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A. Ostapenko

C. Chern

S. Parsanejad

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LEHIGH UNIVERSITY

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STRENGTH FORMULAS FOR DESIGN OF STEEL PLATE GIRDERS

by

Alexis Ostapenko[1]
Chingmiin Chern[2]
Siamak Parsanejad[3]

ABSTRACT

Design formulas are presented for evaluating the ultimate strength of transversely stiffened plate girder panels under bending, shear, or a combination of shear and bending. The plate girder may be homogeneous or hybrid with a symmetrical or unsymmetrical cross section. The formulas were evolved from a study of the numerical data obtained using the analytical methods previously developed in the course of this research. The ultimate strength of a panel is obtained as a sum of the contributions by the web buckling strength (beam action), the web post-buckling strength (tension field action), and the flange flexural strength (frame action). The formulas may be used directly in a load factor design approach or serve as a basis for a working stress design method. A tentative recommendation is made for precluding the development of fatigue cracks due to the back-and-forth deflection of the web plate.

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[1] Professor of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania.

[2] Assistant Professor of Civil Engineering, North Dakota State University, Fargo, North Dakota. Formerly, Research Assistant at Lehigh University, Bethlehem, Pennsylvania.

[3] Research Assistant, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania.

1. INTRODUCTION

The considerable post-buckling strength of plate girder webs has been tacitly recognized in design by using lower factors of safety against buckling than against yielding. However, tests showed that the relationship between the ultimate and buckling strengths is not proportional. Thus, a consistent margin against the ultimate strength cannot be achieved by using a constant factor of safety in conjunction with the buckling strength - - - the ultimate strength must be evaluated as such.[3]

Basler offered a plausible theory[3,4,5] which gave good agreement with tests[6] and was accepted by AISC as the method for designing plate girders in buildings.[2] A slightly modified version of this theory was also incorporated in the load factor method proposed under auspices of AISI for designing steel highway bridges.[21] Recently, this method has been accepted by AASHO for use in practice.[1] Further developments of the ultimate strength theory were made, among others, by Fujii[13] and Rockey and Skaloud[17] who included the effect of flanges on the strength of the web plate. All these theories are based on the development of a failure mechanism by the plate girder panel.

Djubek proposed that the maximum web stress in the post-buckling range of deformations remain under the yield level. From a series of theoretical computations he established the stress amplification factors for various panel proportions to be used with the buckling stress.[12]

All of this work has been concerned with symmetrical plate girders, that is, girders having the neutral axis at the mid-depth of the girder web. Also, none of these theories have a continuous description of the girder failure mode for a variable combination of shear and moment. To compensate for these deficiencies, a new approach was developed by Chern and Ostapenko.[8,9,10] This method was also successfully extended to longitudinally stiffened plate girders[14] and confirmed by additional tests.[11,18]

Since for an efficient application the method requires use of a computer, it is hardly suitable for manual calculations. To overcome this difficulty, the numerical computer output was utilized to develop simplified formulas for practical use. So far, this approximation has been successful only for transversely stiffened plate girders and the resultant formulas are described in this report. They can be used directly in a load factor design approach or serve as a basis for a working stress design method.

Design Conditions - A typical plate girder panel is shown in Fig. 1(a). The cross section is unsymmetrical and, for the sake of discussion, the smaller top flange is assumed to be subjected to compression and the larger bottom flange to tension. A larger portion of the web plate is thus under compression. The internal forces acting in the panel are defined at the mid-length as moment M and shear V. As indicated in the moment diagram, Fig. 1(c), a greater moment M_{max} is developed at one end of the panel and it also is taken into consideration in design.

Since for a particular arrangement of loads on a plate girder the moment in a panel is directly proportional to the shear in it, it is convenient to define the moment in terms of the shear span ratio μ.

$$M = \mu b V \tag{1}$$

where

$$\mu = \frac{M}{b V} \tag{2}$$

Analogously to the theory presented in References 8, 9 and 10 three force conditions are considered here: pure bending ($V = 0$), pure shear ($M = 0$, but $M_{max} \neq 0$), and a combination of shear and bending ($M \neq 0$, $V \neq 0$). In the following, the strength formulas for these cases are described separately and then their application is illustrated with numerical examples.

Design of Stiffeners - This report does not deal with the design of stiffeners since no additional studies on their strength were conducted by the project. Current recommendations for proportioning stiffeners in panels designed for ultimate strength are considered to be adequate.[1,2]

2. BENDING STRENGTH

It has been found that the ultimate capacity of a plate girder panel subjected to pure bending is limited by the strength of the compression or tension flange rather than by the buckling of the web plate, although the web plate after buckling does not contribute to the strength of the panel as much as it would if it were flat.

The effective cross section of the plate girder panel after web buckling can be visualized to have the compression flange column composed of the flange itself and a portion of the web plate. A method of analysis based on such an assumption is presented in Ref. 9. Good correlation was obtained with test results on symmetrical, unsymmetrical, hybrid and homogeneous plate girders. Although the generality of this method is very attractive, it was desirable to compare it with the popular method developed by Basler and Thürlimann [3] which has been already accepted by AISC [2] and AASHO [1].

A series of sample computation showed that Basler's method agreed quite well for symmetrical homogeneous girders and also that its familiar and relatively simple formula could be modified to apply to unsymmetrical and hybrid girders.* Such a modified version of the Basler formula is given here.

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*The approximate adaptation to unsymmetrical girders made in Refs. 1 and 21 is not sufficiently accurate for the general case, but rather, applicable only to plate girders of slenderness ratio less than approximately 200. The method for hybrid girders given in Ref. 20 is limited to symmetrical girders having the web which does not buckle before yielding.

The original Basler formula established for symmetrical girders is [3]

$$M_u = \frac{I}{y}\sigma_{cf} \left[1 - 0.0005 \frac{A_w}{A_f} \left(\frac{b}{t} - 5.7 \sqrt{\frac{E}{\sigma_{cf}}} \right)\right] \tag{3}$$

where I is the moment of inertia of the cross section, A_w = bt is the area of the web, $A_f = A_{fc} = A_{ft}$ is the flange area of a symmetrical girder, E is the modulus of elasticity, σ_{cf} is the critical stress of the compression flange, and $y = y_c$, b and t are indicated in Fig. 1b. A plausible extension of this formula to unsymmetrical sections can be made by assuming that the effect of the buckled web may be evaluated as that for a symmetrical section whose total depth is equal to the double of the web portion under compression in the unsymmetrical section. The following replacements are then to be made in Eq. 3:

$$A_w = 2y_c t \quad \text{and} \quad b/t = 2y_c/t \tag{4}$$

Design of hybrid girders requires consideration of different material properties of the web and flanges. Of particular interest is the case when the web is of lower strength than the flanges. The following formula which also incorporates a modification for unsymmetrical girders is proposed here:

$$M_u = \frac{I}{y_c}\sigma_{cf} \left\{ \frac{\sigma_{yw}}{\sigma_{cf}} \left[\frac{I_w}{I} - 0.002 \frac{y_c t}{A_{fc}} \left(\frac{y_c}{t} - 2.85 \sqrt{\frac{E}{\sigma_{yw}}} \right)\right] + \left(1 - \frac{I_w}{I}\right) \right\} \tag{5}$$

in which

$$\left(\frac{y_c}{t} - 2.85 \sqrt{\frac{E}{\sigma_{yw}}} \right) \geqq 0 \quad \text{and} \quad \sigma_{yw} \leqq \sigma_{cf} \tag{6}$$

Due to a lack of research information, set $\sigma_{yw} = \sigma_{cf}$ when $\sigma_{yw} > \sigma_{cf}$.

The additional notation in Eq. 5 is

I_w = moment of inertia of the web plate about the centroidal axis of the whole section.

σ_{yw} = yield stress of the web.

σ_{cf} = critical stress of the compression flange due to lateral or local buckling.

1) Lateral buckling $\left(\frac{2c_c}{d_c} \leq 12 + \frac{L}{2c_c}\right)$:

$$\sigma_{cf} = \left(1 - \frac{\lambda_L^2}{4}\right) \sigma_{yc} \qquad (7a)$$

for $o < \lambda_L < \sqrt{2}$

or $$\sigma_{cf} = \frac{1}{\lambda_L^2} \sigma_{yc} \qquad (7b)$$

for $\lambda_L \geq \sqrt{2}$

and $$\lambda_L = L \sqrt{\frac{\sigma_{yc}}{E\pi^2} \cdot \frac{A_{fc} + (1/3)\, y_c t}{I_f}} \qquad (7c)$$

where

c_c = half width of the compression flange

d_c = thickness of the compression flange

L = unbraced length of the compression flange

I_f = moment of inertia of the compression flange about the vertical axis

σ_{yc} = yield stress of the compression flange

$L < \frac{4c_c^2}{d_c} - 24c_c$

2) Local (Torsional) buckling $\left(\frac{2c_c}{d_c} > 12 + \frac{L}{2c_c}\right)$:

$$\sigma_{cf} = [\,1 - 0.53\,(\,\lambda_t - 0.45)^{1.36}]\,\sigma_{yc}$$

$$\text{for} \qquad 0.45 < \lambda_t < \sqrt{2} \tag{8a}$$

or

$$\sigma_{cf} = \frac{1}{\lambda_t^2}\,\sigma_{yc}$$

$$\text{for} \qquad \lambda_t \geqslant \sqrt{2} \tag{8b}$$

where

$$\lambda_t = \frac{c_c}{d_c}\sqrt{\frac{12(1-\nu^2)\,\sigma_{yc}}{0.425\,\pi^2 E}} \tag{8c}$$

According to Eq. 5 the plate girder strength is assumed to consist of two contributions. The first, as given by the expression in brackets, is the contribution of the web plate up to the point of yielding in the web. This term is nothing else but the Basler formula (Eq. 3) with the critical flange stress replaced by the web yield stress and modifications made for unsymmetrical sections. Since the strength of the panel is not exhausted at this point, the second term in parentheses reflects the total moment contributed by the flanges.

Equation 5 is on one hand somewhat unconservative since its composition assumes that the neutral axis remains at its original position in spite of redistribution of the web stresses due to buckling and yielding. On the other hand it is conservative since it neglects the contribution to the moment from the increases in web stresses, especially in the tension zone. Since the equation gives very good correlation with experimental results and a more rigorous approach,[9] these two effects apparently cancel each other.

For symmetrical ($b = 2y_c$) and homogeneous ($\sigma_{yw} = \sigma_{cf}$) girders Eq. 5 reduces to the original Eq. 3.

