1970

Analysis of shingle joints, June 1970.

Suresh Desai

J. W. Fisher

LEHIGH UNIVERSITY

OFFICE OF RESEARCH

ANALYSIS OF SHINGLE JOINTS

by

Suresh Desai

J. W. Fisher

Fritz Engineering Laboratory Report No. 340.5

SUMMARY

Although shingle joints have been extensively used for a long time, particularly in bridges, it is only recently that systematic effort has been directed towards their study. Mathematical models have been suggested for predicting load partition for the linear elastic range and their validity demonstrated through tests on large full scale shingle joints. However, no analytical solution is so far available for the inelastic range.

In the present study, a mathematical model for the shingle joint in bearing is developed and used to predict the complete force-displacement relationship up to the ultimate load. Earlier tests indicated that it is reasonable to assume that the various plies of the main and the lap plates act as a unit and that the transfer of load takes place by shear on two planes only. Using this assumption and the force-displacement relationship for the plate with holes and for the fasteners, equilibrium and compatibility equations are written down in essentially the same form as for the butt joints. This set of non-linear simultaneous equations are solved by iteration on the computer.

A comparison with the results of two tests indicates good agreement. The maximum error between the ultimate load computed from the program and the results of the modified joints is 8.5%. The error is attributed to (i) Ignoring the influence of the transverse stresses in the wide test joints, (ii) Uncertainty of the value of the ultimate deformation of the fastener Δ_{ult}, which has a considerable influence on the value of ultimate load when this load is reached through fastener failure. A series of tests is now under way to examine the validity of the suggested analysis for a wide range of parameters.

ANALYSIS OF SHINGLE JOINTS

by

Suresh Desai

J. W. Fisher

State Project No. 736-01-21

This research was conducted by
Fritz Engineering Laboratory
Lehigh University
for
LOUISIANA DEPARTMENT OF HIGHWAYS
Research and Development Section
In Cooperation with
U. S. Department of Transportation
FEDERAL HIGHWAY ADMINISTRATION

Fritz Engineering Laboratory

Department of Civil Engineering

Lehigh University

Bethlehem, Pennsylvania

June 1970

Fritz Engineering Laboratory Report No. 340.5

ABSTRACT

This report presents the development of a mathematical model for the solution of shingle joints loaded into the inelastic range. After making simplifying assumptions in accordance with the available test data on shingle joints, the analytical model developed earlier for the butt joint is generalized and extended to obtain load-partition in shingle joints loaded beyond major slip. A computer program has been written which provides complete force-displacement relationships for both the plates and the fasteners for the entire range of loading of a bearing type joint. For most shingle joints encountered in practice, this program can be used to predict load-partition beyond major slip when fasteners are in bearing and shear so that the transfer of load by friction may be ignored. A comparison with the available test data indicates agreement within 8.5% of the analytical solution.

SUMMARY

Although shingle joints have been extensively used for a long time, particularly in bridges, it is only recently that systematic effort has been directed towards their study. Mathematical models have been suggested for predicting load partition for the linear elastic range and their validity demonstrated through tests on large full scale shingle joints. However, no analytical solution is so far available for the inelastic range.

In the present study, a mathematical model for the shingle joint in bearing is developed and used to predict the complete force-displacement relationship up to the ultimate load. Earlier tests indicated that it is reasonable to assume that the various plies of the main and the lap plates act as a unit and that the transfer of load takes place by shear on two planes only. Using this assumption and the force-displacement relationship for the plate with holes and for the fasteners, equilibrium and compatibility equations are written down in essentially the same form as for the butt joints. This set of non-linear simultaneous equations are solved by iteration on the computer.

A comparison with the results of two tests indicates good agreement. A series of tests is now under way to examine the validity of the suggested analysis for a wide range of parameters.

IMPLEMENTATION STATEMENT

Recent tests on shingle joints have already revealed the inadequacy of the existing procedures for the analysis and design of shingle joints. A clear need is, therefore, indicated for better procedures to solve this problem.

Once the validity of the model suggested in this report has been demonstrated thru a series of tests now under way, design criteria will be developed based on the analysis of a large number of shingle joints with different parameters. These criteria will provide a more rational basis for the analysis and design of shingle joints and will ensure a better utilization of material in a joint resulting in economy.

TABLE OF CONTENTS

1. INTRODUCTION

Although shingle joints have been extensively used for a long time, particularly in bridges, it is only recently that systematic effort has been directed towards their study. Mathematical models have been suggested for predicting load partition for the linear elastic range and their validity demonstrated through tests on large full scale shingle joints.[1,2] However, no analytical solution is so far available for the inelastic range.

In the present study, a mathematical model for the shingle joint in bearing is developed and used to predict the complete force-displacement relationship up to the ultimate load. This mathematical model is a generalization of the model developed earlier for the butt joints.[3,4] This generalization is achieved through certain simplifying assumptions regarding the joint behavior so that butt joints constitute only a special class of shingle joints. Using the force-displacement relationship for the plate with holes and for the fasteners, equilibrium and compatibility equations are written down in essentially the same form as for the butt joints. This set of non-linear simultaneous equations are solved by iteration on the computer.

2. ANALYTICAL SOLUTION

Analytical expressions describing the force-displacement relationship for perforated plates of uniform width and for various types of fasteners have been developed from an extensive series of tests.[3,4] Using these relationships, a mathematical model was developed and used successfully to predict load-partition in butt joints.[3] This model is extended here for the more general case of shingle joints. Thus, the butt joint model constitutes only a special case of the generalized mathematical model for shingle joints.

The analytical solution is based on the following major assumptions. Some of these assumptions formed a basis of the butt joint model and have already been discussed at length.[3] Other assumptions characteristic only of the generalized model were partly the result of observations made on the large test joints.[1,2]

1. The analysis is essentially developed for joints containing only one longitudinal line of fasteners with all the holes of the same diameter and lying on one straight line as illustrated in Fig. 1. However, wider joints with more than one longitudinal line of fasteners may be assumed to be cut into longitudinal slices each of which may be analyzed independently provided of course, that the fasteners are uniformly distributed and that none of these strips violate any of the assumptions listed here.

2. Each individual ply of the joint is of homogeneous material and of uniform width. Thus plies of different width or with different mechanical properties are admissible.

3. The transfer of load between the lap plate and the main plate takes place only on the two planes common to the lap and the main plate as illustrated in Fig. 2. Thus no relative movement between the various plies of the lap plate or between the various plies of the main plate is considered. Each segment of the lap plate or the main plate between consecutive fasteners is assumed to function as a unit with properties which are aggregate of the constituent plies.

4. The fasteners transmit most of the applied load by shear and bearing. The frictional forces if present, are treated as shear and bearing. As already pointed out, this assumption is valid for real joints with some clearance in the holes, at the high loads subsequent to the major slip, which is really the area of interest in this study.[3] At the critical sections very little frictional force exists because of the inelastic fastener deformations and separation of the plies.

5. The top and bottom lap plates are combined into a single plate of variable thickness similar in appearance to the main plate. The average fastener characteristics for this combined condition are also used. This idealization results in regions of variable length which has uniform plate properties within each region.

The force-displacement relationship for plies of uniform width as well as for the fasteners are those empirically developed in Ref. 4.

Once the force-displacement characteristics of the plate segments and the fasteners are available, the solution of the problem can be obtained by any of the numerous approaches of structural analysis. However, the algorithm developed below is particularly convenient for shingle joints and has been also adopted in the computer program.

Fig. 3 illustrates the idealized transfer of load between the main and the lap plates through the fasteners in accordance with the assumptions outlined earlier. Fasteners 1 to n are numbered from left and the plate segments 1 to n+1 are numbered such that the ith segment lies to the left of the ith fastener. P_i and Q_i are respectively forces in the ith main plate segment and ith lap plate segment and R_i is the shear force in the ith fastener. The corresponding displacements are denoted by p_i, q_i and Δ_i respectively.

The equilibrium equations can now be written down as

$$P_{i+1} = P_i - R_i \qquad \text{for } i = 1, n-1 \qquad (1)$$

$$Q_{i+1} = Q_i + R_i \qquad \text{for } i - 1, n-1 \qquad (2)$$

$$\text{and } P_G = \sum_{i=1}^{n} R_i \qquad (3)$$

The compatibility condition can be written for each pair of adjacent segments as

$$\Delta_i + q_{i+1} = \Delta_{i+1} + p_{i+1}$$

$$\text{or} \quad f(R_i) + \Psi(Q_i) = f(R_{i+1}) + \phi(Q_{i+1})$$

$$\text{for } i = 1, n-1 \qquad (4)$$

The number of variables in the above equations is 3n-1 which consists of 2n-2 plate forces, n fastener forces and the applied load P_G.(3n-2) equations are provided by the set (1) to (4). One more equation is provided by the nature of the problem. If the solution at a load less than the ultimate load is desired, P_G is given. If the ultimate load is to be determined, depending on the mode of failure either P_G is known as the ultimate load of the plate section or R_1 is known as the ultimate load of the fastener.

Solution to this set of non-linear equations was obtained on the computer. The details of the computer program are described in an appendix which appears at the end of this report.

3. COMPARISON WITH TEST RESULTS

The computer program was run with the geometry and the experimentally determined properties of the plate material and the fasteners of the modified joints.[1,2] Strips of plates containing only a single row of fasteners were analyzed and the following results were obtained. In all cases, the end fasteners failed.

Modified Joint	Test Load Kips	Computed Load, Kips	Error
Bolted	3,545	3,683	+ 3.9
Riveted	2,800	3,102	+ 8.5

A comparison of the analytical predictions and the test results of load partition in the main and lap plates of the modified bolted joint is shown in Fig. 4A and 4B. Results for the modified riveted joint are shown in Fig. 5A and 5B. The analytical results indicate a good agreement with the test results throughout the entire length of the joint despite all the simplifying assumptions of the mathematical model.

