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COMPRESSIVE STRENGTH OF CONCRETE IN FLEXURE AS  
DETERMINED FROM TESTS OF REINFORCED BEAMS\*

by Willis A. Slater\*\* and Inge Lyse\*\*\*

The Joint Committee on Concrete and Reinforced Concrete first recommended in 1909 a maximum working stress for concrete in compression of 32.5% of the strength of concrete determined on 8 by 16-in. control cylinders 28 days old. The recommended working stress in the steel was 16 000 lb. per sq.in. These stresses were based on the use of the straight line formula for the design of reinforced concrete beams. The recommendation appears to provide a factor of safety of approximately 3 for compressive stresses and 2.5 for tensile stresses. Test beams proportioned to meet these conditions have always failed in tension in the reinforcement with a factor of safety between 2.5 and 3. However, tests on beams with sufficient reinforcement to develop compression failure of the concrete have shown factors of safety considerably larger. This fact was first brought out by Dr. Fritz Emperger in "Beton und Eisen" in 1903. Later the "German Committee for Reinforced Concrete" initiated a more extensive investigation on the same subject. The tests sponsored by the Committee were carried out under the direction of C. Bach and Otto Graf and a report was published in Heft 19 of "Deutscher Ausschuss für Eisenbeton" in 1912. The results of Emperger's as well as of Bach's and Graf's studies have been plotted in Fig. 1.

\* Abstract of paper published in Journal of the American Concrete Institute.

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The control specimens used in these tests were 30 cm. (11.8 in.) cubes. In order to compare the results on the bases of ordinary 6 by 12-in, cylinders the ratios of the computed beam strength to the cylinder strength are shown at the right hand side of the diagram. For convenience this ratio is termed the beam-cylinder strength ratio.

A similar study of results from different sources in the United States was published in a paper by Slater and Zipprodt in the Proceedings of the American Concrete Institute in 1920. The essential results from these results are shown in Fig. 2.

Fig. 1 and 2 show that for all these tests the strength of the concrete computed by the straight line formula was considerably greater than the strength of the concrete as shown by tests of cubes or cylinders. The tests also indicate that the beam-cylinder strength ratio increased as the strength of the concrete decreased. The variation of this ratio with the strength of the concrete was less marked for the tests reported by Bach and Graf than for the others.

These working stresses recommended in 1909 were quite generally used until in 1921 and 1924 reports of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete raised the maximum working stress for concrete in flexure to 40% of the cylinder strength. This working stress, however, has not been universally accepted.

In order to study this subject more fully an investigation was made at the Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pa. from January to March 1930. This investigation in-

cluded two series of tests, one in which with beams of equal dimensions the cylinder strength of the concrete ranged from about 1400 to 5800 lb. per sq.in., and another series in which for two different strengths of concrete (2800 and 4000 lb. per sq.in.) the depth to the center of gravity of the reinforcement varied from 4 to 14 inches. A total of 36 beams was made. All beams were 8 in. wide and 11 ft. long. The percentage of reinforcement varied from 2.1 to 5.6 insuring against failure in tension of the longitudinal reinforcement. Web reinforcement was used to guard against diagonal tension failure in the web. For beams having an effective depth of 10 in., or more, vertical stirrups were used, while for beams of 8 in. or less inclined bars were used as web reinforcement. The design of the beams is shown in Fig. 3. Rail steel reinforcement was used which had a yield point of approximately 65 000 lb. per sq.in. and an ultimate strength of 106 000 lb. per sq.in. Three beams of each kind were made for each group. The control specimens consisted of 6 by 12-in. cylinders and three were made with each beam.

The aggregate combination used in the concrete mixes consisted of 40% sand and 60% gravel. The gravel was graded so as to contain 40% between No. 4 and 3/8-in. sieves and 60% between 3/8-in. and 3/4-in. sieves. The different concrete mixes were designed for a consistency corresponding to a slump of 3 to 6 in. This consistency was obtained by maintaining the amount of water constant at 40 gal. per cubic yard of concrete for the different water-cement ratios used.

The 28 days strength results of the 6 by 12-in. control

cylinders are shown in Fig. 4. In Fig. 5 the 28-day cylinder strengths have been plotted against the cement content per cubic yard of concrete. It will be seen from this figure that the strength of the concrete increased proportionally to the increase in cement content.

A fair degree of uniformity of the concrete as measured by the strength of the control cylinders was secured throughout the series. The maximum variation from the average in any group of beams was 12.9% and the average of the maximum variations for all the groups was 6.4%.

The reinforcement was welded into a rigid unit by means of electric arc welding before placing in the forms. This proved very effective in maintaining the correct position of the reinforcement during the placing of concrete in the beams. Fig. 6 shows the fabricated reinforcement for beams of different depths.

The control cylinders as well as the beams were generally left in the forms for 48 hours after making. The forms were then removed and the specimens placed in a moist room of 100% humidity and a temperature of approximately 70°F until a 28-day age was reached. The specimens were then removed from the moist room and tested to failure. The control cylinders were tested in compression in the ordinary manner. Deformation measurements were taken on at least one cylinder from each beam in order to determine the modulus of elasticity of the concrete. The beams were supported in the testing machine at 9 in. from each end, making a span of 9 ft. 6 in. between centers of supports. The

load was applied at points 21 in. on either side of the center of the beam. This arrangement of loads produced a constant moment in the central 42-in. portion of the concrete beam. Fig. 7 shows the arrangement during the testing of one of the beams.

Deformation measurements were taken on the concrete near the top of the beam and at the elevation of center of gravity of the reinforcement, and in some cases near the neutral axis of the beam. The center deflection was measured on all the beams.

All beams except two failed in compression of the concrete between the loading points. Generally the failure was very close to the center. Beams of low strength concrete gave a gradual failure while beams of higher strength concrete broke suddenly without previous warning. Two beams failed in diagonal tension at a load nearly equal to that of the companion specimens failing in compression. The uniformity in strength results obtained from the tests of three beams in each group was found to be very good. The greatest of the maximum variations for any group was 13%, and the average of the maximum variations for all groups was 8.2%.

In Fig. 8 the beam-cylinder strength ratios have been plotted against the strengths of the control cylinders. Three different values of  $n$  were used in the computations of the ratios, (1) the values specified by the American Concrete Institute, (2) the values specified in the Joint Committee report of 1924, (3) the values determined from the deformation measurements on control cylinders and on coupons from the reinforcement steel used in the tests. The upper curves of Fig. 8 represent the values of the beam-cylinder strength ratio when the ordinary straight line form-

ula is used for computing the stresses. It is noted that the beam-cylinder strength ratios are considerably above 1.0 for all the groups included. The ratios vary from a low value of about 1.40 for concrete having a strength of 5800 lb. per sq.in. to as high as 2.2 for concrete having a strength of 1400 lb. per sq.in. The lower curve in Fig. 8 represents the beam-cylinder strength ratios when a parabolic formula is used for computing the compressive stress. In this curve the beam-cylinder strength ratios vary from about 1.5 for concrete having a strength of 1400 lb. per sq.in. to 1.0 for concrete having a strength of 5800 lb. per sq.in.

