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STRUCTURAL ANALYSIS

OF

HOT-METAL LADLES

by

KNUD-ENDRE KNUDSEN

An Abridgment of a Dissertation

Presented to the Graduate Faculty
of Lehigh University
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STRESSES IN HOT METAL LADLES

By K. E. KNUDSEN, WM. M. MUNSE and B. G. JOHNSTON

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... this report describes the results of a research investigation sponsored and financed by the Association of Iron and Steel Engineers for the steel industry ...

▲ THE designation "Hot-Metal Ladle," when used in this report, will mean a vessel, lined with refractory material, used for conveying molten metal.

During the extensive growth of the steel industry there has been a consistent trend to increase the capacity of hot-metal ladles. Larger furnaces and the convenience of pouring the whole furnace charge into one ladle have combined to make this increase necessary. At the same time the allowable load on existing ladle cranes and supporting structures is limited, thereby rendering any decrease in ladle dead weight directly applicable to increased capacity for molten metal. The introduction of the welded type ladle in 1932 opened new possibilities for decrease in dead weight (2).^{*} Oval shaped welded ladles came into use as an expedient to increase capacity without interference with height clearances and hook distances originally determined for the round riveted ladles.

Thus, larger ladles are being built, new shapes introduced, and the ratio of dead weight to ladle capacity is forced down. This continual development gives significance to the application of more rational and accurate stress analysis procedures in order to maintain the required safety and dependability of hot-metal ladles. Little design information is available in the technical literature. The design procedures used by the different ladle manufacturers apparently have given completely satisfactory ladles. However, these procedures are in general based on assumptions which have not necessarily been verified by tests.

A program of experimental and theoretical stress analyses of hot-metal ladles was therefore adopted by the Association of Iron and Steel Engineers as a part of the Standardization Committee's postwar program. Fritz Engineering Laboratory of Lehigh University undertook the task of investigating the structural behavior of such ladles, initial work starting on June 15,

1946. The general program was determined at a committee meeting in August 1946. It was decided to test 3 models based on prototypes of 150-ton net capacity. Both riveted and welded construction was considered, and a model of an oval ladle was included. Additional variables were: number and size of stiffener rings, size of trunnions, angle of tilt, amount of load, and distance between the points of support on the trunnion pins. The important problems involving stresses due to temperature differentials caused by the molten metal are not considered in this investigation.

A progress report was presented before the annual convention of the AISE in Pittsburgh, September 1947. The tests were completed in December 1947, and the experimental program, procedures and results are described in detail in a separate test report (1). The present report will therefore cover the conclusions only, illustrated by typical experimental data.

The interest and advice given by Mr. Ingvald Madsen, Research Engineer of AISE; Mr. F. E. Kling, chairman of the ladle design committee, together with other members of the committee, were essential factors in the planning and execution of the program. The valuable help of Mr. Paul Kaar, engineer of tests; and Mr. Kenneth Harpel, foreman, is acknowledged, together with the valuable help of the many student assistants in working up the strain gage data and preparing test result curves.

LADLE MODELS

The ladle types chosen for this investigation represent three general types in use in the mills. They also reflect different developments in ladle design. Ladle "A" is a round riveted ladle, Ladle "B" is an oval welded ladle, and Ladle "C" is a round welded type with a dished bottom. All three specimens are 1/5 scale models of

Presented before AISE Annual Convention, Cleveland, Ohio, September 28, 1948

^{*} Numbers refer to bibliography at end of report.

were made long enough to permit study of the effect of hook spreading, as required by the test program.

Some simplifications were made in the model design, as compared with actual ladles. The slag spout and the pouring device were eliminated. Special joints for facilitating the transport of the ladles were not considered. The additional trunnion pins sometimes used when the ladle is placed in stands were disregarded. It is believed that the elimination of the slag spout at the lip of the ladle is the only significant deviation of those here mentioned. The test results indicate that the effect of the lip ring on the structural behavior of the ladle is greater than emphasized in present design procedures. The serious discontinuity introduced in this ring by the slag spout seems therefore to deserve special consideration. Tests with such spouts added on the models were proposed at one step in the development of the test program but were not adopted.

Ladle "A" is shown in Figure 1. Attention is called to the 1/16 in. additional plate on the bottom, similar plates on actual ladles being provided to protect against heat radiation during the pouring operation. Ladle "A" was tested with two different size trunnion pairs, also with or without a stiffener band combined with the small pair of trunnions. Figure 2 gives details of ladle "B." The oval shape of the ladle is obtained by inserting

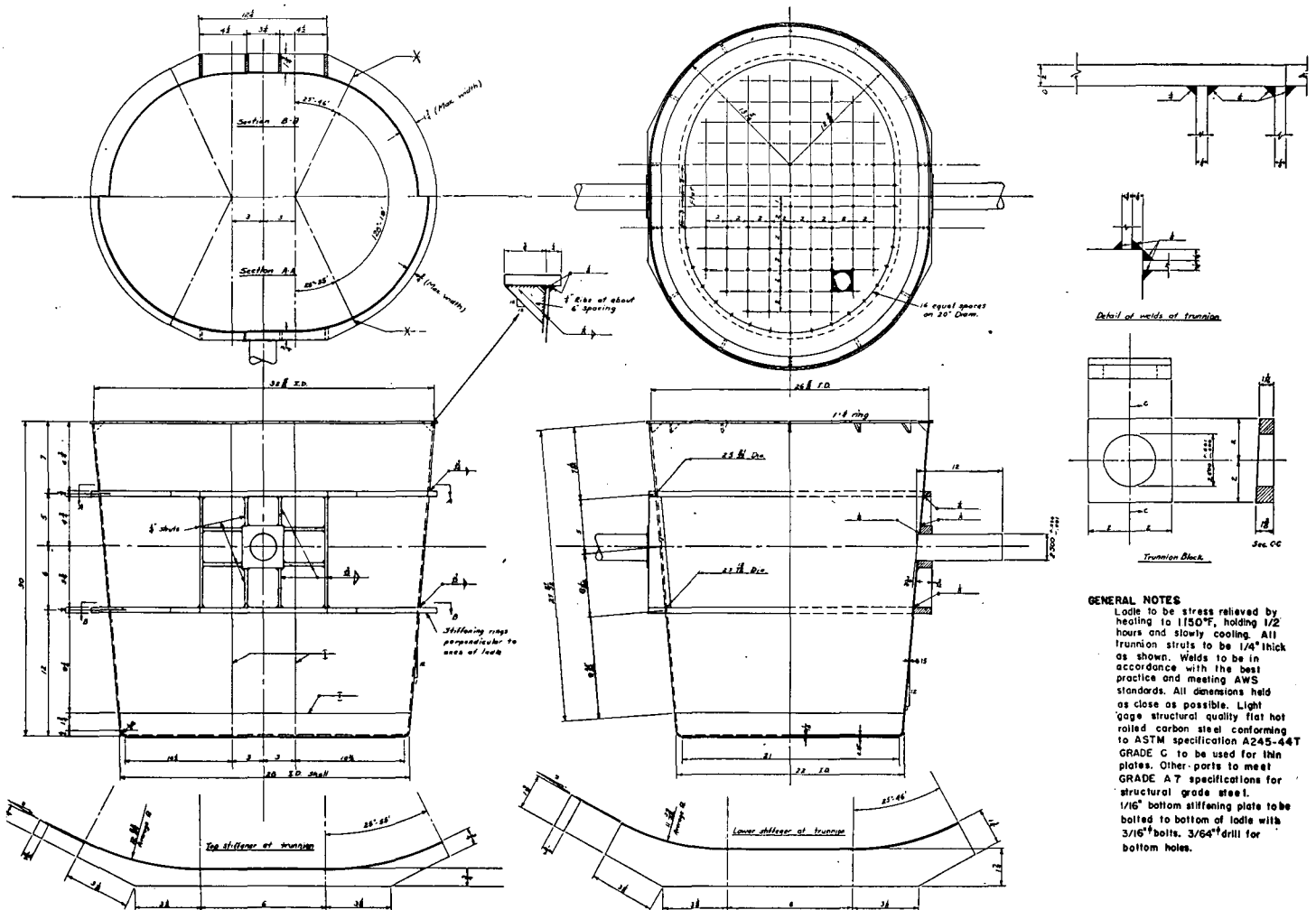
a straight middle section in the sides between the semi-circular parts of the cross section. Ladle "B" has a flat bottom with protection plate as in the case of Ladle "A." On Ladle "C," Figure 3, the bottom is dished and has no reinforcing plate. The stiffener rings are of equal size and comparatively small due to the heavier middle section of the shell between the rings. The trunnions are built up of plates with no ribs of the type used on Ladles "A" and "B," as may be seen from Figures 4, 5, and 6.

To conform with general practice in ladle manufacturing, the models were made of structural carbon steel conforming to ASTM Specification A-7 for heavier material, and to ASTM Specification A 245-44T, Grade C, for the thin plates. The models were stress-relieved for one half hour at 1150 F after fabrication. Before testing, the ladles were lined with fireclay in a way simulating the fire-brick lining used in actual ladles. The thickness of the fireclay was 3/4 in. on the sides and 1 in. on the bottom, slightly more on Ladle "A."

TEST PROGRAM AND RESULTS

In the various tests, strains and deflections were measured at the points indicated in Figure 7 for Ladle

Figure 2 — Ladle "B" is a one-fifth scale model of a 150-ton welded oval ladle.



"A." The locations for the other ladles were similar. The strains were measured by means of bonded electric wire resistance gages, mounted on identical locations on the inside and outside ladle surfaces. The principal stresses and their directions, as well as the horizontal and vertical stress components, were computed from the measured strains. Horizontal deflections of the sides and vertical deflections of the bottoms were measured by means of mechanical deflection dials reading to 1/1000 of one inch. Figure 8 gives an over-all picture of the test set-up.

Actual ladles are loaded with molten metal weighing at an average 420 lb per cu ft. Due to obvious inconveniences in handling molten metal, mercury was used as loading agent in the laboratory tests.

A summary of the complete testing program is presented in Table II. The table lists the testing conditions for all tests on each ladle, and indicates the measurements taken in each case. The variables included in the test program may be found from a study of Table II, and will be pointed out in connection with the discussion of their effect on the structural behavior of the ladles. The complete set of data is given in the unpublished test report, as previously mentioned. The AISE may be consulted if some of this data should be desired.

The general structural behavior of the ladle models

deviates considerably from what is often assumed as a basis for design. A description of this behavior will be given before the effect of the test variables is discussed.

Figures 9 and 10 show representative deflection results. They were obtained for ladle "A" under conditions explained in the figures. Figure 9 shows the general tendency of the horizontal cross section to become oval in shape due to the loading, the trunnion regions moving inward, and the regions between trunnions moving outward. Zero deflection is found approximately 45 degrees from the trunnion lines. The side deflections, as shown on vertical cross sections in Figure 10, are zero at the bottom and increase nearly linearly to the maximum at the lip. Some irregularity is caused near the trunnion pin by the concentrated hook reaction, especially for the larger hook distance. The bottom deflection gives the picture of a partially restrained circular plate under uniform load.

The stresses in the stiffener rings are primarily bending stresses, hence these stresses should be proportional to the change in curvature which may be visualized in Figure 9. This is confirmed by the stress measurements, as shown in Figures 11 and 12, for the outside and inside of the three stiffener rings on ladle "A." On each ring, the maximum stresses occur at the trunnion line and midway between the trunnions. The ring stresses are

Figure 3 — Ladle "C" is a one-fifth scale model of a 150-ton welded round ladle.

