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European Column Studies

# COMPARISON OF TESTING METHODS FOR HEAVY COLUMNS

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> by N. Tebedge P. Marek L. Tall

Fritz Engineering Laboratory Report No. 351.2

European Column Studies

#### COMPARISON OF TESTING METHODS

FOR HEAVY COLUMNS

by

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Paul Marek

Lambert Tall

This study has been carried out as part of an investigation jointly sponsored by the European Convention of Constructional Steelwork Associations, National Science Foundation, and the Welding Research Council. Technical guidance was provided by the Task Group 11 of the Column Research Council.

> Fritz Engineering Laboratory Lehigh University Bethlehem, Pennsylvania

> > October, 1969

Fritz Engineering Laboratory Report No. 351.2

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#### ABSTRACT

This report describes an experimental study on different column testing methods for medium and heavy columns and their effects on test results. Two methods were investigated, namely, the old Lehigh method and the European Convention method, and a new Lehigh method was proposed.

Tests were performed on seven 12WF161(A36 grade steel) pinned-end columns having a slenderness ratio of 50. The specimens were prepared from a single unstraightened rolled piece. Supplementary tests (residual stress measurement, stub column and tension coupon) were also made. The instrumentation and testing procedure used for each method are fully discussed.

This report includes complete experimental data from the column tests and supplementary tests which are of use for further theoretical analyses. The results provided consist of initial geometric measurements of cross-sectional dimensions and out-ofstraightness along the length, and the test data after

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loading, namely, lateral deflections at various levels (about weak and strong axes), end rotations, angles of twist, fiber strains at different locations, and overall shortening during the loading.

As a result of this study on testing methods, instrumentation and supplementary tests, the following recommendations were made:

- The ECCSA\* method, required for the European Convention column studies, was clarified and additional measurements were suggested.
- (2) A new procedure for the testing of medium and heavy columns is proposed. This method requires geometrical alignment with respect to the center of column flanges and a dynamic loading with constant strain rate. Only one point on the static curve close to the ultimate column strength is required to be recorded. A method of deriving the static column curve from the dynamic curve and one test point on the static curve is suggested.

\*European Convention for Constructional Steel Associations.

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#### 1. INTRODUCTION

#### 1.1 Objective

The ultimate objective in this study of column strength and column testing is to evaluate different testing procedures used especially for medium and heavy column shapes, different instrumentations and alignments, to correlate the test results with theoretical predictions, and finally, to propose a new testing procedure, alignment and instrumentation for column testing.

#### 1.2 The Pinned-End Column

A column may be defined as a member subjected to a compressive load through the centroid and whose length is considerably larger than its crosssectional dimensions. Columns may have different end conditions, ranging from full restraint to zero restraint in rotation and warping. Under any type of end condition, no translation of the end of the column is allowed to occur relative to the load.

Most column investigators in the past have used the pinned-end condition for column testing for a number of reasons. Under pinned-end conditions the critical stress condition exists at about the midheight section thus making the section of interest remote from the boundary and, therefore, not influenced by any end effects. For the same effective slenderness ratio, the pinned-end condition requires the use of a shorter column length than the fixed-end condition.

The pinned-end column is regarded as the basic column, although it does not exist in actual structures. It is the member to which the strength of all other columns is referred. Until methods for the design of structures as a whole come into use, the design of columns will continue to be based on the strength of the simple pinned-end column.

#### 1.3 Experiments on Columns

In testing column specimens, the experimental results form a wide scatter band instead of a well-defined relationship between strength and slenderness ratio. This is due to imperfections in

the experimental conditions, such as end conditions, initial out-of-straightness, eccentricities of load, lateral loads, as well as to residual stresses and nonhomogeneity of the material. To understand column behavior, there is a need to isolate the effects of these factors.

For pinned-end conditions it is essential that friction virtually be eliminated since a small amount of end constraint will cause an appreciable increase in the column strength. Several schemes have been used to provide the required pin condition. Some of the different basic types of end fixtures used by column investigators are shown in Fig. 1.<sup>(1)</sup> The end fixtures differ from each other in that they are either "position-fixed" or "direction-fixed" at the ends.<sup>(2)</sup> The other basic differences are with respect to their maximum carrying capacity and effective column length.

Probably the best way to reduce friction is by the use of a relatively large hardened cylindrical surface bearing on a flat hardened surface. Even if an indentation should occur under heavy load, rotation will be virtually frictionless. Plastic

indentation, however, is not desirable. Another interesting feature about the cylindrical fixtures is that the effective column length can be made equal to the actual length of the column by designing the fixtures so that the center of the cylinder is located on the center line at the end of the column.<sup>(3)</sup> Effectively, the column acts as pinned-end about one axis (usually the weak axis) and is fixed-end about the other.

A schematic diagram of the end fixtures used at Fritz Engineering Laboratory is shown in Fig. 2. The fixtures have a maximum capacity of 2.5 million pounds.<sup>(4)</sup> Description of the fixture and its performances as a "pin" is given in Ref. 3.

#### 1.4 Testing Procedure

In column tests, as in other stability tests, the response of a column is influenced by the loading device used. The common types of loading are the gravity, deformation and pressure types. The resulting "load-deflection characteristics" of each loading system are not alike.<sup>(5)</sup>

The oldest form of testing device used for columns was the gravity type. The load-deformation characteristics for such a system are simple and can be represented by a series of straight lines parallel to the deformation axis. Later, the screw-type testing machine became a common laboratory apparatus. Such a loading device has the advantage of providing an accurately defined load-deflection characteristic, where the slope of this characteristic depends on the elastic response of the load system. As higher capacity of loading machines became needed, the hydraulic-type testing machine was utilized. Such a loading device, however, does not have an easily defined load-deflection characteristic and depends on the properties of hydraulic system, leakage, temperature and other similar factors.

#### 1.5 Recording of Results

In experimental investigations of column strength it is common practice to represent deflections of the column as a function of the axially applied load even though in ideal cases there will be no deflections up to the critical load. The experimental column will always begin to deflect with the beginning of loading owing to various kinds of imperfections.

The behavior of test columns under load is determined with the assistance of measurements of lateral deflections at various levels, rotations at ends, strains at characteristic points and angles of twist.

The lateral deflections measured both about the weak and strong axis can be used:

- to determine additional moments produced due to deflection
- to check the predicted deflection curve
- to check end rotations

The strain measured using strain gages located at points of particular interest can be used:

- to show strain distribution along critical sections for checking original hypotheses
- to indicate initial eccentricity
- to determine the curvature of the column at various load levels

- to indicate the location of initial yielding and the corresponding load.

Finally, the angle of twist is measured to have complete data which may be of use for the theoretical prediction of column strengths when torsion effects are to be considered. Column tests with free warping at end sections are also possible when using end fixtures such as shown in Fig. 1(h).

The instrumentation for measurements has changed drastically in the past few years due to progress made on measuring techniques and devices. It is now possible to obtain automatic recordings for all measurements in the form of plots. Such recordings have been found to be convenient and more precise than the manual readings used before. There is now also the possibility of recording all measurements automatically which may then be analyzed directly using the computer.

As heavy shapes are used increasingly more in today's structures, tests on heavy columns will soon become of considerable importance. This requires the use of high-capacity testing machines and end fixtures. Since repetition of such tests to allow statistical

evaluation is very expensive, special care should be taken for the testing procedure, instrumentation and recording of results.

#### 2. PREPARATION OF SPECIMENS

#### 2.1 Specimens

A total number of eight straight specimens were obtained from a single rolled shape 12WF161 having a total length of 125 ft. Table 1 shows the properties of the material (A36 steel) as indicated in the mill test.

Figure 3 shows the specimen on the cooling bed after cutting with the hot saw. No cold-straightening was allowed in order to avoid redistribution of residual stresses.

Each individual column was designated by a number (01, 02, 03, 04, 05, 06, 07). For the purpose of identification each column was marked FRONT and BACK conforming to the original rolled piece, also TOP and BOTTOM. Thus, the relationship of each piece to the original length was known.

From the eight available specimens, each having a length of about 15 ft., seven were used for

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pinned-end column testing. The remaining one was used for supplementary tests. Figure 4 shows the layout of the test specimens. The columns were then cut to a length of 13' - 4" using a cold saw and milled at both ends. Milling was performed perpendicular to the end portions of the columns. For columns initially not straight, the milled surfaces may not therefore be parallel to each other, but will be perpendicular to the center line at the ends. Base plates were welded at each end of the specimen using 1/4 inch welds by matching the center of the web to the center of the plate.

The study of the behavior of a column requires supplementary tests of tensile coupon, residual stress and stub column. The description and results from the supplementary tests are discussed in the following sections.

