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# THE RELATION BETWEEN QUALITY AND ECONOMY OF CONCRETE

by Inge Lyse\*

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## SYNOPSIS

This paper presents a summary of the inter-relation between the strength, permeability, durability, fire resistance and volume changes of concrete. It also submits a study of the relation between the strength of the concrete and the economy of plain and reinforced concrete members. It explains how a rational economical study is made possible by the constant water content theory, and how such factors as prices of cement and aggregate, strength quality of cement and gradation of aggregates affect the economy of concrete mixes. A study has also been made of the economy of reinforced concrete members subjected to compression and flexure, and results are presented which show the inter-relation between the strength of the concrete, the yield-point strength of the reinforcement and the economy of the member.

## INTRODUCTION

The relation between quality and economy of plain and reinforced concrete members has long been a doubtful question.

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It has been pointed out by different authors that a rich concrete mix will usually result in a more economical reinforced concrete structure than will a lean mix. As early as 1907 Professor Talbot<sup>(1)</sup> pointed out that the richer concrete mix was more economical for columns of a given strength than was a leaner mix. Professor Gillespie<sup>(2)</sup> and Professor Withey<sup>(3)</sup> have also stated that the richer concrete mix is the more economical for reinforced concrete columns and that an increase in richness of the concrete mix was more economical for adding strength to the column than an increase in the percentage of longitudinal reinforcement. Mr. A.R. Lord<sup>(4)</sup> recently presented data which show that a considerable saving was obtained when the strength of the concrete was 3000 instead of 2000 lb. per sq.in. Dr. H. Olsen<sup>(5)</sup> made a study of the effect on the economy of a structure when higher working stresses were used in the design. He concluded that the higher working stress (which requires stronger concrete and higher yield-point strength of the reinforcement) resulted in a lower cost of the structure.

No rational study, however, has previously been given to the direct relation between the strength of the concrete and the economy of the concrete member. This study has been made possible by the establishment of the constant water requirement for concrete of a given consistency regardless of the richness of the mix (6).

The unit cost of concrete may therefore be expressed in terms of the cement content, and since the strength is a function of the cement content; the unit cost is definitely related to the strength of the concrete.

#### NOTATION

The notation adopted for use in this paper is as follows:

- A = coefficient in strength equation
- $A_c$  = area of concrete (gross area)
- $A_s$  = area of longitudinal reinforcement
- $A_s'$  = equivalent area of spiral reinforcement
- a = aggregate content
- a' = change in aggregate content
- B = coefficient in strength equation
- c = cement content
- c' = change in cement content
- D = density of concrete, or portion of solids
- d = effective depth of beam
- E = cost per unit of strength
- F = total load on structural member
- $f_c'$  = cylinder strength of concrete
- $f_s$  = yield-point stress of longitudinal reinforcement
- $f_s'$  = yield-point stress of spiral reinforcement
- $\epsilon_a$  = specific gravity of aggregates
- $\epsilon_c$  = specific gravity of cement

- $\epsilon_p$  = specific gravity of cement paste
- $\epsilon_w$  = specific gravity of water
- $j$  = ratio in beam equation
- $K$  = coefficient in strength equation
- $k$  = ratio in beam equation
- $k'$  = effectiveness ratio of spiral reinforcement,  
that is, the ratio between the strength added  
by the spiral and that added by the same amount  
of longitudinal reinforcement.
- $M$  = bending moment at maximum load
- $n$  = ratio of modulus of elasticity of steel to that  
of concrete
- $P$  = cost per unit volume
- $p$  = percentage of reinforcement
- $p_a$  = price of aggregates
- $p_c$  = price of cement
- $p_c'$  = price of concrete
- $p_s$  = price of reinforcement
- $S$  = strength
- $s$  = factor of safety
- $V$  = volume of concrete
- $V_a$  = absolute volume of aggregates
- $V_c$  = absolute volume of cement
- $V_g$  = absolute volume of coarse aggregate

- $V_s$  = absolute volume of sand
- $V_w$  = volume of water
- $v$  = voids in concrete, air voids plus water
- $w$  = water content
- $y$  = the product  $kj$
- $Z$  = thickness of protective cover

HISTORICAL DEVELOPMENT

The first rational study of the factors affecting the strength of cement mortars was made by R. Feret<sup>(7)</sup>. He found that the strength of the mortars was determined by the amount of cement per unit of voids (air voids + water) in the mortar and also by the amount of cement per unit of cement plus voids. Feret's experimental evaluation of the relationship was:

$$S = K \left( \frac{c}{1 - v_s} \right)^2 \quad (1)$$

where  $K$  depends upon the materials and the conditions of the test.

Assuming that the same relationship would hold for concrete, the equation becomes:

$$S = K \left( \frac{c}{1 - v_s - v_g} \right)^2 = K \left( \frac{c}{1 - v_a} \right)^2 \quad (2)$$

Professor M. O. Withey<sup>(8)</sup> in 1914 published results which showed a straight line relationship between the strength and the cement-void ratio of concrete. Professor Withey's relation may be expressed by the equation:

$$S = A + B \cdot \frac{C}{V} \quad (3)$$

In 1918 Professor Duff A. Abrams<sup>(9)</sup> published his water-cement ratio relationship and presented the following equation for plastic and workable concrete mixes:

$$S = \frac{A}{B \frac{w}{c}} \quad (4)$$

For average laboratory conditions Professor Abrams evaluated the constants in the equation:

$$S = \frac{14,000}{7 \frac{w}{c}} \quad (5)$$

Since the first publication of the water-cement ratio a great many investigators have contributed a vast amount of information on the relationship between the strength and the water-cement ratio of the mix. The outstanding contributors are F. R. McMillan<sup>(10)</sup> and Professor Otto Graf<sup>(11)</sup> of Germany. Professor Graf expanded Abrams' water-cement ratio relationship by the use of a factor which represented the strength quality of the cement used.

At the University of Illinois experimentation on the fundamental relationships for the strength of concrete had been carried out for many years under the able leadership of Professor A. N. Talbot. In 1923 the results of these studies were published<sup>(12)</sup> by Professors Talbot and Richart. The relationship for strength was expressed as a function of the cement-space ratio or the cement-void ratio. The equation given for average conditions was:

$$S = \frac{32,000}{(1 + \frac{v}{c})^{2.5}} = 32,000 \left( \frac{c}{c+v} \right)^{2.5} \quad (6)$$

It is noted that the Illinois results check Feret's results for mortar quite well. However, the cement-void and the cement-space relations never received the attention which was stimulated by Abrams' water-cement ratio.

A study of the relationships for the strength of concrete presented above, reveals that in all cases the cement and the voids or the water, played an important role in the equations. There is therefore no fundamental difference between the several relationships obtained. Since for plastic and workable mixes the amount of air voids in properly placed concrete is generally less than one per cent, the difference between the voids and the water content becomes so small that the void-cement ratio is very nearly equal to the water-cement ratio. Professor Slater<sup>(13)</sup> has shown how well the Talbot-Richart void-cement ratio curve agrees with Abrams' water-cement ratio curve when the voids are equal to the water content.

The water-cement ratio relationship is at present generally recognized as the criterion for the strength of concrete.

In 1930 Mr. R. L. Bertin<sup>(14)</sup> suggested the use of the specific gravity of the cement paste as a measure of the strength of the concrete. He found that the Abrams water-cement ratio curve became very nearly a straight line when the strength was plotted against the specific gravity of the cement paste.