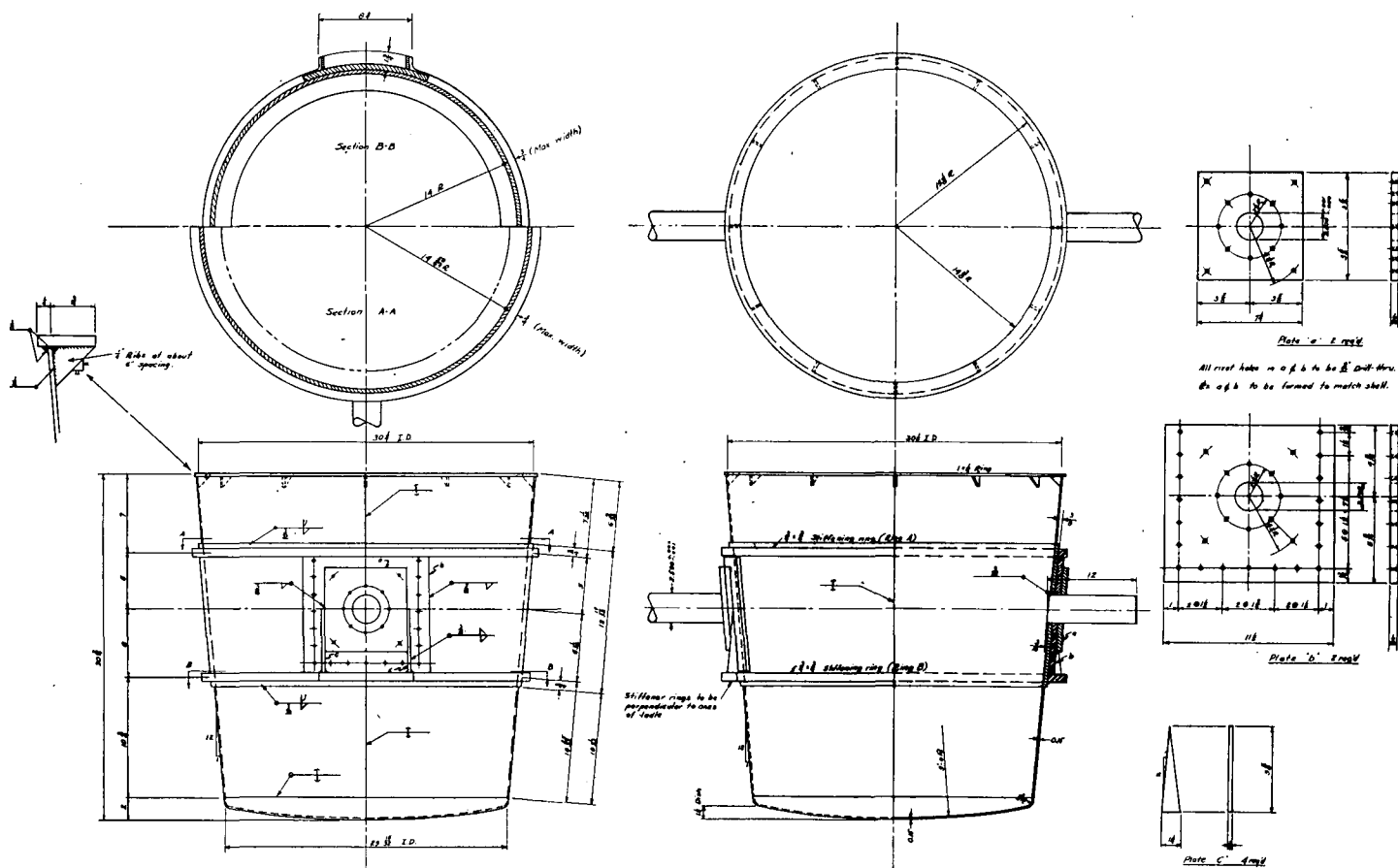


TABLE II
SUMMARY OF THE TEST PROGRAM

Entry No.	Model	Variables							Measurements		
		Loading agent	Trunnion size, in.	Other structural variables	(1) Load amount, per cent	(2) Hook dist, in.	Tilt angle, degrees	(3) Lining thickness, in.	Strains	Deflections	
										Side	Bottom
1	Ladle "A"	Mercury	8 x 8 $\frac{1}{4}$	With band	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
2		Mercury	8 x 8 $\frac{1}{4}$	Without band	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
3		Mercury	16 x 8 $\frac{1}{4}$	Without band	25	2 $\frac{1}{2}$	0	$\frac{1}{2}$.	.	x
4		Mercury	16 x 8 $\frac{1}{4}$	Without band	50	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
5		Mercury	16 x 8 $\frac{1}{4}$	Without band	75	2 $\frac{1}{2}$	0	$\frac{1}{2}$.	.	x
6		Mercury	16 x 8 $\frac{1}{4}$	Without band	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
7		Mercury	16 x 8 $\frac{1}{4}$	Without band	100	7 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
8	Ladle "B"	Water	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	100	2 $\frac{1}{2}$	0	None	x	.	.
9		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	7.2	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	x
10		Water	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	x
11		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	25	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	x
12		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	50	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	x
13		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	75	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	.
14		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
15		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	75	3 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
16		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	75	5 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
17		Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	75	7 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	.
18	Mercury	12 $\frac{1}{4}$ x 10 $\frac{1}{2}$	91	(4)	0	$\frac{1}{2}$	x	.	.	
19	Ladle "C"	Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	7.7	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
20		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	50	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	x	x
21		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	90	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
22		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	90	3 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
23		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	90	7 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
24		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$.	x	x
25		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	100	7 $\frac{1}{2}$	0	$\frac{1}{2}$.	x	x
26		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	98	2 $\frac{1}{2}$	± 6	$\frac{1}{2}$	x	.	.
27		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	82	2 $\frac{1}{2}$	± 20	$\frac{1}{2}$	x	.	.
28		Water	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	With both rings	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
29		Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	Less lower ring	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.
30	Mercury	10 $\frac{1}{8}$ x 11 $\frac{1}{2}$	Less both rings	100	2 $\frac{1}{2}$	0	$\frac{1}{2}$	x	.	.	

- (1) Loads given in volume per cent of load when liquid level is 3 in. from the lip.
- (2) Hook distance is measured from the inside of the shell.
- (3) Lining thicknesses given in comparison with unworn lining on actual ladles.
- (4) In this test the ladle was supported in stands.

largest on the lip ring and smallest on the lower stiffener, as are the deflections.

The normal stresses in the side shell are considerably less than the stiffener ring stresses, except near the juncture with the bottom plate. The same is the case for shear stresses in the side shell. Figure 13, giving the distribution of vertical normal stresses on ladle "B," clearly shows the high local stress peak near the bottom. These high stresses are similar to the discontinuity stresses produced near the heads of pressure vessels. This stress problem has been studied by the design division of the Pressure Vessel Research Committee of the Welding Research Council, and by several others. Although these stresses sometimes exceed the yield point, experience from actual ladles seems to show that they do not endanger the safety of the ladle. The high discontinuity stresses also occur on the bottom-plate side of the juncture. Except in this narrow region, the flat bottoms sustain mainly a high and nearly uniform bending moment throughout, while in the dished bottom smaller membrane stresses prevail.

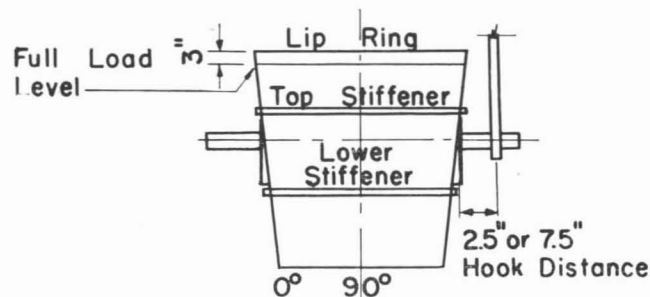
The general description of the structural behavior of the ladle models holds for all tests on all three models, and constitutes perhaps the most important result of this investigation. Actual ladles cover a large field of

types and variations in design. The tests made will therefore not give the complete picture for any of them, but the general behavior as described is believed to be common for all types. The experimental program did, however, consider some of the more important factors in ladle design. The variables included may be divided into two groups, of which the first covers the variation in testing conditions, such as amount of load, distance between the hook supports, and angle of tilt. The other group includes structural variables; riveted versus welded construction, round versus oval shape, flat or dished bottom, trunnion assembly size, and type and number of shell stiffener rings. The effect of these variables will be discussed in the order mentioned.

The effect of amount of load on the deflections and stresses is partly demonstrated by Figures 9 to 15, and so is the effect of variation in the distance between the two supporting hooks. As a rule, stress and deflection data for the stiffener rings increase in almost linear proportion to both load and hook distance, as shown in Figures 16 and 17, respectively. Since the load distribution varies with amount of load, direct proportionality between load and stresses is not obvious. Table III shows the critical stresses in the rings for the smallest and the largest hook distance. A similar, but much less

TABLE III
EXPERIMENTAL STRESSES, PSI

Locations			1	2	3	4	5	6	7	8	9	10
			Full load — 2.5 in. hook distance							Full load — 7.5 in. hook distance		
0°: At trunnions 90°: Between trunnions o : Outside surface i : Inside surface			Ladle "A"			Ladle "B"	Ladle "C"			Ladle "A"	Ladle "B"	Ladle "C"
			8 x 8 in Trunnions	8 x 8 in. Trunnions plus spacer band	8 x 16 in. Trunnions		With both stiffeners	Less lower stiffeners	No stiffeners	8 x 16 in. Trunnions		Both stiffeners
Lip ring	0°	o	-10,800	- 8,100	- 6,900	+ 9,200	- 9,200	-11,200	-16,700	-12,200	+17,900	-18,500
		i	+28,400	+22,800	+23,400		+ Yield	+ Yield	+ Yield	+33,600		+ Yield
ring	90°	o	+ 6,100	+ 4,900	+ 4,000	+ 2,800	+ 3,500	+ 4,700	+10,100	+ 7,700	+ 5,700	+ 6,000
		i	-20,900	-15,500	-18,000	- 6,300	- 5,900	- 7,700	-17,200	-23,800	- 7,500	- 9,500
Top stiffener	0°	o	- 6,500	- 5,400	- 2,200	-10,000	- 3,100	- 3,400	- 3,200	- 4,400	-22,100	- 6,700
		i	+ 3,600	+ 1,200	+ 1,600	+ 3,600	+ 3,700	+ 4,300	+ 4,100	+ 3,400	+ 6,800	+ 7,300
stiffener	90°	o	+ 4,400	+ 3,700	+ 3,500	+ 3,400	+ 5,300	+ 6,700	+ 1,700	+ 5,600	+ 6,400	+ 9,300
		i	- 2,800	- 1,800	- 2,000		- 3,400	- 4,600	- 2,700	- 3,100	- 4,200	- 5,700
Lower stiffener	0°	o	- 2,500	- 1,600	- 1,000	+ 1,600	- 400	+ 100	- 100	- 1,300	+ 3,100	- 200
		i	+ 2,000	+ 1,900	+ 400	- 100	+ 100	- 100	+ 100	- 700	+ 300	- 300
stiffener	90°	o	+ 4,700	+ 4,600	+ 4,200	+ 4,300	+ 4,200	+ 2,000	+ 3,600	+ 7,600	+ 8,400	+ 4,400
		i	- 2,300	- 1,300	- 1,900	- 3,300	- 2,300	- 1,300	- 2,700	- 4,200	- 7,000	- 4,400
Ladle dead weight, without lining, lb			280	350	310	250	340	333	325	310	250	340



the trunnions are one third as great in the riveted ladle as in either of the welded ladles. The experimental stresses, essentials of which are given in Table III, show no consistent difference between riveted and welded construction. The structural efficiency of the ladles should be compared on a dead weight basis, since maximum live load within the ladle crane capacity limit is of prime interest. These weights are therefore given on the bottom of Table III.

Ladle "B" is an oval ladle, while "A" and "C" have circular cross sections. The deflections show the same trend for all three ladles, and the stresses do not allow any conclusions as to preference. It is believed that the small degree of ellipticity commonly used has but a minor influence upon the structural behavior.

consistent variation is obtained for the side shell. Bottom stresses and deflections also increase with the load, but less rapidly than in the rings. This should be expected, because membrane stresses prevail in the bottom when the deflections become larger than the plate thickness. Bottom stresses and deflections show very little response to an increase of the hook distance. One test was made with a ladle supported in stands, resting on the underside of the trunnion assemblies. This condition is essentially equivalent to a smaller hook distance, and the results fall in line with those given above.

Ladle "C" was tested in tilted positions up to 20 degrees, with no increase of the stresses. At more than 20 degrees, the stresses started to decrease due to out-pour of the liquid load.

Ladle model "A" is riveted, while the two others represent welded construction. The three ladles differ also in other ways, and the test results are therefore not directly comparable. As an average, under equivalent conditions, the magnitude of the side deflections are about three times as large for the riveted ladle as for either of the two welded ones. This is at least partly explained by the fact that the average ratios of the bending stiffness of the stiffener rings above and below

Figure 4 — Trunnion assembly for ladle "A."



A study of the results from the tests of the flat bottoms on ladles "A" and "B" and the dished bottom on ladle "C" allows definite conclusions in favor of the dished type. As mentioned earlier, the dished bottom plate carries the load mainly through membrane stresses. The resulting deflection, Figure 15, is therefore only one sixth to one seventh of those developed in the flat bottoms, as shown in Figure 10. The figures also show a different deflected shape for the dished bottom, which does not have the maximum deflection at the center point. The only resemblance between the two types is the insensibility to variation in hook distance. The flat bottom plates sustain high bending stresses, while the dished shape shows tensile stresses throughout its thickness of about one fourth of the magnitude of the measured stresses on the flat bottoms. The discontinuity stresses both in the side shell and in the bottom plate near the knuckle is even more reduced when using a dished bottom.

Ladle "A," the riveted model, was tested with two sizes of trunnion assemblies, 8 and 16 in. wide, covering 34 and 68 degrees of the ladle circumference, respectively. The effect of the wider trunnions is to decrease the ring stresses, as seen from columns 1 and 3 of Table III. The reduction is largest on the lip ring and

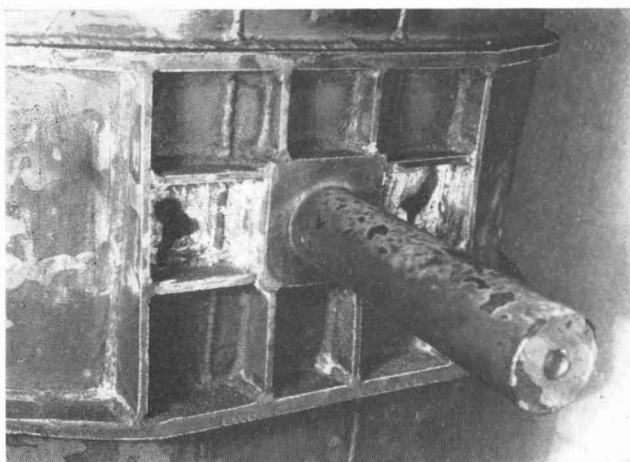


Figure 5 — Trunnion assembly for ladle "B."

on the top stiffener near the trunnion, and averages 43 per cent. The side deflections with the wide trunnions are about 75 per cent of those obtained with the narrow type. The bottom stresses and deflections stay practically unchanged.

Columns 1 and 2 in Table III give information on the effect of the addition of a spacer band on ladle "A." This $\frac{1}{4} \times 8\frac{1}{4}$ in. band, which is shown in Figure 1, is riveted to the sides of the trunnions, but not connected with the ladle shell. It is intended to carry most of the bending moment from trunnion to trunnion, thus relieving the regular reinforcing rings. Table III shows an average reduction in ring stresses with 26 per cent of those obtained without the spacer band. The additional weight of ladle steel material due to the band is 25 per cent. Stresses in the side shell are also decreased, although not altogether consistently. The bottom stresses are not affected by the addition of the spacer band. A comparison of columns 2 and 5 in Table III seems to indicate that the same advantage may be



Figure 6 — Trunnion assembly for ladle "C."

obtained in a cheaper way by adding the equivalent steel weight to the ladle shell thickness between the top and lower stiffeners. Higher lip ring stress is registered in the latter case. Deflection tests show, on the other hand, that the heavier shell more effectively resists the local bending effects caused by the moments introduced through the trunnions.

