

1963

On the fatigue strength of welded plate girders, Ph.D. Dissertation, November 1963

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ON THE FATIGUE STRENGTH OF WELDED PLATE GIRDERS

B. T. Yen

Fritz Engineering Laboratory Report No. 303.1

Submitted to
The Welded Plate Girder Project Subcommittee
of
Welding Research Council
for Approval for Publication

ON THE FATIGUE STRENGTH OF WELDED PLATE GIRDERS

by

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Fritz Engineering Laboratory Report No. 303.1

November, 1963

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S Y N O P S I S

The objective of this report is to formulate and analyze problems concerning the strength of welded plate girders subjected to repeated loading. To begin, a distinction is made between the analysis of the fatigue strength of girders and that of metals and weldments. Then, in reviewing the behavior of girder webs, it is pointed out that web plates deflect laterally under load. Fluctuation of web plates is thus a consequence of repeated application of load, and fluctuating plate bending stresses may cause fatigue cracking of girder webs along their boundaries. Whereas it is not possible to predict mathematically the occurrence of fatigue cracks by bending of the web plate, evidence of such a phenomenon was indeed obtained from an experimental investigation on two full-size girders with slender webs. The girders were primarily subjected to high shear with a load range of half-maximum to maximum. Effects of repeated loading on stresses and deflections were small, and web cracks propagated slowly. Cracks of a few inches long at panel boundaries were observed to have little influence on the load-carrying capacity of girders. Furthermore, repairs of cracks were proven successful. The results of this investigation are compared with current design limitations and permissible stresses are recommended. Further investigations are suggested to substantiate the obtained results.

1. I N T R O D U C T I O N

Plate girders used as main structural members of buildings and bridges have long been designed to meet both strength and stability requirements. Careful consideration has always been given to the slenderness of the girder web. Because the true static strength of a girder was not known, the criterion used for practical design was the computed web buckling load even though attainment of this load did not necessarily exhaust the strength of a plate girder. Recently, a theory has been developed providing a reliable method of evaluating the static carrying capacity of plate girders^(1,2,3) and consequently furnishing a more reasonable basis for design. Plate girders may now be designed with webs more slender than those permitted in the past.⁽⁴⁾

Highway and railway bridge girders are subjected to repeated loads. The problem of fatigue, therefore, has been an important one for the design of welded bridge girders to avoid cracking and failure. With the advent of more slender webs for girders, it becomes necessary to review the fatigue strength accordingly.

For the evaluation of fatigue strength of welded plate girders, it is essential to know the basic fatigue characteristics of girder materials and weldments. In engineering practice, the approach has been to compile data concerning the behavior of different metals and weldments under various stress conditions and environments. Reference 5 gives a summary of the results of numerous

investigations, together with an extensive list of references. Reference 6 contains a review of steel weldments and a bibliography.

Under a stable environment and a given stress pattern, the fatigue property of a metal can be expressed through three variables: the maximum stress (S_{\max}), the minimum stress (S_{\min}), and the fatigue life in number of cycles (N). Therefore, results of investigations on a metal can be presented graphically on a three-dimensional diagram with the three variables as coordinates. Figure 1, with stress coordinates expressed as a function of the static ultimate strength of the material (S_u), schematically illustrates such a diagram. A point on the curvilinear surface of this diagram indicates the fatigue life corresponding to a given maximum stress and a given minimum stress. (For example, for point A, $S_{\max} = 0.8 S_u$, $S_{\min} = 0$ and $N = 10^5$ cycles). When the minimum stress is kept constant, the fatigue life varies with the magnitude of the maximum stress. A corresponding curve from Fig. 1 is the so-called "S-N curve", Fig. 2, which is drawn for $S_{\min} = 0$. On the other hand, for a particular fatigue life N , the relationship between the maximum and the minimum stresses may be expressed by another two-dimensional diagram obtained from Fig. 1. This is, in effect, a "modified Goodman diagram" or an "AWS-WRC diagram", Fig. 3, where N equals 10^5 cycles.

In actuality, the determination of an S_{\max} - S_{\min} - N diagram or the two-dimensional diagrams involves considerable difficulties. The scattering of test results may be so great that no clearly

defined relationship can be determined between the stresses and the fatigue life. Then, a statistical analysis of data should be made leading to a fourth variable, the probability of failure. In an S_{\max} - S_{\min} - N diagram these would be a set of curvilinear surfaces each corresponding to a certain probability of failure. For lack of sufficient data, this statistical consideration has often been disregarded in the past.

The fatigue characteristics of weldments are even more complicated than those of metals. In addition to the aspects stated before, the geometry and the configuration of a weldment must be considered because they affect the state of stress at any given point. Furthermore, welding creates a heat-affected zone in the parent metal where the metallurgical properties are modified by the weld metal and the heat input. This, in general, leads to a lower fatigue strength of a weldment.⁽⁵⁾ In compiling fatigue data, sample weldments are usually subjected to simple loading patterns, such as fillet-welded lap joints under direct tension, butt-welded plates under bending, etc. There are many frequently used configurations of weldments and various basic loading patterns. To construct fatigue strength diagrams for all types of weldments of different metals under all possible loading conditions, is a broad and time consuming task which has been a constant goal in fatigue investigation.⁽⁶⁾

Assuming that the fatigue characteristics of metals and weldments can be obtained, the fatigue strength of a structural member could be determined if the stresses at critical locations could be

computed. If there were a reliable method of analysis which correctly described the stresses, then a strength prediction should be possible. For example, the fatigue strength of a fillet-welded, built-up I-beam under repeated flexural loading could be regarded to be the same as that of a welded tee joint under the same loading condition. This would be due to the fact that the joint represents the critical detail of the member under consideration.

However, methods of precisely describing the stresses in a structural member under given loads are not always available. When plate girders were designed to exclude the possibility of "web buckling", stresses could be predicted with simple beam theory. Consequently, the estimation of fatigue strength of welded plate girders has been regarded as identical to that of welded I-beams. Butt-welded flange plates, partial length cover plates, flange-web junctions, transverse stiffener attachments, and other structural sub-assemblies at which the local stress distributions are not clearly known have been locations of fatigue cracking and thus the key points of study. They will remain as main features of investigations for the near future. In addition, new problems are encountered if girder webs are permitted to be stressed into the post-buckling range. Are the slender webs which are permitted for static loading also adequate for repeated loading? Are welded plate girders more susceptible to fatigue failure than before? What is the influence of "tension field action" on fatigue strength of slender web plate girders? To begin the study of these problems, a brief review of a girder's static strength follows.

II. BEHAVIOR OF PLATE GIRDER WEBS

2.1 WEB BUCKLING AND STATIC STRENGTH

Measurements show that webs of plate girders are seldom plane and that sudden buckling of webs under load is generally non-existent. (7,8,9,10) Under an applied bending moment in the plane of the girder, a web which is not initially plane deflects laterally. Measured cross-sectional configurations of a girder subjected to pure bending and the corresponding flexural stresses are shown in Fig. 4. The applied moments are expressed in terms of the yield moment, M_y -- the moment at which yielding begins at the extreme fiber of a flange. The stresses are lower than predicted in the web and higher in the compression flange, indicating a redistribution of stress from the web to the compression flange as a result of lateral deflection of the web. By considering this phenomenon, the static strength of girders in bending is estimated to be that of an imaginary column consisting of the compression flange and a portion of the web. (1) The slenderness of the web affects the column strength but the web buckling load is no longer significant.

When shear is applied to a girder web which has an initial deflection, again the deflection changes gradually with load. Redistribution of stresses to sustain high shear forces is accomplished through a so-called "tension field action" (2) which is comparable to the load-carrying action of a Pratt truss (Fig. 5). The shear force

in a truss panel is carried by the tensional diagonal. As long as the neighboring vertical struts and chord members provide suitable anchorage, a diagonal is able to sustain a static tensile force of $\sigma_y A$, where σ_y is the yield point of the truss material and A the cross-sectional area of the member. Analogously, if the vertical stiffeners and the flanges of a girder can provide an adequate anchorage, a portion of the girder web can sustain tensile stresses up to yielding or develop tension field action. Because a plate girder is a deep beam by nature, it is assumed that the web resists shear according to the beam theory up to the web buckling stress and thereafter by tension field action. By taking into account panel dimensions, assuming a tension field geometry, and considering yield stress limitations, the shear strength of a girder panel can be evaluated.⁽²⁾ For example the shear strength of structural carbon steel girders with different geometrical configurations are given in Fig. 6 in terms of the plastic shear strength of webs. For girders with the same ratio of stiffener spacing to web depth (aspect ratio = $\alpha = a/b = \text{constant}$), the shear strength decreases with increasing web slenderness ratio ($\beta = b/t$). For the same web depth to thickness ratio ($\beta = \text{constant}$), the static strength increases with decreasing aspect ratio. These relationships have been confirmed by experimental investigations.^(7,8)

The utilization of the above-described post-buckling strength often creates stresses higher than those permitted by the web buckling theory. From the viewpoint of fatigue, cracks conceivably

might be expected to initiate at a lower number of cycles of load application than for girders designed on the basis of web buckling. Since a tension field requires anchorage, would cracks occur near panel corners where the tension field is anchored? Since web plates deflect laterally under load, would the fluctuating of the web be of consequence under repeated loading? These are problems not encountered in welded I-beams but are unique to welded plate girders. They are directly tied to the deflections of a web; thus, analysis of these deflections is made next.

2.2 LATERAL DEFLECTION OF GIRDER WEBS

The magnitude of lateral deflections of girder webs under membrane force can be of the order of the web thickness. (7,8,9,10) The relationship between such "large" deflections and the membrane forces may be obtained by considering the equilibrium of a plate element in the deflected position. If the coordinate system (x, y, z) and the forces per unit length (N_x , N_y , N_{xy}) acting in the middle plane of a small element are as shown in Fig. 7, the equilibrium equations give the following: (11)

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (1)$$

$$\left. \begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} &= 0 \end{aligned} \right\} \quad (2)$$

With four unknowns (w , N_x , N_y , and N_{xy}) but only three equations, another relationship is required. This is provided through the compatibility equation:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \quad (3)$$

in which

$$\left. \begin{aligned} \epsilon_x &= \frac{1}{Et} (N_x - \nu N_y) \\ \epsilon_y &= \frac{1}{Et} (N_y - \nu N_x) \\ \gamma_{xy} &= \frac{1}{Gt} N_{xy} \end{aligned} \right\} \quad (4)$$

In the above equations,

w = a function defining the web deflection

D = flexural rigidity of the web plate = $\frac{Et^3}{12(1-\nu^2)}$
(t is the plate thickness)

E = Young's modulus

G = Shear modulus = $\frac{E}{2(1+\nu)}$

ν = Poisson's ratio, and

$\epsilon_x, \epsilon_y, \gamma_{xy}$ = the strains corresponding to the unit forces $N_x,$
 $N_y, N_{xy}.$

In addition, the boundary conditions of the web plate must be defined to solve for the deflections. A closed solution of these equations can be obtained for a limited number of cases. In any event, a solution is of significance only if it takes into account the initial deflection of the web, and this is not incorporated in the above equations.

One way of solving the equations is to assume an approximate deflection shape fulfilling applicable boundary conditions and then to determine the magnitude of the deflection at a given load. Hence, a deflection surface of a rectangular web plate represented by the trigonometric series

$$w = \sum_{1}^{\infty} \sum_{1}^{\infty} C_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \quad (5)$$

implies a simply supported edge condition, whereas the shape

$$w = \sum_{1}^{\infty} \sum_{1}^{\infty} C_{mn} \left(1 - \cos \frac{m \pi x}{a}\right) \left(1 - \cos \frac{n \pi y}{b}\right) \quad (6)$$

is for a web with built-in edges. The coefficients C_{mn} are a measure of the deflection magnitude and are to be determined for each individual case.

Another approximate method of obtaining load-deflection relationships is by considering the strain energy of the web plate.⁽¹¹⁾

For example, the case of a simply-supported web panel under edge compression has been investigated,⁽¹²⁾ and the result is shown in Fig. 8. In this figure the applied strain ϵ_a is expressed in terms of the web buckling strain ϵ_{cr} , or $X = \epsilon_a / |\epsilon_{cr}|$, whereas the initial and final lateral deflections are nondimensionalized as Y_i and Y . The deflection configuration is assumed to remain constant, only the magnitude changes with the applied strain. It can be seen that only when the initial deflection is zero ($Y_i = 0$ at $X = 0$) does the web remain plane below the critical compressive strain of $X = -1$. With an initial deflection, the web deflection increases with increasing compressive strain. Lateral deflections of girder webs always exist when applied strains are above the web buckling values.

Figure 8 indicates that there is a definite web deflection position corresponding to each applied load intensity on a girder. In other words, a fluctuating load intensity causes corresponding fluctuations of the web plate.

However, Fig. 8 or even a solution of Eqs. 1 to 4 using an assumed deflection shape gives only a qualitative indication of the web deflection. The difficulty of obtaining a quantitative solution for the web deflection is not so much due to the complexity of the equations. Rather, it is due to the uncertainty of physical conditions essential for computation. First, the initial deflected shape of a web is by no means predicted from present day knowledge. It can only be known by direct measurement after a girder is fabricated.

These measured magnitudes differ from girder to girder, as indicated by the data of Table 1.^(7,8) Secondly, the determination of the restraint which is offered to the web panel boundary usually is complicated and is not clearly defined. Factors that contribute to the complicated nature of the boundary restraint are the stiffener and the flange sizes, the weld sizes, and the rigidity offered by the neighboring panels. Thirdly, the loading condition along the panel boundary does not remain constant when the web deflects under bending moment or under tension field action. Finally, the web deflection often changes its configuration with the change of the applied load magnitude. All these uncertainties must be evaluated for a mathematical computation of web deflection. Approximations, therefore, are to be made for deflection estimation (Section 3.2).

2.3 WEB BENDING STRESSES

The effects of lateral web deflection are two fold. Besides affecting the primary membrane stresses in the web, they also create plate bending stresses across the web thickness. These two sets of stresses and their resultant stresses are shown on a small plate element in Figs. 9a, 9b, and 9c, respectively. The membrane stresses act uniformly over the entire thickness of the web whereas the plate bending stresses vary linearly. Even though both sets of stresses change with varying web deflection, the magnitude of these changes

are not proportional to each other. For girders with slender webs, the web bending stresses may have much higher magnitudes than the membrane stresses. Nevertheless, because the summation of the plate bending stresses over the thickness is zero, they have no effect on the static carrying capacity of a girder and are justifiably disregarded in evaluating the static strength. As will be seen later, they cannot be neglected in fatigue considerations.

If the web deflection of a girder web is described by a function w for a given load, the web bending stresses are derived through the ordinary plate flexural equations:

$$\left. \begin{aligned} \bar{\sigma}_x &= -\frac{Ez}{1-\nu^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ \bar{\sigma}_y &= -\frac{Ez}{1-\nu^2} \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \end{aligned} \right\} \quad (7)$$

where $\bar{\sigma}_x$ and $\bar{\sigma}_y$ are the web bending stresses in the x and y direction (Fig. 9) and z is the ordinate perpendicular to the middle plane of the web. The corresponding shear stress is

$$\bar{\tau}_{xy} = -2Gz \frac{\partial^2 w}{\partial x \partial y} \quad (8)$$

where the constants E , G , and ν are as defined before.

Since the fluctuating loads cause corresponding fluctuations of the web plates and each deflected position of the web has its own set of plate bending stresses as described by Eq. 7, a plate girder under repeated loading is subjected to repeated web bending stresses. The analysis of their influence on the fatigue strength of a girder relies upon the evaluation of their magnitude. Unfortunately, due to inadequate knowledge of the initial and final web deflection shapes, no true values of plate bending stresses can be obtained at this time. As a result, only a qualitative examination is possible at most. Such an examination is made in the next chapter.

III. CONSIDERATION OF WEB STRESSES IN FATIGUE

3.1 STRESSES IN A WEB

From previous discussions, it is clear that there are two groups of stresses in a girder web: the membrane stresses and the plate bending stresses. To the former belong the web buckling stresses, the tension field stresses, and the primary bending stresses, in contrast to the secondary, plate bending stresses. As has been mentioned before, the buckling stress of a web is not significant in the determination of the static strength of a girder. Because of the nonexistence of the web buckling phenomenon, and because of the web fluctuation under applied loads, the web buckling stress is also not significant in fatigue consideration. Instead, residual stresses in welded plate girders play an important role along with other membrane and plate bending stresses.

Residual stresses in a girder are stresses created as a result of plastic deformation by welding, cold bending, or other fabrication methods. Although no measurements of residual stresses have been made on the type of girders considered in this study, experimentally obtained data on plain welded plates give an indication of their distribution.⁽¹³⁾ Based on these results a probable distribution of residual stresses in the longitudinal direction of

a structural carbon steel, welded plate girder section of 50 x 1/4 in. web and 12 x 3/4 in. flanges is constructed in Fig. 10. The stress magnitude is very high at the flange-to-web fillet weld but reduces rapidly with increasing distance from the weld. Along the welds the residual stress is tensile, hence superposition of any small additional tensile stresses may possibly bring the sum of stresses to the level of yielding. Therefore, residual stresses are probable one of the main reasons causing early fatigue failure of some welded beams.

With the knowledge that residual stresses are high along the web boundary, it is only logical to examine other web stresses to see their distribution also along the web boundary. First, it has been pointed out that the tension field stresses are anchored at the panel boundary. Secondly, it has also been indicated that the primary compressive bending stresses are higher along the flange-to-web junction than in the web (Fig. 4). Thirdly, in considering the deflection of girder webs under load, it may be expected that the plate bending stresses are also higher at the boundary where the curvature of plate bending is, in general, highest. Finally, in addition to all these stress conditions, the weldments along the web edges are more vulnerable to fatigue cracking than is the base metal in the web. Taking all the above into consideration, it is most likely that fatigue cracks will occur at the web boundary rather than in the web. Consequently, when stress examinations are made in the subsequent sections, attention will only be directed to the panel boundary.

3.2 STRESS APPROXIMATIONS

To gain some idea as to the order of magnitude of the plate bending stresses along a panel boundary, an approximate deflection shape must first be assumed. Since its evaluation by a mathematical process (Section 2.2) necessitates assuming an equation to describe the final shape (such as Eq. 5 which is approximate at best), a direct assumption of the deflection shape (with the guidance of experimental results) may provide a more reliable estimate. As an example, the deflection configuration (w) of a rectangular panel under high shear is approximated as depicted in Fig. 11, which is based on the results of actual measurements on full-size girders.⁽⁷⁾ The configuration is assumed antisymmetrical about the center of the panel where the deflection is highest. One of the assigned cartesian coordinate axes (x) is coincident with the tension diagonal of the panel. No matter what the boundary conditions of the web panel may be, the magnitude and the slope of the deflection curve must vanish when approaching the corners. A simple, nondimensional equation fulfilling these conditions is

$$\omega(\xi) = (1 - \xi)^{2n}(1 + \xi)^{2n} = (1 - \xi^2)^{2n} \quad (9)$$

with the exponential constant n open for variation to fit individual cases. The nondimensionalized deflection parameter ω and length parameter ξ are expressed in terms of panel dimensions a , b , and the magnitude of maximum deflection w_0 at the panel center (Fig. 11).

$$\left. \begin{aligned} \xi(x) &= x / \sqrt{a^2 + b^2} \\ \omega(\xi) &= w(x) / w_0 \end{aligned} \right\} \quad (10)$$

In most cases the relatively heavy flange plates offer full restraint to the web plate whereas the much lighter transverse stiffeners have little rigidity for plate bending. Thus the highest plate bending stresses would be expected to occur at the flange-to-web junctions. By assuming that the deflection curve for a partial web strip perpendicular to a flange (in the shaded portion of Fig. 11) is that of a fixed-end beam subjected to equal and opposite end moments (upper right of Fig. 11), the plate bending moment at the junction is

$$M = 6EIw_0 \frac{\omega(\xi)}{l^2} \quad (11)$$

with I = moment of inertia of the web strip and $l = (1 - \xi)b/2$.

