1939

The structural significance of stress, Civil Engineering, Vol. 9 (1939), pp. 291-294, Reprint No. 43(39-2)

B. G. Johnston

FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

THE STRUCTURAL SIGNIFICANCE OF STRESS

by Bruce Johnston

Associate Member of the American Society of Civil Engineers. Assistant Professor of Civil Engineering and Assistant Director, in charge of research, at the Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.

INTRODUCTION

The structural engineer is confronted today with questions involving revision of working stresses, selection of factors of safety, and modification of familiar methods of analysis and design. It is the purpose of this article to correlate some of the factors which are fundamental to a study of these questions.

Primary attention will be given to the behavior of structural steel members loaded statically. The interrelation between the following subjects will be discussed:

1. Stress analyses and the state of stress.
2. Yielding of materials and yielding of structures.
3. Experimental research and the load-history of a structure.
4. Working stresses and the factor of safety in design.

A simple case will illustrate the relations under discussion. A steel beam may be designed for a working stress of 20,000 p.s.i. on the basis of the usual "beam formula", i.e., stress equals moment divided by section modulus. The "factor of safety" in this case might be thought to be equal to the yield-point stress of the material divided by the working stress of 20,000 p.s.i. In the case of structural steel with a yield point of 33,000 p.s.i. the computed factor of safety would thus be 1.65. A more exact analysis of local stresses under bearing blocks or in the fillets might show these stresses to be above the yield point of the material. Naturally this does not mean that there is no factor of safety. Actually, if the beam in question is put in a testing machine and loaded it will not yield as a structural unit until the computed stress by the beam formula is well above the yield-point stress of 33,000 p.s.i. The real factor of safety defined as the load ratio between the "general yielding" and the design load, will be nearer 2 than 1.65.

STRESS ANALYSES AND THE STATE OF STRESS

In studying the fundamental relations between the state of stress and the physical properties of a ductile material it is essential to consider the three-dimensional character of stress.

Imagine a very small cube cut from the interior of any member under load as shown in Fig. 1a. The cube is imagined to be microscopic in size so that the resultant stress may be considered as uniform over each plane face. In general there may be a different resultant stress on each of these three faces, with equal and oppositely directed stresses on the three faces hidden from view. Each of these three stresses may be resolved into three components parallel with the x, y, z, axes. The determination of these nine components of stress at every point in a structural member in accordance with the conditions of static equilibrium, the equations of continuity or compatability, and the boundary conditions constitute a stress analysis for a mathematically idealized material. Solutions of typical problems may be found in standard treatises on the theory of elasticity. It is also possible to determine by photoelasticity the stress distribution in a Bakelite model which is made to simulate the actual structure. Photoelastic studies have usually been two-dimensional but the extension to the three-dimensional problem recently has been made possible[1]. Stresses on the surface of actual structures may be explored by means of strain-rosettes. The stresses in the interior of dams have been determined by casting electric telemeters or electric strain gages into the concrete. A variety of analogies are available for experimentally determining stresses for certain special problems.

It will be assumed here that by one of the foregoing methods a stress analysis has been obtained. What is the significance of the stress analysis to the engineer? To answer this question it is necessary to begin by relating the state of stress to the initial elastic failure of the material.

---

1. THE FUNDAMENTALS OF THREE-DIMENSIONAL PHOTOELASTICITY M. Hetenyi, A.S.M.E. Journal of Applied Mechanics Vol. 5, No. 4, p. A-149.

As a preliminary to the study of the yielding of materials it is convenient to simplify the three-dimensional state of stress illustrated in Fig. 1a. It is always possible to determine a new orientation of the direction of the axes of the cube such that the shearing components will vanish, leaving only the normal stresses σ_1, σ_2, and σ_3 as shown in Fig. 1b. One of these principal stresses will be a maximum stress and another will be a minimum.

Now consider an octahedron constructed by connecting the centers of each face of a cube which is oriented in the principal directions as shown in Fig. 2. It may be shown[2] that no matter what state of stress exists at a particular location, the normal stresses on each face of the octahedron are identical and equal to:

$$\sigma_n = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Likewise, the magnitudes of the shear stress on each octahedral face are identical and equal to:

$$\tau_n = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}$$

Any state of stress thus reduces to a very simple concept which has particular significance when the initial plastic yielding of a ductile material under any state of stress is to be determined.

- -

2. THEORIES OF STRENGTH by A. Nadai, A.S.M.E. Journal of Applied Mechanics, Vol.1, No.3, July 1933, pp.111-129

THE RELATION BETWEEN STATE OF STRESS AND THE PHYSICAL PROPERTIES AND FAILURE OF MATERIALS

Concepts regarding all of the physical properties of a material are usually based on the behavior of the material under one-directional loading in a tension or compression test. In the actual structure the stress is usually not one-directional and the physical properties of the material as usually conceived may no longer hold.