Tension Flange Failure. When the tension portion of the web is sufficiently larger than the compression portion, the bending capacity of the panel may go up to the plastic moment M_p. Due to some uncertainties in the behavior of a very slender web, it is more conservatively assumed that the capacity is limited by the yielding of the tension flange.

$$M_u = \frac{I}{y_t} \sigma_{yt} \left[1 - \frac{I_w}{I} \left(1 - \frac{\sigma_{yw}}{\sigma_{yt}}\right) \right] \qquad (9)$$

When it is uncertain whether the compression or the tension flange failure controls the design, both, Eq. 5 and 9 should be used to determine which one gives the smaller ultimate moment.

3. SHEAR STRENGTH

The ultimate strength of a plate girder panel subjected to pure shear is assumed according to Ref. 8 to consist of three contributions: the buckling strength of the web V_τ (beam action), the post-buckling strength of the web V_σ (tension field action) which leads to the formation of a tension diagonal in the web, and a contribution resulting from the resistance of the flanges to the change of the panel from a rectangular to a parallelogram shape V_f (frame action).

$$V_u = V_\tau + V_\sigma + V_f \tag{10}$$

These three strength contributions are shown in Fig. 2.

A parametrical study of the numerical output from a computer program based on the method of Ref. 8 showed that the three individual contributions could be computed with adequate accuracy from relatively simple formulas suitable for manual computations.

Beam Action. - Beam action contribution V_τ is the shear buckling strength of the web and is given by the product of the web area and the shear buckling stress:

$$V_\tau = A_w \tau_{cr} = bt\tau_{cr} \tag{11}$$

In order to conveniently define τ_{cr} in the elastic, inelastic, and strain-hardening ranges, the following non-dimensional shear buckling parameter is introduced:

$$\lambda_v = \frac{b}{t}\sqrt{\frac{12\ (1-\nu^2)}{\sqrt{3}\ \pi^2\ E}\ \frac{\sigma_{yw}}{k_v}} \quad = \quad 0.8\,\frac{D}{t_w}\sqrt{\frac{\sigma_{yw}}{E K_\tau}} \qquad (12)$$

where:

ν = Poisson's ratio

σ_{yw} = yield stress of the web

E = modulus of elasticity

k_v = plate buckling coefficient in the elastic range

Assuming the web plate to be fixed at the flanges and pinned at the stiffeners, k_v is computed as follows:

$$k_v = \frac{5.34}{\alpha^2} + \frac{6.55}{\alpha} - 13.71 + 14.10\alpha \qquad (13a)$$

$$\text{for } \alpha < 1.0$$

$\left(K_\tau = 5 + \frac{5}{\alpha^2}\right)$

or

$$k_v = 8.98 + \frac{6.18}{\alpha^2} - \frac{2.88}{\alpha^3} \qquad (13b)$$

$$\text{for } \alpha \geq 1.0$$

$4 + \frac{5.34}{\alpha^2}$

With the shear yielding stress

$$\tau_y = \sigma_{yw}/\sqrt{3} \quad = \quad 0.58\,\sigma_{yw} \qquad (14)$$

the shear buckling stress τ_{cr} is given then as a function of λ_v only

$$\tau_{cr} = \left[1 + 4.3\ (0.58 - \lambda_v)^{1.56}\right] \tau_y \quad = 0.58\ \sigma_{yw} \tag{15a}$$

for $\lambda_v \leq 0.58$
(strain-hardening range)

$$\tau_{cr} = \left[1 - 0.615\ (\lambda_v - 0.58)^{1.18}\right] \tau_y \quad = \sigma_{yw}\left[0.58 - 0.359\ (\lambda_v - 0.58)^{1.18}\right] \tag{15b}$$

for $0.58 < \lambda_v \leq \sqrt{2}$
(inelastic range)

$$\tau_{cr} = \frac{1}{\lambda_v^2}\ \tau_y \quad = \frac{0.58}{\lambda_v^2}\ \sigma_{yw} \tag{15c}$$

for $\lambda_v > \sqrt{2}$
(elastic range)

These are the same relationships as were used in the theory.[8] They are shown in Fig. 3.

Tension Field Action. - The tension field stresses are assumed to develop in the pattern shown by the middle sketch of Fig. 2. The inclined band has the maximum intensity which in combination with the stresses at buckling may not exceed the yielding condition.

According to Ref. 8, the tension field action contribution V_σ is a function of the aspect ratio α, web slenderness ratio (b/t), and the material yield stress σ_{yw}. Since the method of Ref. 8 requires the use of a digital computer, an analysis of the computer output was performed for various combinations of geometry and material properties. It was found possible to separate the effects of α and λ_v on V_σ as shown in Fig. 4. Thus, V_σ can be given as a function of α and λ_v in three ranges of λ_v.

$$V_\sigma = 0 \tag{16a}$$

for $\lambda_v \leq 0.58$

$$V_\sigma = \frac{0.6\,\lambda_v - 0.348}{\sqrt{\alpha^2 + 1.6}}\, V_p \tag{16b}$$

for $0.58 < \lambda_v \leq \sqrt{2}$

$$V_\sigma = \frac{0.9 - 0.787/\lambda_v^2}{\sqrt{\alpha^2 + 1.6}}\, V_p \tag{16c}$$

for $\lambda_v > \sqrt{2}$

where

$$V_p = A_w \frac{\sigma_{yw}}{\sqrt{3}} = bt\,\frac{\sigma_{yw}}{\sqrt{3}} \tag{17}$$

is the plastic shear force.

Frame Action. - The frame action contribution V_f is the resistance of the flanges to the distortion of the panel from a rectangle into a parallelogram. The maximum frame action shear is assumed to be reached when the mechanism shown by the right sketch of Fig. 2 is formed. Because the continuity of the web provides sufficient rigidity to, essentially, preclude rotation of the transverse stiffeners, plastic hinges are assumed to form in the flanges.

According to the method of Ref. 8 a portion of the web plate is assumed to act with the flanges. For the sake of simplification, the contribution by the web plate is neglected here. Since the frame action contribution to the ultimate panel strength is, in most cases, about 10%, this assumption does not introduce any appreciable error in the final result. Thus, V_f is given by

$$V_f = \frac{2}{a} (m_c + m_t) \tag{18a}$$

where m_c and m_t are the plastic moments of the compression and tension flanges, respectively. For flanges consisting of rectangular plates, this equation can be rewritten by

$$V_f = \frac{1}{2a} (\sigma_{yc} A_{fc} d_c + \sigma_{yt} A_{ft} d_t) \tag{18b}$$

where σ_{yc}, A_{fc}, d_c and σ_{yt}, A_{ft}, d_t are, respectively, the yield stress, the area, and the thickness of the compression flange and the tension flange.

Design Formulas. - Substitution of the three contributions into Eq.10 from Eqs. 11, 16 and 18 gives the following design formulas for the ultimate shear strength. It is assumed that the yield stress of both flanges is the same, $\sigma_{yc} = \sigma_{yt} = \sigma_{yf}$.

$$V_u = V_p \left\{ \left[1 + 4.3 \ (0.58 - \lambda_v)^{1.56} \right] + \frac{\sqrt{3}}{2} \frac{A_{fc} d_c + A_{ft} d_t}{a A_w} \frac{\sigma_{yf}}{\sigma_{yw}} \right\} \tag{19a}$$

for $\lambda_v \leq 0.58$
(strain-hardening range)

$$\text{or } V_u = V_p\left\{\left[1 - 0.615\ (\lambda_v - 0.58)^{1.18}\right] + \frac{0.6\ \lambda_v - 0.348}{\sqrt{\alpha^2 + 1.6}}\right. \tag{19b}$$

$$\left. + \frac{\sqrt{3}}{2}\,\frac{A_{fc}\ d_c + A_{ft}\ d_t}{a\ A_w}\ \frac{\sigma_{yf}}{\sigma_{yw}}\right\}$$

for $0.58 < \lambda_v \leq \sqrt{2}$
(inelastic range)

$$\text{or } V_u = V_p\left\{\frac{1}{\lambda_v^2} + \frac{0.9 - 0.787/\lambda_v^2}{\sqrt{\alpha^2 + 1.6}} + \frac{\sqrt{3}}{2}\,\frac{A_{fc}\ d_c + A_{ft}\ d_t}{a\ A_w}\ \frac{\sigma_{yf}}{\sigma_{yw}}\right\} \tag{19c}$$

for $\lambda_v > \sqrt{2}$
(elastic range)

For steel with $\nu = 0.3$ and $E = 29{,}000$ ksi, Eq. 12 gives

$$\lambda_v = 0.00465\ \frac{b}{t}\sqrt{\frac{\sigma_{yw}}{k_v}} \tag{20}$$

σ_{yw} is in ksi and k_v is obtained from Eq. 13.

Comparison with Test Results and Other Methods. - A comparison of the proposed formulas, Basler's,[4] and Fujii's[13] methods with the available test results are shown by the cumulative distribution curves in Fig. 5. It is seen that the correlation of the proposed formulas with the test results is within 10%. Basler's method gives less than 10% deviation for about 65% of the tests and larger deviations (up to 33%) for about 35% of the tests. Since Fujii's method does not apply to unsymmetrical plate girders, two sets of computations were made. The thin dashed line denoted by "S" in the figure represents the Fujii's case when both flanges were assumed to be of the smaller flange size, and the thin solid line denoted by "L" represents the case of assuming both flanges as the larger flange. It is seen that Fujii's method gives good correlation with tests for symmetrical girders, but the method is ambiguous when applied to unsymmetrical girders.

End Panel. - Full development of the tension field capacity requires that the neighboring panels be sufficiently strong to anchor it. This means that either the panel at the end of a girder should have a very strong end stiffener capable of resisting the horizontal component of the tension field force, or that the panel should be designed to develop only the buckling strength. The latter approach is recommended here. Thus, the shear capacity of the end panel is to be computed from Eq.10 with $V_\sigma = 0$.

The resultant shear capacity of the end panel is greater than that specified by AASHO[1] and AISC[2] because of two reasons: 1) the web plate is assumed to be fixed at the flanges and simply supported at the stiffeners rather than simply supported at all edges, and 2) the frame action shear V_f is included.

4. STRENGTH UNDER BENDING AND SHEAR

The strength of a plate girder panel under various combinations of shear and moment can be described by the interaction curve Q_5-Q_4-Q_1-Q_2-Q_3 shown in Fig.6 . The ordinate gives the shear non-dimensionalized with respect to the ultimate value for the pure shear case, and the abscissa is the moment non-dimensionalized with respect to the ultimate moment for pure bending. The right and left parts are, respectively, for the larger portion of the web plate under compression and tension as indicated by the small sketches under the diagram.

