCTION

This report contains the results of an experimental study on the behavior of as-rolled ASTM A7 steel wide-flange beam-columns. The study was prompted by the need to obtain information on the strength and deflection characteristics of beam-columns for the development of plastic design methods. The experiments have been performed over a period of about twelve years, and some of the results have already been reported in Refs. 1 to 8. This report presents a summary of the whole experimental program.

A total of 42 full scale beam-column experiments were performed. Of these, 32 tests were made on laterally unsupported members (T-Series tests) and 10 tests were made on laterally supported members (A-Series tests). In each test the axial force and the end moments were applied independently of each other, and testing was continued until after the member failed by the unloading of the variable load parameter. In most of the tests, bending was about the strong axis of the member.

By choosing specimens of varying sizes and lengths, by varying the ratios of the applied end moments, and by bracing some of the columns against lateral-torsional buckling, the results of the experiments furnished complete or partial answers to the following questions:

1) What are the effects of slenderness ratio, loading condition, and axial force on the maximum moments which can be carried by a beam-column?

2) Is it possible to obtain reasonable correlation between the experimental results and various "exact" and "empirical" theories used for predicting beam-column strength?

3) Can the load-versus-deformation relationships be adequately predicted by theory?

4) By what mechanism do steel wide-flange beam-columns ultimately fail?

In the ensuing report first the experiments and the principal variables will be discussed. Then the test setups will be briefly described, and finally the results will be examined in connection with the questions raised above.

II. COMPARISON WITH OTHER BEAM-COLUMN EXPERIMENTS

Following is a brief discussion on the major beam-column experiments reported in the literature:

In the 1920's, several series of column tests on rolled, built-up, and tied sections were conducted at the University of Wisconsin.[9] These columns were loaded eccentrically in such a way that equal end eccentricities caused single curvature deformation about the strong axis. The objective of these tests was to compare the behavior of the "actual" column with the "ideal" column. The effects of crookedness, imperfections of rolling, and other effects causing variations in the strength of the member were considered in the evaluation of the results.

Johnston and Cheney[10] reported on ninety-six column experiments conducted at Lehigh University in the early 1940's. Tests were made on both axially loaded and eccentrically loaded columns. Both weak and strong axis tests were performed. The slenderness ratio and the eccentricity were the principal variables. The experiments were carried out in an effort to make a comparison with the current column formulas of the AISC for concentrically and eccentrically loaded columns.

In 1955 Campus and Massonnet[11] described ninety-five column tests performed in Belgium. These columns were subjected to eccentric compressive loads with equal, unequal, and opposite eccentricities about the strong axis. These loading conditions are similar to case a, c, and d loadings shown in Fig. 1. The tests were performed in order to develop simple design (interaction type) formulas, provide an exact theoretical analysis of columns under oblique compression, and to make a comparison with present European column specifications.

At Cornell University in 1958, Mason, Fisher, and Winter[12] conducted twenty-four eccentrically loaded hinged-end column tests. The tests were made on welded hat-shaped sections with bending about a minor axis parallel to the flanges. The loading condition was similar to the case "c" loading of Fig. 1. Because of the geometry of the section, the influence of lateral-torsional buckling was eliminated.

The column experiments of this report compared to those just described are unique in that bending moment and axial force were applied independently. Also, a greater number of variables were investigated.

The main objective of the experiments was to investigate the behavior of typical American rolled wide-flange column shapes used in continuous frames. The results of the investigation have been used specifically to advance and to verify the methods of plastic design.

III. DESCRIPTION OF THE EXPERIMENTAL PROGRAM

The experimental program consisted of 42 full scale beam-column tests. The general distribution of the principal test parameters is given in Table I. The cross sectional properties (that is, the area A, section modulus S, the plastic modulus Z, the major and the minor radii of gyration r_x and r_y), and the column lengths are shown in Table II. Table III lists the yield strength σ_y, the yield load $P_y = A\,\sigma_y$, the plastic moment $M_p = Z\,\sigma_y$, and the modulus of elasticity E. The member size, the loading condition, the slenderness ratio, the maximum axial force, and the maximum applied end moment are tabulated in Table IV. The information in these four tables contains all the data necessary to visualize the scope of the test program.

III.1 Material

The test columns were rolled from ASTM A7 steel. The columns were tested in an "as-delivered" condition, and no attempt was made to eliminate rolling or cold bending residual stresses by annealing. The magnitude and distribution of the rolling residual stresses was determined for the 8WF31 and

4WF13 sections in an independent study.[7] It was found that these sections contained considerable compressive residual stresses at the flange tips. The average magnitude of these stresses was approximately 30% of the yield stress. In a theoretical study[5] it was shown that these residual stresses cause a distinct reduction of column strength. The experimental results, as will be shown later, proved this theory to be correct. It was concluded[5] that the ultimate strength behavior of as-rolled beam-columns cannot be predicted rationally unless the cooling residual stresses are taken into consideration. The effect of the cold bending residual stresses, the presence of which was evidenced on the test specimens by the yield lines in the vicinity of the gag points, seemed to be only local.

The yield stress and the modulus of elasticity were determined for each heat of the T-Series tests, and the values for these properties are listed in Table III. These properties were obtained from tensile coupon tests at the very start of the testing program (around 1950), and the yield stress does not include the effect of strain rate.[13] The static yield stress is therefore somewhat smaller than the values listed. For the A-Series tests, coupons were cut from the web and the flanges of each column after the completion of the beam-column test, and the values of σ_y listed in Table III are static yield stress values. In addition, several stub columns were cut from the unyielded parts of the columns for this series to check the yield stress level obtained from the tensile coupons and to determine the magnitude of the maximum compressive residual stress. The residual stress magnitude was approximately that obtained in the earlier study.

III.2 Force Application

For the majority of the tests a predetermined axial force was applied first to the column. This axial force was kept constant while end bending moments were applied by hydraulic jacks through a lever arm until failure occurred. The column was considered to have failed when it could resist no additional bending moment.

The process of loading was reversed for tests T-8 and T-22, where the moment was held constant and the axial load was varied from zero to its maximum value.

The direction of bending was in the plane of the web for almost all of the tests (that is, strong axis bending).

The only exceptions to this were tests T-25 and T-27 where bending was about the weak axis.

In the first series of tests (T-Series) no intermediate lateral bracing was used. Because of this, most of the strong-axis bending tests failed by inelastic lateral-torsional buckling. In order to eliminate the effects of this type of buckling, intermittent lateral bracing was used for the last ten experiments (A-Series). The main purpose of these experiments was to determine the moment-end rotation characteristics when failure occurs due to excessive bending in the plane of the applied moments.

The end conditions of the test setup were such that the column was pin ended in the plane of the applied moments. Because of the knife-edge end fixtures, the columns were essentially fixed ended in the plane perpendicular to the plane of bending. The degree of fixity was such that the effective length of the column was approximately 0.6 L in this plane.

III.3 Member Sizes

Most of the columns used in the experiments were 8WF31 or 4WF13 rolled sections. These members were chosen because they are geometrically similar typical column sections. Only one 8WF40 section (test T-2) was tested. Three tests were performed on the 8B13 section (8 in. x 4 in. nominal size) which is a typical beam-type section.

The cross sectional dimensions of each test column were measured and the computed values of the cross sectional properties A, S, Z, r_x, and r_y are tabulated in Table II. For a small number of the tests these dimensions were not available, and therefore only Handbook[14] values are listed.

Column lengths of 6 to 16 ft. were chosen for the tests, giving slenderness ratios from 21 to 111.

III.4 Loading Conditions

The axial force was applied concentrically for all of the tests. The various combinations of end moments are shown in Fig. 1. These various loading conditions are designated as condition "a", "b", "c", "d", and "e".

Loading condition "a" is that loading where two equal end moments were applied in the same sense, causing double curvature deformation. Three tests were done for this loading case.

In loading condition "b", the two end moments were applied in such a way that the slope on one column end was zero. This loading condition simulates a fixed-end column in the lower story of a rigid frame. Eleven case "b" tests were performed.

For condition "c" two equal end moments were applied in opposite directions resulting in single curvature deformation. For this loading case, eight tests were conducted.

Loading condition "d" has moment applied at only one end. A total of fifteen tests were completed using this loading case, including all the A-Series tests.

In condition "e" loading, only axial force was present. Five experiments were made. The results have already been reported in connection with other axially loaded column tests.[7,13]

The magnitudes of the axial force, as well as the loading condition, the slenderness ratio and the maximum applied bending moment are shown in Table III. The axial load varied from 10 to 80% of the yield load P_y.

The outline of the whole testing program is illustrated in Table I. In this tabulation the relationship between each test is shown with regard to the four principal variables (axial force ratio, slenderness ratio, loading case, and member size). For example, test T-13 was subjected to $P = 0.12\ P_y$, its slenderness ratio was 55, the test was a loading condition "d" case, and the member was an 8WF31 shape. Comparison could be made with test A-3, for which all conditions except the magnitude of the axial force were the same as for test T-13.

IV. TEST APPARATUS AND EXPERIMENTAL PROCEDURE

IV.1 Test Setup for the T-Series Tests

For the tests in the first experiments the test apparatus described by Beedle, Ready, and Johnston[1] was used. Figure 2 shows a typical set of forces applied to the column by the test apparatus. Loading condition "a" is shown. The concentric axial load P was applied through knife-edges by a testing machine. The end moments were applied by means of forces F, acting through rigidly fixed lever arms attached to the ends of the column. The end moment forces were produced by tension-compression hydraulic jacks connected to the test frames. Any lateral thrust H which was developed was taken up by lateral tie rods attached to the end fixtures.

Reaction support for the end moment forces F and lateral thrust H was provided by the test frame shown in Fig. 3. The frame consisted of four rigidly braced columns made to accommodate test columns up to 16 ft. in length. The Partial End View shows the system used to apply moments. A dynamometer was connected in series with the hydraulic jack to measure the end moment producing force.

The whole test frame assembly, including the test column, could be picked up by a crane and placed in the testing machine. For the T-Series tests, an 800,000 lb. capacity mechanical screw-type testing machine was used to apply the axial load.

The photograph in Fig. 8 shows how the axial load was applied to the upper end of the column. The end fixtures received the load from two 8 in. long knife-edges spaced 13 inches apart. One of the knife-edges is visible in Fig. 8. The knife-edges were parallel to the strong axis of the column for bending about this axis. At the top on both sides of the center roller, wedge blocks are visible. These have been installed to prevent rotation of the column end about the weak axis.

IV.2 Test Setup for the A-Series Tests

The test setup for the A-Series tests utilized the 10 ft. wide testing space of a 5,000,000 lb. capacity hydraulic testing machine as shown in Fig. 6. In this setup the need for a testing frame was eliminated by using the frame of the testing machine for support. The same end fixture assemblies and the same moment application arrangements were used as for the

T-Series tests. Reaction support was provided by the vertical frame of the testing machine, anchorages in the floor, and various structural steel members. The force application is identical to that shown in Fig. 2. A sketch of the test setup is shown in Fig. 4. The photograph in Fig. 6 shows the whole assembled setup.

Lateral bracing systems were provided for all the A-Series tests in order to prevent lateral-torsional buckling. The bracing was spaced in accordance with the provisions of Chapter 6.3 in the Commentary on Plastic Design in Steel.[15] The number of bracing systems required varied from one to three. The details of a lateral bracing system are shown in Fig. 5. Channels were clamped against the flange tips on both sides of the column at the center line of the position requiring lateral support by tie rods bolted to the channels. These tie rods were extended parallel to the X-axis of the section, fastening to a roller arrangement at the vertical face of the testing machine. The photograph in Fig. 7 shows two lateral bracing systems installed on a column. The tie rods were tensioned uniformly on both sides of the column to hold the column in alignment and prevent any twisting. As the column deflected under load, the bracing system followed the column, allowing freedom of movement in the Y-direction but rigidly restraining the column from movement in the X-direction at the bracing points. The bracing system was checked at every load increment to assure freedom of movement in the plane of bending. The locations of the lateral braces are listed in Table V for all ten A-Series tests.

IV.3 Instrumentation

To record the behavior of the column under load, several measuring techniques were employed. Lateral and transverse deflections were measured by taunt-wire and mirror gages[1] for tests T-1 to T-5. Transverse or strong direction deflections for tests T-6 to T-32 were measured by means of Ames dials attached to a deflection gage rig. The lateral deflections for tests T-6 to T-32 were measured by dial gage arrangements attached to the flange tips along the length of the columns.[3] For the A-Series tests, lateral deflections were measured using the transit shown in Fig. 6 and a metal tipped scale held against the flange tips and web of the column at various levels along the column height. Transverse deflections for the A-Series were measured by dial gages attached to a fixed rig with fine wire stretched between the plunger of the gage and a magnetic base attached to the flange of the column. Measurements were taken at four locations along the column length.

Level bars were mounted on support brackets which were welded to the base plates at the top and bottom of the test column to indicate angle changes at the column ends about the X-axis. As the specimen rotated, a micrometer screw in the level bar was adjusted to center the level bubble that was fastened on the bar. A dial gage mounted on one end of the level bar recorded the movement or rotation of the bar over a fixed gage length as the column end rotated. For loading condition "b", the level bar at one end was used as a guide for "fixing" the column end against rotation.

Strain gages (SR-4 type) were mounted on various sections of the test specimen depending on the test conditions. The strain gage data served as a check on the other measurements, such as the point of inflection for loading condition "a" and the strain distributions across the section and along the member. These gages were also used as a means of checking the initial alignment.

Prior to testing, a whitewash coating was applied to the specimen so that when yielding was reached the whitewash would flake off with the mill scale, leaving a visual yield pattern.

Other measurements recorded were the axial load as indicated by the testing machine and end moment forces measured by dynamometers. These measurements denote the overall strength of the column.

IV.4 Test Procedures

The test procedure for each test was as follows:

a) The preliminary work consisted of the measurement of the column dimensions, the calculation of the expected failure loads, and the preparation of control curves.

b) The next step consisted of erecting the test fixtures, placing of the column in the testing machine, installation of the instrumentation, and aligning the column.

c) After the predetermined axial load was applied to the column, increments of bending moment were applied. After each increment of moment, sufficient time was allowed for the system to come to a complete rest before the readings of force and deformation were taken. This was especially observed after yielding was initiated. In this way all strain rate effects were eliminated, and the readings represented a static condition. In the inelastic range increments of deflection rather than increments of load were used.

d) Loading was usually continued until the column was so far deformed that it could not maintain its axial force or until it was evident that the knife-edges did not rotate freely. In all tests at least some unloading of the variable force parameter was observed before termination of the test.