The failure of the material will now be considered, and two types of failure will be distinguished. In a simple tension test if the material elongates considerably after initial yielding before it finally fractures it is said to be a ductile material. The initial plastic yielding will be termed elastic failure. A material which breaks suddenly in a simple tension test with little or no elongation or reduction in area is termed a brittle material. Nadai[3] and others have shown, however, that ductility and brittleness as exhibited by the type of fracture and the ability to withstand permanent elongation without fracture depend not only on the nature of the material but on the state of stress as well.

The theory of elastic failure which to date gives the best evaluation of initial plastic yielding in a ductile material under a state of combined stress is usually called the Von Mises-Hencky theory. It is also called the shear-strain energy

- -

3. PLASTICITY by A. Nadai, Engineering Societies Monographs p.55

theory of failure but may be defined without reference to strain energy as follows[2]: "Plastic yielding in a ductile material will result when the octahedral shear stress reaches a certain limiting value." Let this limit of octahedral shear stress be denoted by τ_{ny}. In a tension test with tensile stress σ_1 acting in one direction only the octahedral shearing stress $\tau_n = 0.47\sigma_1$. If $\sigma_{y.p.} =$ 33,000 p.s.i. one may therefore state that elastic failure or plastic yielding in mild structural steel will occur when the octahedral shear stress $\tau_{ny} = 0.47 \times 33{,}000 = 15{,}500$ p.s.i. It should be kept in mind that the octahedral shear stress is usually not the maximum shear stress.

Fig. 3 shows the stress-strain relations and initial yielding of structural steel as evaluated by this theory for three stress combinations in addition to simple tension. For the state of uniform all-around tension the load at initial plastic yielding is theoretically infinite. It is experimentally impractical to produce a uniform all-around tension but the conclusion is not inconsistent with the theory because in such a state of stress we may expect no plastic yielding since no shearing stress is present. The "ductile" material would exhibit under a state of all-around tension the characteristics of a brittle material. Brittle failures of this type may sometimes be caused by zones of three-directional tension set up as internal stresses due to cooling.

The yielding of material under a state of uniform combined stress has been discussed. What is the relation between the yielding of the material and the general yielding of the structural member? A tension member of uniform cross section such as an eyebar or a hollow tube in torsion are two of the rate instances in which a state of uniform stress critically affects an entire structural member. In such cases one may expect the member to yield and fail at loads corresponding to the yield-point stress and ultimate strength of the material. In other structures or structural units, such as beams, columns, rigid frames, or floor slabs it is well known that general yielding of the structure does not coincide with the load at which the material at some particular point passes the yield point.

THE LOAD-HISTORY OF THE STRUCTURE

The load-history of a material or of a structure is the complete record of its behavior from initial load to final failure. The load-history of the material is usually recorded by the stress-strain graph of a standard tension test together with data regarding elongation, reduction in area, etc. The load-history of the structure as a whole may be studied by plotting the load against a deflection or deformation which is associated with the overall behavior of the structure as shown in Fig. 4. In a few special instances such as the bending of a beam or twisting of a circular rod the load-history of the structure may be calculated analytically from that of the material by the theory of plasticity[3]. Usually the structure is studied experimentally in the testing machine of the laboratory or under dead-weight loading in the field. On this subject Hardy Cross states[4]: "The interpretation of stress analysis makes absolutely necessary a clear idea of the action of the structural part up to the stage at which rupture is conceivable."

The experimentally determined load histories of a standard tension test and the large scale test of a structural unit have qualitative similarities. The following subdivisions may be made for each case:

1. Initial readjustments of grips or bearings at low load.
2. The elastic range, wherein load is proportional to deformation.
3. The proportional limit.
4. The yielding of bar or structure.
5. Maximum ultimate load.
6. Final fracture or complete failure.

A typical load-deformation curve such as shown in Fig. 4 gives a graphical record of these stages in the load-history of the structure.

---

4. LIMITATIONS AND APPLICATIONS OF STRUCTURAL ANALYSIS Hardy Cross, Part II, Engineering News-Record, October 24, 1935, Vol. 115, No. 17, p. 571.

In the elastic range the stress distribution on the surface of the structure may be determined experimentally by strain-rosette readings[5,6]. The proportional limit may be determined approximately as the point at which the load-deformation graph deviates from a straight line. Between the proportional limit and the general yielding of a steel structure local yielding may be noted by the flaking off of mill scale along well defined lines which indicate the intersection between planes of maximum shear and the surface of the structure.

The general yielding of the structure is particularly important as it is the reference point of a real factor of safety. It usually represents a limit beyond which the structure will no longer usefully serve its original purpose. Methods for determining this "useful limit" correspond in some cases to those used in determining the yield point of a material in a simple tension test. Four methods will be outlined here, the useful limit of the structure in each case being the load at which:

- -

5. William R. Osgood, DETERMINATION OF PRINCIPAL STRESSES FROM STRAINS ON FOUR INTERSECTING GAGE LINES 45° APART Bureau of Standards Journal of Research (R.P. No.861) Vol. 15, December 1935

6. R. D. Mindlin, THE EQUIANGULAR STRAIN-ROSETTE Civil Engineering, Vol. 8, No. 8, August 1939, p.546

1. A limiting amount of permanent deformation has taken place.

2. The slope of the load-deformation curve is smaller than the slope of the original tangent by an arbitrarily determined ratio.

