1959

Commentary on plastic design in steel: general provision and experimental verification, (Progress Report 2), Proc. ASCE, 85 (EM3), p. 27, (1959), Reprint No. 141

Fritz Lab

Welded Continuous Frames and Their Components

Interim Report No. 33

COMMENTARY ON PLASTIC DESIGN

(Theoretical Considerations--Experimental Verification--Design Guides)

This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Institute of Research, Lehigh University
Column Research Council (Advisory)
Office of Naval Research (Contract no 61003)
Bureau of Ships
Bureau of Yards and Docks

Project Staff: L. S. Beedle, G. C. Driscoll, J. W. Fisher, T. V. Galambos, R. L. Ketter, T. Kusuda, L. W. Lu, J. A. Santos, B. Thurlimann

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

Fritz Laboratory Report No. 205.53
Draft 3, February, 1958

TABLE OF CONTENTS

SYNOPSIS

This report aims to justify and to document the applicability of plastic analysis to design in structural steel. Theoretical considerations involved in the plastic theory and in certain secondary design problems are given. Experimental verification is provided. Approximations in the form of "design guides" are suggested.

A separate and comparison report will illustrate the procedures of the plastic method with specific design examples, and will supply information to supplement clauses in a specification for plastic design.

1. INTRODUCTION

1.1 Objective

The evaluation of a considerable amount of research work has demonstrated the applicability of plastic analysis to structural design. The justification for plastic design is that it results in an over-all balanced design with known factor of safety, it holds promise for a more economical use of material, and compared with elastic methods it is a simpler design office technique for the types of construction later described.

In considering the practical application of plastic design, it is considered that three documents eventually will be required.

(1) A commentary giving the background of theory and tests, together with such design approximations as are appropriate.

(2) A manual of plastic design, containing design examples.

(3) A specification

It is the purpose of this report to meet the need of item (1) above. It will constitute a justification and a documentation of the applicability of plastic analysis to design in structural steel. Theoretical considerations involved in the plastic theory and in certain secondary design problems are given. Experimental verification is given, and approximations in the form of "design guides" are suggested.

Work on the manual of design examples (Item 2) is underway in the offices of the American Institute of Steel Construction. A specification (Item 3) would follow. As a means of implementing the use of plastic design, a paragraph (or paragraphs) could be added to present specifications allowing the designer to proportion certain structures according to the plastic method of structural analysis so long as he demonstrates that he conforms to the recommended procedures. An alternate specification might then be developed to which specific reference would be made thereafter.

1.2 Structural Design

In selecting the members for a steel frame structure it is necessary first to make a general analysis of structural strength and, second, to examine certain details (usually covered by codes or specifications) to assure that local failure does not occur before the structure performs its intended function.

The structural strength or design load of a steel frame may be determined or controlled by a number of factors, any one of which may actually constitute a "Limit of Structural Usefulness":

1. Attainment of a hypothetical yield-point stress (ignoring stress concentrations)
2. Attainment of maximum (plastic) strength
3. Large deflections
4. Instability
5. Brittle fracture
6. Fatigue (endurance limit)

Item 1 in conjunction with Items 4 and 6 has, for many years, been the basis for structural design, which uses the "working stress" concept. Certain provisions also are included in standard specifications which are intended to insure that the capacity is not limited by one of the other "limits of structural usefulness".

1.3 Plastic Design

Strictly speaking, a design based on any one of the six factors listed above could be referred to as a "Limit Design", although the term usually has been applied to determination of maximum strength according to Items 2 and 4. "PLASTIC DESIGN", as an aspect of limit design, embraces primarily Item 2 (attainment of maximum plastic strength) as applied to continuous beams and frames. It is, first, a design on the basis of the maximum load the structure will carry, as determined from an analysis of strength in the plastic range (i.e., plastic analysis). Secondly, it involves the consideration -- by rules or formulas -- of certain "limitations", "restrictions", or "modifications", without which the structure might not attain this theoretical maximum strength. Many of these limitations are present in conventional design, while others are inherently associated with plastic behavior. <u>The unique feature of plastic design is that the ultimate load rather than the yield stress is regarded as the design criterion</u>. Whereas elastic design is performed by assuming <u>working</u> loads and a <u>working</u> unit stress, plastic design is based on <u>ultimate</u> loads and <u>ultimate</u> (or capacity) moments.

Maximum load computations for continuous frames are based on the assumption that "plastic hinge moments" are developed at critical points in the structure and maintained during the subsequent loading. Thus design criteria for the stability of details, which merely guard against elastic buckling, require re-examination in plastic design where plastic buckling must be controlled. Deflection may constitute a second design criterion but is of no greater significance than in other design specifications.

The maximum load determined by plastic analysis may thus be thought of as an "ideal maximum", as there is the possibility that one or more factors may operate to make it impossible of attainment, while other factors will operate to increase this maximum load. However, certain "plastic parameters" are now sufficiently substantiated so that design guides may be suggested in order that engineers can enjoy the advantages inherent in the method.

A Hungarian, Gabor Kazinszy, first applied these concepts to the design of some apartment-type buildings in 1914.(1.1) (1.2) Early tests were made in Germany by Maier-Leibnitz.(1.3) Van den Broeck,(1.4) Baker,(1.5) Roderick,(1.6) Horne,(1.5, 1.7) Heyman,(1.8) Greenberg,(1.9) Prager,(1.9) Symonds,(1.10) Neal,(1.11) Winter,(1.12) and Johnston(1.12) have all made contributions to the plastic theory of structures. Plastic design is already a part of certain specifications and engineers are now making use of it.

R E F E R E N C E S

Chapter 1

1.1 G KAZINSZY, "Kiserletek Befalazott Tartokkal" ("Experiments with Clamped Girders"), Betonszemle, 2 (4) p 68, (1914); 2(5) p 83 (1914); 2 (6) p 101 (1914).

1.2 N J HOFF, Discussion of Ref 1.12, Welding Journal 33 (1), 14-s (1954).

1.3 MAIER-LEIBNITZ, "Contribution to the Problem of Ultimate Carrying Capacity of Simple and Continuous Beams of Structural Steel and Timber", Die Bautechnik, 1(6) (1927).

1.4 J A VAN DEN BROEK, Theory of Limit Design, John Wiley and Sons, 1948.

1.5 J F BAKER, M R HORNE, J HEYMAN, The Steel Skelton, Vol II: Plastic Behavior and Design, Cambridge University Press, Cambridge, England, 1956.

1.6 J W RODERICK, I H PHILLIPPS, "Carrying Capacity of Simply Supported Mild Steel Beams", Engineering Structures, Academic Press, N.Y. 1950, p 9.

1.7 M R HORNE, "A Moment Distribution Method for the Analysis and Design of Structures by the Plastic Theory". Proc. Inst. Civ. Engineers, p 51, April 1954.

1.8 J HEYMAN, "Plastic Design of Portal Frames", Cambridge University Press, Cambridge, England, 1957.

1.9 H J GREENBERG, W PRAGER, ASCE Transactions, Vol. 117, 1952, p 447.

1.10 P S SYMONDS, B G NEAL, "Recent Progress in the Plastic Methods of Structural Analysis", J. Franklin Inst., 252, 383-407, 469-492 (1951)

1.11 B G NEAL, "The Plastic Methods of Structural Analysis", John Wiley and Sons, New York, 1957.

1.12 G WINTER, "Trends in Steel Design and Research", Building Research Congress (1951) Div I, Part II, pp 81-88.

1.13 B G JOHNSTON, C H YANG, L S BEEDLE, "An Evaluation of Plastic Analysis as Applied to Structural Design", Welding Journal, 32(5), 224-s (1953).

2. BASIC PRINCIPLES

2.1 Behavior of Material and Structural Elements

Plastic design is based on an important property of structural steel, namely its ductility. This ductility may be seen by examining a stress-strain curve obtained from a simple tension or compression test. This curve consists (for practical purposes) of two straight lines as shown in Fig 2.1. Up to the yield stress level the material is elastic. After the yield stress has been reached the strain increases greatly without any further increase of the stress. From this it follows that, if a section is subjected to bending moment, the section has a considerable reserve in strength beyond the value of the moment which produces a maximum fiber stress equal to the yield stress.

Fig 2.2 shows the stress distribution at five stages as bending moment is applied to a member of rectangular cross-section. The moment-curvature relationship for this beam is shown in Fig 2.3; the numbered points correspond to the five stages in Fig 2.2. Stage 2 corresponds to the yield moment M_y and stage 5 corresponds to the plastic moment, M_p. The exact shape of the moment-curvature diagram between stages 2 and 5 depends on the cross-sectional shape, but the moment rapidly approaches the value of the full plastic moment corresponding to stress distribution 5 (Fig 2.2). The "rapid approach" to this limiting plastic moment is the essence of the simple plastic theory. In most calculations the moment-curvature relationship is approximated by two straight lines as shown partially dotted in Fig 2.3.

The process of successive yielding of fibers as bending moment is increased (Stage 2 to Stage 5 of Fig 2.2) is called plastification of the cross section. The plastic hinge thus formed permits redistribution of moments in statically indeterminate frames. At the section(s) where yielding occurs, relatively large rotations are possible without a significant increase or decrease of moments; in other words, "plastic hinges" develop. Thus, further increases of the loads are carried by other parts of the structure, until a sufficient number of plastic hinges are formed so that the structure at this point starts to behave as a mechanism. Thereafter deflections would increase rapidly while the loads remained practically constant. In other words, the ultimate load has been reached.

In summary, a structure will reach its ultimate load as determined by simple plastic theory only if the sections or connections where plastic hinges are to form attain the predicted moment and subsequently are able to undergo sufficiently large rotations. An exception, of course, is the plastic hinge which forms last, for which no inelastic rotation is required after the plastic moment has been reached.

2.2 Plastic Theory

(a) Conditions

In the elastic analysis of an indeterminate structure one must consider three conditions:

1. Continuity - the deflected shape is assumed to be a continuous curve, and thus "continuity equations" may be formulated.

2. Equilibrium - Summation of forces (and moments) is equal to zero.

3. Limiting Stress (or moment) - In elastic analysis the limiting moment is the yield moment.

In plastic analysis three similar conditions (or modifications thereof) must be considered. With regard to continuity, the situation is just the reverse: theoretically plastic hinges interrupt continuity, so the requirement is that sufficient plastic hinges form to allow the structure (or part of it) to deform as a mechanism. This could be termed a mechanism condition. The equilibrium condition is the same as in elastic analysis. Instead of initial yield, the limit of usefulness is the attainment of plastic hinge moments, not only at one cross section but at each of the critical sections; this will be termed a plastic moment condition. A corollary to this is the obvious fact that moments in excess of the plastic bending strength could not be resisted. The three conditions that must be satisfied in plastic analysis are, therefore,

1. Mechanism Condition
2. Equilibrium Condition
3. Plastic Moment Condition

(b) Introduction to Methods of Analysis

When all three of the above necessary conditions are satisfied (Equilibrium, Plastic Moment, and Mechanism), then the resulting analysis for ultimate load is correct because two basic theorems are satisfied. These are the "Lower Bound" and the "Upper Bound" Theorems which are as follows:

Lower Bound Theorem: A load corresponding to an equilibrium moment diagram in which $M \leq M_p$ and with arbitrarily assumed values for the redundants is smaller than or at best equal to the true ultimate load.

Upper Bound Theorem: A load computed on the basis of an assumed mechanism will always be greater than or at best equal to the ultimate load.

These theorems, proved in Ref 2.1, have been illustrated and discussed in Ref 2.2.

There are numerous methods by which a continuous steel structure may be analyzed for maximum strength. In the semi-graphical ("statical" or "equilibrium") method, an equilibrium moment diagram is drawn such that the moment is nowhere greater than M_p. It thus automatically satisfies the lower bound theorem. The resulting ultimate load is correct only if sufficient plastic hinges were assumed to create a mechanism (thus satisfying the upper bound theorem). In the mechanism method a mechanism is assumed and the resulting virtual work equations are solved for ultimate load. This value is correct only if the plastic moment condition is also satisfied.

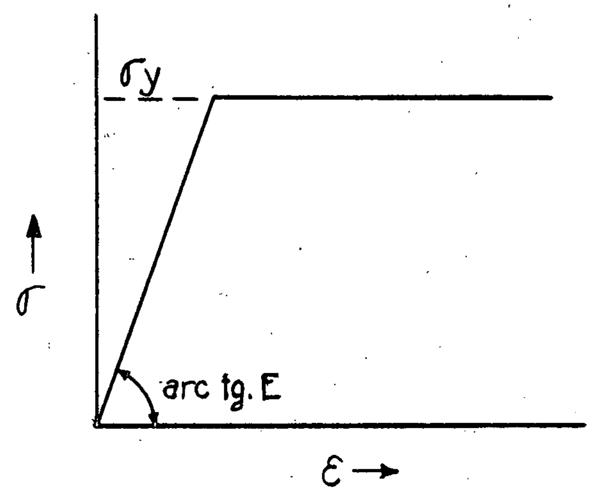
Other methods of analysis are available and still others may be developed in the future. A specification need not direct that any particular one of these methods be used, but it should call for an analysis giving an accurate measure of maximum strength.

