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Fritz Engineering Laboratory Report No. 290.6

Welded Built-Up and Rolled Heat Treated T-1 Steel Columns

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ABSTRACT

The effect of residual stress on the buckling strength of centrally loaded columns of structural steel is studied. Numerical methods of computing the tangent modulus and the reduced modulus column curves of non-dimensionalized stress versus non-dimensionalized slenderness ratio are presented for pinned-end columns. The methods can be applied to columns containing residual stress of any distribution. Box, H, tee and equal leg angle cross sections are considered.

Numerical results are obtained for two wide flange columns, 8WF31, and 27WF94, containing idealized residual stress distributions of various magnitudes.

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1. INTRODUCTION

The strength of a column may be defined by either of two criteria; the bifurcation (or buckling) load, and the ultimate load. The buckling load may be defined as the load at which the theoretically straight column is indifferent to its deflected position. The ultimate load is the maximum load a column can carry. Although no such perfectly straight member exists in practice, the buckling strength is the most fundamental characteristic of the compression member on which the strength of practical compression members is dependent, and the uncertainties such as initial out-of-straightness are best taken care of for design purposes by the factor of safety.[(1)] The buckling strength of a steel column depends mainly on the slenderness ratio, the sectional properties of the column, the stress-strain relationship of the material, the residual stress distribution in the cross section[(1)(2)(3)(4)].

Residual stresses were introduced in the past decade as the main factor influencing the strength of centrally loaded columns.[(3)(4)(5)(6)(7)] Residual stresses are observed in almost all structural steel members. They are formed as a result of plastic deformations which take place during or after various fabrication procedures. For instance, rolled sections contain residual stress due to uneven cooling after rolling.[(3)(8)] There are several other causes by which the residual stress may be formed, such as cold straightening, shearing, flame cutting, and welding. The residual stress formed by the welding process is usually larger in magnitude than that existing in rolled shapes, or those due to shearing and flame cutting and consequently welding residual stresses have more effect on buckling strength[3][7].

Since welded built-up members are being used more frequently in steel construction due to economy and convenience, it is quite important to have a knowledge of the strength of these members.

When a column containing residual stress is subjected to thrust, it will behave elastically until the thrust reaches a certain value which causes yielding in some portion of the cross section. Under a thrust less than this value and for certain geometrical limitations, the column may buckle elastically. The residual stress has no effect for this elastic buckling* and thus the buckling stress is equal to the Euler stress.

When the thrust exceeds this value, some parts of the cross section start to yield due to the presence of compressive residual stresses. Thereafter, the cross section consists of elastic parts and plastic parts** and the buckling under this loading is called "inelastic buckling" in this report.

In the literature the computation of buckling strength has been made mainly in the form of column curve of stress versus slenderness ratio because of the convenience of computation.

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* In general, residual stress does have some effect on the elastic buckling of a column[9][10]; however, in this report only the buckling due to excessive bending in the direction parallel to a symmetrical axis of the cross section is considered, in which case residual stress plays no role.

**The word plastic is used, since structural steel can be considered as an elastic - perfectly plastic material[11][12].

This has been carried out by either of two methods: one is an analytical method based on the assumed or measured residual stress distribution in the cross section and the other is an approximate method based on the stress-strain relationship of the cross section containing residual stresses[3][13][14]. However, because of the difficulty encountered in computation, the analytical method was made only for a column with a simplified residual stress distribution such as the triangular distribution for rolled shapes and the rectangular distribution for welded shapes in the flanges; the residual stress present in the web plate was mostly neglected.[15]

Residual stresses have been measured on a wide variety of welded plates and on welded built-up shapes of various steels[16][17][18][19][20]. The results obtained are significantly different compared to the idealized patterns of residual stress assumed for the analyses of column strength in the literature. Further, a significant difference in the patterns of residual stress distribution existed depending on material and on geometry of the shapes. The analytical approach to computing column curves seems to be quite difficult and does not cover a wide variety of residual stress patterns. A numerical method of computation is considered in this report, with the help of a digital computer.

This report presents a method of compute column curves by means of a digital computer for structural steel columns containing residual stress. The analysis includes the preparation of equations for column curves based on the tangent modulus and the reduced modulus concepts of column buckling so that the upper and lower limits of the ultimate strength[1] of a straight and centrally loaded steel column are obtained over a wide range of slenderness ratios.

2. BUCKLING STRENGTH OF COLUMNS

2.1 Assumptions

The following assumptions are the basis for the subsequent analysis:

(1) The column is initially straight and free of imperfections. The deflection due to the moment from partial yielding* is small so that the column is straight before buckling even in the inelastic domain.

(2) The external load is applied to the centroid of the cross section, causing uniform strain over the cross section, and along the whole length.

(3) The mechanical properties, and the residual stress, are uniform along the whole length of the column for each fiber.

(4) The cross section has at least one axis of symmetry and does not change along the length of the column.

(5) The residual stress is constant in the thickness direction of the wall of the cross section. The distribution of residual stress is symmetric about the axis of symmetry of the cross section.

*The partial yielding is due to the presence of residual stress in the case of a column with homogeneous material.

2.2 Buckling Strength of Columns

The differential equation of equilibrium of a slightly buckled column is well known(1)(2)(10)(14)(21) and can be written in the following form,

$$B \frac{d^4u}{dz^4} + P \frac{d^2u}{dz^2} = 0 \tag{1}$$

where z is the co-ordinate along the length of a column and u denotes the deflection. P is the axial load at which the column is going to buckle. The bending rigidity of the column, B, is defined by the product of modulus of elasticity, E, and moment of inertia, I, of the cross section for an elastic column, thus

$$B = EI \tag{2}$$

For an inelastic column, the bending rigidity is defined by either of two theories; the tangent modulus theory or the reduced modulus theory. The tangent modulus theory defines the rigidity by the following expression with the tangent modulus of elasticity

$$B = \int_A E_t \, y^2 \, dA \tag{3}$$

where y is the distance from the neutral axis to a fiber in the cross section and A denotes the area. The tangent modulus of elasticity, E_t, is not a constant for a column containing residual stress, since it is a function of strain at each fiber, which is not uniform but it is shown by the sum of residual strain and the strain due to loading.

The reduced modulus theory defines the rigidity by the following equation

$$B = \int_{A_l} E_t\, y^2\, dA + \int_{A_u} E\, y^2\, dA \tag{4}$$

where A_l and A_u denote the area respectively, where strain is increasing and the area where strain is decreasing. The neutral axis is determined by the condition that no increment of axial load occurs at the instant of buckling, thus

$$\int_{A_l} E_t\,(y-Y_o)\,dA + \int_{A_u} E\,(y-Y_o)\,dA = 0 \tag{5}$$

where Y_o is the coordinate of the neutral axis.

The expression for the buckling strength of a column is obtained as the characteristic value of Equation 1. For the analysis of a column with residual stress and loaded into the inelastic range of the material, it is not, in general, practical to solve for the buckling load, but it is easier to solve for the critical length under a known loading. The expression for the buckling of a pinned-end column is[10][21].

$$L_{cr} = \pi \sqrt{\frac{B}{P}} \tag{6}$$

where L_{cr} is the length of a pinned-end column which is going to buckle under a loading P. Since the bending rigidity is a function of loading and of residual stress distribution in the inelastic range, the explicit numerical solutions need more consideration.

3. NUMERICAL COMPUTATION OF COLUMN CURVES

The numerical method of computation is considered for steel columns of box-, H-, tee-, and equal-leg angle-sections, containing residual stress. The method, by its nature, is applicable for columns with any kind of residual stress distribution under the given assumptions and it is suitable for computation by a digital computer.

In addition to those in Article 2.1, an assumption that the stress strain relationship of steel is elastic-perfectly plastic is made for the following analyses. Because of this assumed stress strain relationship of steel, the bending rigidity, B, at the instant of buckling becomes simply the product of modulus of elasticity and the effective moment of inertia, I_e

$$B = EI_e \tag{7}$$

The tangent modulus theory defines the effective moment of inertia by the moment of inertia of the cross section which remains elastic, and the reduced modulus theory defines it by that of the cross section which consists of the elastic part in the loading region of the cross section and the whole part in the unloading region at the instant of buckling.

In order to compute the non-dimensionalized column curves, and using Eq. 7, Eq. 6 may be changed to

$$\frac{1}{\pi} \cdot \frac{L}{r} \cdot \sqrt{\frac{\sigma_Y}{E}} = \sqrt{\frac{I_e}{I} \cdot \frac{\sigma_Y}{\sigma_{cr}}} \tag{8}$$

where, r and I are the radius of gyration and the moment of inertia of the cross section, respectively. σ_{cr} is the buckling strength defined by the axial load divided by the area of the cross section. The left hand term of the above equation is the non-dimensionalized slenderness ratio.

The analysis is made in such a way that the results are presented in the form of column curves of stress versus slenderness ratio. Only the analysis for box-columns, including H-columns as a special case, is described in this article and the analyses for other sections are given in Appendix A.

3.1 Tangent Modulus Column Curve

Since two axes of symmetry exist in a box-section, Eq. 8 holds for flexural buckling both in the x- and y-direction. For the buckling in the y-direction, Eq. 8 can be rewritten in the following form with subscript x for both the moment of inertia and the radius of gyration denoting them as values about the x-axis.

$$\frac{1}{\pi} \cdot \frac{L_x}{r_x} \cdot \sqrt{\frac{\sigma_Y}{E}} = \sqrt{\frac{I_{xe}}{I_x} \cdot \frac{\sigma_Y}{\sigma_{cr}}} \tag{9}$$

Similarly for buckling in the x-direction, the following equation is obtained

$$\frac{1}{\pi} \cdot \frac{L_y}{r_y} \cdot \sqrt{\frac{\sigma_Y}{E}} = \sqrt{\frac{I_{ye}}{I_y} \cdot \frac{\sigma_Y}{\sigma_{cr}}} \tag{10}$$

where the subscript y denotes that the values are about the y-axis.

On specifying a strain due to the external load which is distributed uniformly in the cross section, the load which causes the specified strain may be computed from the stress distribution in the cross section. The effective moment of inertia I_{xe} and I_{ye} under the load are also determined. The slenderness ratio is a function of the external load and the moment of inertia as seen in Eq. 8, and consequently it is a function of the specified strain.

A cross section of a box-column may be considered as consisting of small segments as shown in Fig. 1, with the change of the residual stress distribution inside each segment linear in the tangential direction. The magnitudes of residual stresses or strains are assumed to be known at the boundaries of the segments. Since the cross section has two axes of symmetry, it is enough to consider only a quarter of it. A half of the flange consists of n small segments of the same size. Similarly a half of the web consists of m segments. Numbers are given, 1 to n, to the segments for the flange part, and 1 to m for the web as shown in Fig. 1. It is noted that the H-shape is a special case of the box-shape when $\overline{b}$ is equal to zero.

The strain due to external load at which the column is going to buckle is denoted as ε_{cr}. Since the residual stresses are in equilibrium in the cross section, the external load may be computed by the sum of stress at each segment

$$P = 4 \sum_{n,m} \sigma_s \Delta A \qquad (11)$$

where ΔA is the area of each segment and σ_s is the average stress in the segment. $\sum_{n,m}$ denotes the summation of segments throughout a quarter part of the cross section. The average stress σ_s can be computed from the residual strains at the edges of the segment and uniform external strain ϵ_{cr}. The strains at the edges are

$$\begin{aligned} \epsilon_{i-1} &= \epsilon_{r,i-1} + \epsilon_{cr} \\ \epsilon_{i} &= \epsilon_{r,i} + \epsilon_{cr} \end{aligned} \tag{12}$$

where ϵ_r denotes the residual strain and the subscripts i and i-1 refer to the values at both edges of segment i as shown in Fig. 1. Then the average stress in the segment can be obtained by simple arithmetic in a form non-dimensionalized by the yield stress σ_Y, depending on the value of strain at the edges.

$$\begin{aligned} \frac{\sigma_s}{\sigma_Y} &= 1 && \text{for } \epsilon_i \geq \epsilon_Y && \epsilon_{i-1} \geq \epsilon_Y \\ &= \frac{\epsilon_i + \epsilon_{i-1}}{2} && \text{for } \epsilon_i < \epsilon_Y && \epsilon_{i-1} < \epsilon_Y \\ &= \frac{2\epsilon_i - \epsilon_{i-1}^2 - 1}{2\epsilon_Y(\epsilon_i - \epsilon_{i-1})} && \text{for } \epsilon_i \geq \epsilon_Y && \epsilon_{i-1} < \epsilon_Y \\ &= \frac{2\epsilon_{i-1} - \epsilon_i^2 - 1}{2\epsilon_Y(\epsilon_{i-1} - \epsilon_i)} && \text{for } \epsilon_i < \epsilon_Y && \epsilon_{i-1} \geq \epsilon_Y \end{aligned} \tag{13}$$

Then the critical stress non-dimensionalized by using the yield stress can be computed as

$$\frac{\sigma_{cr}}{\sigma_Y} = \frac{4}{A} \sum_{n,m} \left(\frac{\sigma_s}{\sigma_Y} \right) \cdot \Delta A \tag{14}$$

where A is the cross sectional area.

The moment of inertia of the cross section about the x- and y- axes, I_x and I_y, can be computed as

$$I_x = 4 \left[\sum_{n,m} y^2 \Delta A + \sum_{n,m} \frac{\Delta A}{12} (\Delta y)^2 \right] \tag{15}$$

$$I_y = 4 \left[\sum_{n,m} x^2 \Delta A + \sum_{n,m} \frac{\Delta A}{12} (\Delta x)^2 \right] \tag{16}$$

where x and y are the coordinates at the center of each segment. Δx and Δy are the dimensions of each segment in the x- and y- directions, and ΔA is its area. Figure 1 shows the detail of those notations. The moment of inertia of the cross section which remains elastic can be computed similarly,

$$I_{xe} = 4 \left[\sum_{n,m} y_e^2 \ \Delta A_e + \sum_{n,m} \frac{\Delta A_e}{12} (\Delta y_e)^2 \right] \tag{17}$$

$$I_{ye} = 4 \left[\sum_{n,m} x_e^2 \ \Delta A_e + \sum_{n,m} \frac{\Delta A_e}{12} (\Delta x_e)^2 \right] \tag{18}$$

where x_e and y_e are the coordinates at the center of the elastic part, of which the dimensions are Δx_e and Δy_e in each segment. ΔA_e is the area which remains elastic in each segment.

With notations b, $\bar{b}$, d, t, and w for the dimensions of the cross section as shown in Fig. 1, the following relationships hold true for a segment in the flange

$$\Delta x = \frac{b}{2n}, \quad \Delta y = t, \quad \Delta A = \frac{bt}{2n} \tag{19}$$

and in the web

$$\Delta x = w, \quad \Delta y = \frac{d}{2m}, \quad \Delta A = \frac{dw}{2m} \tag{20}$$

With these relationships, the non-dimensionalized stress, Eq. 14, can be written for the convenience of computation as

$$\frac{\sigma_{cr}}{\sigma_Y} = \frac{2}{A}\left[\frac{bt}{n}\cdot\sum_{i=1}^{n}\left(\frac{\sigma_s}{\sigma_Y}\right)_i + \frac{dw}{m}\cdot\sum_{i=1}^{m}\left(\frac{\sigma_x}{\sigma_Y}\right)_i\right] \tag{21}$$

When yielding penetrates partially into a segment, the dimensions of the elastic part and the coordinates at the center of the part can be obtained by the following equations:

for a flange segment

$$\left.\begin{aligned}
\Delta x_{e,i} &= \frac{\epsilon_{i-1}}{\epsilon_i - \epsilon_{i-1}}\ \Delta x, \\
x_{e,i} &= \Delta x\ (i-1) + \frac{\Delta x_{e,i}}{2} \\
&\quad \text{for}\quad \epsilon_i \geq \epsilon_Y,\quad \epsilon_{i-1} < \epsilon_Y \\
\Delta x_{e,i} &= \frac{\epsilon_{i-1} - 1}{\epsilon_{i-1} - \epsilon_1}\ \Delta x, \\
x_{e,i} &= \Delta x\ i - \frac{\Delta x_{e,i}}{2} \\
&\quad \text{for}\ \epsilon_i < \epsilon_Y,\ \epsilon_{i-1} \geq \epsilon_Y
\end{aligned}\right\} \tag{22}$$

and for a web segment

$$\left.\begin{aligned}
\Delta y_{e,i} &= \frac{\epsilon_i - 1}{\epsilon_i - \epsilon_{i-1}} \cdot \Delta y, \\
y_{e,i} &= \Delta y\ (i-1) + \frac{\Delta y_{e,i}}{2} \\
&\quad \text{for } \epsilon_i \geq \epsilon_Y, \quad \epsilon_{i-1} < \epsilon_Y \\
\Delta y_{e,i} &= \frac{\epsilon_{i-1} - 1}{\epsilon_{i-1} - \epsilon_i}\ \Delta y, \\
y_{e,i} &= \Delta y\ i - \frac{\Delta y_{e,i}}{2} \\
&\quad \text{for } \epsilon_i < \epsilon_Y, \quad \epsilon_{i-1} \geq \epsilon_Y
\end{aligned}\right\} \tag{23}$$

where the subscript i shows the values at the i-th segment. Then the moment of inertia about the x-axis can be computed as follows in the non-dimensionalized form

$$\frac{I_{xe}}{I_x} = \frac{1}{I_x}\left\{(3d^2+6dt+4t^2)\left(\frac{bt}{6n}\right)\sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right) + \left(\frac{wd^3}{24m^3}\right)\sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right)\left[12\left(\frac{y_{e,i}}{\Delta y}\right)^2 + \frac{\Delta y_{e,i}}{\Delta y}\right]\right\} \tag{24}$$

and similarly about the y-axis

$$\frac{I_{ye}}{I_y} = \frac{1}{I_y}\left\{(3\bar{b}^2 + 6\bar{b}w + 4w^2)\left(\frac{wd}{6m}\right)\sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right) + \left(\frac{tb^3}{24n^3}\right)\sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right)\left[12\left(\frac{x_{e,i}}{\Delta x}\right)^2 + \left(\frac{\Delta x_{e,i}}{\Delta x}\right)^2\right]\right\} \tag{25}$$

With a specified strain ϵ_{cr} due to external loading, and with the known residual stress distribution, Eq. 21 gives the critical stress corresponding to the critical strain. Equation 9, together with Eqs. 21 and 24, gives the non-dimensionalized critical slenderness ratio of a column which is going to buckle in the y-direction under the action of the stress. Similarly Eq. 10, together with Eqs. 21 and 25 gives the ratio for buckling in the x-direction. By varying the strain, ϵ_{cr}, from a small value to a large value at a certain increment, a column curve of stress versus slenderness ratio can be obtained.