Bertin's equation for the strength of concrete is:

$$S = A \cdot \rho_p - B \quad (7)$$

In 1931 it was shown<sup>(15)</sup> that when the strength of the concrete was plotted against the reciprocal of the water-cement ratio, that is the cement-water ratio, both the Abrams and the Talbot-Richart curves for plastic and workable mixes gave an approximately straight line relation. The relationship between the strength and the cement-water ratio of concrete has since been studied more fully and it has been shown that within the range of practical concrete mixes the straight line relation serves very well. It has also been shown that when the water content in a unit of concrete remains constant and the cement and the aggregate contents are the only variables, the strength increases in direct proportion to the increase in the cement content, or in other words, in direct proportion to an increase in the cement-water ratio. The relationship between strength and the cement-water ratio of concrete is therefore given by the formula:

$$S = A + B \cdot \frac{C}{W} \quad (8)$$

This equation holds only within a certain range, which, however, covers all practical concrete mixes. For extremely lean as well as extremely rich mixes, the straight line relationship does not apply. This leads to the conclusion that for practical mixes the strength of the concrete is determined by the concentration of cement particles in a unit of water which may be expressed

as follows: Above a minimum number of cement particles necessary to give workability and binding strength to concrete, the strength of the concrete increases in direct proportion to the increase in number of cement particles per unit of water. Several French technical papers have also pointed out this straight line relation between the cement-water ratio of the paste and the strength of the concrete<sup>(16)</sup>.

Furthermore, it has been shown<sup>(6)</sup> that for a given type and gradation of aggregates the consistency of the concrete remains nearly constant as long as the water content per unit of concrete remains the same. Thus for concrete of equal consistency the strength equation becomes:

$$S = A + B/w.c = A + K.c \quad (9)$$

The strength of the concrete is here a function of the cement content in such a way that the strength increases in direct proportion to the increase in the cement content. Equation (9) is very convenient for the design and control of concrete mixes<sup>(17)</sup> and also gives the foundation for the rational study of the economy of concrete structures.

#### RELATION BETWEEN STRENGTH AND OTHER QUALITIES OF CONCRETE

It has been shown above that the compressive strength of concrete increases directly in proportion to the increase in the cement-water ratio. Gonnerman and Shuman<sup>(18)</sup> have shown that the factors which determine the compressive strength also ascertain the tensile and flexural strength. They found that the tensile

and flexural strengths of concrete were determined by the water-cement ratio of the paste in much the same manner as the compressive strength. It may therefore be concluded that the strength properties of concrete are primarily determined by the cement-water ratio.

The permeability of concrete has also been primarily dependent upon the same factors as those determining the strength of the concrete<sup>(19)</sup>. Fig. 1 shows the relation between the strength and permeability of concretes, each of which contained a different brand of cement. It is noted that the permeability decreases in direct proportion to the increase in strength of the concrete. Mr. McMillan<sup>(20)</sup> has also shown that for a given brand of cement and a given curing of the concrete, the permeability decreases with the increase in strength.

Professor Scholer<sup>(21)</sup> has demonstrated that the durability of concrete is to a large extent determined by the strength of the concrete. The relation between the number of cycles of freezing and thawing which will produce an initial disintegration<sup>(22)</sup> and the cement-water ratio of the concrete is shown in Fig. 2. The number of cycles increases directly with the increase in the cement-water ratio. This means that the resistance to initial disintegration increases in direct proportion to the increase in the concentration of cement in the cement paste used. The same law, therefore, which applies to the strength of concrete also applies to the resistance of initial disintegration.

The fire resistance of concrete has lately been studied quite extensively<sup>(23)</sup> and the outstanding results of these studies are presented in Fig. 3. It is noted from this figure that the fire resistance of the concrete increases in direct proportion to the increase in cement content, both for ordinary aggregates and for the light weight aggregate, Haydite. It is also seen that the concrete containing light weight aggregate shows a considerably higher fire resistance for the same cement content than does concrete containing ordinary aggregates, and that the fire resistance increases more rapidly with the increase in cement content for concrete containing light weight aggregates than for concrete containing ordinary aggregates.

From these results it may be concluded that the factors which give high strength will also give high impermeability, greater resistance to freezing and thawing, and high fire resistance.

On the other hand, the volume changes due to soaking and drying of the concrete will increase with the increase in cement content<sup>(13)</sup>. Therefore, the greater the richness of the concrete mix, the greater is the volume change. The properties of the concrete which produce high strength will therefore not necessarily produce low volume changes.

THE ECONOMY OF PLAIN CONCRETE

The constant water content theory enables us to make a rational study of the relationship between the economy of different concrete mixes. For mixes having a constant water content the amount of solids in the concrete remains the same regardless of richness of mix so that the change in cement content is accompanied by a similar change in the aggregate content. Thus the following relation is found:

$$\frac{c}{\rho_c} = \frac{a}{\rho_a} \text{ or } \frac{a}{c} = \frac{\rho_a}{\rho_c} \quad (10)$$

Setting up the equation for the volume of the concrete:

$$V = V_c + V_a + V_w = \frac{c}{\rho_c} + \frac{a}{\rho_a} + V_w = D + V_w \quad (11)$$

since  $\frac{c}{\rho_c} + \frac{a}{\rho_a} = D$ , from which:  $a = D \cdot \rho_a - c \cdot \frac{\rho_a}{\rho_c}$

The price of the concrete materials is:

$$P = p_c c + p_a a \quad (12)$$

or substituting for a:

$$P = p_c c + p_a (D \rho_a - c \cdot \frac{\rho_a}{\rho_c}) = p_a D \cdot \rho_a + c \cdot (p_c - \frac{\rho_a}{\rho_c} p_a) \quad (13)$$

Thus the cost of the concrete materials is a direct function of the cement content for given prices on aggregates and cement. By means of equation (13) the cost of any mix may be determined when the prices of the materials, the density of the mix and the specific gravities of the aggregates and the cement are known. Since for ordinary concrete materials  $\rho_a =$  about 2.65

and  $g_c =$  about 3.10, the equation is reduced to:

$$P = 2.65D.p_a + (p_c - 0.85p_a).c \quad (14)$$

If one cubic yard be used as unit of concrete, the equation becomes when  $p_c$  and  $p_a$  are given in cents per pound:

$$P = 4460D.p_a + (p_c - 0.85p_a)c \quad (15)$$

where  $c$  is given in pounds per cubic yard of concrete. Thus for given prices of the material and for a given consistency of the concrete, the cost of the materials per cubic yard of concrete is directly related to the variation in the cement content.

It has previously been shown that the strength for concrete of constant water content is given by the formula:

$$S = A + Kc \quad \text{or} \quad c = \frac{S-A}{K} \quad (16)$$

Both the cost of the materials and strength of the concrete are therefore given as a direct function of the cement content. The cost expressed in terms of the strength becomes when  $p_a$  and  $p_c$  are given in cents per pound:

$$P = 4460D.p_a + (p_c - 0.85p_a)\frac{S-A}{K} \quad (17)$$

For plain concrete members loaded directly in compression the load-carrying capacity is directly proportional to the strength of the concrete. The size of the concrete member for a given load is therefore determined by the strength of the concrete. The cross section area of the member is given by:

$$A_c = \frac{F}{s.f_c'} \quad (18)$$

For a unit length of the member the volume of concrete is proportional to the cross-sectional area. Thus the volume of concrete required to carry a given load is inversely proportional to the strength of the concrete. The price of concrete per unit of strength is therefore a measure of the economy of the mix. The price per unit of strength is given by the equation:

$$E = \frac{P}{S} = \frac{1}{S} (4460D \cdot p_c + (p_c - 0.85p_a) \frac{S-A}{K}) \quad (19)$$

Fig. 4 shows the relation between the cost per cubic yard of concrete per 1000 lb. per sq.in. of strength and the strength of the concrete for given materials and conditions of test and for five different prices of the cement. The decrease in cost for a given load with the increase in strength of the concrete is very great. A change in the cement content, which will produce a corresponding change in the strength, will easily offset small differences in the price of the cement. This figure is a strong evidence for increased economy with increased strength of the concrete. It should furthermore be kept in mind that the concrete of high strength has other advantages as compared with concrete of low strength, namely, higher resistance to leakage, to fire and to the action of freezing and thawing. The only disadvantage incorporated in rich concrete over lean mixes is the larger volume changes.