Finally, the effect of removal of stiffener rings was investigated on ladle "C." Some of the test stresses are recorded in columns 5 to 7 in Table III. It should be remembered in interpreting these results that ladle "C" has a heavy middle shell section and correspondingly weak rings. Removal of the lower stiffener ring increases the stresses on the top stiffener by 23 per cent and the lip ring stress by 27 per cent on an average. With both the top and lower stiffeners removed, the lip ring stresses increase to an average of 143 per cent above those recorded with both stiffeners in place. No test has been made with only the top stiffener removed, but the tests made indicate that the top stiffener is comparatively more efficient than the lower stiffener. The deflection tests, Figure 15, also point to this conclusion, since the deformation at the top ring is considerably larger than at the other rings.

DESIGN RECOMMENDATIONS AND STRESS ANALYSIS

The conclusions which can be drawn from the test results, as presented in the previous chapter, are incorporated in the design recommendations below, which also summarize recommendations made in various articles on ladle design. Statements which are referred to articles and not to parts of this investigation should be regarded as the opinion of the author of the reference.

Material — Ordinary low-carbon steel is recommended for hot-metal ladle construction. The temperature effects and high cost discourage the use of high-strength alloys as a means of reducing the dead weight. For the trunnions, a 3 per cent nickel, low carbon steel is sometimes advised as being stronger and more resistant to spills of metal and slag (4). Reference 6 recommends forged steel ASTM A-235 Class B for the trunnions.

For trunnion pins and other parts for which reliable design methods allow the use of high stresses, the im-

portance of a smooth finish of the entire surface is emphasized (3).

Main ladle proportions—The ladles are usually shaped like a frustum of a cone with a side slope, or increase in radius, of one inch per 12 to 15 inches of the height.

The quasi-elliptical or oval cross section shape, introduced to increase the capacity under existing mill conditions, is usually obtained by inserting a straight shell section at the trunnions. By making these middle sections slightly curved, lining conditions are improved (5). The ratio of minimum to maximum cross section width usually falls between 1 and $\frac{3}{4}$. On the oval ladle model this ratio is 0.82. No noticeable difference was found in structural behavior between the circular and the oval ladle models. The round shape is preferred because of considerations of manufacturing cost and lining conditions.

In Figure 18, the ladle height and diameter, or average diameter for oval shapes, is given as a function of the carrying capacity for the ladles listed in Table I. A similar diagram for capacities up to 100 tons is given in Reference (3). It is seen that the height is slightly larger than the diameter up to approximately 12-foot height, or 125-ton capacity. Above that tonnage, the height is smaller than the diameter. In Reference (4) it is advised that the diameter only should be increased above 125-ton capacity in order to keep down the head of metal during pouring.

The location of the center of gravity of a fully loaded ladle of 12-foot height should be about 15 inches below the trunnion pin center line, and correspondingly placed

Figure 7 — The general location of the gages are shown in this drawing for ladle "A." Similar locations were used on the other ladles.

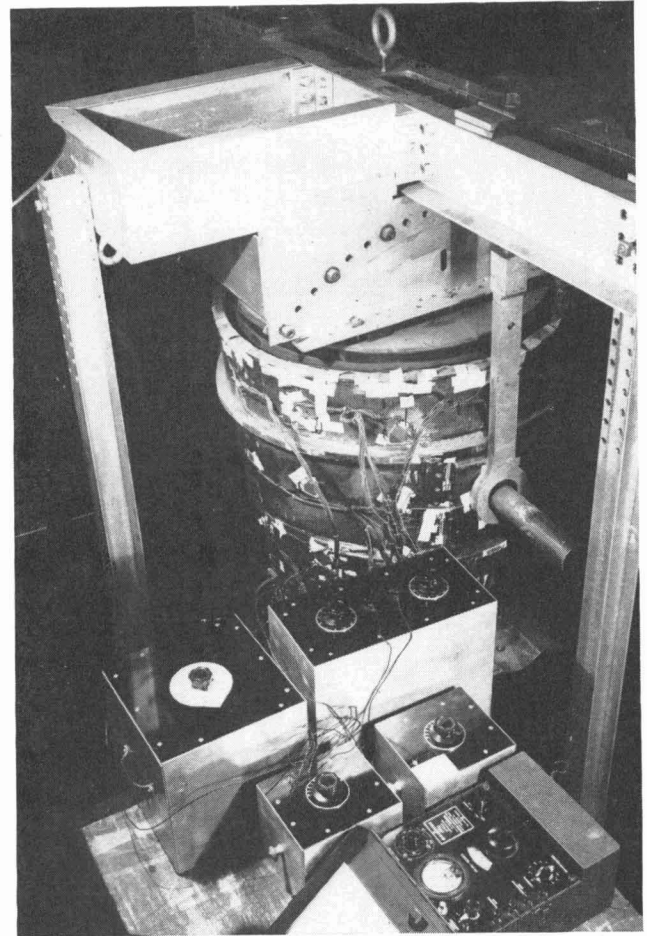
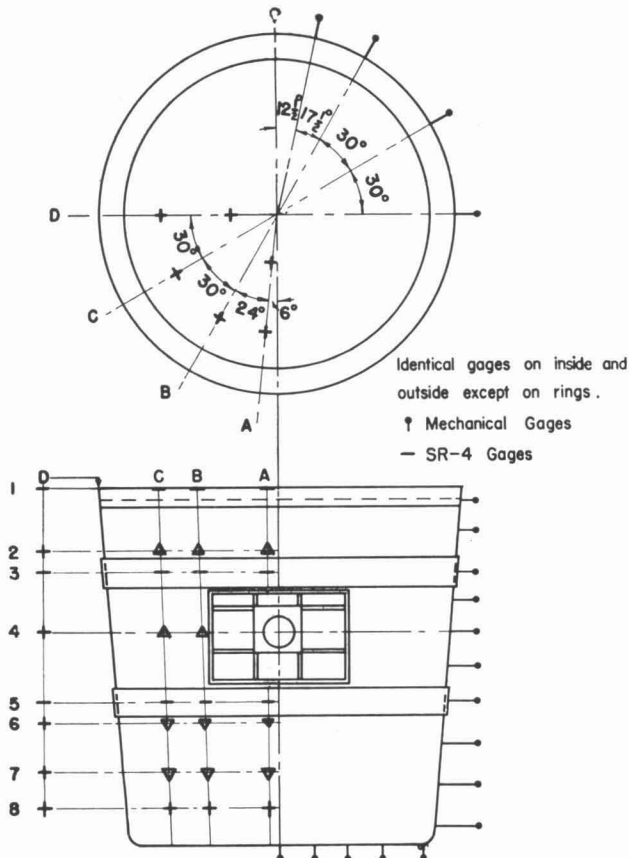


Figure 8 — In this setup for the strain tests, the equipment for measuring the strains is seen in the foreground. The ladle is seen in the background in the vertical position. Tape seen on the ladle is used to keep the leads from the electric strain gages in place.

for other size ladles (4). A method for calculating the location of the center of gravity and the tipping moment is given in Reference (10). In Reference (6) it is recommended that the center of gravity be $d = 6 + 0.09 C$ inches below the trunnion center line, where C is the ladle capacity in net tons. If this formula is used, lock bars to prevent tilting should not be required.

Type of construction — The two commonly used types of ladle construction, riveted and electric arc welded, were both represented in the experimental investigation. The riveted model underwent much larger deflections than the welded ones under equivalent conditions, but this was at least partly due to the lighter design of the riveted model. The experimental stresses showed no consistent difference between the two types.

The steel weight of the riveted ladle model was 280 lb as compared to 250 and 340 lb for the two welded models (Table III). In actual ladle design, the older riveted types, without spouts, lining and stopper rings, weigh 21 to 27 per cent of the rated capacity. The same percentage for similar welded types of more than 50-ton capacity is only 16 to 17 per cent. Welding gives lighter ladles for capacities above 60-70 tons. Considering the weight of slag, lining, etc., welded construction allows an increase in capacity above that of riveted construction by about 7 per cent for a 150-ton ladle (4).

The joints in the shell should be butt welded. All welding should be specified in accordance with the

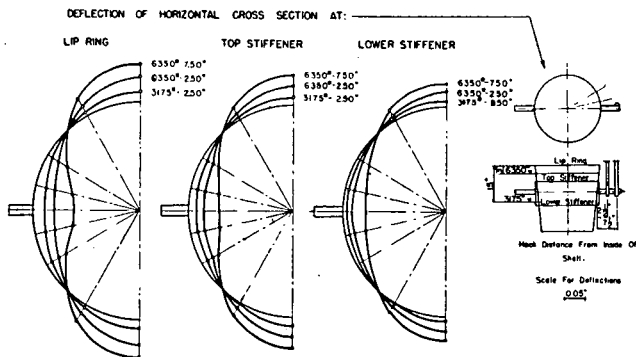
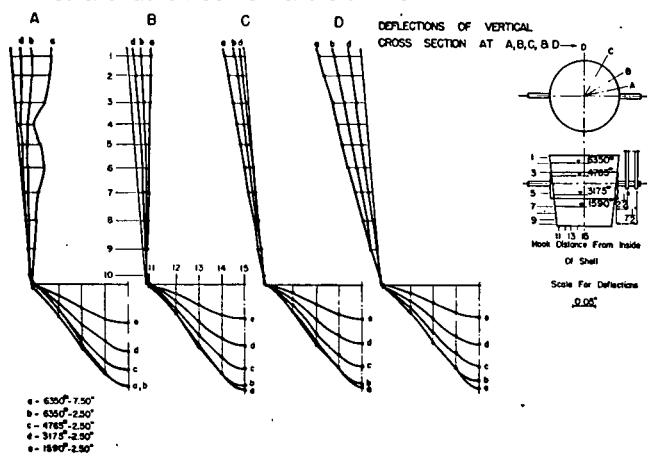


Figure 9 — The deflections for ladle "A" through horizontal planes are given for various loads and for two positions of the ladle hook on the trunnion.

Figure 10 — The deflections for ladle "A" through vertical planes are given for various loads and for two positions of the ladle hook on the trunnion.



applicable codes of the American Welding Society. The opinions on the necessity of stress relieving after welding seem to be divided. Most manufacturers, it is believed, stress relieve the ladles at 1200-1300 F for one hour per inch thickness of the heaviest material.

Design assumptions — The design assumptions should be in agreement with the general structural behavior observed during the tests. The ladle sides are found to deflect inward in their full height in the trunnion region, and outward in the region midway between the trunnions. The deflections are nearly zero at the bottom juncture, increasing approximately linearly towards the maximum deflections at the lip ring. A circular ladle cross section will, accordingly, tend to become elliptical under load, the amount of ellipticity increasing from bottom to lip. Vertical lines on the side shell located about 45 degrees from the trunnions experience no deflections. Thus, the maximum side deflections at each level occur along vertical lines through the trunnions and half-way between the trunnions, the absolute maximum being obtained at these locations on the lip ring.

The highest stresses on the reinforcing rings were obtained at the same locations, and followed the same pattern as the maximum deflections. High stresses were also measured in the bottom plates, and discontinuity peak stresses were found in a narrow region on both sides of the side-bottom juncture.

At these critical locations, as well as on other parts of the ladle, the most unfavorable condition is obtained

with full liquid load and the longest distance between the supporting hooks. Tilting of the originally fully loaded ladle has no increasing effect upon the stresses. It follows that, in practical design, the ladle should be assumed loaded to the slag spout level with liquid metal, and the hook spacing should be chosen no larger than required by operating conditions.

Dead weight assumptions — The specific weight of molten metal varies with the grade of the steel. A suitable design value is 420 lb per cu ft. As mentioned earlier, the net dead weight of the ladle is close to 1/4 of the rated capacity for riveted ladles, and 1/6 for welded types of more than 50-ton capacity. Spout, stopper, etc, will increase these fractions by 10-15 per cent. The weight of the fire-brick lining will vary with the preferred thickness. Common lining thicknesses are 9-14 in. on the bottom, and 9 in. decreasing to 7 in. on the sides from the bottom up. For ladles of about 150-ton net capacity, the lining weight is ordinarily 80-85 per cent of the total steel dead weight.

Trunnions — The trunnions on ladles of 100-ton capacity or more are usually of welded construction. Castings are now mostly used for smaller ladles. The trunnions should be connected to the stiffener rings which are usually provided above and below the trunnions in order to minimize local shell stresses due to the trunnion reaction. Tests show a beneficial effect of comparatively wide trunnions.

Figure 18 shows recommended trunnion pin diameters for different ladle capacities. Reference (6) advises that the pin diameter should not be reduced where it fits into the trunnion plate, and if the pin diameter is increased at that location, a large fillet radius should be provided. The stresses in the trunnion pins can be calculated by means of conventional methods.

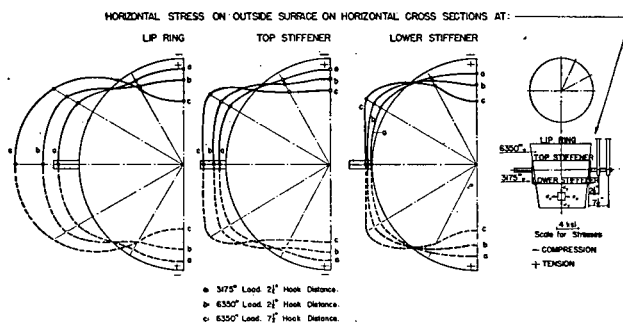
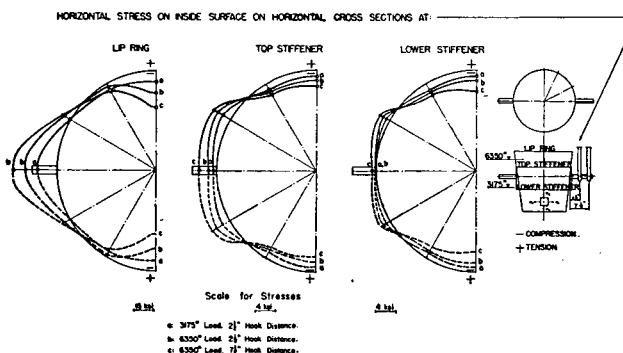


Figure 11 — The horizontal stresses on the outside of the ladle "A" are given for several loadings and hook distances for the ladle in the vertical position.

Figure 12 — The horizontal stresses on the inside of the ladle "A" are given for several loadings and hook distances for the ladle in the vertical position.