3.2 Reduced Modulus Column Curve

The procedure followed in computing a column curve based on the reduced modulus concept is similar to that based on the tangent modulus concept. The difference involved is that under the reduced modulus concept, the neutral axis does not remain at the axis of symmetry of the cross section at the instant of buckling even for the cross section with two axes of symmetry with symmetric distribution of residual stress; instead the location of the neutral axis is a function of loading. This is due to the difference of modulus of elasticity in the loading and in the unloading portion of the cross section where yielding has penetrated.

Assuming buckling in the negative y-direction, the neutral axis is parallel to the x-axis and will be such that y is positive. The distance between the neutral axis and the axis of symmetry is denoted by Y_o, which is shown in Fig. 2(a). The distance Y_o has to be determined to satisfy Eq. 5, which can be written in the form of a summation of stress at each segment,

$$\sum_{A_u} (y-Y_o)\ \Delta A + \sum_{A_l} (y_e - Y_o)\ \Delta A_e = 0 \tag{26}$$

where $\sum_{A_u}$ and $\sum_{A_l}$ denote summation of area of each segment throughout the unloading region and the loading zone, respectively. Equation 26 may be rewritten for the box cross section considered here,

$$\frac{bt}{2}\left(\frac{d+t}{2} - Y_o\right) + \frac{wd}{2m} \sum_{i=k+1}^{m} (y_i - Y_o) + w \sum_{i=1}^{k-1} (y_{e,i} - Y_o)\ \Delta y_{e,i}$$
$$- t\left(\frac{d+t}{2} + Y_o\right) \sum_{i=1}^{n} \Delta x_{e,i} - w \sum_{i=1}^{m} (y_{e,i} + Y_o)\ \Delta y_{e,i} + S_k = 0 \tag{27}$$

where k is the number of a segment where the neutral axis passes through as shown in Fig. 2(a). The last term in the left hand side of Eq. 27, S_k, is the stress acting in the segment k. The expression of S_k depends on the penetration of yielding into the segment, and is considered in Appendix B. Equation 27 is valid when the resulting neutral axis is located in the web, namely under the following condition,

$$Y_o \leqq \frac{d}{2}$$

When the resulting neutral axis is located in the flange, Eq. 27 must be modified, thus,

$$\frac{b}{4}\left(\frac{d}{2} + t - Y_o\right)^2 - \frac{1}{2}\left(Y_o - \frac{d}{2}\right)^2 \sum_{i=1}^{n} \Delta x_{e,i} + w \sum_{i=1}^{m} (y_{e,i} - Y_o)\Delta y_{e,i}$$
$$- t\left(\frac{d+t}{2} + Y_o\right) \sum_{i=1}^{n} \Delta x_{e,i} \tag{28}$$

for

$$\frac{d}{2} \leqq Y_o \leqq \frac{d}{2} + t$$

After the neutral axis is determined, the moment of inertia of the cross section, which consists of all the parts in the unloading section of the cross section and the parts remaining elastic in the loading section, can be computed by the following equations in the non-dimensionalized form

$$\frac{I_{xe}}{I_x} = \frac{1}{I_x}\left\{ bt\left[\left(\frac{d+t}{2} - Y_o\right)^2 + \frac{t^2}{12}\right] + \frac{bt}{n}\left[\left(\frac{d+t}{2} + Y_o\right)^2 + \frac{t^2}{12}\right] \cdot \right.$$

$$\cdot \sum_{i=1}^{n}\left(\frac{\Delta_{xe,i}}{\Delta x}\right) + \frac{d^3 w}{48m^3}\left\{ \sum_{i=k+1}^{m}\left[12\left(\frac{Y_i - Y_o}{\Delta y}\right)^2 + 1\right]\right.$$

$$+ \sum_{i-1}^{k-1}\left[12\left(\frac{y_{e,i} - Y_o}{\Delta y}\right)^2 + \left(\frac{\Delta y_{e,i}}{\Delta y}\right)^2\right]\left(\frac{\Delta y_{e,i}}{\Delta y}\right) \tag{29}$$

$$\left.\left. + \sum_{i=1}^{m}\left[12\left(\frac{Y_{e,i} + Y_o}{\Delta y}\right)^2 + \left(\frac{\Delta y_{e,i}}{\Delta y}\right)^2\right]\left(\frac{\Delta y_{e,i}}{\Delta y}\right)\right\} + 2I_k \right\}$$

when

$$Y_o \leqq \frac{d}{2}$$

and

$$\frac{I_{xe}}{I_x} = \frac{1}{I_x}\left\{ \frac{2b}{3}\left(\frac{d}{2} + t - Y_o\right)^3 + \frac{b}{3n}\left(Y_o - \frac{d}{2}\right)^3 \sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right) + \frac{bt}{n} \cdot \right.$$

$$\cdot \left[\left(\frac{d+t}{2} + Y_o\right)^2 + \frac{t2}{12}\right] \sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right) + \frac{d^3 w}{48m^3}\left\{ \sum_{i=k+1}^{n}\left[12\left(\frac{y_i - Y_o}{\Delta y}\right) + 1\right]\right.$$

$$+ \sum_{i=1}^{k-1}\left[12\left(\frac{y_{e,i} - Y_o}{\Delta y}\right)^2 + \left(\frac{\Delta y_{e,i}}{\Delta y}\right)^2\right]\left(\frac{\Delta y_{e,i}}{\Delta y}\right) \tag{30}$$

$$\left.\left. + \sum_{i=1}^{m}\left[12\left(\frac{y_{e,i} + Y_o}{\Delta y}\right)^2 + \left(\frac{\Delta y_{e,i}}{\Delta y}\right)^2\right]\left(\frac{\Delta y_{e,i}}{\Delta y}\right)\right\} + 2I_k \right\}$$

when

$$\frac{d}{2} \leqq Y_o \leqq \frac{d}{2} + t$$

where I_k is the contribution to moment of inertia of segment k, of which the expressions are shown in Appendix B, and depending on the penetration of yielding into the segment.

Similarly as for the tangent modulus curve, a column curve for the buckling in the y-direction can be computed from Eq. 9 substituting Eqs. 14 and either Eqs. 29 or 30.

A column curve for the buckling in the x-direction is also obtained by a procedure similar to that for buckling in the y-direction. The expression of the effective moment of inertia, however, is different depending on the location of the neutral axis, inside or outside of the web.

The equations which determine the location of the neutral axis become the following, depending on the resulting location of the neutral axis:

$$\frac{wd}{2}\left(\frac{\overline{b}+w}{2} - x_o\right) + \frac{bt}{2n}\sum_{i=k+1}^{n}(x_i - x_o) + t\sum_{i=1}^{k-1}(x_{e,i} - x_o)\Delta x_{e,i}$$
$$- w\left(\frac{\overline{b}+w}{2} + x_o\right)\sum_{i=1}^{m}\Delta y_{e,i} - t\sum_{i=1}^{n}(x_{e,i} + x_o)\Delta x_{e,i} + S_k = 0 \qquad (31)$$

when

$$x_o \leqq \frac{\overline{b}}{2}$$

$$\frac{d}{4}\left(\frac{\bar{b}}{2} + w - x_o\right)^2 - \frac{1}{2}\left(x_o - \frac{\bar{b}}{2}\right)^2 \sum_{i=1}^{m} \Delta y_{e,i} + \frac{bt}{2n} \sum_{i=k+1}^{n} (x_i - x_o)$$

$$+ t \sum_{i=1}^{k-1} (x_{e,i} - x_o)\, \Delta x_{e,i} - w\left(\frac{\bar{b}+w}{2} + x_o\right) \sum_{i=1}^{m} \Delta y_{e,i} - t \sum_{i=1}^{n} \cdot \qquad (32)$$

$$\cdot\, (x_{e,i} + x_o)\, \Delta x_{e,i} + S_k = 0$$

when

$$\frac{\bar{b}}{2} < x_o \leqq \frac{\bar{b}}{2} + w$$

and

$$w\left(\frac{\bar{b} + w}{2} - x_o\right) \sum_{i=1}^{m} \Delta y_{e,i} + \frac{bt}{2n} \sum_{i=k+1}^{n} (x_i - x_o) + t \sum_{i=1}^{k-1} (x_{e,i} - x_o) \cdot$$

$$\cdot\, \Delta x_{e,i} - w\left(\frac{\bar{b} + w}{2} + x_o\right) \sum_{i=1}^{m} \Delta y_{e,i} - t \sum_{i=1}^{n} (x_{e,i} + x_o)\, \Delta x_{e,i} \qquad (33)$$

$$+ S_k = 0$$

when

$$\frac{\bar{b}}{2} + w < x_o \leqq \frac{b}{2}$$

where X_o is the distance between the neutral axis and the axis of symmetry.

In a manner similar for that of Eq. 29, equations by which the effective moment of inertia of the cross section can be calculated are derived as follows, depending on the location of the neutral axis,

$$
\begin{aligned}
\frac{I_{ye}}{I_y} = \frac{1}{I_y} \Bigg\{ & dw \left[\left(\frac{\bar{b}+w}{2} - \chi_o \right)^2 + \frac{w^2}{12} \right] + \frac{dw}{m} \left[\left(\frac{\bar{b}+w}{2} + \chi_o \right)^2 + \frac{w^2}{12} \right] \sum_{i+1}^{m} \left(\frac{\Delta y_{e,i}}{\Delta y} \right) \\
& + \frac{b^3 t}{48 n^3} \Bigg\{ \sum_{i=k+1}^{n} \left[12 \left(\frac{\chi_i - \chi_o}{\Delta x} \right)^2 + 1 \right] + \sum_{i=1}^{k-1} \left[12 \left(\frac{\chi_{e,i} - \chi_o}{\Delta x} \right)^2 + \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right)^2 \right] \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right) \\
& + \sum_{i=1}^{n} \left[12 \left(\frac{\chi_{e,i} + \chi_o}{\Delta x} \right)^2 + \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right)^2 \right] \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right) \Bigg\} + 2 I_k \Bigg\}
\end{aligned}
\tag{34}
$$

when

$$\chi_o \leq \frac{\bar{b}}{2}$$

$$
\begin{aligned}
\frac{I_{ye}}{I_y} = \frac{1}{I_y} \Bigg\{ & \frac{2d}{3} \left(\frac{\bar{b}}{2} + w - \chi_o \right)^3 + \frac{d}{3m} \left(\chi_o - \frac{\bar{b}}{2} \right)^3 \Sigma \left(\frac{\Delta y_{e,i}}{\Delta y} \right) + \frac{dw}{m} \bullet \\
& \bullet \left[\left(\frac{\bar{b}+w}{2} + \chi_o \right)^2 + \frac{w^2}{12} \right] \sum_{i=1}^{m} \left(\frac{\Delta y_{e,i}}{\Delta y} \right) + \frac{b^3 t}{48 n^3} \Bigg\{ \sum_{i-1}^{k-1} \left[12 \left(\frac{\chi_i - \chi_o}{\Delta x} \right)^2 + 1 \right] \\
& + \sum_{i=1}^{k-1} \left[12 \left(\frac{\chi_{e,i} - \chi_o}{\Delta x} \right)^2 + \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right)^2 \right] \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right) + \sum_{i=1}^{n} \left[12 \left(\frac{\chi_{e,i} + \chi_o}{\Delta x} \right)^2 \right. \\
& \left. + \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right)^2 \right] \left(\frac{\Delta \chi_{e,i}}{\Delta x} \right) \Bigg\} + 2 I_k \Bigg\}
\end{aligned}
\tag{35}
$$

when

$$\frac{\bar{b}}{2} < \chi_o \leq \frac{\bar{b}}{2} + w$$

$$
\begin{aligned}
\frac{I_{ye}}{I_y} = \frac{1}{I_y}\Bigg\{ & \frac{wd}{m}\left[\left(\frac{\bar{b}+w}{2} - \chi_o\right)^2 + \frac{w^2}{12}\right]\sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right) + \frac{dw}{m}\left[\left(\frac{\bar{b}+w}{2} + \chi_o\right)^2 + \frac{w2}{12}\right] \cdot \\
& \cdot \sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right) + \frac{b^3 t}{48n^3}\Bigg\{ \sum_{l=k+1}^{n}\left[12\left(\frac{\chi_i - \chi_o}{\Delta x}\right)^2 + 1\right] + \sum_{i=1}^{k-1}\Bigg[12 \cdot \\
& \cdot \left(\frac{\chi_{e,i} - \chi_o}{\Delta x}\right)^2 + \left(\frac{\Delta\chi_{e,i}}{\Delta x}\right)^2\Bigg]\left(\frac{\Delta\chi_{e,i}}{\Delta x}\right) \\
& + \sum_{i=1}^{n}\left[12\left(\frac{\chi_{e,i} + \chi_o}{\Delta x}\right)^2 + \frac{\Delta\chi_{e,i}{}^2}{\Delta x}\right]\left(\frac{\Delta\chi_{e,i}}{\Delta x}\right)\Bigg\} + 2I_k\Bigg\}
\end{aligned}
\tag{36}
$$

when

$$\frac{\bar{b}}{2} + w < \chi_o$$

A column curve can be computed, as above, from Eq. 9 substituting Eq. 14 and one of the Eqs. 34, 35, and 36.

4. NUMERICAL EXAMPLES

Numerical computation was carried out by a digital computer.* Although any distribution of residual stress can be considered, only idealized patterns of residual stress distribution in H-shapes were used (as shown in Fig. 3) for the illustration of column curves. The triangular distribution of Fig. 3a is close to the patterns found in rolled shapes(3)(5)(20) and a similar pattern was assumed to predict the strength of centrally loaded columns of rolled shapes(3)(6); the pattern of Fig. 3b resembles the pattern in the welded shapes, among which the residual stress pattern in welded T-1 shapes is the closest(6)(18)(19)(22).

Figure 4 shows the tangent modulus column curves of non-dimensionalized stress against non-dimensionalized slenderness ratio for columns with cross sections of 8WF31 and 27WF94 containing residual stresses of the triangular patterns. Similarly, Fig. 5 presents the tangent modulus curves for the same columns with welding type residual stress patterns. The curves were computed for various values of the compressive residual stresses. The discontinuities of the column curves for the sections with welding type residual stress patterns are due to simultaneous yielding over a large portion of the cross section due to the assumed residual stress pattern. The column curves for strong axis bending are slightly different for the two cross sections, 8WF31, and 27WF94,

*The descriptions of the programs are given in Appendix C.

while the difference of the curves for weak axis bending is insignificant for the two columns.

For the assumed residual stress patterns of Fig. 3a, the larger the compressive residual stress is, the greater the reduction of strength for the entire elastic-plastic buckling. The same is true for the assumed residual stress pattern of Fig. 3b except for columns of small slenderness ratio.

The reduced modulus column curves were computed for the same column shapes as used for the tangent modulus curves, and the results are shown in Figs. 6 and 7; Fig. 6 for the triangular distribution and Fig. 7 for the welding-type distribution. The general shape of the column curves based on the reduced modulus concept is similar to those found in curves based on the tangent modulus concept. The only difference in the general pattern of the column curves is that there is a visible difference in the reduced modulus column curves of 8WF31 and 27WF94 for weak axis bending as seen in Figs. 6b and 7b, while no visible difference resulted in the corresponding tangent modulus curves. This is due to the relatively large contribution of the web to the moment of inertia when strain reversal is taken into account.

Both the tangent modulus and the reduced modulus column curves are plotted in Figs. 8 and 9 for 8WF31 columns with various residual stress distribution. As can be seen in the figures, the difference between the tangent modulus load and the reduced modulus load depends largely on slenderness ratio. Both the tangent modulus load and the reduced modulus load coincide with the Euler load for a column with a large slenderness ratio. Generally, the difference first appears between the loads at a certain slenderness ratio, and it increases with decreasing slenderness ratio reaching the maximum point. After the maximum point is passed, the difference between the loads becomes small and the loads coincide again for a column with zero slenderness ratio.

The strength of steel columns is often represented by the tangent modulus loads taking into consideration residual stresses[3][6][10][14]. The comparison of the reduced modulus curves and the tangent modulus curves of Figs. 8 and 9, however, suggests that some caution has to be paid in some cases. For example, the strength of an 8WF31 column containing residual stress of $\sigma_{rc} = \frac{1}{4}\sigma_Y$ with the distribution pattern of Fig. 3b cannot be more than the tangnet modulus load, because of unavoidable initial crookedness and eccentricity, when its non-dimensionalized slenderness ratio is larger than 0.7, (Figure 9b). For a shorter column, on the other hand, there is so large a difference between the reduced modulus load and the tangent modulus load that it may be possible that the strength exceeds the tangent modulus load.[14] When the difference is large, as found in a case of a column with welding type residual stresses and bent about the weak axis, an analysis of ultimate strength of the column would be made preferably in addition to the tangent modulus analysis[15].

5. SUMMARY

The report has presented numerical methods of computing the tangent modulus and the reduced modulus column curves of non-dimensionalized stress versus non-dimensionalized slenderness ratio such that the lower and the upper limits of the ultimate strength of a straight and centrally loaded column are obtained. These limits are defined by the tangent modulus and reduced modulus, respectively. Equations were developed for the flexural buckling of pinned-end columns of structural steel containing residual stress of any distribution. Among the shapes considered, H, box, tee, and equal leg angle, equations for the reduced modulus curves were developed only for H and box columns. Numerical results were obtained by a digital computer for 8WF31 and 27WF94 columns with idealized residual stress distribution of various magnitudes. It was found that the difference between the reduced modulus load and the tangent modulus load depends largely on the slenderness ratio of the column and the residual stress pattern. It was found in the numerical results that there might be a large difference between the reduced modulus and the tangent modulus loads for a welded built-up column of medium to small slenderness ratio so that the ultimate strength of a practical column of this range may exceed the tangent modulus load.