In Fig. 5 the only variable introduced was that of the price of the aggregates. It is noted that a change in the price of the aggregates has a greater effect upon the economy of the

mix than has a corresponding change in price of the cement. This is due to the relatively large quantities of aggregates in a unit of concrete. It is noted also that the effect of the price of the aggregates on the economy of the mix is more prominent for lean than for rich mixes. The economy of the concrete increased very much with the increase in strength of the concrete regardless of the price of the aggregates. The strength relation for the concrete used in Fig. 4 and 5 was:

$$S = -2570 + 3600.c/w$$

In order to study the effect of the strength-giving qualities of the materials on the cost of the concrete, three different strength relations were considered. In Fig. 6 the strengths have been plotted against the cement-water ratio for concretes containing three different cements. The strengths for a given cement-water ratio were greatly different for the different cements. In Fig. 7 the cost per cubic yard of concrete per 1000 lb. per sq.in. strength has been plotted against the strength of the concrete for these three strength relations. The cost is seen to be slightly higher for the cement-giving the lower strength, but the difference is not marked. A slight increase in the richness of the concrete would easily offset the small differences in cost due to the strength qualities of the cements.

In Fig. 8 the variation in cost with strength of concrete is shown for different densities of the concrete. In preparing this figure it was assumed that the strength of the concrete increased in direct proportion to the increase in the cement-water



ratio of the paste, regardless of the gradation of the aggregates and the consequent variation in the density of the concrete. The density of the concrete was varied from as high as 0.85 to as low as 0.75. It is noted that these variations in the density produced only slight differences in the cost of the concrete.

The outstanding result of this study of plain concrete is the great increase in economy with the increase in the strength of the concrete.

In the practical application of the above study the cost of mixing, placing, curing and possible finishing of the concrete and also the cost of formwork and labor contribute to the total cost of the final structure. However, the cost of preparing, handling and curing the concrete is nearly the same for rich and for lean mixes. The cost of formwork, labor, and finish may be somewhat different for the different concretes, since the cross-sectional area of the member changes with the strength of the concrete and is greater for lean than for rich mixes. However, in most cases the differences in the cost of these items are so small that for studies of the nature presented in this paper they may be neglected.

#### ECONOMY OF REINFORCED CONCRETE COLUMNS

In a reinforced concrete member the cost per linear length is made up of the cost of the concrete, the cost of the reinforcement, the cost of forms and the cost of possible finish. Where the

total area of the member is in compression, both the concrete area and the steel area will contribute their full load-carrying capacity. The strength of such a member is made up of the individual strengths of the concrete and the reinforcement. The effectiveness of the concrete in a reinforced concrete column has been found to be approximately 85 per cent<sup>(24)</sup> of the cylinder strength. Thus for a tied reinforced concrete column the ultimate strength is given by:

$$S = 0.85f'_c \cdot A_c + f_s A_s \quad (20)$$

For a spirally reinforced concrete column the ultimate strength is given by:

$$S = 0.85f'_c A_c + f_s A_s + k' \cdot f'_s A'_s \quad (21)$$

If the load carried by the spiral is equal to, or less than the load carried by the protective concrete shell outside the spiral, the spiral will not add to the strength of the column. The effectiveness\* ratio,  $k'$ , becomes zero and the formula for the spirally reinforced columns becomes equal to the formula for the tied column. In the following study the strength contributed by the spiral reinforcement has been assumed to be equal to or less than the strength of the concrete shell.

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\* The effectiveness is here measured in terms of the strength added by the spiral in excess of the strength added by the protective shell.

In making an economical study of a reinforced concrete column, the following cost equation is used:

$$P = p'_c \cdot A_c + p_s \cdot A_s + \text{Forms} + \text{Finish} \quad (22)$$

where P is final cost per unit of length of column. Since the cost of forms and finish does not vary appreciably with the size of the columns used, it has been neglected in the following study. The first term on the right-hand side of the equation is studied under plain concrete except that the cost is 17-1/2 per cent greater than for plain concrete, due to the fact that only 0.85 of the cylinder strength is available in the reinforced column.

The most economical column is the one having the lowest cost per unit of strength. The unit cost of concrete is:

$$\frac{p'_c \cdot A_c}{0.85 f'_c \cdot A_c}$$

and the unit cost of steel is:

$$\frac{p_s \cdot A_s}{f_s \cdot A_s}$$

In which the steel area  $A_s$  in the ~~numerator~~<sup>numerator</sup> and denominator is equal for columns having welded reinforcement, but in which the steel area in the ~~numerator~~<sup>numerator</sup> is about 1.20 times the area in the denominator for columns having spliced reinforcement. Thus for columns having welded reinforcement we have that  $\frac{p'_c}{0.85 f'_c}$  and  $\frac{p_s}{f_s}$

are unit cost of concrete and steel respectively, per unit of length. In order to have a lower unit cost for reinforcement than for concrete,  $\frac{p_s}{f_s}$  must be less than  $\frac{p'_c}{0.85f'_c}$ . Since  $p_s$  is usually given in terms of tons and  $p'_c$  in terms of cubic yards, it is necessary to relate these terms.

1 cu.yd. steel = 490 lb. x 27 = 13,200 lb. = 6.6 tons.  
 Thus  $\frac{p_s}{f_s}$  must be less than  $\frac{1}{6.6} \cdot \frac{p'_c}{0.85f'_c} = \frac{p'_c}{5.6f'_c}$ , or the price  $p_s$  must be less than  $\frac{1}{5.6} \cdot \frac{f_s}{f'_c} \cdot p'_c = 0.18 \frac{f_s}{f'_c} p'_c$ . For  $f_s = 45,000$  and  $f'_c = 2000$ ,  $p_s$  must be less than  $0.18 \frac{45000}{2000} p'_c = 4.05 p'_c$ . For  $f_s = 80,000$  and  $f'_c = 2000$ ,  $p_s$  must be less than  $0.18 \frac{80000}{2000} p'_c = 7.20 p'_c$ . In Fig. 9 the values of the cost ratio are given for several yield-point strengths of the reinforcement and for several strengths of the concrete.

For columns having spliced reinforcement the cost ratio becomes, when the amount of splicing is 20 per cent of the theoretical amount of reinforcement:

$$p_s = \frac{1}{1.20} \cdot 0.18 \frac{f_s}{f'_c} \cdot p'_c = 0.15 \frac{f_s}{f'_c} \cdot p'_c$$

Under plain concrete it was pointed out that the most economical concrete mix was that which produced the greatest strength. It is noted from Fig. 9 that the greater the yield-point of the reinforcement, and the lower the strength of the concrete, the most chance there is that the reinforcement may

be more economical than the concrete for the carrying of a definite load.

By means of this relationship between the cost of steel and concrete it is a simple matter to compute the most economical reinforced concrete column.

If a reinforced concrete column contains spiral reinforcement which will give strength in excess of the strength of the protective shell, a study of the economy of the spiral can be made in the same manner as shown above for the longitudinal reinforcement. However, the economy of the spiral reinforcement may be considered more directly. If the cost of the spiral reinforcement shall be less than the cost of the longitudinal reinforcement per unit of load carried, the cost ratio between the spiral and the longitudinal reinforcement must be less than the effectiveness ratio of the spiral. Thus, if the effectiveness ratio of the spiral is 1.0, the cost of the spiral reinforcement in place must be less than the cost of the equivalent percentage of longitudinal reinforcement if a more economical column is desired.

#### ECONOMY OF REINFORCED CONCRETE BEAMS

It is obvious that in reinforced concrete slabs and rectangular beams the most economic design results when both the steel and the concrete are fully utilized, that is, when the maximum stress in the steel corresponds to its yield-point stress and the maximum stress in the concrete corresponds to its ultimate

strength, providing the bending moment governs the size. If both materials were not fully utilized the steel would reach its yield-point stress before the concrete reached its ultimate strength, or vice versa, and the depth or the amount of reinforcement would be greater than that necessary for its load-carrying capacity. The depth of the section or the amount of reinforcement could be adjusted until the steel and concrete stresses were balanced without affecting the strength of the section.

