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FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

WEB CRIPPLING OF STEEL BEAMS

by Inge Lyse* and H.J. Godfrey**

INTRODUCTION

In order to study the stability of the web in structural steel beams under a concentrated load, a cooperative investigation was undertaken by the Bethlehem Steel Company and its subsidiary, the McClintic-Marshall Corporation, and the Fritz Engineering Laboratory of Lehigh University. The Bethlehem Steel Company designed and supplied the beams, and the Fritz Engineering Laboratory carried out the testing program.

To distinguish the failure of the web under a concentrated load from interior web buckling, the failure has been termed "web crippling" throughout this report. Web crippling is principally a local failure produced by excessive compressive stresses. The problem of preventing web crippling is most common for structural beams supported on seat angles. The aim of this study was to establish the stress at which the beams yielded due to web crippling.

ACKNOWLEDGMENT

A theoretical study of this problem was made by Professor Joseph B. Reynolds of Lehigh University. This study is included in the appendix of this report.

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PROGRAM

Since only a few beams were available for this investigation, these tests were more of a preliminary than of a final nature.

The program included the testing of six Bethlehem B 22-58 rolled beams, four of which were cut from the same section and two from another section. The information for these beams is given in Table I, and the loading arrangement is shown in Fig. 1 and 2. The h/t ratio of these beams was about 52. Three of the beams were designed to fail at the point of application of the load and the other three to fail at the supports. The nominal bearing lengths were 7 and 11 inches for the center failures, and 3-1/2 and 5-1/2 inches for the end failures. The bearing lengths of 3-1/2 and 5-1/2 inches correspond to seat angles of 4 and 6 inches respectively.

In order to prevent failure due to end twisting, steel plates were welded to the bottom flange at each end of the beam. In each end plate, at a height even with the top flange, there was a slot which served as a guide for bars welded to the center of the top flange. This arrangement, as shown in Fig. 3, allowed the top flange to deflect vertically but restrained it from moving laterally.

DESCRIPTION OF TESTS

All the beams were supported by rollers and loaded at the center of the span. A roller was also used for the application of the load for beams No. 1 and 2, but a spherical bearing block was employed for this purpose in the testing of the remaining four beams. Steel bearing plates of the proper length were used to transfer the load from the rollers and bearing block to the beam. The load was generally applied in increments of 20,000 lb. at low loads and 10,000 lb. at higher loads, at a speed of 0.05 in. per minute.

Vertical deflections and strain gage observations were taken on all beams. Lateral web deflections were measured only on Beam No. 1. The location of gage lengths and deflection gages is shown in Fig. 1 and 2. Except for Beam No. 1, all beams were whitewashed before testing.

RESULTS OF TESTS

The vertical deflections of Beam No. 1 are shown in Fig. 4, from which it is noted that considerable settling of the supports took place at low loads. The net deflection of the beam as shown by the curve, $C - \frac{W + E}{2}$, shows a uniform rate of deflection up to a load of 190,000 lb., at which an increase in the rate of deflection took place. The deflections near the supports also increased

at this load. Furthermore, Fig. 5 shows that the strains at point N3 increase greatly at a load of 190,000 lb., indicating that the actual yielding of the beam occurred at this load. At point N-9, which is immediately above the flange, the strain curves indicate that the yield point of the material in the web was reached at considerably lower loads. However, this local yielding did not have any effect on the beam as a whole. The lateral web deflections gave no definite information of the behavior of the beam, thus they have been omitted from this report.

Strain lines in the web appeared at both supports at a load of 115,000 lb., and at the center at a load of 200,000 lb. The beam continued to take load until a maximum of 202,000 lb. was reached at which time the web crippled at the end, as shown in Fig. 6.

In computing the actual bearing length at the root of the web it was assumed that the stress was distributed through the flange on an angle of 45° . The total bearing length thus equals the length of the bearing plate plus twice the thickness of the flange at the root of the web. The thickness of the flange was measured by micrometers at a point next to the fillet as indicated in Table I. The length of the bearing plate was $3\text{-}1/2$ inches at the supports where the beam was designed to fail. The compressive stress at the root of the web at the yield point of the beam was

50,500 lb. per sq.in., which compares very well with the average tensile yield-point stress of 49,000 lb. per sq. in. for the web material. The full yield-point stress of the web was thus utilized in this beam.

Beam No. 2 was designed to fail at the loading point where the length of the bearing plate was seven inches. The vertical deflections of this beam are presented in Fig. 7. Since the failure of this beam occurred at the loading point no definite indication of yielding could be obtained from the deflection curves. The strain gage results as shown in Fig. 8 show that the strains near the root of the web (NC1) increased at a high rate above a load of 120,000 lb. The strains measured at the center of the web (NC3) indicated a yielding at a load of 160,000 lb. The latter value was taken as the yield-point load of the beam.

Strain lines appeared at the supports at a load of 100,000 lb. and at the loading point at 140,000 lb. The beam continued to take load until a maximum of 223,000 lb. was reached, at which time the web crippled at the loading point as shown in Fig. 9.

At the yield point of the beam the maximum compressive stress at the root of the web was 49,000 lb. per sq.in., or the same as the tensile yield-point stress of the web material.

Beam No. 3 was designed to fail at the supports where the length of the bearing plates was 5-1/2 inches. To prevent failure at the center, stiffeners were welded to both sides of the web at this point.

The vertical deflection curves in Fig. 10 indicate a definite yielding of the beam at a load of 230,000 lb. The strain curves, shown in Fig. 11, indicate yielding at a load of 220,000 lb. which was taken as the yield point of the beam.

The first strain lines appeared in the web near the center at a load of 60,000 lb. The strain lines continued to form and at a load of 110,000 lb. they appeared in the web at the supports. The web showed an indication of crippling at a load of 220,000 lb., and at the maximum of 231,500 lb. the web did cripple at one of the supports, as shown in Fig. 12.

The maximum compressive stress at the root of the web was 41,000 lb. per sq.in. at the yield point of the beam. This is considerably less than the tensile yield-point stress of 50,000 lb. per sq.in. for the material in the web.

As Beam No. 4 was designed to fail at the center, stiffeners were welded to the web at the supports. The length of the bearing plate at the center of this beam was eleven inches. As this beam failed at the center, the vertical deflection curves shown in Fig. 13 gave no indication of yielding of the beam up to a load of 260,000 lb. However, the strain curves given in Fig. 14 indicate a yielding at a load of 210,000 lb. which was taken as the yield-point load.

The first strain lines appeared in the web at the supports at a load of 90,000 lb. and at 120,000 lb. they appeared under the loading point. The beam continued to take load until a maximum of 264,400 lb. was reached, at which time the web crippled at the center of the beam as shown in Fig. 15.

At the yield-point load the maximum compressive stress at the root of the web was 43,200 lb. per sq.in., which is considerably less than the tensile yield-point stress of the web material.

Attention is called to the fact that both beams No. 3 and 4 yielded at stresses considerably below the yield point of the material. The explanation offered for this behavior is the fact that the bearing lengths of these two beams were so long that it was impossible to maintain an even stress distribution over the entire

bearing distance. The stress was probably concentrated over a part of the bearing length and developed high local stresses which caused the material to yield.

Beam No. 5 was similar to Beam No. 1, having a bearing length of 3-1/2 inches at the supports where it was designed to fail. The vertical deflections shown in Fig. 16 have an increase in rate at a load of 160,000 lb. The strain curves shown in Fig. 17 and 18 also show that the beam yielded at this load.

The first strain lines appeared in the web at the supports at a load of 60,000 lb. and at the loading point at a load of 130,000 lb. A slight crippling of the web at the support was noted at 190,000 lb., and at a load of 205,000 lb. the web crippled as shown in Fig. 19.

At the yield-point load of 160,000 lb. the computed stress at the root of the web was 41,900 lb. per sq.in., which compares favorably with the yield-point stress of 44,660 lb. per sq.in. for the web material.

Beam No. 6 was similar to Beam No. 2, having a bearing length of seven inches at the loading point. As this beam failed at the center, the vertical deflections, as shown in Fig. 20, indicate that there was no yielding of the beam as a whole. The strain curves, however, as presented in Fig. 21, indicate a bending tendency in the web at a load of 80,000 lb. The rate of strains increased at a load of 150,000 lb. which was taken as the yield-point of the beam.

The first strain lines appeared at the center at a load of 48,000 lb. and at the supports at 70,000 lb. The beam continued to take load until a maximum of 209,500 lb. was reached and at this load the web crippled under the center as shown in Fig. 22.

At the yield-point load of the beam the maximum compressive stress at the root of the web was 45,000 lb. per sq.in. which is very close to the yield-point stress of 44,600 lb. per sq.in. for the web material.

As both the flexural and shearing stresses were low in all of these beams, there was no indication of yielding due to these stresses.

SUMMARY

Since only six beams were tested in this investigation, no general conclusions can be drawn. However, from the results obtained, the following is indicated.

1. The appearance of the first strain lines have no relation to the yielding of the beam as a whole but are just an indication of high local stress.

2. For short bearing lengths, either at the support or loading point, the compressive stress at the root of the web at the yield point of the beam was found to correspond very well with the tensile yield-point stress of the material.

3. For longer bearing lengths, the stress is probably not evenly distributed over its entire length. The concentrated stresses may cause the material to yield before the entire bearing area is fully utilized.

4. In all these beams, the yielding was caused by the compressive stresses at the root of the web.

5. The following formula for compressive stress at the root of the web is recommended for short bearing lengths.

$$\text{Compressive Stress} = S = \frac{R}{t(A+N)}$$

where:

R = load on bearing plate

t = thickness of web

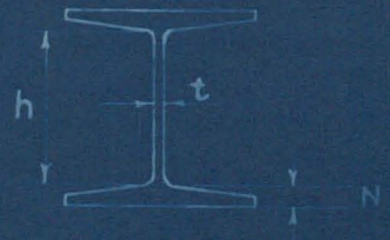
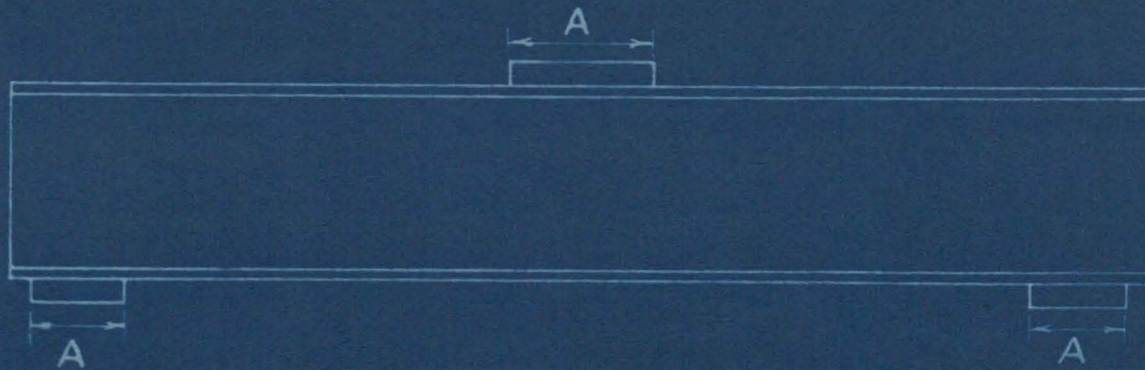
A = length of bearing plate

N = thickness of flange at edge of fillet

June 22, 1933

TABLE I - END CRIPPLING

Beam	Web Thickness inches t	Bearing Length inches $A + 2N$	Bearing Area sq. in.	Yield-Point Load lb.	Compressive Yield-Point Stress lb./sq. in.	Tensile Yield-Point Stress of Material lb./sq. in.	Ultimate Load lb.
CT-1	0.397	4.74	1.88	190,000	50,500	49,000	202,000
CT-2	0.397	8.24	3.27	160,000	49,000	49,000	223,000
CT-3	0.397	6.74	2.68	220,000	41,000	50,000	231,500
CT-4	0.397	12.24	4.86	210,000	43,200	50,000	264,400
CT-5	0.407	4.69	1.91	160,000	41,900	44,660	205,000
CT-6	0.407	8.17	3.33	150,000	45,000	44,660	209,500