#### 2.2 Tension Coupon Tests

The mechanical properties of the material were obtained from tension coupon tests conducted in accordance with the ASTM Specification. A total number of three coupons were tested; two from the

flange and one from the web. Figure 5 shows a schematic diagram of the dimensions and the location of the coupons with respect to the cross section.

The two coupons from the flange were tested in a 300,000 lb. hydraulic universal testing machine and the coupon from the web in a 120,000 lb. mechanical screw type universal testing machine. The load-elongation curve was plotted automatically. The gage length used was 8 inches.

The static yield stress<sup>(6)</sup> was obtained after the testing machine was stopped at a strain of 0.005 in./ in. The results of the tensile coupon tests are summarized in Table 1.

It was noted that the coupons taken from the flange did not exhibit a "flat" yield plateau, whereas the web coupon had a "flat" yield plateau. Figure 6 shows the flanges and web coupon test results superimposed. Notice also the slight positive modulus in the yield region of the flange coupon while the web coupon has the "flat" yield region usually observed in A36 tensile coupon.<sup>(7)</sup> 351.2

#### 2.3 Residual Stress Measurement

The residual stress magnitude and distribution was measured by the method of "sectioning"<sup>(8)</sup> using the gage length of 10 in. Figure 7 shows the residual stress pattern. The edges have compressive residual stress varying from 6 to 18 ksi with an average value of about 13 ksi; and the web has an average of 14 ksi in tension.

One noteworthy aspect of the pattern of residual stress distribution is the considerable difference in residual stresses for the two flanges. This may be due to the positioning of the specimen on the cooling bed. During cooling, the upper flanges of the specimens may have, for instance, been exposed to a different air circulation (Fig. 3).

#### 2.4 Stub Column Test

Two stub column tests were made on sections from the same piece from which the actual column specimen was cut. A stub column may be defined as a column long enough to retain the original magnitude of residual stress in the section and short enough to prevent any premature failure occurring before the yield load of the section is obtained.<sup>(9)</sup>

A stub column test is performed in order to obtain an average stress-strain curve for the complete cross section which takes into account the effects of residual stress. The proportional limit, the elastic modulus, and tangent modulus are the important data furnished by the curve. Using prepared charts, <sup>(10)</sup> where a simplified residual stress pattern and a homogeneous material are assumed, column strength may be predicted directly from stub column test results.

The stub column specimens were tested in a 5,000,000 lb. hydraulic universal testing machine. Figure 8 shows the instrumentation of the stub column. Four SR-4 electric strain gages at the flange tips at mid-height were used for alignment. The alignment of the specimen was made at loads not exceeding one-third of the expected yield stress level, this being an estimate of the proportional limit based on the measured residual stress distribution. A constant check was made of the whitewash on the specimen to detect any premature yielding. The alignment was considered satisfactory when the deviation of any of the four strain gage readings did not exceed 5% of the average value at the maximum alignment load.

Two 1/10,000 inch dial gages and an electrical clip gage were mounted along the middle line of the flanges at opposite sides of the specimen to measure strain over the 10 inch gage length (Fig. 9). The original magnitude of residual stress is not disturbed within this gage length.

Two methods of loading were tried to obtain the stress-strain curve. The first method dealt with making a point to point plot of the static curve. The static points from the proportional limit to the point near to the yield stress level were obtained by maintaining the applied load until no increase in strain is observed. The static points for the remaining portion of the curve were obtained by keeping the crosshead movement constant until the load is stabilized. This was obtained by closing the loading valve until the increase in deformation and the decrease in load approached zero. A load-relaxation diagram was plotted as schematically shown in Fig. 10 for each point. The average time required for stabilization was about 15 minutes. Figure 11 shows the results of this test.

In the second method the specimen was loaded continuously with only one stop made at the yield plateau to determine the static yield stress level. A loading rate of 1.42 ksi\* per minute was used in the elastic range and the same valve setting was used throughout the test. The result from this test is shown in Fig. 12.

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For both stub columns flaking of the whitewash was observed at 800 kips, and the flaking at the end of the test is shown in Fig. 8(b).

For both methods recordings were made using both the automatic X-Y plotter (Fig. 13) and manual recording.

The usual procedure in evaluating the stub column test results is to use a yield stress level criteria defined by the stress at 0.005 in./in. strain.<sup>(9)</sup> Using this criteria, the static yield stress is found to be 27.5 ksi and 27.6 ksi from the two tests, both of which indicate a very close correlation to the yield stress determined by tensile coupons, 27.1 ksi.

\*This is equivalent to 1 kp/sq mm per min. which is the required loading rate on column tests by the European Convention for Constructional Steel Associations. (11)

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#### 3. COLUMN TESTING

#### 3.1 Design of Tests

A total number of seven 12WF161 pinned-end columns each having a slenderness ratio of 50 were tested in order to make a comparison of different testing procedures, instrumentation, alignment and some other variables. The shape 12WF161 was used since it is the shape almost identical to the European shape HEM340 which will be tested under the program European Column Studies.<sup>(11)</sup> The slenderness ratio of 50 was selected out of the two slenderness ratios 50 and 95 to be used under the same program. This choice of slenderness ratio was made for a number of reasons:

- the shorter length is economical from the material point of view
  - more columns could be obtained from one single rolled length (a maximum length of about 125 ft. is possible for 12WF161)
  - relatively straight columns can be obtained if short columns are used since no cold straightening is allowed.

The experimental testing procedure as well as the results obtained are discussed below.

3.2 Initial Measurements

Initial geometric imperfections in axially loaded columns affect the column strength. Thus, initial measurement of the geometric characteristics of a column is an important step in column testing. Initial measurements were made for all columns of the crosssectional dimensions and out-of-straightness.

Cross-sectional measurements were obtained to determine the variation between the actual dimensions of the section and the nominal handbook dimensions. Measurement of the initial out-of-straightness will be used in the evaluation of the results of the tests.

Cross-Sectional Dimensions

Figure 14 shows all cross-sectional dimensions measured at five locations: the two ends, the quarter, the middle, and the three-quarter points. The measuring tools used were:

Thickness and depth - Vernier Caliper  $(\frac{1}{1000}$  in. sensitivity)

Web Thickness - depth micrometer  $\frac{1}{1000}$  in. sensitivity, Fig. 15 shows the determination of the web thickness.

The recorded dimensions and the calculated cross-sectional areas are given in Table 2. The percentage variation of cross-sectional areas and dimensions with respect to handbook values are given in Table 3.

Initial Out-of-Straightness

The initial out-of-straightness of each specimen was measured at nine levels, each spaced at one-eighth of the column length. Measurements were taken in the two planes of symmetry of the section.

Figure 16(a) shows the method for measuring initial out-of-straightness using the theodolite and the movable carpenters frame square with a strip scale attached to it.

Out-of-straightness (x) about the weak axis (Fig. 16(b)) is obtained from four readings - one with reference to each tip surface of the flange. For the

theoretical evaluation, the values for each flange may be used separately. The average of the four readings is taken as the out-of-straightness of the whole section.

Out-of-straightness (y) about the strong axis is obtained from two readings as shown in Fig. 16(c) where the average of the two readings is also plotted. Similarly, the separate values may be used for theoretical evaluation. All measurements taken were within an accuracy of 1/1000 inch.

A plot of the readings obtained for all columns tested is shown in Figs. 17(a) to (d) for weak axis measurements and in Figs. 18(a) to (c) for strong axis measurements. In both figures, the complete form used for taking initial measurements is shown only for column No. 01 and for the remaining columns the average values only are given. The initial out-of-straightness for the weak axis ranged from a minimum eccentricity ratio e/b of 0.010 for column No. 05 to a maximum of 0.029 for column No. 04.

#### 3.3 Prediction

A computer program was used to predict the tangent modulus curves for both the weak and strong axes.

The results from tension coupon tests and residual stress measurements were used for prediction. An equilibrated and symmetrical residual stress distribution derived from the measured values was used on the cross section, divided into a sufficient number of finite area meshes for the numerical computation. The cross-sectional dimensions, material properties and residual stresses were assumed constant along the full column length. The result obtained is shown in Fig. 19.

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#### 3.4 Alignment

A proper alignment of the column before testing is another important step to be fulfilled in column testing. In the test, two methods of alignment have been used.

The first method, developed at Fritz Laboratory and now known as the "Old Lehigh method", requires the column to be centered such that some established criterion is satisfied. The alignment is based on the four strain gages at each end of the specimen and at midheight. The alignment is considered satisfactory if the deviation of any of the four corner gage readings does not exceed five percent of their

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average value at maximum alignment load. The criterion is applied at each end of the three control sections.