V. DISCUSSION OF THE TEST RESULTS

The principal results of each experiment can be presented by the following information.

a) The maximum bending moment which the column supported besides the constant axial force, or, as was the case for tests T-8 and T-22, the maximum axial force which was carried besides the constant end moment. These maximum forces are listed in Table IV. In this table both the absolute values of the forces as well as the non-dimensional ratios P/P_y and M_o/M_p are given.* These forces represent the ultimate load of the column.

b) The load deformation curves, such as the moment-versus-end slope or the moment-versus-deflection curves. Several of the curves are shown in Figs. 13 and 14. Since a subsequent report will deal with the comparison between the experimental load-deflection curves and curves determined by theory, these relationships will not be discussed here in

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*In Table IV the bracketed values of P and P/P_y refer to the cases where the axial load was the variable. All of these, with the exception of tests T-8 and T-22, were axially loaded columns.

great detail. It should be noted that the moment-rotation curves represent the most important results of the experiments.

c) Observations on the types and causes of failure. For the most efficient utilization of the column, failure should be triggered by excessive bending in the plane of the moments. It is therefore desirable to know under what conditions the more detrimental effects of local and lateral-torsional buckling would initiate failure.

V.1 Influence of the Axial Force and the Length

Several test results are plotted on the rectangular coordinate system formed by the axial force (P/P_y) and maximum bending moment (M_o/M_p) in Fig. 12. The experimental points represent the coordinates of the maximum axial force and bending moment for columns subjected to case "b" loading. The circles denote a slenderness ratio of 55 and the triangles a slenderness ratio of 111. For any one slenderness ratio it is seen that the moment carrying capacity decreases as the axial force is increased. A comparison of the two slenderness ratios shows that the more slender columns are weaker.

The same conclusions were reached for all the other loading conditions.

V.2 Influence of Lateral-Torsional Buckling

In Fig. 13 the moment-versus-end slope curves of two identical columns are shown. Both are 4WF13 columns 16 ft. long ($L/r_x = 111$) and both are subjected to case "d" loading. The axial force is approximately the same on each column. Test A-7 was provided with appropriate lateral bracing, whereas test T-31 was not braced. It is seen from Fig. 13 that the unbraced column is weaker (despite its somewhat smaller axial force) and that it possesses a smaller rotation capacity.* Unfortunately these two tests were the only two for which comparison could be made. The differences between braced and unbraced columns would have shown up more drastically for higher axial load ratios.

The effects of lateral-torsional buckling (that is, reduction in strength and rotation-capacity) are most pronounced for case "c" loading. As pointed out in Ref. 8, lateral-torsional buckling should be prevented by bracing if the column is to perform in its most efficient manner. It

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*The difference between the slopes of the curves in the elastic range is due to the differences in the axial force and the material and cross-sectional properties.

can be seen in the last column in Table IV that in the T-Series tests (unbraced columns) the most prevalent type of failure was lateral-torsional buckling. In Ref. 6 it is shown that the effect of residual stresses on lateral-torsional buckling is very pronounced. A typical lateral-torsional buckling failure is shown in the photograph of Fig. 9.

V.3 Influence of Local Buckling

The 8WF31 and 4WF13 tests where failure was by bending plus local buckling show that for a rather stiff member ($L/r_x=55$) and a low axial load ratio failure will be of this type for loading conditions a, b, and d. As the slenderness ratio is increased, reducing the stiffness, failure takes place due to lateral-torsional buckling.

Tests A-8 and A-9, which were performed on 8B13 members having relatively slender flanges and webs, failed by lateral-torsional buckling between the lateral braces followed by local buckling. Typical local buckling of the compression flange is shown in Fig. 11.

It should be noted here that local and/or lateral-torsional buckling always occurred, even if the members were made up of stocky plate elements and if lateral bracing was present. The purpose of the lateral bracing was not to completely eliminate these effects, but to postpone them until after the column starts to unload, and failure has already taken place due to excessive bending in the plane of applied moments. This was the case for all but two of the A-Series tests. The photograph shown in Fig. 10 shows a typical column which failed in this manner.

V.4 Influence of Member Size

The influence of member size cannot be easily separated from the effects of local and lateral-torsional buckling. This can be seen by comparing the two curves in Fig. 14, where the M-θ curves for test T-13 (8WF31) and A-9(8B13) are shown. The condition of loading and the axial force ratio were equal for both columns with only a slight difference in slenderness ratio. The 8B13 column, having less resistance to lateral-torsional and local buckling, failed earlier.

Ketter and Beedle[(2)] have compared the behavior of 8WF31 and 4WF13 sections. These comparisons indicated that the size of the member has little influence on the test results in the elastic range, but in the inelastic range this variable can have some effect. If the cooling residual stresses are not of nearly the same magnitude or if the loading condition is not favorable, then member size will have some influence on the test results.

Theoretically[5] the only difference between the strength of various size columns is due to the difference of the shape factor if failure is due to excessive bending.

V.5 Influence of Loading Condition

The influence of the loading condition is illustrated in Fig. 15 where test points are plotted for $P/P_y = 0.12$ for slenderness ratios of 55 (circles) and 111 (triangles). The abscissa in this figure is the maximum end moment (M_o/M_p) and the ordinate is β, the ratio of the larger end moment to the smaller one. Also shown on the ordinate are the values of β for the loading conditions "c", "d", and "a". Case "b" is not shown, since the ratio β is not defined precisely. It would fall however in the neighborhood of $\beta = -0.5$.

From Fig. 15 it can be seen that the loading condition has a definite influence on column strength and that case "c" loading represents the most severe loading.

VI. COMPARISON OF TEST RESULTS WITH THEORY

The principal purpose of the experiments described herein was to provide experimental verifications to various theories which were proposed to predict the strength of wide-flange beam-columns. In the following section the test results will be compared to two "exact" theories and to two widely used empirical interaction equations.

VI.1 Comparison with a Theory which Assumes Failure by Excessive Bending in the Plane of the Applied Moments

Strength interaction curves between the axial force, the maximum end moments, the slenderness ratio, and the moment ratio are given in Refs. 16 and 5 for as-rolled wide-flange columns bent about the major principal axis. The construction of these curves consisted of determining the moment-end slope relations of beam-columns by numerical integration of the moment-curvature curves.[4] The interaction curves represent the maximum moment which a given length column can support if a given axial force is present. The theoretical work underlying the interaction curves was based on the assumption that failure will take place by excessive bending in the plane of the applied moments. It was further assumed that a maximum compressive residual stress of 30% of the yield stress (which was assumed to be 33 ksi) was present at the flange tips.*

The theoretical conditions of the analysis were fulfilled by all the tests which did not fail by lateral-torsional buckling. For those tests which failed by this type of buckling, the theory gives a lower bound to the column strength. The theoretically determined ultimate loads are shown in Table VI for all the tests in the column marked (1). In most cases the maximum bending moment is listed. Exceptions are tests T-8 and T-22, where the bending moment was constant and the axial force was the variable.

Since the yield stress was not equal to 33 ksi for any of the experiments, the slenderness ratio was adjusted to account for this by the following formula[5]:

$$(L/r)_{adj} = (L/r)\sqrt{\frac{\sigma_y}{33}} \qquad \text{.......(1)}$$

No particular difficulty was encountered in determining the

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*The assumed residual stress pattern is shown in Fig. 7.12 of Ref. 8.

theoretical values for loading cases "a", "c", and "d", because interaction curves were already available[16,5,8]. Since the end moment ratio for case "b" loading (one end fixed, see Fig. 1) is not clearly defined, the theoretical results were computed for $\beta = -0.5$, giving only an approximation to the true situation.

Graphical comparisons between theory and experiment are provided by Figs. 16 and 17. In Fig. 16 interaction curves are shown for L/r = 55 and L/r = 111 for loading case "d". The solid lines represent the theoretical curve and the points (circles for L/r = 55 and triangles for L/r = 111) represent test results. An examination of Fig. 16 shows that correlation between theory and experiment is excellent. Another type of correlation is shown in Fig. 17 where the ratio of the maximum experimental load to the maximum theoretical load is shown, using the slenderness ratio as an abscissa. The various loading cases are indicated by different symbols. This figure shows the correlation for most of the tests is quite good, except for the columns which failed by lateral-torsional buckling. This was especially true for case "c" loading and for long unbraced columns.

The comparisons in Figs. 16 and 17 show that the theory is capable of predicting column strength very well if lateral-torsional buckling is prevented by bracing. For all loading cases besides case "c" (which is the most serious loading condition) the theory can predict the strength quite well even if failure is by lateral-torsional buckling.

VI.2 Comparison with a Theory which Considers Lateral-Torsional Buckling

A theory taking account of the effects of lateral-torsional buckling is presented in Ref. 6. The theory has so far been applied to case "c" loading, and therefore the theoretical values computed by it are only shown for this loading case in Table VI (Column marked (2)). A comparison between theory and experiment shown in Fig. 18 indicates that correlation is excellent.

VI.3 Comparison with Empirical Interaction Equations

In design practice usually empirically determined interaction equations are used to determine the adequacy of a given column to support the loads imposed on it. One such interaction equation is the following (Eq. 5.15 in Ref. 17).

$$\frac{P}{P_u} + \frac{M_o}{M_u} = 1.0 \qquad \text{.......(2)}$$

In this equation P and M_o are the maximum axial force and end moment supported by the column, respectively. The term P_u represents the maximum axial load which can be carried by the column if no bending moments are present, and it can be computed by the tangent modulus theory. A formula suggested by the Column Research Council (Eq. 2.9, Ref. 17) has been used in this comparison; the smaller value furnished by the two following equations was used:

$$\frac{P_u}{P_y} = 1 - \frac{\sigma_y}{4\pi^2 E} (L/r_x)^2$$

or

$$\frac{P_u}{P_y} = 1 - \frac{\sigma_y}{4\pi^2 E} (0.6L/r_y)^2 \qquad \text{.......(3)}$$

The first of these equations is for strong axis buckling, and as such it is applicable for all braced columns (A-Series), and the second formula represents weak axis buckling.

The term M_u is the maximum moment which can be supported by the column if no axial force is present. For the braced columns the value of $M_u = M_p$, the full plastic moment. For columns where there is a possibility of lateral buckling, M_u will always be less than M_p. The methods suggested by the

Column Research Council (Chapter IV, Ref. 17) were used in the computations.*

The correlations between the ratios of the experimental maximum load and the maximum load determined by Eq. 2 are shown in Fig. 19. It is seen that Eq. 2 represents essentially a lower bound to the actual column strength, and that it does not seem economical to use this equation for other than loading case "c". The formula can over estimate column strength by as much as 50%.

In order to estimate column strength more economically, the following interaction formula has been suggested by the Column Research Council (Eq. 5.14, Ref. 17):

$$\frac{P}{P_u} + \frac{M_{eq}}{M_u(1-P/P_E)} = 1.0 \qquad ....(4)$$

The terms P, P_u, and M_u in Eq. (4) have the same meaning as in Eq. (2); P_E is the Euler load in the plane of bending, and it is equal to:

$$\frac{P_E}{P_y} = \frac{\pi^2 E}{\sigma_y} \quad \frac{1}{(L/r_x)^2} \qquad ....(5)$$

and M_{eq} is an equivalent bending moment defined by Eq. 6:

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*Ideal moments were computed first by using Eq.4.8 in Ref. 17. These moments were then reduced by the CRC column formula to account for premature yielding caused by residual stresses.

$$\frac{M_{eq}}{M_p} = \frac{M_o}{M_p}\sqrt{0.3(1 + \beta^2) + 0.4\beta} \quad ....(6)$$

The correlation between Eq. 4 and the experiments is shown in Fig. 20. Good correlation exists for most of the experiments.

As a final comparison, results of the axially loaded tests (loading case "e") are compared with the Eqs. 3 in Fig. 21. Again, reasonable correlation is seen to exist.

VII. SUMMARY AND CONCLUSIONS

The experiments discussed in this report were performed to study the inelastic behavior of wide-flange beam-columns and to provide an experimental basis for theories used in the plastic design of steel frames. The conclusions reached in this study are the following:

(1) The primary cause of failure of beam-columns subjected to axial force and bending moment is lateral-torsional buckling if the columns are not externally braced (Table IV).

(2) The largest reduction in strength and rotation capacity occurs for longer columns and for columns which are loaded such that the moments cause single curvature deformation (Figs. 12 and 15).

(3) Theoretical interaction curves[16,5] for maximum loads at failure due to excessive bending in the plane of the applied moments can adequately predict the strength of braced columns and of unbraced columns which are subjected to a moment

ratio of less than zero.* (Fig. 17.)

(4) The experiments in the A-Series tests (braced columns) have shown that lateral bracing spaced in accordance with the plastic design beam-bracing rule[15] will prevent lateral-torsional buckling failure. It should be noted that the actual bending moment distribution (that is, including the moments caused by the deflection) must be used in the computations.

(5) It was shown that strength and rotation capacity increase as the length and axial force on the column is decreased (Fig. 12).

(6) The effect of loading condition is such that the strength of the columns increases as the moment ratio is decreased from $\beta = +1.0$ to its minimum value of $\beta = -1.0$ (Fig. 15).

(7) A comparison with an "exact" theory which incorporates the effects of lateral-torsional buckling has shown that good correlation exists between theory and experiment (Fig. 18).

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*Since lateral-torsional buckling also reduces rotation capacity, it has been recommended[8] that in plastic design all columns bent about the strong axis should be braced.

(8) The results of the experiments have indicated that a linear interaction equation (Eq. 2) will usually underestimate the strength of a beam-column (Fig. 19).

(9) A comparison with the Column Research Council interaction equation (Eq. 4) has shown that this equation can predict beam-column strength with sufficient accuracy to warrant its use in design (Fig. 20).

(10) The Column Research Council basic column strength equation (Eq. 3) shows good correlation with experiment (Fig. 21).

(11) Studies (which will be published in a subsequent report) have shown that the complete elastic-plastic load-deformation behavior of beam-columns can be predicted by theory.

This experimental study has substantiated recently developed elastic-plastic theories which recognize the importance played by the residual stresses. The verification of these methods by the relatively simple experiments described herein should lead to their application to the more complex loading cases encountered in the design of multi-story frames and thus aid the extension of plastic design methods to these structures.