The estimated plate bending stresses are then given by

$$\begin{aligned} \bar{\sigma}_y &= \frac{M}{I} \cdot \frac{t}{2} \\ &= \frac{t}{2I} \cdot 6EIw_0 (1 - \xi^2)^{2n} \cdot \frac{4}{b^2(1 - \xi)^2} \\ &= 12E \left[(1 - \xi)^{n-1} (1 + \xi)^n \right]^2 w_0 t / b^2 \quad (12) \end{aligned}$$

or

$$\bar{\sigma}_y = 12E \left[(1 - \xi)^{n-1} (1 + \xi)^n \right]^2 \gamma / \beta^2 \quad (13)$$

where t is the web thickness, $\beta = b/t$, and $\gamma = w_o/t$.

For the evaluation of $\bar{\sigma}_y$, the exponential constant n must be known. Again, based on actual measurements, an empirical value can be determined. The measured deflection along a diagonal of a girder panel (girder G6 of Ref. 7) is plotted in Fig. 12. This deflection curve can be approximated by the dotted-line curve which is described by Eq. 9 using $n = 1.2$. With $n = 1.2$, and for a girder web with a slenderness ratio $\beta = 260$ and a web deflection four times the web thickness ($\gamma = 4$), the plate bending stresses given by Eq. 13 are plotted in Fig. 13. These are stresses at a flange-to-web junction and are perpendicular to the flange. It can be seen that the estimated stress is 36 ksi at the third point along the flange ($\xi = 1/3$). Such a stress intensity is as high as the yield point of the structural carbon steel. In fact, the maximum estimated bending stress in the present example is 46 ksi.

It must be pointed out that, although the assumptions on the deflection curve and boundary restraint may not be true, the computation nevertheless indicates the order of magnitude of the so-called "secondary bending stress". A plate bending stress as high as the membrane stress certainly plays an important part in the fatigue strength of a girder.

Along with the plate bending stresses, the membrane stresses also must be estimated. Instead of carrying through a tedious computation employing an assumed deflection shape, once more assumptions are made by analyzing experimental results. In the case of a girder subjected to high shear, measurements have been made on webs to obtain strains at different shear loads.⁽⁷⁾ As is illustrated by the stress vectors at panel centers in Fig. 14, the principal tensile stresses in the region of a tension field are higher than those computed by beam theory (solid arrows versus dashed-lines). Outside the tension field, near the flanges, however, the stresses by measurements differ only slightly from those by computations. Regardless of how a tension field is anchored or how high the stress magnitudes are along a stiffener, the experimental results alone justify the assumption that for girders under high shear load the membrane stresses near the flange-to-web junction can be approximated by using the beam theory:

As pointed out earlier, the distribution of membrane stresses has been investigated for a simply-supported plane web under edge compression by considering strain energy.⁽¹²⁾ Computed results agree with actual measurements, examples of the latter being given in Fig. 4. In a cross section, in order to balance the applied bending moment, the stresses in the compression flange and near the weld must be higher than that computed by the beam theory. Since they are compressive stresses, the increase of their magnitude compensates for some amount of plate bending stresses and gives a beneficial

effect to the fatigue endurance of the welded flange-to-web junction. Concerning fatigue, therefore, it will be on the conservative side to ignore this beneficial effect by assuming that the girder primary bending stresses at a flange-to-web joint conform to the beam theory.

3.3 COMBINATION OF STRESSES

The evaluation of various stress components is for the purpose of comparing them with the fatigue characteristics of metals and weldments to estimate fatigue strength. A number of theories exists, which attempt to define the relationship between a system of stress components in an actual member and the fatigue properties of a specific type of stress pattern in a mode.⁽⁵⁾ For example, the principal stress theory combines these stress components into an equivalent principal stress σ_e , Eq. 14, and compares it with the fatigue properties of a model in which this principal stress alone is acting.

$$\sigma_e = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (14)$$

If a point at the flange-to-web junction of a girder is considered, the primary bending stress σ_b and the residual stress σ_r are the stresses acting in the longitudinal direction whereas the plate bending stress $\bar{\sigma}_y$ is perpendicular to them. The magnitude of the

equivalent principal stress at this point under a given load is then

$$\sigma_e = \frac{\sigma_b + \sigma_r + \bar{\sigma}_y}{2} + \sqrt{\left(\frac{\sigma_b + \sigma_r - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \quad (15)$$

When stress components at critical points of the flange-to-web junctions are known for both the maximum and the minimum values of the applied load, corresponding maximum and minimum equivalent principal stresses can be calculated. These stresses (S_{\max} and S_{\min}) are then compared with fatigue characteristics for fatigue life prediction.

However, even if the relationship between stresses and the fatigue properties are proven satisfactory, and the fatigue properties well defined, stress components in girders are not known exactly. Analyses so far give only qualitative indications of stress distributions but not quantitative values for strength prediction. Such a situation naturally leads to the consideration of some experimental investigations. Actual testing has been made and is to be described next.

IV. F A T I G U E T E S T S O F W E L D E D P L A T E G I R D E R S

With the static strength of girders now determined,^(1,2,3) design rules have been set up accordingly.⁽⁴⁾ Without yet knowing the precise stress distribution, a short cut to engineering application is to investigate experimentally the applicability of these established design rules for static loading to girders subjected to repeated loading. Besides serving as "acceptance-type" tests, the experiments may be regarded as pilot tests for more complete investigations of fatigue behavior. Whether or not girder webs fluctuate under repeated loads can be observed, and the effects of such fluctuations as well as those of tension field action can be detected. In effect, it can be determined whether or not these effects constitute any more critical situation than do the other factors that are inherent in welded plate girders, such as butt-welded flange plates, partial length cover plates, flange-web junctions, transverse stiffener attachments, and other structural details.

4.1 GIRDER SPECIMENS, SETUP AND LOADING

Tests were made on two welded plate girders. The two girder specimens almost duplicated full-size, structural carbon steel welded girders which developed tension fields in static strength

tests (Ref. 7, girders G6 and G7). By so doing, direct comparison of static and fatigue behavior could be made while the effects of steel property, girder geometry, residual stress, and specimen size as variables were minimized.

The girders are shown in Fig. 15. Both girders were 40 feet long and had 12" x 1" continuous flanges. Cover plates at reaction points added to the stiffness of the girders to keep vertical deflections within the limitations of the test equipment. The 50-inch deep webs were composed of 3/8-inch plates at the ends and 3/16-inch plates in the central test sections. Slender webs were used to emphasize the effect of tension field action. The only difference between the two girders was the spacing of intermediate stiffeners in the test sections. Girder F1 had stiffeners 75-inches apart thus having an aspect ratio (α) equal to 1.5. Girder F2, with $\alpha = 1.0$, had three identical panels of 50-inch length. Statically, therefore, girder F2 was stronger than girder F1.

As shown by the shear diagram in Fig. 16 the various test panels were subjected to high, uniform shear forces which were introduced through the loading points at the girder ends and the supports at quarter points (Fig. 15). The loading configuration was such that the magnitudes of bending moments were low in the test section and that a shear failure would occur if a static test were conducted. An overall view of the test setup is shown as Fig. 17.

The determination of test loads was different from the usual procedure. In fatigue testing of metals and weldments, the common reference for loads applied on a specimen is the stress magnitude in the specimen. Usually, stress intensity is directly proportional to the applied load; in which case the specifying of a stress also determines the load intensity. But stresses in a girder web differ from point to point and their relationship with load is not known exactly. Such a situation, coupled with the inability to predict the location of cracks, made it more logical to specify average shear stresses in the web for shear considerations than to specify stresses at a particular point in a girder. Because average shear stresses are directly proportional to applied loads-- which can be expressed in terms of the static strength of girders-- it is possible to assign stresses or loads in terms of the static strength.

If a diagram as Fig. 18a is constructed with its coordinates in units of forces, a reference is obtained for the determination of test loads. It is emphasized that this reference basis is quite arbitrary, but it did provide a starting point for planning the experiment where there was no other suitable reference available. The diagram is similar in appearance to a modified Goodman Diagram (Fig. 3) with P_u (the static strength of a girder) corresponding to S_u (the static tensile strength of a metal). Assuming that the diagram is valid for a fatigue life of 2,000,000 cycles, point A was selected by applying a factor of safety of 33/18 to the girder's

static strength. The ratio 33/18 is the factor of safety used in highway bridge design. For girder F1, a somewhat higher value of $0.65 P_u$ was assumed for point A, where the factor of safety is 33/20.

Now, for a true modified Goodman Diagram, any load range between the two inclined lines in the diagram should result in the same fatigue life. This idea was used in planning these tests. Though it should not matter whether a range of 0 to P_w or from half-maximum to maximum were chosen, the latter range was used to better approximate actual field conditions. Thus, the maximum loads were 83.4 and 70.6 percent of their ultimate values for girders F1 and F2, respectively (Figs. 18a and 18c).

These loads and other reference values are summarized in Table 2. The web buckling stresses and loads were computed assuming simply-supported edges for the panels. Since the maximum loads would be about three times as high as the buckling loads, large lateral deflections would be expected (Refer to Fig. 8 and its related discussion).

4.2 TESTING SEQUENCE, CRACK DEVELOPMENT AND REPAIR

A static test up to the maximum fatigue load was performed on each girder prior to the application of the repeated loading of 250 cycles per minute. Strain measurements were taken before the

fatigue test and also at specified intervals during the cyclic testing to examine the effect of fatigue loading. The testing sequences for the two girders are shown in Fig. 19 and are reviewed here. The results of these tests are summarized in Table 3 and will be discussed later.

Girder F2

Girder F2, statically the stronger one, was tested first. No fatigue crack was observed anywhere in the test section before 2,000,000 cycles, a commonly used fatigue life reference. At 2,000,000 cycles, a pair of hair cracks were detected at the web toe of the fillet weld along a stiffener. These cracks were only visible on one side of the web, thus were less than 3/16-inch deep. The first observations of cracks are marked on Fig. 19 and the crack locations are sketched in Fig. 20.

After placing weld beads over the cracks (without any other preparation), cyclic loading was resumed with the previous load range of 46.5 kips to 93 kips. This attempted repair was not successful since some hair cracks became visible either through the weld beads or along the edges of them shortly after loading. At 2,500,000 cycles, these cracks had penetrated through the web and started to propagate both upward and downward. Also, a new crack of a similar nature had developed along the same stiffener but eighteen inches above. Before excessive propagation of cracks, the test was stopped. The stiffener and the cracks along it were isolated by welding a pair of stiffeners

on each side, as shown in Fig. 20, detail A.

Girder F2 as reinforced thus had one original panel with an aspect ratio α of 1.0, two stronger panels with $\alpha = 0.85$ (Table 3), and two isolated portions with $\alpha = 0.15$. A new load range of 55 kips to 110 kips was applied which was originally planned for the girder if no cracking occurred at 2,000,000 cycles (110 kips being the maximum load possible with the equipment). Under this new load range, a new crack was discovered when an additional 580,000 cycles had elapsed, (a total of 3,080,000 cycles in different ranges). It was located near the longitudinal axis of the girder and at the web toe of the fillet weld along a stiffener (Fig. 20, detail B). Again, the crack was only visible from one side of the web. Later, at about 3,108,000 total cycles, another small crack developed just above the previous one. These cracks were observed for a while and the test terminated at 3,277,000 cycles because of cracks at the ends of the cover plates outside of the test section.

From the experience that repair by simple welding over a crack without other preparation did not prevent propagation of cracks - whereas isolation by stiffeners stopped crack growth - it is confirmed that the former is not an adequate repair measure.

Girder F1

This specimen had two panels 75-inches long, with an aspect ratio $\alpha = 1.5$. For the given web slenderness ratio, this aspect ratio

is beyond the limit specified for building girders (see Sec. 5.1). As compared with girder F2, the static strength was thus lower. Although the load range of 44 kips to 88 kips was smaller than that for F2, the maximum load actually was higher in terms of the girder's static strength: 83.4 percent for F1 and 70.6 percent for F2. This condition of longer panel and higher load percentage was to cause more serious web deflections in F1 than in F2. Therefore crack occurrence was expected to be much earlier for this girder than the previous one.

At 330,000 cycles a crack was noticed at the web toe of the fillet weld along the top flange (Fig. 21 detail A). It was a few inches long and visible from both sides of the girder. Because of the relatively rapid rate of propagation of this particular crack (see later discussion), the test was brought to a stop. No decrease of load was observed before stopping, nor was any damage detected other than the crack.

Similar to the procedure used before on girder F2, the failed part was isolated by adding a pair of stiffeners to permit further testing. Prior to reinforcing, and in order to insure the soundness of repair as a result of the experience with girder F2, the metal around the crack was first removed and weld beads deposited, resulting in a heavier fillet weld in this area. With the reinforcing stiffeners, the new panel had an aspect ratio of 1.1 as compared to the neighboring original panel of 1.5 (Table 3).

Testing was resumed after repair. At 1,850,000 cycles, load application was discontinued because of propagation of another crack. This crack appeared on one side of the web along the center-line stiffener at 1,200,000 cycles and then penetrated the web, growing to the stage of Fig.21, detail B.

Again repair was made, this time only by removal of metal around the crack and by weld deposits without reinforcing stiffeners. The first crack in the girder after this repair appeared along the top flange at 2,330,000 cycles (Fig. 21, detail C). As all other cracks, it was located at the web toe of the fillet weld and was visible at first only on one side of the web. With an increasing number of cycles, it penetrated through the web and branched out gradually, generally perpendicular to the tension field. Other hair cracks of similar shape also developed in the same general area and gradually joined each other. By 3,780,000 cycles, a small crack began to appear along the nearby stiffener (Fig. 21, detail C). From then on, while the cracks grew, many new hair cracks could be detected along the boundaries of the long panel, in the vicinity of large web deflections. The final failure occurred at 4,077,000 cycles when the crack branching out from the top flange joined the crack along the stiffener, forming a long break of the web (dashed-line arrows in detail C). This constitutes a considerable reserve of strength (and thus safety) from the initiation of crack to the final failure of the panel.

4.3 EFFECTS OF REPEATED LOADING ON DEFLECTIONS AND STRESSES

On each girder, measurements were made to examine the deflections and strains. Results of measurements at various load magnitudes are similar to those obtained in the investigation of the static strength of plate girders.⁽⁷⁾ Hardly any change of strain was noticed as a result of repeated loading. Since the results were similar for both girders, the discussion for one applies also to the other.

Girder Deflection

One simple way of detecting effects of fatigue loading on the overall performance and strength of a girder is to examine the relationship between the girder deflection and the number of load cycles applied on the girder. Such a relationship is revealed in Fig. 22 for girder F1 which suffered more damage and repair than girder F2. The vertical deflections were recorded at one end of the girder under maximum loads. For reference, the occurrence of cracks and repairs are also indicated.

It has been stated before that no decrease of load was observed when the first crack of the girder was detected, even though the rate of propagation was relatively rapid. From the deflection versus load cycle curve of Fig. 22 it can be seen that there was no increase of deflection where the crack was first observed and for a considerable period thereafter. This suggests that fatigue cracks of a few inches in the web of a girder do not affect the

performance or the strength of the girder. Only when an appreciable amount of cracking was observed did the deflection increase slightly. The final, sudden increase of deflection for this girder corresponded to a complete "tearing" of the web across a tension field strip depriving it of its shear carrying ability (Fig. 21).

Membrane Stresses in Web

Even though the strength of the two girders did not change when web cracks grew to a few inches, it still might be possible that the stress or strain patterns would be influenced. If any detectable change in web strain resulted from cyclic loadings, the principal membrane stresses would change accordingly. The recorded changes were so small that they are regarded as of little significance. For example, principal membrane stresses at points along a panel centerline ($x = +37.5$) of girder F1 are shown in Fig. 23 for three different cycle numbers (0 , 10^6 , and 1.85×10^6). All stresses were recorded under the maximum load of 88 kips. Between zero and 1,000,000 cycles, cracking occurred in the left test panel of the girder and repair was made; at 1,200,000 cycles a crack initiated along the centerline stiffener and grew to about 15-inches at 1,850,000 (Fig. 21). Yet only very slight changes in the magnitudes and orientations of these principal stresses were found among the three stages of testing, as are indicated by the nearly identical lengths and directions of the stress vectors in the three diagrams in the figure. In other words, changes in the tension field due to hair cracks in a web panel cannot be clearly measured, if any changes do occur.

Naturally, when cracks grow to a stage where they hinder the tension field action and hence reduce the girder strength, the principal membrane stresses are changed. But the magnitudes of these stresses at such a stage are of little significance.

Web Deflections

Web behavior under repeated loading was one of the main observations. Lateral movements of some web points along vertical cross sections were measured. Deflected cross-sectional shapes at given loads are approximated by connecting positions of respective web points in Figs. 24 and 25. Figure 24 shows two cross sections of girder F2 under various loads; Fig. 25 is a sketch of the test section of girder F1 showing the relative web position between zero and the maximum load. Both figures have an enlarged scale for deflection. As was expected, deflections were large relative to the web thickness, the highest magnitude being about three quarters of an inch for girder F1 and two-fifths of an inch for girder F2 under their respective maximum loads.

In Fig. 24, the deflected web positions are for the maximum and the minimum loads. It is evident from this figure that the web fluctuated considerably back and forth between the two positions under the repeated maximum and minimum loads. With an amplitude of about one-fifth of an inch, this fluctuating phenomenon was clearly visible during testing.

In spite of fluctuations of such magnitudes and in spite of the formation of cracks along panel boundaries, the deflection of a girder web under a given load remained practically unchanged throughout a test. This is borne out by the two almost identical deflection shapes under maximum loads in each of the two cross sections of Fig. 24. The approximate shapes with heavier lines correspond to loads before any cyclic loading, and the adjacent shapes with thinner lines are obtained after application of two million cycles ($x = 0$) or after fatigue testing was completed ($x = +50$).

4.4 EVALUATION OF WEB STRESSES

Direct measurement of strains on a web permits the evaluation of web stresses at the points of measurements, so long as the strains are less than the yield-point value. At other locations on the web, approximations have to be made using web deflections.

If the initial web deflection is negligible or disregarded, Eq. 13 gives approximate plate bending stresses along rigid flange-to-web joints. For girder F1, the web slenderness ratio was $\beta = 50 \div \left(\frac{3}{16}\right) = 267$, and a nondimensionalized maximum web deflection in the center of the right-hand panel at station $+37 \frac{1}{2}$ was $\gamma = 0.74 \div \left(\frac{3}{16}\right) = 4$ (See Table 4 for web deflection magnitudes). Thus the estimated plate bending stresses for the panel would be as discussed in Section 3.2 and depicted in Fig. 13.