Depending on the relative magnitude of shear and moment and on the direction of the moment, the ultimate strength of the panel may be controlled by one of the following three conditions: 1) the shear strength reduced by bending (web failure---portion Q_4-Q_1-Q_2), 2) the bending strength reduced by shear and limited by the compression flange failure (portion Q_2-Q_3), and 3) the bending strength reduced by shear and limited by the yielding of the tension flange (portion Q_5-Q_4). The mechanisms of failure are indicated by the insert sketches. The individual contributions due to beam, tension field and frame actions are shown schematically by separate areas in the interaction diagram. The design procedure recommended here is to compute the ultimate strength for each applicable strength condition and use the lower value as the controlling one.

Web Failure. (Curve Q_4-Q_1-Q_2 in Fig. 6) - As the insert in Fig. 6 shows, the panel strength is obtained as a sum of buckling (beam action), post-buckling (tension field action), and flange flexural (frame action) contributions.

This is analogous to the case of pure shear, except that now each contribution is affected by the presence of bending moment.

$$V_{uc} = V_{\tau c} + V_{\sigma c} + V_{fc} \tag{21}$$

where subscript c indicates the combination of shear and bending forces.

The beam action contribution is

$$V_{\tau c} = \tau_c \, A_w \tag{22}$$

τ_c is the shear buckling stress of the web subjected to shear and bending stresses as shown in Fig. 7 . It may be computed with adequate accuracy from the following interaction equation

$$\left(\frac{\tau_c}{\tau_{cr}}\right)^2 + \cancel{\frac{1 + R}{2}\left(\frac{\sigma_c}{\sigma_{cr}}\right)} + \frac{1 - R}{2}\left(\frac{\sigma_c}{\sigma_{cr}}\right)^2 = 1.0 \tag{23}$$

where $R = -1$

τ_{cr} = shear buckling stress for pure shear according to Eqs. 15

σ_c = buckling stress at the extreme compression fiber of the web for combined loading (Fig. 7)

σ_{cr} = buckling stress under pure bending according to Eq. 24

R = ratio of the maximum tensile stress (or minimum compressive stress) to the maximum compressive stress (see Fig. 7). R is negative when the stress is tensile.

The buckling stress under pure bending, σ_{cr}, is to be computed from the following equations which are analogous to Eqs. 15 used for τ_{cr}:

$$\sigma_{cr} = \sigma_{yw} \tag{24a}$$

for $\lambda_b \leq 0.58$ 0.65
(yielding)

$\sigma_{yw}\left[0.0926(\lambda_b - 5.62)^2 - 0.178\right]$

$$\sigma_{cr} = \left[1 - 0.615\ (\lambda_b - 0.58)^{1.18}\right] \sigma_{yw} \tag{24b}$$

for $0.58 < \lambda_b \leq \sqrt{2}$ 0.65 < < 1.5
(inelastic range)

or

$$\sigma_{cr} = \frac{1}{\lambda_b^2}\ \sigma_{yw} \tag{24c}$$

$\frac{1}{\lambda_b^2}\sigma_{yw}$

for $\lambda_b > \sqrt{2}$ > 1.5
(elastic range)

where

$= 1.05\,\frac{b}{t}\sqrt{\frac{\sigma_{yw}}{E k_b}}$

$$\lambda_b = \frac{b}{t}\sqrt{\frac{12\ (1-\nu^2)}{\pi^2\ \ E}\ \frac{\sigma_{yw}}{k_b}} = 1.314\lambda_v\sqrt{\frac{k_v}{k_b}} \tag{25}$$

k_b, the plate buckling coefficient is conservatively obtained by assuming $\alpha = \infty$ and deriving the following formula by curve fitting

$$k_b = 13.54 - 15.64\ R + 13.32\ R^2 + 3.38\ R^3 \tag{26}$$

for $(-1.5) \leqslant R \leqslant (0.5)$

Since σ_c is directly related to τ_c by

$\alpha > 2/3 \quad k = 24$

$\alpha < 2/3 \quad k = 24 + 73(2/3 - \alpha)^2$

$\alpha = 0.5 \quad 26$

$$\sigma_c = (\mu\ b\ A_w\ y_c/I)\ \tau_c \tag{27}$$

Equation 23 can be solved for τ_c

$$\tau_c = \tau_{cr} \frac{\sqrt{F^2(3-R)^2 + 16} - (1+R)\,F}{2\,[2 + (1-R)\,F^2]} \tag{28}$$

where

$$F = \frac{\mu\ b\ y_c\ A_w}{I} \cdot \frac{\tau_{cr}}{\sigma_{cr}} \tag{29}$$

Experimental evidence shows that full plastic moment and shear force can be developed for low b/t. In view of this, it is tentatively recommended here not to consider interaction whenever λ_v and λ_b are less than 0.58. Then, $\sigma_c = \sigma_{yw}$ and $\tau_c = \tau_{cr}$. + Beam Actio

The tension field action contribution to the web strength was found from numerical computer results to vary only about 2% due to the application of bending. Therefore, it is assumed that

$$V_{\sigma c} = V_\sigma \tag{30}$$

where V_σ is computed using Eqs. 16.

The frame action contribution is usually quite small in ordinary welded plate girders (see Fig. 6). Thus, it would be quite adequate to use an approximate reduction factor to consider the effect of axial force in the flanges instead of performing exact computations of Ref. 10. The effect of bending on the frame action is assumed to be the same as on the shear buckling stress.

$$V_{fc} = \left(0.01 + \frac{\tau_c}{\tau_{cr}}\right) V_f \tag{31}$$

where V_f is from Eq.18 (the constant 0.01 serves to simplify computations when the strength is limited by the failure of the compression flange).

The ultimate shear is then obtained by adding the results of Eqs.22, 30 and 31 according to Eq. 21.

<u>Compression Flange Failure</u>. (Curve Q_2-Q_3 in Fig. 6) - In this range of moment-shear combinations, the compression flange fails before the web strength can be fully developed. Thus, bending is now the principal loading parameter. However, it is still convenient to define the panel strength in terms of shear given as a sum of the beam, tension field and frame action contributions.

The beam action and frame action contributions are computed from Eqs. 22 and 31.

However, the tension field action does not fully develop and a special study was needed to arrive at an acceptably simple formula for its computation. The formula finally selected on the basis of a parametrical study of the numerical computer output of the method of Ref.10 is

$$V'_{\sigma c} = \frac{(A_{fc} + 30\ t^2)\ (\sigma_{cf} - \sigma_c) - \mu\ V_{fc}}{B\left(\frac{V_p}{V_\sigma}\right)\left(\frac{180}{b/t}\right)\sqrt{\frac{33}{\sigma_{yw}}\ \frac{b}{y_c}} + \mu} \geqq 0 \tag{32}$$

where

$$B = 0.338\lambda_v - 0.196 \tag{33a}$$

$$\text{for } 0.58 \leqq \lambda_v \leqq \sqrt{2}$$

or

$$B = 0.235\lambda_v - 0.05 \tag{33b}$$

$$\text{for } \lambda_v > \sqrt{2}$$

λ_v is given by Eq. 12 and σ_{yw} is in ksi.

Besides the composition of Eq. 32, the parameter which was developed from the numerical output is B. Figure 8 shows a plot of B versus λ_v. The points give the values of B obtained by equating the ultimate strength expressed by the design formula to the theoretical ultimate strength at the point of transition from web failure to compression flange failure; each point represents a particular panel. A least squares fit through the plotted points was used to find the expression for B when $\lambda_v > \sqrt{2}$, Eq. 33b. Since for $0.58 < \lambda_v \leq \sqrt{2}$, $V'_{\sigma c}$ represents only a small portion of the total shear strength, the effect of B would be negligible. Thus, a straight-line approximation is made, Eq. 33a.

The ultimate panel shear causing failure of the compression flange is given by the sum of the values from Eqs. 22, 31 and 32.

$$V_{uc} = V_{\tau c} + V_{fc} + V'_{\sigma c} \tag{34}$$

and the corresponding panel moment is

$$M_{uc} = \mu b \, V_{uc} \leq M_u \tag{35}$$

Equation 35 indicates that M_{uc} should not exceed M_u. Since the nature of the approximations involved in the evaluation of $V'_{\sigma c}$ could lead to an unrealistic condition of M_{uc}, computed from Eqs. 34 and 35, being less than M_u for the case of pure bending, the constant 0.01 was introduced into Eq. 31 to preclude this situation. The result is illustrated in the right lower corner of the interaction diagram of Fig. 9.

Ultimate Strength Under Bending and Shear. - Since the specific combinations of moment and shear which are controlling for the web or compression flange failure modes are not defined, it is necessary to check both modes and select the one which gives a lower capacity.

A typical interaction diagram based on the above derived formulas is shown in Fig. 9. A ray emanating from the origin represents an increasing load for a particular moment-shear combination. Two intersection points with the interaction curves are shown, one due to web failure and the other due to the compression flange failure. The smaller shear is to be selected as the controlling ultimate shear. For the two rays shown, the controlling cases are indicated with the heavy dots.

Maximum Panel Moment. - Since in a panel under combined forces the moment at one end of the panel is greater than the mid-panel moment used in the analysis (Fig. 1), it may happen that the strength will be controlled by this maximum panel moment M_{max}. This is particularly true for panels with large aspect ratios.

A reasonable and sufficiently accurate approach appears to be a requirement that the maximum panel moment be below the moment which would cause failure under pure bending. Thus,

$$V'_u = \frac{M_u}{b\ (\mu + \frac{1}{2}\ \alpha)} \tag{36}$$

where M_u is the smaller value of Eq.5 or Eq.9.*

Tension Flange Yielding. - As indicated by the vertical line marked "Flange Yielding" in Fig. 9 , the check by Eq.36 also covers the case when the panel strength is limited by the yielding of the tension flange. This criterion may be somewhat conservative for sections with low b/t (compact sections) or in cases when most of the web is in tension and essentially full plastic moment may be attained (see the left-most curves in Figs. 6 and 9). It is left to the judgement of the designer when he would want to take advantage of the additional panel capacity due to plastification.

*A more refined check on the compression flange capacity is (with M_u from Eq.5)

$$V'_u = \frac{M_u}{b\ (\mu + \frac{1}{2}\ \alpha)}\ \frac{\sigma_{yc}}{\sigma_{cf}} \tag{37}$$

5. CONSIDERATION OF FATIGUE

Many tests as well as a comprehensive study conducted specifically for this purpose[19] showed that initial out-of-plane deflections of the web plate have no detrimental effect on the ultimate strength of girders subjected to static loads. However, when the load application is repeated many times as is the case for bridge and crane girders, fatigue cracks may develop in the web due to the lateral flexing ("breathing") of the web at each load application. Initial deflections and the amount of stressing beyond the buckling stress level of the web plate appear to be the principal factors influencing the development of these fatigue cracks.[16] Since both of these factors are functions of the web slenderness ratio b/t, a recommendation was made to limit the web slenderness ratio to a specific value.[1, 21]

This b/t limitation was critically reviewed in Reference 15 in the light of additional tests on unsymmetrical plate girders. It was found to be somewhat conservative for conventionally proportioned girders but is endorsed here until more research is conducted.

$$\frac{2\,y_c}{t} \leq \frac{1,150}{\sqrt{\sigma_{yw}}} \geqslant \frac{b}{t} \qquad (38)$$

where σ_{yw} is in ksi.