6. ACKNOWLEDGEMENTS

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7. NOMENCLATURE

A	area of a cross section
A_l	loading area in a cross section
A_u	unloading area in a cross section
B	bending rigidity
b	width of a flange
$\bar{b}$	width between inside faces of webs of a box section
d	depth of a cross section
E	modulus of elasticity
E_t	tangent modulus of elasticity
I	moment of inertia
I_e	effective moment of inertia of a cross section
I_k	contribution of moment of inertia by the segment k
I_x	moment of inertia about x-axis
I_{xe}	effective moment of inertia about x-axis
I_y	moment of inertia about y-axis
I_{ye}	effective moment of inertia about y-axis
i	a number used as a subscript
k	the number of a segment where a neutral axis passes
L_{cr}	critical length of a pinned-end column
L_x	critical length of a pinned-end column bent on the x-axis
L_y	critical length of a pinned-end column bent on the y-axis
m	number of segments in a web
n	number of segments in a flange
p	load
$S_{e,i}$	distance as shown in Fig. 10
S_k	stress acting in the segment k

t	thickness of a flange plate
u	displacement
w	thickness of a web plate
X_o	x-coordinate of a neutral axis
x_e	coordinate x of the center of the elastic part in a segment
Y_o	y-coordinate of a neutral axis
y_e	coordinate y of the center of the elastic part in a segment
x,y,z	cartesian coordinate
ϵ	strain
ϵ_{cr}	strain a buckling load
ϵ_r	residual strain
ϵ_Y	yield strain
σ_{cr}	buckling stress
σ_s	average stress in a segment
σ_y	yield stress
A	area of a segment
A_e	area of elastic part in a segment
ΔS	length of a segment along the centerline of plates
$\Delta S_{e,i}$	length of the elastic part of a segment along S.
Δx	length of a segment in the x-direction
Δx_e	length of the elastic part of a segment in the x-direction
Δy	length of a segment in the y-direction
Δy_e	length of the elastic part of a segment in the y-direction
$\sum_{n,m}$	summation of all the segment in a flange and in a web

PROGRAM LANGUAGE

AREA	A
AVESTRESS	$\sigma_{cr} = P/A$
B	b
BI	$\bar{b}$
DA	d, $\bar{b}$
DAA	$1/2\cdot(d+t)$, $1/2\cdot(\bar{b}+w)$
DB	d, $\bar{b}$
EINERTIA	I_{xe}, I_{ye}
ELN	$\Delta y_e/\Delta y$, $\Delta x_e/\Delta x$
ELNF	$\Delta x_e/\Delta x$
EXINERTIA	I_{xe}
EYINERTIA	I_{ye}
FINALSTRAIN	the maximum strain to which a column curve is computed
IB	$\frac{t^2}{12}$, $\frac{W^2}{12}$
IK	I_k, name of a subprogram to compute I_k increment of strain at which interval critical slenderness ratios are computed
INER	name of a subprogram to compute location of neutral axis and the effective moment of inertia
ISTRAIN	the minimum strain from which a column curve is computed
J, JA	sequence numbers
KX, KY, KZ	sequence numbers to be used in determining location of neutral

L	d
LENGTH	Δx, Δy
M	$(d/2+t)/100\Delta y$, $b/200\Delta x$
NA, NB	n, m
NF	n
NW	m
RF	residual strains in a flange (ϵ_r)
RW	residual strains in a web (ϵ_r)
SELN	$\sum_i (\Delta y_{e,i}/\Delta y)$, $\sum_i (\Delta x_{e,i}/\Delta x)$
SF	$\sum_{i=1}^{n} \sigma_s$
SFEL	$\sum_{i=1}^{n} (\Delta x_{e,i}/\Delta x)$
SK	S_k, name of a subprogram to compute S_k
SRN	ϵ_{i-1}
SRNA	ϵ_i
STRAIN	ϵ, ϵ_{cr}
STRESS	σ_s, $\sum_i \sigma_{s,i}$
SW	$\sum_{i=1}^{m} \sigma_s$
SWEL	$\sum_{i=1}^{m} (\Delta y_{e,i}/\Delta y)$
SWA, SWB, SWC, SWD	variables for switching name of a subprogram to compute average stress and penetration of yielding at each segment
S1, S2	variables used in determining a neutral axis and the effective moment of inertia

T	t
TA	t, w
TB	t
TLN	$Y_e/\Delta y$, $X_e/\Delta x$
TLNF	$x_e/\Delta x$
W	w
XINERTIA	I_x
XLENGTH	L_x
XMFACTOR	$\pi r_x/\sqrt{\epsilon_Y}$
XSLENDRATIO	$\frac{1}{\pi} \frac{L}{r_x} \sqrt{\epsilon_Y}$
YIELD STRAIN	ϵ_Y
YINERTIA	I_y
YLENGTH	L_y
YMFACTOR	$\pi r_y/\sqrt{\epsilon_Y}$
YSLENDRATIO	$\frac{1}{\pi} \frac{L}{r_y} \sqrt{\epsilon_Y}$

8. APPENDICES

APPENDIX A: BUCKLING STRENGTH OF COLUMNS WITH TEE AND EQUAL LEG ANGLE CROSS SECTIONS

The procedure for computing the column curve for tee and equal leg angle cross sections is the same as for the Box- and H-columns described in Chapter 3. Only the final forms of equations for the tangent modulus column curves are presented here, so that the column curves are computed simply by substituting these results into Eq. 9. The cross section considered are shown in Fig. 10 together with the dimensions, coordinate axes and numbering system to the segments.

1. TEE-COLUMNS

$$\frac{\sigma_{cr}}{\sigma_r} = \frac{1}{A}\left[\frac{bt}{n}\sum_{i=1}^{n}\left(\frac{\sigma_s}{\sigma_Y}\right)_i + \frac{dw}{m}\sum_{i=1}^{m}\left(\frac{\sigma_s}{\sigma_Y}\right)_i\right] \tag{A.1}$$

$$A = bt + dw \tag{A.2}$$

$$Y_o = -\frac{\frac{d^2w}{m^2}\sum_{n,m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right)\left(\frac{s_{e,i}}{\Delta y}\right)}{\frac{bt}{n}\sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right) + \frac{dw}{m}\sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right)} \tag{A.3}$$

in which $s_{e,i}$, is the distance from the junction of flange and web to the center of elastic part of the i-th segment.

$$\frac{I_{ex}}{I_x} = \frac{1}{I_x}\left\{\frac{bt}{n}\left(Y_o^2 + \frac{t2}{12}\right)\sum_{i=1}^{n}\left(\frac{\Delta x_{e,i}}{\Delta x}\right) + \frac{d^3w}{m^3}\sum_{i=1}^{m}\left(\frac{\Delta y_{e,i}}{\Delta y}\right)\left[\left(\frac{y_{e,i}}{\Delta y}\right)^2 + \frac{1}{12}\cdot\left(\frac{\Delta y_{e,i}}{\Delta y}\right)^2\right]\right\} \tag{A.4}$$

2. COLUMNS OF EQUAL LEG ANGLE

$$\frac{\sigma_{cr}}{\sigma_Y} = \frac{2}{n} \sum_{i=1}^{n} \left(\frac{\sigma_s}{\sigma_Y}\right)_i \tag{A.5}$$

$$A = 2bt \tag{A.6}$$

$$Y_o = \frac{b}{2\sqrt{2}} + \frac{b \sum_{i=1}^{n} \left(\frac{\Delta S_{e,i}}{\Delta S}\right)\left(1 - \frac{2}{b} S_{e,i}\right)}{2\sqrt{2} \sum_{i=1}^{n} \left(\frac{\Delta S_{e,i}}{\Delta S}\right)} \tag{A.7}$$

$$\frac{I_{xe}}{I_x} = \frac{1}{I_x} \frac{b^3 t}{n^3} \sum_{i=1}^{n} \left(\frac{\Delta S_{e,i}}{\Delta S}\right)\left(\frac{S_{e,i}}{\Delta S}\right)^2 \tag{A.8}$$

where $S_{e,i}$ is the distance from the corner to the center of elastic part of the i-th segment.

APPENDIX B: SUMMARY OF STRESS AND MOMENT OF INERTIA CONTRIBUTED BY A SEGMENT CUT BY THE NEUTRAL AXIS

The expression for the stress caused by the buckling, and present in the segment where the neutral axis passes, depends on the penetration of yielding into the segment. Similarly, the contribution of the segment to the bending regidity depends on the penetration of yielding. This appendix summarizes the expressions for the stress and the moment of inertia.

For buckling about the x-axis:

(1) $\epsilon_k < \epsilon_Y, \ \epsilon_{k-1} < \epsilon_Y$

$$S_k = w\,(y_k - Y_o)\ \Delta y \tag{B.1}$$

$$I_k = w\left\{(y_k - Y_o)^2 + \frac{\Delta y^2}{12}\right\}\ \Delta y \tag{B.2}$$

(2) $\epsilon_k < \epsilon_Y, \ \epsilon_{k-1} \geqq \epsilon_Y, \ y_{e,k} - \dfrac{\Delta y_{e,k}}{2} - Y_o < 0$

$$S_k = w\,(y_{e,k} - Y_o)\ \Delta y_{e,k} \tag{B.3}$$

$$I_k = w\ \Delta y_{e,k}\left[(y_{e,k} - Y_o)^2 + \frac{\Delta y_{e,k}^{\ 2}}{12}\right] \tag{B.4}$$

(3) $\epsilon_k < \epsilon_Y, \ \epsilon_{k-1} \geqq \epsilon_Y, \ y_{e,k} - \dfrac{\Delta y_{e,k}}{2} - Y_o \geqq 0$

$$S_k = \frac{w}{2}\left(y_k + \frac{\Delta y}{2} - Y_o\right)^2 \tag{B.5}$$

$$I_k = \frac{w}{3}\left(y_k + \frac{\Delta y}{2} - Y_o\right)^3 \tag{B.6}$$

(4) $\epsilon_k \geq \epsilon_Y, \ \epsilon_{k-1} < \epsilon_Y, \ y_{e,k} + \frac{\Delta y_{e,k}}{2} - Y_o \geq 0$

(The same as Case 1)

(5) $\epsilon_k \geq \epsilon_Y, \ \epsilon_{k-1} < \epsilon_Y, \ y_{e,k} + \frac{\Delta y_{e,k}}{2} - Y_o < 0$

$$S_k = w \ \Delta y \left[(y_k - Y_o)^2 + \frac{\Delta y^2}{12} \right] - \frac{w}{2} (Y_o - Y_{e,k} - \frac{\Delta y_{e,k}}{2})^2 \qquad (B.7)$$

$$I_k = w \ \Delta y \left[(y_k - Y_o)^2 + \frac{\Delta y^2}{12} \right] - \frac{w}{3} (Y_o - y_{e,k} - \frac{\Delta y_{e,k}}{2})^3 \qquad (B.8)$$

(6) $\epsilon_k \geqq \epsilon_Y, \ \epsilon_{k-1} \geqq \epsilon_Y$

(The same as Case 3)

Replacing Y_o, Y and w by X_o, X, and t, the equations of S_k and I_k are obtained for buckling about the y-axis.

APPENDIX C: COMPUTER PROGRAMS

Based on the analyses of Chapter 3, programs for a digital computer were prepared with WIZ language[23] as summarized in Table 1. A representative flow diagram is shown in Fig. 11 for computation of the tangent modulus curves of H and Box columns. All of the programs prepared are shown in Programs Nos. 1, 2 and 3.

The basic nomenclature used in the program is listed in Chapter 7.

All programs prepared provided memory spaces of up to 100 for the number of segments in a flange and in a web. The number is sufficient for practical distributions of residual stress. The location of the neutral axis has to be determined for the computation of column curves based on the reduced modulus concept. The program is prepared to determine the location with an accuracy of 1/100 of a half width of a flange and a half depth of a web for the buckling of a box column in the x- and y-direction respectively.

The prepared programs include manual controlling instruction such that, when the pinned-end length of columns is needed in addition to the non-dimensionalized slenderness ratio, it can be obtained by pressing the console switch No. 1 of the GE 225 computer.

1. Data Input

Identical data cards can be used for computation of both the tangent modulus and the reduced modulus curves for H and box columns, while for computation of tee columns, only the data for dimensions of the cross section are different. Sign convention is such that compressive stresses and strains are positive and tensile stresses and strains are negative.

The sequence of data input is as follows:

(a) <u>The Yield Strain</u>

The yield strain (in/in) has to be placed first when console switch no. 1 is pressed by operator. Otherwise, the data is not necessary.

EXAMPLE: For a steel with yield stress of 100 ksi and the modulus of 30,000 ksi, the data is

3.3333333-3

(b) <u>Three Data on Critical Strains</u>

The data on strains due to loading at which the column is going to buckle must be put in. It is necessary to compute at several strains over a wide range with certain increments in order to draw a column curve. The data consist of three numbers; initial strain, increment, and final strain. All data have to be non-dimensionalized by the yield strain, ϵ_Y.

EXAMPLE: Critical slenderness ratios to be computed for a range of critical strains from 0.3 to 1.2 of the yield strain with a interval of 0.05 will have data as shown

.3 .05 1.2

(c) Two data on Number of Segments

The number of segments in half of the flange must be put in, followed by the number of segments in the web (for a tee column) or a half of the web (for a H and box column).

EXAMPLE: Referring to Fig. 1 or to Fig. 10, if n is equal to 4 and m is equal to 2, the data will be,

4 2

(d) Data on Dimensions of Cross Section (5 Data for an H and Box Column and 4 Data for a Tee Column)

Data concerning the dimensions of the cross section must be put in as the fourth group of data. The sequence is b, $\bar{b}$, t, d, and w for an H and box column and b, t, d, and w for a tee column, which refers to Fig. 1 and Fig. 10, respectively. Although any unit can be used as long as the same unit is used for all dimensions, a unit of one inch may be preferable.

EXAMPLE: (1) 8WF31 8. 0 .433 7.134 .144

(2) ST 4WF15.5 8. .433 3.567 .288

NOTE: the width of the web plate for 8WF31 is not .288 but a half of this.

(e) Data on Residual Strains

The last group of data is the amount of residual strains present at the edge of segments. Data must be non-dimensionalized by the yield strain of the material. The data have to be arranged in a sequence such that the residual strain at one edge of segment 1 in the flange is first, and then followed by that at the edge between segments 1 and 2, that between segments 2 and 3 and so on, till that at the extreme edge of the flange. The similar set-up of data for the web follows from the segment 1 to the segment m. Thus a total number of (n+1) data for the flange and (m+1) data for the web are necessary, resulting in a grand total of (n+m+2) data.

It is noted that the residual strain distribution has to be so arranged that the residual stress is in equilibrium.

EXAMPLE: (1) No residual stress (N=4, m=2)

0 0 0 0 0 0 0 0

(2) Idealized Residual Stress Distribution of Fig. 3.b, $\sigma_{rc}/\sigma_y = 0.25$ (n=5, m=5)

-1. -.375 .25 .25 .25 .25 .25 .25

.25 .25 -.375 -1.

(f) Other Sets of Data

Other sets of data arranged in the same manner as (b) through (e) can be followed directly as many times as necessary. A "WIZ DATA" card and an "END OF DATA" card followed with two empty cards are necessary at the front and at the end of the data, respectively.

An example of typical data set up is summarized as follows:

```
WIZ DATA CARD
3.3333333-3
.3  .05  1.2
4  2
8.  0  .433  7.134  .144
0  0  0  0  0  0  0  0
.75  .05  2.
8.  0  .433  7.134  .144
-1.  -.375  .25  .25  .25  .25
.25  .25  .25  .25  -.375  -1.
END OF DATA
(Two Empty Cards)
```

2. Output

Example of outputs are shown in Program No. 4 computed by the program based on the tangent modulus concept with the data shown above.

Following the title of the program, all the input data are printed out so that any error in punching the data may be detected.

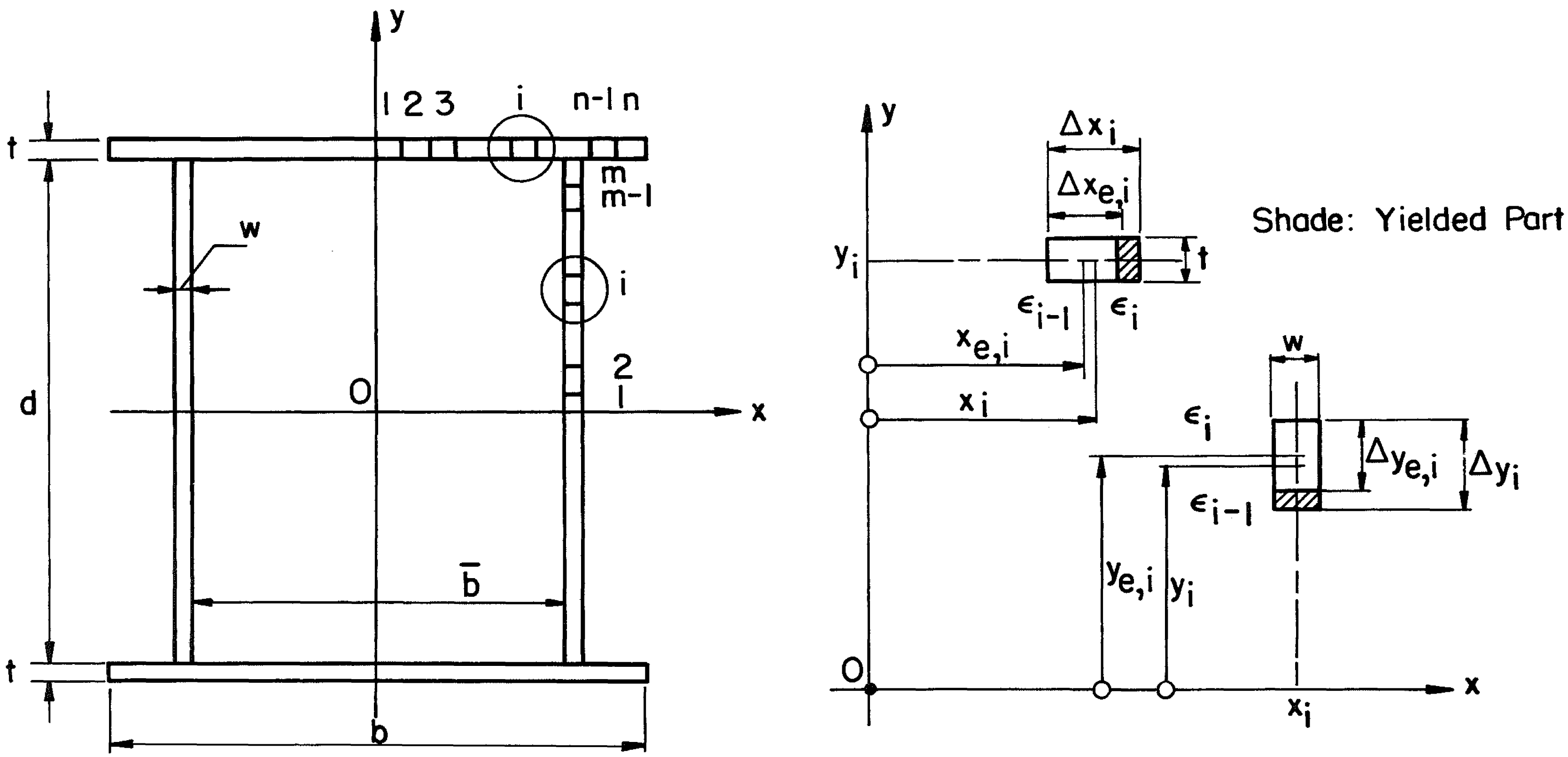
The area of the cross section and moment of inertia on the x-axis and on the y-axis are printed, below the title "CALCULATED RESULTS". The units are square inches for the area and in^4 for the moments of inertia, provided a unit of one inch is used for the dimensions of the cross section. Next follows the computed results for a column curve. The output consists of 6 columns, under the headings, "STRAIN", "AVESTRESS", "XSLENDRATIO", "YSLENDRATIO", "XLENGTH", AND "YLENGTH". These headings mean the strain and the stress at which column is going to buckle, the critical slenderness ratio for the strong axis and for the weak axis bending and the critical pinned end length of the column for the strong axis and for the weak axis bending, respectively. The strain, stress, and slenderness ratios are non-dimensionalized. Plotting the average stress against the slenderness ratio, the column curve is obtained.