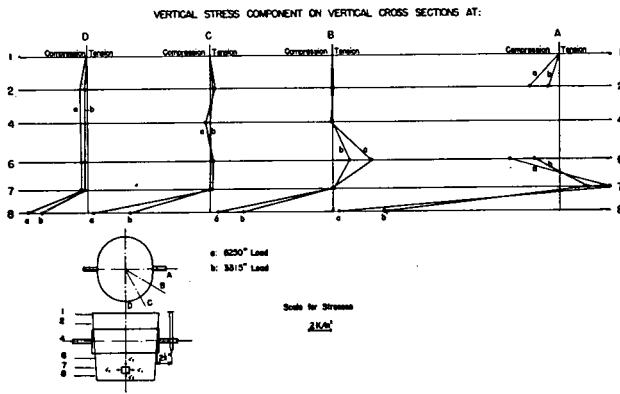


Figure 13 — Outside stresses through vertical planes of ladle "B" for two loads with a hook distance of 2½ in. from the inside of the shell.

Side shell — The thicknesses commonly used for ladle side shells are within 10 per cent either way from those shown in Figure 18. In order to obtain comparable stresses in the model and prototype, the model shell thicknesses were made as small as possible, considering available materials. The model corresponds to a ¾ in. thickness on the 150-ton capacity prototype. The stresses in the model, loaded with mercury, should be about 3/5 of the stress in the corresponding prototype, loaded with molten metal. No severe stresses were observed, except near the bottom juncture, as shown in Figure 19, which records the average of the outside and inside side shell and bottom stresses for a section through ladle "C." Practical operating considerations probably require a thicker ladle shell than required by stress analysis, and the former will therefore be the determining factor in selection of the shell thickness, at least for ladles up to 150-ton capacity.

The problem of a strict analytical determination of the discontinuity stresses in the side shell near the bottom juncture is a very complicated one in the case of a ladle. The trunnion reaction caused a non-uniform distribution of these stresses around the ladle circumference, with a peak directly below the trunnions. Even on flat bottoms, a small knuckle radius is ordinarily provided, which further complicates the analysis. On riveted ladles, the lap-joint and the discontinuous connection add to the inadequacy of conventional stress calculation methods. Most disturbing, however, is the effect of the very large deflections of flat ladle bottoms.

A complete theoretical attack on the problem (not yet finished for use in practical design) is given in Reference (11). The problem is treated in a more practical way in Reference (12), and also in Timoshenko's "Theory of Plates and Shells," Chapter XI. The procedure below is based on the two latter references. Development of the formulas may be found in Reference (13).

With notation as explained in the nomenclature, and disregarding all the disturbing factors mentioned above, an impression of the magnitude of the discontinuity stresses may be obtained by the following procedure. Let:

$$\beta^4 = \frac{3(1-\nu^2)}{r^2 t^2} = \frac{2.73}{r^2 t^2} \dots \dots \dots (1)$$

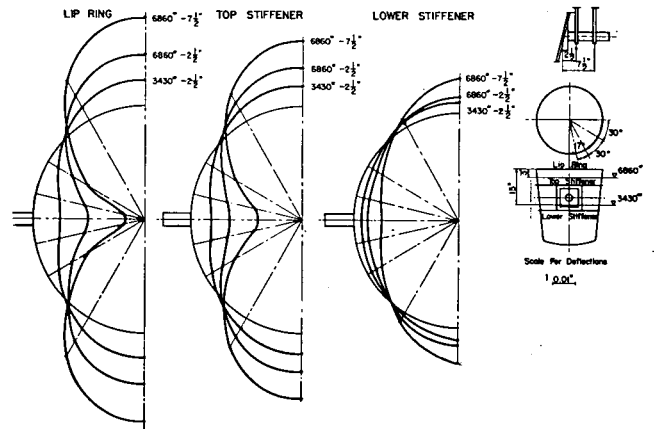


Figure 14 — The deflections through horizontal planes of ladle "C" are given for no-tilt of the ladle.

The moment per inch of the circumference acting on the bottom end of the side shell is then, with Poisson's ratio taken as 0.3:

$$M_o = (-pr) \frac{0.525 rt + 0.425 \beta h^3}{4.200 t + \beta^3 h^3 r} \dots \dots \dots (2a)$$

A positive moment caused tension at the outside shell surface. The radial end shear at the same location, positive when acting inward on the side shell, is:

$$P_o = \frac{0.425 p}{\beta} - \beta M_o \dots \dots \dots (3a)$$

The moment acting in the vertical direction per inch width of the shell at any point at a distance y from the bottom juncture is:

$$M_y = M_o \phi + \frac{P_o}{\beta} \xi \dots \dots \dots (4)$$

and in the circumferential, or horizontal, direction:

$$M_x = 0.3 M_y \dots \dots \dots (5)$$

The functions

$$\left. \begin{aligned} \phi &= e^{-\beta y} (\cos \beta y + \sin y) \\ \xi &= e^{-\beta y} \sin \beta y \\ \psi &= e^{-\beta y} (\cos \beta y - \sin \beta y) \\ \theta &= e^{-\beta y} \cos \beta y \end{aligned} \right\} \dots \dots \dots (6)$$

where e is the base of the natural logarithms, are tabulated for values of βy between zero and 7.0 on page 394 of Timoshenko's "Theory of Plates and Shells," First Edition. Considering both the ordinary membrane stresses and the discontinuity effect, the total stresses in the shell near the flat bottom are then:

$$\sigma_y = \frac{pr}{2t} \pm \frac{6M_y}{t^2} \dots \dots \dots (7)$$

in the longitudinal direction, and:

$$\sigma_x = \frac{pr}{t} - \frac{2\beta r}{t} \left[\beta M_o \psi + P_o \theta \right] \pm \frac{6M_x}{t^2} \dots \dots (8)$$

in the circumferential direction.

From the table referred to above it may be seen that the discontinuity stresses decrease rapidly with increasing distance from the bottom juncture. Beyond $\beta y = \pi$, or, using Equation (1) and $\nu = 0.3$,

$$y = 2.44 \sqrt{rt} \dots \dots \dots (9)$$

TABLE IV
COMPARISON OF EXPERIMENTAL AND COMPUTED NORMAL STRESSES IN THE SIDE SHELL NEAR THE BOTTOM JUNCTURE MIDWAY BETWEEN THE TRUNNIONS — FULL MERCURY LOAD

Ladle	Distance from juncture (in.)	σ_x : horizontal σ_y : vertical		Experimental stresses (psi)	Computed stresses (psi)	Ratio: $\frac{\text{computed}}{\text{experimental}}$
		o: outside i: Inside				
"A" Flat bottom	3.43	σ_x	o	+1,600	+ 800	0.50
			i	+ 0	+ 0	1.00
		σ_y	o	+2,200	+1,800	0.82
			i	-1,300	- 700	0.54
"B" Flat bottom	1.43	σ_x	o	-8,100	-8,200	1.01
			i	-5,600	-6,000	1.07
		σ_y	o	-4,600	-3,100	0.67
			i	+4,400	+4,100	0.93
"C" Dished bottom	0.10 (d=1.90)	σ_x	o	-2,500	-8,200	3.28
			i	-4,500	-9,200	2.04
		σ_y	o	+2,100	+2,000	0.95
			i	-1,200	-1,100	0.92

These stresses are without any practical concern. The calculation above, on the other hand, will also show that the maximum stress for ordinary ladle dimensions exceeds the yield point. The maximum stress will, in most cases, occur in the longitudinal direction for $y=0$, and can then be computed by putting $M_y = M_o$ in Equation (7).

It can be shown, that the most favorable bottom shape from a combined economy and stress analysis point of view is an ellipsoid with a depth of about one quarter of the bottom radius. However, a shallower bottom, as generally used for ladles, in comparison with a flat bottom will greatly reduce the discontinuity stresses. The stress calculation in case of a dished bottom follows the same procedure and is subject to the same reservations outlined above for flat heads. Only the expressions (2a) and (3a) will be changed. Assuming that the ordinary two-radius dished bottom deviates

little from the ellipsoidal shape, and that the knuckle radius is comparatively large, these expressions are substituted by

$$M_o = \left(-P_o \right) \left(\frac{h^{1.5} - t^{1.5}}{2 \beta (h^2 + t^2) h^{0.5}} \right) \dots \dots \dots (2b)$$

$$P_o = \left(\frac{p}{2 \beta d^2 h} \right) \left(\frac{r^2 t + 1.7 (h-t) d^2}{4 + \frac{3h^2 - t^2}{h^2 + t^2} \left[\frac{h^{1.5} - t^{1.5}}{h^{1.5}} \right]} \right) \dots (3b)$$

For bottom thickness h equal to the side shell thickness t the expressions become

$$M_o = 0 \dots \dots \dots (2c)$$

$$P_o = \frac{pr^2}{8 \beta d^2} \dots \dots \dots (3c)$$

The tests of the ladles were not specifically designed

Figure 15 — The deflections through vertical planes of ladle "C" are given for no-tilt of the ladle.

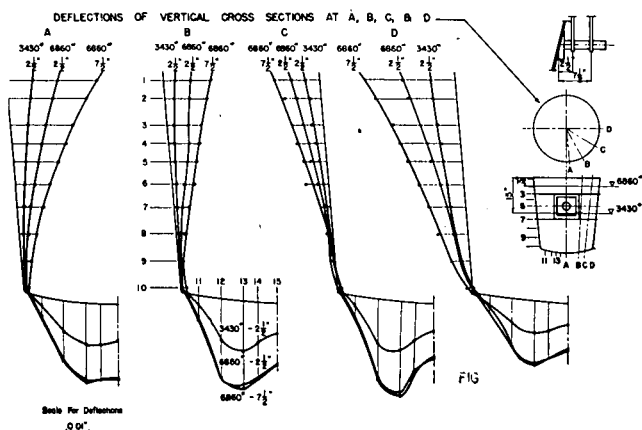
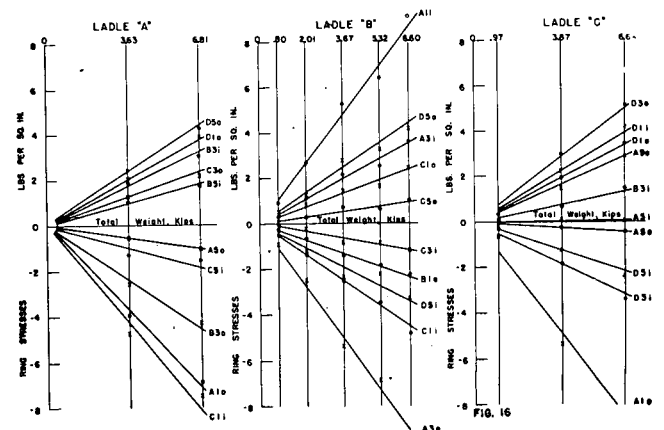


Figure 16 — The measured ring stress for different weights in the ladle indicate that they increase directly with the load.



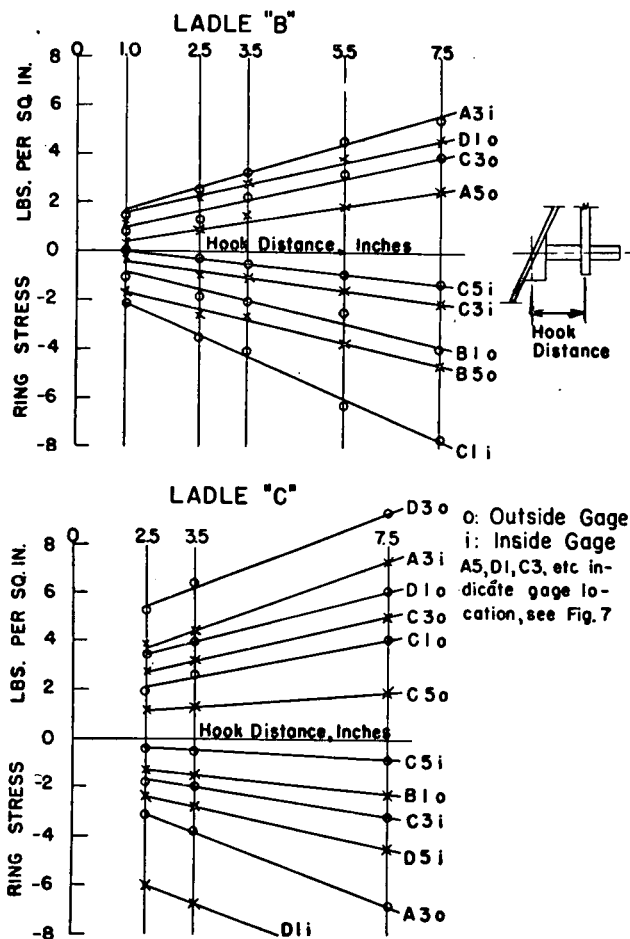


Figure 17 — The ring stresses increased directly with the moment arm of the ladle hook on the trunnion.

to investigate the problem of discontinuity stresses, which in themselves constitutes a large field of research. The deviations of a ladle from the ideal cases covered by the above formulas are also too many and significant to allow a close check with experimental stresses. A comparison, Table IV, is therefore carried out only for one gage point on each ladle located near the bottom juncture midway between the trunnions where the disturbances are smallest. The discrepancy in Table IV for the horizontal stress on ladle "C" may be explained by the location of the gage, which was mounted very close to the reinforced bottom butt weld. Also, the expressions (2b) and (2c) are not exact.

The corresponding experimental discontinuity stresses directly below the trunnions are two to four times as large as those midway between the trunnions. The calculated maximum stress at the juncture ($y=0$) is far above the yield point for ladles "A" and "B" with the flat bottoms, and 10,300 psi for ladle "C" with a dished bottom.