It should be noted, however, that Eq. 38 may be unconservative for girder panels with stiffeners of high torsional rigidity, such as heavy bearing stiffeners and stiffeners with closed sections. Until more research is conducted, it is tentatively recommended here that the web panels adjoining such torsionally rigid stiffeners, be checked <u>not to buckle under a load equal to about 1.1 of the working load</u>.

6. GIRDERS WITHOUT INTERMEDIATE STIFFENERS

Plate girders having stiffeners only at the supports (bearing stiffeners) and possibly under heavy concentrated loads are of considerable economic interest. As described in References 8 and 10, such plate girders may be safely and accurately designed by neglecting the contribution of the tension field action (post-buckling strength) whenever the panel aspect ratio α exceeds 3.0. Thus,

$$V_{uc} = V_{\tau c} + V_{fc} \tag{39}$$

where $V_{\tau c}$ and V_{fc} are given by Eqs. 22 and 31, respectively, or by Eqs. 11 and 18 for the case when $M = 0$. It is important that such long panels be always checked for the maximum panel moment according to Eq. 36.

Plate girders without intermediate stiffeners and subjected to uniformly distributed static loading, such as roof girders, have attracted attention of engineers in Sweden.(7) A temporary design specification was developed primarily on the basis of experimental work. A comparison of Eq. 39 with this specification for a few sample girders showed that for most cases Eq. 39 was more conservative than the specification rules.

7. DESIGN PROCEDURE AND NUMERICAL EXAMPLE

The sequences for the computation of the ultimate strength of a panel subjected to pure bending, pure shear or a combination of bending and shear are shown schematically by block diagrams in Figs. 10a, 10b and 11, respectively. The following numerical example illustrates the procedure in detail.

Example

Given is a bridge girder panel with the following dimensions (in inches) and material properties:

Panel length : $a = 126.0$

Panel depth : $b = 84.0$

Compression flange: $2c_c \times d_c = 27.0 \times 2.5$

Tension flange : $2c_t \times d_t = 27.0 \times 1.75$

Web : $b \times t = 84 \times 7/16$

Unbraced length of the compression flange: $L = 126.0$

Yield stress of the compression flange : $\sigma_{yc} = 100.0$ ksi

tension flange : $\sigma_{yt} = 100.0$ ksi

web : $\sigma_{yw} = 36.0$ ksi

Cross-sectional properties: $I = 229{,}000.0$ in.^{4}, $I_w = 22{,}750.0$ in.^{4}, $I_f = 4{,}100$ in.^{4}, $A_{fc} = 67.5$ in.^{2}, $A_{ft} = 47.2$ in.^{2}, $A_w = 36.75$ in.^{2}, $y_c = 36.4$ in., $y_t = 47.6$ in.

Non-dimensional parameters: $\alpha = a/b = 1.5$, $b/t = 192$, $2y_c/t = 166$, $R =- (y_t/y_c) =- 1.28$

Find:

The ultimate panel strength for:

(a) Pure bending

(b) Pure shear

(c) Combination of bending and shear with $\mu = M/bV = 14.0$

Check fatigue requirement:

$$\text{Eq. 38:} \quad \frac{1,150}{\sqrt{\sigma_{yw}}} = \frac{1,150}{\sqrt{36}} = 192 = b/t$$

$> 2y_c/t$, therefore, O.K.

(a) Bending Strength

Compression Flange Failure:

Check: $2\,c_c/d_c = 27.0/2.5 = 10.8$

$12 + L/(2c_c) = 12 + 126.0/27 = \underline{16.7 > 10.8}$

Thus, lateral buckling of the compression flange governs

$$\text{Eq. 7c:} \quad \lambda_L = 126\sqrt{\frac{100}{29,000\,\pi^2}\left(\frac{67.5 + (1/3)(36.4)(7/16)}{4,100}\right)} = 0.314 < \sqrt{2}$$

$$\text{Eq. 7a:} \quad \sigma_{cf} = 100(1 - 0.314^2/4) = 97.5 \text{ ksi}$$

Check: $\sigma_{yw} = 36.0$ ksi $< \sigma_{cf} = 97.5$ ksi. Thus, use $\sigma_{yw} = 36.0$ ksi

Check: $y_c/t - 2.85\sqrt{E/\sigma_{yw}} = (166/2) - 2.85\sqrt{(29,000/36)} = 2.0 > 0$ O.K.

$$\text{Eq. 5:} \quad (M_u)_c = \frac{229,000}{36.4}(97.5)\left\{1 - \frac{22,750}{229,000} + \frac{36}{97.5}\left[\frac{22,750}{229,000} - 0.002\left(\frac{36.4(7/16)}{67.5}\right)(2)\right]\right\} = 575,000 \text{ kip-in.}$$

$(M_u)_c = 575,000$ kip-in.

Yielding of Tension Flange:

$$\text{Eq. 9:} \quad (M_u)_t = \frac{229,000}{47.6}(100)\left[1 - \frac{22,750}{229,000}\left(1 - \frac{36}{100}\right)\right] = 450,000 \text{ kip-in.}$$

$(M_u)_t < (M_u)_c$, thus, yielding of the tension flange governs:

$\underline{M_u = 450,000 \text{ kip-in.}}$

(b) Shear Strength

For $\alpha = 1.5 > 1.0$

Eq. 13b: $k_v = 8.98 + 6.18/1.5^2 - 2.88/1.5^3 = 10.88$

Eq. 17: $V_p = 36(36.75)/\sqrt{3} = 763$ kips

Eq. 12: $\lambda_v = 192 \sqrt{\dfrac{12(1 - 0.3^2)}{29{,}000\,\pi^2\,\sqrt{3}}\;\dfrac{36}{10.88}} = \underline{1.64 > \sqrt{2}}$

Eq. 19c: $V_u = 763 \left\{ \dfrac{1}{(1.64)^2} + \left[\dfrac{0.9 - 0.787/(1.64)^2}{\sqrt{1.5^2 + 1.6}}\right] + \dfrac{\sqrt{3}}{2} \cdot \dfrac{67.5(2.5) + (47.2)(1.75)}{126(36.75)} \left(\dfrac{100}{36}\right) \right\}$

$= 763\ (0.372 + 0.310 + 0.127) = \underset{(V_\tau)}{284} + \underset{(V_\sigma)}{237} + \underset{(V_f)}{97} = 618$ kips

$\underline{V_u = 618 \text{ kips}}$

(c) Combined Shear and Bending Strength

Web Failure:

For $R = -1.28$

Eq. 26: $k_b = 13.54 - 15.64(-1.28) + 13.32(-1.28)^2 + 3.38(-1.28)^3 = 48.24$

Eq. 25: $\lambda_b = 1.314(1.64) \sqrt{\dfrac{10.88}{48.24}} = 1.03, \qquad 0.58 < 1.03 < \sqrt{2}$

Eq. 24b: $\sigma_{cr} = 36[1 - 0.615(1.03 - 0.58)^{1.18}] = 27.4$ ksi

For $\lambda_v = 1.64 > \sqrt{2}$

Eq. 15c: $\tau_{cr} = \dfrac{36}{\sqrt{3}\ (1.64)^2} = 7.72$ ksi

Eq. 29: $F = \dfrac{14(84)(36.4)(36.75)}{229{,}000}\ \dfrac{7.72}{27.4} = 1.94$

Eq. 28: $\tau_c = 7.72\ \dfrac{\sqrt{(1.94)^2(3 + 1.28)^2 + 16} - (1 - 1.28)(1.94)}{2[2 + (1 + 1.28)(1.94)^2]} = 3.56$ ksi

Eq. 22: $V_{\tau c} = 3.56(36.75) = 131$ kips

Eq. 30: $V_{\sigma c} = V_{\sigma} = 237$ kips

Eq. 31: $V_{fc} = 97(0.01 + \frac{3.56}{7.72}) = 45.6$ kips

Eq. 21: $(V_{uc})_w = 131 + 237 + 45.6 = 413.6$ kips

$$\underline{(V_{uc})_w = 413.6 \text{ kips}}$$

Compression Flange Failure

For $\lambda_v = 1.64 > \sqrt{2}$

Eq. 33b: $B = 0.235(1.64) - 0.05 = 0.335$

Eq. 27: $\sigma_c = 3.56(1.94)(27.4/7.72) = 24.4$ ksi

Eq. 32: $$V'_{\sigma c} = \frac{[67.5 + 30(7/16)^2](97.5 - 24.4) - 14(45.6)}{0.335\left(\frac{763}{237}\right)\left(\frac{180}{192}\right)\sqrt{\frac{33(84)}{36(36.4)}} + 14} = 305 \text{ kips}$$

Eq. 34: $(V_{uc})_c = 131 + 305 + 45.6 = 481.6$ kips

$$(V_{uc})_c = 481.6 \text{ kips}$$

Since $(V_{uc})_w < (V_{uc})_c$, $\underline{V_{uc} = 413.6 \text{ kips}}$

Eq. 35: $M_{uc} = 14(84)(413.6) = 486{,}000$ kip-in.

$$\underline{M_{uc} = 486{,}000 \text{ kip-in.}}$$

Check Maximum Panel Moment

Eq. ~~6~~ 5 (or from(a)): $(M_u)_c = 575{,}000$ kip-in.

Eq. 9 (or from(a)): $(M_u)_t = 450{,}000$ kip-in.

Since $(M_u)_c > (M_u)_t$, use $(M_u)_t$

Eq. 36: The ultimate shear $V'_u = \frac{450{,}000}{84(14 + (1/2)1.5)} = 363$ kips

Since $V'_u < V_{uc}$, (363 $<$ 413.6), the ultimate capacity of the panel under combined loads ($\mu = 14$) is

$$\underline{V_{uc} = V'_u = 363 \text{ kips}}$$

and (Eq. 35): $M_{uc} = 14(84)(363) = 427{,}000$ kip-in.

$$\underline{M_{uc} = 427{,}000 \text{kip-in.}}$$

(d) Summary of Results

(a) Pure Bending: -------------------------- $M_u = 450{,}000$ kip-in.

(b) Pure Shear: ---------------------------- $V_u = 618$ kips

(c) Combined Bending and Shear ($\mu = \frac{M}{bV} = 14$): $V_{uc} = 363$ kips

$M_{uc} = 427{,}000$ kip-in.