The variations in the assumed values of  $n$  had a small effect on the beam-cylinder strength ratio for computations made by the straight line formula and a smaller effect for computations made by the parabolic formula.

The beam-cylinder strength ratios for beams with different depths and two different strengths of concrete (2800 and 4000 lb. per sq.in.) have been plotted in Fig. 9. The beams with concrete of 2800 lb. per sq.in. indicated a slight increase in the ratio with the increase in the depth of the beams. However, the increase was very small and not entirely consistent. The beams having concrete of 4000 lb. per sq.in. showed the same beam-cylinder strength ratio for all the different depths of beams used. This seems to justify the conclusion that the effect of the variation in the depth on the beam-cylinder strength ratio was negligible.

In Fig. 10 the stresses determined from the strains observed at the level of the center of gravity of the reinforce-

ment are shown as ordinates, and the stresses computed by the straight line formula, using  $n$  as specified by the American Concrete Institute, are shown as abscissas. The plotted points represent the results from this series of tests and the solid straight lines are the graphs of the equations given in the figure. These equations represent closely the relation between observed and computed stresses in an extensive series of tests carried out by the United States Geological Survey under the direction of Richard L. Humphrey about 25 years ago. On the whole the observed stresses agree very well with the computed stresses in the steel, rather more closely, in fact, than in the tests by the Geological Survey.

Fig. 11 shows the relation between the so-called observed compressive stresses in the beams and the stresses computed by means of the straight line formula. The values of  $n$  used in these computations were according to the specification for the American Concrete Institute's building code. For the strains observed at the extreme fiber of the beam at different loads, corresponding stresses were taken from the stress-strain curve for the control cylinders. These are the so-called observed stresses for the beams. The dotted line gives the condition for equal computed and observed stresses. It is noted that for stresses up to the strength of the cylinders there is a close agreement between the observed and the computed stresses. The indication is that in a beam at the maximum load the computed compressive stresses are in agreement with the stresses for corresponding strains in the cylinders up to a height above the neutral axis at which the stress is approximately equal to the cylinder strength. For points above

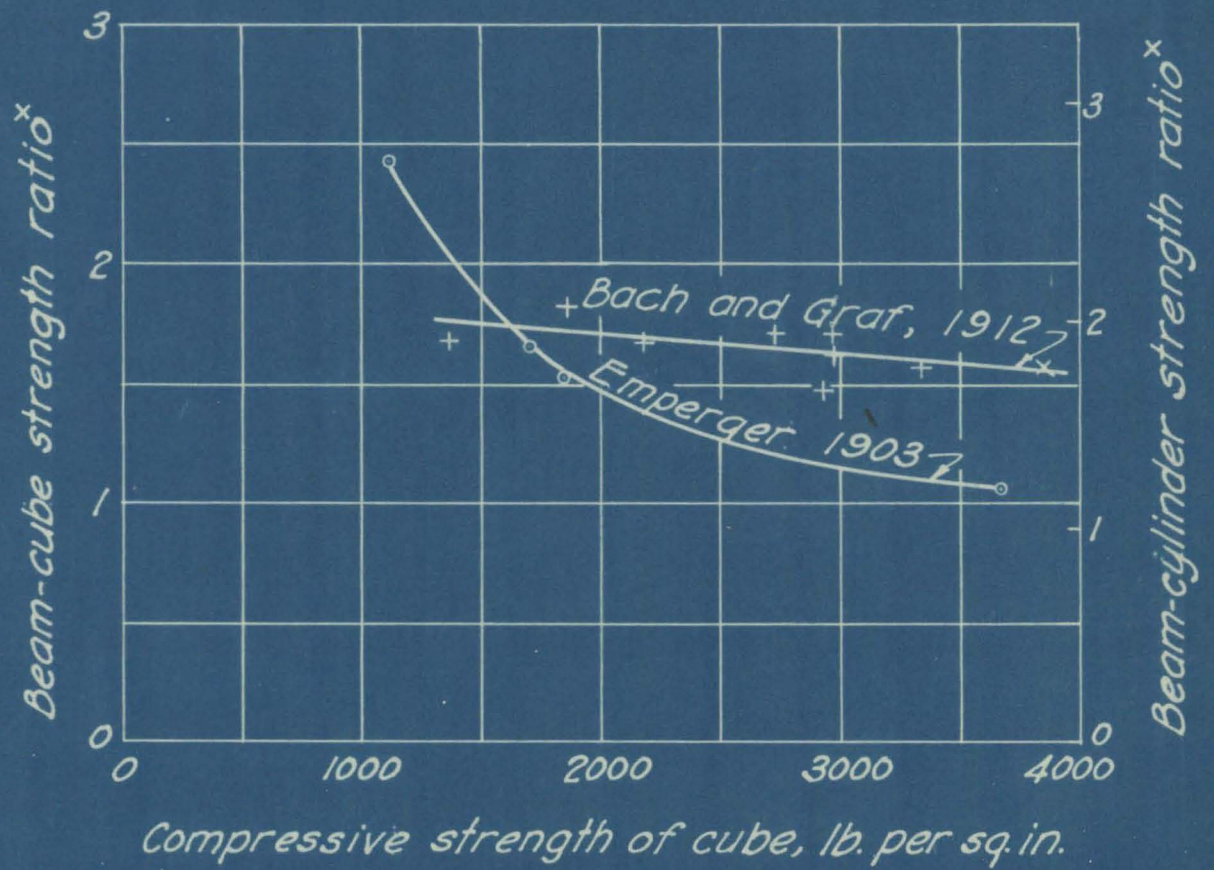


this height the stresses in the beam are not known, but computations indicated that they must be somewhat greater than the cylinder strength in order to produce equilibrium between the internal and external moments.

The average deflection for the different groups of beams of five different strengths of concrete in which the size of the beams remained constant is shown in Fig. 12. For comparison, deflections computed from a formula proposed by Prof. G. A. Maney have also been shown. The agreement between the computed and the observed deflections is very good and indicates that an intimate relation exists between the measured deformations of a reinforced concrete beam and the deflections. It will also be noted from Fig. 12 that the total observed deflections near the maximum load were practically equal for all the beams included in this series. In order for this to be true the deformations in the concrete must also have been nearly the same at maximum load for all groups. This was found to be approximately the case.

The test results also indicated that for design computations of stresses in the reinforcement of concrete beams the value of  $j$  might well be taken as a constant regardless of the values of  $p$  and  $n$ .

These tests indicate that the maximum compression stresses in the beams were approximately equal to the stresses computed by the straight line formula, until a load was reached at which the maximum stress was equal to the strength of the control cylinder. In addition they show that at failure of the beam the stresses computed by the straight line formula were from 40 to 120% greater than the strength of the control cylinders.