REFERENCES

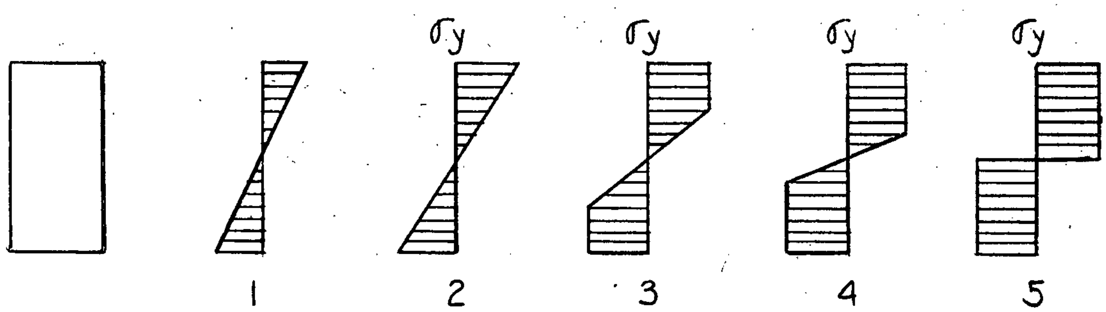
Chapter 2

2.1 H J GREENBERG, W PRAEGER, "Limit Design of Beams and Frames", ASCE Transactions, V 117, p 447, 1952.

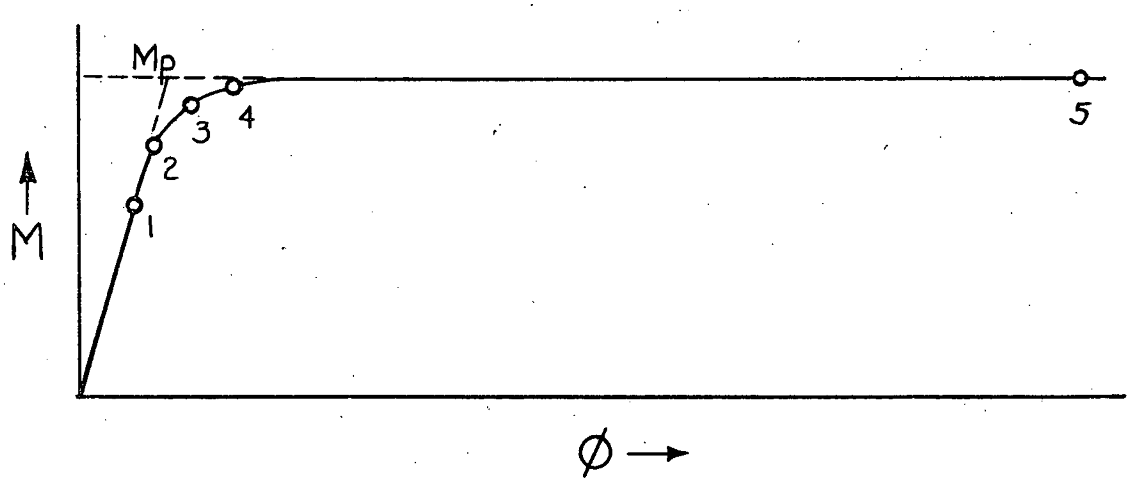
2.2 L S BEEDLE, R L KETTER, B THURLIMANN, "Plastic Design in Structural Steel", Lehigh University-AISC publications, September 1955.



Stress Strain Curve for Uniaxial Tension or Compression 2.1



Successive Stages of Stress Distributions 2.2



Moment-curvature Relationship for Beam in Bending 2.3

3. ANALYSIS

When completed, this chapter will first contain a detailed listing of the important assumptions that are a part of the simple plastic theory. A simple example will then be given of the use of the STATICAL method of analysis and a second example will illustrate the MECHANISM method.

It is not planned that this chapter will give complete design information. That is the purpose of other documents, in particular the AISC Manual, "Plastic Design in Steel". The examples will only serve to illustrate the basic principles discussed in the previous chapter.

4. GENERAL PROVISIONS

4.1 Introduction

This chapter will discuss some of the basic conditions that should be satisfied before a plastic design procedure may be set up. This includes questions regarding types of construction, materials, structural ductility (avoidance of brittle failure), the yield stress level to be used, the plastic moment, the loads and forces that would be considered as applied to the structure, and the load factor.

The particular "provision" will be given first, followed by pertinent discussion.

4.2 Types of Construction

> The following types of construction are suitable for plastic design:
>
> (a) Continuous Beams
> (b) One and two-story, single- and multi-span continuous type building frames
> (c) Multi-story tier buildings with sidesway prevented by walls and/or diagonal bracing.
> (d) Structures required to absorb dynamic load (bomb blast, etc.)

Plastic design is not recommended as a substitute for elastic design for structures that are essentially pin-connected. It is intended for structures which depend upon continuity for their ability to carry the computed ultimate load.

The necessary continuity may be achieved by welding, riveting or bolting. The background and justification for design guides for the use of such connecting devices is discussed elsewhere in this report (See Chapter 7).

4.3 Materials

> Material with the characteristics of ASTM A7 steel for bridges and buildings should be used, with modifications, when needed, to insure weldability and ductility at lowest expected service temperature.

It is not the intent to specify any one steel, but to indicate that the important property that is required of a material is ductility. Many of the high strength steels exhibit stress-strain characteristics similar to those of structural grade steel except with a higher yield stress level. It is reasonable to expect that plastic design may be applied to structures in which such steels are used, providing they meet design guides similar to those suggested in this report, but appropriate to the particular material.

4.4 Structural Ductility

> Fabrication processes should be such as to retain ductility. Sheared edges and punched holes in tension flanges should not be permitted. Sub-punched and reamed holes for connecting devices would be satisfactory if the reaming removes the cold-worked material.
>
> In design, triaxial states of tensile stress set up by geometrical restraints should be avoided.

This provision together with Art 4.3 assures that brittle failure will not prevent the formation of a plastic hinge. The assumption of ductility is an equally important aspect of elastic design and numerous design assumptions rely upon it.

In plastic design the engineer should be guided by the same principles that govern the proper design of an all-welded structure designed by the older methods, since ductility is of equal importance to both. Thus the proper material must be specified to meet the appropriate service conditions, the fabrication and workmanship must meet high standards, and design details should be such that the material is as free to deform as possible.(4.3)

With respect to fabrication, due to the severe cold working involved, punched holes and sheared edges should not be permitted in parts that might be subjected to the yield stress in tension at ultimate load. Punched holes would be permitted if followed by sufficient reaming to remove the cold-worked material. This limitation is not as severe as might seem to be the case; if punched holes are required for erection, they can often be located in regions that would not be subject to large tensile forces (for example, near the web center or, in tier buildings, on the bottom flange at the ends of beams.) In Ref 4.5 the effect of various edge conditions on the brittle failure of steel has been studied.

4.5 Yield Stress Level

For ASTM A-7 steel,
Normal stress, $\sigma_y = 33.0$ ksi
Shear stress, $\tau_y = 19.0$ ksi

A yield stress level of 33.0 ksi corresponds to the minimum yield point permitted in a mill-type acceptance test of A-7 steel. It is not for that reason, however, that it is suggested as a reasonable value to be used in the subsequent computation of the plastic moment. Use of this quantity is justified because it is very close to the average basic yield stress level of this same material under sustained loading.

The mill-type test differs from the test conducted in the laboratory because of a number of factors, one of the most important of which is strain rate. An extensive investigation into the yield stress level has been conducted(4.1) using as the test specimen a complete cross-section of a rolled WF shape. The loading was carried out in a manner that simulates "static" loading. By such a test procedure it was possible to include such effects as differences in web and flange strength, strain rate, and size, since representative cross sections from the very smallest to the largest rolled shapes were included in the program.

The investigation showed that the most probable value of the yield stress level is 34.1 ksi, with variations ranging from 24.6 ksi to 43.0 ksi. (According to the usual acceptance-type test, the most probable value of the yield stress level would be 42.6 ksi.). Fig 4.1 shows the histogram of the ratio of yield stress level according to a stub column test as compared with a mill-type acceptance test. Fig. 4.2 shows an "average" stress-strain curve for A-7 steel.

While 33.0 ksi is the minimum yield stress permitted in acceptance tests, as pointed out above, it represents about the average basic yield stress level of this material. Thus the factor of safety includes the possibility of variation below this average value, because the design is actually based on an average, not a minimum. This situation has always existed in the design of simple beams, and therefore represents no departure from past practice.

If some other material is being considered for an application in plastic design the proper approach would be to select a representative series of "stub columns" and test them in a controlled manner.

4.6 Plastic Moment

$$\boxed{\begin{aligned} M_p &= \sigma_y Z \qquad \ldots (4.1) \\ \sigma_y &= \text{yield stress level} \\ Z &= \text{plastic modulus*} \end{aligned}}$$

As pointed out in Article 2.1, the formation of plastic hinges is of basic importance to plastic design. Fig 2.3 shows the characteristic moment-rotation curve of a beam under bending, and the moment at "stage 5" shown in Figs 2.2 and 2.3 is called the plastic moment. It is computed according to Eq (4.1).

Z, the plastic modulus, is defined as the combined statical moments about the neutral axis of the cross-sectional areas above and below that axis. Appendix 1 contains the plastic modulus for rolled WF shapes and I shapes, arranged as an "economy" table.

As will be evident in Chapter 5, it is frequently observed in

---

*See Appendix 1 for WF and I shapes

tests that the moment-deformation behavior is not exactly like that shown in Fig 2.3 (See Fig 5.4, for example). Because of strain-hardening, the resisting moment is greater than the value computed according to Eq (4.1) (a reserve that in some cases will not be attained). Further, any theory that would attempt to take this additional reserve in strength into account would lead to undue complications.

For a material whose characteristics are not similar to A7 steel, but which exhibits continuous strain-hardening, it might be desirable to arrive at a semi-empirical value for the "plastic hinge" moment. Research would have to be carried out on the particular material (including bending tests and tests of indeterminate structures) to arrive at a suitable approximation.

4.7 Loads and Forces

> The loads and forces to be provided for (allowable loads and forces) should be those that are customary for the particular type of construction. Members are selected on the basis of their plastic strength to resist the most critical loading condition.
>
> $$P_u = F P_w \qquad \ldots (4.2)$$
>
> F = Load factor
> P_u = Ultimate load
> P_w = Allowable (working) load
>
> Loading is assumed to be static and proportional.

The use of plastic design does not involve any changes in the nature of the loading on the structure. The structure is designed to support the same service loads as at present. The difference is that members are selected so that the structure will just support the computed ultimate load, P_u, whereas in elastic design the members are so selected that a certain critical unit stress will not be exceeded at service load, P_w.

It is assumed that the loading is static and proportional even for the ordinary fluctuations of load found in buildings. For unusual conditions, deflection stability would be investigated. (See Art 6.5).

The loading conditions that would be investigated (for building construction) are:

1) Total live load plus dead load
2) Live load plus dead load plus wind or earthquake acting from the left*
3) Live load plus dead load plus wind or earthquake acting from the left and then right* in the case of unsymmetrical structures.

4.8 Load Factor

Dead load plus live load, F = 1.85
Dead load plus live load plus wind or earthquake, F = 1.40

As indicated by Eq (4.2) in Art 4.7, in order to determine the ultimate loads to be used in plastic design the expected or working loads are multiplied by a factor called the "load factor". Sections

---
*In the case of multi-story buildings, wind is taken by walls or separate bracing systems.

are then selected which will just support this factored load.

The philosophy by which the load factor is selected is as follows: If the present elastic design of a simply supported beam is satisfactory, then an indeterminate structure should be designed with the same safety factor against ultimate load. There would certainly be no point in requiring any greater margin of safety simply because the structure is redundant.

The load factor of a simple beam is equal to the ratio of the ultimate load, P_u, divided by the working load, P_w; thus $F = P_u/P_w$. In a simple beam the bending moment varies linearly with the load, or

$$F = \frac{P_u}{P_w} = \frac{M_p}{M_w}$$

Substituting the known values for M_p and M_w,

$$F = \frac{M_p}{M_w} = \frac{\sigma_y Z}{\sigma_w S} = \frac{33}{20} f = 1.65 f \qquad \ldots (4.3)$$

The magnitude of the load factor is thus dependent upon the shape factor, a quantity that will vary with different cross-sectional shapes. The magnitude of the shape factor is given by

$$f = Z/S \qquad \ldots (4.4)$$

The variation of the shape factor for WF shapes used as beams, for WF shapes used as columns, and for I shapes is shown in Fig. 4.3. For WF shapes normally used as beams (and listed in "Section Economy" table in AISC MANUAL, STEEL CONSTRUCTION)[4.4] the shape factor varies from 1.10 to 1.18 with an average value of 1.134 and a mode of 1.12. For WF shapes normally used as columns (members that appear in the "column tables" of the AISC MANUAL "STEEL CONSTRUCTION"),[4.4] the shape factor varies from 1.10 to 1.23 with an average value of 1.137 and a mode of 1.115. The shape factor distribution of American Standard I beams is shown in the lower portion of Fig 4.3. The minimum is 1.14 and the maximum is 1.23, the average being 1.18.

The following table shows the possible values of the load factor, depending on the choice of the shape factor.

SHAPE FACTOR	FACTOR OF SAFETY*	LOAD FACTOR
1.10 -- Minimum value	1.65	1.81
1.12 -- Mode for WF beams	1.65	$\boxed{1.85}$
1.14 -- Average for WF beams and Columns	1.65	1.88
1.18 -- Average for American Standard I beams	1.65	1.95
1.23 -- Maximum value	1.65	2.03

The two most likely values for the load factor are 1.85 and 1.88. The former is selected because it represents the shape factor that will recur most frequently in beams. The number 1.88 implies an accuracy in our knowledge of the general problem of safety that is not justified.

In the case of gravity loading in combination with wind or earthquake forces, elastic design specifications normally permit a one-third increase in stresses. Consistent with this allowance, the

----
* Yield stress divided by working stress

value of F for combined dead, live, and wind loading would be $3/4 \times 1.85 \cong 1.40$.

Although safety is achieved in plastic design by multiplying the working loads by a term called a load factor, it is important to recognize that many other items can vary in addition to the load. Possible sources of "error" in design are:

(1) overrun in computed dead loads
(2) future increases in live loads
(3) loss of section due to corrosion
(4) approximations in analysis
(5) underrun in dimensions
(6) underrun in physical properties
(7) inadequate design theory
(8) errors in distribution of load
(9) errors in fabrication and erection
(10) presence of residual stresses and stress-concentrations

Depending upon the type of structure and the use intended, the variation due to one "error" may be larger than another. While one might arrive at a new "factor of safety" by determining the magnitude of possible errors and making a statistical estimate, such a change should also be reflected in the allowable stresses permitted in elastic design. Therefore, all of the possible variations have been combined into a "load factor" which assures that a plastically designed structure will have the same degree of safety as does a simply-supported beam designed by elastic methods.

Although it is recognized that factors other than variation in load enter into the problem of structural safety, the term "load factor" serves to emphasize that the final test of the suitability of a structure rests upon its ability to support the load.