Bottom plate — A dished bottom is recommended as advantageous in several ways as compared to the flat type. The dished bottom is usually of the shallow dish type tank head, shaped like a spherical segment, with a smaller knuckle radius giving an arch tangent to the head flange (sometimes called torispherical). The maximum deflections of the flat plates were six to seven times as large as the maximum measured for the dished bottom, and exceeded the thickness of the bottom

including the cover plate. The maximum deflection of the dished bottom was only about 0.2 of its thickness, and the latter also gave smaller over-all stresses. The discontinuity peak stresses at the juncture of side and bottom are greatly reduced when a dished bottom is used, especially if a large knuckle radius is provided. Flat bottoms usually become semi-dished after long service, indicating that the yield point is exceeded in some parts of the plate. The dished type provides inter-locking of the bottom fire-bricks, decreasing the danger of damage due to the liquid flotation force. One practical advantage of using a flat bottom is that the ladle may be set directly on the ground without danger of tilting.

The bottom should be furnished with a removable cover plate for protection against heat radiation during the pouring process. The cover plate is spaced away from the bottom by means of washers, and can be renewed or removed to allow inspection. The cover plate extends over the whole bottom or parts of it, according to the pouring schedule applied (4). A similar plate is also recommended underneath the slag spouts, extending down to the lower stiffener ring (6). Pouring openings in the bottom should be reinforced like holes on pressure vessels.

Common thicknesses of the main bottom plate, flat or dished, are shown in Figure 18. The thickness is usually a little larger than for the side shell, but may vary up to 20 per cent in either direction from those indicated in the figure. The tests showed that the bottom stresses depend only upon the depth of the liquid load, and that an analytical determination of the plate thickness must include both bending and membrane stresses.

For flat bottom plates, such a stress computation may be based upon the diagrams in Figure 20, reproduced from Timoshenko's "Theory of Plates and Shells," First Edition, pages 340 and 341. A ladle bottom edge is not completely fixed, as assumed in the diagrams. The strain measurements on the bottom were few (Figure 7), and do not coincide with the location of the stresses given in Figure 20. Although disregarded in actual design, the presence of the cover plate on the ladle model impedes a verification of the computed stresses. Using the added thickness of the main plate and cover, the diagram gives a maximum deflection of 0.17 in. as compared to 0.21 in. measured. The calculated model stresses at the center for full load are +22,100 psi at the outside and -11,700 psi at the inside surface. The corresponding experimental stresses at 0.32 times the bottom radius from the center point were +18,900 psi and -14,600 psi. The plate connection efficiency factor and the smaller actual degree of edge restraint add to give a too low calculated deflection and bending stress, and subtract in their effect on the calculated membrane stress. The maximum computed stress at center, which occurs on the outside surface, is believed to be a good criterion of the required plate thickness. The edge stresses will be smaller than given by this method due to imperfect clamping.

For a ladle of 150-ton capacity the empirical diagram in Figure 18 indicates a 1.3 in. thick bottom plate. With other dimensions as given in Figure 18, the stress in the bottom plate is found to be +33,200 psi and

—18,000 psi at the outside and inside surface at center, respectively. The experience that flat bottoms will undergo permanent deformation during service thus seems verified by stress calculation. Whether yielding should be tolerated as in the past, or be prevented by increasing the thickness or using a dished plate, will be a matter of judgment.

ASME's Boiler Construction Code of 1946, Sections I and VIII, for Power Boilers and Unfired Pressure Vessels, and also the joint API-ASME 1938 Code for Unfired Pressure Vessels for Petroleum Liquids and Gases, require a minimum flat head thickness h inches according to the formula

$$h = d \sqrt{\frac{cp}{s}} \dots \dots \dots (10)$$

where

- d = plate diameter, or shortest span, in.
- p = maximum load, psi
- s = allowable unit working stress, psi
- $c = 0.25$ for butt welded circumferential bottom joint
- $c = 0.30$ for lap-riveted circumferential bottom joint

For the 150-ton prototype with 1.3 in. bottom plate, this formula gives 16,000 psi maximum stress. The formula thus leads to thicknesses of the order now ordinarily used, but fails to warn against the actual high stresses.

The codes mentioned above also give a formula for the required thickness of dished heads:

$$h = \frac{5 p R}{6 s} \dots \dots \dots (11)$$

where the symbols have the meaning as explained for Equation (10). However, the codes restrict the radius of curvature, R , to be smaller than the bottom diameter. For hot-metal ladles this radius is usually much larger, and a too small thickness therefore results from this

formula. Assuming radii of bottom curvature proportional to the one used on the model (Figure 3), Equation (11) gives a maximum bottom stress of about 15,000 psi for ladles of ordinary dimensions as shown in Figure 18. The actual stresses are larger. Thus, for the dished bottom model, Equation (11) gives $s_{max} = 3,700$ psi, where 8,100 psi was measured near the bottom center.

The discontinuity stresses near the bottom edge are of a magnitude similar to those earlier discussed for the side shell. These stresses can only be reduced to below the yield point by using a dished bottom, and therefore were not discussed in connection with the design of flat bottoms.

A stress analysis procedure for dished bottoms will not be attempted here. The problem is discussed in the article "Stresses in Dished Heads of Pressure Vessels" by C. O. Rhys, ASME Trans. 1931. When finished, the work of reference (11) will probably give the most complete answer.

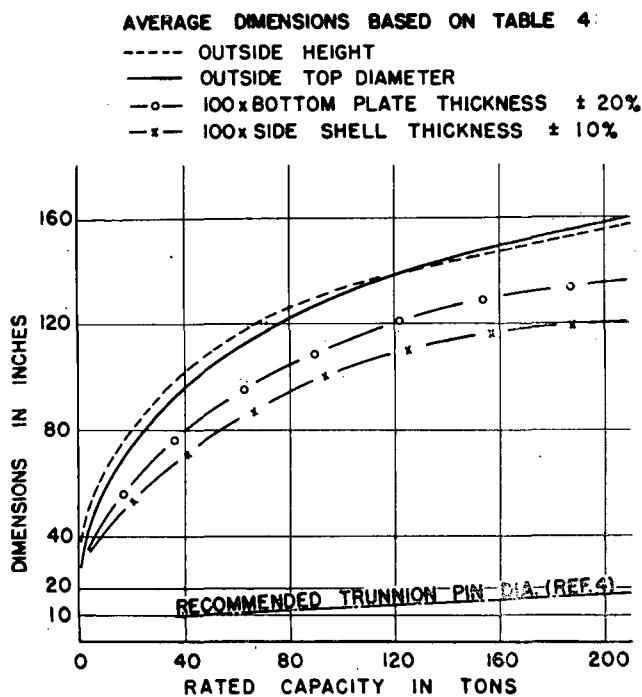
Reinforcing rings — The ladle models all had stiffener rings at the lip and above and below the trunnions. Investigation of the effect of the two trunnion rings indicated that the most efficient use of ring material is obtained at the location above the trunnions. Tests alone would suggest emphasizing the lip ring in ladle design. Practical experience shows, however, that the lip ring often suffers great damage from the hot slag and metal splash during service. The safety of a ladle should therefore not be made dependent on the presence of this ring. Also, the actual load carrying contribution of the lip ring may be lessened because of the discontinuity introduced by the spouts, which were eliminated on the models. It is therefore not recommended to follow the test indication towards an increase of lip ring dimensions. In order to meet the condition after long time service, the stresses should be calculated both considering and disregarding the lip ring.

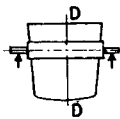
Reinforcing of the shell against bending and twisting moments is sometimes accomplished by other means than the stiffener rings discussed above. A wide stiffener band, connected to the trunnions but not to the shell, reduced the experimental stresses. However, a similar result seems obtainable by adding the equivalent steel material to the regular stiffener rings or to the shell thickness between the trunnion rings. Reinforcing by means of a heavy middle shell section and rather small rings, as on one of the ladle models, was found to give a very rigid ladle with small deflections, but the stresses were not correspondingly reduced.

The locations and conditions for the maximum stiffener ring stresses are discussed earlier. It follows that, in designing the rings, it is sufficient to calculate the stresses at the trunnion line and midway between the trunnions on each ring for full liquid load and the largest hook spacing which may occur during operation.

The adjacent rings above and below the trunnion have an increased stiffness by virtue of their connection to the trunnion assemblies. Even if the trunnion assemblies are connected to the side shell only, as in the case of ladle "A" (Figure 1), the rings will be somewhat restrained in the neighborhood of the trunnions. Due to this effect, the two trunnion rings will ordinarily obtain the maximum stress midway between the trunnions, as may be seen from Table III. The increase in ring stiffness at the trunnions is not easily calculated.

Figure 18 — The variation of ladle dimensions for various capacity ladles is given in this diagram.





Section D-D

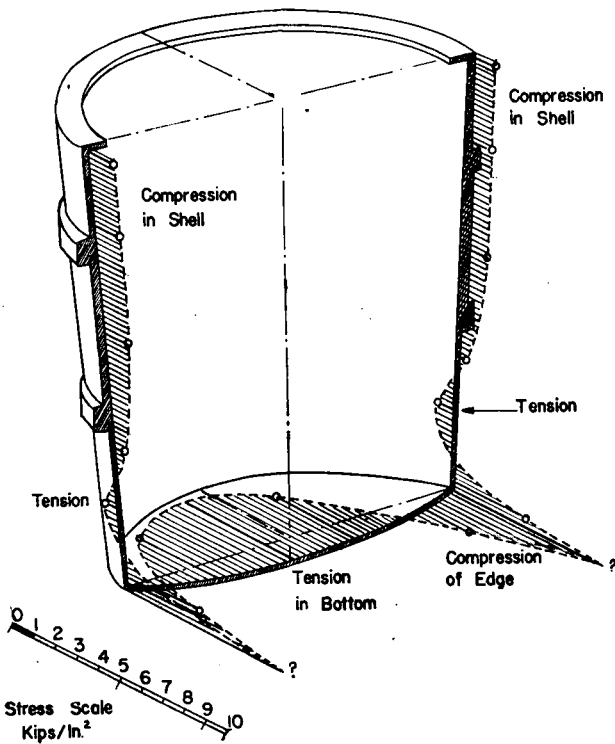


Figure 19 — Graphical view of horizontal membrane stresses on a vertical cross section halfway between the trunnions is here given for ladle "C" for full load, with no tilt of the ladle, and with a hook distance of $2\frac{1}{2}$ in.

If the degree of restraint is obviously large, as for ladle "C" (Figure 3), the problem is of no consequence since the stresses midway between the trunnions will govern. The same is the case when wide trunnions are used: Rows 1 and 3 of Table III show that an increase of trunnion width from 34 to 68 degrees of the ladle circumference moves the critical section of the top stiffener from the trunnion line to midway between the trunnions. Without the increased strength due to the trunnions, all three rings would obtain maximum stress at the trunnion line. As earlier recommended, the trunnions should therefore be made comparatively wide, and be connected directly to the stiffener rings. Following this recommendation, it is necessary and sufficient to calculate the stresses at the following locations:

Lip ring (Ring 1): 0 and 90 degrees

Top stiffener (Ring 2): 90 degrees

Lower stiffener (Ring 3): 90 degrees

The designations are explained on Table III.

An exact analysis of the stresses in a hot-metal ladle, if possible, would be extremely complicated. A practical procedure must compromise between exactness and simplicity. The semi-rational method proposed emphasizes the latter. A more theoretical attack was attempted, using methods outlined in the article "Stresses in a Reinforced Monocoque Cylinder Under Concentrated Symmetric Transverse Loads," by N. J. Hoff, *Journal of Applied Mechanics*, Vol. 11, No. 4. Probably due to neglecting the bending stiffness of the

side shell, this method gives unsatisfactory numerical results for hot-metal ladles.

The method given below is partly based upon general trends of the experimental results, which limits the range of its usefulness to ladles of types not basically different from those tested. Details in development of the procedure are avoided here, and are given in Reference (13).

The following are the assumptions on which the stress analysis procedure is based:

1. The deflections of the ladle side increase linearly, from zero at the bottom juncture, to a maximum at the lip.
2. The rings adjacent to the trunnion assemblies are subjected to concentrated inward transverse loads, as shown in Figure 21. When the lip ring is disregarded these loads are the only ring loads, and must therefore, acting separately, cause deflections in accordance with assumption 1.
3. The lip ring deflects in agreement with assumption 1 due to a loading imposed by a side shell shear flow q psi which, on a horizontal section through the ladle, is distributed according to

$$q = a_2 \sin 2\phi + a_4 \sin 4\phi \dots \dots \dots (12)$$

where a_2 and a_4 are different in each of the shell fields between two rings or between the lower stiffener and the bottom. The angle ϕ is measured as shown in Figure 23.

4. The stiffener rings would experience no stress if the ladle were supported on an inward extension of the trunnion pins at points determined experimentally (Figure 23). The first assumption is in agreement with the test experience, demonstrated in Figures 10 and 15, that the ladle side deflects inward at all levels in the trunnion region. It was sometimes assumed in ladle design that the lower stiffener deflected outward in that region due to the moment introduced through the trunnion pin. The tests showed that this effect is of negligible importance.

The two concentrated loads P on each ring at each trunnion represent a simplification of the actual distribution of the load imposed upon the ring by the shear flow in the side shell. Assuming a triangular resultant load over the width of the trunnion assemblies, Figure 22, and P -loads are one third of the total trunnion width apart. This assumption gives stresses for different trunnion widths which are in good agreement with the experimental results. The use of concentrated loads on the rings adjacent to the trunnion assemblies, and a distributed loading on the lip ring, is most readily explained in connection with the fourth assumption. The radial deflections of a circular ring of uniform moment of inertia subjected to concentrated loads as explained are

$$\Delta_r = \frac{Pr^3}{2EI} \left[\frac{\pi}{2} - \alpha + (2 - \cos \alpha) \sin \alpha - \frac{4}{\pi} (\alpha \sin \alpha + \cos \alpha) \right]$$

at the trunnions (0 degrees), and

$$\Delta_r' = \frac{Pr^3}{2EI} \left[\frac{4}{\pi} (\alpha \sin \alpha + \cos \alpha) - 1 - \sin^2 \alpha \right]$$

midway between the trunnions (90 degrees), an increase in radius being taken as positive. The fore-

going equations may be obtained from Reference (14), page 153. Assumption 2 then leads to the following relation between the loads P_2 on the top stiffener and P_3 on the lower stiffener:

$$P_3 = \frac{B_3}{B_2} P_2 \dots \dots \dots (13)$$

where B is the product of the ring bending stiffness and the distance of the ring from the bottom. For the lip ring,

$$B_1 = \frac{EI_1}{r_1^3} h_1$$

and for the top and lower stiffener the subscripts 2 and 3, respectively, are used.