VIII. ACKNOWLEDGMENTS

This study is part of a general investigation "Welded Continuous Frames and Their Components" currently being carried out at the Fritz Engineering Laboratory of the Civil Engineering Department of Lehigh University under the general direction of Lynn S. Beedle. The investigation is sponsored jointly by the Welding Research Council, and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, the American Iron and Steel Institute, Lehigh University Institute of Research, the Office of Naval Research, the Bureau of Ships, and the Bureau of Yards and Docks. The Column Research Council acts in an advisory capacity.

The authors express their thanks to all those who over the past 15 years had a hand in conducting the experiments, giving advice, and guiding in the preparation of this report. Special recognition goes to K. Harpel, laboratory foreman, and to his group of technicians, without whose help the performance of the experiments would have been impossible.

IX. NOTATIONS

A = Cross sectional area

E = Young's modulus of elasticity

L = Length of column between base plates

L/r = Slenderness ratio

L/r_x = Strong axis slenderness ratio

L/r_y = Weak axis slenderness ratio

M = Moment

$M_{eq.}$ = Equivalent end bending moment

M_o = Applied end bending moment

M_p = Full plastic moment of a cross section

M_u = Maximum moment which can be carried in the absence of axial force

P = Axial force applied to the column

P_E = Euler load in the plane of bending

P_u = Collapse load for the column centrally loaded for buckling in the unrestrained plane

P_y = $A\sigma_y$ = Axial force causing uniform yielding of the whole cross section

S = Section modulus

Z = Plastic modulus

r_x = Radius of gyration about the x-axis

r_y = Radius of gyration about the y-axis

β = Ratio of $(M_o)_{top}$ to $(M_o)_{bottom}$ where $(M_o)_{top} > (M_o)_{bottom}$

σ_y = Yield stress

θ = End rotation

X. TABLES AND FIGURES

TABLE 1. TESTING PROGRAM

Test No.	P/P_y						L/r						Loading Case					Member Size			
	.12	.30	.50	.60	.67	.83	20	28	41	55	84	111	a	b	c	d	e	4 WF 13	8 WF 31	8 WF 40	8 B 13
T-1	x						x									x			x		
T-2	x						x									x				x	
T-3			x							x				x					x		
T-4	x									x				x					x		
T-5						x				x				x					x		
T-6		x										x		x				x			
T-7		x										x		x				x			
T-8				*						x					x				x		
T-9	x											x		x				x			
T-10			x									x		x				x			
T-11						*				x							x		x		
T-12	x									x					x				x		
T-13	x									x						x			x		
T-14		x								x			x						x		
T-15						*			x								x		x		
T-16	x								x						x				x		
T-17	x									x				x				x			
T-18						*		x									x		x		
T-19	x							x							x				x		
T-20	x									x					x			x			
T-21			x							x				x				x			
T-22				*						x				x				x			
T-23	x										x					x		x			
T-24	x										x			x				x			
T-25					*						x						x		x		
T-26	x										x				x			x			
T-27			*								x				x				x		
T-28						*					x						x	x			
T-29	x										x		x					x			
T-30	x											x	x					x			
T-31	x											x				x		x			
T-32	x											x			x			x			
A-1		x									x					x		x			
A-2					x					x						x			x		
A-3		x								x						x			x		
A-4			x							x						x			x		
A-5		x										x				x		x			
A-6			x									x				x		x			
A-7	x											x				x		x			
A-8		x								x						x					x
A-9	x									x						x					x
A-10				x						x						x					x

* Variable, Maximum Ratio Given

TABLE II. CROSS-SECTION PROPERTIES

Test No.	Section	A sq. in.	S cu. in.	Z cu. in.	r_x in.	r_y in.	L in.
T-1	8 WF 31	9.17	27.7	30.8	3.50	2.00	72.0
T-2	8 WF 40	11.69	35.4	39.7	3.54	2.02	72.0
T-3	8 WF 31	9.10	27.0	30.3	3.46	2.01	192.0
T-4	8 WF 31	9.21	27.6	30.7	3.48	2.00	192.0
T-5	8 WF 31	9.12*	27.4*	30.4*	3.47*	2.01*	192.0
T-6	4 WF 13	3.73	5.35	6.13	1.72	1.03	192.1
T-7	4 WF 13	3.82	5.53	6.34	1.73	1.04	192.1
T-8	8 WF 31	9.16	27.6	30.6	3.49	2.00	192.0
T-9	4 WF 13	3.72	5.37	6.15	1.73	1.03	191.9
T-10	4 WF 13	3.76	5.42	6.20	1.73	1.03	191.9
T-11	8 WF 31	9.23	27.8	30.8	3.48	2.03	192.9
T-12	8 WF 31	9.21	27.7	30.7	3.48	2.00	192.0
T-13	8 WF 31	9.26	28.0	31.0	3.50	2.01	192.0
T-14	8 WF 31	9.13	27.4	30.2	3.48	2.00	192.0
T-15	8 WF 31	9.16	27.6	30.6	3.49	2.00	144.0
T-16	8 WF 31	9.14	27.6	30.6	3.49	2.00	144.0
T-17	4 WF 13	3.94	5.65	6.49	1.72	1.04	96.0
T-18	8 WF 31	9.38	28.2	31.2	3.48	2.02	96.0
T-19	8 WF 31	9.32	28.1	31.2	3.49	2.01	96.0
T-20	4 WF 13	3.99	5.62	6.55	1.71	1.04	96.0
T-21	4 WF 13	4.02	5.74	6.51	1.72	1.02	96.0
T-22	4 WF 13	3.92	5.62	6.45	1.72	1.04	96.0
T-23	4 WF 13	4.09	5.90	6.68	1.73	1.03	144.0
T-24	4 WF 13	4.05	5.87	6.65	1.74	1.03	144.0
T-25	8 WF 31	9.22	9.2*y	13.86*y	3.47*	2.01*	153.5
T-26	4 WF 13	3.82*	5.45*	6.3*	1.72	0.99*	144.0
T-27	8 WF 31	8.97	9.2*y	13.86*y	3.47*	2.01*	144.0
T-28	4 WF 13	3.82*	5.45*	6.3*	1.72*	0.99*	144.0
T-29	4 WF 13	3.82*	5.45*	6.3*	1.72*	0.99*	144.0
T-30	4 WF 13	3.82*	5.45*	6.3*	1.72	0.99*	192.0
T-31	4 WF 13	3.82*	5.45*	6.3*	1.72*	0.99*	192.0
T-32	4 WF 13	3.82*	5.45*	6.3*	1.72*	0.99*	192.0
A-1	4 WF 13	3.86	5.54	6.36	1.72	1.04	144.0
A-2	8 WF 31	9.15	27.5	30.5	3.48	2.01	192.0
A-3	8 WF 31	9.17	27.6	30.6	3.48	2.01	192.0
A-4	8 WF 31	9.10	27.4	30.3	3.48	2.01	192.0
A-5	4 WF 13	3.87	5.56	6.38	1.73	1.04	191.9
A-6	4 WF 13	3.87	5.54	6.36	1.72	1.03	191.9
A-7	4 WF 13	3.79	5.44	6.23	1.71	1.04	191.9
A-8	8 B 13	3.96	10.4	11.9	3.24	0.85	168.0
A-9	8 B 13	4.13	10.9	12.5	3.24	0.87	168.0
A-10	8 B 13	4.01	10.5	12.1	3.24	0.85	168.0

* Handbook values
y About the weak axis

TABLE III. MATERIAL PROPERTIES

Test No.	σ_y ksi	P_y kips	M_p in.-kip	E ksi x 10^3
T-1	39.9	365.7	1227	29.9
T-2	37.5	438.4	1489	28.3
T-3	39.9	363.2	1209	29.9
T-4	39.9	367.3	1226	29.9
T-5	39.9	363.9*	1213*	29.9
T-6	39.5	147.4	242	29.2
T-7	39.5	151.1	250	29.2
T-8	39.9	365.4	1221	29.9
T-9	39.5	147.1	243	29.2
T-10	39.5	148.3	243	29.2
T-11	39.9	368.4	1229	29.9
T-12	39.9	367.6	1225	29.9
T-13	39.9	369.5	1238	29.9
T-14	39.9	364.3	1206	29.9
T-15	39.9	365.4	1219	29.9
T-16	39.9	364.8	1220	29.9
T-17	39.5	155.6	256	29.2
T-18	39.9	374.1	1246	29.9
T-19	39.9	371.8	1244	29.9
T-20	39.5	157.4	259	29.2
T-21	39.5	158.4	257	29.2
T-22	39.5	154.7	255	29.2
T-23	39.5	161.6	264	29.2
T-24	39.5	160.1	263	29.2
T-25	39.9	367.9	553*y	29.9
T-26	39.5	150.9*	249*	29.2
T-27	39.9	347.7	553*y	29.9
T-28	39.5	150.9*	249*	29.2
T-29	39.5	150.9*	249*	29.2
T-30	39.5	150.9*	249*	29.2
T-31	39.5	150.9*	249*	29.2
T-32	39.5	150.9*	249*	29.2
A-1	37.7	146	240	28.5
A-2	36.8	337	1123	30.6
A-3	36.8	338	1127	30.6
A-4	36.8	335	1115	30.6
A-5	35.0	136	223	29.4
A-6	35.0	136	223	29.4
A-7	35.0	133	218	29.4
A-8	41.2	164	490	30.1
A-9	41.2	170	515	30.1
A-10	41.2	165	498	30.1

* Calculated From Handbook Values
y About the Weak Axis

TABLE IV. EXPERIMENTAL RESULTS

Test No.	Section	Loading Condition	L/r_x	P kips	P/P_y
T-1	8 WF 31	d	20.6	47.6	0.130
T-2	8 WF 40	d	20.3	65	0.148
T-3	8 WF 31	b	55.5	180	0.496
T-4	8 WF 31	b	55.2	44.9	0.122
T-5	8 WF 31	b	55.3	287.6	0.790
T-6	4 WF 13	b	111.7	40	0.271
T-7	4 WF 13	b	111.0	40	0.265
T-8	8 WF 31	c	55.0	(215)	(0.588)
T-9	4 WF 13	b	110.9	15	0.102
T-10	4 WF 13	b	110.9	75	0.506
T-11	8 WF 31	e	55.2	(317.5)	(0.862)
T-12	8 WF 31	c	55.2	44.9	0.122
T-13	8 WF 31	d	54.9	44.9	0.122
T-14	8 WF 31	a	55.2	83.7	0.230
T-15	8 WF 31	e	41.3	(310)	(0.848)
T-16	8 WF 31	c	41.3	44.9	0.123
T-17	4 WF 13	b	55.8	18.4	0.118
T-18	8 WF 31	e	27.6	(330)	(0.382)
T-19	8 WF 31	c	27.5	44.9	0.121
T-20	4 WF 13	c	56.1	18.4	0.117
T-21	4 WF 13	b	55.8	74.3	0.468
T-22	4 WF 13	b	55.8	(94)	(0.608)
T-23	4 WF 13	d	83.2	18.4	0.114
T-24	4 WF 13	b	82.8	18.4	0.115
T-25	8 WF 31	e	76.4y	(256)	(0.696)
T-26	4 WF 13	c	83.7	18.4	0.122
T-27	8 WF 31	c	71.6y	180	0.503
T-28	4 WF 13	e	83.7	(118)	(0.782)
T-29	4 WF 13	a	83.7	18.4	0.122
T-30	4 WF 13	a	111.6	18.4	0.122
T-31	4 WF 13	d	111.6	18.4	0.122
T-32	4 WF 13	c	111.6	18.4	0.122
A-1	4 WF 13	d	83.6	47.4	0.325
A-2	8 WF 31	d	55.2	218	0.647
A-3	8 WF 31	d	55.2	110	0.325
A-4	8 WF 31	d	55.2	163	0.487
A-5	4 WF 13	d	110.0	45	0.331
A-6	4 WF 13	d	111.6	68	0.500
A-7	4 WF 13	d	112.3	21	0.158
A-8	8 B 13	d	51.8	49	0.299
A-9	8 B 13	d	51.8	20.4	0.120
A-10	8 B 13	d	51.8	99.2	0.600

y About The Weak Axis

() Maximum Axial Load, Which Is The Load Variable For The Test

TABLE IV. EXPERIMENTAL RESULTS (Cont'd)

Test No.	(Mo) max. in.-kip	$\frac{Mo}{Mp}$ Max.	Type of Failure
T-1	1266	1.032	B
T-2	1685	1.132	B
T-3	714	0.591	LTB+LB
T-4	1164	0.949	B+LB
T-5	194	0.160	LTB+LB
T-6	142	0.587	LTB
T-7	161	0.644	LTB
T-8	190	0.156	LTB
T-9	212	0.872	LTB
T-10	53	0.216	LTB
T-11	--	--	LTB
T-12	934	0.762	LTB
T-13	1258	1.016	B+LB
T-14	1058	0.900	B+LB
T-15	--	--	LTB
T-16	917	0.752	LTB
T-17	219	0.856	B+LB
T-18	--	--	LTB
T-19	966	0.776	LTB
T-20	199	0.768	LTB
T-21	111	0.432	B+LB
T-22	98	0.384	LTB
T-23	246	0.932	LTB
T-24	276	1.049	LTB+LB
T-25	--	--	B
T-26	180	0.723	LTB
T-27	116	0.210	B
T-28	--	--	LTB
T-29	278	1.116	LTB
T-30	242	0.972	LTB
T-31	208	0.835	LTB
T-32	160	0.642	LTB
A-1	174	0.725	B
A-2	412	0.367	B
A-3	917	0.814	B
A-4	669	0.600	B
A-5	104	0.466	B
A-6	31	0.141	B
A-7	193	0.884	B
A-8	382	0.779	LTB
A-9	497	0.964	LB+LTB
A-10	228	0.458	B

B - Bending
LTB - Lateral-Torsional Buckling
LB - Local Buckling

TABLE V. LOCATION OF LATERAL BRACES FOR A-SERIES TESTS

Test No.	Location of First Brace From Column Base in.	Location of Second Brace From Column Base in.	Location of Third Brace From Column Base in.
A-1	33.12	76.25	---
A-2	71.38	---	---
A-3	67.50	---	---
A-4	67.00	---	---
A-5	36.38	72.75	---
A-6	36.38	72.75	---
A-7	36.38	72.75	---
A-8	30.00	64.25	100.50
A-9	30.00	60.00	98.25
A-10	30.00	60.00	98.25