REFERENCES

Chapter 4

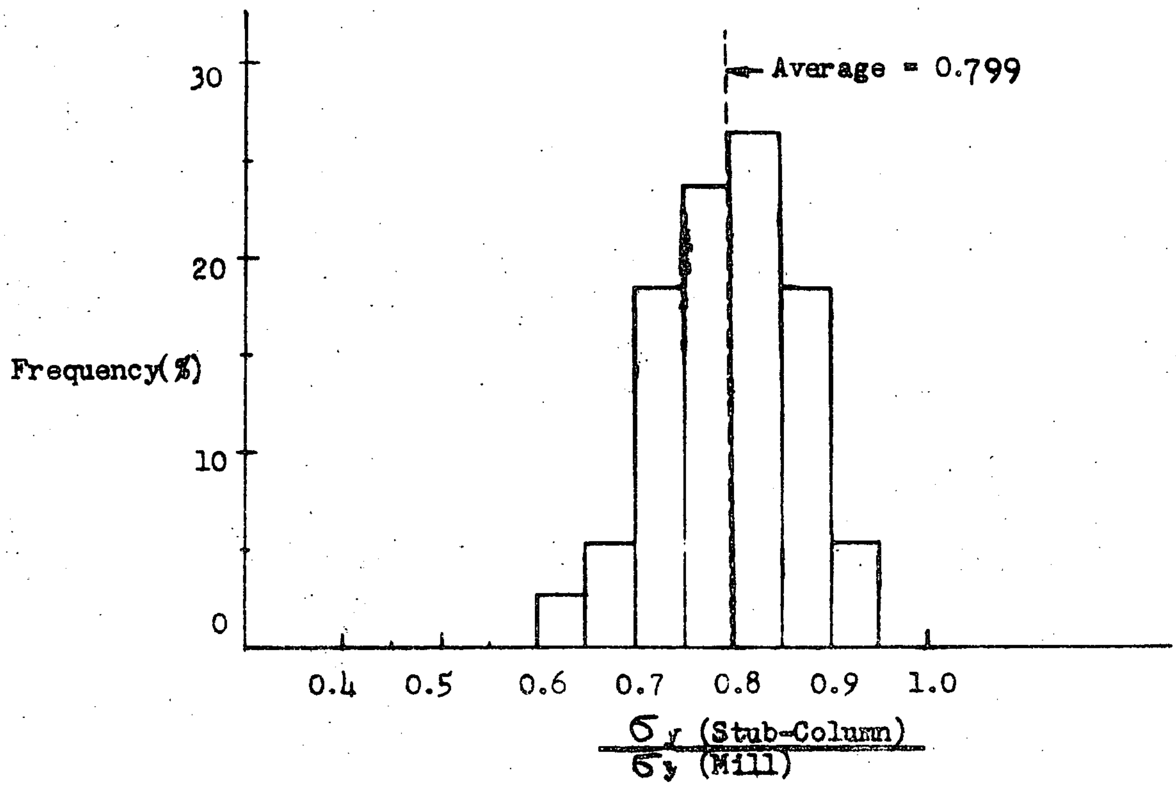
4.1 L S BEEDLE, A W HUBER, "Residual Stress and the Compressive Properties of Steel", A Summary Report, F. L. Report 220A.27, July, 1957.

4.2 A W HUBER, "Residual Stress Measurement", F. L. Report 220A.17, March 1955.

4.3 B THURLIMANN, "Modification to Plastic Theory", Proc. AISC National Engineering Conference, p 50, 1956.

4.4 AISC MANUAL, "STEEL CONSTRUCTION"

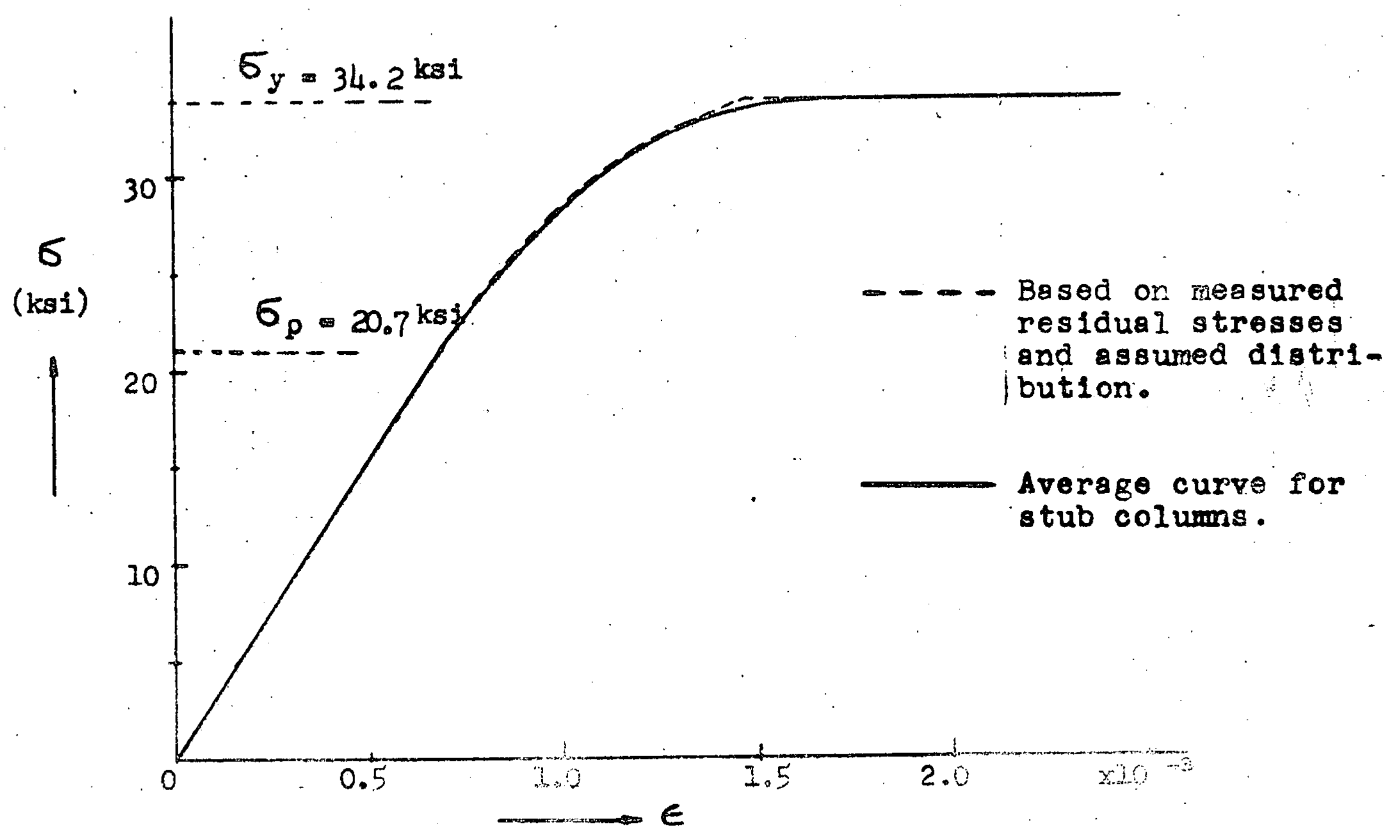
4.5 L A HARRIS, N M NEWMARK, "The Effect of Fabrication Processes on The Brittle Behavior of Steel", University of Illinois, 1957.



FREQUENCY DISTRIBUTION OF RATIO

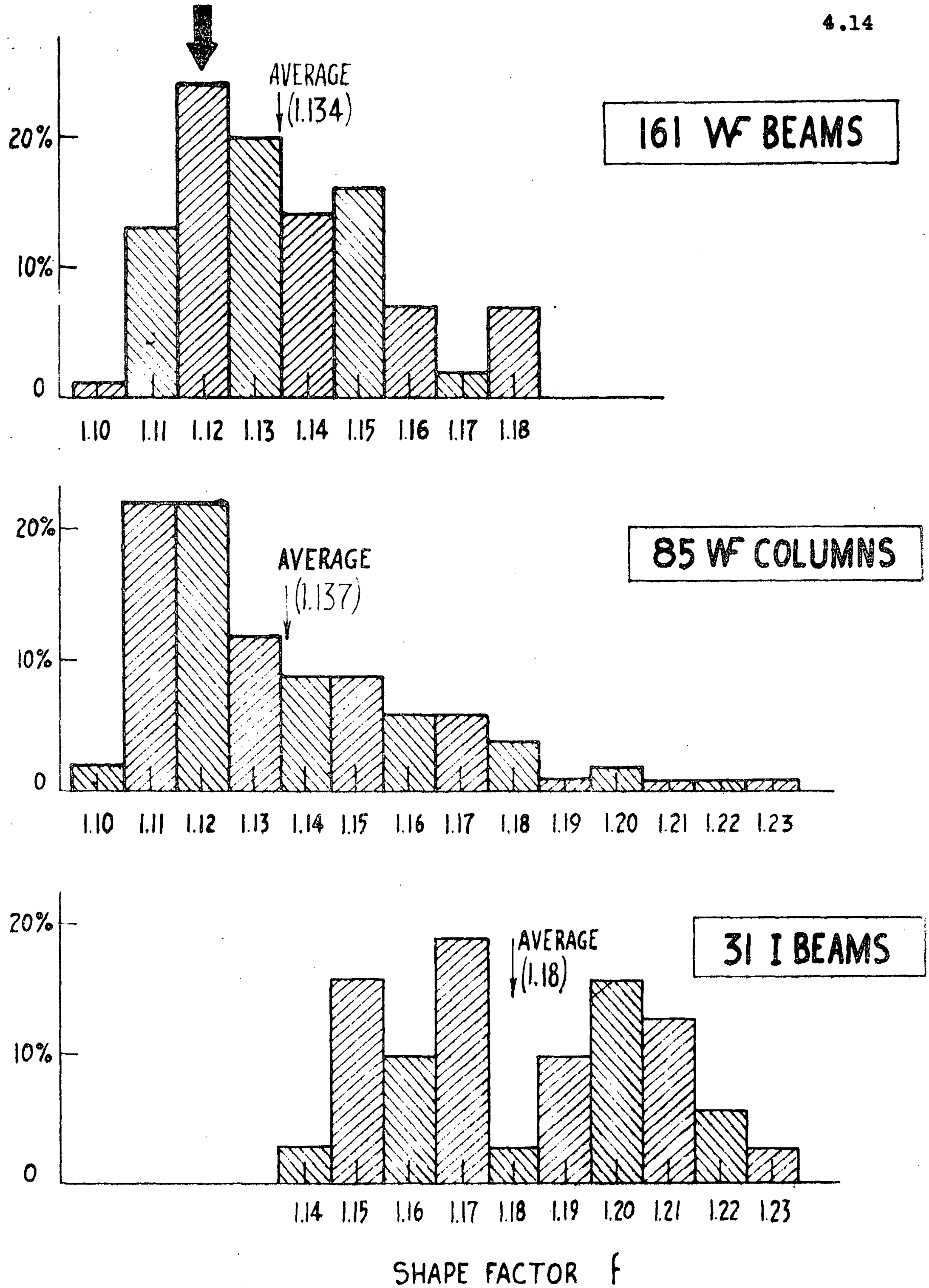
$\frac{\sigma_y \text{ (Stub-Column)}}{\sigma_y \text{ (Mill)}}$ (Group 5)

4.1



AVERAGE σ - ϵ CURVE FOR STUB COLUMNS
(Group 7)

4.2


AVERAGE
(1.134)
161 WF BEAMS
20%
10%
0
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18
AVERAGE
(1.137)
85 WF COLUMNS
20%
10%
0
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19 1.20 1.21 1.22 1.23
AVERAGE
(1.18)
31 I BEAMS
20%
10%
0
1.14 1.15 1.16 1.17 1.18 1.19 1.20 1.21 1.22 1.23
SHAPE FACTOR f

5. VERIFICATION OF PLASTIC THEORY

It is the purpose of this chapter to show that the actual behavior of structures under test verifies the predictions of plastic theory. Art 5.1 will be concerned with demonstrating that structural steel contains the ductility assumed, and that plastic hinges will form and allow the necessary redistribution of moment. Article 5.2 will present the results of continuous beam tests, and finally Art 5.3 will show how tests of rigid frames verify plastic theory.

5.1 BASIC CONCEPTS

1. Ductility of Steel

Fig 5.1 shows the tensile stress-strain curve obtained from two coupons cut from two separate locations of an 8WF40 beam. It is typical of the behavior of ASTM A7 steel. Fig 5.2 represents a conventionalized "average" stress-strain curve for structural steel. It has been obtained from the considerable number of compression and tension tests conducted at Fritz Laboratory for the purpose of evaluating the structural tests performed. Compression tests are of interest in connection with the plastic behavior of structures because, of course, one half of the cross section is yielded in compression. The steel deforms plastically about 15 times the strain at the elastic limit and then commences to strain harden. Although the data is plotted well into the strain-hardening range, the strains shown are still considerably less than those at ultimate strength ("tensile strength).* The compressive and the tensile stress-strain relationships are quite similar. In fact the properties in compression are practically identical with those in tension.

---

* ASTM-A7 requires an elongation in 2-in of not less than 24%, an elongation that is more than 200 times the maximum elastic value.

From Fig 5.2 it may be concluded that structural steel coupons do have the necessary ductility as assumed in plastic analysis.

2. The Plastic Moment and the Plastic Hinge

As a demonstration that the plastic moment is attained through plastification of the cross section, Fig 5.3a shows a typical M-Ø curve obtained from a beam in pure bending.(5.1) The dotted line is the idealized curve and the solid line through the circles shows the results of a test. The theoretical stress distributions (according to the simple plastic theory) at different stages of bending are shown in Fig 5.3(b). Below these in Fig 5.3(c) are shown the corresponding stress distributions as determined from SR-4 gage measurements. It will be seen that plastification of the cross section does occur, and that the bending moment corresponding to this condition is the full plastic moment as computed from the equation $M_p = \sigma_y Z$.

Although there will be inevitable minor variations from the result shown in Fig 5.3(a) the many tests conducted on rolled shapes indicate that most hot-rolled wide-flange beams will develop the strength predicted by the plastic theory and that a plastic hinge (characterized by rotation at near-constant moment) does actually form.

It is true that a somewhat unrealistic loading condition has been taken. "Pure moment" is a condition not likely to be encountered in actual structures, but it represents the most severe loading condition insofar as the plastic behavior of a beam is concerned. Usually there will be a gradient in moment, as when a single concentrated load is applied to a beam. In such a case the deformation tends to be concentrated under the load point (the point of maximum moment). Because the plastic deformation is more localized, the strain-hardening region is reached at a lesser deflection; consequently, the beam tends to develop a moment greater than the plastic moment. Typical of the behavior of a beam under moment gradient is that shown in Fig 5.4.[5.2] The beam continues to increase in load-carrying capacity as the deformation is continued.

Thus strain-hardening improves the moment-carrying capacity of a beam. Although it is neglected in the simple plastic theory (except for checking a beam for stability against buckling) this additional reserve strength is still present in most ordinary structures, and this contributes to a greater than assumed actual factor of safety.

3. Redistribution of Moment

From the previous section it is seen that plastic hinges may be depended upon to form at connections and at concentrated load points. This development of this plastic moment is one of the sources of reserve strength in structural steel. Another source is the redistribution of moment in continuous structures.

In Fig 5.5 is shown a picture of the redistribution process -- as predicted theoretically and as obtained experimentally. A test was made on a continuous beam to simulate the condition of third-point loading on a fixed-ended span;[4.3] thus experimental data was available to compare with the theoretical predictions. The fixed-ended beam and its various components are shown in four stages:

Stage 1 -- near the computed elastic limit.