However, the initial deflection of this web panel being three-fifths of an inch is too large to be negligible in the stress computation. The stresses of Fig. 13 are consequently much too inaccurate. To obtain more realistic estimates, let it be assumed that no web stresses exist in the initial shape, (that is, disregard the effects of the girder weight). Any web plate bending stress in a girder under applied load is then due to the relative deflection of the web with respect to its initial position. In the present tests the web deflections were recorded for a few vertical cross sections. The relative deflections can be deduced easily. If an equation is found to fit these relative deflections, the plate bending stresses at flange-to-web joints may be approximated. Thus, by assuming a web strip perpendicular to a flange as a cantilever beam with a deflection curve

$$w = C_1 y^3 + C_2 y^2 \quad (16)$$

where w is the relative lateral web deflection, C_1 and C_2 are numerical coefficients to be determined through actual deflections, and y is the vertical distance from the boundary point interested, the web bending stresses may be estimated from the following formulas:

$$\bar{\sigma}_y = \frac{M}{I} \cdot \frac{t}{2} = EI \frac{d^2 w}{dy^2} \cdot \frac{t}{2I} \quad (17)$$

or

$$\bar{\sigma}_y = C_2 Et \quad (18)$$

The elastic plate bending stresses so computed at the flange (see Appendix) are 50 ksi and 12 ksi for the maximum and the minimum loads, respectively, at the cross section $x = -18 \frac{3}{4}$ of girder F1 (Fig. 25, location A, just to the left of the centerline). It is, therefore, not surprising that a crack developed in that vicinity (Fig. 21) after only 330,000 cycles of load application and propagated at a relatively rapid rate.

At the top flange-to-web junction of the cross section $x = +56 \frac{1}{4}$ of girder F1, Fig. 25, these web bending stresses are estimated as 34 ksi and 15 ksi, respectively, for the applied maximum and minimum loads. In addition, membrane stresses caused by the bending moment and shear force in the girder must be considered. From the discussion in Sec. 3.2, primary stresses may be approximated by using the beam formulas. Under the maximum load, magnitudes of these membrane stresses are computed to be 7 ksi for bending and 8 ksi for shear (see Appendix). If the residual stress along the flange-to-web joint is assumed to be about half the yield point of the girder material, that is, 18 ksi, then the magnitudes of the maximum equivalent principal stresses at this point are, by Eq. 15, 39 ksi for the maximum load and 23 ksi for the minimum load. Thus, with a stress range of 23 ksi to 39 ksi, hair cracks appearing at about two million cycles are to be expected (see for example, Fig. 3 of Ref. 14). Actual cracking first occurred nearby at 2,330,000 cycles (Fig. 21, detail C). Shortly after, numerous hair cracks were observed in that region. The significance of this stress

evaluation and comparison to actual cracking is that the effects of plate bending as well as those of residual stresses in fatigue cracking are as expected. Crack initiation can be predicted if stresses are known and the fatigue properties of weldments are available.

Cracks also developed along vertical stiffeners of the test girders at various load cycles. Such is the consequence of the fact that web-to-stiffener junctions are not simply-supported joints. If the restraint which a stiffener offers to a web is close to the built-in condition, then the assumptions of Sec. 3.2 also apply for web plate bending stress estimation. Simple expressions similar to Eq. 16 may also be regarded as adequate and used with measured deflections for the stresses. But the actual restraint to a web at its boundary depends not only on the stiffeners but also on the conditions of neighboring panels. It may be quite different from the built-in case. Adding to this is the uncertainty of primary stress distribution under tension field action. A quantitative approximation analogous to that for flange-to-web joints therefore does not seem justified.

Qualitatively, the development of cracks along stiffeners confirmed the concept that web deflection and boundary restraint play an important role in fatigue considerations. For girder F1, the lateral movement of the web in both panels was excessive in terms of the web thickness (Fig. 25). Because the heavy flanges provided more restraint to the web than the two-sided stiffener and because the

deflected shape had a sharp reverse curve at the flange, the plate bending stress at the flange was higher than at the stiffener. The first crack thus occurred along the top flange (Fig. 21, detail A). The addition of reinforcing stiffeners increased the restraint of the stiffener-to-web joint at the girder centerline. While the deflection in the right hand panel remained almost unchanged, more plate bending stresses were thus introduced along the stiffener and caused cracking there (Fig. 21, detail B). Subsequent repair by welding only reduced the magnitude of deflection nearby. As more load cycles were applied, cracks then developed further to the right along the flange in the same manner as the first crack. Had the repair of crack B by welding reduced deflections greatly in the right hand panel so that the shorter left panel ($\alpha = 1.1$) were comparatively more critical, a crack probably would have developed in the shorter panel.

Similar reasoning applies to the other girder, F2, and to the comparison between girders F1 and F2. If all other conditions are identical and the deflections equal, the higher the boundary restraint, the higher will be the plate bending stress, and the shorter the fatigue life. This, in fact, is implied by Eq. 13. For a given β and a given boundary restraint, the larger the web deflection (γ), the higher will be the plate bending stress ($\bar{\sigma}_y$). Even though the actual stresses are not known, this relationship provides a means of comparison and may lend itself to practical application.

4.5 DISCUSSION OF TEST RESULTS

From the analytical study and the results of the experiments, it is confirmed that slender webs of plate girders fluctuate laterally under repeated loads. The consequence of this behavior is the fatigue cracking of webs along their boundaries. Because stresses have not been evaluated exactly, prediction of crack occurrence is not yet possible through the consideration of fatigue properties of the girder material and weldments. However, it has been observed that, in general, a crack caused by plate bending propagates very slowly. When a girder web is subjected to repeated loading causing fluctuation, a crack appears first only on the tension side of the plate, then penetrates through the web while propagating. For girder F1, which had large web deflections under a load range of $0.41 P_u$ to $0.83 P_u$, the rate of propagation of crack B (Fig. 21) was in the order of one inch per 50,000 cycles.

As has been pointed out in the discussion of girder deflection (see Fig. 22), a crack of a few inches length caused by plate bending hardly affects the overall girder behavior. In other words, a web bending crack does not have a drastic effect on the strength of a girder. This is quite different from the situation of fatigue cracking of a tension flange or other simple structural elements where the stress carrying area is reduced and the stress intensity is increased rapidly by the crack. For the two test girders which were subjected mainly to shear, no effect on girder behavior was

detected before the bending cracks interfered with the tension field action and branched out into the middle of the web plate. Because tension field action alone did not exhibit any adverse effects, and plate bending stresses are not direct consequences of tension field action but rather the result of web deflections, it may be said that the influence of tension field action on the fatigue strength of welded plate girders is not direct.

Admittedly, the combined effect of plate bending cracking and its interference with tension field action will finally bring about failure of the girder by reducing the load carrying-capacity of the girder. However, a load creating high plate bending stresses and high tension field forces is usually a "high" load in terms of the girder's static strength ($0.71 P_u$ and $0.83 P_u$ for girders F1 and F2, respectively). If a working load is defined as $P_w = 18 P_u / 33$, then the maximum loads were $1.30 P_w$ and $1.53 P_w$, respectively). Under high loads, other modes of failure which are common to welded beams may occur prior to plate bending cracking. (This was in effect the case of girder F2 which finally failed by a tension flange crack at the end of a partial-length cover plate). A comparison between the magnitudes of the applied stresses on the two girders and their static strengths is made in the following chapter on design considerations.

V. ON DESIGN LIMITS AND
FURTHER INVESTIGATIONS

5.1 DESIGN LIMITATIONS

The final goal of the fatigue strength investigation on welded plate girders is to set up a guide for the safe and economical design of bridge girders. Since the fatigue strength of girders is not known, a logical way of establishing design criteria for fatigue is to adopt and adjust the limits set by static considerations to assure that girders can attain a specified fatigue life. This was one of the reasons which led to the testing of girders F1 and F2.

Based on the static strength of girders in shear and a factor of safety 33/18 (which is commonly used for highway bridge design), permissible average shear stresses in bridge girder webs are suggested and are shown in Fig. 26. In the figure, the ordinate is the shear stress, the abscissa is the web slenderness ratio β , and the suggested permissible average shear stresses are plotted for selected values of the aspect ratio α . These suggested stresses are, in essence, those permitted for building-girders⁽⁴⁾ but incorporate a factor of safety 33/18 instead of 33/20. (For comparison, the shear stresses permitted by current specifications for highway bridges⁽¹⁵⁾ are also shown for some values of α and β). For a given girder geometry, the applied static web shear stresses must not be

higher than the corresponding values in the figure. If, in addition, girders conforming to this stress limitation can sustain 2,000,000 cycles of load application without fatigue cracking, the stress limitation of Fig. 24 can then be used as a guide for the fatigue design of girders under high shear.

To investigate the adequacy of these suggested design limits, the applied stresses of the two full-size girders are shown in the figure for comparison. Girder F1 had a web slenderness ratio $\beta = 265$ and two panels with $\alpha = 1.5$ (Fig. 21 and Table 3). The applied average web shear stress was 9.3 ksi at the maximum load, compared to the permissible value of 6.0 ksi. A crack was observed at 330,000 cycles in one panel; the other panel stood for 1,200,000 cycles before any cracks developed. If the left panel after repair is also considered, the suggested permissible shear stress is 6.8 ksi. This panel sustained almost 3 million cycles without cracking.

Girder F2 (Fig. 20) had three panels of $\alpha = 1.0$. Again the applied maximum shear is higher than the permissible, being 9.8 ksi versus 7.5 ksi, respectively. Two of the three panels endured 2,000,000 cycles before observation of hair cracks; the third 3,080,000 cycles. The two short panels of $\alpha = 0.85$ (after reinforcing) never had any cracks at all.

For further discussion, the applied shear stresses for each particular panel are expressed in terms of the corresponding suggested permissible values, and are plotted against the number of cycles to

crack initiation in Fig. 27. (In addition, the results of another investigator⁽¹⁶⁾ are also indicated for comparison. These results were obtained on girders with a total depth of 16 inches). From this figure, it is seen that all panels of the two girders except those two with $\alpha = 1.5$ satisfied the commonly used fatigue requirement of 2,000,000 cycles. Those two panels of girder F1 had quite excessive initial web deflections before loading and the fluctuations of web during testing were quite severe (Table 4 and Fig. 26). To safeguard against excessive web deflections, a limit⁽⁴⁾ has been suggested, arbitrarily correlating the web slenderness ratio and the panel aspect ratio.

$$\alpha \leq \left(\frac{260}{\beta} \right)^2 \leq 3.0 \quad (19)$$

For girders with slender webs (high β), the panel length is not permitted to be long (low α). In the case of the two test girders, the maximum permitted aspect ratio is thus about 1.0. The panels with $\alpha = 1.5$ far exceeded this limit. A reduction in panel length to $\alpha = 1.0$, as for girder F2, brought the cycle numbers to crack initiation within specifications even with an over-stress of 53 per cent (Fig. 25).

Figure 27 is plotted for the initiation of cracks and incorporates, through τ_w , a factor of safety 33/18 against static strength. From the results of investigations on the two girders (Table 3), it is known that cracking propagates very slowly and that cracks a few inches long in the web scarcely affect the girder's

strength. For example, for crack C in the second panel of girder F1, about 1,750,000 cycles were applied to the girder without any drastic effects. Such a condition thus provides ample time for the detection and repair of cracks. Since it has been proven that repairs can be successful, it is with confidence that the suggested permissible stresses can be adopted in lieu of the current values.

5.2 SUGGESTIONS ON FURTHER INVESTIGATIONS

Although the results of these fatigue tests on two girders are encouraging, it is nevertheless necessary to substantiate them by further investigations. While an analytical solution is not yet possible due to uncertainties of physical conditions, further experimental investigations are suggested.

To include as many influencing factors as possible while still maintaining a clear distinction among them, full-size girders are preferable. Dismissed, then, are the effects of specimen size on fatigue, the reproduction of actual residual stresses, and the difference of web deflections on full-size and model girders. Girder geometry (α, β , flange and stiffener size), loading condition (bending, shear, or their combination), and load magnitude are factors to be investigated.

For a systematic determination of the influence of the factors of investigation on the web fluctuation it is suggested that

reference be made to Figs. 26 and 27. In Fig. 26 the limits of geometry specified by Eq. 19, line A, are shown for the control of web deflections. Because part of the very nature of the suggested investigation is an acceptance testing, it is recommended that the parameters under investigation be selected according to this line. If girders so designed are adequate for a specified fatigue life of crack initiation (such as 2,000,000 cycles), then girders with more sturdy webs will also be adequate. For different fatigue lives, a relation may be obtained through the use of Fig. 27.

However, stresses given by Fig. 26 are only for shear considerations. The effects of bending on deflections also must be compared with Eq. 19 to investigate its adequacy, or to establish a new limit. It is suggested, again, the static strength of girders be used as the basis for experimental study. If girders conform to Eq. 19 and are subject to a load magnitude of $18 P_u / 33 = 0.55 P_u = P_w$ without cracking prior to 2,000,000 cycles; the limits by the equation may be used as a design guide.

Further studies are needed to investigate the relative importance of web fluctuation on the fatigue strength as compared to those of other factors inherent in welded plate girders. In this respect, an investigation on the initial lateral deflection of girder webs and their influence on girder strength is necessary. The results of such an investigation will be helpful for setting up limits for web slenderness ratio and for initial out-of-straightness of webs. Too strict

rules will result in girders which are essentially beams without utilizing tension field action. A proper limitation will permit web fluctuation in girders which will have equal fatigue resistance against cracking by the web fluctuation and by the effects of partial length cover plates, attachment of stiffeners, and other inherent factors.

VI. S U M M A R Y

The following is a summary of the findings of this study:

1. The fatigue strength of welded plate girders cannot be regarded as simply equivalent to that of welded I-beams. Large lateral deflections of webs occur in slender web plate girders and repeated loads cause fluctuation of the web in that direction. Fluctuating plate bending stresses are thus created, sometimes with magnitudes along web boundaries as high as the yield point of the girder material. When these stresses are superimposed on the primary bending and shear stresses of the girder and on the tensile residual stresses which are also highest along web boundaries, the possibility of fatigue cracking at flange-to-web junction and along stiffeners is increased.
2. Actual tests on slender web plate girders show that cracks do occur along web boundaries, forming first on one side (confirming the bending action due to web deflection) and later penetrating through the plate thickness. When lateral web deflections of the order of the web thickness are present and the boundary restraint is close to the fixed-end condition, cracks will initiate at those points.

3. By using assumed web deflections which are based on actual measurements and by assuming magnitudes of residual stresses along web boundaries, it is able to approximate stresses which show that fatigue cracking at otherwise low-stressed regions is not surprising.
4. The rates of crack propagation are slow, being in the order of one inch per 50,000 cycles even when girders are subjected to loads as high as 83 per cent of the static strength or 53 per cent above the working load.
5. Crack lengths of up to eight to ten inches caused no decrease of the load-carrying capacity of girders, nor resulted in any increase in girder deflections. The fundamental reason for this is that the type of cracks which developed in these girders was not so much the result of primary stresses through which the girder resists loading, but was the consequence of what might be termed "secondary stresses" associated with lateral deflection.
6. After initiation of a crack, a girder can sustain a large number of load cycles until failure. For example, girder F1 which cracked at 2,330,000 cycles continued for another 1,750,000 cycles before it failed. Such a great "post-cracking" fatigue life lends itself to easy inspection and repair with no loss of safety.

7. Although accurate evaluation of the fatigue strength of plate girders is not yet possible, the findings of this investigation nevertheless indicate encouragingly the possibility of using the static strength as a design reference. Every panel with proportions permitted by even the most liberal specifications withstood at least 2,000,000 cycles of stress at levels 30 per cent greater than the allowable static stresses on a half-maximum to maximum basis.

8. Accordingly, design limitations analogous to that for building-girders⁽⁴⁾ are suggested for bridge girders in Fig. 26. With further recommended experimental investigations, these limits would insure that no fatigue cracking would occur along web boundaries prior to 2,000,000 cycles of load application.

Table 1 INITIAL LATERAL WEB DEFLECTIONS OF
WELDED PLATE GIRDERS

(Web Depth = 50 inches)

$\beta = \frac{\text{Web Depth}}{\text{Web Thickness}}$	$\alpha = \frac{\text{Panel Length}}{\text{Web Depth}}$	Max. Deflection (in.)	Girder No. (7,8)
388	1.5	0.456	G5
	0.75	0.243	
388	1.5	0.278	G4
	0.75	0.160	
382	3.0	0.474	G9
	1.5	0.642	
265	1.5	0.608	F1
263	1.0	0.215	F2
259	1.5	0.448	G6
255	1.0	0.358	G7
254	3.0	0.122	G8
	1.5	0.425	
185	1.5	0.141	G1
	0.75	0.054	
185	1.5	0.157	G2
	0.75	0.080	
185	1.5	0.160	G3
	0.75	0.101	
131	1.5	0.066	E1
128	1.5	0.054	E4
	0.75	0.064	
128	1.5	0.127	E5
	0.75	0.068	
128	1.0	0.222	H2-T1
128	0.5	0.161	H2-T2
127	3.0	0.241	H1-T1
127	1.5	0.087	H1-T2
99	3.0	0.030	E2
	1.5	0.101	

Table 2 TESTS ON SLENDER-WEB WELDED GIRDERS

GIRDER	F1	F2
Panel length, a	75 in.	50 in.
Web depth, b	50 in.	50 in.
Web thickness, t	0.189 in.	0.190 in.
Critical web stress, τ_{cr}	2.72 ksi	3.61 ksi
Critical load, P_{cr}	25.7 kips	34.3 kips
Static strength, P_u	105.7 kips	131.3 kips
Maximum load, P_{max}	88 kips = 0.83 P_u	93 kips = 0.71 P_u
Minimum load, P_{min}	44 kips = 0.41 P_u	46.5 kips = 0.35 P_u
Frequency, f	250 cpm	250 cpm

Table 3 SUMMARY OF TEST RESULTS

Girder		F1			F2				
Web Slenderness,		263			265				
Test Panel		1	2	1A	1	2	3	1A	2A
Aspect Ratio, α		1.5	1.5	1.1	1.0	1.0	1.0	0.85	0.85
Max. Shear Stresses (ksi)	Applied	9.3	9.3	9.3	9.8	9.8	11.6 ^(a)	11.6	11.6
	τ_w ^(b)	6.0	6.0	6.8	7.5	7.5	7.5	7.7	7.7
Ratio-Applied / τ_w		1.53	1.53	1.37	1.30	1.30	1.55	1.51	1.51
First Crack (Cycles)		330,000	1,200,000	--	2,000,000	2,000,000	3,080,000	--	--
Additional Cycles		--	2,807,000	3,740,000	500,000	500,000	197,000	770,000	770,000
Failure		--	4,077,000 ^(c)	--	-- ^(d)	--	--	--	--
Test Stopped		(See 1A)	4,077,000	4,077,000	(See 1A)	(See 2A)	3,277,000	3,277,000	3,277,000

(a) First 2,000,000 cycles at 9.8 ksi

$$(b) \tau_w = \frac{18 P_u}{33 A_w}$$

(c) Failure not by first crack

(d) Failure outside of test panels

Table 4 WEB DEFLECTIONS, GIRDER F1

Maximum Load, 88 kips

	$x = -56\frac{1}{4}$	$-37\frac{1}{2}$	$-18\frac{3}{4}$	$+18\frac{3}{4}$	$+37\frac{1}{2}$	$+56\frac{1}{4}$
y = +21	-007	-058	-112	+004	+048	+059
+15	-023	-338	-328	+025	+287	+342
+ 9	-212	-568	-125	+220	+642	+471
0	-499	-177	+484	+738	+740	+106
- 9	-174	+411	+455	+674	+126	-251
-15	+085	+364	+232	+285	-135	-214
-21	+060	+094	+053	+043	-051	-063

Minimum Load, 44 kips

y = +21	-049	-048	-028	+024	+060	+030
+15	-156	-210	-053	+130	+273	+193
+ 9	-236	-228	+150	+366	+536	+353
0	-143	+218	+523	+700	+655	+267
- 9	+146	+475	+413	+529	+240	-051
-15	+167	+317	+215	+233	+010	-127
-21	+042	+075	+055	+058	-027	-052

Initial, 0 kips

y = +21	-046	-005	0	+035	+041	-023
+15	-061	+021	+071	+191	+193	+035
+ 9	+042	+154	+262	+419	+414	+188
0	+185	+464	+508	+631	+594	+326
- 9	+169	+420	+387	+421	+331	+142
-15	+083	+333	+220	+174	+111	-005
-21	+010	+052	+061	+047	+003	-029

Note: All web deflections are in thousandths of an inch.