\*Ratio computed compressive stress in beam at maximum load to compressive stress of cube or cylinder.

Fig. 1 - Foreign tests of heavily reinforced concrete beams



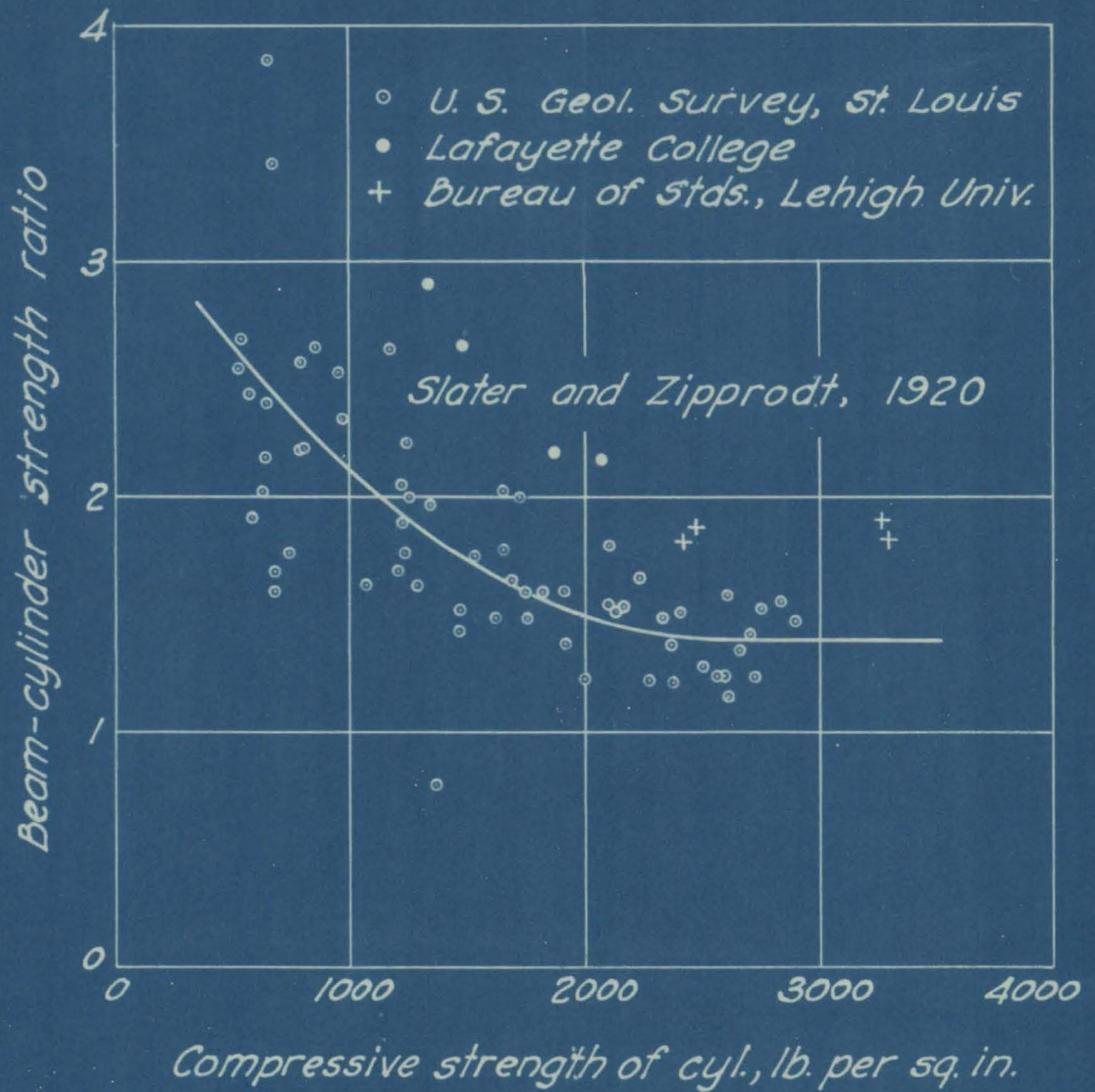


Fig. 2 - Tests of heavily reinforced concrete beams made in United States



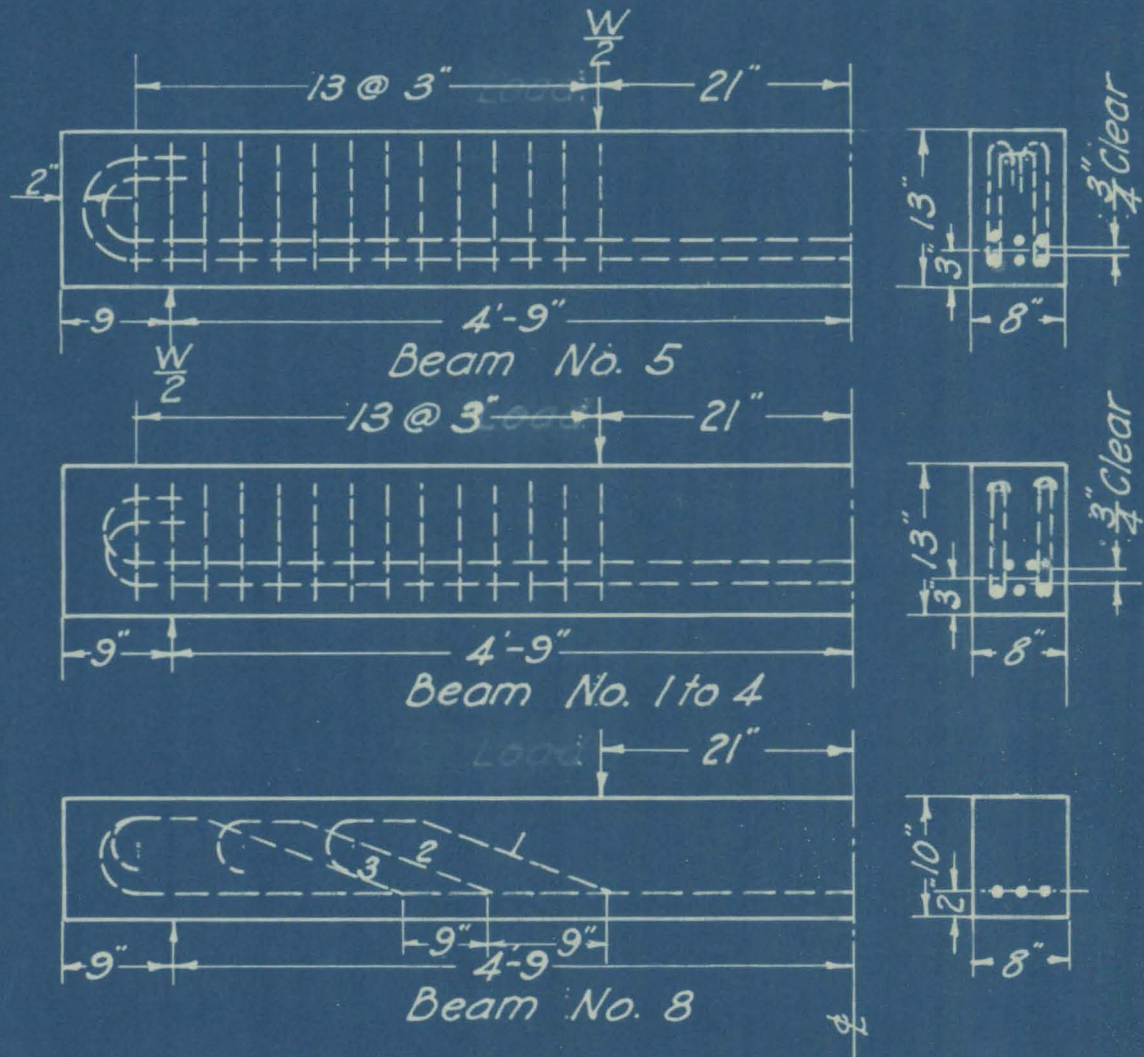