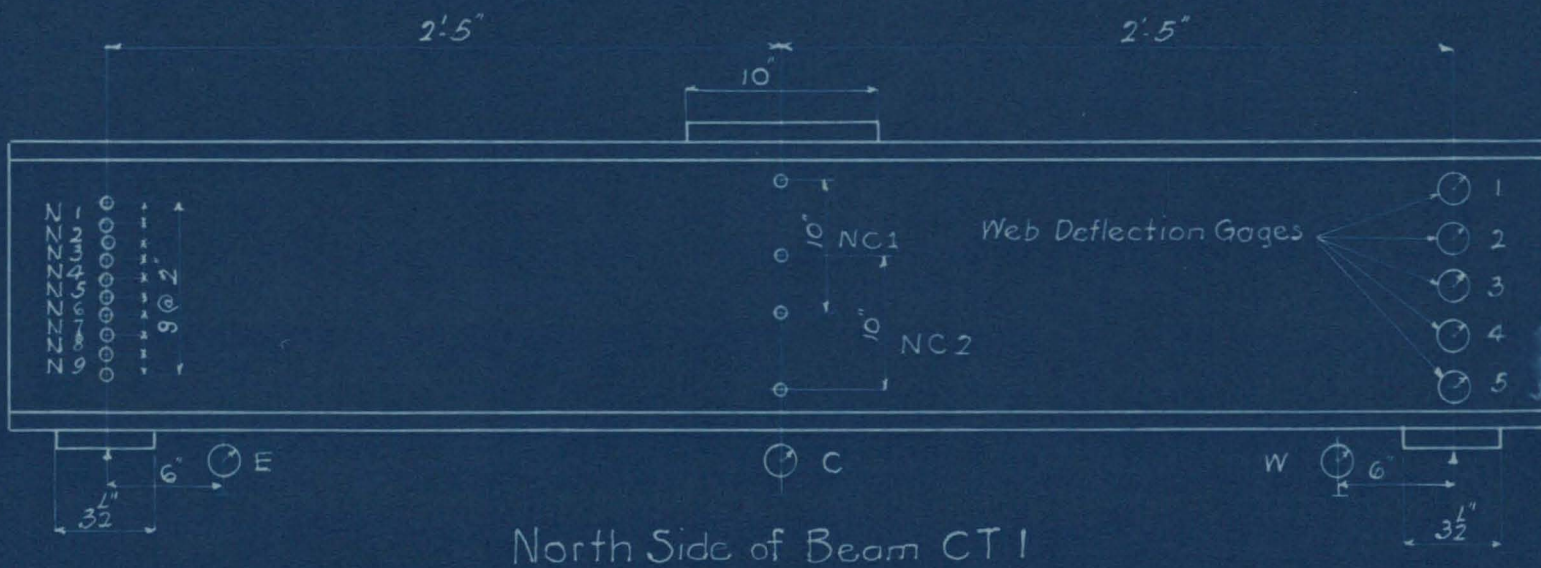
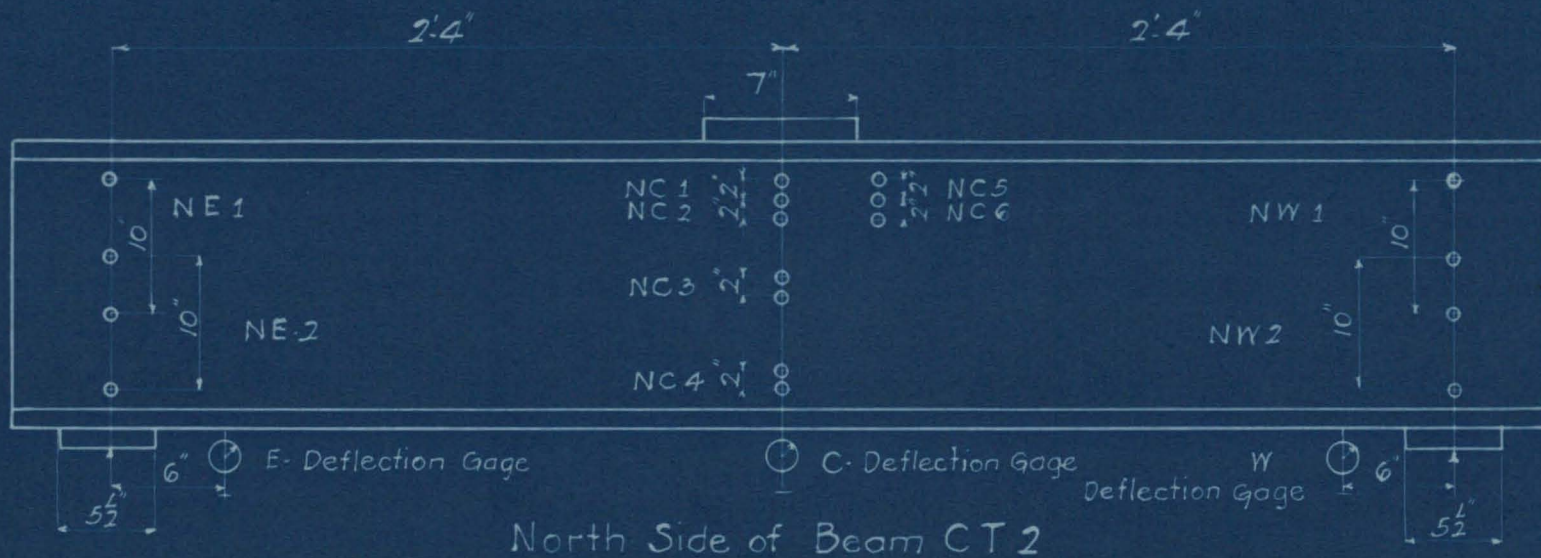


Fig.1 Beams CT 1 and CT 2 Showing Position of Gage Lengths and Deflection Gages

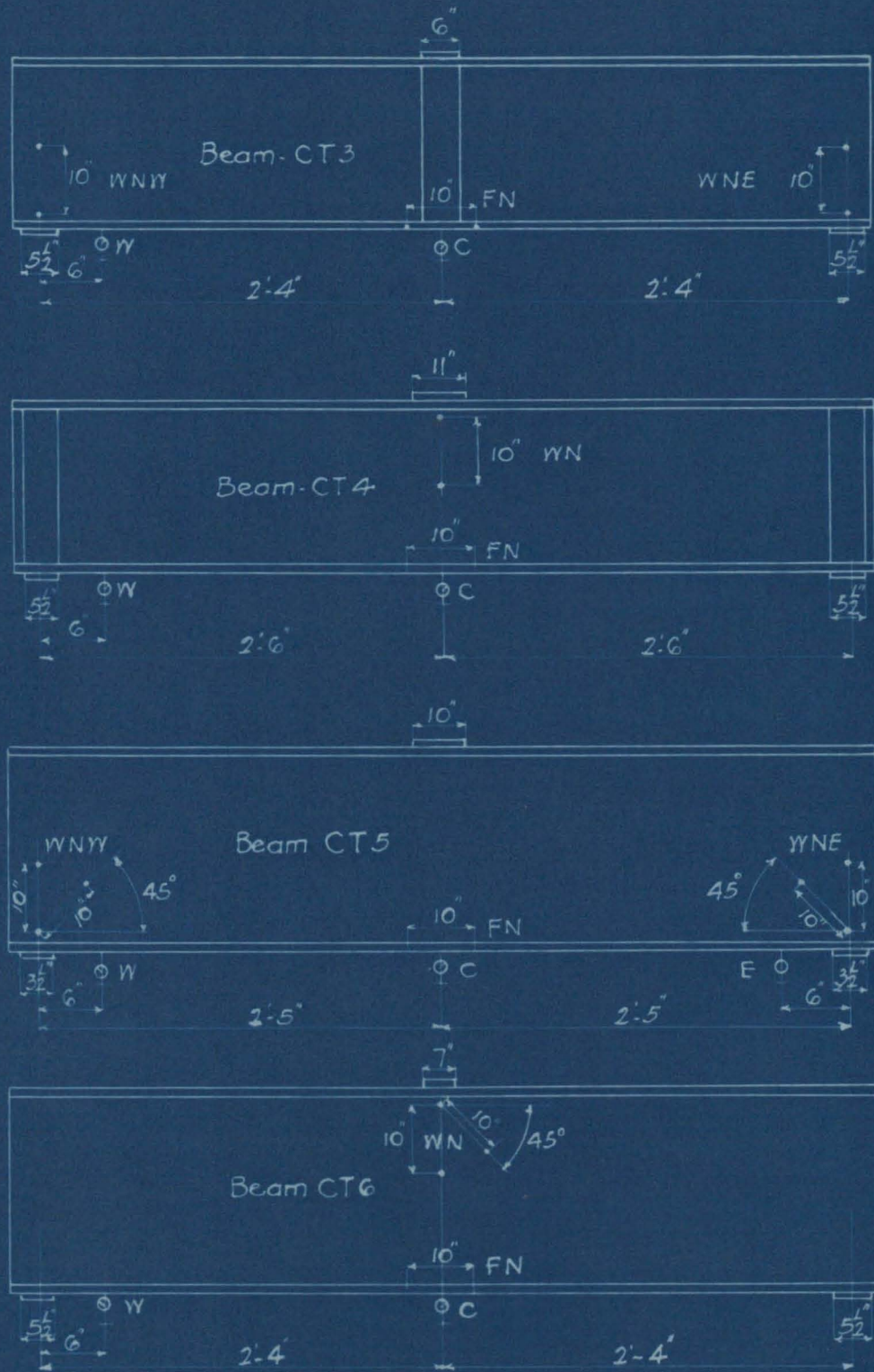


Fig 2 Beams CT3+CT6 Showing Position of Gage Lengths and Deflection Gages.



Fig. 3 - Beam CT-5, Showing End Plate for the
Prevention of End Twisting

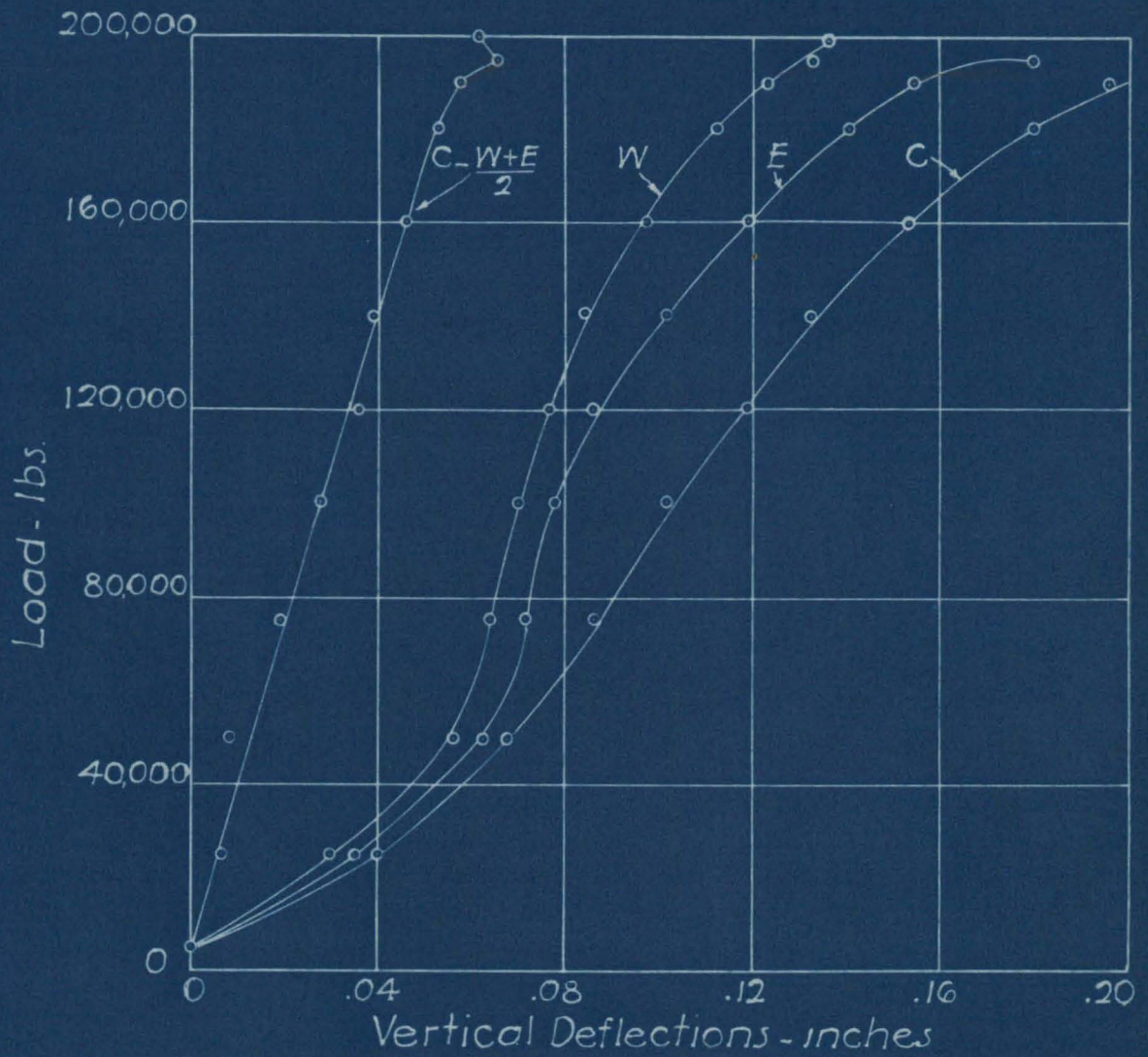


Fig. 4. Vertical Deflections of Beam CT-1
 Ultimate Load - 202,000 lbs. Tested July 26, 1932.