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10. APPENDIX II. - NOTATION

A_{fc}	Area of the compression flange.
A_{ft}	Area of the tension flange.
$A_w=bt$	Area of the web.
B	Parameter defined by Eq. 33.
E	Modulus of elasticity (Young's modulus).
F	Factor defined by Eq. 29.
I	Moment of inertia of the girder cross section.
I_f	Moment of inertia of the compression flange about the vertical axis.
I_w	Moment of inertia of the web about the centroidal axis of the whole cross section.
L	Unbraced length of the compression flange.
M	Design moment at mid-panel.
M_{max}	Maximum moment in the panel.
M_p	Plastic moment of the panel.
M_u	Ultimate moment of the panel under pure bending.
M_{uc}	Ultimate moment of the panel under combined forces.
M_y	Moment causing yielding of the tension flange.
R	Ratio of the maximum tensile stress (or minimum compressive stress) to the maximum compressive stress of the web (negative when the stress is tensile).
V	Design shear at mid-panel.
V_f	Frame action shear under pure shear.
V_{fc}	Frame action shear under combined forces.
V_p	Plastic shear of the web.

V_σ Tension field action shear under pure shear.

$V_{\sigma c}$ Tension field action shear under combined forces.

$V'_{\sigma c}$ Incomplete tension field action shear under combined forces.

V_τ Beam action shear under pure shear.

$V_{\tau c}$ Beam action shear under combined forces.

V_u Ultimate shear strength of the panel under pure shear.

V'_u Ultimate shear controlled by the maximum panel moment.

V_{uc} Ultimate shear strength of the panel under combined forces.

a Panel length.

b Panel depth.

c_c Half width of the compression flange.

c_t Half width of the tension flange.

d_c Thickness of the compression flange.

d_t Thickness of the tension flange.

k_v Plate buckling coefficient for pure shear (Eq. 13a and 13b).

k_b Plate buckling coefficient for pure bending (Eq. 26).

t Web thickness.

y_c Distance from the centroidal axis to the compression edge of the web.

y_t Distance from the centroidal axis to the tension edge of the web.

$\alpha = a/b$ Aspect ratio.

λ_b Web buckling parameter for bending, Eq. 25.

λ_v Web buckling parameter for shear, Eq. 12.

λ_L Lateral buckling parameter, Eq.7c.

λ_t Local (torsional) buckling parameter, Eq. 8c.

$\mu = \frac{M}{bV}$ Shear span ratio.

ν Poisson's ratio.

σ_c	Bending buckling stress at the extreme compression fiber of the web.
σ_{cf}	Buckling stress of the compression flange.
σ_{cr}	Web buckling stress under pure bending.
σ_{yc}	Yield stress of the compression flange.
σ_{yt}	Yield stress of the tension flange.
σ_{yw}	Yield stress of the web.
τ_c	Shear buckling stress under combined forces.
τ_{cr}	Shear buckling stress under pure shear.
τ_y	Shear yielding stress.