The skin shear is set up by the tendency for relative tangential movement between a ring and the adjacent side shell when the ring deflects. The skin shear must therefore be zero at $\phi = 0$ degrees and $\phi = 90$ degrees (Figure 23), where the ring deflections are purely radial. The shear distribution must also be symmetrical about the two axes of symmetry of a horizontal ladle cross section. A sine-series, using only the two first terms as in Equation (12), will satisfy these requirements. The additional terms cause negligible ring stresses, and are dropped.

The resultant skin shear load on a ring equals the difference in shear in the two shell fields adjacent to the ring. Writing

$$c_{22} = \Gamma_2 a_{22} - \Gamma_1 a_{12} \dots \dots \dots (14)$$

etc., where the first subscript refers to the ring number and the second subscript to the sine series term number,

Figure 20 — The deflections and stresses in a uniformly loaded circular plate with a clamped edge is given in this figure as presented by Timoshenko's "Theory of Plates and Shells."

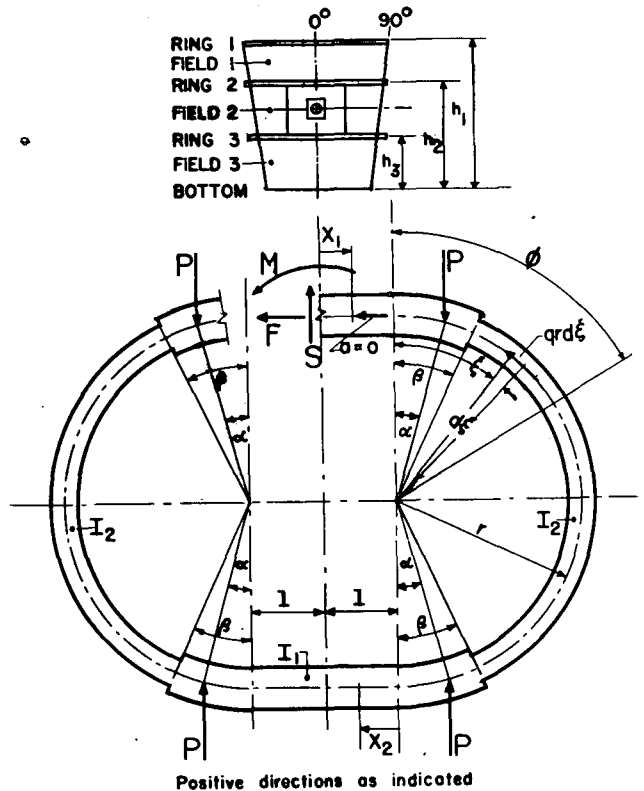
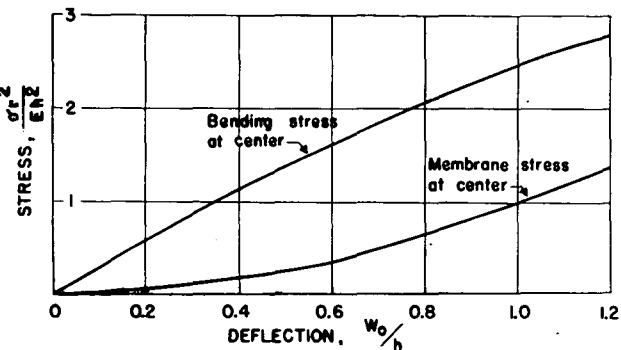
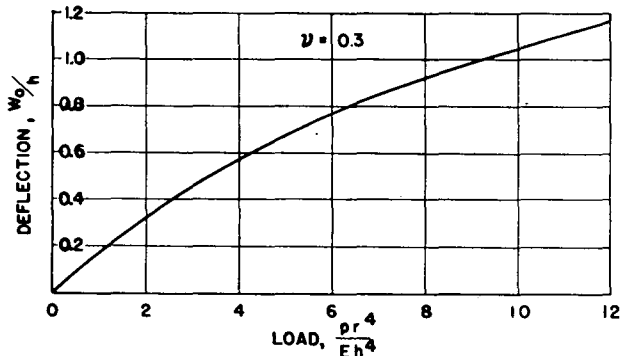


Figure 21 — The free body diagram which illustrates forces acting on the ladle are given in this figure.

the loadings over an arc $d\xi$ of the circumference of the rings are, avoiding details in the development,

$$\left. \begin{aligned} q_1 r_1 d\xi &= c_{12} \sin 2\xi + c_{14} \sin 4\xi \\ q_2 r_2 d\xi &= c_{22} \sin 2\xi + c_{24} \sin 4\xi \\ q_3 r_3 d\xi &= c_{32} \sin 2\xi + c_{34} \sin 4\xi \end{aligned} \right\} \dots \dots \dots (15)$$

For simplicity, these loadings are assumed to act at the neutral axis of the rings. (Figure 21.)

The fourth assumption listed above is in fair agreement with the deflection pictures, Figures 10 and 15. With the assumed P -loads and shear loadings, Equation (15), the equations expressing straight-line deflection of the sides at the trunnion line and midway between the trunnions may be written (13):

$$\left. \begin{aligned} c_{22} &= \frac{B_2}{B_1} c_{12} - P_2 g(\alpha) \\ c_{32} &= \frac{B_3}{B_1} c_{12} - P_3 g(\alpha) \\ c_{24} &= \frac{B_2}{B_1} c_{14} - P_2 f(\alpha) \\ c_{34} &= \frac{B_3}{B_1} c_{14} - P_3 f(\alpha) \end{aligned} \right\} \dots \dots \dots (16)$$

where $g(\alpha)$ and $f(\alpha)$ are given by Equations (24) and (25).

Figure 16 shows that the ring stresses are nearly proportional to the amount of load. Figure 17 indicates a linear relationship between the ring stresses and the hook distance. If the stress diagrams in Figure 17 are extended to the left, they will all cut the horizontal axis $\sigma = 0$ at or near one point, located approximately a distance $r_t - r \sin 45$ degrees inside the ladle shell.

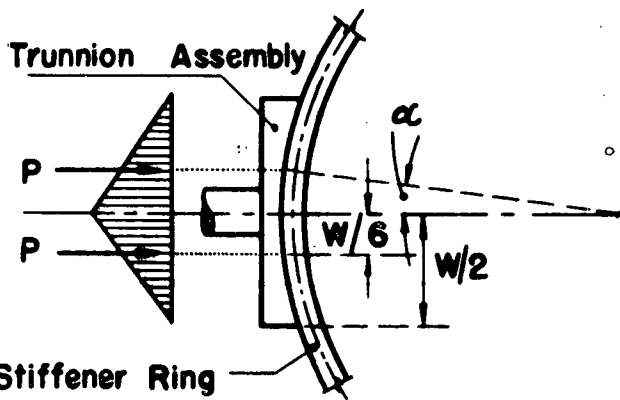


Figure 22 — A triangular resultant load is assumed to act over the width of the trunnion assemblies.

Thus, if the ladle were supported at such points, the ring stresses would become zero. Furthermore, the straight stress lines in Figure 17 show that the ring stresses are directly proportional with the distance "a," Figure 23, from the points discussed above to the actual point of support on the trunnion pins.

A free body diagram, obtained by cutting along section m-m, Figure 23, is shown in Figure 24. If the ladle were supported at a point on the vertical line through the intersection of section m-m and the bottom plate, the free body with zero ring stresses would be in equilibrium. Now, while moving the point of support the distance "a" to its actual location, ring stresses and the shell shear "S" on the cut section are set up. Including the horizontal normal shell forces in the components N_i of the ring forces at the cut section, equilibrium of the free body requires the moment about the lower end of the cut section to be zero:

$$\frac{W}{2} a - 2N_1 h_1 - 2N_2 h_2 - 2N_3 h_3 = 0 \dots \dots \dots (17)$$

A similar free body diagram of the ladle cut along n-n, Figure 23, furnishes another moment equilibrium condition:

$$2N_1 h_1 + 2N_2 h_2 + 2N_3 h_3 = 0 \dots \dots \dots (18)$$

Consistent with the assumed ring loadings it may be shown that,

$$N_2 = \frac{\sqrt{2}}{6} c_{22} + \frac{4\sqrt{2}}{30} c_{24} - P_2 \dots \dots \dots (19)$$

$$N_2 = -\frac{\sqrt{2}}{6} c_{22} + \frac{4\sqrt{2}}{30} c_{24} \dots \dots \dots (20)$$

Similar expressions hold for rings 1 and 3.

The development of suitable formulas for the ring loadings P_2 , P_3 and the six c-coefficients now reduce to a matter of solving Equations (16) to (20). The resulting expressions are given in Equations (26) to (28).

Summarized, the stress analysis procedure for circular ladles includes the following steps. Use the auxiliary expressions:

$$a = r_t - 0.7071 r + e \dots \dots \dots (21)$$

$$B_1 = \frac{EI_1}{r_1^3} h_1, B_2 = \frac{EI_2}{r_2^3} h_2, B_3 = \frac{EI_3}{r_3^3} h_3 \dots \dots \dots (22)$$

$$k = \left(\frac{Wa}{4} \right) \left(\frac{1}{B_1 h_1 + B_2 h_2 + B_3 h_3} \right) \dots \dots \dots (23)$$

$$g(\alpha) = 4.5 [0.5708 - \alpha + (2 - \sin \alpha - \cos \alpha) \sin \alpha] \dots \dots \dots (24)$$

$$f(\alpha) = 225 [2.5708 - \alpha + (2 + \sin \alpha - \cos \alpha) \sin \alpha - 2.5464 (\alpha \sin \alpha + \cos \alpha)] \dots \dots \dots (25)$$

to determine the ring loadings:

$$P_2 = \left(\frac{Wa}{4} \right) \left(\frac{B_2}{B_2 h_2 + B_3 h_3} \right) \dots \dots \dots (26)$$

$$P_3 = \frac{B_3}{B_2} P_2$$

$$c_{12} = -g(\alpha) B_1 k$$

$$c_{22} = -g(\alpha) [B_2 k - P_2] \dots \dots \dots (27)$$

$$c_{32} = -g(\alpha) [B_3 k - P_3]$$

$$c_{14} = -f(\alpha) B_1 k$$

$$c_{24} = -f(\alpha) [B_2 k - P_2] \dots \dots \dots (28)$$

$$c_{34} = -f(\alpha) [B_3 k - P_3]$$

The moments and normal forces in the stiffener rings are then,

In the lip ring, at 0 degrees:

$$M_1 = \frac{r_1}{60} (10c_{12} + c_{14}) \dots \dots \dots (29a)$$

$$F_1 = -\frac{r_1}{15} (10c_{12} + 4c_{14}) \dots \dots \dots (30)$$

In the lip ring, at 90 degrees:

$$M_1 = -\frac{r_1}{60} (10c_{12} - c_{14}) \dots \dots \dots (31a)$$

$$F_1 = \frac{1}{15} (10c_{12} - 4c_{14}) \dots \dots \dots (32)$$

In the top stiffener, at 90 degrees:

$$M_2 = -\frac{r_2}{60} (10c_{22} - c_{24}) + P_2 r_2 [1 - 0.6366 (\alpha \sin \alpha + \cos \alpha)] \dots \dots \dots (33a)$$

$$F_2 = \frac{1}{15} (10c_{22} - 4c_{24}) - P_2 \dots \dots \dots (34)$$

In the lower stiffener, at 90 degrees:

$$M_3 = -\frac{r_3}{60} (10c_{32} - c_{34}) + P_3 r_3 [1 - 0.6366 (\alpha \sin \alpha + \cos \alpha)] \dots \dots \dots (35a)$$

$$F_3 = \frac{1}{15} (10c_{32} - 4c_{34}) - P_3 \dots \dots \dots (36)$$

The sign convention applied above is shown in Figure 21. Tensile normal forces and moments causing tension at the outer ring surface are positive. The total ring stresses are finally found by means of the well-known formulas

$$\sigma_o = \frac{F}{A} + \frac{M}{S_o} \dots \dots \dots (37)$$

$$\sigma_i = \frac{F}{A} - \frac{M}{S_i}$$

TABLE V
COMPARISON OF COMPUTED AND EXPERIMENTAL STIFFENER RING STRESSES

Locations 0°: At trunnions 90°: Between trunnions o: Outside surface i: Inside surface		Full load — 2.5 in. hook distance					
		Ladle "A"				Ladle "B"	
		8 x 8 in. Trunnions		8 x 16 in. Trunnions			
		Computed psi	Measured psi	Computed psi	Measured psi	Computed psi	Measured psi
Lip ring	0° o	- 7,200	-10,800	- 6,600	- 6,900	- 3,900	+ 9,200
	90° i	+21,900	+28,400	+20,000	+23,400	+ 7,900	+ 9,200
Top stiffener	90° o	+ 5,300	+ 6,100	+ 5,300	+ 4,000	+ 4,100	+ 2,800
	90° i	-14,100	-20,900	-14,000	-18,000	- 7,600	- 6,300
Lower stiffener	90° o	+ 5,100	+ 4,400	+ 4,800	+ 3,500	+ 4,300	+ 3,400
	90° i	- 3,200	- 2,800	- 3,000	- 2,000	- 3,100	- 3,100
	90° o	+ 4,500	+ 4,700	+ 4,000	+ 4,200	+ 5,400	+ 4,300
	90° i	- 2,400	- 2,300	- 2,600	- 1,900	- 5,100	- 3,300