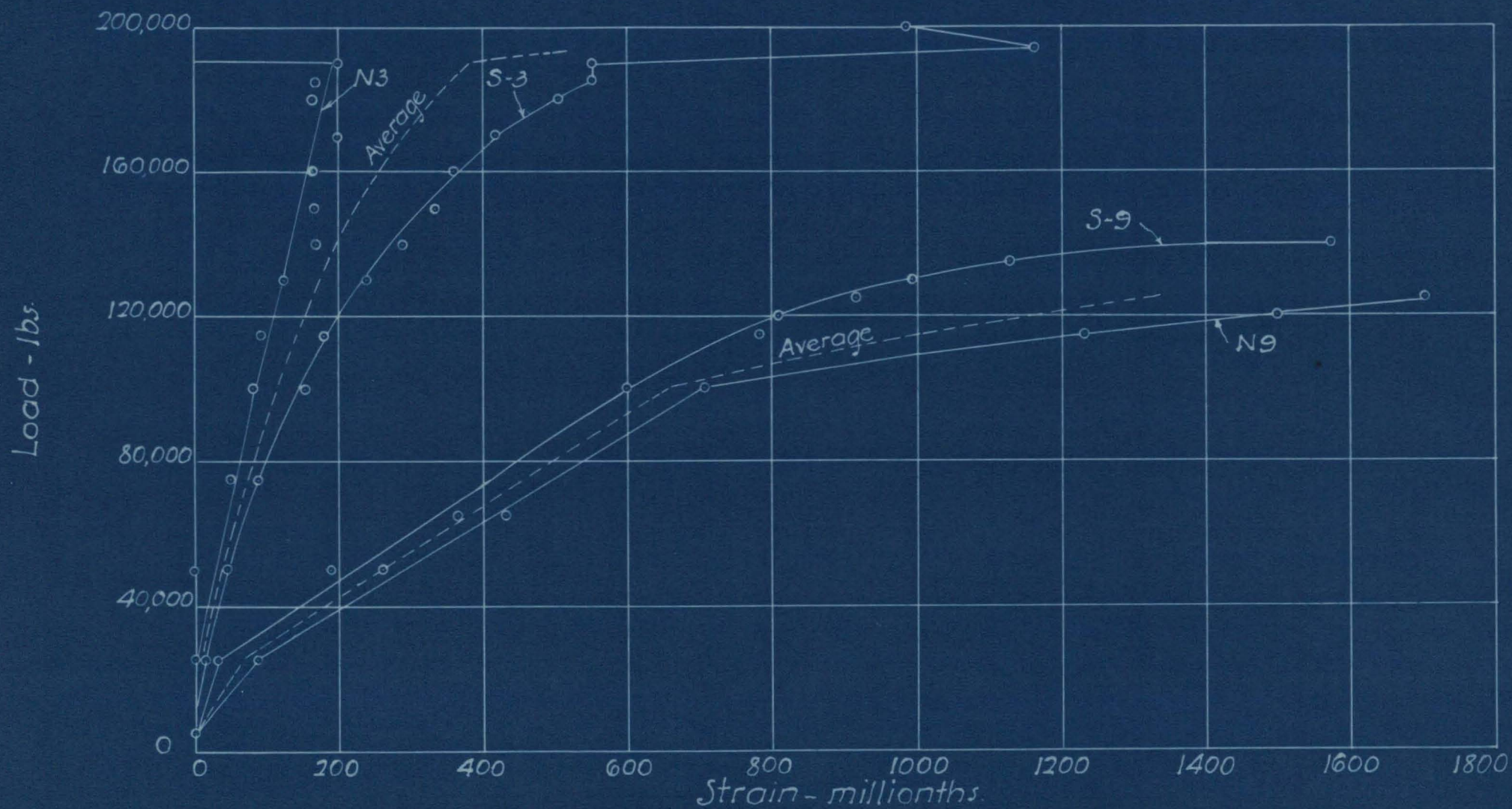


Fig. 5: Compressive Strains in Web of Beam CT-1

Tested July 26, 1932.



Fig. 6 - Beam CT-1, Showing Failure at the Support
Due to Web Crippling

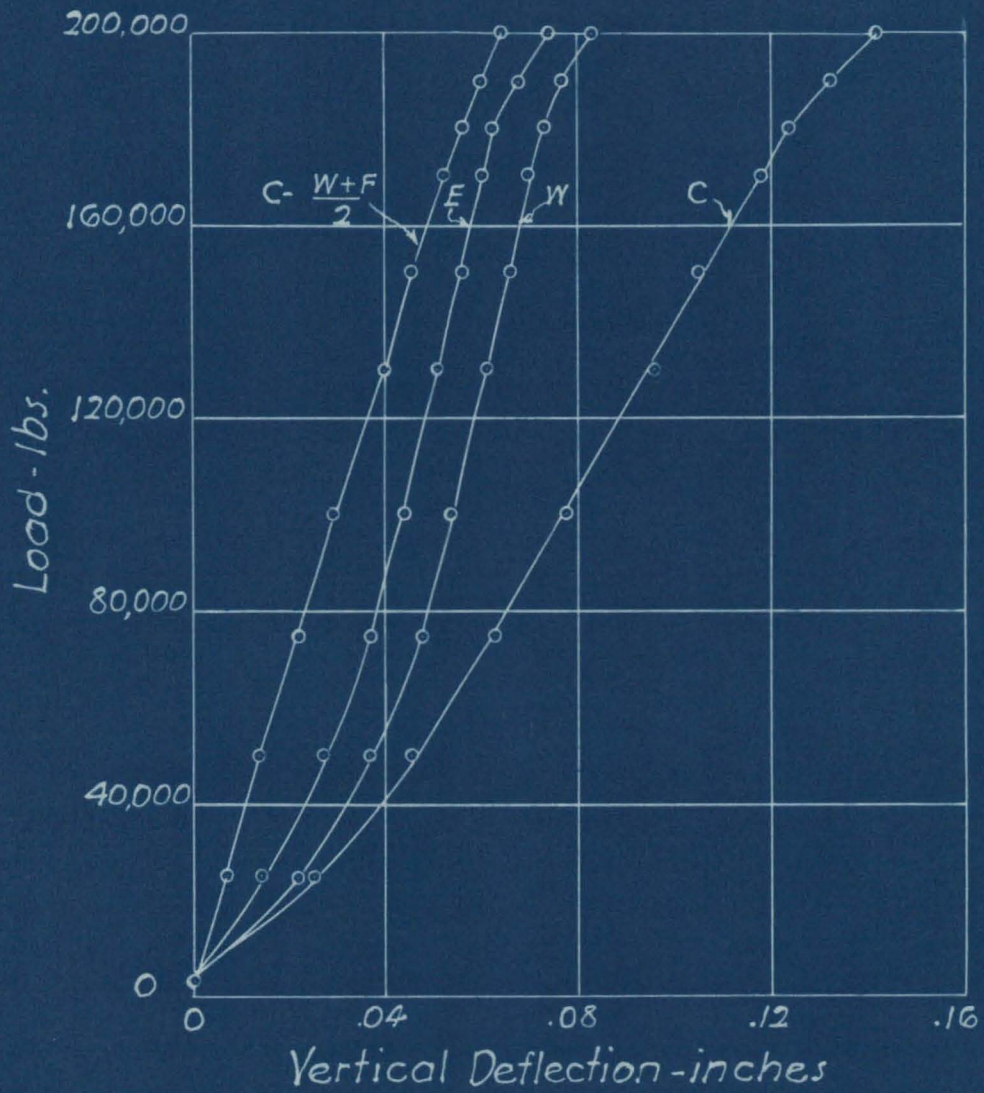


Fig. 7- Vertical Deflections of Beam CT-2
 Ultimate Load-223,000 lbs. Tested July 28, 1932.

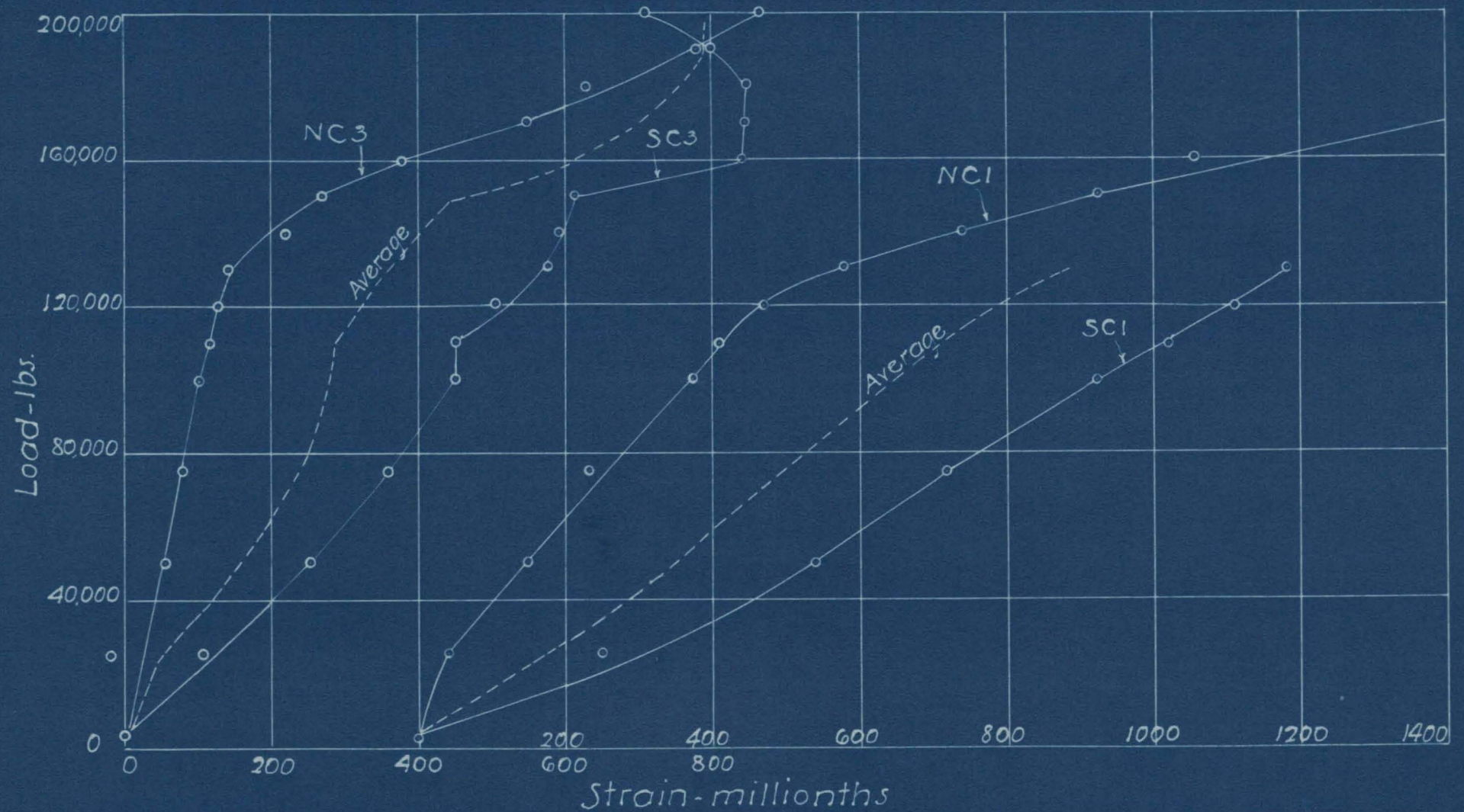


Fig. 8 Compressive Strains in Web of Beam CT-2 Tested July 28, 1932.