Stage 2 -- after the plastic hinge has formed at the ends and the load has increased towards its ultimate value.

Stage 3 -- when the theoretical ultimate load is first reached, and

Stage 4 -- after deformation has been continued through an arbitrary additional displacement.

The figure shows (a) the loading, (b) the deflected shape at the four phases, (c) the moment diagram, (d) the load-deflection curve, and the moment curvature relationship near the ends (e) and at the center (f).

In the elastic range (stage 1) it will be seen that the beam behaves just as assumed by the theory, the moment at the center being one-half the moment at the fixed ends. (Figs 5.5c, 5.5e, 5.5f). As the moment at the ends approaches the yield moment, the curvature, ϕ, commences to increase more rapidly, a plastic hinge begins to form (Fig 5.5e). Because of this "hinge action", the additional moments due to increase in load are distributed between the ends and the center in a different ratio beyond the elastic range than before. As long as the beam is elastic the increase in moment at the center corresponding to a load increment is one-half the increase at the ends. However, after a plastic hinge forms at the ends, most of the increase of moment occurs at the center; the moment increment at the ends is small. (Fig 5.5e, 5.5f) This is the process known as redistribution of moment.

As a result of plastification at the ends, the beam actually behaves somewhat more flexibly than before (Fig 5.5d). At Stage 2 the elastic moment capacity near the center is practically exhausted. It is quite evident from Fig 5.5 that substantially all of the moment capacity has been absorbed by the time Stage 3 is reached (ultimate load). Beyond this, the beam simply deforms as a mechanism with the moment diagram remaining largely unchanged, the plastic hinges at the ends and center rotating further.

Clear evidence is therefore available that redistribution of moment occurs through the formation of plastic hinges, allowing the structure to reach (and usually exceed) its theoretical ultimate load.

5.2 CONTINUOUS BEAMS

Fig 5.6 shows the results of continuous beam tests in which the members were fabricated from rolled sections. The structure and loading are shown to scale at the left. Next, the size of member (or members) is indicated. To the right is a bar graph on which is plotted the percent of predicted ultimate strength exhibited by the test structure. (A test result plotted to the "100%" line shows that the structure reached the load predicted by the simple plastic theory.) The solid portion of the bar chart represents the reserve strength beyond the elastic limit since the end of the "open" portion of each bar graph is the computed elastic limit (on a non-dimensional basis) and the end of the solid portion is the observed maximum strength. A typical continuous beam test (No. 5 in Fig 5.6) is shown in Fig 5.7.

Particularly remarkable among the continuous beam tests of Fig 5.6 is one conducted by Maier Leibnitz[5.4] shown as the next to the last structure. In this experiment, prior to applying the vertical load, he raised the center support until the allowable working stress was just reached, with the result that application of the first increment of external load was, in fact, an overload. In spite of this, the computed ultimate load was attained. The observed ultimate load in this test was within 3% of that of the two structures shown immediately in Fig 6.

The continuous beams shown in Fig 5.8 were tested to show that members of otherwise inadequate strength may be cover plated to achieve the desired load-carrying capacity.

5.3 FRAME TESTS

The structure shown in Fig 5.9 is typical of some of the frames tested in this country as part of the experimental verification of plastic theory. The span is 40-ft and the frame was fabricated of 12WF36 rolled shapes.

Figs 5.10, 5.11, 5.12, and 5.13 show frames tested both in this country and abroad, and represent some of the structures which have been tested to maximum load capacity prior to 1958. Good agreement is observed except for those cases in which strain-hardening accounted for an increase. In Fig 5.10 testing of the fourth frame was interrupted in order that the fifth test might be carried out on the same structure but with a different proportion of horizontal to vertical load.

In view of the notable agreement between plastic theory and the results of these tests, the applicability of the plastic method to structural design is justified.

REFERENCES

Chapter 5

5-1 Luxion, W. & Johnston, B. G. "PLASTIC BEHAVIOR OF WIDE FLANGE BEAMS, Weld. Jour. 27(11), November 1948.

5-2 Driscoll, G. C. & Beedle, L. S. "THE PLASTIC BEHAVIOR OF STRUCTURAL MEMBERS AND FRAMES", Weld. Jour. 36(6) p 275-s, June 1957.

5-3 Yang, C. H., Beedle, L. S. & Johnston, B. G. "RESIDUAL STRESS AND THE COMPRESSIVE STRENGTH OF STEEL BEAMS", Weld. Jour. 31(4) April, 1952, p 205-s

5-4 Maier-Leibnitz "CONTRIBUTION TO THE PROBLEM OF ULTIMATE CARRYING CAPACITY OF SIMPLE AND CONTINUOUS BEAMS OF STRUCTURAL STEEL AND TIMBER", Die Bautechnik, 1, 6 (1947), Abstract available as Fritz Laboratory Report No. 205.15.

5-5 Hall, W. J. & Newmark, N. M. "SHEAR DEFLECTION OF WIDE FLANGE BEAMS IN THE PLASTIC RANGE", Trans. ASCE, 122, 666 (1957)

5-6 Popov, E. P., Willis, J. A. "PLASTIC DESIGN OF COVER PLATED CONTINUOUS BEAMS", ASCE Jour., 84(EM-1) 149s-1 (1958)

5-7 Ruzek, J., Knudsen, K. E., Johnston, E. R., & Beedle, L. S. "WELDED PORTAL FRAMES TESTED TO COLLAPSE", Welding Journal, 33(9), September 1954.

5-8 Schilling, C. G., Schutz, F. W. & Beedle, L. S. "BEHAVIOR OF WELDED SINGLE SPAN FRAMES UNDER COMBINED LOADING"

5-9 Driscoll, G. C., Lu, L. W. "TEST OF TWO-SPAN GABLED PORTAL FRAME", Fritz Laboratory Report on preparation.

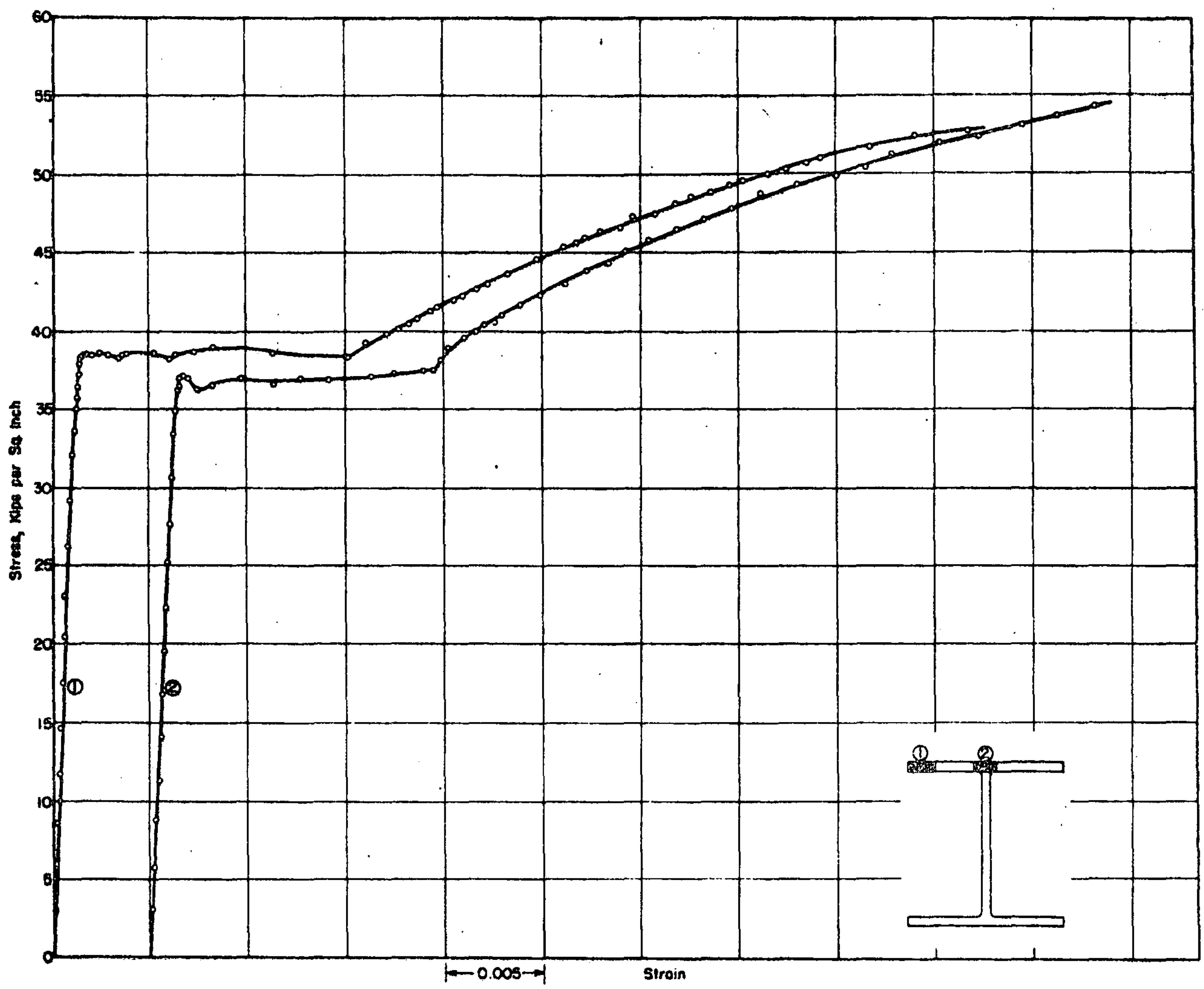
5-10 Nelson, H. M., Wright, D. T., & Dolphin, J. W. "DEMONSTRATIONS OF PLASTIC BEHAVIOR OF STEEL FRAMES", ASCE Proceedings, Paper 1390, October 1957.

5-11 Baker, J. F. & Roderick, J. W. "TESTS ON FULL-SCALE PORTAL FRAMES", Inst. C. E. (Br.), January 1952.

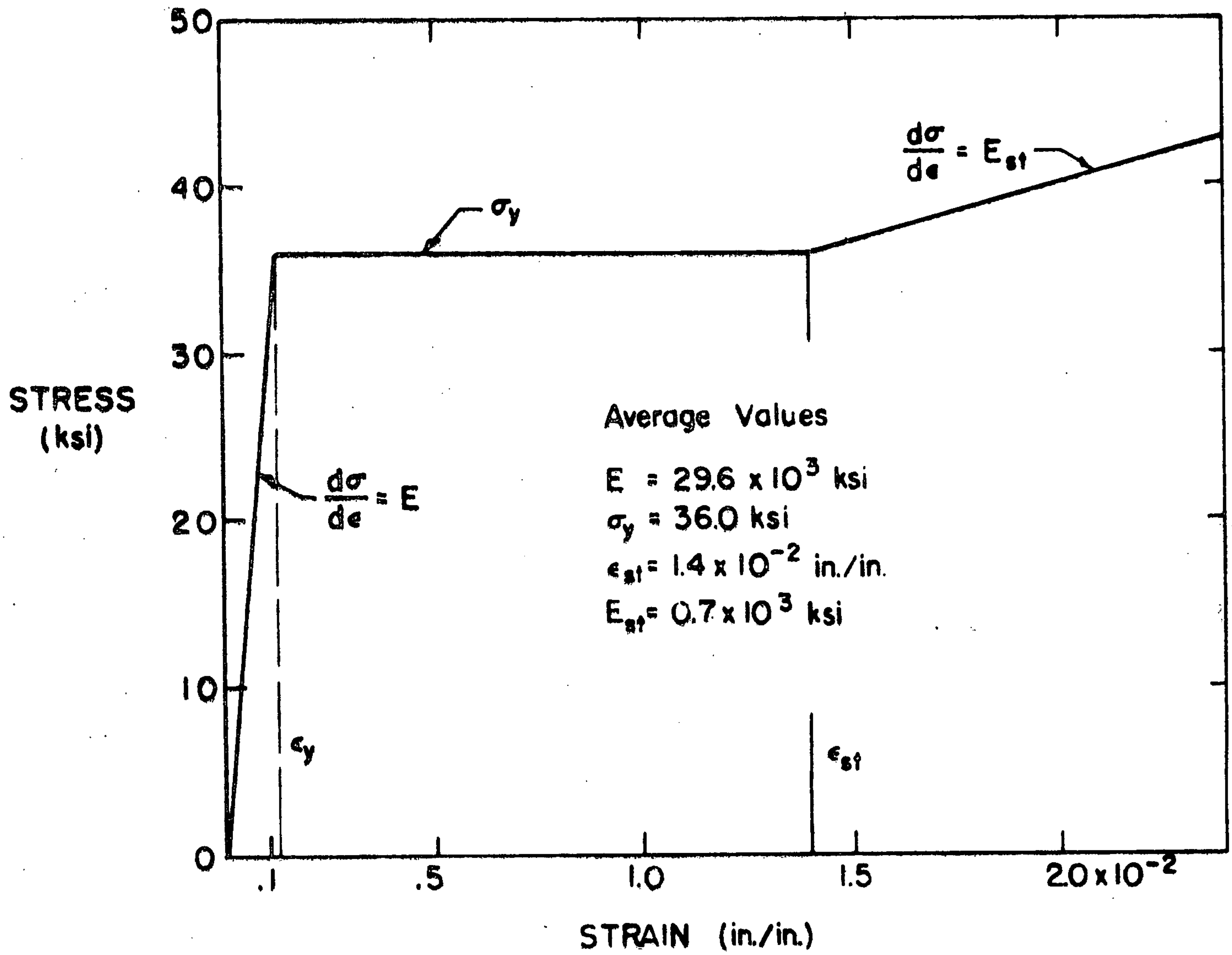
5-12 Baker, J. F., & Eichhoff, K. G. "THE BEHAVIOR OF SAW TOOTH PORTAL FRAMES", Conference on the Correlation between Calculated and Observed Stresses and Displacements in Structures. Institute of Civil Engineers, 1955, p. 107.

5-13 Baker, J. F., & Eickhoff, K. G. "A TEST ON A PITCHED ROOF PORTAL", Br. Weld. Res. Assn. Report No. FE 1/35, January 1954.