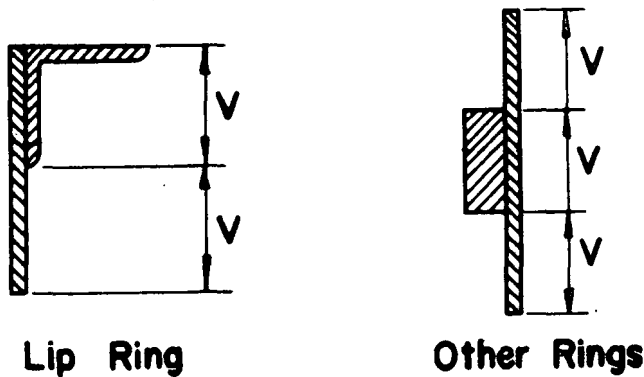
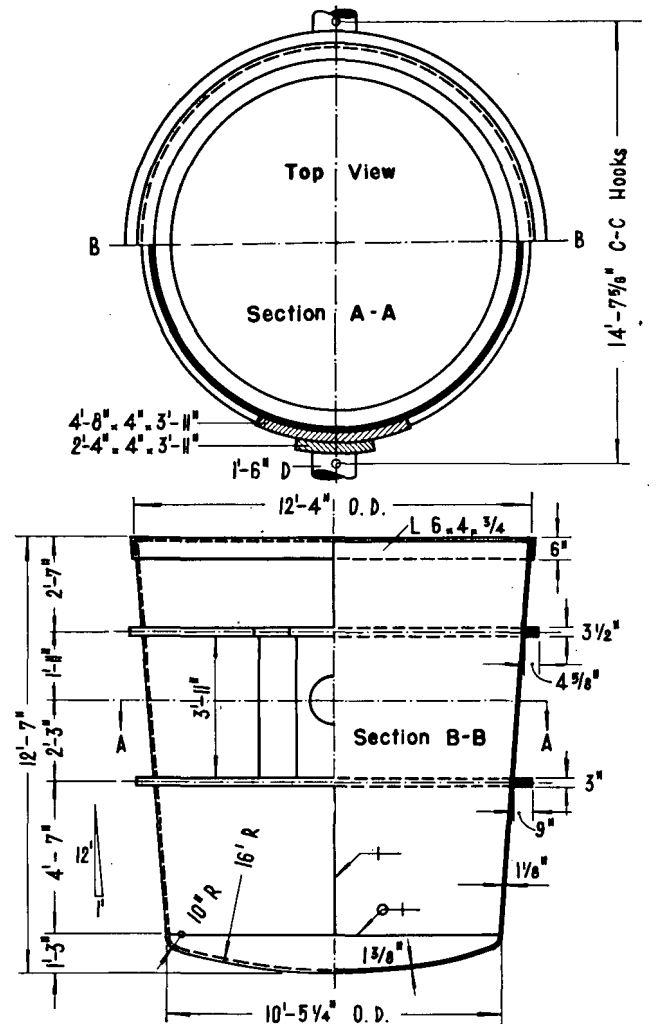


Figure 25 — The shell width which acts with the ring is given in this diagram.

The discontinuity stresses in the side shell may be found by means of Equations (1) to (9), and other stresses checked by conventional methods.

Figure 26 — The diagram shows the dimensions of the ladle used in the numerical example.



however, it represents a distinct improvement over methods now in use.

The ring stresses listed in Tables III and V correspond to about 2/5 of the stresses for a prototype ladle loaded with molten metal.

NUMERICAL EXAMPLE

For a numerical stress analysis example, a round, welded ladle of 150-ton capacity is chosen, Figure 26. The ladle is meant to illustrate the procedure only, and does not necessarily represent a desirable design. Ladle height, top diameter, plate thicknesses and trunnion pin diameter are chosen in accordance with the empirical trend, Figure 18. The ladle capacity may be checked by means of the formula

$$W_c = \pi \gamma r^2 H + \frac{\pi \gamma}{3} (r_H^3 - 3r_H r^2 + 2r^3) \dots \dots \dots (40)$$

where the radii should be taken to the inside of the lining, which here is assumed to average 8 in. in thickness. With a maximum depth of 9 ft-6 in. of liquid metal, and $\gamma = 420$ lb per cu ft, Equation (39) gives $W = 150$ tons.

Only the stiffener ring stresses will be calculated here.

TABLE V — Continued
COMPARISON OF COMPUTED AND EXPERIMENTAL STIFFENER RING STRESSES

Locations 0° At trunnions 90°: Between trunnions o: Outside surface i: Inside surface		Full load — 2.5 in. hook distance						Full load — 7.5 in. hook distance					
		Ladle "C"						Ladle "A"		Ladle "B"		Ladle "C"	
		With both stiffeners		Less lower stiffeners		Less both stiffeners		8 x 16 in. Trunnions				Both stiffeners	
		Com- puted psi	Meas- ured psi	Com- puted psi	Meas- ured psi	Com- puted psi	Meas- ured psi	Com- puted psi	Meas- ured psi	Com- puted psi	Meas- ured psi	Com- puted psi	Meas- ured psi
Lip ring	0° o	-5,300	-9,200	-8,700	-11,200	-18,900	-16,700	-11,000	-12,200	-7,200		-8,800	-18,500
	0° i	+11,100	+Yield	+17,100	+Yield	+37,400	+Yield	+33,300	+33,600	+14,800	+17,900	+18,500	+Yield
Lip ring	90° o	+3,900	+3,500	+6,300	+4,700	+13,800	+10,100	+8,800	+7,700	+7,600	+5,700	+6,500	+6,000
	90° i	-8,100	-5,900	-11,400	-7,700	-24,900	-17,200	-23,900	-29,800	-14,100	-7,500	-13,500	-9,500
Top stiffener	90° o	+8,200	+5,300	+8,400	+6,700	+5,900	+1,700	+8,000	+5,600	+8,000	+6,400	+13,600	+9,300
	90° i	-5,200	-3,400	-5,000	-4,600	-6,200	-2,700	-5,000	-3,100	-5,700	-4,200	-8,600	-5,700
Lower stiffener	90° o	+4,800	+4,200	+1,600	+2,000	+3,500	+3,600	+6,500	+7,600	+10,100	+8,400	+8,000	+4,400
	90° i	-3,000	-2,300	-1,600	-1,300	-3,700	-2,700	-4,300	-4,200	-9,500	-7,000	-5,000	-4,400

The dimensions given in Figure 26 yield

$$\begin{aligned}
 I_1 &= 10.4 \text{ in.}^4 & I_2 &= 86.5 \text{ in.}^4 & I_3 &= 372.0 \text{ in.}^4 \\
 S_{10} &= 8.4 \text{ in.}^3 & S_{20} &= 24.6 \text{ in.}^3 & S_{30} &= 63.3 \text{ in.}^3 \\
 S_{1i} &= 3.8 \text{ in.}^3 & S_{2i} &= 38.9 \text{ in.}^3 & S_{3i} &= 87.7 \text{ in.}^3 \\
 A_1 &= 20.4 \text{ in.}^2 & A_2 &= 28.0 \text{ in.}^2 & A_3 &= 37.1 \text{ in.}^2 \\
 h_1 &= 136 \text{ in.} & h_2 &= 105 \text{ in.} & h_3 &= 55 \text{ in.} \\
 r_1 &= 73.52 \text{ in.} & r_2 &= 71.32 \text{ in.} & r_3 &= 69.17 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 t &= 1.125 \text{ in.} & E &= 30,000,000 \text{ psi} \\
 h &= 1.375 \text{ in.} & w &= 56.0 \text{ in.}
 \end{aligned}$$

$$e = 16.1 \text{ in.} \quad \sin \alpha = \frac{w}{6r_t} = 0.1300$$

$$\begin{aligned}
 r &= 62.06 \text{ in.} & \cos \alpha &= 0.9915 \\
 r_t &= 71.75 \text{ in.} & \alpha &= 0.1304
 \end{aligned}$$

$$\begin{aligned}
 \text{Capacity, Equation (39), } W_c &= 300,000 \text{ lb} \\
 \text{Steel dead weight } W_s &= 0.19W_c = 57,000 \text{ lb} \\
 \text{Lining } W_L &= 0.85W_s = 48,000 \text{ lb} \\
 W &= 405,000 \text{ lb}
 \end{aligned}$$

$$\begin{aligned}
 \text{Equation (21): } a &= 43.13 \text{ in.} \\
 \text{Equation (22): } B_1 &= 106,000 \text{ lb} & B_2 &= 751,000 \text{ lb} \\
 & & B_3 &= 1,855,000 \text{ lb}
 \end{aligned}$$

$$\begin{aligned}
 \text{Equation (23): } k &= 0.0224 \\
 \text{Equation (24): } g(\alpha) &= 2.496 \\
 \text{Equation (25): } f(\alpha) &= 4.590
 \end{aligned}$$

The ring loadings are:

$$\begin{aligned}
 \text{Equations (26): } P_2 &= 18,100 \text{ lb} & P_3 &= 44,800 \text{ lb} \\
 \text{Equations (27):} & & & \\
 c_{12} &= -5,900 \text{ lb} & c_{22} &= 3,300 \text{ lb} & c_{32} &= 8,200 \text{ lb} \\
 \text{Equations (28):} & & & \\
 c_{14} &= -10,800 \text{ lb} & c_{24} &= 6,200 \text{ lb} & c_{34} &= 15,100 \text{ lb}
 \end{aligned}$$

The moments and normal forces in the rings due to these loadings become:

$$\begin{aligned}
 \text{Lip ring, 0 degrees:} & \\
 \text{Equation (29a): } M_1 &= -85,000 \text{ lb in.} \\
 \text{Equation (30): } F_1 &= 7,000 \text{ lb} \\
 \text{Lip ring, 90 degrees:} & \\
 \text{Equation (31a) } M_1 &= 59,000 \text{ lb in.} \\
 \text{Equation (32): } F_1 &= -1,000 \text{ lb} \\
 \text{Top stiffener, 90 degrees:} & \\
 \text{Equation (33a): } M_2 &= 430,000 \text{ lb in.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Equation (34): } F_2 &= -14,000 \text{ lb} \\
 \text{Lower stiffener, 90 degrees:} & \\
 \text{Equation (35a): } M_3 &= 1,033,000 \text{ lb in.} \\
 \text{Equation (36): } F_3 &= -35,000 \text{ lb} \\
 \text{By disregarding the lip ring} & \\
 c_{12} = c_{22} = c_{32} = c_{14} = c_{24} = c_{34} &= 0 \\
 \text{and} &
 \end{aligned}$$

Figure 27 — The graph shows numerical values of $g(\alpha)$ and $f(\alpha)$ as functions of the ratio of trunnion width w to the side shell radius, r_t at the trunnion level.

$$\begin{aligned}
 g(\alpha) &= 4.5 [0.5708 - \alpha + (2 - \sin \alpha - \cos \alpha) \sin \alpha] \\
 f(\alpha) &= 225 [2.5708 - \alpha + (2 + \sin \alpha - \cos \alpha) \sin \alpha \\
 &\quad - 2.5464 (\alpha \sin \alpha + \cos \alpha)]
 \end{aligned}$$

$$\text{where } \alpha = \sin^{-1} \frac{w}{r_t}$$

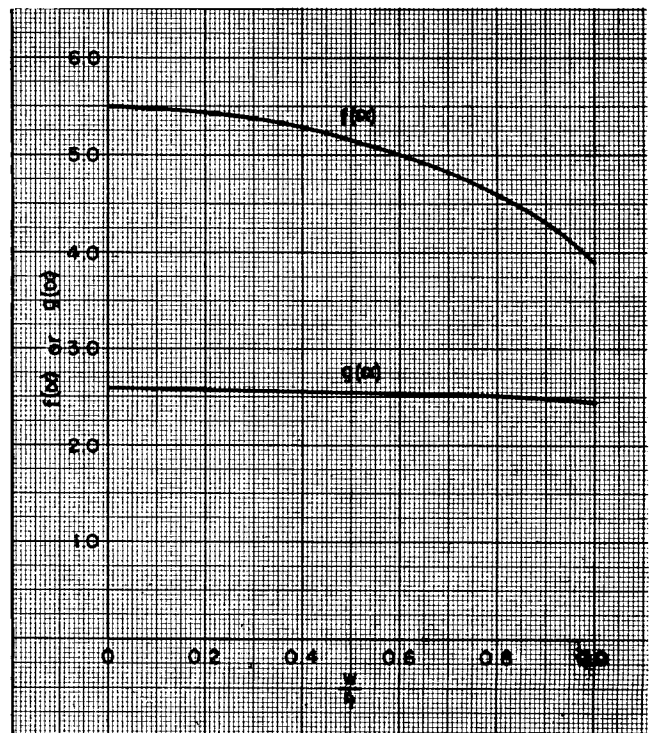


TABLE VI — CALCULATED CRITICAL RING STRESSES
IN LADLE, FIGURE 26

Ring	Surface	Maximum stress with lip ring, psi	Maximum stress without lip ring, psi
Lip ring	o	- 9,800
	i	+22,700
Top stiffener	o	+17,000	+18,200
	i	-11,600	-12,500
Lower stiffener	o	+15,400	+16,300
	i	-12,700	-13,900

$P_2 = 18,100$ lb $P_3 = 44,800$ lb
as before the corresponding moments and normal forces in the stiffeners are, Top stiffener, 90 degrees:

Equation (33a): $M_2 = 463,000$ lb in.

Equation (34): $F_2 = -18,000$ lb

Lower stiffener, 90 degrees

Equation (35a): $M_3 = 1,109,000$ lb in.

Equation (36): $F_3 = -45,000$ lb

The unit stresses in the two cases, i.e. including and disregarding the lip ring, are now calculated by means of Equations (37). Table VI gives the resulting maximum stresses in each ring. The lip ring usually is considerably weaker than the other stiffeners and will show stresses which are nearly proportional to the depth of the ring. To avoid high stresses, as obtained in this example, the lip ring should be made even narrower than shown in Figure 26. Experience shows, on the other hand, that a yield stress in the lip ring does not endanger the safety of the ladle.