3. The point at which the load-deformation curve has the highest rate of change of curvature. This may be determined approximately by the construction shown in Fig.4.

4. A point which depends not only on the initial behavior but on the ultimate strength as well and which is obtained by the other construction shown in Fig. 4.

All of these methods have advantages and disadvantages. Method 1 is the simplest and gives a well defined value but depends on specifying an arbitrary allowable deformation. Method 2 depends on specifying an arbitrary slope and also has the disadvantage of not giving a well defined point if the material or structure yields slowly. Method 2 is quite satisfactory if there is a sharp break in the curve. Method 3 is ideal in obtaining very nearly the sharpest break in the curve and is especially adapted to types of failure in which there is a secondary region above the yield point where the deformation is nearly proportional to the strain. The last method, No. 4, has the advantage of giving some weight to the ultimate strength of the material or structure. Methods 3 and 4 have recently been tried out on a research project at the Fritz Laboratory[7] and have the advantage that they do not depend on any arbitrary slopes or deformations. Method 4 is the most definite and least arbitrary of all but may define a limit of structural usefulness somewhat above the actual yield point in cases where the ultimate strength is high. In any actual research program satisfactory results should be obtained by the consistent use of any one method which is judged to be best suited to the particular program in question.

FACTORS WHICH INFLUENCE THE LOAD-HISTORY OF A STRUCTURE

The load-history and useful limit of a structure in a static load test will be affected by one or more of the following factors:

1. The physical properties of the material used.
2. The state of stress in various parts of the structure.
3. The degree to which maximum stresses are localized and the structural importance of this location.
4. The stability of the structure and its component parts.

The following specific illustrations will demonstrate the influence of these factors.

---

7. HIGH YIELD-POINT STEELS AS TENSION REINFORCEMENT IN BEAMS, Bruce Johnston and Kenneth Cox (unpublished February 1939).

<u>Stress Concentrations</u> - It is well known that stress concentrations due to sharp reentry corners or holes in structural members cause high localized stresses. Fig. 5 shows the results of tension tests[3] on polished steel specimens all having the same net section. The apparent yield point has been raised in the grooved bars because local constriction is prevented by the shape and by surrounding low stressed areas. The nominal ultimate strength is also raised because necking down is prevented since planes of slip cannot develop freely. The notched bars, however, would undergo much less deformation before fracture occurred and the fracture would be characteristic of a brittle rather than a ductile material. These bars would be particularly poor in resisting impact loads or repeated stress.

Stress concentrations have a similar effect in bending. A steel bar having two circular grooves and dimensions as shown in Fig. 6 was loaded as a cantilever beam. The load-deformation diagram deviated almost imperceptibly from a straight line when the stress concentrations in the groove as determined photoelastically[8] reached the yield point of the material. But the beam did not yield as a structure until a load 53 per cent above that calculated with the stress concentration effect included or 28 per cent above the load calculated by ordinary

---

8. M. M. Frocht, FACTORS OF STRESS CONCENTRATION PHOTOELASTICALLY DETERMINED
A.S.M.E. Journal of Applied Mechanics, Vol. 2, No. 2, June 1935, p.A-67

beam theory neglecting stress concentrations entirely. This higher strength is partly due to the low stressed areas on either side of the notch and partly due to the inherent reserve strength of a beam in bending which will be discussed as the next topic.

Effect of Cross-Sectional Shape and Distribution of Load - The general yield or "useful limit" of an elastically stable beam always occurs at a load higher than the computed load at which the material in the extreme fibers passes the yield-point stress of the material[9]. For beams of equal section modulus loaded at the center with a single concentrated load the increase in useful limit is nearly one hundred per cent for a circular beam, over fifty per cent for a rectangular beam, and is between fifteen and forty per cent for I-beam sections of various proportion. Plastic yielding for beams loaded at the center commences at a very localized region at the top and bottom of the center of the beam. In the solid beams there is a large reserve of material but in the I-beam section most of the effective material in the cross section is immediately stressed above the yield point.

Beams loaded at the third-points or uniformly have slightly lower useful limits than corresponding beams with center loading. In the third-point loading all the extreme

---

9. MODERN STRESS THEORIES. Discussion by E. Mirabelli of a paper by A. V. Karpov. Transactions of the Am. Soc. C. E., Vol. 102, Fig. 48, p. 1401, 1937

fibers between the two load-points pass the yield point at the same moment, thereby affecting at equal stress a greater percentage of the material than in the case of center loading.

The shape factor may be illustrated in torsion by comparing the moment-twist curves of a solid and hollow bar having the same polar moment of inertia. In the hollow round bar all of the material is in a nearly uniform state of stress and initial yielding of material results at once in the general yielding of the structure. In the solid bar there is a reserve elastic region after the outer fibers have exceeded their yield stress locally and as a result of the general yield strength is raised.