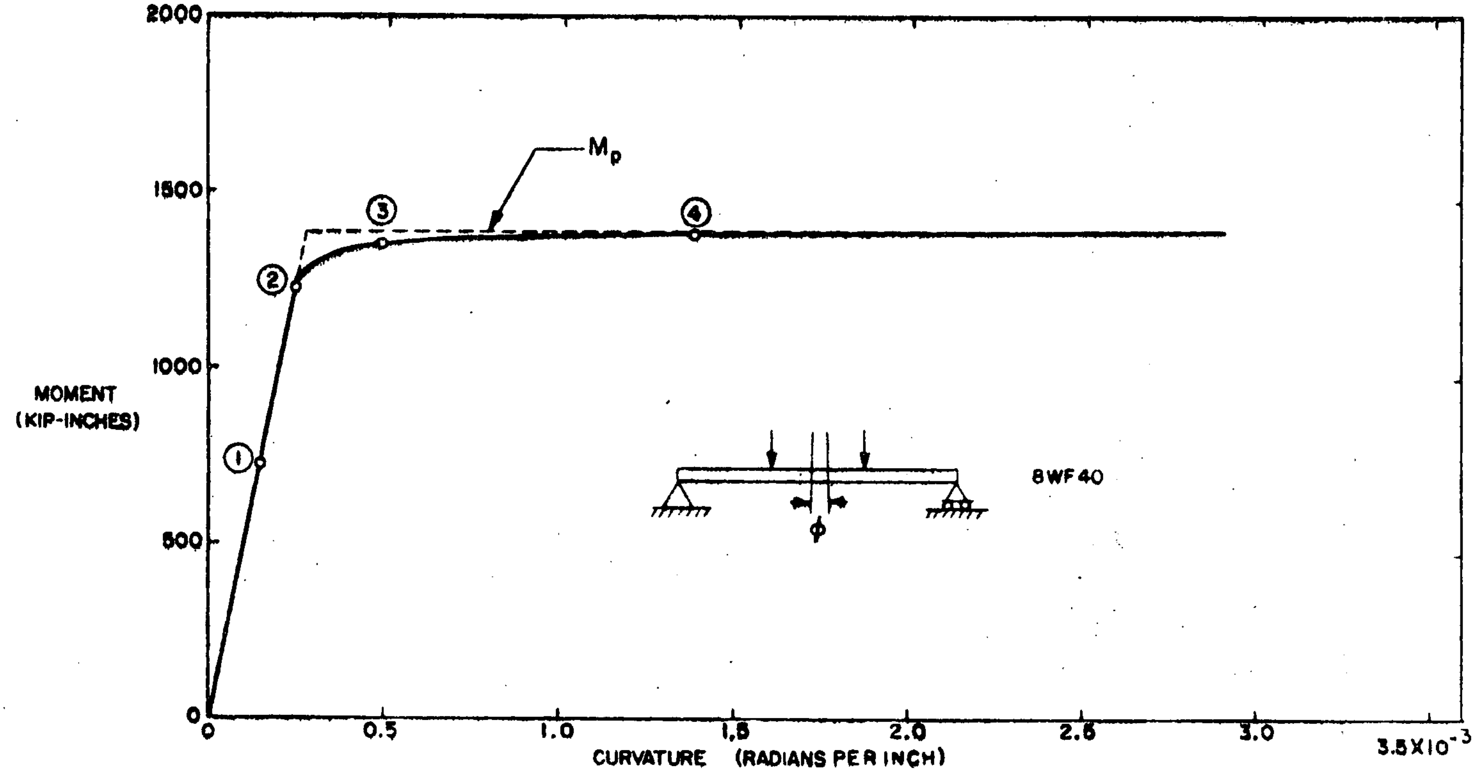
5-14 Blessey, W. E. A letter report from Mr. Walter E. Blessey, Tulane University, February 7, 1958.



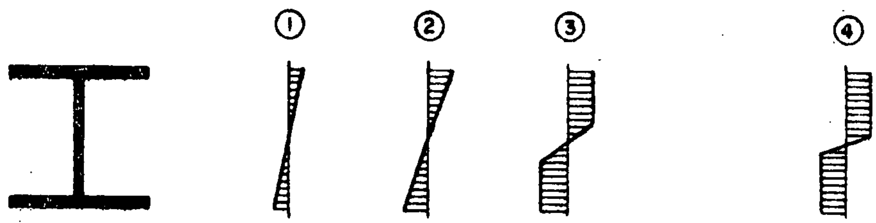
60
55
50
45
40
35
30
25
20
15
10
5
0
Stress, Kips per Sq Inch
0.005
Strain
①
②



50
40
30
20
10
0
STRESS
(ksi)
$\frac{d\sigma}{d\epsilon} = E_{st}$
σ_y
$\frac{d\sigma}{d\epsilon} = E$
ϵ_y
ϵ_{st}
Average Values
$E = 29.6 \times 10^3$ ksi
$\sigma_y = 36.0$ ksi
$\epsilon_{st} = 1.4 \times 10^{-2}$ in./in.
$E_{st} = 0.7 \times 10^3$ ksi
.1
.5
1.0
1.5
2.0×10^{-2}
STRAIN (in./in.)

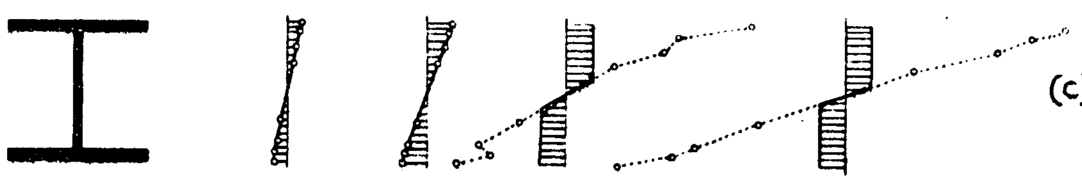


(a)



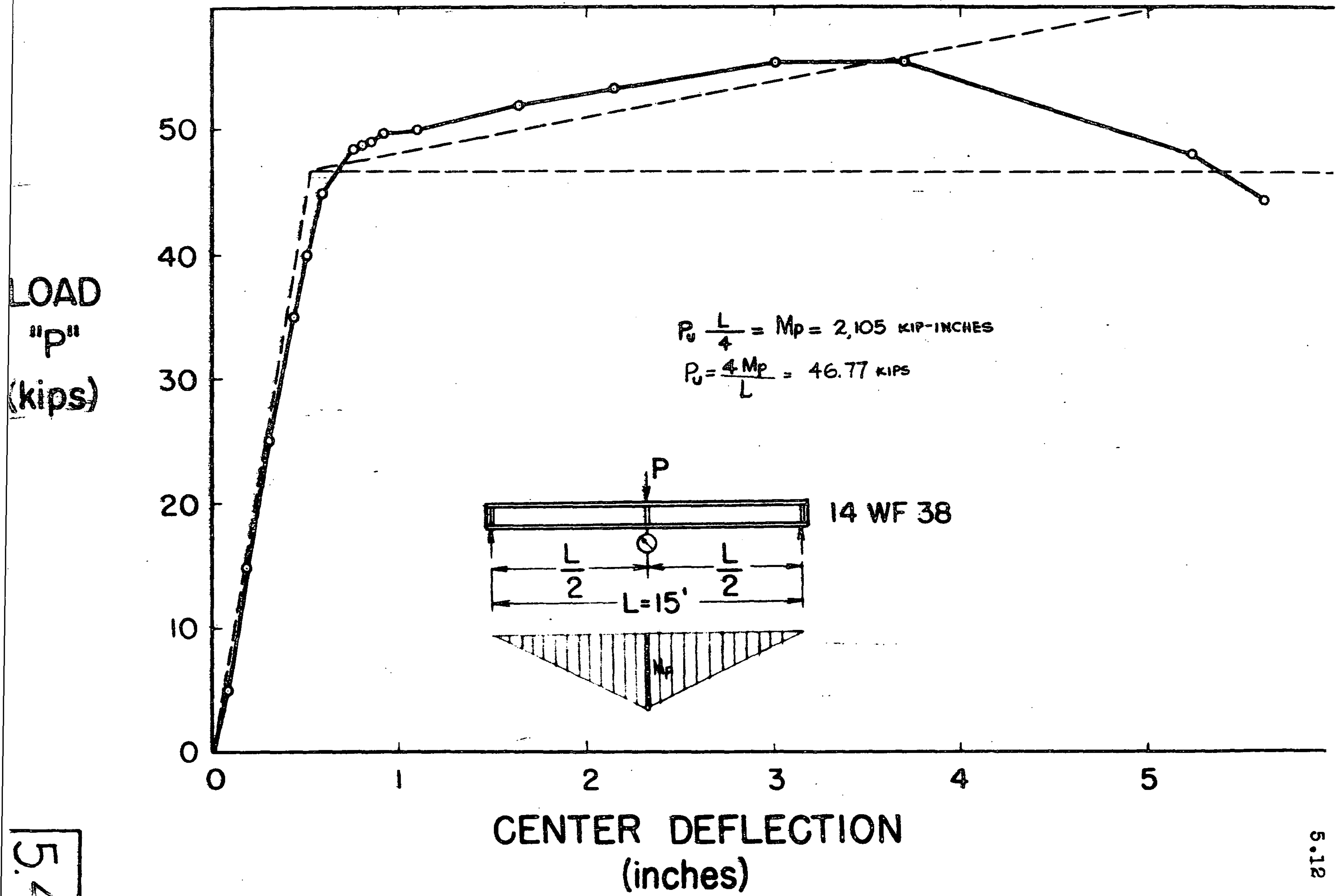
(b)

THEORETICAL STRESS DISTRIBUTION

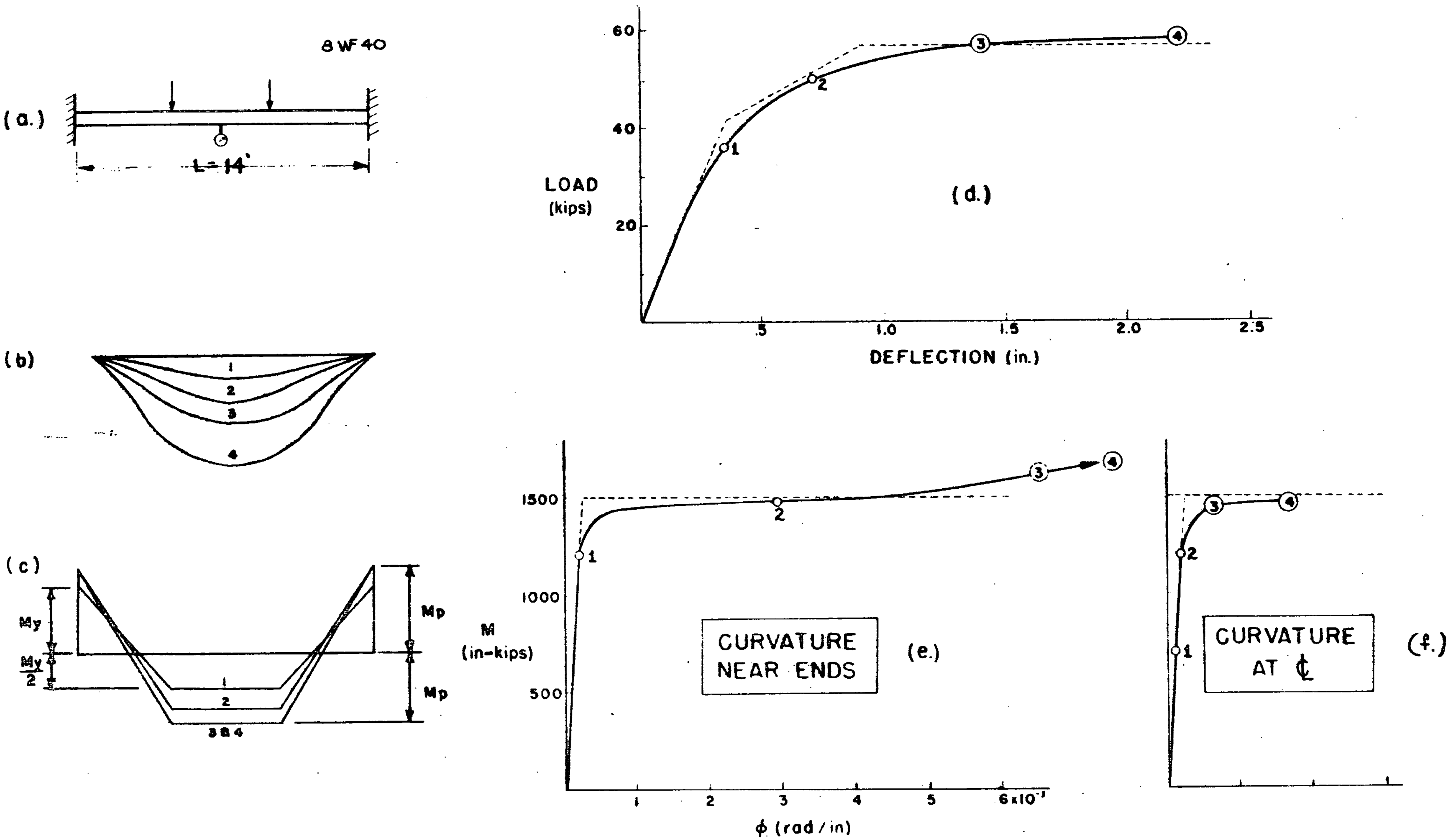


(c)

STRESS DISTRIBUTION COMPUTED FROM MEASURED STRAINS


LOAD "P" (kips)
0
10
20
30
40
50
CENTER DEFLECTION (inches)
0
1
2
3
4
5
$P_u \frac{L}{4} = M_p = 2{,}105$ KIP-INCHES
$P_u = \frac{4 M_p}{L} = 46.77$ KIPS
P
14 WF 38
$\frac{L}{2}$
$\frac{L}{2}$
L=15'
M_p