Fig. 3 - Design of Reinforced Concrete Beams  
 Note : All web reinforcement is  $\frac{3}{8}$ " dia.



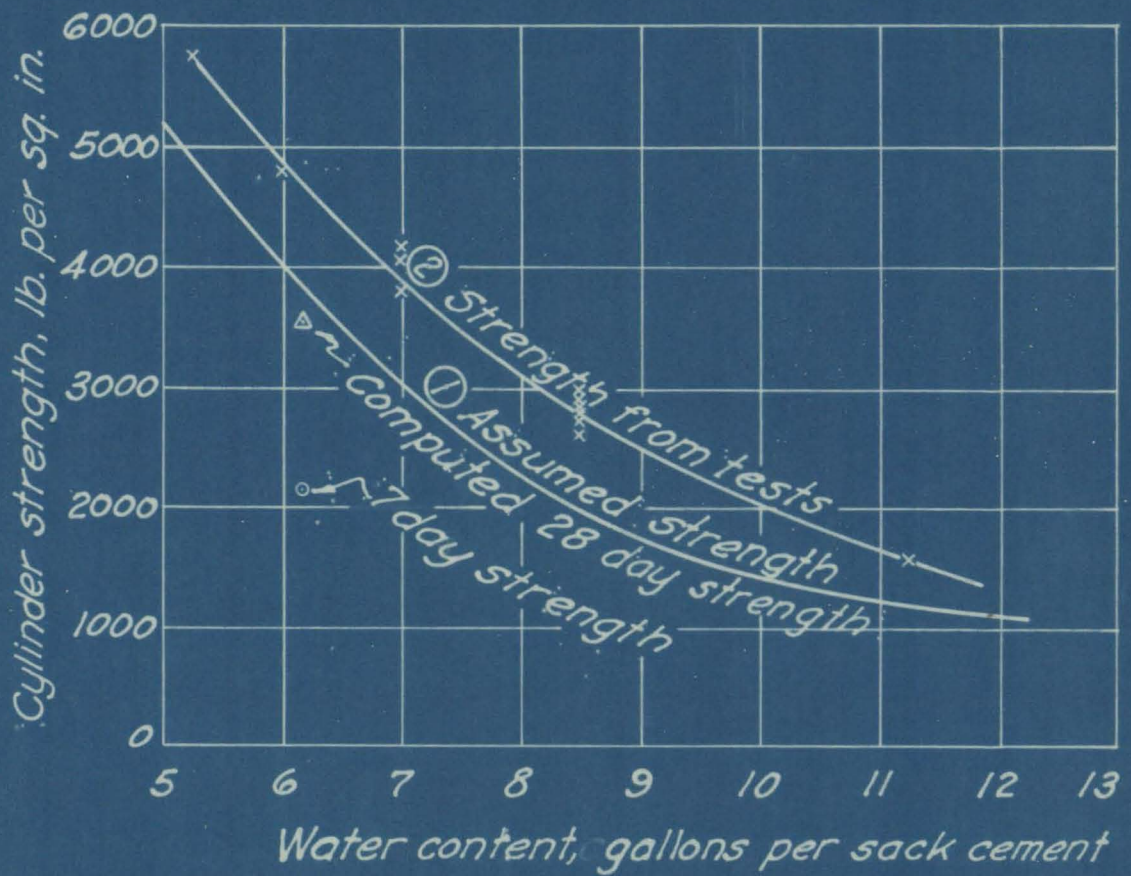


Fig. 4 - Water-cement-strength ratio,  
 ① Assumed for design and  
 ② Secured from tests.

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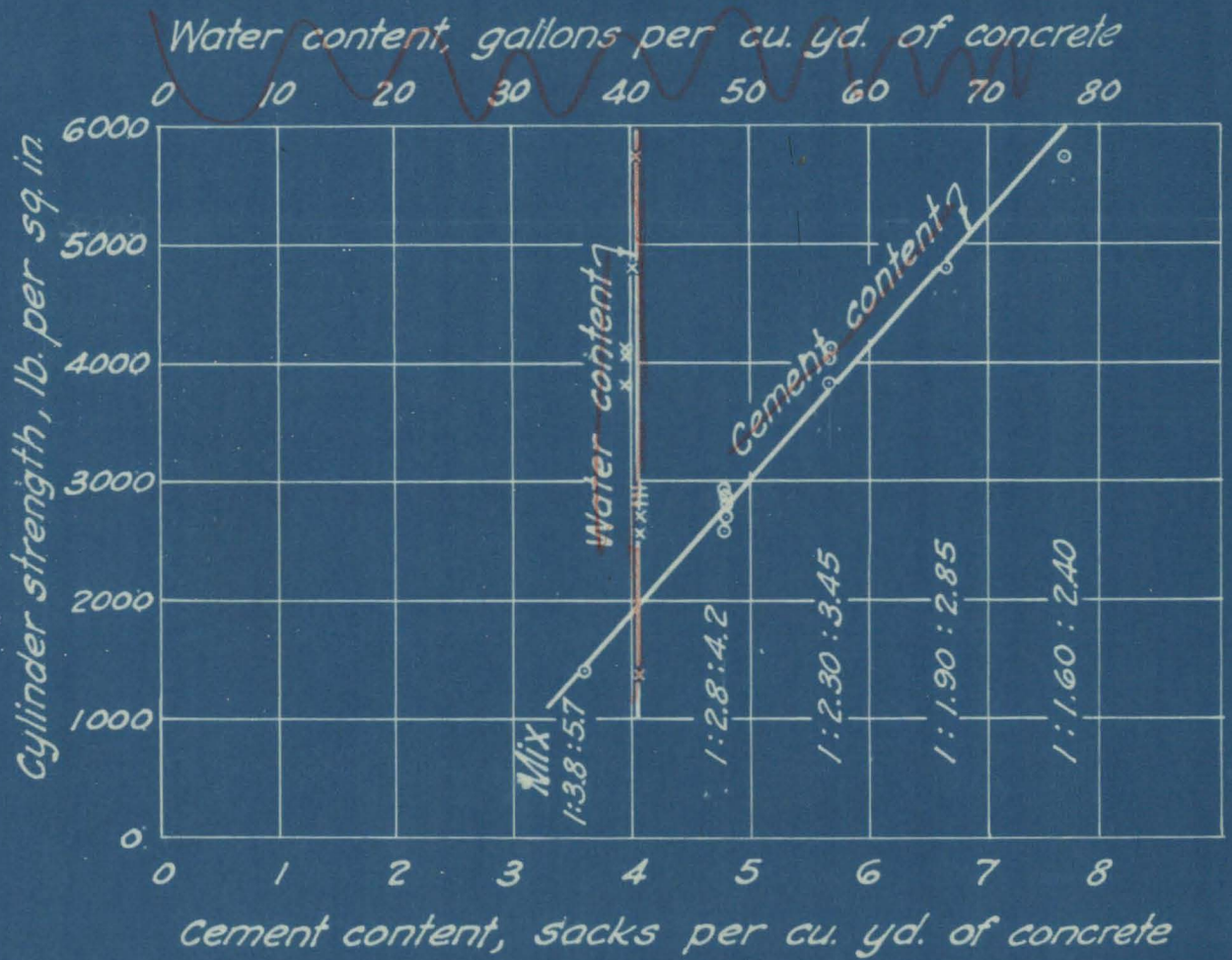