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Out of the seven columns, one column (column No. 01) was aligned according to the "Old Lehigh method", the rest were aligned geometrically with respect to the center of web which is according to the ECCSA\* method which requires the alignment to be through the real center of gravity even if the section shows a dissymmetry due to unusual tolerances.<sup>(11)</sup>

#### 3.5 Instrumentation

The instrumentation used gave not only all test data required by ECCSA and the "Old Lehigh method", but also different additional measurements for the purpose of comparison and completion of the test results.

The most important records needed are the load versus deflection curves, the measurements of strain in characteristic points, angle of twist and the end rotations. The set-up and the instrumentation is shown

<sup>\*</sup>European Convention of Constructional Steel Associations, Sec. Ref. 11.

in Fig. 20 and is described in detail below. The experience with recording, as well as with the different forms of measurements, is summarized in Section 5.3 where final recommendations for future testing are also formulated.

#### Applied Load

The magnitude of applied load was obtained from the dial indicator of the 5,000,000 lb. hydraulic universal testing machine. The load was continuously plotted for all columns (except Column 01) using the X-Y plotter which is shown in Fig. 13, by connecting to the mechanism of the load indicator. The machine was originally calibrated, and then checked again at the end of the tests using a dynamometer (Fig. 21).

#### Lateral Deflections

Lateral deflections about the weak and strong axes were measured using strip scales (transit), potentiometer and dial gages.

Lateral deflections about the weak axis were measured from strip scales (graduated to one hundreth of an inch) attached at nine levels, each spaced at oneeighth of the column length, and read with a theodolite. To check if the theodolite is maintaining a fixed position, the cross-line was frequently checked to see if it matches a fixed reference point established at a region which is not disturbed by the testing. Similarly, the end-fixtures were also checked for any possible slip that may have occurred during the test. A coat of whitewash was applied at possible slip surfaces as a check.

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Lateral deflections about the weak axis were measured also with 4 inch and 1 3/8 inch potentiometers (with sensitivities of 4/1000 inch and 1/1000 inch respectively, Fig. 22) attached at five levels each spaced one-fourth of the column length. The deflections were continuously recorded on a multichannel oscillograph (Fig. 23) in a form of a deflection versus time plot. A typical recording obtained from such a recorder is shown in Fig. 24.

The midheight-deflection curve was continuously recorded using the X-Y plotter (Fig. 13).

Lateral deflections about the strong axis were

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measured with 1/1000 inch dial gages attached to the center of flange at mid-height and at the two ends.

All potentiometers and dial gages were fixed to the testing machine, and the wires were attached to small screws tapped at the columns at one end and weights suspended at the other end.

Strain Measurement

The strains at selected points and three sections of the columns (mid-height and the two ends) were measured with SR-4 electric strain gages, Type A-1. The strains were measured with digital type indicators for the columns tested to obtain the static curve.

End Rotations

End rotations were measured using mechanical and electrical rotation gages. The mechanical rotation  $gage^{(12)}$  is used by mounting the level bar on support bracket welded to the base plate and the top plate of the column (Fig. 25). Angle changes are measured by centering the level bubble by adjusting the micrometer screw. A vertical dial gage attached to the end of the level bar 351.2

gives an indication of the rotation of the bar over a gage length of 20 inches.

In the electrical rotation gage, <sup>(13)</sup> rotations are measured in the form of bending strains induced in a thin strip from which a heavy pendulum is suspended. One end of the strip is attached to the pendulum and the other end fixed to a round bar which is rigidly connected to a bracket welded to the base plate. As the column rotates, the weight tends to maintain its vertical position, and bending strains are induced in the strip of steel (Fig. 26). It has been shown<sup>(13)</sup> that the strain at any location is proportional to the end rotation.

Angle of Twist

Measurement of angle of twist may require the use of a complicated instrumentation if more accurate readings are desired. The instrumentation for measuring angle of twist in these tests was simplified greatly by taking the measurements of deflections causing the predominant twist and neglecting all other displacements considered to be of secondary importance. The differential lateral deflection of the flanges about the weak axis is considered to be the primary indication of twist. Figure 27 shows how the angle of twist may be obtained from such measurements. Measurement using potentiometers were taken at mid-height and the two ends (Section 4.8). Measurements at the ends were taken for the purpose of using a reference.

#### Overall Shortening

The overall shortening was obtained by measuring the cross-head movement using a dial gage, graduated to 1/10,000 inch. The dial gage was attached to a bar fixed on the testing machine. An aluminum rod was used to transmit the cross-head movement to the dial gage (Fig. 28). Since the dial gage was located at a remote location making it inconvenient to make a frequent reading, a TV camera was used to obtain the readings at the floor level (Fig. 29). Another reason for using the TV camera was to obtain a simultaneous reading of the cross-head movement and the applied load, the purpose of which is described below.

Progress of Yielding

As the column was loaded, a qualitative picture of the yielding pattern was seen from the flaking of the mill scale as detected by the cracking of the whitewash (hydrated lime) painted on the specimen. Figure 30 shows a recording of the whitewash cracking pattern for Column No. 01. From this figure, it is seen that yielding occurred at a load of 800 kips. The progression of yielding can be traced further by referring to the contour lines.

#### 3.6 Testing Procedure

After a careful alignment was completed, the test was started with an initial load of about 1/20 to 1/15 of the estimated ultimate load capacity of the column. This was done to preserve the alignment established at the beginning of the test. At this load all measuring devices were adjusted for initial readings. The testing was proceeded by loading the column progressively. Depending on the manner of loading and recording, the column curve may either be the static or a dynamic one. A further discussion on the mode of loading and the determination of the column curve is discussed in Section 5.6.

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In this study three column testing procedures were performed and these are explained in the following sections:

Old Lehigh Method

The testing procedure practiced in Fritz Engineering Laboratory<sup>(1)</sup> gives the static curve by making a point-by-point plot of the load-deflection curve. The load is applied in appropriate increments as estimated by the load deflection curve. Readings are taken when the load and the strains are stabilized. Column No. Ol was tested using this procedure and the dynamic P-A curve was not recorded.

The dial gage used for overall shortening (Section 4.5) was simultaneously used for observing stabilization. The single criterion for stabilization can best be defined by plotting the load change and cross-head movement versus time. As shown in Fig. 10, after some time, both values may be assumed as constant and static readings may be taken.
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New Method

The alternative method is essentially similar to the Old Lehigh method in the manner of loading but differs in performing the alignment. This method too, deals with determining the static curve. This method uses simpler alignment and also acknowledges the difficulties in determining the static load when using a hydraulic type testing machine. The problems encountered in the static curve when using the hydraulic type testing machine following the criterion of stabilization adapted by the Old Lehigh method are discussed in Section 5.6.

The manner of determining the static curve will not follow a fixed criterion as the old method but depends on the state of loading. Whenever it seems appropriate, the approach maintaining the applied load until the strains are stabilized, henceforth referred to as "horizontal approach", may also be used as a criterion for determining a point for the static curve.

The ECCSA Method

Column No. 05 was tested using the ECCSA method where a continuous load at a prescribed rate is

applied and the load and deflections are recorded automatically. The rate of loading used was 1 kp/sq. mm per minute (1.42 ksi/min.). This rate is established when the column is still within the elastic range and the resulting value setting is kept fixed until the end of the column testing. All required data are recorded automatically using the X-Y plotter and the multichannel oscillograph.

#### 4. COLUMN TEST RESULTS

## 4.1 Deflection Curves

Lateral deflections about the weak axis measured from strip scales at nine levels read with a theodolite were recorded. A plot of the deflected shape and the corresponding load causing the deflection for all columns (except Column No. 05 which was tested under continuous loading) is shown in Fig. 31(a) and 31(b). The load versus deflection curves for every eighth division is shown in Fig. 32 for Column No. 01 which were obtained from strip scale measurements.

## 4.2 Mid-Height Deflection

Lateral weak-axis deflections at mid-height were obtained from deflection curves (Section 4.1) and also were measured using a 1/1000 inch dial gage for No. 01 and a 4 inch stroke potentiometer for the remaining columns are shown in Figs. 33(a) to (d). 351.2

## 4.3 Ultimate Dynamic Load

The ultimate dynamic load for Column No. 01 was indicated by the stopping of the follower pointer of the dial on the testing machine. For the rest of the columns the dynamic load-deflection curves were recorded automatically using the X-Y plotter (Figs. 33(b) to (d)). For Column No. 01 a point by point plot of the loaddeflection curve was made and the static column curve was obtained by joining the points with straight lines. To provide a smooth curve closer points were used.

# 4.4 Static Column Curves

The static curve for the remaining columns were obtained by joining with a smooth curve the points obtained when the load and strains are stabilized. The initial out-of-straightness ratio as well as the rates of loading used are also indicated for each column.