It has been shown<sup>(25)</sup> that the compressive strength of the concrete in flexure is considerably greater than that in direct compression when straight line stress distribution is used in the computation. The maximum compressive stress in reinforced beams corresponded to approximately 1.5 times the cylinder strength for concrete of strengths of more than 2000 lb. per sq.in. and this value has been used in the following study. It has also been shown<sup>(25)</sup> that the location of the neutral axis for beams of different strengths of concrete does not vary greatly. This is due to the fact that the location of the neutral axis is not only determined by the ratio of the moduli of elasticity of steel and of concrete, but also by the percentage of reinforcement. The position of the neutral axis is given by  $k = \frac{\sqrt{2pn + (pn)^2} - pn}{2}$ . Since an increase in the strength of the concrete will decrease the ratio  $n$  but require an increase in the percentage  $p$ , the position of the neutral axis is only slightly affected. The location of

the neutral axis has therefore been taken as fixed in the following study. Per unit width of slab the ordinary straight line stress distribution gives:

$$1.5f'_c = \frac{2M}{k.j.d^2} = \frac{2M}{y.d^2} \quad (23)$$

$$\text{or } d^2 = \frac{2M}{1.5.yf'_c} = \frac{4}{3} \frac{M}{y.f'_c}$$

$$d = 2 \sqrt{\frac{M}{3.y.f'_c}} \quad (24)$$

The effective depth of the beam thus decreases inversely with the square root of the cylinder strength. Since the cost equation per unit length of a reinforced concrete beam of unit width is:

$$P = d.p'_c + A_s p_s + Z.p'_c + \text{Forms} + \text{Finish} \quad (25)$$

the cost of the slab for a given condition decreases with the decrease in the effective depth. If the cost of forms and finish be considered equal for different depths, the variation in the strength of the concrete affects only the first and third terms of the equation. Neglecting for the present, the cost of the protective cover,  $Z.p'_c$ , a study will be given to the first term:

$$dp'_c = p'_c \cdot 2 \cdot \sqrt{\frac{M}{3.y.f'_c}} = \frac{2p'_c \cdot \sqrt{\frac{M}{3.y}}}{\sqrt{f'_c}}$$

For a given moment,  $M$ , the cost of the concrete in the effective depth of the section is seen to vary inversely with the square root of the strength of the concrete. The cost of the protective cover,  $Z.p'_c$ , will increase with the strength of the concrete

and the amount of steel will have to be increased in order to prevent failure in the reinforcement. The amount of reinforcing steel is given by:

$$A_s = \frac{M}{j \cdot d \cdot f_s} = \frac{M}{2 \cdot j \cdot \sqrt{\frac{M}{3 \cdot y \cdot f'_c}} \cdot f_s} = \frac{\sqrt{3 \cdot M \cdot y \cdot f'_c}}{2 \cdot j \cdot f_s} \quad (26)$$

The amount of reinforcement will thus vary directly with the square root of the strength of the concrete. Since the relative increase in the amount of steel is equal to the relative decrease in the volume of effective concrete, the total cost of the section may either increase or decrease with the increase in strength of the concrete, depending upon the relative costs and amounts of concrete and steel. Generally the decrease in cost of the concrete will more than offset the increase in the cost of the steel and of the protective cover. In order to illustrate the relation between the variation in the cost of the section and the strengths of the concrete for given conditions Fig. 10 has been prepared. In this figure relative costs of the section for different strengths of the concrete are shown for the following conditions.

Mult. = 100,000 lb-in. per inch of width

$f_s = 40,000$  lb. per sq.in.

$j = 0.82$

$y = k \cdot j = 0.44$

$D = 0.82$



$$p_c' = 4460D \cdot p_a + (p_c - 0.85p_a) \frac{S-A}{K} = 3860p_a + (p_c - 0.85p_a) \frac{S+2570}{12}$$

in which:

$$p_a = 0.10 \text{ cents per pound, and } p_c = 0.50 \text{ cents per pound}$$

Then:

$$p_c' = 366 + 0.415 \frac{S+2570}{12} = 366 + 89 + 0.0346S = 455 + 0.0346S, \text{ cents per cubic yard.}$$

$$p_s = 3-1/2 \text{ cents per pound, and } Z = 1.0 \text{ in.}$$

It is noted from Fig. 10 that the relative cost of the section decreases slightly with the increase in the strength of the concrete. The decrease, however, is very small when compared with concrete in direct compression, indicating that the flexural concrete member is very little affected by the variation in the strength of the concrete. If other factors than the flexural stresses govern the size of the section, the above study is of no value in judging the economy of the member.

Since the yield-point strength of the reinforcement is fully utilized at the ultimate strength of the reinforced concrete beam or column, it becomes the criterion for the economy of the reinforcement. The economy of the reinforcement is therefore directly proportional to the ratio between the cost of the steel and its yield-point stress, that is:  $E = \frac{P_g}{P_s}$ .

In this paper no study has been given to such important items as the reduction in dead load produced by the use of smaller sections and the increase in floor space gained by any reduction in the size of the columns. For given conditions, the economic effect of these items may be estimated directly by the use of the ordinary design formulas.

### CONCLUSIONS

1. The constant water content for concretes of a given consistency presents a basis for a rational study of the relation between the quality and the economy of the concrete.

2. The impermeability, the durability and the fire resistance, as well as the strength of concrete are determined primarily by the cement-water ratio of the paste.

3. The economy of a plain concrete member designed to carry a given load increases markedly with the increase in the strength of the concrete.

4. The ordinary variations in strength qualities of the cement as well as the density of the concrete have slight effect upon the economy of the concrete.

5. The economy of reinforced concrete columns carrying a given load increases with the increase in strength of the concrete used.

6. If for columns having welded reinforcement, the cost of the longitudinal steel (per ton) is  $0.18 \frac{f_s}{f_c}$  times the cost of

the concrete (per cubic yard), the strength of the column may be increased at equal cost by the use of either steel or concrete. If the cost of the steel is greater than this ratio, the most economical column is obtained by the use of a minimum amount of reinforcement. For spliced reinforcement the cost ratio is about  $0.15 \frac{f_s}{f'_c}$ .

7. When the flexural stresses govern the size of a reinforced concrete member, the economy of the member is only slightly affected by the strength of the concrete used.

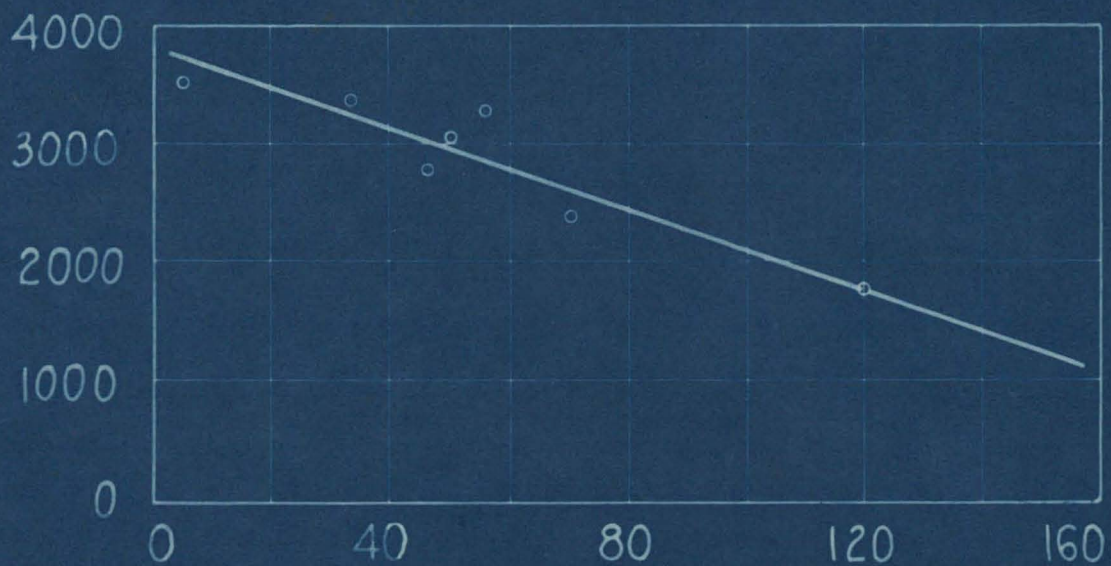
8. The economy of the reinforcement is directly proportional to the ratio between the cost of the steel and its yield-point stress.