The discrepancy between the analytical prediction and test results could be attributed to several factors.

1. Some uncertainty was introduced by the stagger in the fastener pattern on the joints which necessitated additional assumptions about the cross sectional properties of the strips.

2. The transverse stresses which are present in wider

joints are not taken into account in the mathematical model. In the joints tested, the contribution of transverse stresses was probably more significant due to the stagger of the fasteners.

3. The assumption that both the shear planes are critical is not quite accurate and this results in fastener failure before the full resistance is developed.

4. There is some uncertainty about the value of the ultimate deformation Δ_{ult} of the fastener. The ultimate load for fastener failure is quite sensitive to this value and the value used in analysis may be different from that attained in the tests. The greater discrepancy in the case of riveted joint may thus be attributed to the greater variation of Δ_{ult} for the rivets.

5. An examination of Figs. 4A, 4B, 4C and 4D reveals that the greatest discrepancy in the fastener forces appears to occur between two consecutive fasteners between which a plate is discontinuous. Obviously treating a group of plates as one unit leads to a large error at such discontinuities.

It must be pointed out that the tests referred to above were of an exploratory character. A series of tests is planned to verify the validity of the proposed mathematical model for a wide range of parameters.

4. SUMMARY AND CONCLUSIONS

The mathematical model for shingle joints in bearing is essentially a generalization of the butt joint model developed by Fisher and Rumpf in Ref. 3. The force-displacement relationships for fasteners and for perforated plates of uniform width developed there are utilized in the generalized model. The only important additional assumption required was that the transfer of load occurs only on two planes as shown in Fig. 2 so that each segment of the main plate or the lap plate between consecutive fasteners function as a unit with properties which are aggregate of the constituent plies.

The maximum error between the ultimate load computed from the program and the results of the modified joints is 8.5%. The error is attributed to (i) Ignoring the influence of the transverse stresses in the wide test joints, (ii) Uncertainty of the value of the ultimate deformation of the fastener Δ_{ult}, which has a considerable influence on the value of ultimate load when this load is reached through fastener failure.

5. ACKNOWLEDGMENTS

This study has been carried out as a part of the research project on "Studies on Simulated Bridge Joints" being conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. Professor Lynn S. Beedle is Director of the Laboratory and Professor David A. VanHorn is Head of the Department.

The project is sponsored by the Louisiana Department of Highways in cooperation with the U. S. Department of Transportation - Bureau of Public Roads. Technical guidance has been provided by the Research Council on Riveted and Bolted Structural Joints through an advisory committee under the chairmanship of Mr. T. W. Spilman.

The help provided by Messrs. S. N. S. Iyengar, E. Power and C. Yilmaz is sincerely appreciated. Thanks are also extended to Mrs. Charlotte Yost for typing the manuscript and to Mr. Jack Gera for the drafting.

6. APPENDIX

The computer program solves by iteration the set of equations described in Chapter 2. The program can predict the ultimate load and the load-partition at the ultimate load or for any load smaller than the ultimate load.

In order to simplify programming, the following restrictions have been imposed.

(1) All the plies of the main plate are of uniform width and have identical material properties. Similar restrictions apply to the lap plate.

(2) All the fasteners have identical load-displacement relationships. However, with only small modifications it would be possible to take into account (i) fasteners with different properties (ii) fasteners in single shear and (iii) fasteners in multiple shear as in knife joints.

(3) Only the more generally encountered parameters will be considered. For example, the plate areas should change gradually. Referring to Fig. 2, main plate area should either decrease or stay constant and the lap plate area should either increase or stay constant while proceeding from left to right. Further, the ultimate strength of the first segment of the main plate should be less than or equal to the ultimate strength of the (n+1)th segment of the lap plate.

The program can be broken up into five principal segments:

(1) Read and compute the problem parameters.

(2) Determine whether the end fastener deformation or the applied load is to be held constant. For constant end fastener deformation, go to (3) and for constant applied load, go to (4)

(3) Iterate, holding end fastener deformation at its maximum value and change the applied load starting from its highest value which is the plate failure load. The failure here is in the end fastener. The corresponding load is the ultimate load.

(4) Iterate, holding the applied load constant and vary the deformation of the first fastener starting from its first value. This segment of the program also provides the solution when the failure is in the first segment of the main plate. This condition is determined from (2) and then the applied load is held constant at the value $\sigma_u A_n$ where σ_u is the ultimate stress and A_n is the net area in the first segment of the main plate.

(5) Output the results.

The operations are described in a logical flow chart. See Fig. 6. The following additional symbols are used.

P_{ult} = Ultimate load of the joint

P_i' = Ultimate load of the main plate in the i^{th} segment

P_m = Modified value for the applied load

Q_i' = Ultimate load of lap plate in the i^{th} segment

Δ_{ult} = Ultimate deformation of one fastener

Δ_m = Modified value for the deformation of the first fastener

ϵ = A small number to check convergence of the solution

The functional relationships between the forces P_i, Q_i and R_i and the displacements p_i, q_i and Δ_i are given by