# 4.5 Strong-Axis Lateral Deflections

Lateral deflectoins about the strong axis measured using 1/1000 inch dial extensometers at three levels are shown in Figs. 34(a) and (b). The magnitudes of the deflection are relatively small, and may be regarded as negligible.

## 4.6 Fiber Strains

For Column No. 01 fiber strains were measured using electrical strain gages at five levels, each spaced one-fourth of the column length. The results are shown in Fig. 35(a) and (b). For the rest of the columns the strains were measured at three levels (midheight and the two ends). Figure 36(a) and (b) shows measured strains at mid-heights of all columns.

# 4.7 End Rotations

End rotations at both ends measured using mechanical and electrical rotation gages are shown for all columns except Column No. 05. For the purpose of comparison, the measuring devices, the readings, from each type gage were plotted on the same page. The results are shown in Figs. 37(a) and (b).

# 4.8 Angles of Twist

The angles of twist measured at three levels are shown in Figs. 38(a) and (b). The net angle of twist at mid-height may be obtained by taking the angles at both ends into consideration.

# 4.9 Overall Shortening

Figure 39 shows the load versus overall shortening curves. It should be noted that the overall shortening recorded is the summation of three forms of deformations; axial shortening of the specimen, deformation of bearing plates and end-fixtures, and second-order effects due to large deflection.

#### 5. DISCUSSION

#### 5.1 Test Specimen

The test specimens were prepared from a single rolled piece in order to reduce the number of variable parameters, such as material, geometry and residual stresses, to a possible minimum. For the same reason, the specimens were not allowed to be coldstraightened. This may reduce the initial out-ofstraightness but would also redistribute the residual stresses. The effect of straightening is not discussed in this study.

Cooling conditions, such as the type of cooling bed and position of specimen on the cooling bed, influence the final residual stress distribution, and this may be one reason for the slightly unsymmetrical distribution of residual stresses measured for the shape in this study (Fig. 7). Such unsymmetrical distribution of residual stresses may be considered equivalent to some initial eccentricity imposed on the test specimen.

The column ends may not always be machined

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to have parallel surfaces, since milling is usually performed with reference to the end portions of the columns. Such deviations are difficult to measure or check, but would be expected to significantly influence the column strength. Even though the alignment is accomplished on strictly geometrical basis, the alignment may be improved by adjusting the leveling plates at the sensitive cross-head of the testing machine. For extreme cases it may be recommended to use the four strain gages at the flange tips at mid-height of the column and use the differences in readings as an indication for adjusting of end plates.

Such diagrams of strains near end sections as in Fig. 35 may indicate that restraining moments exist at the top and bottom pins (in the form of friction and eccentricity). The curvatures at the respective positions may be used to determine the restraining moments. Assuming that the strains vary linearly through the depth, the curvature may be computed from the readings of the strain gages located at the same elevation but opposite to each other. The curvature is equal to the quotient of the

differences between the strains at two points opposite each other in a particular cross section divided by the distance between the points, measured perpendicular to the bending axis.

## 5.2 Supplementary Tests

The purpose for conducting supplementary tests including residual stress measurement is to determine the basic properties of the specimen material so as to enable evaluation of theoretical predictions of column strengths.

#### Tension Tests

To determine more exact values of the mechanical properties for such a section\* it may be advisable to conduct tensile tests on test pieces taken from a number of specified points through the thickness of the flanges and the web.<sup>(14)</sup> The discrepancy in values and the differences in the characteristic stress-strain curves shown in Fig. 6 strongly suggest that tension tests should be conducted

\*The 12WF161 shape is regarded as a heavy shape in European practice.

on more specimens taken at characteristic locations.

As shown in Table 1, the difference between mill tests and ASTM standard tension tests may be very significant. To allow theoretical analysis, the tension test results should not be omitted and mill test information may be used only as informative values.

Residual Stress Measurement

For heavy shapes further measurement of residual stress through the thickness may also be required if more accurate data for theoretical evaluation is desired. The measurement can be obtained by "slicing"<sup>(15)</sup> the elements after a complete "sectioning" has been performed.

Stub Column Tests

The purpose for carrying out a stub column test is to determine the tangent-modulus load from the stress-strain curve of the specimen to predict the column strength. For columns of intermediate slenderness ratio, the curve from the proportional limit on would then be of greater importance. To make this portion of the curve smooth, the test points should be closely spaced. If the "static" curve is to be plotted, a time from 20 to 30 minutes would be required for each test point. Furthermore, there is a possibility of not obtaining the exact static curve, since it depends on the type of machine and the manner of loading used. These problems may be reduced if the dynamic curve is used (Fig. 11). The dynamic curve may be obtained in a much shorter time and with an even smoothness. The problem associated with this approach, however, is determining the "static" curve.

The static and dynamic curves start to branch off after the commencement of yielding which is at the proportional limit on the stress-strain curve. This is because effect of strain rate is not very significant in the elastic range. A static point at the yield plateau may be obtained by stopping the machine or using the relationship between strain rate and yield stress developed in Ref. 16. The static curve between these two points may be determined using the method developed by Cozzone and Melcon. <sup>(17)</sup> This method, however, depends solely on the geometric shape of the curve, since it is accomplished by means of "affine transformation", and thus may not provide the true static curve. In actual case,

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the static curve should be dependent, mainly, on the ratio of the yielded to the total area since the effect of strain rate is significant after commencement of yielding (Fig. 40). A discussion on determining the static curve from a recorded dynamic curve is presented in Appendix 1.

While such refinements to stub column testing are being contemplated, there are some basic points not clarified concerning the importance of the stub column test.

The stub column test by itself is not sufficient to make a prediction on column strength since the relationship between the effective tangent modulus and the corresponding effective area needs to be known beforehand.  $^{(10)}$  But if the pattern of the residual stress distribution is assumed to be known and the section is homogeneous the column strength may be predicted accurately using the procedure developed in Ref. 10.

The tangent modulus and the effective momentof-inertia may also be determined from tension coupon tests and residual stress measurement, without conducting a stub

column test, from which column strength may be predicted. Figure 19 shows the tangent modulus curves predicted from such supplementary tests.

To avoid a virtual repetition of supplementary tests it may be necessary to use either a stub column test or tension coupon tests and residual stress measurement. The choice may have to be dictated by the reliability of test, economy and saving in time.

## 5.3 Column Testing

The comparative study was designed to allow comparison of different testing procedures for heavy columns and to clarify some problems in instrumentation and recording. The study was encouraged since sufficient references, experience and data about testing of heavy columns was not presently available. The major reasons for this study may be summarized as:

- to gain experience with the European testing procedure (18)
- to obtain data for correlation of test results from the old Lehigh method and the European method which differ in basic aspects

to obtain test data for heavy rolled shape (the shape 12WF161 is almost identical to the heaviest rolled shape in Europe which currently is being experimentally studied under the program European Column Studies)
to recommend a new testing procedure for

heavy shapes for use at Fritz Engineering Laboratory.

### Initial Measurements

The extension of initial measurements for cross-sectional dimensions and out-of-straightness should correspond to the accuracy and coverage of other complementary tests (mechanical properties, residual stress and stub column tests). This would give satisfactory data of equivalent importance for corresponding theoretical investigation. The variation in cross-sectional area and shape and the out-of-straightness directly affect the column strength. In general, small imperfections result in significant reductions of the ultimate load.

# Alignment

The alignment of a column is the most important

step to be carried out before testing the column. Basically, there are two systems for aligning pinned-end columns. The first method is to align the column carefully such that the absolute maximum load which the pinned-end column can carry can be attained. The second method is simply to align geometrically with respect to some reference point on the cross-section.

The first method (old Lehigh method) has problems associated with it in satisfying the criterion (Sect. 4.4) and are summarized as follows:

- it is time consuming (Table 4)
- it is difficult or sometimes impossible to satisfy the criteria especially for long columns with large out-of-straightness
- the maximum alignment load is not a clearly defined load, instead, it requires a certain degree of judgement for its determination, since it depends on the proportional limit and the degree of accuarcy of the alignment.

The geometric alignment, on the other hand, is very simple and time saving since the end plates can easily be welded with reference to any desired reference point on the cross section. Consequently, the end plates can be positioned with reference to the centerline of the testing machine without much difficulty. Another attractive feature about geometric alignment is its conformity to practical conditions employed in steelwork construction.