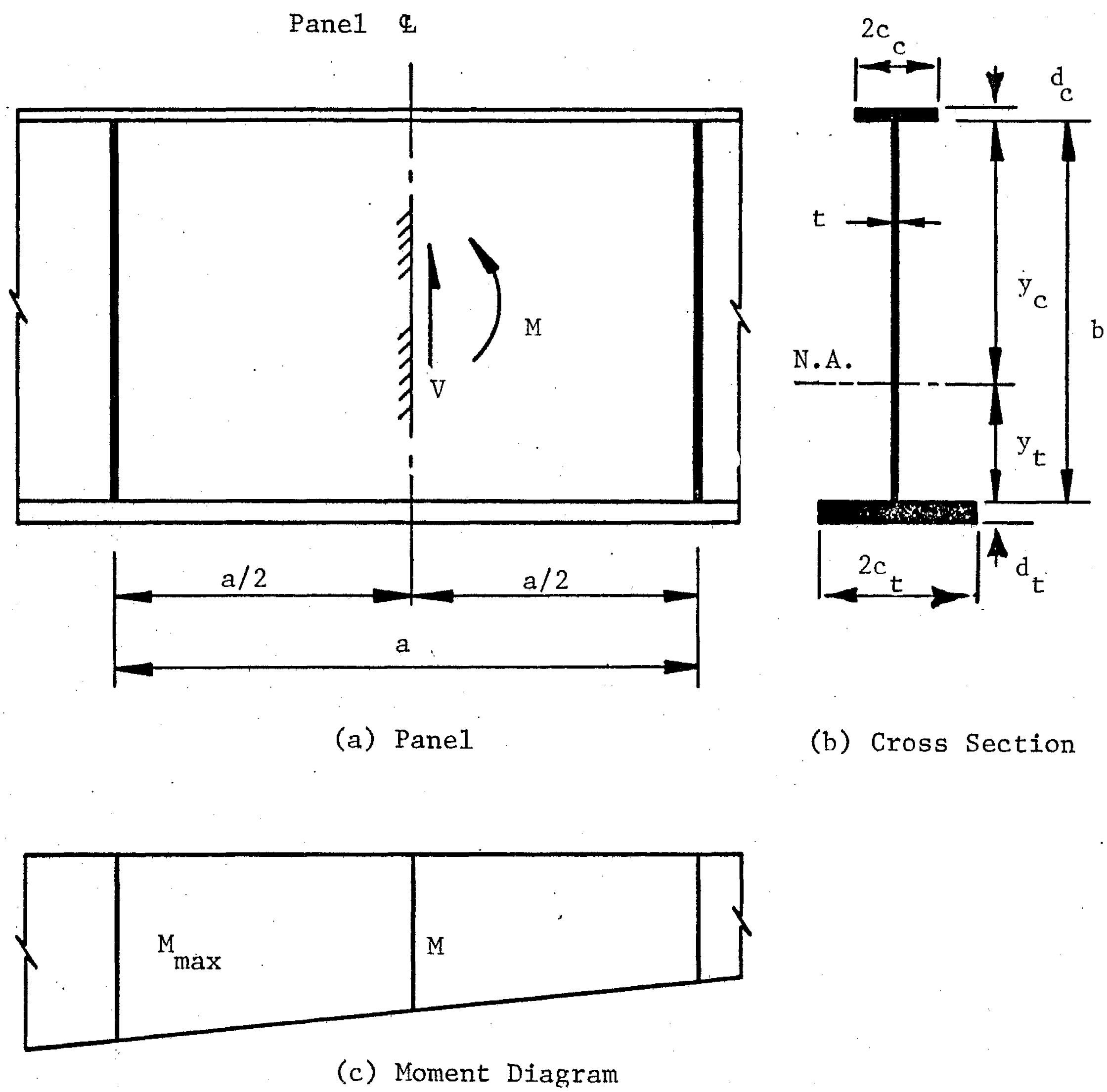


Fig. 1 Plate Girder Panel With Design Forces

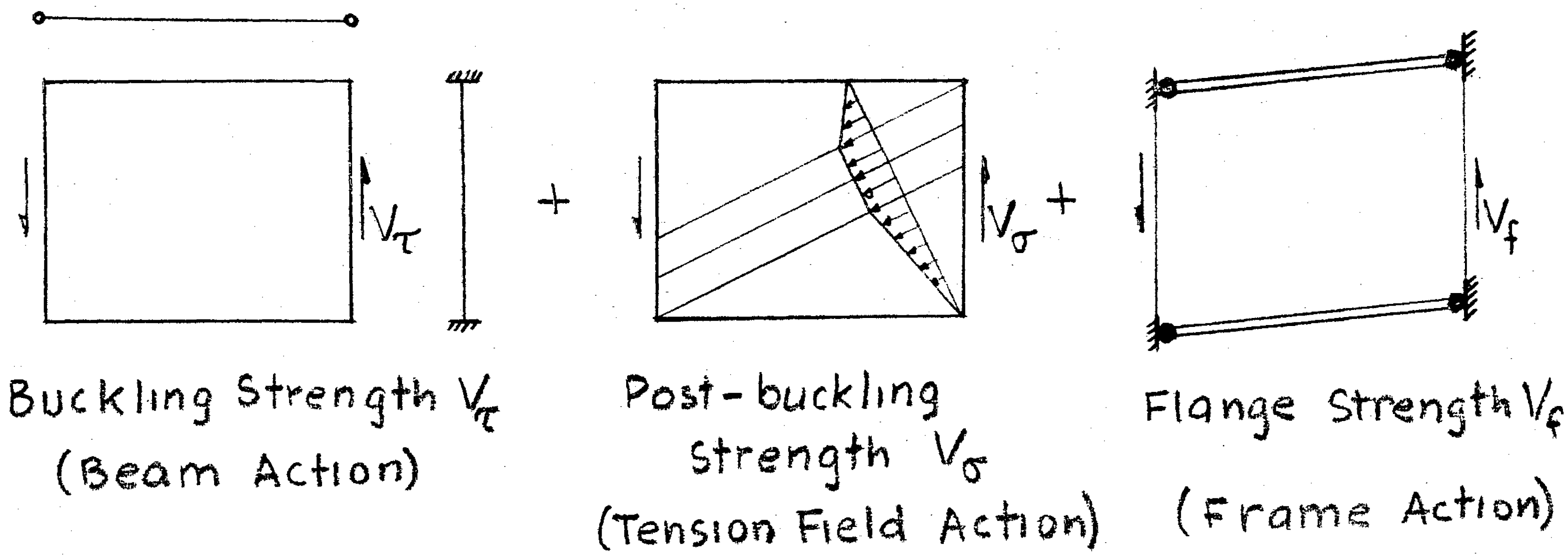


Fig. 2 Panel Strength Under Pure Shear

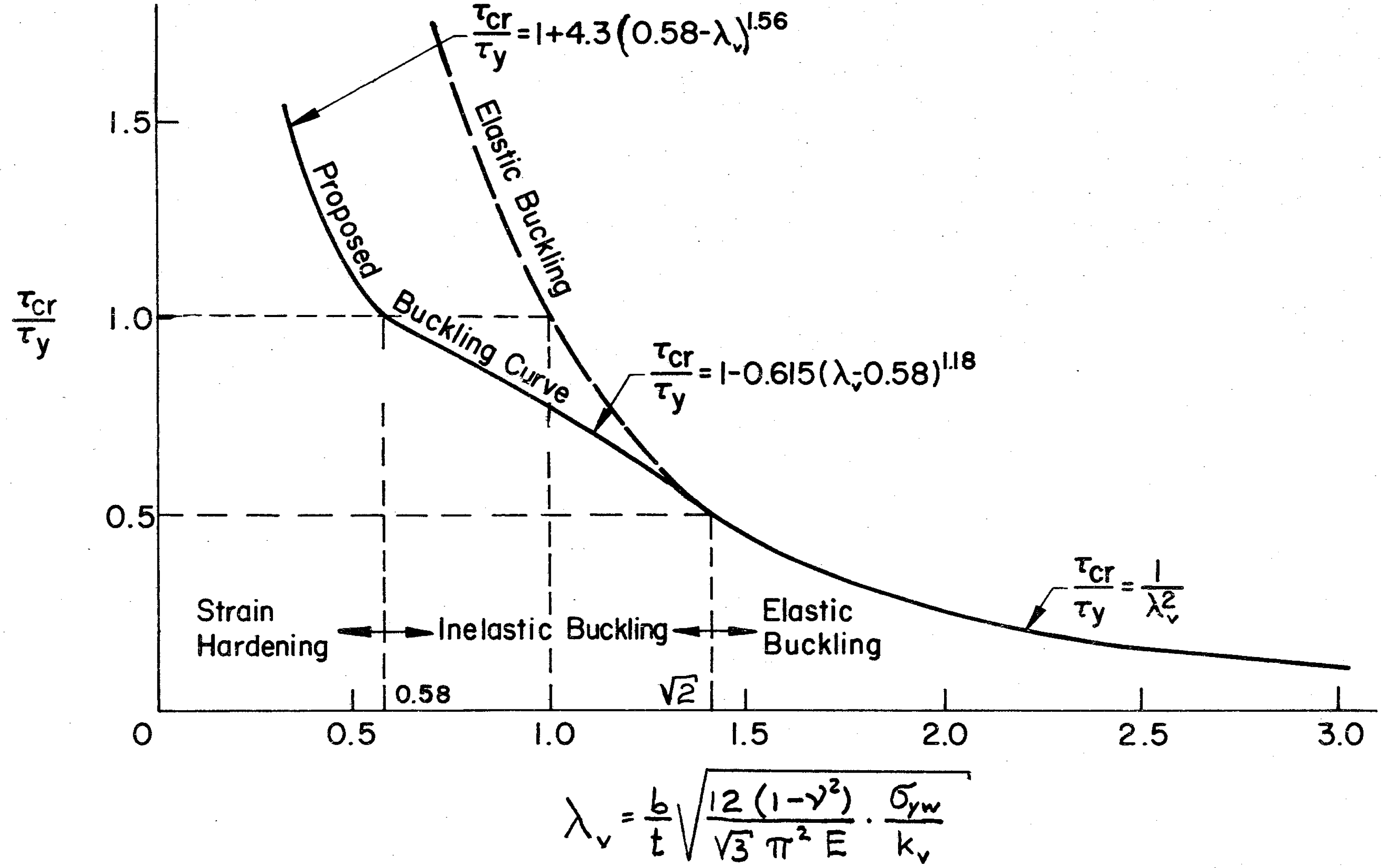


Fig. 3 Buckling Strength of the Web Under Pure Shear

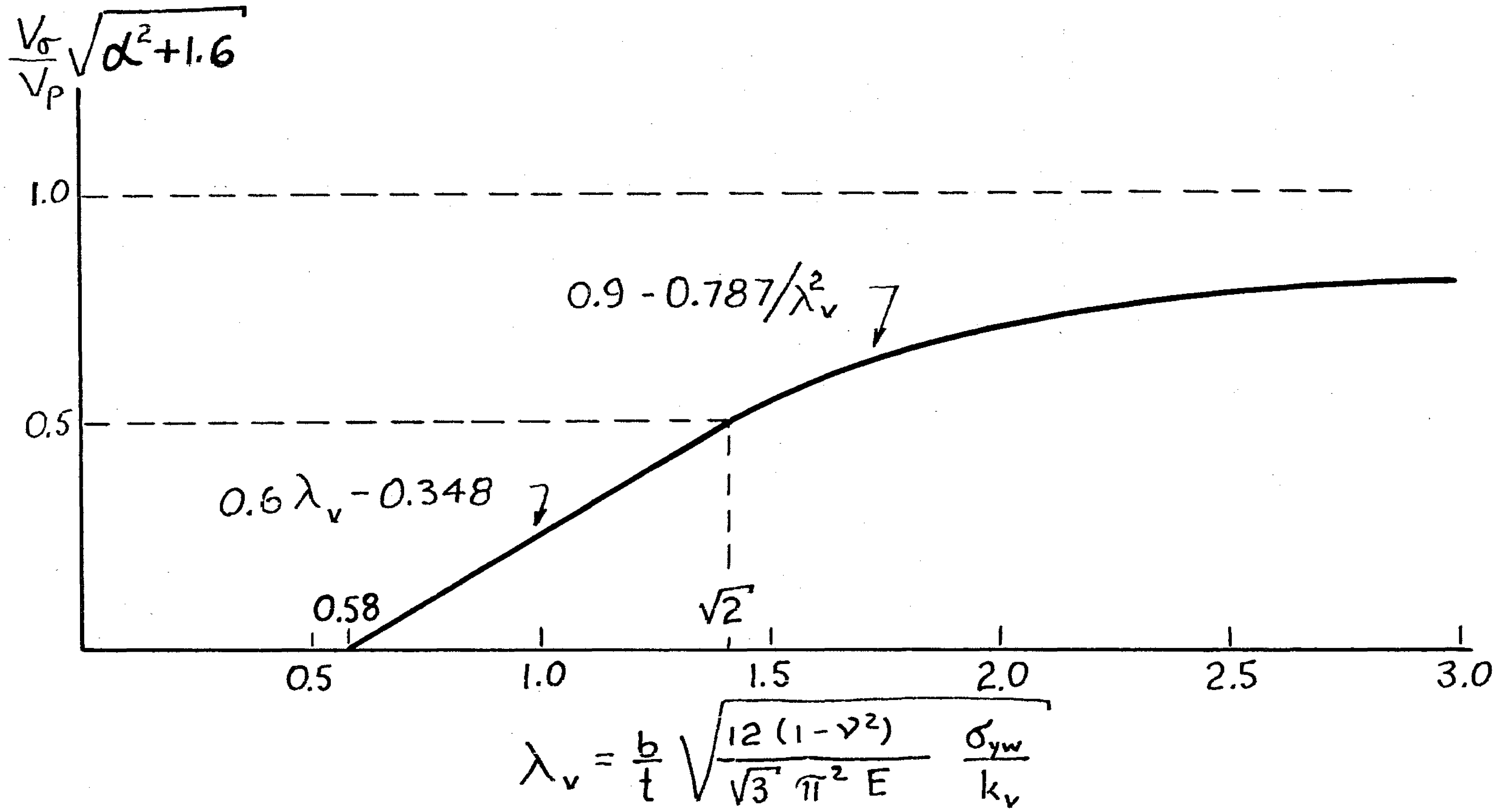


Fig. 4 Post-buckling Web Strength Under Pure Shear

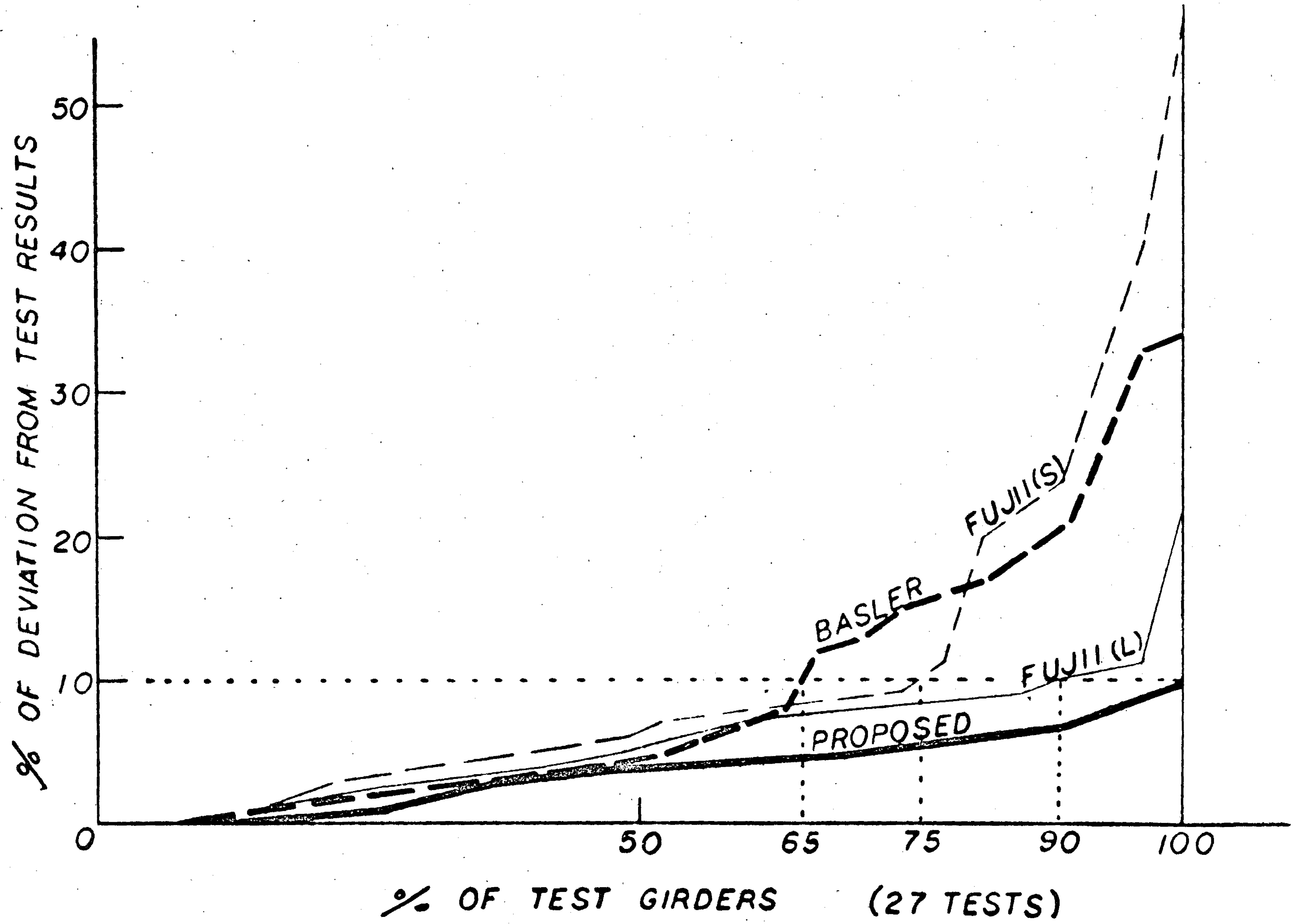


Fig. 5 Comparison of Proposed Design Formulas and Basler's and Fujii's Methods with Test Results