The influence of both the shape and load factors may be stated in another more general way, i.e., the useful limit or general yield point and the shape of the load-deformation curve after yielding depend on the relative percentage of material in which the yield point is exceeded and the rate at which this percentage changes. The most effective shapes in the elastic region are the most ineffective after the maximum stress passes the yield point. This is the natural result of putting as much material as possible in the regions of highest stress.

<u>Other Factors</u> - If the structure is statically indeterminate other factors affect the load-history of the structure. The complete yielding of one part of an indeterminate structure may still leave a stable structure. As an example one may consider a frame consisting of two columns and a horizontal beam attached to the columns by angle connections. After the connections started to yield the load deformation curve would continue on a new slope but the structure would still be quite safe as it would be in a state of transition from a continuous frame over into a statically determinate system consisting of two columns and a simply supported beam between.

One example in the field of buckling will illustrate another possibility. If plate girders with vertical stiffeners were made with webs thinner than are allowed now one could design a web which would buckle at extremely low loads. Although the web would quickly become useless in compression it would still be able to carry tensile forces from the top of one vertical stiffener to the bottom of the next. The intermediate stiffeners would serve as compression struts and the girder would as a result behave as a truss instead of a beam. Aeronautical engineers consider this fact in designing wing girders.

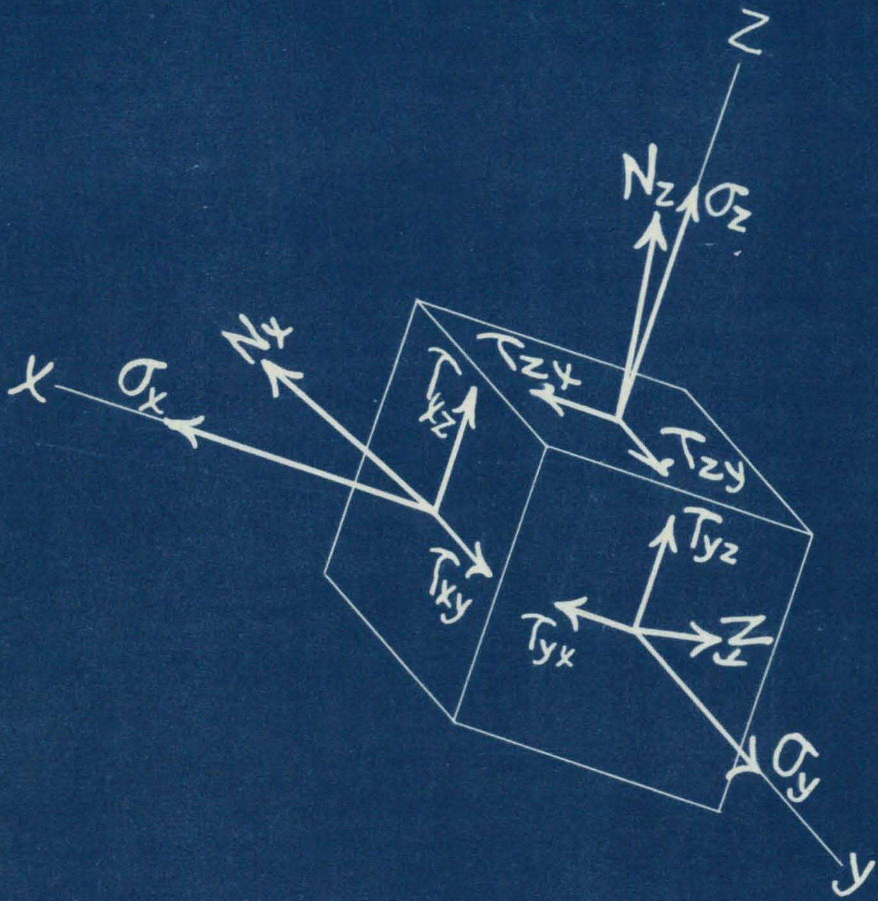
WORKING STRESSES AND DESIGN

Examples have been given to illustrate some of the many factors which affect the strength of a structure. What is really desired is a safe structure and every type of structure behaves to a certain extent by laws determined by its own peculiar characteristics. To repeat - the real factor of safety in any structure is not the ratio between calculated working stress and yield-point stress but rather the load ratio between working design load and load at the limit of structural usefulness. In connection with present design methods the important problem in every case is to specify the correct allowable working stresses to give a safe load ratio. In some cases it may be necessary to also specify the manner or degree of precision with which the stresses are to be computed. This may be important because the calculated stresses may be an approximation which depend for their exactness entirely on the method of computation used.