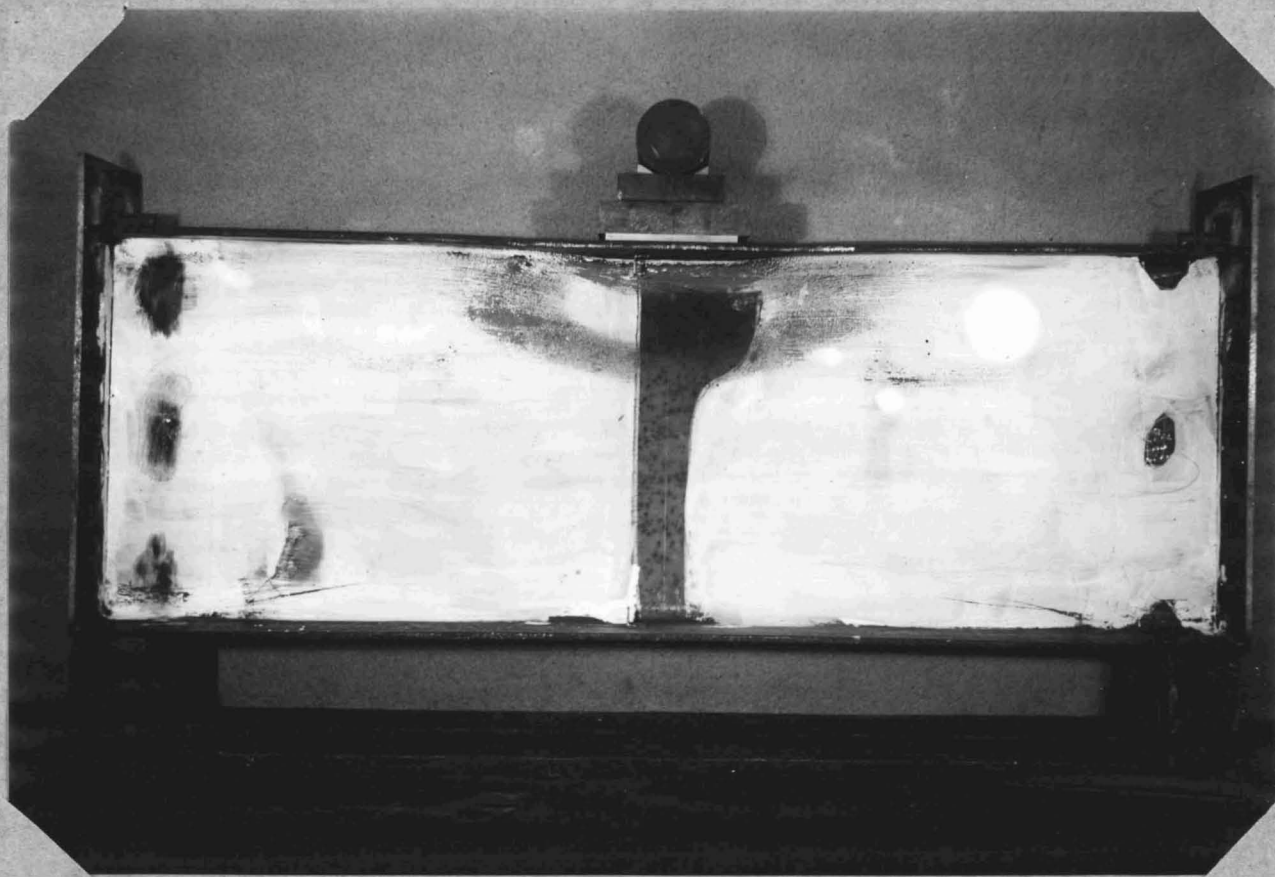


Fig. 9 - CT-2, Showing Failure at the Center Due to Web Crippling

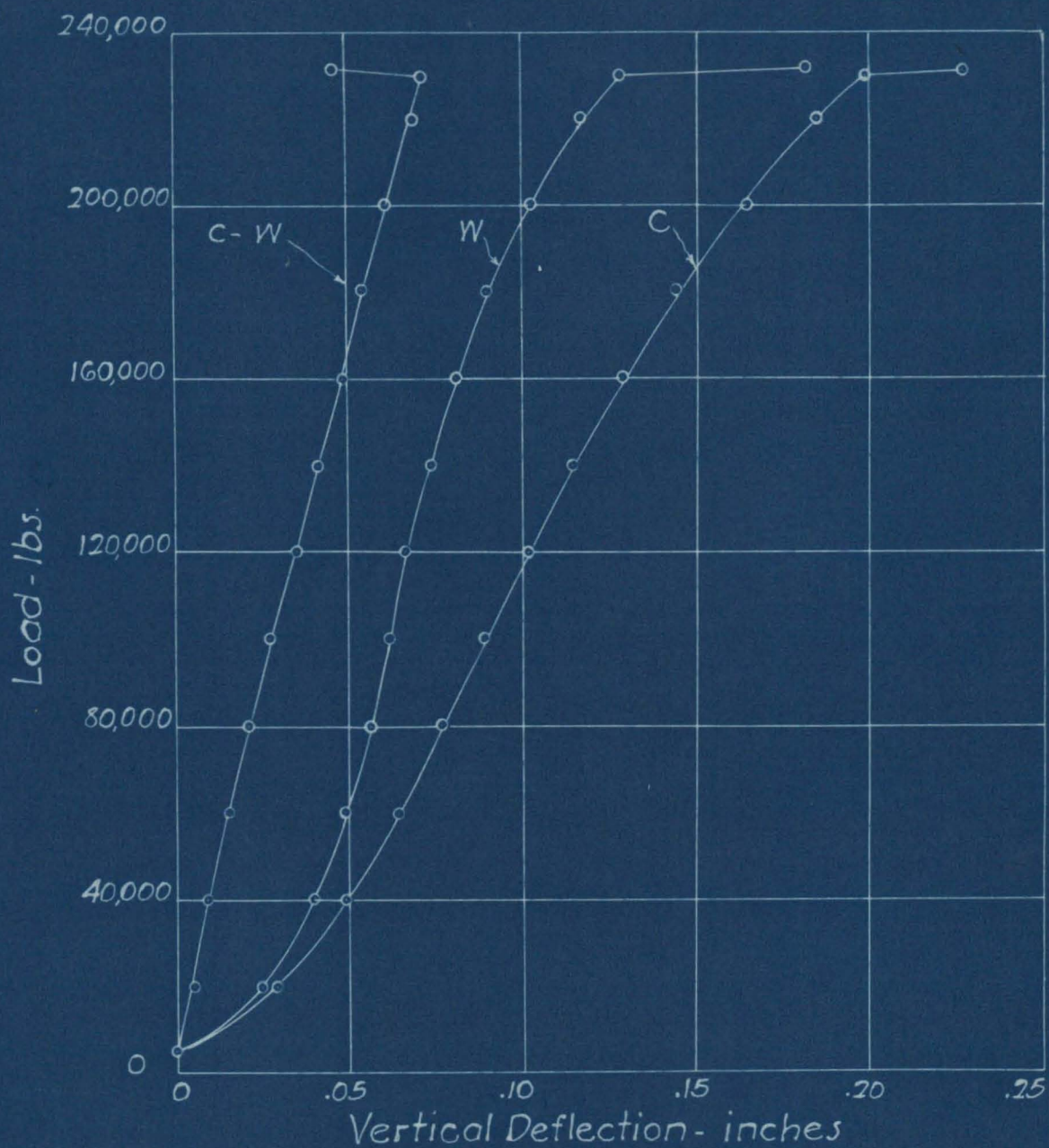


Fig. 10 Vertical Deflections of Beam CT-3
 Ultimate Load 231,500 lbs. Tested Nov. 29, 1932

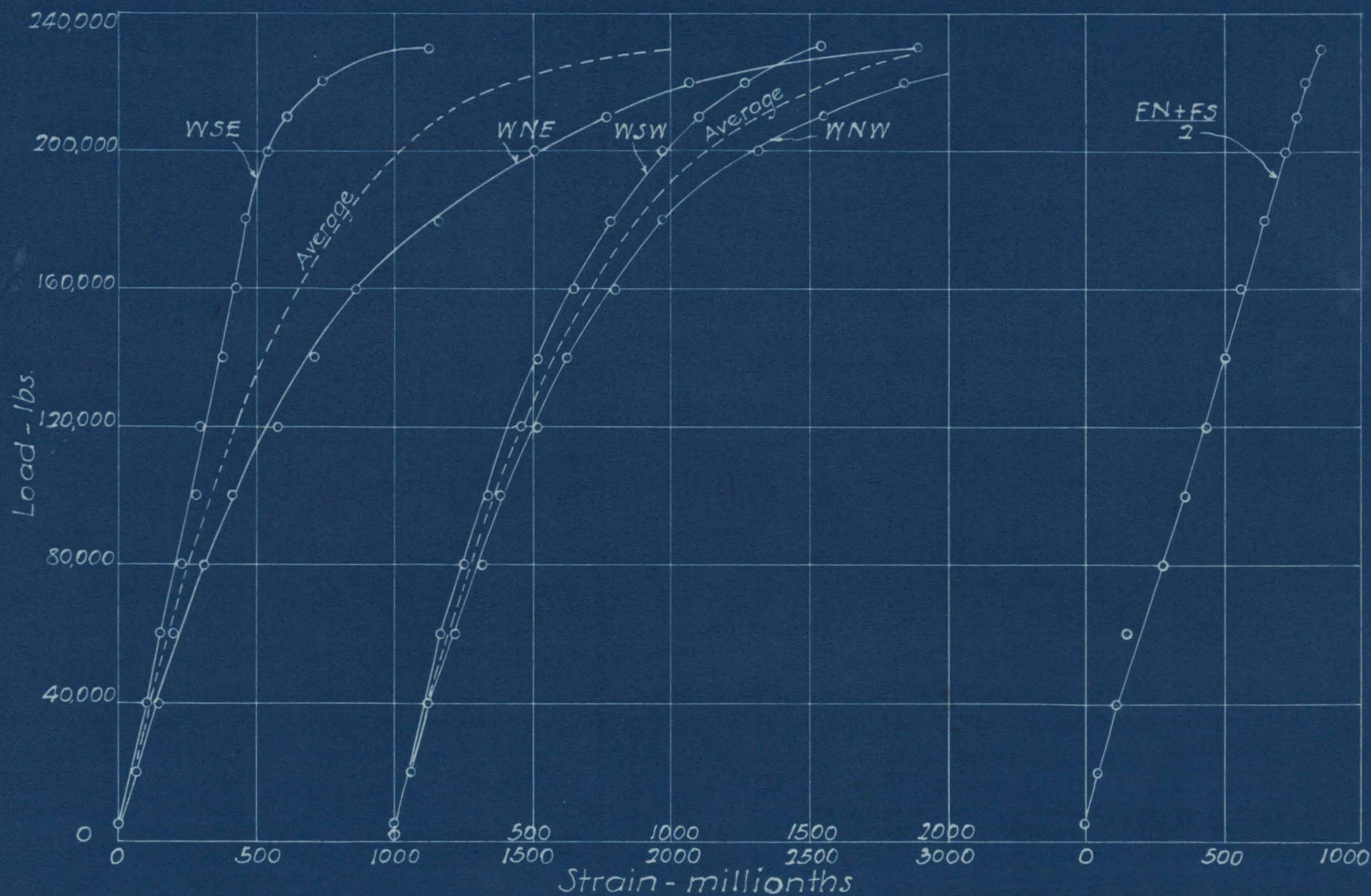


Fig. II. Strains in Web and Flange of Beam CT-3

Tested Nov. 29, 1932.