TABLE VI. COMPARISON OF EXPERIMENT WITH THEORY

Test No.	Loading Condition	(Mo/Mp) max. Experimental	(Mo/Mp) max. Theoretical (1)	(Mo/Mp) max. Theoretical (2)	(P/Py) max. Experimental	(P/Py) max. Theoretical (1)	(P/Py) max. Theoretical (2)
T-1	d	1.032	.956				
T-2	d	1.132	.942				
T-3	b	.591	.56				
T-4	b	.949	.955				
T-5	b	.160	.15				
T-6	b	.587	.70				
T-7	b	.644	.70				
T-8	c	(.156)			.588	0.71	0.71
T-9	b	.872	.95				
T-10	b	.216	.16				
T-12	c	.762	.845	.72			
T-13	d	1.016	.960				
T-14	a	.900	.955				
T-16	c	.752	.920	.75			
T-17	b	.856	.956				
T-19	c	.776	.916	.78			
T-20	c	.768	.850	.79			
T-21	b	.432	.590				
T-22	b	(.384)			.608	0.65	
T-23	d	.932	.960				
T-24	b	1.049	.950				
T-26	c	.723	.790	.68			
T-27	c	.210	.458				
T-29	a	1.116	.955				
T-30	a	.972	.955				
T-31	d	.835	.930				
T-32	c	.642	.665	.62			
A-1	d	.725	.661				
A-2	d	.367	.345				
A-3	d	.814	.755				
A-4	d	.600	.550				
A-5	d	.466	.484				
A-6	d	.141	.130				
A-7	d	.884	.872				
A-8	d	.779	.773				
A-9	d	.964	.960				
A-10	d	.458	.403				

( ) Held Constant
(1) Assumed Failure By Excessive Bending in the Plane of the Applied Moments
(2) Assumed Failure by Lateral-Torsional Buckling