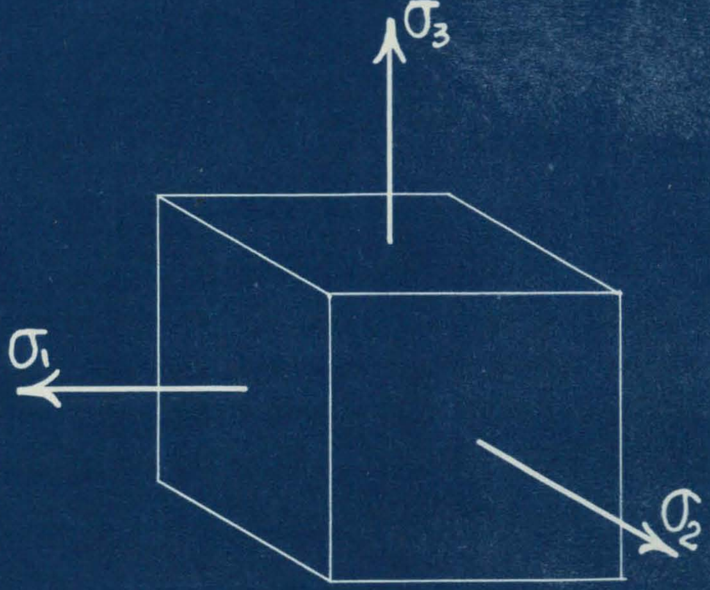
In unusual design problems requiring special stress analyses by methods of elasticity or photoelasticity consideration must be given not only to the magnitude of the maximum stresses but to their location, state of combination, and probable effect on the behavior of the structure as a whole. No definite working stresses can be specified in such cases.

The principal problem of the structural designer is that of proportioning a structure which will be both economical and safe; and which will give satisfactory service in these respects throughout its useful life. New types of structures such as concrete shell domes, steel rigid frames, and welded structures of all kinds are coming into use. These require experimental and theoretical research in order to evaluate the allowable loads and allowable computed stresses. The factors which have been discussed as well as other practical questions such as expected corrosion, repetition of stress, expected life of structure, ahd hazard to human life - all have a bearing on the selection of the proper allowable unit stress to determine a load factor which will result in a safe and enduring structure.