$$p_i = \phi(P_i) \quad , \quad P_i = \phi'(p_i)$$

$$q_i = \Psi(Q_i) \quad , \quad Q_i = \Psi'(q_i)$$

$$\Delta_i = f(R_i) \quad , \quad R_i = f'(\Delta_i)$$

A complete listing of the program in Fortran is reproduced here.

```
      PROGRAM INELSH(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)
C     THIS PROGRAM ANALYZES BUTT JOINTS FOR THE ENTIRE RANGE OF ELASTIC
C     AND INELASTIC PHASES. ANY TYPE OF JOINT GEOMETRY CAN BE ANALYZED
C     INCLUDING VARIABLE PLATE AREAS SO THAT SHINGLE JOINTS CONSTITUTE
C     ONLY A SPECIAL CASE.
C     LIST OF ARRAYS USED IN THE PROGRAM
C     AGL : GROSS AREA OF THE LAP PLATE
C     AGM : GROSS AREA OF THE MAIN PLATE
C     AGRL: GROSS AREA OF THE LAP PLATE IN A REGION
C     AGRM: GROSS AREA OF THE MAIN PLATE IN A REGION
C     ANL : NET AREA OF THE LAP PLATE
C     ANM : NET AREA OF THE MAIN PLATE
C     ANRL: NET AREA OF THE LAP PLATE IN A REGION
C     ANRM: NET AREA OF THE MAIN PLATE IN A REGION
C     APL : APPLIED FORCE FOR WHICH THE JOINT IS BEING ANALYZED
C     DF  : DEFORMATION OF FASTENER
C     NFR : NUMBER OF FASTENERS IN A REGION
C     P   : PITCH
C     PR  : PITCH IN A REGION
C     PFL : FORCES IN THE LAP PLATE
C     PFM : FORCES IN THE MAIN PLATE
C     RF  : RESISTANCE OF FASTENER
C     SL  : STRAIN IN THE LAP PLATE
C     SM  : STRAIN IN THE MAIN PLATE
C     ULL : ULTIMATE LOAD OF THE LAP PLATE
C     ULM : ULTIMATE LOAD OF THE MAIN PLATE
C     YDL : YIELD DEFORMATION OF THE LAP PLATE
C     YDM : YIELD DEFORMATION OF THE MAIN PLATE
C     YLL : YIELD LOAD OF THE LAP PLATE
C     YLM : YIELD LOAD OF THE MAIN PLATE
C     OTHER SYMBOLS USED IN THE PROGRAM
C     ANAS : THE RATIO AN/AS
C     CHD  : CHANGE IN DEFORMATION AT THE END OF PREVIOUS ITERATION
C     CHL  : CHANGE IN LOAD AT THE END OF PREVIOUS ITERATION
C     DFF  : DEFORMATION OF THE FIRST FASTENER
C     DULT : ULTIMATE DEFORMATION OF THE FASTENER
C     EPS  : CONVERGENCE CRITERION
C     FA   : AREA OF THE FASTENER
C     FD   : DIAMETER OF THE FASTENER
C     FPL  : FASTENER PARAMETER LAMBDA
C     FPM  : FASTENER PARAMETER MU
C     G    : GAGE
C     HD   : DIAMETER OF THE HOLE
C     NF   : TOTAL NUMBER OF FASTENERS
C     NITER: MAXIMUM NUMBER OF ITERATIONS
C     NPZ  : TOTAL NUMBER OF PLATE ZONES=NF+1
C     NREG : NUMBER OF REGIONS
C     RULT : ULTIMATE RESISTANCE OF THE FASTENER
C     SRF  : SUM OF THE RESISTANCE OF THE FASTENERS
C     SUL  : ULTIMATE STRESS OF THE LAP PLATE
C     SUM  : ULTIMATE STRESS OF THE MAIN PLATE
C     TSA  : TOTAL SHEAR AREA
C     UNIP : UNIFORM PITCH
C     YM   : YOUNGS MODULUS
      COMMON/A1/ANM(50),AGM(50),ANL(50),AGL(50),YLM(50),YLL(50),APL(50)
      COMMON/A2/ULM(50),ULL(50),YDM(50),YDL(50),RF(50),DF(50)
      COMMON/A3/SM(50),SL(50),PFM(50),PFL(50),P(50),PDL(50),PDM(50)
```

```
      COMMON/A4/NFR(10),ANRM(10),AGRM(10),ANRL(10),AGRL(10)
      COMMON/A5/ELAS,PLAS,NITER,COEF,EPS,YM,IN,IO,NLOAD
      COMMON/A6/NF,NREG,SYM,SUM,SYL,SUL,G,FD,RULT,DULT,FPL,FPM,UNIP
      DATA ELAS,PLAS,FLAG,NITER,COEF,EPS,YM,IN,IO/2HE ,2HPL,1H*,50,
     .0.66667,1.0,30000.0,1, 2/
  100 READ(IN,11)NF,NREG,SYM,SUM,SYL,SUL,G,FD,RULT,DULT,FPL,FPM,UNIP
      IF(NF)200,200,110
  110 READ(IN,12)((NFR(I)),I=1,NREG),NLOAD
      READ(IN,13)((ANRM(I),AGRM(I),ANRL(I),AGRL(I)),I=1,NREG)
      IF(UNIP)120,120,130
  120 READ(IN,13)(P(I),I=2,NF)  $  GO TO 150
  130 DO 140 I=2,NF
  140 P(I)=UNIP
  150 IF(NLOAD-1)170,170,160
  160 READ(IN,13)((APL(I)),I=2,NLOAD)  $  GO TO 180
  170 NLOAD=1
  180 CONTINUE
      CALL SHING
      GO TO 100
  200 CONTINUE
   11 FORMAT(2I5,14F5.0)
   12 FORMAT(16I5)
   13 FORMAT(10F8.0)
      CALL EXIT
      END
      SUBROUTINE SHING
      COMMON/A1/ANM(50),AGM(50),ANL(50),AGL(50),YLM(50),YLL(50),APL(50)
      COMMON/A2/ULM(50),ULL(50),YDM(50),YDL(50),RF(50),DF(50)
      COMMON/A3/SM(50),SL(50),PFM(50),PFL(50),P(50),PDL(50),PDM(50)
      COMMON/A4/NFR(10),ANRM(10),AGRM(10),ANRL(10),AGRL(10)
      COMMON/A5/ELAS,PLAS,NITER,COEF,EPS,YM,IN,IO,NLOAD
      COMMON/A6/NF,NREG,SYM,SUM,SYL,SUL,G,FD,RULT,DULT,FPL,FPM,UNIP
      FR(X)=RULT*(1.0-EXP(-FPM*X))**FPL
      PSM(X)=-X*HM*ALOG(1.0-(STR-SYM)/SDM)
      PSL(X)=-X*HL*ALOG(1.0-(STR-SYL)/SDL)
      RNF=NF  $  FA=3.14159265*FD*FD/4.  $  TSA=RNF*FA*2.  $  NPZ=NF+1
      HD=FD+0.0625  $  SDM=SUM-SYM  $  SDL=SUL-SYL  $  TEM=(G-HD)/G
      HM=TEM/SDM  $  HL=TEM/SDL  $  K=0  $  NFM1=NF-1
      DO 110 I=1,NREG
      NFI=NFR(I)  $  ANMI=ANRM(I)  $  AGMI=AGRM(I)  $  ANLI=ANRL(I)
      AGLI=AGRL(I)  $  ULMI=SUM*ANMI  $  ULLI=SUL*ANLI
      YLMI=SYM*ANMI  $  YLLI=SYL*ANLI
      DO 100 JJ=1,NFI  $  J=K+JJ  $  JP=J+1
      ULM(J)=ULMI  $  ULL(JP)=ULLI  $  YLM(J)=YLMI  $  YLL(JP)=YLLI
      ANM(J)=ANMI  $  AGM(J)=AGMI  $  ANL(JP)=ANLI  $  AGL(JP)=AGLI
  100 CONTINUE
  110 K=K+NFI  $  TEM=SYM/YM  $  TEL=SYL/YM
      DO 115 I=2,NF  $  YDM(I)=P(I)*TEM  $  YDL(I)=P(I)*TEL
  115 CONTINUE
      ANM(NPZ)=AGM(NPZ)=ANL(1)=AGL(1)=ULM(NPZ)=ULL(1)=YLL(1)=YDM(1)=0.0
      YDM(NPZ)=YDL(1)=YDL(NPZ)=P(1)=P(NPZ)=0.0  $  YLM(NPZ)=YLM(NF)
      IF(ULM(1)-ULL(NPZ))120,120,116
  116 NPZ2=NPZ/2
      DO 118 I=1,NPZ2  $  J=NPZ-I
      TEM= P(I)  $   P(I)= P(J)  $   P(J)=TEM
      TEM=ANM(I)  $  ANM(I)=ANM(J)  $  ANM(J)=TEM
      TEM=ANL(I)  $  ANL(I)=ANL(J)  $  ANL(J)=TEM
      TEM=AGM(I)  $  AGM(I)=AGM(J)  $  AGM(J)=TEM
      TEM=AGL(I)  $  AGL(I)=AGL(J)  $  AGL(J)=TEM
      TEM=ULL(I)  $  ULL(I)=ULL(J)  $  ULL(J)=TEM
      TEM=ULM(I)  $  ULM(I)=ULM(J)  $  ULM(J)=TEM
      TEM=YLM(I)  $  YLM(I)=YLM(J)  $  YLM(J)=TEM
      TEM=YDM(I)  $  YDM(I)=YDM(J)  $  YDM(J)=TEM
      TEM=YLL(I)  $  YLL(I)=YLL(J)  $  YLL(J)=TEM
      TEM=YDL(I)  $  YDL(I)=YDL(J)  $  YDL(J)=TEM
  118 CONTINUE
```

```
  120 WRITE(IO,10)   $   WRITE(IO,14)
       ANAS=ANM(1)/TSA   $   ALOAD=AMAX=ULM(1)   $   AMIN=0.
      WRITE(IO,15)NF,NREG,FD,FA,RULT,DULT,FPL,FPM
      WRITE(IO,21)   $   WRITE(IO,16)
      WRITE(IO,17)SUM,SYM,SUL,SYL,G,ANAS
      WRITE(IO,21)   $   WRITE(IO,19)