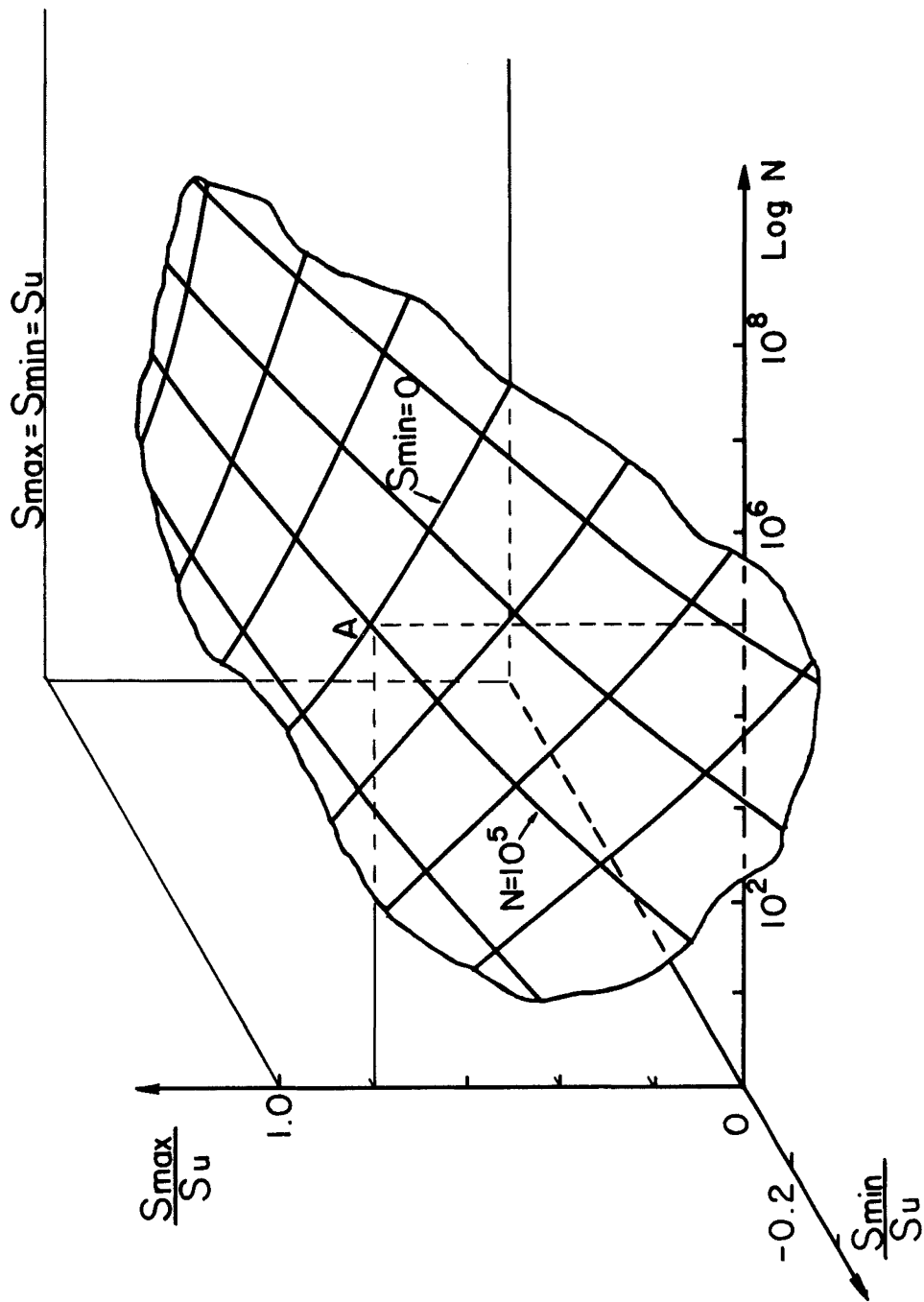


Fig. 1 $S_{max} - S_{min} - N$ Surface

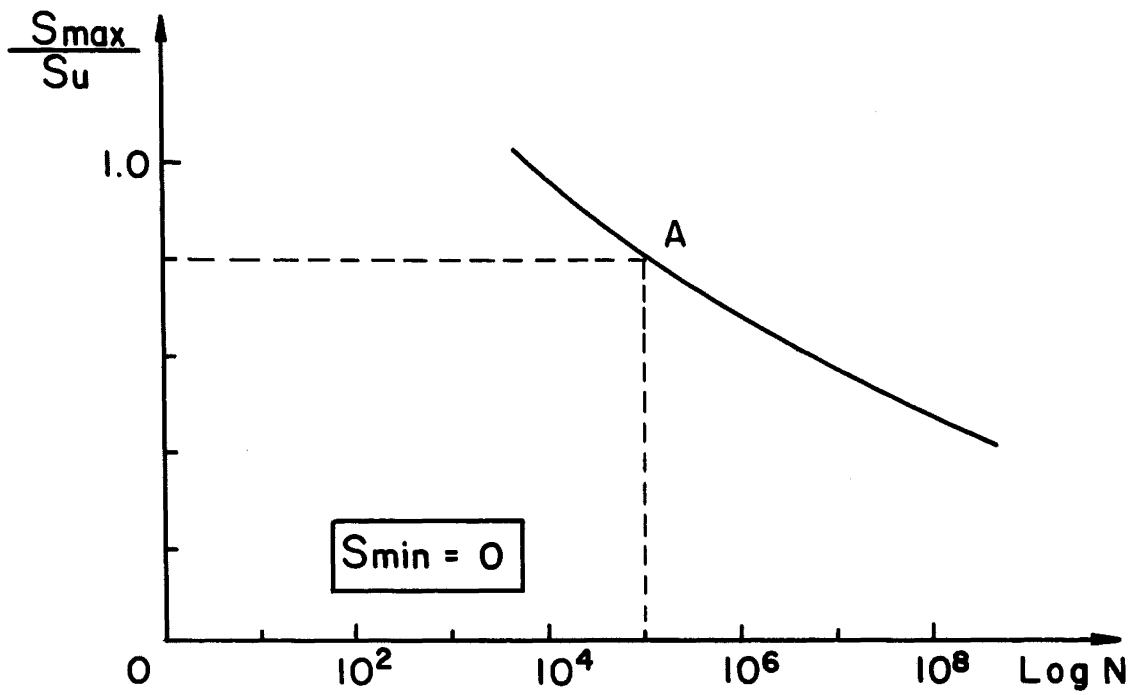


Fig. 2 S-N Curve

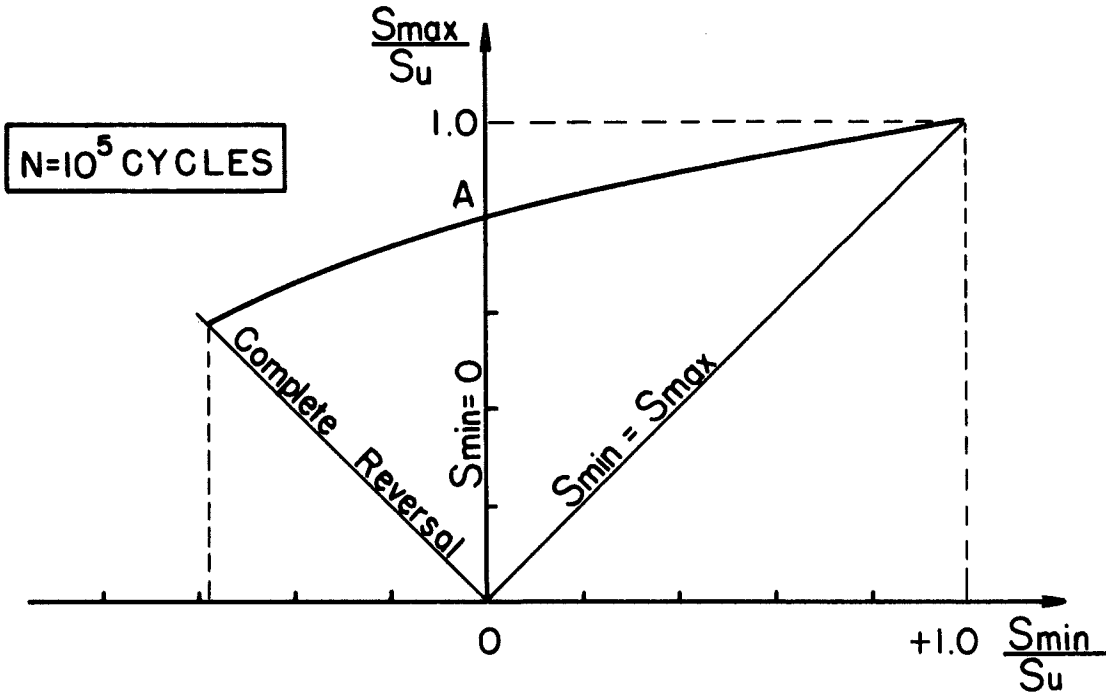


Fig. 3

AWS-WRC Diagram

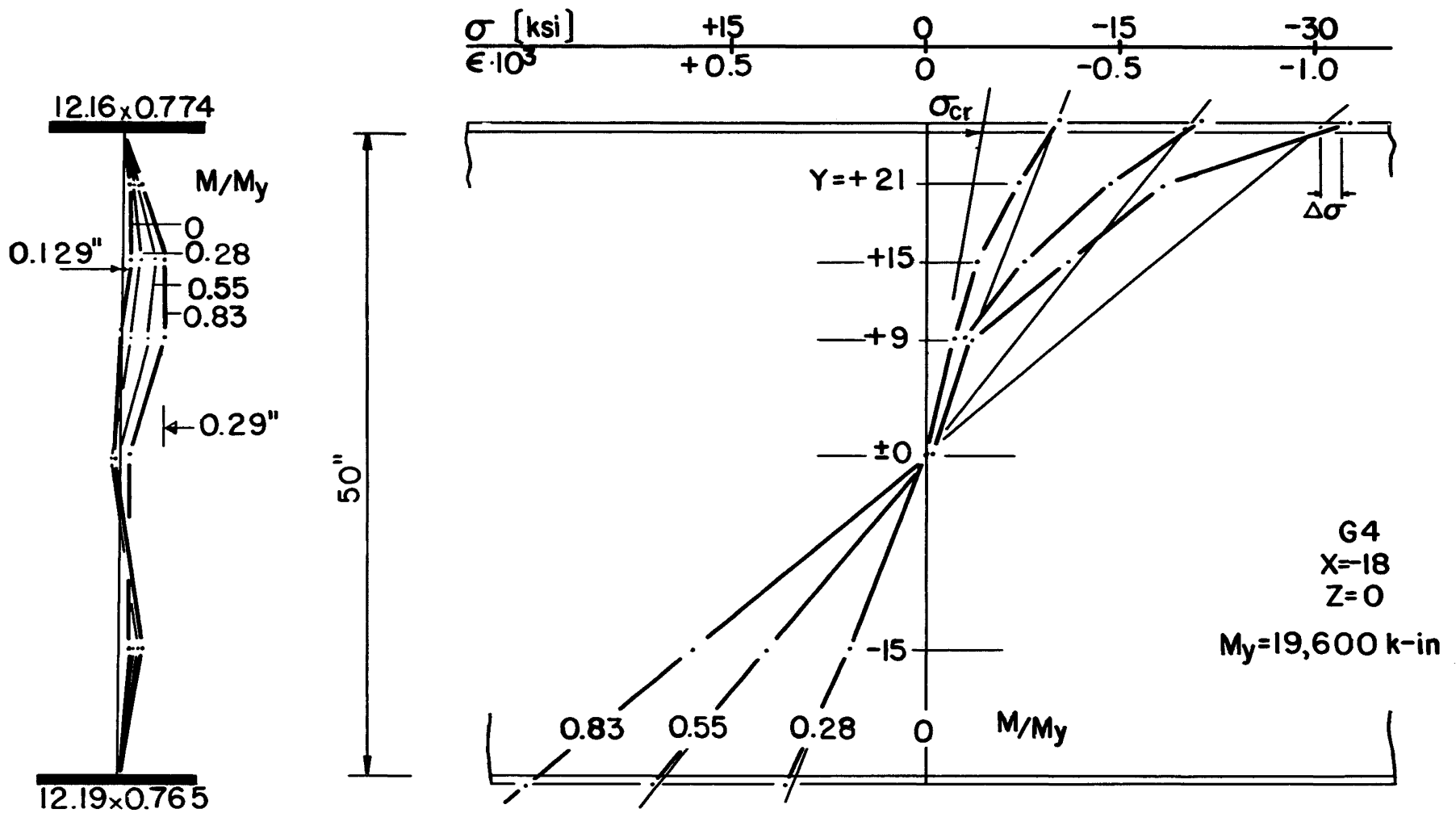
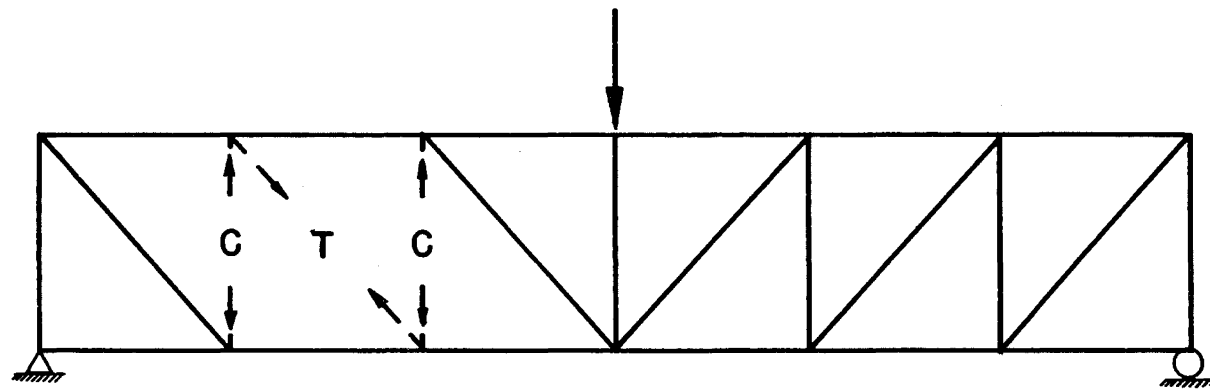