Fig. 5 - Relation of strength to cement content and water content for concrete of constant consistency.

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BETHLEHEM, PA.

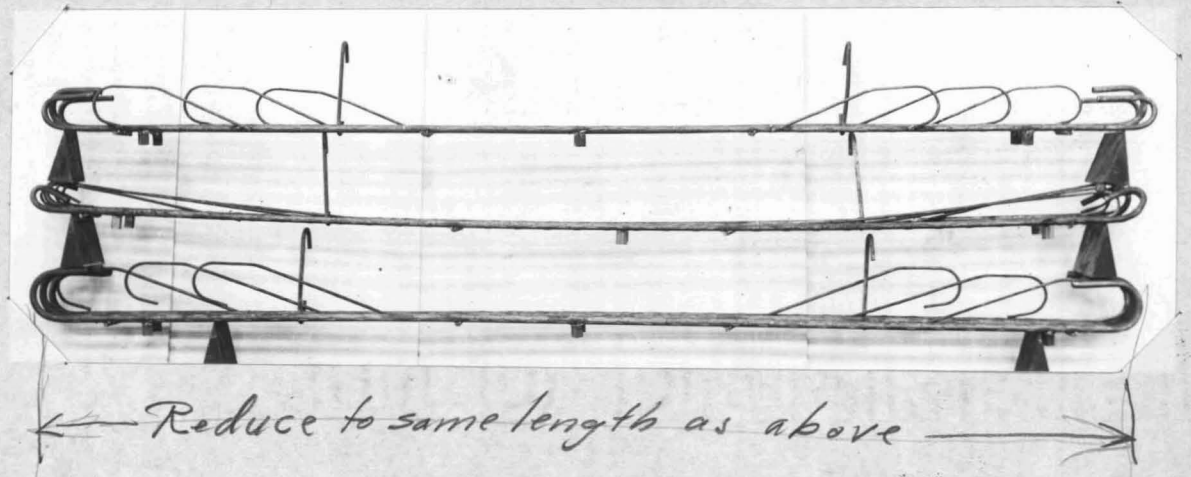
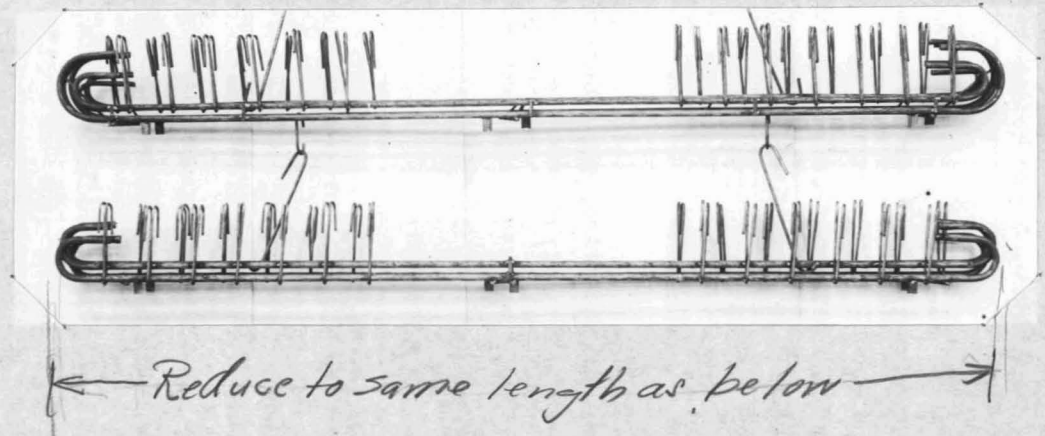


Fig. 6. Reinforcement for different beams.