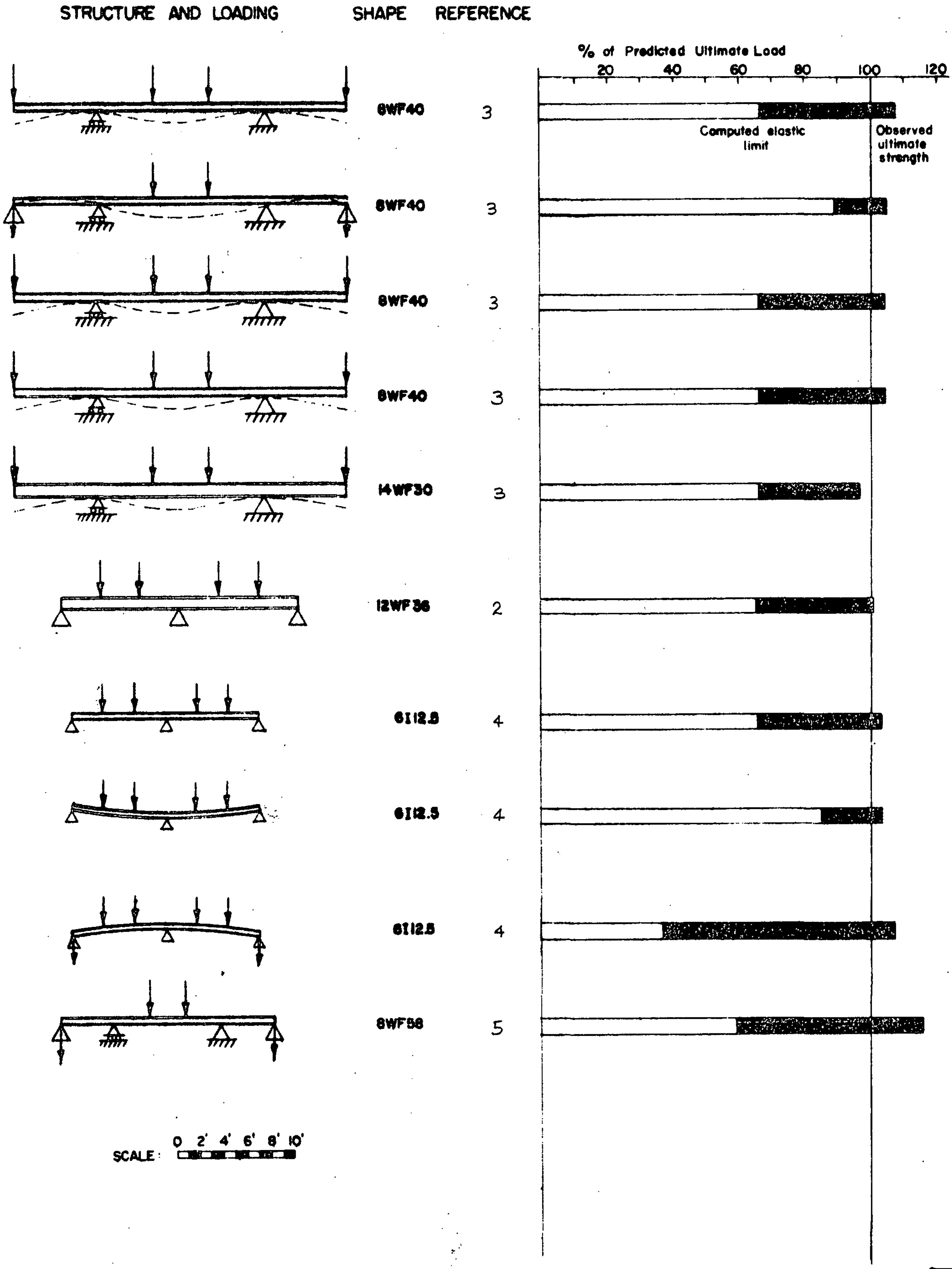
5.4


8 WF 40
(a.)
L = 14'
(b)
1
2
3
4
(c)
My
My/2
Mp
Mp
1
2
3 & 4
LOAD (kips)
60
40
20
1
2
3
4
(d.)
.5
1.0
1.5
2.0
2.5
DEFLECTION (in.)
M (in-kips)
1500
1000
500
CURVATURE NEAR ENDS
(e.)
1
2
3
4
5
6 x10⁻³
φ (rad / in)
CURVATURE AT ℄
(f.)

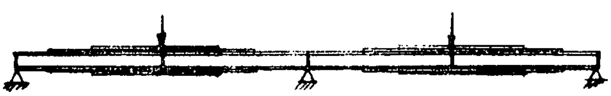
5.13

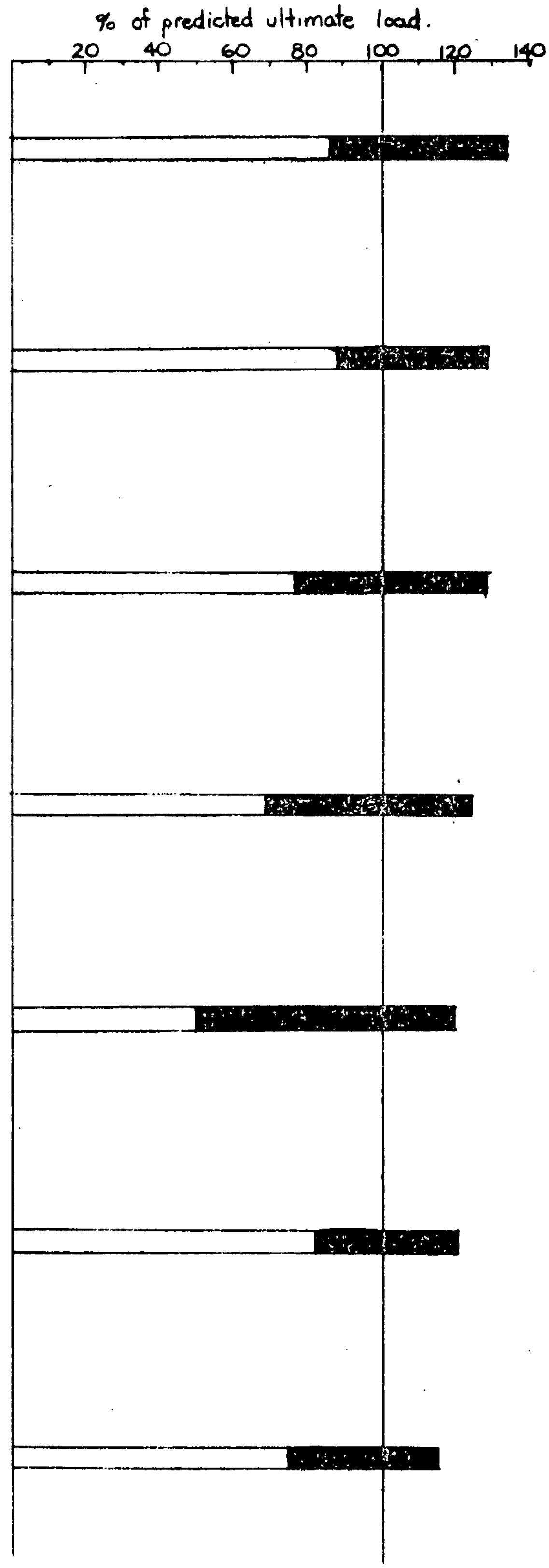
5.5

CONTINUOUS BEAMS

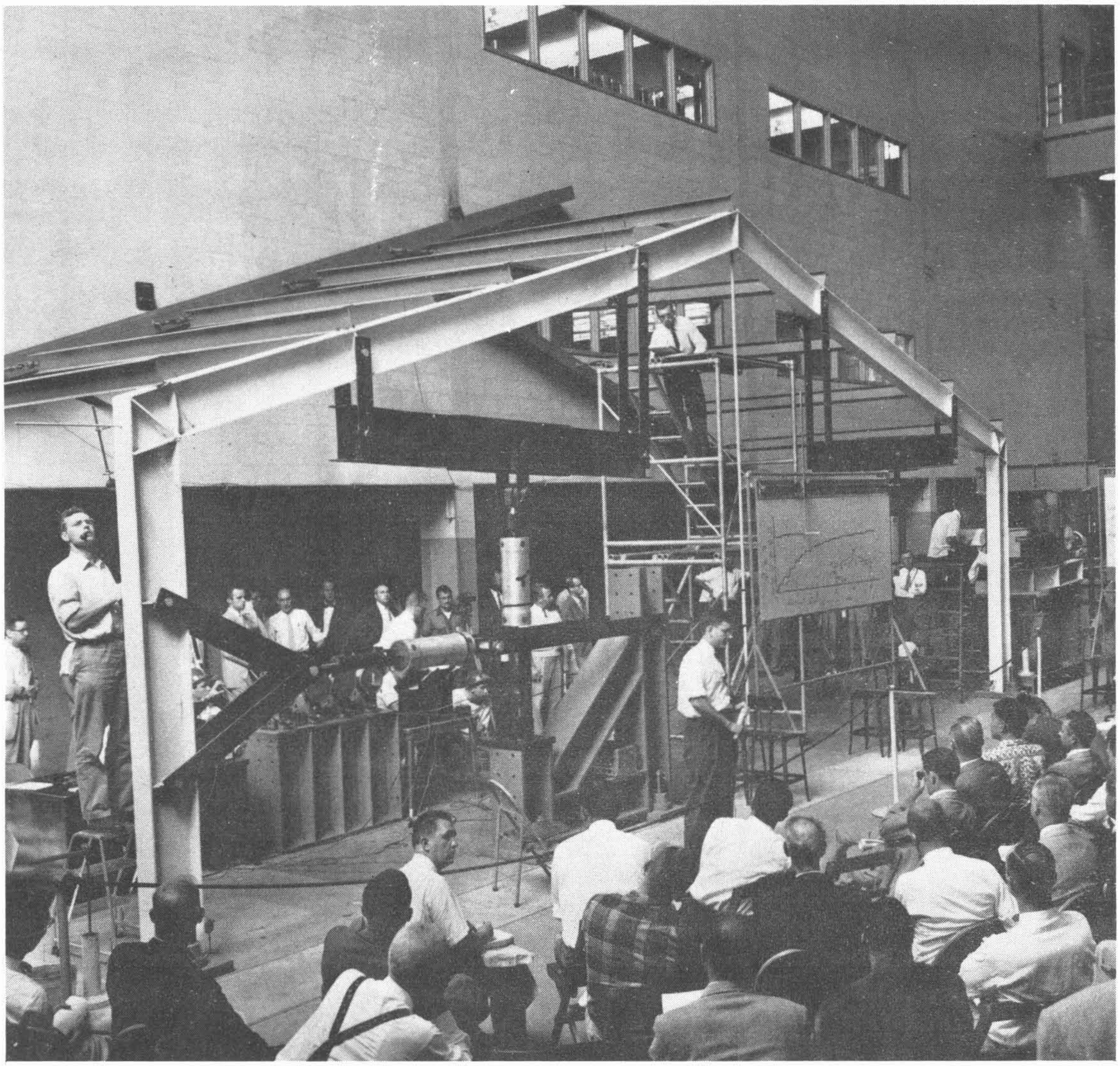


Test of a Continuous Beam 14 WF 30

STRUCTURE AND LOADING	SHAPE	REF.
	5 I 10	6
	5 I 10	6
	6 I 12.5	6
	5 I 10	6
	4 I 7.7	6
	5 I 10	6
	5 I 10	6



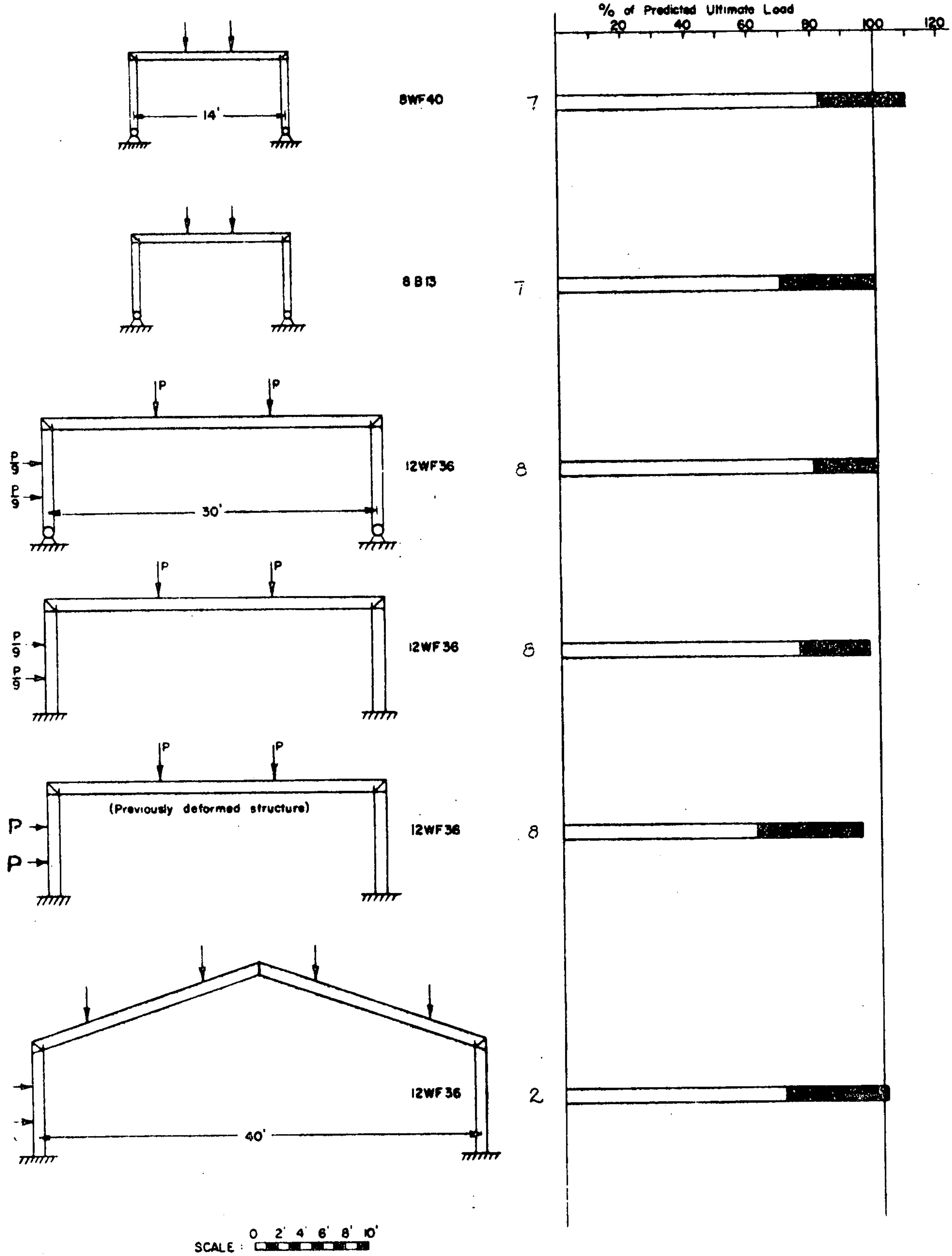
SCALE: 