- - - - -



(a) Stress Components in any Direction



(b) Principal Stresses and Planes of Zero Shear

FIG. 1 GENERAL STATE OF STRESS

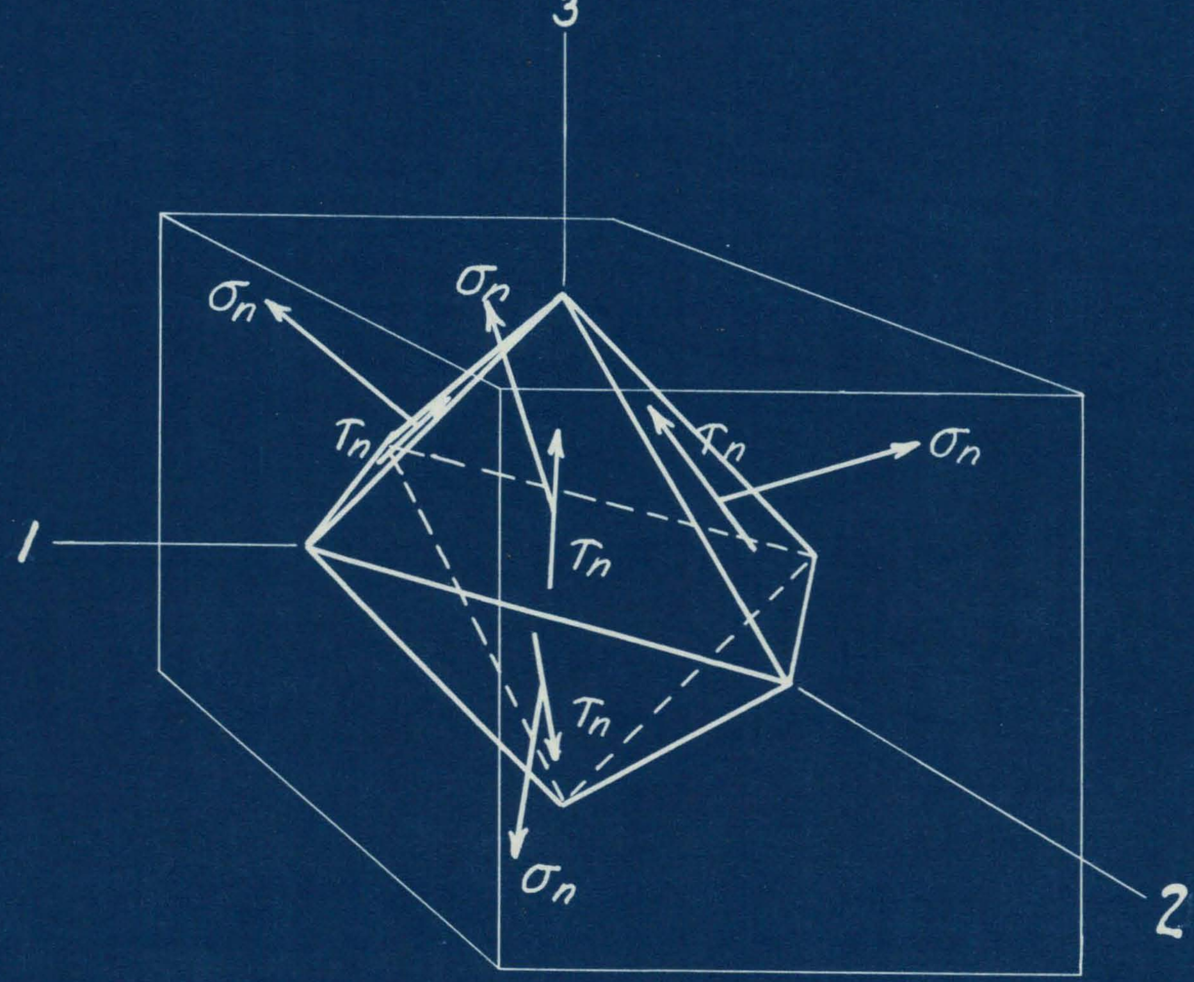


FIG.2 THE OCTAHEDRAL STRESSES

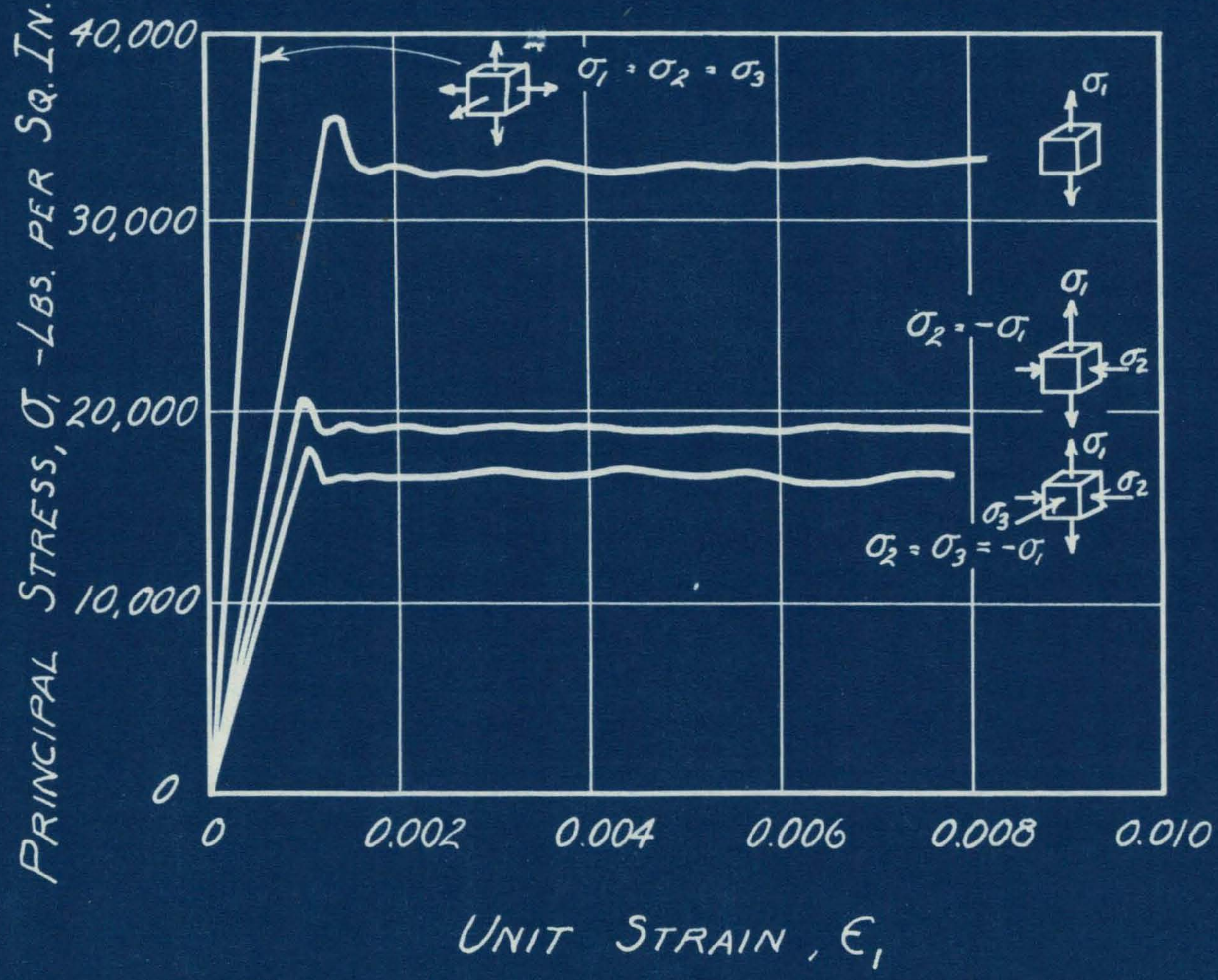


FIG. 3 RELATION BETWEEN MAXIMUM PRINCIPAL STRESS AND CORRESPONDING STRAIN FOR STRUCTURAL STEEL UNDER DIFFERENT STRESS COMBINATIONS.