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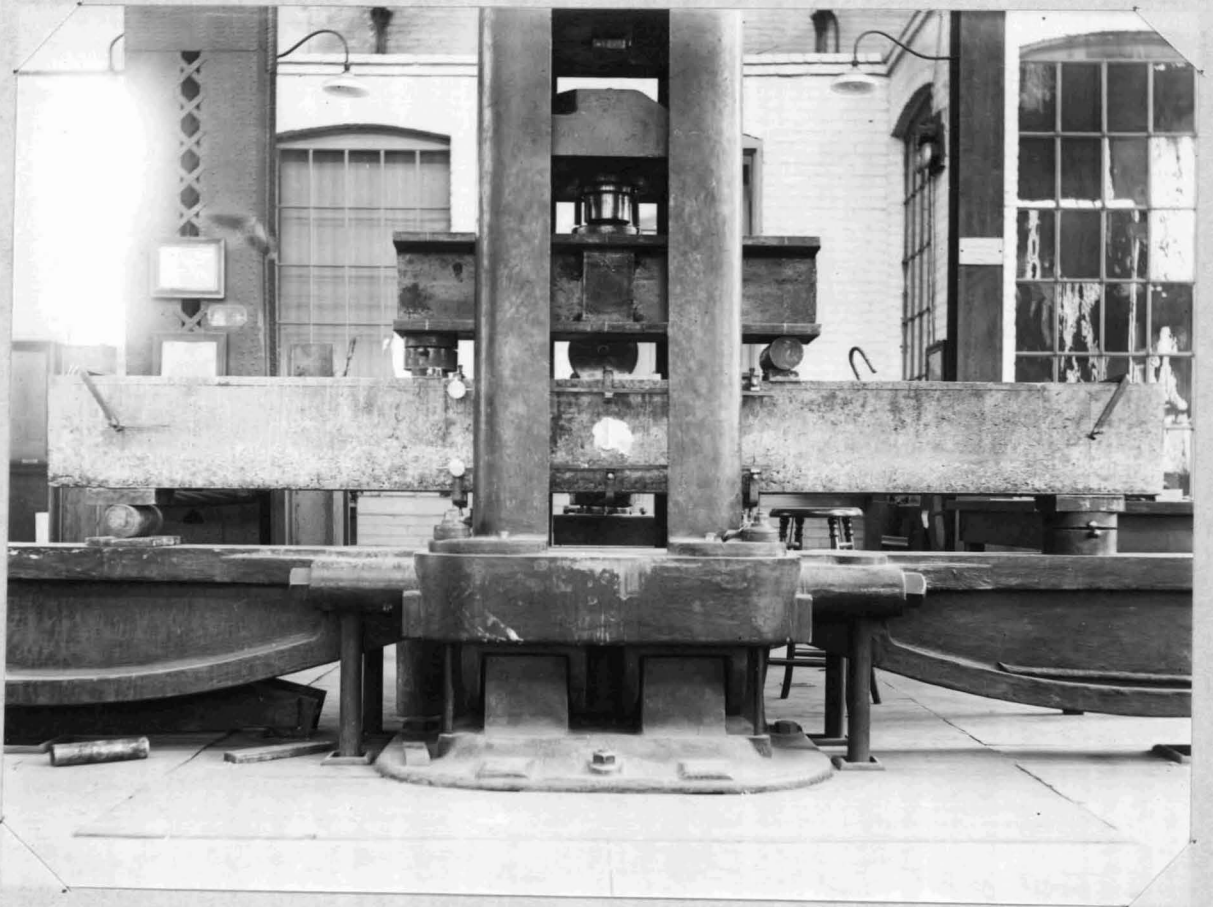


Fig. 70 Arrangement for testing <sup>of</sup> Beams.



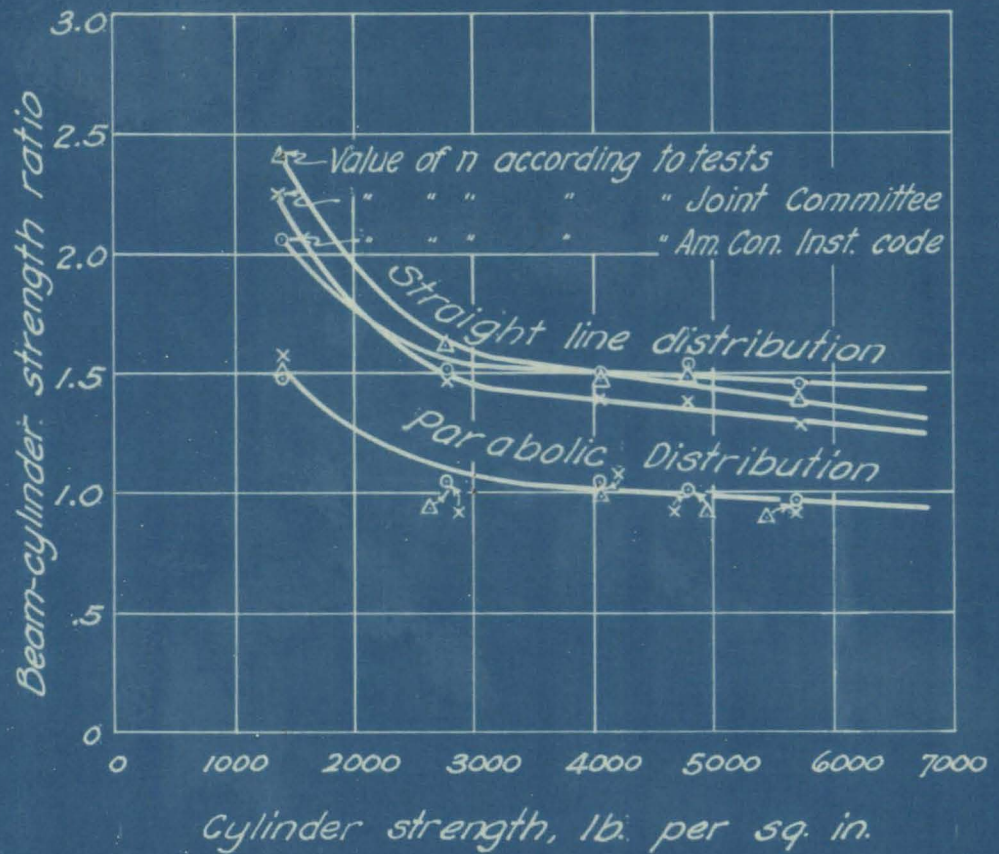


Fig. 8 - Relation of beam-cylinder strength ratio to strength of concrete.

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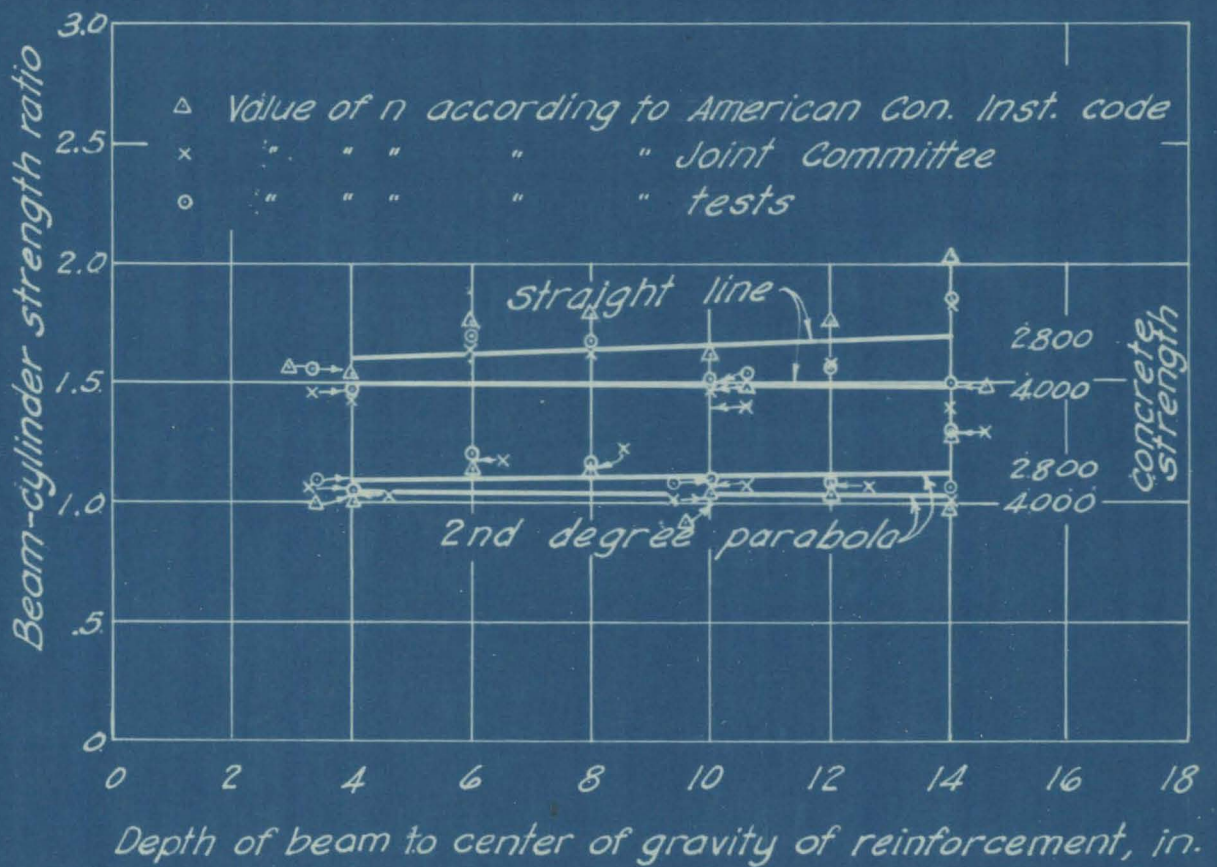