Fig. 4

Measured Web Deflections and Flexural Stresses



PRATT TRUSS

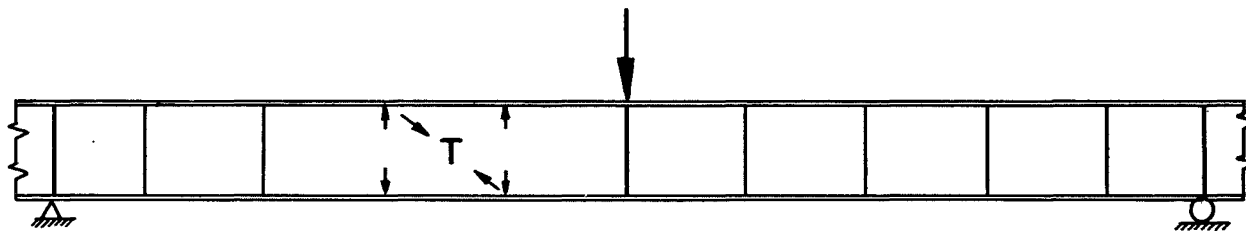


PLATE GIRDER

Fig. 5

Truss Analogy of Plate Girders

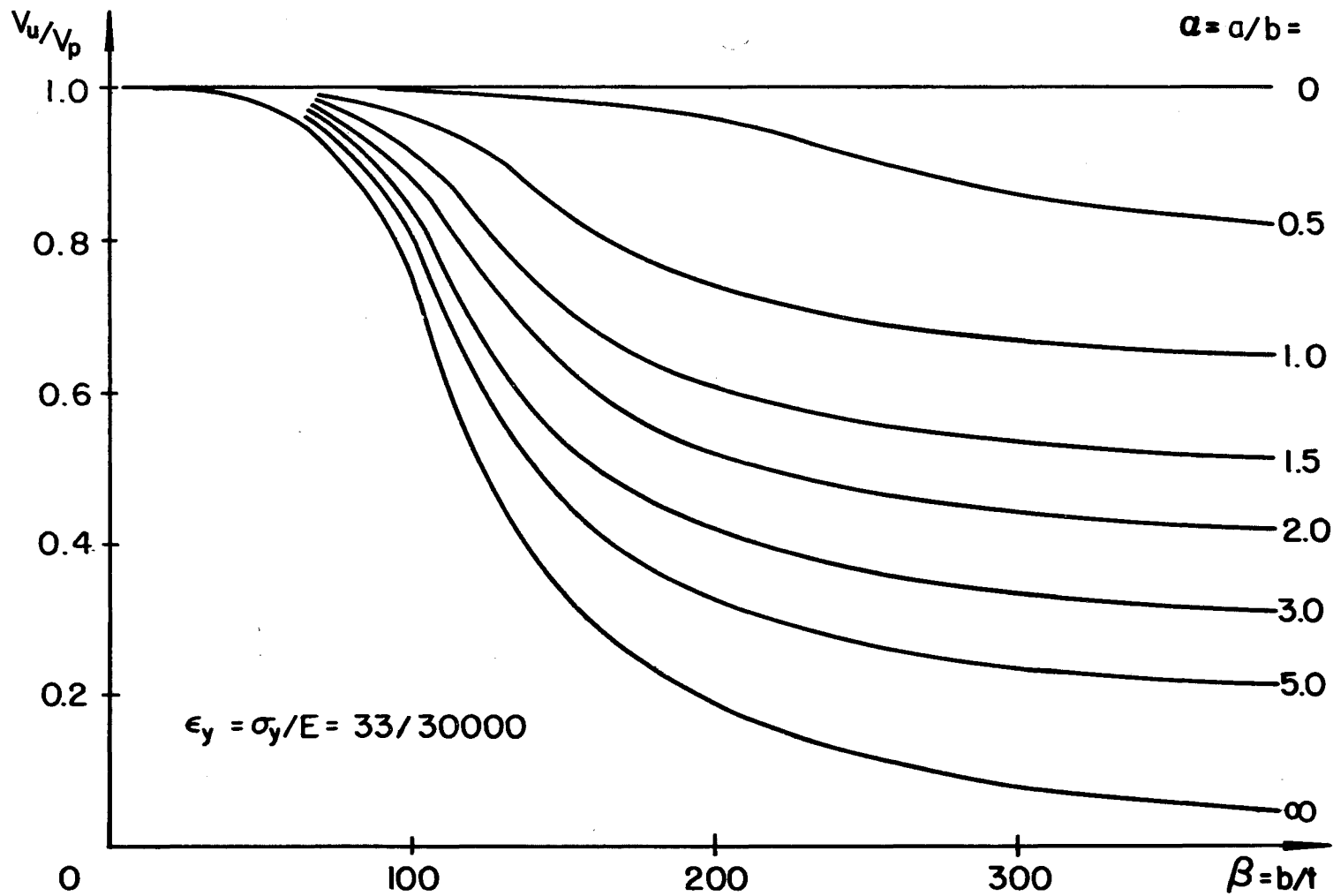


Fig. 6

Shear Strength of Plate Girders

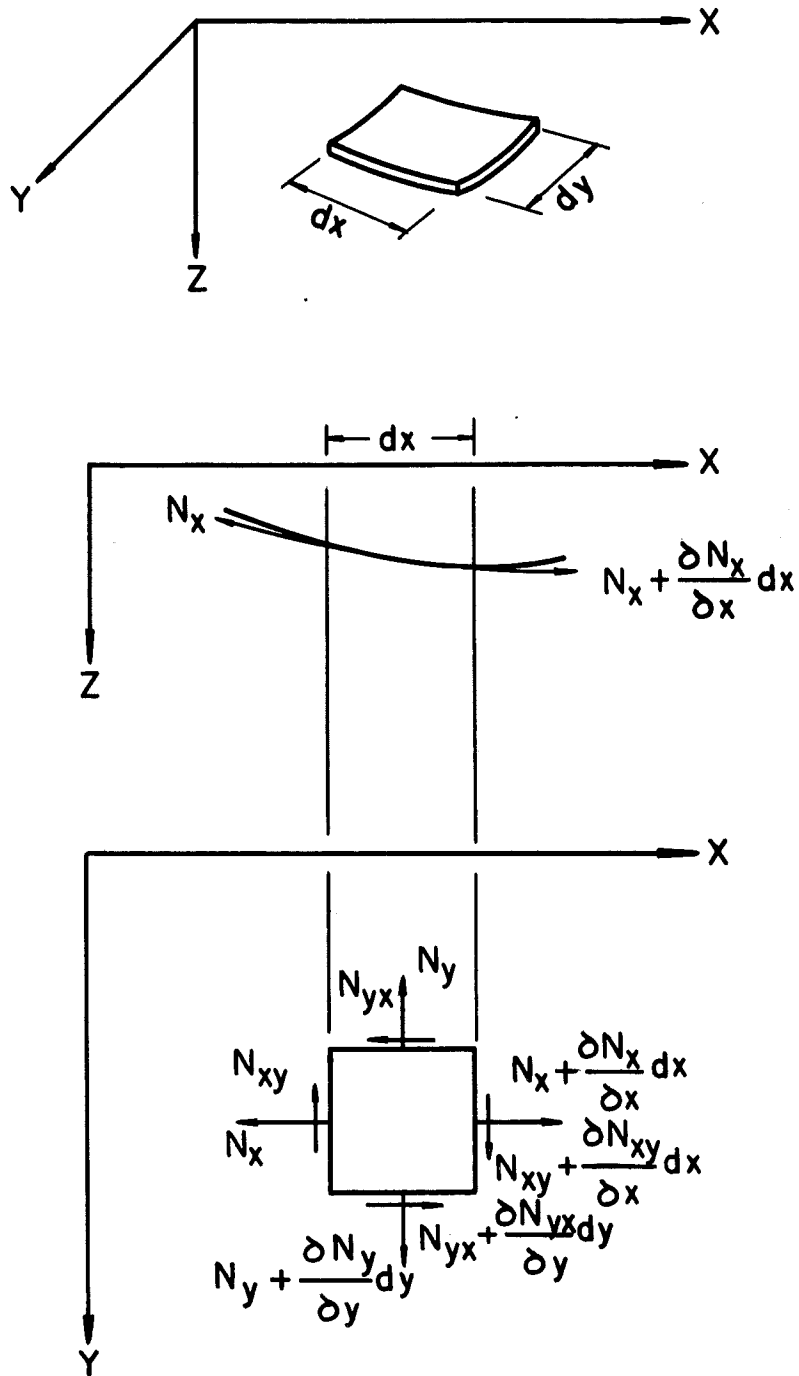


Fig. 7

Plate Element and Membrane Forces

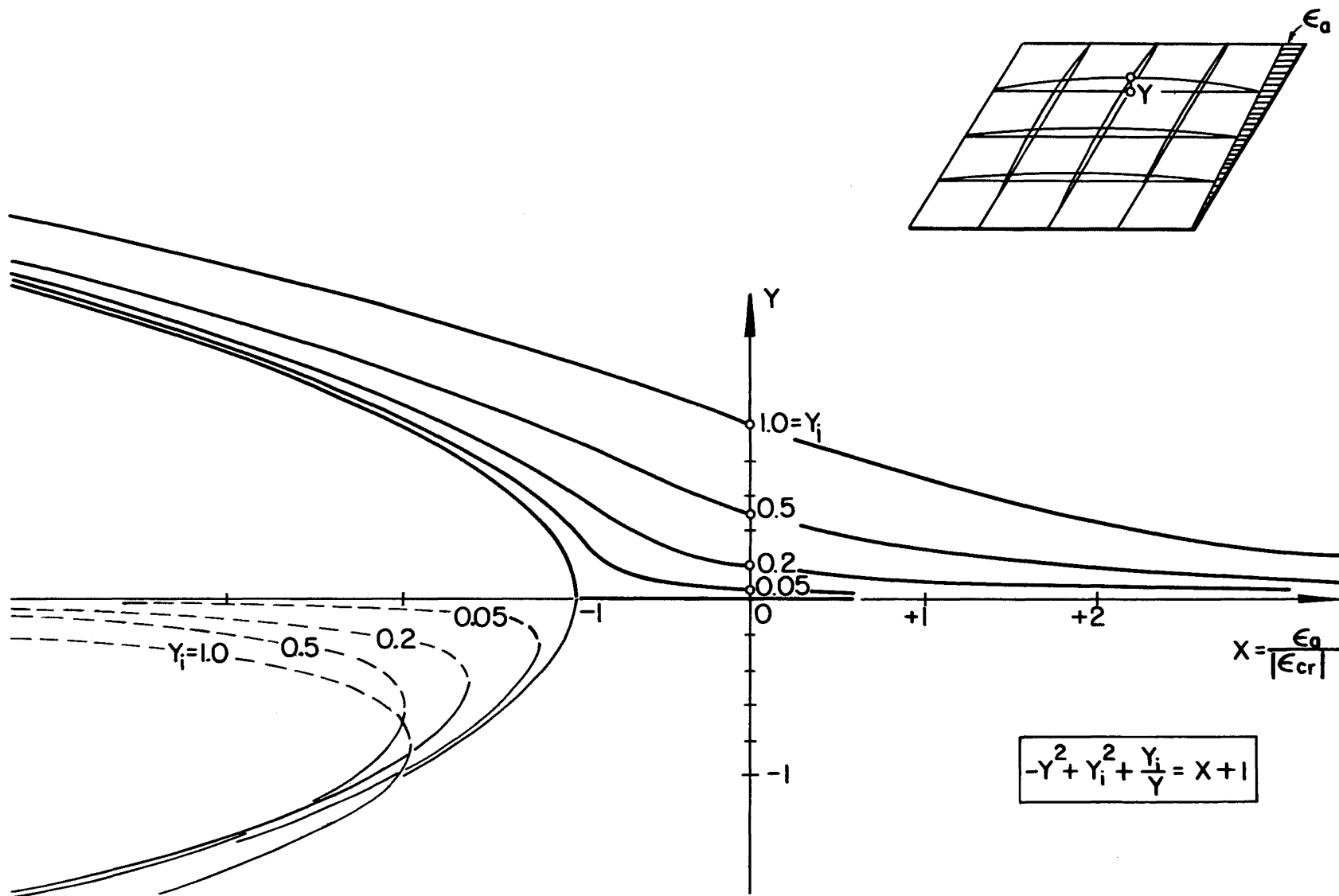


Fig. 8

Lateral Deflection vs Applied Strain

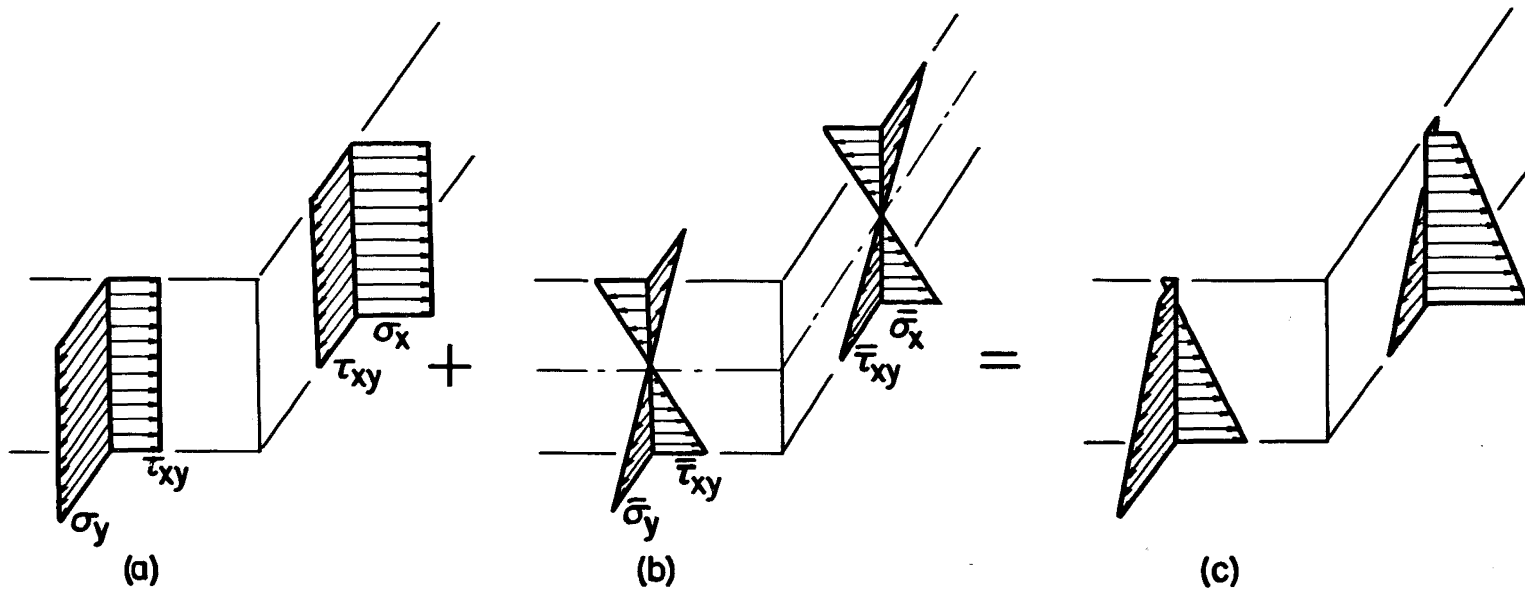
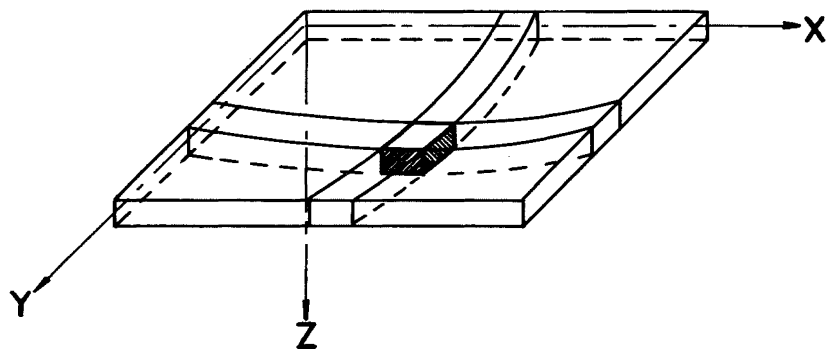