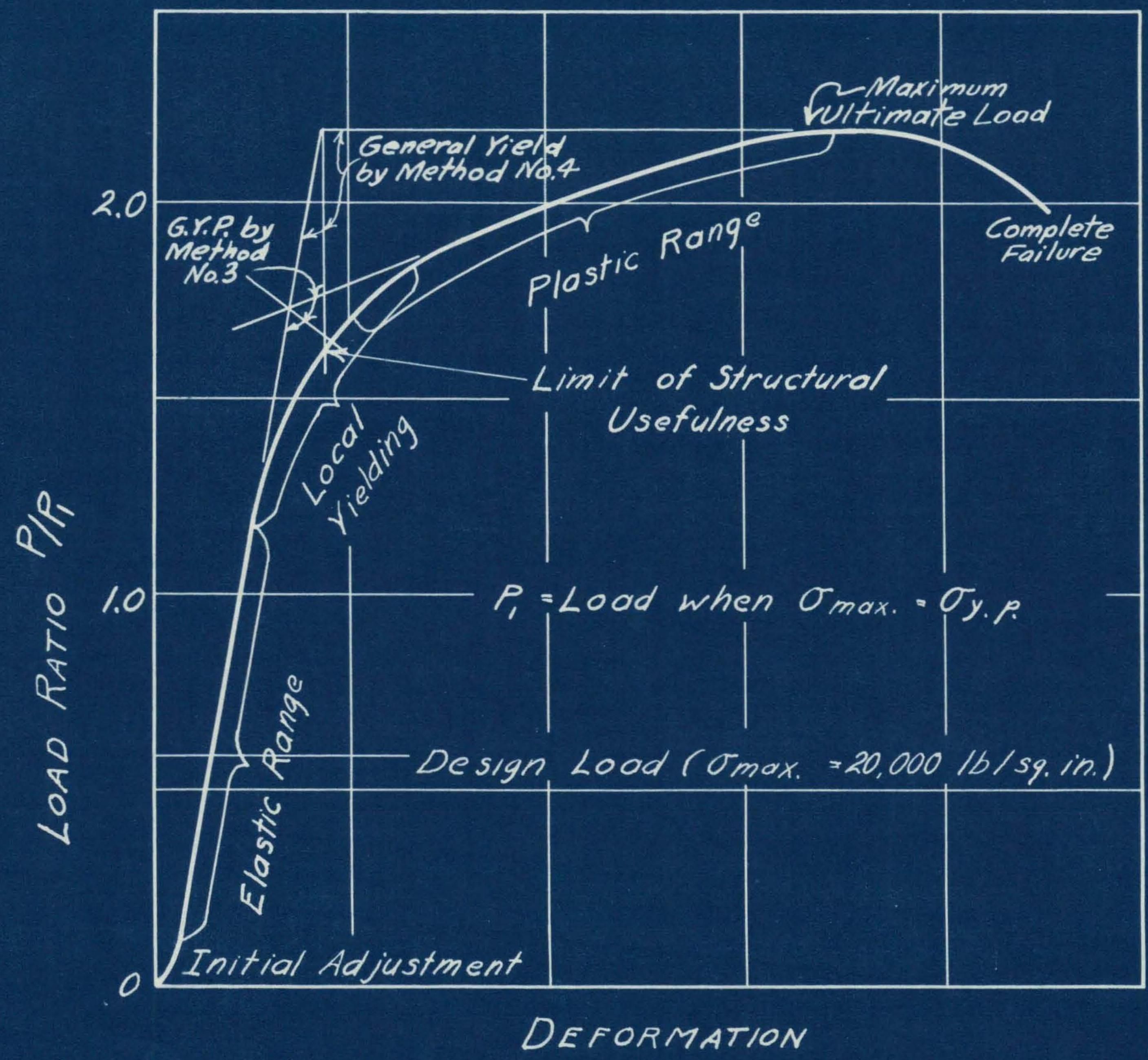


FIG. 4 TYPICAL LOAD DEFORMATION CURVE FOR A STRUCTURE

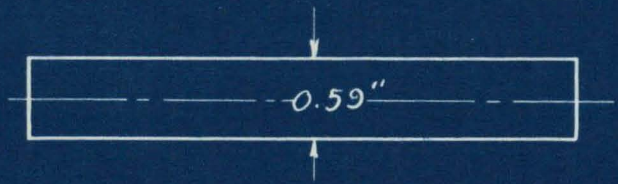
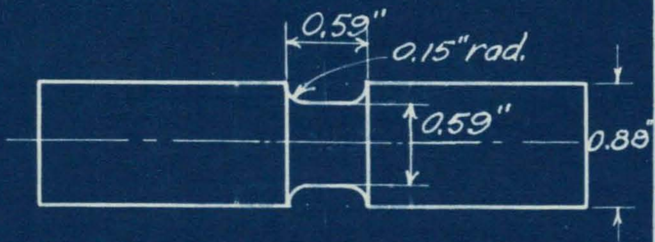
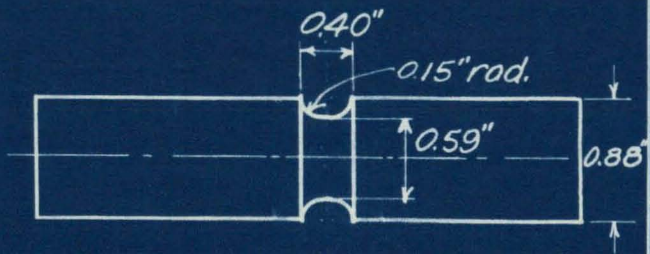
TENSILE SPECIMEN	UNANNEALED			ANNEALED		
	First Yielding	Upper Y. P.	Lower Y. P.	First Yielding	Upper Y. P.	Lower Y. P.
	Kips per Square Inch					
	—	48.2	42.2	—	43.2	40.2
	43.4	45.4	44.2	36.6	42.2	41.8
	42.4	52.2	51.4	38.1	50.1	49.3