      WRITE(IO,20)((I,AGRM(I),ANRM(I),AGRL(I),ANRL(I)),I=1,NREG)
      WRITE(IO,21)
      WRITE(IO,31)P(1),ULM(1),ULL(1),
     .((I,P(I+1),ULM(I+1),ULL(I+1)),I=1,NF)

C     CALCULATING PLATE AND FASTENER FORCES

  130 DO 900 ILOAD=1,NLOAD
      DMAX=DFF=DULT   $   DMIN=0.   $   ITER=0
      IF(ILOAD-1)204,204,134
  134 ALOAD=APL(ILOAD)
      IF(ALOAD-ULTL)402,136,136
  136 WRITE(IO,26)ALOAD   $   GO TO 900
  204 EPS=0.001*ALOAD   $   DO 206 I=1,NPZ
  206 SM(I)=SL(I)=ELAS
      PFM(1)=ALOAD   $   DF(1)=DFF
      DO 242 I=1,NFM1   $   J=I+1
      RF(I)=FR(DF(I))   $   PFM(J)=PFM(I)-RF(I)   $   PFL(J)=PFL(I)+RF(I)
  208 IF(PFM(J))462,462,210
  210 IF(PFM(J)-YLM(J))212,212,214
  212 PDM(J)=YDM(J)*PFM(J)/YLM(J)   $   GO TO 218
  214 IF(PFM(J)-ULM(J))216,368,368
  216 STR=PFM(J)/ANM(J)   $   PDM(J)=YDM(J)+PSM(P(J))
  218 IF(PFL(J)-ULL(J))220,462,462
  220 IF(PFL(J)-YLL(J))222,222,224
  222 PDL(J)=YDL(J)*PFL(J)/YLL(J)   $   GO TO 226
  224 STR=PFL(J)/ANL(J)   $   PDL(J)=YDL(J)+PSL(P(J))
  226 DF(J)=DF(I)-PDM(J)+PDL(J)   $   IF(DF(J))368,368,228
  228 IF(DF(J)-DULT)240,240,462
  240 CONTINUE
  242 CONTINUE
      RF(NF)=FR(DF(NF))$PFM(NPZ)=PFM(NF)-RF(NF)$PFL(NPZ)=PFL(NF)+RF(NF)
      ITER=ITER+1   $   IF(PFM(NPZ))452,504,368
  302 IF(ITER-NITER)304,304,502
  304 DO 306 I=1,NPZ
  306 RF(I)=DF(I)=PDM(I)=PDL(I)=PFM(I)=PFL(I)=0.0
      ITER=ITER+1
      PFM(1)=ALOAD   $   DF(1)=DFF
      DO 342 I=1,NFM1   $   J=I+1
      RF(I)=FR(DF(I))   $   PFM(J)=PFM(I)-RF(I)   $   PFL(J)=PFL(I)+RF(I)
  308 IF(PFM(J))362,362,310
  310 IF(PFM(J)-YLM(J))312,312,314
  312 PDM(J)=YDM(J)*PFM(J)/YLM(J)   $   GO TO 318
  314 IF(PFM(J)-ULM(J))316,368,368
  316 STR=PFM(J)/ANM(J)   $   PDM(J)=YDM(J)+PSM(P(J))
  318 IF(PFL(J)-ULL(J))320,362,362
  320 IF(PFL(J)-YLL(J))322,322,324
  322 PDL(J)=YDL(J)*PFL(J)/YLL(J)   $   GO TO 326
  324 STR=PFL(J)/ANL(J)   $   PDL(J)=YDL(J)+PSL(P(J))
  326 DF(J)=DF(I)-PDM(J)+PDL(J)   $   IF(DF(J))368,368,328
  328 IF(DF(J)-DULT)340,340,362
  340 CONTINUE
  342 CONTINUE
      RF(NF)=FR(DF(NF))$PFM(NPZ)=PFM(NF)-RF(NF)$PFL(NPZ)=PFL(NF)+RF(NF)
      RES=PFM(NPZ)   $   ARES=ABS(RES)
      IF(ARES-EPS)504,504,360
  360 IF(RES)362,504,368
  362 AMIN=ALOAD   $   GO TO 370
  368 AMAX=ALOAD
  370 ALOAD=(AMAX+AMIN)/2.   $   GO TO 302
```

```
402 IF(ITER-NITER)404,404,502
404 DO 406 I=1,NPZ
406 RF(I)=DF(I)=PDM(I)=PDL(I)=PFM(I)=PFL(I)=0.0
    ITER=ITER+1
    PFM(1)=ALOAD  $  DF(1)=DFF
    DO 442 I=1,NFM1  $  J=I+1
    RF(I)=FR(DF(I))  $  PFM(J)=PFM(I)-RF(I)  $  PFL(J)=PFL(I)+RF(I)
408 IF(PFM(J))462,462,410
410 IF(PFM(J)-YLM(J))412,412,414
412 PDM(J)=YDM(J)*PFM(J)/YLM(J)  $  GO TO 418
414 IF(PFM(J)-ULM(J))416,468,468
416 STR=PFM(J)/ANM(J)  $  PDM(J)=YDM(J)+PSM(P(J))
418 IF(PFL(J)-ULL(J))420,462,462
420 IF(PFL(J)-YLL(J))422,422,424
422 PDL(J)=YDL(J)*PFL(J)/YLL(J)  $  GO TO 426
424 STR=PFL(J)/ANL(J)  $  PDL(J)=YDL(J)+PSL(P(J))
426 DF(J)=DF(I)-PDM(J)+PDL(J)  $  IF(DF(J))468,468,428
428 IF(DF(J)-DULT)440,440,462
440 CONTINUE
442 CONTINUE
    RF(NF)=FR(DF(NF))$PFM(NPZ)=PFM(NF)-RF(NF)$PFL(NPZ)=PFL(NF)+RF(NF)
    RES=PFM(NPZ)  $  ARES=ABS(RES)
    IF(ARES-EPS)508,508,460
460 IF(RES)462,508,468
462 DMAX=DFF  $  GO TO 470
468 DMIN=DFF
470 DFF=(DMAX+DMIN)/2.  $  GO TO 402

C   OUTPUT OF RESULTS

502 WRITE(IO,10)  $  WRITE(IO,25)  $  GO TO 508
504 ULTL=ALOAD
508 WRITE(IO,10)
    DO 514 I=1,NF
    IF(PFM(I)-YLM(I))514,514,512
512 SM(I)=PLAS
514 CONTINUE
    DO 520 I=2,NPZ
    IF(PFL(I)-YLL(I))520,520,518
518 SL(I)=PLAS
520 CONTINUE
    WRITE(IO,22)
    DO 530 I=1,NF
    ASS=RF(I)/2./FA
    WRITE(IO,24)PFM(I),PFL(I),PDM(I),SM(I),PDL(I),SL(I)
530 WRITE(IO,23)I,RF(I),DF(I),ASS
    WRITE(IO,24)PFM(NPZ),PFL(NPZ),PDM(NPZ),SM(NPZ),PDL(NPZ),SL(NPZ)
    K=1
    WRITE(IO,27)
    DO 550 I=1,NREG
    NFI=NFR(I)+K-1  $  TSR=0.0  $  TEM=NFR(I)
    DO 540 J=K,NFI
540 TSR=TSR+RF(J)
    AFF=TSR/TEM  $  AFS=AFF/2.0  $  ASS=AFS/FA
    WRITE(IO,29)I,NFR(I),AFF,AFS,ASS
550 K=NFI
    AFF=(ALOAD-RES)/RNF  $  AFS=AFF/2.0  $  ASS=AFS/FA
    WRITE(IO,30)NF,AFF,AFS,ASS
    WRITE(IO,28) ITER
900 CONTINUE
 10 FORMAT(1H1)
 14 FORMAT(10X,*FASTENER DATA*,//,10X,*NUMBER  REGIONS  DIAMETER*,
   13X,*AREA    ULTIMATE   ULTIMATE    LAMBDA    MU*,/,29X,*IN.*,
   25X,*SQ.IN.   STRENGTH  DEFORMATION*,/,48X,*KIPS*,8X,*IN.*)
 15 FORMAT(I14,I8,F12.3,F9.3,F9.1,F12.3,F10.2,F8.2)
 16 FORMAT(10X,*PLATE DATA*,//,10X,
```

```
     1*MAIN PLATE  MAIN PLATE  LAP PLATE  *,
     2*LAP PLATE  GAGE  RATIO*,/,11X,*ULTIMATE     YIELD     ULTIMATE*,
     35X,*YIELD     IN.    AN/AS*,/,11X,8HSTRENGTH,4X,3(8HSTRENGTH,3X),
     4 /,13X,*KSI*,9X,*KSI*,9X,*KSI*,8X,*KSI*)
17   FORMAT(F17.1,2F12.1,F11.1,F8.1,F7.3)
19   FORMAT(10X,*REGION  *,2(10HMAIN PLATE,2X),2(9HLAP PLATE,3X)/
    .18X,2(24HGROSS AREA    NET AREA    )/19X,4(7HSQ. IN.,5X))
20   FORMAT(I14,F11.2,3F12.2)
21   FORMAT(///)
22   FORMAT(10X,*FORCES AND DEFORMATIONS IN PLATES AND FASTENERS*,//,
    .10X,2(10HFASTENER  ),*MAIN PLATE  LAP PLATE    FASTENER     *,
    .*MAIN PLATE    LAP PLATE    FASTENER*/11X,6HNUMBER,4X,2(5HFORCE,6X
    .),8H FORCE   ,3(13H  DEFORMATION),4X,6HSTRESS/22X,3(4HKIPS,7X),
    .2X,3(3HIN.,10X),3HKSI/)
23   FORMAT(I15,F12.2,23X,F12.6,30X,F8.2)
24   FORMAT(27X,F11.2,F12.2,14X,2(F10.6,2X,A2))
25   FORMAT(10X,*NO CONVERGENCE. RESULTS OF LAST ITERATION--*//)
26   FORMAT(10X,*APPLIED LOAD OF*,F9.2,* KIPS EXCEEDS ULTIMATE LOAD*)
27   FORMAT(//,10X,*REGION  NUMBER OF  *,2(16HAVERAGE FASTENER,2X),
    .13HAVERAGE SHEAR,/,18X,*FASTENERS     FORCE KIPS        *,
    .*SHEAR KIPS        STRESS  KSI*)
28   FORMAT(//10X,*E--ELASTIC  PL--PLASTIC  NO.OF ITERATIONS=*,I4)
29   FORMAT(I14,I9,2F17.2,F16.2)
30   FORMAT(/,10X,*COMPLETE*,/,11X,*JOINT*,I7,2F17.2,F16.2)
31   FORMAT(10X,16HFASTENER    PITCH,2(16H    ULTIMATE LOAD)/11X,
    .45HNUMBER       IN.       MAIN  PLATE        LAP PLATE/33X,2(4HKIPS,12X)
    ./F26.3,F13.2,F16.2/(I15/F26.3,F13.2,F16.2))
     RETURN
     END
```