It should be mentioned, however, that a new variable is introduced for sections with the center of gravity not at the center of web. Practical considerations prohibit use of the center of gravity as a reference point. The best centering would then be with respect to the flanges, since the web has little effect on buckling about the weak axis. This reference point may be located at the mid-point of the line connecting the two centers of flanges. Still another feature about the center of flanges, according to the rolled shape considered in this study, is that its position on the cross-section is usually nearer to the center of gravity than the center of the web. This is indicated in Fig. 41 which shows a plot of the computed results obtained from the measurements made at each end for all seven columns. Figure 42 shows the variation of the three reference points along the length for one of the columns.

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The effects of the two methods of alignments on column strength is shown in Fig. 43. The figure shows a plot of the ultimate static strength of each column versus its corresponding ratio of initial out-ofstraightness. For the column aligned according to the old Lehigh method the ultimate load was very close to the predicted tangent modulus load (Fig. 19) even though it has a considerably large initial out-of-straightness. For the columns aligned geometrically, the ultimate loads were below the predicted tangent modulus load. The differences increase as the initial out-of-straightness Note, however, that this comparison is made increases. on the ultimate strengths of the experimental columns and the tangent modulus load for the ideally straight column.

## 5.4 Testing Procedure

Loading of the column in a testing machine is always conducted under some rate of loading which causes the difference between static and dynamic  $P-\Delta$  curves correlated to the static and dynamic yield stresses. The experimental curve is, therefore, influenced by the rate of loading. Two types of column curves can be obtained from column testing; the dynamic and the static curve which may be defined as the dynamic curve at "zero" rate

of loading. This is one basic explanation for the difference of the two methods used in this investigation. Evaluation for the old Lehigh method is based on the static measurements while the European procedure uses only the dynamic loading completely neglecting the static equilibrium.

To obtain the "static" curve there are some factors to be considered. According to the old Lehigh method, the static curve is determined when the load carried by the column shows no further decrease in magnitude while maintaining the cross-head movement fixed. This, for example, is rather easy to satisfy if a mechanical type machine is used since the cross-head can be held fixed in position. The contrary is true if a hydraulic type machine is used, since leakage of oil, change in oil temperature and other factors which always are inherent during normal working conditions make it rather difficult to maintain the cross-head movement. Maintaining the load is usually simpler when using a hydraulic type machine. Therefore, the definition for determining the static curve should take into consideration the type of machine used and the manner of loading imposed.

In general, the effect of the rate of loading

is noticeable after yielding on some fibers starts and becomes more noticeable as the yielding progresses. Therefore, the preferable manner of loading depends on the state of the column. For a hydraulic type testing machine, the portion of the static curve up to the ultimate load can be found more accurately by maintaining the load. The curve obtained will always be higher or may match the "true" curve. Figure 44 shows the possible range of error when using the "horizontal" approach which varies from 0.25 to 0.5 percent on the unconservative side. This approach, however, has the disadvantage that it cannot be applied after the ultimate load is reached (unless the load is lowered well below the static curve and then maintained), also it requires a much longer period of time for stabilization especially for loads very close to the ultimate. Figure 45 shows the stabilization time required as the applied load approaches the ultimate load for Column No. 07. Note that all curves don't show a complete stabilization, and further increments of displacements were considered negligible. The applied load may be determined if it is greater or smaller than the ultimate load by observing the rate in increase of the cross-head movement. If an inflection point on the curve deflection versus time is indicated, stabilization will occur. In general, since

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the stable region of the column curve is usually of prime importance in engineering design, the "horizontal" approach may thus be used effectively.

The "vertical" approach (that is by maintaining the cross-head movement) may not give as accurate a static curve as the other approach if a hydraulic type machine is used. Under normal conditions an asymptotic load (Fig. 46) would not be observed. The possible range of error depends on the condition of the testing machine. The continuous drop of the load while maintaining the cross-head movement is not only due to oil leakage, but could also be due to creep at bearing surfaces such as the cover plates and also friction at bearing surfaces. For the columns tested in this study the error was in the order of magnitude of one percent (Fig. 47).

It is customary to plot the  $P-\Delta$  curve using the "total" mid-height deflection. The "net" mid-height deflection should be used for a correct plot. The difference in these deflections is the mean lateral deflection at the ends which occurs simultaneously with end rotations. The results obtained using the "net" and "total" deflection is shown for each tested column in Figs. 48(a) to (d). Note that the over-all P- $\Delta$  curve is not significantly altered even though the differences in deflections are considerable, especially for large deformations. It is therefore recommended to use the "total" deflection for the P- $\Delta$  plot whenever such refinements are not justified.

Figure 49 shows the static P- $\Delta$  curves for all columns using the "net" mid-height deflections. Note that these columns geometrically aligned and having identical initial out-of-straightness show similar results thus forming very narrow bands. For instance, Column Nos. 03, 04 and 07 with eccentricity ratio of about 0.028 show very identical results, so also Column Nos. 02 and 06. The numerical results are summarized in Table 5.

While the technique and precision in column testing is being improved some objective questions which may alter the whole testing procedure seem to be yet unanswered. These questions may be summarized as:

> What actually would simulate more the actual manner of loading on a column of a structure? Static or dynamic loading? It is also possible

that for some structures the dynamic approach should be used while the static is for some

others. Then, how should the categorization be carried out?

If the P-A curve for a static loading would be required, should it be obtained from a plot of static points? Or should it be derived from the dynamic curve?

What should be the appropriate testing approach to determine a static point? Maintaining the deflection or maintaining the load?

If the dynamic curve would be sufficient, what should be the rate of loading to use?

In an attempt to find a solution to these problems, it is recommended for the new Lehigh method to use an "interrupted" dymanic loading with only one interruption. The dynamic curve will be plotted until the ultimate load is reached immediately after which the static load will be recorded using the "vertical" approach. After the static load is recorded the test will be resumed using the value setting established originally until the desired configuration has been attained. A sketch of the complete  $P-\Delta$  curve resulting from such a test will be similar to that shown in Fig. 50.

Such a procedure will present the dynamic curve and the main information about the ultimate static load which should be sufficient for statistical evaluation and for comparison with theoretical predictions. 6. SUMMARY AND CONCLUSIONS

The purpose of this experimental study was to investigate and to compare different column testing procedures for medium and heavy pinned-end columns. The main subjects of interest were the alignment and the manner of loading. The following methods were considered:

Old Lehigh Method - Alignment with respect to stresses at three levels; static column curve.

 Different loading rates and two different approaches were investigated to obtain the static curve.

ECCSA Method - Geometric alignment and dynamic column curve.

New Lehigh Method - (Proposed)

- Geometric alignment and interrupted dynamic loading; the static curve derived from dynamic column curve and one static point.

A total of seven 12WF161 (A36 grade steel)

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prepared from one rolled piece and no straightening was allowed. Before the columns were tested geometric measurements and supplementary tests were performed.

Based on the experience and test results the following recommendations and conclusions can be stated:

- Testing of heavy columns requires a well-developed testing procedure, more complete in instrumentation and supplementary tests, than for light sized columns. This is to avoid very expensive replications required for statistical evaluation, and to allow more accurate correlations with theoretical analysis.
- 2. Measurement of cross-sectional dimensions at closer points along the length (which is possible to include individually in the computer program for predictions) and the respective initial out-of-straightnesses both about the weak and strong axes are of considerable importance. Measuring techniques providing better accuracy were developed and are described in this paper. Measurement of initial twist which may also be required, was not considered.
- 3. For heavy columns, the mechanical properties of the material may not only be different for the web and

flanges, but may also vary significantly through the thickness. It would be, therefore, recommended for heavier shapes to conduct coupon tests on test pieces taken from a number of specified points throughout the thickness and to use these results for theoretical predictions. Mechanical properties of the material from a mill test, generally, may differ very much compared to coupon test results (Table 1).

- 4. The magnitude and distribution of residual stresses is required to make a theoretical prediction and for correlation with test results. Residual stresses may be measured using the method of sectioning and slicing or may be obtained from previous studies on heavy shapes.
- 5. Column strength with zero initial out-of-straightness may be predicted from stub column test results and using charts<sup>(10)</sup> where a simplified residual stress pattern, homogeneous and ideal elastic-plastic material are assumed. If a more accurate column strength prediction is required using stub column test results, additional information of the residual stress distribution and mechanical properties across the section especially for medium and heavy shapes are required. But if such information is already available, column strength may

be directly predicted analytically and stub column test would not be required.