9. TABLE, FIGURES AND COMPUTER PROGRAMS

Table 1 LIST OF COMPUTER PROGRAMS

Program No.	Program Identification*	Cross Section	Buckling Concept	Bucklg. Mode
1	290-3-2	H and Box	Tangent Modulus	Flexural Bucklg. on both princ. axes
2	290-3-6	Tee	Tangent Modulus	Flexural Bucklg.
3	290-3-3	H and Box	Reduced Modulus	Flexural Bucklg. on both Princ. axes
4	--	H and Box	Example of Output	

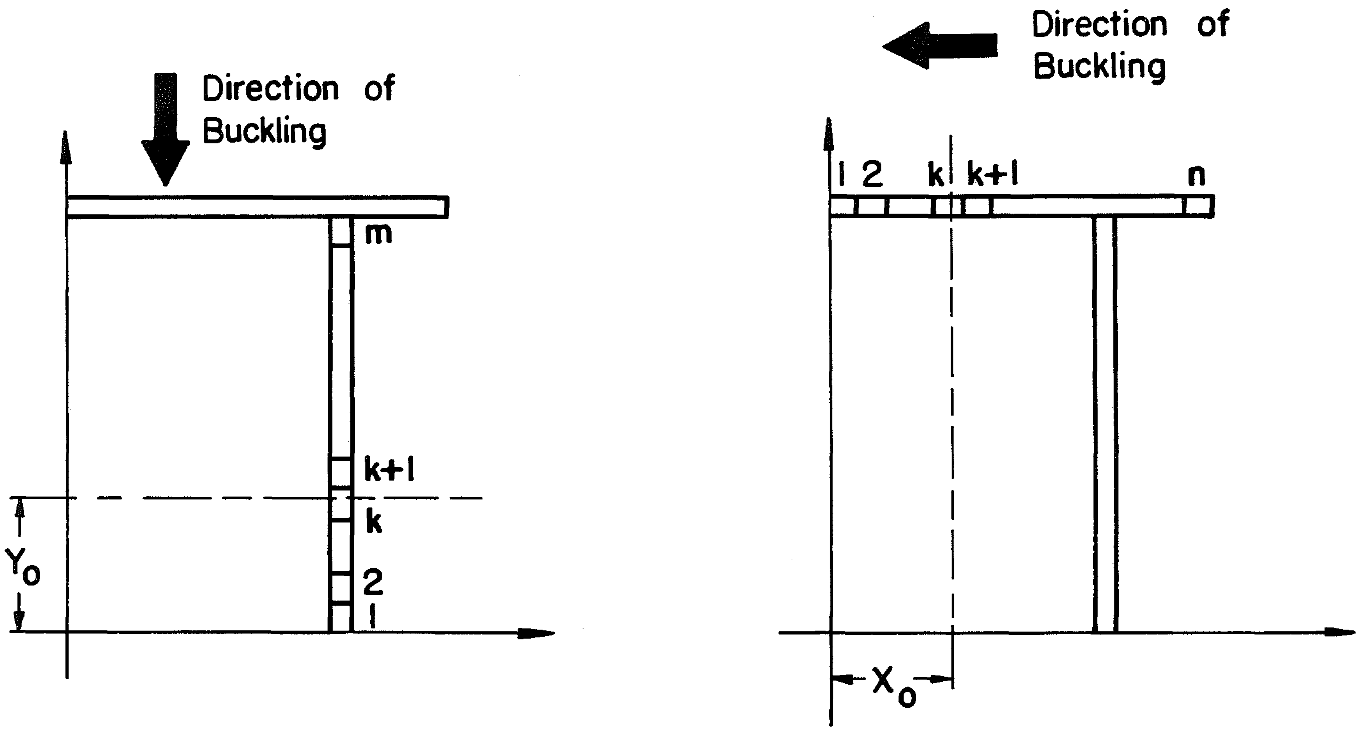
* Programs are identified and stored at Fritz Laboratory, Lehigh University.



(a) Dimension of Cross Section

(b) Detail of Segments

Fig. 1 DIMENSIONS OF BOX CROSS SECTION



(a) Buckling in the Y-Direction

(b) Buckling in the X-Direction

Fig. 2 LOCATION OF NEUTRAL AXIS

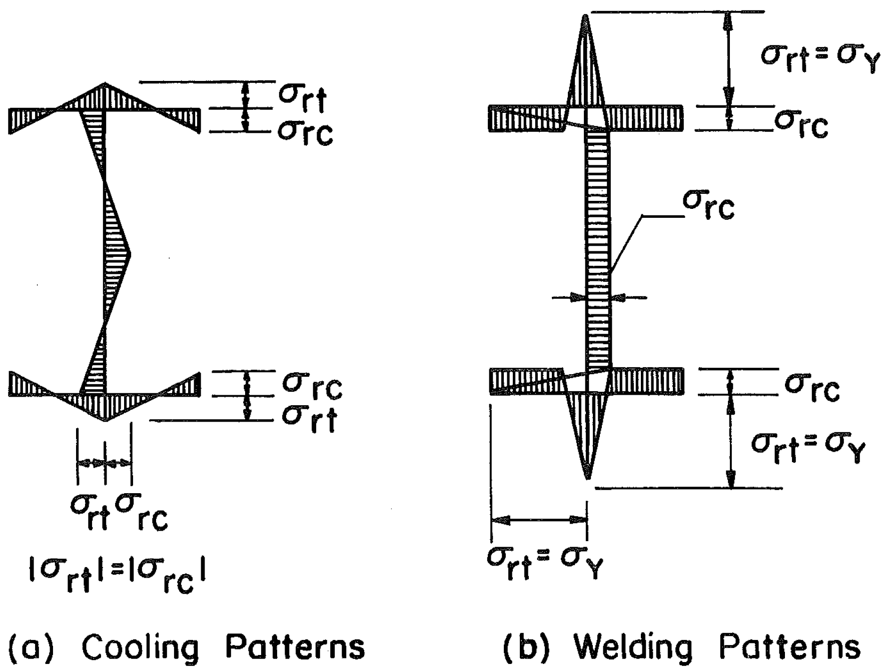
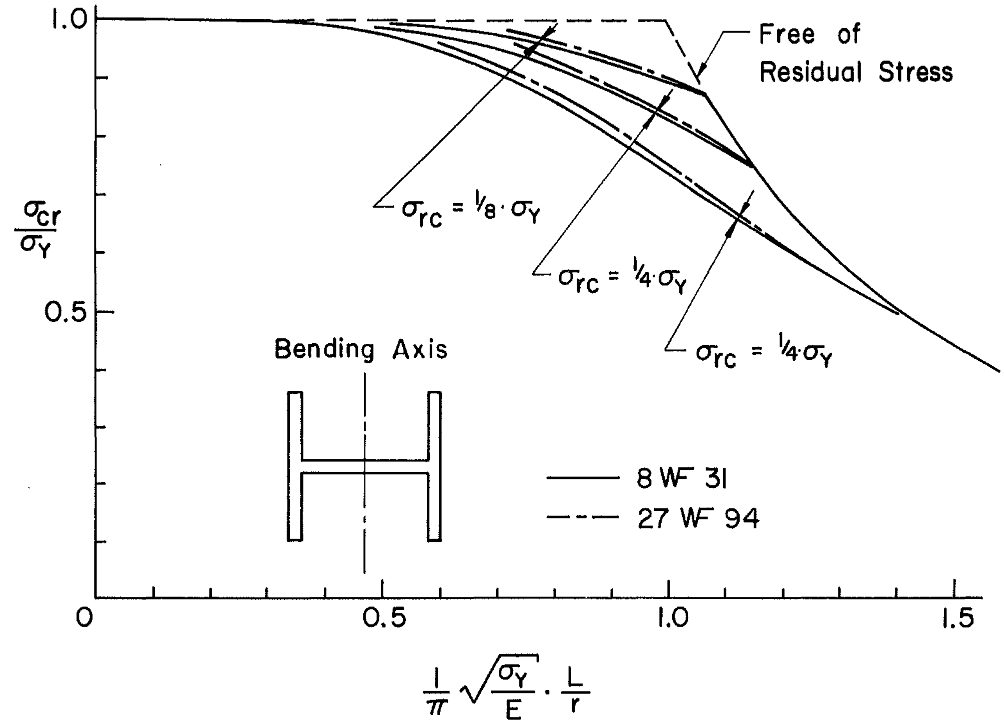
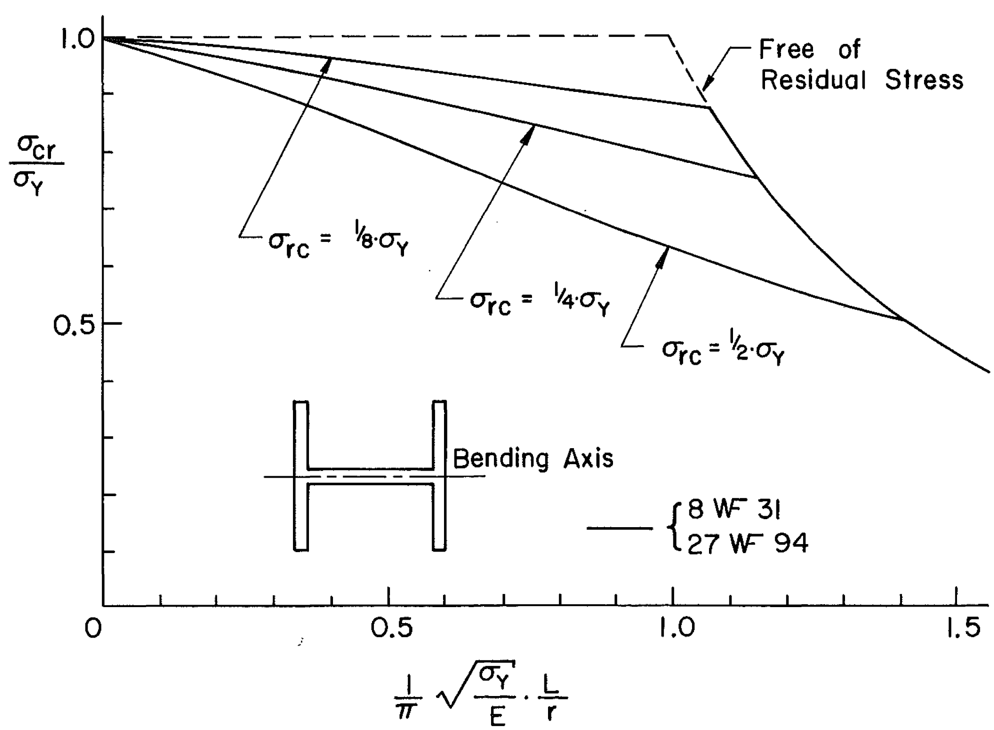


Fig. 3 IDEALIZED RESIDUAL STRESS PATTERNS FOR ILLUSTRATIONS OF COLUMN CURVES

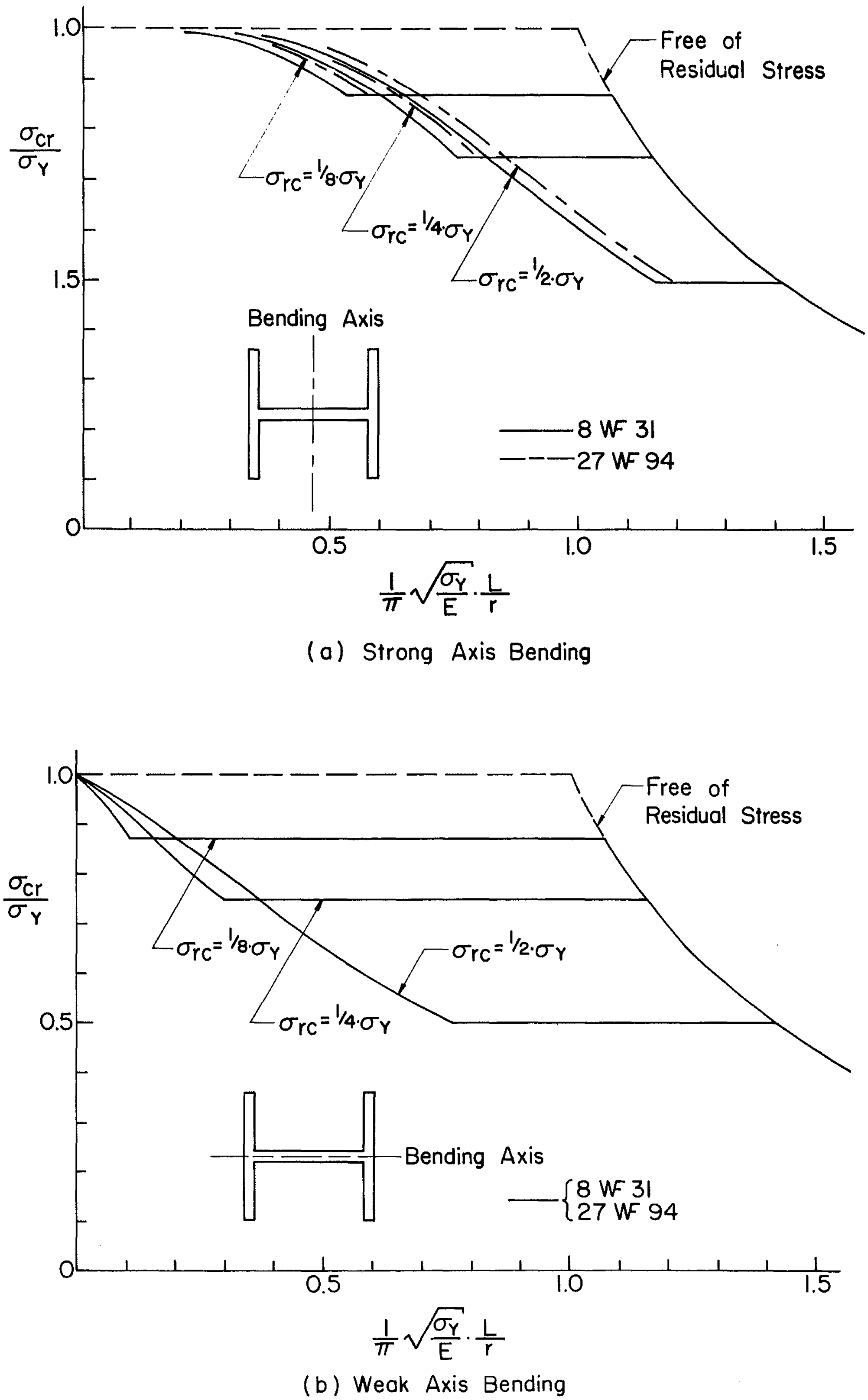


(a) Strong Axis Bending



(b) Weak Axis Bending

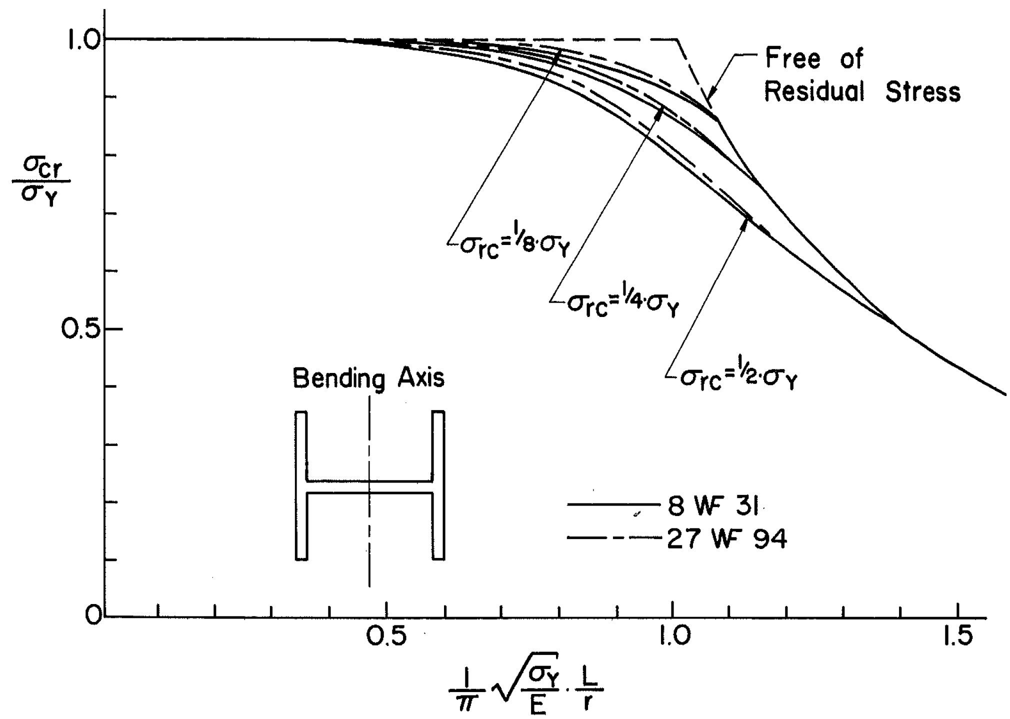
Fig. 4 TANGNET MODULUS CURVES OF ROLLED COLUMNS (WITH RESIDUAL STRESS OF FIG. 3(a))