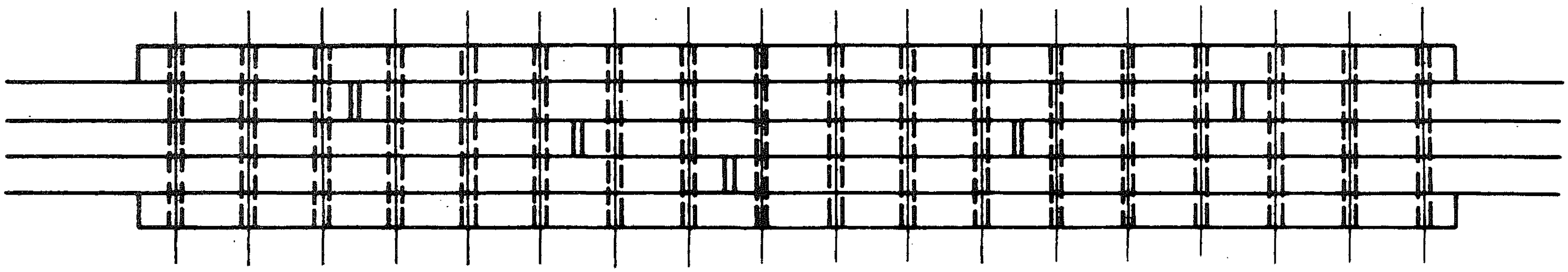


FIG. 1: A TYPICAL SHINGLE JOINT

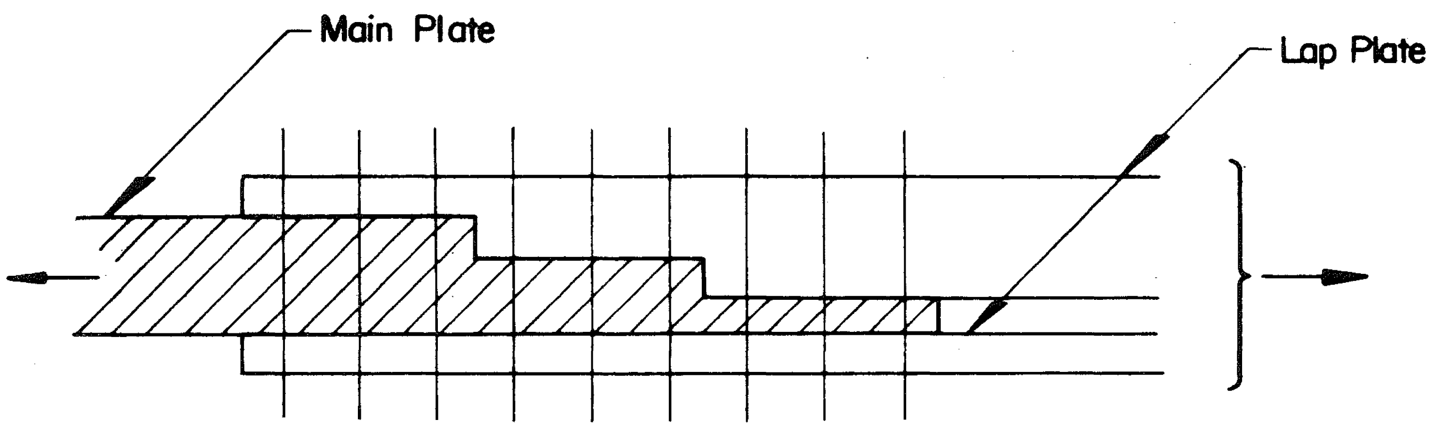
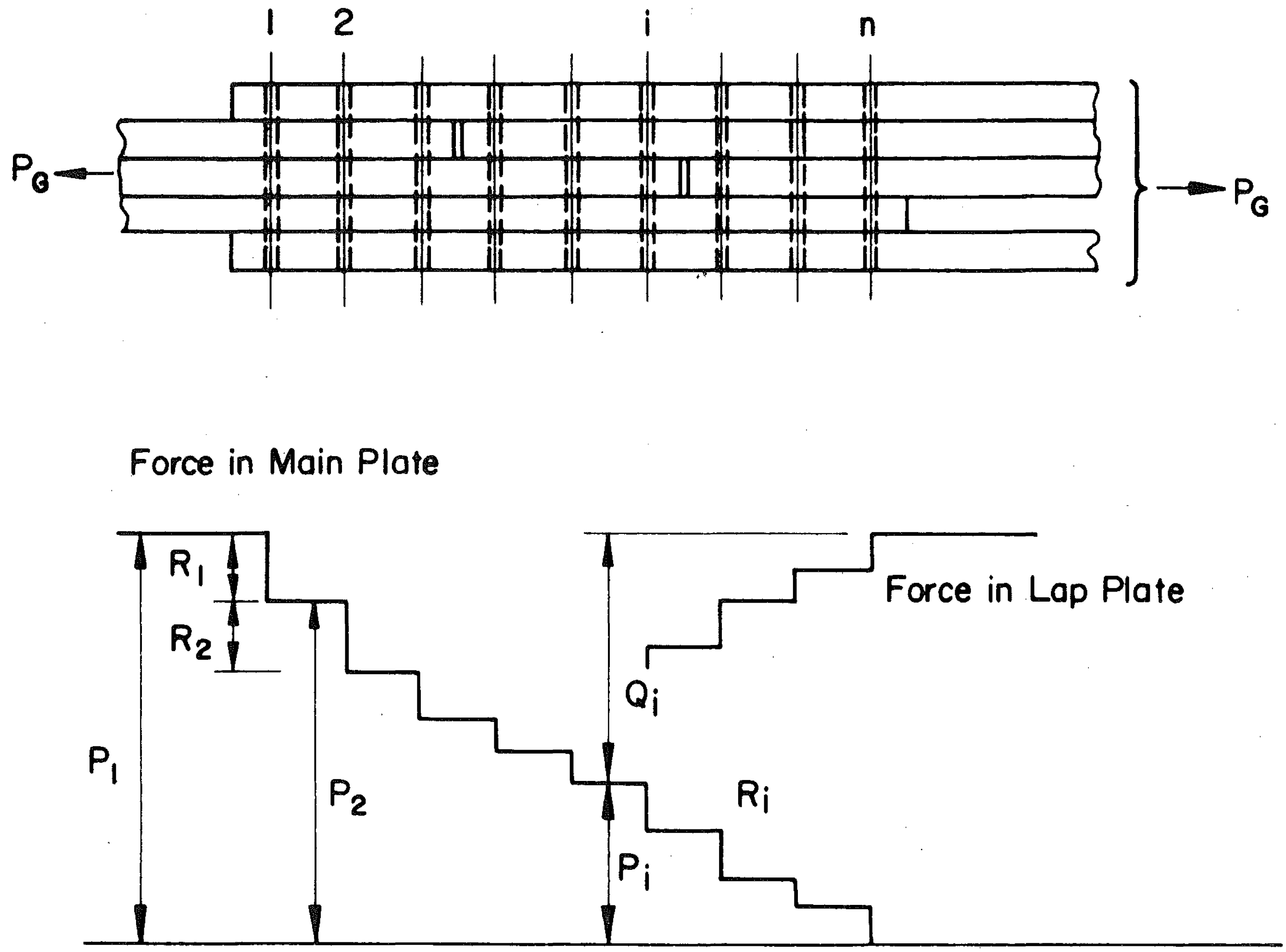