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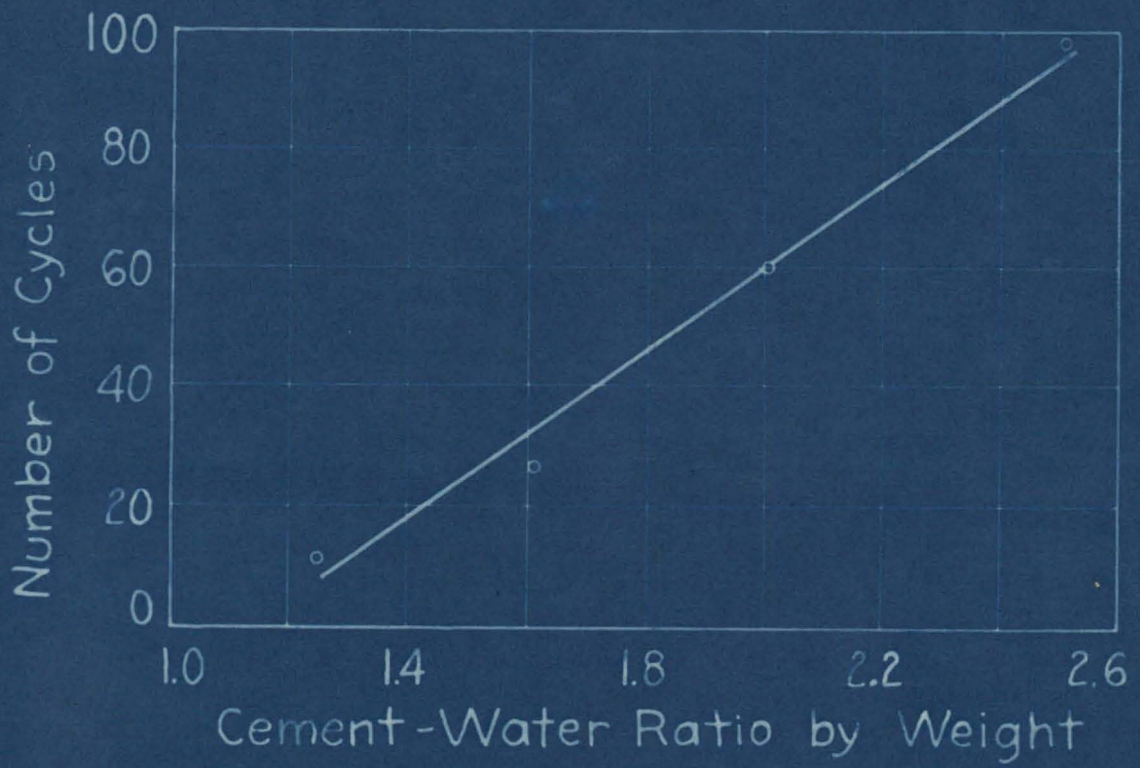
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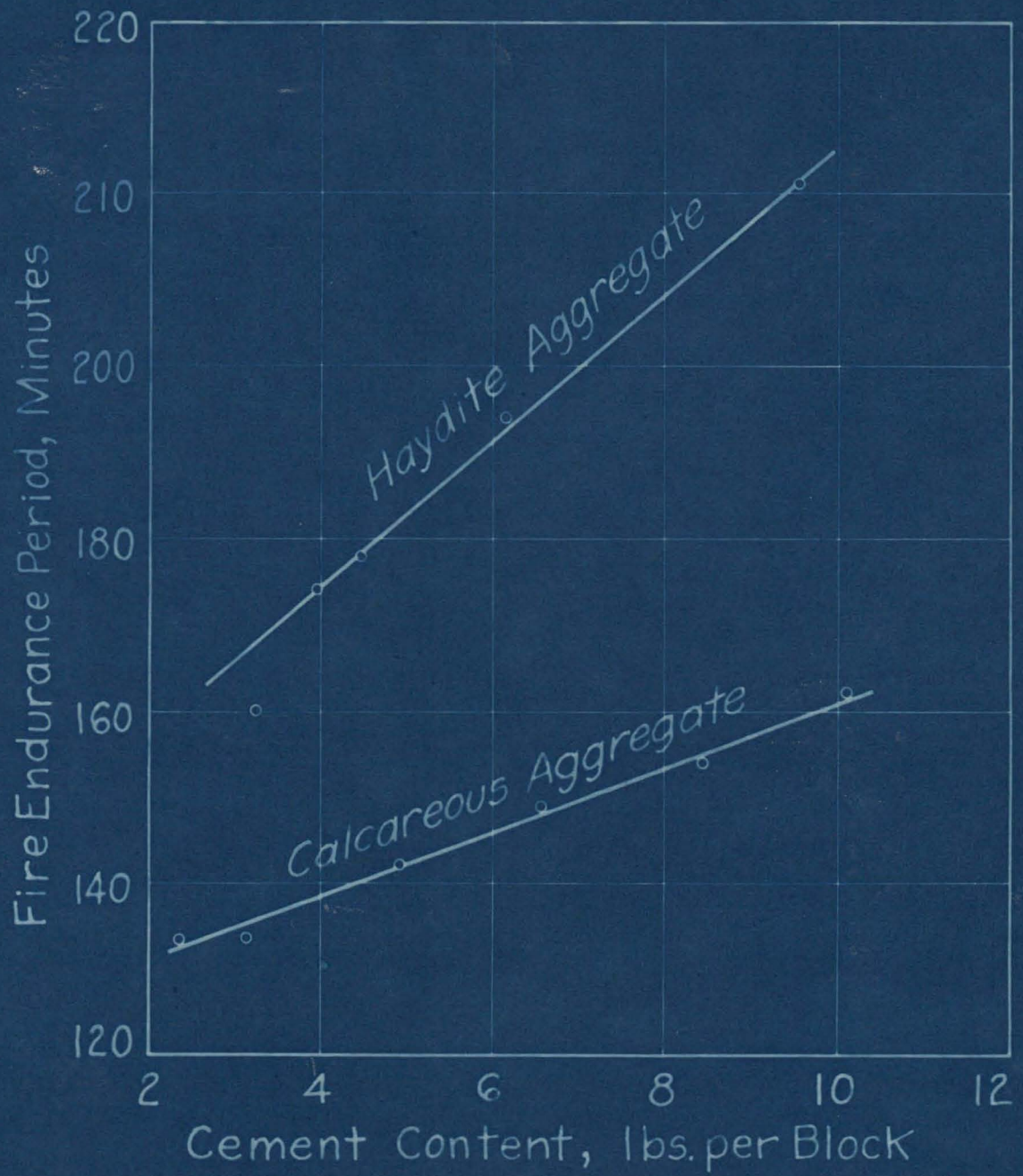
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Compressive Strength,  
lb. per sq. in.