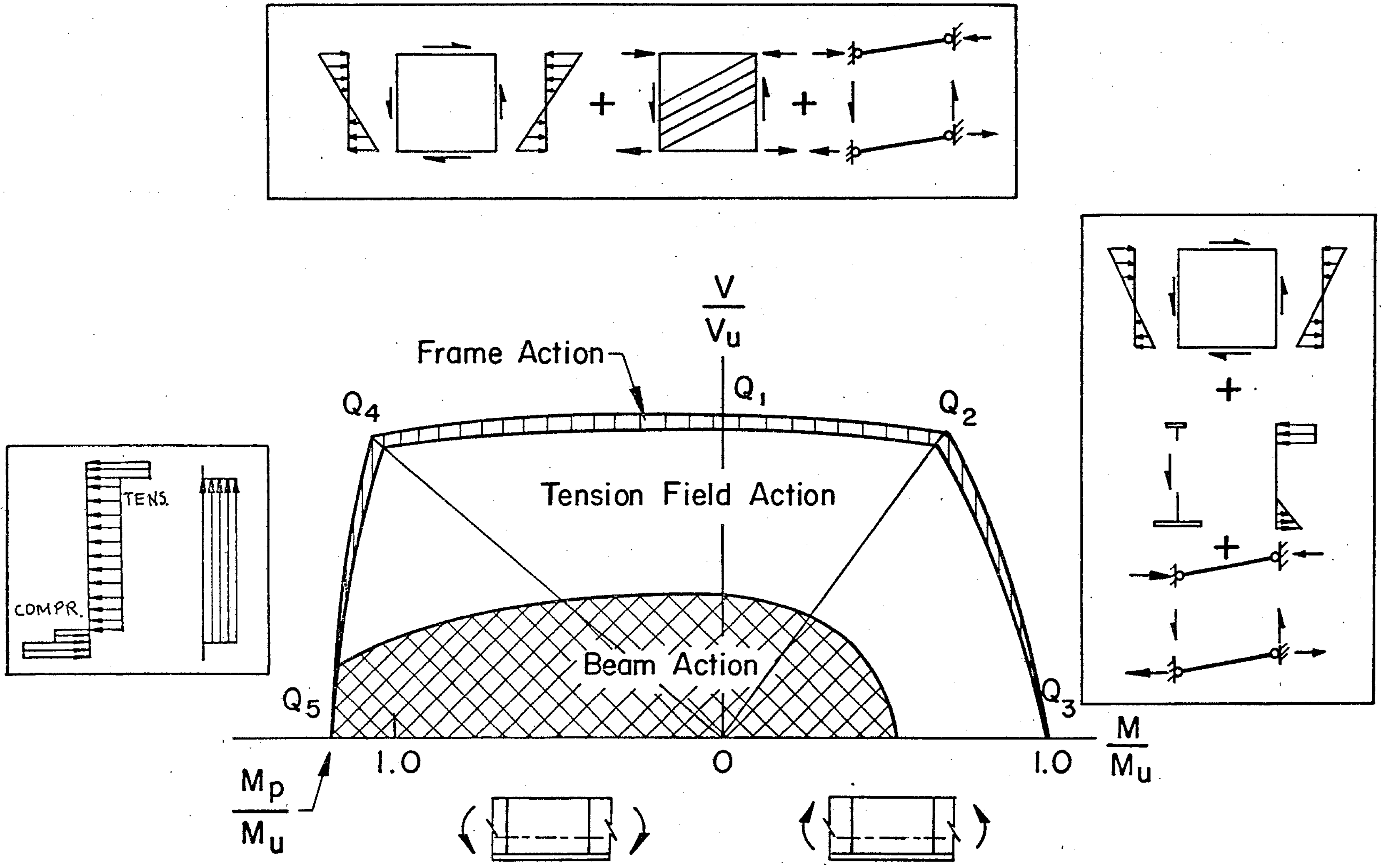


Fig. 6 Schematic Interaction Diagram and Failure Modes

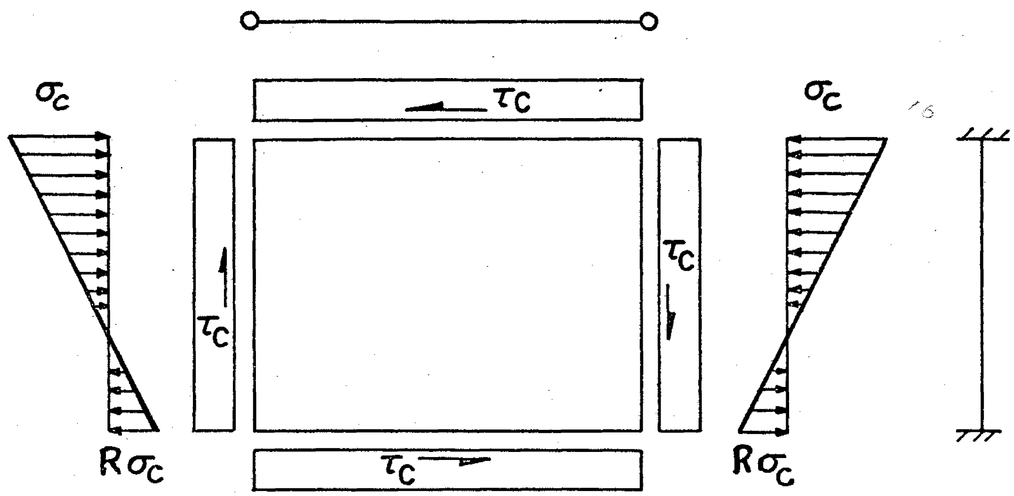


Fig. 7 Web Stresses Under Combined Loads

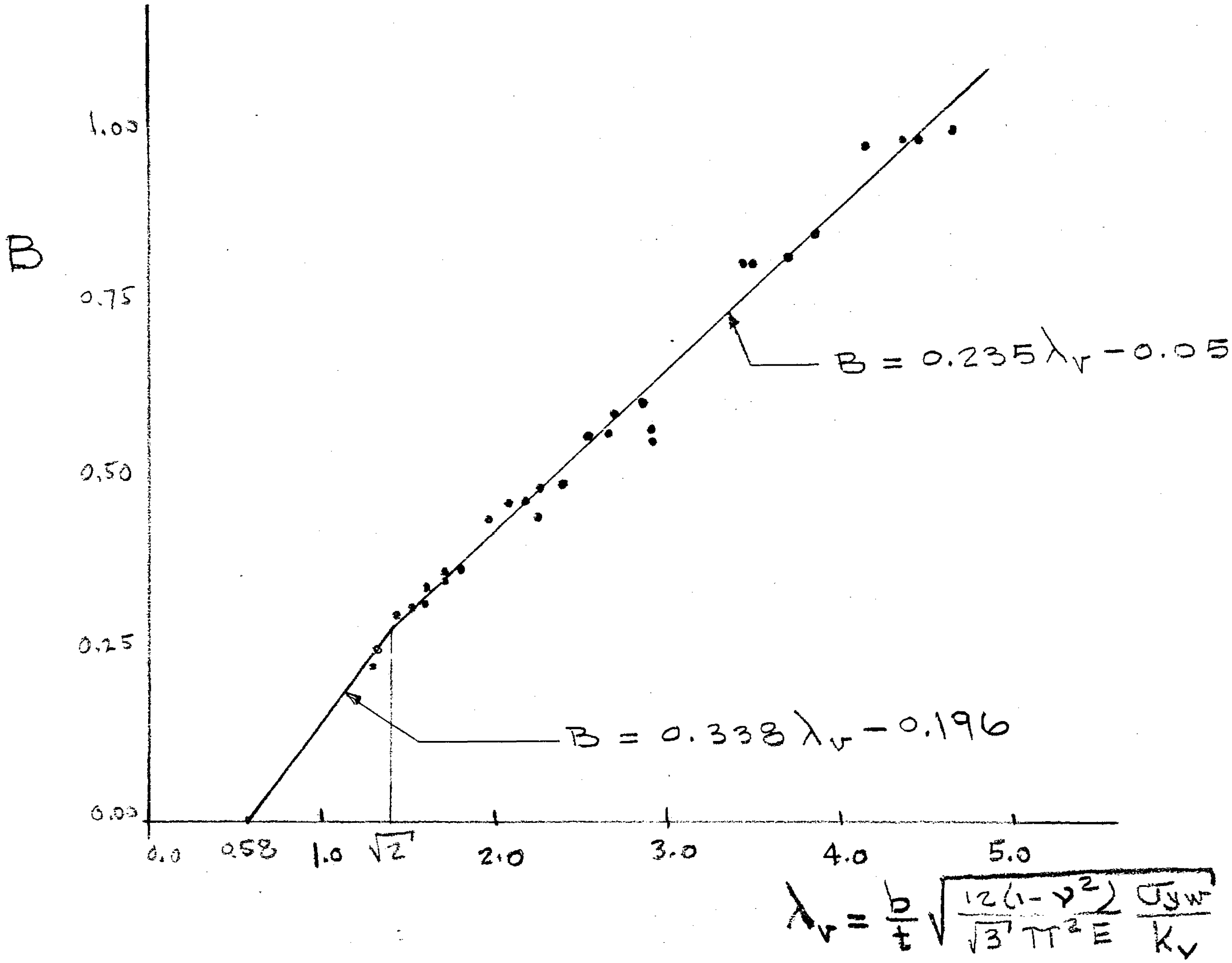


Fig. 8 Plot of Parameter B (Combined Forces)