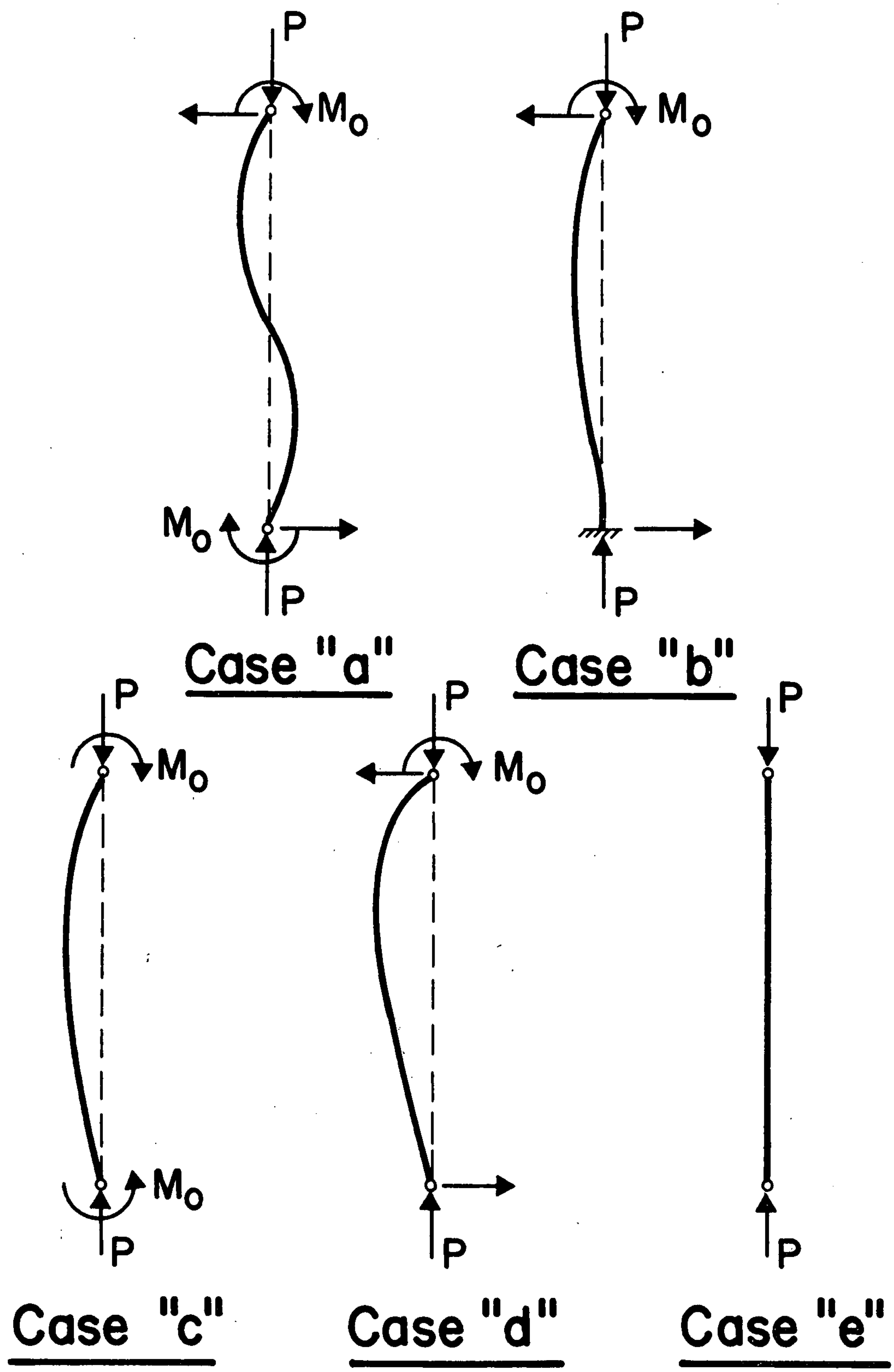


FIG. 1 LOADING CONDITIONS

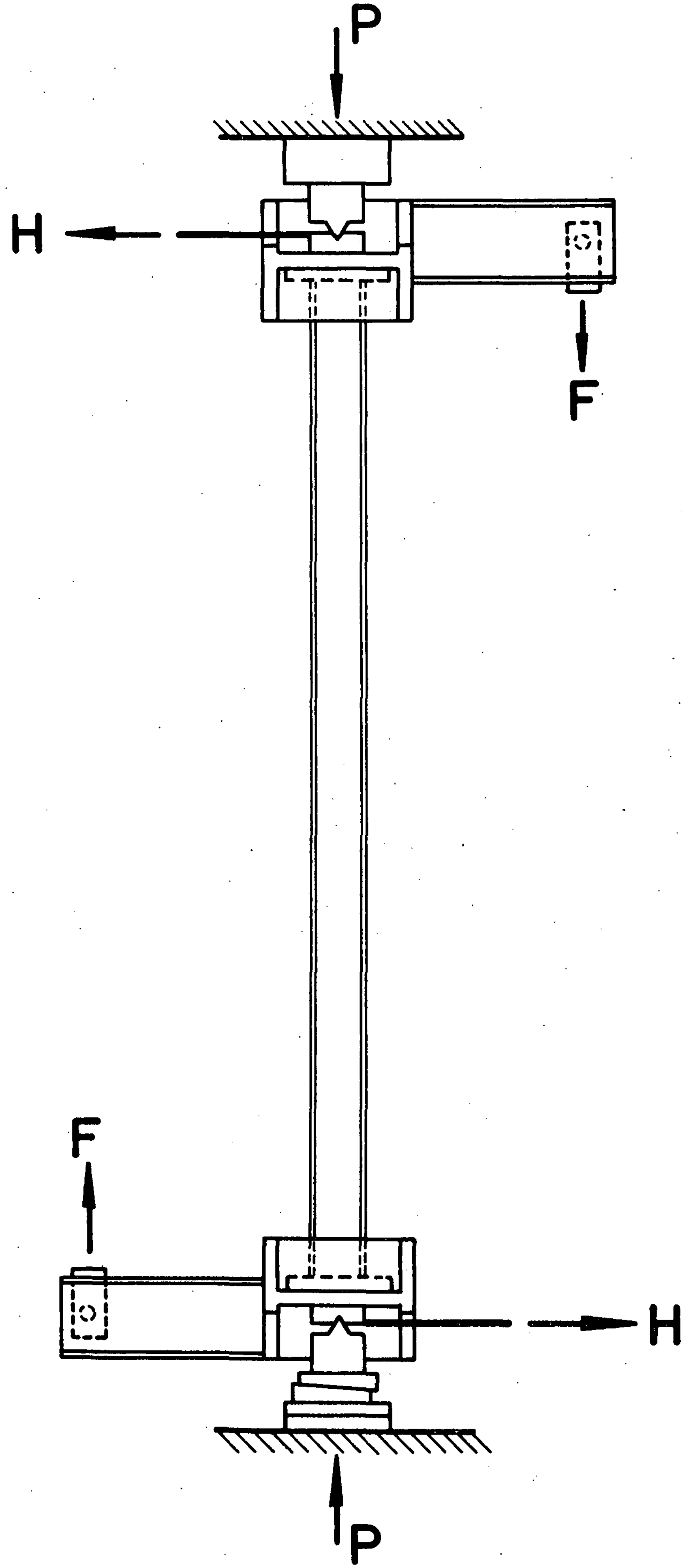


FIG. 2 LOADS APPLIED TO THE TEST COLUMN

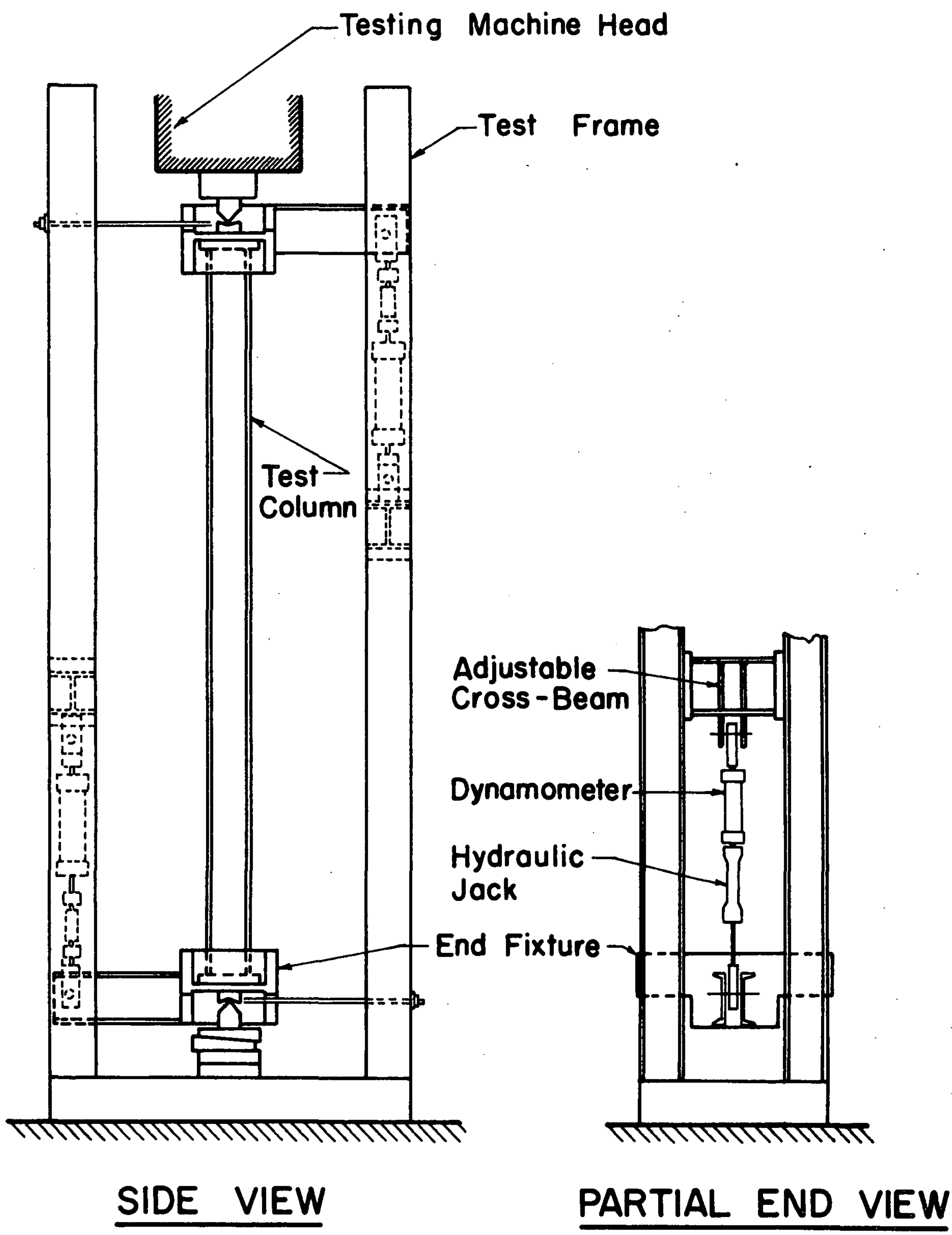


FIG. 3 COLUMN TEST FRAME FOR THE T-SERIES TESTS

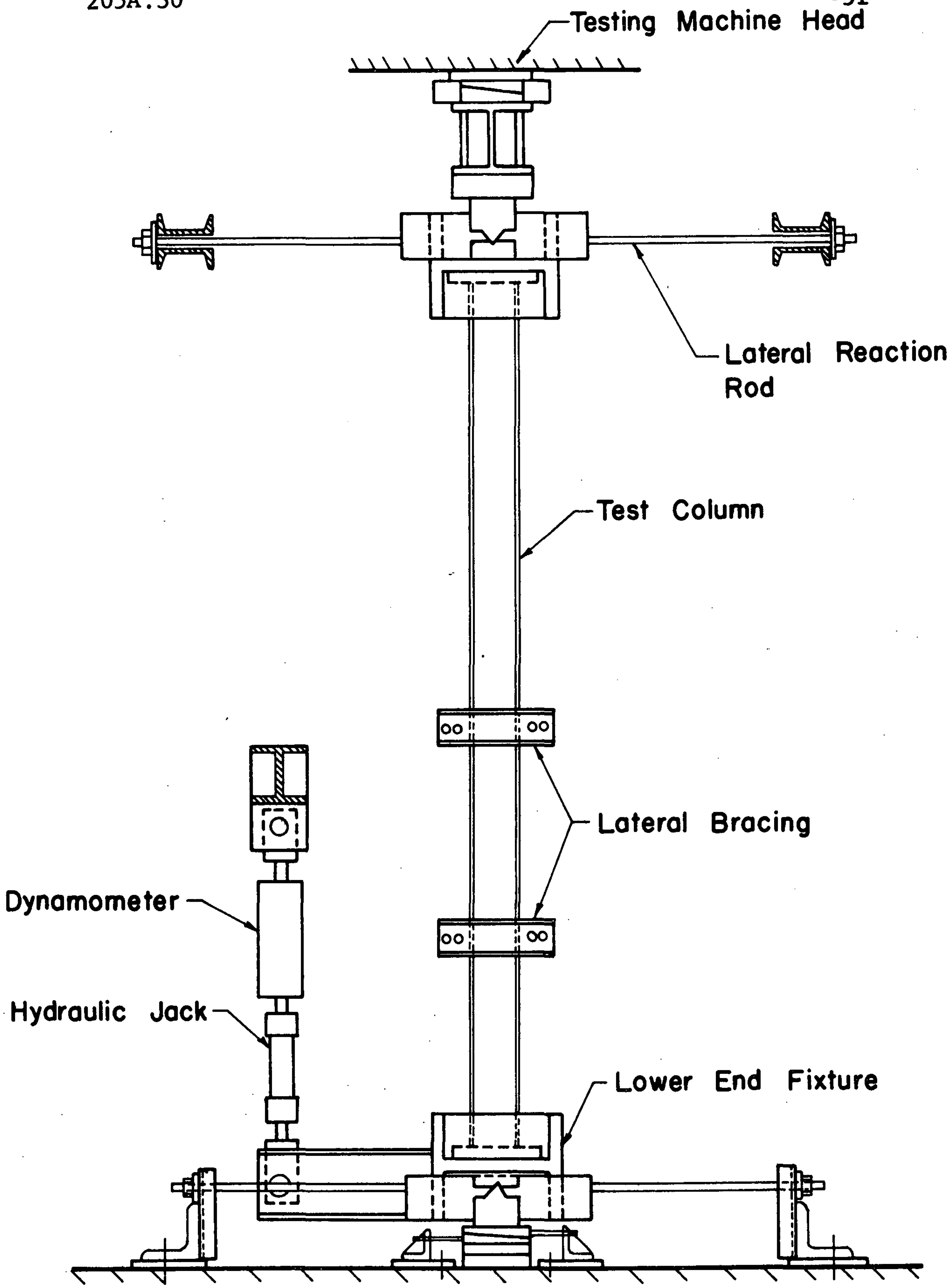


FIG. 4 TEST ASSEMBLY FOR THE A-SERIES TESTS

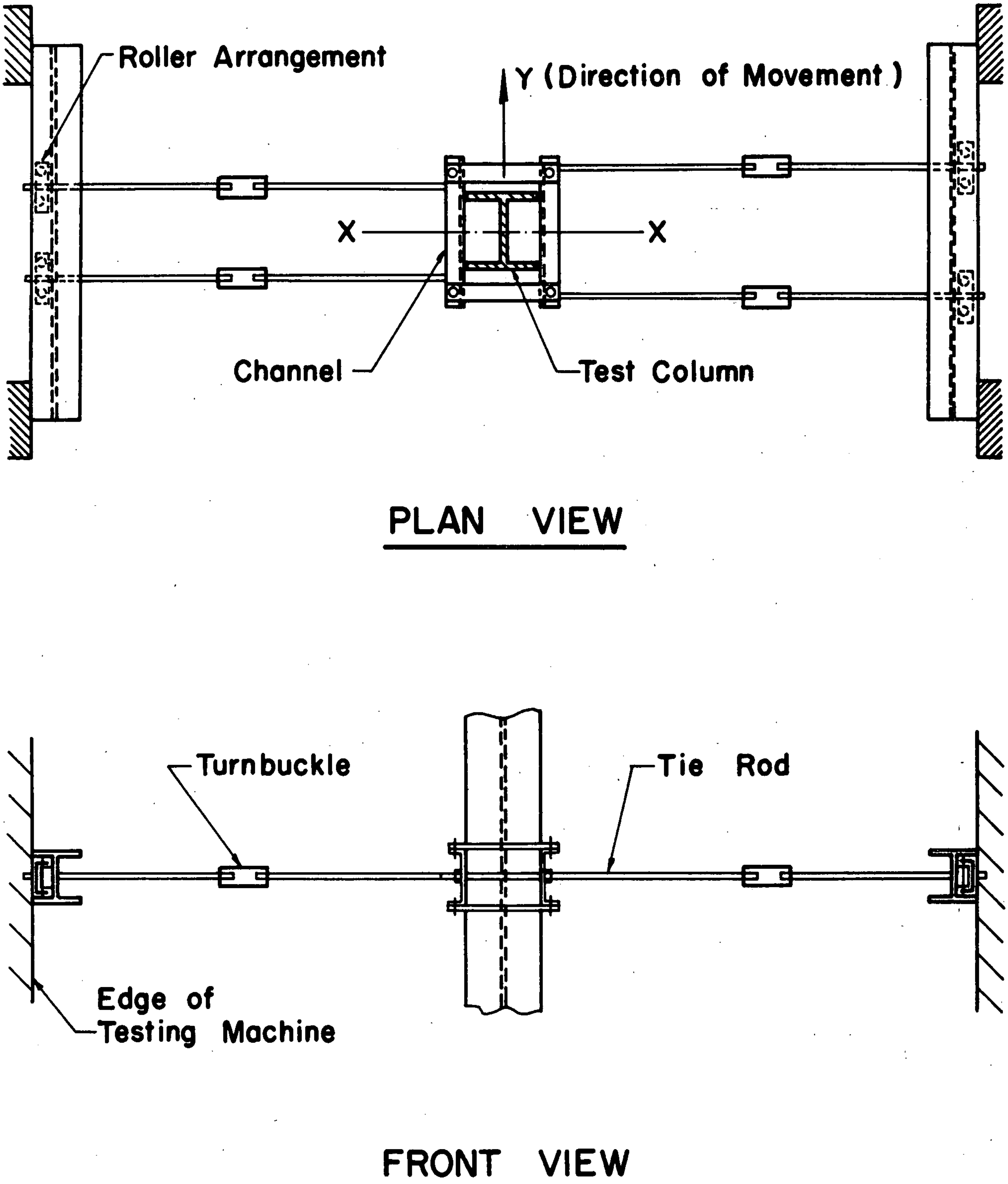


FIG. 5 LATERAL BRACING SYSTEM FOR THE A-SERIES TESTS

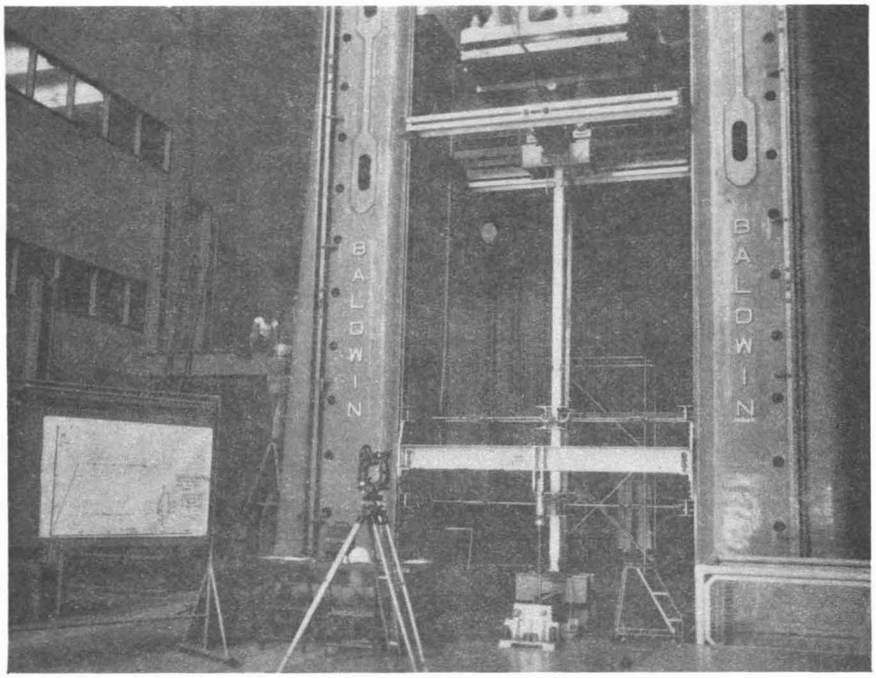


FIG. 6 TEST SETUP FOR THE A-SERIES TESTS