Fig. 2 - Relation of beam-cylinder strength ratio to depth of beam.

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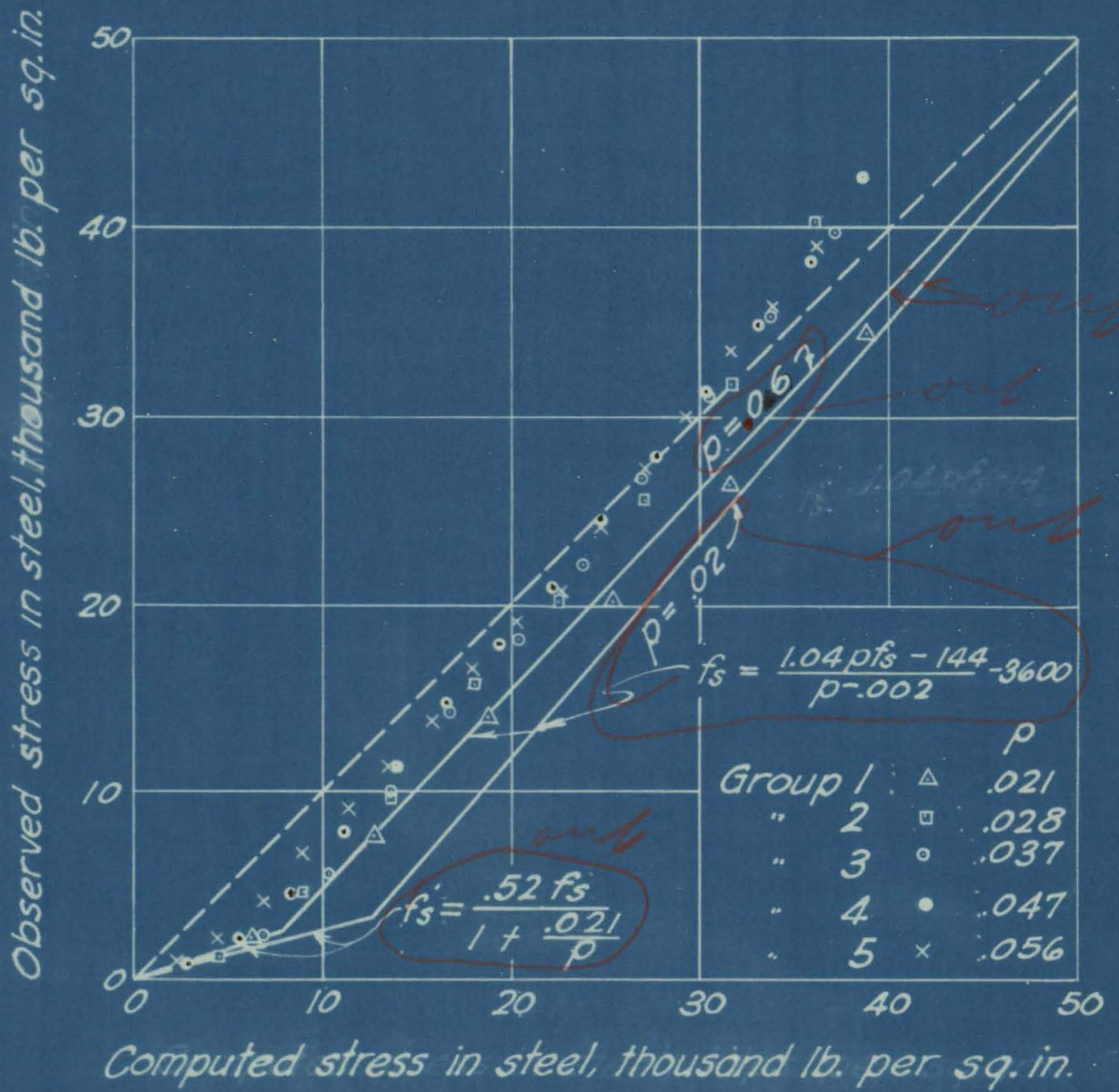


Fig. 10 - Relation between observed stresses in steel and stresses computed according to A. C. I. specifications:

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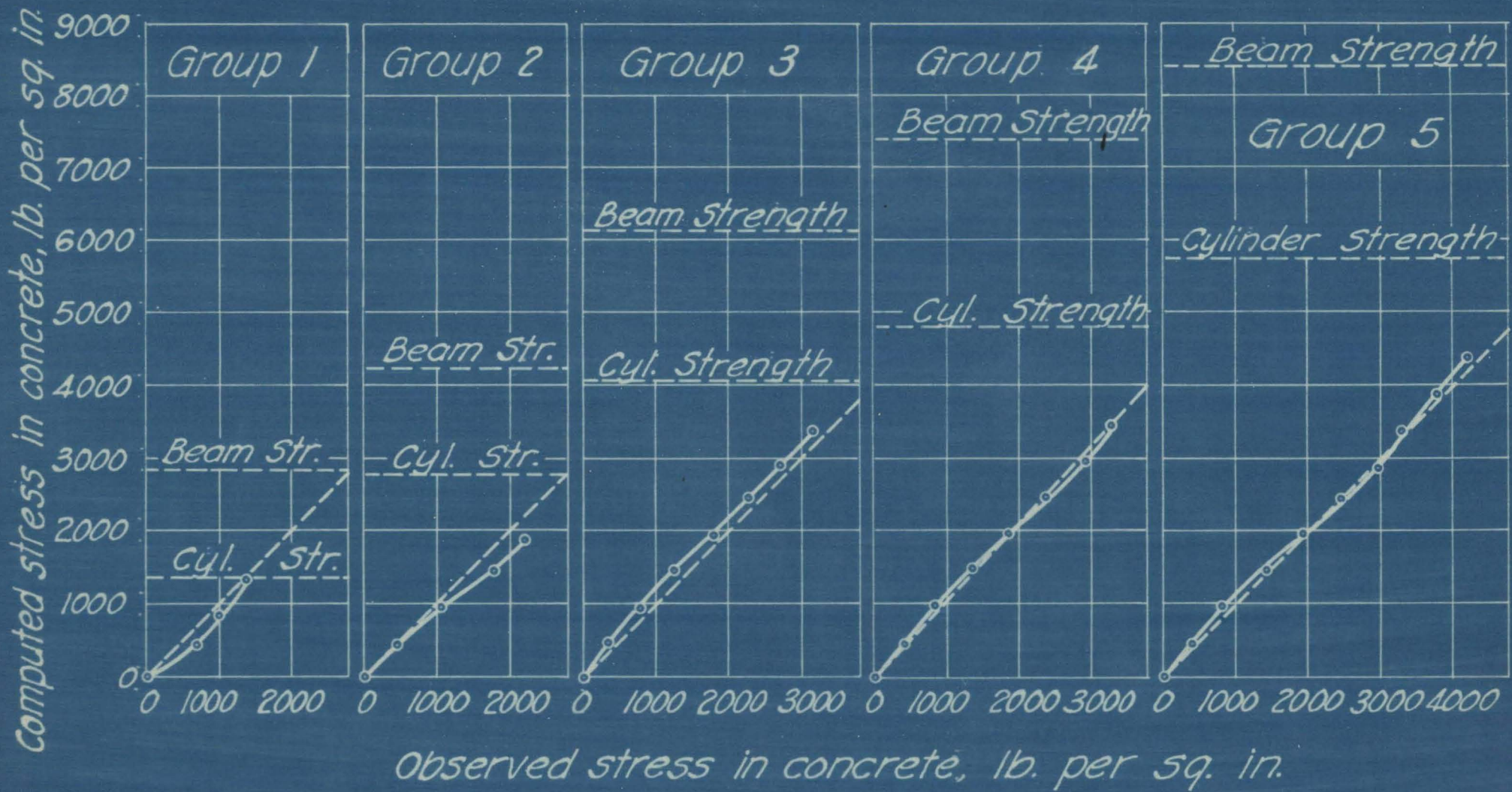


Fig. 11 - Relation between observed stresses in concrete and stresses computed according to A. C. I. specifications.

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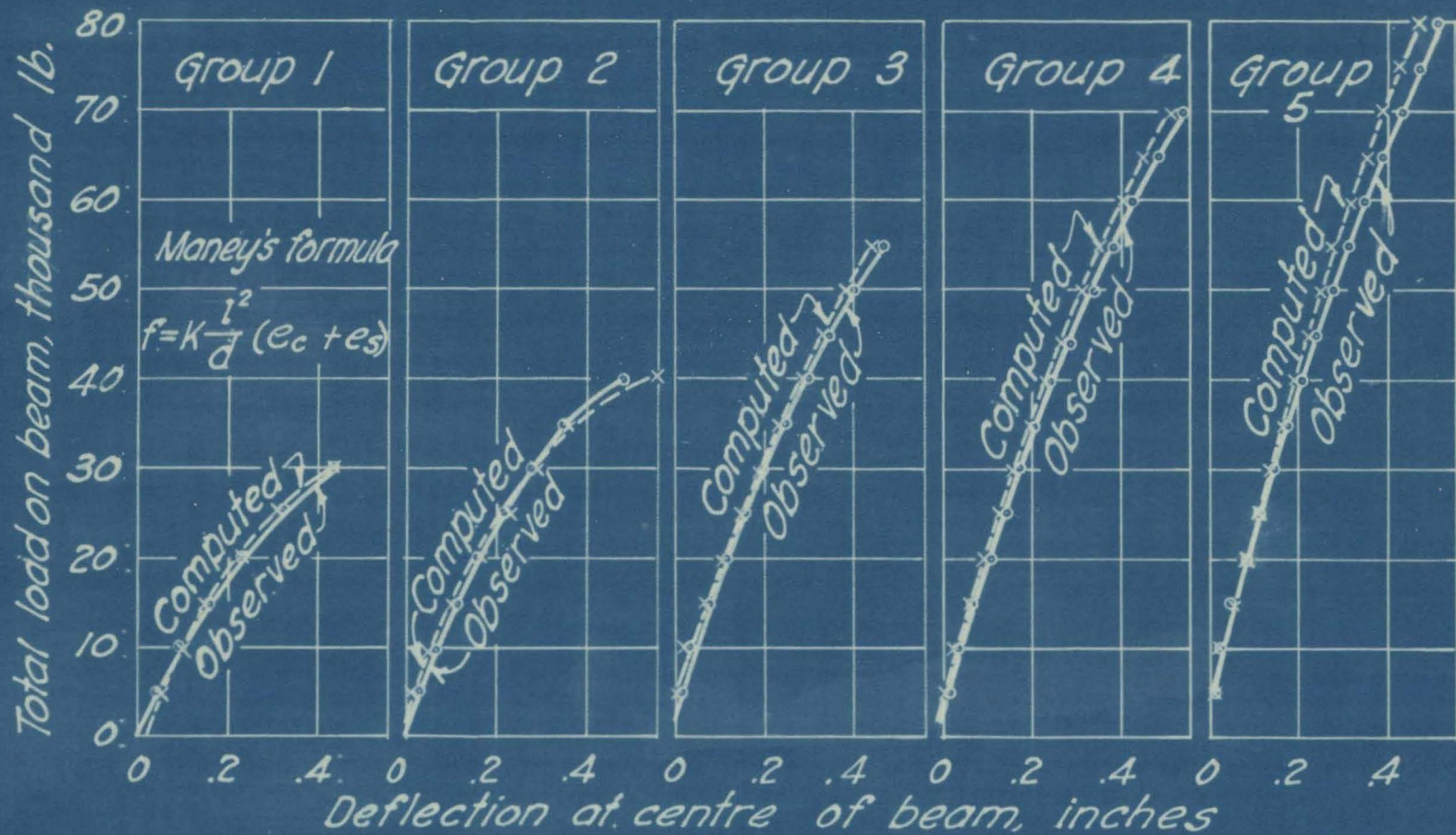


Fig. 12 - Relation between observed deflections of beams and deflections computed by Maney's formula.