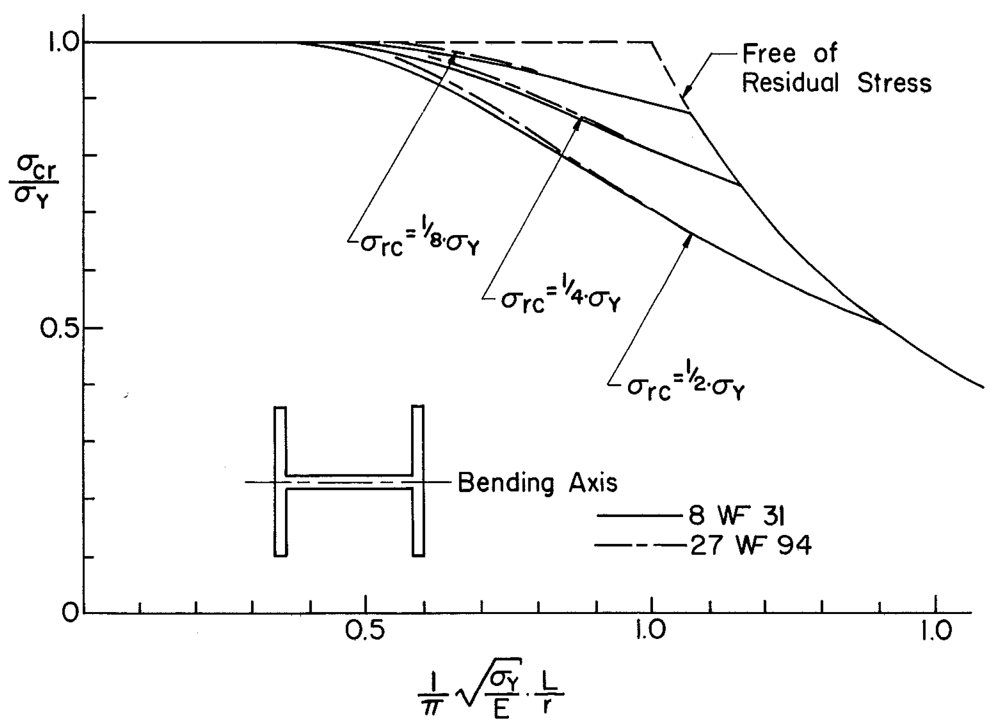
(a) Strong Axis Bending

(b) Weak Axis Bending

Fig. 5 TANGENT MODULUS CURVES OF WELDED COLUMNS (WITH RESIDUAL STRESS OF FIG. 3(b))

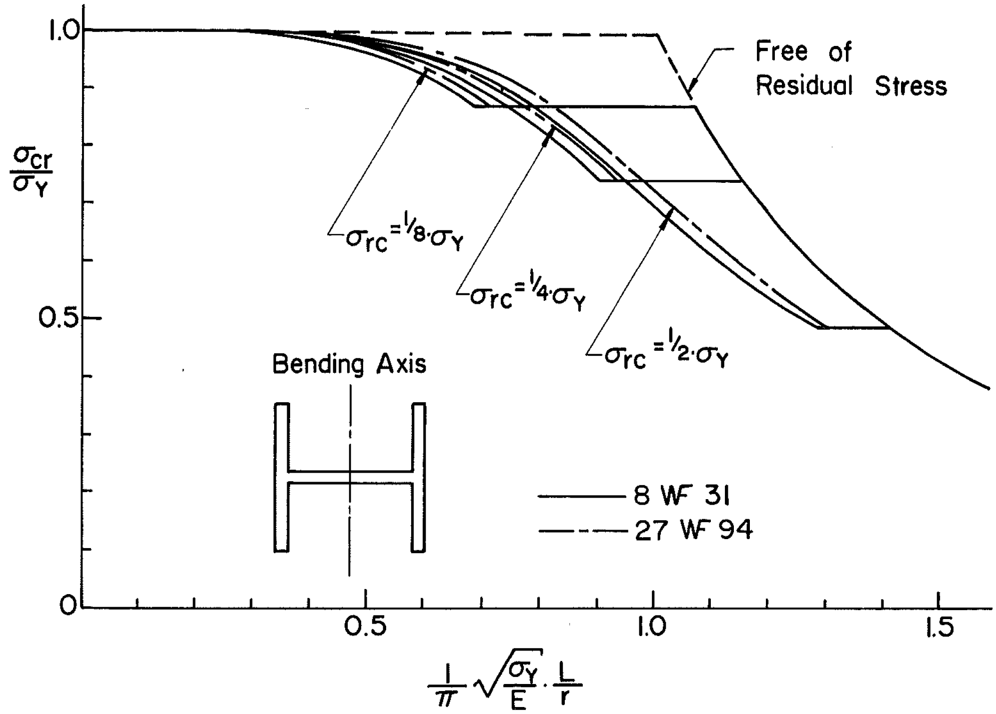


(a) Strong Axis Bending

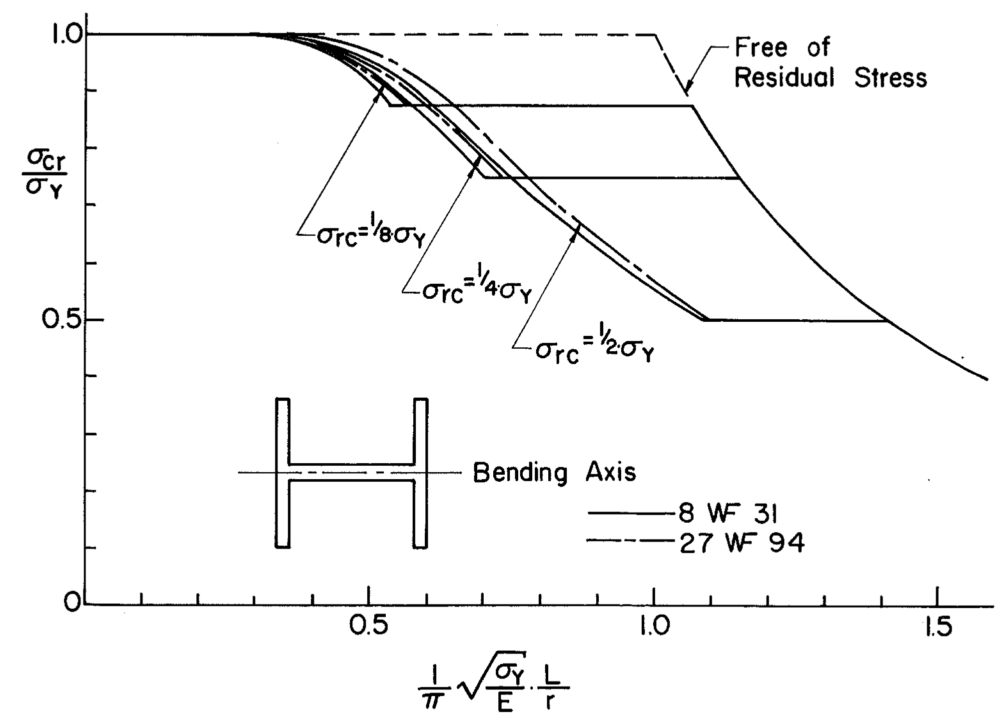


(b) Weak Axis Bending

Fig. 6 REDUCED MODULUS CURVES OF ROLLED COLUMNS (WITH RESIDUAL STRESS OF FIG. 3(a))

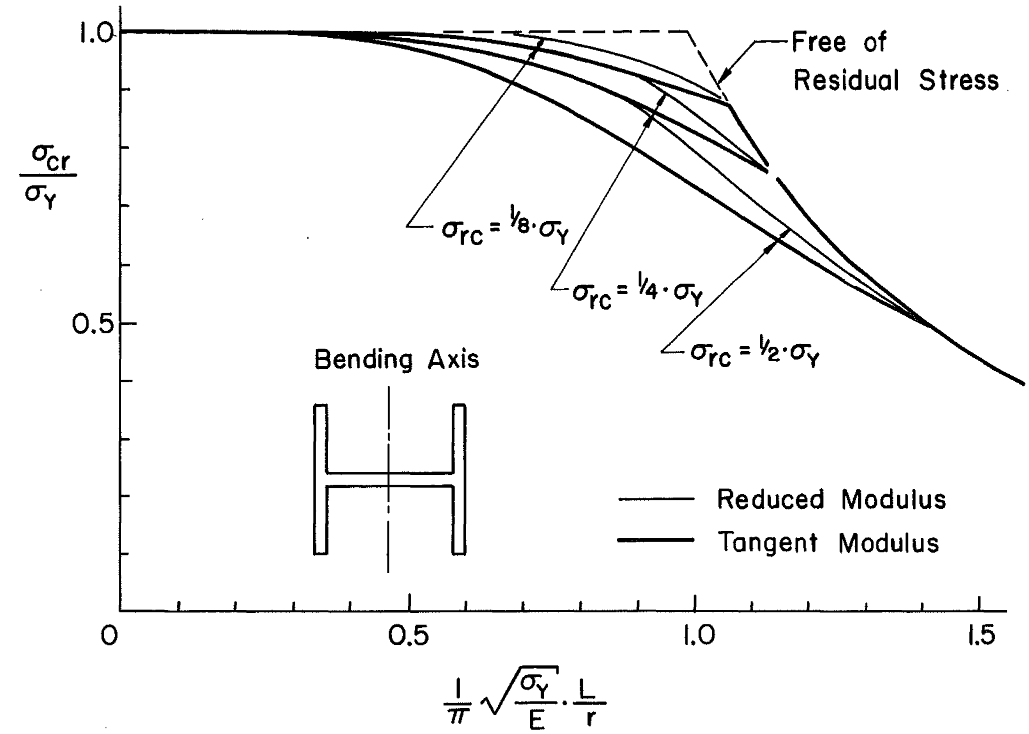


(a) Strong Axis Bending

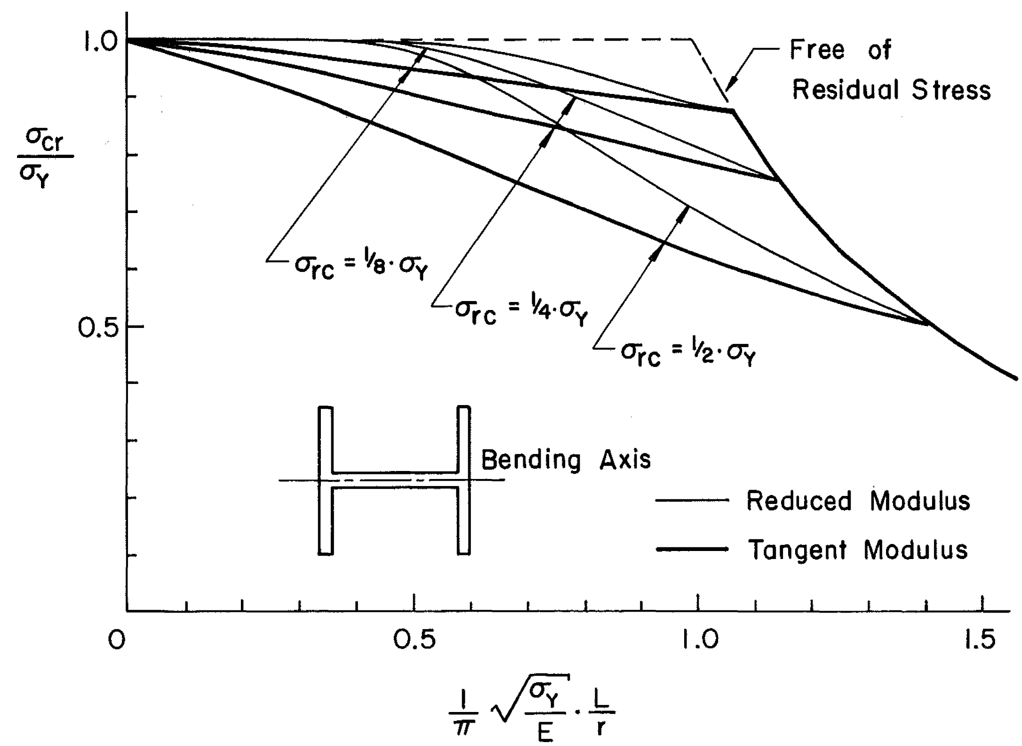


(b) Weak Axis Bending

Fig. 7 REDUCED MODULUS CURVES OF WELDED COLUMNS (WITH RESIDUAL STRESS OF FIG. 3(b))

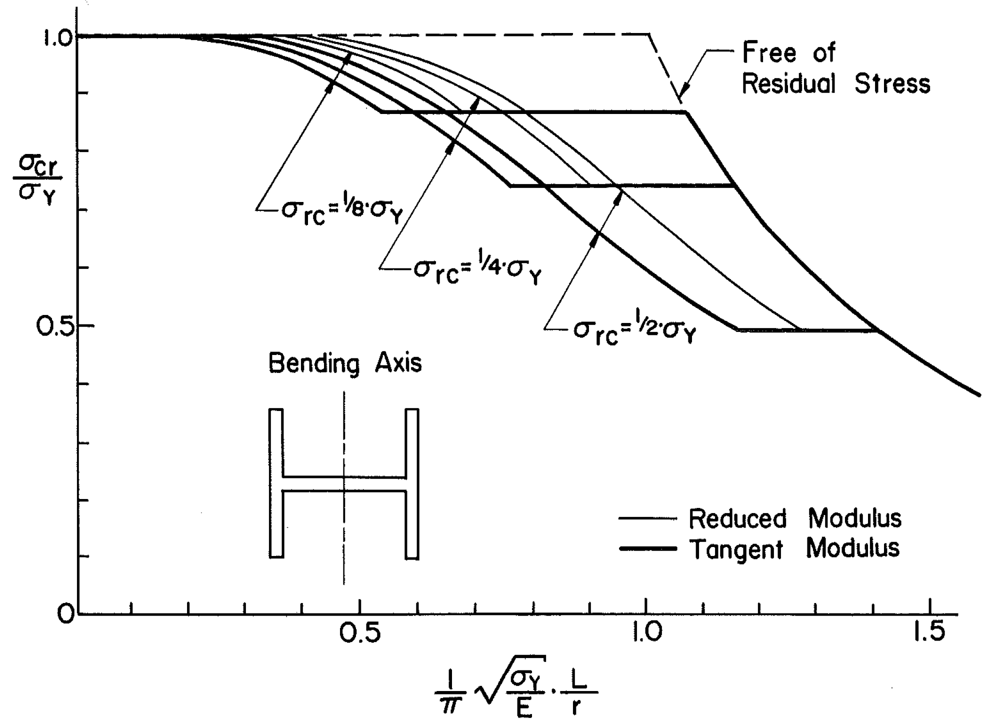


(a) Strong Axis Bending

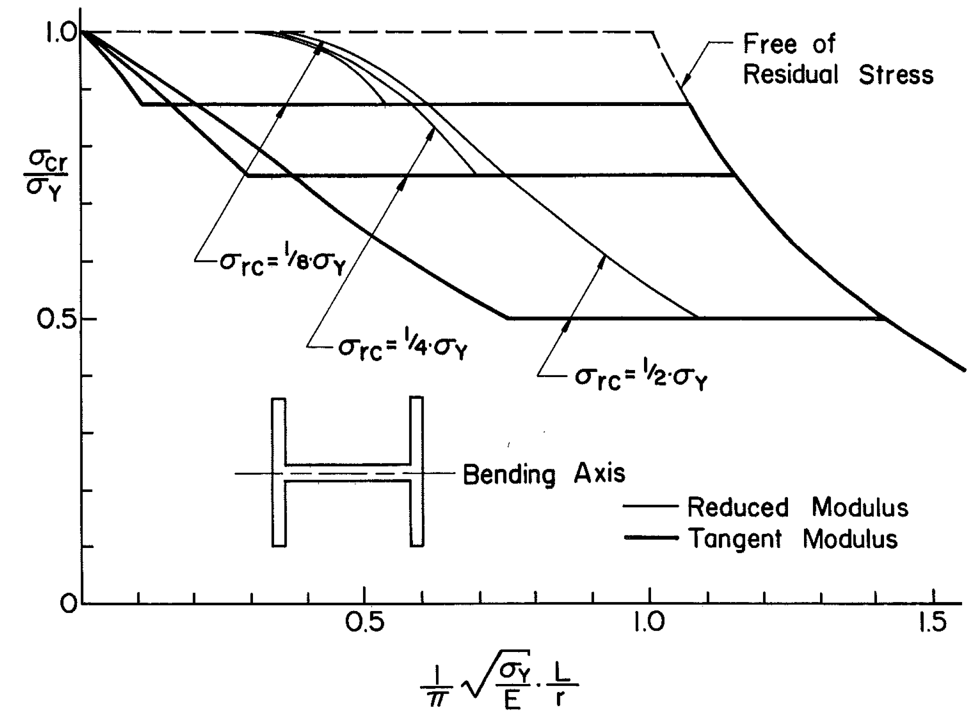


(b) Weak Axis Bending

Fig. 8 COLUMN CURVES OF ROLLED 8WF31
(WITH RESIDUAL STRESS OF FIG. 3(a))