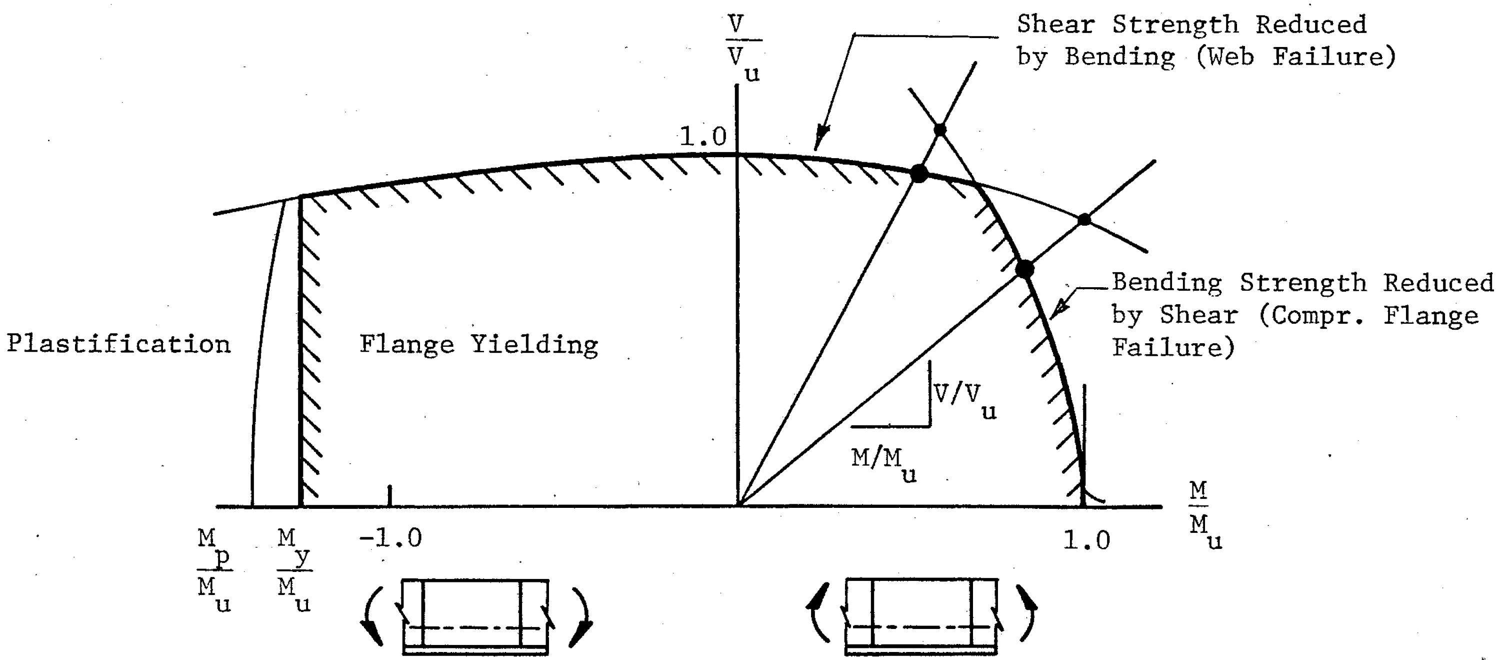
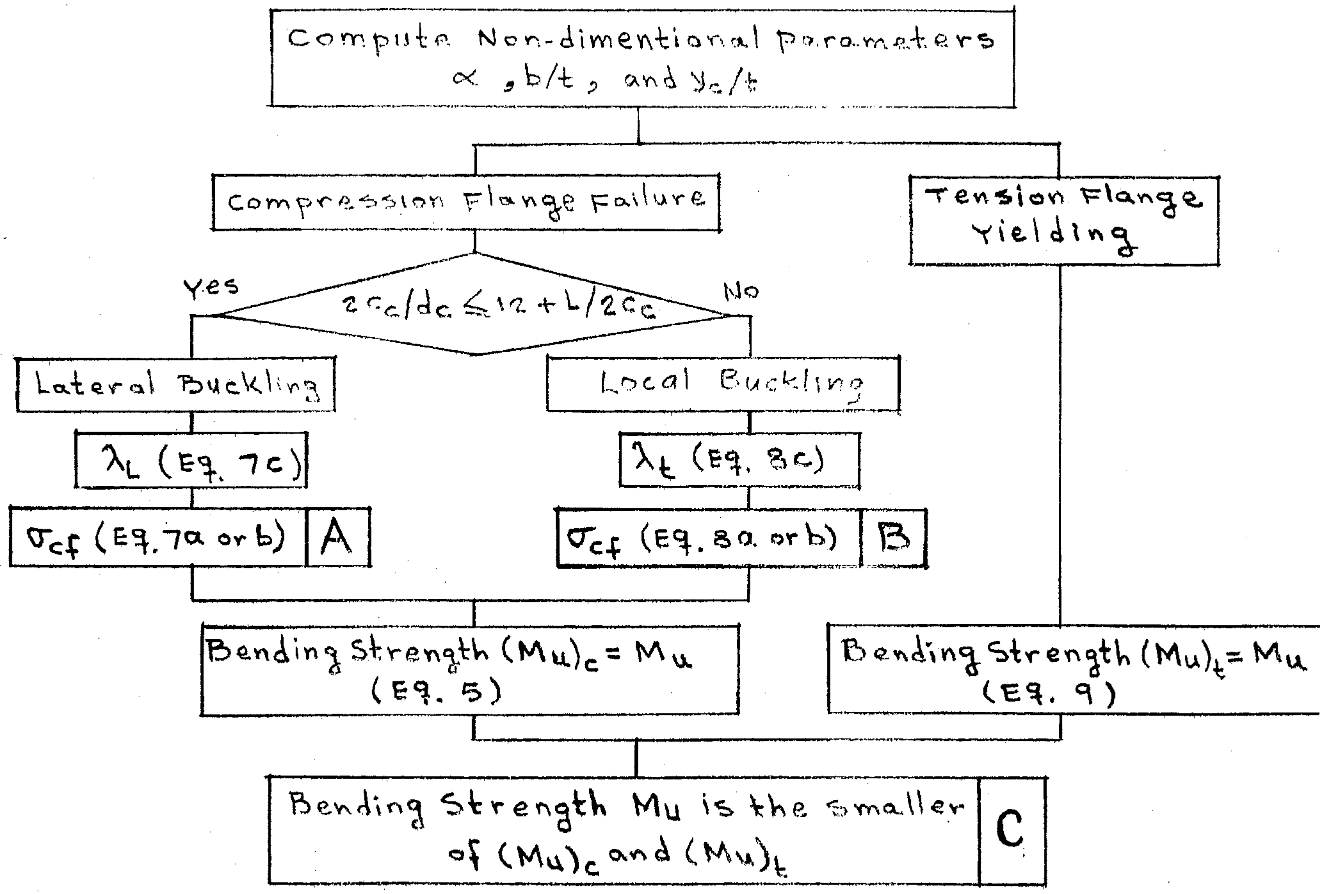
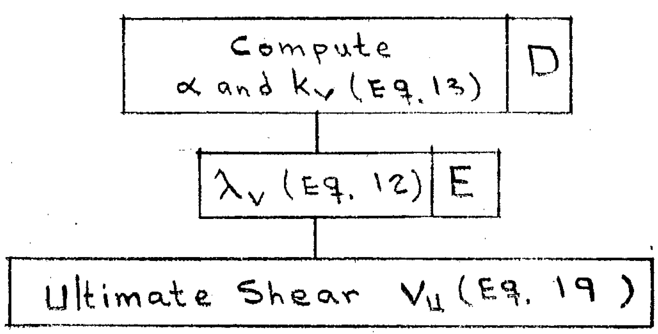


Fig. 9 Design Interaction Diagram



(a) Bending Strength



(b) Shear Strength

Note: Letters A, B, C, D and E designate operations the results of which are also used in Fig. 11.

Fig. 10 Computational Sequence for Bending and Shear Strengths

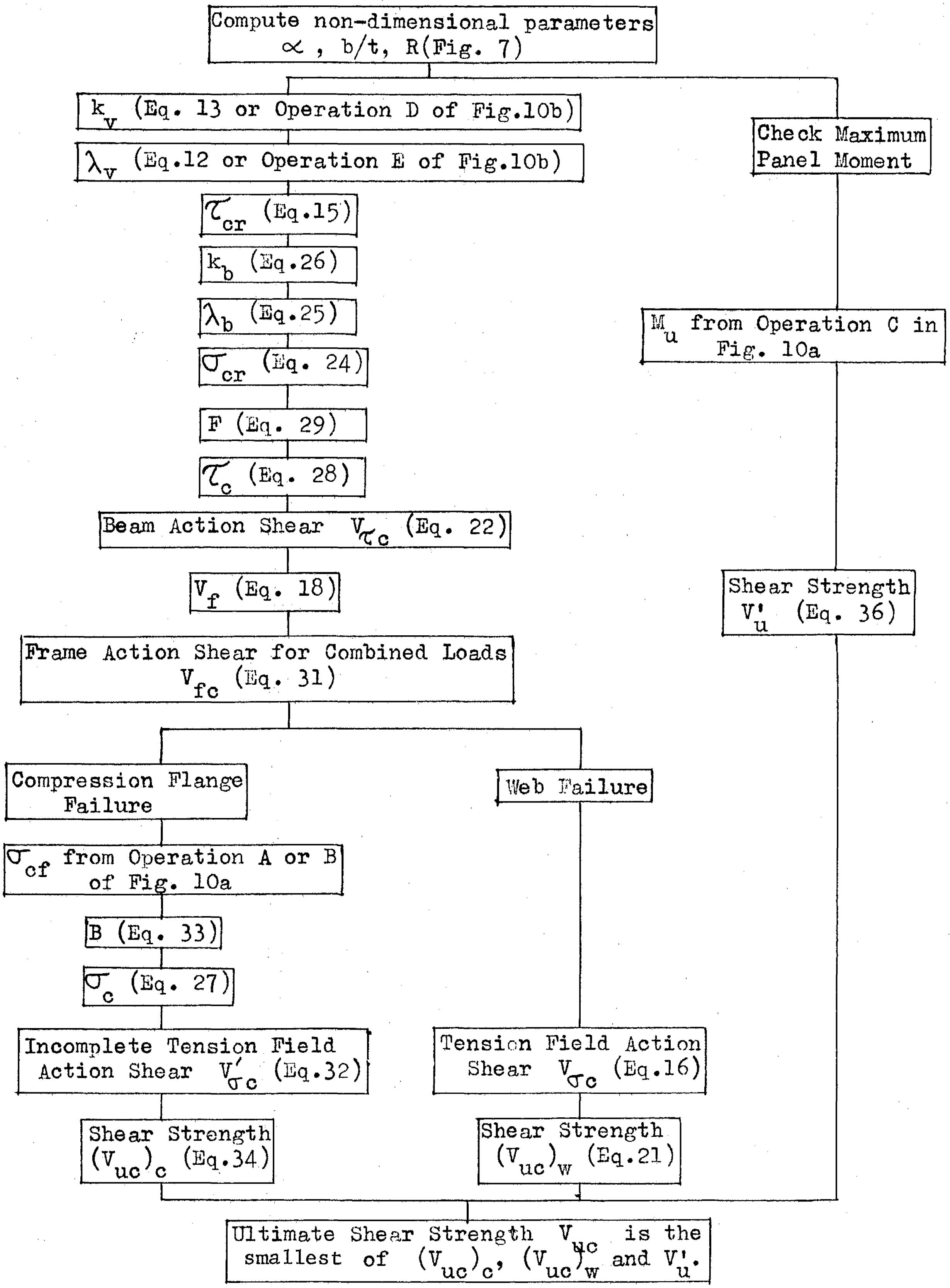


Fig. 11 Computational Sequence for Combination of Shear and Bending