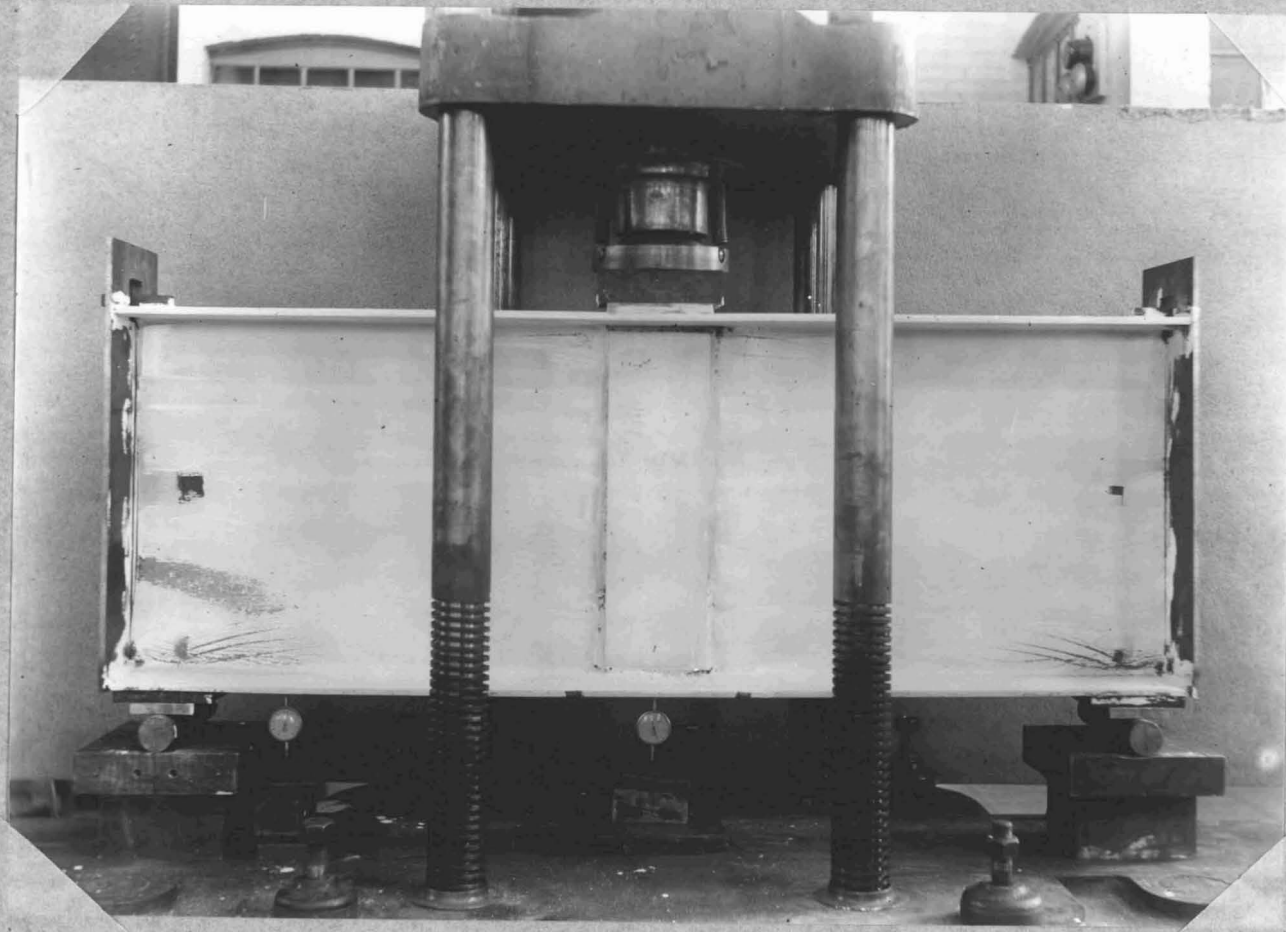


Fig. 12 - Beam CT-3, Showing Failure at the Supports Due to Web Crippling

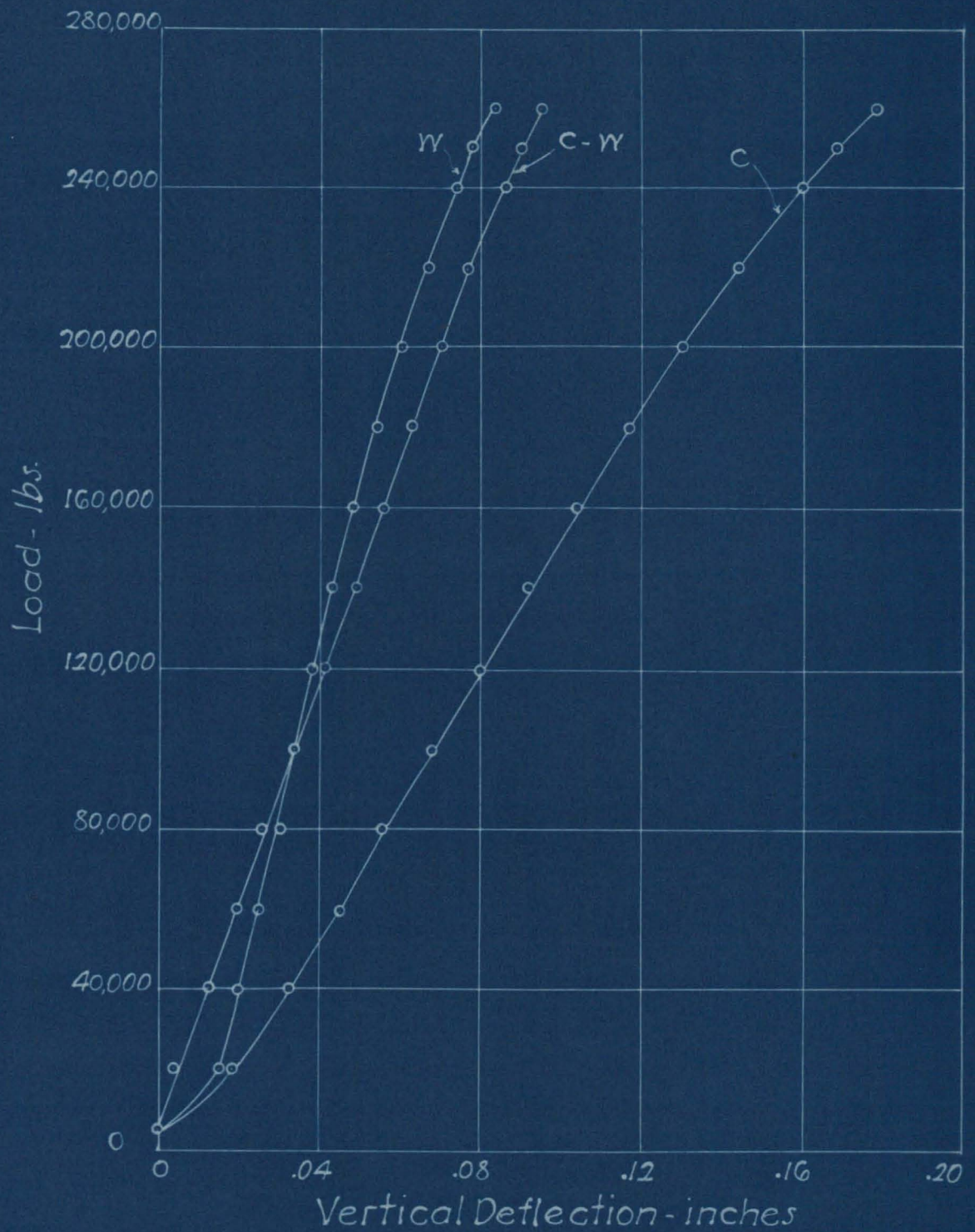


Fig. 13 Vertical Deflections of Beam CT-4
 Ultimate Load - 264,400 lbs. Tested Nov. 29, 1932.

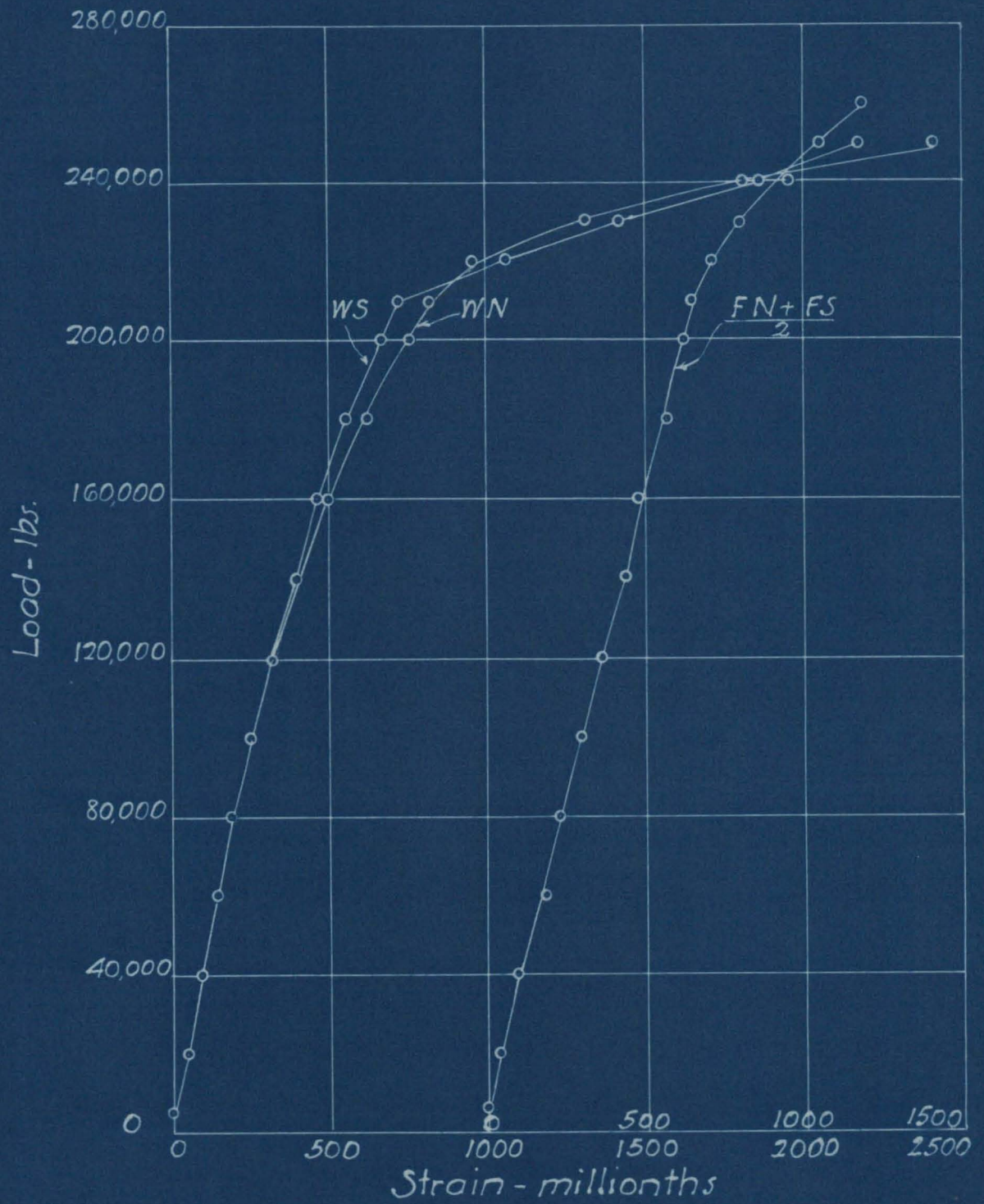


Fig. 14 - Strains in Web and Flange of Beam CT-4 Tested Nov. 29, 1932.