Fig. 9

Membrane and Plate Bending Stresses

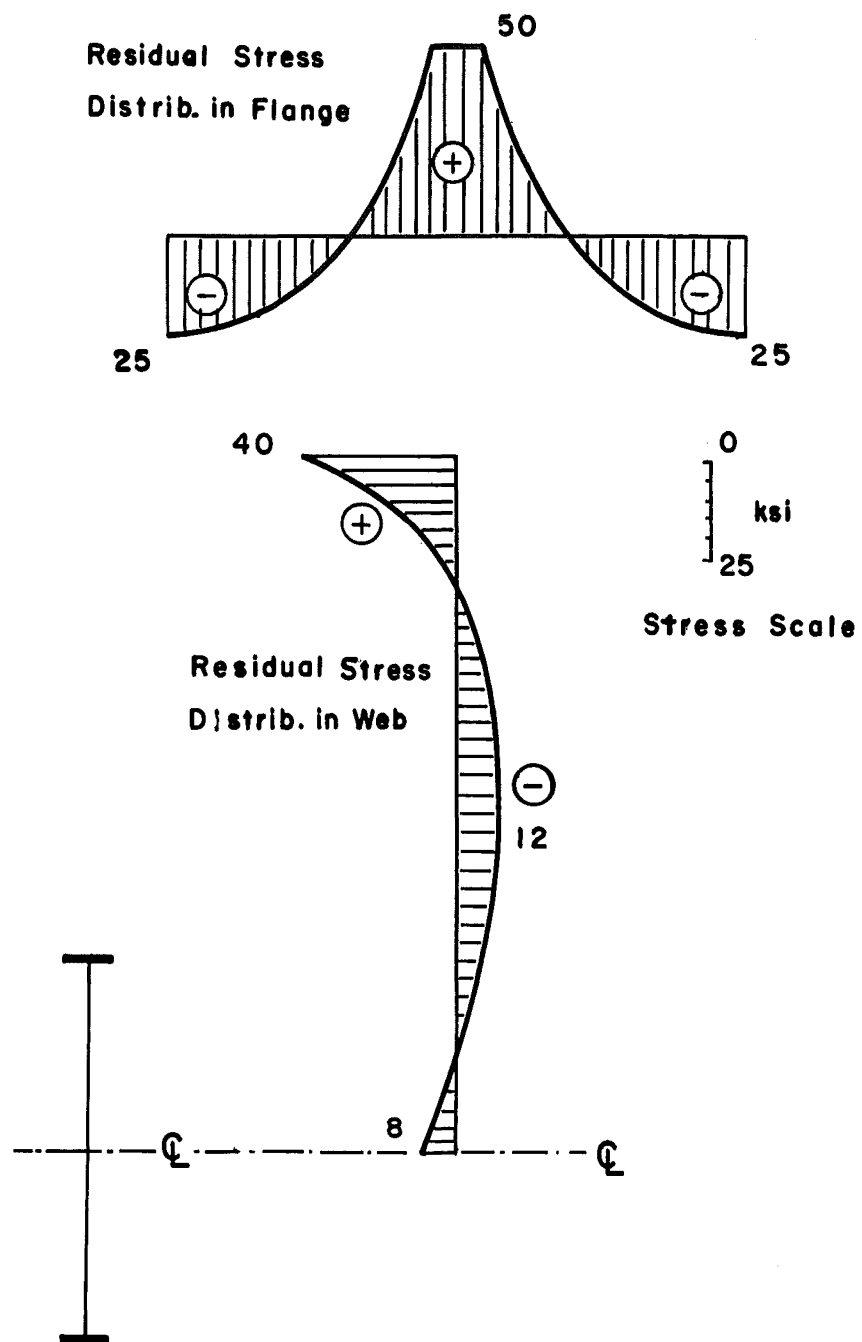


Fig. 10

Probable Distribution of Residual Stresses

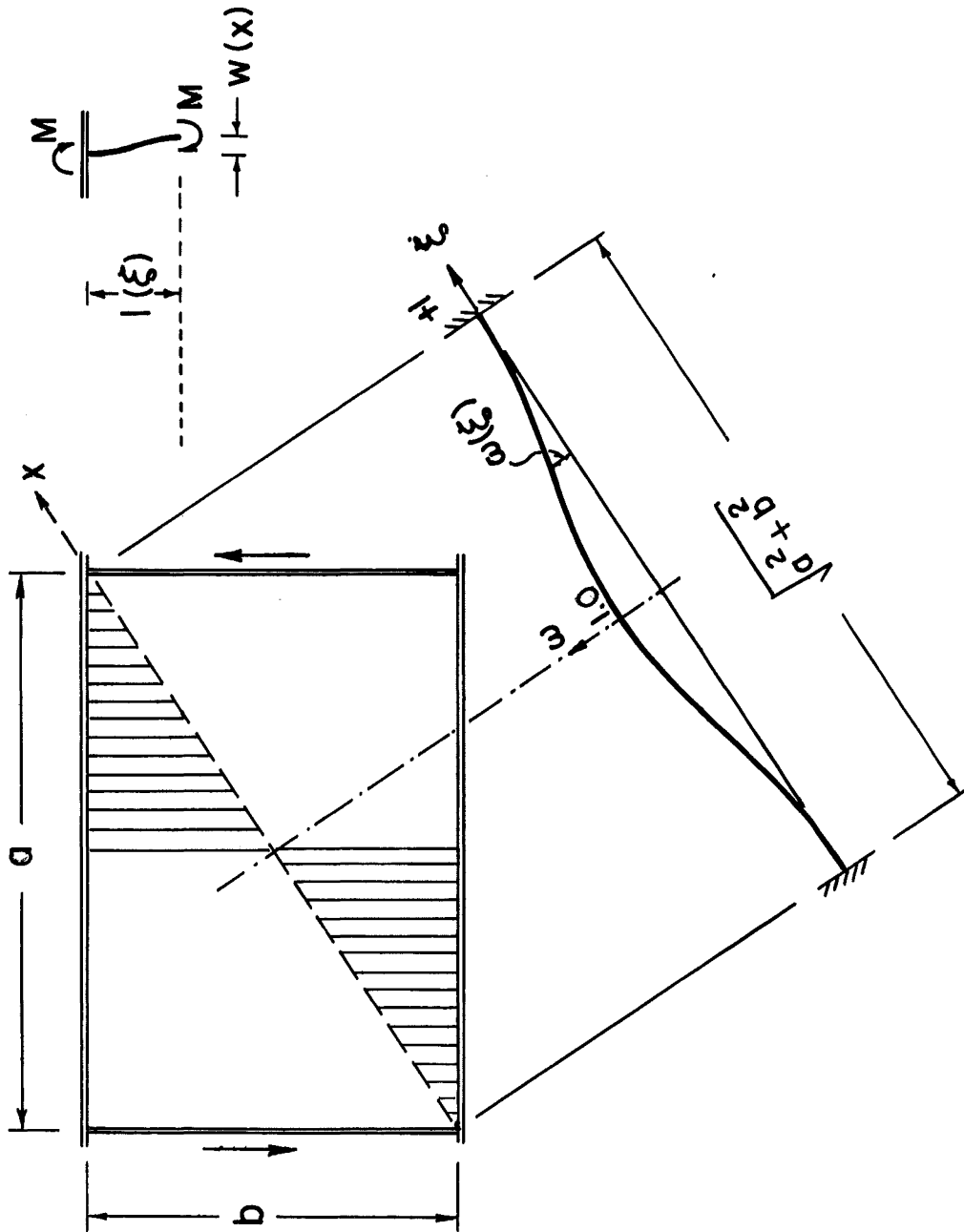


Fig. 11 Assumed Web Deflection

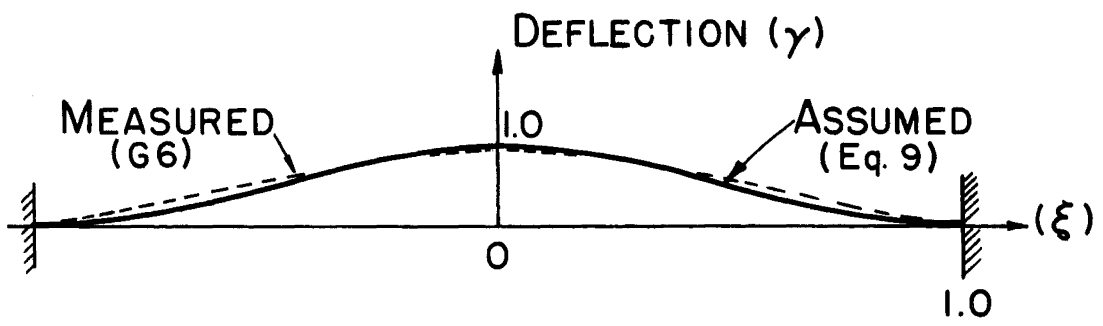


Fig. 12

Assumed and Measured Deflection Curve

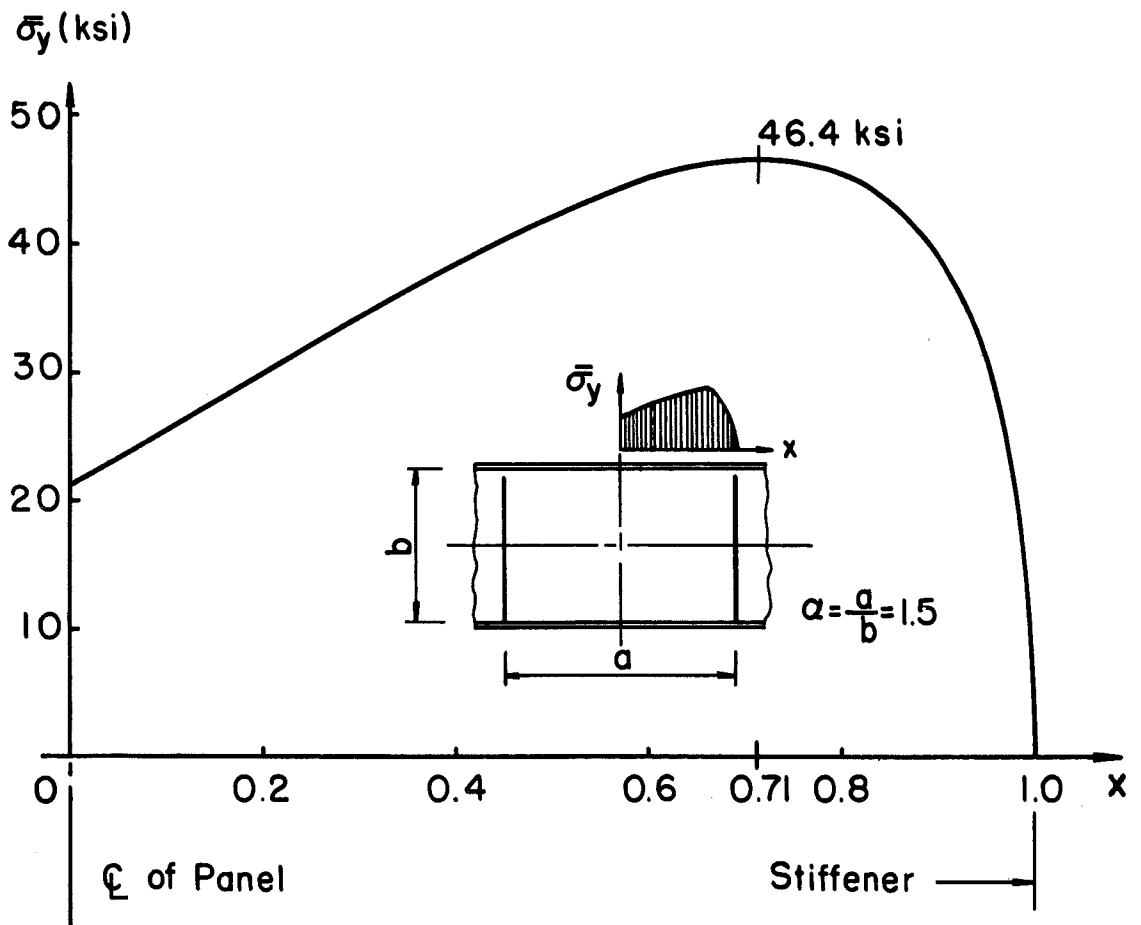


Fig. 13

Plate Bending Stresses Along a Flange

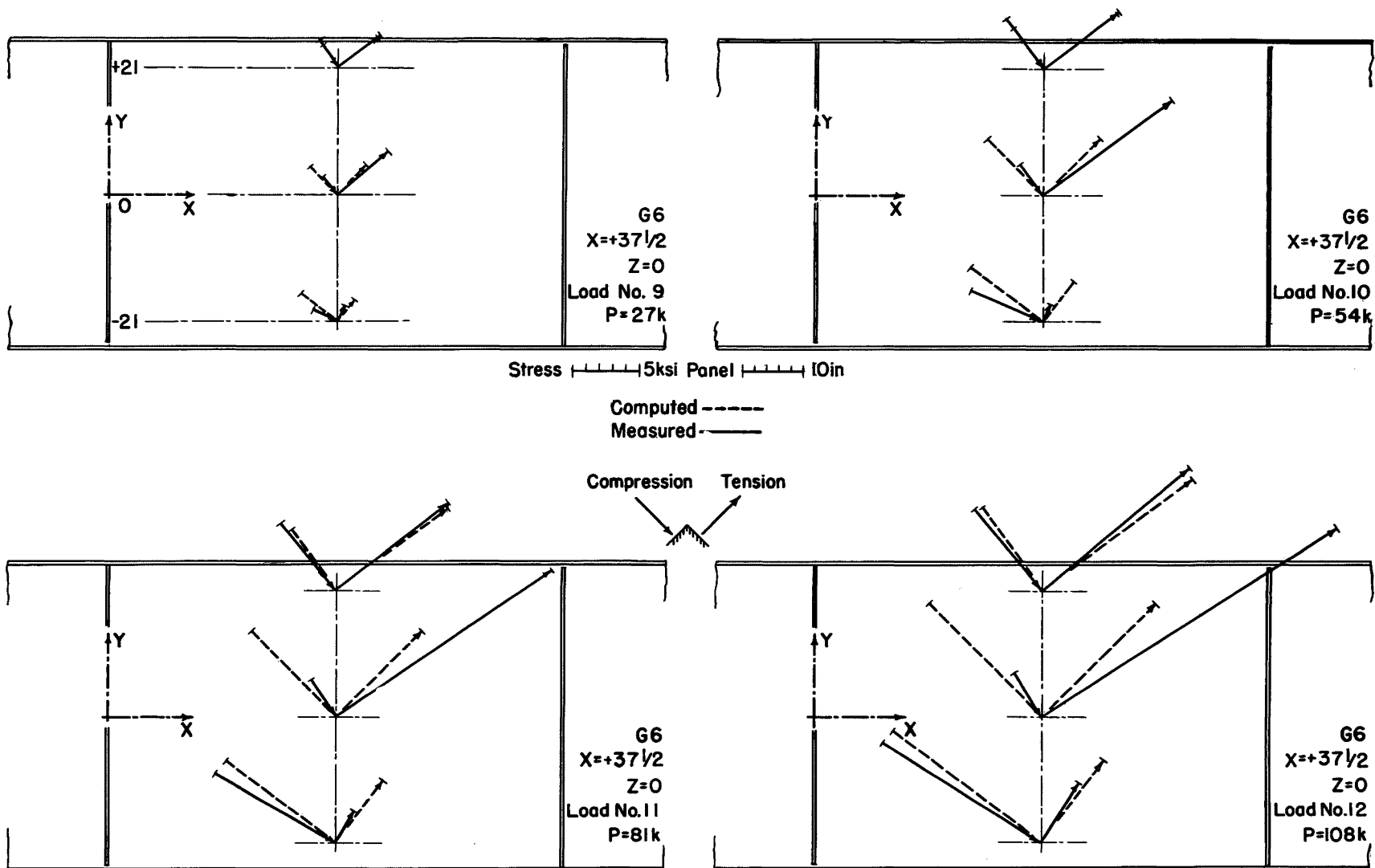


Fig. 14

Principal Stresses in Web of Girder G6

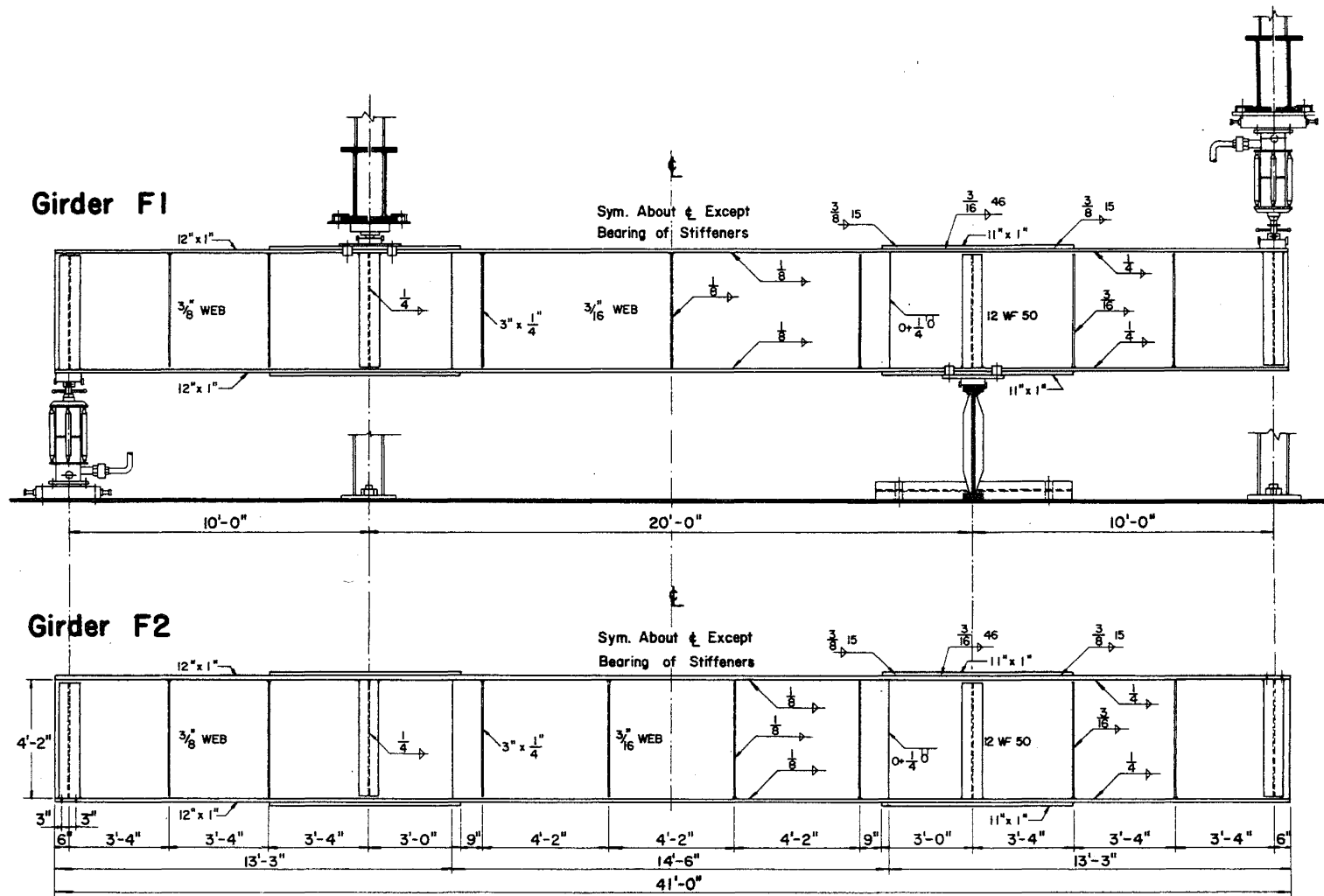


Fig. 15 Test Girders & Setup

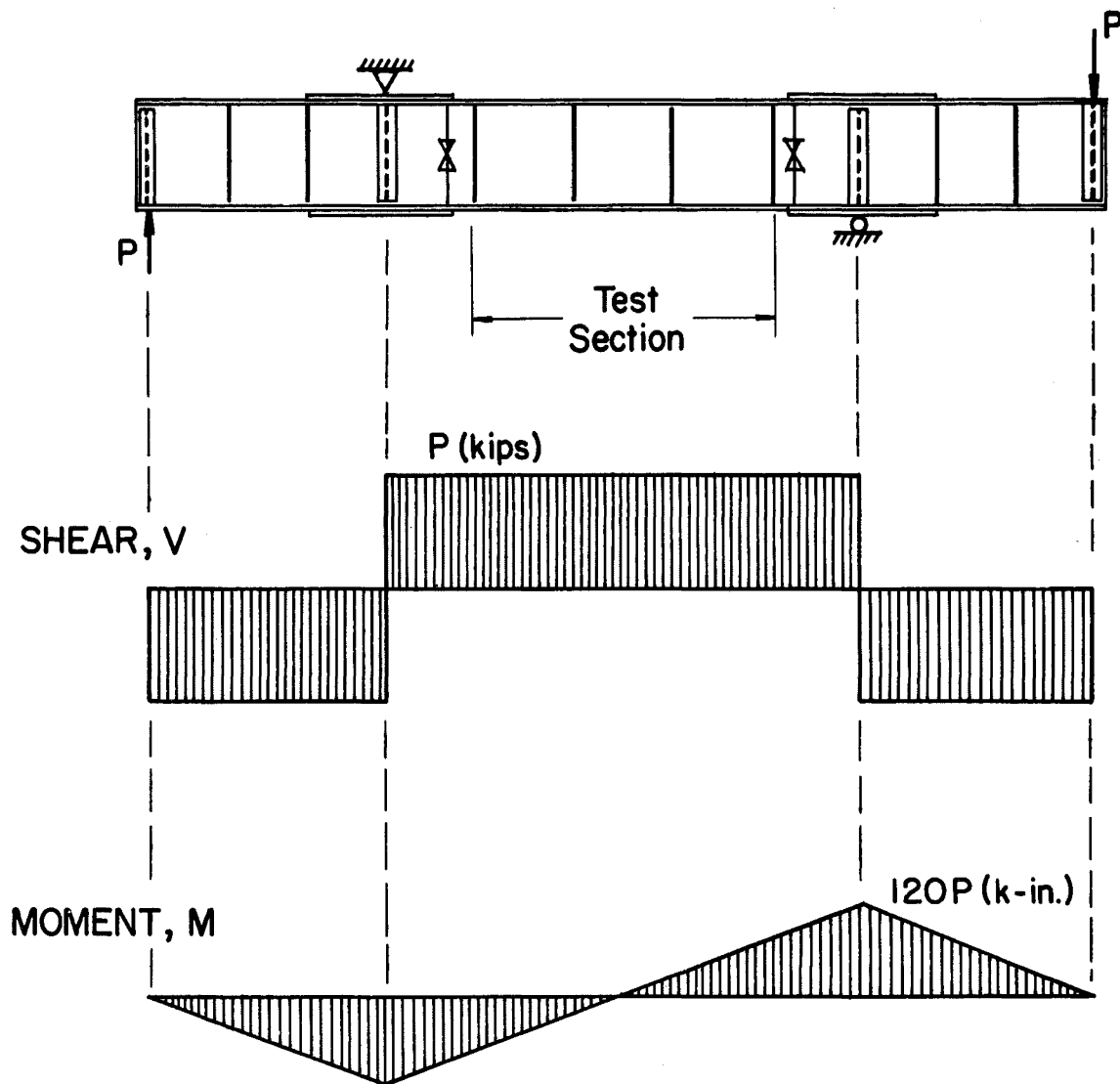


Fig. 16

Moment and Shear Diagram

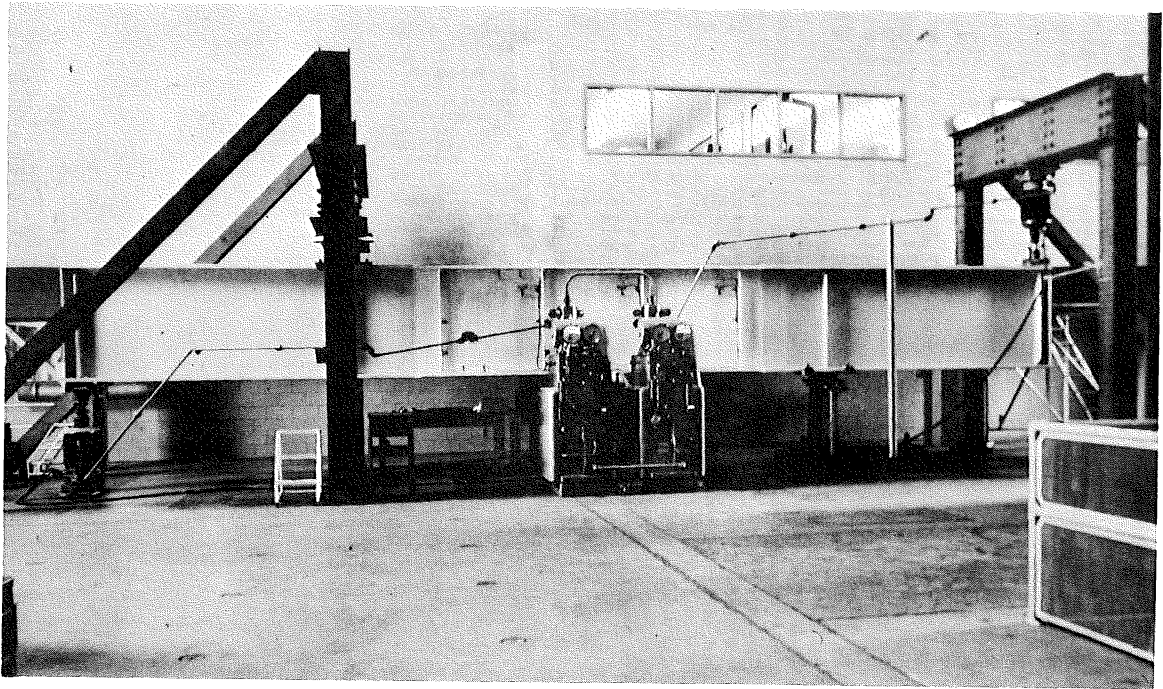


Fig. 17 Overall View of Test Setup

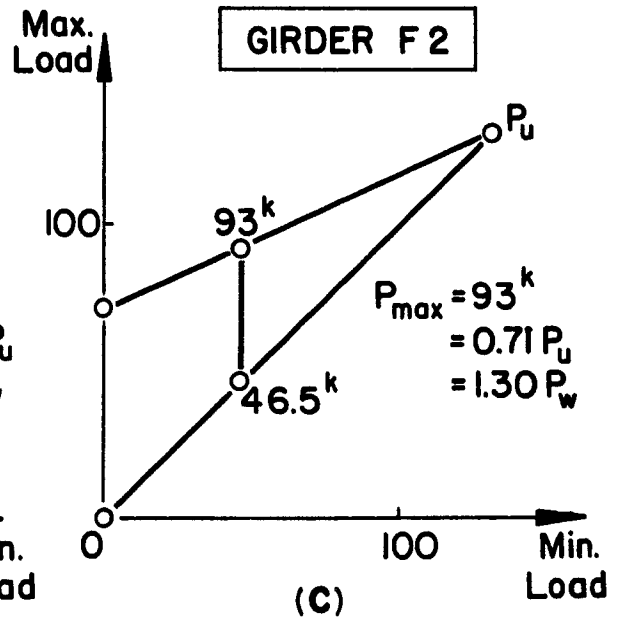
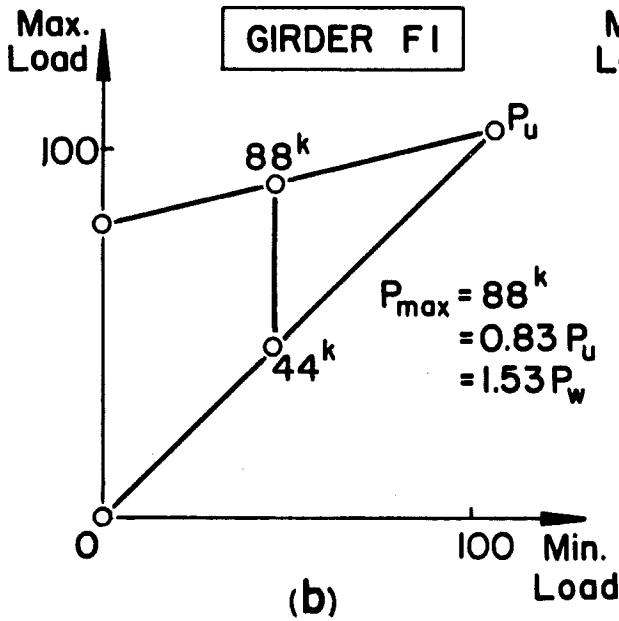
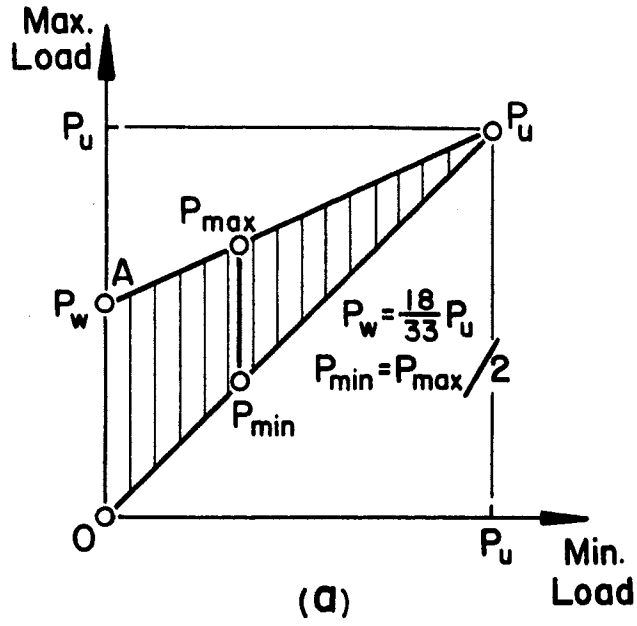