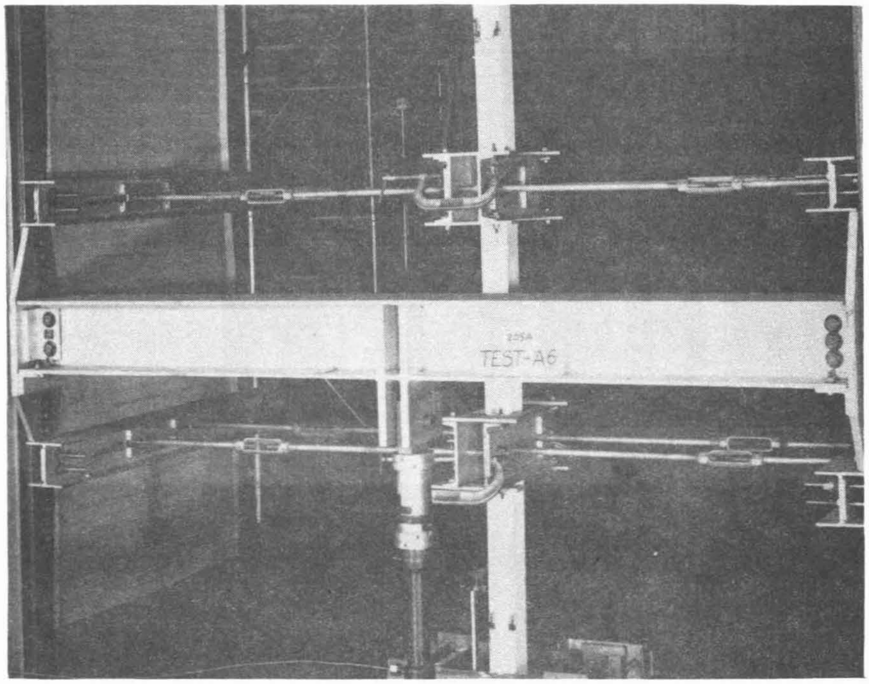


FIG. 7 LATERAL BRACING ARRANGEMENT

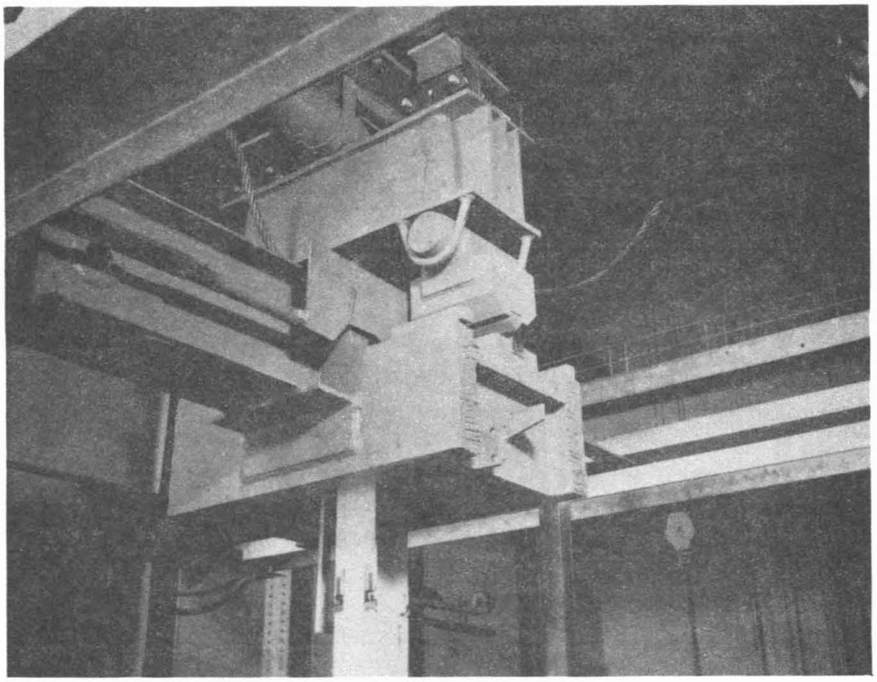


FIG. 8 UPPER END FIXTURE

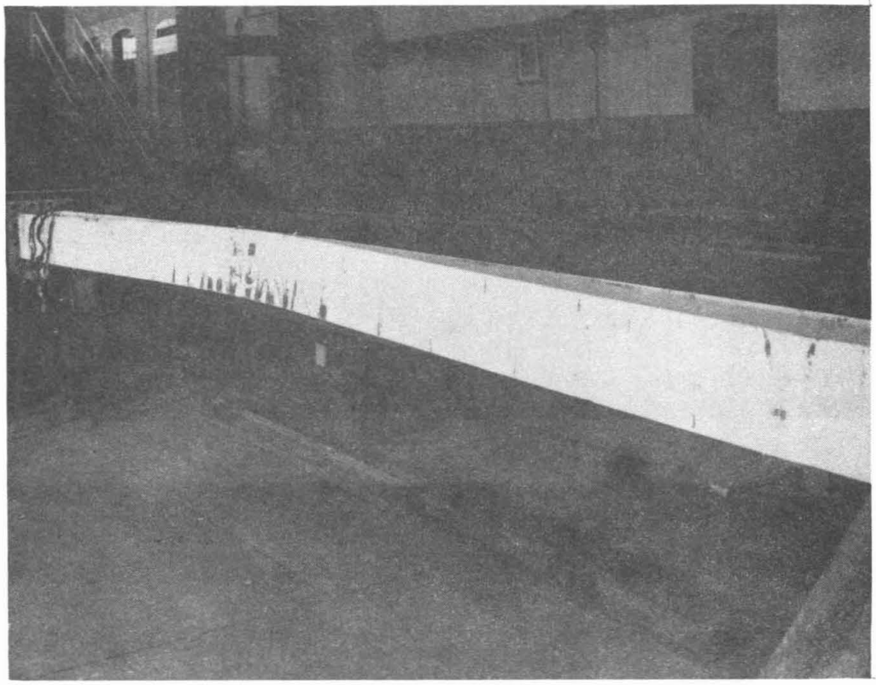


FIG. 9 LATERAL-TORSIONAL BUCKLING FAILURE

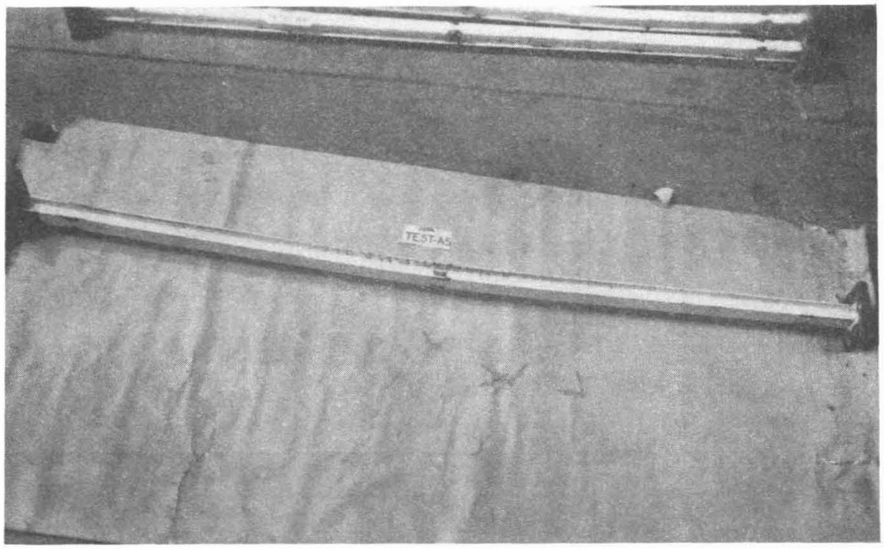


FIG. 10 EXCESSIVE BENDING FAILURE

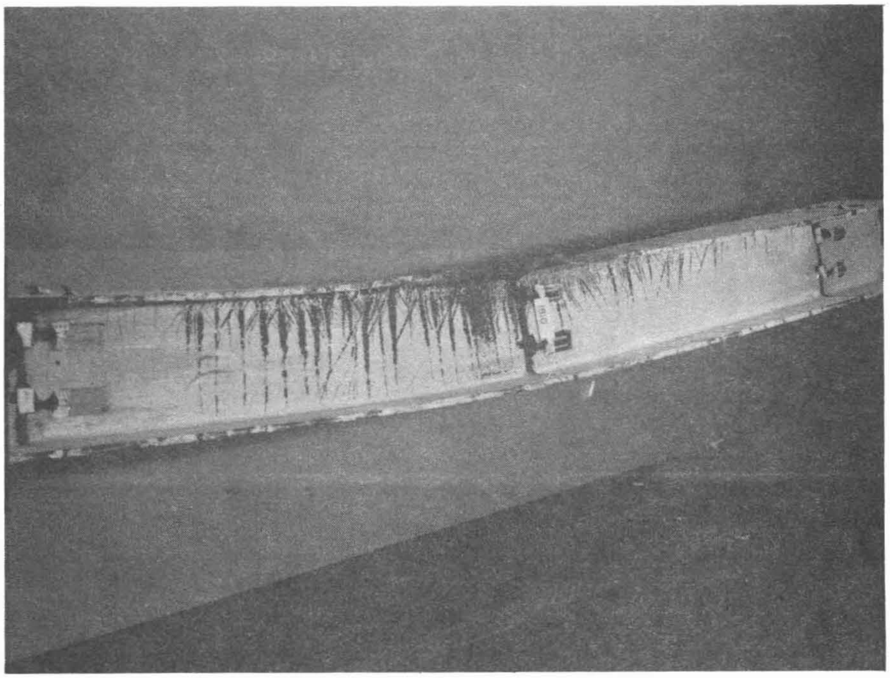


FIG. 11 LATERAL-TORSIONAL BUCKLING PLUS LOCAL BUCKLING FAILURE

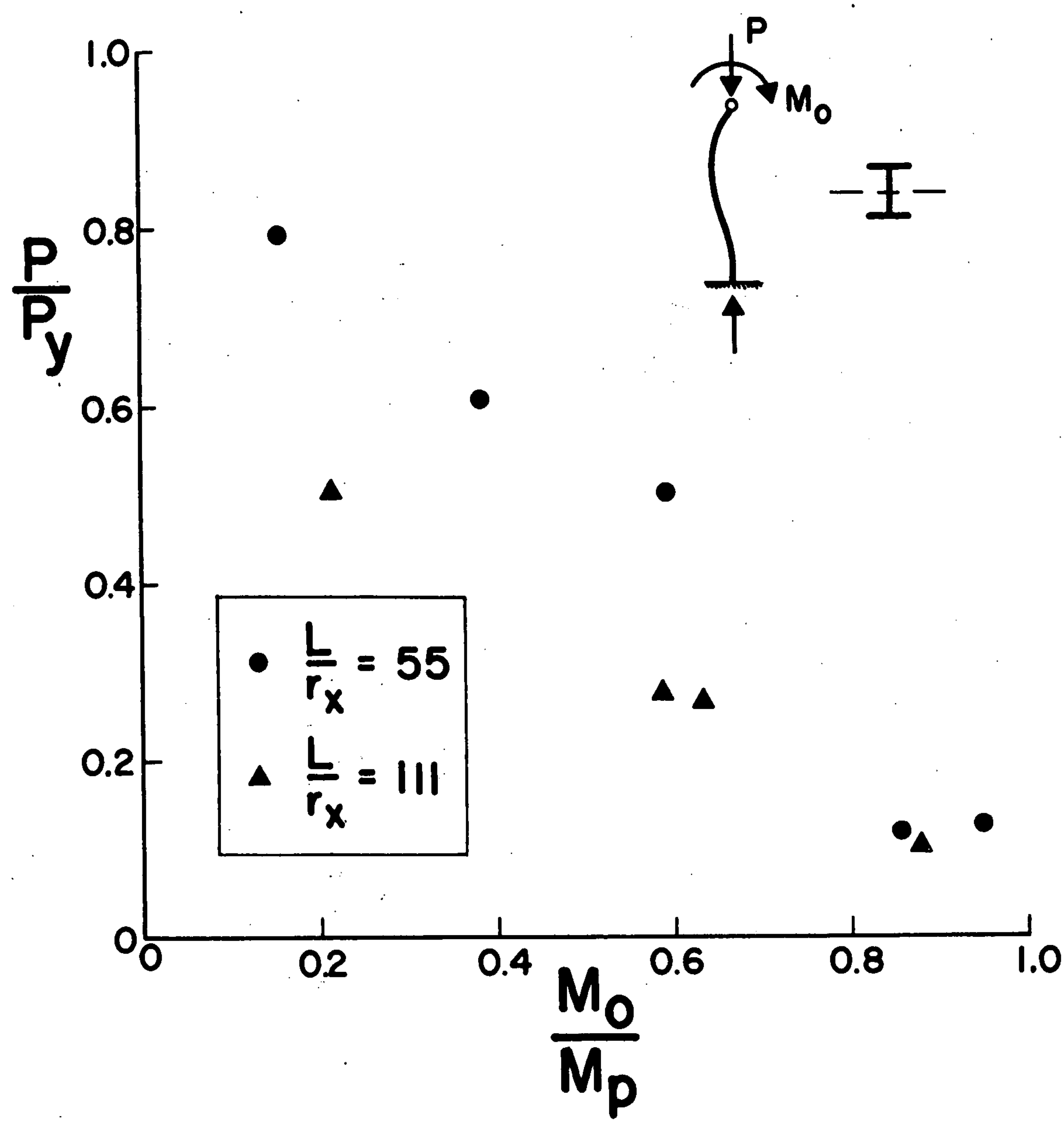


FIG. 12 INFLUENCE OF AXIAL FORCE AND END MOMENT

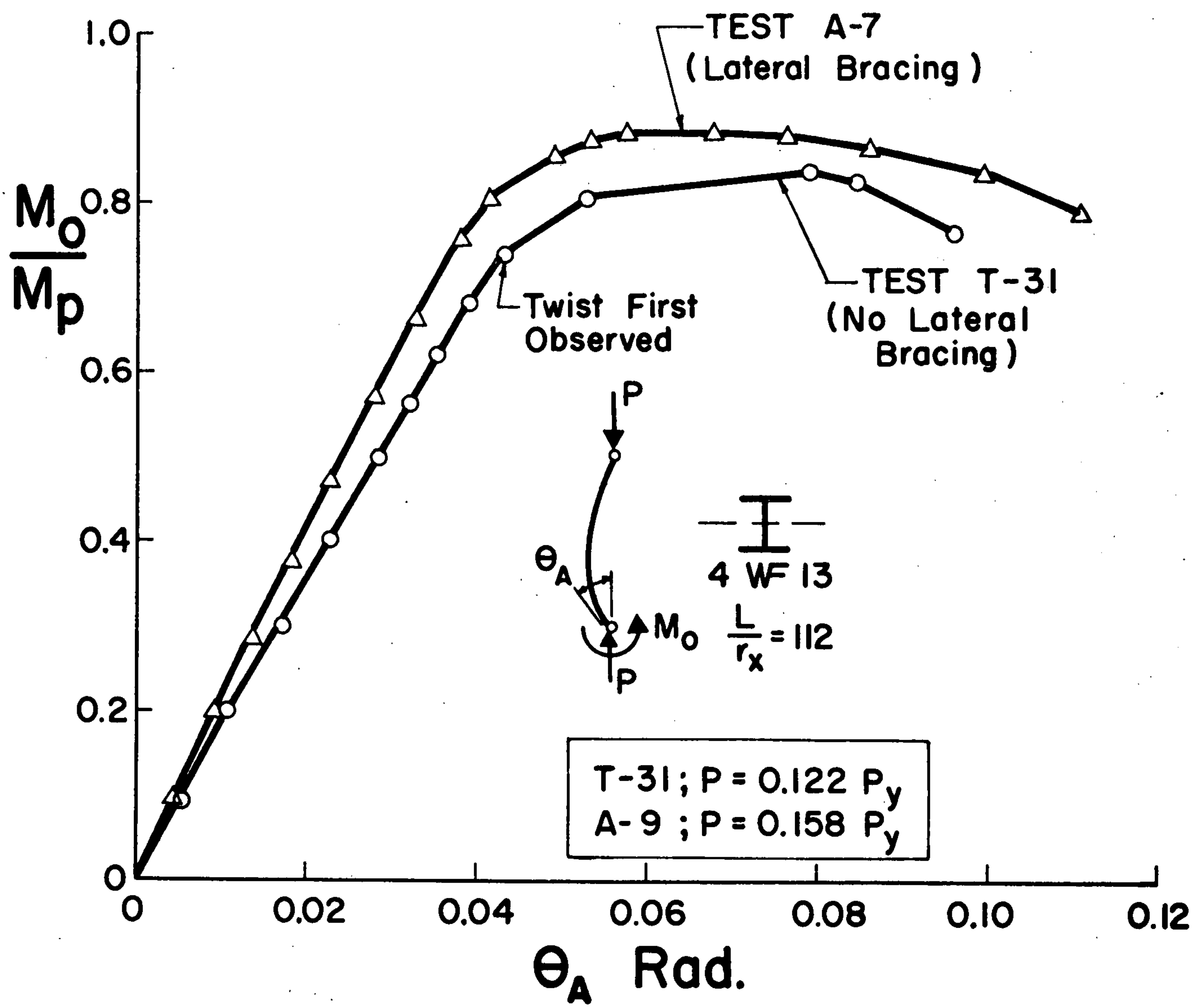