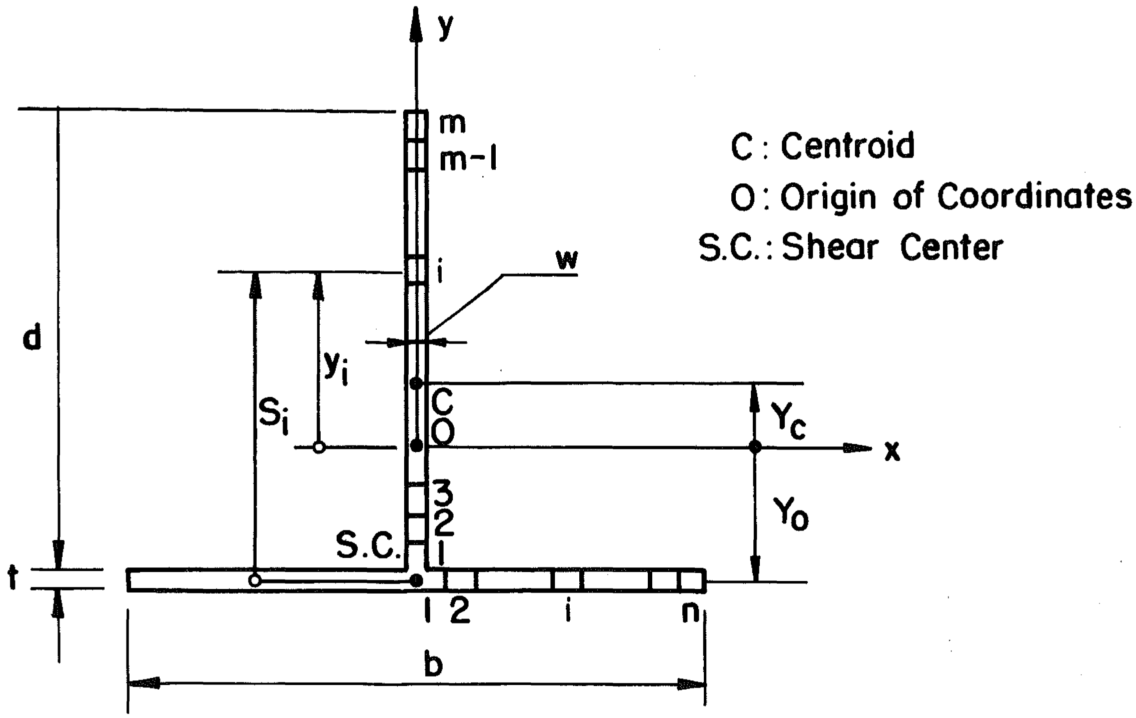


(a) Strong Axis Bending

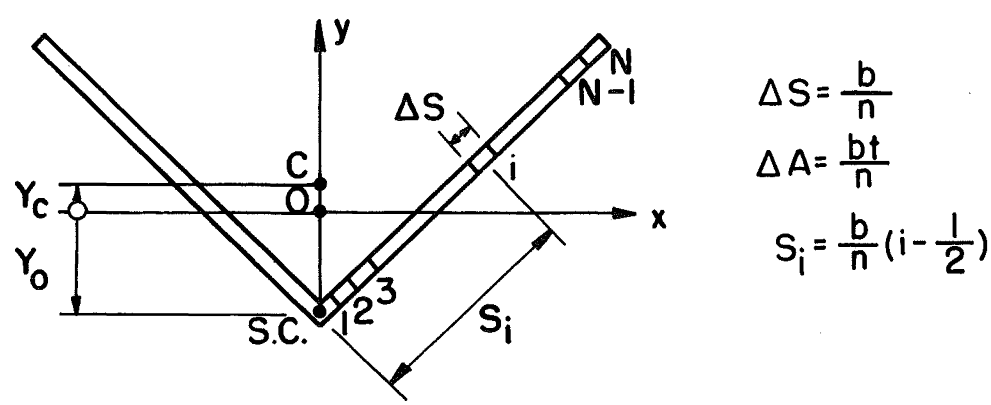


(b) Weak Axis Bending

Fig. 9 COLUMN CURVES OF WELDED 8WF31
(WITH RESIDUAL STRESS OF FIG. 3(b))

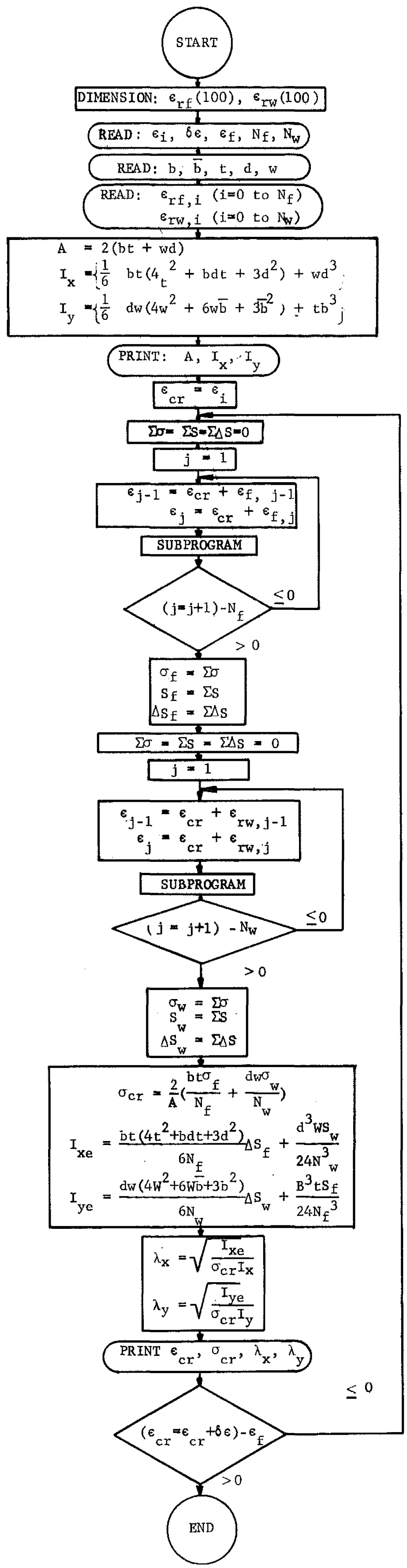


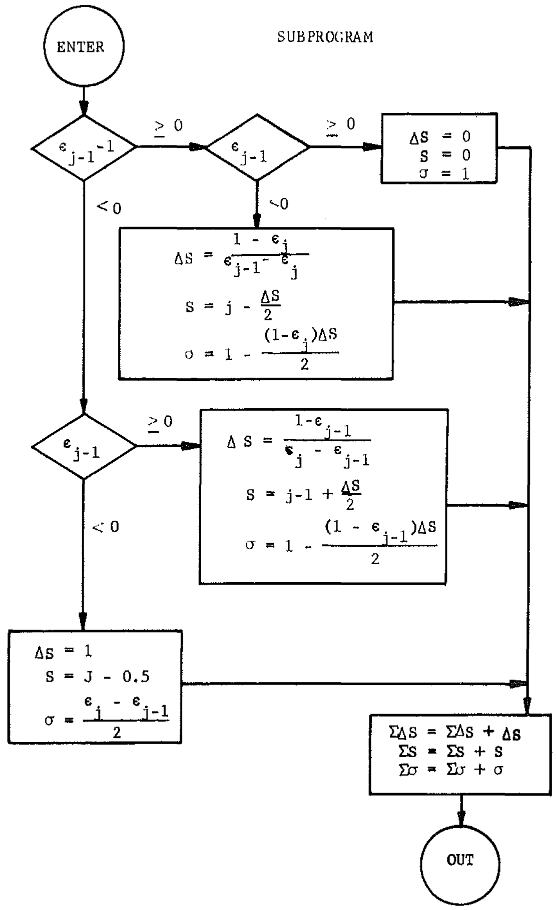
(a) Tee Section



(b) Equal Leg Angle Section

Fig. 10 DIMENSIONS OF TEE AND ANGLE SECTIONS





NOTATION

- N_f : NUMBER OF SEGMENTS IN FLANGE
- N_w : NUMBER OF SEGMENTS IN WEB
- S : DISTANCE FROM ONE OF THE PRINCIPAL AXES TO THE CENTER OF ELASTIC PART IN A SEGMENT
- ΔS : WIDTH OF ELASTIC PART IN A SEGMENT ALONG THE MIDDLE PLANE OF THE PLATE
- f : A SUBSCRIPT DENOTING PROPERTIES OF FLANGE
- i,j : SUBSCRIPTS FOR SEQUENCE NUMBERS
- W : A SUBSCRIPT DENOTING PROPERTIES OF WEB
- ϵ_{cr}: CRITICAL STRAIN
- ϵ_i : INITIAL VALUE OF CRITICAL STRAIN
- ϵ_f : FINAL VALUE OF CRITICAL STRAIN
- $\delta\epsilon$: INCREMENT OF CRITICAL STRAIN

Fig. 11 FLOW DIAGRAM FOR FLEXURAL BUCKLING OF BOX- AND H-COLUMNS CONTAINING RESIDUAL STRESS

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005 2082 FUMIO NISHINO   COLUMN BUCKLING
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                                                                                   PAGE 1.        DEC 21 66

 #      SEQ  LABL TYP                   STATEMENT                     C ZERO      NOT 0      PLUS      MINUS      ELSE

001.  0000            $ 290-3-3                                        [     ]    [     ]   [     ]   [     ]   [     ]
002.  0010            $ CALCULATION OF COLUMN CURVE BASED ON           [     ]    [     ]   [     ]   [     ]   [     ]
003.  0020            $ REDUCED MODULUS CONCEPT                        [     ]    [     ]   [     ]   [     ]   [     ]
004.  0030            $ DECEMBER, 1966     BY FUMIO NISHINO            [     ]    [     ]   [     ]   [     ]   [     ]
005.  0040          D  RF[100], RW[100], SUB.[14], INER.[22],         C
   .  0050             SK.[22], IK.[15], FLNF[100], ELN[100],         C
   .  0060             TLN[100], TLNF[100]                             [     ]    [     ]   [     ]   [     ]   [     ]
006.  0070             SWT.[1]                                         [READ]    [     ]   [     ]   [     ]   [     ]
007.  0080          CRDYIELDSTRAIN                                     [     ]    [     ]   [     ]   [     ]   [     ]
008.  0090  READ    CRDISTRAIN                                         [     ]    [     ]   [     ]   [FIN ]   [     ]
009.  0100          CRDINCREMENT,FINALSTRAIN                           [     ]    [     ]   [     ]   [     ]   [     ]
010.  0110          CRDNF,NW                                           [     ]    [     ]   [     ]   [     ]   [     ]
011.  0120          CRDB,BI,T,L,W                                      [     ]    [     ]   [     ]   [     ]   [     ]
012.  0130          SL CALCULATION OF COLUMN CURVE BASED ON REDUCED C
   .  0140             MODULUS CONCEPT, FLEXURAL BUCKLING, BOX- AND C
   .  0150             H-COLUMNS [290-3-3]                             [     ]    [     ]   [     ]   [     ]   [     ]
013.  0160          PL                                                 [     ]    [     ]   [     ]   [     ]   [     ]
014.  0170          PL         BY    FUMIO NISHINO                     [     ]    [     ]   [     ]   [     ]   [     ]
015.  0180          PV ,,                                              [     ]    [     ]   [     ]   [     ]   [     ]
016.  0190          PL    INPUT PARAMETERS                             [     ]    [     ]   [     ]   [     ]   [     ]
017.  0200          PVL, NF, NW,, B, BI, T, L, W                       [     ]    [     ]   [     ]   [     ]   [     ]
018.  0210          PVL ISTRAIN, INCREMENT, FINALSTRAIN,               [     ]    [     ]   [     ]   [     ]   [     ]
019.  0220          PL    RESIDUAL STRESSES IN THE FLANGE              [     ]    [     ]   [     ]   [     ]   [     ]
020.  0230          PV                                                 [     ]    [     ]   [     ]   [     ]   [     ]
021.  0240             J=0                                             [     ]    [     ]   [     ]   [     ]   [     ]
022.  0250  *       CRDRF[J]                                           [     ]    [     ]   [     ]   [     ]   [     ]
023.  0260          PV SKIP,RF[J]                                      [     ]    [     ]   [     ]   [     ]   [     ]
024.  0270             [J=J+1]-NF                                      [*    ]    [     ]   [     ]   [*    ]   [     ]
025.  0280          PV ,                                               [     ]    [     ]   [     ]   [     ]   [     ]
026.  0290          PL    RESIDUAL STRESSES IN THE WEB                 [     ]    [     ]   [     ]   [     ]   [     ]
027.  0300          PV                                                 [     ]    [     ]   [     ]   [     ]   [     ]
028.  0310             J=0                                             [     ]    [     ]   [     ]   [     ]   [     ]
029.  0320  *       CRDRW[J]                                           [     ]    [     ]   [     ]   [     ]   [     ]
030.  0330          PV SKIP,RW[J]                                      [     ]    [     ]   [     ]   [     ]   [     ]
031.  0340             [J=J+1]-NW                                      [*    ]    [     ]   [     ]   [*    ]   [     ]
032.  0350             AREA=2*[B*T+W*L]                                [     ]    [     ]   [     ]   [     ]   [     ]
033.  0360             XINERTIA=[B*T*[4*T*T+6*L*T+3*L*L]+W*L*L*L]/6    [     ]    [     ]   [     ]   [     ]   [     ]
034.  0370             YINERTIA=[L*W*[4*W*W+6*W*BI+3*BI*BI]+T*B*B*  C
   .  0380                      B]/6                                   [     ]    [     ]   [     ]   [     ]   [     ]
035.  0390             SWT.[1]                                         [1    ]    [     ]   [     ]   [     ]   [     ]
036.  0400             XMFACTOR=3.1415926*SQRT.[XINERTIA/AREA/      C
   .  0410                YIELDSTRAIN]                                 [     ]    [     ]   [     ]   [     ]   [     ]
037.  0420             YMFACTOR=3.1415926*SQRT.[YINERTIA/AREA/      C
   .  0430                YIELDSTRAIN]                                 [     ]    [     ]   [     ]   [     ]   [     ]
038.  0440  1       PV ,,                                              [     ]    [     ]   [     ]   [     ]   [     ]
039.  0450          PL    CALCULATED RESULTS                           [     ]    [     ]   [     ]   [     ]   [     ]
040.  0460          PVL,, AREA, XINERTIA, YINERTIA,,                   [     ]    [     ]   [     ]   [     ]   [     ]
041.  0470             SWT.[1]                                         [2    ]    [     ]   [     ]   [     ]   [     ]
042.  0480          PL       STRAIN         AVESTRESS       XSLENDRATIO C
   .  0490                YSLENDRATIO                       XLENGTH    C
   .  0500                  YLENGTH                                    [     ]    [     ]   [     ]   [     ]   [3    ]
043.  0510  2       PL       STRAIN         AVESTRESS       XSLENDRATIO C
   .  0520                YSLENDRATIO                                  [     ]    [     ]   [     ]   [     ]   [     ]
044.  0530  3       PL                                                 [     ]    [     ]   [     ]   [     ]   [     ]
045.  0540             STRAIN=ISTRAIN                                  [     ]    [     ]   [     ]   [     ]   [     ]


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 #      SEQ  LABL TYP                   STATEMENT                     C ZERO      NOT 0      PLUS      MINUS      ELSE

046.  0550  **         KX=KY=0                                         [     ]    [     ]   [     ]   [     ]   [     ]
047.  0560             STRESS=SELN=J=0                                 [     ]    [     ]   [     ]   [     ]   [     ]
048.  0570  *          JA=J+1                                          [     ]    [     ]   [     ]   [     ]   [     ]
049.  0580             SRN=STRAIN+RF[J]                                [     ]    [     ]   [     ]   [     ]   [     ]
050.  0590             SRNA=STRAIN+RF[JA]                              [     ]    [     ]   [     ]   [     ]   [SUB ]
051.  0600             ELNF[JA]=ELN, TLNF[JA]=TLN                      [     ]    [     ]   [     ]   [     ]   [     ]
052.  0610             [J=JA]-NF                                       [     ]    [     ]   [     ]   [*    ]   [     ]
053.  0620             SF=STRESS, SFEL=SELN                            [     ]    [     ]   [     ]   [     ]   [     ]
054.  0630             STRESS=SELN=J=0                                 [     ]    [     ]   [     ]   [     ]   [     ]
055.  0640  *          JA=J+1                                          [     ]    [     ]   [     ]   [     ]   [     ]
056.  0650             SRN=STRAIN+RW[J]                                [     ]    [     ]   [     ]   [     ]   [     ]
057.  0660             SRNA=STRAIN+RW[JA]                              [     ]    [     ]   [     ]   [     ]   [SUB ]
058.  0670             ELN[JA]=ELN, TLN[JA]=TLN                        [     ]    [     ]   [     ]   [     ]   [     ]
059.  0680             [J=JA]-NW                                       [     ]    [     ]   [     ]   [*    ]   [     ]
060.  0690             SW=STRESS, SWEL=SELN                            [     ]    [     ]   [     ]   [     ]   [     ]
061.  0700             KZ=KY,SWA=0, NA=NW, NB=NF                       [     ]    [     ]   [     ]   [     ]   [     ]
062.  0710             DA=L, DB=B, DAA=.5*L+.5*T, TA=W, TB=T           [     ]    [     ]   [     ]   [     ]   [     ]
063.  0720             IB=T*T/12., SEL=SFEL, LENGTH=.5*L/NW            [     ]    [     ]   [     ]   [     ]   [     ]
064.  0730             M=[.5*L+T]/LENGTH/100.                          [     ]    [     ]   [     ]   [     ]   [INER]
065.  0740             EXINERTIA=EINERTIA/XINERTIA                     [     ]    [     ]   [     ]   [     ]   [     ]
066.  0750             KY=KZ                                           [     ]    [     ]   [     ]   [     ]   [     ]
067.  0760             J=NF                                            [     ]    [     ]   [     ]   [     ]   [     ]
068.  0770  *          TLN[J]=TLNF[J], ELN[J]=ELNF[J]                  [     ]    [     ]   [     ]   [     ]   [     ]
069.  0780             J=J-1                                           [     ]    [     ]   [*    ]   [     ]   [     ]
070.  0790             KZ=KX, SWA=1, NA=NF, NB=NW                      [     ]    [     ]   [     ]   [     ]   [     ]
071.  0800             DA=BI, DB=L, DAA=.5*BI+.5*W, TA=T, TB=W         [     ]    [     ]   [     ]   [     ]   [     ]
072.  0810             LENGTH=.5*B/NF                                  [     ]    [     ]   [     ]   [     ]   [     ]
073.  0820             IB=W*W/12., SEL=SWEL, M=NF/100.                 [     ]    [     ]   [     ]   [     ]   [INER]
074.  0830             EYINERTIA=EINERTIA/YINERTIA                     [     ]    [     ]   [     ]   [     ]   [     ]
075.  0840             KX=KZ                                           [     ]    [     ]   [     ]   [     ]   [     ]
076.  0850             AVESTRESS=2.*[SF*T*B/NF+SW*W*L/NW]/AREA         [     ]    [     ]   [     ]   [     ]   [     ]
077.  0860             XSLENDRATIO=SQRT.[EXINERTIA/AVESTRESS]          [     ]    [     ]   [     ]   [     ]   [     ]
078.  0870             YSLENDRATIO=SQRT.[EYINERTIA/AVESTRESS]          [     ]    [     ]   [     ]   [     ]   [     ]
079.  0880             SWT.[1]                                         [4    ]    [     ]   [     ]   [     ]   [     ]
080.  0890          PV STRAIN,AVESTRESS,XSLENDRATIO,YSLENDRATIO,SKIPC
   .  0900             ,XSLENDRATIO*XMFACTOR,YSLENDRATIO*YMFACTOR      [     ]    [     ]   [     ]   [     ]   [5    ]
081.  0910  4       PV STRAIN,AVESTRESS,XSLENDRATIO,YSLENDRATIO        [     ]    [     ]   [     ]   [     ]   [     ]
082.  0920  5          [STRAIN=STRAIN+INCREMENT]-FINALSTRAIN-.0001     [     ]    [     ]   [     ]   [**   ]   [READ]
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                                                                          PAGE 3.      DEC 21 66