Fig. 18

Diagram for Determination of Test Loads

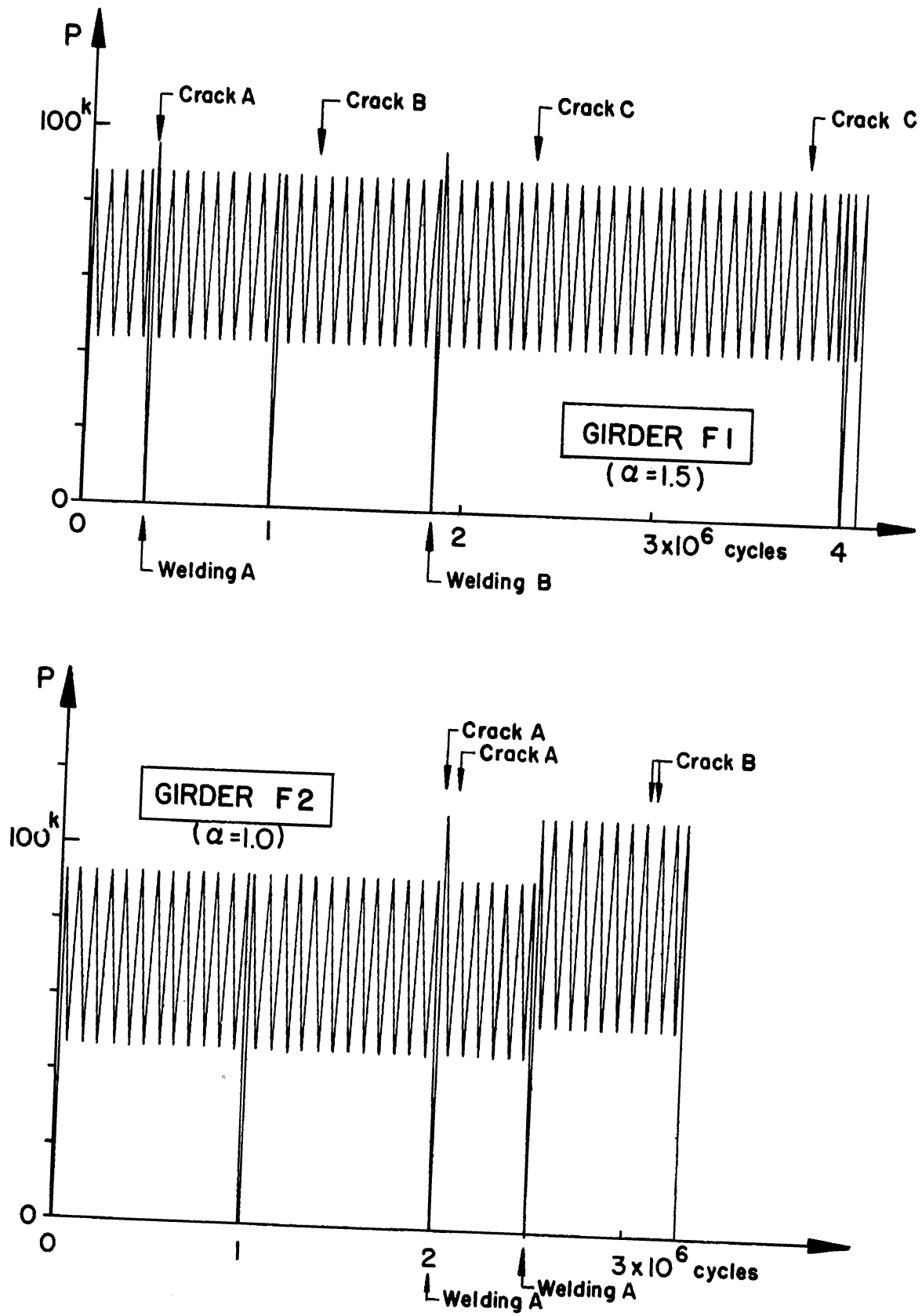


Fig. 19 Test Sequences

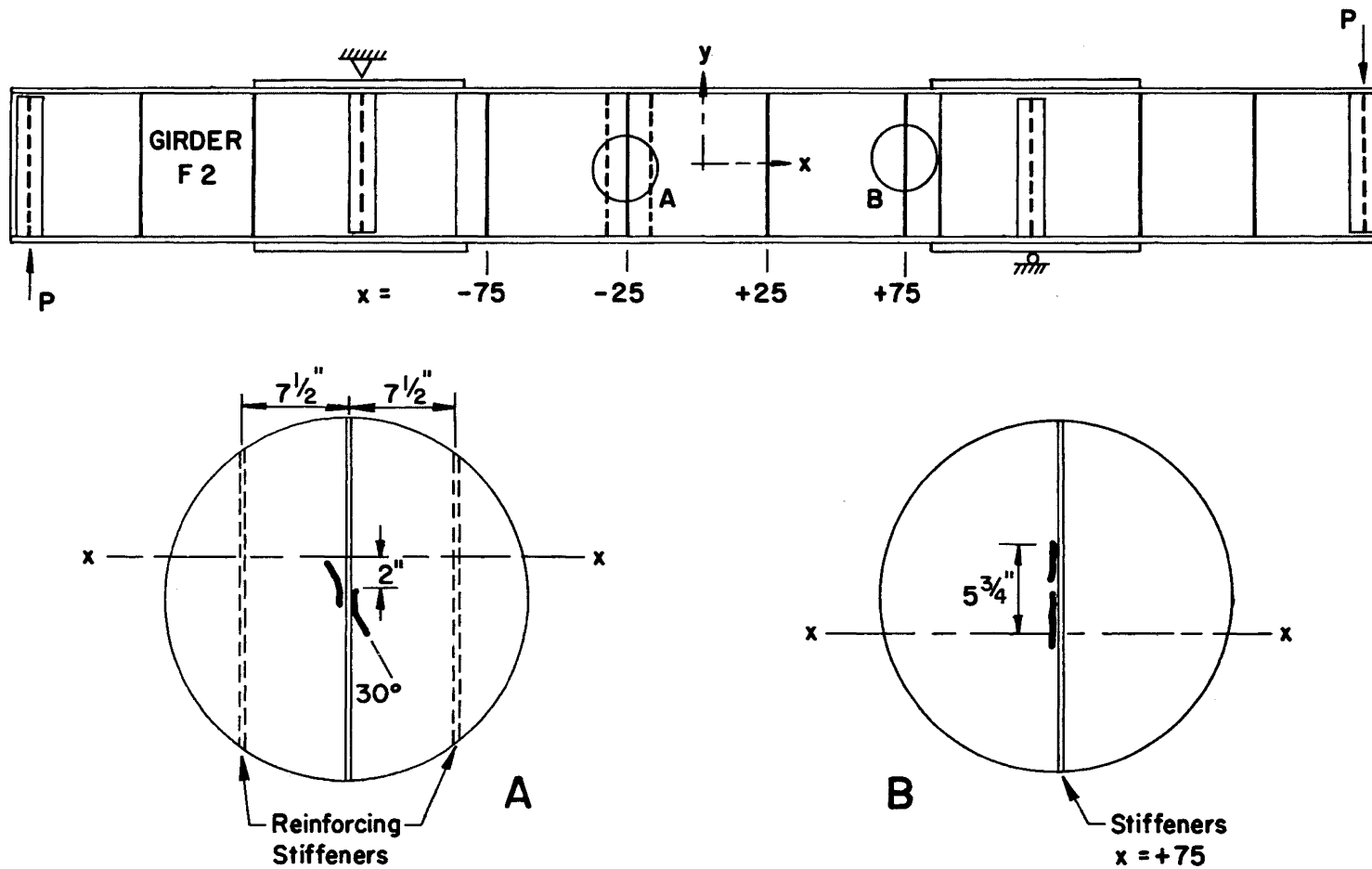


Fig. 20

Crack Locations and Repairs, Girder F2

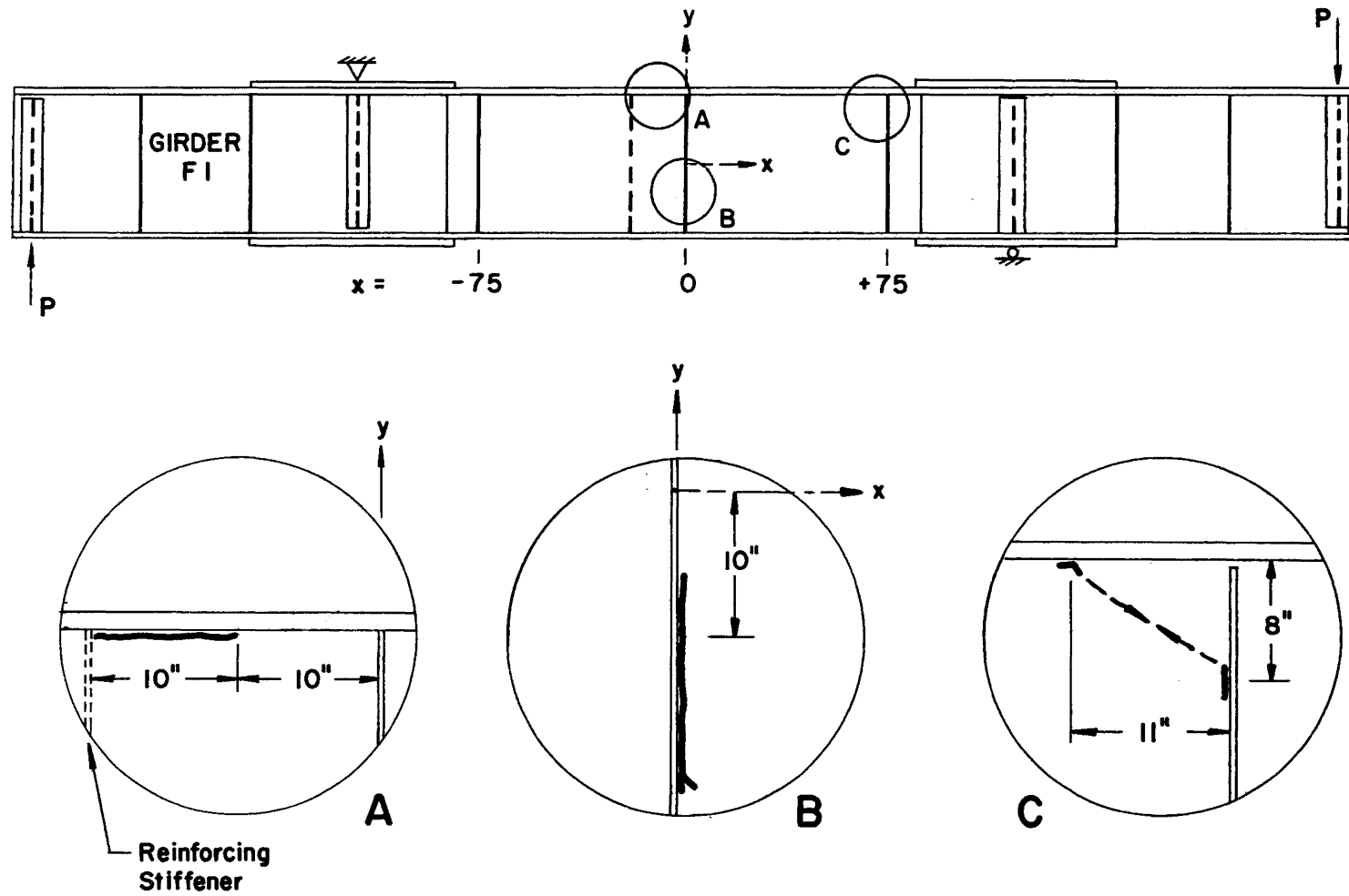


Fig. 21

Crack Location and Repairs, Girder F1

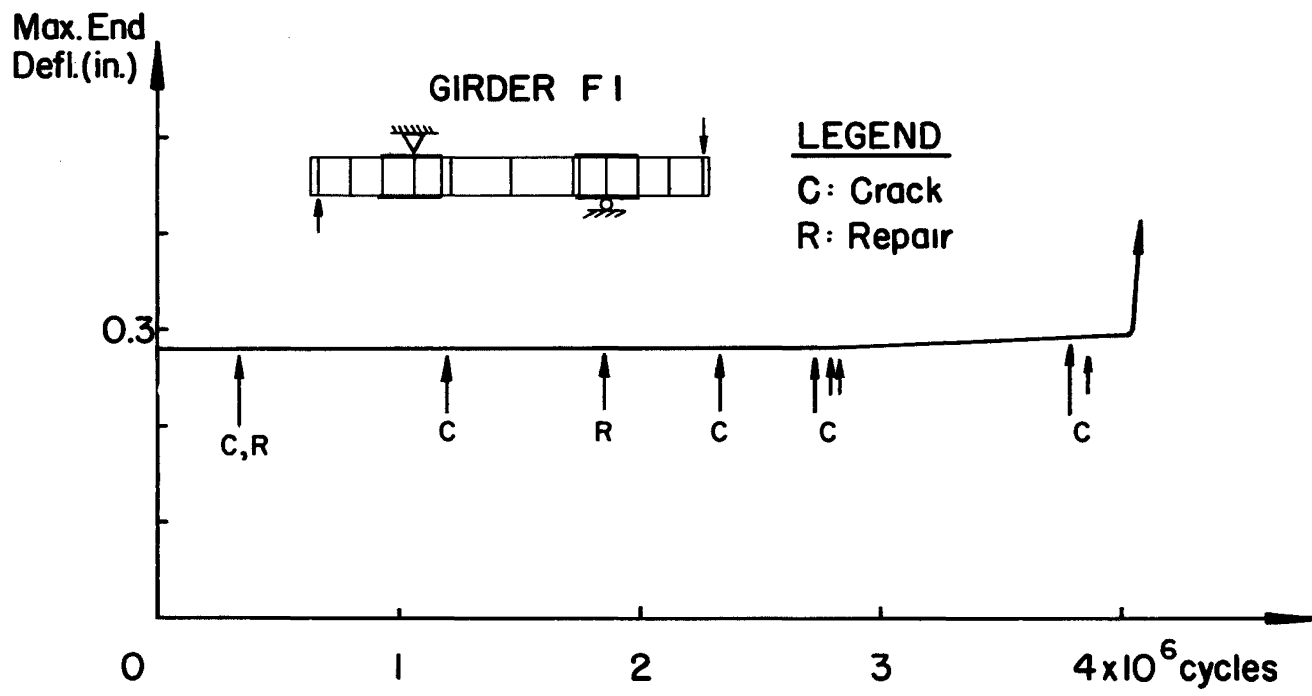


Fig. 22 Maximum Girder Deflection

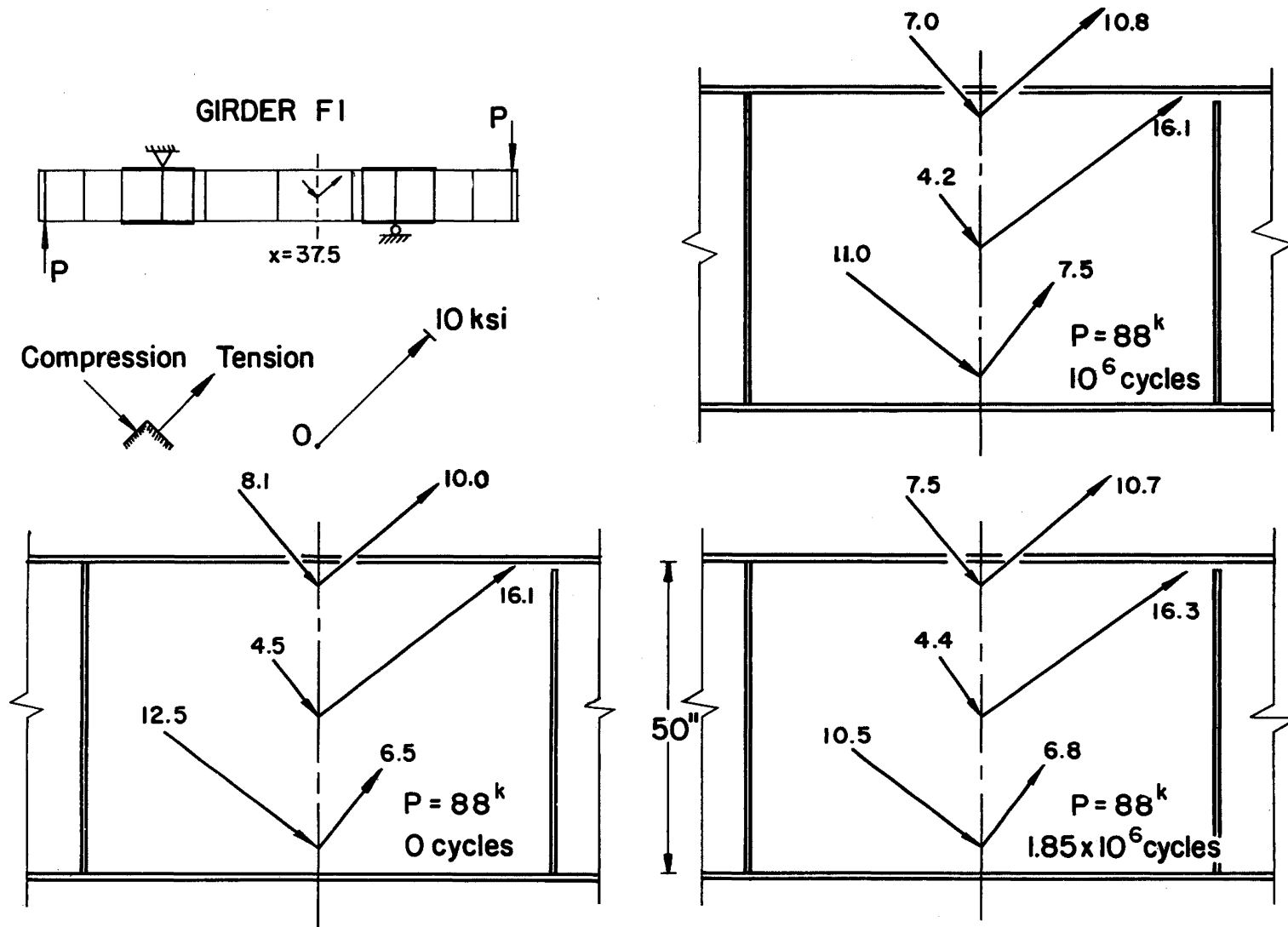


Fig. 23

Principal Stresses in Girder Web

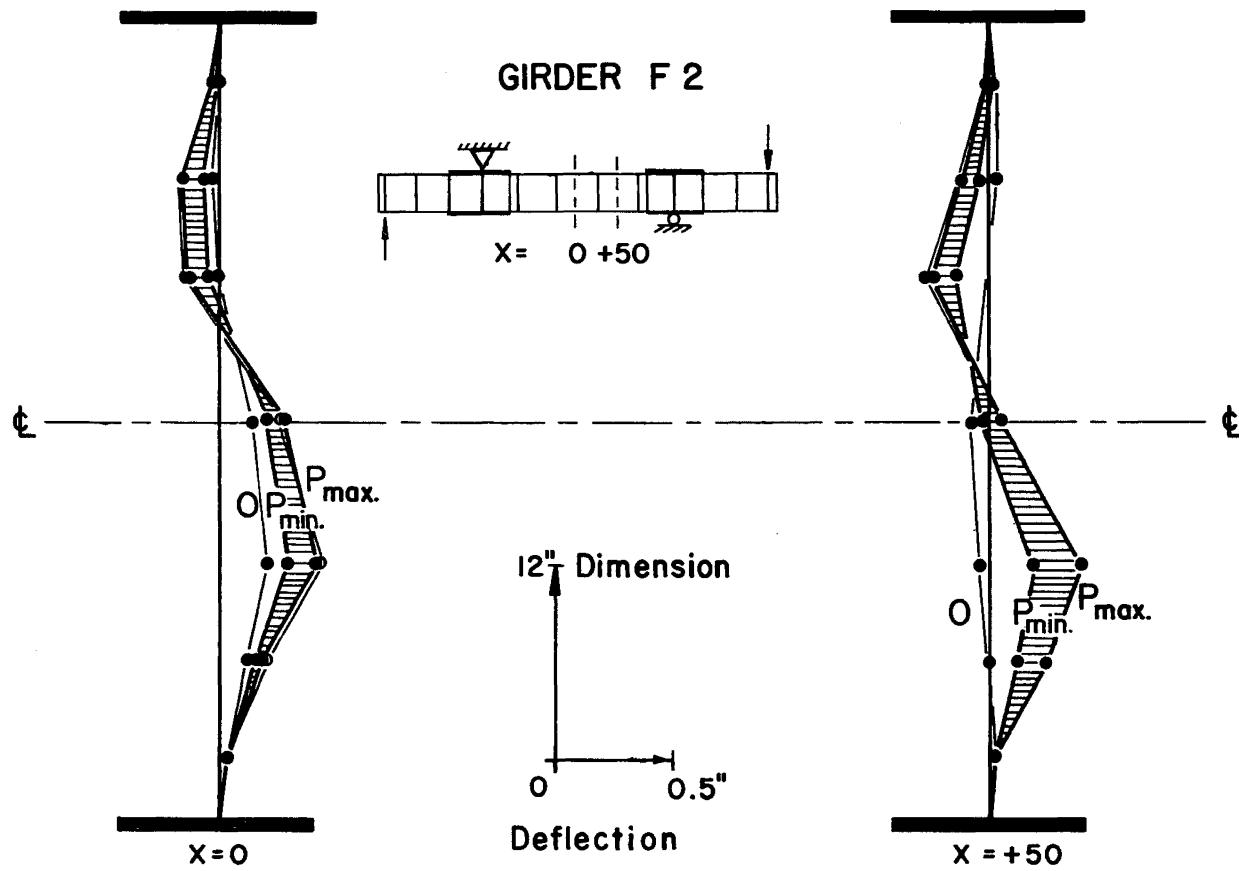


Fig. 24

Web Deflections, Girder F2

GIRDER F1

0-88 kips

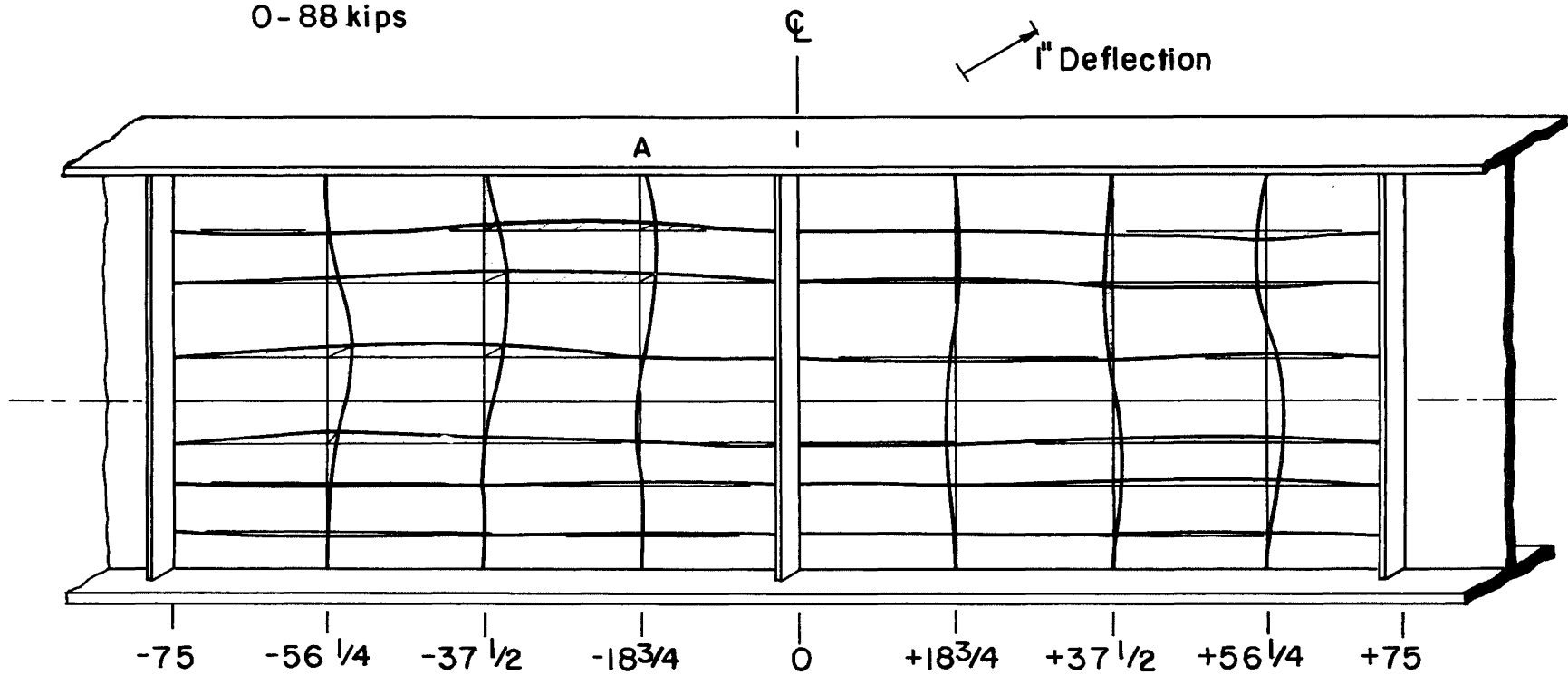


Fig. 25

Relative Web Deflection, Girder F1

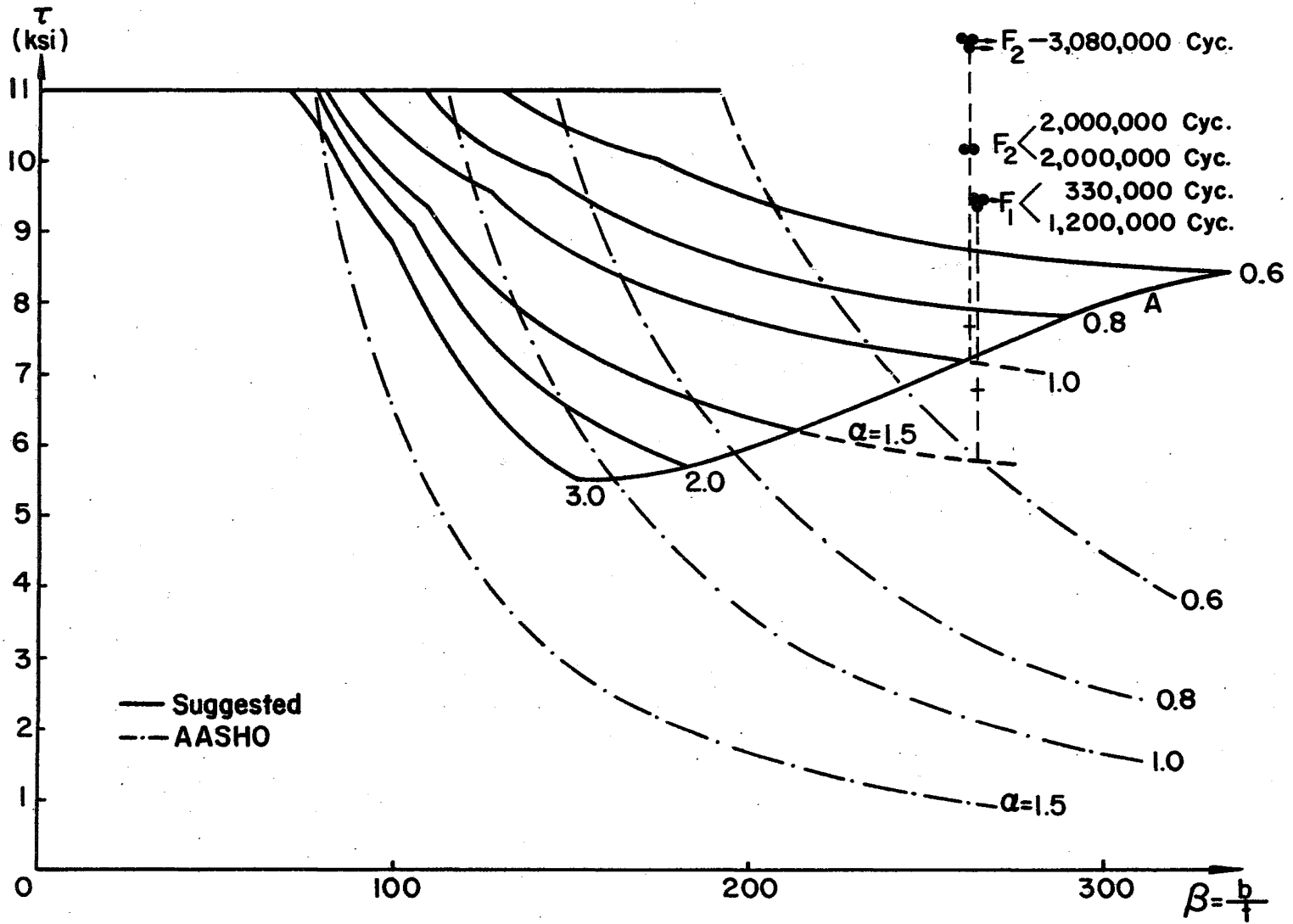


Fig. 26

Permissible Shear Stresses in Web

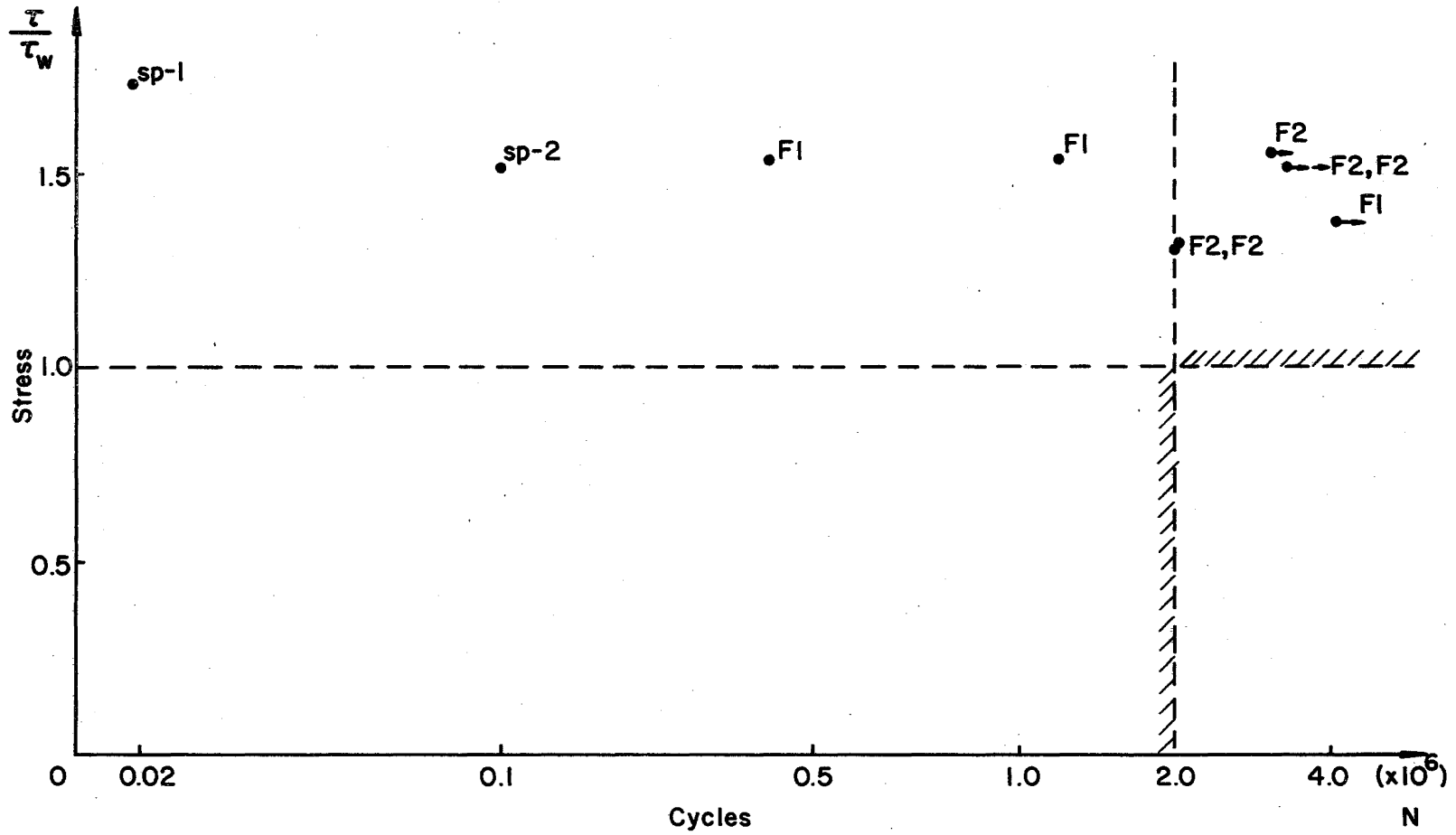


Fig. 27

Shear Stress vs Load Cycles

A P P E N D I X

1. Plate Bending Stresses

Girder F1, At $x = -18\frac{3}{4}$, $y = +25$

At 88 kips, from Table 2

$$\begin{cases} y = 4 & w = 0.112 - 0 = 0.112 \\ y = 10 & w = 0.328 + 0.071 = 0.399 \end{cases}$$

By substituting into $w = C_1 y^3 + C_2 y^2$ (16)

$$\begin{cases} 0.112 = 64C_1 + 16C_2 \\ 0.399 = 1000C_1 + 100C_2 \end{cases} \quad C_2 = 0.009$$

From $\bar{\sigma}_y = C_2 E t$ (18)

$$\bar{\sigma}_y = 0.009 \times 29.6 \times 10^6 \times 0.189 = 50 \text{ ksi}$$

At 44 kips

$$\begin{cases} y = 4 & w = 0.028 - 0 = 0.028 \\ y = 10 & w = 0.053 + 0.071 = 0.124 \end{cases}$$

$$\begin{cases} 0.028 = 64C_1 + 16C_2 \\ 0.124 = 1000C_1 + 100C_2 \end{cases} \quad C_2 = 0.0021$$

$$\bar{\sigma}_y = 0.0021 \times 29.6 \times 0.189 = 12 \text{ ksi}$$

2. Equivalent Principal Stresses

Girder F1, At $x = +56\frac{1}{4}$, $y = +25$, before 2,000,000 cycles

a. Plate Bending Stresses:

At 88 kips

$$\begin{cases} y = 4 & w = 0.076 \\ y = 10 & w = 0.289 \end{cases} \quad C_2 = 0.006$$

$$\bar{\sigma}_y = 0.006 \times 29.6 \times 10^6 \times 0.189 = 34 \text{ ksi}$$

At 44 kips

$$\left. \begin{array}{l} y = 4 \quad w = 0.035 \\ y = 10 \quad w = 0.143 \end{array} \right\} C_2 = 0.0027$$

$$\bar{\sigma}_y = 0.0027 \times 29.6 \times 10^3 \times 0.189 = 15 \text{ ksi}$$

b. Membrane Stresses by Beam Theory