- 6. Different stub column testing procedures were investigated and compared. To obtain the static curve the "horizontal" approach (maintaining load) would be more preferable for a hydraulic type testing machine. If the measured residual stress distribution is available, the testing procedure can be simplified using a dynamic curve and one static point after the yield plateau is reached. The static curve is to be plotted through the static test point using relationship described in Appendix 1. This simplified method, to obtain the static curve, may be considered as accurate as the usual procedure and is not time consuming.
- 7. The stub column test should be used for heavy shapes only if direct analytical prediction cannot be made to allow comparison.
- 8. The test results for medium and heavy columns are greatly influenced by the alignment method used. The "stress criterion" alignment used in the old Lehigh method was introduced to reduce the effects of initial out-ofstraightness, but it also increased the ultimate load. Such alignment is not only tedious and time-consuming,

but does not correspond to the behavior of a compression member in an actual structure. Also from a statistical point of view, this method depends on uncontrolled variable - the end moments.

- 9. After comparing different alternatives of alignments (Old Lehigh method, geometrical alignment - center of flanges) the geometrical alignment is recommended, using the center of flange as a reference point. Such a method is very simple and not time-consuming. The boundary conditions are kept the same and are easily included in theoretical predictions.
- 10. The results from column tests using different testing methods are often not directly comparable. One of the main reasons is the mode of loading. Some testing methods use dynamic loading and the static curve is not recorded at all, whereas, some other methods are based on the static curve and only the ultimate dynamic load is recorded. To allow comparison on column test results the mode and the rate of loading must be comparable.
- 11. The investigation of loading of a column in an actual structure will not give a single answer; some loadings may be considered as static loads (dead load and live load) and some as dynamic loads (wind, earthquake, etc.).

It is therefore recommended to obtain from a column test both curves, static and dynamic. The proposed new testing procedure may be considered as a compromise between static and dynamic testing methods. It also takes advantage of past experience on initial measurements, alignment, and instrumentation. The dynamic loading with constant "strain rate" and continuous recording of data is used up to the ultimate load where a static reading is taken, and dynamic loading is then resumed. The static column curve is derived from the dynamic curve using the relationship between dynamic and static yield stresses.

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## 7. ACKNOWLEDGEMENTS

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This paper presents the results of an experimental study of column testing methods. The investigation is one phase of a major research program on European Column Studies.

The investigation was conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The European Convention of Constructional Steelwork Associations, National Science Foundation, and the Welding Research Council jointly sponsor the study. The specimens were fabricated by the Bethlehem Steel Corporation, and thanks are due to that company and its personnel who provided the selection of specimens with special care.

The guidance of Task Group 11 of the Column Research Council, under the chairmanship of Duiliu Sfintesco, is gratefully acknowledged.

Thanks are due to K. R. Harpel, Laboratory Superintendent, and his staff for the preparation of the test specimens, to R. N. Sopko, Laboratory Photographer, for the photographs and to W. A. Gast, for help in reducing the data. Special thanks are due to Mrs. Sharon Balogh for the preparation of the drawings and to Miss Joanne Mies for her care in typing the manuscript. 8. APPENDIX 1

# Evaluation of Static Curve from a Dynamic Curve

Experiment has shown that if a continuous rate of loading is applied on either a stub column or regular column a dynamic curve different from the static curve is obtained. This is because the effect of strain rate has a considerable influence during plastic deformation, whereas it is not very significant during elastic deformation. For theoretical evaluation of test results, however, the static curve is usually required. In this Appendix, the approach for deriving the static curve from a recorded dynamic curve is discussed and a simplified method is proposed.

The difference between static and dynamic column curves is dependent on the difference between the static and dynamic yield stresses of the material. The effects of strain rate on the yield stress of structural steels was investigated in Ref. 16. The main conclusions are:

> a) The dynamic yield stress level is influenced by the speed of testing, size of specimen,

testing machine and the shape of the column.

- b) There is no simple relationship between crosshead speed (or mid-height deflection) and strain rate.
- c) The static yield stress level is a property of steel independent of size of specimen, testing procedure and testing machine.
- d) The dynamic yield stress ratio increases rapidly at low strain rates and very slowly at the higher rates; it decreases with increase in static yield stress level.
- e) An average curve relating the difference  $(\sigma_{yd} \sigma_{ys})$  and strain rate is proposed to predict the static yield stress level of a specimen from a standard tensile coupon test.

The relationship between static and dynamic curves is not simple even for a tension coupon. To investigate the relationship between static and dynamic column curves consideration of more variables would be required. The main variables are; residual stresses, geographical variation of mechanical properties through

the section of the specimen, nonuniform yielding along the column length and nonuniform strain rate, due to loading.

In some testing method practices, the static curve is obtained directly (Old Lehigh Method<sup>(1)</sup>). The ECCSA method,<sup>(11)</sup> on the other hand, is concerned with obtaining the dynamic curve and the static curve is not required. For the proposed new Lehigh method, one static point close to the ultimate strength is available and the derivation of static curve from dynamic curve is required.

A method depending solely on the geometric shape of the curve was developed by Cozzone and Melcon<sup>(17)</sup> as shown in Fig. 40(a). This method, however, may not provide the true static curve. In actual case the static and dynamic curves start to branch off after the commencement of yielding which is at the stress level when a fiber of the column is stressed beyond the proportional limit. The deviation will increase with increasing ratio  $A_y/A$ ; where  $A_y$  is the area of the yielded part of the cross-section and A is the total area.

In determining the static curve from the recorded dynamic curve the following assumptions are made:

a) The material has the property of elastic-plastic behavior (Fig. 40(b)).
b) The test member yields with constant ratio of A<sub>v</sub>/A along the length.

Assumption (b) is valid for the case of stub column but not necessarily for a column. But such variations as nonuniform yielding along the column length have already been taken into consideration in the recorded dynamic curve. In a more refined analysis such variation should also be included in the correction process. Since similar variations are neglected, the introduction of this refinement here is not warranted.

At the ultimate column strength both the static and dynamic values are recorded. The difference is,

 $\Delta P = P_{ud} - P_{us} \qquad (1)$   $P_{ud} = ultimate dynamic load$   $P_{us} = ultimate static load$ 

The dynamic load at any point between the proportional limit and the ultimate load is,

$$P_{d} = P_{s} + A_{y} (\sigma_{yd} - \sigma_{ys}), \qquad (2)$$

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where

where

 $P_d = dynamic load$   $P_s = static load$   $A_y = yielded area of the cross section$   $\sigma_{yd} = dynamic yield stress$  $\sigma_{ys} = static yield stress$ 

Assuming  $A \cong A$  at a point close to the ultimate load, the recorded dynamic load is,

$$P_d = P_s + \Delta P$$

Then,

$$(\sigma_{yd} - \sigma_{ys}) = \frac{\Delta P}{A}$$
 (4)

Substituting Eq. (4) into Eq. (2), the static load will be,

$$P_{s} = P_{d} - A_{y} \frac{\Delta P}{A}$$
(5)

Note that the values  $P_d$ ,  $\Delta P$  and A are known from test results and initial measurements. The area  $A_y$ can be computed using the same procedure as in evaluating the effective area when determining the tangent modulus load where the residual stress distribution and the yield stress of the material are taken into consideration. A sketch of the complete P- $\Delta$  curve resulting from such test and computation will be similar to that shown in Fig. 40(c).
9. TABLES AND FIGURES

		ASTM ST	ON TEST		
NO.	Location	Static Load stress, σ (psi) <sup>y</sup>	Ultimate Tensile Stress σ <sub>u</sub> , (psi)	Percent Elongation (%)	
1	Web	28,837	62,536	30.95	
2	Flange	27,128	61,470	34.08	
3	Flange	25,454	61,410	33.75	

Table	1:	MECHANICAL	PROPERTIES	OF	SPECIMEN

MILL	Min.	33,372	62,591	32
TEST	Max.	35,703	67 <b>,</b> 767	33

#### TABLE 2: CROSS SECTION DIMENSION

Top -

Bottom ∐-5

2

-3

4



 Proj. No:
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 Steel Grade:
 A36

 Shape:
 12 WF161

 Col. No:
 O1

 Length:
 13'-4 ½"

 Recd:
 P.M. & N.T.