FIG. 2: ASSUMED SHEAR PLANES



IDEALIZED LOAD TRANSFER DIAGRAM

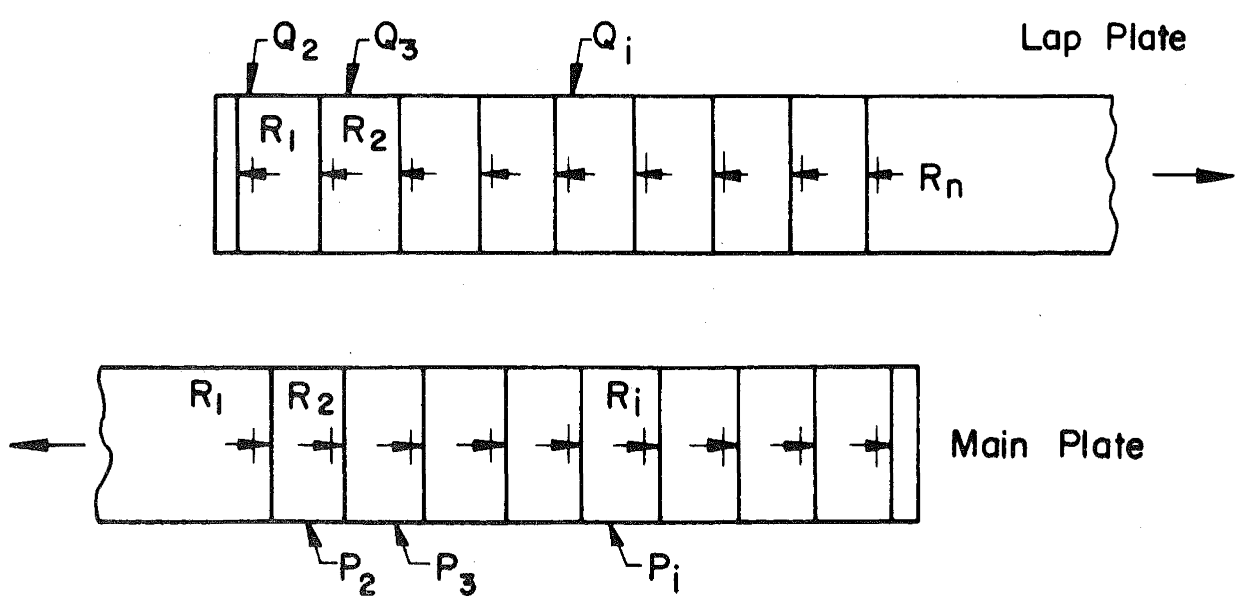


FIG. 3: IDEALIZED LOAD TRANSFER

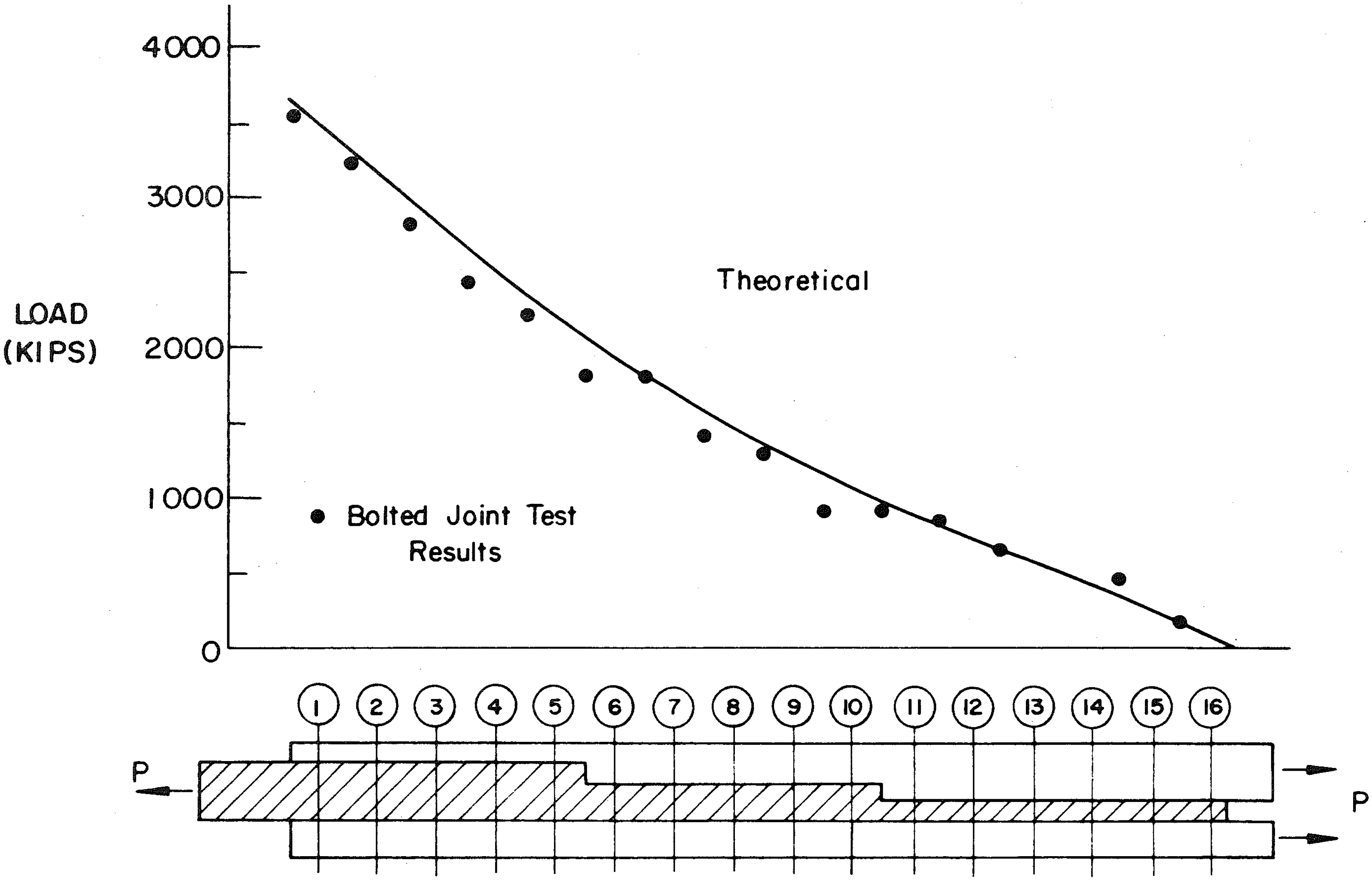


FIG. 4A LOAD PARTITION IN THE MAIN PLATE OF THE BOLTED JOINT
AT ULTIMATE LOAD

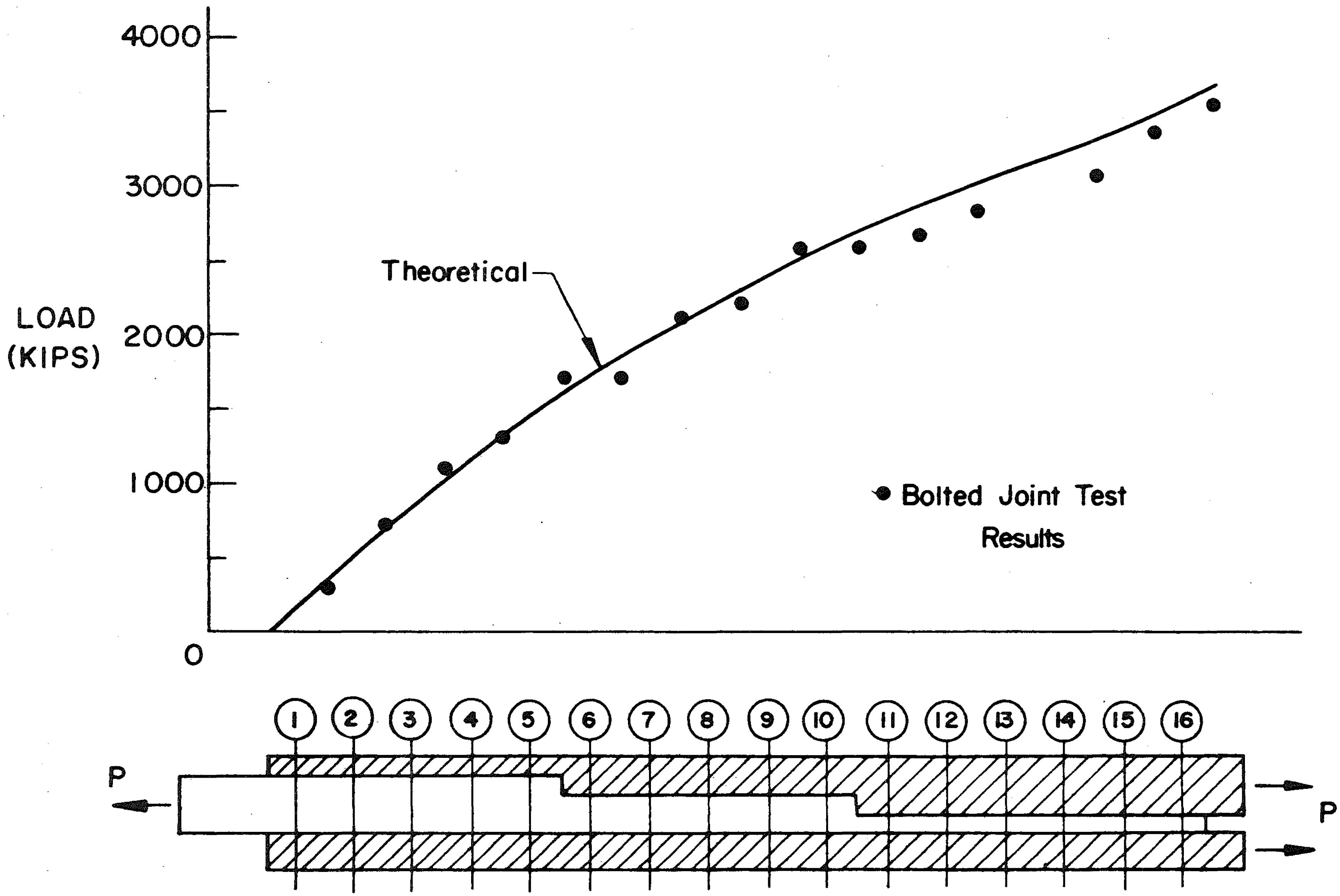


FIG. 4B LOAD PARTITION IN THE LAP PLATE OF THE BOLTED JOINT
AT ULTIMATE LOAD

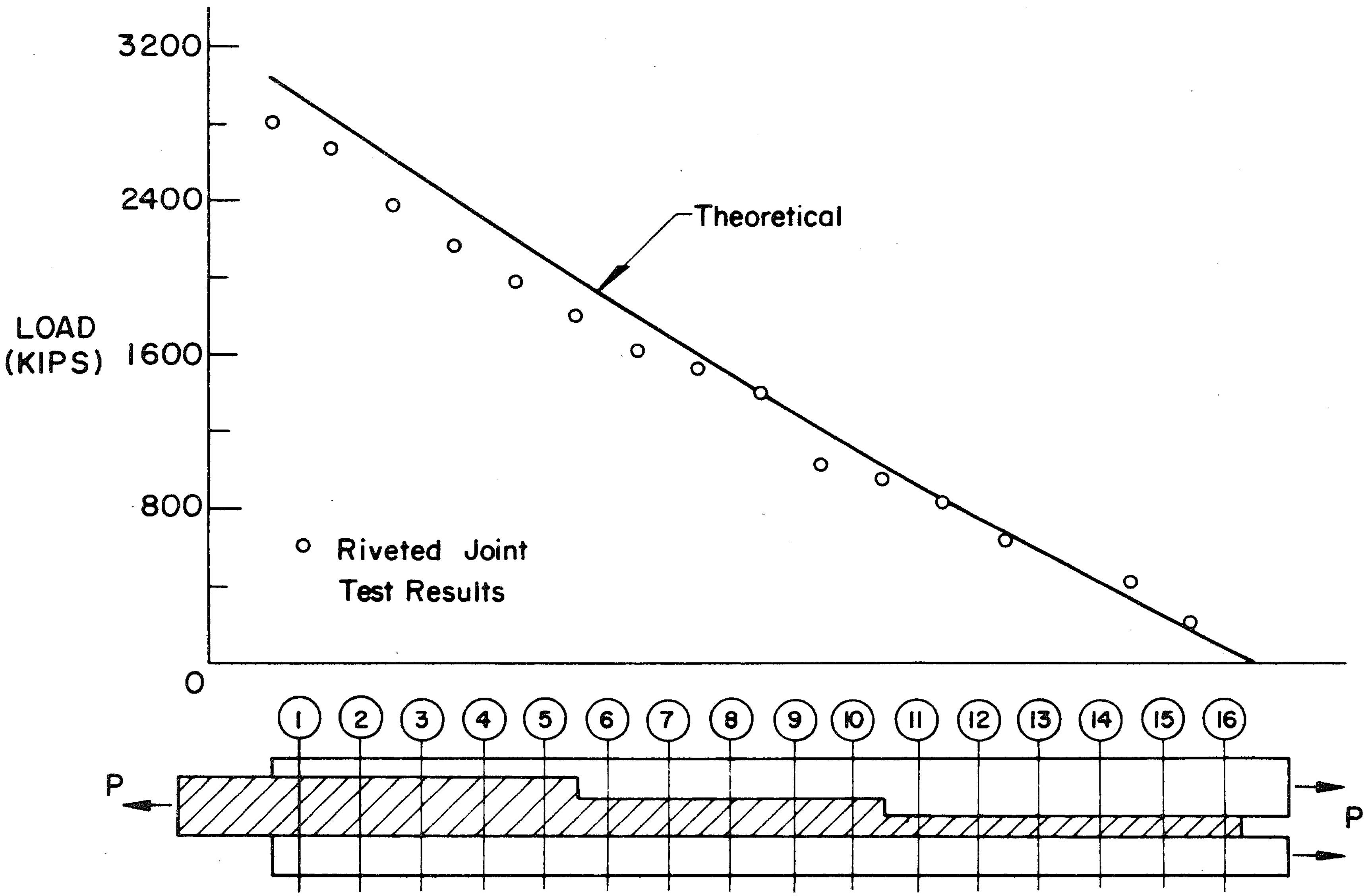


FIG. 5A LOAD PARTITION IN THE MAIN PLATE OF THE RIVETED JOINT AT ULTIMATE LOAD