$$I = 17,565 \text{ in.}^4, \quad Q = 312 \text{ in.}^3$$

At 88 kips

$$M = 88 \times 56\frac{1}{4} = 4950 \text{ kip-in.} \quad V = 88 \text{ kips}$$

$$\sigma_b = \frac{MC}{I} = \frac{4950 \times 25}{17,565} = 7 \text{ ksi}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{88 \times 312}{17,565 \times 0.189} = 8 \text{ ksi}$$

At 44 kips

$$\sigma_b = 3.5 \text{ ksi}, \quad \tau_{xy} = 4 \text{ ksi}$$

c. Combined Stresses by Principal Stress Theory

$$\sigma_e = \frac{\sigma_b + \sigma_r + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_b + \sigma_r - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (15)$$

$$\sigma_r = 18 \text{ ksi (assumed)}$$

At 88 kips

$$\sigma_e = \frac{7 + 18 + 34}{2} + \sqrt{\left(\frac{7 + 18 - 34}{2}\right)^2 + 8^2} = 39 \text{ ksi}$$

At 44 kips

$$\sigma_e = \frac{3.5 + 18 + 15}{2} + \sqrt{\left(\frac{3.5 + 18 - 15}{2}\right)^2 + 4^2} = 23 \text{ ksi}$$

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A C K N O W L E D G E M E N T S

The work of this report is a part of the research on welded plate girders conducted at Fritz Engineering Laboratory, Civil Engineering Department, Lehigh University, Bethlehem, Pennsylvania. Professor William J. Eney is Head of the Laboratory and Dr. Lynn S. Beedle is the Director.

The research is jointly sponsored by the American Institute of Steel Construction, the Pennsylvania Department of Highways, the U. S. Department of Commerce - Bureau of Public Roads, the American Iron and Steel Institute, and the Welding Research Council. It is supervised by the Welded Plate Girder Project Subcommittee of the Welding Research Council. The support of the sponsors and the continued interest and guidance of the committee members to the project are gratefully acknowledged.

The author is greatly indebted to Dr. Beedle and Dr. Konrad Basler, present and former director of the project respectively, for their encouragement, advice and suggestions. Special thanks are due Messrs. P. B. Cooper, J. A. Mueller, and H. S. Lew for their assistance. Also thanks are due Messrs. R. Sopko and H. A. Izquierdo for the preparation of drawings and Miss Grace E. Mann for her patience in typing the report.