 #     SEQ  LABL TYP               STATEMENT                      C ZERO    NOT 0     PLUS     MINUS     ELSE

063.  1010  SUB .   SRN-1.                                         [    ]   [    ]   [    ]   [12  ]   [    ]   SUB .
064.  1020          SRNA-1.                                        [    ]   [    ]   [    ]   [11  ]   [    ]   SUB .
065.  1030          ELN=TLN=0, S=1.                                [    ]   [    ]   [    ]   [    ]   [14  ]   SUB .
066.  1040  11      ELN=[1.-SRNA]/[SRN-SRNA]                       [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
067.  1050          TLN=J+1.-.5*ELN                                [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
068.  1060          S=1.+[SRNA-1.]*.5*ELN                          [    ]   [    ]   [    ]   [    ]   [14  ]   SUB .
069.  1070  12      SRNA-1.                                        [    ]   [    ]   [    ]   [13  ]   [    ]   SUB .
090.  1080          ELN=[1.-SRN]/[SRNA-SRN]                        [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
091.  1090          TLN=J+.5*ELN                                   [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
092.  1100          S=1.+[SRN-1.]*.5*ELN                           [    ]   [    ]   [    ]   [    ]   [14  ]   SUB .
093.  1110  13      ELN=1.                                         [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
094.  1120          TLN=J+.5                                       [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
095.  1130          S=.5*[SRN+SRNA]                                [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
096.  1140  14      STRESS=STRESS+S                                [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
097.  1150          SELN=SELN+ELN                                  [    ]   [    ]   [    ]   [    ]   [.   ]   SUB .
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098.  2010  INER.   $ DETERMINATION OF N. AXIS                     [    ]   [    ]   [    ]   [    ]   [    ]   INER.
099.  2020  **      KZ-100                                         [22  ]   [    ]   [22  ]   [    ]   [    ]   INER.
100.                SWT.[100]                                      [    ]   [    ]   [    ]   [    ]   [    ]   INER.
101.  2030          Z=KZ*M        $ LOC. OF N. AXIS                [    ]   [    ]   [    ]   [    ]   [    ]   INER.
102.  2040          K=INT.[Z+1.]                                   [    ]   [    ]   [    ]   [    ]   [    ]   INER.
103.  2050          S1=0                                           [    ]   [    ]   [    ]   [    ]   [    ]   INER.
104.  2060          J=NA                                           [    ]   [    ]   [    ]   [    ]   [    ]   INER.
105.  2070  *       S1=S1-[TLN[J]+Z]*ELN[J]                        [    ]   [    ]   [    ]   [    ]   [    ]   INER.
106.  2080          J-K                                            [11  ]   [    ]   [    ]   [10  ]   [    ]   INER.
107.  2090          S1=S1+[J-.5-Z]                                 [    ]   [    ]   [    ]   [    ]   [11  ]   INER.
108.  2100  10      S1=S1+[TLN[J]-Z]*ELN[J]                        [    ]   [    ]   [    ]   [    ]   [    ]   INER.
109.  2110  11      J=J-1                                          [    ]   [    ]   [*   ]   [    ]   [    ]   INER.
110.  2120          KZ=KZ+1                                        [    ]   [    ]   [    ]   [    ]   [    ]   INER.
111.  2130          SWB=Z*LENGTH-.5*DA                             [    ]   [    ]   [12  ]   [    ]   [    ]   INER.
112.  2140          S2=.5*DB*TB*[DAA-Z*LENGTH]                     [    ]   [    ]   [    ]   [    ]   [15  ]   INER.
113.  2150  12      SWA                                            [13  ]   [    ]   [    ]   [    ]   [    ]   INER.
114.  2160          SWC=SWB-W                                      [    ]   [    ]   [14  ]   [    ]   [    ]   INER.
115.  2170  13      S2=.25*DB*[.5*DA+TB-Z*LENGTH]**2-.25*[.5*DA- C
   .  2180          Z*LENGTH]**2*SEL*DB/NB                         [    ]   [    ]   [    ]   [    ]   [15  ]   INER.
116.  2190  14      S2=TB*[DAA-Z*LENGTH]*SEL*.5*DB/NB              [    ]   [    ]   [    ]   [    ]   [    ]   INER.
117.  2200  15      S2-TB*[DAA+Z*LENGTH]*SEL*.5*DB/NB+           C
   .  2210            TA*LENGTH**2*S1+SK.[K]-1.E-6                 [    ]   [    ]   [**  ]   [    ]   [    ]   INER.
118.  2220          KZ=KZ-1                                        [    ]   [    ]   [    ]   [    ]   [    ]   INER.
119.  2230          $ DETERMINATION OF M. OF I.                    [    ]   [    ]   [    ]   [    ]   [    ]   INER.
120.  2240          S1=0                                           [    ]   [    ]   [    ]   [    ]   [    ]   INER.
121.  2250          J=NA                                           [    ]   [    ]   [    ]   [    ]   [    ]   INER.
122.  2260  *       S1=S1+[12.*[TLN[J]+Z]**2+ELN[J]**2]*ELN[J]     [    ]   [    ]   [    ]   [    ]   [    ]   INER.
123.  2270          J-K                                            [17  ]   [    ]   [    ]   [16  ]   [    ]   INER.
124.  2280          S1=S1+12.*[J-.5-Z]**2+1.                       [    ]   [    ]   [    ]   [    ]   [17  ]   INER.
125.  2290  16      S1=S1+[12.*[TLN[J]-Z]**2+ELN[J]**2]*ELN[J]     [    ]   [    ]   [    ]   [    ]   [    ]   INER.
126.  2300  17      J=J-1                                          [    ]   [    ]   [*   ]   [    ]   [    ]   INER.
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                                                                          PAGE 4.      DEC 21 66

 #     SEQ  LABL TYP               STATEMENT                      C ZERO    NOT 0     PLUS     MINUS     ELSE

127.  2310          SWB                                            [    ]   [    ]   [18  ]   [    ]   [    ]   INER.
128.  2320          S2=DB*TB*[[DAA-Z*LENGTH]**2+IB]                [    ]   [    ]   [    ]   [    ]   [21  ]   INER.
129.  2330  18      SWA                                            [19  ]   [    ]   [    ]   [    ]   [    ]   INER.
130.  2340          SWC                                            [    ]   [    ]   [20  ]   [    ]   [    ]   INER.
131.  2350  19      S2=DB/3.*[2.*[.5*DA+TB-Z*LENGTH]**3+          C
   .  2360          [Z*LENGTH-.5*DA]**3*SEL/NB]                    [    ]   [    ]   [    ]   [    ]   [21  ]   INER.
132.  2370  20      S2=TB*DB*[[DAA-Z*LENGTH]**2+IB]*SEL/NB         [    ]   [    ]   [    ]   [    ]   [    ]   INER.
133.  2380  21      EINERTIA=S2+DB*TB*[[DAA+Z*LENGTH]**2+IB]*SEL/C
   .  2390          NE+TA*LENGTH**3/6.*S1+2*IK.[K]                 [    ]   [    ]   [    ]   [    ]   [ .  ]   INER.
134.  2400  22      EINERTIA=0                                     [    ]   [    ]   [    ]   [    ]   [ .  ]   INER.
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135.  3010  SK      K-NA                                           [    ]   [    ]   [22  ]   [    ]   [    ]   SK  .
136.  3020          SWA                                            [    ]   [14  ]   [    ]   [    ]   [    ]   SK  .
137.  3030          SRN=STRAIN+RW[K-1], SRNA=STRAIN+RW[K]          [    ]   [    ]   [    ]   [    ]   [15  ]   SK  .
138.  3040  14      SRN=STRAIN+RF[K-1], SRNA=STRAIN+RF[K]          [    ]   [    ]   [    ]   [    ]   [    ]   SK  .
139.  3050  15      SRNA-1                                         [    ]   [    ]   [    ]   [18  ]   [    ]   SK  .
140.  3060          SRN-1.                                         [    ]   [    ]   [    ]   [16  ]   [    ]   SK  .
141.  3070          SWD=12                                         [    ]   [    ]   [    ]   [    ]   [12  ]   SK  .
142.  3080  16      TLN[K]+ELN[K]*.5-Z                             [    ]   [    ]   [    ]   [17  ]   [    ]   SK  .
143.  3090          SWD=10                                         [    ]   [    ]   [    ]   [    ]   [10  ]   SK  .
144.  3100  17      SWD=13                                         [    ]   [    ]   [    ]   [    ]   [13  ]   SK  .
145.  3110  18      SRN-1.                                         [    ]   [    ]   [    ]   [20  ]   [    ]   SK  .
146.  3120          TLN[K]-ELN[K]*.5-Z                             [    ]   [    ]   [    ]   [19  ]   [    ]   SK  .
147.  3130          SWD=12                                         [    ]   [    ]   [    ]   [    ]   [12  ]   SK  .
148.  3140  19      SWD=11                                         [    ]   [    ]   [    ]   [    ]   [11  ]   SK  .
149.  3150  20      SWD=10                                         [    ]   [    ]   [    ]   [    ]   [    ]   SK  .
150.  3160  10      SK[1]=K-.5-Z                                   [    ]   [    ]   [    ]   [    ]   [21  ]   SK  .
151.  3170  11      SK[1]=[TLN[K]-Z]*ELN[K]                        [    ]   [    ]   [    ]   [    ]   [21  ]   SK  .
152.  3180  12      SK[1]=.5*[K-Z]**2                              [    ]   [    ]   [    ]   [    ]   [21  ]   SK  .
153.  3190  13      SK[1]=K-.5-Z-.5*[Z-TLN[K]-ELN[K]*.5]**2        [    ]   [    ]   [    ]   [    ]   [    ]   SK  .
154.  3200  21      SK[1]=SK[1]*TA*LENGTH**2                       [    ]   [    ]   [    ]   [    ]   [.   ]   SK  .
155.  3210  22      SK[1]=0                                        [    ]   [    ]   [    ]   [    ]   [.   ]   SK  .
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                                                                                        PAGE 5.        DEC 21 66

  #      SEQ   LABL TYP                 STATEMENT                         C ZERO    NOT 0     PLUS     MINUS     ELSE

156.    4010   IK   .   K-NA                                              [    ]   [    ]   [15  ]   [    ]   [SWD ]   IK  .
157.    4020   10          IK[1]=[K-.5-Z]**2+1./12.                       [    ]   [    ]   [    ]   [    ]   [14  ]   IK  .
158.    4030   11          IK[1]=[[TLN[K]-Z]**2+ELN[K]**2/12.]*ELN[K]     [    ]   [    ]   [    ]   [    ]   [14  ]   IK  .
159.    4040   12          IK[1]=[K-Z]**3/3                               [    ]   [    ]   [    ]   [    ]   [14  ]   IK  .
160.    4050   13          IK[1]=[K-.5-Z]**2+1./12.                     C
   .    4060               -[Z-TLN[K]-.5*ELN[K]]**3/3                     [    ]   [    ]   [    ]   [    ]   [    ]   IK  .
161.    4070   14          IK[1]=IK[1]*TA*LENGTH**3                       [    ]   [    ]   [    ]   [    ]   [.   ]   IK  .
162.    4080   15          IK[1]=0                                        [    ]   [    ]   [    ]   [    ]   [.   ]   IK  .
163.    4090   FIN  PV ,,,                                                [    ]   [    ]   [    ]   [    ]   [    ]   IK  .
164.    4100        PL END OF PROGRAM                                     [    ]   [    ]   [    ]   [    ]   [    ]   IK  .
165.    4110        END                                                   [    ]   [    ]   [    ]   [    ]   [    ]   IK  .
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*****SYMBOL TABLE*****
              AREA                     AVESTRESS                B
              BI                       DA                       DB
              DAA                      ELNF                     ELN
              EXINERTIA                EINERTIA                 EYINERTIA
              FIN                      FINALSTRAIN              INER
              IK                       ISTRAIN                  INCREMENT
              IB                      *INT                      J
              JA                       KX                       KY
              KZ                       K                        L
              LENGTH                   M                        NF
              NW                       NA                       NB
              RF                       RW                       READ
              SUB                      SK                       SWT
              SKIP                     SQRT                     STRAIN
              TLN                      TLNF                     T
              STRESS                   SELN                     SRN
              SRNA                     SF                       SFEL
              SW                       SWEL                     SWA
              TA                       TB                       SEL
              S                        S1                       SWB
              W                        S2                       SWC
              SWD                      XINERTIA                 XMFACTOR
              XSLENDRATIO              YIELDSTRAIN              YINERTIA
              YMFACTCR                 YSLENDRATIO              Z

UNUSED MEMORY FROM [OCTAL]  10034 TC [OCTAL]  14603.
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005 2082 FUMIO NISHINC        COLUMN BUCKLING
DEC 20 66        12 36.4
                                                                                   PAGE 1.       DEC 20 66

 #     SEQ  LABL TYP                STATEMENT                           C ZERO    NOT 0     PLUS     MINUS     ELSE

001.  0000            $ 290-3-6                                          [      ]  [      ]  [      ]  [      ]  [      ]
002.  0010            $ CALCULATION OF COLUMN CURVE BASED ON             [      ]  [      ]  [      ]  [      ]  [      ]
003.  0020            $ TANGENT MODULUS CONCEPT                          [      ]  [      ]  [      ]  [      ]  [      ]
004.  0030            $ FLEXURAL BUCKLING, TEE COLUMN                    [      ]  [      ]  [      ]  [      ]  [      ]
005.  0040            $ DECEMBER,  1966  BY FUMIO NISHINO                [      ]  [      ]  [      ]  [      ]  [      ]
006.  0050         D  RF[100], RW[100], TLN[100], ELN[100].SUB.[14]      [      ]  [      ]  [      ]  [      ]  [      ]
007.  0060            SWT.[1]                                            [READ]    [      ]  [      ]  [      ]  [      ]
008.  0070         CRDYIELDSTRAIN                                        [      ]  [      ]  [      ]  [      ]  [      ]
009.  0080  READ   CRDISTRAIN                                            [      ]  [      ]  [      ]  [FIN  ]   [      ]
010.  0090         CRDINCREMENT,FINALSTRAIN                              [      ]  [      ]  [      ]  [      ]  [      ]
011.  0100         CRDNF,NW                                              [      ]  [      ]  [      ]  [      ]  [      ]
012.  0110         CRDB,T,L,W                                            [      ]  [      ]  [      ]  [      ]  [      ]
013.  0120         SL CALCULATION OF TANGENT MODULUS COLUMN CURVE,  C
   .  0130            FLEXURAL BUCKLING, TEE-COLUMNS                     [      ]  [      ]  [      ]  [      ]  [      ]
014.  0140         PL                                                    [      ]  [      ]  [      ]  [      ]  [      ]
015.  0150         PL  BY  FUMIO NISHINO                                 [      ]  [      ]  [      ]  [      ]  [      ]
016.  0160         PV ,,                                                 [      ]  [      ]  [      ]  [      ]  [      ]
017.  0170         PL   INPUT PARAMETERS                                 [      ]  [      ]  [      ]  [      ]  [      ]
018.  0180         PVL, NF, NW,, B,T,L,W                                 [      ]  [      ]  [      ]  [      ]  [      ]
019.  0190            SWT.[1]                                            [6     ]  [      ]  [      ]  [      ]  [      ]
020.  0200         PVLYIELDSTRAIN                                        [      ]  [      ]  [      ]  [      ]  [      ]
021.  0210  6      PVL ISTRAIN, INCREMENT, FINALSTRAIN,                  [      ]  [      ]  [      ]  [      ]  [      ]
022.  0220         PL   RESIDUAL STRESSES IN THE FLANGE                  [      ]  [      ]  [      ]  [      ]  [      ]
023.  0230         PV                                                    [      ]  [      ]  [      ]  [      ]  [      ]
024.  0240            J=0                                                [      ]  [      ]  [      ]  [      ]  [      ]
025.  0250  *      CRDRF[J]                                              [      ]  [      ]  [      ]  [      ]  [      ]
026.  0260         PV SKIP,RF[J]                                         [      ]  [      ]  [      ]  [      ]  [      ]
027.  0270            [J=J+1]-NF                                         [*     ]  [      ]  [      ]  [*     ]  [      ]
028.  0280         PV ,                                                  [      ]  [      ]  [      ]  [      ]  [      ]
029.  0290         PL   RESIDUAL STRESSES IN THE WEB                     [      ]  [      ]  [      ]  [      ]  [      ]
030.  0300         PV                                                    [      ]  [      ]  [      ]  [      ]  [      ]
031.  0310            J=0                                                [      ]  [      ]  [      ]  [      ]  [      ]
032.  0320  *      CRDRW[J]                                              [      ]  [      ]  [      ]  [      ]  [      ]
033.  0330         PV SKIP,RW[J]                                         [      ]  [      ]  [      ]  [      ]  [      ]
034.  0340            [J=J+1]-NW                                         [*     ]  [      ]  [      ]  [*     ]  [      ]
035.  0350            AREA=B*T+W*L                                       [      ]  [      ]  [      ]  [      ]  [      ]
036.  0360            YE=-.5*[L+T]*W*L/AREA                              [      ]  [      ]  [      ]  [      ]  [      ]
037.  0370            XINERTIA=[B*T**3+W*L**3]/12.+B*T*YE**2+       C
   .  0380               W*L*[.5*[L+T]+YE]**2                            [      ]  [      ]  [      ]  [      ]  [      ]
038.  0390            SWT.[1]                                            [1     ]  [      ]  [      ]  [      ]  [      ]
039.  0400            XMFACTOR=3.1415926*SQRT.[XINERTIA/AREA/        C
   .  0410               YIELDSTRAIN]                                    [      ]  [      ]  [      ]  [      ]  [      ]
040.  0420  1      PV ,,                                                 [      ]  [      ]  [      ]  [      ]  [      ]
041.  0430         PL   CALCULATED RESULTS                               [      ]  [      ]  [      ]  [      ]  [      ]
042.  0440         PVL, SKIP, AREA, XINERTIA,,                           [      ]  [      ]  [      ]  [      ]  [      ]
043.  0450            SWT.[1]                                            [2     ]  [      ]  [      ]  [      ]  [      ]
044.  0460         PL                     STRAIN       AVESTRESS    C
   .  0470               XSLENDRATIO                   XLENGTH           [      ]  [      ]  [      ]  [      ]  [3     ]
045.  0480  2      PL                     STRAIN       AVESTRESS    C
   .  0490                XSLENDRATIO                                    [      ]  [      ]  [      ]  [      ]  [      ]
046.  0500  3      PL                                                    [      ]  [      ]  [      ]  [      ]  [      ]
047.  0510            STRAIN=ISTRAIN                                     [      ]  [      ]  [      ]  [      ]  [      ]
048.  520   **        STRESS=SELN=0                                      [      ]  [      ]  [      ]  [      ]  [      ]
049.  0530            J=1, STWA=1                                        [      ]  [      ]  [      ]  [      ]  [      ]
050.  0540  *         SRN=STRAIN+RF[J-1]                                 [      ]  [      ]  [      ]  [      ]  [      ]