SUMMARY

Structural tests have been made of three 1/5 scale models of 150-net ton capacity hot-metal ladle prototypes. The results of the experimental and theoretical study are, in brief:

1. The structural behavior of hot-metal ladles deviates considerably from what has often earlier been assumed.
2. Under load, the ladle side deflections vary approximately linearly from zero at the bottom to the maximum at the lip. This deflection is inward in the trunnion region and outward between the trunnions.
3. The stiffener rings obtain the highest stresses at the trunnion line and midway between the trunnions. The lip ring carries the largest stresses, and the lower stiffener the smallest.
4. The side shell suffers only small stresses, except near the bottom juncture, where large discontinuity stresses are found.
5. Flat bottoms have very large deflections, and the stresses both at center and edge are likely to exceed the yield point.
6. Dished bottoms show moderate deflections and stresses, and are superior to the flat types.
7. Stresses and deflections increase nearly linearly with both amount of load and distance between the supporting ladle crane hooks, except for bottom

8. Tilting of the ladle does not increase the stresses.
9. No major difference in structural behavior was found between riveted and welded ladles.
10. No major difference in structural behavior was found between circular and oval ladles.
11. Wider trunnion assemblies tend to reduce the stiffener ring stresses.
12. A spacer band or a heavy middle shell portion reduces the stiffener ring stresses approximately in proportion to the dead weight added by these members.
13. Although the load-carrying contribution of the stiffener rings increase with their distance from the bottom, the lip ring should not be emphasized in ladle design due to its exposure to damage. Design stresses should be calculated both considering and disregarding the lip ring.
14. Stress analysis procedures which give a fair agreement with the experimental results are proposed for stiffener rings, flat bottom plates, and the bottom-side juncture discontinuity stresses. A numerical design example is given. When finished, the work of the design division of the Pressure Vessel Research Committee may furnish a design procedure for dished bottoms.
15. Summaries of design practices and recommendations as given in various articles on hot-metal ladles are incorporated in this report.

NOMENCLATURE

Subscripts

$i = 1, 2, 3$ for rings and shell fields as shown in Figure 21.

o = outside ring surface.

i = inside ring surface.

Roman alphabet

- a Moment arm of trunnion reaction. (Figures 23 and 24.)
- A_i Area of effective stiffener ring cross sections.
- A_i' Net areas of stiffener ring cross sections, including only that shell in contact with ring.
- B_i Stiffener ring constants given by Equation (22).
- c_{ij} Coefficients of shell shear flow. Equation (14). First subscript refers to ring number, second to sine series term number.
- d Depth of dished head.
- e Distance along trunnion center line from center of supporting ladle crane hook to middle of side shell. (Figure 23.)
- E Youngs modulus.
- $f(\alpha)$ A function of α given by Equation (25).
- F_i Normal force in rings.
- $g(\alpha)$ A function of α given by Equation (24).
- h Bottom plate thickness, cover plate excluded.
- h_i Vertical distance from rings to bottom juncture.
- H Total maximum depth of liquid metal.
- H_i Head of liquid metal above rings.
- I_i Moment of inertia of rings.
- k An auxiliary constant given by Equation (23).
- l Half the length of straight shell section of oval ladle. (Figure 21.)

M_i	Moment in rings.
M_o	Moment per inch along juncture of side shell and bottom.
M_x	Moment per inch of vertical section through side shell, or circumferential section through bottom plate.
M_y	Moment per inch of horizontal section through side shell or radial section through bottom plate.
N_i	Components of normal ring force at $\phi = 45$ degrees. Equation (19), (Figure 24).
N_i'	Components of normal ring force at $\phi = 45$ degrees. Equation (20).
p	Liquid pressure per unit area of bottom plate.
P_o	Transverse shear on side shell per inch of juncture between side shell and bottom.
P_i	Concentrated ring loads. (Figure 21.)
q_i	Shear flow per unit length of horizontal sections through side shell.
r	Bottom plate radius.
r_i	Radii of neutral axes of rings.
r_H	"Wet" radius at liquid metal surface.
r_t	Shell radius at trunnion level.
R	Radius of curvature of hemispherically dished bottom.
s	Slope of ladle side.
S_i	Section modulus of rings.
t	Side shell thickness.
u	Constant for oval ladles. Equation (38).
v_i	Width of stiffener rings. (Figure 25.)
V_i	Liquid pressure per inch of circumference of rings. Equation (39).
w	Width of trunnion assembly. (Figure 22.)
W	Total weight of loaded ladle.
W_c	Ladle capacity. Equation (40).
y	Distance along side shell meridian from bottom juncture.
<i>Greek alphabet</i>	
α	Half angle between concentrated ring loads. (Figure 22.)
β	Side shell constant. Equation (1).
γ	Specific weight of liquid metal.
ν	Poisson's ratio, taken as 0.3.
ξ	In ring analysis, an angle, Figure 21.
π	= 3.1416.
σ	Normal stress per unit area.
ϕ	In ring analysis, an angle, Figure 21.
ϕ	} In shell analysis, functions are given by Equation (6).
ξ	
ψ	
θ	

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1. "Structural Tests of Hot-Metal Ladles," by K. E. KNUDSEN, W. H. MUNSE and B. G. JOHNSTON. Report to AISE, Standardization Committee, January 1948. Unpublished. Available for loan at Fritz Laboratory Lehigh University.
The results of a study of the structural behavior of hot-metal ladles are offered. Three 1/5 scale steel model ladles were tested for strains and deflections under static loads, disregarding the temperature effects of the molten metal. The models were: a round riveted ladle, an oval welded ladle, and a round welded ladle, all based on a prototype of 150-ton net capacity. The test results are given in the form of deflection diagrams, stress diagrams, and tables. The principal stresses as well as horizontal and vertical stress components, as presented, have been evaluated from the measured strains.
2. "History and Use of Welded Ladles," by F. L. LINDEMUTH, A.I.M.M.E. Iron and Steel Division, Open Hearth Comm., Open Hearth Proc. 1938, pp. 27-32.

Gives list of manufacturers, number and types of welded ladles made since 1932 and where located. Discusses savings in weight and some causes of cracks in the plates.

3. "Ladle Design and Service," by J. H. HRUSKA, "Blast Furnace and Steel Plant" v. 19, 1931, pp. 673-677 and 836-838.

Discusses the design and maintenance of steelworks ladles. Information is given concerning types of slag taken from ladles used for different classes of steel; the properties of ladle linings; the principal dimensions of refractory parts; the chemical composition of stopper parts and ladle nozzles; discusses methods of drying linings; emphasizes the importance of supporting parts.

4. "Welded Ladles for Open-Hearth Service," by F. L. LINDEMUTH, "Steel" v. 109, 1941, pp. 74-77.

Presents data on the design of round and elliptical welded steel ladles with capacities of 50-175 tons of molten steel. It is pointed out that with welded construction the weight of the ladle is about one-sixth that of the contents, whereas for riveted ladles the weight ratio is 1:4.

5. "Welded Ladles Help Increase Steel Output," by G. R. REISS, "Steel" v. 112, 1943, pp. 112, 115.

By fabricating ladles from welded steel plates weight was reduced by over one-third, obtaining greater capacity without increasing total load on cranes.

6. "Standard Design Specifications for Welded Steel Ladles," Carnegie-Illinois Steel Corp., Operating Dept., Engineering Division. S. P. No. 2, Jan. 10, 1945. Unpublished.

7. "Lining and Scheduling Ladles for Maximum Life," by L. R. BERNER, A.I.M.M.E., Proc. of Open Hearth Comm., 1945, p. 153.

8. "Hot Metal Ladles of Spherical Shape," by F. L. KLING, "Iron Age" Aug. 1, 1929, p. 284.

Description of the "Kling Ladle."

9. "Blast Furnace Practice (1929)," by F. CLEMENTS. Vol. 2: Design of Plant and Equipment, pp. 401-403.

Illustrated descriptions of hot-metal ladles and cars.

10. "Ermittlungen Des Kippmomentes Und Der Kippkraft Einer Giesspfanne," by B. OSANN, "Giesserei-Zeitung" v. 22, 1925, pp. 44-46.

Determination of the tipping moment and tipping force of a ladle. Calculation of center of gravity; behavior of ladle at interruption of peration; tipping force at different tipping positions.

11. "The Basic Elastic Theory of Vessel Heads Under Internal Pressure," by G. W. WATTS and W. R. BURROWS, Standard Oil Co., Indiana, March 1949, p. 55, ASME Journal of Applied Mechanics, vol. 16, No. 1.

Means are outlined for calculating stresses and deformations at or near the junctures of vessel shells with flat, ellipsoidal and other type heads.

12. "The State of Stress in Full Heads of Pressure Vessels," by W. M. COATES, ASME Trans. v. 52, 1930, APM-52-12.

13. "Structural Analysis of Hot-Metal Ladles," by K. E. KNUDSEN, October 1948. Dissertation. Unpublished. Available for loan at Fritz Laboratory, Lehigh University.

Presents the development of the formulas given in this report and discusses other methods of attacking the stress analysis problem.

14. "Formulas for Stress and Strain (1943)," by R. J. ROARK, McGraw-Hill Book Co.

DISCUSSION

PRESENTED BY

- L. LARSON, Chief Engineer, Corrigan-McKinney Plant; Republic Steel Corp., Cleveland, Ohio
- F. LINDEMUTH, Chief Engineer, William B. Pollock Co., Youngstown, Ohio
- B. G. JOHNSTON, Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.
- K. E. KNUDSEN, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.
- W. H. MURSCH, Staff Engineer, Carnegie-Illinois Steel Corp., Pittsburgh, Pa.
- W. W. TRINKS, Associated Engineers, Pittsburgh, Pa.
- J. A. EVANS, Assistant Chief Draftsman, Republic Steel Corp., Canton, Ohio

Leonard Larson: I believe that this program, as outlined and summarized in Mr. Knudsen's paper, is a fundamental step toward a clearer concept of what takes place in a ladle under load. For many of us, there have

been too many fluid spots in attempting to analyze stresses in ladles. In many cases we have probably treated these blind spots with our best judgment, which at times may not have been very sound. The fact that there have been so few mishaps with hot metal ladles indicates that at least we have exercised our ignorance on the safe side.

Tests to date have been made without hot metal in the ladle, so we still have one major blind spot, namely influence of heat from hot metal on ladle behavior.

F. Lindemuth: The ladle models had complete lip rings for retaining the lining at the top of the shells. In the tests these lip rings supplied a large percentage of the strength of the models. Mr. Knudsen has said in his discussion that in actual practice this lip ring should not be included in any strength calculations, and this point should be emphasized.

Member: I make a few ladles and I am quite interested in whether there are any records of failures. In other words, with all of these figures that we have, can we apply those to any ladles that have failed, except for heating and overloading? I know they are overloading ladles. We design a ladle for 180 tons and the first thing you know they have added to the top and it is a 225-ton ladle.

Bruce Johnston: We think it is of considerable interest, that all of the design procedures we were able to lay our hands on do not even give the correct sign of the stress. That is, where there actually is compression stress, the design procedure may indicate tension stress.

The method that Mr. Knudsen has developed in every case gives the correct sign of the stress and in most cases is within 10 or 20 per cent of the magnitude, so with these formulas you can be confident in the design of the ladle and know that the rings, at least, will not be overstressed.

Member: A question that comes into our operations a good many times is what are the steel companies to accept in the way of a factor of safety in ladles?

I think the answer to that, as far as we have figured out, is that we used to build the riveted ladles with a factor of safety of ten to fifteen and then, when cranes were being overloaded the users wanted lighter ladles so the steel companies began to drop their factor of safety.

My question is, how far can you carry that? It seems to me this committee should make some recommendations on what the factor of safety should be in a ladle.

Leonard Larson: I think the answer to that is arbitrary. That is, there would be as many opinions as there are individuals discussing it. However, certainly their ability to use better judgment has been sharpened by more information.

K. E. Knudsen: The last discussion mentioned the change from riveted to welded ladles. I should have mentioned, of course, we just have one riveted ladle and just two welded ones, so there is no broad basis for conclusions, but we did not find any basic difference in structural behavior.

Member: I think the answer to that is that you were working on the basis of a modern riveted ladle and a modern welded one. However, you go back ten or fifteen years and examine some of the riveted ladles we built ten or twenty years ago and you will find the factor of safety is much higher all of the way through.

K. E. Knudsen: That question, too, could not be decided from these tests at all. You have to take into consideration the temperature effect, of which we know very little.

Bruce Johnston: In connection with the factor of safety, I might say that you cannot know what your factor of safety is unless you know what your stresses are. Therefore, this report should provide a step toward a real factor of safety instead of a "factor of ignorance."

Leonard Larson: In considering factor of safety as related to objective of reducing excess weight in hot metal ladles by more intelligent design, it seems obvious to me that we must continue to provide for some abuse, and normal deterioration due to wear and tear. Where inspection and maintenance are below par, a higher factor of safety is necessary as compared with a higher level of inspection and maintenance. It is difficult to draw a definite line between economy and safety, where so many variables may be involved.

Wm. Mursch: In our office there are a good many specifications coming through for ladles. You look at these different specifications and drawings and see the different types of ribs and thicknesses of plates specified on these drawings for the same size ladles and you wonder which is the correct design.

In other words, for a 150-ton ladle one plant will have a bottom plate specified maybe an inch and an eighth thick and the same size ladle from another plant will specify an inch and a half thickness. The result is that we have never had any real criterion to tell what is a good design or a poor design. The question is always raised, which is right and which is wrong. Now, these formulas which have been presented in this paper will help in checking and in getting a ladle which is designed more theoretically correct than what we have had in the past.

W. W. Trinks: I would like to ask whether, in the test ladles, there was a lining?

K. E. Knudsen: Yes, we had one on all tests with a thickness of about $\frac{3}{4}$ in. on the side, and one in. on the bottom.

W. W. Trinks: This whole investigation was originally instituted, for the purpose of finding out whether welded ladles could be made lighter than riveted ladles. Did you reach a conclusion on that?

K. E. Knudsen: It was not the specific purpose; it was one of the purposes of the investigation. Although we had riveted and welded ladles, we did not find any major difference in stress behavior between the two.

J. A. Evans: Did you check the stresses by any known methods, such as strain gages or photo-elastic devices?

K. E. Knudsen: We used bonded electric strain gages.

Bruce Johnston: How many strain gage readings did you make?

K. E. Knudsen: On each ladle we had approximately 150 of these gages and we took in all about 14,000 readings measured by means of these electric strain gages.

Bruce Johnston: This last table he showed was the check between the strain readings and the method of analysis. Mr. Knudsen tried over thirty different empirical assumptions in arriving at the best check of all three ladles.