 Date:
 2/9/69

Section	<b>.</b>	<b>b</b>	he	hi	<sup>†</sup> fr	† br			C fr	с <sub>br</sub>	Area
Section	11 <u>R</u>	nr	Df	DP	† fl	t bl	w†	wБ	6f1	ср	(in.2)
I	13.765	13.829	12.613	12.596	1.464 1.460	1.440 1460	.978	.993	5.789 5.846	5.804 5.799	47,432
2	13.771	13.856	12.604	12.59,4	1.464 1.447	1.432 1.460	.973	.990	5.785 5.845	5.815 5.789	47.266
3	13.795	13.818	12.603	12.577	1.469 1.452	1.432 1.471	.975	.982	5.773 5.855	5.801 5.794	47.323
4	13.780	13.832	12,618	12.596	1.471 1.451	1.427 1.439	1.034	1.006	5.784 5.800	5.804 5.786	47.615
5	13.763	13.845	12.608	12.598	1.472 1.455	1.433 1.436	1.025	1.006	5.788 5.795	5.800 5.792	47.599
Average	13.775	13.836	12.609	12.592	1.461	1.443	0.997	0,995	5,806	5.798	47.450

See Fig. 14 for Notation

Table	2
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2 (Continued)

7

										0	Col. O2
Section	h,	h,	ba	bh	† fr	tfr	Ws	WL	с <sub>іг</sub>	° <sub>br</sub>	Area
Section		1 <sup></sup>		- 0	tfl	t <sub>bl</sub>		D	c <sub>fl</sub>	с <sub>bl</sub>	(in <sup>2</sup> )
1			<u></u>		1,463	1.450			5.824	5,840	47.4152
l l	13.816	13.796	12.628	12.607	1.438	1.465	0.991	0.977	5.813	5.790	
2					1.465	1.442			5.842	5.855	47.728
۲	13.817	13.810	12.660	12,623	1.428	⊥.468	1.041	0.984	5.777	5,784	
7					1.465	1.439			5.806	5,825	47.261
5	13.825	13,798	12,620	12.600	1.429	1.465	0.995	0.967	5,819	5.808	
	ĺ				1,474	1.435			5.838	5.848	47,493
7	13.844	13,786	12.656	12.600	1.429	1.462	1.023	0.969	5,795	5,783	
5					1.470	1.440			5.824	5.828	47.435
5	13.845	13.765	12.645	12,609	1.425	1.461	1.013	0.975	5.808	5.806	
Average	13.829	13,791	12,6418	12.6078	1.4486	1.4482	1.0126	0.9744	5.8146	5.8167	47.41.9

					_						Col. 03
Section	h,	h	be	Ъъ	† <sub>fr</sub>	tfr	We	W.	с <sub>ír</sub>	с <sub>br</sub>	Area
Section	·· X	i "r	-1	~ D	† <sub>fl</sub>	†bl		"D	C <sub>fl</sub>	с <sub>р</sub>	(in <sup>2</sup> .)
1			1		1,475	1.449	]		5.870	5.866	46.959
	<u>13.837</u>	13.800	12,615	12,596	1.429	1.485	0.936	0,929	5,809	5,801	
2					1,473	1,454			5,857	5.853	46.811
<u> </u>	13.860	13.808	12,615	12.597	1.435	1,475	0.916	0.920	5,842	5.824	
٦			ł		1.480	1.447			5.838	5,838	47.011
	13,841	13,812	12.618	12.595	1.420	1.470	0.938	0.957	5.842	5.800	
4					1,475	1.452			5.824	5.852	46.501
	13.833	13.788	12,605	12.600	1.424	1.472	0.926	0.874	5.855	5.874	
5					1.480	1,465			5.823	5,824	47.2029
	13.838	13,782	12,605	12.608_	1.418	1.470	0.958	0.958	5.824	5.826	
Average	13.842	13.798	12.612	12.599	1,451	1.464	0.935	0,928	5,838	5.836	46.903

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Table 2 (Continued)

										[	Col. 04
Section	h a	h.	be	bu	† fr	<sup>†</sup> fr	Ws	V # W L	<sup>C</sup> fr	c <sub>br</sub>	Area
Section		''r		- 0	tfl	<sup>†</sup> bl	- • T	"D	c <sub>fl</sub>	cbl	(in²)
1			1		1.460	1.459		Ì	5.789	5.826	47.215
L	13.780	13.820	12,540	12.601	1.427	1.475	0.969	0.983	5.782	5.792	
2					1.453	1.460			5,818	5.859	47.118
<u> </u>	13,795	13.847	12.590	12.600	1.420	1.465	0.990	0.952	5.782	5.789	
7					1,455	1.464			5.878	5.920	47.196
	13.800	13.799	12.575	12,600	1.424	1.470	0.969	0.979	5.728	5.701	
4					1.469	1,465			5.943	5.993	47.494
	13.830	13,801	12.604	12,600	1.424	1.458	0.982	1.007	5.679	5.600	
5				1	1.460	1.478			5,930	5,980	47.497
	13,835	13.822	12.591	12,595	1.435	1,474	0,977	0.982	5.684	5.633	
Average	13.808	13.818	12,580	12.599	1.443	1.469	0.977	0.981	5.801	5.809	47.333

Col. 05

								-			01.00
Section	h,	hr	bf	bh	<sup>†</sup> fr	† <sub>fr</sub>	Wf	Wh	C <sub>fr</sub>	с <sub>br</sub>	Area
	^				†fl	†bℓ			C <sub>f</sub> l	cbl	(in?)
					1.480	1.485			5.800	5.800	47.590
	13,844	13.760	12,610	12,607	1.424	1,465	0.984	0.981	5.826	5.826	
2				ł	1.460	1.460			5.782	5.805	47.268
۲	13,890	13,760	12.620	12.635	1.420	1.454	0.977	0,968	5.861	5.852	]
<b>र</b>					1.455	1.465		1	5.795	5.803	47.425
	13,886	13.785	12,602	12,630	1.422	1,460	0.981	0.999	5.826	5.828	]
1					1.450	1.459			5,788	5.848	47.151
-	13.825	13.776	12.596	12.636	1.440	1.470	0.969	0.949	5,839	5.839	
5					1.453	1.455			5.805	5,855	47.447
5	13.855	13.765	12,600	12.625	1.439	1.480	0.977	0.987	5.818	5.783	]
Average		1									
L	13.860	113.769	12.606	12.627	11,044	1.465	0.978	0.977	5.814	5.825	47.362

351.2

Table 2 (Continued)

										[	Col. 06
Section	h a	h.,	be	b.	† fr	† fr	Wr	W.L	<sup>C</sup> fr	° <sub>br</sub>	Area
Section	"1	1.14		° D	† <sub>fl</sub>	tbl	- • · I	"D	Cfl	cbl	(in <sup>2</sup> )
1			1		1.458	1.428	_		5.800	5.798	47.205
	13.760	13.810	12.615	12.574	1.448	1,458	1.004	0.967	5.811	5,809	
2					1.465	1,425			5.804	5.797	47.209
<u>د</u>	13.792	13.835	12.610	12.599	1.448	1.460	0.966	0.989	5,840	5.813	ļ
٦					1.470	1.425		Į	5,793	5.805	47.287
5	13.808	13,816	12.614	12.610	1.456	1.453	0.971	0.988	5.850	5,817	
4					1,465 L	1.424			5.787	5,793	47.191
	13.810	13.840	12.607	12,605	1.450	1.455	0,968	0.985	5,852	5,827	
5					1.454	1,424			5.780	5.797	47.189
5	13.800	13.815	12.608	12,620	1.448	1.460	0.983	0.977	5.845	5.846	
Average	13.794	13.823	12.611	12.602	1.456	1.441	0.978	0.981	5.816	5.810	47.362

										(	Col. 07
Section	h a	h.,	he	br	<sup>†</sup> fr	† fr	Wc	W	<sup>C</sup> fr	c <sub>br</sub>	Area
Section	··· <u>/</u>	''r		° D	† <sub>fl</sub>	<sup>†</sup> bl		"D	c <sub>fl</sub>	cbl	(in <sup>2</sup> )
1					3.475	_1.425			5.780	5.788	47.662
	13.766	13.831	12,590	12.600	1,453	1.474	0.986	1.029	5.824	_5,833	
2	- - -	-			1,465	1.420			5,794	5.782	47.206
	13,782	13.845	12.600	12.605	1.449	1.470	0.970	0.979	5.836	5.844	
7					1.462	1.420			5,805	5.779	47.197
5	13,787	13.832	12.602	12.603	1.443	1.465	0.965	0.998	5,832	5.826	]
4	-				1.464	1.422			5.792	5.781	47.194
4	13,800	13,840	12.586	12.615	1.445	1,472	0.962	0.986	5.832	5.848	
5			Γ		1.465	1.415			5.794	5.794	47.089
່ວ	13,790	13.841	12.580	12.616	1.448	1.470	0.956	0.980	5.830	5.842	]
Average	13,785	13.838	12,591	12,608	1.457	1.487	0.968	0.994	5,812	5.812	47.754