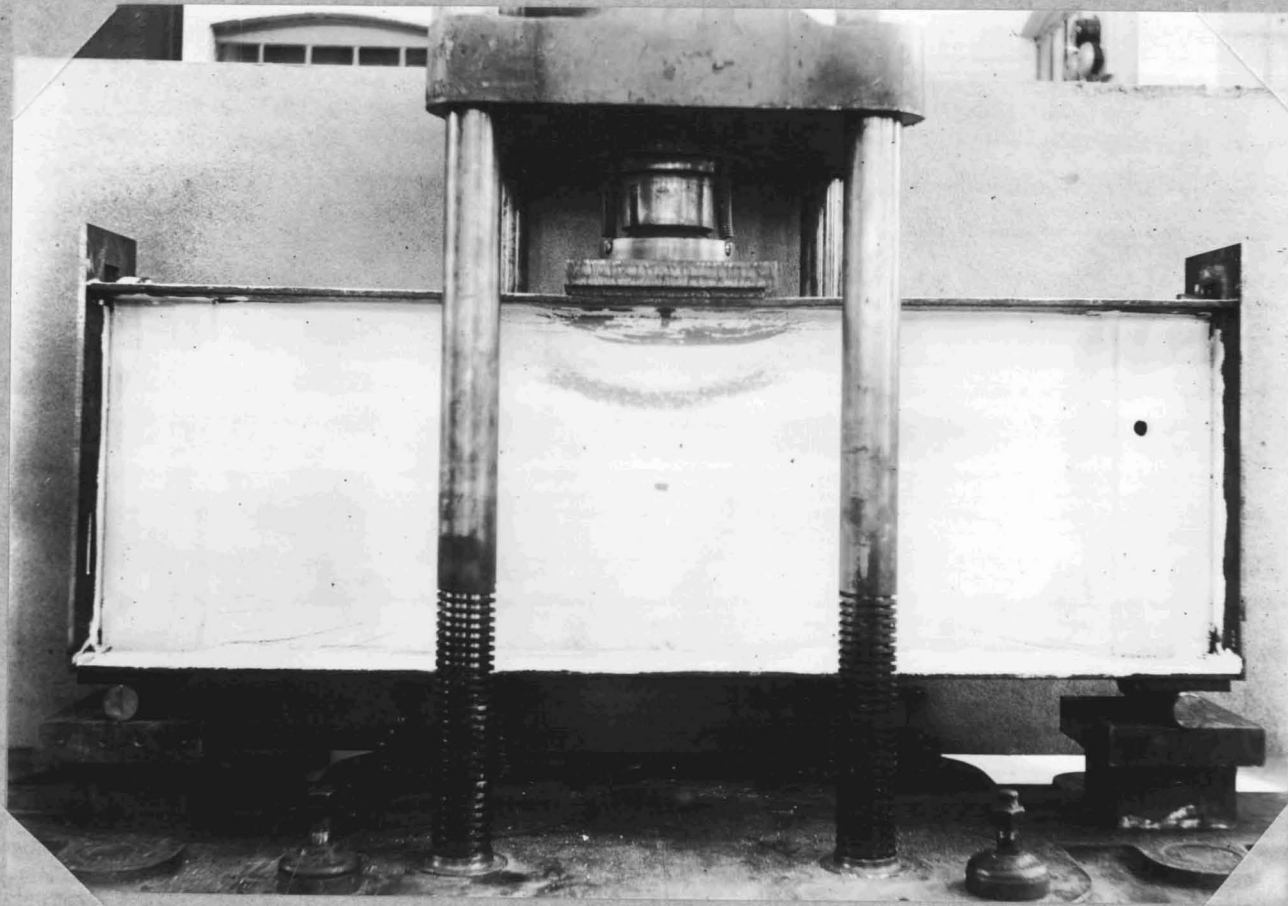


Fig. 15 - Beam CT-4, Showing Failure at the Center Due to Web Crippling

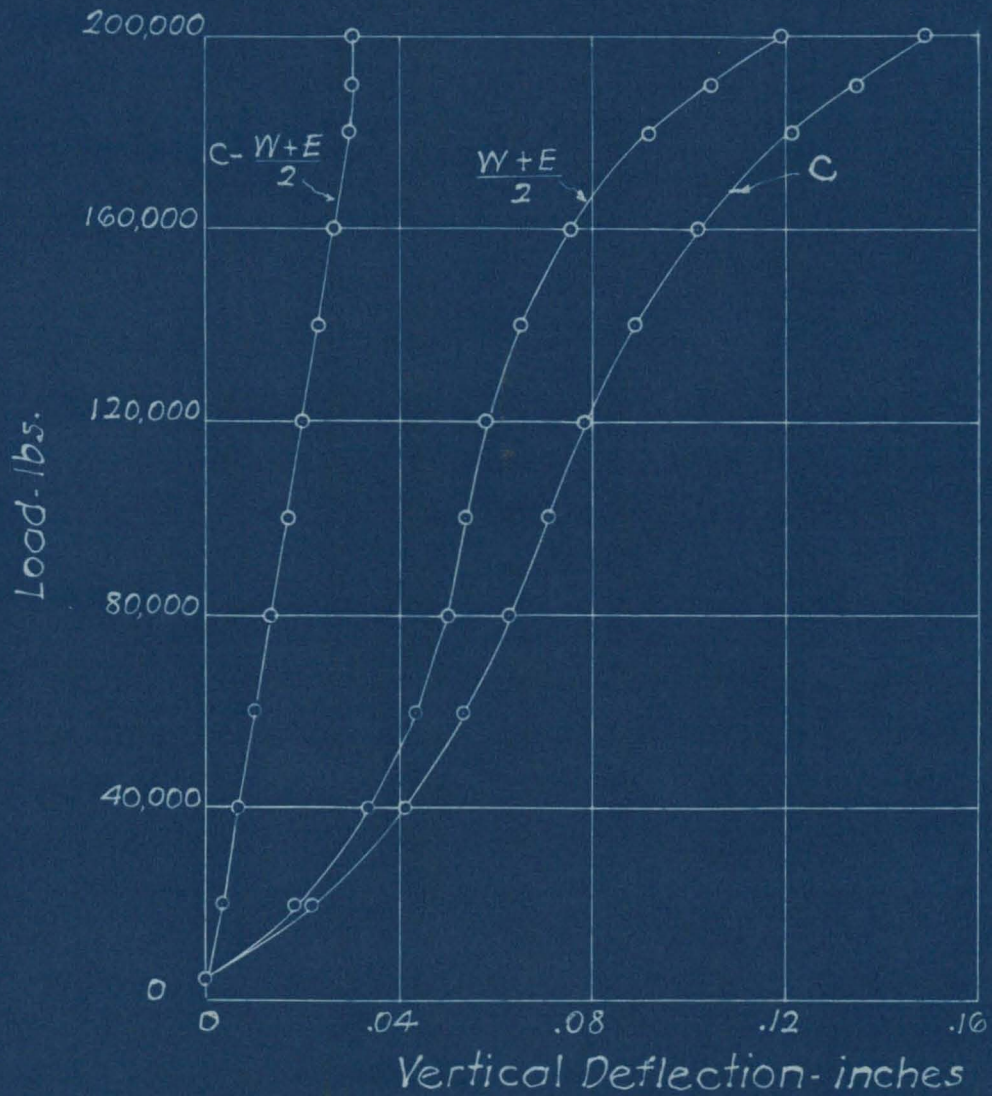


Fig. 16 Vertical Deflections of Beam CT-5
 Ultimate Load - 205,000 lbs. Tested Nov. 30, 1932.

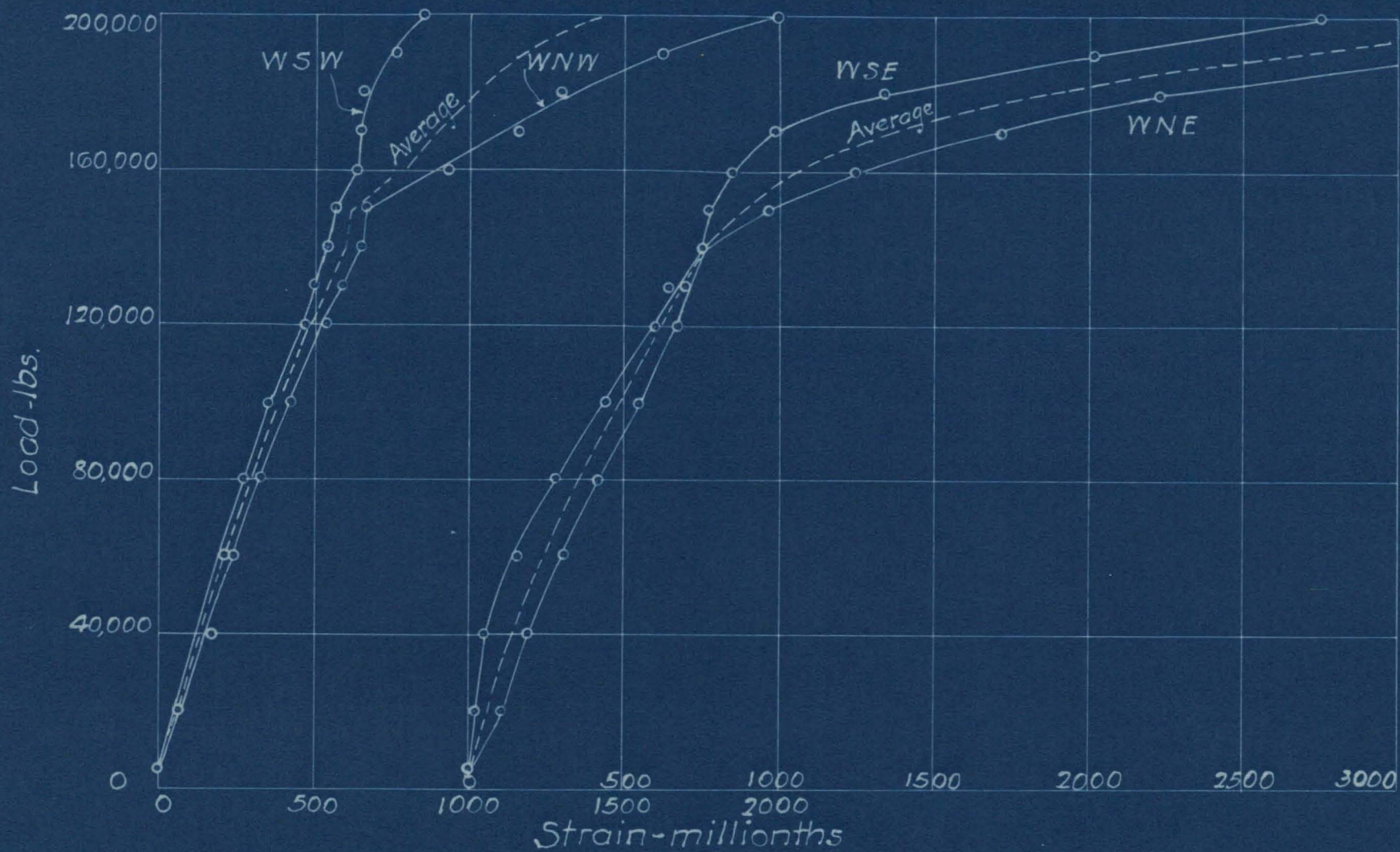


Fig.17-Compressive Strains in Web of Beam CT-5 Tested Nov.30,1932.

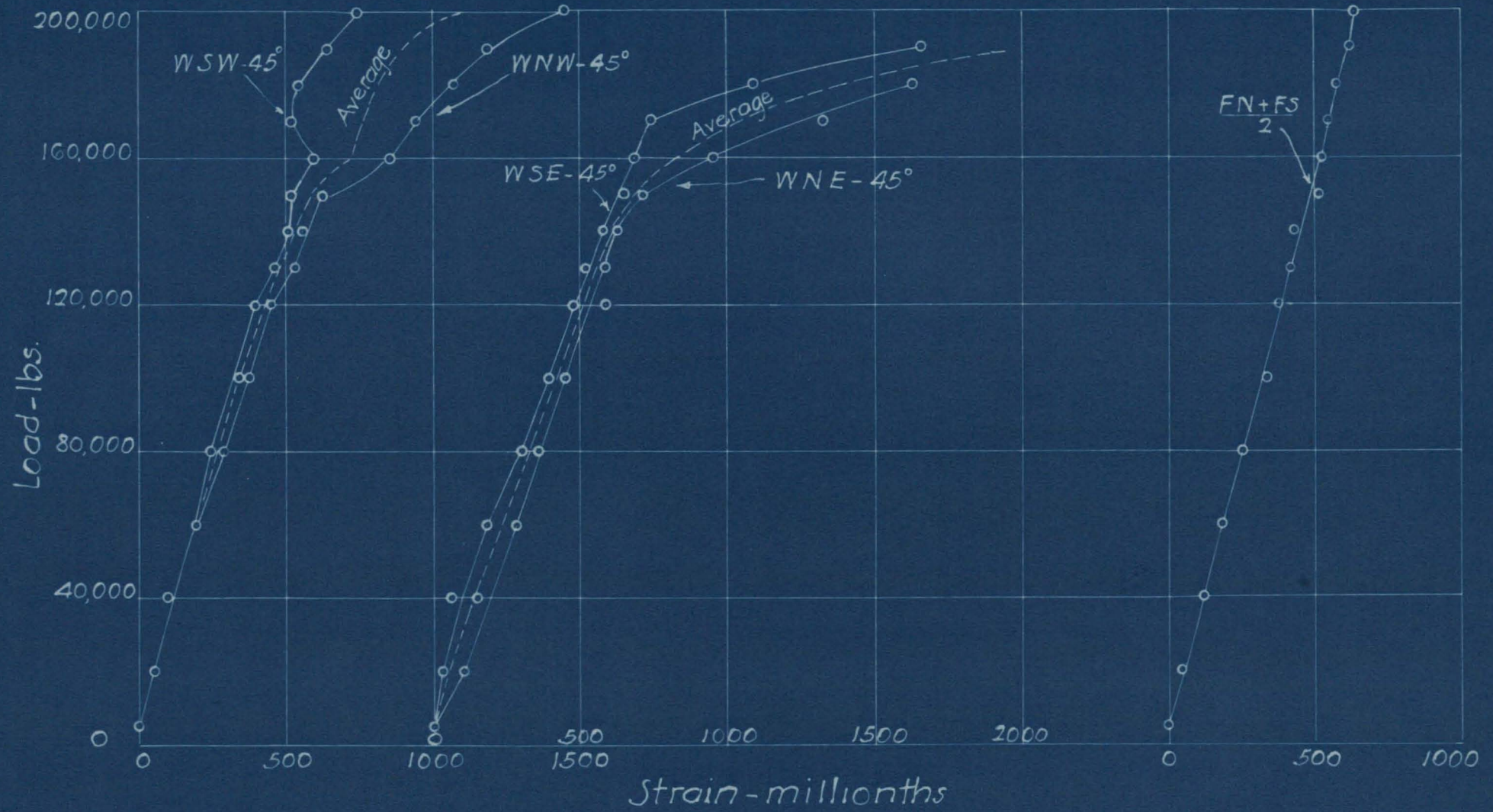


Fig.18-Strains in Web and Flange of Beam CT5

Tested Nov.30-1932.