                                                                                   PAGE 2.       DEC 20 66

 #     SEQ  LABL TYP                STATEMENT                           C ZERO    NOT 0     PLUS     MINUS     ELSE

051.  0550            SRNA=STRAIN+RF[J]                                  [      ]  [      ]  [      ]  [      ]  [SUB  ]
052.  0560            [J=J+1]-NF                                         [*     ]  [      ]  [      ]  [*     ]  [      ]
053.  0570            SF=STRESS                                          [      ]  [      ]  [      ]  [      ]  [      ]
054.  0580            SFEL=SELN                                          [      ]  [      ]  [      ]  [      ]  [      ]
055.  0590            STRESS=SELN=STLN=SWTL=0                            [      ]  [      ]  [      ]  [      ]  [      ]
056.  0600            J=1, STWA=0                                        [      ]  [      ]  [      ]  [      ]  [      ]
057.  0610  *         SRN=STRAIN+RW[J-1]                                 [      ]  [      ]  [      ]  [      ]  [      ]
058.  0620            SRNA=STRAIN+RW[J]                                  [      ]  [      ]  [      ]  [      ]  [SUB  ]
059.  0630            [J=J+1]-NW                                         [*     ]  [      ]  [      ]  [*     ]  [      ]
060.  0640            SW=STRESS                                          [      ]  [      ]  [      ]  [      ]  [      ]
061.  0650            SWEL=SELN                                          [      ]  [7     ]  [      ]  [      ]  [      ]
062.  0660            SFEL                                               [      ]  [7     ]  [      ]  [      ]  [      ]
063.  0670            XSLENDRATIO=0                                      [      ]  [      ]  [      ]  [      ]  [4     ]
064.  0680  7         YP=-W*[L/NW]**2*STLN/                         C
   .  0690              [B*T/NF*SFEL+L*W/NW*SWEL]                        [      ]  [      ]  [      ]  [      ]  [      ]
065.  0700            J=1                                                [      ]  [      ]  [      ]  [      ]  [      ]
066.  0710  *         SWTL=ELN[J]*[[TLN[J]+NW/L*YP]**2+             C
   .  0720                 ELN[J]**2/12.]+SWTL                           [      ]  [      ]  [      ]  [      ]  [      ]
067.  0730            [J=J+1]-NW                                         [*     ]  [      ]  [      ]  [*     ]  [      ]
068.  0740            EXINERTIA=[YP*YP+T*T/12.]*SFEL*B*T/NF/        C
   .  0750              XINERTIA+SWTL*W*[L/NW]**3/XINERTIA               [      ]  [      ]  [      ]  [      ]  [      ]
069.  0760            AVESTRESS=SF*B*T/NF/AREA+SW*L*W/NW/AREA            [      ]  [      ]  [      ]  [      ]  [      ]
070.  0770            XSLENDRATIO=SQRT.[EXINERTIA/AVESTRESS]             [      ]  [      ]  [      ]  [      ]  [      ]
071.  0780            SWT.[1]                                            [4     ]  [      ]  [      ]  [      ]  [      ]
072.  0790         PV SKIP,STRAIN,AVESTRESS,XSLENDRATIO,SKIP,       C
   .  0800               XSLENDRATIO*XMFACTOR                            [      ]  [      ]  [      ]  [      ]  [5     ]
073.  0810  4      PV SKIP,STRAIN,AVESTRESS,XSLENDRATIO                  [      ]  [      ]  [      ]  [      ]  [      ]
074.  0820  5         [STRAIN=STRAIN+INCREMENT]-FINALSTRAIN-.0001        [      ]  [      ]  [      ]  [**    ]  [READ]


075.  0830  SUB ,     SRN-1.                                             [      ]  [      ]  [      ]  [12    ]  [      ]   SUB .
076.  0840            SRNA-1.                                            [      ]  [      ]  [      ]  [11    ]  [      ]   SUB .
077.  0850            ELN=TLN=0,S=1.                                     [      ]  [      ]  [      ]  [      ]  [14    ]   SUB .
078.  0860  11        ELN=[1.-SRNA]/[SRN-SRNA]                           [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
079.  0870            TLN=J-.5*ELN                                       [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
080.  0880            S=1.+[SRNA-1.]*.5*ELN                              [      ]  [      ]  [      ]  [      ]  [14    ]   SUB .
081.  0890  12        SRNA-1.                                            [      ]  [      ]  [      ]  [13    ]  [      ]   SUB .
082.  0900            ELN=[1.-SRN]/[SRNA-SRN]                            [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
083.  0910            TLN=J-1.+.5*ELN                                    [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
084.  0920            S=1.+[SRN-1.]*.5*ELN                               [      ]  [      ]  [      ]  [      ]  [14    ]   SUB .
085.  0930  13        ELN=1.                                             [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
086.  0940            TLN=J-.5                                           [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
087.  0950            S=.5*[SRN+SRNA]                                    [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
088.  0960  14        STRESS=STRESS+S                                    [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
089.  0970            SELN=SELN+ELN                                      [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
090.  0980            STWA                                               [      ]  [.     ]  [      ]  [      ]  [      ]   SUB .
091.  0990            ELN[J]=ELN                                         [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
092.  1000            TLN[J]=TLN+.5*T*NW/L                               [      ]  [      ]  [      ]  [      ]  [      ]   SUB .
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                                                                             PAGE 3.      DEC 20 66
 #     SEQ  LABL TYP               STATEMENT                C ZERO    NOT 0     PLUS     MINUS     ELSE

093.  1010          STLN=STLN+ELN*TLN[J]                    [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
094.  1020  FIN  PV ,,                                      [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
095.  1030       PL END  OF  PROGRAM                        [    ]   [    ]   [    ]   [    ]   [    ]   SUB .
096.  1040       ENDS END OF PROGRAM                        [    ]   [    ]   [    ]   [    ]   [    ]   SUB .



*****SYMBOL TABLE*****
                AREA                             AVESTRESS                        B
                ELN                              EXINERTIA                        FIN
                FINALSTRAIN                      ISTRAIN                          INCREMENT
                J                                L                                NF
                NW                               RF                               RW
                READ                             SUB                              SWT
                SKIP                             SQRT                             STRAIN
                STRESS                           TLN                              T
                SELN                             STWA                             SRN
                SRNA                             SF                               SFEL
                STLN                             SWTL                             SW
                SWEL                             S                                W
                XINERTIA                         XMFACTOR                         XSLENDRATIO
                YIELDSTRAIN                      YE                               YP

UNUSED MEMORY FROM [OCTAL]  06232 TO [OCTAL]  15713.
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005 2082 FUMIO NISHINO  COLUMN BUCKLING
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                                                                       PAGE 1.      DEC 21 66

 #    SEQ  LABL TYP              STATEMENT                          C ZERO   NOT 0   PLUS   MINUS   ELSE

001.  0011          $ 290-3-2
002.  0015          $  CALCULATION OF COLUMN CURVE BASED ON
003.  0020          $TANGENT MODULUS CONCEPT
004.  0030          $ AUG. 1966  BY  FUMIO NISHINO
005.  0040        D  XF[100],XW[100]
006.  0050  READ CRD ISTRAIN,INCREMENT,FINALSTRAIN
007.  0060          STRAIN=ISTRAIN                                                          [FIN ]
008.  0070       CRD NF,NW,B,BI,T,L,W
009.  0080          J=1
010.  0090  *    CRD XF[J]
011.  0100          [J=J+1]-NF                                  [*  ]                      [*  ]
012.  0110          J=1
013.  0120  **   CRD XW[J]
014.  0130          [J=J+1]-NW                                  [** ]                      [** ]
015.  0140       SL CALCULATION OF COLUMN CURVE BASED ON TANGENT C
   .  0150          MODULUS CONCEPT   BY FUMIO NISHINO
016.  0160       PV ,,
017.  0170       PL   INPUT PARAMETERS
018.  0180       PVL, NF, NW, B, BI, T, L, W
019.  0190       PVL ISTRAIN, INCREMENT, FINALSTRAIN,
020.  0200       PL   RESIDUAL STRESSES IN THE FLANGE
021.  0210       PV ,
022.  0220          J=1
023.  0230  *    PV XF(J)
024.  0240          [J=J+1]-NF                                  [*  ]                      [*  ]
025.  0250       PV ,
026.  0260       PL   RESIDUAL STRESSES IN THE WEB
027.  0270       PV ,
028.  0280          J=1
029.  0290  **   PV XW[J]
030.  0300          [J=J+1]-NW                                  [** ]                      [** ]
031.  0310          AREA=2*[B*T+W*L]
032.  0320          XINERTIA=[B*T*[4*T*T+6*L*T+3*L*L]+W*W*L*L]/6
033.  0330          YINERTIA=[L*W*[4*W*W+6*W*BI+3*BI*BI]+T*B*B*  C
   .  0340                   BI/6
034.  0350       PV ,,
035.  0360       PL   CALCULATED RESULTS
036.  0370       PVL,, AREA, XINERTIA, YINERTIA,,
037.  0380       PL    STRAIN        AVESTRESS       XSLENDRATIO   C
   .  0390            YSLENDRATIO    EXINERTIA       EYINERTIA
038.  0400  20      SF=0 $SUM OF STRAINS IN FLANGE
039.  0410          SCFX=0 $ SUM OF CCEFF OF FLANGE X
040.  0420          SCFY=0 $ SUM OF CCEFF OF FLANGE Y
041.  0430          J=1
042.  0440  *       FSTRAIN=XF[J]+STRAIN $ STRAIN IN FLANGE
043.  0450          FSTRAIN-1 $TEST FCR YIELD                   [11 ]                      [11 ]
044.  0460          FSTRAIN=1
045.  0470          CF=0 $CCEFF IN THE FLANGE                                                      [12 ]
046.  0480  11      CF=1
047.  0490  12      SF=SF+FSTRAIN
048.  0500          SCFX=SCFX+CF
049.  0510          SCFY=SCFY+CF*[12*J*J-12*J+4]
050.  0520          [J=J+1]-NF                                  [*  ]                      [*  ]
051.  0530          SW=0 $ SUM OF STRAINS IN WEB
052.  0540          SCWX=0 $SUM OF COEFF IN WEB X

                                                                       PAGE 2.      DEC 21 66

 #    SEQ  LABL TYP              STATEMENT                          C ZERO   NOT 0   PLUS   MINUS   ELSE

053.  0550          SCWY=0 $ SUM OF CCEFF IN WEB Y
054.  0560          J=1
055.  0570  **      WSTRAIN=XW[J]+STRAIN $ STRAIN IN WEB
056.  0580          WSTRAIN-1 $ TEST FOR YIELD                  [13 ]                      [13 ]
057.  0590          WSTRAIN=1
058.  0600          CW=0 $ COEFF IN WEB                                                            [14 ]
059.  0610  13      CW=1
060.  0620  14      SW=SW+WSTRAIN
061.  0630          SCWX=SCWX+CW*[12*J*J-12*J+4]
062.  0640          SCWY=SCWY+CW
063.  0650          [J=J+1]-NW                                  [** ]                      [** ]
064.  0660          AVESTRESS=2*B*T*SF/[NF*AREA]+               C
   .  0670          2*L*W*SW/[NW*AREA] $  AVERAGE STRESS
065.  0680          EXINERTIA=B*T*[4*T*T+6*L*T+3*L*L]*SCFX/[6*NF*C
   .  0690            XINERTIA]+W*L*L*L*SCWX/[24*NW*NW*NW*     C
   .  0700            XINERTIA] $EFFECTIVE M OF I, X
066.  0710          EYINERTIA=L*W*[4*W*W+6*W*BI+3*BI*BI]+SCWY/   C
   .  0720            [6*NW*YINERTIA]+T*B*B*B*SCFY/[24*NF*NF*    C
   .  0730            NF*YINERTIA] $EFFECTIVE M OF I, Y
067.  0740          XSLENDRATIO=SQRT.[EXINERTIA/AVESTRESS]*3.1416C
   .  0741          *SQRT.[29600.0/34.5]
068.  0750          YSLENDRATIO=SQRT.[EYINERTIA/AVESTRESS]*3.1416C
   .  0751          *SQRT.[29600.0/34.5]
069.  0760       PV ,STRAIN,AVESTRESS,XSLENDRATIO,YSLENDRATIO    C
   .  0761          , EXINERTIA, EYINERTIA
070.  0790          [STRAIN=STRAIN+INCREMENT]-FINAL STRAIN-.0001                           [20 ]  [READ]
071.  0800  FIN  PV ,,
072.  0810       PL END OF PROGRAM
073.  0820       END $ END OF PROGRAM

*****SYMBOL TABLE*****
           AREA              AVESTRESS          B
           BI                CF                 CW
           EXINERTIA         EYINERTIA          FINALSTRAIN
           FIN               FSTRAIN            ISTRAIN
           INCREMENT         J                  L
           NF                NW                 READ
           STRAIN            SF                 SCFX
           SCFY              SW                 SCWX
           T                 SCWY               SQRT
           W                 WSTRAIN            XF
           XW                XINERTIA           XSLENDRATIO
           YI...             YSLENDRATIO

UNUSED MEMORY FROM ...   ...45 TO [OCTAL]  16677.
```

CALCULATION OF COLUMN CURVE BASED ON TANGENT MODULUS CONCEPT   BY FUMIO NISHINO

INPUT PARAMETERS

```
2.0000000+00 $ NF    2.0000000+00 $ NW    8.0000000+00 $ B  0.0000000+00 $ BI    4.3300000-01 $ T  7.1340000+00 $ L
1.4400000-01 $ W
6.5000000-01 $ ISTRAIN  5.0000000-02 $ INCREMENT   1.2000000+00 $ FINALSTRAIN
```

RESIDUAL STRESSES IN THE FLANGE

```
-1.8830689-01
 3.0000000-01
```

RESIDUAL STRESSES IN THE WEB

```
-1.8830000-01
-1.8830000-01
```

CALCULATED RESULTS

8.9825920+00 $ AREA 1.0799554+02 $ XINERTIA   3.6963535+01 $ YINERTIA

STRAIN	AVESTRESS	XSLENDRATIO	YSLENDRATIO	EXINERTIA	EYINERTIA
6.5000000-01	6.5000204-01	1.1413789+02	1.1413789+02	1.0000000+00	1.0000000+00
7.0000000-01	7.0000204-01	1.0998603+02	1.0998603+02	1.0000000+00	1.0000000+00
7.5000000-01	7.3072030-01	7.9130993+01	3.8110954+01	5.4034361-01	1.2533617-01
8.0000000-01	7.6143856-01	7.7518395+01	3.7334297+01	5.4034361-01	1.2533617-01
8.5000000-01	7.9215682-01	7.6000530+01	3.6603265+01	5.4034361-01	1.2533617-01
9.0000000-01	8.2287508-01	7.4568473+01	3.5913560+01	5.4034361-01	1.2533617-01
9.5000000-01	8.5359334-01	7.3214432+01	3.5261428+01	5.4034361-01	1.2533617-01
1.0000000+00	8.8431160-01	7.1931571+01	3.4643579+01	5.4034361-01	1.2533617-01
1.0500000+00	9.1502986-01	7.0713864+01	3.4057109+01	5.4034361-01	1.2533617-01
1.1000000+00	9.4574812-01	6.9555978+01	3.3499449+01	5.4034361-01	1.2533617-01
1.1500000+00	9.7646638-01	6.8453169+01	3.2968316+01	5.4034361-01	1.2533617-01
1.2000000+00	1.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00

#END OF DATA# CARD READ
STATEMENT 006 WAS BEING EXECUTED.

DEC 21 66   21 52.9

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