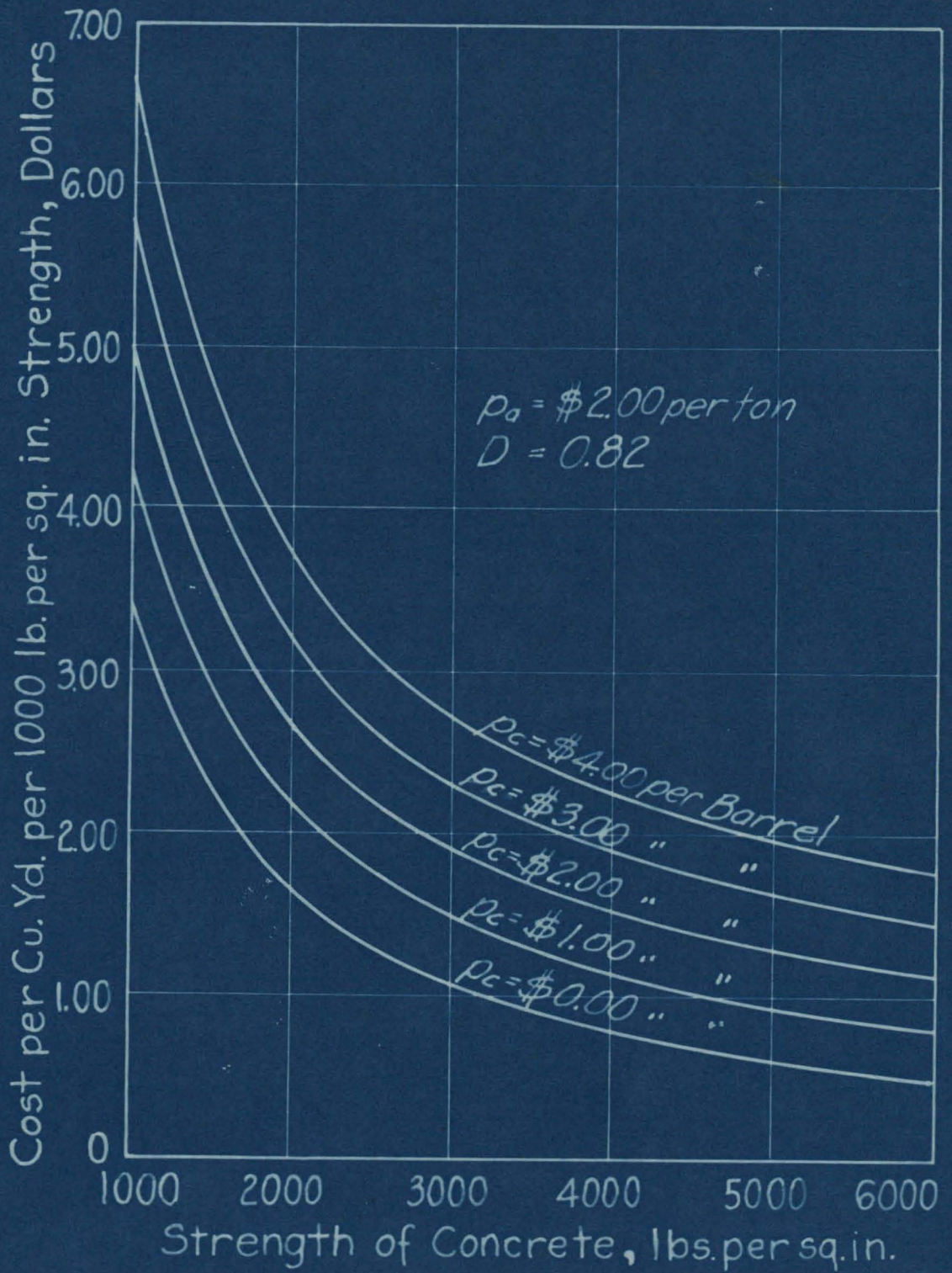


Average Leakage, c.c. per hr per sq. ft.









This graph shows the relationship between the strength of concrete and the cost per cubic yard for different cement prices. The cost per cubic yard increases as the strength of concrete increases, and the cost also increases as the cement price increases. The graph is based on the following assumptions:

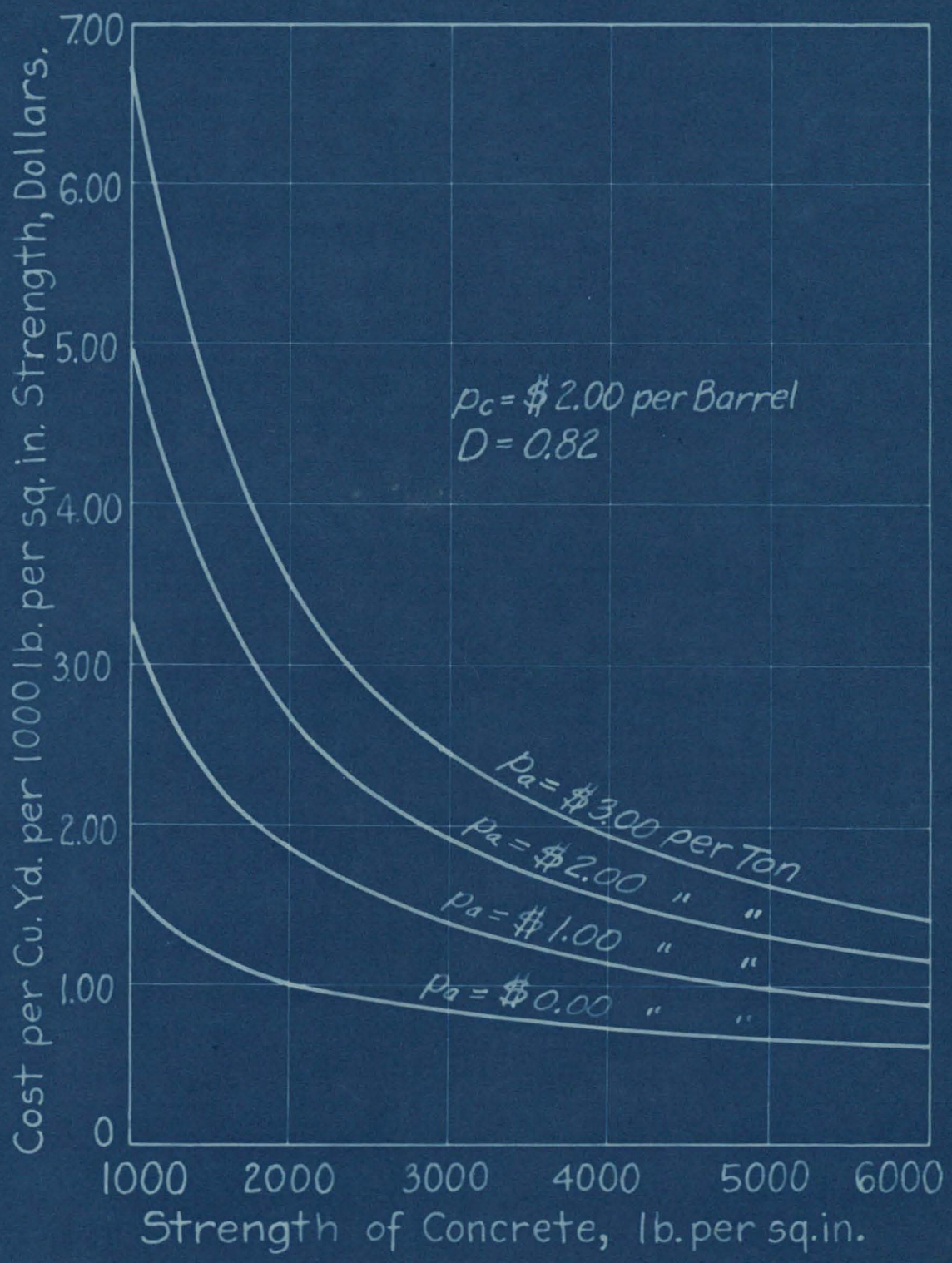


Figure 10. Cost per Cu. Yd. per 1000 lb. per sq. in. Strength of Concrete, lb. per sq. in. for different prices of aggregate per ton.  $p_c = \$2.00$  per barrel,  $D = 0.82$ .