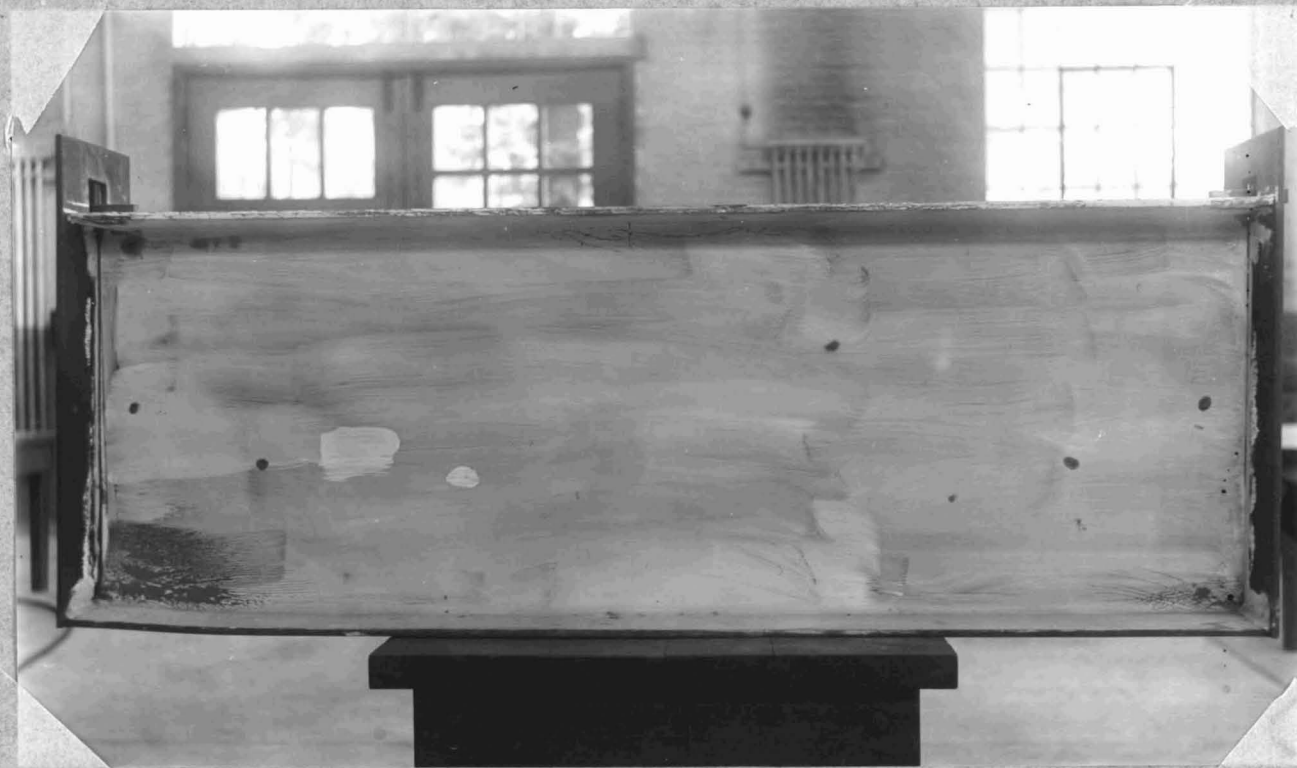


Fig. 19 - Beam CT-5, Showing Failure at the Supports Due to Web Crippling

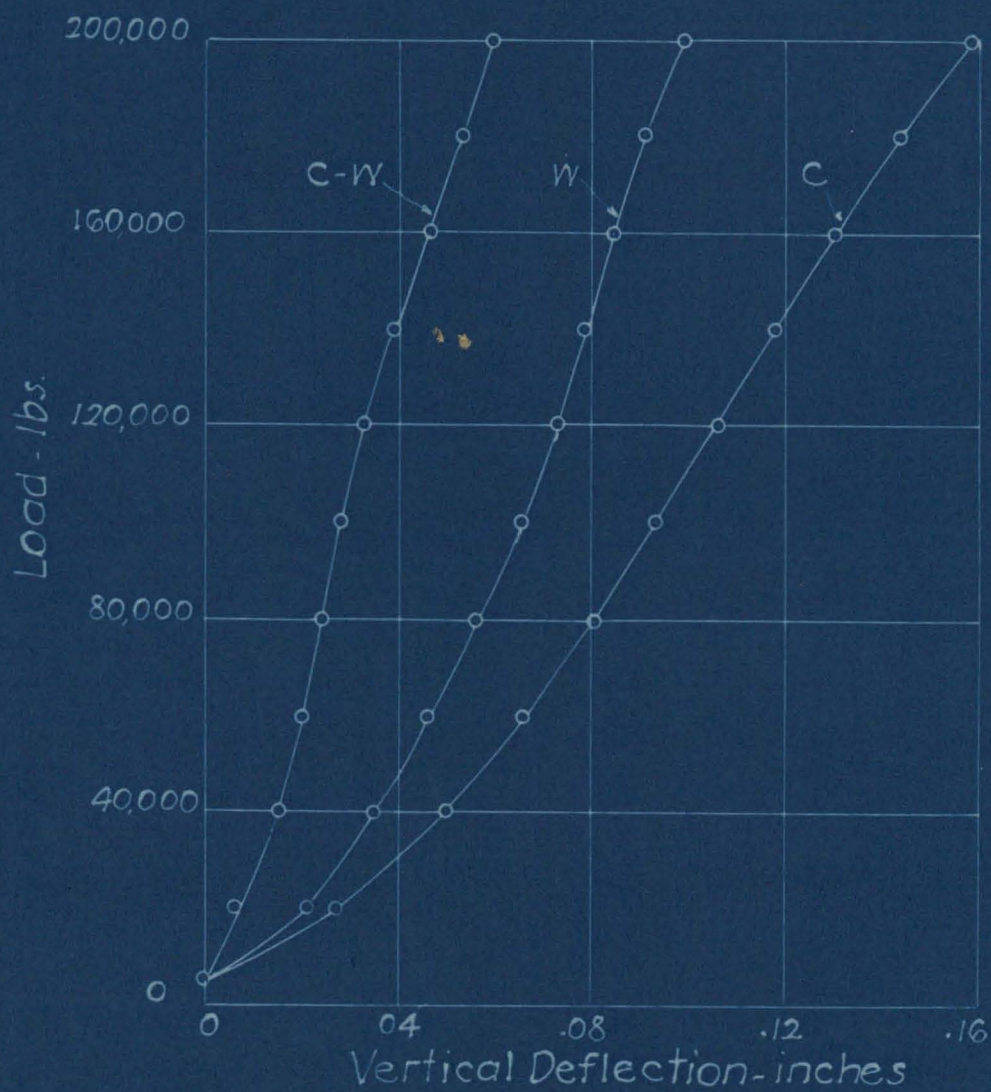


Fig. 20-Vertical Deflections of Beam CT-6
 Ultimate Load-209,500 lbs. Tested Nov. 28, 1932.

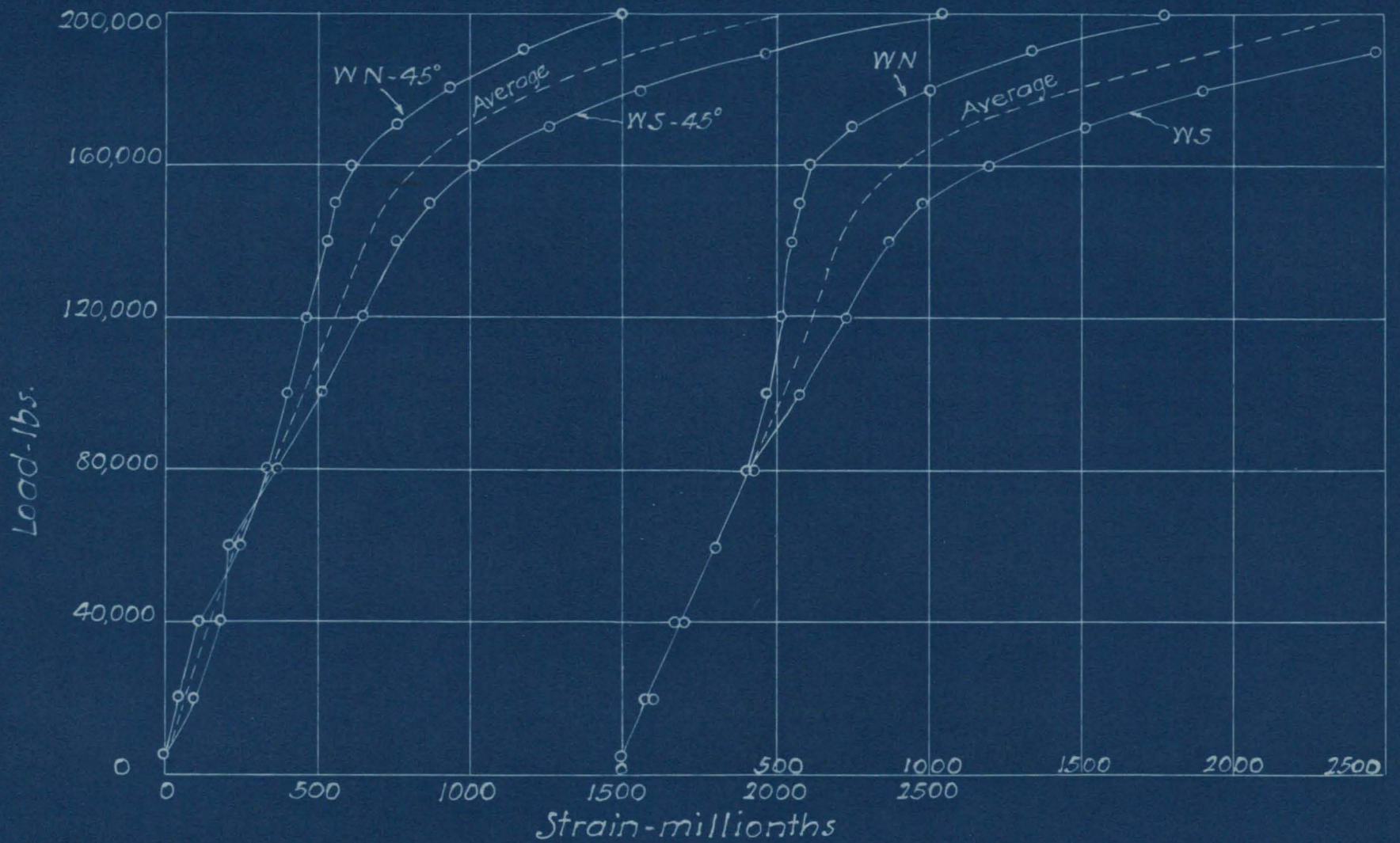


Fig. 21 - Compressive Strains in Web of Beam CT-6 Tested Nov 28, 1932.

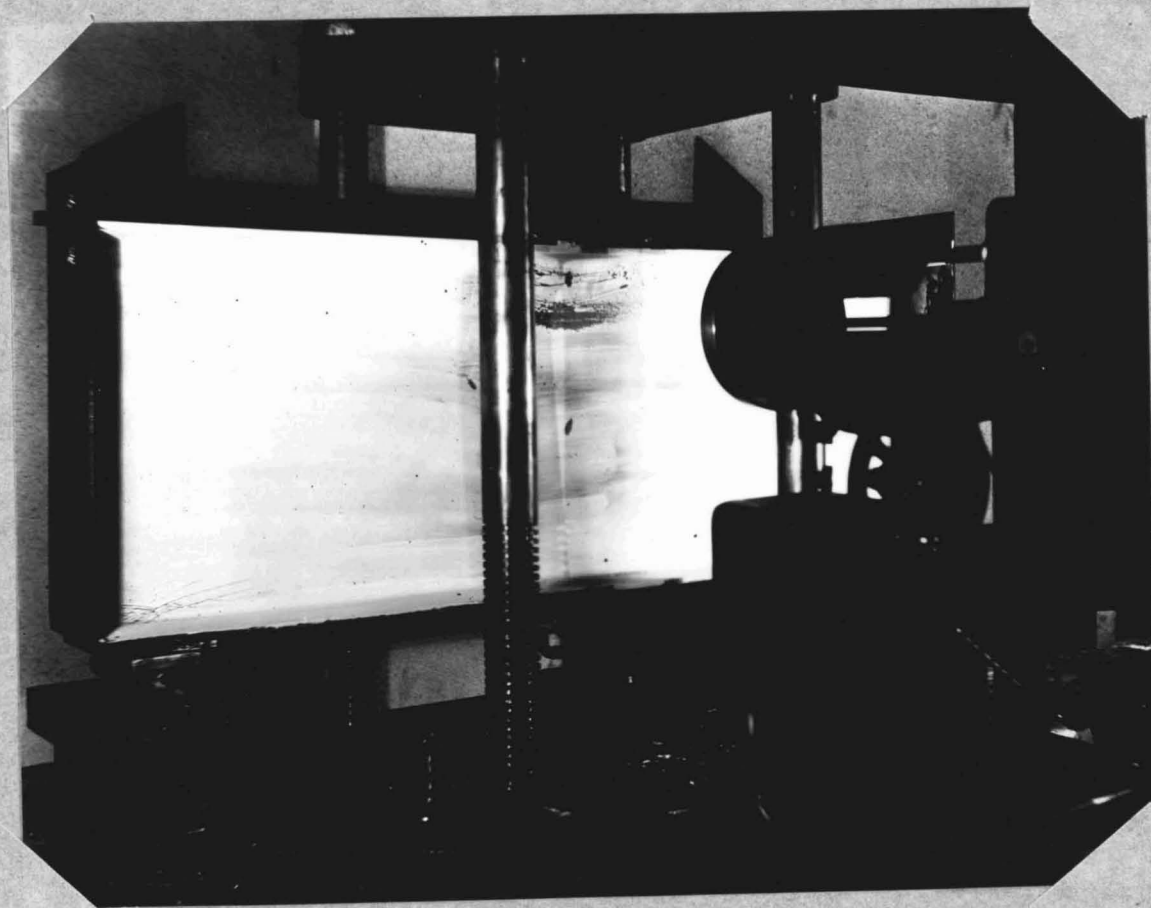


Fig. 22 - Beam CT-6, Showing Failure at the Center Due to Web Crippling

APPENDIX

by

Joseph B. Reynolds*

A mathematical solution that gives results closely approximating those found by experiment to exist in a deep horizontal I-beam loaded over a short portion of its upper length, has been found⁺. In applying this solution we assume the deep web of the beam as a continuous thin plate of infinite extent from one straight boundary (the upper edge). This is, in effect, assuming that the extra width represented by the upper flange does little more than help hold the web in a vertical plane and that the stresses and strains are so small towards the bottom of the web that they differ little from the values they would have in an infinite thin vertical plate at corresponding points.