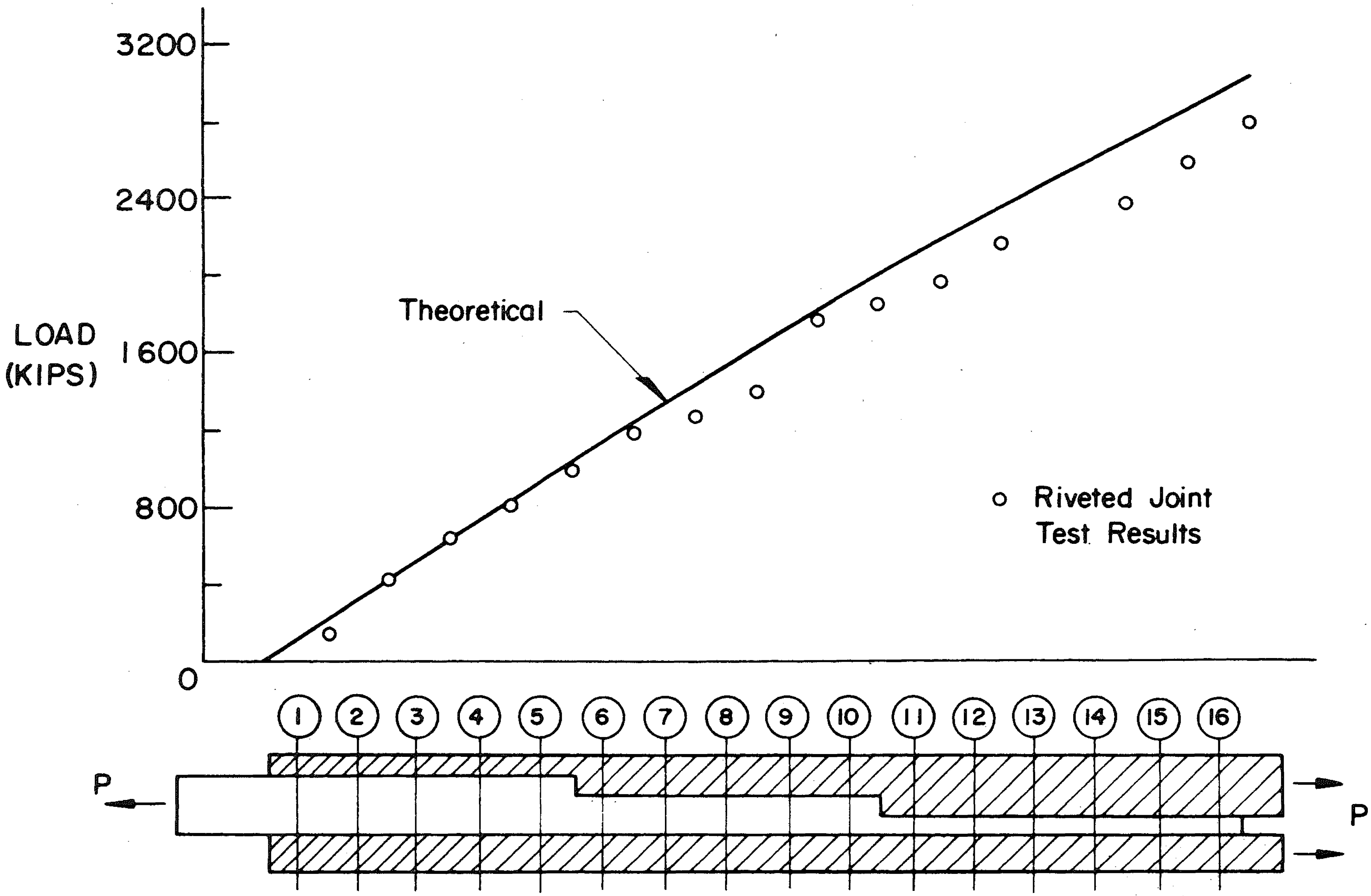


FIG. 5B LOAD PARTITION IN THE LAP PLATE OF THE RIVETED JOINT
AT ULTIMATE LOAD

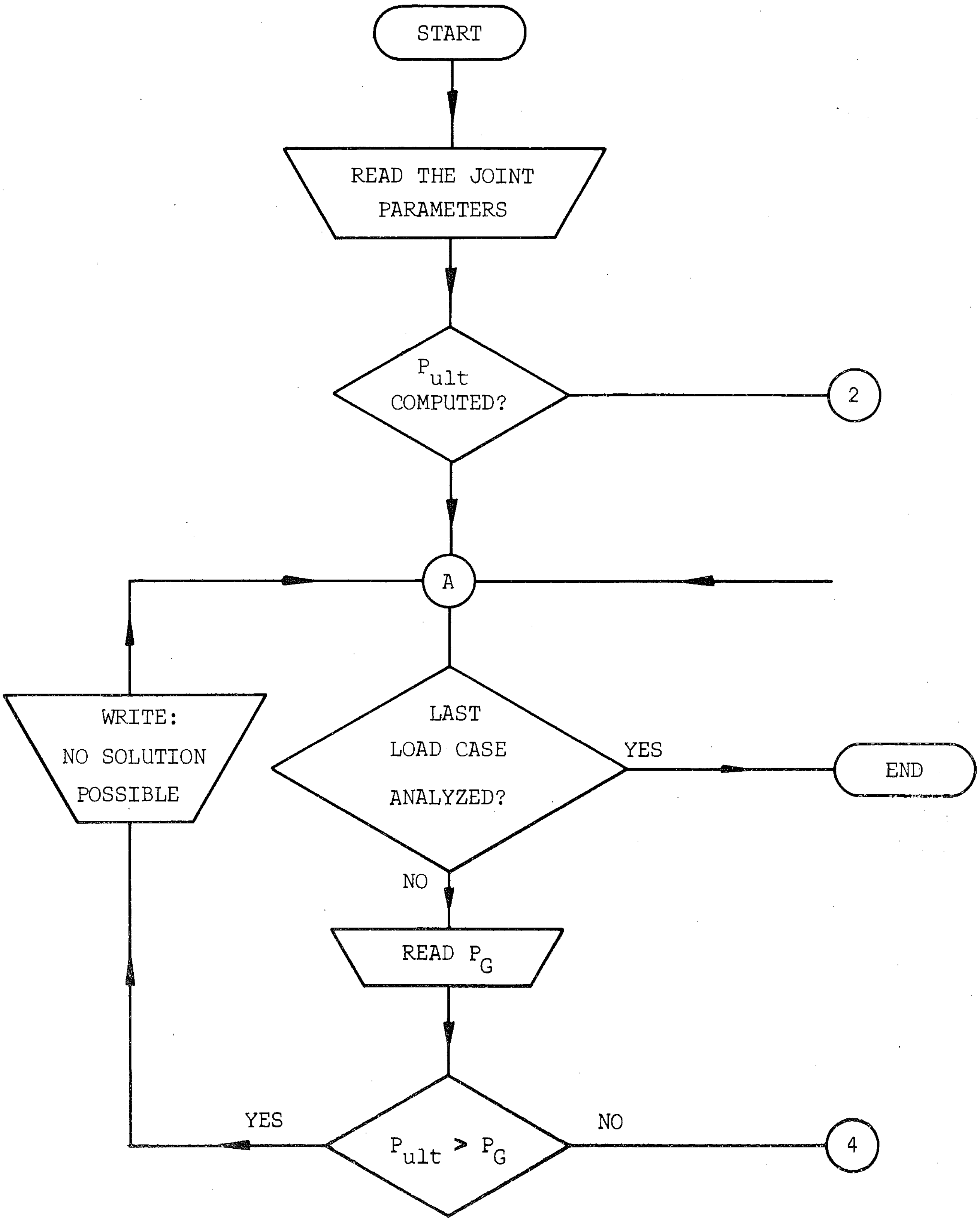


FIG. 6A FLOW CHART FOR THE COMPUTER PROGRAM

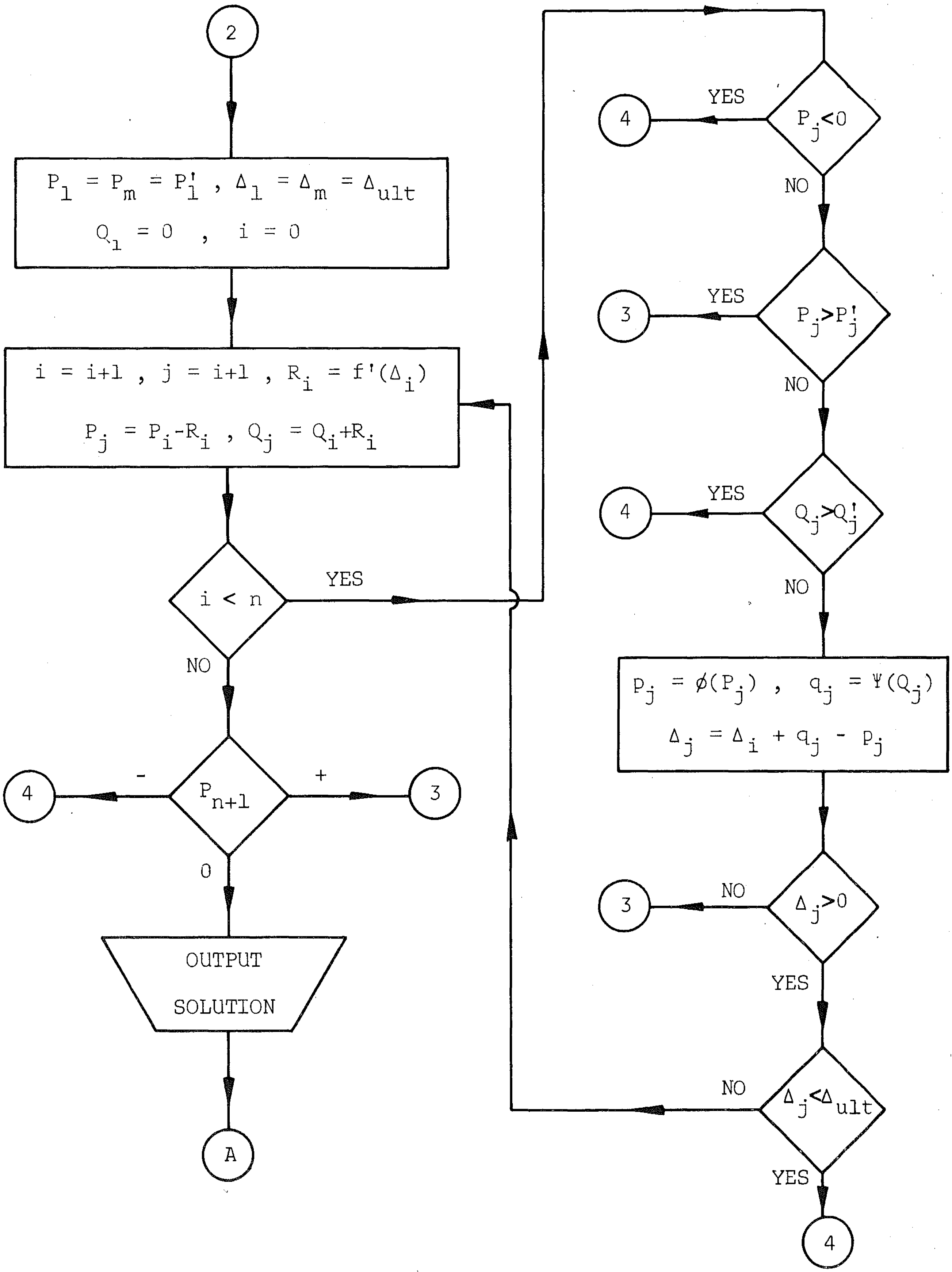


FIG. 6B FLOW CHART FOR THE COMPUTER PROGRAM

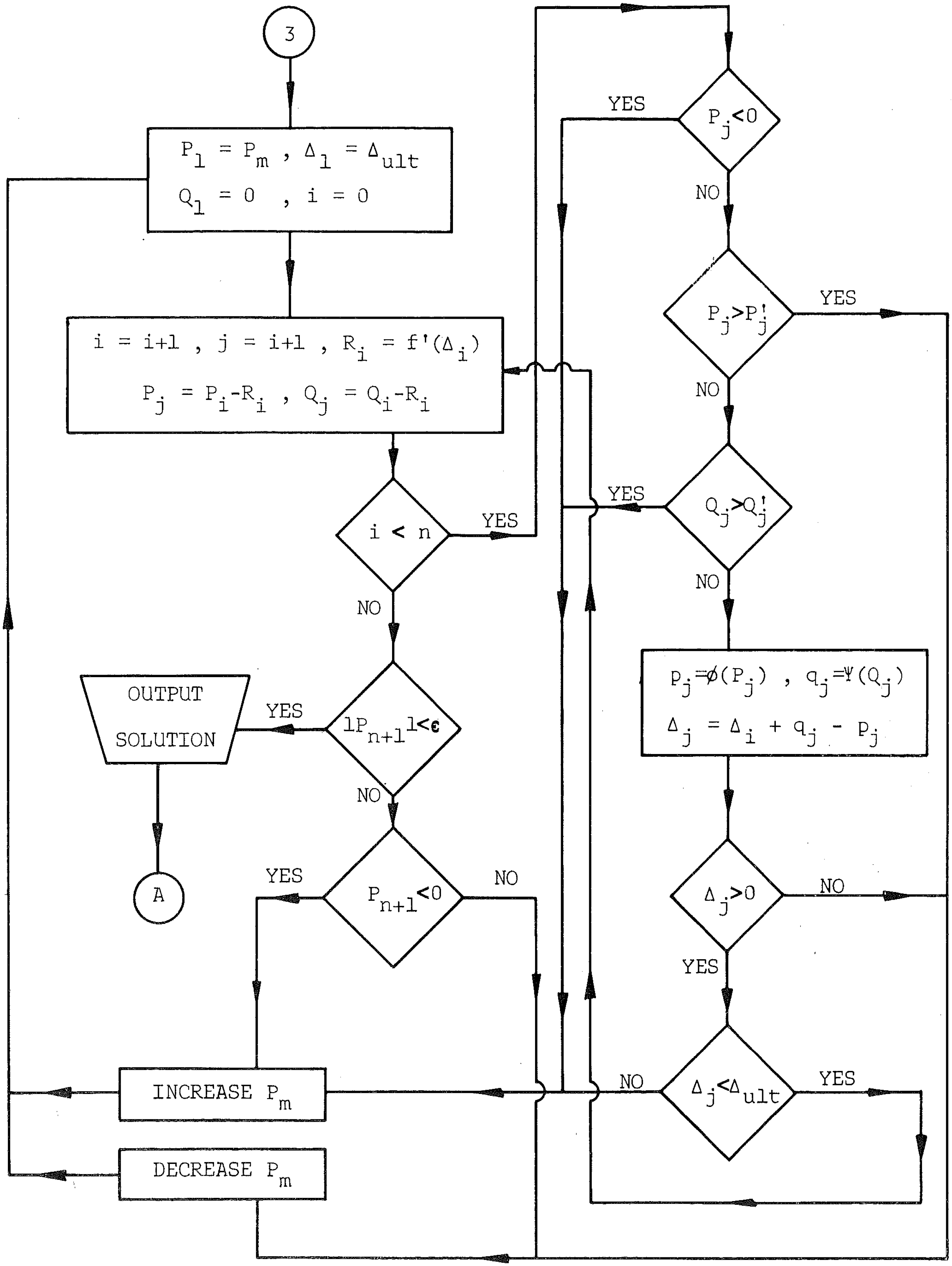


FIG. 6C FLOW CHART FOR THE COMPUTER PROGRAM

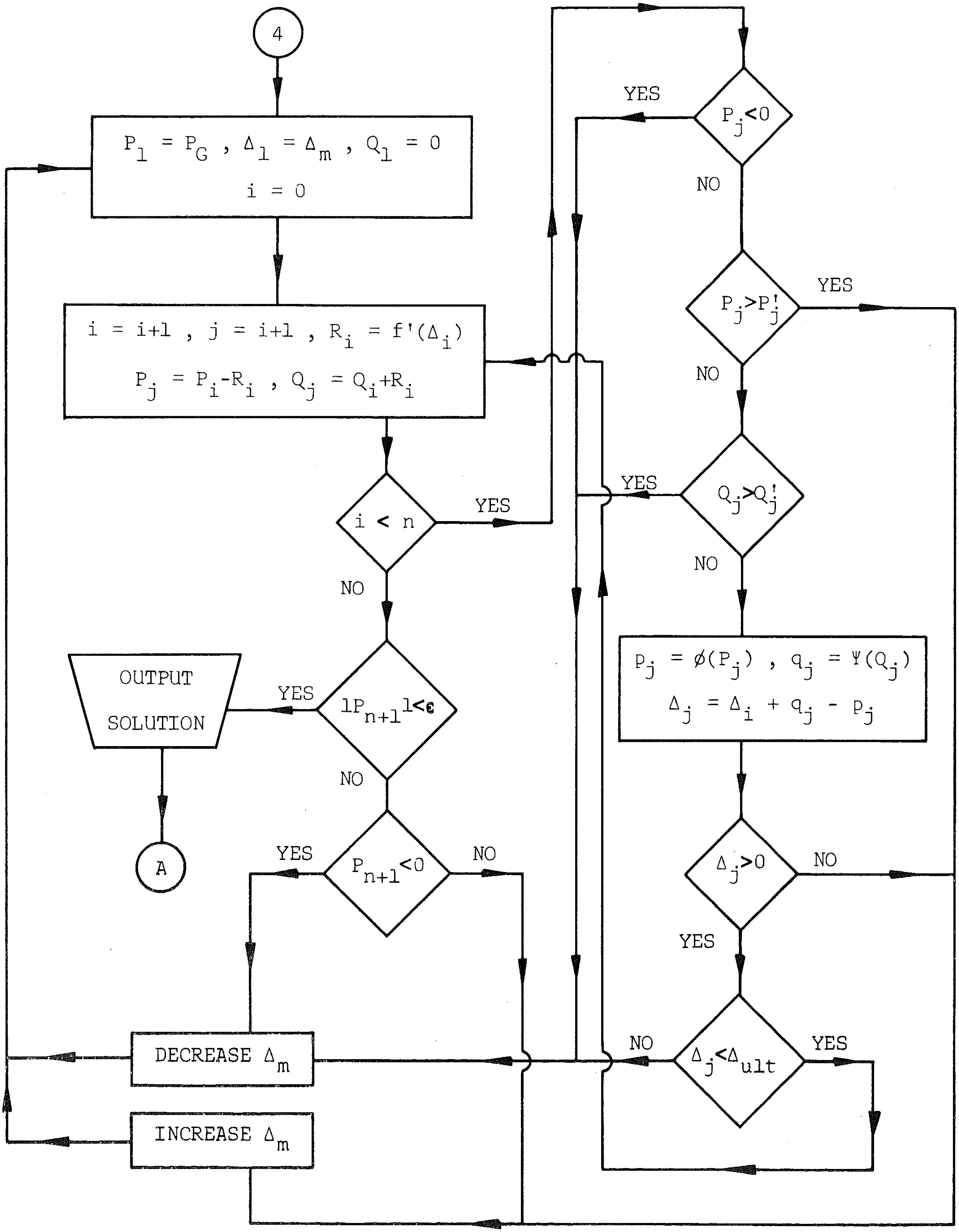


FIG. 6D FLOW CHART FOR THE COMPUTER PROGRAM

8. REFERENCES

1. Fisher, J. W. and Yoshida, N.
LARGE BOLTED AND RIVETED SHINGLE SPLICES, Journal of the Structural Division, ASCE (To be published in 1970)

2. Rivera, U. and Fisher, J. W.
ULTIMATE STRENGTH OF SHINGLE JOINTS, Fritz Engineering Laboratory Report No. 340.4, April 1970

3. Fisher, J. W. and Rumpf, J. L.
ANALYSIS OF BOLTED BUTT JOINTS, Journal of the Structural Division, ASCE, Vol. 91, ST5 (Oct. 1965)

4. Fisher, J. W.
BEHAVIOR OF FASTENERS AND PLATES WITH HOLES, Journal of the Structural Division, ASCE, Vol. 91, ST6 (Dec. 1965)