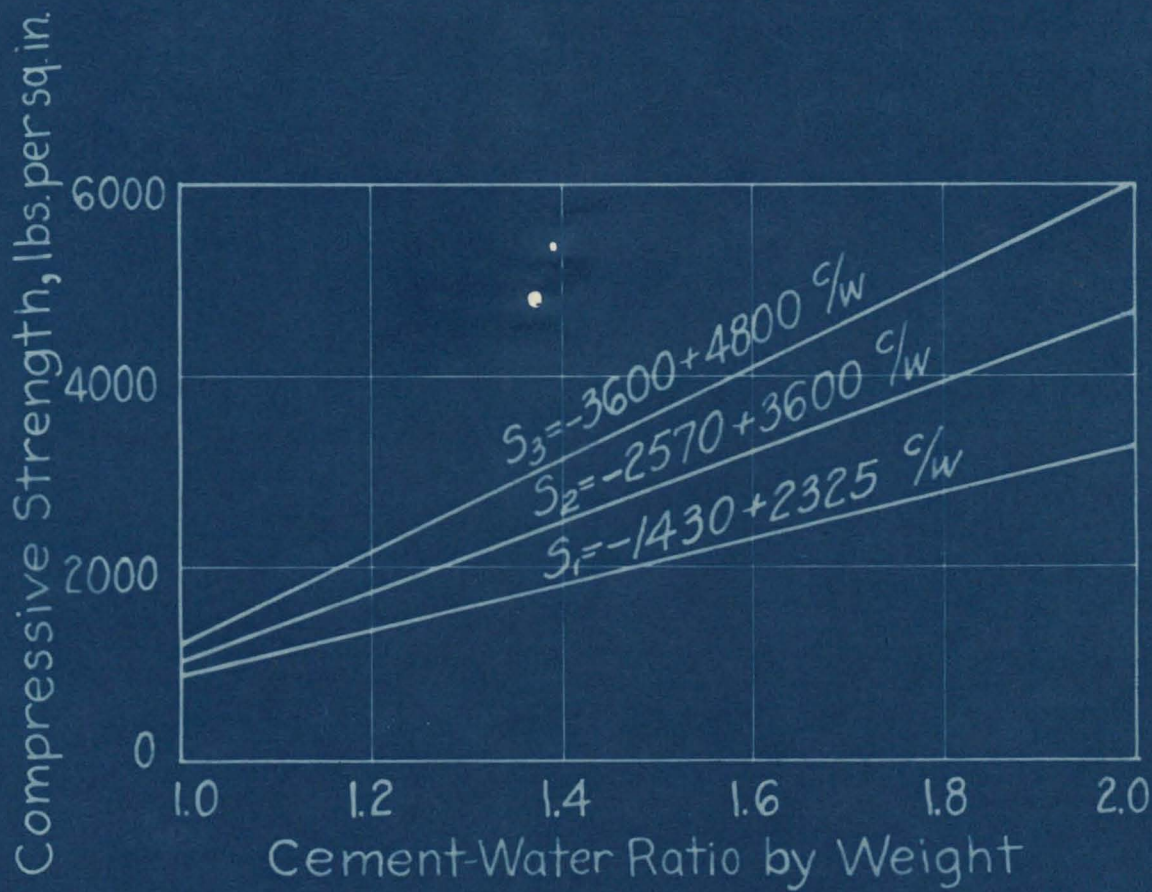
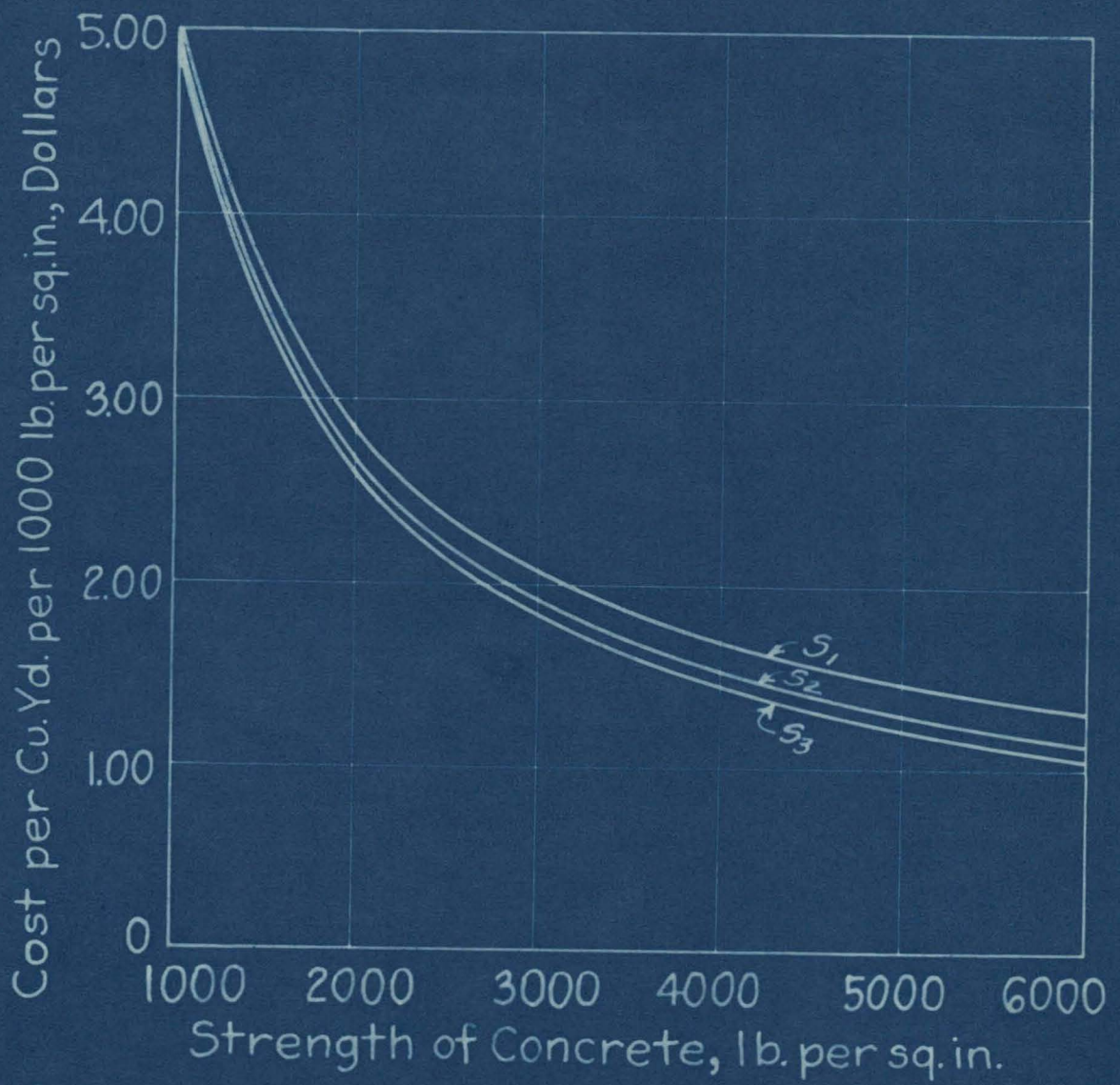
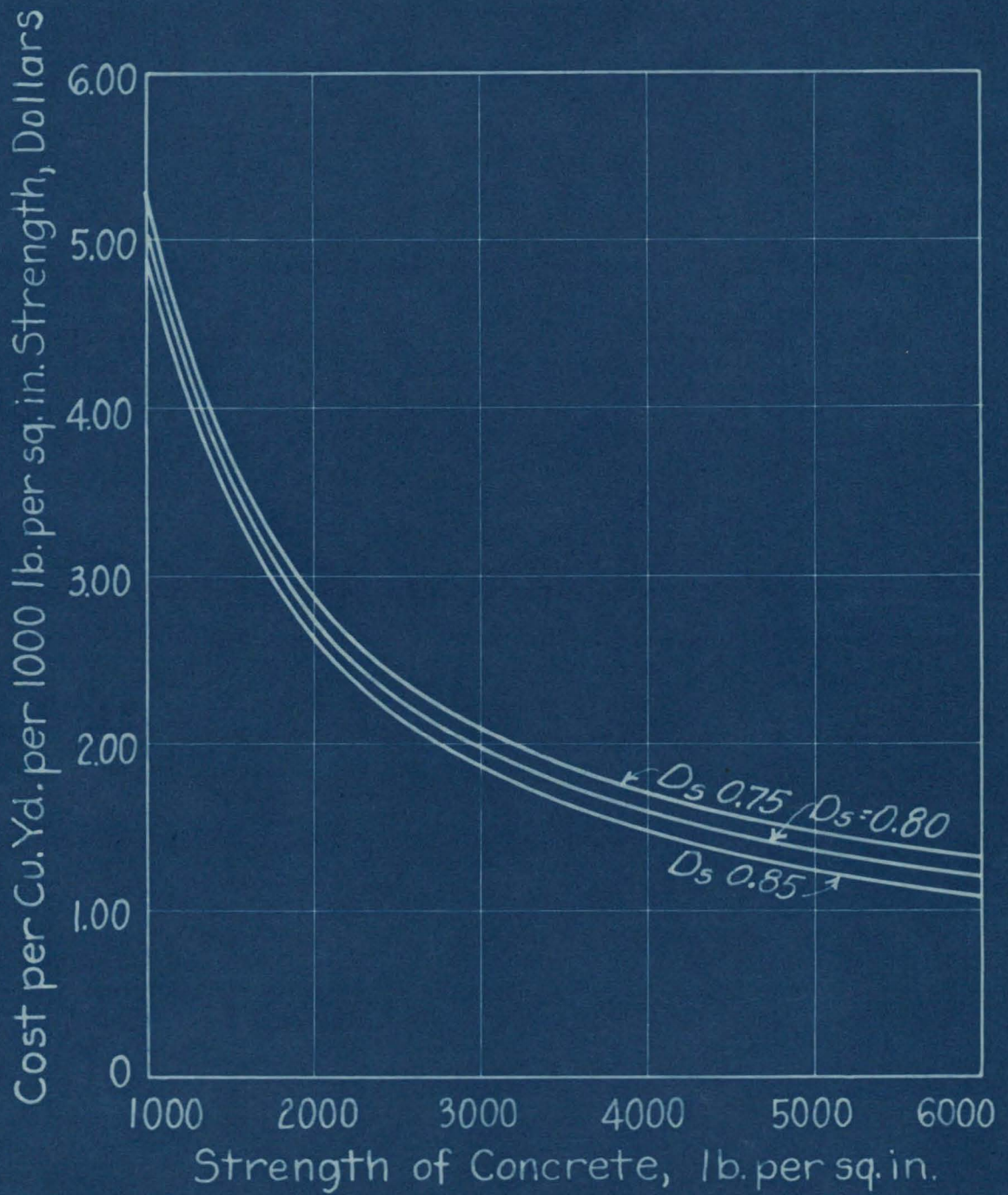