FIG. 13 INFLUENCE OF LATERAL-TORSIONAL BUCKLING

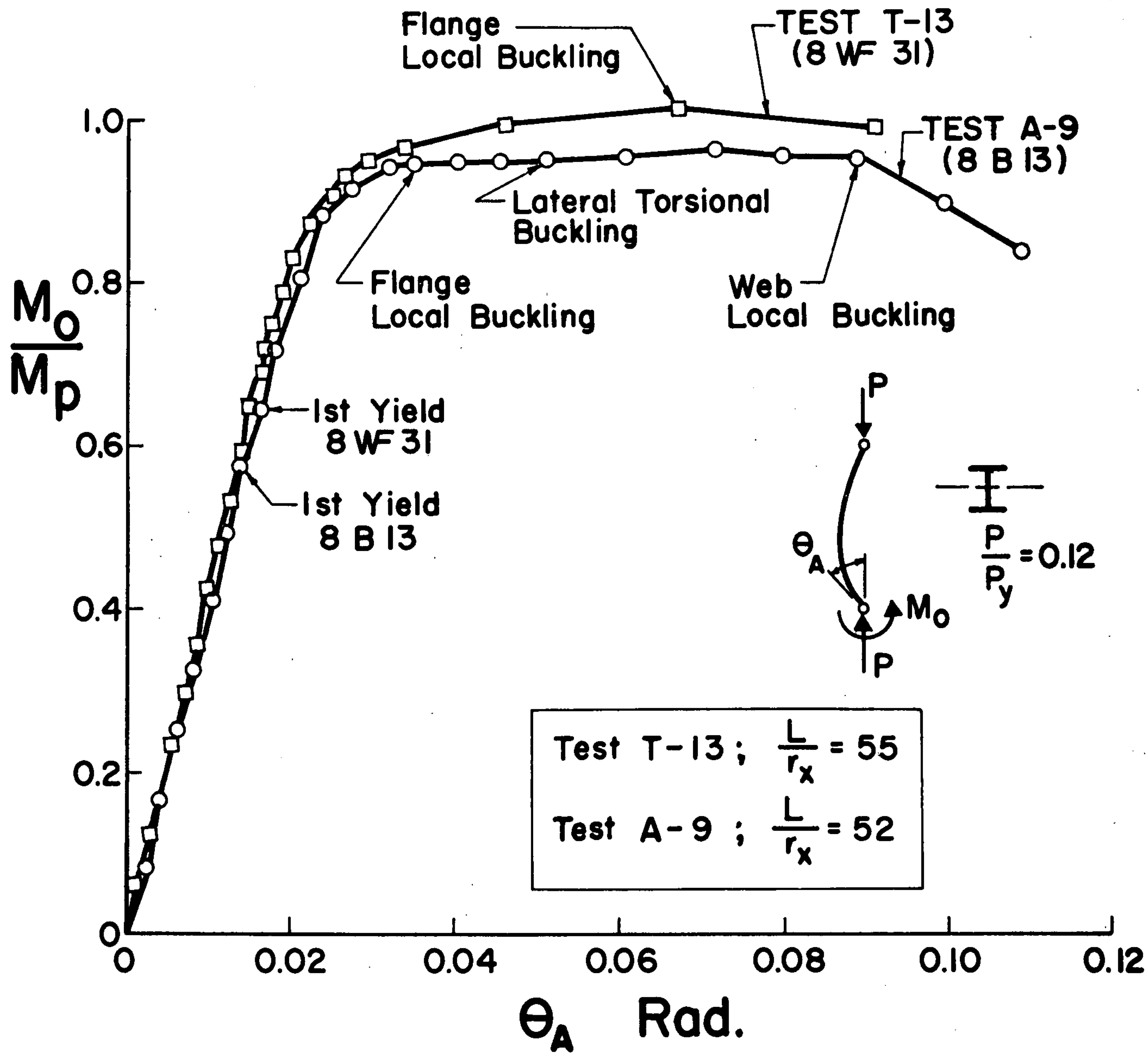


FIG. 14 INFLUENCE OF MEMBER SIZE

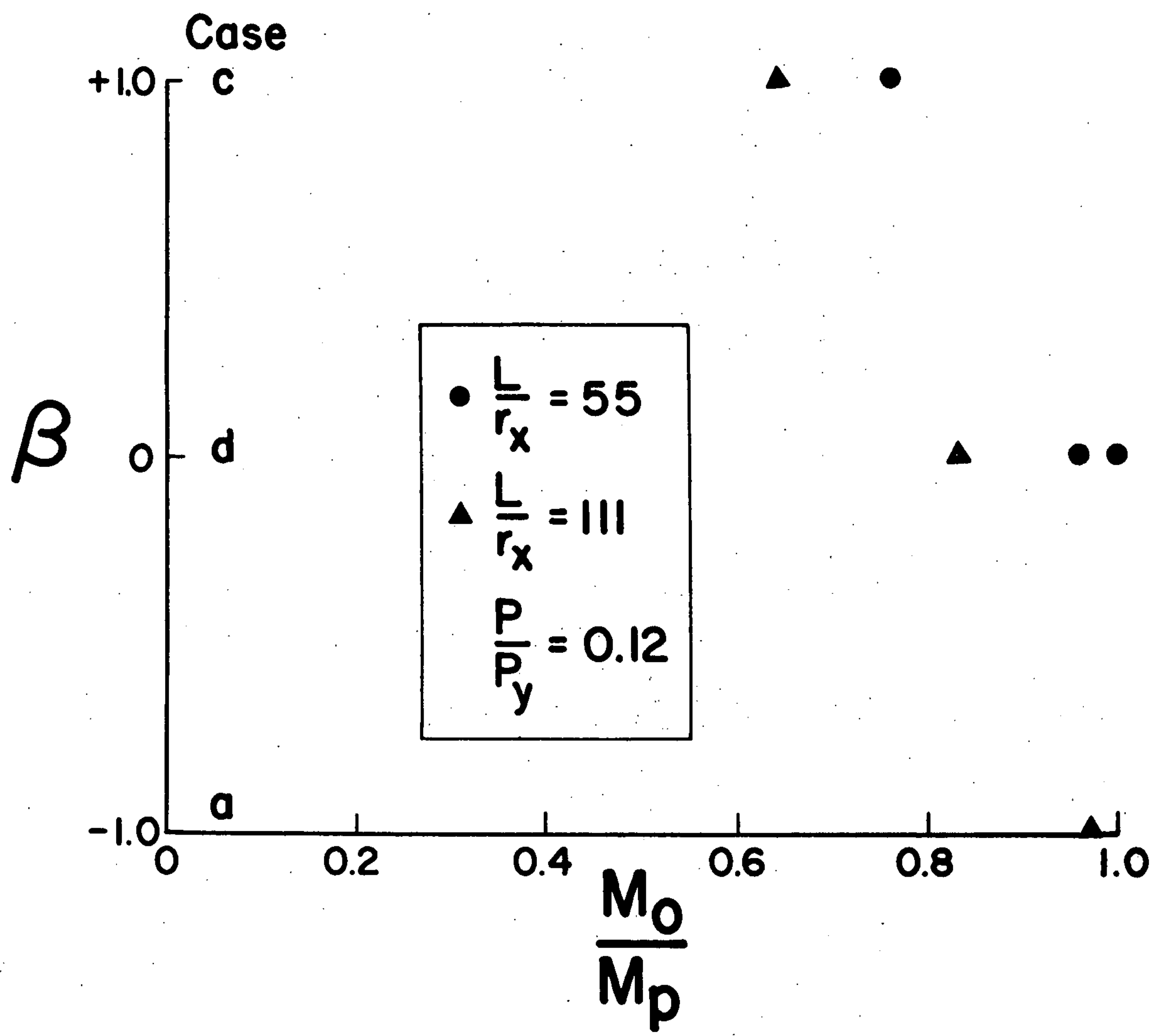


FIG. 15 INFLUENCE OF LOADING CONDITION

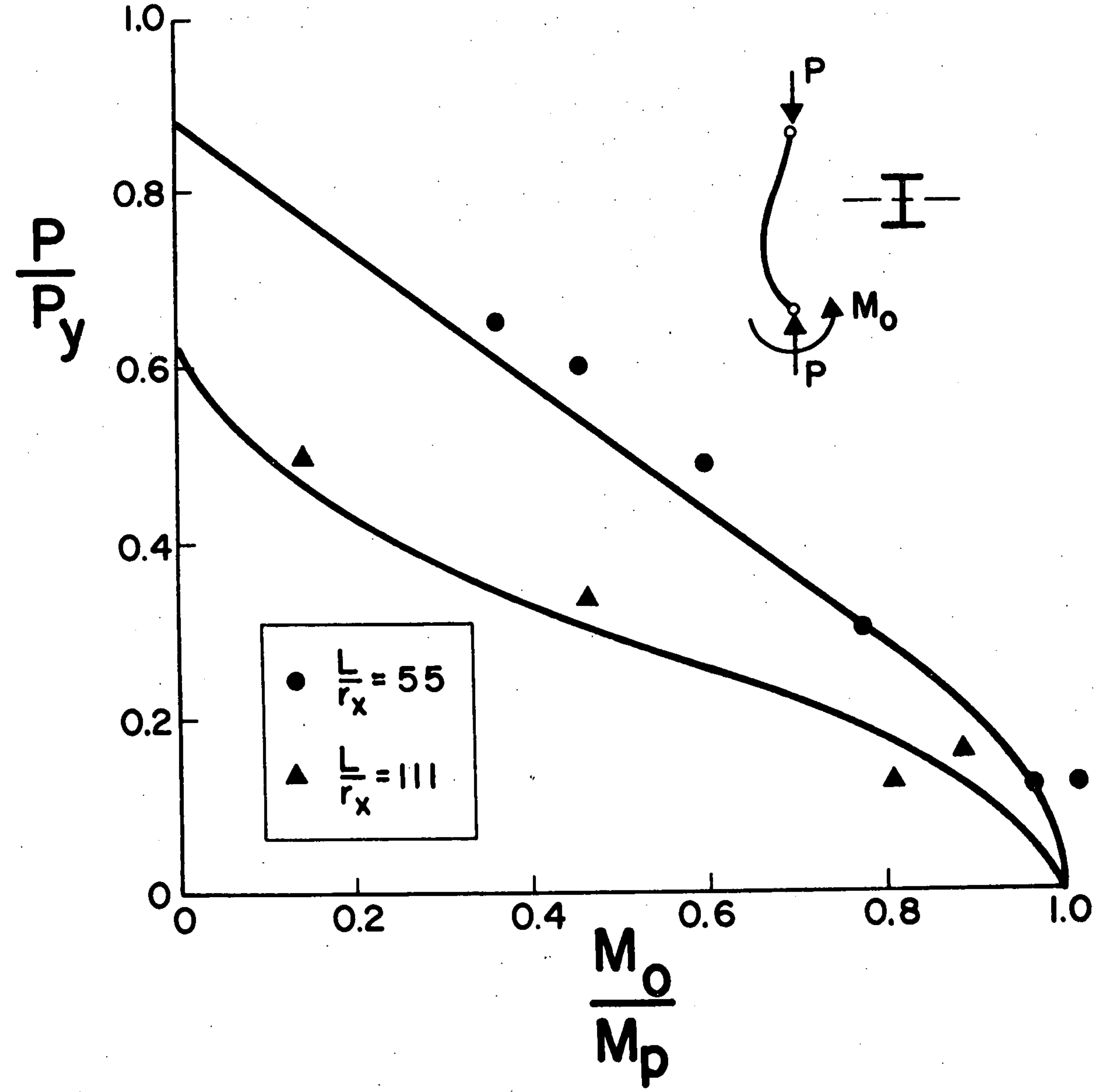


FIG. 16 COMPARISON OF EXPERIMENT WITH THEORY (CONDITION "d"; FAILURE BY EXCESSIVE BENDING)

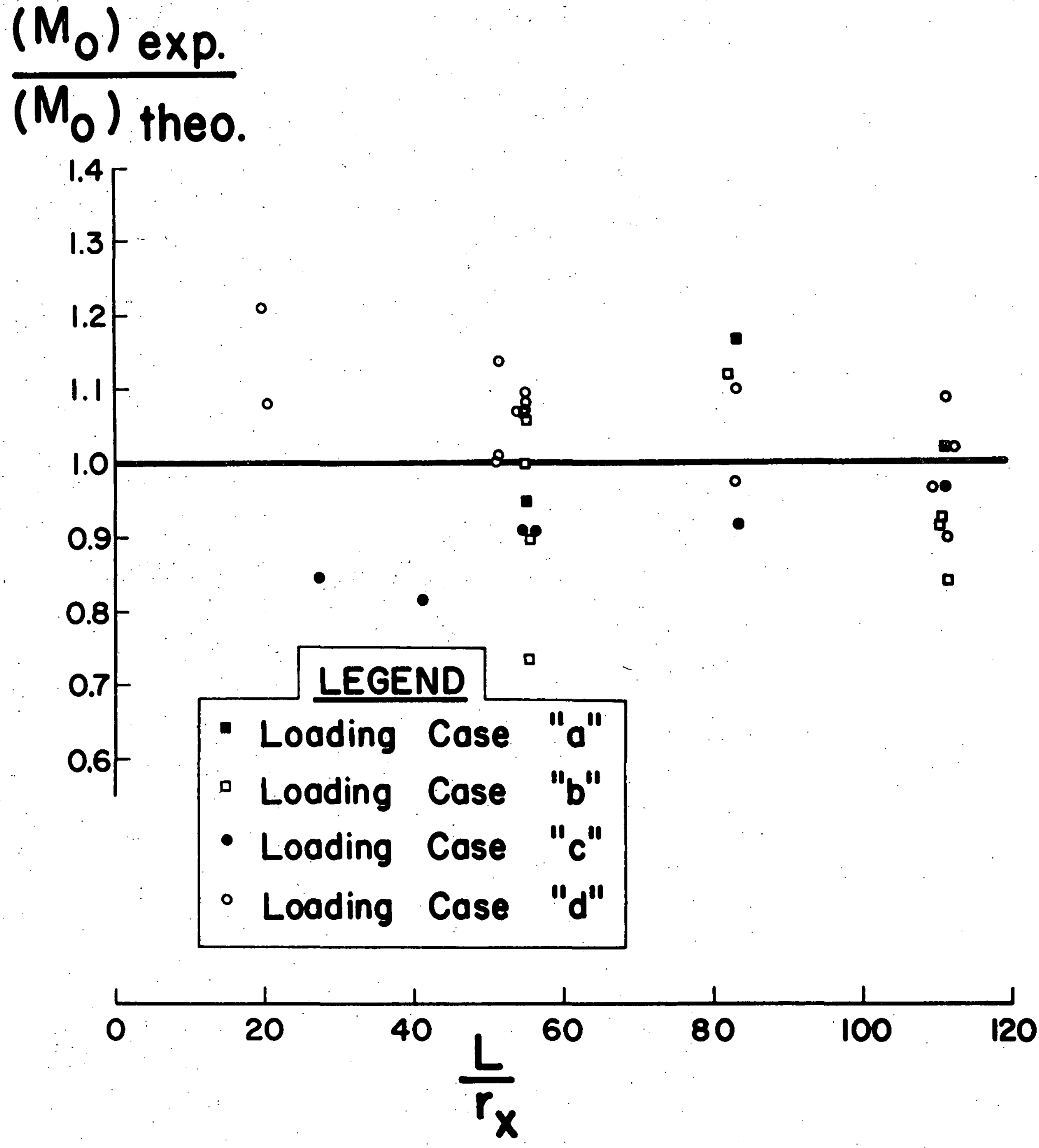


FIG. 17 COMPARISON OF MAXIMUM EXPERIMENTAL LOAD WITH MAXIMUM THEORETICAL LOAD FOR FAILURE BY EXCESSIVE BENDING from plastic curv.