FIG. 5 TENSION TESTS OF GROOVED BARS (NADAI)

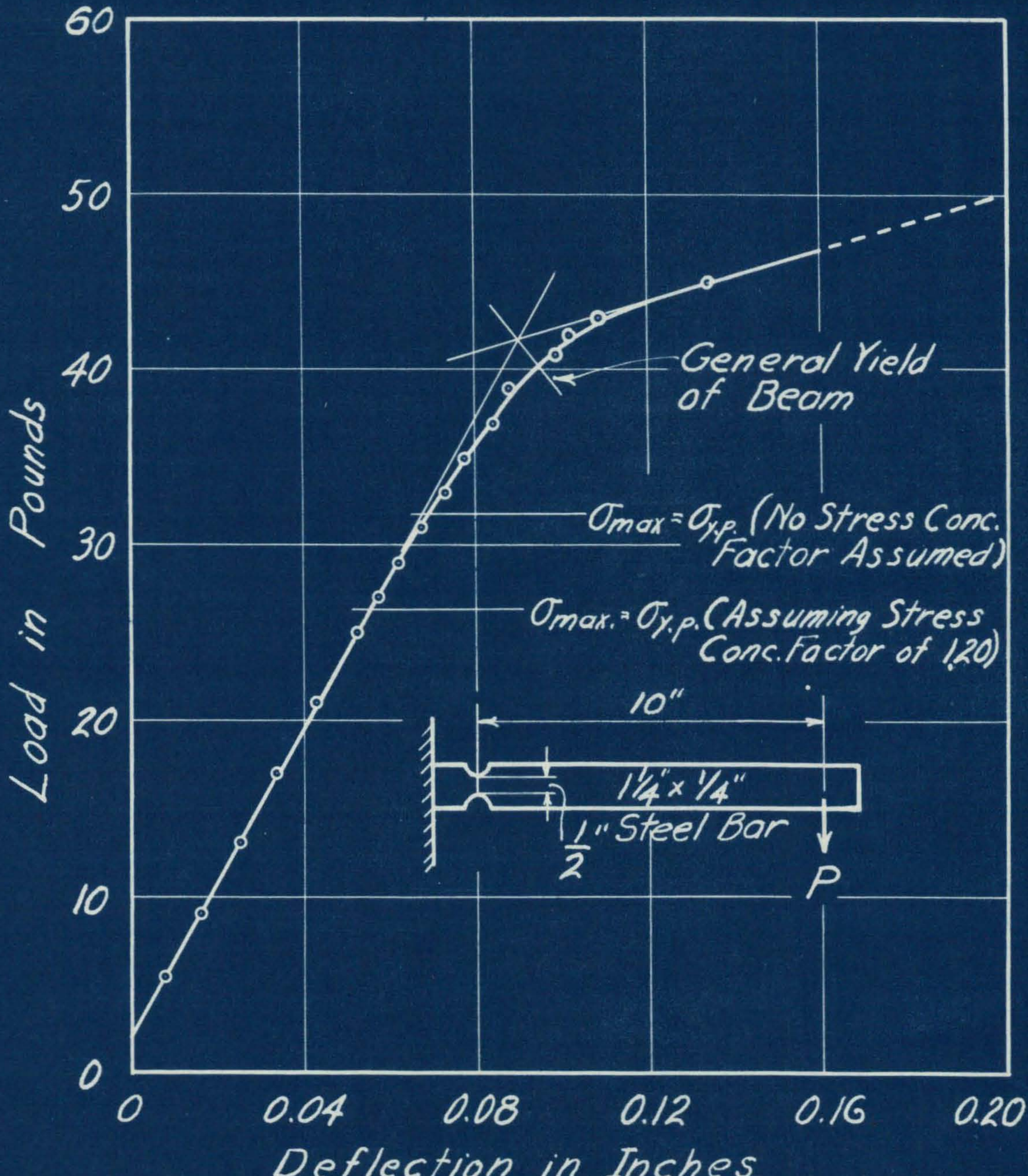


FIG. 6 RELATION BETWEEN LOAD AND DEFLECTION FOR CANTILEVER BEAM WITH GROOVE