### Table 3 DIMENSIONS OF TEST SPECIMENS

Col		Area, A,	Depth of Flange		Web	
No.		in. <sup>2</sup>	Section, d	width,b	Thickness,t	Thickness, w
01	Measured	47.45	13.806	12.600	1.450	0.996
	% Variation	+0.1	-0.5	+0.7	-1.6	+10.1
02	Measured	47.42	13.810	12.625	1.448	0.994
	% Variation	+0.1	-0.5	+0.9	-2.6	+9.8
03	Measured	46.90	13.820	12.606	1,457	0.932
	% Variation	-1.0	-0.4	+0.7	-2.0	+3.0
04	Measured	47.33	13.813	12.589	1.456	0.979
	% Variation	-0.1	-0.5	+0.6	-2,0	+8.2
05	Measured	47.36	13.815	12.617	1.455	0.977
0.5	% Variation	0	-0.5	+0.8	-2.1	+8.0
06	Measured	47.36	13.809	12.607	1.449	0.979
	% Variation	0	-0.5	+0.7	-2.5	+8.2
07	Measured	47.75	13.811	12.600	1.472	0.981
	% Variation	-0.8	-0.5	+0.7	-0.9	+8.4
Har	ndbook Values	47.38	13.88	12.515	1.486	0.905

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#### Table 4 SUMMARY OF COMPARISON OF TESTING METHODS

Method	Typical Column Curve	Loading	Testing Time	Accuracy of Static Curve	Remarks
01d L.U.	P	Static	4-6 hrs.	0.5 to 1.0% (for hydraulic testing machine)	- Time Consuming - Dynamic Curve not available except P <sub>ud</sub> .
Alternative Old L.U.	P	Static	4-6 hrs.	"Horizontal" approach 0.25-0.50% "Vertical" approach 0.5 - 1.0%	<ul> <li>Time Consuming</li> <li>Dynamic curve not available except</li> <li><sup>P</sup>ud</li> <li>Slightly more accurate Static Curve.</li> </ul>
ECCSA	P	Dynamic	15-20 min.	Static Curve not available.	Static Curve not available,
New L.U.	P	Semi- Dynamic	30-40 min.	0.5 - 1.0%	Only the ultimate Static Point available.

### LOADING

## ALIGNMENT

Method	Aligning Time	Remarks		
old L.U.	4-5 hrs.	5% max. deviation from uniform stress at three levels.		
ECCSA	30 min.	Center of Web.		
New L.U.	30 min.	Center of Flanges.		

### Table 5 SUMMARY OF TEST RESULTS

Col. No.	Method of Testing	Max. Initial Eccentricity (inch)		Rate of Loading	Ultimate Load, P	$\frac{P}{\frac{ult}{p \times x}} \times 100$	Mid-height at P <sub>ult</sub>	Remarks
		Weak Axis	Strong Axis	(ksi/min)	(kips)	у	(inch)	
01.	Old L.U.	0.25	0.08	1.42	1154	84.6	0.52	Col. buckled opposite to initial curve.
				static	1084	79.5		
05	F.C.C.S.A.	0.13	0.035	1.42	1170	85.8	0.52	
04	Alternative L.U.	0.36	0.08	1.42	940	68.9	1.65	
				static	902	66.1		
02	Alternative L.U.	0.19	0.075	2.50*	1000	73.4	1.2	*Four different rates were tried 0.25, 0.50, 1.42, 2.50 ksi/min
				static	950	69.6		
06	Alternative L.U.	0.18	0.04	0.5	990	72.6	1.16	
				static	952	69.8	1.04	
07	Alternative	0.35	0.06	1.42	946	69.4	1.34 🕔	Load maintained
	L.U.			static	900	66.0		
03	Alternative L.U.	0.35 0.1		1.42	968	7.1.0	1.54 1.85	Deflection maintained
			0.11	static	916	67.2		



\R

Oil —— Pressure

Rollers

R

R

Free Warping End

Rollers

Cylinder

Fig. 1 Basic Types of End Fixtures

-75



#### Fig. 2 Standard Column End Fixture at Fritz Engineering Laboratory

-76







Fig. 4







### Section A-A

Fig. 5

Location of Coupons with Respect to the Cross Section



Fig. 6 Stress-Strain Curves from Tension Coupon Tests

-79



Fig. 7 Residual Stresses in 12WF161



a) Instrumentation



b) End of Test



Fig. 9 The Dial Gage and Electrical Clip Gage Over 10 inch Gage Length of Stub Column





Fig. 11 Stub Column Test Result Using "Horizontal" and "Vertical" Approaches -83



#### Fig. 12 Stub Column Test Result Using the Dynamic Curve and One Static Point

-84









Fig. 15 Determination of Web-Thickness Using the Depth Micrometer





## Fig. 17(a) Measurement of Initial Out-of-Straightness (Weak Axis)

88-







Fig. 17(b) Measurement of Initial Out-of-Straightness (Weak Axis), Continued





Col. 04

Front Back

4.40

2.40

4.24

2.22

4.06

2.03

3.92

1.86

3.80

⊥.76

4.00

1.90

2.10

4.05

2.20

4.18

2.32

4.28

2.38

4.34

2.43

4.38

2.42

4.40

2.36

4,36

2.18

4.16

1.96

3.93

Right

Left



Col. 05

Front Back

Right



0.34

0.16



-91

STRONG AXIS, INITIAL OUT-OF-STRAIGHTNESS



Fig. 18(a) Measurement of Initial Out-of-Straightness (Strong Axis)

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# Fig. 18(b) Measurement of Initial Out-of-Straightness (Strong Axis), Continued

-93



Fig. 18(c) Measurement of Initial Out-of-Straightness (Strong Axis), Continued

351.2

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Fig. 19 Tangent Modulus Curves Based on Tension Coupon and Residual Stress Measurements -95



a) Beginning of Test



b) End of Test

#### Fig. 20 Set-Up for Column Testing

351.2

-96



Fig. 21 The Dynamometer in the 5,000,000 lb. Universal Testing Machine



Fig. 22 The 4 inch Stroke Potentiometer



Fig. 23

Strain Indicators and the Multichannel Oscillograph



Fig. 24 Typical Load-Deflection Time Recordings From Multichannel Oscillograph -99

351.2





Fig. 25

The Mechanical Rotation Gage

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ELECTRICAL ROTATION GAGE



Fig. 26 The Electrical Rotation Gage


Fig. 27 Deflection Measurements to Determine Angles of Twist



Fig. 28

Dial Gage to Measure Overall Shortening Located Near Top End Fixture



Fig. 29

Simultaneous Reading of the Load Dial and the 1/10,000 inch Extensometer as Seen Through the TV Screen.



# Fig. 30

## Whitewash Cracking Pattern



Fig. 31(a) Lateral Deflections of Columns Measured by Theodolite and Strip Scales

-<sup>868</sup>.

Col. 06

3

2

-950--924--302-

1

(MINOR AXIS)



(MINOR AXIS)

Fig. 31(b)

Lateral Deflections of Columns Measured by Theodolite and Strip Scales (Continued)



Fig. 32 Load-Deflection Curves at Various Levels

-107



Fig. 33(a) Load-Mid-Height Deflection Curve for Col. 01



Fig. 33(b) Load-Mid-Height Deflection Curves from the X-Y Plotter



Fig. 33(c) Load-Mid-Height Deflection Curves from the X-Y Plotter (Continued)





Fig. 33(d) Load-Mid-Height Deflection Curves from the X-Y Plotter (Continued)



Fig. 34(b) Strong Axis Deflections at Three Levels (Continued)

-113



STRONG AXIS DEFLECTION (IN.)

-112





Fig. 35(a) Fiber Strains at Five Levels for Column No. 01



# Fig. 35(b) Fiber Strains at Five Levels for Column No. 01 (Continued)

-115





Fig. 36(a) Fiber Strains at Mid-Heights of Columns



Fig. 36(b) Fiber Strains at Mid-Heights of Columns (Continued)

-117



Fig. 37(a) End Rotations of Columns



Fig. 37(b) End Rotations of Columns (Continued)







Fig. 38(a) Angles of Twist at Three Levels

-120



Fig. 38(b) Angles of Twist at Three Levels (Continued)

351.2



Fig. 39

Load Versus Overall Shortening Curves









Derivation of the Static Curve from a Dynamic Curve



Fig. 4l

Comparison of Geometric Centers of Flanges and Webs with the Centroids at Ends of Specimen



Fig. 42 Variation of Geometric Centers Along the Length of Col. 06

-125





# Fig. 44

"Horizontal" and "Vertical" Approaches to Determine the Static Curve



## Fig. 45 Relaxation Curve for "Horizontal Approach"

-128





Fig. 47 Range of Error Using the "Vertical" Approach for Static Curve

-130





Fig. 48(b) Static Load-Deflection Curves of Columns (Continued)



Fig. 48(c) Static Load-Deflection Curves of Columns (Continued)



Fig. 48(d) Static Load-Deflection Curves of Columns (Continued)



### Fig. 49 Load Versus Net Mid-Height Deflection Curves of Columns

-135



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