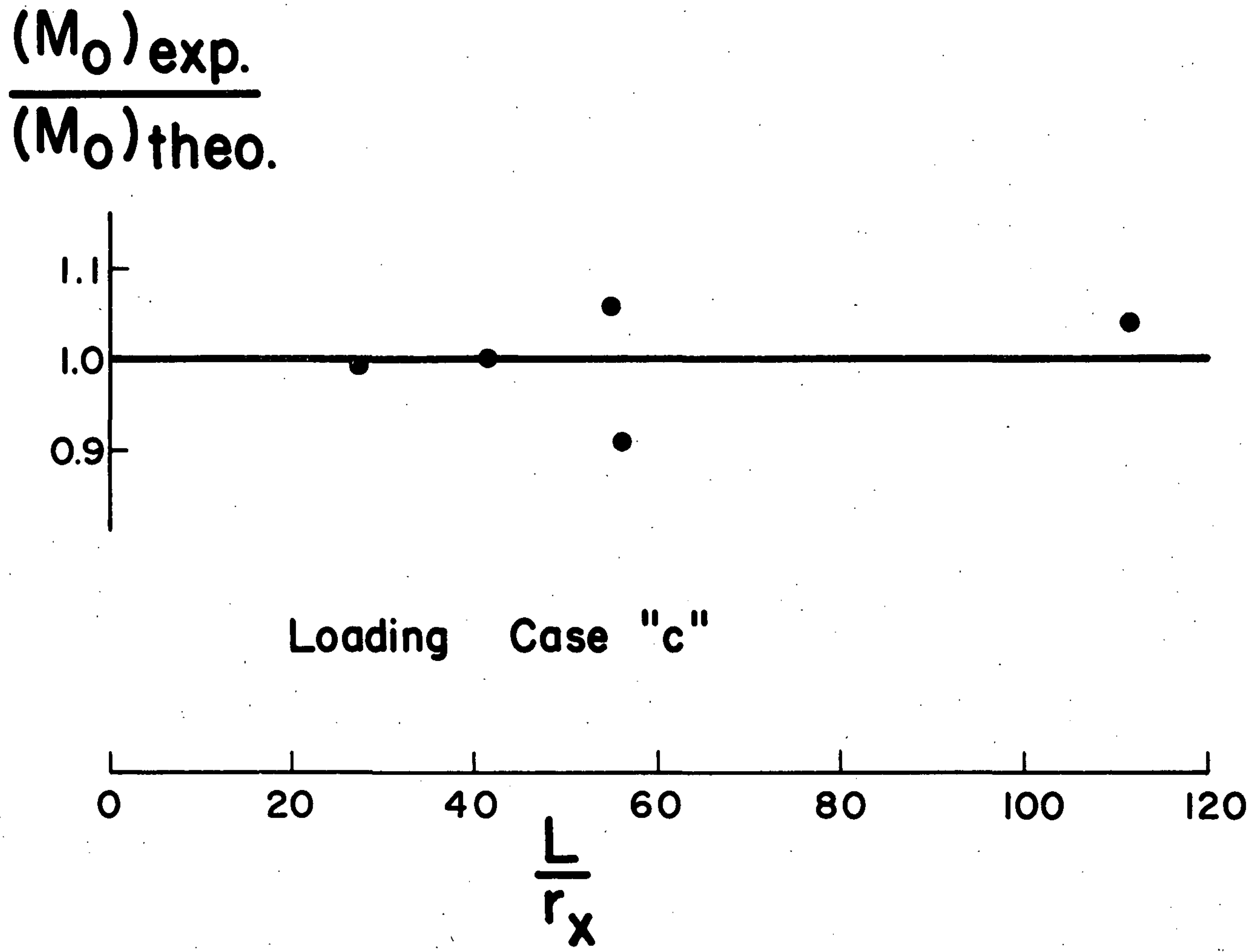


FIG. 18 COMPARISON OF MAXIMUM EXPERIMENTAL LOAD WITH MAXIMUM THEORETICAL LOAD FOR FAILURE BY LATERAL-TORSIONAL BUCKLING

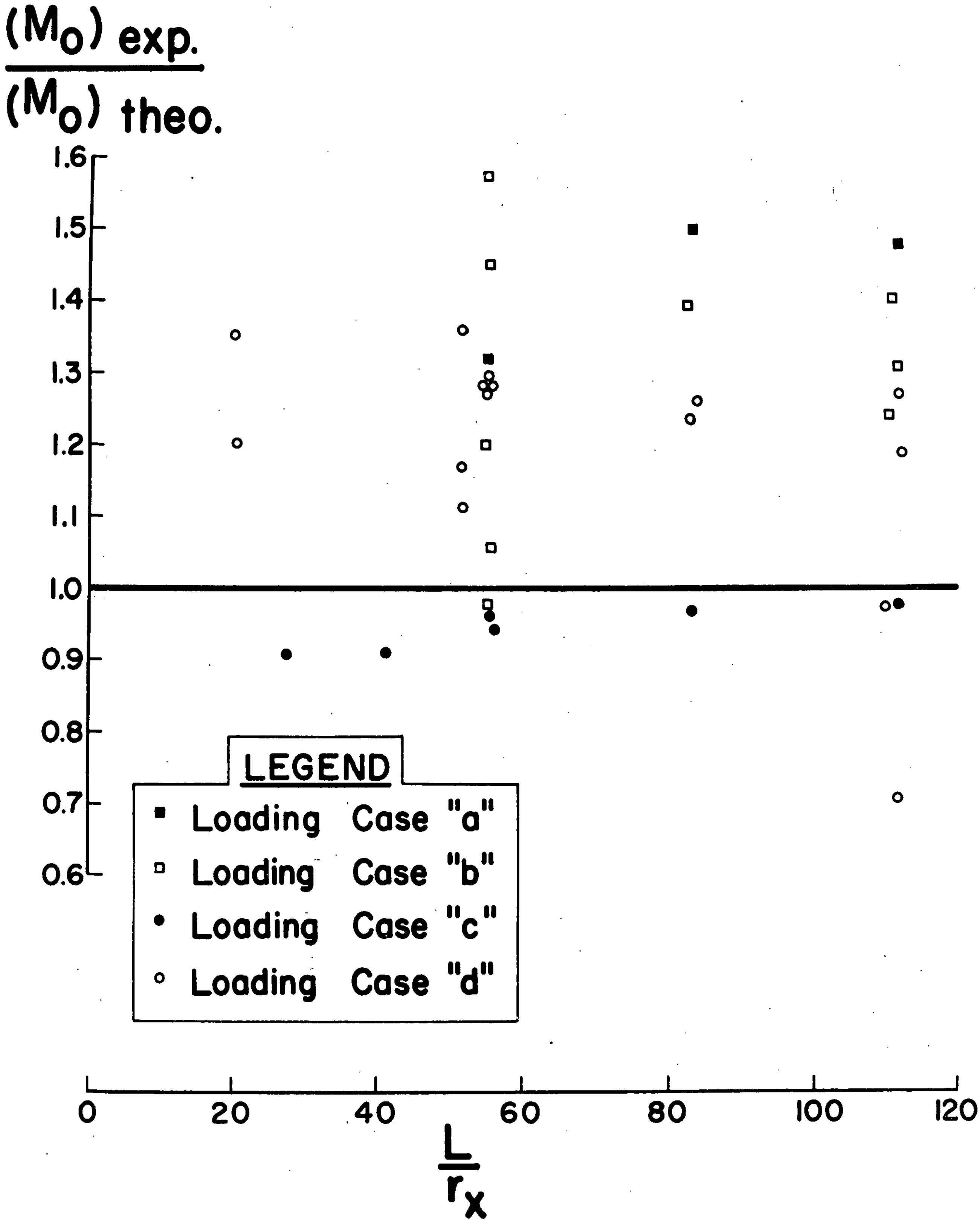
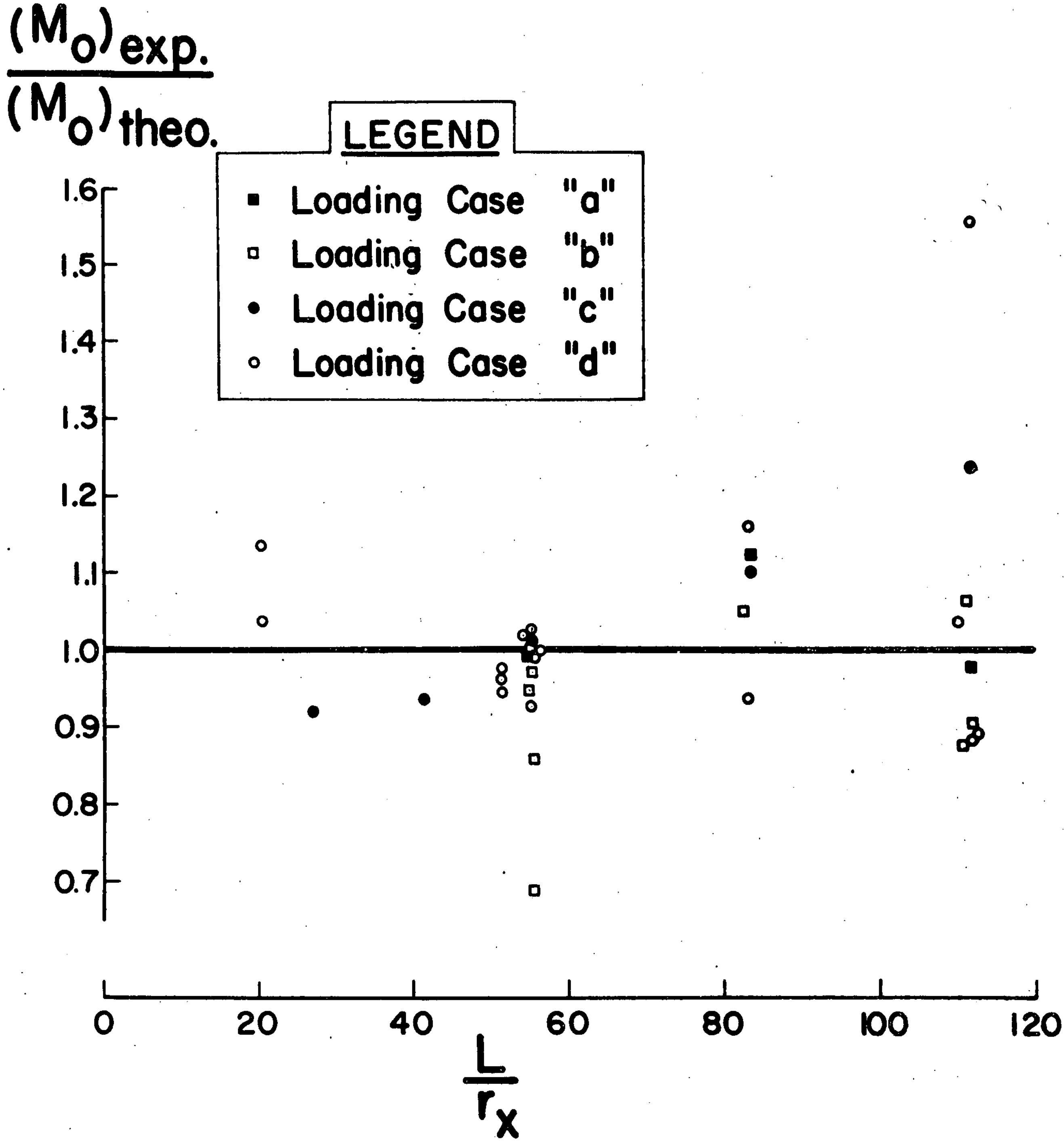


FIG. 19 COMPARISON OF MAXIMUM EXPERIMENTAL LOAD WITH MAXIMUM COMPUTED LOAD BY EQ. (2)



crc

FIG. 20 COMPARISON OF MAXIMUM EXPERIMENTAL LOAD WITH MAXIMUM COMPUTED LOAD BY EQ. (4)

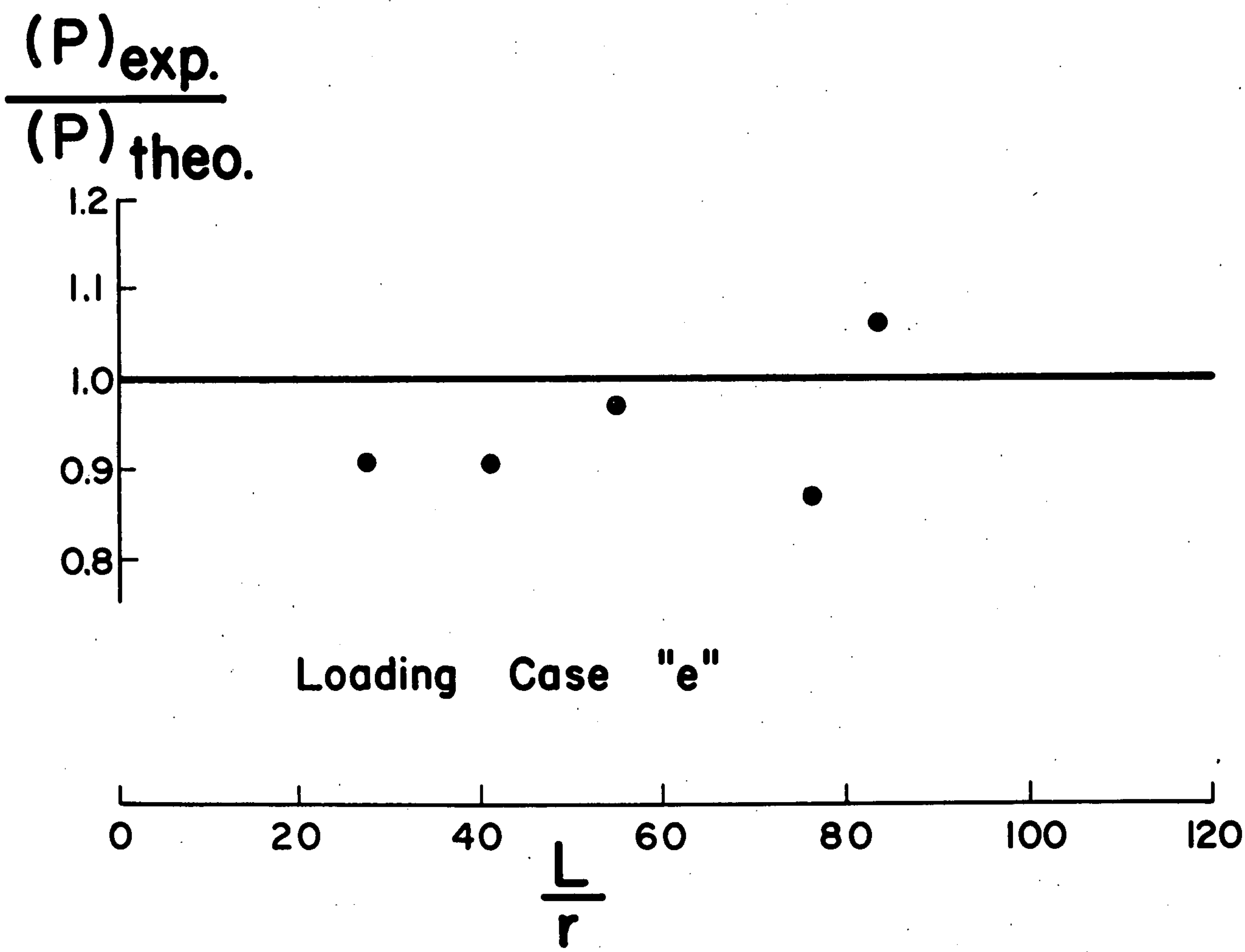


FIG. 21 COMPARISON OF MAXIMUM EXPERIMENTAL LOAD WITH MAXIMUM COMPUTED LOAD BY EQ. (3)

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