The theory assumes that the stresses and strains dealt with are the averages over the thickness of the web, that the faces of the web are free from shear and that fibres perpendicular to these faces are free from stress. The notation used is as follows:

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+ See article by J. H. Michell, London Math. Proc. Vol. 34,
p. 134.

d = length of loaded span

$e_r, e_\theta, e_{r\theta}, e_n, e_t$ = strains corresponding in sense
to the stresses $S_r, S_\theta, S_{r\theta}, S_n, S_t$

E = Young's modulus

F = stress function of r, θ, r', θ'

G = shear modulus, $2(1+\mu)G = E$

k = constant

P = applied load

r, r' = radii vectors of points $P(r, \theta), (r', \theta')$ in neutral
plane of web

S_n = normal stress, tension in fibers normal to a
curved section of the web

S_t = tangential stress, tension in fibers parallel
to the tangent to curve in the
neutral plane of the web and in
the section determined by this curve

$S_{tn} =)$
 $S'_{tn} =)$ Tangential shear on curved section

S_r, S'_r = radial stress, tension in radial fibers

S_θ, S'_θ = peripheral stress, tension in fibers on
circular arcs about origin of coordinates

$S_{r\theta}, S'_{r\theta}$ = peripheral shear, shear on sections perpendicular
to radius

θ, θ' = vectoral angles

μ = Poisson's ratio

$2v$ = angle between radii vectors r, r'

The stresses can be obtained from the stress function by means of the equations:

$$S_r = \frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \theta^2}$$

$$(1) \quad S_\theta = \frac{\partial^2 F}{\partial r^2}$$

$$S_{r\theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F}{\partial \theta} \right)$$

The strains are connected with the stresses by the relations:

$$e_r = \frac{1}{E} (S_r - \mu S_\theta)$$

$$(2) \quad e_\theta = \frac{1}{E} (S_\theta - \mu S_r)$$

$$e_{r\theta} = \frac{2(1 + \mu)}{E} S_{r\theta} = \frac{1}{G} S_{r\theta}$$

For a curve cutting the radius vector r at an angle ϕ we have:

$$S_t = S_r \cos^2 \phi + 2S_{r\theta} \sin \phi \cos \phi + S_\theta \sin^2 \phi$$

$$(3) \quad S_n = S_r \sin^2 \phi - 2S_{r\theta} \sin \phi \cos \phi + S_\theta \cos^2 \phi$$

$$S_{tn} = (S_\theta - S_r) \sin \phi \cos \phi + S_{r\theta} (\cos^2 \phi - \sin^2 \phi)$$

Finally the stress function must satisfy the differential equation:

$$(4) \quad \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) F = 0$$

The solution we are interested in is given by superposing the two stress systems given by the stress functions $kr^2\theta$ and $-kr'^2\theta'$, both of which satisfy (4). Thus:

$$(5) \quad F = k(r^2\theta - r'^2\theta')$$

The origins of the two sets of coordinates (r, θ) and (r', θ') are two points O and O' on the straight boundary of the plate (Fig.1).

The formulas (1) applied to (5) give the two stress systems:

$$(6) \quad \begin{array}{ll} S_r = 2k\theta & S'_r = -2k\theta' \\ S_\theta = 2k\theta & S'_\theta = -2k\theta' \\ S_{r\theta} = -k & S'_{r\theta} = k \end{array}$$

Along the line $O'A$, $\theta = \theta' = 0$; so that the radial tension $S_r + S'_r = 0$ on this part of the boundary. On the same line the normal stress $S_\theta + S'_\theta = 0$ and also the shear $S_{r\theta} + S'_{r\theta} = 0$. Hence the boundary along $O'A$ is free of stress. For the line OB , $\theta = \theta' = \pi$, and this part of the boundary is found to be free of stress.

For the part of the boundary, OO' , $\theta = 0$, $\theta' = \pi$; so that for it the radial tension, $S_r + S'_r = -2k\pi$, the normal tension, $S_\theta + S'_\theta = -2k\pi$ and the shear, $S_{r\theta} + S'_{r\theta} = 0$. Hence this portion OO' of the boundary is under constant normal pressure $2k\pi$ per square inch, for this stress function to yield the solution. When this is the external loading on OO' this part of the boundary is under constant tangential stress also. In other words, O and O' tend to move towards each other. If the thickness of the plate is t and the total

applied load P , we must have $2k\pi = P/td$ where $OO' = d$, and therefore:

$$(7) \quad k = \frac{P}{2\pi td}$$

We next refer the combined stress system to the bisectors of the angles between r and r' at P , Fig. 1. For this purpose the primed system must be rotated through an angle $-v$ by use of equations (3) and the other system through an angle $+v$ where the angle $OP O' = 2v$. Adding the results thus obtained we find:

$$(8) \quad \begin{aligned} S_t &= -2k[2v - \sin 2v] = -\frac{P}{\pi td}[2v - \sin 2v] \\ S_n &= -2k[2v + \sin 2v] = -\frac{P}{\pi td}[2v + \sin 2v] \end{aligned}$$

$$S_{tn} = 0$$

Equations (8) show that these bisectors are tangents to the lines of principal stress at P . Since tangents to ellipses with O and O' as foci bisect the external angles between r and r' and tangents to hyperbolas with O and O' as foci bisect the internal angles between r and r' the lines of stress are evidently confocal hyperbolas and ellipses. These curves are shown in Fig. 2.

The stresses S_t and S_n have, as seen by (8), the same magnitude for all points for which v has the same value. Such points lie on a circular arc passing through O and O' subtending an angle $2\pi - 4v$ at the center of the circle. If the elastic limit is reached at a limiting value of S_n the region so

affected would be expected to be bounded approximately by the arc of a circle. This has been found to be the case*.

If S'_{tn} is the shear tangential to a curve through P making an angle θ with the lines of principal stress, we have by the third of equations (3) since $S_{tn} = 0$.

$$(9) \quad S'_{tn} = (S_t - S_n) \sin\theta \cos\theta = 2k \sin 2v \sin 2\theta$$

The shear is, therefore, greatest for a given value of v along lines at 45° to the lines of principal stress. Two pairs of these lines are shown on Fig. 2. Failure in shear would be expected to follow such lines. Experiment has demonstrated that it does so*.

Values of v were computed from the formula:

$$(10) \quad \cos 2v = \frac{r^2 + r'^2 - d^2}{2rr'}$$

The values on the right being obtained from measurement.

From equations of the form of (2) it was found that:

$$(11) \quad S_n = \frac{E}{1 - \mu^2} (\epsilon_n + \mu \epsilon_t), \quad S_t = \frac{E}{1 - \mu^2} (\mu \epsilon_n + \epsilon_t) \quad \checkmark$$

which were used to determine observed values of S_n and S_t , the strains ϵ_n and ϵ_t being taken from the specimen under test.

The theoretical stresses were computed by means of equations (8) values of v being known from (10).

* In additions to the results of this test see an article by G. Mesmer Bd I Technische Mechanik und Thermodynamik p. 85 et seq.

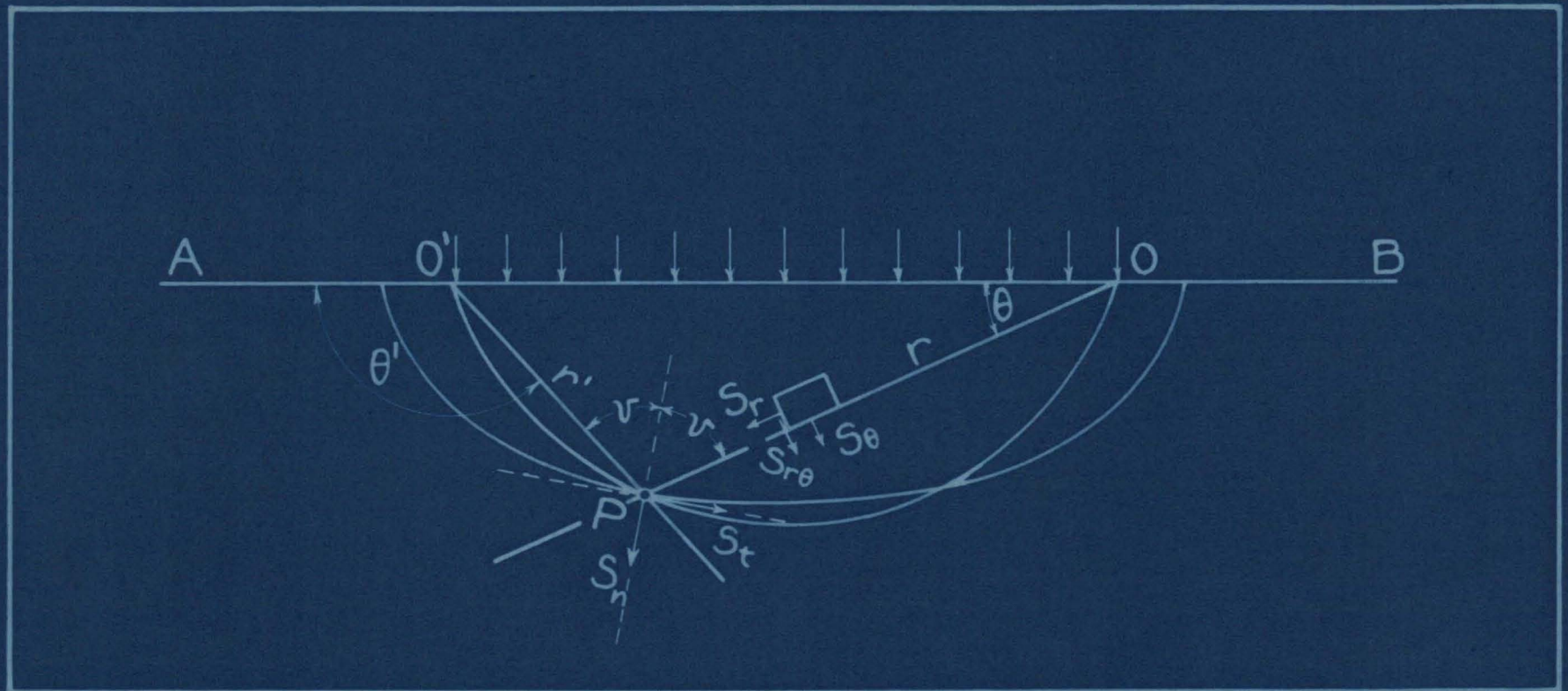


Fig.1 Illustration of Stress Distribution

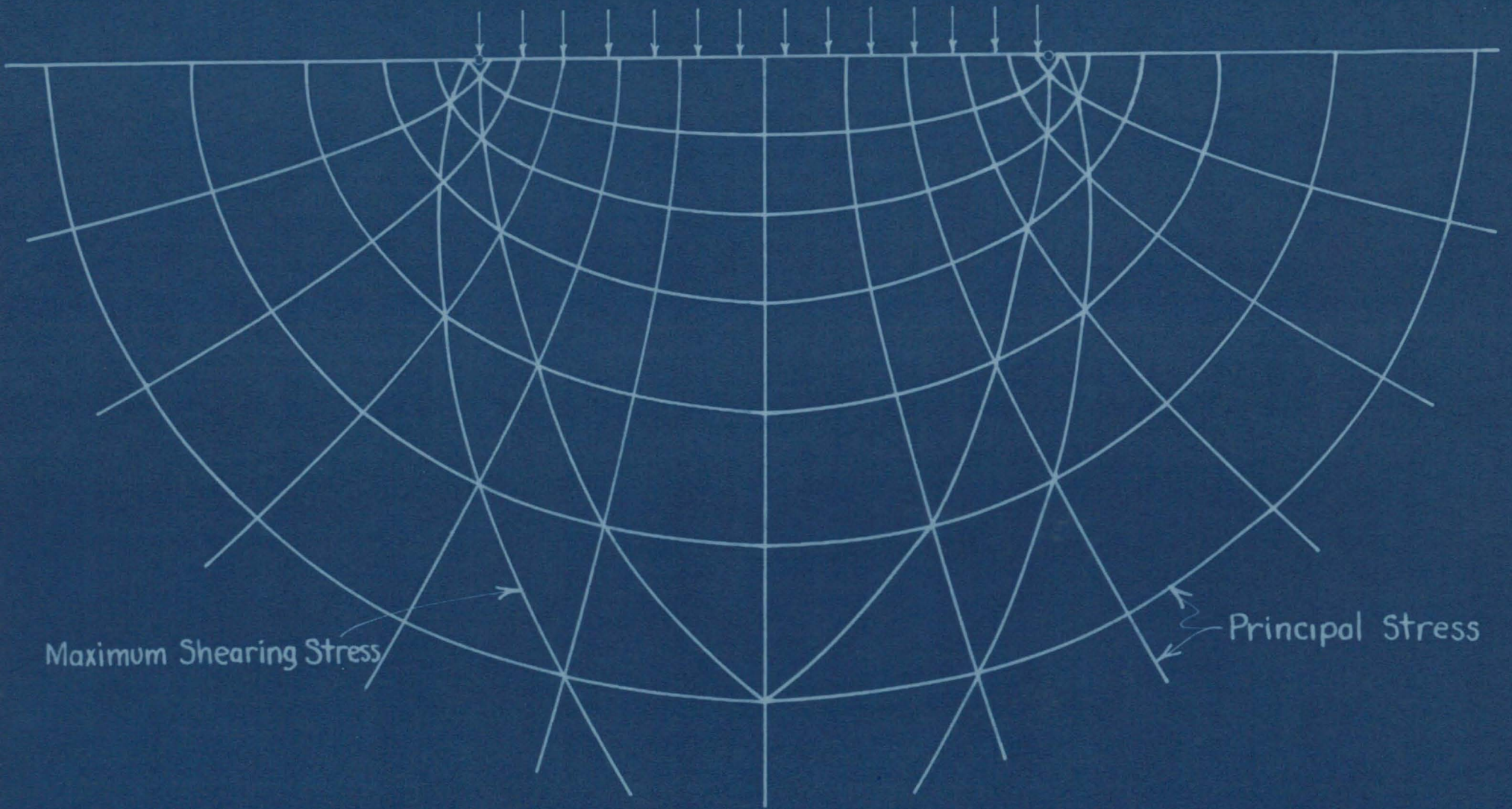


Fig.2 Lines of Principal Stress and Lines of Maximum Shearing Stress