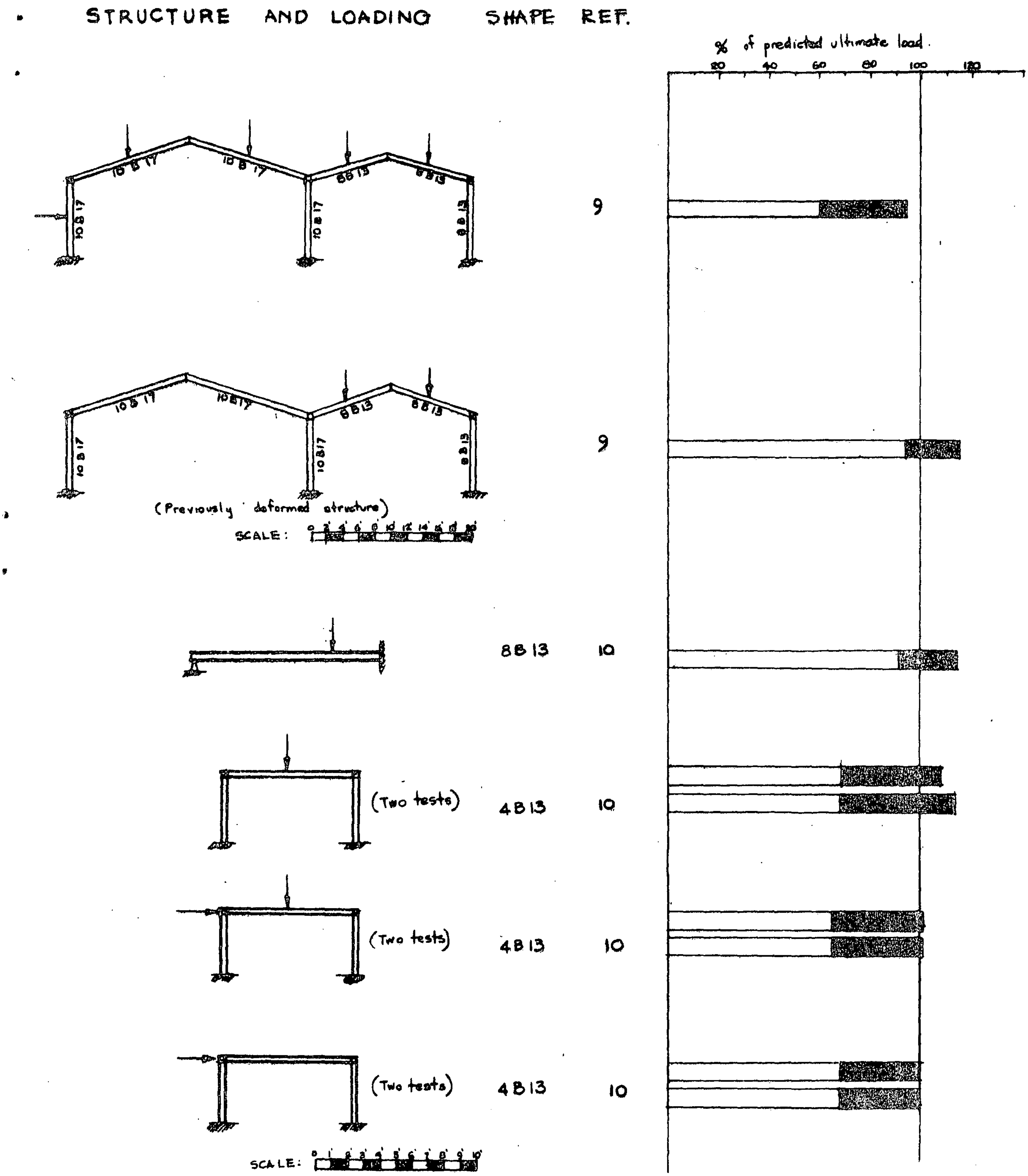


Test of a Gable Frame

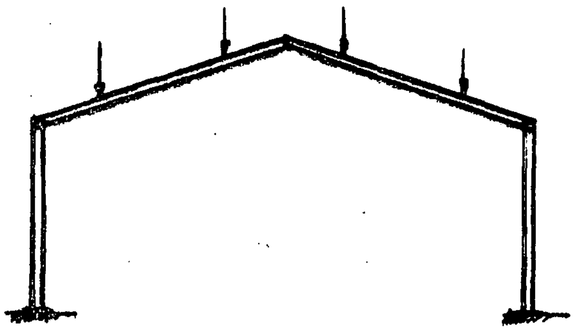
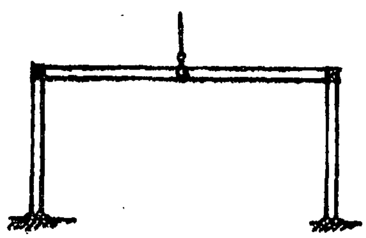
FRAMES

STRUCTURE AND LOADING SHAPE REFERENCE

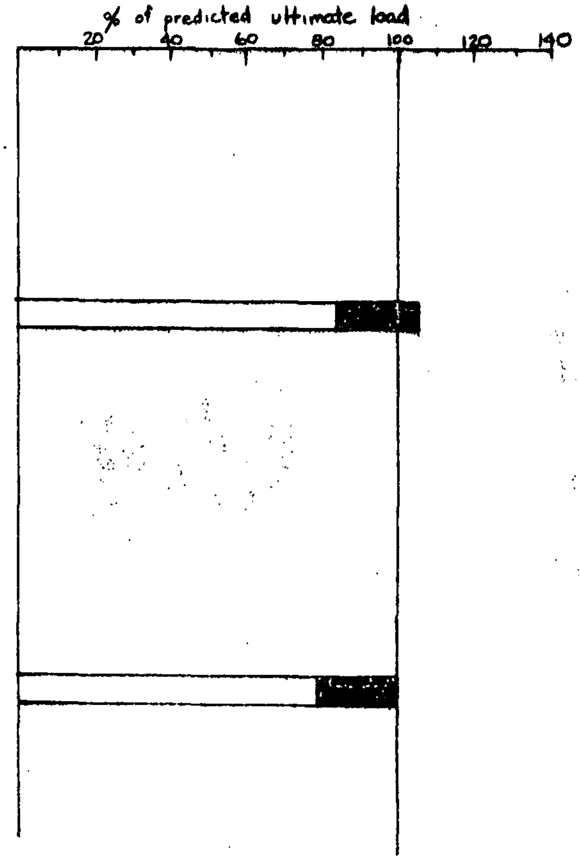



STRUCTURE AND LOADING
SHAPE
REF.
% of predicted ultimate load
20
40
60
80
100
120
10 B 17
10 B 17
8 B 13
8 B 13
10 B 17
10 B 17
8 B 13
9
10 B 17
10 B 17
8 B 13
8 B 13
10 B 17
10 B 17
8 B 13
9
(Previously deformed structure)
SCALE:
8 B 13
10
(Two tests)
4 B 13
10
(Two tests)
4 B 13
10
(Two tests)
4 B 13
10
SCALE:

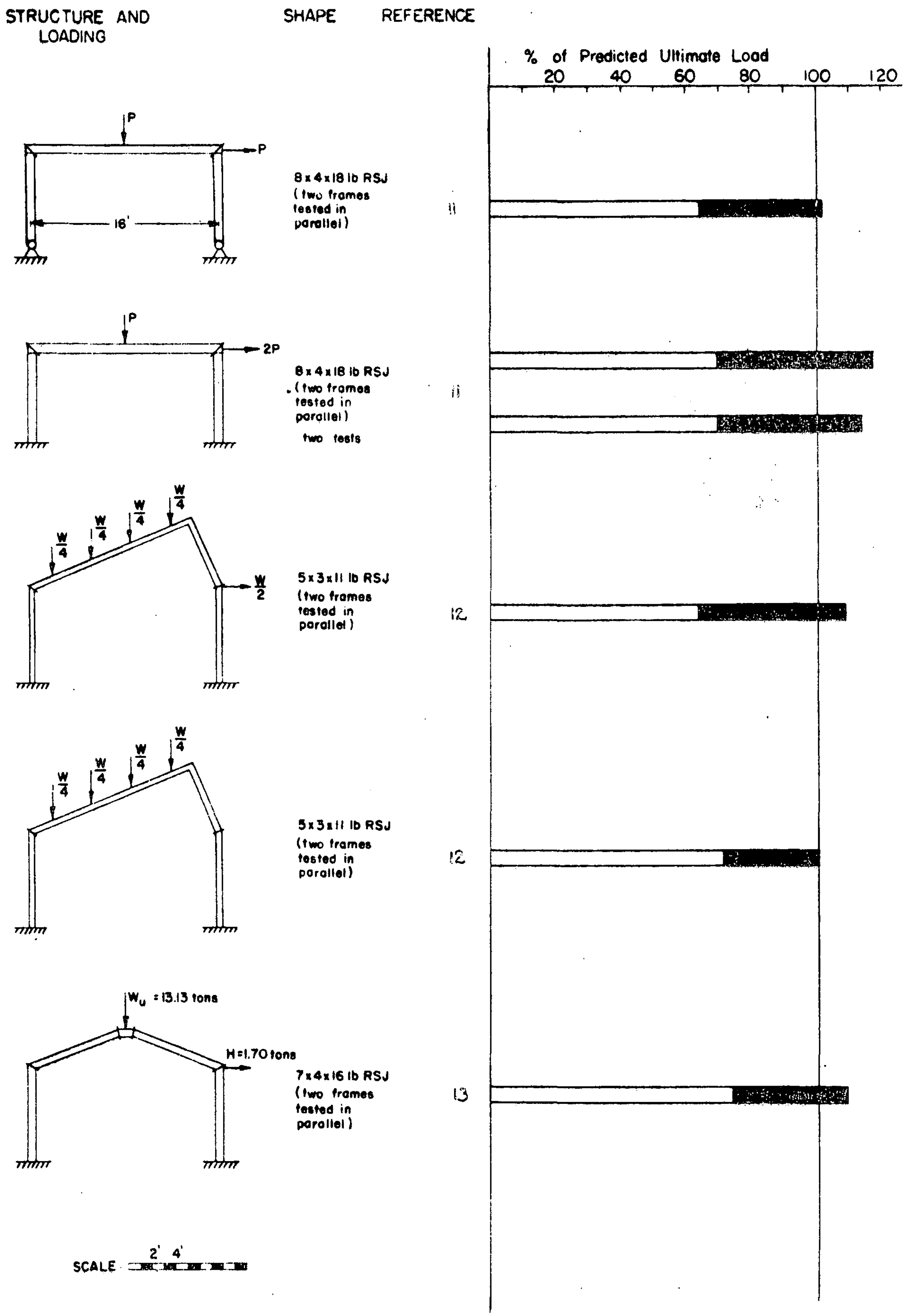
5.11

STRUCTURE AND LOADING	SHAPE	REF.
	8WF17	10
	12WF36	14

SCALE: 0 1 2 3 4 5 6 7 8 9 10'



FRAMES


STRUCTURE AND LOADING
SHAPE
REFERENCE
% of Predicted Ultimate Load
20
40
60
80
100
120
P
P
16'
8x4x18 lb RSJ (two frames tested in parallel)
11
P
2P
8x4x18 lb RSJ (two frames tested in parallel) two tests
11
W/4
W/4
W/4
W/4
W/2
5x3x11 lb RSJ (two frames tested in parallel)
12
W/4
W/4
W/4
W/4
5x3x11 lb RSJ (two frames tested in parallel)
12
W_u = 13.13 tons
H=1.70 tons
7x4x16 lb RSJ (two frames tested in parallel)
13
SCALE 2' 4'

6. SECONDARY DESIGN CONSIDERATIONS

6.1 SHEAR FORCE (Article to be prepared)

This article will show that the influence of shear force under certain conditions is to cause the deflection to start to increase at a more rapid rate and at a load that is less than the yield load neglecting the influence of shear forces.

A brief outline of the theoretical solution to the problem will be given and the results of tests will be shown.

A design approximation has been developed to indicate when shear force will be critical. It is of the form,

$$V_{max} = 18.000\ wd$$

where V is the shear force in pounds, w is the web thickness in inches, and d is the section depth in inches. This approximation will be derived and examples will be given to illustrate its use.

6.2 LOCAL BUCKLING (Article to be prepared)

Based on work recently completed, this article will summarize the essential steps in the theoretical analysis of the buckling of flanges and webs of wide flange shapes, when failure takes place in the inelastic range. By requiring that an element sustain the yield load until the strain reaches strain-hardening, a solution to the problem is achieved.

The results of correlating tests will be presented, followed by the presentation of a design guide to assure that compressive strains may reach and even exceed ϵ_{st} without premature failure ($b/t \leq 17$, $d/w \leq 55$).

This article will review the work that has been done on repeated loading as it applies to plastically design structues. Pertinent test results will be shown.

6.3 LATERAL BUCKLING (Article to be prepared)

Although tentative solutions to the problem of lateral buckling of beams in the plastic range have been prepared, the problem is not completely solved. This article will summarize the theoretical and experimental work that has been done and will indicate the procedures that seem most appropriate in view of present state of knowledge. Design approximations and illustrative examples will be shown.

6.4 AXIAL FORCE AND COLUMN BUCKLING

The simple plastic theory assumes that the full plastic hinge value is available in all members, and that failure of a frame (in the sense that a mechanism is formed) is not preceeded by column instability. This article will present methods of determining the amount by which the presence of axial force tends to reduce the magnitude of the effective plastic moment, and will indicate the solutions that have been obtained to the problem of a column loaded with axial force and end moments. These solutions will be compared with tests carried out both in this country and abroad and will suggest a design apprpximation based on these findings.

As in the case of lateral buckling, the entire problem is not completely solved. However, solutions are available for certain practical loading conditions. These will be presented, and the areas of remaining work will be outlined.

6.5 REPEATED LOADING (Article to be prepared)

In a large majority of practical cases, the ultimate load is not influenced by the fact that there is some variation in the loading. The number of cycles may be small or the magnitude of variation in an individual load may be inconsequential. There may be cases, however, where a major part of the loading may fluctuate for a considerable number of cycles.

Chapter 7

C O N N E C T I O N S

(To be completed)

Connections must be proportioned in such a way that they will transmit the plastic moment and have adequate rotation capacity. Theoretical and experimental studies have shown that connections can be fabricated economically and still meet these requirements.

This chapter will first outline the specific requirements that different types of connections must fulfill. Next the various connection types will be discussed, and in each case the method of analysis will be given, experimental correlation will be shown, and a suggested guide for proportioning the element will be presented. The methods will be illustrated by examples.

The following types of connections will be considered:

(1) Straight corner connections

(2) Haunched connections (tapered and curved)

(3) Beam-to-column connections

(4) Beam-girder connections

(5) Miscellaneous building connections

(6) Splices

(7) Details with regard to welding

(8) Details with regard to bolting

Chapter 8

MISCELLANEOUS REQUIREMENTS

8.1 DEFLECTIONS (To be completed)

This article will review some of the methods for computing deflections for structures designed by the plastic method. Although the determination of the load-vs-deflection relationship is too complicated to be of practical value, by making certain simplifications with regard to the M-ϕ relationship, the problem may be reduced to one that is no more complicated than that of computing the elastic deflection of a frame once the moment diagram is known.

Comparison will be made between computed deflections and those observed in tests.