Fig. 6 - Strength curves for 3 types of concrete





*Handwritten note:* This curve is similar to that in the book for concrete.

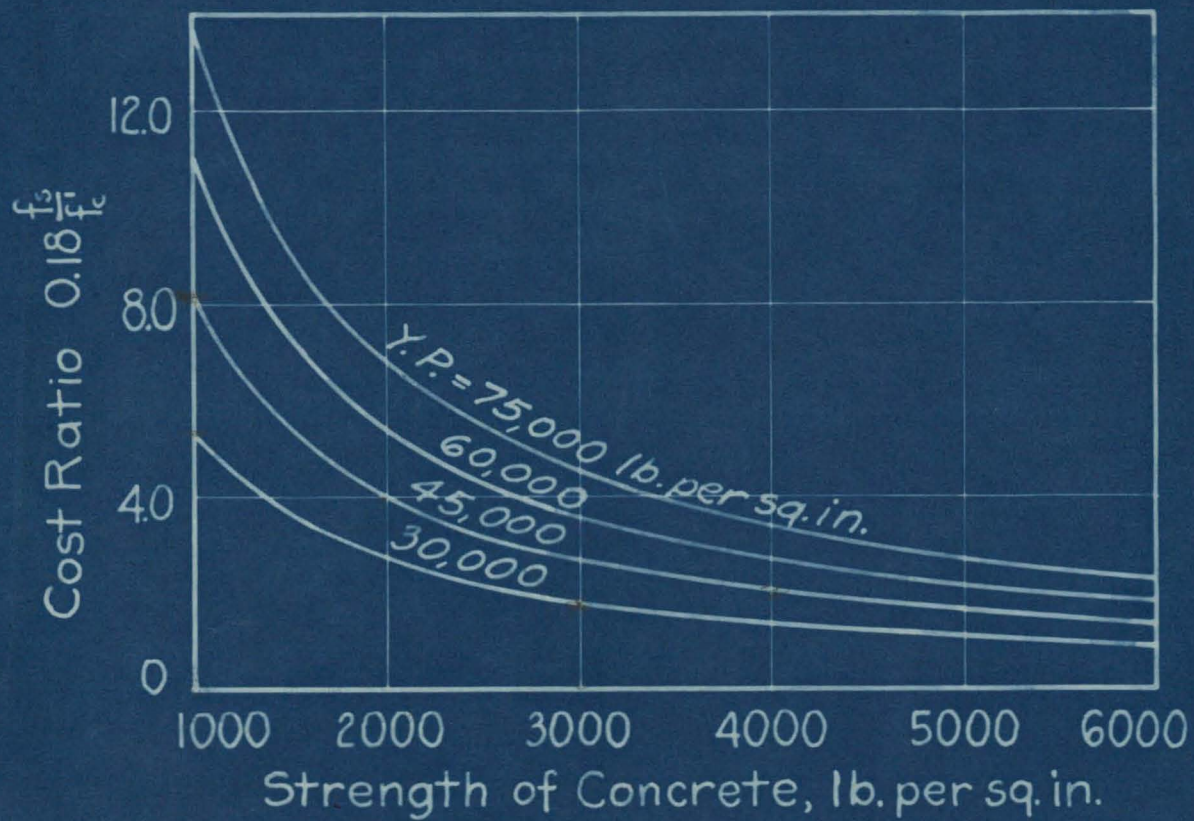


Fig. 9 Relation between Strength of Concrete and Economic Cost Ratio for Reinforced Concrete Columns.

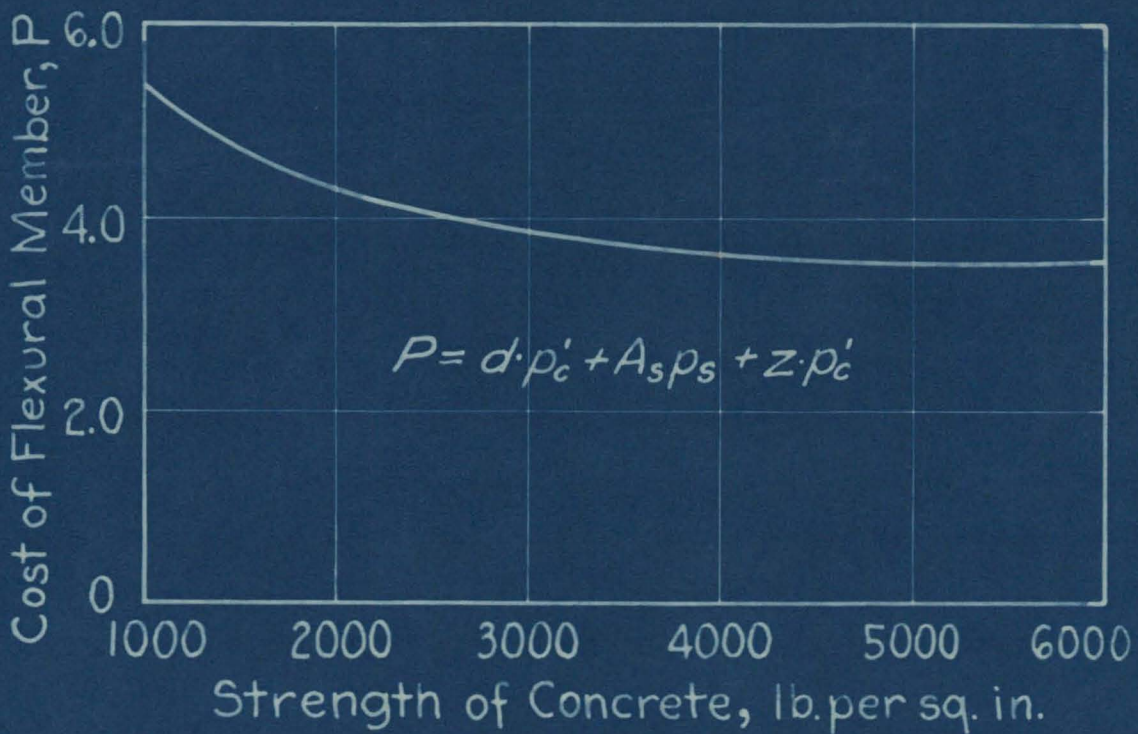


Fig. 10. Relation between Strength of Concrete and Cost of Reinforced Concrete Flexural Member for given Conditions.

F = 3.07 - .07p