This article will also discuss the importance of deflection as a design requirement. Actually, it is a secondary one, since the important function of a structure is to carry load. Comparison of deflections at working load for plastically-designed structures will be made with deflections of conventionally-designed structures (simple-beam design). Such a comparison leads to the conclusion that a structure designed plastically will usually deflect no more at working load than a structure designed elastically according to current specifications.

D E F I N I T I O N S

LIMIT LOAD -- The load under which a structure reaches a defined limit of structural usefulness.

ULTIMATE LOAD -- The load attained when a sufficient number of yield zones have formed to permit the structure to deform plastically without further increase in load. It is the largest load a structure will support, when such factors as instability and fracture are excluded.

LIMIT DESIGN -- A design based on any chosen limit of structural usefulness.

ELASTIC DESIGN -- A design method which defines the limit of structural usefulness as the load at which a calculated stress equal to the yield point of the material is first attained at any point (usually disregarding local stress raisers).

PLASTIC DESIGN -- A design method which defines the limit of structural usefulness as the "ultimate load". (The term, "plastic" comes from the fact that the ultimate load is computed from a knowledge of the strength of steel in the plastic range)

FACTOR OF SAFETY -- As used in conventional elastic design, it is a factor by which the yield stress is divided to determine a working or allowable stress for the most highly stressed fibre.

LOAD FACTOR -- In plastic design, a factor by which the working load is multiplied to determine the ultimate load. This choice of terms serves to emphasize the reliance upon load-carrying capacity of the structure rather than upon stress.

YIELD MOMENT -- In a member subjected to bending, the moment at which an outer fibre first attains yield point stress.

PLASTIC MOMENT -- Resisting moment of a fully-yielded cross-section.

PLASTIFICATION -- Gradual penetration of yield stress from the outer fibre towards the centroid of a section under increasing moment. Plastification is complete when the plastic moment, M_p, is attained.

PLASTIC MODULUS -- The resisting modulus of a completely yielded cross section. It is the combined statical moments about the neutral axis of the cross-sectional areas above and below that axis.

SHAPE FACTOR -- The ratio M_p/M_y , or Z/S, for a cross-section.

PLASTIC HINGE -- A yielded zone which forms in a beam when the plastic moment is applied. The beam rotates as if hinged, except that it is restrained by the moment M_p.

HINGE ANGLE (H) -- The angle of rotation through which a yielded segment of a beam must sustain its plastic moment value.

<u>ROTATION CAPACITY</u> -- The angular rotation which a given cross-sectional shape can sustain at the plastic moment value without prior local failure.

<u>MECHANISM</u> -- An articulated system able to deform without a finite increase in load. It is used in the special sense that the linkage may include real hinges and/or plastic hinges.

<u>REDISTRIBUTION OF MOMENT</u> -- A process which results in the successive formation of plastic hinges until the ultimate load is reached. As a result of the formation of plastic hinge, less-highly stressed portions of a structure may carry increased moments.

<u>PROPORTIONAL LOADING</u> -- All loads increase in a constant ratio, one to the other, and without repetition, to a maximum value.

NOMENCLATURE

A = Area of cross-section

A_f = Area of two flanges of WF shape, $A_f = 2bt$

A_p = Cross-sectional area of plate

A_w = Area of web, $A_w = wd$

a = Column height to span length ratio of gable frame (column height = aL).

= Distance between centroids of cross-sectional areas above and below neutral axis.

= Distance from end of cantilever to critical section of beam.

b = Flange width

= Breadth of rectangular cross section.

= Roof rise to span length ratio of gable frame (roof rise = bL).

C = Overturning moment parameter (windward side)

C_f = Correction factor due to end fixity (restraint) for determining critical length for lateral buckling.

c = Distance from neutral axis to the extreme fiber.

D = Overturning moment parameter (leeward) side.

d = Depth of section

d_f = Distance between centers of two flanges.

d_p = Distance between two cover plates.

d_w = Web depth of WF shape $(d - 2t)$

E = Young's modulus of elasticity

E_{st} = Strain-hardening modulus = $\frac{d\sigma}{d\epsilon_{st}}$

E_t = Tangent modulus

e = Eccentricity

F = Load factor of safety

f = Shape factor = $\frac{M_p}{M_y} = \frac{Z}{S}$

G = Modulus of elasticity in shear.

G_{st} = Modulus of elasticity in shear at onset of strain-hardening.

G_t = Tangent modulus in shear.

g_A = Moment ratio in adjacent segment.

H = Hinge angle required at a plastic hinge.

= Horizontal reaction

H_B = Portion of hinge angle that occurs in critical (buckling) segment of beam.

h = Story height in multi-story frame.

I = Moment of inertia; subscripts denote axis.

= Number of redundants remaining in a structure at ultimate load.

I_e = Moment of inertia of elastic part of cross-section.

I_p = Moment of inertia of plastic part of cross-section.

I_w = Warping constant

K = Torsion constant

KL = Effective (pin-end) length of column. K = Euler length factor

k = Distance from flange face to end of fillet

= Plastic moment ratio

= Stiffness factor of a beam

L = Span length

= Actual column length

= Length of bar

L_B = Length of buckling (critical) segment

ℓ = Length of segment (slope-deflection equation)

L_{cr} = Critical length for lateral buckling

L_ℓ, L_s = Critical length (with $C = 1.0$) of adjacent spans; subscripts ℓ and s denote larger and shorter critical lengths, respectively.

ΔL = Equivalent length of connection

= Length of plastic hinge

M = Bending moment

= Number of plastic hinges necessary to form a mechanism

M_{cr} = Critical moment for lateral buckling of a beam

M_h = Moment at the haunch point

M_{max} = Maximum moment

M_o = Column end moment; a useful maximum moment

M_p = Plastic moment

M_{pc} = Plastic hinge moment modified to include the effect of axial compression.

M_{ps} = Plastic hinge moment modified to include effect of shear force

M_s = Maximum moment of a simply-supported beam

M_w = Moment at working (service) load

M_y = Moment at which yield point is reached in flexure.

M_{yc} = Moment at which initial outer fiber yield occurs when axial thrust is present.

N = Number of possible plastic hinges.

= Normal force.

n = Number of possible independent mechanisms

= Shift of neutral axis.

P = Concentrated load.

P_e = Euler buckling load

P_{max} = Maximum load

P_s = Stabilizing ("shakedown") load.

P_t = Tangent modulus load.

P_u = Ultimate load (theoretical)

P_w = Working (allowable) load

P_y = Axial load corresponding to yield stress level; $P = A\sigma_y$. Load on beam when yield point is reached in flexure.

Q = b/a = roof rise $\div$ column height

R = Rotation capacity

= Radius of curved haunch

r = Radius of gyration; subscripts denote flexure axis

S = Section modulus, I/c

S_e = Section modulus of elastic part of cross section

s = Length of compression flange of haunch

T = Force. Horizontal load applied at eaves which produces overturning moment about the base of the structure equivalent to that of horizontal distributed load.

t = Flange thickness; subscripts c and t denote compression and tension.

t_s = Stiffener thickness.

t_{tr} = Transverse stiffener thickness

V = Shear force

V_{max} = Maximum allowable shear force.

u,v,w = Displacements in x, y, and z directions.

W = Total distributed load

W_E = External work due to virtual displacement

W_I = Internal work due to virtual displacement

w = Distributed load per unit of length

= Web thickness

X = Number of redundancies in original structure.

x = Longitudinal coordinate

= Distance to position of plastic hinge under distributed load

y = Transverse coordinate

y_o = Ordinate to furthest still-elastic fiber

= Distance from midheight to neutral axis

$\bar{y}$ = Distance from neutral axis to centroid of half-area

Z = Plastic modulus, $Z = \frac{M_p}{\sigma_y}$

Z_e = Plastic modulus of elastic portion of cross-section

Z_p = Plastic modulus of plastic portion of cross-section

Z_t = Trial value of Z, neglecting axial force

z = Lateral coordinate

α = Central angle between points of tangency of curved connection

β = Angle between two non-parallel flanges

Δ = Virtual displacement

δ = Deflection. Subscripts u, w, y denote deflection at ultimate, working, and yield load respectively.

ϵ = Strain

ϵ_{max} = Elongation at fracture

ϵ_{st} = Strain at strain-hardening

ϵ_y = Strain corresponding to theoretical onset of plastic yielding

θ = Measured angle change, rotation

= Mechanism angle

μ = Poisson's ratio

ρ = Radius of curvature

σ = Normal stress

σ_{ly} = Lower yield point

σ_p = Proportional limit

σ_r = Residual stress

σ_{ult} = Ultimate tensile strength of material

σ_{uy} = Upper yield point

σ_w = Allowable (working) stress

σ_y = Yield stress level

τ = Shear stress

τ_y = Shear yield stress

ϕ = Rotation per unit length, or average unit rotation; curvature.

ϕ_p = M_p/EI

ϕ_{st} = Curvature at strain-hardening

ϕ_y = Curvature corresponding to first yield in flexure.

PLASTIC MODULUS TABLE

1255.0	36 WF 300
1167.0	36 WF 280
1076.0	36 WF 260
1008.0	36 WF 245
942.7	36 WF 230
918.2	33 WF 240
869.3	14 WF 426
836.2	33 WF 220
803.0	14 WF 398
767.2	36 WF 194
754.4	33 WF 200
737.3	14 WF 370
733.9	30 WF 210
716.9	36 WF 182
666.7	36 WF 170
659.6	30 WF 190
623.3	36 WF 160
611.5	14 WF 314
593.0	30 WF 172
592.2	14 WF 320
579.8	36 WF 150
558.3	33 WF 152
556.9	27 WF 177
551.6	14 WF 287
513.2	33 WF 141
504.3	27 WF 160
502.4	14 WF 264
466.0	33 WF 130
464.5	14 WF 246
463.7	24 WF 160
452.0	27 WF 145
445.4	14 WF 237
436.7	30 WF 132
427.2	14 WF 228
416.0	24 WF 145
408.0	14 WF 219
407.4	30 WF 124
391.7	14 WF 211
377.6	30 WF 116
373.6	14 WF 202
369.2	24 WF 130
357.0	21 WF 142
355.1	14 WF 193
345.5	30 WF 108
342.8	27 WF 114
337.5	14 WF 184
336.6	24 WF 120
321.3	14 WF 176
317.8	21 WF 127
311.5	12 WF 190
307.7	24 WF 110
304.4	27 WF 102
302.9	14 WF 167
298.0	24 I 120
286.3	14 WF 158
278.0	21 WF 112
278.3	24 WF 100
277.7	27 WF 94
273.0	24 I 105.9
270.2	14 WF 150
259.2	12 WF 161
254.8	14 WF 142
253.0	24 WF 94
247.9	18 WF 114
242.7	14 WF 136
238.8	24 I 100
226.5	18 WF 105
226.3	21 WF 96
225.9	14 WF 127
224.0	24 WF 84
220.5	24 I 90
210.9	14 WF 119
209.7	12 WF 133
206.0	18 WF 96
203.0	24 I 79.9
200.1	24 WF 76
196.0	14 WF 111
192.0	20 I 95
191.6	21 WF 82
186.4	12 WF 120
186.0	16 WF 96
181.0	14 WF 103
177.6	18 WF 85
177.3	20 I 85
172.1	21 WF 73
169.0	16 WF 88
166.6	14 WF 95
163.4	12 WF 106
160.5	18 WF 77
159.8	21 WF 68
151.8	12 WF 99
151.5	20 I 75
151.3	14 WF 87
147.5	10 WF 112
145.5	16 WF 78
145.4	14 WF 84
144.7	18 WF 70
144.1	21 WF 62
140.2	12 WF 92
137.3	20 I 65.4
134.0	14 WF 78
131.8	18 WF 64
131.6	16 WF 71
130.1	10 WF 100
129.1	12 WF 85
125.6	14 WF 74
123.8	18 I 70
122.6	18 WF 60
119.3	12 WF 79
117.9	16 WF 64
114.8	14 WF 68
114.4	10 WF 89
111.6	18 WF 55
108.1	12 WF 72
106.2	16 WF 58
103.5	18 I 54.7
102.4	14 WF 61
100.8	18 WF 50
97.7	10 WF 77
97.0	12 WF 65
92.7	16 WF 50
90.7	10 WF 72
87.1	14 WF 53
86.5	12 WF 58
82.8	10 WF 66
82.0	16 WF 45
78.51	14 WF 43
78.16	12 WF 53
76.5	15 I 50
75.1	10 WF 60
72.7	16 WF 40
72.57	12 WF 50
70.1	8 WF 67
69.65	14 WF 43
68.6	15 I 42.9
67.0	10 WF 54
64.88	12 WF 45
63.9	16 WF 36
61.49	14 WF 38
60.65	12 I 50
60.3	10 WF 49
59.9	8 WF 58
57.6	12 WF 40
54.95	10 WF 45
54.5	14 WF 34
52.45	12 I 40.8
51.42	12 WF 36
49.0	8 WF 48
47.1	14 WF 30
46.95	10 WF 39
44.37	12 I 35
43.96	12 WF 31
41.58	12 I 31.8
39.9	8 WF 40
38.8	10 WF 33
37.97	12 WF 27
35.16	10 I 35
34.70	8 WF 35
34.70	10 WF 29
32.8	8 M 34.3
30.4	8 WF 31
29.5	10 WF 25
29.35	12 B 22
28.04	10 I 25.4
27.1	8 WF 28
24.78	12 B 19
24.1	10 WF 21
23.42	8 M 24
23.1	8 WF 24
21.56	10 B 19
20.61	12 B 16.5
19.15	8 I 23
19.1	8 WF 20
19.03	6 WF 25
18.63	10 B 17
17.86	6 M 25
17.46	8 M 20
17.39	12 B 14
16.34	8 I 18.4
15.97	10 B 15
15.8	8 WF 17
15.70	8 M 17
15.04	6 WF 20
14.56	6 M 20
14.37	7 I 20
14.21	12 Jr 11
13.59	8 B 15
12.13	10 B 11.5
11.94	7 I 15.3
11.61	6 B 16
11.35	8 B 13
11.35	5 WF 18.5
11.29	6 WF 15.5
11.06	5 M 18.9
10.49	6 I 17.25
9.62	5 WF 16
9.23	10 Jr 9
8.85	8 B 10
8.36	6 I 12.5
8.25	6 B 12
7.37	5 I 14.7
6.29	4 WF 13
6.11	4 M 13
5.70	6 B 8.5
5.57	5 I 10
5.44	8 Jr 6.5
4.03	7 Jr 5.5
4.02	4 I 9.5
3.46	4 I 7.7
2.82	6 Jr 4.4
2.31	3 I 7.5
1.93	3 I 5.7